

# CAN EXCHANGE RATES FORECAST COMMODITY PRICES?

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**Abstract.** This paper demonstrates that “commodity currency” exchange rates have remarkably robust power in predicting future global commodity prices, both in-sample and out-of-sample. A critical element of our in-sample approach is to allow for structural breaks, endemic to empirical exchange rate models, by implementing the approach of Rossi (2005b). Aside from its practical implications, our forecasting results provide perhaps the most convincing evidence to date that the exchange rate depends on the present value of identifiable exogenous fundamentals. We also find that the reverse relationship holds; that is, that commodity prices Granger-cause exchange rates. However, consistent with the vast post-Meese-Rogoff (1983a,b) literature on forecasting exchange rates, we find that the reverse forecasting regression does not survive out-of-sample testing. We argue, however, that it is quite plausible that exchange rates will be better predictors of exogenous commodity prices than vice-versa, because the exchange rate is fundamentally forward looking. Therefore, following Campbell and Shiller (1987) and Engel and West (2005), the exchange rate is likely to embody important information about future commodity price movements well beyond what econometricians can capture with simple time series models. In contrast, prices for most commodities are extremely sensitive to small shocks to current demand and supply, and are therefore likely to be less forward looking.

J.E.L. Codes: C52, C53, F31, F47.

Key words: Exchange rates, forecasting, commodity prices, random walk.

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## 1. INTRODUCTION

This paper demonstrates that the exchange rates of a number of small commodity exporters have remarkably robust forecasting power over global commodity prices. The relationship holds both in-sample and out-of-sample. It holds when non-dollar major currency cross exchange rates are used, as well as when one assumes that the key variables are stationary with high persistence. Overall, the evidence constitutes strong support for models in which the exchange rate depends on the present value of global commodity prices, a considerable improvement over previous findings based on regressing one set of endogenous variables on another set of endogenous variables. We also find that commodity prices Granger-cause exchange rates in-sample, assuming one employs suitable methods to allow for structural breaks. However, this relationship is not robust out-of-sample.

We argue that the apparent disconnect between the forward and reverse regressions can be traced to the fact that the exchange rate is fundamentally a forward-looking variable that likely embodies information about future commodity price movements that cannot easily be captured by simple time series models. In contrast, commodity prices tend to be quite sensitive to current conditions because both demand and supply are typically quite inelastic. In addition, financial markets for commodities tend to be far less developed than for the exchange rate. As a result, commodities tend to be less of a barometer of future conditions than are exchange rates.<sup>1</sup>

Our laboratory here is that of the “commodity currencies.” These include the Australian,

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<sup>1</sup>The existing literature provides only scant empirical evidence that economic fundamentals can consistently explain movements in major OECD floating exchange rates, let alone actually forecast them, at least at horizons of one year or less. Meese and Rogoff’s (1983a,b, 1988) finding that economic models are useless in predicting exchange rate changes remains an outstanding challenge for international macroeconomists, although some potential explanations have been put forward. Engel and West (2005), for example, argue that it is not surprising that a random walk forecast outperforms fundamental-based models, as in a rational expectation present-value model, if the fundamentals are  $I(1)$  and the discount factor is near one, exchange rate should behave as a near-random walk. See also Rossi (2005a, 2006) for alternative explanations. Engel, Mark and West (2007) and Rogoff and Stavreklava (2008) offer discussions of the recent evidence.

Canadian, and New Zealand dollars, as well the South African rand and the Chilean peso. For all of these floating currencies, price fluctuations in world commodity markets represent exogenous terms-of-trade shocks that impact a significant share of their country's exports. By adopting testing procedures that are robust to parameter instabilities, we not only overturn the previous exchange rate-fundamental disconnect findings, we uncover an empirical regularity that has potentially important practical implications to a wide range of developing countries.<sup>2</sup>

We are not the first to test present value models of exchange rate determination by running a reverse regression. Campbell and Shiller (1987), and more recently in Engel and West (2005), show that because the nominal exchange rate reflects expectations of future changes in its economic fundamentals, it should help predict them. However, previous tests have employed standard macroeconomic fundamentals such as interest rates, output and money supplies that are plagued by issues of endogeneity, rendering causal interpretation impossible.<sup>3</sup> This problem can be finessed for the commodity currencies, at least for one important determinant, the world price for an index of their major commodity exports. We hasten to emphasize that while our results provide strong support for the proposition that exchange rates depend on the present expected value of commodity prices, they do not necessarily lend support to any of the various popular monetary models of exchange rate determination (e.g., Dornbusch 1976). Our results can equally well be rationalized in a model with fully flexible prices as in a model with highly sticky prices.

Even after so finessing the exogeneity problem, disentangling the dynamic causality between

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<sup>2</sup>Disentangling the dynamic relationship between the exchange rate and its fundamentals is complicated by the possibility that this relationship may not be stable over time. Mark (2001) states, "... ultimately, the reason boils down to the failure to find a time-invariant relationship between the exchange rate and the fundamentals." See also Rossi (2006).

<sup>3</sup>This problem is well-stated in the conclusion of Engel and West (2005), "Exchange rates might Granger-cause money supplies because monetary policy makers react to the exchange rate in setting the money supply. In other words, the present-value models are not the only models that imply Granger causality from exchange rates to other economic fundamentals."

exchange rates and commodity prices is still complicated by the possibility of parameter instability, which confounds traditional Granger-causality regressions. After, and only after, controlling for the instabilities we find prevalent in these data series, we uncover robust in-sample evidence that exchange rates predict world commodity price movements. Individual commodity currencies Granger-cause their corresponding country-specific commodity price indices, and can also be combined to predict movements in the aggregate world market price index.

As a further test of the robustness of our results to parameter instability, we also look at out-of-sample commodity price forecasting. Here we find that exchange rate-based forecasts deliver substantial improvements over the random walk benchmark in mean square forecast error comparisons. We even find that individual commodity currencies can help forecast important individual commodity prices as well, such as that of gold and nickel, most likely due to their correlation with the global commodity price index. As one may be concerned that the strong ties global commodity markets have with the U.S. dollar may induce endogeneity in our data, we also conduct robustness checks using exchange rates relative to the British pound and the Japanese yen.<sup>4</sup> We further consider longer-horizon predictability and alternative regression specifications as additional robustness checks.

To summarize, we obtain the robust yet theoretically sensible finding that commodity exchange rates are not only connected to their fundamentals, they can actually forecast global commodity price movements. Since world commodity prices are essentially exogenous to these small countries' exchange rates, our finding breaks free of the reverse causality problem afflicting previous research efforts, and provides direct support for a present-value model of nominal exchange rate

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<sup>4</sup>For example, since commodities are mostly priced in dollars, one could argue that global commodity demands and thus their prices would go down when the dollar is strong. In addition to the yen and the pound cross rates, we also used the nominal effective exchange rates for the commodity economies and reached the same qualitative results.

determination. Although our results are not surprising from a theoretical standpoint, their empirical robustness is, especially in view of the decades of negative findings that dominate the empirical exchange rate literature. As an important by-product of our analysis, we develop a novel approach for predicting future commodity price movements, an approach that may prove to have practical value as many commodities lack deep and liquid forward markets.

## 2. BACKGROUND AND DATA DESCRIPTION

Although the commodity currency phenomenon may extend to a broader set of developing countries, our study focuses on five small commodity-exporting economies with a sufficiently long history of market-based floating exchange rates, and explores the dynamic relationship between exchange rates and world commodity prices.

As shown in Appendix Table A1, Australia, Canada, Chile, New Zealand, and South Africa produce a variety of primary commodity products, from agricultural and mineral to energy-related goods. Together, commodities represent between a quarter and well over a half of each of these countries' total export earnings. Even though for certain key products, these countries may have some degree of market power (e.g. New Zealand supplies close to half of the total world exports of lamb and mutton), on the whole, due to their relatively small sizes in the *overall* global commodity market, these countries are price takers for the vast majority of their commodity exports.<sup>5</sup> As such, global commodity price fluctuations serve as an easily-observable and *exogenous* shock to these countries' exchange rates.

From a theoretical standpoint, exchange rate responses to terms-of-trade shocks can operate

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<sup>5</sup>In 1999, for example, Australia represents less than 5 percent of the total world commodity exports, Canada about 9 percent, and New Zealand 1 percent. Furthermore, substitution across various commodities also mitigates the market power these countries have, even within the specific market they appear to dominate. See Chen and Rogoff (2003) for a more detailed discussion and analyses.

through several well-understood channels, such as the income effect and the Balassa-Samuelson channel.<sup>6</sup> In practice, however, sound theories rarely translate into robust empirical support; moreover, for most OECD countries, it is extremely difficult to actually identify an exogenous measure of terms-of-trade. Since exogenous world commodity prices can be observed daily from the few centralized exchanges, the commodity currencies we study overcome these concerns, and thus offer a unique laboratory for testing exchange rate theories.

The exchange rate theories we want to assess are those in which the nominal exchange rate is viewed as an asset price (e.g. Obstfeld and Rogoff 1996, p.529). The asset-pricing approach encompasses a variety of structural models that relate the nominal exchange rate  $s_t$  to its fundamentals  $f_t$  and to its expected future value, giving rise to a present-value relation between the nominal exchange rate and the discounted sum of its expected future fundamentals<sup>7</sup>:

$$s_t = \gamma \sum_{j=0}^{\infty} \psi^j E_t(f_{t+j}|I_t) \quad (1)$$

where  $\psi$  and  $\gamma$  are parameters dictated by the specific structural model, and  $E_t$  is the expectation operator given information  $I_t$ . It is this present-value equation that shows that exchange rate  $s$  should Granger-cause its fundamentals  $f$ .

While the present-value representation is well accepted from a theoretical standpoint, there is so far little convincing empirical support for it in the exchange rate literature. The difficulty lies in the actual testing, as the standard exchange rate fundamentals considered in the literature are essentially all endogenous and jointly determined with exchange rates in equilibrium. They may also directly react to exchange rate movements through policy responses. When  $f$  is not exogenous,

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<sup>6</sup>See, for example, Chen and Rogoff (2003), and Chs. 4 and 9 in Obstfeld and Rogoff (1996).

<sup>7</sup>The transversality or "no-bubbles" condition is imposed here.

a positive finding that exchange rate  $s$  Granger-causes fundamental  $f$  could simply be the result of endogenous response or reverse causality, and is thus observationally equivalent to a present-value model. For instance, exchange rates Granger-causing money supply or interest changes may simply be the result of monetary policy responses to exchange rate fluctuations, as would be the case with a Taylor interest rate rule that targets CPI-inflation. Exchange rate changes may also precede inflation movements if prices are sticky and pass-through is gradual. As such, positive Granger-causality results for these standard fundamentals are difficult to interpret and cannot be taken as evidence for the present-value framework, *unless* the fundamental under consideration is clearly exogenous to exchange rate movements. Commodity prices are a unique exchange rate fundamental for these countries because the causality is clear, and a direct testing of the present-value theoretical approach is thus feasible.<sup>8</sup> In addition, since these countries all experienced major changes in policy regimes and/or market conditions (such as the adoption of an inflation target), we also emphasize the importance of allowing for time-varying parameters.

For this reason, our empirical finding that exchange rates predict exogenous world commodity prices represents a significant step forward in the empirical literature. As we are going to show, given the present value model (1) we should expect that exchange rates predict exogenous world commodity prices even if commodity prices do not predict future exchange rates. Suppose, for example, that commodity price changes are driven by a variable  $X_t$  that is perfectly forecastable and known to all market participants but not to econometricians:  $\Delta cp_t = X_t$ . The example is extreme, but there are plausible cases where it may not be a bad approximation to reality. For example, commodity prices may depend in part on fairly predictable factors, such as world population growth,

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<sup>8</sup>Amano and van Norden (1993), Chen and Rogoff (2003, 2006), and Cashin, Cespedes, and Sahay (2004), for example, establish commodity prices as an exchange rate fundamental for these commodity currencies

as well as cobweb cycles that are predictable by market participants' expertise but are not easily described by simple time series models. Thus, there may be patterns in commodity pricing that could be exploited by knowledgeable market participants but not by the econometrician. Such factors are totally extraneous to exchange rate dynamics. Note that econometricians omitting such variables may likely find parameter instabilities, such as those that we indeed detect in our regressions.

To make the example really stark, let's assume that the (known) sequence  $\{X_\tau\}_{\tau=t,t+1,\dots}$  is generated by a random number generator. Note that someone who does not know  $\{X_\tau\}_{\tau=t,t+1,\dots}$  will not be able to forecast commodity prices even though they are perfectly forecastable by market participants. Since commodity prices are perfectly forecastable by the markets, (1) and  $f_t = cp_t$  imply:

$$\Delta s_{t+1} = \gamma \sum_{j=1}^{\infty} \psi^j \Delta cp_{t+j} + z_{t+1}. \quad (2)$$

where  $z_t$  are other shocks determining exchange rates in equilibrium independently of commodity prices.

Note that  $\Delta cp_t$  will be of no use for the econometrician in forecasting  $\Delta s_{t+1}$ , nor will it be of any use in forecasting  $\Delta cp_{t+1}$ . But  $\Delta s_t$  will be useful in forecasting  $\Delta cp_{t+1}$ , because it embodies information about  $X_{t+1}$ . This asymmetry is indeed perfectly consistent with our empirical findings in Section 3 below in terms of out-of-sample forecasts. We find exchange rates to forecast commodity prices well, but not vice versa.<sup>9</sup> Our results follow directly from the fact that exchange rates are a strongly forward looking variable and do not directly depend on the variables explaining commodity prices. They depend on them only through the net present value relationship.

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<sup>9</sup>The point of having  $X_t$  generated by a random number generator is to produce the simplest case where using past exchange rates and commodity prices is not going to help forecast  $X$ .



We examine the dynamic relationship between exchange rates and commodity prices both in terms of Granger-causality and out-of-sample forecasting ability.<sup>10</sup> We regard these two tests as important alternative approaches to evaluating the predictive content of a variable. The in-sample tests take advantage of the full sample size and thus are likely to have higher power, while the out-of-sample forecast procedure may prove more practical as it mimics the data constraint of real-time forecasting.<sup>11</sup> We use quarterly data over the following time-periods: Australia (from 1984:1 to 2005:4), Canada (from 1973:1 to 2005:4), Chile (from 1989:3 to 2005:4), New Zealand (from 1987:1 to 2005:4), and South Africa (from 1994:1 to 2005:4).<sup>12</sup> For each commodity economy, we aggregate the relevant dollar spot prices in the world commodity markets to construct country-specific, export-earnings-weighted commodity price indices (labeled “ $cp$ ”).<sup>13</sup> For nominal exchange rates (“ $s$ ”), we use the end-of-period U.S. dollar rates from the IFS for the majority of our analyses. We also consider cross rates relative to the Japanese yen and the British pound as a robustness check. We use the *All Commodities and Oil Index* in US dollars (“ $cp^W$ ”) from the IMF to measure movements in the overall aggregate world commodity markets. It is a world export-earnings-weighted price index for over forty products traded on various exchanges.<sup>14</sup>

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<sup>10</sup>Previous studies on commodity currencies emphasize the strong contemporaneous causal relationship from commodity prices to exchange rates. There has been little success in finding stable dynamic relationships in various exchange rate forecasting exercises (see Chen (2005), for example.)

<sup>11</sup>Note that all data are available in real-time and are never revised. As is well-known in the literature, in-sample predictive tests and out-of-sample forecasting tests can and often provide different conclusions, which could result from their differences in the treatment of time-varying parameters, the possibility of over-fitting, sample sizes, and other biases...etc. See Inoue and Kilian (2004). We do not promote one over the other here, but recognize the trade-offs.

<sup>12</sup>Canada began floating its currency in 1970, and Australia and New Zealand abandoned their exchange rate pegs in 1983 and 1985 respectively. For Chile and South Africa, our sample periods are chosen a bit more arbitrarily: Chile operated under a crawling peg for most of the 1990s, and the starting point for South Africa roughly corresponds to the end of apartheid. We note that we also conducted all the analyses presented in this paper using monthly data up to the end of 2007 for the subset of countries that we have all the data. The results are qualitatively similar and are available upon request.

<sup>13</sup>Individual commodity price data are collected from the IMF, Global Financial Database, the Bank of Canada, and the Reserve Bank of New Zealand. Appendix Table A1 provides the country-specific weights used to aggregate individual world commodity prices into country-specific indices.

<sup>14</sup>The IMF series underwent significant revisions in recent years (see IMF’s website for details). We splice the series

As standard unit root tests cannot reject that these series contain unit roots, we proceed to analyze the data in first-differences, which we denote with a preceding  $\Delta$ .<sup>15</sup> In Section 4, we present an alternative predictive regression specification that is robust to the possibility that the autoregressive roots in these data may not be exactly one, although very close to it (i.e. they are "local-to-unity"). We see that our findings are robust to these different assumptions. In addition, we note that even in the individual data series, we observe strong evidence of structural breaks, found mostly in early 2000's.<sup>16</sup> This finding foreshadows one of our major conclusions that controlling for parameter instabilities is crucial in analyzing the exchange rate-fundamental connection.

### 3. EXCHANGE RATES AND COMMODITY PRICES: WHICH PREDICTS WHICH?

In this section, we analyze the dynamic relationship between nominal exchange rates and commodity prices by looking at both in-sample predictive content and out-of-sample forecasting ability. We first examine whether the exchange rate can explain future movements in commodity prices, as a test of the present-value theoretical approach. Following the Meese-Rogoff (1983a,b) literature, we next look at the reverse analyses of exchange rate predictability by commodity prices.

Using Rossi's (2005b) procedure that is robust to time-varying parameters, we first see that individual exchange rates Granger-cause movements in their corresponding country-specific commodity price indices, and that this predictive content translates to superior out-of-sample forecast

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in order to have a relatively consistent series going as far back as possible (to 1980).

<sup>15</sup>Here we do not consider cointegration but first differences since we are not testing any specific models. Chen and Rogoff (2003) showed that, in analyzing real exchange rates, DOLS estimates of cointegrated models and estimates of models in differences produce very similar results. (From a practical point of view, real exchange rates and nominal ones behave very similarly.) Chen (2005) examines commodity-priced augmented monetary models in the cointegration framework.

<sup>16</sup>A more detailed analysis of the time series properties of these series, as well as the other fundamentals typically used in the canonical exchange rate literature, are not included in this draft but are available upon request.

performance relative to a random walk benchmark. We then look into multivariate analyses using several exchange rates and forecast combinations. We find these commodity currencies together forecast price fluctuations in the aggregate world commodity market quite well. Figures 1 and 2 present a quick visual preview to this key finding. World commodity price forecasts based on the exchange rates - whether entered jointly in a multivariate model or individually under a forecast combination approach - track the actual data dramatically better than the random walk.

Concerning the reverse exercise of forecasting exchange rates, addressing parameter instability again plays a crucial role in uncovering evidence for in-sample exchange rate predictability from commodity prices. The out-of-sample analyses, however, show essentially no evidence of exchange rate forecastability, suggesting the reverse regression may be more fragile.

All the analyses in this section are based on U.S. dollar exchange rates. Later, we will demonstrate the robustness of our results by looking at different anchor-currencies, longer-horizon predictive regressions robust to “local-to-unity” regressors, and implications for individual commodity markets. Appendix 2 provides an overview of the time series methods that we use.

**3.1. Can Exchange Rates Predict Commodity Prices?.** We first investigate the empirical evidence on Granger-causality, using both the traditional testing procedure and one that is robust to parameter instability. We demonstrate the prevalence of structural breaks and emphasize the importance of controlling for them. Our benchmark Granger-causality analyses below include one lag each of the explanatory and dependent variables, though our findings are robust to the inclusion of additional lags.<sup>17</sup>

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<sup>17</sup>Additional lags are mostly found to be insignificant based on the BIC criterion.

**In-Sample Granger-Causality (GC) Tests.** Present value models of exchange rate determination imply that exchange rates must Granger-cause fundamentals. In other words, ignoring issues of parameter instabilities, we should reject the null hypothesis that  $\beta_0 = \beta_1 = 0$  in the regression:<sup>18</sup>

$$E_t \Delta cp_{t+1} = \beta_0 + \beta_1 \Delta s_t + \beta_2 \Delta cp_t \quad (3)$$

Panel A in Table 1 reports the results based on the above standard Granger-causality regression for the five exchange rates and their corresponding commodity price indices. All variables are first differenced, and the estimations are heteroskedasticity and serial correlation-consistent.<sup>19</sup> The table reports the p-values for the tests, so a number below 0.05 implies evidence in favor of Granger-causality (at the 5% level). We note that overall, traditional Granger-causality tests find essentially no evidence for exchange rates Granger-causing commodity prices.

An important drawback in these Granger-causality regressions is that they do not take into account potential parameter instabilities. We find that structural breaks are a serious concern not only theoretically as discussed above, but also empirically as observed in the individual time series data under consideration.<sup>20</sup> Table 2 reports results from the parameter instability test, based on Andrews (1993), for the bivariate Granger-causality regressions. We observe strong evidence of time-varying parameters in several of these relationships. As such, we next consider the joint null hypothesis that  $\beta_{0t} = \beta_0 = 0$  and  $\beta_{1t} = \beta_1 = 0$  by using Rossi's (2005b) *Exp - W\** test, in the

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<sup>18</sup>We note that the qualitative results are the same if one tests for only  $\beta_1 = 0$ . Our choice here is more consistent with the driftless random walk benchmark commonly used in the exchange rate literature. Our finding is also robust to the inclusion of additional lags, or even the exclusion, of  $\Delta cp_t$ .

<sup>19</sup>Results are based on the Newey and West (1987) procedure with bandwidth  $T^{1/3}$  (where  $T$  is the sample size.)

<sup>20</sup>Results from structural break analyses using Andrews' (1993) QLR test and Rossi's (2005b) *Exp-W\** test are available upon request.

following regression setup:<sup>21</sup>

$$E_t \Delta cp_{t+1} = \beta_{0t} + \beta_{1t} \Delta s_t + \beta_{2t} \Delta cp_t \quad (4)$$

Table 3, Panel A shows that this test of Granger-causality, which is robust to time-varying parameters, indicates strong evidence in favor of a time-varying relationship between exchange rates and commodity prices.

INSERT TABLES 1, 2 AND 3 HERE

**Out-of-Sample Forecasts.** We now ask whether in-sample Granger-causality translates into out-of-sample forecasting ability. We adopt a rolling forecast scheme based on eq. (3), without the lagged dependent variable  $\Delta cp_t$ , and test for forecast encompassing relative to a random walk ( $E_t \Delta cp_{t+1} = 0$ ). Here we present results based on a random walk benchmark due to its significance in the exchange rate literature.<sup>22</sup> Specifically, we use a rolling window with size equal to half of the total sample size to estimate the model parameters and generate one-quarter ahead forecasts recursively (what we call “model-based forecasts”).<sup>23</sup> Table 4 reports the difference between the associated mean square forecast errors (MSFE) from this procedure and the MSFE of the random walk benchmark, after re-scaling by a measure of their variability.<sup>24</sup> A negative number indicates that the model outperforms a driftless random walk, and for proper inference, we use Clark

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<sup>21</sup>See Appendix 2 for a detailed description of Rossi’s (2005b) test. In addition, we tested only  $\beta_{1t} = \beta_1 = 0$  and confirmed that our positive Granger-causality findings are not the result of random walk fundamentals with time-varying drifts.

<sup>22</sup>Our findings are robust to the inclusion of lagged dependent variables in both forecast specifications, as one may do in the spirit of an out-of-sample Granger-causality test. We also extend the comparison to a random walk with drift, and find similar results.

<sup>23</sup>Rolling forecasts are robust to the presence of time-varying parameters and have the advantage of not making any assumption as to the nature of the time variation in the data. We implement rolling, rather than recursive, forecasts as the former adapt more quickly to possible time variation.

<sup>24</sup>This procedure produces a statistic similar to the standard Diebold and Mariano (1995) test statistic.

and McCracken’s (2001) “ENCNEW” test of equal MSFEs to compare these nested models. A rejection of the null hypothesis, which we indicate with asterisks, implies that the regressor contains out-of-sample forecasting power for the dependent variable.

Panel A in Table 4 shows that exchange rates help forecast commodity prices, even out-of-sample. The exchange rate-based models outperform a random walk in forecasting changes in world commodity prices, and this result is quite robust across the five countries.<sup>25</sup> The strong evidence of commodity price predictability in both in-sample and out-of-sample tests is quite remarkable, given the widely documented pattern in various forecasting literature that in-sample predictive ability often fails to deliver out-of-sample success.<sup>26</sup>

INSERT TABLE 4 HERE

### 3.2. Can Exchange Rates Predict Aggregate World Commodity Price Movements?

**Multivariate Predictions and Forecast Combinations.** Having found that individual exchange rates can forecast the price movements of its associated country’s commodity export basket, we next consider whether combining the information from all of our commodity currencies can help predict price fluctuations in the aggregate world commodity market. For the world market index, we use the IMF’s “All Commodities and Oil Index” ( $cp^W$ ) discussed earlier.<sup>27</sup> We will show that forecasts of commodity prices improve by combining multiple commodity currencies as opposed to using only a single bilateral exchange rate. This makes sense because, a priori, one would expect that global commodity prices depend mainly on global shocks, whereas commodity currency exchange rates also depend on country specific shocks, in addition to depending on global shocks

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<sup>25</sup>We note that the sample size for South Africa, being quite a bit shorter than the other countries, may not be sufficient for meaningful testing of out-of-sample forecast power.

<sup>26</sup>Because exchange rates are available at extremely high frequencies, and because they are not subject to revisions, our analysis is immune to the critique that we are not looking at real time data forecasts.

<sup>27</sup>The index only goes back to 1980, so the sample size we are able to analyze is shorter in this exercise for Canada.

(mainly through commodity prices.) Thus, a weighted average of commodity currencies should, in principle, average out some of the country specific shocks and produce a better forecast of future global commodity prices.

We first look at the in-sample predictability of the world price index and consider multivariate Granger-causality regressions using the three longest exchange rate series (South Africa and Chile are excluded to preserve a larger sample size):

$$E_t \Delta cp_{t+1}^W = \beta_0 + \beta_{11} \Delta s_t^{AUS} + \beta_{12} \Delta s_t^{CAN} + \beta_{13} \Delta s_t^{NZ} + \beta_2 \Delta cp_t^W \quad (5)$$

Panels A through C in Table 5 reveal results consistent with our earlier findings using single currencies. This time, traditional Granger-causality tests suggest that the commodity currencies have predictive power (panel A), and controlling for time-varying parameters reinforces the evidence in favor of the three exchange rates jointly predicting the aggregate commodity price index (panel C).

We next extend the analysis to look at out-of-sample forecasts. We consider two approaches: multivariate forecast and combination of univariate forecasts. The multivariate forecast uses the same three exchange rates as in equation (5) above to implement the rolling regression forecast procedure described in the previous section. We again use Clark and McCracken’s (2001) “ENC-NEW” test to evaluate the model’s forecast performance relative to a random walk forecast. Table 5 Panel D shows that using the three commodity currencies together, we can forecast the world commodity price index significantly better than a random walk at the 1% level. This forecast power is also quite apparent when we plot the exchange rates-based forecasts along with the actual realized changes of the (log) global commodity price index in Figure 1. The random walk forecast

is simply the x-axis (forecasting no change). We see that overall, the commodity currency-based forecasts track the actual world price series quite well, and fit strikingly better than a random walk.<sup>28</sup>

INSERT TABLE 5 AND FIGURE 1 HERE

We next consider forecast combination, which is an alternative way to exploit the information content in the various exchange rates. The approach involves computing a weighted average of different forecasts, each obtained from using a single exchange rate. That is, we first estimate the following three regressions and generate one-step ahead world commodity price forecasts, again using the rolling procedure:

$$E_t \Delta c p_{t+1}^{W,i} = \beta_{0,i} + \beta_{1,i} \Delta s_t^i \quad \text{where } i = AUS, CAN, NZ \quad (6)$$

While there are different methods to weigh the individual forecasts, it is well known that simple combination schemes tend to work best (Stock and Watson 2003 and Timmermann 2006.) We consider equal weighting here, and compare our out-of-sample forecast of future global commodity prices,  $(\Delta c p_{t+1}^{W,AUS} + \Delta c p_{t+1}^{W,CAN} + \Delta c p_{t+1}^{W,NZ}) / 3$ , with the random walk forecast. We report the result in Table 5 Panel E. Again, we observe that the MSFE difference is negative and significant, indicating that the commodity price forecasts constructed from individual exchange rate-based forecasts outperform the random walk.<sup>29</sup> This finding is illustrated graphically in Figure 2, which plots the forecasted global commodity price obtained via forecast combination, along with the actual data (both in log differences). The random walk forecast, of no change, is the x-axis. The

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<sup>28</sup>We can improved the forecast performance of the model even more by further including lagged commodity prices in the forecast specifications.

<sup>29</sup>To judge the significance of forecast combinations, we used critical values based on Diebold and Mariano (1995).



figure shows that the combined forecast tracks the actual world price series much better than the random walk.

INSERT FIGURE 2 HERE

Finally, as a robustness check, we also examine whether each individual exchange rate series by itself can predict the global market price index. We note that this exercise is perhaps more a test to see whether there is strong co-movement amongst individual commodity price series, rather than based on any structural model. The first lines (labeled " $s_t$  GC  $cp_{t+1}$ ") in Table 6 report results for the predictive performance of each country-specific exchange rates. Remarkably, the finding that exchange rates predict world commodity prices appears extremely robust: the Australian, Canadian, and New Zealand exchange rates each have predictive power for the aggregate global commodity price index, and they also all outperform a random walk in out-of-sample forecasts.

INSERT TABLE 6 HERE

**3.3. Can Commodity Prices Predict Exchange Rates?.** Having found strong and robust evidence that exchange rates can Granger-cause and forecast out-of-sample future commodity prices, we now consider the reverse exercise of forecasting these exchange rates. First, we show positive in-sample results by allowing for structural breaks. In terms of out-of-sample forecasting ability, however, commodity currencies exhibit the same Meese-Rogoff puzzle as other major currencies studied in the literature; none of the fundamentals, including commodity prices, consistently forecasts exchange rate movements better than a random walk.<sup>30</sup>

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<sup>30</sup>We conducted, but excluded from this draft, the same analyses presented in Tables 1-4 using the standard exchange rate fundamentals as well. (These include the short-run interest rate differential, the long-run interest rate differential, the inflation rate differential, the log real GDP differential, and the log money stock differential between the relevant country-pairs.) We observe exactly the Meese-Rogoff puzzle, consistent with findings in the literature.

The lower panels (Panel B) in Tables 1-4, and Table 6 present results on exchange rate predictability by commodity prices. We first consider whether commodity prices Granger-cause nominal exchange rate changes, using standard tests that ignore the possibility of parameter instability. We look for rejection of the null hypothesis that the  $\beta_0 = \beta_1 = 0$  in the following regression:

$$E_t \Delta s_{t+1} = \beta_0 + \beta_1 \Delta cp_t + \beta_2 \Delta s_t \quad (7)$$

Similarly to the results in Panel A, Table 1 Panel B shows that traditional Granger-causality tests do not find any evidence that commodity prices Granger-cause exchange rates. We do find strong evidence of instabilities in the regressions, however, as seen in Table 2 Panel B. We then test the joint null hypothesis of  $\beta_{0t} = \beta_0 = 0$  and  $\beta_{1t} = \beta_1 = 0$ , using Rossi's (2005b) *Exp - W\** test in the following regression:

$$E_t \Delta s_{t+1} = \beta_{0t} + \beta_{1t} \Delta cp_t + \beta_2 \Delta s_t \quad (8)$$

Results in Table 3, Panel B, show that when looking at in-sample Granger-causality, exchange rates are predictable by their country-specific commodity price indices, once we allow for time-varying parameters. This is a very promising result given previous failures to connect the exchange rate and its fundamentals dynamically. We note that there does not appear to be significant differences between using exchange rates to predict commodity prices or vice versa, when we look at in-sample Granger-causality regressions robust to parameter instability.

The major difference between the two directions comes from comparing out-of-sample forecasting ability. Comparing Panel B to Panel A in Table 4, we see that there are hardly any negative

numbers in Panel B, giving us exactly the Meese-Rogoff stylized fact. We note the same pattern in Table 6, where individual exchange rates forecast aggregate world commodity price index better than a random walk, but world commodity price index does not help forecast exchange rates. This asymmetry in forecastability can be the result of many factors, ranging from potential non-linearities to the relative depth of the exchange rate markets, which may contribute to the exchange rates being more closely approximated by a random walk than commodity prices. Our favored explanation, though, as already discussed, is that the exchange rate likely contains valuable market information on the future evolution of commodity price fundamentals that cannot be easily captured by an econometrician. The reverse regression, however, is much less powerful, because commodity prices tend to be extremely sensitive to current shocks, given the very low short-term elasticities of both demand and supply.

#### 4. ROBUSTNESS ANALYSES

The previous section shows strong evidence that the U.S. dollar-based exchange rates of the five commodity-exporters can forecast price movements in global commodity markets. This novel finding raises some questions as well as potentially interesting implications, which we explore in this section. First, we consider whether this dynamic connection between movements in the currencies and in the commodity prices may result from a “dollar effect”, as both are priced in U.S. dollars. In addition, we consider an alternative predictive regression specification that is robust to highly persistent regressors, and examine longer-horizon predictions, up to two years ahead. Lastly, since we observe such robust results that exchange rate-based models can forecast various aggregated commodity price indices, we explore whether the forecastability extends to the prices of individual commodity products.

**4.1. Alternative Benchmark Currencies.** Since commodity products are priced in dollars, there may be some endogeneity induced by our use of dollar cross rates in the analyses above. For instance, one could imagine that when the dollar is strong, global demand for dollar-priced commodities would decline, inducing a drop in the associated commodity prices. Any aggregate uncertainty about the U.S. dollar may also simultaneously affect commodity prices and the value of the dollar (relative to the commodity currencies.) To remove this potential reverse causality or endogeneity, this section re-examines the predictive Granger-causality regressions and out-of-sample forecast exercises using exchange rates vis-a-vis the British pound and the Japanese yen. Table 7 reports results parallel to those in Tables 1-4. Panels A and B report the p-values for the Granger-causality and Andrews' (1993) QLR tests for the predictive regressions. Panel C shows predictability results robust to parameter instabilities, using Rossi's (2005b)  $Exp - W^*$  test. Lastly, Panel D reports the relative MSFEs from comparing exchange rate-based models to the random walk in out-of-sample forecasts.

Overall, we see that our earlier conclusions are quite robust under the two alternative benchmark currencies. We first observe that ignoring structural breaks, none of the traditional Granger-causality tests in Panel A rejects the null hypothesis that there is no relationship between exchange rates and commodity prices. However, as before, we uncover substantial instabilities in such regressions (Panel B), found mostly around 2002. When such instability is taken into account, we see extremely strong evidence in favor of Granger-causality. In particular, we see the evidence is slightly stronger when we use exchange rates to predict the commodity price indices than the other way around. Panel D shows that the predictive power of exchange rates for future commodity prices carries over to out-of-sample forecasts as well. The results based on the British pound are strongly supportive of the forecasting power of exchange rates for future commodity prices, whereas

those for the yen are somewhat weaker.<sup>31</sup>

INSERT TABLE 7 HERE

**4.2. Highly Persistent Regressors and Long-Horizon Predictability.** We have analyzed the dynamic connections between nominal exchange rates and fundamentals using data in first-differences thus far. This approach is consistent with the view that the series contain unit roots, which both has overwhelming empirical support and is theoretically sensible.<sup>32</sup> In this section, we consider an alternative specification and inference procedure that is robust to the possibility that the largest autoregressive (AR) roots in these series may not be *exactly* one, despite being very close to one. That is, we model the regressors in the predictive regressions as highly persistent and use tests statistics based on local-to-unity asymptotics.<sup>33</sup> We consider the robustness of our main findings (in Section 3) to this form of high persistence in the regressors, and also to longer-horizon predictive analyses. Results below show that our earlier findings are very robust.

We focus on three countries only: Australia, Canada, and New Zealand, as they have longer sample periods which are necessary for more meaningful testing of long-horizon predictability. Letting  $s_t$  and  $cp_t$  denote the levels of nominal exchange rate and fundamental (commodity prices) at time  $t$ , the short horizon exchange rate predictive regression can be expressed as follows:

$$\Delta s_{t+1} = \mu_1 + \beta cp_t + \gamma \Delta s_t + \epsilon_{1,t+1} \quad (9)$$

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<sup>31</sup>Using monthly data up to the end of 2007 for a subset of countries we have the full set of data, we observe strong predictability of commodity prices, both in- and out-of-sample, using nominal effective exchange rates. This is another indication that "the dollar effect" is not dominating our findings.

<sup>32</sup>See Obstfeld and Rogoff (1996), Mark (2001), for example. A not-for-publication appendix providing detailed empirical analyses on the time series properties of the fundamentals we consider is available upon request.

<sup>33</sup>See Elliott (1998), Campbell and Yogo (2006), for example. The local-to-unity asymptotics allows us to obtain reliable small sample approximations to the distribution of the test statistics when, empirically, the largest root is close to unity, and conveniently avoids problems arising from pre-test bias.

$$b(L)^{-1}(1 - \rho L)cp_{t+1} = \mu_2 + \epsilon_{2,t+1}$$

where  $\epsilon_{1,t+1}$  and  $\epsilon_{2,t+1}$  are assumed to be contemporaneously but not serially correlated, and  $\rho$  is assumed to be “local-to-unity” (very close to 1). The inference procedure robust to highly persistent regressors for this short-horizon predictive regressions is based on Campbell and Yogo (2006).

Assuming the same stochastic process for  $cp_t$  above, the corresponding long-horizon regression can be expressed as:<sup>34</sup>

$$\sum_{j=1}^h \Delta s_{t+j} = \beta_h cp_t + \lambda \Delta s_t + \xi_{t,h} \quad (10)$$

The long horizon regression analyses are based on Rossi’s (2007) procedure, which consists of inverting Elliott, Rothenberg and Stock’s (1995) test in the first stage, and adopting Campbell and Yogo’s (2006) test in the second stage.

For the reverse direction - using exchange rates to predict commodity prices - the regression robust to highly persistent regressor can be specified as:

$$\sum_{j=1}^h \Delta cp_{t+j} = \beta_h s_t + \lambda \Delta cp_t + \xi_{t,h} \quad (11)$$

where  $s_t$  would then be assumed to “highly persistent”:

$$b(L)^{-1}(1 - \rho L)s_{t+1} = \mu_1 + \epsilon_{2,t+1}$$

Table 8 reports the 95% confidence intervals for  $\beta$  estimated from (9) in the rows with “ $h =$

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<sup>34</sup>Regression (9) includes the lagged endogenous variable, where we assume  $|\gamma| < 1$ . The formula in Rossi (2007) has to be modified to take this into account. Her expression (4.14) becomes:  $\beta_h = \beta \sum_{j=1}^h \rho^{j-1} (1 - \gamma)^{-1}$ , and the confidence interval follows straightforwardly from this. Direct calculations show that  $\lambda \equiv h \sum_{j=1}^h \gamma^j$ .

1" (one quarter-ahead forecast), and confidence intervals for  $\beta_h$  estimated from (10) and (11) in the rows under " $h = 4$ " and " $h = 8$ ", for one- and two-year-ahead forecasts, respectively.<sup>35</sup> When the confidence intervals do not contain zero, we consider them as evidence in favor of predictive ability. The table shows that the predictability at long horizons is quite strong, both from exchange rates to commodity prices and vice-versa (with the exception of predicting the Australian commodity price index). This supports our earlier findings, based on first-differenced specifications, that the in-sample dynamic connection between commodity prices and exchange rates is very strong and robust.<sup>36</sup>

INSERT TABLE 8 HERE

**4.3. Individual Commodity Prices.** Having found robust evidence that exchange rate-based models can forecast various aggregated commodity price indices, we now explore whether the forecasting ability extends to some individual commodity products of interest. Positive results may have significant policy implications and could be very useful to the many commodity-exporting countries that care about forecasting price movements in world commodity markets. While it would be interesting to compare exchange rate-based commodity price forecasts with other methods (such as using commodity forwards), we do not explore the comparisons here. We note that some commodity markets do not have forwards, in which case exchange rate-based forecasts may be one viable alternative to gauge future market movements.

We consider both in-sample Granger-causality regressions (robust to parameter instability) and out-of-sample forecasts, focusing on the connection between a selected group of mineral products

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<sup>35</sup>We note the  $h = 1$  case is just a special case of the other two.

<sup>36</sup>We also conducted additional analyses using standard fundamentals, although these are highly endogenous, as we have noted. In the interest of space, we do not report the full table here. Overall, we find that for most countries and most fundamentals, we are able to reject the null hypothesis of no predictability (i.e. most confidence intervals exclude zero).

and the exchange rates of Australia, Canada, and New Zealand.<sup>37</sup> Panel A in Table 9 reports the p-values from regression (4), and Panel B the p-values from regression (8) for gold, silver, and nickel. Asterisks denote rejections at conventional significance levels. We find that all three exchange rates have predictive ability for price movements of silver and gold, and the price of nickel is predictable by the Australian and New Zealand exchange rates as well. Evidence for the reverse, i.e. for gold or silver prices to Granger-cause exchange rates, appears weaker.

#### INSERT TABLE 9

Table 10 reports the difference between the MSFE of the model and that of the random walk benchmark, after re-scaling by a measure of their variability. The forecasts are conducted using the same rolling procedure described in Section 3. The null hypothesis in Panel A is that individual commodity prices are best forecasted by a random walk, while the alternative hypothesis is that exchange rate-based models outperform a random walk commodity price forecast. Asterisks denote rejections of the random walk based on Clark and McCracken's (2001) "ENCNEW" test, which indicate that exchange rates provide significant predictive content for forecasting commodity prices out-of-sample. Panel B report similar results for the reverse exercise of exchange rate forecast, comparing individual commodity price-based forecasts with a random walk. We see from Table 10 that nickel prices appear to be forecastable by all three exchange rates. Interestingly, the in-sample predictability of silver prices did not carry over out-of-sample.

#### INSERT TABLE 10 HERE

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<sup>37</sup>Unreported results show that for almost all commodities and countries, we do not reject the null hypothesis that the parameter is insignificant in either regression (7) or regression (3). However, when we take into account the possibility that the parameter is time-varying, the results change substantially.



While we only explore a small set of commodity products in these simple bilateral tests, we consider these findings to be very promising. They provide a simple and viable method of forecasting price movements in individual commodity markets where alternative indicators may not be available. We leave more comprehensive explorations, such as using the multivariate forecast procedure discussed in Section 3 on a broader set of commodity products, to future research.

## 5. CONCLUSION

This paper investigates the dynamic relationship between commodity price movements and exchange rate fluctuations. After controlling for time-varying parameters, we not only find robust a relationship, we also uncover a surprising finding that exchange rates are very useful in forecasting future commodity prices. From a technical perspective, because our approach is robust to parameter instabilities and because commodity prices are essentially exogenous to the exchange rates we consider, our findings can be given a causal interpretation and thus represent a substantial advance over the existing literature. We are able in particular to overcome the greatest difficulty in testing single-equation, reduced-form exchange rate models, namely, that the standard fundamentals may be endogenous and that omitted variables may lead to parameter instabilities. For these reasons, we argue that commodity currencies offer an ideal laboratory for cutting-edge work on exchange rate models. There simply is no other instance of such a consistently clear and identifiable shock as world commodity prices.

Our results are robust to multivariate regressions, alternative benchmark currencies, forecast combinations, highly persistent (local-to-unit root) regressors, and longer-horizon predictions. One might eventually extend the approach to look at countries that have few or no commodities, such as most of Asia, to see if commodity prices affect the value of their currencies, and if their currency

fluctuations may offer predictive power for, say, oil prices. Our findings raise a broader question of whether one can exploit information from the nexus of global foreign exchange markets to predict commodity prices. We do not attempt such analysis here, and leave it for future research.

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## 7. TABLES

**Table 1. Bivariate Granger-Causality Tests**

	AUS	NZ	CAN	CHI	SA
A. P-values of $H_0 : \beta_0 = \beta_1 = 0$ in $\Delta cp_{t+1} = \beta_0 + \beta_1 \Delta s_t + \beta_2 \Delta cp_t$	0.51	0.12	0.03**	0.10	0.29
B. P-values of $H_0 : \beta_0 = \beta_1 = 0$ in $\Delta s_{t+1} = \beta_0 + \beta_1 \Delta cp_t + \beta_2 \Delta s_t$	0.25	0.93	0.06*	0.17	0.35

Note: The table reports p-values for the Granger-causality test. Asterisks mark rejection at the 1% (\*\*\*) , 5% (\*\*), and 10% (\*) significance levels respectively, indicating evidence of Granger-causality.

**Table 2. Andrews' (1993) QLR Test for Instabilities**

	AUS	NZ	CAN	CHI	SA
A. P-values for stability of $(\beta_{0t}, \beta_{1t})$ in: $\Delta cp_{t+1} = \beta_{0t} + \beta_{1t} \Delta s_t + \beta_2 \Delta cp_t$	0*** (2002:1)	1	0.47	0.08* (2002:4)	0*** (2003:3)
B. P-values for stability of $(\beta_{0t}, \beta_{1t})$ in: $\Delta s_{t+1} = \beta_{0t} + \beta_{1t} \Delta cp_t + \beta_2 \Delta s_t$	0*** (2002:1)	0*** (2002:3)	0.23	0*** (2002:4)	0*** (2003:3)

Note: The table reports p-values for Andrew's (1993) QLR test of parameter stability. Asterisks mark rejection at the 1% (\*\*\*) , 5% (\*\*), and 10% (\*) significance levels respectively, indicating evidence of instability. When the test rejects the null hypothesis of parameter stability, the estimated break-dates are reported in the parentheses.

**Table 3. Granger-Causality Tests Robust to Instabilities,  
Rossi (2005b)**

	AUS	NZ	CAN	CHI	SA
A. P-values for $H_0 : \beta_t = \beta = 0$ in $\Delta cp_{t+1} = \beta_{0t} + \beta_{1t}\Delta s_t + \beta_2\Delta cp_t$	0.09*	0.34	0.10*	0***	0***
B. P-values for $H_0 : \beta_t = \beta = 0$ in $\Delta s_{t+1} = \beta_{0t} + \beta_{1t}\Delta cp_t + \beta_2\Delta s_t$	0***	0***	0.19	0***	0***

Note: The table reports p-values for testing the null of no Granger-causality that are robust to parameter instabilities. Asterisks mark rejection at the 1% (\*\*\*), 5% (\*\*), and 10% (\*) significance levels respectively, indicating evidence in favor of Granger-causality.

**Table 4. Tests for Out-of-Sample Forecasting Ability**

	AUS	NZ	CAN	CHI	SA
A. MSFE difference between the model: $E_t \Delta cp_{t+1} = \beta_{0t} + \beta_{1t} \Delta s_t$					
and the random walk: $E_t \Delta cp_{t+1} = 0$					
	-1.40***	-0.22***	-0.39***	-0.52***	0.55
B. MSFE difference between the model: $E_t \Delta s_{t+1} = \beta_{0t} + \beta_{1t} \Delta cp_t$					
and the random walk: $E_t \Delta s_{t+1} = 0$					
	0.50	0.81	0.01**	0.42	0.55

Note. The table reports re-scaled MSFE differences between the model and the random walk

forecasts. Negative values imply that the model forecasts better than the random walk.

Asterisks denote rejections of the null hypothesis that random walk is better in favor of the alternative hypothesis that the fundamental-based model is better at 1% (\*\*\*), 5% (\*\*), and 10% (\*) significance levels, respectively, using Clark and McCracken's (2001) critical values.



**Table 5. Exchange Rates and the Aggregate Global Commodity Price Index**

Panel A. Multivariate Granger-Causality Tests
0***
Panel B. Andrews' (1993) QLR Test for Instabilities
0.29
Panel C. Multivariate Granger-Causality Tests Robust to Instabilities, Rossi (2005b)
0***
Panel D. Out-of-Sample Forecasting Ability
-1.76***
Panel E. Forecast Combination
-2.4***

Notes: The table reports results from various tests using the AUS, NZ and CAN exchange rates to jointly predict aggregate global future commodity prices ( $cp^W$ ).

Panels A-C report the p-values, and Panels D and E report the differences between the model-based forecasts and Random Walk forecasts. \*\*\* indicates significance at the 1% level.

**Table 6. Aggregate Global Commodity Price Index  
and Individual Exchange Rates**

	AUS	NZ	CAN
Panel A. Granger-Causality Tests			
$s_t$ GC $cp_{t+1}^W$	0.06**	0***	0***
$cp_t^W$ GC $s_{t+1}$	0.41	0.17	0.81
Panel B. Andrews' (1993) QLR Test for Instabilities			
$s_t$ GC $cp_{t+1}^W$	0.33	0.77	0.25
	--	--	--
$cp_t^W$ GC $s_{t+1}$	0***	0***	0***
	(2001:3)	(2001:3)	(2001:3)
Panel C. Granger-Causality Tests Robust to Instabilities, Rossi (2005b)			
$s_t$ GC $cp_{t+1}^W$	0.05**	0.05**	0***
$cp_t^W$ GC $s_{t+1}$	0***	0***	0***
Panel D. Out-of-Sample Forecasting Ability			
$s_t \Rightarrow cp_{t+1}^W$	-2.04***	-2.11***	-0.17***
$cp_t^W \Rightarrow s_{t+1}$	0.88	0.89***	1.52

Note. Panels A-C report p-values for tests for  $\beta_0 = \beta_1 = 0$  based on two regressions:

(i)  $\Delta cp_{t+1}^W = \beta_0 + \beta_1 \Delta s_t + \beta_2 \Delta cp_t^W$  (labeled  $s_t$  GC  $cp_{t+1}^W$ ) and (ii)  $\Delta s_{t+1} = \beta_0 + \beta_1 \Delta cp_t^W + \beta_2 \Delta s_t$  (labeled  $cp_t^W$  GC  $s_{t+1}$ ). Estimated break-dates are reported in parentheses. Panel D

reports the differences between model-based out-of-sample forecasts and the RW forecasts,

where the model is  $E_t \Delta y_{t+1} = \beta_0 + \beta_1 \Delta x_t$  (labeled  $x \Rightarrow y$ ). Asterisks indicate significance

levels at 1% (\*\*\*), 5% (\*\*), and 10% (\*) respectively.

Table 7. Alternative Benchmark Currencies

	U.K. pound					Japanese yen				
	AUS	NZ	CAN	CHI	SA	AUS	NZ	CAN	CHI	SA
Panel A. Multivariate Granger-Causality Tests										
$s_t$ GC $cp_{t+1}$	0.33	0.63	0.20	0.07*	0.36	0.45	0.42	0.21	0.02**	0.42
$cp_t$ GC $s_{t+1}$	0.48	0.36	0.37	0.30	0.22	0.77	0.52	0.12	0.17	0.03**
Panel B. Andrews' (1993) QLR Test for Instabilities										
$s_t$ GC $cp_{t+1}$	0.03**	0***	0.45	0***	0***	0.01***	0.04**	0.87	0.03**	0***
	(2002:1)	--	--	(2002:4)	(2003:3)	(2002:1)	(2002:3)	--	(2002:4)	(2003:3)
$cp_t$ GC $s_{t+1}$	0	0.04**	1	0***	0.03**	0.08*	0.08*	0.62	0***	0.56
	(2002:1)	--	--	(2002:4)	(2003:3)	(2002:1)	(2002:3)	--	(2002:4)	--
Panel C. Granger-Causality Tests Robust to Instabilities, Rossi (2005b)										
$s_t$ GC $cp_{t+1}$	0.15	0***	0.03**	0***	0***	0.41	0***	0.43	0***	0***
$cp_t$ GC $s_{t+1}$	0.12	0***	1	0***	0***	0.39	0.15	0.04**	0***	0.01***
Panel D. Out-of-Sample Forecasting Ability										
$s_t \Rightarrow cp_{t+1}$	-1.21***	0.63**	-0.17	-0.51**	0.04	-1.56***	0.79	-1.06	-0.42	-0.00
$cp_t \Rightarrow s_{t+1}$	1.20	0.77**	1.52	1.10**	0.21***	2.02	-0.24	1.27	0.23	0.26***

Note. Panels A-C report p-values for tests of  $\beta_0 = \beta_1 = 0$  based on two regressions: (i)  $\Delta cp_{t+1} = \beta_0$

$+ \beta_1 \Delta s_t + \beta_2 \Delta cp_t$  (labeled  $s_t$  GC  $cp_{t+1}$ ) and (ii)  $\Delta s_{t+1} = \beta_0 + \beta_1 \Delta cp_t + \beta_2 \Delta s_t$  (labeled  $cp_t$  GC  $s_{t+1}$ ).

Estimated break-dates are reported in parentheses. Panel D reports the differences between model-based out-

of-sample forecasts and the RW forecasts, where the model is  $E_t \Delta y_{t+1} = \beta_0 + \beta_1 \Delta x_t$  (labeled  $x \Rightarrow y$ ).

Asterisks indicate 1% (\*\*\*), 5% (\*\*), and 10% (\*) significance levels.

**Table 8. Short- and Long-Horizon Predictive Regressions**  
**(Robust to Highly Persistent Regressors)**

A. Confidence Interval for $\beta_h$ in: $E_t \sum_{j=1}^h \Delta c p_{t+j} = \beta_h s_t + \gamma \Delta c p_t$			
h:	1	4	8
AUS	(-0.01;0.01)	(-0.02;0.02)	(-0.02;0.02)
NZ	(-0.06;-0.05)	(-0.12;-0.16)	(-0.13;-0.22)
CAN	(0.18;0.19)	(0.24;0.33)	(0.24;0.35)
CHI	(0.44;0.53)	(0.54;0.89)	(0.54;0.92)
SA	(0.05;0.08)	(0.06;0.14)	(0.06;0.15)
B. Confidence Interval for $\beta_h$ in: $E_t \sum_{j=1}^h \Delta s_{t+j} = \beta_h c p_t + \gamma \Delta s_t$			
h:	1	4	8
AUS	(0.22;0.24)	(0.44;0.72)	(0.47;1.04)
NZ	(0.15;0.18)	(0.25;0.46)	(0.26;0.56)
CAN	(0.010;0.016)	(0.0;0.04)	(0.02;0.05)
CHI	(-0.06;-0.03)	(-0.08;-0.07)	(-0.08;-0.07)
SA	(0.14;0.19)	(0.17;0.32)	(0.17;0.34)

Note. The table reports confidence intervals for the long horizon regression parameter

$\beta_h$  at different horizons  $h$ .

**Table 9. Granger-Causality Tests Robust to Instabilities  
for Individual Commodity Prices, Rossi (2005b)**

Commodity:	P-values of $H_0 : \beta_t = \beta = 0$ in:					
	A. $\Delta cp_{t+1} = \beta_{0t} + \beta_{1t}\Delta s_t + \beta_2\Delta cp_t$			B. $\Delta s_{t+1} = \beta_{0t} + \beta_{1t}\Delta cp_t + \beta_2\Delta s_t$		
	AUS	NZ	CAN	AUS	NZ	CAN
Silver	0***	0.05*	0.01***	0.64	0.03**	0.06*
Gold	0***	0***	0.01***	1.00	0.31	0.04**
Nickel	0.02**	0***	0.31	0.80	0.02**	1.00

Note. The table reports p-values for testing the null hypothesis of Granger-causality robust to instability.

Asterisks mark rejection at the 1% (\*\*\*), 5% (\*\*), and 10% (\*) significance levels, respectively.

Panel A tests whether exchange rates Granger-causes commodity prices, and panel B tests if commodity prices Granger-causes exchange rates.

**Table 10. Tests for Out-of-Sample Forecasting Ability**

Commodity:	A. $\Delta s_t \Rightarrow \Delta cp_{t+1}$			B. $\Delta cp_t \Rightarrow \Delta s_{t+1}$		
	AUS	NZ	CAN	AUS	NZ	CAN
Silver	2.09	1.48	1.39	0.54	0.50*	0.39
Gold	2.61	0.40***	2.08	0.71	0.33*	0.07*
Nickel	-0.76**	-0.84**	0.13***	0.90	1.34	0.80

Note. The table reports re-scaled MSFE differences between the model and the random walk forecasts. Negative values imply that the model forecasts better than the random walk.

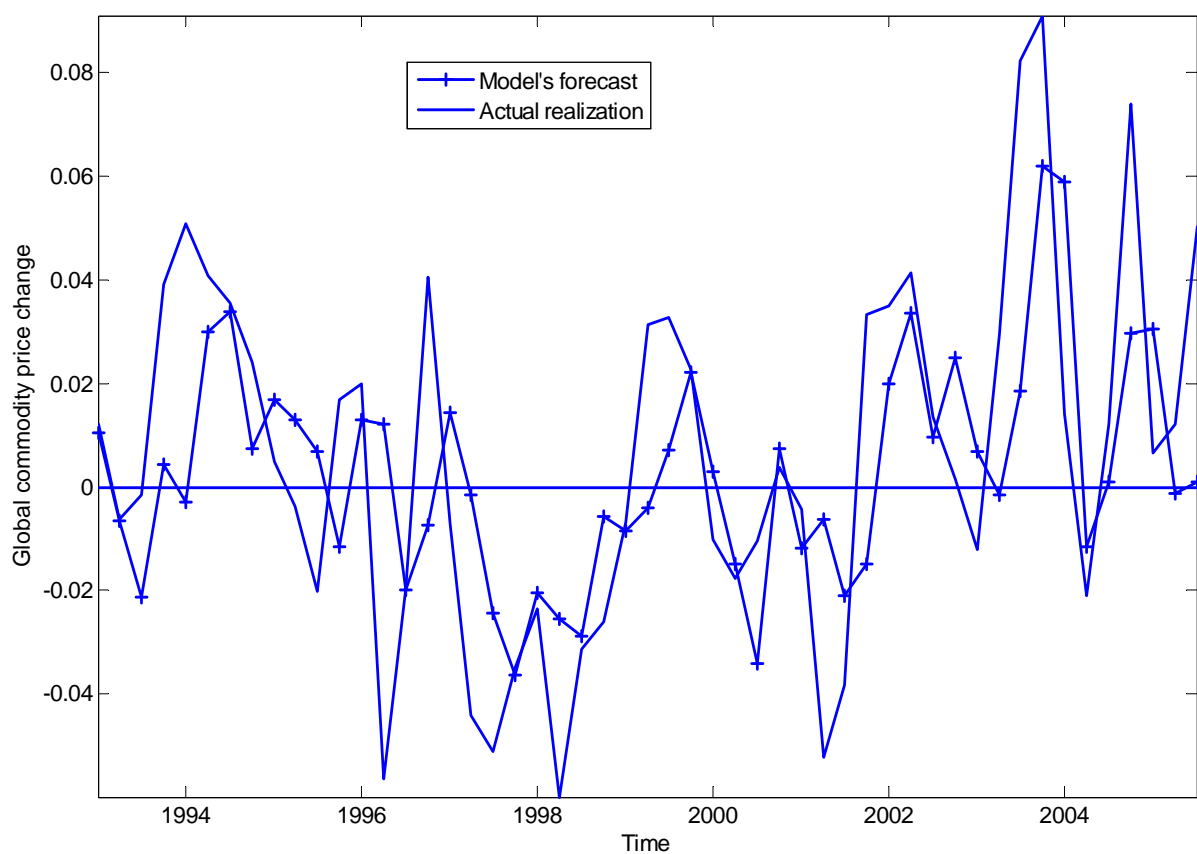
Asterisks denote rejections of the null hypothesis that random walk is better in favor of the alternative hypothesis that the fundamental-based model is better at 1% (\*\*\*), 5% (\*\*), and 10% (\*) significance levels, respectively, using Clark and McCracken's (2001) critical values.

$x \Rightarrow y$  indicates using  $x$  to forecast  $y$  out-of-sample. Panel A compares forecasts of the model  $E_t \Delta cp_{t+1} = \beta_{0t} + \beta_{1t} \Delta s_t$  and those of the random walk ( $E_t \Delta cp_{t+1} = 0$ ).

Panel B compares forecasts of the model  $E_t \Delta s_{t+1} = \beta_{0t} + \beta_{1t} \Delta cp_t$  and those of the random walk ( $E_t \Delta s_{t+1} = 0$ ).

**Figure 1. Forecasting Aggregate Global Commodity Price with Multiple Exchange Rates**

$$\text{Model : } E_t \Delta cp_{t+1}^W = \beta_0 + \beta_{11} \Delta s_t^{AUS} + \beta_{12} \Delta s_t^{CAN} + \beta_{13} \Delta s_t^{NZ}$$

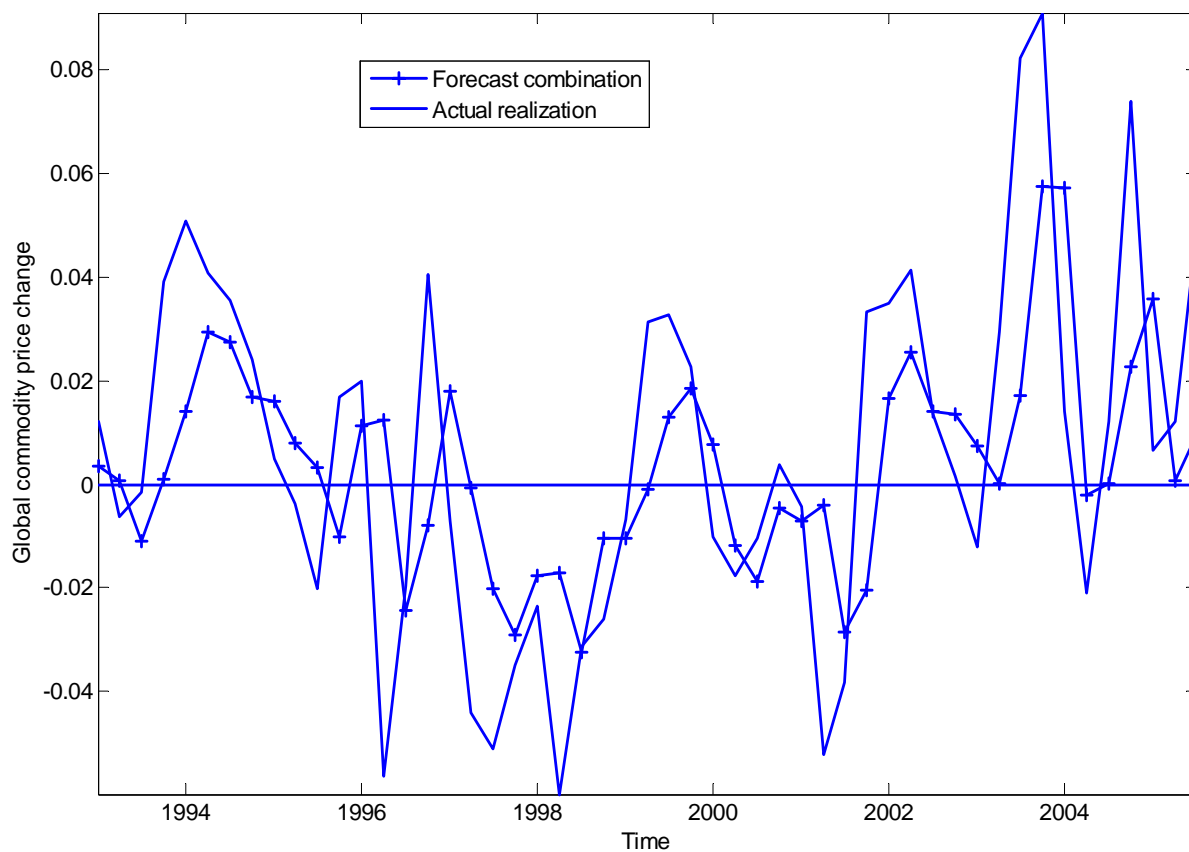


Note. The figure plots: the realized change in the global commodity price level (labeled “Actual realization”), and their forecasts based on: 1) exchange rates (labeled “Model’s forecast”) and 2) the random walk (the “X-axis”)

**Figure 2. Forecasting Aggregate Global Commodity Price Using Forecast Combination:**

$$\text{Model: } (\Delta cp_{t+1}^{W,AUS} + \Delta cp_{t+1}^{W,CAN} + \Delta cp_{t+1}^{W,NZ})/3,$$

$$\text{where } E_t \Delta cp_{t+1}^{W,i} = \beta_{0,i} + \beta_{1,i} \Delta s_t^i, \quad i = AUS, CAN, NZ$$



Note. The figure plots: the realized change in the global commodity price level (labeled “Actual realization”), and their forecasts based on: 1) the three exchange rates (labeled “Forecast combination”) and 2) the random walk (the “X-axis”)



## 8. APPENDIX 1. COMPOSITION OF THE COMMODITY PRICE INDICES

**Table A1. Weights**

Australia		Canada		New Zealand		South Africa	
1983Q1-2005Q4		1972Q1-2005Q4		1986Q1-2005Q4		1994Q1-2005Q4	
Product	Wt.	Product	Wt.	Product	Wt.	Product	Wt.
Wheat	8.3	Aluminum	5	Aluminum	8.3	Coal	22
Beef	7.9	Beef	7.8	Apples	3.1	Gold	48
Wool	4.1	Canola	1.2	Beef	9.4	Platinum	30
Cotton	2.8	Coal	1.8	Butter	6.5		
Sugar	2.5	Copper	2	Casein	6.7		
Barley	1.9	Corn	0.5	Cheese	8.3		
Canola	1	Crude Oil	21.4	Fish	6.7		
Rice	0.5	Fish	1.3	Kiwi	3.7		
Aluminum	8.1	Gold	2.3	Lamb	12.5	Chile	
Copper	2.8	Hogs	1.8	Logs	3.5	1989Q1-2005Q4	
Nickel	2.6	Lumber	13.6	Pulp	3.1	Product	Wt.
Zinc	1.5	Nat. Gas	10.7	Sawn Timber	4.6	Copper	100
Lead	0.7	Newsprint	7.7	Skim MP	3.7		
Coking coal	14.7	Nickel	2.4	Skins	1.6		
Steaming coal	9.7	Potash	1.6	Wholemeal MP	10.6		
Gold	9.4	Pulp	12.8	Wool	7.7		
Iron ore	9.3	Silver	0.3				
Alumina	7.4	Wheat	3.4				
LNG	4.8	Zinc	2.3				

## 9. APPENDIX 2: TIME SERIES METHODS

This section provides a description of the test statistics used in this paper. Let the model be:

$$y_t = x'_{t-1}\beta_t + \varepsilon_t, \quad t = 1, \dots, T, \quad \text{where } x_{t-1} \text{ is a } p \times 1 \text{ vector of explanatory variables.}^{38}$$

**9.1. Granger-causality tests.** Traditional Granger-causality regressions assume that the parameter  $\beta_t = \beta$ ; that is,  $\beta$  is constant. They are implemented as:

$$GC : W_T = T \left( \hat{\beta} - 0 \right)' \hat{V}_\beta^{-1} \left( \hat{\beta} - 0 \right),$$

where  $\hat{V}_\beta$  is a consistent estimate of the covariance of  $\hat{\beta}$ . For example,  $\hat{V}_\beta = S_{xx}^{-1} \hat{S} S_{xx}^{-1}$ ,  $S_{xx} \equiv \frac{1}{T-1} \sum_{t=1}^{T-1} x_{t-1} x'_{t-1}$ ,

$$\hat{S} = \left( \frac{1}{T} \sum_{t=2}^T x_{t-1} \hat{\varepsilon}_t \hat{\varepsilon}_t' x'_{t-1} \right) + \sum_{j=2}^{T-1} \left( 1 - \left| \frac{j}{T^{1/3}} \right| \right) \left( \frac{1}{T} \sum_{t=j+1}^T x_{t-1} \hat{\varepsilon}_t \hat{\varepsilon}_{t-j}' x'_{t-1-j} \right), \quad (12)$$

$\hat{\varepsilon}_t \equiv y_t - x'_{t-1} \hat{\beta}$ , and  $\hat{\beta}$  is the full-sample OLS estimator:

$$\hat{\beta} = \left( \frac{1}{T} \sum_{t=1}^{T-1} x_{t-1} x'_{t-1} \right)^{-1} \left( \frac{1}{T} \sum_{t=1}^{T-1} x_{t-1} y_t \right)^{-1}.$$

Under the null hypothesis of no Granger-causality ( $\beta = 0$ ),  $W_T$  is a chi-square distribution with  $p$  degrees of freedom. If there is no serial correlation in the data, only the first component in (12) is relevant.

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<sup>38</sup>The Granger-causality test described below is valid under the following assumptions: (i)  $\{y_t, x_t\}$  are stationary and ergodic, (ii)  $E(x_t x_t')$  is nonsingular, (iii)  $E(x_t \varepsilon_t) = 0$  and (iv)  $\{x_t \varepsilon_t\}$  satisfies Gordin's condition (p. 405, Hayashi, 2000) and its long-run variance is non-singular. Condition (iii) allows the data to be serially correlated, but rules out endogeneity. Rossi (2005b) relaxes these conditions.

**9.2. Rossi (2005b).** Rossi (2005b) shows that traditional Granger-causality tests above may fail in the presence of parameter instabilities. She therefore develops optimal tests for model selection between two nested models in the presence of underlying parameter instabilities in the data. The procedures are based on testing jointly the significance of additional variables that are present only under the largest model and their stability over time.<sup>39</sup> She is interested in testing whether the variable  $x_t$  has no predictive content for  $y_t$  in the situation where the parameter  $\beta_t$  might be time-varying. Among the various forms of instabilities that she considers, we focus on the case in which  $\beta_t$  may shift from  $\beta$  to  $\bar{\beta} \neq \beta$  at some unknown point in time.

The test is implemented as follows. Suppose the shift happens at a particular point in time  $\tau$ .

Let  $\widehat{\beta}_{1\tau}$  and  $\widehat{\beta}_{2\tau}$  denote the OLS estimators before and after the time of the shift:

$$\begin{aligned}\widehat{\beta}_{1\tau} &= \left( \frac{1}{\tau} \sum_{t=1}^{\tau-1} x_{t-1} x'_{t-1} \right)^{-1} \left( \frac{1}{\tau} \sum_{t=1}^{\tau-1} x_{t-1} y_t \right)^{-1}, \\ \widehat{\beta}_{2\tau} &= \left( \frac{1}{T-\tau} \sum_{t=\tau}^{T-1} x_{t-1} x'_{t-1} \right)^{-1} \left( \frac{1}{T-\tau} \sum_{t=\tau}^{T-1} x_{t-1} y_t \right)^{-1}.\end{aligned}$$

The test builds on two components:  $\frac{\tau}{T} \widehat{\beta}_{1\tau} + (1 - \frac{\tau}{T}) \widehat{\beta}_{2\tau}$  and  $\widehat{\beta}_{1\tau} - \widehat{\beta}_{2\tau}$ . The first is simply the full-sample estimate of the parameter,  $\frac{\tau}{T} \widehat{\beta}_{1\tau} + (1 - \frac{\tau}{T}) \widehat{\beta}_{2\tau} = \widehat{\beta}$ ; a test on whether this component is zero is able to detect situations in which the parameter is constant but different from zero. However, if the regressor Granger-causes the dependent variable in such a way that the parameter changes but the average of the estimates equals zero, then the first component would not be able to detect such situations. The second component is introduced to perform that task. It is the difference of

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<sup>39</sup>Rossi (2005b) considered the general case of testing possibly nonlinear restrictions in models estimated with General Method of Moments. Here, we provide a short description in the simple case of no Granger-causality restrictions in models whose parameters are consistently estimated with Ordinary Least Squares (OLS), like the Granger-causality regressions implemented in this paper. She also considers the case of tests on subsets of parameters, that is the case where  $y_t = x'_{t-1} \beta_t + z'_{t-1} \delta + \varepsilon_t$  and the researcher is interested in testing only whether  $x_t$  Granger-causes  $y_t$ .

the parameters estimated in the two sub-samples; a test on whether this component is zero is able to detect situations in which the parameter changes at time  $\tau$ . The test statistic is the following:

$$Exp - W_T^* = \frac{1}{T} \sum_{\tau=[0.15T]}^{[0.85T]} \frac{1}{0.7} \exp\left(\frac{1}{2}\right) \left( \begin{array}{cc} (\widehat{\beta}_{1\tau} - \widehat{\beta}_{2\tau})' & \left(\frac{\tau}{T}\widehat{\beta}_{1\tau} + (1 - \frac{\tau}{T})\widehat{\beta}_{2\tau}\right)' \end{array} \right) \widehat{V}^{-1} \left( \begin{array}{c} (\widehat{\beta}_{1\tau} - \widehat{\beta}_{2\tau}) \\ \left(\frac{\tau}{T}\widehat{\beta}_{1\tau} + (1 - \frac{\tau}{T})\widehat{\beta}_{2\tau}\right) \end{array} \right)$$

where  $\widehat{V} = \begin{pmatrix} \frac{\tau}{T}S'_{xx}\widehat{S}_1^{-1}S_{xx} & 0 \\ 0 & \frac{T-\tau}{T}S'_{xx}\widehat{S}_2^{-1}S_{xx} \end{pmatrix},$

$$\widehat{S}_1 = \left( \frac{1}{\tau} \sum_{t=2}^{\tau} x_{t-1}\widehat{\varepsilon}_t\widehat{\varepsilon}'_t x'_{t-1} \right) + \sum_{j=2}^{\tau-1} \left( 1 - \left| \frac{j}{\tau^{1/3}} \right| \right) \left( \frac{1}{\tau} \sum_{t=j+1}^{\tau} x_{t-1}\widehat{\varepsilon}_t\widehat{\varepsilon}'_{t-j} x'_{t-1-j} \right), \quad (13)$$

$$\widehat{S}_1 = \left( \frac{1}{T-\tau} \sum_{t=\tau+1}^{T-\tau} x_{t-1}\widehat{\varepsilon}_t\widehat{\varepsilon}'_t x'_{t-1} \right) + \sum_{j=\tau+1}^{T-\tau} \left( 1 - \left| \frac{j}{(T-\tau)^{1/3}} \right| \right) \left( \frac{1}{T-\tau} \sum_{t=j+1}^{T-\tau} x_{t-1}\widehat{\varepsilon}_t\widehat{\varepsilon}'_{t-j} x'_{t-1-j} \right). \quad (14)$$

Under the joint null hypothesis of no Granger-causality and no time-variation in the parameters ( $\beta_t = \beta = 0$ ),  $Exp - W_T^*$  has a distribution whose critical values are tabulated in Rossi's (2005b) Table B1. If there is no serial correlation in the data, only the first component in (13) and (14) is relevant.

**9.3. Tests of out-of-sample rolling MSFE comparisons.** To compare the out-of-sample forecasting ability of:

$$Model : y_t = x'_{t-1}\beta_t + \varepsilon_t \quad (15)$$

$$Random Walk : y_t = \varepsilon_t, \quad (16)$$

we generate a sequence of 1-step-ahead forecasts of  $y_{t+1}$  using a rolling out-of-sample procedure. The procedure involves dividing the sample of size  $T$  into an in-sample window of size  $m$  and an out-of-sample window of size  $n = T - m - \tau + 1$ . The in-sample window at time  $t$  contains observations indexed  $t - m + 1, \dots, t$ . We let  $f_t(\hat{\beta}_t)$  be the time- $t$  forecast for  $y_t$  produced by estimating the model over the in-sample window at time  $t$ , with  $\hat{\beta}_t = \left( \sum_{s=t-m+1}^{t-1} x_s x_s' \right)^{-1} \sum_{s=t-m+1}^{t-1} x_s y_{s+1}$  indicating the parameter estimate; we let  $f_t^{RW}$  denote the forecast of the random walk (that is,  $f_t^{RW} = 0$ ).

To compare the out-of-sample predictive ability of (15) and (16), Diebold and Mariano (1995), West (1996) suggest focusing on:

$$d_t \equiv \left( y_t - f_t(\hat{\beta}_t) \right)^2 - \left( y_t - f_t^{RW} \right)^2 \quad (17)$$

They show that the sample average of  $d_t$ , appropriately re-scaled, has an asymptotic standard Normal distribution. However, this is not the case when the models are nested, as in our case. Clark and McCracken's (2001) show that, under the null hypothesis that the model is (16), the tests of Diebold and Mariano (1995) and West (1996) do not have a Normal distribution. They propose a new statistic, ENCNEW, which is the following:

$$ENCNEW = n \frac{\left[ \frac{1}{n} \sum_{t=m+1}^T \left( \left( y_t - f_t(\hat{\beta}_t) \right)^2 - \left( y_t - f_t(\hat{\beta}_t) \right) \left( y_t - f_t^{RW} \right) \right) \right]}{\left[ \frac{1}{n} \sum_{t=m+1}^T \left( \left( y_t - f_t^{RW} \right)^2 - \frac{1}{n} \sum_{t=m+1}^T \left( y_t - f_t^{RW} \right)^2 \right)^2 \right]}$$

Its limiting distribution is non-standard, and critical values are provided in Clark and McCracken (2001). Clark and West (2006) propose a correction to (17) that results in an approximately normally distributed test statistic.