The Gains from Openness: Trade, Multinational Production, and Diffusion[*]

*** PRELIMINARY AND INCOMPLETE ***

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April 22, 2008

Abstract

In this paper we quantify the role played by trade, multinational production (MP), and diffusion of ideas in generating gains from "openness". We extend the Eaton and Kortum (2002) model of trade by introducing MP and diffusion of ideas. A key contribution is to model the simultaneous role of trade, MP, and diffusion, and explore some of the interactions among these different channels. Both trade and MP are substitutes with diffusion, but the relationship among trade and MP is more complex. Trade and MP are alternative ways to serve a foreign market, which makes them substitutes, but we also allow for complementarity by having MP rely on imports of intermediate goods from the home country. We use trade and MP data to estimate the model and quantify the gains from openness, trade, MP and diffusion.

[*]We benefited from comments by participants at the Annual Meeting of the Society for Economic Dynamics (2007), the Annual Conference on Macro and Development at U. Pittsburgh (2007), and seminars at New York University, FRB New York, Pennsylvania State University, and University of Texas-Austin. We have also benefited from comments and suggestions from Russell Cooper, Alexander Monge-Naranjo, Joris Pinkse, and Jim Tybout. All errors are our own.

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1 Introduction

Our goal in this paper is to quantify the gains from openness, and the role played by trade, multinational production (MP) and diffusion in generating those gains. To do so, we extend the Eaton and Kortum (2002) model of trade by introducing MP and diffusion of ideas. We estimate the model to match certain key facts of the trade and MP data and use the resulting model to calculate the joint as well as the separate gains from trade, MP and diffusion.

Most attempts to quantify the gains from trade use theories where there is no MP or diffusion (e.g., Eaton and Kortum, 2002; Alvarez and Lucas, 2007; Waugh, 2007), while recent attempts to quantify the gains from MP are based on models that do not allow for trade or diffusion (e.g., Ramondo, 2006; Burstein and Monge-Naranjo, 2007; McGrattan and Precott, 2007). Similarly, studies on the gains from diffusion of ideas typically ignore both trade and MP (Eaton and Kortum, 1999, Klenow and Rodríguez-Clare, 2005).[1]

Considering each of these channels separately, however, may understate or overstate the associated gains depending on the existence of significant sources of complementarity or substitutability among them. Suppose that MP depends on the ability of foreign affiliates to import certain inputs from their home country. In this case, shutting down trade would also decrease MP and generate losses beyond those calculated in models with trade but no MP. Alternatively, trade and MP may behave as substitutes because they are competing ways of serving foreign markets. In this case, shutting down trade would generate smaller losses than in models with only trade because MP would partially replace the lost trade.

Another way to look at this problem is by noting that we do not know whether the gains from trade (GT), the gains from multinational production (GMP), and the gains from diffusion (GD), can be added to compute the overall gains from openness (GO). This depends crucially on the interaction between trade, MP, and diffusion flows: if they behave as substitutes then $GO < GT + GMP + GD$, while if they behave as complements then $GO > GT + GMP + GD$.

The literature has typically modeled trade and "horizontal" FDI as substitutes in the context

[1] One exception is Rodríguez-Clare (2007), who evaluates the contribution of trade and diffusion of ideas to the overall gains from openness using a Ricardian model that incorporates both of these channels (along the lines of Krugman, 1979, and Eaton and Kortum, 2006). Another exception is Garetto (2007), who develops a model with both trade and vertical MP.

of the "proximity-concentration" trade-off: firms choose to either serve a foreign market by exporting or opening an affiliate there (Brainard, 1997, Markusen and Venables, 1997, Helpman, Melitz, and Yeaple, 2004). On the other hand, the literature has modeled trade and "vertical" FDI as complements: foreign affiliates rely on intermediate goods imported from their parent firms to produce goods that are consumed in other markets (Markusen, 1984, Grossman and Helpman, 1985, Antras, 2003).

The empirical evidence appears consistent with both of these views. Starting with Lipsey and Weiss (1981), the literature has found that exports and outward FDI are positively correlated across destination countries. Subsequent studies that used more detailed data at the industry or product level, and at the firm level conclude that MP and trade flows in *intermediate inputs*, often conducted within the firm, are complements, while MP and trade flows in *final goods* are substitutes (Belderbos and Sleuwaegen, 1988, Bloningen, 2001, Head and Ries, 2001, Head, Ries, and Spencer, 2004).

Additionally, the empirical evidence points to large intra-firm trade flows related to "vertical" multinational activities. This is specially true among rich countries where imported inputs from the home country are large relative to total revenues of foreign affiliates (Bernard, Jensen, and Schott, 2005, Hanson, Mataloni, and Slaughter, 2003, and Alfaro and Charlton, 2007). Furthermore, even among rich countries, foreign subsidiares of multinationals often sell a sizable part of their output outside of the host country. For example, around 30% of total sales of US affiliates in Europe are not done in the host country (Bloningen, 2005).

This paper presents a general-equilibrium, multi-country, Ricardian model of trade, MP, and technology diffusion that aims to capture key facts of the data on trade and MP flows across countries. The model has two sectors: tradable intermediate goods, and non-tradable consumption goods. Goods can be produced with three sets of constant-returns-to-scale technologies: national technologies that can only be used for domestic production (but not MP); multinational technologies that can be used for domestic production and also MP; and global technologies that are available for production in any country. In particular, firms producing tradable goods with country i's multinational technologies can serve market n by producing in i and exporting to n (trade), by producing directly in country n (MP), or by producing in some third ("bridge") country and then exporting to country n (BMP for "bridge MP").[2] In our model, trade and

[2]We avoid refering to this type of MP as vertical MP because the main motivation for bridge MP in our model

MP are subject to country-pair specific costs of the "iceberg" type.[3]

The multiplicity of choices regarding how to serve a foreign market makes trade and MP substitutes: arm-length trade and MP are alternative ways of serving a foreign market. However, the possibility of BMP creates complementarities between trade and MP: the decision by country i of serving market n producing in a third country l generates a trade flow from l to n associated with MP from i to l. We further introduce a more direct source of complementarity between trade and MP by assuming that affiliate plants can import some of their inputs of production from their home country; in our empirical application we think of this as intra-firm trade. Hence, when country i serves market n through MP, there is a trade flow in intermediate inputs from country i associated with it. Thus, even in a world without BMP as described above, our model generates complementarities between trade and MP.

We estimate the parameters of the model by matching simulated and observed moments. We use data on bilateral trade and MP flows for a set of OECD countries, as well as data on intra-firm trade flows for U.S. multinationals and foreign multinationals operating in the U.S. We follow Eaton and Kortum (2002) in using price data to estimate trade costs. A key concern in our estimation is that the empirical evidence reveals a strong positive correlation between bilateral flows of trade and MP, even controlling for destination and source country size. This correlation between trade and MP *flows* can be due to at least two reasons: a positive correlation between trade and MP *costs* (i.e. across country-pairs lower trade costs are associated with lower MP costs), and complementarities between trade and MP.[4] At present, the data available does not allow for a clean identification of the parameter governing the strength of the complementarity between trade and MP. This is something on which we are currently working. Reasonable parameters suggest that the forces of substitution and complementarity cancel out.

We use the estimated model to compute the joint gains from trade, MP and diffusion; we think of these as the overall gains from openness. We also compute the separate gains from these three channels. Our preliminary results suggest that $GT + GMP + GD > GO$, mainly because trade and MP are substitutes with diffusion. In future work we plan to explore ways to

is to avoid trade costs rather than allocating the different elements of the production process across locations according to their comparative advantage.

[3]In contrast to some recent models (e.g., Helpman, Melitz and Yeaple, 2004), our model has no fixed costs of production, no fixed costs of exporting or MP, and no firm-level heterogeneity. In this sense, there is no clear concept of a "firm" or a "multinational" in our model.

[4]This point is made clearly in Head and Riess (2004).

model complementarity between diffusion and trade.

The paper is organized as follows. Section 2 presents the model and the equilibrium. Section 3 presents model's estimation and welfare calculations. Section 4 concludes.

2 Model

We extend Eaton and Kortum's (2002) model of trade to incorporate MP, "intra-firm" trade, and diffusion of ideas in a multi-country, general equilibrium set-up. Our model is Ricardian with a continuum of tradable intermediate goods and non-tradable final goods, produced under constant-returns-to-scale (as in Alvarez and Lucas, 2007). We adopt the probabilistic representation of technologies as first introduced by Eaton and Kortum (2002), but we enrich it to incorporate MP and diffusion.

Labor is the only factor of production, and consumers in country $i \in \{1, ..., I)$ are endowed with L_i units of it. There is a continuum of non-tradable consumption goods indexed by $v \in [0, 1]$, and consumers have CES preferences over these goods with elasticity σ. We now present the basic assumptions regarding how these goods are produced when there is no trade and no multinational production (MP). In the following subsection we explain how we incorporate trade, MP, and diffusion of ideas into the model.

2.1 Production Structure in an Isolated Economy

There is a continuum of tradable intermediate goods indexed by $u \in [0, 1]$ that are used to produce a "composite" intermediate good with a CES production function:[5]

$$Q_m = [\int_0^1 q(u)^{\frac{\sigma-1}{\sigma}} du]^{\frac{\sigma}{\sigma-1}}.$$

Each intermediate good u is produced using this composite intermediate good and labor, with a Cobb-Douglas production function with labor share β. It is convenient to think of an "input bundle" produced from labor and the composite intermediate good that is in turn used to

[5]Without loss of generality, we assume that the elasticity of substitution between intermediate goods is the same as for consumption goods. This parameter plays no role in the analysis.

produce each of the intermediate goods. In country i, the cost of this input bundle is $c_i = Bw_i^{\beta}p_{mi}^{1-\beta}$, where w_i is the wage, p_{mi} is the price index associated to Q_m in country i, and $B \equiv \beta^{-\beta}(1-\beta)^{\beta-1}$.

Each intermediate good is produced from this input bundle with constant-returns technologies that differ across goods and countries. As in Eaton and Kortum (2002), we use a stochastic representation of technologies with some additional structure to allow, later on, for MP and diffusion of ideas in addition to trade. In each country there are three types of technologies to produce intermediate goods: national technologies, multinational technologies, and global technologies. In a closed economy there is no difference between these different technologies; differences emerge when we open up economies to trade, MP, and diffusion.

Following Alvarez and Lucas (2007), we distinguish technologies across goods and countries by modeling cost rather than productivity parameters. In particular, $x_i^N(u)$, $x_i^M(u)$, and $x_i^G(u)$ denote the cost parameters associated with the national, multinational and global technologies, respectively, to produce intermediate good u, in country i.[6] Thus, the unit cost of production of intermediate good u in country i produced with country $i's$ national (multinational, global) technology is $x_i^N(u)^{\theta}c_i$ ($x_i^M(u)^{\theta}c_i$, $x_i^G(u)^{\theta}c_i$), where $\theta > 0$ is a common parameter that regulates the variability of the cost structure across goods and countries.

Similarly to intermediate goods, non-tradable consumption goods are produced from labor and the composite intermediate good with a Cobb-Douglas production function with labor share α. The input bundle for consumption goods has unit cost $c_i^{NT} = Aw_i^{\alpha}p_{mi}^{1-\alpha}$, in country i.[7] The (stochastic) cost parameters associated with national, multinational, and global technologies, for consumption goods, are $z_i^N(v)$, $z_i^M(v)$, and $z_i^G(v)$, respectively. As for intermediate goods, the unit cost of production for good v in country i produced with country $i's$ national (multinational, global) technology is $z_i^N(v)^{\theta}c_i^{NT}$ ($z_i^M(v)^{\theta}c_i^{NT}$, $z_i^G(v)^{\theta}c_i^{NT}$).[8]

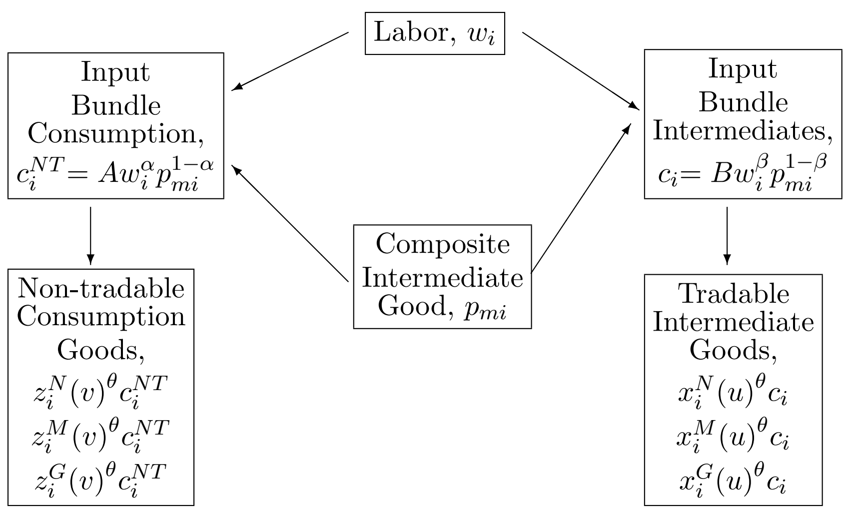
Figure 1 illustrates the cost structure in the closed economy.

[6]The production function for an intermediate good u is $q(u) = x^{-\theta}(u)l(u)^{\beta}Q_m(u)^{1-\beta}$, where $l(u)$ and $Q_m(u)$ are labor and the composite intermediate input used in the production of intermediate good u, and x represents the cost parameter associated with the national, multinational, or global technology for this good.

[7]$A \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$

[8]The production function for a consumption good v is given by $q(v) = z^{-\theta}(v)l(v)^{\beta}Q_m(v)^{1-\beta}$, where $l(v)$ and $Q_m(v)$ are labor and the composite intermediate good used in the production of consumption good v, and z represents the cost parameter associated with the national, multinational, or global technology for this good.

Figure 1: Cost Structure in the Closed Economy



Following Alvarez and Lucas (2007), we assume that $x_i^N(u)$, $x_i^M(u)$, $x_i^G(u)$, for all i and $u \in [0,1]$, are independently drawn from an exponential distribution with parameters λ_i^N, λ_i^M, and λ_i^G, respectively. The cost parameters for non-tradable goods, $z_i^N(v)$, $z_i^M(v)$, and $z_i^G(v)$, are also independently drawn from exponential distributions with the same parameters λ_i^N, λ_i^M, and λ_i^G, respectively. We refer to λ_i^N $(\lambda_i^M, \lambda_i^G)$ as the "stock" of national (multinational, global) "ideas" in country i.[9]

2.2 Trade and Multinational Production

Allowing for trade but without MP and diffusion, this model collapses to the Alvarez and Lucas (2007) version of Eaton and Kortum (2002), with the parameter of the exponential distribution in country i given by $\widetilde{\lambda}_i \equiv \lambda_i^N + \lambda_i^M + \lambda_i^G$. To add MP and diffusion, it is important to keep track of the countries where technologies originate, where goods are produced, and where goods are consumed. To do so, we will in general use subscript n to denote the country where the good is consumed, l for the country where the good is produced, and i for the country where the technology originates.

[9] This interpretation of λ as a stock of ideas can be understood as follows. Imagine that there are λ ideas for each good (each associated with a cost parameter) and that these ideas are independently drawn from an exponential distribution with parameter 1. Then, from the properties of the exponential distribution it follows that the distribution of the best technology is exponential with parameter λ.

For each consumption good v and each intermediate good u there are $3I$ technologies available: I national technologies, I multinational technologies, and I global technologies. The main difference between these technologies is that a country's national technologies can only be used for production in that country ($i = l$), whereas its multinational and global technologies can be used in any other country ($i \neq l$).

Since consumption goods are non-tradable goods, we must have $l = n$ with any technology. If we have $i \neq n$, then there is MP by i in n.

For intermediate goods, since they are tradable goods, it is possible to have $l \neq n$. Notice that, by definition, production with national technologies must satisfy $i = l$; if $l \neq n$ there is trade from i to n. For multinational technologies, if $i \neq l = n$ then there is MP by i in n; if $i \neq l \neq n$ then there is "bridge" MP by i in l to serve n (we should observe MP flows from i to l, and trade flows from l to n). Finally, for production with global technologies, if $i = l \neq n$, there is trade from i to n, whereas if $i \neq l = n$, country n produces the good with global ideas from i.[10]

Consider an intermediate good u produced in country l. This good can be produced with the national technology from country l or with a multinational or global technology from any other country. If produced with the national technology, then the unit production cost of this good would be $x_l^N(u)^\theta c_l$. If produced with the global technology from country i, then the unit production cost would be $x_i^G(u)^\theta c_l$. Finally, if produced with the multinational technology from country i, the unit production cost would be $x_i^M(u)^\theta c_{li}$, where c_{li} is the unit cost of the *multinational input bundle* required by country i to produce in country l. The unit cost c_{li} will in general be different than c_l, as we discuss below. This generates a difference between multinational and global technologies: global technologies from country i can be used for production in country l using the domestic input bundle with cost c_l, whereas multinational technologies require the use of a multinational input bundle with cost c_{li}.

Trade is subject to "iceberg" transportation costs, with one unit of a good shipped from country l resulting in $k_{nl} \leq 1$ units arriving to country n. We assume that $k_{nn} = 1$ and that the

[10] Global ideas can also be used for MP, but this never happens in equilibrium because – given our assumptions below – MP is dominated by either domestic production or trade. (MP occurs with multinational ideas precisely because local production is, by assumption, not an option). Hence, $i \neq l \neq n$ is ruled out. This assumption plays a role in the welfare gains calculations: we consider the scenario in which there is MP but no diffusion as one in which a country's global ideas can only be used abroad through MP.

triangular inequality holds (i.e., $k_{nl} \geq k_{nj}k_{jl}$ for all n, l, j).

The input bundle required by MP, i.e. the *multinational input bundle*, combines the input bundle from the home country (i.e., the country where the technology originates), and the host country (i.e., the country where production takes place). The home country input bundle must be shipped to the host country, and this implies paying the corresponding transportation cost: the cost of the home country input bundle for MP by country i in country l is then c_i/k_{li}. The host country input bundle has cost c_l, and incurs an "iceberg" type efficiency loss of $h_{li} < 1$ associated with using an idea from i to produce in l.[11] This entails a unit cost c_l/h_{li}. Combining the costs of home and host country input bundles into a CES aggregator, we get the unit cost of the *multinational input bundle* by i in l,

$$c_{li} = \left[(1-a)\left(\frac{c_l}{h_{li}}\right)^{1-\rho} + a\left(\frac{c_i}{k_{li}}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}, \tag{1}$$

where $a \in [0,1]$ and $\rho > 1$.[12] (Note that $c_{ll} = c_l$.) The parameter ρ indicates the degree of complementarity between input bundles from the home and host countries. It is a key parameter for our estimated welfare gains.

We assume that MP for consumption goods uses only the host country input bundle (as if $a = 0$ in 1). Thus, the unit cost of the *multinational input bundle* for non-tradable consumption goods produced by country i in n is simply $c_{ni}^{NT} = c_n^{NT}/h_{ni}$.

2.3 Prices

Since intermediate goods enter symmetrically the production of the composite intermediate good, we can label intermediate goods by their productivity in the following way:

$$x = (x_1^N, x_1^M, x_1^G, ..., x_I^N, x_I^M, x_I^G).$$

[11]Similarly to trade costs, we assume that $h_{ll} = 1$. However, we do not impose the triangular inequality for these efficiency losses. This is because, in contrast to trade, there is no possibility of arbitrage in MP: MP by i in l cannot be done by combining MP by i in some country j and then by MP from j in l.

[12]Note that $c_{li} \geq \min\{c_l, c_i/k_{li}\}$. This implies that local production of exports (weakly) dominates MP.

Since all technologies entail constant returns to scale, the price of intermediate good x in country n is simply the minimum unit cost of producing this good among all the alternatives,

$$p_n(x) = \min\{\min_i\{(x_i^N)^\theta \frac{c_i}{k_{ni}}\}, \min_l\{(x^G)^\theta \frac{c_l}{k_{nl}}\}, \min_{i,l}\{(x_i^M)^\theta \frac{c_{li}}{k_{nl}}\}\}, \tag{2}$$

where $x^G = \min_i x_i^G$ is the cost parameter associated with the best global technology for this good. Note that this expression includes the possibilities of trade in goods produced with national technologies (first term), and trade in goods produced with global technologies (second term). The third term includes the following possibilities: goods produced with multinational technologies in i ($l = i$) and exported to n, i.e. arms-length trade; goods produced with multinational technologies from i in n ($l = n$); and goods produced in l with multinational technologies from i and shipped to n ($l \neq i, n$).[13]

Regarding consumption goods, a country consumes goods produced with its own national technologies, as well as with foreign global and multinational technologies. Letting

$$z = (z_1^N, z_1^M, z_1^G, ..., z_I^N, z_I^M, z_I^G),$$

the price of good z in country n is

$$p_n^{NT}(z) = \min\{\left(z_n^N\right)^\theta c_n^{NT}, \left(z^G\right)^\theta c_n^{NT}, \min_i\{(z_i^M)^\theta c_{ni}^{NT}\}\},$$

where $z^G = \min_i z_i^G$ is the cost parameter associated with the best global technology for this good. The first term corresponds to the cost of domestic production with the national technology, the second term corresponds to the cost of domestic production with the best global technology, and the final term corresponds to the cost of MP with the multinational technology from country i.

[13] Note that if $i = n$ and $l \neq i$, then country i produces in l with its multinational technologies, and then imports the final good back home.

2.4 Equilibrium

Letting $\widetilde{c}_{ni} \equiv \min_l \{c_{li}/k_{nl}\}$ and $\widetilde{c}_n \equiv \min_l \{c_l/k_{nl}\}$, then equation (2) becomes

$$p_n(x) = \min\{\min_i\{(x_i^N)^\theta \frac{c_i}{k_{ni}}\}, (x^G)^\theta \widetilde{c}_n, \min_i\{(x_i^M)^\theta \widetilde{c}_{ni}\}\}.$$

Given the properties of the exponential distribution, $p_n^{1/\theta}$ is distributed exponentially with parameter

$$\psi_n \equiv \sum_i (\psi_{ni}^N + \psi_{ni}^M) + \psi_n^G,$$

where

$$\psi_{ni}^N = (c_i/k_{ni})^{-1/\theta} \lambda_i^N, \quad \psi_{ni}^M = \widetilde{c}_{ni}^{-1/\theta} \lambda_i^M, \quad \psi_n^G = \widetilde{c}_n^{-1/\theta} \lambda^G,$$

and

$$\lambda^G = \sum_i \lambda_i^G.$$

The price index for the composite intermediate good in country n is given by

$$p_{mn} = C\psi_n^{-\theta}, \tag{3}$$

where C is a constant.[14]

Regarding consumption goods, a similar procedure shows that the price index of the consumption CES aggregate in country n is given by

$$p_n = C\zeta_n^{-\theta}, \tag{4}$$

where ζ_n plays the same role for consumption goods as ψ_n for intermediate goods, with

$$\zeta_n \equiv \zeta_{nn}^N + \sum \zeta_{ni}^M + \zeta_n^G,$$

where

$$\zeta_{nn} = \left(c_n^{NT}\right)^{-1/\theta} (\lambda_n^N + \lambda^G), \quad \zeta_{ni}^M = \left(c_n^{NT}\right)^{-1/\theta} \lambda_i^M, \quad \zeta_n^G = \left(c_n^{NT}\right)^{-1/\theta} \lambda^G.$$

[14] $C \equiv \Gamma(1+\theta-\sigma)^{1/1-\sigma}$ where $\Gamma()$ is the Gamma function evaluated at $1+\theta-\sigma > 0$.

As shown by Eaton and Kortum (2002), the average price charged by any country l in country n is the same. Moreover, by the properties of the exponential distribution we know that a share $s_{nl}^{T} \equiv \psi_{nl}^{N}/\psi_n$ of intermediate goods bought by country n will be produced by country l with national technologies. Thus, letting X_n^T denote total spending on intermediates by country n, then

$$s_{nl}^{N} X_n^T \tag{5}$$

is the value of of intermediate goods produced with national technologies in country l that are exported to country n. Similarly, $\frac{\psi_n^G}{\psi_n} X_n^T$ is the value of intermediate goods bought by n that are produced with global technologies. These goods could be produced domestically or imported from any country $l \in \arg\min_j \{c_j/k_{nj}\}$. To proceed, let y_{nl}^G be the share of total spending by country n on goods produced with global technologies that are produced in country l (and then shipped to country n). Clearly, $\sum_l y_{nl}^G = 1$. In equilibrium, the following "complementary slackness" conditions must hold:

$$\begin{array}{lcl} c_l/k_{nl} > \widetilde{c}_n & \Longrightarrow & y_{nl}^G = 0, \\ y_{nl}^G > 0 & \Longrightarrow & c_l/k_{nl} = \widetilde{c}_n. \end{array} \tag{6}$$

Letting $s_{nl}^G \equiv y_{nl}^G \frac{\psi_n^G}{\psi_n}$, then imports by country n of goods produced in country l with global technologies are

$$s_{nl}^G X_n^T. \tag{7}$$

Finally, $\frac{\psi_{ni}^M}{\psi_n} X_n^T$ is the value of intermediate goods bought by n that are produced with multinational technologies from i. But note that these goods could be produced in any country $l \in \arg\min_j(\widetilde{c}_{ji}/k_{nj})$. Let y_{nli}^M be the share of total spending by country n on goods produced with country i multinational technologies that are produced in country l (and then shipped to country n). Clearly, $\sum_l y_{nli}^M = 1$. Note that if there were only MP, then $y_{nli}^M = 0$ for all $l \neq n$ and $y_{nni}^M = 1$, while if there were no vertical MP, then $y_{nli}^M = 0$ for all $l \neq i, n$. On the other hand, if MP were not feasible, then $y_{nii}^M = 1$ for all n, i. In equilibrium the following "complementary slackness" conditions must hold:

$$\begin{array}{lcl} c_{li}/k_{nl} > \widetilde{c}_{ni} & \Longrightarrow & y_{nli}^M = 0, \\ y_{nli}^M > 0 & \Longrightarrow & c_{li}/k_{nl} = \widetilde{c}_{ni}. \end{array} \tag{8}$$

The value of MP by i in l for n is $s_{nli}^{M} X_n^T$, where $s_{nli}^{M} \equiv y_{nli}^{M} \psi_{ni}^{M} / \psi_n$. Summing up over i yields the total imports by country n from l of intermediate goods produced with multinational technologies,

$$\sum_i s_{nli}^{M} X_n^T. \tag{9}$$

To calculate the observed imports we need to add the input bundle imported for MP. To do so, we first need to get an expression for total MP in intermediates by i in l, $X_{li}^{T,MP}$. Summing up over all destination countries n, this is

$$X_{li}^{T,MP} = \sum_n s_{nli}^{M} X_n^T.$$

Let ω_{li} be the cost share of the home input bundle for the production of any intermediate good in country l by multinationals from country i. This is

$$\omega_{li} = \frac{a \left(c_i / k_{li}\right)^{1-\rho}}{(1-a)\left(c_l / h_{li}\right)^{1-\rho} + a \left(c_i / k_{li}\right)^{1-\rho}}.$$

Imports associated with MP by i in l are then

$$\omega_{li} \sum_n s_{nli}^{M} X_n^T. \tag{10}$$

It is important to note that even if ω_{li} were equal to zero, MP by i in l would use imported intermediate goods. This is because MP uses the domestic input bundle, which is produced with labor and both domestic and imported intermediate goods. The term in expression (10) refers to an additional source of imports from country i associated with MP by i in l. That is, these imports are associated *only* with MP and not with domestic production. In the quantitative section below we will think of MP by i in l as being done by country i multinationals, and we will think of the imports in (10) as "intra-firm" trade, even though there is nothing in the model that implies that these imports will be done inside the firm.

Adding up terms in expressions (5), (7), (9), and (10) yields total imports by n from $i \neq n$,

$$M_{ni} \equiv s_{ni}^{N} X_n^T + s_{ni}^{G} X_n^T + \sum_j s_{nij}^{M} X_n^T + \omega_{ni} \sum_j s_{jni}^{M} X_j^T$$

Aggregate imports for country n are simply $M_n = \sum_{i \neq n} M_{ni}$.

We can now explain why we have national, multinational and global technologies in the model. Global technologies are added to model diffusion, and play a role in the estimation of the model, but are not crucial for the main theoretical results. National technologies are more important. Without them, we would observe zero arms-length exports of intermediate goods from i to n when $\widetilde{c}_{ni} < c_i/k_{ni}$. Adding technologies that are not amenable to MP makes it easier to avoid this implication. Moreover, as long as MP costs are low relative to trade costs (i.e., h_{ni} is low relative to k_{ni}), then we will also avoid zeroes in MP for intermediate goods from i to n.

Let $\eta \equiv (1-\alpha)/\beta$. Total spending on final goods by country n is $X_n = w_n L_n$, while it can be shown that total spending on tradable intermediate goods is $X_n^T = \eta X_n$.[15] Total imports by n from i are

$$M_{ni} = \eta \left(s_{ni}^N + s_{ni}^G + \sum_j s_{nij}^M \right) w_n L_n + \eta \omega_{ni} \sum_j s_{jni}^M w_j L_j. \tag{11}$$

Trade balance conditions close the model, determining equilibrium wages for each country.[16] Trade balance for country n entails total imports equal to total exports, or

$$\sum_{i \neq n} M_{ni} = \sum_{i \neq n} M_{in}. \tag{12}$$

Finally, the total value of MP by i in n is given by the value of MP for intermediate goods, $X_{ni}^{T,MP}$, as derived above; plus the corresponding value for consumption goods, $X_{ni}^{NT,MP}$. Since these goods are non-tradable, we simply need to derive an expression for the share of goods $v \in [0,1]$ bought by country n that are produced with multinational technologies from country i. Again, from the properties of the exponential distribution, this is given by $s_{ni}^{NT,M} \equiv \zeta_{ni}^M / \zeta_n$. Thus, $X_{ni}^{NT,MP} \equiv s_{ni}^{NT} X_n$, and the total value of MP by country i in country n is

$$X_{ni}^{MP} \equiv X_{ni}^{T,MP} + X_{ni}^{NT,MP} = \sum_j s_{jni}^M X_j^T + s_{ni}^{NT,M} X_n,$$

[15] This result follows from assuming Cobb-Douglas production functions for both intermediate and final goods (see Appendix for the proof).

[16] We use the following normalization: $\sum_{i=1}^{I} w_i L_i = 1$.

or

$$X_{ni}^{MP} = \eta \sum_j s_{jni}^M w_j L_j + s_{ni}^{NT} w_n L_n. \tag{13}$$

2.5 Observed Trade and MP shares

Using (11), imports from $i \neq n$ by country n as a share of absorption of intermediates in country n, is given by

$$D_{ni} \equiv \frac{M_{ni}}{X_n^T} = s_{ni}^N + s_{ni}^G + \sum_j s_{nij}^M + \omega_{ni} \sum_j s_{jni}^M \frac{w_j L_j}{w_n L_n}. \tag{14}$$

Analogously, MP for $n \neq i$, as a share of total absorption or GDP, is

$$D_{ni}^M \equiv \frac{X_{ni}^{MP}}{X_n} = \eta \sum_j s_{jni}^M \frac{w_j L_j}{w_n L_n} + s_{ni}^{NT,M}. \tag{15}$$

For the estimation procedure it is convenient to further normalize trade shares by $D_{ii}^T \equiv 1 - \sum_{n \neq i} D_{in}$ and MP shares by $D_{ii} \equiv 1 - \eta \sum_{n \neq i} D_{in}$.[17] Thus, we focus on the following normalized shares

$$\tau_{ni} \equiv \frac{D_{ni}}{D_{ii}^T}, \tag{16}$$

and

$$\tau_{ni}^M \equiv \frac{D_{ni}^M}{D_{ii}}. \tag{17}$$

It is worth noting that the normalized trade shares τ_{ni} would be equal to one in a model with no diffusion (i.e., no global technologies) if there were no trade costs (i.e., $k_{ni} = 1$ all n, i). Normalized trade shares will be lower than one in our model both because of trade costs and because of MP and diffusion.

[17] An alternative but equivalent way to define D_{ii}^T is as $D_{ii}^T = X_{ii}^T / X_i^T$, where $X_{ii}^T \equiv X_i^T - M_i$ is spending on locally produced intermediates. Similarly, $D_{ii} = X_{ii}/X_i$, where $X_{ii} \equiv X_i - M_i$ denotes the value added corresponding to final goods produced in country n. It is easy to show that $X_{ii}^T / X_i^T = 1 - \sum_{n \neq i} D_{ni}$ while $X_{ii}/X_i = 1 - \eta \sum_{n \neq i} D_{ni}$.

2.6 A symmetric example

To gain intuition on the workings of the model, we consider the case of symmetric countries ($L_i = L$) and symmetric trade and MP costs, $k_{nl} = k$ and $h_{nl} = h$ for all $l \neq n$, with $k < h < 1$. This is a case that can be solved analytically, and, more importantly, the basic intuition carries to the asymmetric case.

Wages, costs, and prices are equalized across countries: $w_n = w$, $c_n = c$, $c_n^{NT} = c^{NT}$, $p_{mn} = p_m$, and $p_n = p$. This implies that $\widetilde{c}_n = c < c/k$ and $y_{nl}^G = 0$ for $l \neq n$: there is no trade in goods produced with global technologies. Moreover, the multinational input bundle collapses to $c_{li} = c/m$ for tradable goods, and $c_{nl}^{NT} = c^{NT}/h$ for non-tradable goods, for all $n \neq l$, where

$$m \equiv \left[(1-a)h^{\rho-1} + ak^{\rho-1}\right]^{\frac{1}{\rho-1}} . \tag{18}$$

It is easy to see that $h > k$ implies that $m > k$, and hence $y_{nli}^M = 0$ for all $n \neq l$: there is no trade in goods produced with multinational technologies. Thus, in a symmetric world, there is no BMP.[18] Shutting down this source of complementarity allows us to better highlight the complementarity between trade and MP coming from the possibility of using the home country input bundle when doing MP (i.e. the role of the parameter ρ in equation 1).

From (3), and $c = w^\beta p_m^{1-\beta}$, we get

$$p_m = C^{1/\beta}\left[\lambda^N + (I-1)k^{1/\theta}\lambda^N + \lambda^M + (I-1)m^{1/\theta}\lambda^M + I\lambda^G\right]^{-\theta/\beta} w. \tag{19}$$

Intuitively, the term inside the brackets captures the total stock of ideas, with foreign national ideas discounted by trade costs, $k^{1/\theta}$, and foreign multinational ideas discounted by MP costs, $m^{1/\theta}$. In other words, $\lambda^N + (I-1)k^{1/\theta}\lambda^N$ is the effective stock of national ideas used by each country through domestic production and imports; $\lambda^M + (I-1)m^{1/\theta}\lambda^M$ is the effective stock of multinational ideas used by each country through domestic production and MP; and $I\lambda^G$ is the stock of global ideas used through domestic production. Note that if $h > k$ then $m^{1/\theta} > k^{1/\theta}$, so foreign MP ideas are discounted by less than foreign national ideas.

[18] Note that if $h < k$, there would be no MP in intermediate goods. By virtue of the distinction between national and multinational technologies, even when trade is more costly than MP, $k < h$, there is trade between countries.

Using (4) and (19), the final goods' price index is

$$p = C^{1+\eta}\left[\widetilde{\lambda}+(I-1)\left(h^{1/\theta}\lambda^{M}+\lambda^{G}\right)\right]^{-\theta} \cdot\left[\widetilde{\lambda}+(I-1)\left(k^{1/\theta}\lambda^{N}+m^{1/\theta}\lambda^{M}+\lambda^{G}\right)\right]^{-\eta\theta} w, \quad (20)$$

where $\widetilde{\lambda} \equiv \lambda^N + \lambda^M + \lambda^G$ is the total stock of ideas originated in each country. We can compute the gains from openness GO (i.e., the increase in welfare from isolation to benchmark) by comparing the associated real wage levels, w/p. Since wages are equal across countries, they can be normalized to one, so we can just compare prices across different scenarios. The price index for the benchmark is given by (20), whereas the analogous result with no trade, no MP and no diffusion is obtained by letting $k \to 0$ and $h \to 0$ in (20) and suppressing the "diffusion" term $(I-1)\lambda^G$. This yields

$$p_{ISOL} = C^{1+\eta}\widetilde{\lambda}^{-\theta(1+\eta)}.$$

The gains from openness $(\widetilde{GO})$ are given by

$$\widetilde{GO} = \frac{p_{ISOL}}{p} = \left[\frac{\widetilde{\lambda}+(I-1)\left(h^{1/\theta}\lambda^{M}+\lambda^{G}\right)}{\widetilde{\lambda}}\right]^{\theta} \cdot\left[\frac{\widetilde{\lambda}+(I-1)\left(k^{1/\theta}\lambda^{N}+m^{1/\theta}\lambda^{M}+\lambda^{G}\right)}{\widetilde{\lambda}}\right]^{\eta\theta}, \quad (21)$$

or, $GO = \ln(\widetilde{GO})$. (Below we follow this notation so that expressions for gains with a ~ represent proportional gains.) The first term on the right-hand-side captures the gains from MP and diffusion for non-tradable goods, whereas the second term captures the gains from trade, MP and diffusion for tradable goods. It is clear that GO increases with h and k: the lower MP or trade costs, the larger the gains from openness.

We calculate gains from trade by computing the gains of moving from isolation to only trade (no diffusion and MP), GT. Analogously, we calculate gains from MP by computing the gains of moving from isolation to only MP (no diffusion and trade), GMP. Finally, we compute the gains from diffusion GD in a similar way.

We first derive the price index when there is only trade (no MP and diffusion). From (20), by setting $m^{1/\theta} = h = 0$, and allowing multinational and global ideas to be used for domestic production and trade, we get:

$$p_T = C^{1+\eta}\widetilde{\lambda}^{-\theta}\left[\widetilde{\lambda}(1+(I-1)k^{1/\theta})\right]^{-\eta\theta}.$$

Gains from trade are then given by

$$\widetilde{GT} = \frac{p_{ISOL}}{p_T} = \left[1+(I-1)k^{1/\theta}\right]^{\eta\theta}.$$

Not surprisingly, GT increases with k.

Similarly, the gains from diffusion (increase in real wage from isolation to only diffusion) are

$$\widetilde{GD} = \frac{p_{ISOL}}{p_D} = \left[\frac{\widetilde{\lambda}+(I-1)\lambda^G}{\widetilde{\lambda}}\right]^{\theta(1+\eta)},$$

while the gains from MP (increase in real wage from isolation to only MP) are

$$\widetilde{GMP} = \frac{p_{ISOL}}{p_{MP}} = \left[\frac{\widetilde{\lambda}+(I-1)h^{1/\theta}(\lambda^M+\lambda^G)}{\widetilde{\lambda}}\right]^{\theta}\left[\frac{\widetilde{\lambda}+(I-1)\widetilde{m}^{1/\theta}(\lambda^M+\lambda^G)}{\widetilde{\lambda}}\right]^{\theta\eta},$$

where $\widetilde{m} = (1-a)^{\frac{1}{\rho-1}}h$ is the MP cost adjustment under no trade ($k=0$). Note that since there is no diffusion, global ideas behave exactly like multinational ideas: a country's global ideas can only be used abroad through MP.

We also compute the joint gains from trade and MP (increase in real wage from isolation to trade and MP):

$$\begin{aligned}\widetilde{GTMP} &= \frac{p_{ISOL}}{p_{TMP}} = \left[\frac{\widetilde{\lambda}+(I-1)h^{1/\theta}(\lambda^M+\lambda^G)}{\widetilde{\lambda}}\right]^{\theta} \\ &\cdot\left[\frac{\widetilde{\lambda}+(I-1)\left(k^{1/\theta}\lambda^N+m^{1/\theta}(\lambda^M+\lambda^G)\right)}{\widetilde{\lambda}}\right]^{\eta\theta}.\end{aligned}$$

Finally, it is useful to calculate the gains from trade given by moving from a situation with only MP and diffusion, to the benchmark, denoted by GT'. The final goods' price index under no trade is obtained by letting $k \to 0$ in (20):

$$p_{k\to 0} = C^{1+\eta}\left[\widetilde{\lambda} + (I-1)\left(h^{1/\theta}\lambda^M + \lambda^G\right)\right]^{-\theta}\left[\widetilde{\lambda} + (I-1)\left(\widetilde{m}^{1/\theta}\lambda^M + \lambda^G\right)\right]^{-\theta\eta}.$$

Thus,

$$\widetilde{GT}' = \frac{p_{k\to 0}}{p} = \left[\frac{\widetilde{\lambda} + (I-1)\left(k^{1/\theta}\lambda^N + m^{1/\theta}\lambda^M + \lambda^G\right)}{\widetilde{\lambda} + (I-1)\left(\widetilde{m}^{1/\theta}\lambda^M + \lambda^G\right)}\right]^{\eta\theta}.$$

2.6.1 Substitution vs. Complementarity

First, note that $GO < GD + GTMP$, a reflection of the fact that diffusion and trade/MP are substitutes. This substitutability arises because diffusion, trade and MP are different ways of sharing ideas across countries: once diffusion is available, then trade and MP are less valuable, and once trade and MP are available, then diffusion is less valuable.

The key role of ρ in generating complementarity between trade and MP can be seen by noting that when $\rho \to 1$ then $\widetilde{m} \to 0$. Using the results above, this implies that for low ρ we must have $GTMP > GT + GMP$: trade and MP behave as complements. Conversely, when $\rho \to \infty$, then $m \to h$ and $\widetilde{m} \to h$. This implies that for high ρ we have $GTMP < GT + GMP$: trade and MP behave as substitutes.

More generally, the relationship between $GTMP$ and $GT + GMP$ depends on the elasticity of substitution ρ and the technology parameter θ. In particular, if $\rho - 1 > 1/\theta$, then $GTMP < GT + GMP$, so that trade and MP are net substitutes (see the proof in the Appendix). The intuition is the following. While $\rho - 1$ governs the effect of trade costs on trade flows in Armington or Krugman models, $1/\theta$ has analogous role in Ricardian models. Thus, this condition says that MP and trade are substitutes if the effect of trade costs on "intra-firm" trade flows is larger than their effect on "arms-length" Ricardian trade flows.

Alternatively, the role of ρ and θ in generating complementarity between trade and MP can be seen by analyzing the change in trade and MP flows when their costs change. In particular, when h goes up, MP increases, and arm-length trade decreases. Simultaneously, there are two effects on "intra-firm" trade. On the one hand, higher h shifts production towards using more host country inputs: the higher the elasticity of substitution ρ, the stronger the switch towards the local input bundle. On the other hand, since MP increases, both the use of home as well as host country input bundles increases: the lower θ, the stronger this effect.

Formally, we can see these effects of a change in h by computing trade shares in a symmetric world under no diffusion. From equation (14) and (15), we get trade shares for any pair of countries (with $i \neq n$), respectively:

$$D = s^T + \omega s^M, \tag{22}$$

where s^T corresponds to "arms-length" trade (in this case trade in goods produced with national technologies), s^M corresponds to MP shares in the tradable sector, and ωs^M corresponds to "intra-firm" trade. Using (18), we can re-write $\omega = a(k/m)^{\rho-1}$, and further replacing in (22), we get:

$$\begin{aligned} s^T &= \frac{k^{1/\theta}\lambda^N}{\widetilde{\lambda} + (I-1)(k^{1/\theta}\lambda^N + m^{1/\theta}\lambda^M)} \\ \omega s^M &= a\left(\frac{k}{m}\right)^{\rho-1} \frac{m^{1/\theta}\lambda^M}{\widetilde{\lambda} + (I-1)(k^{1/\theta}\lambda^N + m^{1/\theta}\lambda^M)}. \end{aligned}$$

It is clear that s^T decreases with h (and increases with k): "arms-length" trade is a substitute for MP. However, "intra-firm" trade might increase or decrease: the share ω decreases with h (and increases with k), while s^M increases with h.[19] The first effect dominates when ρ is sufficiently high: if $\rho - 1 > 1/\theta$, then $d\omega s^M/dh < 0$, and trade and MP are net substitutes.[20] Notice that even without any "arms-length" trade the model can generate substitutability between trade and MP.

3 Quantitative analysis

In relating the model to the data, we think of MP by i in n as the gross value of production in country n by multinationals with home country i, and imports of the home-country input

[19]
$$\frac{d\log \omega s^M}{d\log h} = \left[\frac{1}{\theta} - (\rho-1)\right]\frac{d\log m}{d\log h} - \frac{1}{\theta}(I-1)\frac{m^{1/\theta}\lambda^M}{\widetilde{\lambda} + (I-1)(k^{1/\theta}\lambda^N + m^{1/\theta}\lambda^M)}\frac{d\log m}{d\log h},$$
where $d\log m/d\log h > 0$.

[20] Note that an increase in k can either increase or decrease s^M; the condition for $ds^M/dk < 0$ is stronger than $\rho - 1 > 1/\theta$.

bundle by multinationals as "intra-firm" trade. We use manufacturing trade and price data, gross production value in manufacturing, MP data, GDP data, and intra-firm imports by multinational affiliates, to estimate the model using a simulation-based procedure. We then use these estimates to calculate gains from openness.

3.1 Data Description

We restrict our analysis to the set of nineteen OECD countries considered by Eaton and Kortum (2002): Australia, Austria, Belgium/Luxemburg, Canada, Denmark, Spain, Finland, France, United Kingdom, Germany, Greece, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Sweden, United States. For bilateral variables, we have 342 observations, each corresponding to one country-pair. Depending on availability, our observations are for 1990, an average over the period 1990-2002, or for the late 1990s.

We use data on trade flows from country i to country n in the manufacturing sector (our proxy for the tradable sector). These data are from the STAN data set for OECD countries, for both 1990 and an average over 1990-2002 (see below). This is the empirical counterpart of bilateral trade flows, M_{ni}, in our model.

Our measure of bilateral MP flows is gross production value (both for domestic sales and for exports) of multinational affiliates from i in n. We have data for this variable for all sectors combined as averages over 1990-2002. The main source of these data is UNCTAD and the Globalization data set for the OECD (see Ramondo, 2006, for a detailed description). This is the empirical counterpart of bilateral MP flows, X_{ni}^{MP}, in our model.

We need to normalize trade and MP flows as indicated by (16) and (17), respectively. Thus, we need to calculate X_i, D_{ii}, X_i^T, and D_{ii}^T, from the data. We compute total expenditure as $X_i = GDP_i + IM_i - EX_i$, where IM_i refers to imports into country i from the remaining 18 OECD countries in the sample, and EX_i refers to total exports from country i to the rest of the world. We calculate country i's share of domestic sales as $D_{ii} = (GDP_i - EX_i)/X_i$. Analogously, we calculate X_i^T and D_{ii}^T using data on gross production, exports, and imports from the OECD for country i, in the manufacturing sector only. Data on countries' GDP is from the World Bank (WDI), total exports and imports are from Feenstra (NBER-United Nations Trade Data), manufacturing gross production, exports, and imports are from the STAN database, for the period 1990-2002. Combining the values for M_{ni}, X_{ni}^{MP}, X_i, D_{ii}, X_i^T, and D_{ii}^T, we then obtain the empirical counterparts of the normalized bilateral trade and MP flows, τ_{ni} and τ_{ni}^M, in our model.

As explained above, we think of intra-firm trade as the empirical counterpart of the imports of the home-country input bundle by multinational affiliates in the model. We only have data on intra-firm trade involving the United States as either the source or destination country. We combine data on intra-firm exports from the United States to affiliates of U.S. multinationals in foreign countries with data on imports done by affiliates of foreign multinationals located in the United States from their parent firms. These data is from the Bureau of Economic Analysis (years 1999 to 2002). This is the empirical counterpart of imports of intermediate goods associated with MP, $\eta\omega_{ni}\sum_j s_{jni}^M w_j L_j$, in our model, but this is only available when the U.S. is the destination or source country.

To construct a measure of bilateral trade costs k_{ni}, we follow Eaton and Kortum (2002) in using international price data for 50 manufacturing products from the United Nations International Comparison Program 1990 benchmark study. For each good u, and each country pair i and n, we compute the logarithm of relative prices, $r_{ni}(u) \equiv \log p_n(u) - \log p_i(u)$, and pick the second highest (for possible measurement error) as a measure of trade costs.[21] In particular, our trade cost measure is given by $\log k_{ni} = -max_{2u} r_{ni}(u)$.[22]

We need an empirical counterpart for the model variable L_i. This variable captures the total number of "equipped-efficiency units" available for production, so employment must be adjusted to account for human and physical capital available per worker. Our preliminary strategy is to use data on R&D employment as a proxy for L_i (from World Development Indicators for 1990). Countries with a higher share of employment in R&D are treated as being larger, which is most likely a good approximation because those countries should also have more equipped-efficiency units per worker. Moreover, one would expect that those countries have larger stocks of ideas.

[21] The logic is that for goods that country n actually imports from i, we must have $p_n(u)/p_i(u) = 1/k_{ni}$, whereas for goods that are not imported we must have $p_n(u)/p_i(u) \leq 1/k_{ni}$. This implies that if i exports something to n then $1/k_{ni} = \max p_n(u)/p_i(u)$.

[22] Eaton and Kortum (2002) calculate $\log p_i/(p_n k_{ni})$ as $D_{ni} = max_{2u} r_{ni}(u) - (1/50)\sum_{u=1}^{50} r_{ni}(u)$. Using this measure as a proxy for trade costs yields very similar results to the ones using k_{ni}. An alternative measure for trade costs that we use is the residual of regressing (log of) k_{ni} on (log of) bilateral distance, source, and host country dummies. Again, results are very similar to the ones using directly k_{ni}.

3.2 Estimation Procedure

Our procedure is to estimate the vector of the model's parameters (except ρ, see below) by matching moments from the data with moments from simulations of the model. We choose moments that are relevant to the workings of our model. That is, we choose moments that are informative about the model's parameters.

To reduce the number of parameters to be estimated, our preliminary strategy is to make two assumptions: first, we assume that the stock of ideas relative to the labor force is the same across countries, $\lambda_i^N + \lambda_i^M + \lambda_i^G = \phi L_i$, and second, we assume that $\lambda_i^M = \delta_M \phi L_i$ and $\lambda_i^G = \delta_G \phi L_i$ for all i for some common parameters δ_M and δ_G. These two assumptions imply that we only need to estimate two parameters: δ_M and δ_G, since ϕ will not affect any of the variables of interest for our analysis.[23]

We assume that bilateral MP costs h_{ni}, are related to trade costs k_{ni} according to

$$h_{ni} = k_{ni} + \gamma \varepsilon_{ni}(1 - k_{ni}), \tag{23}$$

where ε_{ni} is independently drawn from the uniform distribution with support $[0, 1]$ and $\gamma \in [0, 1]$. These assumptions imply that $h_{ni} \in [k_{ni}; 1]$, and that the correlation between h_{ni} and k_{ni} is regulated by γ.[24] In particular, higher γ implies lower correlation between trade and MP costs, and viceversa.

The model's relevant parameters are $\delta_M, \delta_G, \theta, \gamma, \rho, a, \alpha, \beta, \phi$. For the labor share in the tradable sector (β), and non-tradable sector (α), we use 0.5 and 0.75, respectively, as calibrated by Alvarez and Lucas (2007). We normalize $\phi = 1$. Unfortunately, the limited data we have at this point for intra-firm trade (only with US as origin or destination) is not enough to estimate the elasticity of substitution (ρ) between home and host country inputs for multinationals. We will thus conduct estimation and counterfactual exercises for three different values of ρ, namely $\rho = 2$, $\rho = 5$ and $\rho = 8$. Hence, we end up with a vector of five model's parameters to estimate

[23] In future work, we plan to estimate $\lambda_i^N + \lambda_i^M + \lambda_i^G$ and L_i using a combination of GDP data (which corresponds to $w_i L_i$ in the model), income per capita (which corresponds to w_i), R&D employment (which will help to pin down $\phi_i L_i$), and either total outward MP or prices for tradable and non-tradable goods (as in Alvarez and Lucas, 2007).

[24] In future work, we plan to allow for a more general functional form, adding a parameter d such that $h_{ni} = dk_{ni} + \gamma \varepsilon_{ni}(1 - dk_{ni})$.

$\Delta = [\delta_M, \delta_G, \theta, \gamma, a]$ in each of these three cases. These parameters correspond to the share of multinational technologies in the total stock (δ^M), and the share of global ideas in the total stock (δ^G), the variability of costs for tradable and non-tradable goods (θ), the importance of the random component of MP costs in (23) (γ), and the parameter a in the CES cost function for MP in equation (1).

3.2.1 Moments

We choose to match five moments that, according to our model, are key to identify the five parameters of interest. They are:

1. average normalized bilateral trade shares, τ_{ni}, across country pairs;
2. average normalized bilateral MP shares, τ_{in}^{M}, across country pairs;
3. correlation coefficient between bilateral trade and MP shares, $COR(\tau_{ni};\tau_{ni}^{M})$, across country pairs;
4. OLS coefficient on trade costs in the following "gravity" regression:

$$\log \tau_{ni} = b \log k_{ni} + S_i + H_n + v_{ni}, \tag{24}$$

where S_i and H_n are two sets of source and host country fixed effects, respectively;

5. average imports of affiliates from i to n as share of total MP sales from i in n:

$$\widetilde{\omega}_{ni} = \frac{\eta \omega_{ni} \sum_j s_{jni}^{M} w_j L_j}{\eta \sum_j s_{jni}^{M} w_j L_j + s_{ni}^{NT} w_n L_n}, \tag{25}$$

for $n = US$ or $i = US$

Table 1 below summarizes the moments from the data. Normalized trade and MP shares are calculated as an average over the nineties, for each country-pair. Trade costs k_{ni} in (24) are calculated from prices for manufacturing products across OECD countries, for 1990. Consistently, in equation (24), we use data for normalized manufacturing trade flows in 1990. Data on bilateral imports of affiliates needed to calculate (25) are averages over 1999-2003, for each country-pair with $n = US$ or $i = US$.

Moments	Data	$\rho = 2$	$\rho = 5$	$\rho = 8$
Average normalized trade share τ_{ni}	0.033	0.033	0.036	0.033
	(0.06)			
Average normalized MP share τ_{ni}^M	0.025	0.025	0.024	0.025
	(0.05)			
Correlation $(\tau_{ni}; \tau_{ni}^M)$	0.70	0.70	0.70	0.70
OLS coefficient $b^{\dagger}$	4.69	4.70	4.78	4.71
	(0.36)			
Average imported inputs' share (US) $\widetilde{\omega}_{ni}$	0.074	0.074	0.076	0.075
	(0.07)			

(†): Equation 24; Standard Errors for data moments, in parenthesis.

Table 1: Moments: Data and Model.

Even though these five moments jointly identify the five parameters we want to estimate (given some value for ρ), some moments are more responsive to some parameters than others. Intuitively, one can think that the share of multinational technologies and global technologies, δ_M and δ_G, are pinned down by average trade and MP flows, τ_{ni} and τ_{ni}^M (moments 1 and 2). Higher δ_M implies more MP (and more "intra-firm" trade), while higher δ_G implies less trade and MP. On the other hand, the correlation between trade and MP flows (moment 3) is determined in the model both by the correlation between trade and MP costs, which is linked to γ, and by the complementarity between trade and MP costs, which is linked to the elasticity of substitution ρ.

To understand the role of the OLS coefficient b in (24), recall that Eaton and Kortum (2002) run a regression like that in (24) but without the source and host country fixed effects, and show that the resulting coefficient is an unbiased estimate of $1/\theta$ in their model. We add source and host country fixed effects to the regression as mandated by the model given the presence of MP and diffusion. But since total trade flows are the sum of arms-length and intra-firm, the estimated coefficient b is now affected by the way in which intra-firm trade responds to trade costs. This is determined by ρ in our model. We also have to take into account that MP costs h_{ni} indirectly affect trade flows. Since h_{ni} are part of the residual ν_{ni} in (24), the positive correlation between k_{ni} and h_{ni} will lower b. All this implies that b (moment 4) helps to pin down several parameters: θ, ρ, and γ.

Finally, moment 5 helps to pin down the CES parameter a in the cost of MP.

For a given ρ and a set of parameter values Δ, matrix of trade costs k_{ni}, matrix of random draws ε_{ni} (ε matrix), vector of country sizes L_n, we can compute the equilibrium of the model and generate a simulated data set with 361 observations (one for each country-pair, including the domestic pairs) for each of the following variables: MP costs h_{ni}, normalized trade shares τ_{ni}, normalized MP shares τ_{ni}^M, intra-firm trade shares $\widetilde{\omega}_{ni}$. The algorithm used to compute the equilibrium builds on the one in Rodríguez-Clare (2007), which in turn builds on the one developed by Alvarez and Lucas (2007) (see the Appendix for a description).

For the data generated in this way we can then compute the 5 moments enumerated above. This implies that for a given ρ and for each set of parameter values Δ and for each ε matrix we can compute a vector of simulated moments which we denote by $MOM_s(\rho, \Delta, \varepsilon)$. We use a simulated method of moments procedure in which we estimate the model parameters by minimizing

$$\Delta^*(\rho) = \arg\min_{\Delta} \left[MOM_d - \sum_{\varepsilon \in \Omega} MOM_s(\rho, \Delta, \varepsilon) \right]' I \left[MOM_d - \sum_{\varepsilon \in \Omega} MOM_s(\rho, \Delta, \varepsilon) \right].$$

Here Ω is the set of ε matrices used for different simulations, I is the identity matrix and MOM_d is the vector of moments from the data.[25] In this preliminary estimation, we report parameters' estimates using only one ε matrix. Table 1 above reports the moments associated with estimates for different values of ρ.

3.3 Estimation Results

Results of parameter estimates are reported in Table 2 for $\rho = 2$, $\rho = 5$, and $\rho = 8$. Additionally, this table shows the implied statistics for MP costs and trade costs.

[25] Note that we have as many moments as number of parameters to estimate. Thus, using the identity matrix as optimal weighting matrix does not affect estimates.

Parameter	$\rho = 2$	$\rho = 5$	$\rho = 8$	Definition
δ_G	0.031	0.025	0.033	share of global technologies
δ_M	0.11	0.12	0.15	share of multinational technologies
θ	0.21	0.21	0.216	variability of costs
γ	0.79	0.66	0.65	parameter affecting correlation between trade and MP costs in (23)
a	0.55	0.76	0.84	weight of Home intermediate input bundle in (1)
E_k		0.60		mean trade cost
σ_k		0.17		s.e. trade cost
E_h	0.77	0.74	0.74	mean MP cost
σ_h	0.18	0.19	0.19	s.e. MP cost
COR_{kh}	0.70	0.79	0.80	correlation trade and MP costs

Table 2: Parameters' Estimates.

The estimate of θ does not vary significantly with ρ. Its value is higher than Eaton and Kortum's (2002) central result of $\theta = 0.12$, but within the range of their estimates, $[0.08, 0.28]$.[26] The difference between our results and Eaton and Kortum's is due to the presence of MP and diffusion, which leads to an intercept in the gravity equation that affects the estimated OLS coefficient b in (24).[27]

The results in Table 2 also show that lower values of ρ imply higher values of γ. This implies that, as argued in the Introduction, there are different ways to generate the observed positive correlation between trade and MP flows across country pairs: either high complementarity or

[26] Our estimate of θ is virtually the same as the one obtained by Rodríguez-Clare (2007).

[27] With no diffusion, Eaton and Kortum (2002) are able to recover θ from a OLS gravity equation without intercept. Rodríguez-Clare (2007) shows that such intercept arises from the inclusion of diffusion on top of trade as a way to share ideas. With a very different methodology, Rodríguez-Clare estimates $\theta = 0.22$.

a high positive correlation between trade and MP costs. In particular, with $\rho = 2$ we get $\gamma = 0.79$, and the correlation between k and h across country pairs is 0.7, whereas with $\rho = 8$ we get $\gamma = 0.65$, and the correlation between k and h is 0.8.

Moments	Data	$\rho = 2$	$\rho = 5$	$\rho = 8$
Variation Coef. for τ_{ni}	1.85	1.17	1.14	1.18
Variation Coef. for τ_{ni}^M	1.90	2.01	1.87	1.84

Table 3: "Out-of-sample" Moments: Data and Model.

Table 3 above shows the goodness of fit for two moments not included in the estimation matching procedure: variation coefficients for trade and MP (normalizes) shares. The model does well in predicting the variation in normalized MP shares across country pairs, but the implied variation in normalized trade shares is lower than in the data.

Table 4 shows the model's fit with the data regarding aggregate quantities: exports, imports, outward MP and inward MP. The predictions of the model correspond to averages over ten simulations (i.e., ten draws of the ε matrix) for $\rho = 8$ (results are very similar for the other values of ρ). The first column shows the correlation between the model and the data in levels, while the second column shows this correlation as shares of GDP. We see that the model performs well in terms of aggregate levels. It also does well regarding exports and imports relative to size, but not as well in terms of outward and inward MP when adjusted for size.

Figure 2, Panel A, shows outward MP as a share of GDP for the model and the data, against the model's GDP, $w_i L_i$. The model appears to do well except for some small countries that either have a very high (The Netherlands) or very low (e.g., Spain and New Zealand) outward MP relative to size. Panel B is analogous to Panel B except that it shows inward rather than outward MP. Again, the model does well except for several small countries with very low ratios of inward MP relative to their size. In general, the model's failings in this area may be because of our proportionality assumption and our choice of R&D employment as a measure of the model's L_i. We will explore alternative ways to measure λ_i and L_i in future work. We also plan to investigate how well the model does regarding the prevalence of bridge MP in the data.

	Levels	GDP shares
Exports	0.92	0.62
Imports	0.92	0.64
Outward MP	0.81	0.27
Inward MP	0.96	0.17

Table 4: Correlations between model and data

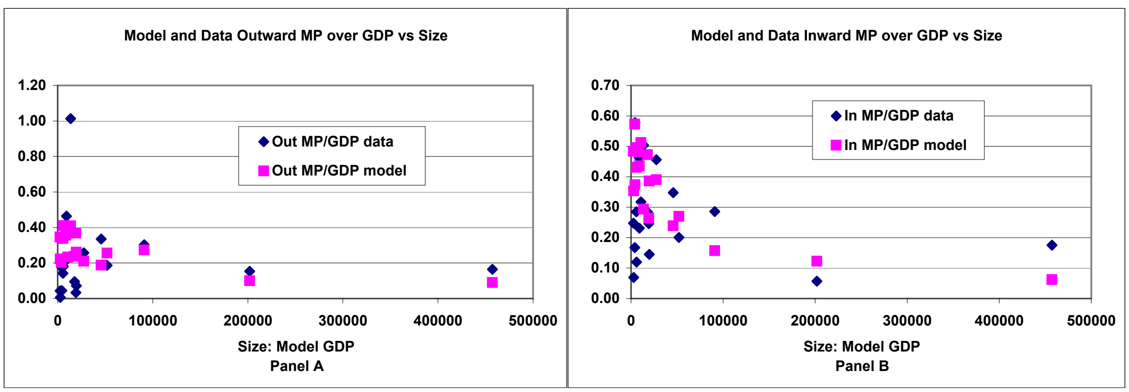


Figure 2: The figure shows two scatter plots, by OECD(19) country. Panel A shows the model's implied outward MP relative to GDP (vertical axis) calculated from an average over ten simulations of the model, and data. Analogously, Panel B shows inward MP over GDP (vertical axis). The horizontal axis is model's GDP ($w_i L_i$).

3.4 Gains from Openness

Gains from openness, trade, MP and diffusion are given by changes in real wages in terms of the final consumption good: w_i/p_i. We calculate real wages under five counterfactual scenarios: (1) isolation, (2) trade but no MP and no diffusion, (3) MP but no trade and no diffusion, (4) diffusion but no trade and no MP, and (5) trade and MP but no diffusion. Scenario (1) entails $k_{ni} = h_{ni} = 0$ (no trade, no MP), and $\lambda^G = \lambda_i^G$ (no diffusion); scenario (2) entails $h_{ni} = 0$ and $\lambda^G = \lambda_i^G$ (no MP and no diffusion), but $k_{ni} > 0$; scenario (3) entails $k_{ni} = 0$ and $\lambda^G = \lambda_i^G$ (no trade and no diffusion), but $h_{ni} > 0$; scenario (4) entails $k_{ni} = h_{ni} = 0$ (no trade and no MP) and $\delta_G > 0$; and scenario (5) entails and $\lambda^G = \lambda_i^G$ (no diffusion) and $h_{ni}, k_{ni} > 0$.

We present gains from openness, trade, MP, and diffusion, for the benchmark values of trade costs, MP costs, and share of global technologies estimated above, for 19 OECD countries. Table 5 shows these calculations for the three values of ρ and the corresponding values of the parameters estimated above (see Table 2). The implied gains from openness are large: log gains of around 0.5 imply percentage gains of 65% on average for the 19 countries in our sample. Of course, these gains will be much larger for the smaller countries, as we show below when we report gains for individual countries.

Interestingly, the gains from trade implied by the model are smaller than the gains from MP, which in turn are smaller than the gains from diffusion. The reason is that MP flows are actually higher than trade flows. For example, total inward MP flows are more than double the total imports in the data. This could seem contradictory with the finding of a small share of multinational technologies (i.e., $\delta_M < 15\%$). But there are two forces that make MP larger than trade (in the model) in spite of the low share of technologies that allow MP: first, MP costs are lower than trade costs ($E_h = 0.74 > E_k = 0.6$), and second, MP is feasible for non-tradable goods. Similarly, the gains from diffusion are large in spite of a low share of global technologies (i.e., $\delta_G < 2.5\%$) because of the absence of any costs of diffusion and because of the presence of diffusion in both tradable and non-tradable goods.

In all cases, trade and diffusion behave as substitutes with diffusion: $GO < GD + GMPT$. The difference can be big. For the intermediate value of ρ, for example, the percentage gains from openness are 63%, whereas the added percentage gains from diffusion and trade and MP combined ($\exp(GD + GMPT)$) are 103%.

Turning to the relationship between trade and MP, Table 5 that they behave as substitutes for the intermediate and high value of ρ: $GMPT < GT + GMP$, although the difference is not high. In contrast, for $\rho = 2$ we see that trade and MP behave as complements: $GMPT > GT + GMP$, but just barely so. As a result, it is always the case that the three flows behave as substitutes in the sense that $GO < GD + GT + GMP$. The difference is particularly high for $\rho = 8$, for which the added percentage gains from diffusion, trade and MP are 140%.

It is interesting to ask why it is that even for $\rho = 2$ the complementarity between trade and

MP is weak, in the sense that $GMPT$ is just barely higher than $GT + GMP$. The reason is the relatively small levels of intra-firm trade, which we are using to discipline the parameter a for each ρ. In particular, when ρ falls from 8 to 2 we have to decrease a from 0.84 to 0.55, and this weakens the higher complementarity associated with a lower ρ.

Turning to the gains from trade given the presence of MP and diffusion, GT', we see that - as one would expect - it increases with the degree of complementarity between trade and MP. It is interesting to compare this measure of gains to the gains associated with Eaton and Kortum (2002), which we associate with GT under $\theta = 0.12$ and label GT_{EK}. Table 5 shows that GT_{EK} = 0.021 (or 2.1%) a bit lower than Eaton and Kortum's actual estimated gains of 3.5%. There are two sources of differences between GT' and GT_{EK}. First, the fact that there is diffusion and MP in our model, and that in general these flows behave as substitutes with trade, implies that GT' will tend to be lower than GT and GT_{EK}. Second, the higher value of $\theta = 0.21$ that we estimate in comparison with Eaton and Kortum's $\theta = 0.12$ will increase GT' and GT over GT_{EK}. We see that the latter effect dominates, so that $GT' > GT_{EK}$.

Table 6 shows gains of moving from isolation to baseline values of trade costs, MP costs, and share of global technologies, under the five scenarios described above (GO, GT, GMP, GD, and $GTMP$), for each country in our sample for $\rho = 8$. Countries are ordered by size (according to total R&D employment). Indeed, gains from openness are related to size: larger countries have lower gains than smaller countries. Moreover, for all countries diffusion, trade and MP behaves as substitute in the sense that $GD + GT + GMP > GO$. Notice that a country like Finland, which represents 1% of total R&D employment, has $GO = 0.68$, which imply percentage gains of 97%. This is approximately half of the sum of the separate gains from diffusion, trade and MP are 192%. Of course, this difference between GO and $GD + GT + GMP$ is lower for lower values of ρ.

	$\log(w/p) - \log(w^{iso}/p^{iso})$		
From isolation to:	$\rho = 2$	$\rho = 5$	$\rho = 8$
trade, MP, and diffusion (GO)	0.52	0.49	0.55
only trade (GT)	0.14	0.14	0.15
only MP (GMP)	0.27	0.28	0.33
only diffusion (GD)	0.36	0.32	0.38
only trade and MP (GTMP)	0.42	0.38	0.43
only trade with $\theta = 0.12$ (GT_{EK})	0.02	0.02	0.02
trade given MP and diffusion (GT')	0.07	0.06	0.05

Table 5: Gains from Openness: benchmark (average OECD)

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	GO	GT	GMP	GD	GTMP	GT'	GT_{EK}	$L/\sum L$
			$\left[\log(w'/p'^{iso}/p^{iso})\right] * 100$					(%)
New Zealand	96	29	65	70	82	9	5	0.3
Greece	79	14	50	63	59	4	1	0.4
Portugal	70	19	30	57	46	7	3	0.5
Norway	78	16	60	51	69	5	2	0.6
Denmark	71	21	40	51	56	8	4	0.6
Austria	74	22	48	49	63	9	4	0.7
Belgium/Luxemburg	66	23	38	41	56	11	5	0.9
Finland	62	20	32	41	49	10	4	1.0
Sweden	61	17	43	33	55	8	3	1.3
Netherlands	47	12	22	33	32	6	1	1.4
Spain	39	9	18	26	25	5	1	1.9
Australia	45	14	26	23	37	8	2	2.3
Italy	40	12	20	22	31	7	2	2.4
Canada	30	9	12	17	21	6	1.1	3.3
United Kingdom	18	4	6	12	10	3	0.4	5.2
France	22	7	10	11	16	5	1.2	5.5
Germany	15	5	5	8	10	4	0.7	8.4
Japan	6	1	3	3	5	1	0.1	22
United States	4	1	2	1	3	1	0.1	41

Countries sorted by R&D employment.

Table 6: Gains from Openness, by country.

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A Proofs

First we prove that for the symmetric example analyzed in Section 2.3, if $\rho - 1 > 1/\theta$, and $h > k$, then $GTMP < GT + GMP$.

Proof: Let $\widetilde{\lambda} = \lambda + \lambda^M + \lambda^G$. Recall that $\widetilde{GTMP}$, $\widetilde{GT}$, and $\widetilde{GMP}$ are given by:

$$\begin{aligned}
\widetilde{GTMP} &= \frac{p_{ISOL}}{p_{TMP}} = \left[\frac{\widetilde{\lambda} + (I-1)h^{1/\theta}(\lambda^M + \lambda^G)}{\widetilde{\lambda}}\right]^{\theta} \cdot \left[\frac{\widetilde{\lambda} + (I-1)\left(k^{1/\theta}\lambda^N + m^{1/\theta}(\lambda^M + \lambda^G)\right)}{\widetilde{\lambda}}\right]^{\eta\theta}, \\
\widetilde{GT} &= \frac{p_{ISOL}}{p_T} = \left[1 + (I-1)k^{1/\theta}\right]^{\eta\theta}, \\
\widetilde{GMP} &= \frac{p_{ISOL}}{p_{MP}} = \left[\frac{\widetilde{\lambda} + (I-1)h^{1/\theta}(\lambda^M + \lambda^G)}{\widetilde{\lambda}}\right]^{\theta} \left[\frac{\widetilde{\lambda} + (I-1)\widetilde{m}^{1/\theta}(\lambda^M + \lambda^G)}{\widetilde{\lambda}}\right]^{\theta\eta}.
\end{aligned}$$

We find sufficient conditions under which $\widetilde{GT} \cdot \widetilde{GMP} > \widetilde{GTMP}$.

$$\begin{aligned}
\left[1 + (I-1)k^{1/\theta}\right] \cdot \left[\widetilde{\lambda} + (I-1)\widetilde{m}^{1/\theta}(\lambda^M + \lambda^G)\right] &> \widetilde{\lambda} + (I-1)(k^{1/\theta}\lambda^N + m^{1/\theta}(\lambda^M + \lambda^G)) \\
\widetilde{m}^{1/\theta} + k^{1/\theta} + (I-1)(k\widetilde{m})^{1/\theta} &> m^{1/\theta}.
\end{aligned}$$

For the above inequality to hold it is sufficient that:

$$\widetilde{m}^{1/\theta} + k^{1/\theta} > m^{1/\theta}.$$

Recall that $m \equiv \left[(1-a)h^{\rho-1} + ak^{\rho-1}\right]^{\frac{1}{\rho-1}}$, and $\widetilde{m} \equiv (1-a)^{\frac{1}{\rho-1}}h$. Thus, replacing these expressions in the inequality above, and rearranging we get:

$$\left[((1-a)^{\frac{1}{\rho-1}}h)^{1/\theta} + k^{1/\theta}\right]^{\theta} > \left[((1-a)^{\frac{1}{\rho-1}}h)^{\rho-1} + ak^{\rho-1}\right]^{\frac{1}{\rho-1}}.$$

For $k \leq 1$, $h \leq 1$, and $a \leq 1$, if $1/\theta < \rho - 1$, then the inequality above holds and $\widetilde{GTMP} < \widetilde{GT} \cdot \widetilde{GMP}$. □

Second, we prove that $X_n^T = [(1-\alpha)/\beta]\, X_n$.

Proof: Let Z_n be total quantity of the input bundle produced in country n.[28] Let Q_{mn} be the total quantity of the composite intermediate good used to produce Z_n, Q_{fn} the total quantity of the composite intermediate good used to produce consumption goods, and $Q_n = Q_{mn} + Q_{fn}$ the total quantity of the composite intermediate good produced in n. Let L_{mn} be the total quantity of labor used to produce intermediate goods, and L_{fn} the total quantity of labor used to produce final (consumption) goods. It must be that $L_n = L_{mn} + L_{fn}$. Note that $p_{mn}Q_n$ is the total cost of the intermediate goods used in production in country n, so $p_{mn}Q_n = X_n^T$. We first calculate the total cost of the intermediate goods produced in country n. This includes the total cost of the domestic input bundle for intermediates,

$$w_n L_{mn} + p_{mn} Q_{mn} = c_n Z_n,$$

plus the intra-firm imports of foreign multinationals located in n,

$$\sum_{i \neq n} \omega_{ni} \sum_j s_{jni}^M p_{mj} Q_j,$$

minus the exports of the domestic input bundle for intermediates to country $n's$ subsidiaries abroad,

$$\sum_{i \neq n} \omega_{in} \sum_j s_{jin}^M p_{mj} Q_j.$$

Hence, the total cost of intermediate goods produced in country n is

$$w_n L_{mn} + p_{mn} Q_{mn} + \sum_{i \neq n} \omega_{ni} \sum_j s_{jni}^M p_{mj} Q_j - \sum_{i \neq n} \omega_{in} \sum_j s_{jin}^M p_{mj} Q_j.$$

Second, we calculate the total value of intermediate goods produced in country n. This is composed of the value of sales (domestic plus exports) using national technologies, $\sum_j s_{jn}^T p_{mj} Q_j$, plus the value of sales (domestic plus exports through VMP) using domestic and foreign multinational technologies, $\sum_i \sum_j s_{jni}^M p_{mj} Q_j$,

$$\sum_j s_{jn}^T p_{mj} Q_j + \sum_i \sum_j s_{jni}^M p_{mj} Q_j.$$

[28] What is the relationship between $c_n Z_n$ and X_n^T? $c_n Z_n$ is the total cost of the input bundle produced in n, which is used to produce intermediate goods in country n, and by country n multinationals abroad. X_n^T is total spending on intermediate goods in n, which does not include the cost of labor used to produce the input bundle.

In equilibrium, we must have these two things equal, hence

$$\begin{aligned}
& w_n L_{mn} + p_{mn} Q_{mn} + \sum_{i \neq n} \omega_{ni} \sum_j s^M_{jni} p_{mj} Q_j - \sum_{i \neq n} \omega_{in} \sum_j s^M_{jin} p_{mj} Q_j \\
= \quad & \sum_j s^T_{jn} p_{mj} Q_j + \sum_i \sum_j s^M_{jni} p_{mj} Q_j.
\end{aligned}$$

The trade balance condition is imports equal exports, or

$$\sum_{i \neq n} M_{ni} = \sum_{i \neq n} M_{in},$$

with

$$\begin{aligned}
M_{ni} &= \left(s^T_{ni} + \sum_j s^M_{nij} \right) p_{mn} Q_n + \omega_{ni} \sum_j s^M_{jni} p_{mj} Q_j, \\
M_{in} &= \left(s^T_{in} + \sum_j s^M_{inj} \right) p_{mi} Q_i + \omega_{in} \sum_j s^M_{jin} p_{mj} Q_j.
\end{aligned}$$

We have

$$\begin{aligned}
w_n L_{mn} + p_{mn} Q_{mn} + \sum_{i \neq n} \omega_{ni} \sum_j s^M_{jni} p_{mj} Q_j - \sum_{i \neq n} \omega_{in} \sum_j s^M_{jin} p_{mj} Q_j &= \\
\left(s^T_{nn} + \sum_i s^M_{nni} \right) p_{mn} Q_n &+ \sum_{j \neq n} \left(s_{jn} p_{mj} Q_j + \sum_i s^M_{jni} p_{mj} Q_j \right) \\
w_n L_{mn} + p_{mn} Q_{mn} + \sum_{i \neq n} \omega_{ni} \sum_j s^M_{jni} p_{mj} Q_j - \sum_{i \neq n} \omega_{in} \sum_j s^M_{jin} p_{mj} Q_j &= \\
\left(s^T_{nn} + \sum_i s^M_{nni} \right) p_{mn} Q_n &+ \sum_{j \neq n} \left(M_{jn} - \omega_{jn} \sum_l s^M_{ljn} p_{ml} Q_l \right) \\
w_n L_{mn} + p_{mn} Q_{mn} + \sum_{i \neq n} \omega_{ni} \sum_j s^M_{jni} p_{mj} Q_j &= \\
\left(s^T_{nn} + \sum_i s^M_{nni} \right) p_{mn} Q_n &+ \sum_{j \neq n} M_{jn}.
\end{aligned}$$

From the trade balance condition, we then have

$$\begin{aligned}
w_n L_{mn} + p_{mn} Q_{mn} + \sum_{i \neq n} \omega_{ni} \sum_j s^M_{jni} p_{mj} Q_j &= \left(s^T_{nn} + \sum_i s^M_{nni} \right) p_{mn} Q_n + \sum_{i \neq n} M_{ni} \\
w_n L_{mn} + p_{mn} Q_n &= \left(s^T_{nn} + \sum_i s^M_{nni} \right) p_{mn} Q_n + \sum_{i \neq n} \left(M_{ni} - \omega_{ni} \sum_j s^M_{jni} p_{mj} Q_j \right) \\
w_n L_{mn} + p_{mn} Q_n &= \left(s^T_{nn} + \sum_i s^M_{nni} \right) p_{mn} Q_n + \sum_{i \neq n} \left(s^T_{ni} + \sum_j s^M_{nij} \right) p_{mn} Q_n \\
w_n L_{mn} + p_{mn} Q_n &= \left(\sum_i s^T_{ni} + \sum_i \sum_j s^M_{nij} \right) p_{mn} Q_n .
\end{aligned}$$

But, we know that $s^T_{nl} \equiv \psi_{nl}/\psi_n$, and $s^M_{nli} \equiv y_{nli} \psi^M_{ni}/\psi_n$. Hence,

$$\sum_i s^T_{ni} + \sum_i \sum_j s^M_{nij} = \frac{\sum_i \psi_{ni} + \sum_i \sum_j y_{nij} \psi^M_{nj}}{\psi_n} = \frac{\sum_i \psi_{ni} + \sum_j \left(\sum_i y_{nij} \right) \psi^M_{nj}}{\psi_n}.$$

Given $\sum_i y_{nij} = 1$, we have

$$\sum_i s^T_{ni} + \sum_i \sum_j s^M_{nij} = \frac{\sum_i \psi_{ni} + \sum_j \psi^M_{nj}}{\psi_n} = \frac{\sum_i (\psi_{ni} + \psi^M_{ni})}{\psi_n} = 1,$$

where the last equality follows from $\psi_n \equiv \sum_i (\psi_{ni} + \psi^M_{ni})$. Thus,

$$w_n L_{mn} + p_{mn} Q_{mn} = p_{mn} Q_n. \tag{26}$$

We know that

$$\frac{L_{fn}}{Q_{fn}} = \left(\frac{\alpha}{1-\alpha} \right) \frac{p_{mn}}{w_n}, \tag{27}$$

and

$$\frac{L_{mn}}{Q_{mn}} = \left(\frac{\beta}{1-\beta} \right) \frac{p_{mn}}{w_n}. \tag{28}$$

Plugging 28 into 26 we get

$$\left(\frac{\beta}{1-\beta} \right) p_{mn} Q_{mn} + p_{mn} Q_{mn} = p_{mn} Q_n,$$

from which we get

$$Q_{mn} = (1 - \beta)\, Q_n.$$

Using $Q_{fm} + Q_{mn} = Q_n$, we then get

$$Q_{fn} = \beta Q_n. \tag{29}$$

Plugging $Q_{mn} = (1 - \beta)\, Q_n$ back into (26), we get

$$w_n L_{mn} = \beta p_{mn} Q_n.$$

Using $L_{mn} + L_{fn} = L_n$, we then get

$$w_n (L_n - L_{fn}) = \beta p_{mn} Q_n. \tag{30}$$

From (27) and (29), we get

$$w_n L_{fn} = \left(\frac{\alpha}{1 - \alpha} \right) \beta p_{mn} Q_n.$$

Using (30), we then have

$$L_{fn} = \left(\frac{\alpha}{1 - \alpha} \right) (L_n - L_{fn}),$$

and hence

$$L_{fn} = \alpha L_n.$$

Plugging into (30), we get

$$(1 - \alpha) w_n L_n = \beta p_{mn} Q_n,$$

or

$$X_n^T = \left(\frac{1 - \alpha}{\beta} \right) w_n L_n.$$

$\square$

B Algorithm

We now explain the algorithm to solve for the equilibrium. Given a matrix Y with elements y_{ni} then one can solve the system forgetting about the complementary slackness conditions in (8) by following an extension of the algorithm in Alvarez and Lucas (2007). This is as follows: first, there is a function $p_m(w)$ that solves for the vector of prices p_m given the vector of wages w. Second, there is a mapping $w' = T(w; Y)$ whose fixed point, $w = F(Y)$, gives the equilibrium wages given Y.

The final step is to solve for the equilibrium Y. Let $C_T(Y)$ be matrix with typical element c_i/k_{ni} associated with Y and let $C_{MP}(Y)$ be the matrix with typical element c_{ni} associated with Y. Let $M(Y)$ be a matrix with typical element given by $\chi(c_i(Y)/k_{ni} \leq c_{ni}(Y))$ (where $\chi(A) = 1$ if the statement A is true and $\chi(A) = 0$ otherwise). Finally, let $\Gamma(Y)$ be a matrix with typical element given by

$$\gamma_{ni}(Y) = \frac{\min\{c_i(Y)/k_{ni}, c_{ni}(Y)\}}{y_{ni}c_i(Y)/k_{ni} + (1 - y_{ni})c_{ni}}.$$

We use a mapping $Y' = H(Y) = Y \cdot \Gamma(Y) + M(Y) \cdot (I - \Gamma(Y))$, where I is a NxN matrix of ones and where the operation $A \cdot B$ is the entry-wise or Hadamard matrix multiplication,

Note that if $\widetilde{Y}$ is a fixed point of $H(Y)$ then $\Gamma(Y) = I$, which implies that Y satisfies the complementary slackness conditions in (8). The algorithm to find the equilibrium Y is to start with $y_{ni} = 0$ for all n, i and then iterate on $Y' = H(Y)$ until all the elements of $\Gamma(x)$ are sufficiently close to one.