Deflation in Durable Goods Markets: an Empirical Model of the Tokyo Condominium Market*

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Abstract

Throughout the 1990s, the supply of new condominiums in Tokyo significantly increased while prices persistently fell. This paper investigates whether the market power of condominium developers is a factor in explaining the outcome in this market and whether there is a relationship between production cost trend and the degree of market power that the developers were able to exercise. In order to respond to these questions, a dynamic durable goods oligopoly model of the condominium market—one incorporating time-variant costs and a secondary market—is constructed and structurally estimated using a nested GMM procedure. On the basis of estimates and counterfactual experiments using the estimated model, the following results are obtained. First, the data provide no evidence that firms in the primary market have substantial market power in this industry. Second, the counterfactual experiment provides evidence that inflationary and deflationary expectations on production cost trends have asymmetric effects to the market power of condominium producers: the increase in their markup when cost inflation is anticipated is significantly higher than the decrease in the markup when the same magnitude of cost deflation is anticipated.

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1 Introduction

Throughout the 1990s, the supply of new condominiums in Tokyo increased significantly while prices of condominiums fell persistently. In 1994, the annual supply of condominium units surged from 8,000 units to 20,200 units and has maintained an increasing trend ever since. Meanwhile, the average rate of decrease in condominium prices is 5.2 percent. It has been believed that a major cause of this deflationary price trend is land price depreciation—such as has taken place since the burst of the asset price bubble in the Japanese economy, inasmuch as land is the most expensive factor of condominium production. At the same time, the top 15 firms have maintained control of about one-half of the market, despite the entrance of new developers. This fact may suggest that those top firms exercise market power, and, therefore, the price fall of condominiums may not only be due to the cost reduction but also to the change in their markup. This paper investigates how much market power the large developers exercised in the Tokyo condominium market and how the deflationary cost trend affects the market power of durable goods producers.

In the durable goods market, with focus on a secondary market such as the condominium market, the degree of market power possessed by producers is worth investigating further because theoretical predictions are ambiguous. When there is no secondary market, it has been well known since Coase (1972) that rational expectations of consumers can erode the market power of a durable good producer by generating competition with one's past self as well as one's future self, unless one can commit to a future price path; buyers will not purchase a product at their valuation of the product since they correctly anticipate the producer's incentive to cut its price in the future.[1]

When there is a secondary market, however, theoretical predictions are inconclusive regarding the degree of market power because of the several competing effects of the secondary market on the primary market. On one hand, the secondary market provides more varieties of imperfect substitutes to newly produced goods within and across time, and thus it reduces the market power of producers. On the other hand, the secondary market provides an opportunity for owners of durable goods to replace current holdings more easily, and, thus, it increases the demand for new goods and market power. Furthermore, given future resale possibilities, consumers may prefer the goods that yield higher resale values rather than simply inexpensive goods. Hence, the degree of

[1]This stark conjecture by Coase—that monopoly results in a perfectly competitive outcome—was later formally proven by several researchers, among them Stokey (1981), Bulow (1982) and Gul et al. (1986).

market power of producers depends on which of those effects dominates in a market.

In order to measure the degree of market power in the Tokyo condominium market and investigate the relationship between the market power and cost deflation, we construct a dynamic Cournot oligopoly model based on Esteban (2001) that explicitly incorporates the secondary market. In our model, the products are differentiated by vintage, and producers face a cost function that varies across time. Using building-level supply data, the proposed model is structurally estimated via a nested GMM procedure, in which, following Rust (1987), Markov perfect Nash equilibrium of the model is solved for at each iteration. Using estimated parameter values, we performed several counterfactual experiments.

There are few empirical studies on supply dynamics of durable goods. Notable exceptions are Ramey (1989), who studies the Coase problem—inability of the monopolistic producer to gain a profit—in the US automobile market, and Esteban and Shum (2006) who extend Esteban's model into a vertically differentiated product framework. They find that secondary markets allow firms to exploit their asymmetries. A producer of goods at the high end of the quality spectrum tends to produce more, as this can hurt the profits of producer of low-quality products without seriously hurting his or her own profit; this is because his or her product will act more as a substitute for newly produced low-quality goods once it reaches the secondary market.[2] In contrast to Esteban and Shum, our model is not restricted to vertical differentiation; thus, the problem is not formulated as a linear quadratic but as a more general dynamic problem.

Relative to literature on durable goods supply, there is a growing empirical literature on demand dynamics of consumer durables where the supply dynamics are a given, beginning with Melnikov (2000). He models the consumer's purchase behavior as an optimal stopping problem and develops an estimation procedure by extending the framework of Berry et al. (1995). Subsequent works on this framework include Nair (2004), Gowrisankaran and Rysman (2007), Gordon (2007), Carranza (2007) and Schiraldi (2007); all of them incorporate a dynamic programming algorithm to solve the consumer's optimal stopping problem following Rust (1987) in their estimation. All but Schiraldi give no explicit consideration of the presence of the secondary market.

The relationship between the production cost and market power of a durable goods producer

[2]Porter and Sattler (1999) point out that there exists similar benefits of a secondhand market for durable goods producers as in the differentiated product market. The number of units sold is increased by giving low-valuation consumers a chance to obtain durable goods, and by reducing the incentive of producers to cut prices in order to sell to those low-valuation consumers.

is investigated by Kahn (1986). He suggests that increasing production costs mitigates the Coase problem inasmuch as buyers believe correctly that producers benefit from spreading production over time, and, thus, will not cut prices in the near future.[3] Based on Kahn's intuition, it is expected that, faced with rising costs, producers will have incentives to produce now rather than in the future. Correspondingly, consumers will correctly believe that producers will not cut prices in the future. On the contrary, deflationary costs provide firms with incentives to postpone production in order to cut costs.

Durability is also considered an important characteristic in the literature on housing supply. Within that literature, the closest approach to the one used in our paper is the investment approach, whereby consumers consider a residential building as an investment good, the price of which is determined by the present discount value of the rental price (i.e., the price of the service derived from the structure).[4] The rental price is a function of the level of the existing housing stock. Consumers, therefore, do not explicitly make distinctions between vintages. Homebuilders maximize the present discount value of the profit by taking the price as given. Poterba (1984) and Topel and Rosen (1988) are the examples in this strand. They all assume a competitive market, something rationalized by the fact that new construction is a small part of the existing stock; consequently, builders do not have any control over it. In our application, inasmuch as there seems to exist a distinction between newly constructed units and old units in the minds of consumers, we allow for strategic interactions between builders. To our knowledge, this is the first attempt at considering the housing market in an oligopolistic framework using an investment-based approach. One of a few micro level empirical studies on housing supply is Rosenthal (1999), who tests the efficiency market hypothesis that implies perfect competition. He finds that the deviation between new building prices and construction costs disappears faster than the time required for construction, indicating that the builders of single-family housing in Vancouver do not have excess profit opportunities. This paper approaches the housing market in a more structural manner, both on the demand and supply sides.

[3] Bulow (1982) points out that the capacity constraint might work similarly to increasing costs in an infinite horizon framework. Karp and Perloff (1996) endogenize the technology choice made by a monopolist, thus showing that he or she is able to benefit from an inferior technology as it allows him or her to credibly commit to low production. Kutsoati and Zabojnik (2005) also find that there exists an incentive for a durable goods monopolist seller to adopt an inferior technology using a model of technology selection where "learning-by-doing" is present.

[4] The alternative is the urban spatial approach, which considers an equilibrium wherein the stock of housing always equals the size of the urban population. Under such conditions, the supply of housing is equal to the inflow of new persons (i.e., the increase in the population). Land is defined here as a distinct input and its price is endogenously determined by the housing stock.

The results are summarized as follows: First, there is no evidence that firms in the primary market have substantial market power in this industry—thus, the imperfect competition does not contribute to the observed deflation of prices and increased output. Second, the inflation in the production costs strengthens the market power of the condominium producers, whereas deflation of the production costs exacerbates the erosion of the market power derived from the durability of condominiums; increase in markup when cost inflation is anticipated is significantly higher than decrease in markup when the same magnitude of cost deflation is anticipated.

Given that the impact of the housing market on the overall economy is substantial, understanding the relationship between market power and the cost trend in the housing market is important for the policymakers. For example, to accurately measure the effect of a new housing policy—for instance, that concerning a preferential tax system—one needs to understand the degree of difference in the response of producers to policy changes under different cost phases. More generally, a knowledge of the impact of market power in different cost trends in the durable goods industry is valuable for understanding the impact of changes in factor prices and scheduling revision of the tax code or analyzing merger cases.

This paper proceeds in the following manner. The next section gives a description of the Tokyo condominium market. Section 3 introduces a dynamic oligopoly model of condominium suppliers. Section 4 explains the estimation method for the model. Section 5 reports the estimation results followed by the simulation results. The last section concludes.

2 The Tokyo Condominium Market

2.1 Definition of the Market and the Product

This paper studies the market for newly constructed condominiums in the Tokyo metropolitan area. A condominium is defined as multi-unit housing that consists of five or more units, with three stories or more, and with a steel-reinforced concrete structure. In this paper, we only consider those units that are developed by private companies.[5] We define the market as encompassing 23 central

[5]The public entity known as the Japan Public Housing Corporation (JPHC) has been the alternative seller of condominiums and has provided both rental housing and housing for sale since 1955. Its average annual national supply of housing units for sale of all types was approximately 13,000 units. Those units are excluded from our analysis, as the influence of this entity on the Tokyo market is likely to be insignificant. Additionally, given the growing trend toward privatization and the abundance of housing in urban areas, it retreated from the sales business

districts in Tokyo. It is 621.45 square kilometers in size, and, in 1995, consisted of about 3.5 million households.[6] Throughout the 1990s, population growth was moderate within this area, while the number of households grew at an annual rate of 1 percent. This growth is mostly accounted for by an increase in single-person households. Condominiums have become an increasingly common form of housing in the area—as of 1998, multi-unit housing owned by individuals accounted for 20 percent of all housing in the Tokyo metropolitan area. As of 2001, more than half of all households purchasing new housing chose a condominium unit rather than a single detached house. This ratio was about 45 percent on average throughout the 1990s.[7]

Based on the national tax law as of 1999, the statutory useful life of a condominium unit is 47 years. In reality, most existing condominiums are said to require either large scale repair or rebuilding after about 30 years. However, the physical and quality depreciation of a condominium depends on the maintenance quality over the years. Thus, the vintage may not be the best proxy for quality. This fact motivates us to incorporate other characteristics in the demand specification at the estimation stage, which is described in section 4.[8]

2.2 The Market Environment

Figure 1 summarizes the various price indices relating to the housing market between 1984 and 2002, taking 1995 as the base year. The expansion of the economy started in 1986. During the subsequent four years, the annual real GDP growth rate in Japan was about 5 percent. The burst of the asset price bubble started at the stock market during 1990 and the real GDP growth rate in the following decade was about 1 percent, on average. The burst of the land market bubble gradually prevailed in 1991, a year later than the stock market crash. By that time, the residential land prices in Tokyo had risen by 122 percent from their 1986 level. Since the burst, land prices have been consistently decreasing. In the face of the devastated economy and declining asset prices,

in 1999.

[6]The corresponding statistics for New York City in 2004 are as follows: an area of 785 square kilometers (approximately 303 square miles), encompassing 3 million households and 8.1 million people.

[7]These data are from the Tokyo Metropolitan Government Bureau of Housing (2004):"Tokyo Housing White Paper—Fiscal Year 2003" (in Japanese) and the Mizuho Corporate Bank Industry Survey Division (2003): "Mizuho Industry Survey: An Overview of the Condominium Market in the Tokyo Metropolitan Area" (in Japanese).

[8]In 2002, units that were older than 30 years constituted 6 percent of the total condominiums in Tokyo. The increasing proportion of aged stock for condominiums is becoming a regulatory concern for safety reasons. In particular, condominiums built before 1981 were designed under a weaker regulation code, and, thus, do not satisfy current building standards. In many cases, the re-building of condominiums has proven difficult, as the law requires an approval of re-building plans by four-fifths of the owners of units in the building.

quite a few companies had to sell their unused lots, some of which were suitable for condominium construction: for all condominiums built between 1995 and 2000 in central areas of Tokyo, about 60 percent of sites were formerly owned by the corporate sector. The prices of new condominiums in Tokyo showed almost identical movement with land prices through 1993, presumably because land was the most expensive factor in condominium production.[9] Meanwhile, construction work price deflators reflecting the material and labor costs in the construction industry displayed a gradual increase, but the movement was very modest compared with the fluctuation in land-related prices. The rental index (not shown in Figure 1) exhibited a slight decrease over time, but again, the change has been mild compared with that for land prices, and might reflect the fact that land prices have not been fully adjusted for non-bubble prices.[10] After 1993, land price series and new condominium price series started to show a divergence. Land prices depreciated 1 to 5 percent more rapidly than condominium prices between 1993 and 2000. It might suggest the presence of market power. In other words, the decline in factor price may not be fully reflected in sales prices. Alternative explanations can be that the construction costs become more expensive relative to land prices, and, thus, the total cost falls slower than the land prices.[11] Furthermore, technological innovation made it possible to build taller condominiums, and, thus, the average size of land and the average cost of land for each unit decreased. Nevertheless, in this paper, we focus on the possibility that the market outcome is dictated by the market power.

2.3 The Industry

Condominium construction involves at least two types of firms: developers and construction companies. Developers acquire land, plan condominium development projects and place the order for construction with the construction companies. Developers either sell the units directly to consumers or through dealer companies. In this study, developers are assumed to produce and sell condominium units directly to consumers and the role of construction companies is abstracted away.

[9] The greater part of condominium ownership included sectional ownership of land.

[10] Under an efficient market assumption, the theoretical price of an asset is the present discount value of the expected flow of income gain from the asset. Thus, the value of a house has to be equal to the present discount value of a future rental stream. For example, if the expected rent is fixed to today's level, the land price is proportional to the rent. This means that the land price and rental price indices must be identical.

[11] It may not be applicable, however, to condominium construction costs since there were news reports that the large contractors took orders at very low prices facing decreasing profitable orders from the public sector during the 1990s.

One of the contributors to the surge in the supply in 1994 was new entrants to the market. The transition in market participants is summarized in Table 1. The number of active firms was 111 in 1993, which increased to 205 in 1994, and has stayed around 230 since then. For each year, about 13-27 percent of firms appeared in the data set only once over the sample period of seven years. Those firms or individuals seemed to take advantage of the market expansion to sell their unused land. At the same time, the top 15 firms maintained a market share of more than 50 percent during the period of analysis, despite the large number of entries.

On average, condominium construction takes 15 months.[12] The average time lag in the sample between construction and sales is about 198 days and about 10 percent of the properties are sold before the completion of construction. It is common to divide the units from one project into groups and sell them at different phases. Our dataset is organized by the sales phases. While the actual production decisions tend to be made well ahead of sales time, we assume that transaction and production choices occur at the same time.

2.4 The Secondary Markets

It is said that the resale housing market in Japan is less developed than its counterpart in North America. The trade volume in the secondary market has been estimated at 50,000 units annually in the Tokyo metropolitan area alone. This accounts for 3 percent of the condominium stock in the area.[13]

Three factors are commonly recognized as sources of the low volume of transactions in the secondary market. First, there is a strong preference towards newly constructed housing in Japan. According to the survey of housing demand conducted by the Ministry of Land, Infrastructure and Transport, more than 50 percent of households preferred newly built housing in 2003. Second, there are substantial information asymmetry problems in the Japanese real estate market. Real estate brokers tend to be small scale and specialize in specific areas; they are thus inclined to monopolize local information. This leads to higher search costs for potential buyers and higher opportunity costs derived from vacancies for owners or potential sellers. Shimizu et al. (2004) focus

[12]This data is according to Maeda, Susumu (2005): "The Outlook for the Market for Houses Built for Sale—Inventory of Condominiums," the Japanese Economy Insight, Mizuho Research Institute.

[13]The percentage of aged home transactions in all home transactions in Japan was 11.8 percent in 2001. It is exceptionally small compared with the corresponding figures in the US (76.1 percent), the UK (88.2 percent) and France (71.4 percent).

on this point and find that there would be substantial cost savings if there were an information agency that provided relevant information to all (potential) market participants on all properties for resale, for almost zero cost. Although this suggests that information asymmetry may be important in this market, this paper maintains the assumption of a perfectly competitive secondary market. Third, the transactions cost is rather high. There are various fees in housing markets including real estate acquisition tax, the national registration tax, stamp duty, capital gains tax if the sales are for replacement, brokerage fees, as well as the opportunity costs for sellers mentioned above. Additionally, Kanemoto (1997) points out that the favorable loan treatment for homebuyers of newly constructed houses by the Japan Housing Loan Corporation increases the relative cost of purchasing aged housing.[14] Nevertheless we maintain the assumption that the secondary market is perfect.

3 Model of the Condominium Market

This section gives a description of the dynamic oligopoly model for the primary market for condominiums. The model is constructed based on the discrete-time semidurable goods oligopoly model of Esteban (2001), wherein both firms and consumers are forward looking. The behavior of consumers is modeled using a multinomial logit framework but the model incorporates the dynamics arising from durability.[15] Firms are quantity-setting oligopolists facing a macro cost shock and stochastically evolving fringe competitors.

3.1 The Environment and the Transition of the States

Condominiums are durable and are assumed to last for D periods. Newly constructed condominiums are traded in the oligopoly market, whereas older condominiums are traded in competitive secondary markets. Thus, the producers do not have direct control over the outcome in secondary markets. The condominium units are differentiated by vintage, implying that they are homogeneous within the same vintage. There are three types of state variables in this model: the stock of condominiums, $\vec{s}_t$, the macro cost shock, $\tilde{c}_t$, and the supply of fringe competitors, x_t.

[14] Japan Housing Loan Corporation was a government affiliated and the largest single mortgage lender in Japan. The corporation is privatized in 2006.

[15] This modeling approach is employed by Berkovec (1985) with respect to car consumption.

In the market, there exist J firms producing and selling durable condominiums. Firms are indexed by j and are assumed to be homogenous. A typical firm j produces q_{jt} units of condominiums at time t. Besides J firms, there are fringe competitors who take the price as a given. They collectively produce x_t units at time t. It is assumed that x_t evolves due to an AR(1) process (i.e., $x_t = \overline{x} + \vartheta x_{t-1} + \xi_t$,where ξ_t is distributed mean 0 and is finite variance σ^2_{ξ}.).[16] Therefore, x_t eventually converges to the steady-state level, x^{ss}. If current x_t is below x^{ss}, it would be in a growth phase. The condominium market for each vintage clears for each period. Thus, all existing units are transacted in the secondary market of each vintage until they reach age D. A condominium unit depreciates at an annual rate of $1 - \delta$ before age D. Note that units above age D stay in the market, but as part of an outside alternative. In other words, after age D, the specific links of used units with new or younger units are lost. Stock of age d is expressed as:

$$s_t^d = \delta \left(\sum_{j=1}^{J} q_{j,t-d} + x_{t-d} \right), d = 1...D.$$

Firm j incurs cost to produce $q_{j,t}$ according to the quadratic cost function:

$$C(q_{j,t}^0, \tilde{c}_t) = (\bar{c}_1 + \tilde{c}_t) q_{j,t} + \bar{c}_2 q_{j,t}^2, \tag{1}$$

where $\bar{c}_1$ and $\bar{c}_2$ are constants, while $\tilde{c}_t$ is stochastic, following an $AR(1)$ process to capture macro shocks to the market. Formally, it is expressed as $\tilde{c}_{t+1} = \rho \tilde{c}_t + \eta_{t+1}$, where $\rho \in (0, 1)$ is the persistence parameter and η_{t+1} is white noise (i.e., independently and identically distributed over time with mean zero and a finite variance, σ^2_η). The cost function is common among the J firms, which observe $\tilde{c}_t$ when making production decisions.

Let $\vec{q_t}$ be a vector consisting of the production of the J firms. The above specifications on stock, exogenous production and cost determine the law of motion, as follows:

[16]This treatment of exogenous competitors is similar to that of exporters in the US automaker model used by Esteban and Shum (2006). However, they assumed the stochastic process to be a random walk without drift.

$$\begin{bmatrix} s^1_{t+1} \\ s^2_{t+1} \\ \vdots \\ s^D_{t+1} \\ x_{t+1} \\ \widetilde{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{0}_0 \\ \overline{x} \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 & \mathbf{0}_1 \\ \mathbf{0}_2 & \mathbf{B}_2 \end{bmatrix} \begin{bmatrix} s^1_t \\ s^2_t \\ \vdots \\ s^D_t \\ x_t \\ \widetilde{c}_t \end{bmatrix} + \begin{bmatrix} \delta & \ldots & \delta \\ \vdots & \vdots & \vdots \\ 0 & \ldots & 0 \\ 0 & \ldots & 0 \end{bmatrix} \vec{q}_t + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \eta_{t+1} + \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \end{bmatrix} \xi_{t+1}. \quad (2)$$

Note $\mathbf{B}_1(i,k) = \begin{cases} 0 & \text{if } k = i+1, \\ \delta & \text{otherwise }, \end{cases}$ and $\mathbf{B}_2 = \begin{bmatrix} \vartheta & 0 \\ 0 & \rho \end{bmatrix}$. $\mathbf{0}_0$ is a $D \times 1$ vector of zeros, $\mathbf{0}_1$ is a $D \times 2$ matrix of zeros and $\mathbf{0}_2$ is a $2 \times D$ matrix of zeros. For notational convenience, we denote the vector of the state variables as $\vec{S}_t = [\vec{s}_t' \; x_t \; \tilde{c}_t]'$, where $\vec{s}_t$ is defined by $[s^1_t, s^2_t, ...s^D_t]'$.

3.2 Consumers

The decision of consumers about condominium purchases are modeled using a discrete-choice logit framework. There are M consumers in the market and they are indexed by i. For each period, a typical consumer i purchases, at most, one unit from a set of condominiums, of age $0, 1, ...D$. The age 0 product is traded in the primary market while older units are traded in a competitive secondary market. The owner of a new condominium unit can sell it in the secondary market after holding it for at least a year. The owner of a condominium unit of age D receives a terminal value $\overline{p}$ at the end of the year. The product is indexed by its age, denoted by d. The outside alternative is denoted by $d = n$, and it includes the choice of purchasing single-unit housing, condominiums older than D, or not buying any type of housing (i.e., renting).

The flow utility of consumer i from purchasing a good of vintage d at time t is given by the following quasi-linear form:

$$u_{it}(d) = \begin{cases} g(d) - \alpha p^d_t + e^d_{it} & \text{if } d = 0, 1, ...D, \\ e^d_{it} & \text{if } d = n, \end{cases} \quad (3)$$

where $g(.)$ is a function of the age of the product (d) and measures the quality of the product, p^d_t is the price of the product, and e^d_{it} captures the heterogeneity of consumers that is unobservable by the econometrician. These follow some zero mean finite-variance distributions independently across time and age. Let $\mathbf{J} = [1, ..., J]$ be a set of active firms in the market and $\mathbf{D} = [0, 1, ...D]$ a set of available vintages for those products. Consumer i maximizes the sum of his or her present discounted utility flow by making choice (d) from set $\mathbf{D+1}$. Given the consumer's time discount factor, β, the problem for the consumer i is given by:

$$\max_{\{d_\tau\}_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} u_{i\tau}(d). \tag{4}$$

This problem involves cumbersome dynamic programming. However, it is known that the problem can be simplified to a static one by assuming that there are no transaction costs. Berkovec (1985) and Esteban and Shum (2006) show that problem (4) can be replaced by:

$$\max_{d \in \mathbf{D+1}} UG_{it}(d),$$

where the utility gain $UG_{it}(d)$ is defined by:

$$UG_{it}(d) = \begin{cases} g(d) - \alpha ECC_t^d + e_{it}^d & \text{if } d \leq D, \\ e_{it}^d & \text{if } d = n. \end{cases}$$

The expected capital cost ECC_t^d is defined by:

$$ECC_t^d = \begin{cases} p_t^d - \beta p_{t+1}^{d+1} & \text{if } d < D, \\ p_t^d - \beta \overline{p} & \text{if } d = D, \end{cases}$$

where $\overline{p}$ is the terminal or scrap value of the condominium when it reaches age D in period $t+1$. It implies that a consumer's dynamic decision is equivalent to a comparison of the utility gains from the choices available in the period. The utility gain consists of terms $g(d)$, the benefit from the consumption of the goods for the given period, and $p_t^d - \beta p_{t+1}^{d+1}$, the implicit rental price under the assumption of no transaction costs. Here, consumers have perfect foresight, such that $E_t(p_{t+k}^{d+k}) = p_{t+k}^{d+k}$ for all t and $k = 0, 1, D+1$. This assumption is relaxed at the estimation stage.[17]

The unobserved heterogeneity of consumers, e_{it}^d, is assumed to be identically and independently distributed with respect to the type I extreme value distribution across consumers (i), vintage(d) and time (t). Integrating ϵ_{it}^d then yields a market share equation for each vintage as follows:

[17] Berkovec (1985) considers stochastic breakdown and the possibility of scrap for automobiles. Correspondingly, the expected capital cost takes these possible events into consideration. In the case of condominiums, because a complete breakdown is seldom observed, we disregard this possibility.

$$\mu_t^d(p_t, p_{t+1}) = \begin{cases} \frac{\exp\left(g(d)-\alpha ECC_t^d\right)}{1+\sum\limits_{d'=0}^{D} \exp\left(g(d)-\alpha ECC_t^{d'}\right)} & \text{for } d \leq D, \\ \frac{1}{1+\sum\limits_{d'=0}^{D} \exp\left(g(d)-\alpha ECC_t^{d'}\right)} & \text{for } d = n\ . \end{cases} \tag{5}$$

Applying the transformation method of Berry (1994), the logarithms of μ_t^d and μ_t^n are taken and their differences are given by the following expression:

$$\ln \mu_t^d(p_t, p_{t+1}) - \ln \mu_t^n(p_t, p_{t+1}) = g(d) - \alpha ECC_t^d, \tag{6}$$

$$= g(d) - \alpha p_t^d + \alpha\beta p_{t+1}^{d+1}, \tag{7}$$

for $t = 1, ...T, d = 0, 1, ...D$. Note that the market share of each type is defined by:

$$\mu_t^d = \begin{cases} \frac{x_t+\sum_j q_{jt}}{M} & \text{if } d = 0, \\ \frac{s_t^d}{M} & \text{if } d = 1, 2, ...D \\ 1 - \frac{\left(\sum_j q_{jt}+\sum_d s_t^d+x_t\right)}{M} & \text{if } d = n. \end{cases} \tag{8}$$

Iterating over the future expected capital cost ECC, together with some manipulations, yields the following expression for the price of each new unit produced by firm j:

$$p_t^0 = \frac{1}{\alpha}\left[\sum_{d=0}^{D} \beta^d(\ln \mu_{t+d}^n - \ln \mu_{t+d}^d + g(d))\right] + \beta^{D+1}\overline{p}_{t+D+1}, \tag{9}$$

$$= P^0(\vec{s}_t, x_t, x_{t+1}, ..., x_{t+D}, \vec{q}_t, \vec{q}_{t+1}, ...\vec{q}_{t+D}). \tag{10}$$

It shows that the price of each new product depends not only on today's production $(\vec{q}_t, x_t)$, but also on that of the future $(\vec{q}_{t+k}, x_{t+k},\ k = 1, ...D)$ and of the past $(\vec{s}_t)$, through the outside market share μ_{t+d}^n.

Given this inverse demand function, the description of a firm's problem is given in the next section.

3.3 Firms

Firms are competing in a Cournot quantity setting game. Condominium development requires a long period of planning and it is difficult to make quick adjustments in terms of the number of units being supplied once a development plan is approved by the authorities. Thus, it is reasonable to consider the production level as a strategic variable of a firm. Given the inverse demand function of a new product (9) and the cost function (1), firm j chooses the level of production to maximize its present discounted profit stream:

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} E_t \left[p_\tau^0 q_{j\tau} - C(q_{j\tau}, \widetilde{c}_\tau) \right]. \tag{11}$$

Because of the dependence of new condominium prices on the current, future and past production of the entire condominium stock (i.e., of all firms), any given firm's production strategy may depend on the entire history of its production. The convenient assumption is to allow the production plans of all firms at time t to depend only on the stock of condominiums that is actively being traded in the market at any given time. This assumption corresponds to the concept of a Markov perfect Nash equilibrium, which is a subgame perfect equilibrium where actions are only functions of payoff-relevant state variables, as defined in Maskin and Tirole (1988a, 1988b). In the current problem, the payoff-relevant variables are the state variables ($\vec{S}_t$), as defined in section 3.1.

Formally a firm's problem is given by:

$$\max_{q_{jt}^0} \sum_{\tau=t}^{\infty} \beta^{\tau-t} E_t \left[p_\tau^0 q_{j\tau} - C(q_{j\tau}, \widetilde{c}_\tau) \right], \tag{12}$$

subject to (2) and

$$q_{jt} = h_j(\vec{S}_t), \tag{13}$$

and

$$q_{jt} \leq M - \sum_{d'=1}^{D} s_t^{d'} - x_t - \sum_{j' \neq j} q_{j't}, \tag{14}$$

given

$$q_{j't} = h_{j'}(\vec{S}_t),\ \ j' = 1, 2, ...j-1, j+1, ...J, \tag{15}$$

where $h_l(\cdot)$ is the stationary policy function for firm l. The constraints (13) and (15) ensure that the solution is a Markov perfect Nash equilibrium. The expectation operator in the infinite sum in problem (12) is over the η_s and ξ_s, $s = t, t+1, \ldots$.. The constraint (14) restricts the choice of production so there is no oversupply. At equilibrium, the policy functions that rational firms use to forecast future production, both their own and that of competitors, coincides with the optimal policy for each. Note that, in this case, the equilibrium strategy is time consistent.

The problem stated by equations (12) to (15) gives the following Bellman equation:

$$V_j(\vec{S}_t) = \max_{q_{jt}} \left[E\pi_{jt}(\vec{S}_t, q_{jt}, \vec{q}_{-jt}, \{\vec{q}_\tau\}_{\tau=t+1}^{D}) + \beta EV_j(\vec{S}_{t+1}|\vec{q}_t) \right], \tag{16}$$

subject to (2), and (13) for $j = 1, ..., J$. $\vec{q}_{-jt}$ denotes a vector of production at time t, for all firms but j. It can be further simplified as follows:

$$V_j(\vec{S}_t) = \max_{q_{jt}} \left[E\pi_{jt}(\vec{S}_t, q_{jt}, \vec{q}_{-jt}, \{H(\vec{S}_\tau)\}_{\tau=t+1}^{D}) + \beta EV_j(\vec{S}_{t+1}) \right], \tag{17}$$

where the vector $H(\vec{S}_\tau) = [h_1(\vec{S}_\tau), ..., h_J(\vec{S}_\tau)]'$ stands for the vector of (expected) future production given the state $\vec{S}_\tau$. To obtain tractability and overcome the computational burden, we focus only on symmetric equilibria, so that $h_j(\vec{S}_\tau) = h(\vec{S}_\tau)$ for all j.

3.4 Discussions about Some Assumptions

In this section, we discuss four assumptions that, while important in implementing the estimation, are certainly not innocuous in other respects. First, products are differentiated only by vintage. Thus, condominiums are homogeneous within the same vintage and the quality of the product in a given vintage is constant for each given time. Although the data suggests that each year there exists great variation in the characteristics of new condominiums, and that those characteristics change over time, this assumption is nonetheless maintained, as the focus of the current paper is on the durability of condominiums.[18] This simplification implies that firms take the quality of each rival's product as a given; likewise, that they consider a stated quality as being the same as their own.

Second, firms are homogeneous. This restriction, together with the first assumption, greatly reduces the dimensionality of the problem by allowing a structure wherein the policy function only depends on common variables (i.e. total stock, exogenous production and macro cost shock), rather than also on firm-specific variables. It also enables us to impose a symmetric equilibrium when solving the model. If firm-specific variables are included in the set of state variables, the dimension of the problem grows with the number of the firms, and the problem becomes intractable. The gain from these assumptions is that we are only required to solve the problem for a single agent and do

[18] Treating the products of oligopolistic firms and the product of fringe firms is unlikely to be problematic. The estimation of the probability that a unit is provided by a fringe firm, controlling for characteristics and year effects, using the probit indicates that there are no substantial differences between the products of two types of firms.

not have to worry about multiple equilibria. A drawback from this restriction is that the model does not explain the variation in the production level across firms, something which is observed in the data. Instead, this is dealt with using idiosyncratic production errors, as described in section 4.

Third, the terminal value of the condominium unit is fixed. This assumption permits us to obtain an analytical expression for the inverse demand function in a very simple manner. There are two shortcomings, however. First, we get a high price elasticity of demand and a low sensitivity of price to output, inasmuch as the terminal value does not depend on the stock or production. Second, it is likely that $\tilde{c}$ is correlated with $\overline{p}$, because of the certainty that the value of the physical building depreciates over time; thus, the price gets much closer to the land price as it ages. Nevertheless, it is difficult to infer the relationship between these variables unless we impose further structure on them, as we do not directly observe $\tilde{c}$.

Fourth, $\tilde{c}$ is treated as being exogenous. Hence, the cost, mainly as reflecting the land price, is not allowed to be endogenous; if the project involves the development of a large community, large-scale condominium construction could raise the value of the land.

4 Estimation

The set of structural parameters in the model described above is $\Theta = [\bar{x}, \vartheta, \sigma_{\xi}^2, \alpha, \beta, \delta, \bar{p}, \{g(d)|d = 0, 1, ...D\}, \bar{c}_1, \bar{c}_2, \rho, \sigma_{\eta}^2]$. This section describes the estimation strategy of those parameters in three steps. The third step involves the dynamic programming algorithm in the standard GMM procedure following Rust (1987). Note that various estimation approaches for dynamic games have been recently developed; among others, Hotz et al. (1994), Aguirregabiria and Mira (2002) and Bajari et al. (2007) are computationally less expensive compared with the nested fixed point approach. However, two features of our model—a continuous choice variable and a state variable that is common to all agents but unobservable to econometrician—do not easily allow the direct application of those methods.

4.1 Data

The data for this study are obtained from two sources: primary market data for the years from 1990 to 2000 are taken from the yearly publication "Condominium Apartment Market Trends," as constructed by the Real Estate Economic Institute; and secondary market data are taken from periodical advertisements entitled "Weekly Housing Information," for the years 1992 to 2002, as published by Recruit Co., Ltd..

The unit of observation in the first dataset corresponds to a group of units in one development project that are sold at the same sales timing, called a phase. While 26 units on average are sold in a phase from one project, one phase could contain as many as 319 units. The data include the names of buildings, their addresses, the closest train stations, distances to stations, the names of developers, the names of builders, as well as other characteristics. Some of those variables are summarized in Table 2. The fifth column reports the mean of the variables weighted by the number of units to grasp the distribution of the variables in terms of units. This table displays the large variety of characteristics in the sample, as is common in any real estate data at the micro level. As described in the previous section, the model imposes that all products within the same vintage are homogeneous. However, we took advantage of the richness of the microlevel data.

The second set of data is organized by unit. Two datasets are merged using common information such as the names of buildings and addresses. However, for each given time, the majority of condominiums are not traded; furthermore, the "Weekly Housing Information" advertisements do not cover all the properties on the market; 27 percent of the observations have corresponding secondary market data. For these reasons, prices for unobserved units are imputed using a linear regression of prices for each age on variables for various characteristics using data on observed units. Appendix B describes the method in detail.

The last two sections in Table 2 report the summary statistics for imputed prices. Prices are adjusted for inflation using a GDP deflator. The base year is 1995. In order to obtain numerical stability in the nested algorithm, prices are re-scaled by one millionth, and the units are re-scaled by one thousandth.

Our model classifies firms into two types: oligopolistic firms and fringe firms. The firms are selected by the ranking of the cumulative production during the sample period. The estimation is performed for the models of the a five-firm oligopoly (model I) and a 10-firm oligopoly(model II).[19]

4.2 Fixed Parameters

Some parameters that are fixed in the estimation procedure are summarized in Table 3; these are fixed in order to implement the estimation.

Two parameters that dictate durability in the model—the lifespan of a condominium unit, D, and the depreciation factor, δ —are fixed for computational reasons. The value of D is fixed at one calendar year; thus, a condominium unit lasts in the market for two years, in order to reduce the dimensionality. Note that as D increases, the number of vintages included in the state vector increases accordingly. To see the consequence of this treatment, the production paths of monopolists over a period of 25 years for $D = 1$ and $D = 2$ are simulated using the same parameter values. The results are shown in Figure 2. The diagram indicates that there are no substantial differences in the nature of the two series. Although the same parameter values are used, additions to the vintage increase the value of a steady state; thus, we see the difference in the levels of production. Since what is important for the estimation and for the purpose of this study is the property of the series, this treatment does not cause any substantial differences in the results.[20] It is unlikely that a condominium unit physically depreciates over the first two years of its life. However, we set its annual depreciation rate, $1 - \delta$, at 0.01 for two reasons. First, since the precise data of the stock of condominiums are unavailable, the parameter value cannot be estimated. Second, the numerical stability of the nested dynamic programming algorithm requires $1 - \delta$ to be strictly greater than zero.[21]

The common discount factor for firms and consumers is fixed at $\beta = 0.975$, which reflects the interest rate during this period and follows the convention found in the IO literature. It is known, in general, that the discount factor tends to be collinear to other parameters in a dynamic model, and, thus, it is difficult to identify.

As discussed in the previous section, to obtain an analytical expression of the inverse demand

[19] Top 10 firms are Daikyo, Mitsui, Recruit Cosmos, Sumitomo, Towa, Cesar, Marubeni, Asahi Construction Dia Construction, and Nomura Real Estate. These are ordered by the value of the cumulative production. The top five firms were within the top 15 for nine consecutive years between 1992 and 2000.

[20] For the initial value of this simulation, we used the value corresponding to the 1991 observations for stock and exogenous production, and the calibrated value for the macro cost shock. The method of calculation for the initial value of the macro cost shock is described in Appendix C.

[21] Note that the optimal policy does not substantially change at each set of state variables when δ is reduced further.

function, the price of a two-year-old unit, $\overline{p}$, is considered constant. In the estimation, it is fixed at 42.3 million yen, which corresponds to the weighted average (imputed) price of a two-year-old unit between 1994 and 2002.[22]

The cost parameter $\bar{c}_1$ is set at 24.71 million yen, which is equivalent to 61 percent of the projected cost for 2002.[23] This parameter is thought of as constituting the steady state of the level of the constant portion of the marginal cost. For numerical optimization, we restrict the sum of $\bar{c}_1$ and the macro shock, $\tilde{c}_t$, so that it is bounded below by zero. If this parameter is to be estimated, the range of $\tilde{c}_t$ must be adjusted for each iteration, something that increases the computation time. The variance of the macro cost shock, σ_η^2, is normalized at unity, as the policy function is very insensitive to this parameter. The market size, M, is fixed at $3,514,000$, which is equal to the number of households in the area in 1995, a figure obtained from the census data. Thus, outside alternatives include not only condominiums older than two years but also all types of housing, inclusive of single-unit ownership and "no purchase." "No purchase" is equivalent to rental housing.

Given these fixed parameters, the set of structural parameters to be estimated is reduced to $\Theta = [\bar{x}, \vartheta, \sigma_\xi^2, \alpha, g(0), g(1), \bar{c}_2, \rho]$. In the next subsection, structural errors are introduced. Subsequently, the three-step estimation procedure is described.

4.3 Econometric Model

To carry out a statistical inference of the model, unobservable stochastic terms must be introduced so that variations observed in the data are generated by the model.

The key equations for the estimation are the market share equations (6) and the equilibrium production rule (13). For the demand-side relationship, the assumption about a consumer's expectation (i.e., perfect foresight) is relaxed, and a rational expectation is assumed instead. Specifically, the price of product aged $d+1$ at time $t+1$ can be written as follows:[24]

$$p_{t+1}^{d+1} = E_t(p_{t+1}^{d+1}|\Omega_t) + \nu_{t,t+1}^{d+1}, \tag{18}$$

[22]The simple average of the imputed unit price for the same period was 45.52 million yen, with a standard deviation of 29.18 million yen.

[23]Based on an estimate by the industry analyst, non-land costs (i.e., construction cost and sales service cost) accounts for 61 percent of total cost per unit. For the determination of this parameter, the total cost for 2002 is projected by setting the average margin to 10 percent for 2000 and applying the growth rate of each cost index.

[24]Note that the product that was aged d at time t becomes age $d+1$ at $t+1$.

where Ω_t is the information available at time t and $\nu_{t,t+1}^{d+1}$ is the forecast error for vintage d.

For the supply side, an error λ_{jt} for firm j at time t is introduced; thus, the relation between the observed data and the optimal production rule can be written as:

$$q_{jt} = h(\vec{S}_t) + \lambda_{jt}, j = 1, ..., J, \tag{19}$$

where it is assumed that λ_{jt} is unobserved by any firm when making a decision, and that it is independently and identically distributed as $N(0, \sigma_\lambda^2)$ across firms and time. This implies that a producer integrates out not only its own production errors, but also those of its rivals when solving the problem (17) although they are not state variables. Note that λ_{jt}s do not affect the equilibrium policy function but still allow for the heterogeneity in the realized production. This change in the assumption adds one more parameter to estimate, σ_λ^2.[25] The assumption that there may be unexpected adjustments in production at the time of planning may sound restrictive. However, it is observed that, in some cases, condominium developers purchase condominium buildings from other developers. Hence λ_{jt} can be thought of as constituting such adjustments.

Additionally, the forecast error $\nu_{t,t+1}^{d+1}$ and λ_{jt} are assumed to be independent across time, firms and vintages. With this assumption, the introduction of $\nu_{t,t+1}^{d+1}$ does not change the problem for the producer.[26]

Using forecast errors as the basis for an estimation is not a common approach in the empirical discrete choice literature, which assumes the existence of unobserved heterogeneity. In this model, the time-invariant heterogeneity is captured by the term $g(d)$, while time-variant heterogeneity cannot be introduced, as it will not be consistent with the dynamic problem solved by the producers unless $\nu_{t,t+1}^{d+1}$ is treated as another state variable. This is not feasible because of computational difficulties. An alternative structure is the introduction of measurement errors. However, as the equilibrium production rule (19) is not linear for state variables measured using past errors, the construction of the GMM objective function requires an integration of all past errors. This is not available, however, given the current computational ability.

[25] One of the advantages of the GMM procedure over other methods, such as the maximum likelihood estimation, is that it does not require a parametric assumption of the error term. However, in this model, a parametric assumption is required, as the current price and profit depends on ω_{jt}, and each firm solves its own profit maximization problem with regard to expectation.

[26] It is because the forecast errors are entered additively to the expected price function that the expected current period profit function is identical to the one without $\nu_{t,t+1}^{d+1}$, so long as it is independent of q or λ.

The First Step—Estimation of x_t Process($\bar{x}, \vartheta, \sigma_\xi^2$)

The evolution of x_t, the production level of fringe competitors, is estimated using data from 1992 to 2000, by regressing it on its lagged variable (i.e. x_{t-1}). From the residuals, we obtain an estimate for σ_ξ^2. The variable x_t is constructed for each model by subtracting the aggregated production of oligopolistic firms from the total production in each year.

The Second Step—Estimation of the Demand Parameters ($\alpha, g(0), g(1)$)

Since the model treats all condominiums of the same vintage as homogeneous, the corresponding data are aggregated by year. Inasmuch as the aggregation makes the number of observations too few for reasonable estimation, we employ sales phase data (i.e., the lowest level of aggregation) to estimate the parameters α, $g(0)$ and $g(1)$. Thus, the available variables at this stage are the averages of the characteristics and prices for the group of units that are sold by a particular firm at a particular location in a given sales phase. We index the unit of observation by k and let K denote the total number of groups of newly produced units. To capture quality variations across products as produced by different firms, a characteristic vector, $\vec{X}_{k,t}^d$, is introduced.

Introducing forecast errors $\nu_{k,t,t+1}^{d+1}$ and using phase-level data modify equation (6) as follows:

$$\begin{aligned}\ln \mu_{k,t}^d - \ln \mu_t^n &= \vec{X}_{k,t}^d \Gamma - \alpha(p_{k,t}^d - \beta E p_{k,t+1}^{d+1}), \\ &= \vec{X}_{k,t}^d \Gamma - \alpha(p_{k,t}^d - \beta p_{k,t+1}^{d+1} - \beta \nu_{k,t,t+1}^{d+1}), \\ &= \vec{X}_{k,t}^d \Gamma - \alpha CC_{k,t}^d + \omega_{k,t+1}^{d+1}, k = 1, ...K, d = 0, 1, t = 1, ...T, \qquad (20)\end{aligned}$$

where $CC_{k,t}^d (= p_{k,t}^d - \beta p_{k,t+1}^{d+1})$ denotes the realized capital cost of a unit in group k of age d at time t.

As the disturbance ($\omega_{k,t+1}^{d+1} = \alpha\beta\nu_{k,t+1}^{d+1}$) is due to a forecast error, its definition gives the orthogonality condition as below (i.e., given the information set at time t, the expected error on the forecast is zero). Note that the number of observations increases proportionally to D because each new condominium will be one-year-old stock in the next period. Between ages, only prices and the amount of stock vary, since other observed characteristics do not change over time:

$$\begin{aligned}E(\omega_{k,t+1}^{d+1} | \Omega_t) &= 0 \\ E(y_{k,t} \cdot \omega_{k,t+1}^{d+1}) &= 0, \qquad (21)\end{aligned}$$

where $y_{k,t}$ consists of variables that are known at time t. Note that Ω_t cannot include $q_{k,t}$, as it is not known when consumers make their choices. The vector $y_{k,t}$ includes a constant and some characteristic variables. Consistent estimation of the demand parameters, (α, Γ), can be obtained using a GMM estimator. The parameters for the next estimation step, $g(0)$,and $g(1)$, are obtained by calculating the mean characteristic vector for each vintage across k and t ($\vec{X}^d = \sum\sum_{kt} \vec{X}^d_{k,t}$), and by evaluating $\hat{g(d)} = \vec{X}^d\hat{\Gamma}$ for $d = 0, 1$. By doing this, $g(d)$ becomes fixed to the mean of the quality for vintage d across time.

The Third Step—Estimation of the Supply Parameters

Given the estimates from the previous steps, we estimate the cost-related parameters, $(\rho, \sigma^2_\lambda, \bar{c}_2)$, by estimating eq.(19), using the nested GMM procedure. In this model, where the parametric form of the policy function is unknown, our data-matching procedure utilizes a function approximation technique. Note that the alternative method such as utilizing equilibrium conditions cannot avoid obtaining a solution of the dynamic programming problem because current price is a function of future productions. Given the variables $\vec{z}_{jt}$, which are orthogonal to λ_{jt}, we are able to obtain the moment condition:

$$E(\vec{z}'_{jt} \cdot \lambda_{jt}) = 0. \tag{22}$$

Under the assumptions for λ_{jt} , the instruments are the constant, lagged production for two periods with the exception of its own, and exogenous production (i.e., $z_{jt} = [1, q_{-j,t-1}, q_{-j,t-2}, x_t]$). The distribution assumptions for λ_{jt} give another moment restriction, based on the second moment for λ_{jt}, namely:

$$E\left[\vec{z}'_{jt}[\lambda^2_{jt} - \sigma^2_\lambda]\right] = 0. \tag{23}$$

The stacking conditions (22) and (23), together yield $E(Z_{jt} * \Lambda_{jt}) = 0$, where Z_{jt} is the block diagonal matrix. Its sample analogue is given by:

$$\Upsilon_s = \frac{1}{TJ}\sum_{t=1}^{T}\sum_{j=1}^{J} Z_{jt} * \Lambda_{jt}. \tag{24}$$

For each evaluation of the set of parameter values, the firms' dynamic programming problem has to be solved, as the function $h(\cdot)$ is a function of parameters $(\rho, \sigma^2_\lambda, \bar{c}_2)$.

The GMM criterion function (24) is, however, not available due to an initial condition problem—one of the state variables, $\tilde{c}$, is unobservable and serially correlated. The feasible objective function is obtained by integrating out the sequence of $\tilde{c}$ from (24) using the density of $\tilde{c}$. Nevertheless, as none of $\tilde{c}$ are observable, the serial dependence of $\tilde{c}$ requires a further assumption on its initial value (or terminal value). In this application, the terminal value $\tilde{c}_T$ is assumed to be nonstochastic and fixed to the value based on informal information on the cost of condominium production in the late 1990s. See Appendix C for how we calibrated this value. The feasible moment condition is thus given by:

$$\Upsilon_{si} = \frac{1}{J}\sum_{j=1}^{J}\int \cdots \int Z_{jt} * \Lambda_{jt}(\vec{c}) f(\vec{c}|c_T) d\vec{c}. \tag{25}$$

Given a positive definite weighting matrix $\hat{\Xi}_s$, the GMM estimator minimizes $\Upsilon'_{si}\hat{\Xi}_s\Upsilon_{si}$. In the first-stage estimation, we used the inverse of the squared instrument matrix as $\hat{\Xi}_s$. The results reported in this paper involve the optimal GMM estimator, which uses a consistent estimator for $E(\Upsilon_{si} * \Upsilon'_{si})$ as a weighting matrix.

4.4 Identification

As explained in the first and the second estimation steps, identifications of parameters for the process of production by fringe competitors and the demand system are obtained using cross-sectional and time-series variations of observables: market shares, observed capital costs, and aggregate production by fringe competitors. However, the identification in the third step is not trivial.[27] Although the model in this paper fully specifies the parametric form of the return function, those assumptions alone do not guarantee identification. Stated more formally, the objective function for the estimation (i.e., the optimally weighted quadratic form of the GMM conditions) must be reasonably sensitive to changes in the parameter values.

In order to gain some ideas about its sensitivity to the parameter values in which we are interested, we present simulated production paths of a monopolist for different values of $\bar{c}_2$ and ρ in Figure 3. The initial value of each simulation is set at the observed value for 1991, and each run consists of 10,000 simulations over nine periods. The panel on the left shows that the increase in $\bar{c}_2$, the coefficient of the quadratic term in the cost function, decreases the production at each period but does not greatly change the shape of the path. Thus, it determines the level of the optimal production. The panel on the right indicates that a low value of ρ, which implies a smaller persistence of the macro cost shock, $\tilde{c}$, generates a hump-shaped path by making $\tilde{c}$ reach

[27] For a more precise discussion of nonparametrical identification of the dynamic Markov decision problem, see Rust (1994).

its steady-state level faster. Hence, the peak of the production occurs later, as ρ increases. At a higher level of ρ, the peak is not realized within nine periods. Therefore, observed variations of the level of production can identify $\tilde{c}$, and observed shapes of the production paths can identify ρ. The variance of idiosyncratic production shock, σ_λ^2, can be identified by cross-sectional variations in production because all heterogeneity among the firms are summarized in λ_{jt} in the model.

5 Results

5.1 The Parameter Estimates

The empirical results are reported in Tables 4, 5 and 6, respectively, for each estimation step.

The process for x_t is estimated for model I (a five-firm oligopoly) and model II (a 10-firm oligopoly). For both models, the AR(1) coefficient, ϑ, is positive and significantly less than one, as seen in Table 4. The constant term, however, is positive, although not significant. These parameter estimates imply that the process of x_t gradually converges towards a positive steady-state value, x^{ss}, which is estimated to be $32,200$ units with model I; and $31,160$ units with model II.

The demand system is estimated using demand data for the period 1994-1999. This period includes the year during which the consumption tax rate changed and a new tax preferential system for homebuyers was implemented; both are likely to have had a large impact on housing purchase behavior. Nevertheless, we assume that all consumers and firms anticipated these events at the beginning of the period.[28] All observations appear twice in the estimation in the dataset, as any given year's new condominiums become one-year-old units the following year, with the amount depreciating by $1-\delta$. Overall, the dataset consists of 10,113 observations.

Columns (i) and (ii) in Table 5 report the estimates for the demand parameters, by OLS and GMM, respectively. Since imputed prices are used as the values of the condominium stock, standard errors must be adjusted for the noise caused by imputation. The correction is performed using the bootstrap method. Possible endogeneity for expected capital cost is dealt with using the

[28] The consumption tax was raised from 3 to 5 percent in April 1997, in order to compensate for the fiscal loss from the income tax cut of 1994. Note that the consumption tax is imposed on any consumption expenditure, inclusive of residential buildings while the value of the land is not subject to it. As a part of its economic stimulus package, in 1999, the government extended the existing tax preferential system to include mortgage payments on housing loans. The change, which went into effect in 1999 as planned, increased the maximum tax benefit from 1.7 million yen to 5.9 million yen, a 245 percent increase.

log of its height and the log of the distance from the nearest train station. Both are known at the time of purchase by potential buyers, and correlate with the value of the property; the distance from the nearest train station is negatively correlated with the land prices, while the height of the building is positively correlated with the production costs. The coefficient for expected capital cost, $-\alpha$, is estimated to be negative and significant for both model specifications, although the magnitude's absolute value is larger when instruments are used, indicating that the forecast error, ω_{jt}, causes a bias toward zero. The negative value of $-\alpha$ suggests that consumers prefer a good with a lower capital cost, as expected. The tests for relevance of instruments (the canonical correlations likelihood-ratio test), endogeneity, and the overidentification restriction show that the adopted instruments are acceptable. Dummy variables for age, which measure the quality of each vintage after controlling for other characteristic variables, are negative and significant for all ages and for all specifications, implying that, relative to the outside alternative, consumers value condominium units less. Among condominiums, new condominiums are valued more than older condominiums, as is shown by the larger estimated coefficient of the age-0 dummy relative to the age-1 dummy. Having obtained estimated parameters for the age dummies and the other characteristic variables, the vintage quality parameters $g(d), d = 0, 1$ are calculated using the mean values for all characteristic variables. These are reported in Table 5, and represent the average valuations of consumers for condominium units for each vintage relative to outside goods. For the two specifications, the rankings of these two parameters by age are the same as that for the age dummies.

The cost parameters estimated in the third step are reported in Table 6. Firm-level data for five- and 10-oligopolistic firms over seven periods (from 1994 to 2000) are used. The estimates for ρ indicate that the macro cost shock is a stationary process for both specifications. These estimates from models I and II, imply that the linear coefficient of the cost function, $\bar{c}_1 + \tilde{c}_t$, is deflating on average, at 2.4 percent and 1.5 percent, respectively. The estimated value of $\bar{c}_2$ is positive and significant, confirming that this industry's technology has decreasing returns to scale. This result reflects the fact that, relative to small firms, large developers are more apt to construct costly units such as large-scale buildings and high-rise complexes. This implies that large condominium developers have an incentive to spread production across time, as predicted by Kahn (1986); thus, they have some ability to commit to a future production plan.

5.2 The Numerical Solution of the Model

In this section, the solution of the producers' problem with the estimated parameters is presented. The solution for the model is obtained using a policy function iteration algorithm, that utilizes a function approximation technique, known as the collocation method, which is described in Appendix A. The nature of the solution is the same for all values of J (the number of oligopolistic firms). Consequently, the result reported in this section is based on model II ($J = 10$).

The panels in Figure 4 display the contour maps of the resulting policy function corresponding to eq.(19), the value function, and the price of a new condominium as a function of the macro cost shock, $\tilde{c}_t$, and the production of fringe competitors, x_t, at the steady-state stock level, ($s_t = s^{ss} = 31.16$).

Both the policy and value functions decreases for all of the state variables for the age-1 stock, s_t, the fringe competitors, x_t, and the macro cost shock, $\tilde{c}_t$. To understand the nature of the optimal production policy, it is useful to break down the states based on the values of the exogenous state variables, $\tilde{c}_t$ and x_t, relative to their steady states. For instance, if $\tilde{c}_t$ is above zero, the process shows a decreasing trend; thus, the state describes a deflationary period for $\tilde{c}_t$. If x_t is below the steady-state value, the process shows an increasing trend and the state describes a growth period. Table 7 reports the elasticities of the policy function with respect to the state variables for the different exogenous state phases. For example, when the macro cost shock is deflationary and the exogenous competitor is growing, a one-percent change in the one-year-old stock results in a 0.02 percent change in production. From this analysis, three more properties of the policy function are derived.

First, as measured by elasticities, the policy is more responsive to the production of the fringe competitors, x_t, than to the one-year-old stock,s_t; This is true for all states. For example, a 1 percent increase in x_t leads to a decrease of between a 0.07 percent and 0.33 percent in production, while a 1 percent increase in s_t leads to roughly a 0.02 percent decrease in production. The intuition behind this result is that, inasmuch as it is a part of current production, x_t influences the market longer through future stock than through one-year-old stock.

Second, production is more responsive to exogenous state variables when the cost is in a deflationary period ($\tilde{c} > 0$) and exogenous production is contracting ($x > 31.16$). Conversely, production is less responsive when the macro cost shock is appreciating and exogenous production is growing.

This reflects the effect of consumers' expectations; since the policy function is a decreasing function of both $\tilde{c}$ and x, cost inflation and growth in exogenous production imply lower production in the future. As a result, consumers are convinced that there will not be a drastic price cut in the future. Consequently, producers do not have to respond to changes in the market environment so much. As a result, adjustment towards the steady state is slower. With the opposite scenario, where cost depreciates and the production of fringe competitors contracts, producers have to respond relatively more, as consumers expect greater production and lower prices. Therefore, the convergence to the steady state occurs more rapidly than with the inflationary phase. This property corresponds to the result in Kahn (1986), wherein a decreasing return-to-scale cost function helps firms to credibly implement a low production plan, although in this case, the cost varies over time. Furthermore, in the deflationary phase of the macro cost shock, consumers correctly expect a future price cut, on account of which, producers quickly lose their market power. This point is investigated further in the next section.

Third, the response to the one-year-old stock does not vary a great deal with respect to the exogenous variables. This is partly due to the fact that, in the next period, the stock will move from the market to the outside alternative; consequently, the stock does not have a direct impact on the future market.

5.3 Simulations

In this section, several simulation results are presented to show the dynamics in the market. Unless otherwise noted, all simulations consist of over 10,000 independent seven-period simulations of the dynamic model. For the initial condition of s_t and x_t, the actual observations for 1994 are used. For $\tilde{c}_t$, which cannot be observed directly, we set $\tilde{c}_{2000}$ as the calibrated value of $\tilde{c}$, based on industry information and the estimated parameter value.

The Predictive Power of the Model

Table 8 compares the simulated statistics and the observations for total production, new condominium prices and the production of fringe competitors for models I and II. Performance is especially good for the prices. The second last row in the table reports the percentages of time within which the predictions fall within the 15 percent intervals from the observations. In both cases, the percentages for prices are close to 100 percent. For production, however, in models I and II, they are 24.4 percent and 37.7 percent, respectively. The performances with respect to production may indicate the limitations in the assumption that all oligopolistic firms are homogeneous. For the rest of the simulation exercises, the set of parameter estimates for model II is adopted because that model yields higher predictive power. Furthermore, it is statistically more reliable because it is based on more observations than is the case for model I. Additionally, alternative assumptions on the number of oligopolistic firms(J) are explored. These indicate that the predictive power increases with J, suggesting that the market is closer to being competitive.

Market Power and Profits

The mean prediction of markup, evaluated at the marginal cost, is reported in the last raw of Table 8. The average markup is between 0.48 percent and 0.56 percent; these are small values, and they indicate that firms may not possess substantial market power.[29]

Given our quadratic production costs specification, however, markups evaluated at the marginal costs for firms are not indicative of profits. The average profit margin measured using the average costs in simulated data is 8.4 percent and 12.4 percent for models I and II, respectively. These levels of profit margin are comparable to the profit margins reported in the financial statements of developers during the late 1990s, which suggests that the condominium business yields a profit margin of about 10 percent.[30]

The Role of Cost Variations

For the purpose of examining the relationship between production cost trends and the market power of durable goods producers, we compare the markups in a cost increasing phase and a cost decreasing phase. To make a valid comparison, the following two paths are compared. The path of the decreasing phase is generated using the estimated parameter value, and then setting the initial value of the macro cost shock at $\tilde{c}_t = C$, where C has a positive value. To see the effect only of the cost variation trend, the path being compared should initiate from the same marginal cost function at the beginning. Thus, $\bar{c}_1 + \tilde{c}_t$ should be at the same level and $\tilde{c}_t$ should have the same absolute value but with a negative sign; consequently, $\tilde{c}_t = -C$. Producers in both cases then face the same

[29]In general, the model with the estimated parameter values yields a low level of markup across all ranges of states where the problem is solved. Using simulations, we assess to what extent an assumption that firms produce differentiated products can lead to higher markups.

[30]For example, the average operating margin of the firms in the sample were 8 to 12 percent in 1994-2000. The profit breakdown estimate for the unit price around 2000, performed by an industry analyst indicates that the margin was about 10 percent.

speed of convergence in terms of $\tilde{c}_t$. To align the value of the marginal cost, the solution of the producer's problem, where $\bar{c}_1$ equals the sum of the previously set value (24.71 million yen) and twice the value, is set as the point of comparison ($2 \times C$). This solution is used to simulate the path in the cost increasing phase. For the initial value of exogenous production and one-year-old stock, the observed values for 2000 are used in both paths. For the value of C, the terminal value of the macro cost shock obtained in the estimation, $\tilde{c}_{2000}$, is used.

Figure 5 compares the simulated markups over 11 periods; the dotted line indicates the markup for the cost increasing phase and the solid line indicates that for the cost decreasing phase. At the initial point, the markup under the increasing phase is 31 percent higher than that under the decreasing phase. As the time passes, the difference in markups increases. By the 11th period, the markup in the increasing phase is 2.9 times more than that of the decreasing phase. This comparison indicates that the firms have significantly more market power during the cost increasing phase than during the cost decreasing phase.

Underlying these results is the change in the incentive of producers compared with that under a time-invariant cost structure. When cost is increasing, a firm has an incentive to produce more now rather than later, since it will be more costly to produce in the future. On the other hand, when cost is decreasing, a firm has an incentive to postpone production, as it can save on the cost by accruing a lower marginal cost in the future. Forward-looking consumers are aware of these incentives to firms. Therefore, cost increases lead consumers to believe that firms will not flood the market in the future; hence, their willingness to pay does not decrease. Conversely, cost decreases lead to a reduction in a willingness to pay. Hence, the Coase problem of firms becoming less serious during the cost increasing phase, but the problem is worse during the cost decreasing phase. Given that, on average, factor prices in the Tokyo market were on a decreasing trend throughout the 1990s up through the mid 2000s, this result suggests that condominium developers had more difficulty in making profits during the 1990s than during preceding decades.

Factors of Price Deflation between 1994 and 2000

In this section, the contributors to the price deflation between 1994 and 2000 are decomposed using the parameter estimates from model II. The result here, however, should be interpreted with caution given the way oligopolistic firms are selected in the estimation.

For this purpose, we obtain the following simulated price paths: (i) the benchmark price path reported at the beginning of this section; (ii) the price path where $\tilde{c}$, and x_t are fixed at the initial levels for all simulation periods; and (iii) the price path using the actual values of x_t and with a fixed $\tilde{c}$. By comparing these series, we are able to obtain how much of the variation in price is accounted for by increased exogenous competition. Table 9 presents the results of the simulation. Column (iv) reports the percentage of the price deflation since 1994 that is accounted for by the increased competition caused by exogenous fringe competitors.

Note that the contribution of fringe competitors dropped from 45 percent to 13 percent in 1997 and revived to 25 percent in 1998. For those two years, the overall output dropped compared with that for 1996. This shift is likely due to the effects of the consumption tax hike introduced in 1997 and the change in the tax preferential system, which was expected to go into effect in 1999. With the anticipation of the first event that was announced in 1994, forward-looking consumers were apt to engage in last-minute purchases in the period leading up to 1997 and to reduce purchases following the change in the tax rate.[31] In anticipation of the second event, potential buyers had a strong incentive to postpone their purchases so as to benefit from the new system. Thus, prices after 1996 up through 1999 decreased due to the components not accommodated for in the model. In this simulation, all the remaining change is the contribution of $\tilde{c}_t$. Excluding those two years, the contribution of x_t is about 50 percent. One thing to note is the possibility that x_t is correlated with cost factors. It is likely that, generally speaking, competition intensifies as costs decline. Nevertheless, this is beyond the scope of this paper, and is left for future research.

Column (v) reports the price path if oligopolistic firms act as price takers. By comparing this path with the benchmark path (i), the effect of imperfect competition on the market price can be measured. Column (vi) reports these measures in percentage terms; they are very close to zero. On average, the benchmark price is 0.09 percent or 36,000 yen higher than the competitive price, suggesting that the market power was not a key factor in explaining the divergence of the condominium prices and land prices that is observed in Figure 1.

6 Conclusion

This paper examines the market phenomenon of the primary market for condominiums in Tokyo between 1994 and 2000. During this period, increased output and persistent falls in the prices of

[31] For a description of these events, see footnote 28.

condominiums, land, and other factors of production were observed. The main question posed here is whether market power played any role in explaining this outcome.

We focus on the durability of the condominiums and the presence of a secondary market, and developed a dynamic oligopoly model that is based on Esteban (2001). The model incorporates an important feature found in this industry: the persistent factor price variations that affect the dynamics of the market on account of the expectations of all agents. This framework allows for an investigation of the relationship between the trend in production costs and the degree of market power possessed by the durable goods producers.

The structural parameters of the proposed model are estimated using a three-step estimation procedure, one which includes a nested GMM method, in which the algorithm solves the dynamic programming problem of producers for each evaluation of the GMM objective function.

For each estimated set of parameter values, the model yields an optimal policy for oligopolists as a decreasing function for all state variables (i.e., condominium stock, exogenous production and macro cost shock). The optimal policy is the most responsive to the macro cost shock, followed by exogenous competitors as measured by elasticities. Furthermore, the model shows that firms respond to changes in the market environment more drastically during the deflationary phase than during the inflationary phase.

In the estimation and simulation experiments, we find two major results. First, the data provides no evidence that the firms in the primary market had substantial market power in this industry. Contrary to our conjecture, therefore, imperfect competition did not play a role during this period. Second, increasing and decreasing expectations on production cost trends have asymmetric effects to the market power of condominium producers: an increase in their markup when cost increases are anticipated is significantly higher than a decrease in markup when the same magnitude of cost decreases are anticipated.

Those results may call for caution on the part of policymakers when considering the effects of policy instruments, such as modifications in the tax codes and the evaluation of merger cases. It is particularly relevant in recent years because the land prices started to exhibit an inflationary trend in some areas in Tokyo in 2003 and construction costs started to increase in 2005.

References

Aguirregabiria, V. and P. Mira, "Swapping the Nested Fixed Point Algorithm: A Class of Estimators for Discrete Markov Decision Models," *Econometrica* 70 (July 2002), 1519–1543.

Bajari, P., C. L. Benkard and J. Levin, "Estimating Dynamic Models of Imperfect Competition," *Econometrica* 75 (2007), 1331–70.

Benkard, C., "A Dynamic Analysis of the Market for Wide-Bodied Commercial Aircraft," *Review of Economic Studies* 71 (2004), 581–611.

Berkovec, J., "New car sales and used car stocks: a model of the automobile market," *RAND Journal of Economics* 16 (1985), 195–214.

Berry, S., J. Levinsohn and A. Pakes, "Automobile Prices in Market Equilibrium," *Econometrica* 63 (July 1995), 841–890.

Berry, S. T., "Estimating Discrete-choice Model of Product Differentiation," *RAND Journal of Economics* 25 (1994), 242–262.

Bulow, J., "Durable Goods Monopolists," *Journal of Political Economy* 90 (1982), 314–332.

Carranza, J. E., "Estimation of demand for differentiated durable goods," mimeo., University of Wisconsin-Madison, 2007.

Coase, R., "Durability and Monopoly," *Journal of Law and Economics* 15 (1972), 143–149.

Dipasquale, D., "Why don't we know more about housing supply?," *Journal of Real Estate Finance and Economics* 18 (January 1999), 9–23.

Esteban, S., "Equilibrium Dynamics in Semi-Durable Goods Markets," mimeo., Penn State University, 2001.

Esteban, S. and M. Shum, "Durable-Goods Oligopoly with Secondary Markets: the Case of Automobiles," *RAND Journal of Economics* forthcoming (2006).

Gordon, B. R., "A Dynamic Model of Consumer Replacement Cycles in the PC Processor Industry," mimeo., Carnegie Mellon University, 2007.

Gowrisankaran, G. and M. Rysman, "Dynamics of Consumer Demand for New Durable Goods," mimeo., Washington University in St. Louis, 2007.

Gul, F., H. Sonnenschein and R. Wilson, "Foundation of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory* 39 (1986), 155–190.

Hotz, V. J., R. A. Miller, S. Sanders and J. Smith, "A Simulation Estimator for Dynamic Models of Discrete Choice," *Review of Economic Studies* 61 (April 1994), 265–289.

Judd, K. L., *Numerical Methods in Economics* (Cambridge,MA: MIT Press, 1998).

Kahn, C., "The Durable Goods Monopolist and Consistency with Increasing Costs," *Econometrica* 54 (March 1986), 275–294.

Kanemoto, Y., “The housing question in Japan,” *Regional Science and Urban Economics* 27 (1997), 613–641.

Karp, L. S. and J. M. Perloff, “The Optimal Suppression of a Low-Cost Technology by a Durable-Good Monopoly,” *RAND Journal of Economics* 27 (Summer 1996), 346–364.

Kutsoati, E. and J. Zabojnik, “The Effects of Learning-by-Doing on Product Innovation by a Durable Good Monopolist,” *International Journal of Industrial Organization* 23 (Feburuary 2005), 83–108.

Maskin, E. and J. Tirole, “A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs,” *Econometrica* 56 (May 1988a), 546–69.

Maskin, E. and J. Tirole, “A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles,” *Econometrica* 56 (May 1988b), 571–99.

Melnikov, O., “Demand for Differentiated Durable Products: The Case of the U.S. Computer Printer Market,” mimeo., Yale University, 2000.

Miranda, M. and P. L. Fackler, *Applied Computational Economics and Finance* (Cambridge,MA: MIT Press, 2002).

Nair, H., “Intertemporal Price Discrimination with Forward-looking Consumers: Application to the US Market for Console Video-Games.,” mimeo., Stanford University, Graduate School of Business, 2004.

Noguchi, Y. and J. Poterba, *Housing Market in the United States and Japan-A NBER Conference Report,* (Chicago, Illinoi: The University of Chicago Press, 1994).

Ono, H., H. Takatsuji and C. Shimizu, “Development of Hedonic Housing Price Index for Second-hand Condominium Market in Tokyo Metropolitan Area (in Japanese),” Working Paper, Reitaku Instutute of Political Economics and Social Studies, April 2002.

Porter, R. H. and P. Sattler, “Patterns of Trade in the Market for Used Durables: Theory and Evidence,” NBER Working Paper 7149, National Bureau of Economic Research, 1999.

Poterba, J. M., “Tax Subsidies to Owner Occupied Housing: An Asset Market Approach,” *Quarterly Journal of Economics* 99 (1984), 729–752.

Ramey, V. A., “Durable Goods Monopoly Behavior in the Automobile Industry,” Working Paper, Department of Economics, D-008, University of California, San Diego, 1989.

Rosenthal, S. S., “Residential Buildings and the Cost of Construction: New Evidence on the Efficiency of the Housing Market,” *The Review of Economics and Statistics* 81 (May 1999), 288–302.

Rust, J., “Optimal replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher,” *Econometrica* 55 (September 1987), 999–1033.

Rust, J., “Structural Estimation of Markov Decision Process,” in R. Engle and D. McFadden, eds., *Handbook of Econometrics*volume 4, chapter 51 (Elsevier Sicence Publisher B.V., 1994), 3081–3143.

Schiraldi, P., "Automobile Replacement: a Dynamic Structural Approach," mimeo., Boston University, 2007.

Shimizu, C., K. G. Nishimura and Y. Asami, "Search and vacancy costs in the Tokyo housing market: Attempt to measure social costs of imperfect information," *Regional Science and Urban Economics* 16 (Nov 2004), 210–230.

Stokey, N., "Rational Expectations and Durable Goods Pricing," *Bell Journal of Economics* 12 (1981), 112–128.

Topel, R. and S. Rosen, "Housing Investment in the United States," *Journal of Political Economy* 96 (1988), 718–740.

Wooldridge, J. M., *Econometric Analysis of Cross Section and Panel Data* (Cambridge,MA: MIT Press, 2002).

APPENDIX

A Solution Algorithm—the Collocation Method

With few exceptions, the Markov decision problems have no analytical solutions. In such cases, one needs to rely on the numerical solution—which is an approximation of the true solution—in order to understand the dynamics of the model. We describe here in this note one of the solution methods for a discrete-time continuous Markov decision problem, the collocation method. Among the difficulties in solving such problems is the fact that the unknown of the dynamic programming is not a particular variable, but rather consists of two functions, usually known as the value function and the optimal policy function. In collocation methods, this difficulty is overcome by approximating the value function by using a linear combination of prespecified functions, called a basis function, and evaluating it at predetermined state nodes. For details, see Miranda and Fackler (2002).

To simplify the notation, s denotes a vector of state variables and q denotes the choice variable. The function $g(s, q)$ describes the transition of a state vector, given the choice of a firm, in the previous period. The problem can be expressed in the following form:

$$v(s) = \max_q \left[\pi(s, q) + \beta E v\left(g(s, q)\right)\right].$$

The collocation method suggests an approximation of this value function, $V(\cdot)$, with a linear combination of n prespecified functions; these functions are evaluated only at the prespecified n state nodes. More specifically, the function V can be expressed as follows:

$$v(s) \approx \sum_{j=1}^{n} c_j \phi_j(s), \tag{26}$$

where c_j is a scalar and $\phi_j(s)$ is a/the nonlinear function. The numerical analysis theory offers several choices of functional form for $\phi_j(s)$ and the associated state nodes, such as the Chebyshev polynomial basis and node, and the piecewise polynomial splines and nodes. Based on this approximation, we can rewrite the problem as follows:

$\sum\limits_{j=1}^{n} c_j \phi_j(s) = \max\limits_x \pi(s, q, h(g(s, q))...) + \beta \sum\limits_{j=1}^{n} c_j \phi_j(g(s, q))$.

The task then is to obtain the optimal policy function, $h(\cdot)$, and coefficient, $c_j, j = 1, 2, ..., n$..

Once $\phi_j(\cdot)$s and s are selected, the optimal policy and value function can be obtained using the algorithm below. Before describing the algorithm, several notations need to be introduced. Let $\mathbf{s}$ be the vector (or matrix if s is a vector) of interpolation nodes $[s_1, s_2, ...s_n]$. Using $\phi_j(\cdot)$s and s, we can construct a matrix, Φ, in which the ijth element is $\phi_j(s_k)$, where s_k is the kth interpolation node. Note that this matrix will not change over the solution algorithm. Let $\mathbf{c} = [c_1, c_2, ..., c_n]'$ be the vector of the approximation coefficients. Let $\mathbf{v}$ be the column vector $[v(s_1), v(s_2), ..., v(s_n)]'$, where s_k denotes each interpolation node. We can then write (26) in vector notation:

$$\mathbf{v} = \Phi \mathbf{c}.$$

The outer loop solves for the value function approximation (obtaining coefficient value, c_j^*, $j = 1, 2, ...n$), while the inner loop solves for the optimal policy and the associated value function. Note that the superscripts for c and h indicate for the number of iteration steps involved/found in the outer loop in the description below.

Step 1 At the beginning of the program, both the initial guess for the coefficient vector, $\mathbf{c}^0$ (which approximates the value function), and the initial guess for the optimal policy, $h^0(\cdot)$, are determined.

Step 2 Given $\mathbf{c}^i$, the inner loop solves the Bellman equation and the return policy function and the value at the interpolation nodes, s. More specifically, for each interpolation node, s, we obtain q, thus satisfying the Karush–Kuhn–Tucker condition: $\frac{\partial \pi}{\partial q} + \beta \sum_{j=1}^{n} c_j^i \phi_j^{\prime}(g(s,q))\frac{\partial g}{\partial q} = 0$, where $\phi_j^{\prime}$ is the first derivative of ϕ_j. To evaluate this condition, h_i has to be approximated with $\Phi\mathbf{c}^*$. This allows us to gain the expected future production. Note that Φ is the same matrix for the approximation of $\mathbf{v}$. This step yields $q = h^{i+1}(s)$ and the optimal value, $v^{i+1}(s)$.

Step 3 Given $\mathbf{v}^{i+1}$, $\mathbf{c}^i$ can be updated by the following rule: $\mathbf{c}^{i+1} = \Phi^{-1}\mathbf{v}^{i+1}$. Alternatively, it can be updated using Newton's method, which uses the iteration rule,
$\mathbf{c}^{i+1} = \mathbf{c}^i + \left[\Phi - \mathbf{v}^{i+1\prime}\right]^{-1}\left[\Phi\mathbf{c}^i - \mathbf{v}^{i+1}\right]$,
where $\mathbf{v}^{i+1\prime}$ is the Jacobian of $\mathbf{v}^{i+1}$. It goes back to step 2 until $\left\|\mathbf{c}^{i+1} - \mathbf{c}^i\right\|$ reaches a particular level of tolerance.

Note that Miranda and Fackler (2002) provides the MATLAB toolkit, which constructs a vector from the basis function and the corresponding interpolations nodes with the users type of choice.

B Imputing Future Prices in the Secondary Market

As mentioned in section 4.1., not all condominium units are traded every year, and not all of the data for those that were traded are available. Thus, future prices (p^*_{jt+n}) must be imputed from the observed data. We followed two steps, as described in this sequel.

First, the secondary market data from the classified magazines documented in section 4.1. are matched with the primary market data by name and address. Of the entire primary market sample, about 27 percent of the entries correspond to at least one secondary market entry between 1992 and 2002.

Second, we estimate an imputation equation in the following specification. Note that we use prices per square meter instead of unit prices so as to control for size differences:

$$\log(p^*_{jt+n}) = a_0 + a_1 \log(p^*_{jt}) + a_3^{\prime}\mathbf{x} + u_{jt}, \tag{27}$$

where p^*_{jt} is the price of the property when it was sold as new, $\mathbf{x}$ is a vector of the characteristics of the condominiums, inclusive of cohort dummies and transaction year dummies, and u_{jt} is an error term. Note that the OLS estimation of (27) is inclined to be biased, as the error term u_{jt}, is likely to be correlated with regressors due to selection bias. There are at least two potential sources of selection bias. First, prices in the secondary market are only observed if properties are on the market (i.e., if there is incidental truncation). Second, as the sample is drawn from weekly classified magazines, only the subgroup for secondary market transactions is included. To correct these biases, Heckman's two–step method is applied. Table10 reports the estimation of equation (27) for selected variables from the OLS and selection model. For vector $\mathbf{x}$, we include the log of total units sold initially in the same phase, the log of the distance from the nearest train station, the log of the total area that was sold in the same phase, birth cohort dummies, transaction year dummies, vintage dummies, ward dummies, building height dummies, floor plan dummies, and railroad dummies. The selection of variables is based on Ono et al. (2002), who study the hedonic price index using data from the same source as this paper. As our data do not include some information, such as the detailed characteristics of each unit, an initial price p^*_{jt} is included in the regression to control for unobserved quality variation. In both models, the higher the price in the secondary market, the higher the initial price in the primary market. The negative and significant coefficient estimates for the distance from the nearest train station suggests that the future value of the unit is higher if it is closer to a train station. This is because, generally speaking, most people commute to work or school by train and stores tend to be concentrated around train stations. Thus, the distance to a train station measures the degree of convenience. As expected, the estimates of the vintage dummies are all negative and significant, and the magnitude increases monotonically with the vintage, suggesting that older units are less expensive. The transaction year dummies are negative and monotonically decrease with the year. This implies that properties have become less expensive in recent years, something which reflects the overall housing market trend. The birth cohort dummies are positive and increase with the year, until 1996. There is no distinct incident that seems to drive this result. A comparison of models (a) and (b) shows the direction of the bias due to selection. The variables whose coefficients are the most biased are the vintage and year dummies; the coefficients for the birth cohort dummies are underestimated in terms of magnitude. The coefficients for the vintage dummies are underestimated, while those for the transaction year dummies are overestimated.

C Calculation of the Terminal Condition for $\tilde{c}$

The terminal condition for $\tilde{c}$, both for estimations and simulations, is calibrated based on the costs and profit breakdown estimate for the unit price around 2000 performed by an industry analyst.[32]

First, from this information, we know that the average profit per unit was approximately 10 percent around 2000. Since the weighted average price of new condominiums (age zero) in the data for the year 2000 was 4.78 million yen, the average cost of production is set at 4.32 million yen ($= 4.78 * .9$). Second, since the average cost corresponds to the expression in the text, $\tilde{c} + \bar{c}_1 + \bar{c}_2 * q$, and $\bar{c}_1$ is fixed to 24.71, the value of $\tilde{c}^{2000}$ is obtained using average production of five firms for q.

[32] We thank Koichi Hiraga for providing this information.

D Tables and Figures

Table 1: Number of Firms and Concentration Measures

year	(A) number of active firms	(B)number of single appearance [a]	$(C) = (B)/(A)$	5 -firm[b]	10-firm[b]	15-firm[b]	HHI[c]
1992	89	35	0.393	0.383	0.523	0.614	0.046
1993	111	26	0.234	0.402	0.538	0.618	0.062
1994	205	56	0.273	0.309	0.442	0.540	0.032
1995	228	48	0.211	0.293	0.416	0.498	0.027
1996	227	29	0.128	0.297	0.412	0.501	0.037
1997	231	35	0.152	0.306	0.443	0.511	0.036
1998	221	32	0.145	0.259	0.385	0.480	0.026
1999	230	33	0.143	0.312	0.417	0.495	0.032
2000	231	44	0.190	0.317	0.433	0.516	0.030
Total	1,773	338	0.191	0.310	0.433	0.518	0.034

[a] The number of firms that appeared in the dataset only once during the sample period. [b] x-firm concentration ratio is the sum of the market share of the top x firms. [c] Herfindahl Index.

Table 2: Summary Statistics [a]

Variable	Notation	Obs	Mean	Weighted Mean[b]	Std. Dev.	Min	Max
Distance from NTS [c]	*dist*	5,522	701.31	734.44	913.61	0	13,880
Height of the building	*height*	5,522	8.72	9.61	4.39	2	54
Total units for sale in a given phase	q_t	5,522	26.33	–	20.54	1	319
Average size of the units (m^2)	*size*	5,522	66.59	65.35	34.92	20	807
Number of developers	–	5,522	1.21	1.13	0.45	1	4
Whether secondary market data are available	–	5,522	0.27	–	0.45	0	1
Unit Price (0,000 yen)							
Primary market (age 0)	p_t^0	5,522	5,209	4,905	3,221	1,559	73,706
Secondary market (age 1)	p_t^1	5,522	4,749	4,548	2,810	1,316	70,086
Secondary market (age 2)	p_t^2	5,522	4,583	4,380	2,740	1,179	66,686
Production by an oligopolistic firm							
5 firms	q_t	35	1,475		1,033	443	4,054
10 firms	q_t	70	1,025		871	0	4,054

[a] Each observation corresponds to a group of units in one building or one development project sold at the same phase.
[b] Weights are the units in each phase. [c] NTS stands for "nearest train station."

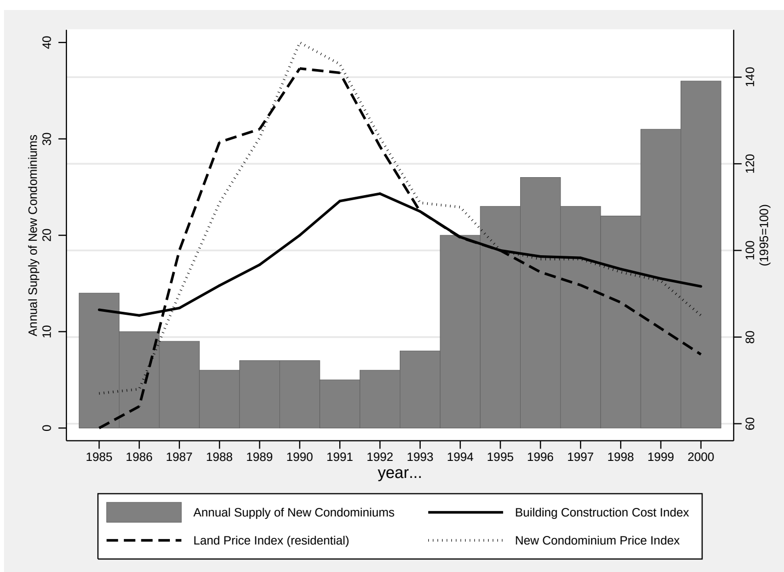
Table 3: Fixed Parameters

Description	Notation	Value	Unit
Common discount rate	β	0.975	
1-period survival rate	δ	0.990	
Scrap price	$\bar{p}$	42.3	million yen
Market size	M	3.514	million households
Number of firms	J	5, 10	
Steady state cost	$\bar{c}_1$	24.710	million yen
Variance of macro cost shock	σ_η^2	1	

Table 4: Parameter Estimates for the Process of x_t: the First-step estimation

	Model I 5 firms	Model II 10 firms
ϑ	0.878	0.897
	$(0.203)^{***}$	$(.1829)^{***}$
$\bar{x}$	3.846	3.200
	(3.189)	(2.228)
σ_ξ^2	3.363	2.631
x_{ss}^a	32.20	31.16
	(63.30)	(75.46)
R^2	0.7692	0.7748
N	9	9

Standard errors are reported in parentheses. Stars refer to the significance level of a t-test. *= significant at 10% level,**=significant at 5% level,***=significant at 1% level. ^{a} The standard error are obtained by the delta method.



Source:The Real Estate Economic Institute, National Condominium Market Trend, 1984-2002, Statistics Bureau,Statistics Yearbook

Figure 1: Key Variables in the Tokyo Condominium Market—1985-2000

Table 5: Demand Parameter Estimates: the Second-step Estimation

	OLS	GMM
α	0.016	0.328
(coefficient for ECC)	(0.002)***	(0.076)***
1(age==0)	-11.067	-22.007
	(0.020)***	(0.426)***
1(age==1)	-11.089	-22.636
	(0.008)***	(0.250)***
log(size)	-0.2194	2.7389
	(0.024)***	(0.673)***
$g(0)$	-11.978	-10.605
	(0.012)***	(0.181)***
$g(1)$	-11.998	-11.233
	(0.012)***	(0.102)***
Instruments		log(height)
		log(distance)
Observations	10,113	10,113
Anderson LR statistic		70.329
(p-val)		0.00
Hansen J		0.08
(p-val)		0.78
Endogeneity		287.56
(p-val)		0.00

Robust standard errors are in parentheses and adjusted for the noise in imputed prices by bootstrap. *significant at 10%; **significant at 5%;***significant at 1%

Table 6: Cost Parameter Estimates: the Third-step Estimation

Parameter	Explanation	Model I 5 firms	Model II 10 firms
ρ	AR(1) Coefficient	0.9443***	0.9552***
		(0.0044)	(0.0012)
$\bar{c}_2$	Cost Parameter	3.5812***	5.8332***
		(0.0159)	(0.0045)
σ^2_λ	Standard Deviation of	0.1664	0.096
	Idiosyncratic Production Shock	(1.2056)	(1.0383)
$\tilde{c}^d_{2000}$	terminal value of $\tilde{c}$	14.5151	10.4544
N		35	70

[a] Robust standard error is given in the parentheses. [b] Stars refer to the significance level of a t-test. *= significant at 10% level,**=significant at 5% level,***=significant at 1% level. [c] Standard error is obtained by the delta method. [d] The detail for this value is described in Appendix C.

Table 7: Responsiveness of Policy Functions to State Variables

	$\tilde{c}$ Inflation		$\tilde{c}$ Deflation	
x_t Growth	s_t	0.023	s_t	0.018
	x_t	0.067	x_t	0.131
	$\tilde{c}_t$	0.037	$\tilde{c}_t$	1.034
x_t Contraction	s_t	0.025	s_t	0.010
	x_t	0.329	x_t	0.184
	$\tilde{c}_t$	0.040	$\tilde{c}_t$	0.803

[a] All figures are measured in elasticities in absolute value.

Table 8: The Model's Performance

	5 firms		10 firms	
	$\sum q_{jt}$	P_t^0	$\sum q_{jt}$	P_t^0
Mean Observation	7.4	49.4	10.3	49.4
(Standard Deviation)	(1.9)	(1.9)	(2.1)	(1.9)
Mean Prediction	5.4	49.5	10.3	49.0
Mean Deviation from Observations [b]	3.5	7.3	3.3	7.8
Mean % of times that prediction falls in 15% interval from observations	24.4	100.0	37.5	100.0
Markup (s.d.)	0.56	(0.0005)	0.48	(0.0007)

[a] The mean is taken over time periods.[b] Let x_t and $\widehat{x}_{tm}$denote the observation in year t and the prediction for year$t$ in m'th draw respectively. The deviation is calculated by following formula for M simulation:$D = \frac{1}{M}\sum_m(\widehat{x}_{tm} - x_t)^2$.

Table 9: Decomposition of Contributors to Price Deflation

Year	(i) Benchmark	(ii) Fix both x_t & $\tilde{c}_t$	(iii) Fix $\tilde{c}_t$	(iv) Contribution (%)	(v) Competitive Price	(vi) Effect of Imperfection(%)
1994	49.88	49.88	49.88	–	–	–
1995	48.97	49.88	49.48	43.63	48.919	0.098
1996	48.23	49.88	49.12	45.65	48.182	0.094
1997	48.41	49.88	49.69	12.63	48.360	0.110
1998	47.95	49.88	49.39	24.96	47.900	0.103
1999	47.10	49.88	48.50	49.74	47.064	0.082
2000	46.60	49.88	47.99	57.63	46.569	0.071
Mean	47.88	49.88	49.03	39.04	47.832	0.093

Table 10: Estimation of Imputation Equation

	(a) OLS	(b)Selection Model		(a) OLS	(b)Selection Model		(a) OLS	(b)Selection Model
			Height dummies:			Transaction year dummies:		
log(price per sq. meter	0.604***	0.611***	6-9 stories	-0.008	-0.008	1993	-0.138**	-0.150***
in the primary market)	(0.02)	(0.02)		(0.006)	(0.006)		(0.045)	(0.045)
log(total units for sale)	-0.003	0.004	10-19 stories	0.001	0.004	1994	-0.199***	-0.223***
	(0.004)	(0.005)		(0.008)	(0.008)		(0.043)	(0.044)
log(distance from NTS)	-0.013***	-0.011***	$>$ 20 stories	0.160***	0.161***	1995	-0.273***	-0.308***
	(0.003)	(0.003)		(0.015)	(0.015)		(0.043)	(0.045)
log(total area for sale)	0.023***	0.022***				1996	-0.258***	-0.304***
	(0.005)	(0.005)	Age dummies:				(0.043)	(0.046)
constant	1.893***	1.786***	age2	-0.035**	-0.025*	1997	-0.278***	-0.335***
	(0.119)	(0.125)		(0.011)	(0.012)		(0.044)	(0.049)
Birth cohort dummies:			age3	-0.094***	-0.073***	1998	-0.292***	-0.360***
1993	0.051***	0.062***		(0.012)	(0.014)		(0.045)	(0.052)
	(0.01)	(0.011)	age4	-0.153***	-0.121***	1999	-0.273***	-0.353***
1994	0.088***	0.101***		(0.013)	(0.018)		(0.046)	(0.055)
	(0.01)	(0.011)	age5	-0.215***	-0.172***	2000	-0.234***	-0.325***
1995	0.093***	0.104***		(0.015)	(0.023)		(0.047)	(0.059)
	(0.012)	(0.012)	age6	-0.263***	-0.208***	2001	-0.197***	-0.299***
1996	0.104***	0.121***		(0.018)	(0.028)		(0.048)	(0.062)
	(0.013)	(0.014)	age7	-0.325***	-0.260***			
1997	0.099***	0.116***		(0.021)	(0.033)			
	(0.015)	(0.016)	age8	-0.398***	-0.322***	N	3119	3119
1998	0.056**	0.069***		(0.024)	(0.038)	ρ		.377
	(0.017)	(0.018)	age9	-0.505***	-0.417***	σ		.128
1999	0.031	0.035		(0.031)	(0.046)	λ		.377
	(0.021)	(0.021)						(.018)

† In both specification, ward dummies and railroad dummies are included. ‡ Selection equation includes distance from the nearest station, total size of area, height of the building, maximum size of unit, year dummies, floor plan dummies, ward dummies, large firm dummies, dealer dummies. Robust standard errors are in parentheses. ***significant at 1% level. **significant at 5% level. *significant at 10% level.

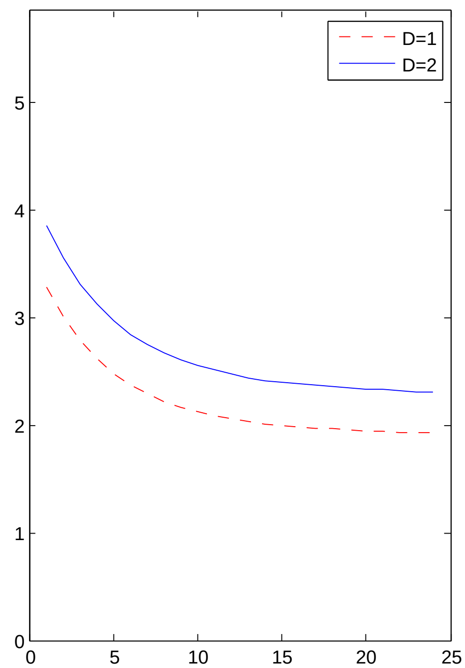


Figure 2: Simulated Production at Different Product Lifespans

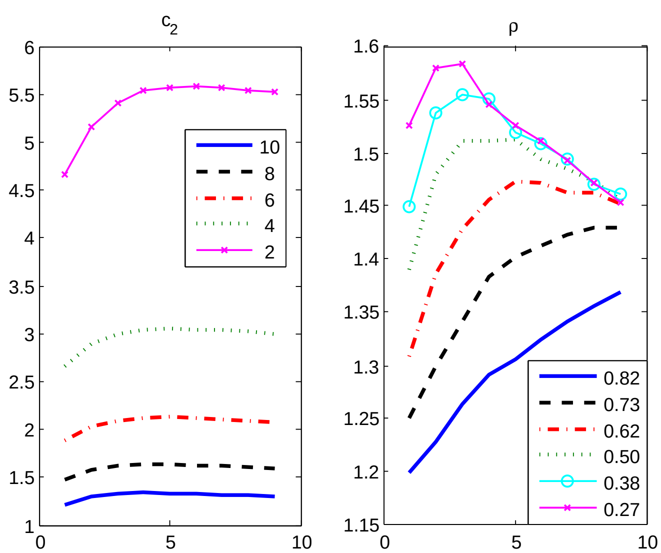


Figure 3: Simulated Production at Different Parameter Values

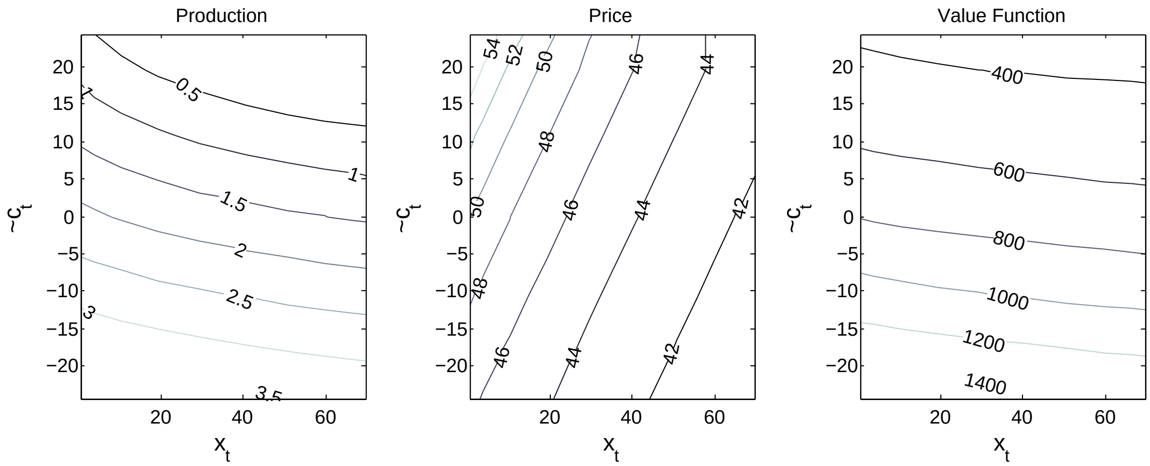


Figure 4: The Solution at $s_t = 31,160$

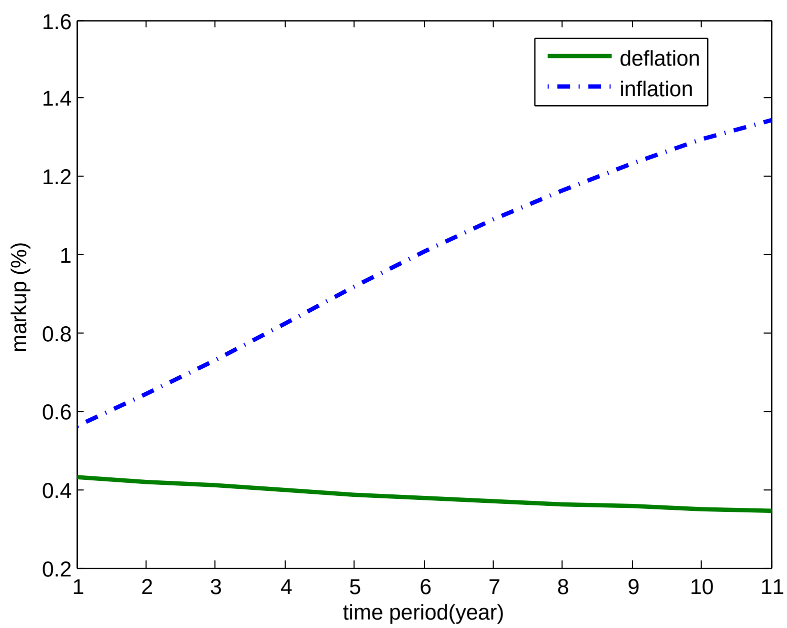


Figure 5: Simulated Markups—Inflation vs. Deflation