Better Factor Portfolios
and Pricing Book-to-Market Characteristics with the Fama-French Factor Model

Gerard Hoberg and Ivo Welch
University of Maryland and Brown University

November 9, 2009

Abstract

This paper suggests forming portfolios by optimizing an objective function instead of by sorting. This is more parsimonious and flexible, and makes better use of the data. Empirically, our paper confirms the Davis, Fama, and French (2000) conjecture that the Daniel and Titman (1997) result was unique to their 1973–1993 sample period. The latter's evidence is obsolete: From 1973–2008, the Fama-French model can price their sort-based incongruence portfolio (spreading HML exposures vs. book-to-market characteristics) almost perfectly. However, we show that it could never price optimized incongruence portfolios.

Moreover, one can also construct optimized benchmark factors in lieu of the original Fama-French benchmark factors. These alternatives have higher Sharpe ratios, and some models based on them can price their own incongruence portfolios.

Tests of asset-pricing models often rely on multidimensional sorting techniques. For example, the common "dependent sorts" technique sorts stocks by market capitalization first into groups of k stocks each. Within each of these groups, stocks are then sorted by a variable of interest, say x. A zero-investment test portfolio is then formed by going long in the stock with the smallest x and short in the stock with the biggest x (i.e., within each group). The next step is to regress (in time-series) the rates of return of the resulting test portfolio on a benchmark factor model. The most prominent is the Fama and French (1993) model, which is based on a book-to-market, a marketcap and a stock-market factor, often augmented by a momentum factor (Jegadeesh and Titman (1993), Carhart (1997)). If the intercept is not zero, the benchmark model is said to be unable to price the stock-return related influence of x.

Sorting techniques have drawbacks. They require an ex-ante choice about the number of groups. They usually do not take advantage of the fact that stocks have different values of x *within* the same portfolio leg. They do not use information that some stocks have higher residual variance than others. And they make it almost impossible to control for more than a few dimensions. Even a dependent 4-dimensional sort on 6,000 stocks has only 9 stocks per portfolio. Independent sorts struggle even more with finding stocks in the extreme corner portfolios. Thus, it is by necessity that researchers often control their test portfolios only for one variable, usually market capitalization.

Our paper suggests replacing sorts as the method for forming portfolios with optimization of an objective function. For example, one can form a portfolio designed to produce reliable inference—it could minimize the expected standard error of alpha, subject to a number of constraints, such as balance with respect to market capitalization and large exposure with respect to the book-to-market ratio. Under a set of assumptions, the portfolio maximizing such an objective function can be the same as a portfolio implicit in the coefficients of the (cross-sectional) Fama-Macbeth tests (Fama (1976)). However, we use the rates of return on some of these portfolios as the dependent variable in a Fama and French (1993) time-series regression with respect to a benchmark factor pricing model. Our test is then whether the alpha of this portfolio is zero.

Empirically, our paper uses optimized portfolios to revive a disagreement about whether the Fama-French model can price the empirical regularity that it was designed to explain—the superior performance of value stocks. Daniel and Titman (1997), henceforth DT, identify stocks that have incongruous characteristics and exposures: either growth firms (low book-to-market characteristics) with high HML exposures, or value firms (high book-to-market characteristics) with low HML exposures. This allows them to determine that stocks' own book-to-market ratios are responsible for their higher returns, and not their HML factor exposures. It suggests that the Fama-French model could not really price stocks with differential book-to-market characteristics—which is what it was built to explain in the first place. In effect, DT argue that book-to-market is a characteristic, not an exposure. The Daniel-Titman paper won the Journal of Finance Brattle prize, and has since been reprinted in a number of volumes.

In a direct response, Davis, Fama, and French (2000), henceforth DFF, reexamined the evidence and showed that the Daniel-Titman sample period (1963–1992) was highly unusual. From 1929 to 1962, and from 1993 to 1997, the sign of alpha reverses. When these additional 39 years of data are included, the overall sample coefficient becomes insignificant. Consequently, the view that the returns of the DT portfolios are due to HML risk-exposures is again consistent with the data. That is, rates of returns of portfolios of firms with book-to-market ratios that are low relative to their HML exposures (and vice-versa) are in line with those predicted by the Fama-French model. Our paper can confirm the DFF inference. Using the same techniques, the DT inference holds only in the DT sample period and not in the DFF sample.

Daniel, Titman, and Wei (2001) argue that the DFF test is flawed, because there was not enough cross-sectional variation to warrant the inclusion of pre-1973 data in tests. This means that the post-1973 sample coefficient remains the relevant test.

However, our paper shows that this defense no longer applies. As of 2008, there is now enough data to show that just including the post-1973 sample is enough to reject the DT interpretation, using their own methods. In our replication, the abnormal performance estimate of their incongruence portfolio is now +1 bp per month (with a T-statistic of $+0.18$) in the 1973–2008 sample period, instead of –24 bp per month (with a T-statistic of -2.37) in the 1973–1993 DFF sample period. Thus, based on the existing sort method, one should agree with the DFF conclusion that the Fama-French model can comfortably explain the performance of stocks with incongruous characteristics.

> *Based on their own testing methods, the DT paper can now be considered obsolete. Its inference has been overturned by newer data since its publication.*

Yet our enhanced portfolio formation methods produce much more powerful tests. We can document that the Fama-French model was never able to price optimized portfolios that maximize the spread between HML exposures and book-to-market characteristics. In a fairly simple specification, our test portfolios underperform the benchmark Fama-French model by –16 bp per month and the momentum-augmented Fama-French model by –29 bp per month (per dollar invested in each leg). This supports the characteristics view over the exposures view. Mispricing that is so high is also economically meaningful. The result is robust. Our inference holds in all sub periods—including even the 1927–1962 sample.

It holds even when we form portfolios that are value-weighted, although the abnormal performance roughly halves. (The test derives power from firms with incongruent book-to-market characteristics and HML factor loadings, which is intrinsically more common in smaller firms. Value-weighting is thus a much more stringent test than what DFF and DT ever required.) It holds outside of Januaries.

Optimized portfolios can also offer alternatives to the sort-based factors in the Fama-French model themselves (the book-to-market factor [aka HML], the size-factor [aka SMB]) and Carhart (the momentum-factor [aka UMD]). Optimized factor portfolio construction is relatively more parsimonious and flexible. The replacement factors also have superior Sharpe ratios. We show that some of these alternative benchmark models can explain their equivalently constructed incongruence portfolios; others fail.

We now describe our data, replicate the earlier results and show that the Fama-French model can price sort-based incongruence portfolios.

I Data

This section primarily explains methods pioneered by the earlier papers that we are copying. We use the same data (CRSP, Compustat, and Ken French's data posted on his website), and follow most of the techniques pioneered by DT and DFF. The only input data construction "novelty" is that we will also use exposures computed from daily stock returns and shrunk via standard Bayesian techniques. Readers familiar with the preceding papers can skip this section.

A Stock Returns

We predict all stock-months' rates of return from July 1929 to December 2008. A stock-month must have had sufficient data in Compustat to calculate a book value and sufficient data in CRSP to obtain a market value, following standard methods outlined in DFF. There are 2,659,242 observations that satisfied this criterion. Our data must further satisfy the following ex-ante criteria:

1. A stock-month must have had at least 3 years of past CRSP stock return data for computing exposures. This eliminates 356,956 observations—mostly (small) recent IPO issuers. This is not only acceptable because this is an ex-ante choice, but also desirable because it avoids the new issues phenomenon documented by Ritter (1991).

2. The stock price must have been at least $1 at the end of the previous month. This eliminates 92,712 observations.

3. A stock-month must have had a positive book value of equity. This eliminates 37,364 firm-months and is our only additional data constraint relative to DT and DFF. The reason is that we want to hold log(B/M) constant in some of our tests. This constraint is not uncommon in other papers in this literature. We have also confirmed that it is not consequential to the inference.[1]

We are left with about 2.2 million stock-months. In the last month of our sample, 12/2008, there were 4,258 stocks before and 3,787 stocks after the screens are applied.

B Factor Exposures and Characteristics

Our paper explores the book-to-market ratio, the market capitalization (firm-size), the own momentum (2 to 13 months lagged stock return performance), and the exposures to four factors. The aggregate factor data was obtained from Ken French's website. The market factor, MFAC, is the rate of return on the value-weighted CRSP index minus the risk-free rate. SMB and HML are the two original Fama-French factors; the UMD factor was added later.

1. **Construction of Fixed-Weight Factors:** Both Daniel and Titman (1997) and Davis, Fama, and French (2000) report that static factors are superior to dynamic factors (from Ken French's website) for the sake of computing exposures that will persist ex-post. Therefore we also first construct these "fixed-weight" factor portfolios. When estimating past exposures, in each December, we hold fixed the portfolio weights in the Fama-French factor portfolio associated with the following July, and then project this portfolio backwards for five years to avoid rebalancing. This fixed weight method prevents other stocks from entering and exiting this portfolio, as would normally occur on an annual basis for the dynamic weight factor portfolios.

2. **Construction of Exposures to Fixed-Weight Factors:** Each December, we compute for each stock the historical factor exposures with respect to these "fixed-weight" portfolios. (The exposures are later used to form portfolios beginning in July of the following year for a period of 12 months.) We found that it makes no difference

[1] In addition, Compustat has since corrected errors, changed coverage slightly, and changed data format. When we ran the tests on the 2005 Compustat data, the results were a few basis points closer to those reported in DFF and DT. The data suggests that these alterations only strengthen the results of the earlier papers.

whether market-betas are recomputed every year, or whether they are recomputed every month.

Specifically, we estimate each stock's exposures to MFAC, SMB, and HML in one multi-variate regression based on a minimum of three and a maximum of five years of historical data. Each regression yields joint estimates of the three factor exposures (on MFAC, SMB, and HML). Following our predecessors, we run the following multi-variate time-series regression separately for each stock

$$R_i - R_f \quad = \quad \mathrm{a}_i \quad + \quad \mathrm{b}_i \cdot \mathrm{MFAC} \quad + \quad \mathrm{s}_i \cdot \mathrm{SMB} \quad + \quad \mathrm{h}_i \cdot \mathrm{HML} \quad + \quad e_i$$

once per year. The resulting "ex-ante" coefficients become input into the sorts and optimizer. They are ex-ante estimates and will be denoted with a hat. We also retain the estimated standard errors of $\hat{\mathrm{a}}_i$ for use in Section IV.F.

We treat UMD differently, because the momentum factor has been hypothesized to matter only for the most recent year. Thus, each stock's exposure to UMD is computed from a regression that adds the UMD factor to the time-series regression just described, but based on one year of data, only. (The other exposures are still taken from the earlier 3–5 year regression, though, not from the 1-year regression with UMD included.) The window is rolling throughout the year, and lagged by one month to avoid the return reversal effect. We do not use the "fixed-weight factor" method for UMD.

In Table 1, we confirm the results in DFF and DT with monthly historical factors. In later tables, we improve our sort and optimizer inputs (the factor exposure estimates) relative to DT and DFF as follows: We use daily stock return data in the time-series regression computing the exposures, and then shrink each of the daily exposure estimates (b_i, s_i and h_i) via the Vasicek (1973) method, as recommended in Elton, Gruber, Brown, and Goetzmann (2003, p.145):

$$\hat{\mathrm{b}}_i \quad = \quad w_i \cdot \hat{\mathrm{b}}_{i,\mathrm{TS}} + (1 - w_i) \cdot \mu_{\mathrm{XS}}$$

$$w_i \quad = \quad 1 - \frac{\mathcal{V}ar_{\mathrm{TS}}(\hat{\mathrm{b}}_i)}{\mathcal{V}ar_{\mathrm{TS}}(\hat{\mathrm{b}}_i) + \mathcal{V}ar_{\mathrm{XS}}(\hat{\mathrm{b}}_i)}$$

where $\hat{\mathrm{b}}_{i,\mathrm{TS}}$ is the familiar ordinary OLS time-series exposure for each firm with associated $\mathcal{V}ar_{\mathrm{TS}}(\hat{\mathrm{b}}_i)$ (the variance of the estimated exposure); and μ_{XS} and $\mathcal{V}ar_{\mathrm{XS}}(\hat{\mathrm{b}}_i)$ are respectively the mean and variance of all exposures in a given month across firms. This shrinkage estimator places more weight on the historical time-series exposure estimate if this estimated exposure has lower estimated variance and when there is a lot of heterogeneity in the cross-section of betas.

We also experimented with some other exposure correction techniques (such as corrections for non-synchronous trading), but these were typically a little worse in predicting future exposures. Thus, it would not have been useful to add their complications to our paper.

Enhanced exposure estimations should generally favor the asset-pricing perspective over the characteristics perspective. We will later show that the daily shrunk exposure estimates indeed do better in predicting *future* monthly exposures after 1963 than the historical (unshrunk or shrunk) monthly exposures. Nevertheless, our results are typically similar regardless of whether factor exposures are computed from daily or monthly returns.

After we have measured the exposures of each stock to these factors, we discard the fixed-weight factors. (They are not used again, nor are they reported in our tables.)

3. **Construction of Zero-Investment Portfolios:** We then form zero-investment portfolios, which can be based on the characteristics and ex-ante exposures (and in Section IV.D on marketcap; and in Section 7 on estimated ex-ante volatility). In the first two tables, we use common sort techniques to do so. In the remainder of the paper, we use our own techniques, which are explained in detail when used.

   After we have formed these portfolios, we discard the historical exposures to these factors, as well as the characteristics data. (They are not used again.)

The above description applies only to the construction of the portfolio investment weights. The reported test results of our paper are based on the time series of the one-month-ahead rates of return of these zero-investment portfolios *after* they have been formed. We focus on the (sometimes UMD-augmented) Fama-French-abnormal alpha, which are the intercepts in Fama-French time-series factor regressions. The rate of return of the portfolio [not net of the risk-free rate, because this is a zero-investment portfolio] is the dependent variable. The three [or four] contemporaneous dynamic FF factors, as obtained from Ken French's website, are the independent variables.

C Timing

We forecast portfolio rates of return beginning in July of a given year, lasting through June of the year after. When we work with characteristics, we assume that the firm's financial statements (book values) are known *without error* six months after the annual statement to which they pertain. (We do not use quarterly data.) We use the market value of equity from the June immediately preceding this twelve month forecast period. These methods are consistent with Fama and French (1992), as well as the more recent papers cited earlier. Own momentum is known after a one month lag (that avoids the well-known return reversal), and is computed on a rolling monthly basis. In some specifications in Section IV.E, we use two own momentum measures, one from -2 to -6 months, the other from -7 to -12 months as inputs into the optimizer. At no point do we use any information in the formation of portfolios that an investor would not have had access to.

D Statistics Reported in Our Tables

Our benchmark model tests closely mirror the DT and DFF tests. We form portfolios that seek to be incongruous in an exposure and its corresponding characteristics, while holding other influences (primarily own marketcap) constant. We then check whether these portfolios have stock returns that cannot be explained by the Fama-French model. Note that the resulting portfolios do not have non-zero loadings on the known risk factors *ex-post*, which is why we must regress the portfolio returns (raw, not net of the risk-free rate) on *all* Fama-French risk factors (HML, SMB, MFAC, and possibly UMD). (Most of our tables do report the average ex-ante $\hat{h}$ of our portfolio for comparison with the ex-post h, which the portfolio formation procedures often seek to maximize.) Our formal tests focus exclusively on the alphas from these regressions. As in DFF, the standard errors are adjusted for heteroskedasticity (White method). Heteroskedasticity is expected, if only because we have different numbers of stocks available in different months. The first two data columns in the tables show the average monthly return of the portfolios and their T-statistics, which is useful background information.

E Perspectives of Asset Pricing Tests

Asset pricing tests require a number of philosophical choices. Our paper follows the standard conventions in this literature.

Conceptually, the easiest way to think about distinguishing between a characteristic and an exposure would be a simple competitive regression, i.e., a Fama-Macbeth test. With both included in the regressions, the regression would tell us how they compete against one another for explaining future stock returns. Unfortunately, this is not a fair test in this context, because exposures are measured with error while characteristics are known. The results would be biased in favor of a characteristics-based interpretation.

Thus, we need to adopt the same time-series FF test approach used in DT and DFF. Its biases are more subtle.

On the one hand, the FF factor model is favorably handicapped. It shall be considered the NULL hypothesis, and it will be up to the alternatives to reject the Fama-French model at the 95% level. Moreover, the FF model is allowed to adjust stock returns via its own ex-post factor returns. That is, when we form a zero-investment portfolio, it must be based only on ex-ante information, and this portfolio has to outperform a model that can adjust returns via factor information that is known ex-post. Put differently, even if an investor were unable to accurately compute the ex-post exposures on a portfolio so as to earn a specific rate of return in accordance with the model, the Fama-French model could still proclaim victory (in that it could price this portfolio).

On the other hand, the FF factor model is also unfavorably handicapped. Its rate of return model has to explain the returns of *any* portfolios formed ex-ante. This gives the experimenter a lot of freedom. If there are five reasonable ways to partition stocks to create market-cap spreads, the FF model has to explain the rates of return on all five partition portfolios. If only one of them rejects, the FF model fails.

We also agree with DFF that the Fama-French model is just that—a model. It is not just statistical significance that matters, but economic significance. Every model fails in data that is plentiful enough. One question is whether the model still offers a good overall description of return patterns.

However, there is not only the more practical task of describing how rates of returns matter, but also the conceptual question of whether stock prices are better seen as exposures (adjusting in anticipation of future returns) or as characteristics (reflecting historical aspects).

II Replicating DT and DFF With Sort Portfolios

Before we move on to empirical results, it is appropriate to give a brief history of the literature that brought us here. The current state of empirical asset pricing traces its heritage back to Banz (1981) [firm-size], Basu (1977) [price-earnings], Keim (1983) and Roll (1983) [Januaries], and Rosenberg, Reid, and Lanstein (1985) [book-to-market]. With a series of papers by Fama and French—beginning with Fama and French (1992) which systematically investigated these effects—this literature took an important leap. Fama and French (1993) proposed a parsimonious model that suggests HML exposures, SMB exposures, and MFAC exposures as a good model to explain stock returns. Jegadeesh and Titman (1993) added momentum to the mix, which in turn gave rise to a common specification, as in Carhart (1997). This is sometimes referred to as the UMD-augmented Fama-French model. In their recent review, Fama and French (2008) add accruals (Sloan (1996), Teoh, Welch, and Wong (1998)) and net stock issues (Ritter (1991)) to the mix, and largely dismiss asset growth and profitability. There are also many other discovered regularities in the "anomalies zoo," e.g. liquidity [Acharya and Pedersen (2005)], individual volatility [Goyal and Santa-Clara (2003)], and turnover [Chordia, Subrahmanyam, and Anshuman (2001)]. However, exposure to the market factor, to the book-to-market HML factor, to the equity-cap SMB factor, and to the up-minus-down UMD momentum remain the staple ingredients of empirical asset pricing models.

It is an important conceptual question where these historical cross-sectional average return differences come from. Davis, Fama, and French (2000), offer three possible reasons:

1. They are statistical spurious anomalies, and/or disappear as soon as investors discover and exploit them.
2. They are common factors in the ICAPM or APT sense (argued not only in Fama and French (1993), but also in Fama and French (1996) and Fama and French (1998)).
3. They are behavioral preferences exhibited by investors.

DFF argue that the first reason can explain many anomalies, including even the small-firm effect. They argue that spurious correlation is however unlikely to explain the book-to-market ratio and momentum—these effects have continued long after they were first documented. As already described in the introduction, the most prominent test to distinguish between the second and third explanation was proposed by Daniel and Titman (1997).[2] Thus, DT's prominence is no surprise. It is also a good reason to reexamine their evidence.

[2]The DFF description is not exhaustive. For example there could also be behavioral factors that are priced in an APT sense, thus blurring the boundary between their second and third categories.

Of course, it is almost impossible to "prove" that an empirical regularity (or behavioral characteristic) is not due to an unknown factor exposure. There are far more stocks than months, plus there is no theoretical reason why factor portfolio investment weights may not vary from period to period. With so many degrees of freedom, one can always construct factors ex-post, whose exposures can explain the alpha of almost any "anomalies."

Consequently, the discipline in constructing factor benchmark models must either come from a theory or from restricting consideration to reasonable ex-ante investment strategies. The justification for studying the specific FF model is that it remains *the* benchmark model today. Its HML factor was constructed specifically to explain the performance of value stocks relative to growth stocks. This is also why it was so perplexing that even this model failed to price value-vs-growth stocks in Daniel and Titman (1997). The next obvious question would then be whether there are good alternative model candidates. Until one can be found, viewing book-to-market as something that is "backward-looking" (such as a behavioral characteristic) rather than "forward-looking" (such as a common factor exposure) cannot be rejected. Absent such a successful factor model, one needs to have much more faith in factor asset-pricing models (i.e., that there are factors not yet known but which will ultimately explain these abnormal stock returns that the characteristics-based behavioral view can so effortlessly explain).

A Monthly Raw Exposures

To replicate the results of our predecessors, we follow the same complex independent sorting procedures. Quoting DFF

> "At the end of June of each year t (1929 to 1996), we allocate the NYSE, AMEX, and Nasdaq stocks in our sample to three size groups (small, medium, or big; S, M, or B) based on their June market capitalization, ME. We allocate stocks in an independent sort to three book-to-market equity (BE/ME) groups (low, medium, or high; L, M, or H) based on BE/ME for December of the preceding year. The break points are the 33rd and 67th ME and BE/ME percentiles for the NYSE firms in the sample. We form nine portfolios (S/L, S/M, S/H, M/L, M/M, M/H, B/L, B/M, and B/H) as the intersections of the three size and the three BE/ME groups. The nine portfolios are each subdivided into three portfolios (Lh, Mh, or Hh) using pre-formation HML slopes. The slopes are estimated with five years (three years minimum) of monthly returns ending in December of year t – 1. Value-weight returns on the portfolios are calculated for July of year t to June of t + 1. Hh–Lh is ((S/L/Hh–S/L/Lh) + (M/L/Hh–M/L/Lh) + (B/L/Hh–B/L/ Lh) + (S/M/Hh–S/M/Lh) +

(M/M/Hh–M/M/Lh) + (B/M/Hh–B/M/Lh) + (S/H/Hh–S/H/Lh) + (M/H/Hh–M/H/Lh) + (B/H/Hh–B/H/Lh))/9."

(There are more issues related to how one handles the scarcity of firms in the corner portfolios, especially early in the sample.) This method produces net portfolios that are neither dominated primarily by small firms nor value-weighted. The portfolios are balanced for marketcap and book-to-market characteristics, but have large exposures to HML. Their HML exposure and book-to-market characteristics are thus incongruous. The economic interpretation of a negative FF benchmark model alpha would be that the firms in (the long legs of) these portfolios have low book-to-market characteristics, but are still viewed by the Fama-French model as having high HML exposures. Thus, according to the model, they should offer relatively high rates of return. If they fail to do so, their alphas would be negative, these portfolios would underperform in the sense of the FF benchmark model, and one would conclude that it is the value/growth characteristics that matter to future stock returns, and not the exposures to the HML factor (in the context of the benchmark model). The converse is the case for the short leg of the strategy, which are value firms with low book-to-market exposures.

[Table 1 here]

Table 1 replicates the main results in DFF's Table IV. The "Overall" sample ranges from 7/1929 to 12/2008 (954 months). The "DT" sample ranges from 7/1973 to 12/1993 (246 months) and was used in the original Daniel and Titman (1997) paper. The "DFF" sample is from 7/1929 to 6/1997 (816 months) and was used in the Davis, Fama, and French (2000) paper. 1963 is a common break, because it was the beginning of the standard Compustat sample used in many papers. 7/1963 is also roughly the half-way point in our sample (408 months vs. 546 months) and encompasses the DT sample. The "1994–" sample begins in 1/1994 and contains 180 months. Thus, it is out-of-sample relative to Daniel and Titman (1997).

Table 1 shows that the data yields results that are very similar to those reported by Davis, Fama, and French (2000). The DT sample results are impressive. The *unadjusted* rate of return of the incongruence portfolio (which has high net log HML exposures but little spread in its investment-weighted net BE/ME characteristic) was a meager –3 bp. This is why the FF model cannot price it—with such a high ex-post h exposure, the portfolio should have performed better. Our alpha point estimate of –30 bp per month is even more pronounced than the –22 bp reported in DFF. (Our three exposures to the Fama-French factors are also similar to those reported in the DFF paper.)

The table also reports the performance of the UMD-augmented Fama-French-Carhart model, which we sometimes abbreviate as "FFC." This model offers similar qualitative inference, but the magnitude of the mispricing is more modest. Another novel piece of information in the table is the average ex-ante $\hat{h}$ exposure of the portfolio, which was used to form the sort portfolios. Not unexpectedly, the realized ex-post h exposure of the incongruency is much lower than the ex-ante $\hat{h}$ exposure. Mismeasurement of h reduces the power of the test, but does not bias the test against the benchmark model.

DFF pointed out the striking contrast that the 20 years of the DT sample (1973 to 1993) were unusual. Again, our results in the DFF sample mimic their's. They report an alpha of –6 bp, similar to the –4 bp that we find in their sample period. Thus, the FF-abnormal alpha of this incongruence portfolio is neither statistically nor economically significant. DFF argue that one should use the longest sample for the most powerful test, and therefore rejects the DT model.

Daniel, Titman, and Wei (2001) take exception to this view. They argue that there is not enough cross-sectional data in years prior to 1973 to disentangle the hypotheses. However, although this is plausible, the evidence no longer support the Daniel-Titman interpretation. We have the additional benefit of another 15 years of "out-of-sample" data, which does not suffer the "few firms" problem. The "DT–2008" row extends the DT sample from 1973 all the way to 2008. Table 1 shows that, with this new data, the Daniel-Titman method would have suggested that the FF model works almost perfectly. The incongruence portfolio has a –1 bp per month abnormal performance, with a T-statistic of –0.09. The momentum-augmented FFC model performs similarly, with an alpha of –7 bp (T-statistic of –0.77). Thus, it is not just in the pre-1973 data that the DT hypothesis fails. It also fails now in the post-1973 data. The reason can be seen by looking at the 1/1994– sample. This is an out-of-sample test relative to DT. Growth firms with high HML exposures have not underperformed but outperformed since 1993. This further backs the DFF perspective. This effect is so strong (+40 bp) that it is statistically significant even though there are only 180 months in the post-1993 sample.

The lack of performance in the post-1973 sample provides the background why our empirical findings in the next few sections are interesting:

> *As of 2008, the evidence and thus the interpretation in Daniel and Titman (1997) is obsolete. The Fama-French model can price sort-based incongruence portfolios on $\hat{h}$ and log(BE/ME) very accurately.*

In much of the rest of the paper, we shall follow the DFF perspective of adopting the "Overall" sample as a benchmark—not because we necessarily disagree with Daniel, Titman, and Wei (2001), but because it presents a stronger challenge to incongruence tests, i.e., a more stringent test of the characteristics hypothesis. This overall sample is in the first row in Table 1. Here, the portfolio's undesirable exposures are indeed low ($b \approx -6\%$, $s \approx 7\%$, $u \approx 7\%$), while its HML exposure is, as desired, a much higher $h \approx 44\%$. Performancewise, the portfolio has an *unadjusted* average net return of +21 bp. After this return is adjusted by the Fama-French model to control for (small) exposures to the market and SMB factors, and to control for the (large) exposure to the HML factor, it has an economically and statistically insignificant +4 bp of abnormal performance. The overall sample portfolio presents a pricing challenge that the FF model passes with flying colors. The FFC model performs similarly well.

Not reported, the net portfolio also retains a small residual positive bias in its log-book-to-market characteristics. This is almost inevitable, despite the attempt to control for the (known) book-to-market characteristic in the sort. After all, within the last sort based on preformation HML, one will still find some small spread in book-to-market characteristics. (These sort-based portfolios also have residual *ex-ante* exposures to other factors, such as the market, that are ignored.) These issues do not cause a bias towards rejecting the FF model, because the FF model receives the opportunity to adjust for exposures *after* the portfolio is formed. Instead, they cause a loss in the power of the test, in effect making it harder to reject the FF model.

B Exposures Shrunk and/or Computed From Daily Stock Returns

Again, the ex-post exposure to HML was much lower than a naïve user of the model would have imagined. The *ex-ante* monthly HML exposure $\hat{h}$ of the overall-sample incongruency portfolios, computed over the 5 years and used in the sort, was 165%. The equivalent realized ex-post exposure h was only 44%. The monthly ex-ante $\hat{h}$ exposures of the incongruency portfolio overestimated their ex-post h exposures by not just a little.[3] The stark discrepancy between ex-ante $\hat{h}$ exposures and ex-post h exposures suggests that it is worthwhile to see if one can improve the accuracy of the exposure estimation. We therefore compute HML exposures that were shrunk via the standard Bayesian procedure suggested by Vasicek (1973), as explained on Page 4. The shrunk exposures can then be used as inputs for the sort procedure instead in lieu of the raw exposures. The upper panel in Table 2 shows the performance of these revised incongruence portfolios. Table 2 shows that shrunk exposures are slightly better predictors of future monthly exposures: The ex-post h is higher in the upper panel of Table 2 than it was in Table 1. **[Table 2 here]**

The lower panel in the table is based on sorts whose inputs experienced one more change: their HML exposures were computed from daily stock returns before they were shrunk. The ex-post performance regressions reported in the tables (i.e., the benchmark models) are always based on monthly stock return data. The table shows that portfolios

[3]Th presence of this noise is not in itself a problem, but it also makes it more likely that their sort portfolio accidentally picks stocks ex-ante out of the cross-section that are not extreme ex-post.

formed based on $\hat{h}$ exposures computed from *daily* stock returns have about the same ex-post h exposures as portfolios formed based on $\hat{h}$ exposures computed from monthly stock returns *in the Overall sample*. However, this hides time-variation: Before 1963, the monthly exposures produce better portfolios than the daily exposures. After 1963, the daily exposures produce much better portfolios. One might therefore want to consult the former as inputs to form incongruence portfolios before 1963, and the latter after 1963. Although our results are similar if we use monthly stock return-based exposures as inputs, the rest of our paper relies mostly on daily exposures.[4]

Using daily exposure estimates to sort should provide more power for tests in the DT 1973–1993 sample period. Yet, Table 2 shows that this diminishes the abnormal performance of the incongruence portfolio in the DT sample. The DT alpha drops from –25 bp to –15 bp, and (just) loses its statistical significance. Thus, we can conclude that—besides the extension of the sample to 2008—the DT evidence was also not as robust as it appeared. The DFF conclusion stands: the FF model prices the incongruence portfolios quite well, except in the DFF sample period.

III Optimized Portfolios

A principal objective of our paper is to offer an alternative to sorts as the method of forming zero-investment portfolios that are suitable for tests of asset-pricing models. We want to find portfolios that are investable based strictly on the same ex-ante, known information that is used in the sorts, and that are designed to have ex-post exposures on some dimensions (e.g., HML exposures), but not on other dimensions (e.g., firm size and book-to-market characteristics).

[4] Even though it would be defensible to use monthly-based HML exposures prior to 1963 and daily-based HML exposures after 1963 in subsequent tests, we stick to daily returns based exposures throughout the sample, because we do not want to be accused of excessive data mining.

Ultimately, the goal will be to produce a test of whether the alpha of this portfolio is different from zero. Therefore, a natural objective function would be[5]

$$\min_{w_i} \sum_i w_i^2 \cdot \widehat{Var(a_i)} \tag{1}$$

subject to the constraints that

$\sum_i w_i$	=	0	The portfolio is zero-investment
$\sum_i w_i \cdot \log(\text{ME}_i)$	=	0	Zero (investment-)weighted firm-size
$\sum_i w_i \cdot \log(\text{BE}_i/\text{ME}_i)$	=	0	Zero weighted book-market characteristic
$\sum_i w_i \cdot \hat{\text{h}}_i$	=	constant	Positive (net) HML exposure

Under the assumption that the residual error variances of stocks are homoskedastic, $\widehat{Var(a_i)}$ is a scalar that does not depend on the stock's own historical return volatility. Thus, it effectively drops out. In this case, the program simplifies to portfolios that are the equivalents of Fama-Macbeth regressions. This was first recognized Fama (1976, Chapter 9, Section 1.C). Fama shows that these portfolios have minimum variance under assumptions similar to those of OLS (e.g., no errors-in-variables, homoskedastic errors, etc).[6]

These OLS portfolios are easy to compute. In a given month, define **X** as an $n \times k$ matrix of n stocks that holds (past) realizations of $k-1$ variables of interest, plus a constant. The **X** matrix is based strictly on the same lagged information as the sorts. The test portfolio to determine whether variable k has marginal power in explaining abnormal returns, holding constant the other variables, are then the k rows in

$$\mathbf{W} \equiv (\mathbf{X}'\,\mathbf{X})^{-1}\,\mathbf{X}' \ .$$

The first row of **W** is a portfolio costing $1 that has no ex-ante loading on any of the included variables. Because the constant is included as the first column, the $k-1$ remaining

[5]Although it would be desirable to examine a maximum "T-statistic" (e.g., forming a portfolio that has the highest average historical alpha with the lowest standard error), such a program does not yield a closed-form portfolio vector. With thousands of stocks in each month as variables that need to be reoptimized, this is not an easy task. Moreover, historical average alphas are unlikely to be very good predictors of future alphas. First moments are less stable and more difficult to predict than second moments.

[6]An earlier draft considered portfolios that minimized the estimated variance of exposures. Again, under a homoskedasticity assumption, this simplifies into OLS portfolios.

portfolios are all zero investment. The second row is a portfolio that has ex-ante loading of 1 on the first x variable and ex-ante loadings of 0 on all of the other variables, and so on. For obvious reasons, we shall call the rows in **W** "OLS Portfolios." (In standard Fama-Macbeth tests, one then obtains each month's k vector of gammas by multiplying the **W** matrix with the one-month-ahead rate of return vector, i.e., $\vec{\gamma} = \mathbf{W}\vec{r}$.) For intuition, Figure 1 provides a numerical illustration of such a portfolio.

[Figure 1 here]

To mimic the plain sort portfolios in Daniel and Titman (1997), the four columns in our basic specification of **X** are the constant, the stock's own log(ME), the stock's own log(BE/ME) ratio, and the stock's own $\hat{\mathrm{h}}$ (its historical HML exposure).

- Including the constant ensures that the incongruence portfolio is zero investment.
- Including the stock's own log(ME) ensures that the weighted-average market cap of stocks in the incongruence portfolio is 0, i.e., that the long and short legs are matched in their log marketcaps.
- Including the stock's own log(BE/ME) ensures that the weighted average value-growth characteristics of the incongruence portfolio is 0, i.e., that the long and short legs are matched in their log book-to-market ratios.
- The incongruence portfolio itself is the final column in **W**, which is the investment weights of a portfolio that has one investment-weighted unit positive exposure on HML.

  Intuitively, this zero-investment marketcap-balanced portfolio is long "growth firms" that have high HML exposures relative to their book-to-market ratios, and short "value firms" that have low HML exposures.[7]

In contrast to sort portfolios, all stocks are (typically) used in an optimized portfolio; stocks that have more of the variable of interest (in our case, $\hat{\mathrm{h}}$ exposure) receive more weight; and there is no need to specify break points ex-ante or to be limited to two or three control variables.

In contrast to the Fama-Macbeth cross-sectional then time-series regression method, we use the rates of return on OLS portfolios as dependent variables in a Fama and French (1993) time-series regression on a benchmark pricing model. This combines the advantage of the linear regression-like approach from Fama-Macbeth (which allows for efficient

[7] In both the exposure variance minimization and return variance minimization techniques, we would ideally like the ex-ante estimated exposure estimates imposed by our constraints also to hold up ex-post (of course, after normalization). In real life, estimation error, regression-to-the-mean, and the generic instability of stock return processes prevent this. This reduces the power of our tests—just as it reduces the power of sort-based tests. It is the same discrepancy between ex-ante $\hat{\mathrm{h}}$ and ex-post h that was in earlier tables.

control of multiple factors) with the advantage of the Fama-French method (in which the mismeasurement of exposures does not bias the results against the factor model.)

The input variables and arbitrage portfolios can be linearly scaled without changing the statistical inferences. Always investing $1 long and $1 short results in the same statistical inference as always investing $2 long and $2 short. This irrelevance does not extend to the magnitude of the coefficient estimates (including the intercept). To make our results more comparable to those in earlier work, we report coefficients with respect to a portfolio that invests $1 dollar in each leg. This can be done by dividing all coefficients (incl. the intercept) by the optimizer's investment in either leg. This is only a reporting normalization to make it easier to interpret economic significance, not an ex-post adjustment.

However, the OLS portfolio objective function is not without drawbacks. First, as noted earlier, in the presence of violations of the standard OLS assumptions, these portfolios are no longer necessarily minimum return variance. (In Section IV.F, we allow for heteroskedastic errors, based on historical own stock return variances.) Second, this method tends to select smaller stocks than the DT and DFF sort techniques. (In Section IV.D, we show that this is not responsible for our differences in inference.)

IV Empirical Results

A A Single Target

[Table 3 here]

Table 3 shows that the optimized incongruence portfolios after 1963 have higher ex-post h exposures than sort portfolios (Tables 1 and 2). This suggests that our test portfolios should have more power. The alphas shows that this worsens the performance of both benchmark models. In the overall sample, the FF model underperforms by –12 bp per month, with a T-statistic of –1.85. This barely misses conventional significance levels. The UMD-augmented FFC model is solidly rejected, however, with economically meaningful abnormal performance of –25 bp and a T-statistic of –3.26. The incongruence portfolios also have significant abnormal performance in DFF's own sample period and in the pre-1963 sample. (There does not seem to be an issue with lack of power before 1963.) However, after 1994, the alpha of this spread portfolio under the FF benchmark is modest.

Not reported, compared to their equivalent incongruence portfolios in Table 1, the portfolio investment weights in the portfolios in Table 3 tend to be less extreme. (This is the case in all tables below, too.) The optimizer has more stocks to work with than do sort methods, which counterbalances its tendency to want to load up more on stocks with more $\hat{h}$ exposures.

B Conflicting Dual Targets

The portfolio formation technique can be easily altered to increase the incongruence between log(BE/ME) characteristics and $\hat{h}$ exposures. We can replace these two variables in the **X** matrix with one variable that is the difference of the two. Computing the plain difference would however give more weight in the difference to the variable that has higher cross-sectional standard deviation. Thus, before we compute the difference, we first standardize both variables by their own cross-sectional means and standard deviations.

[Table 4 here]

The performance of the resulting incongruence portfolios is in Table 4. These incongruence portfolios are even more difficult to price by the benchmark models. The alphas are now solidly negative in all sample periods, regardless of whether the benchmark model is UMD-augmented or not. In the overall sample, the abnormal performance is –18 bp per month (T of –3.19) under the FF benchmark, and –30 bp (T of –5.03) under the FFC benchmark. After 1994, the point estimates are –21 bp and –29 bp, respectively.

C Recalibrated Conflicting Targets

Table 4 shows that the ex-post h is small in the overall sample: Firms with unit-standardized high ex-ante $\hat{h}$ exposures but low unit-standardized book-to-market characteristics tend to have almost zero ex-post h exposures. This means that the test portfolio's incongruence between book-to-market characteristic and $\hat{h}$ exposure is now delivered more by the fact that log(BE/ME) is low than by the fact that $\hat{h}$ is high. This is not a big problem, because it does not favor rejection of the benchmark models. (They are supposed to soak up any ex-post variation in HML.) Nevertheless, such a low exposure is unappealing.

An easy way to lean more on $\hat{h}$ is to "re-target" the portfolio. Instead of subtracting the standardized log(BE/ME) from the standardized $\hat{h}$, we can subtract log(BE/ME) from the standardized $\hat{h}$ amplified by a scaling factor. An arbitrary choice, such as a scaling factor of 2, would do. In our case, we chose this scaling factor to be greater than 1 in a way that takes the historical (i.e., up-to-date) deviation of the ex-post h from a presumed target of $\hat{h}$ of 1 into account.[8] The results are similar in either case.[9]

[Table 5 here]

Table 5 shows that the resulting incongruence portfolios have both higher ex-ante $\hat{h}$ optimizer inputs and higher ex-post h exposures. Thus, the incongruence is now based relatively more on the ex-ante $\hat{h}$ than the log(BE/ME) when compared with Table 4. The abnormal performance of these portfolio is however almost the same as those in Table 4.

D Market Capitalization

As already mentioned, the OLS incongruence portfolios are more aggressive than the DFF and DT incongruence portfolios in tilting towards stocks with smaller market capitalizations. Although the point of our paper is not to fish for a high alpha that can be converted into a profitable trading strategy (which therefore means that high average marketcaps are not as important), it is still interesting to learn whether our results derive from this more aggressive size tilt.

Fortunately, the OLS approach can be tweaked to tilt the incongruence portfolios towards larger firms. Just as the OLS portfolios are the equivalent of an OLS Fama-Macbeth regression, one can create portfolios that are the equivalent of a WLS Fama-Macbeth regression. Here, the weights are based on stocks' marketcaps. Define a weighting matrix Ω, in which the diagonal is the firm's known lagged market capitalization. The equivalent test portfolio is now a row of

$$\mathbf{W}(e) \quad = \quad (\mathbf{X}'\,\Omega^e\mathbf{X})^{-1}\,\mathbf{X}'\Omega^e \ ,$$

where e is an exponent constant between 0 and 1. An exponent of 0 is equal-weighting, i.e., what we had in our earlier tables. An exponent of 1 is full value-weighting (i.e., marketcap-weighting). Incidentally, we know of no equivalent parsimonious procedures that smoothly alter the marketcap-weights of sort-based test portfolios.

A value-weighted portfolio is of course too stark a test. For example, of 4,259 traded stocks in December 2008, the single largest stock had as much marketcap as the bottom 59% of stocks together. Therefore, a test based on a value-weighted portfolio will obviously

[8] We compare the to-date realized coefficients to the to-date targets in *unnormalized* space. For example, if our target was 1 on the normalized ex-ante $\hat{h}$ variable (which had time-varying standard deviations, but typically around 0.4), and we obtain an ex-post h coefficient of 0.1, we know we were 0.3 normalized units off in the past. (This is $0.3/0.4 = 0.75$ in unnormalized h exposures.) We would thus have missed by 0.25 sd-normalized units. We therefore raise our sd-normalized target by a little more (in fact, 1.5 times as much, which here is $0.25 \cdot 1.5 \approx 0.375$). We then rerun the optimizer. This gives us the final portfolio, which is renormalized to \$1 investment.

[9] Note that retargeting can slightly change the portfolio's investment weights. Thus, the original and retargeted portfolios are different.

not be very successful in constructing a spread portfolio with incongruous $\hat{h}$ exposures and log(BE/ME) characteristics.

To assess the market capitalization of incongruence portfolios, we compute in each month the investment-weighted marketcap of the equal-weighted market portfolio and of the value-weighted market portfolio for stocks that satisfied our criteria. For example, in December 2008, these figures were $3.9 billion and $68.8 billion respectively. For a given e weighting factor, we then compute the investment-weighted market capitalization of the incongruence portfolio, and its linear location between the equal- and value-weighted portfolios. For example, an exponent of $e = 47.2\%$ yields a portfolio that has an average investment-weighted marketcap location of 26.7% between the equal- and value-weighted market portfolio. This is the same location as the average location of the sort portfolios in DT and DFF. (We call this portfolio "MFFM" for "matched Fama-French marketcap.") In 12/2008, this 26.7% location corresponded to an investment-weighted marketcap of $21.2 billion. (This corresponds roughly to the 96th percentile firm.) We repeat the calculations for different exponents $e \in [0, 1]$ to get a range of portfolios. We also compute an abnormal alpha for each e weighted portfolio with respect to the momentum-adjusted FF benchmark model.

[Figure 2 here]

Figure 2 plots the FFC alpha and its two-standard deviation bounds as a function of the linear location between equal- and value-weighted portfolio of all stocks. In the overall sample, even incongruence portfolios whose marketcap is about halfway between the equal- and value-weighted stock market are still economically significant. The portfolios' alphas deteriorate with marketcap weighting, but not in an overly sensitive fashion. For the 26.7% location ($e = 47.2\%$), alpha is still more than –20 bp per month.

E Additional Ex-ante Controls

[Table 6 here]

The top panel in Table 6 shows the abnormal performance of three differently value-weighted incongruence portfolios. This table thus provides similar information as the top left panel in Figure 2. The first portfolio is the same as that considered in Table 4 and Table 5. The second portfolio is an incongruence portfolio that has similar marketcap characteristics as the sort portfolio tests in DT and DFF. It has an average location of 26.7% between the equal-weighted and value-weighted market portfolio. The third portfolio is the fully value-weighted incongruence portfolio. The first two portfolios have statistically and economically significant abnormal performance. The third portfolio does not.

So far, we have only used the optimizer to balance out the same variable as our predecessors: market capitalization. Optimization has another advantage that we have not yet used: it can control for more dimensions ex-ante. In our case, we can include market-beta ($\hat{b}$), SMB exposure ($\hat{s}$), UMD exposure ($\hat{u}$), and own momentum (2 to 13 months lagged, denoted $r_{-2,-13}$) as additional controls in the **X** matrix.[10] Note that the justification for ex-ante control of these other portfolio characteristics is the philosophically same as it is for control for market capitalization. Instead of relying merely on the ex-post model to take out the influence of size, the portfolios are themselves tilted to help in this control. To the extent that the exposures do not control well for own momentum, however, the extra balance would be appropriate.

The lower panel in Table 6 shows the abnormal performance of the same three incongruence portfolios as the upper panel, except the incongruence portfolios are now balanced with respect to the aforementioned additional ex-ante characteristics and exposures. The first lines show that more constraints actually dampen the alpha of the unweighted portfolios. However, the bottom lines show that more constraints enhance the alpha of the value-weighted portfolios. The alpha of the incongruence portfolio now remains at –17 bp per month, or about 2 percent per year. Further investigation (not reported) revealed that it is the additional control for ex-ante own momentum (and not for $\hat{s}$, $\hat{b}$, or $\hat{u}$) that is principally responsible for keeping the significance so high. Again, we would not have expected that a test that relies so heavily on value-weighting would have enough power to reject the NULL. The fact that it can do so is surprising.

[Figure 3 here]

Figure 3 plots the abnormal performance of the ex-ante multidimensionally balanced portfolios vis-a-vis the momentum-adjusted FFC benchmark model. The overall, post-1963, and post-1993 sample alphas are now significantly negative, even when the Ω is fully value-weighted. After 1963, the alpha is still more than –20 bp per month.

F Heteroskedasticity

As explained in Section III, the OLS portfolios in effect assume homoskedasticity. Thus, the portfolio construction did not use information about the historical volatility of stocks. Put differently, it ignored the fact that some stocks could have had seemingly more reliable returns ex-ante than others. If this is not transient and thus estimable, ignoring historical stock return volatility would be inefficient. In other words, if we can predict $\widehat{Var(a_i)}$ in (1) with past stock return (or past alpha) variance, we should be able to form incongruence portfolios that are more reliable. Note that tilting away from high volatility firms has a

[10]Recall that power in our model does not come from the in-sample fit of our optimizer. It comes from how good our portfolios are in delivering the out-of-sample portfolio exposures that our target requests. Thus, unlike in an in-sample test, it is quite possible that an additional constraint increases the test power. Adding market exposure can be viewed as improving the prediction by controlling for a previously omitted variable, which improves the OOS prediction, because it delivers a portfolio with better out-of-sample characteristics, more in line with the ($\hat{h}$) target we were after.

similar effect as tilting towards larger firms, because larger firms tend to be less volatile. Value-weighting is thus closely related to heteroskedasticity adjusting.

To explore whether second moments are persistent, we ran a Fama-Macbeth regression in the first 10 years of the sample (1929–1939), predicting ex-post variance with a linear combination of the stock's own ex-ante variance and the cross-sectional average ex-ante variance. Clearly, there are more sophisticated methods, but in this simple framework, the best predictor of future variance in the data put about equal weight on both:

$$\mathcal{V}ar(a_i) \quad \approx \quad 0.5 \cdot \widehat{\mathcal{V}ar(a_i)} + 0.5 \cdot \overline{\mathcal{V}ar(a_i)} + \epsilon_i \ ,$$

where the first term is the historical own variance for stock i, and the second term is the cross-sectional average of i. (A linear combination of own historical volatility and the cross-sectional average volatility is also analogous to the shrinkage of the exposures that we used as inputs into the optimizer.) The 50-50 best rule also held up when we extended the period to the entire sample period.[11]

Substituting such an estimate of the future variance into the optimizer in (1) yields a portfolio optimization program that also has a simple closed-form solution. Table 7 shows the results. As in the case of larger marketcap tilts, the alpha decreases but remains significant. Further investigation reveals that this portfolio is still mostly tilted towards smaller stocks. Thus, this incongruence enhancement does not seem to provide more mileage than its simpler homoskedastic cousin in the construction of incongruence portfolios. **[Table 7 here]**

[11] We denote one firm-year as one observation, and we fit the Fama-French 3 factor model firm by firm in each year using daily return data in this given year, and we extract the standard error of the intercept and square this. We refer to this variable as the "ex-post variance" and it is the dependent variable in the Fama-MacBeth regression we explore. We compute the ex-ante variance of the intercept using the same 3 factor model, but we use daily data over the five years ex-ante (as used in the rest of our study). In particular, for year t ex-post variance, ex-ante variance is computed over the five year period starting in November of year t-6 and ending in November of year t-1 (if available, but a minimum of 3 years). We compute a "candidate predictor" of ex-post variance using various shrinkage factors lambda from zero to one, where a candidate is equal to (cross sectional variance) lambda + (ex_ante_variance) (1-lambda). We then test each candidate using a Fama-Macbeth regression (with no intercept, as the shrinkage method is a linear combination of the ex-ante variance and a constant) and compare candidate R^2 to find the best predictor of ex-post variance. The results (both overall and pre-1940) reach the same conclusion, and a lambda of 0.5 is the best ex-post predictor.

G Calendar Months: January vs. non-Januaries

It is well-known that many empirical anomalies either occur only in January (the small-firm effect), or reverse in January (momentum). Thus, Table 8 splits the sample of all firms into Januaries and non-Januaries. The results are generally consistent with the view that the results are harmed by Januaries before 1963, and unaffected after 1963. In the overall sample, the abnormal performance is negative and significant only in non-Januaries.

[Table 8 here]

H Size and Momentum

We can also use optimized portfolios to take a quick look at the contrast of size characteristics vs. SMB exposures; and the contrast of own momentum characteristics vs. UMD exposures. These are exact analogs to the tests contrasting book-to-market characteristics with HML exposures.

[Table 9 here]

SMB Exposure vs. Market Cap, Extended Controls Table 9 explores whether the FFC model can price large stocks that have *positive* exposures to the small-firm factor—i.e., exposures that are incongruous with their own marketcaps. The table shows that the FF model can price these incongruence portfolios. Thus, market cap can clearly be viewed as an exposure rather than as a characteristic in the context of the FFC model.

[Table 10 here]

UMD Exposure vs. Own Momentum, Extended Controls Table 10 explores whether the FFC model can price stocks that have positive UMD exposures with negative momentum, i.e., exposures that are incongruous with their own momentum.[12] The table shows that momentum should not be viewed as an exposure to UMD. After 1963, the incongruence portfolio has extremely poor abnormal performance. The evidence strongly suggests that momentum is a characteristic, not a factor, in the context of this benchmark model.

[12] The UMD exposures that are used to form portfolio were created from their own 1-year regressions, not from the same 3-5 year regressions that provided us with MFAC, HML, and SMB exposures.

V Fama-French Factor Alternatives

Having concluded our investigation of the performance of the Fama-French model on characteristics/exposure incongruence portfolios, we now investigate whether we can produce better benchmark factors than the sort-based HML, SMB, and UMD factors.

The following manual of how the Fama-French factors are constructed is adopted almost verbatim from Ken French's website (which is itself based on Fama and French (1993)):

HML (high minus low): The six portfolios, which are constructed at the end of each June, are the intersections of 2 portfolios formed on size (market equity, ME) and 3 portfolios formed on the ratio of book equity to market equity (BE/ME). The size break point for year t is the median NYSE market equity at the end of June of year t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME break points are the 30th and 70th NYSE percentiles.

	Median ME	
	Small Value	Big Value
70th BE/ME Percentile		
	Small Neutral	Big Neutral
30th BE/ME Percentile		
	Small Growth	Big Growth

Each of the six portfolios is value-weighted. However, the factor portfolio itself is equal-weighted. It is the average return on the two value portfolios minus the average return on the two growth portfolios,

$$\text{HML} = 1/2 \cdot (\text{Small Value} + \text{Big Value}) - 1/2 \cdot (\text{Small Growth} + \text{Big Growth})$$

SMB (Small Minus Big): is the average return on the three small portfolios minus the average return on the three big portfolios,

$$\begin{aligned}\text{SMB} = \; & 1/3 \cdot (\text{Small Value} + \text{Small Neutral} + \text{Small Growth}) \\ & - 1/3 \cdot (\text{Big Value} + \text{Big Neutral} + \text{Big Growth})\end{aligned}$$

The portfolios are rebalanced once per year (at the end of each June). They include all stocks for which there is market equity data for December of t-1 and June of t, and (positive) book equity data for t-1.

Optimized portfolios can provide a more parsimonious alternative. There is no need to decide on break points. There is no need for value-weighting inside the portfolio and equal-weighting across portfolios to find a factor that is a trade-off between de facto ignoring small firms (if one fully value-weighted) and relying too much on small firms (if one equal-weighted). The HML factor portfolio can control for more than just firm-size (especially own momentum), and/or downweight stocks that have high idiosyncratic stock return volatility. In contrast to the original Fama-French portfolios, optimized factors will tilt more heavily towards stocks that have higher characteristics (e.g., log(BE/ME) for the HML portfolio) within the long and short portfolios. In the Fama-French factors, after a stock is allocated to a portfolio based on its characteristics, its characteristics are ignored. (Fama-French investment weights are determined on the basis of marketcap.)

We entertain four substitute versions:

1. **EW, Balanced:** Factors constructed from an OLS portfolio optimization that hold constant log(ME), log(BE/ME), market-beta (computed from a univariate regression), and own momentum ($r_{-2,-11}$, as in the original UMD construction).

2. **MFFM, Balanced:** Factors constructed from an OLS portfolio optimization that hold constant the same four input variables, but which are weighted to match the FF market cap. (For now, with an e exponent of 0.47, this portfolio has a location of 26.7% between the equal- and value-weighted market portfolio.)

3. **Volat.-adjusted, Balanced:** Factors constructed under the assumption that future $\mathcal{V}ar(a_i) \approx 0.5 \cdot \widehat{\mathcal{V}ar(a_i)} + 0.5 \cdot \overline{\mathcal{V}ar(a_i)}$. These factors are not designed to match the marketcap of the FF benchmark factors.

In all cases, the factors are different rows from the same **W** matrix. (We also experimented with unbalanced factors, but these are more difficult to interpret, because they have much higher correlations among the factors. Thus, we do not report them.)

[Table 11 here]

Table 11 shows statistics for the original Fama-French factors and the factor alternatives. The original HML sort factor has a lower Sharpe ratio than its optimized alternatives. The differences are statistically significant. The original SMB sort factor has a lower Sharpe ratio than two of its three optimized alternatives (again, statistically significant). There is no meaningful economic or statistical difference between the original UMD sort factor and its optimized alternatives.

[Table 12 here]

Table 12 shows the correlation matrices of the factor alternatives. Surprisingly, ex-ante control for the correlation between the UMD factor and other factors is not very successful. The correlations are no lower (but also not generally higher) than that of the sort portfolios.

Table 13 replicates the incongruence tests with respect to benchmark models that are constructed from these alternative factors. Because the factors are different in each model, so are the factor exposures, and thus so are their incongruence test portfolios. The gray lines are the same as those in Table 5 and included for quick comparison with the original FFC model. The factor model that relies on optimized factors that take into account historical stock return volatility by-and-large seems to be able to price its own incongruence test portfolios. In this sense, this factor model can explain what it was constructed to explain—the returns of value firms vis-a-vis growth firms. The "EW, balanced" model performs worse, but at least it still halves the alphas of its incongruence portfolios relative to the Fama-French model. In contrast, the balanced MFFM based factor model (whose factors have similar marketcap characteristics as the FF factors) fails to price its own incongruence portfolio. Together, this suggests that factors that are more tilted towards smaller firms than the Fama-French factors are more likely to be capable of explaining the performance of value over growth stocks. **[Table 13 here]**

VI Summary and Conclusion

Our paper proposed replacing the sorting method in Fama-French time-series tests with a portfolio optimizer. We showed that optimized portfolios could provide for more powerful tests of benchmark models. These established that the Fama-French model cannot price its book-to-market incongruence portfolios. In addition, we showed that more parsimonious alternatives to the Fama-French factors themselves also perform better. The optimized HML and SMB factors have higher Sharpe ratios than the sort-based HML and SMB factors. Some versions of these factors (that however also tilt more towards smaller firms) are capable of explaining their own incongruence portfolios.

References

Acharya, Viral V., and Lasse Heje Pedersen, 2005, Asset pricing with liquidity risk, *The Journal of Finance* 77, 375–410.

Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3–18.

Basu, S, 1977, Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis, *Journal of Finance* 32, 663–82.

Carhart, Mark M, 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Chordia, Tarun, Avanidhar Subrahmanyam, and V. Ravi Anshuman, 2001, Trading activity and expected stock returns, *Journal of Financial Economics* 59, 3–32.

Daniel, Kent, and Sheridan Titman, 1997, Evidence on the characteristics of cross-sectional variation in stock returns, *The Journal of Finance* 52, 1–33.

——— , and K. C. John Wei, 2001, Explaining the cross-section of stock returns in japan: Factors or characteristics, *The Journal of Finance* 56, 743–766.

Davis, James L., Eugene F. Fama, and Kenneth R. French, 2000, Characteristics, covariances, and average returns: 1929 to 1997, *The Journal of Finance* 55, 389–406.

Elton, Edwin, Martin Gruber, Stephen Brown, and William Goetzmann, 2003, *Modern Portfolio Theory and Investment Analysis* (John Wiley and Sons, Inc.: New York).

Fama, Eugene F, 1976, *Foundations of Finance* (Basic Books).

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *The Journal of Finance* 68, 427–465.

——— , 1993, Common risk factors in stock and bond returns, *Journal of Financial Economics* 33, 3–56.

——— , 1996, Multifactor explanations of asset pricing anomalies, *The Journal of Finance* 51, 55–84.

——— , 1998, Value versus growth: The international evidence, *The Journal of Finance* 53, 1975–2000.

——— , 2008, Dissecting anomalies, *The Journal of Finance* 63, forthcoming.

Goyal, Amit, and Pedro Santa-Clara, 2003, Idiosyncratic risk matters!, *The Journal of Finance* 58, 975–1007.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *The Journal of Finance* 48, 65–91.

Keim, Donald B., 1983, Size-related anomalies and stock return seasonality : Further empirical evidence, *Journal of Financial Economics* 12, 13–32.

Ritter, Jay R, 1991, The long-run performance of initial public offerings, *Journal of Finance* 46, 3–27.

Roll, Richard, 1983, Vas ist das: The turn-of-the-year effect and the return premia of small firms, *Journal of Portfolio Management* 9, 18–28.

Rosenberg, B, K Reid, and R Lanstein, 1985, Persuasive evidence of market inefficiency, *Journal of Portfolio Management* 11, 9–17.

Sloan, Richard G., 1996, Do stock prices fully reflect information in accruals and cash flows about future earnigns?, *Journal of Accounting Research* 71, 289–315.

Teoh, Siew Hong, Ivo Welch, and T. J. Wong, 1998, Earnings management and the underperformance of seasoned equity offerings1, *Journal of Financial Economics* 50, 63–99.

Vasicek, Oldrich Alfons, 1973, A note on using cross-sectional information in bayesian estimation on security beta's, *The Journal of Finance* 28, 1233–1239.

Figure 1: Numerical Illustration of Portfolio

For example, consider a situation in which there are 9 stocks with 3 characteristics:

	A	B	C	D	E	F	G	H	I
log(ME)	0	0	0	1	1	1	2	2	2
h	-1	0	1	-1	0	1	-1	0	1
log(BE/ME)	0.5	0.7	0.9	1.1	1.5	1.3	1.9	1.7	2.1

In this example, the correlation between log(BE/ME) and log(ME) is higher (95%) than the correlation between log(BE/ME) and h (21%). The correlation between h and log(ME) is 0. The computed **W** matrix, quoted in percent, is then

	A	B	C	D	E	F	G	H	I
constant	49	44	39	16	−4	21	−32	−7	−27
log(ME)	13	−17	−47	30	−90	60	−43	107	−13
h	−10	0	10	−10	−20	30	−30	20	10
log(BE/ME)	−50	0	50	−50	150	−100	100	−150	50

For example, the final line is a zero-investment portfolio that has zero loading on log(ME) and h, but unit loading on log(BE/ME). To confirm:

- This portfolio is long 3.50 and short 3.50. Thus it is zero investment.
- The investment-weighted log-ME in both its long and short leg is 8.
- The investment-weighted h in both its long and short leg is 0 (which is coincidence).
- The investment-weighted log(BE/ME) is 5.65 long and 4.65 short, for a net unit exposure.

The portfolio loads most heavily long on E and then G, and most heavily short on H and F. Unfortunately, the portfolio's investment weights are only intuitive for those used to inverting matrices mentally and/or thinking in multiple dimensions.

Unfortunately, it would require 27 stocks to illustrate a three-dimensional sort procedure. It is obviously difficult to convey the intuition thereon.

This table will probably be only in the working paper, not the publication.

Table 1: Zero-Investment Portfolios Created By Sorts to Spread Book-to-Market Characteristics, holding own Size and HML exposures constant (with HML Exposures Estimated from Monthly Stock Returns).

				⇓	⇓					
		Unadj. Net		Fama-French-Adjusted						
Sample Period	$\hat{h}$	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Overall	165%	**0.21%***	**(+2.74)**	0.04%	(+0.71)	**−6%,***	**7%,***	**44%,***		42.2%
7/29–12/08, 954 mo				−0.03%	(−0.49)	**−5%,***	**7%,***	**47%,***	**7%***	43.6%
DFF	155%	0.12%	(+1.54)	−0.04%	(−0.65)	**−5%,***	**9%,***	**40%,***		39.5%
7/29–6/97, 816 mo				−0.08%	(−1.18)	**−5%,***	**9%,***	**42%,***	4%	40.0%
DT	157%	−0.03%	(−0.24)	**−0.30%***	**(−3.16)**	1%,	4%,	**49%,***		49.2%
7/73–12/93, 246 mo				**−0.24%***	**(−2.37)**	1%,	3%,	**48%,***	−6%	50.5%
DT–2008	182%	**0.23%***	**(+2.08)**	−0.01%	(−0.09)	−1%,	3%,	**53%,***		51.5%
7/73–12/08, 426 mo				−0.07%	(−0.77)	-0%,	3%,	**54%,***	7%	53.0%
-6/1963	154%	0.20%	(+1.48)	0.04%	(+0.41)	**−6%,***	**15%,***	**38%,***		38.0%
7/29–6/93, 408 mo				−0.03%	(−0.22)	−5%,	**15%,***	**41%,***	7%	39.1%
7/1963-	172%	**0.22%***	**(+2.46)**	0.01%	(+0.18)	−3%,	2%,	**50%,***		50.1%
7/63–12/08, 546 mo				−0.06%	(−0.73)	−2%,	2%,	**52%,***	**7%***	51.8%
1/1994-	217%	**0.57%***	**(+2.98)**	**0.40%***	**(+3.04)**	−5%,	4%,	**56%,***		56.3%
1/94–12/08, 180 mo				0.24%	(+1.65)	1%,	2%,	**60%,***	**15%***	64.3%

Description: This table presents the (ex-post) performance of portfolios formed based on independent sorts of ex-ante firm size, book-to-market ratio, and h exposure. The sorting procedure is identical to that in DFF and described in the text. Both the long and short portfolios are constructed from (three by three [by three]) equal-weighted portfolios, which are themselves value-weighted. The table displays the ex-post performance of this net portfolio. The format of this table is almost identical to the format of Table IV in Davis, Fama, and French (2000), and mostly self-explanatory. b is the ex-post exposure to the market rate of return net of the risk-free rate (MFAC), s to the SMB factor, h to the HML factor, and u to the UMD factor (not used in this table). $\hat{h}$ is the ex-ante HML exposure of each portfolio, used in the sort. Bold-faced numbers with a star denote an absolute T-statistic greater than 1.96. Unlike DFF/DT, we exclude firms with negative book values of equity.

Interpretation: Intuitively, these portfolios are long HML-exposed stocks that are not themselves value stocks. The benchmark FF and FFC models will thus attribute a high expected rate of return to these portfolios, which these non-value stocks should meet.

- The FF model cannot price these portfolios in the DT period. This confirms the DT results and inference.
- If the sample period is extended forward to 2008, the FF model explains the discrepancy portfolio almost perfectly. This confirms the DFF conjecture that the DT results were unusual. It renders the Daniel, Titman, and Wei (2001) response (that there were not enough stocks pre-1973) moot.
- Control for u, the ex-post momentum factor (UMD), alters magnitudes a little, but does not alter inference.

Table 2: V1 Zero-Investment Portfolios Created By Sorts to Spread Book-to-Market Characteristics, holding Own Size and HML exposures constant (with HML Exposures Estimated from Shrunk Monthly and Daily Stock Returns).

		Unadj. Net		Fama-French-Adjusted ⇓	⇓					
Sample Period	ĥ	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Ex-ante h Exposures Are Computed from Monthly Stock Returns										
Overall	136%	**0.21%***	**(+2.70)**	0.03%	(+0.57)	**−6%,***	6%,	**45%,***		43.6%
				−0.04%	(−0.53)	**−4%,***	6%,	**48%,***	**6%***	44.8%
DFF	127%	0.12%	(+1.54)	−0.08%	(−1.19)	**−4%,***	**9%,***	**43%,***	3%	41.3%
DT	135%	−0.04%	(−0.30)	**−0.25%***	**(−2.41)**	1%,	3%,	**48%,***	−6%	50.0%
-6/1963	122%	0.21%	(+1.54)	−0.02%	(−0.13)	−4%,	**15%,***	**42%,***	6%	40.6%
7/1963-	146%	**0.21%***	**(+2.34)**	−0.07%	(−0.84)	−2%,	1%,	**52%,***	**7%***	52.5%
1/1994-	185%	**0.56%***	**(+2.89)**	0.23%	(+1.62)	0%,	0%,	**61%,***	**15%***	65.3%
Ex-ante h Exposures Are Computed from Daily Stock Returns										
Overall	88%	**0.19%***	**(+2.49)**	0.04%	(+0.65)	**−6%,***	−1%,	**44%,***		41.2%
				−0.06%	(−0.91)	−4%,	0%,	**48%,***	**9%***	43.8%
DFF	84%	**0.17%***	**(+2.23)**	−0.06%	(−0.92)	−4%,	6%,	**41%,***	**8%***	38.2%
DT	91%	0.09%	(+0.71)	−0.15%	(−1.81)	−3%,	−3%,	**53%,***	−2%	62.6%
-6/1963	77%	0.18%	(+1.46)	−0.07%	(−0.63)	0%,	**14%,***	**33%,***	**10%***	33.1%
7/1963-	96%	**0.20%***	**(+2.04)**	−0.11%	(−1.34)	1%,	**−7%,***	**63%,***	7%	63.0%
1/1994-	112%	0.26%	(+1.11)	−0.12%	(−0.69)	8%,	**−10%,***	**76%,***	**13%***	70.8%

Description: This table is identical in format, variables, and samples to Table 1, except that the ex-ante exposures used in the sort are shrunk in the upper panel, and computed from daily stock returns and shrunk in the lower subpanel. The (reported) ex-post model exposures b, s, h, and u are always monthly.

Interpretation:

- After 1963, the incongruence portfolios have higher ex-post h exposures when the ex-ante ĥ exposures are computed from daily stock return data.
- Despite its higher h coefficient with daily exposures (53% instead of 48%) in the DFF sample, suggesting more power, the DFF abnormal performance loses significance.
- The FF model can price these sort-based incongruence portfolios.
- When exposures are computed from daily stock returns, they seem more accurate (stable) than when exposures are computed from monthly stock returns (ĥ is always much closer to h.)

(File: shrunksorts)

Table 3: Zero-Investment OLS Portfolios with a Positive HML Exposure Target and a Zero Log B/M Average Target

		Unadj. Net		Fama-French-Adjusted						
				⇓	⇓					
Sample Period	$\hat{h}$	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Overall	105%	0.09%	(+0.94)	-0.12%	(-1.85)	-3%,	**-7%**,*	**56%**,*		47.7%
				-0.25%*	**(-3.26)**	0%,	**-6%**,*	**61%**,*	**12%***	50.8%
DFF	101%	0.05%	(+0.55)	**-0.15%***	**(-2.28)**	-0%,	-3%,	**48%**,*		43.6%
				-0.25%*	**(-3.64)**	1%,	-2%,	**53%**,*	**10%***	45.9%
DT	108%	-0.03%	(-0.16)	**-0.38%***	**(-3.58)**	1%,	-1%,	**68%**,*		56.1%
				-0.43%*	**(-3.92)**	1%,	0%,	**69%**,*	6%	56.7%
-6/1963	94%	-0.02%	(-0.13)	**-0.21%***	**(-2.12)**	6%,	-0%,	**35%**,*		40.8%
				-0.31%*	**(-2.94)**	**8%**,*	1%,	**40%**,*	**10%***	43.2%
7/1963-	112%	0.17%	(+1.33)	-0.14%	(-1.78)	-1%,	-5%,	**76%**,*		60.6%
				-0.27%*	**(-2.73)**	1%,	-5%,	**80%**,*	**13%***	63.5%
1/1994-	126%	0.19%	(+0.65)	-0.10%	(-0.56)	-2%,	-4%,	**89%**,*		67.0%
				-0.27%	(-1.31)	4%,	-7%,	**94%**,*	**16%***	71.2%

Description: This table is identical in format, variables, and sample to the lower panel in Table 2, except that the portfolios are formed by optimization. (The $\hat{h}$ input exposures are daily and Vasicek shrunk.) The optimizer is based on Fama (1976) and generates incongruence portfolios that are the equivalents of Fama-Macbeth regressions. These "OLS" portfolios have no cost ($\sum w_i = 0$), balance out the (investment-weighted) log-firm-size characteristic ($\sum w_i \log(\text{ME}_i) = 0$), and log-book-to-market characteristic ($\sum w_i \log(\text{BE}/\text{ME}_i) = 0$). The investment-weighted *ex-ante* HML exposure is positive ($\sum w_i \hat{h}_i$ = pos constant). This target constant is arbitrary, because the dollar investment in the short and long legs is normalized to \$1/month to make it easier to interpret the magnitude of alpha. The T-statistic is not affected by this normalization.

Interpretation:

- The higher ex-post h coefficients suggest that these portfolios are better in spreading HML exposures than those produced by sorts. This increases the power of the test.
- In the overall sample, the FF model is not rejected at the 5% statistical significance level. The UMD-augmented FFC model is rejected.
- Both benchmark models are rejected in the DFF, DT, and pre-1963 sample periods.

(File: targeth)

Table 4: Zero-Investment OLS Portfolios with a Positive HML Exposure Target and a Negative Log B/M Average Target

		Unadj. Net		Fama-French-Adjusted						
				⇓	⇓					
Sample Period	ĥ	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Overall	61%	-0.18%*	(-2.96)	-0.18%*	(-3.19)	-1%,	5%,	-1%,		0.3%
				-0.30%*	(-5.03)	1%,	5%,*	5%,	12%*	6.8%
DFF	58%	-0.22%*	(-3.45)	-0.21%*	(-3.53)	1%,	5%,	-6%,		1.2%
				-0.34%*	(-5.21)	3%,	6%,*	-0%,	12%*	7.2%
DT	68%	-0.39%*	(-3.75)	-0.53%*	(-5.28)	4%,	13%,*	16%,*		10.2%
				-0.60%*	(-5.74)	4%,	14%,*	17%,*	7%*	12.1%
-6/1963	50%	-0.21%*	(-1.99)	-0.19%*	(-1.97)	5%,	1%,	-15%,*		5.7%
				-0.32%*	(-3.19)	8%,*	2%,	-9%,*	13%*	12.4%
7/1963-	69%	-0.15%*	(-2.22)	-0.29%*	(-4.50)	2%,	15%,*	22%,*		16.6%
				-0.37%*	(-5.58)	3%,	15%,*	24%,*	8%*	20.6%
1/1994-	76%	-0.08%	(-0.58)	-0.21%	(-1.89)	0%,	19%,*	32%,*		29.6%
				-0.29%*	(-2.47)	3%,	18%,*	34%,*	8%*	33.9%

Description: This table is identical in format, variables, and sample to Table 3, except that these zero-investment OLS portfolios maximize the difference between the standardized h exposure and the standardized log(BE/ME), of course holding log(ME) constant.

Interpretation:

- The ex-post h exposure is low. Much of the discrepancy between exposure and characteristic in the incongruence portfolio now comes from low book-to-market characteristics, and not from high h exposures.
- Both the FF model and the FFC are now statistically rejected in all sample periods, with the exception of the FF model post-1994 [a sample with only 180 months]. The underperformance is economically significant, too.

(File: `targethbm`)

Table 5: Recalibrated (2-Step) Zero-Investment OLS Portfolios with a Positive HML Exposure Target and a Negative Log B/M Average Target

		Unadj. Net		Fama-French-Adjusted						
				⇓	⇓					
Sample Period	$\hat{h}$	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Overall	84%	-0.05%	(-0.78)	-0.16%*	(-2.74)	-2%,	1%,	27%,*		19.6%
Overall	83%	-0.06%	(-0.91)	-0.29%*	(-4.51)	1%,	2%,	30%,*	12%*	23.0%
DFF	82%	-0.09%	(-1.31)	-0.19%*	(-3.06)	0%,	1%,	21%,*		14.6%
DFF	81%	-0.10%	(-1.46)	-0.32%*	(-4.70)	2%,	3%,	25%,*	12%*	17.3%
DT	91%	-0.22%	(-1.78)	-0.48%*	(-4.68)	2%,	7%,	43%,*		34.7%
DT	91%	-0.23%	(-1.84)	-0.54%*	(-5.08)	2%,	8%,*	44%,*	6%*	34.8%
-6/1963	74%	-0.12%	(-1.06)	-0.21%*	(-2.14)	6%,*	1%,	10%,*		10.7%
-6/1963	73%	-0.13%	(-1.19)	-0.34%*	(-3.29)	9%,*	3%,	14%,*	13%*	14.2%
7/1963-	92%	-0.01%	(-0.07)	-0.22%*	(-3.26)	0%,	7%,*	47%,*		40.9%
7/1963-	91%	-0.01%	(-0.14)	-0.33%*	(-4.36)	2%,	7%,*	49%,*	10%*	43.7%
1/1994-	97%	0.02%	(+0.13)	-0.17%	(-1.34)	-1%,	11%,*	54%,*		50.7%
1/1994-	97%	0.02%	(+0.11)	-0.29%*	(-2.00)	4%,	9%,*	57%,*	11%*	54.9%

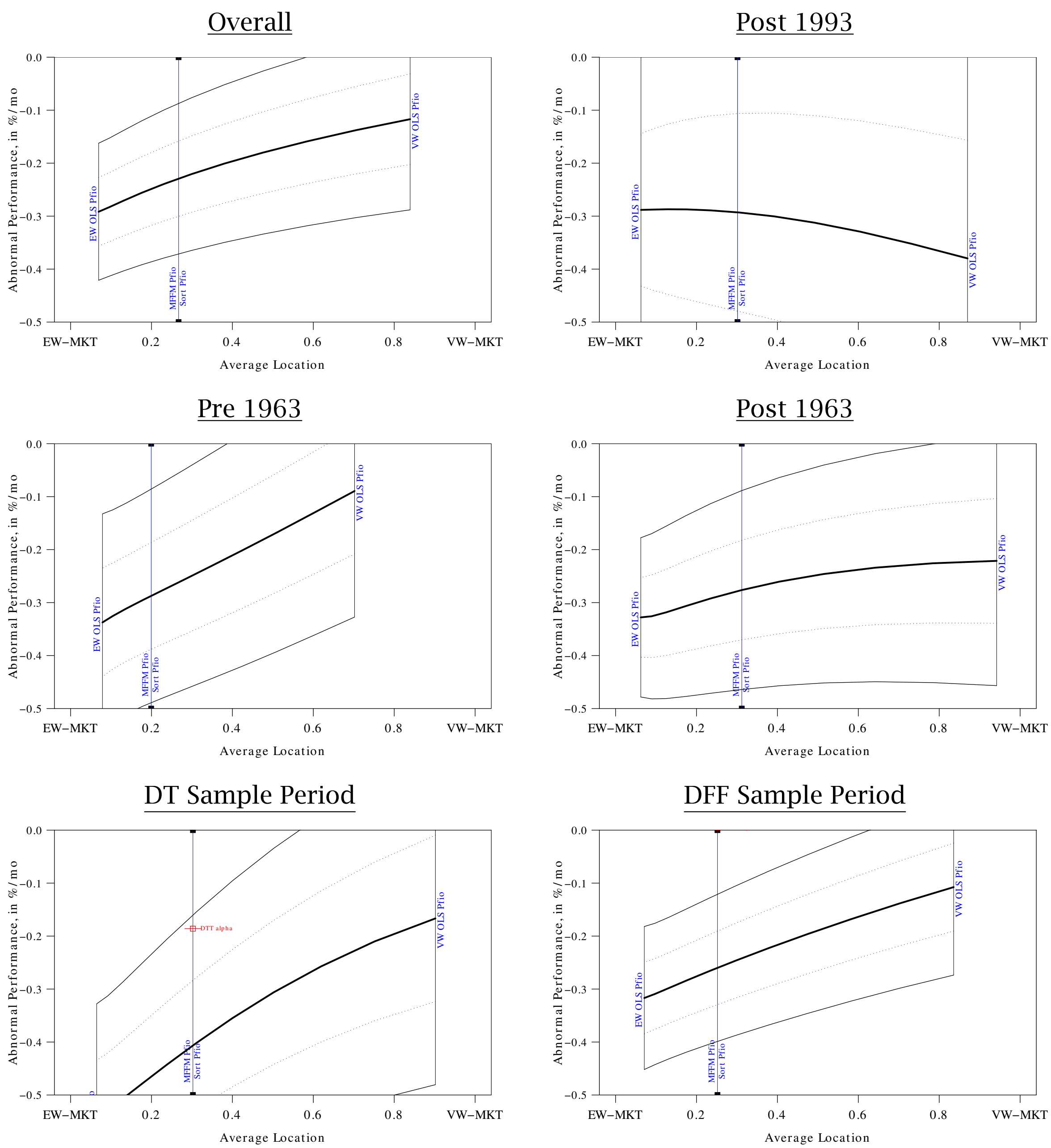
Description: This table is identical in format, variables, and sample to Table 4, except that these OLS portfolios increase the weights of $\hat{h}$ in the difference between the standardized $\hat{h}$ exposure and the standardized log(BE/ME).

Interpretation:

- The 2-step procedure increases the portfolio's ex-ante $\hat{h}$ exposure that is an input into the optimizer. This in turn increases the ex-post h coefficient.
- This has little effect on the ex-post alpha. Both the FF model and the FFC are statistically rejected in most all sample periods. The underperformance is economically significant, too.

(File: `targethbm2`)

Figure 2: Portfolio MarketCap Sensitivity of Alpha



Description: The x-axis is the average investment-weighted marketcap of the incongruence portfolio, quoted in terms of location between the equal-weighted (=0) and the value-weighted (=1) overall stock market. Different average marketcaps arise from different Ω weights (i.e., exponent e's). This is analogous to marketcap-WLS Fama-Macbeth regressions. Depending on this Ω matrix, incongruence portfolios are between about 5% and 80% of the value-weighted stock market portfolio. The y values are the mean and the ***two*** standard deviation bounds of the portfolio alpha.
Interpretation: Portfolios constructed to be as tilted towards larger firms as the DFF/DT sort portfolios still reject the FF model solidly, except in the short 1993- period.

Table 6: WLS Marketcap-Tilted Incongruence Portfolios and Enhanced Balance

		Unadj. Net		Fama-French-Adjusted						
				⇓	⇓					
Sample Period	ĥ	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Spread Portfolio balances only log(ME).										
Equal-Weighted.1Stage	61%	**−0.18%***	**(−2.96)**	**−0.30%***	**(−5.03)**	1%,	**5%***,	5%,	**12%***	6.8%
Equal-Weighted.2Stage	83%	−0.06%	(−0.91)	**−0.29%***	**(−4.51)**	1%,	2%,	**30%***,	**12%***	23.0%
MFFM-Weighted.1Stage	55%	**−0.15%***	**(−2.33)**	**−0.20%***	**(−3.23)**	−3%,	−2%,	−2%,	**12%***	12.8%
MFFM-Weighted.2Stage	80%	−0.06%	(−0.85)	**−0.23%***	**(−3.16)**	−4%,	−5%,	**24%***,	**13%***	17.2%
Value-Weighted.1stage	46%	−0.03%	(−0.42)	−0.07%	(−0.96)	**−5%***,	**−9%***,	1%,	**12%***	12.8%
Value-Weighted.2stage	70%	0.01%	(+0.06)	−0.12%	(−1.37)	**−7%***,	**−11%***,	**20%***,	**14%***	15.2%
Spread Portfolio also balances $\hat{b}$, $\hat{s}$, $\hat{u}$, and $r_{-2,-13}$.										
Equal-Weighted.1Stage	57%	**−0.16%***	**(−2.85)**	**−0.26%***	**(−4.65)**	−1%,	−5%,	**14%***,	**8%***	10.0%
Equal-Weighted.2Stage	76%	−0.08%	(−1.27)	**−0.24%***	**(−4.06)**	**−4%***,	**−6%***,	**32%***,	**8%***	30.6%
MFFM-Weighted.1Stage	51%	**−0.15%***	**(−2.96)**	**−0.22%***	**(−4.12)**	−1%,	**−8%***,	**11%***,	**6%***	9.1%
MFFM-Weighted.2Stage	73%	−0.09%	(−1.49)	**−0.23%***	**(−3.88)**	**−3%***,	**−7%***,	**30%***,	**6%***	28.8%
Value-Weighted.1stage	42%	−0.09%	(−1.46)	**−0.16%***	**(−2.59)**	1%,	**−10%***,	**11%***,	**7%***	7.5%
Value-Weighted.2stage	61%	−0.06%	(−0.84)	**−0.17%***	**(−2.59)**	−2%,	**−7%***,	**25%***,	**5%***	17.1%

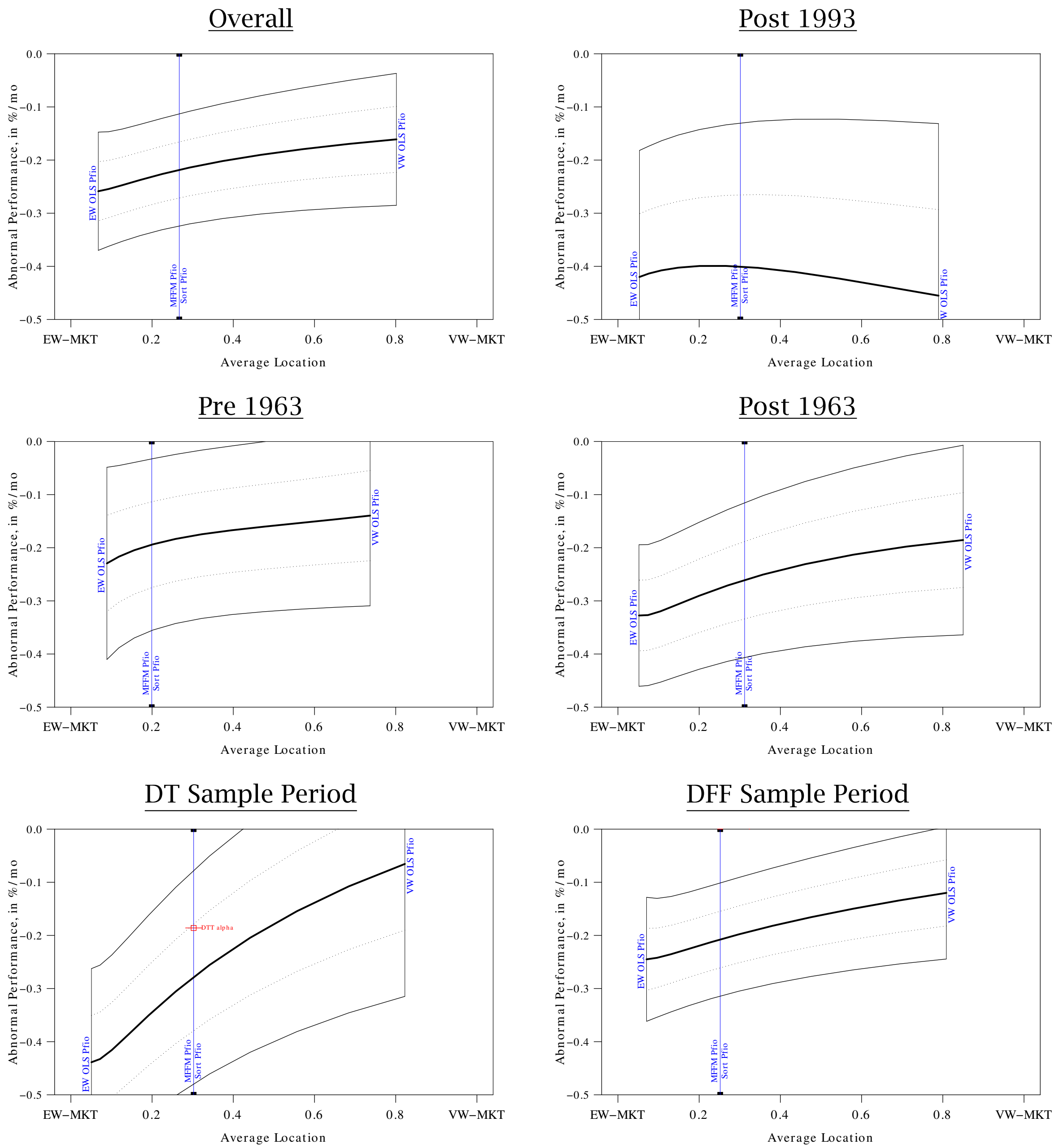
Description: This table is identical in format, variables, and sample to Tables 4 and 5 (indeed, the first two rows are copied), except that MFFM-Weighted and Value-Weighted portfolios tilt more towards stocks with more market cap. The "MFFM" portfolio matches the average market capitalization (location as defined in Figure 2) of the DT and DFF sort-based incongruence portfolios. The "value-weighted" portfolio weights stocks according to their marketcap. In the lower panel, the optimizer also balances the portfolio's ex-ante SMB exposure, MFAC exposure, UMD exposure, and own 1-year momentum (in addition to the firm's own marketcap).

Interpretation:

- The MFFM portfolio cannot be priced by either model, either with or without additional constraints.
- The value-weighted incongruence portfolio without extra balance can be priced by the FF and FFC models.
- When the portfolio is better balanced ex-ante to control for other characteristics—principally, momentum—neither the FF nor the FFC model can price the value-weighted test portfolio.

(File: `vw-constraints`)

Figure 3: Portfolio MarketCap Sensitivity of Alpha,
Enhanced Multidimensional Controls ($\hat{b}$, $\hat{s}$, $\hat{u}$, log(ME), and $r_{-2,-13}$).



Description: This figure replicates Figure 3, but the incongruence portfolios are balanced not only with respect to market cap log(ME), but also with respect to their ex-ante $\hat{b}$, $\hat{s}$, and $\hat{u}$ exposures, and their own momentum ($r_{-2,-13}$).
Interpretation: The FF and FFC models cannot price even the value-weighted incongruence portfolios in the overall, post-1963, and post-1993 samples.

Table 7: Variance-Tilted Incongruence Portfolios (Heteroskedasticity)

		⇓	⇓					
		Unadj. Net		Fama-French-Adjusted				
Sample Period	ĥ	Ave	T-stat	Alpha	T-stat	b s h u		R^2
Overall	73%	-0.06%	(-1.00)	**-0.21%***	**(-3.73)**	**-4%,* -5%,* 32%,* 7%***		34.2%
DFF	70%	-0.07%	(-1.16)	**-0.19%***	**(-3.50)**	**-3%,* -5%,* 27%,* 5%***		26.4%
DT	84%	-0.15%	(-1.37)	**-0.31%***	**(-3.71)**	**-5%,*** -3%, **39%,*** -1%		48.1%
-6/1963	59%	-0.08%	(-0.90)	**-0.21%***	**(-2.69)**	2%, -5%, **20%,* 8%***		18.4%
7/1963-	83%	-0.05%	(-0.57)	**-0.27%***	**(-3.47)**	**-3%,*** -1%, **47%,*** 4%		52.1%
1/1994-	90%	-0.08%	(-0.41)	**-0.38%***	**(-2.56)**	3%, 2%, **62%,*** 10%		62.5%

Description: This table is identical in format, variables, and sample to Table 5, except that the portfolio increases the investment weight on stocks that have lower abnormal expected $\hat{se}(a)$. Stocks with high past return volatility are therefore downweighted. This is equivalent to adjusting for heteroskedasticity.

Interpretation:

- This procedure downweights high volatility stocks. Relative to Table 5, this shrinks both the alpha and its standard error. The inference is weaker, but essentially unchanged.

(File: targetalpha)

Table 8: Recalibrated (2-Step) Zero-Investment OLS Portfolios with a Positive HML Exposure Target and a Negative Log B/M Average Target, in Januaries and non-Januaries

				⇓	⇓					
		Unadj. Net		Fama-French-Adjusted						
Sample Period	ĥ	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Overall All	83%	−0.06%	(−0.91)	**−0.29%***	**(−4.51)**	1%	2%	**30%**	**12%***	23.0%
Overall Jan	80%	0.29%	(+1.04)	0.06%	(+0.25)	2%,	−4%,	**27%***,	**20%***	25.6%
Overall Not-Jan	83%	−0.11%	(−1.60)	**−0.30%***	**(−4.33)**	1%,	2%,	**29%***,	**11%***	20.0%
DFF All	81%	−0.10%	(−1.46)	**−0.32%***	**(−4.70)**	2%	3%	**25%**	**12%***	17.3%
DFF Jan	77%	0.43%	(+1.58)	0.12%	(+0.47)	4%,	-0%,	18%,	**14%***	7.3%
DFF Not-Jan	81%	**−0.17%***	**(−2.35)**	**−0.35%***	**(−4.76)**	2%,	3%,	**24%***,	**12%***	15.1%
DT All	91%	−0.23%	(−1.84)	**−0.54%***	**(−5.08)**	2%	**8%**	**44%**	**6%***	34.8%
DT Jan	91%	0.69%	(+1.35)	−0.61%	(−1.64)	**12%***,	**−19%***,	**57%***,	5%	69.9%
DT Not-Jan	91%	**−0.31%***	**(−2.46)**	**−0.51%***	**(−4.58)**	1%,	**13%***,	**44%***,	5%	30.9%
-6/1963 All	73%	−0.13%	(−1.19)	**−0.34%***	**(−3.29)**	**9%**	3%	**14%**	**13%***	14.2%
-6/1963 Jan	65%	0.11%	(+0.27)	0.61%	(+1.52)	**21%***,	−15%,	−2%,	14%	3.3%
-6/1963 Not-Jan	73%	−0.19%	(−1.68)	**−0.39%***	**(−3.55)**	**9%***,	3%,	**12%***,	**13%***	13.3%
7/1963- All	91%	−0.01%	(−0.14)	**−0.33%***	**(−4.36)**	2%	**7%**	**49%**	**10%***	43.7%
7/1963- Jan	91%	0.43%	(+1.10)	−0.35%	(−1.38)	4%,	−6%,	**59%***,	**16%***	64.2%
7/1963- Not-Jan	91%	−0.05%	(−0.61)	**−0.29%***	**(−3.92)**	1%,	**9%***,	**48%***,	**7%***	41.0%
1/1994- All	97%	0.02%	(+0.11)	**−0.29%***	**(−2.00)**	4%	**9%**	**57%**	**11%***	54.9%
1/1994- Jan	97%	−0.62%	(−0.74)	−0.30%	(−0.68)	−16%,	−5%,	**50%***,	**24%***	72.3%
1/1994- Not-Jan	96%	0.07%	(+0.42)	−0.21%	(−1.52)	3%,	**11%***,	**54%***,	**7%***	53.4%

Description: This table repeats Table 5, but breaks out Januaries vs. other months. Variables and sample periods are defined in Table 1.

Interpretation:

- The failure of the FFC model to price the incongruence portfolios is not due to the January effect.

(File: `jan`)

Table 9: Recalibrated (2-Step) Zero-Investment OLS Portfolios with a Positive SMB Exposure Target and a Positive Log-Marketcap Average Target

				⇓	⇓					
		Unadj. Net		Fama-French-Adjusted						
Sample Period	ĥ	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Overall All	16%	-0.12%	(-1.28)	-0.14%	(-1.66)	23%	16%	-26%	-4%	34.2%
Overall Jan	15%	-0.44%	(-1.42)	-0.39%	(-1.31)	20%*,	10%,	-25%*,	-1%	17.8%
Overall Not-Jan	16%	-0.10%	(-1.08)	-0.10%	(-1.10)	23%*,	16%*,	-27%*,	-5%	34.7%
-6/1963 All	18%	-0.12%	(-1.21)	-0.21%	(-1.92)	12%	3%	-6%	2%	10.3%
-6/1963 Jan	16%	-0.96%*	(-2.45)	-0.34%	(-0.65)	-0%,	-19%,	-6%,	-6%	-3.5%
-6/1963 Not-Jan	18%	-0.09%	(-0.87)	-0.19%	(-1.64)	12%*,	3%,	-6%,	2%	11.2%
7/1963- All	14%	-0.11%	(-0.79)	-0.12%	(-1.06)	36%	26%	-21%	-11%*	52.7%
7/1963- Jan	14%	-0.05%	(-0.12)	-0.56%	(-1.46)	31%*,	24%*,	-21%,	4%	39.9%
7/1963- Not-Jan	14%	-0.11%	(-0.75)	-0.01%	(-0.05)	36%*,	29%*,	-19%*,	-16%*	54.9%
1/1994- All	9%	-0.01%	(-0.03)	0.04%	(+0.15)	51%	44%	-26%	-22%*	66.4%
1/1994- Jan	8%	-0.15%	(-0.15)	-1.07%	(-1.76)	66%*,	66%*,	-3%,	-3%	57.5%
1/1994- Not-Jan	9%	0.01%	(+0.04)	0.23%	(+0.89)	47%*,	44%*,	-31%*,	-28%*	67.6%

Description: This is the analog of Table 8. Instead of contrasting HML exposure vis-a-vis log-book market size, this table contrasts SMB exposure vis-a-vis log-firm size. (Note that the portfolio formation also controls for log(BE).)

Interpretation: Intuitively, these portfolios are long SMB-exposed stocks that are themselves big-firm stocks. The FF model should thus attribute a high expected rate of return to these portfolios.

- The FFC model can price these portfolios statistically, although the magnitude in Januaries is suggestive.

(File: `smb`)

Table 10: Recalibrated (2-Step) Zero-Investment OLS Portfolios with a Positive UMD Exposure Target and a Negative Own Momentum Average Target

		Unadj. Net		Fama-French-Adjusted ⇓	⇓					
Sample Period	ĥ	Ave	T-stat	Alpha	T-stat	b	s	h	u	R^2
Overall All	-4%	**-0.17%***	**(-2.27)**	**-0.21%***	**(-2.55)**	2%	1%	**8%**	-1%	1.8%
Overall Jan	-3%	0.17%	(+0.48)	-0.54%	(-1.41)	7%,	0%,	19%,	-9%	4.8%
Overall Not-Jan	-4%	**-0.19%***	**(-2.42)**	**-0.22%***	**(-2.34)**	2%,	1%,	**9%**,*	0%	1.9%
-6/1963 All	-4%	-0.05%	(-0.45)	0.05%	(+0.49)	2%	3%	-4%	**-19%***	21.6%
-6/1963 Jan	-3%	1.14%	(+1.62)	-0.83%	(-0.93)	5%,	13%,	16%,	-44%	15.7%
-6/1963 Not-Jan	-4%	-0.11%	(-1.01)	0.02%	(+0.21)	3%,	3%,	-4%,	**-18%***	22.8%
7/1963- All	-5%	**-0.26%***	**(-2.60)**	**-0.39%***	**(-3.53)**	-5%	-3%	1%	**17%***	10.1%
7/1963- Jan	-3%	-0.56%	(-1.89)	**-0.58%***	**(-2.14)**	3%,	6%,	-5%,	5%	-6.8%
7/1963- Not-Jan	-5%	**-0.24%***	**(-2.28)**	**-0.45%***	**(-3.84)**	-6%,	-6%,	0%,	**21%***	13.0%
1/1994- All	0%	-0.42%	(-1.65)	**-0.65%***	**(-2.37)**	-8%	-8%	8%	**27%***	19.3%
1/1994- Jan	2%	-0.71%	(-1.29)	**-0.90%***	**(-2.19)**	32%,	4%,	18%,	2%	-10.7%
1/1994- Not-Jan	0%	-0.40%	(-1.48)	**-0.76%***	**(-2.78)**	-6%,	-11%,	12%,	**34%***	23.7%

Description: This is the analog of Tables 8 and 9, but contrasts UMD exposures against own 12-month momentum (with one-month lag). (The portfolio formation also controls for log(ME) and log(BE/ME).)

Interpretation: Intuitively, these portfolios are long UMD-exposed stocks with negative momentum. The FF model should thus attribute a high expected rate of return to these portfolios.

- The FFC model fails in pricing these portfolios in all months. January performance is worse, but not statistically significant, because there are too few observations.

(File: `umd`)

Table 11: Optimized Portfolio Alternatives to FF Factors

HML

	Mean	Sd	Sharpe	Corr
Original-FF	0.43%	3.61%	11.9%	100%
EW,Balanced	0.46%	2.95%	15.5%	79%
MFFM,Balanced	0.36%	2.62%	13.6%	85%
Volat-adj,Balanced	0.52%	3.13%	16.5%	77%

SMB

	Mean	Sd	Sharpe	Corr
Original-FF	0.26%	3.36%	7.8%	100%
EW,Balanced	0.40%	4.38%	9.0%	82%
MFFM,Balanced	0.27%	3.65%	7.4%	90%
Volat-adj,Balanced	0.44%	4.59%	9.6%	80%

UMD

	Mean	Sd	Sharpe	Corr
Original-FF	0.74%	4.72%	15.7%	100%
EW,Balanced	0.74%	4.49%	16.4%	84%
MFFM,Balanced	0.70%	4.25%	16.4%	88%
Volat-adj,Balanced	0.72%	4.63%	15.5%	82%

Description: These are zero-investment factor portfolios (not incongruence portfolios). The first line is the original Fama-French factor, available from Ken French's website. The remaining lines are optimized (OLS) portfolios. All portfolios are balanced. (They are different columns from the same **W** matrix.) The four input characteristics into the optimizer are market-beta, log-market-to-book ratio, marketcap, and own momentum. The "EW" lines are OLS portfolio factors. The "MFFM" lines are WLS portfolio factors whose long and short legs are matched in average market cap to the incongruency factors (now; soon the Fama-French factors themselves). The Volat-adj lines are the equivalents of Table 7, where stocks with more historical variance are downweighted. "Corr" is the correlation with the original equivalent FFC factor. The sample is 7/1929 to 12/2008.

Interpretation:

- The Sharpe ratios of the optimized HML factor portfolios is higher than that of the equivalent original sort-based FF HML factor.
- The Sharpe ratios of some optimized SMB factor portfolios is higher than that of the equivalent original sort-based FF SMB factor.
- There is no improvement for the UMD factor.

todo: add stars for significance of difference from original FF.

(File: appffbase)

Table 12: Own Correlations of Factors

Fama French Factors

	hml	smb	mfac	umd
hml	100%	10%	22%	-40%
smb	10%	100%	33%	-16%
mfac	22%	33%	100%	-35%
umd	-40%	-16%	-35%	100%

MFFM,Balanced

	hml	smb	mfac	umd
hml	100%	-8%	10%	-43%
smb	-8%	100%	39%	-7%
mfac	10%	39%	100%	-22%
umd	-43%	-7%	-22%	100%

EW,Balanced

	hml	smb	mfac	umd
hml	100%	-16%	7%	-46%
smb	-16%	100%	27%	-11%
mfac	7%	27%	100%	-22%
mom	-46%	-11%	-22%	100%

Volat-adj,Balanced

	hml	smb	mfac	umd
hml	100%	-18%	4%	-43%
smb	-18%	100%	26%	-14%
mfac	4%	26%	100%	-21%
umd	-43%	-14%	-21%	100%

Description: These are the same zero-investment factor portfolios from the previous table.

Interpretation:

- By construction, the FF portfolio has HML and SMB mutually balanced.
- Balance with respect to MFAC is not controlled the same way. It is balance with respect to $\hat{b}$, instead.
- The UMD factor (based on own momentum) is tough to balance against HML. Fallen angels become value stocks.
- Maybe we should delete this table.

(File: `appffbasecorr`)

Table 13: Incongruence Test Using Alternative Benchmark Models

				⇓	⇓					
		Unadj. Net		Fama-French or Novel-Factor-Adjusted						
	ĥ	Ave	T-stat	Alpha	T-stat	b'	s'	h'	u'	R^2
Overall	84%	-0.05%	(-0.78)	-0.16%*	(-2.74)	-2%	1%	27%		19.6%
Overall	83%	-0.06%	(-0.91)	-0.29%*	(-4.51)	1%	2%	30%	12%*	23.0%
...EW,balanced	81%	-0.02%	(-0.27)	-0.05%	(-0.78)	-6%*,	16%*,	-1%,		10.5%
...	80%	-0.02%	(-0.32)	-0.16%*	(-2.30)	-5%,	18%*,	5%,	10%*	14.2%
...MFFM,balanced	87%	-0.11%	(-1.45)	-0.13%	(-1.79)	-3%,	14%*,	-1%,		3.8%
...	85%	-0.12%	(-1.51)	-0.30%*	(-3.69)	-1%,	16%*,	9%,	19%*	12.9%
...Volat-adj,balanced	66%	0.03%	(+0.42)	0.01%	(+0.19)	-7%*,	10%*,	2%,		7.0%
...	65%	0.02%	(+0.35)	-0.08%	(-1.33)	-7%*,	12%*,	7%,	9%*	11.0%
-6/1963	73%	-0.13%	(-1.19)	-0.34%*	(-3.29)	9%	3%	14%	13%*	14.2%
...EW,balanced	77%	-0.09%	(-0.79)	-0.09%	(-0.81)	-13%*,	13%*,	6%,	6%	14.7%
...MFFM,balanced	80%	-0.15%	(-1.32)	-0.22%*	(-2.09)	-7%*,	5%,	8%,	20%*	20.2%
...Volat-adj,balanced	57%	-0.08%	(-0.72)	-0.05%	(-0.47)	-14%*,	6%,	7%,	6%	19.4%
7/1963-	91%	-0.01%	(-0.14)	-0.33%*	(-4.36)	2%	7%	49%	10%*	43.7%
...EW,balanced	80%	0.04%	(+0.57)	-0.24%*	(-3.21)	9%*,	24%*,	21%*,	6%*	32.0%
...MFFM,balanced	89%	-0.09%	(-0.88)	-0.41%*	(-3.91)	15%*,	23%*,	29%*,	11%*	26.8%
...Volat-adj,balanced	70%	0.10%	(+1.30)	-0.20%*	(-2.83)	7%*,	20%*,	23%*,	6%*	24.2%
1/1994-	97%	0.02%	(+0.11)	-0.29%*	(-2.00)	4%	9%	57%	11%*	54.9%
...EW,balanced	77%	0.16%	(+1.06)	-0.06%	(-0.50)	17%*,	20%*,	10%,	7%*	47.7%
...MFFM,balanced	96%	0.03%	(+0.10)	-0.35%	(-1.86)	45%*,	28%*,	35%*,	10%*	57.1%
...Volat-adj,balanced	70%	0.25%*	(+1.99)	0.01%	(+0.10)	8%*,	16%*,	16%*,	6%*	21.7%

Description: The gray lines are from Table 5, where the benchmark model is the standard Fama-French model. The remaining lines test benchmark factor models constructed from optimized factor portfolios. The factor portfolios of these alternatives are explained in Table 11. The incongruence tests in this table were reconstructed from scratch. In other words, each line explains its own, different portfolio, constructed to maximize the incongruence between its own h' exposures and the log(BE/ME) characteristics. (The tests are analogous to the tests in Table 5.)

Interpretation:

- A factor model based on a balanced MFFM cannot explain the performance of its incongruence portfolio.
- The equal-weighted balanced performs better.
- The volatility-adjusted balanced factor benchmark models can price its own inconguence test portfolio.
- Conjecture: Factors that use more information from small firms are better.