

# Understanding International Prices: Customers as Capital

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## Abstract

This paper develops a new theory of pricing-to-market driven by sluggish market-shares. Our key innovation is a capital theoretic model of marketing in which relations with the customers are valuable. We discipline the introduced friction using a unique prediction of the model about the low short-run and high long-run price elasticity of international trade flows, consistent with the data. The model accounts for several pricing implications that are puzzling for a large class of theories. The good performance on the quantities side is maintained.(JEL: F41, E32, F31.)

Standard international macroeconomic models, while being successful in accounting for the business cycle dynamics of quantities, have so far failed to account for the movements of international relative prices. In the data three patterns are evident. First, both real export prices<sup>1</sup> and real import prices are highly positively correlated, and both are positively correlated with the real exchange rates. Second, the terms of trade is much less volatile than the real exchange rates.<sup>2</sup> Third, there are large and persistent movements in the real exchange rates. These movements, often interpreted as deviations from the law of one price at the aggregate level, are mimicked by persistent deviations from the law of one price at more disaggregated levels.

Neither real business cycle models nor sticky price models have thus far been able to account for these patterns. In the standard real business cycle model, the real export price is

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<sup>1</sup>Nominal export prices evaluated relative to the domestic price level (measured by the consumer price index [CPI], the CPI for tradable goods, or the producer price index [PPI]).

<sup>2</sup>Consider the most recent real depreciation 2006–2008. The U.S. real effective exchange depreciated between January 2006 and January 2008 by 11%, whereas the terms of trade for manufactured goods increased by only 0.5%. Export and import price indices for manufactured articles both increased by 8.7% and 9.2%, respectively. (Price indices have been pulled out from BLS, and real exchange rate data from IMF IFS Online Database.)

negatively correlated with the real import price and the real exchange rate, the terms of trade is more volatile than the real exchange rate, and while real exchange rates are persistent, the law of one price holds at the disaggregated level. While sticky price models can, under certain assumptions, generate some of these features, they fail to generate anywhere near the persistence of real exchange rates observed in the data.<sup>3</sup>

Our reading of the evidence is that it suggests the presence of frictions that inhibit the flow of tradable goods between countries and break the law of one price. This departure is supported by the micro-level evidence suggesting that exporters are capable of segmenting the markets and price to the market in which they sell. Marston (1990), Knetter (1993), and Goldberg & Knetter (1997) provide evidence that when the real exchange rate depreciates, the price of exported goods systematically rises relative to the price of the similar goods sold at home, regardless how fine the level of disaggregation is. The literature has interpreted this result as evidence that markups on exports, measured relative to domestic costs, tend to systematically rise when the real exchange rate depreciates.

Motivated by the above evidence, our paper proposes a theory in which micro founded frictions result in endogenous market segmentation and deviations from the law of one price of the kind suggested by the literature. The key mechanism is that firms need to build market shares, and this process is costly and time consuming. That inhibits the price arbitrage through quantities traded and in the short-run makes real exchange rate fluctuations endogenously lead to pricing-to-market and varying markups on the exported goods. Quantitatively, due to pricing-to-market, our theoretical economy successfully accounts for the volatility of the terms of trade relative to the real exchange rate, and implies a positive correlation between the real export price, the real import price, and the real exchange rate. Business cycle behavior of quantities is on par with the standard IRBC theory.

The idea of sluggish market shares that we pursue here is not entirely new to economics. In fact, such frictions have been considered a promising avenue since at least the 1980s. Krugman (1986, p. 32), in his seminal contribution to the subject, states: *The best hope of understanding pricing to market seems to come from dynamic models of imperfect competition. At this point, my preferred explanation would stress the roles of [...] the costs of rapidly adjusting the marketing and distribution infrastructure needed to sell some imports,*

<sup>3</sup>See Chari, Kehoe & McGrattan (2002).

*and demand dynamics, resulting from the need of firms to invest in reputation.*

In addition, such frictions find strong support in the anecdotal evidence about international trade relations between firms and in the evidence on firms' market share growth after entry into a foreign market. The anecdotal evidence (H. Hakansson (1982), Turnbull & M. T. Cunningham (1981), and Egan & Mody (1992)), based on surveys with the CEOs, pervasively stresses the importance of long-lasting producer-supplier relationships, high switching costs to new suppliers, and the presence of highly individualized relationships. Evidence on firms' market share growth after entry into a foreign market (Ruhl & Willis (2008)) also supports the view that the buildup of market share takes time. Although dynamic frictions leading to pricing-to-market seemed an attractive avenue for a long time, due to tractability concerns, theoretical treatments of such frameworks are scant. Two notable exceptions are Froot & Klemperer (1989) and Alessandria (2004). To our best knowledge, our model is the first quantitative exploration of the effects of frictions of this type.

We build on the above general ideas, and develop a tractable international business cycle model of market share sluggishness with explicitly formulated micro foundations. In addition, to make our model quantitative, we propose a way to put discipline on the new features of the model by bringing in the data on the discrepancy between the low short-run and high long-run estimates of the price elasticity of trade flows. This discrepancy, well documented in the international trade literature, is often referred to as the *elasticity puzzle* (see Ruhl (2008)). In our framework, the elasticity puzzle is intimately related to the idea of market share sluggishness, which we exploit to calibrate the model and thereby assess its quantitative relevance. In its own right, this appears to be the first attempt to bring this evidence to terms with the Backus, Kehoe & Kydland (1995) strand of international business cycle literature.<sup>4</sup>

The structure of our model is as follows. First, international trade takes place only through matches between buyers (final good producers) and intermediate good producers. Second, intermediate good producers explicitly build their customer base by choosing spending on a broadly interpreted marketing (market research, design and customization of the product, distribution infrastructure, advertising, technical support). Marketing brings in new customers, and each producer, as a state variable, has an endogenous list of customers

<sup>4</sup>Other notable contributions to this topic in terms of business cycle models of a different kind are Ruhl (2008) and Ghironi & Melitz (2005).

to whom he can sell a finite quantity of the good. Because it takes time to bring more customers to this list, the producers face what we term a *market expansion friction*. Due to the bilateral monopoly problem that arises within each match, dock and wholesale prices are determined in the model by *bargaining*.

Market expansion friction and bargaining are the two key features that give rise to a different behavior of prices in our model. First, bargaining makes prices explicitly depend not only on the marginal cost of production, but also on the valuation of the local buyers (final good producers). In particular, export price explicitly depends on the foreign valuation of the domestic good measured in *domestic* consumption units. Second, market expansion friction makes the relative supply of domestic to foreign good in each country sluggish, and when combined with a high assumed elasticity of substitution between these goods, results in scant movements of the valuation (retail price) of the domestic good expressed in *local* consumption units. As a result, when the real exchange rate depreciates in our model, the foreign valuation of the domestic good expressed in the *domestic* consumption units goes up almost one-to-one with the real exchange rate, and goes up relative to the valuation of the same good by the domestic buyers. The extra surplus with the foreign buyers created by that is bargained over by the exporters, which leads to an increased markup on the exported good relative to the markup on the same good sold at home. Markup variability leads to a positive correlation of the real export prices with the real import price, and with the real exchange rate. In addition, just like in the data, fluctuations of the real exchange rate on the aggregate level are closely mimicked by the corresponding deviations from the law of one price on the disaggregated levels.

The main quantitative results of the paper are as follows: (i) relative volatility of the terms of trade to the real exchange rate as low as 26%, (ii) positive correlation of the real export and the real import price, (iii) positive correlation of these prices with the real exchange rate, (iv) low short-run price elasticity of trade flows, and (v) high long-run price elasticity.

**Related literature** Dynamic pricing-to-market models with frictions similar to ours are Krugman (1986) and Froot & Klemperer (1989). In light of these papers, our contribution is to propose a quantitative general equilibrium model in which such frictions endogenously arise from the underlying search and matching frictions. In addition, our paper shows that

this view has the potential to reconcile an international macro approach with static trade theory by accounting for the discrepancy between the measured price elasticities of trade.

The most recent quantitative literature on pricing-to-market includes the papers by Alessandria (2005), Atkeson & Burstein (forthcoming), and Corsetti et al. (2008). The key difference with our paper is that we explore a conceptually different dynamic friction, while these authors explore static market structures and static frictions. For example, in contrast to this literature, in our model permanent shocks do not have permanent effects on prices, and the law of one price is eventually restored. Given the magnitude of the deviations from the law of one price seen in the data, we believe that this property of our model is appealing, as it accords well with the conventional view that arbitrage forces eventually do restore some form of parity.<sup>5</sup>

## I. Three Puzzles for the Standard Model

Here, we set the quantitative goal for our theory by defining the discrepancy between the predictions of standard international macroeconomic model<sup>6</sup> and international price data. We use data for both disaggregated prices and aggregate prices. Our aggregate data is based on H-P-filtered<sup>7</sup> quarterly price data for the time period 1980 to 2005, and our sample includes the time series for the following countries: Belgium, Australia, Canada, France, Germany, Italy, Japan, the Netherlands, United Kingdom, United States, Sweden, and Switzerland. Our disaggregated data are based on the disaggregated producer and wholesale price data for Japan.

### A. Export-Import Price Correlation Puzzle

One of the central predictions of the standard theory for international relative price movements is that the price of the exported goods, evaluated relative to the overall home price level, moves in the opposite direction to the similarly constructed import price. Intuitively, this implication follows from the fact that, by the law of one price, export prices are tied

<sup>5</sup>As Rogoff (1996, p. 647) puts it: *While few empirically literate economists take PPP seriously as a short-term proposition, most instinctively believe in some variant of purchasing power parity as an anchor for long-run real exchange rates.*

<sup>6</sup>Backus et al. (1995). See Stockman & Tesar (1995) for a version of the standard model with non-tradable goods.

<sup>7</sup>Alternative detrending methods of the data, including the band-pass filter, do not change any of the results that follow.

to the prices of domestically-produced and domestically-sold goods, and import prices are tied to corresponding prices abroad, expressed in home units. As a result, whenever the real exchange rate depreciates,<sup>8</sup> import prices rise relative to home prices due to their direct link to the overall foreign price level, and export prices fall relative to home prices, as home prices additionally include the higher priced imports.

To show the above implication formally, we first derive it from a simple model without explicit distinction between tradable and non-tradable goods, and then generalize the results to a model that makes such distinction explicit.

In the standard model without non-tradable goods, the overall home price level measured by the CPI can be approximated by a trade-share-weighted geometric average of the prices of the tradable home good  $d$ , and the tradable foreign good  $f$  (the home-bias toward the local good  $d$  is parameterized by  $1/2 < \omega < 1$ )<sup>9</sup>. Given the formula for the CPI, the definitions of the real export price  $p_x$  and the real import price  $p_m$  of a country (deflated by CPI) are as follows:

$$p_x = \frac{P_d}{CPI} = \frac{P_d}{P_d^\omega P_f^{1-\omega}} = \left(\frac{P_d}{P_f}\right)^{(1-\omega)}, \quad p_m = \frac{P_f}{CPI} = \frac{P_f}{P_d^\omega P_f^{1-\omega}} = \left(\frac{P_f}{P_d}\right)^\omega. \quad (1)$$

From the above formulas, observe that, according to the model, the correlation between  $p_x$  and  $p_m$  must necessarily be negative for all admissible values of  $\omega$ .

To contrast this prediction with the data, we calculate export and import price indices from the import and export price deflators,<sup>10</sup> and then deflate these prices by the all-items CPI index to construct  $p_x$  and  $p_m$ , respectively.<sup>11</sup> Table 1 reports the results. As we can see, the correlations between real export and import prices are highly positive across all 12 OECD countries in our sample, and the values often exceed 0.9. We should also mention that these prices are also quite volatile. Their median volatility relative to the real exchange rate is 0.56 for the real export price and 0.83 for the real import price, respectively.

Next, we verify whether the above results are robust to an explicit distinction between

<sup>8</sup>An increase in the foreign overall price level relative to the overall home price level.

<sup>9</sup>The approximation is exact when the elasticity of substitution between domestic and foreign goods is one. However, unit elasticity is within the range of values commonly used in the literature, and small departures from unity do not matter quantitatively for what follows.

<sup>10</sup>Constructed from the time series for constant- and current-price import and export prices at the national level.

<sup>11</sup>Formal definitions are stated in the Appendix.

tradable and non-tradable goods. For this task, we use a more general constant elasticity of substitution (CES) aggregator,

$$CPI = (v(P_d^\omega P_f^{1-\omega})^{\frac{\mu-1}{\mu}} + (1-v)P_N^{\frac{\mu-1}{\mu}})^{\frac{\mu}{\mu-1}},$$

to have the flexibility of choosing low elasticity of substitution  $\mu$  between tradable and non-tradable goods. Elasticity values most commonly used in the quantitative literature are, in fact, typically significantly below unity.<sup>12</sup>

Straightforward algebraic manipulation applied to the definitions of  $p_x$  and  $p_m$  with the above formula for the CPI imply that, according to the model with non-tradable goods, the following two objects must be negatively correlated:

$$p_m^T \equiv \left[ \frac{1}{v} \left( \frac{P_f}{P} \right)^{\frac{1-\mu}{\mu}} - \frac{(1-v)}{v} \left( \frac{P_f}{P_N} \right)^{\frac{1-\mu}{\mu}} \right]^{\frac{\mu}{1-\mu}} = \left( \frac{P_f}{P_d} \right)^\omega, \quad (2)$$

$$p_x^T \equiv \left[ \frac{1}{v} \left( \frac{P_d}{P} \right)^{\frac{1-\mu}{\mu}} - \frac{(1-v)}{v} \left( \frac{P_d}{P_N} \right)^{\frac{1-\mu}{\mu}} \right]^{\frac{\mu}{1-\mu}} = \left( \frac{P_d}{P_f} \right)^{(1-\omega)}. \quad (3)$$

To contrast the above prediction of the model with the data, we approximate the price of non-tradable goods  $P_N$  by the CPI for housing and services, and similarly as before use all-items CPI to measure  $P$ , and export (import) price deflators to measure  $P_d$  ( $P_f$ ). To generate the time series for  $p_m^T$ ,  $p_x^T$ , we first detrend the time series for  $P_d/P$ ,  $P_d/P_N$  (same for  $P_f$ ) and normalize them so that they oscillate around unity. The parameters  $\mu$  and  $v$  are assumed to be in the range of estimates from the literature and are least favorable to positive correlation ( $v = .6$  is taken from Corsetti et al. (2008) and  $\mu = 0.44$  from Stockman & Tesar (1995)). The results are reported in the last three columns of Table 1. As one can see, the previously reported correlations remain almost intact. The reason behind this result is a high positive correlation and similar volatility of the two objects,  $P_d/P$  and  $P_d/P_N$  (same for  $P_f$ ), which are subtracted in the formula for  $p_x^T$ . The median correlation coefficient between them is as high as 0.98. Because  $1/v \approx 2$  and  $(1-v)/v \approx 1$ , not surprisingly the properties of the time series for  $p_x^T$  and  $p_m^T$  are similar to  $p_x$  and  $p_m$ . We conclude that non-tradable goods cannot account for the export-import price correlation puzzle.

<sup>12</sup>For example, Corsetti et al. (2008) follow Mendoza (1991) and use the elasticity of substitution between tradable and non-tradable goods equal to 0.76, but Stockman & Tesar (1995) report a value as low as 0.44. The share of non-tradable goods  $v$  in the consumer basket oscillates around 50–60%.

Table 1: Correlation of Real Export and Real Import Prices

Country	Correlation					
	$p_x, p_m$	$p_x, x$	$p_m, x$	$p_x^T, p_m^T$	$p_x^T, x$	$p_m^T, x$
Australia	0.57	0.45	0.95	n/a	n/a	n/a
Belgium	0.94	0.72	0.74	0.91	0.64	0.65
Canada	0.71	0.50	0.92	0.72	0.48	0.86
France	0.90	0.61	0.66	0.89	0.60	0.62
Germany	0.62	0.50	0.85	0.47	0.28	0.84
Italy	0.88	0.68	0.72	0.84	0.65	0.67
Japan	0.85	0.92	0.85	0.84	0.90	0.85
Netherlands	0.94	0.76	0.80	0.92	0.75	0.78
Sweden	0.89	0.60	0.74	n/a	n/a	n/a
Switzerland	0.60	0.51	0.83	0.51	0.44	0.86
UK	0.90	0.61	0.79	n/a	n/a	n/a
US	0.75	0.46	0.69	0.68	0.46	0.69
MEDIAN	0.87	0.61	0.80	0.84	0.60	0.78

Notes: Prices as defined in the data section. Statistics based on logged & H-P-filtered quarterly time series. Except for T series, which ends in year 2000, the series range from 1980:1 to 2004:2. Sources are listed in the Appendix.

## B. Terms of Trade Relative Volatility Puzzle

The second firm prediction of the standard theory is about the excess volatility of the terms of trade  $p = \frac{P_f}{P_d}$  (price of imports in terms of exports) relative to the real exchange  $x$ . In this respect, the standard theory predicts that the terms of trade should be exactly equal to the *PPI*-based real exchange rate,<sup>13</sup> and thus exactly as volatile. The reason is that, by the law of one price, the price index of exported goods is equal to the home producer price index and the price index of the imported goods is equal to the foreign country producer price index measured in the home numeraire units. In contrast, in the data export and import prices are highly positively correlated and the terms of trade—defined as their ratio—carries a significantly smaller volatility than the volatility of the CPI or PPI based real exchange rates. This property of the data is illustrated in Table 2.<sup>14</sup> We should note that the terms of trade puzzle is closely related to the export import price correlation puzzle, as by construction its volatility is dampened when these prices are positively rather than negatively correlated.

<sup>13</sup>The PPI-based real exchange rate is the foreign producer price index relative to the home producer price index, when both measured in common numeraire.

<sup>14</sup>When the import price data is cleaned from the influence of the highly volatile crude oil prices—which we do in the later sections for the US—the volatility of the terms of trade relative to the real exchange rate falls even below 1/3.



Table 2: Volatility of Terms of Trade Relative to Real Exchange Rate

Country	Volatility of $p$ relative to $x$ (in %)		
	Price index used to construct <sup>a</sup> $x$		
	CPI all-items	WPI or PPI	None (nominal)
Australia	0.51	0.54	0.60
Belgium	0.57	0.70	0.47
Canada	0.56	0.76	0.61
France	0.80	0.74	0.73
Germany	0.83	0.81	0.80
Italy	0.75	0.79	0.77
Japan	0.52	0.54	0.55
Netherlands	0.52	0.49	0.44
Sweden	0.21	0.21	0.37
Switzerland	0.71	0.68	0.67
UK	0.30	0.32	0.37
US	0.31	0.33	0.28
MEDIAN	0.54	0.61	0.57

Notes: We have constructed trade-weighted exchange rates using weights and bilateral exchange rates for the set of 11 fixed trading partners for each country. The trading partners included in the sample are the countries listed in this table. Statistics are computed from logged and H-P-filtered quarterly time-series for the time period 1980:1-2000.01 ( $\lambda = 1600$ ). Data sources are listed at the end of the paper.

<sup>a</sup>Definitions are stated in the Appendix.

### C. Pricing-to-Market Puzzle

In addition to the aggregate anomalies shown above, there is pervasive direct evidence that the law of one price is systematically violated between countries regardless of the level of disaggregation.<sup>15</sup> Here we document this feature of the data using as an example a sample of the disaggregated price data from the Japanese manufacturing industry.

Our dataset includes quarterly time series for producer/wholesale level price indices for 31 highly disaggregated and highly traded manufactured commodity classifications. For each commodity classification, we have information on the export price of this good when exported (export price EPI) and the when sold on the domestic market (domestic wholesale price DPI).<sup>16</sup>

<sup>15</sup>Our analysis here will be a reminiscent of the incomplete pass-through/pricing-to-market literature that documents related facts using regression analysis. For example, similar analysis to ours can be found in Marston (1990).

<sup>16</sup>Standard PPI or WPI [wholesale price index] series would mix in export prices or import prices, respectively. All these price indices come from the producer survey data and together account for 59% of the total value of Japanese exports and 18% of the total value of domestic shipments (as of year 2000). The complete list of commodity categories can be found in the technical appendix available online. Examples of commodities are: ball bearings, copying machines, silicon wafers, agricultural tractors, etc.

To emphasize the analogy to our aggregate analysis, we construct similar objects to the aggregate real export price indices considered before, but instead computed separately for each single commodity classification. More specifically, for each commodity  $i$ , we divide its export price index (EPI) by the overall Japanese CPI and use the identity relation

$$p_x^i \equiv \frac{EPI_i DPI_i}{DPI_i CPI} \quad (4)$$

to decompose the fluctuations of the real export price of each commodity into two distinct components: (i) the pricing-to-market term  $\frac{EPI_i}{DPI_i}$ —capturing the deviations of the export price of the given commodity from its corresponding home price—and (ii) the residual term  $\frac{DPI_i}{CPI}$ —capturing the deviations of the home price of commodity  $i$  from the overall consumer price index.

Before we discuss any results pertaining to the above decomposition, we should first note that the commodity-level prices  $p_x^i$  exhibit similar patterns as the aggregate data: the median relative volatility of  $p_x^i$  to the real exchange rate is as high as 88%, and the median correlation of  $p_x^i$  with the real exchange rate is as high as 0.82. With our decomposition at hand, we can now look what happens behind the scene.

**Variance driven by pricing-to-market** To measure the contribution of the volatility of each term to the overall volatility of the export price index, we use variance decomposition:

$$\text{median}_i \left( \frac{\text{var}(\frac{EPI_i}{DPI_i})}{\text{var}(\frac{EPI_i}{DPI_i}) + \text{var}(\frac{DPI_i}{CPI})} \right), \quad (5)$$

where  $\text{var}(\cdot)$  in the formula above refers to the logged and H-P-filtered quarterly time series (with smoothing parameter  $\lambda = 1600$ ). In our analysis, we omit the covariance terms, as the two terms actually covary negatively in the data. Clearly, under the law of one price, one should expect that the first term  $\frac{EPI_i}{DPI_i}$  be almost constant, and all the variation in the real export prices  $p_x^i$  come from the fluctuations of the residual term  $\frac{DPI_i}{CPI}$ . The data shows the opposite pattern. The pricing-to-market term  $\frac{EPI_i}{DPI_i}$  carries about 93% of the total volatility, and the residual term  $\frac{DPI_i}{CPI}$  carries only 7%.

**Pricing-to-market related to the real exchange rate** The data also leaves little ambiguity as to which term drives the high positive correlation of real export prices  $p_x^i$  with

the real exchange rate (median=0.82). The median correlation of  $\frac{EPI_i}{DPI_i}$  with the Japanese real exchange rate is as high as 0.84, and the median correlation of the residual term  $\frac{DPI_i}{CPI}$  is actually slightly negative (-0.15).

Our findings are that both aggregate and disaggregated data point to robust pricing patterns for which the standard theory fails to account. We next proceed with the presentation of the model.

## II. Model

The overall structure of the model is similar to Backus et al. (1995) model (BKK, hereafter). Time is discrete,  $t = 0, 1, 2, \dots, \infty$ , and there are two ex-ante symmetric countries labeled *domestic* and *foreign*. Each country is populated by identical and infinitely lived households which supply labor and physical capital, consume goods, trade assets, and accumulate physical capital. Tradable goods are country-specific:  $d$  is produced in the domestic country, and  $f$  in the foreign country. The only source of uncertainty in the economy are country-specific productivity shocks.

Goods are traded at two levels: *wholesale* and *retail*. At the wholesale level, producers of goods ( $d$  at home and  $f$  abroad) sell their respective good to *retailers* from each country.<sup>17</sup> International trade happens only at the wholesale level, and is subject to search and matching frictions. On the retail level, local retailers resell the goods they previously purchased from producers to the local households. For simplicity, retail trade is assumed perfectly competitive.

In terms of notation, we distinguish foreign country-related variables from the domestic ones using an asterisk. The history of shocks up to and including period  $t$  is denoted by  $s^t = (s_0, s_1, \dots, s_t)$ , where the initial realization  $s_0$ , as well as the time invariant probability measure  $\mu$  over the compact shock space  $S$  are assumed given. In the presentation of the model, whenever possible, we exploit symmetry of the two countries and present the model from the domestic country's perspective only.

<sup>17</sup>Retailers should not be interpreted literally as the retail sector. The label is introduced to clearly distinguish the two sides of matching. By retailers we actually mean all other producers who participate in the overall production process – in particular, the retail sector. Distribution of the value added across different types of producers is not critical for the results.

## A. Uncertainty and Production

Each country is assumed to have access to a constant returns to scale production function  $zF(k, l)$  that uses country-specific capital  $k$  and labor  $l$ , and is subjected to a country-specific stochastic technology  $\hat{z} \equiv \log(z)$  following an exogenous AR(1) process

$$\hat{z}(s^t) = \psi \hat{z}(s^{t-1}) + \varepsilon_t, \quad \hat{z}^*(s^t) = \psi \hat{z}^*(s^{t-1}) + \varepsilon_t^*, \quad (6)$$

where  $0 < \psi < 1$  is a common persistence parameter, and  $s_t \equiv (\varepsilon_t, \varepsilon_t^*) \in S$  is an i.i.d. normally distributed random variable with zero mean.

Since the production function is assumed to be constant returns to scale, we summarize the production process by an economy-wide marginal cost  $v$ . Given domestic factor prices  $w, r$  and domestic shock  $z$ , the marginal cost, equal to per unit cost, is given by the following minimization problem:

$$v(s^t) \equiv \min_{k, l} \{w(s^t) l + r(s^t) k \text{ subject to } z(s^t) F(k, l) = 1\}. \quad (7)$$

## B. Households

The problem of the household is standard and identical to a decentralized version of the standard model under complete asset markets.

Each country is populated by a unit measure of identical and infinitely lived households. Households supply production factors to domestic producers, accumulate physical capital, and consume goods. After each history  $s^t$ , the stand-in household chooses the allocation, which consists of the level of consumption  $c$ , investment in physical capital  $i$ , labor supply  $l$ , purchases of tradable goods  $d, f$ , and purchases of a set of one-period  $s_{t+1}$ -contingent bonds  $b(s_{t+1}|s^t)$  to maximize the expected discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \int_{S^t} u(c(s^t), l(s^t)) \mu(ds^t). \quad (8)$$

The preferences over domestic and foreign goods are modeled by the Armington aggregator  $G(d, f)$  with an assumed exogenous elasticity of substitution (Armington elasticity)  $\gamma$ , and

an assumed home-bias parameter  $\omega$ ,

$$G(d, f) = \left( \omega d^{\frac{\gamma-1}{\gamma}} + (1-\omega) f^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}, \quad \gamma \geq 0, \quad \omega > 1/2. \quad (9)$$

Households combine goods  $d$  and  $f$  through the above aggregator into a composite good which they use for consumption and investment into physical capital stock  $k$ , which follows a standard law of motion with constant depreciation rate  $\delta$

$$c(s^t) + i(s^t) = G(d(s^t), f(s^t)), \quad (10)$$

$$k(s^t) = (1-\delta)k(s^{t-1}) + i(s^t), \quad 0 < \delta \leq 1. \quad (11)$$

Asset markets are complete, and the budget constraints of the domestic and foreign households are given by

$$\begin{aligned} & P_d(s^t) d(s^t) + P_f(s^t) f(s^t) + \int_S Q(s_{t+1}|s^t) b(s_{t+1}|s^t) \mu(ds_{t+1}) \\ = & b(s^t) + w(s^t) l(s^t) + r(s^t) k(s^{t-1}) + \Pi(s^t), \quad \text{all } s^t. \end{aligned} \quad (12)$$

$$\begin{aligned} & P_d^*(s^t) d^*(s^t) + P_f^*(s^t) f^*(s^t) + \int_S \frac{x(s^{t+1})}{x(s^t)} Q(s_{t+1}|s^t) b^*(s_{t+1}|s^t) \mu(ds_{t+1}) \\ = & b^*(s^t) + w^*(s^t) l^*(s^t) + r^*(s^t) k^*(s^{t-1}) + \Pi^*(s^t), \quad \text{all } s^t. \end{aligned} \quad (13)$$

In the above formulation of the budget constraints, we assume that the composite consumption good of each country is the numeraire. We do so by normalizing the level of prices in each country so that the resulting ideal CPI price indexes in each country are equal to unity.<sup>18</sup>

The expenditure side of the budget constraints (12) and (13) consist of purchases of domestic and foreign goods and purchases of one-period-forward  $s_{t+1}$ -state contingent bonds. The income side consists of income from maturing bonds purchased at history  $s^{t-1}$ , labor income, rental income from physical capital, and the dividends paid out by home firms. The foreign budget constraint, due to a different numeraire unit, additionally involves a price  $x(s^t)$  that translates the foreign numeraire to the domestic numeraire in the bond purchases term. By definition of the numeraire unit in each country, this price is the real

<sup>18</sup>The ideal-CPI is defined by the lowest cost of acquiring a unit of composite consumption ( $c$  in the domestic country,  $c^*$  in the foreign country)

exchange rate<sup>19</sup>, which integrates the domestic and the foreign asset market into one world asset market.<sup>20</sup>

Summarizing, given the initial values for  $k(s^{-1})$  and  $b(s^{-1}) = 0$ , households choose their allocations to maximize (8) subject to the aggregation constraint (10), the law of motion for physical capital (11), the budget constraint (12), the standard no-Ponzi scheme condition, and the numeraire normalization. In further analysis, we will use the first order conditions that give rise to the demand equations and the foreign and domestic pricing kernels:

$$P_d(s^t) = G_d(s^t), \quad P_f(s^t) = G_f(s^t), \quad (14)$$

$$Q(s_{t+1}|s^t) = \beta \frac{u_c(s^{t+1})}{u_c(s^t)}, \quad (15)$$

$$\frac{x(s^{t+1})}{x(s^t)} Q(s_{t+1}|s^t) = \beta \frac{u_c^*(s^{t+1})}{u_c^*(s^t)},$$

where  $u_l(s^t)$ ,  $u_c(s^t)$ ,  $G_d(s^t)$ ,  $G_f(s^t)$  denote derivatives of the instantaneous utility function and the Armington aggregator function with respect to the subscript arguments.

Iterating backward up to state  $s^0$  on (15) for the domestic and foreign country, we can derive under ex-ante symmetry between countries the real exchange rate,

$$x(s^t) = \frac{u_c^*(s^t)}{u_c(s^t)}. \quad (16)$$

The above equation is called the *efficient risk sharing condition*. It says that a country consumes more (or more precisely, has lower marginal utility of consumption) in a given state and date if and only if its consumption costs less in that state and date.<sup>21</sup>

### C. Producers

Tradable goods  $d$  and  $f$  are country specific and are produced by a unit measure of atomless competitive producers residing in each country. Producers employ local capital and

<sup>19</sup>In the data real exchange rate is measured using fixed-weight CPI rather than ideal CPI indices. Quantitatively, this distinction turns out not to matter in this particular class of models.

<sup>20</sup>Since the foreign budget constraint is expressed in the foreign country numeraire, and so is  $b^*$ , in order to use  $Q$  as the intertemporal price, the term  $x(s^{t+1})b^*(s_{t+1}|s^t)$  first translates the purchase value of the foreign bonds to the domestic country numeraire units, and then  $Q(s_{t+1}|s^t)/x(s^t)$  expresses the price of this purchase again in terms of the foreign numeraire.

<sup>21</sup>This condition is known to imply counterfactual connection of real exchange rate to quantities. Later we will examine robustness of our results to this prediction by considering an alternative modeling assumption.

labor to produce these goods using the technology specific to their country of residence. The unit production cost they face is given by (7).

The novel feature introduced in this paper is that producers need to first match with the retailers in order to sell their goods. Matching is costly and time consuming, and trade involves bargaining. In the next section, we first describe the details of matching and then state the profit maximization problem of the producers. We provide a formal treatment of the bargaining problem in a later section, as it is not essential to define the producer problem.

**List of customers and market shares** To match with retailers, the producers have access to an explicitly formulated *marketing technology* and accumulate a form of capital labeled *marketing capital*,  $m$ . Marketing capital is accumulated separately in each country a producer sells in, and the marketing capital a producer holds in each country relative to other producers, determines the contact probabilities with the searching retailers. For example, an exporter from the domestic country with marketing capital  $m_d^*(s^t)$  in the foreign country attracts a fraction

$$\frac{m_d^*(s^t)}{\bar{m}_d^*(s^t) + \bar{m}_f^*(s^t)} \quad (17)$$

of the searching retailers from the foreign country, where  $\bar{m}_f^*(s^t)$  and  $\bar{m}_d^*(s^t)$  denote the average levels of marketing capital an  $f$  and  $d$  good producer holds in that country. Retailers who join the customer list of this producer,  $H(s^t)$ , will stay on the list until the match is dissolved with exogenous probability  $\delta_H$ .

Formally, given the measure  $h(s^t)$  of searching retailers in a given country, who are *potential customers*, the arrival of *new customers* to the customer list of a given producer is given by

$$\frac{m_d(s^t)}{\bar{m}_d(s^t) + \bar{m}_f(s^t)} h(s^t). \quad (18)$$

We assume that each match with a retailer is long-lasting and is subject to an exogenous destruction rate  $\delta_H$ , and thus the evolution of the endogenous list of customers  $H_d(s^t)$  is described by the following law of motion:

$$H_d(s^t) = (1 - \delta_H)H_d(s^{t-1}) + \frac{m_d(s^t)}{\bar{m}_d(s^t) + \bar{m}_f(s^t)} h(s^t). \quad (19)$$

The size of this list is critical for the producer, as it determines the amount of goods this producer can sell in a given market (country). Specifically, we assume here that in each match, one unit of the good can be traded per period—to reflect the fact that each match is somewhat specific to a particular task at hand.<sup>22</sup> Thus, sales of a given producer cannot exceed the size of the customer list  $H$ . For example, the sales constraint of a producer of good  $d$  in the foreign country with a customer list  $H_d^*(s^t)$  would be given by<sup>23</sup>

$$d^*(s^t) \leq H_d^*(s^t). \quad (20)$$

**Marketing capital** Producers in the model accumulate marketing capital  $m$  to attract searching retailers. Given last period’s level of marketing capital  $m_d(s^{t-1})$  and the current level of instantaneous marketing input  $a_d(s^t)$ , current period marketing capital  $m_d(s^t)$  is given by

$$m_d(s^t) = (1 - \delta_m) m_d(s^{t-1}) + a_d(s^t) - \phi m_d(s^{t-1}) \left( \frac{a_d(s^t)}{m_d(s^{t-1})} - \delta_m \right)^2. \quad (21)$$

The above specification nests two key features: (i) the decreasing returns from the instantaneous marketing input  $a_d(s^t)$  and (ii) the capital-theoretic specification of marketing. These features, parameterized by the *market expansion friction parameter*  $\phi$  and depreciation rate  $\delta_m$ , are intended to capture the idea that marketing-related assets like brand awareness, reputation or distribution network are capital for a firm and the buildup of these assets takes time. As we will later show, this feature gives rise to the disconnect between the short-run and the long-run price elasticity of trade flows and will be critical for the dynamics of export and import prices. We will refer to this feature as a *market expansion friction*.

**Profit maximization** Producers sell goods in the domestic country for the wholesale prices  $p_d$  and in the foreign country for the wholesale export price  $p_x \equiv xp_d^*$  when measured in domestic numeraire. These prices are determined by bargaining with the domestic and

<sup>22</sup>One interpretation could be that each match allows to bring in a different good, and there is Dixit-Stiglitz aggregator on the retail level. In such case, the implied capacity constraint would be continuous rather than discrete (0-1). We can conjecture that the results of the paper would not differ much as long as this capacity constraint would be tight enough—looser/tighter capacity constraints would work similarly to a lower/high value of  $\phi$ . We therefore omit such considerations from the paper.

<sup>23</sup>Due to always positive markups, this condition *always* binds on the simulation path.



foreign retailers. However, to set up the profit maximization of the producers, we can abstract from bargaining at this stage and assume that the prices are taken as given. This is because the producer can perfectly anticipate the outcome of bargaining at every contingency, and cannot strategically influence it beforehand by making a different choice —as we will see later, neither the state variables nor decision variables chosen in the problem below affect the outcome of bargaining.<sup>24</sup>

The instantaneous profit function  $\Pi$  of the producer is determined by the difference between the profit from sales in each market and the total cost of marketing, and is given by:

$$\Pi = (p_d - v)d + (xp_d^* - v)d^* - va_d - xv^*a_d^*. \quad (22)$$

Given the instantaneous profit function  $\Pi$ , our representative producer from the domestic country, who enters period  $t$  in state  $s^t$  with the customer list  $H_d(s^{t-1}), H_d^*(s^{t-1})$  and marketing capital  $m_d(s^{t-1}), m_d^*(s^{t-1})$ , chooses the allocation  $a_d(s^t), a_d^*(s^t), m_d(s^t), m_d^*(s^t), d(s^t), d^*(s^t), H_d(s^t), H_d^*(s^t)$ , to maximize the present discounted stream of future profits given by

$$\max \sum_{\tau=t}^{\infty} \int Q(s^\tau) \Pi(s^\tau) \mu(ds^\tau | s^t), \quad (23)$$

subject to the marketing technology constraints (21), sales constraints (20), and the laws of motion for customer lists (19). The discount factor  $Q(s^t)$  is defined by the recursion on the conditional pricing kernel (15) of the form

$$Q(s^t) = Q(s_t | s^{t-1}) Q(s^{t-1}).$$

## D. Retailers

In each country there is a sector of atomless retailers who purchase goods from producers and resell them in a local competitive market to households. It is assumed that the new retailers who enter into the market must incur the initial search cost  $\chi v$  in order to find a producer with whom they can match and trade. Each match lasts until it exogenously dissolves with a per-period probability  $\delta_H$ . As long as the match lasts, the producer and the

<sup>24</sup>This property follows from the 3 key assumptions of the model: (i) production, marketing and search are all constant returns to scale activities, (ii) search by retailers is subject to zero profit condition (free entry upon paying search cost), (iii) expensed search cost and marketing cost cannot be retrieved by breaking a match (sunk cost of matching).

retailer have an option to trade one unit of the good per period. In equilibrium, the industry dynamics is governed by a free entry and exit condition, which endogenously determines the measure  $h$  of new entrants (searching retailers). Trade between households and retailers takes place in a local competitive market at prices  $P_d$  for good  $d$  and  $P_f$  for good  $f$ . In equilibrium, these prices are given by (14), and throughout the rest of this paper we refer to them as *retail prices* (in contrast to the *wholesale prices*  $p_d, p_f$ ).

In each period, there is a mass of retailers already matched with the producers  $H$  and a mass of new entrants  $h$  (searching retailers). A new entrant, upon paying the up-front search cost  $\chi v$ , meets with probability  $\pi$  a producer from the domestic country and with probability  $1 - \pi$  the producer from the foreign country (selling in the domestic country). The entrant takes this probability as given, but in equilibrium it is determined by the marketing capital levels accumulated by the producers, according to

$$\pi(s^t) = \frac{\bar{m}_d(s^t)}{\bar{m}_d(s^t) + \bar{m}_f(s^t)}. \quad (24)$$

The measures of already matched retailers  $H$  endogenously evolve in each country in consistency with (19).

As we can see from the above formulation of matching probabilities, search by the retailer is guided by the marketing capital accumulated by the producers. This is the essence of our departure from the frictionless competitive market model.

We next proceed with the discussion of the bargaining problem between the producer and the retailer and then set up the zero profit condition governing the entry of new retailers  $h$ .

**Bargaining and wholesale prices** We assume that each retailer bargains with the producer over the total future surplus from a given match. This surplus is split in consistency with Nash bargaining solution with continual renegotiation. Nash bargaining as a surplus splitting rule is an important assumption for our results, but certainly not the only modeling option. However, any departures from this setup can be mapped onto a time-varying Nash bargaining power, and as long as its variation is independent from exchange rate movements or limited in size, our results will remain unchanged.

To set the stage for the bargaining problem, we first need to define the value functions

from the match for the producer and for the retailer. We assume that they trade at history  $s^t$  at some arbitrary wholesale price  $p$ , and in the future they will trade according to an equilibrium price schedule  $p(s^t)$ . The value functions are

$$W_f(p; s^t) = \max\{0, p - x(s^t)v^*(s^t)\} + (1 - \delta_h) E_t Q(s_{t+1}|s^t) W_f(p_f(s^{t+1}); s^{t+1}) \quad (25)$$

$$J_f(p; s^t) = \max\{0, P_f(s^t) - p\} + (1 - \delta_h) E_t Q(s_{t+1}|s^t) J_f(p_f(s^{t+1}); s^{t+1}), \quad (26)$$

where  $W_f$  is the value of the foreign producer selling in the domestic country (importer) and  $J_f$  is the value for the domestic retailer matched with a foreign producer. All values are in domestic country units.

The flow part of the above Bellman equation for the producer is determined by the difference between the wholesale price of the good,  $p$ , and the production cost,  $xv^*$ , whereas for the retailer, it is determined by the difference between the retail price (resell price) of the good  $P_f$  and the wholesale price paid to the producer  $p$ .

With the values from a match at hand, we are now ready to set up the Nash bargaining problem, which imposes the following restriction on the equilibrium schedule of the wholesale prices<sup>25</sup>  $p(s^t)$ , given bargaining power  $\theta$ ,

$$p_f(s^t) \in \arg \max_p \{J_f(p; s^t)^\theta W_f(p; s^t)^{1-\theta}\}, \text{ all } s^t. \quad (27)$$

In the above formulation, the threat-points of both sides are zero, which follows from three assumptions: (i) search cost and marketing cost cannot be retrieved by breaking the match, (ii) there is free entry and exit to retail sector (zero profit condition), (iii) production, marketing and search are all constant returns to scale activities.

The following proposition establishes that with continual renegotiation at every date and state  $s^t$ , the pricing formulas resulting from (27) simply allocate  $\theta$  fraction of the total instantaneous (static) trade surplus given by  $P_f - xv^*$  to the producer and fraction  $1 - \theta$  to the retailer.

**PROPOSITION 1:** *Assume that trade takes place at  $s^t$ . The solution to the bargaining*

<sup>25</sup>Other prices are defined by analogy.

problem stated in (27) is given by

$$p_f(s^t) = \theta P_f(s^t) + (1 - \theta)x(s^t)v^*(s^t). \quad (28)$$

*Outline of Proof.* Let  $S$  be the total discounted surplus from a match. The key to the proof is that at each date and state, the producer and the retailer get a constant fraction of total surplus, i.e.  $W = \theta S$  and  $J = (1 - \theta)S$ . Using  $W = \theta S$  and (25) and subtracting sides gives (28). For a formal statement of proof, see the Appendix.

The intuition behind this result is as follows. Given the continual renegotiation of the price, Nash bargaining implies that in every period the total present discounted value from the match is split in proportion  $\theta, 1 - \theta$  between the producer and the retailer. In particular, from today on this is the case and, for any contingency, from tomorrow on as well. It is impossible to split the surplus from tomorrow onward in any other proportion, and therefore the *static* surplus today must be split in that proportion as well. Since this reasoning holds for all dates and states, the proposition follows.

**Free entry and exit condition** We are now ready to formulate the equilibrium free entry and exit condition governing the measures of searching retailers in each country  $h$ . This condition requires that the expected profit from entry covers the up-front search cost given

$$\pi(s^t)J_d(p_d(s^t); s^t) + (1 - \pi(s^t))J_f(p_d(s^t); s^t) \leq \chi v_i(s^t), \quad \text{with '=' if } h > 0. \quad (29)$$

The left-hand side of the above equation is the expected surplus for the retailer from matching with a producer from the domestic or the foreign country, respectively, and the right-hand side is the search cost incurred to identify such opportunity.

## E. Feasibility and Market Clearing

Equilibrium must satisfy several market clearing conditions and feasibility constraints. The aggregate resource constraint is given by

$$d(s^t) + d^*(s^t) + \sum_{i=d,f} a_i(s^t) + h(s^t)\chi = z(s^t)F(k(s^{t-1}), l(s^t)), \quad \text{all } s^t. \quad (30)$$

It says that the total production in the domestic country  $zF(k, l)$  must be equal to the amount of goods sold in the domestic market  $d(s^t)$ , exported to the foreign country  $d^*(s^t)$ , used in marketing by domestic and foreign producers, and finally, in the distribution of goods at home  $h(s^t)\chi$  (search cost).

Representativeness assumption imposed on equilibrium allocation implies that the average marketing capital is determined by the choices of the representative producer:

$$m_f(s^t) = \bar{m}_f(s^t), \quad m_d(s^t) = \bar{m}_d(s^t), \quad \text{all } s^t. \quad (31)$$

Finally, the contact probability  $\pi(s^t)$  is consistent with the average relative marketing capital accumulated by the producers of each type

$$\pi(s^t) = \frac{\bar{m}_d(s^t)}{\bar{m}_d(s^t) + \bar{m}_f(s^t)}, \quad \text{all } s^t, \quad (32)$$

and the world asset market clears

$$b(s^t) + x(s^t) b^*(s^t) = 0, \quad \text{all } s^t. \quad (33)$$

The formal definition of equilibrium is standard and therefore omitted.

### III. Parameterization

In this section, we describe how we choose functional forms and parameter values. The two key parameters in our model are the elasticity of substitution  $\gamma$  and the marketing friction parameter  $\phi$ . We first describe the data targets we use for these two parameters and then proceed with the description of the remaining targets and parameters.

#### A. The Elasticity of Substitution $\gamma$ and the Marketing Friction $\phi$

To choose the elasticity of substitution and the marketing friction parameter, we use the fact that our model has different predictions for the long-run and the short-run response of imports to the relative price fluctuations. Evidence of a similar discrepancy has been documented for the data and is termed the *elasticity puzzle* in the literature.<sup>26</sup> Below we

<sup>26</sup>See, for example, Ruhl (2008) for a detailed discussion of this puzzle and an overview of the literature.

show how we use long-run and short-run measurements to set calibration targets for these two key parameters.

**Long-run measurement** In our model, when the adjustments of quantities are extended in time, it can be shown that the response of the import ratio  $\frac{f}{d}$  to the relative price of the domestic good  $d$  to the foreign good  $f$  is equal to the elasticity  $\gamma$ . That is, just as in the frictionless model, we have

$$\Delta \log \frac{f}{d} \approx -\gamma \Delta T, \quad (34)$$

where  $\Delta T$  denotes the underlying change in the tariff rate measured in percentage points.<sup>27</sup>

Intuitively, the formula says that in the long-run the market expansion friction is slack, and thus the response of trade to tariff change depends solely on the intrinsic elasticity of substitution between the domestic and the foreign goods. In terms of the estimates of the intrinsic elasticity of substitution in the data, the estimates in the literature range from 6 to about 16. Here we adopt a middle-of-the-pack number of 7.9, reported by Head & Ries (2001).<sup>28</sup>

**Short-run measurement** Over the business cycle, the long-run adjustment of trade flows in response to prices is dampened in our model. This is because in the short-run the market expansion friction limits the instantaneous response of quantities to price fluctuations. Since a similar discrepancy has been identified in the data and our model can replicate it, we use it to quantitatively discipline the value of the market expansion friction parameter  $\phi$ .

To this end, we use our own measurement of the short-run elasticity estimated from the aggregate time series. Specifically, we compute the business cycle volatility of the ratio of imports to domestic absorption of domestic good ( $\approx \frac{f}{d}$  in the model) relative to the volatility of the ratio of the underlying price deflators ( $\approx \frac{p_d}{p_f}$  in the model). We label the ratio of these volatilities the *volatility ratio*<sup>29</sup> and compute it for a cross-section of 16 major

<sup>27</sup>We derive this equation in the technical appendix available online.

<sup>28</sup>Other long-run oriented studies give similar estimates. See, for example, Hummels (2001) or Eaton & Kortum (2002).

<sup>29</sup>To construct the *volatility ratio*, we use series on constant and current price values of imports and domestic absorption, where domestic absorption of domestic good is defined by the sum of domestic expenditures less imports,  $DA = (C + G) + I - IM$ . We next identify the corresponding prices of imports and domestic absorption with their corresponding price deflators (deflators are defined as the ratio of current to constant price values). Denoting the deflator price of domestic absorption of d-good by  $P_{DA}$  and the deflator price of imports by  $P_{IM}$ , the *volatility ratio* is then defined as  $\sigma(\frac{IM}{DA})/\sigma(\frac{P_{DA}}{P_{IM}})$ , where  $\sigma$  refers to the standard

OECD countries.

This methodology of measuring short-run elasticity is motivated by the fact that in a large class of models, the demand for domestic and foreign good is modeled by a CES aggregator (9). In such case, it is straightforward to show that the import ratio is tied to the relative price of domestic and imported goods by

$$\log \frac{f_t}{d_t} = \gamma \log \frac{p_{d,t}}{p_{f,t}} + \log \frac{\omega_t}{1 - \omega_t}. \quad (35)$$

Under normal conditions, i.e., when the supply curve is an upward-sloping function of the price and the supply shocks are uncorrelated with the  $\omega_t$ -demand shocks, we should expect the correlation between  $\log \frac{\omega_t}{1 - \omega_t}$  and  $\log \frac{p_{d,t}}{p_{f,t}}$  to be positive. Then, the *volatility ratio* defined by

$$VR \equiv \sigma(\log \frac{f_t}{d_t}) / \sigma(\log \frac{p_{d,t}}{p_{f,t}}) \quad (36)$$

places an upper bound on the value of the intrinsic price elasticity of trade flows  $\gamma$ , as implied by the following evaluation of (35):

$$\gamma = \sigma(\log \frac{f_t}{d_t}) / \sigma(\log \frac{p_{d,t}}{p_{f,t}} + \frac{1}{\gamma} \log \frac{\omega_t}{1 - \omega_t}) \leq \sigma(\log \frac{f_t}{d_t}) / \sigma(\log \frac{p_{d,t}}{p_{f,t}}) = VR. \quad (37)$$

It will later become clear that for our purposes the upper bound estimate is sufficient. The main results of the paper are only reinforced when lower values of the VR ratio are targeted in calibration.

The computed values of the volatility ratio, shown in Table 3, confirm the low values of the short-run price elasticity of trade flows typically found in the literature.<sup>30</sup> At business cycle frequencies, the median value of the volatility ratio is as low as 0.7 for both H-P-filtered and linearly detrended data. In the model, we use this value as a target for the market expansion parameter  $\phi$ , which, as we describe below, is determined jointly with other parameters.

deviation of the logged and H-P-filtered quarterly time series. Note that our *volatility ratio* places an upper bound on the regression coefficient between the two variables underlying its construction. The regression coefficient, typically used in short-run studies, is the *volatility ratio* rescaled by the correlation coefficient ( $\text{reg}(x, y) = \frac{\sigma_y}{\sigma_x} \rho_{x,y}$ ,  $\rho_{x,y} \in [-1, 1]$ ).

<sup>30</sup>E.g., Blonigen & Wilson (1999) or Reinert & Roland-Holst (1992). In contrast to our approach, this literature uses disaggregated data and regression analysis.

Table 3: Volatility Ratio in a Cross-Section of Countries

Country	Detrending method	
	HP-1600	Linear <sup>a</sup>
Australia	0.94	0.93
Belgium	0.57	0.50
Canada	1.27	0.64
France	0.54	0.73
Germany	0.90	1.16
Italy	0.69	0.46
Japan	0.60	0.43
Netherlands	0.44	0.72
Switzerland	0.71	1.16
Sweden	0.95	0.95
UK	0.65	0.61
US <sup>b</sup>	1.23	1.02
MEDIAN	0.71	0.73

Notes: Based on quarterly time-series, 1980 : 1 – 2000 : 1. Data sources are listed at the end of the paper.

<sup>a</sup>Linear trend subtracted from logged time series.

<sup>b</sup>For the entire postwar period (1959 : 3 – 2004 : 2) this ratio in U.S. is 0.88.

## B. Choice of Parameter Values and Functional Forms

Here, we describe in detail how we choose the functional forms and benchmark parameter values. We report our choices in Table 4.

We assume a constant relative risk aversion (CRRA) utility function and a Cobb-Douglas production function,

$$u(c, l) = \frac{(c^\eta(1-l)^{1-\eta})^{1-\sigma}}{1-\sigma}, \sigma > 0, 0 \leq \eta \leq 1, \quad (38)$$

$$F(k, l) = k^\alpha l^{1-\alpha}. \quad (39)$$

Consider first the parameters that can be selected independently from all other parameters by targeting a single moment from the data. This group includes: (i) the discount factor  $\beta$ , (ii) capital share parameter  $\alpha$ , (iii) depreciation rate of physical capital  $\delta$ , and (iv) Armington elasticity  $\gamma$ . We choose standard value of  $\beta$  to give the average annual risk-free real interest rate of 4%, and a standard value of  $\alpha$  to match the constant share of labor income in GDP of 64%. We follow BKK and choose the value of  $\delta$  to target the investment to GDP ratio of 25%.<sup>31</sup> Following the business cycle literature, we choose the value of  $\sigma$

<sup>31</sup>In the updated data we find a slightly smaller ratio. For example, 20% in the United States, 28% in



equal to 2. Finally, as explained in the previous subsection, we choose the value of  $\gamma$  equal to 7.9. The parameter  $\delta_H$  is arbitrarily chosen to be equal to 0.1 —implying that the matches in the economy last on average 2.5 years (10 quarters). In the technical appendix we present sensitivity analysis that shows that this parameter, except for increasing the persistence of prices, has a negligible effect on the results.

Table 4: Parameter Values in the Model Economies

Model	Parameter Values
Benchmark Model	Preferences and technology: $\beta = 0.99$ , $\sigma = 2.0$ , $\eta = 0.3436$ , $\gamma = 7.9$ , $\omega = 0.5581$ , $\alpha = 0.36$ , $\delta = 0.025$ , Marketing friction: $\theta = 0.40$ , $\chi = 1.38$ , $\delta_m = 0.2$ , $\delta_h = 0.1$ , $\phi = 18.4$ , Productivity process: $\psi = 0.735$ , $var(\epsilon) = 0.0000835$ , $corr(\epsilon, \epsilon^*) = 0.2$
BKK (Standard Model)	Preferences: $\gamma = 0.7$ , $\eta = 0.3385$ , $\omega = 0.945$ , Productivity process: $\psi = 0.91$ , $var(\epsilon) = 0.000037$ , $corr(\epsilon, \epsilon^*) = 0.28$ ; rest as in Benchmark.
Benchmark: Financial Autarky	Preferences: $\omega = 0.5565$ , Marketing friction: $\theta = 0.33$ , $\chi = 1.9$ , $\delta_m = 0.32$ , $\phi = 0.17$ , Productivity process: $\psi = 0.84$ , $var(\epsilon) = 0.00008$ , $corr(\epsilon, \epsilon^*) = 0.4$
BKK with Adjustment Cost	Preferences and technology: $\gamma = 7.9$ , $\eta = 0.3385$ , $\omega = 0.5625$ , $v = 50.0$ , Productivity process: $\psi = 0.91$ , $var(\epsilon) = 0.000037$ , $corr(\epsilon, \epsilon^*) = 0.28$ ; rest as in Benchmark.

The remaining parameters need to be jointly determined because there is no one-to-one mapping between their values and moments in the data. This group includes: (i) the marketing friction parameter  $\phi$ , (ii) the up-front search cost  $\chi$ , (iii) the bargaining power  $\theta$ , (iv) the home-bias  $\omega$ , and (v) the consumption share parameter  $\eta$ . We choose the values of these parameters to target jointly the following moments: (i) median volatility ratio in the OECD of 0.7 as given in Table 3, (ii) producer markups of 10% as estimated by Basu & Fernald (1997), (iii) relative volatility of the real export price  $p_x$  to the real exchange rate  $x$

Japan, 22% in Germany, and 21% in France. The OECD median is close to 20%. We adopt a bit higher number to make the model comparable to the results documented in the literature.

of 37% (U.S. data 1980–2004), (iv) standard value for the share of market activities in total time endowment of households equal to 30%, (v) imports to GDP ratio of 12% (U.S. data 1980–2004), and finally, (vi) the share of marketing expenditures to sales on the industry level of 7% as reported by Lilien & Little (1976) (also Lilien & Weinstein (1984)), and (vii) moments of the productivity process as discussed in the next paragraph.

**Productivity process** We follow a procedure similar to Heathcote & Perri (2004) to back out the total factor productivity (TFP) residuals  $z$  from the data. However, because the model-implied TFP residuals are different from the assumed ones,<sup>32</sup> we modify the correlation and volatility of the assumed disturbances  $\varepsilon, \varepsilon^*$ , and the AR(1) persistence parameter so that the model implied residuals match the following targets from the data: (i) volatility of model-generated TFP residuals of 0.79%, (ii) the correlation of model-generated TFP residuals of 0.3, and (iii) autoregressive coefficient of 0.91. The exact values of these parameters used in the model economies are reported in Table 4.

Finally, we solve the model by taking a second order approximation of the equilibrium conditions as described in Schmitt-Grohe & Uribe (2004).

## IV. Results

In this section, we confront our model’s quantitative predictions with the data.<sup>33</sup> We identify the United States with the domestic country and the aggregate of 18 major OECD countries with the foreign country.<sup>34</sup> Unless otherwise noted, all reported statistics are based on logged and H-P-filtered quarterly time series. The standard model (BKK model), with which we contrast our results, has been parameterized analogously whenever applicable. Table 4 reports parameter choices in the theoretical economies.

**Business cycle implications for international prices** Table 5 reports the business cycle statistics on comovement and relative volatility of international relative prices. As we can see from this table, the benchmark model successfully accounts for the aggregate patterns

<sup>32</sup>Marketing expenditures are not treated as investment in national accounts, which is reflected in measured TFP. See McGrattan & Prescott (2005).

<sup>33</sup>In the technical appendix to the paper (available online), we describe how we map actual national accounting procedures onto our model economy.

<sup>34</sup>Detailed list of countries can be found in the Appendix.

discussed in Section I: (i) real export and real import prices are *positively* correlated (and positively correlated with the real exchange rate), (ii) relative volatility of the terms of trade to the real exchange rate is about 26%, matching the value of 27% for U.S. data after cleaning import price data from the influence of volatile fuel prices,<sup>35</sup> and (iii) producers price-to-market to which they sell—the relative price  $\frac{p_x}{p_d}$  is no longer constant and comoves positively with the real exchange rate. None of these features are reproduced by the original BKK model.

As we can see from Panel C of Table 5, both the BKK model and the benchmark model fail to replicate the volatility of the real exchange rate by an order of magnitude, and both models imply a positive correlation between the real exchange rate and the consumption ratio (the Backus-Smith puzzle). In order to make sure that our results would not go away once the properties of the real exchange rate are accounted for, we follow Heathcote & Perri (2002) and solve our model under financial autarky—described in detail in Section VI. Under this modification, the real exchange rate implied by the model is negatively correlated with the consumption ratio and is about four times more volatile. As we can see from the fourth column of Table 5, all of our results still stand.

**Business cycle implications for quantities** Table 6 reports the statistics on quantities. The benchmark model implies a bit too low international comovement of investment<sup>36</sup> (0.03 model vs. 0.23 data), but it matches the rest of the statistics well. Note that, unlike the standard BKK model, the benchmark model is additionally consistent with the fact that output is more internationally correlated than consumption (data 0.4 output and 0.25 consumption; model 0.35 output 0.23 consumption), addressing the so-called quantity puzzle. Because most of the quantitative discrepancies can be fixed by incorporating additional features (e.g., convex adjustment cost or home production), both models are relatively

<sup>35</sup>To arrive at this estimate, we use the price indices for export and import prices disaggregated to a one-digit SITC level by the BLS. We next remove from the index classification SITC-3 (fuels) from both the export and the import price index. We then measure by how much it reduces the standard deviation of the logged and H-P-filtered overall terms of trade (1983 – 2005) constructed from the BLS price indices. The result is that the volatility of terms of trade falls from about 1.94% with fuels to about 1.32% without fuels. We next obtain the non-fuel statistics for the United States by multiplying the volatility of the terms of trade measured from the deflator prices of exports and imports (as in Table 2) by the correcting ratio derived from the BLS data:  $1.32/1.94 \approx 0.68$ . A slightly larger estimate of about 35% would be obtained from the BLS data directly (the BLS estimate refers to a fixed weight index, not a deflator price).

<sup>36</sup>For the most recent subperiod (1986–2000), Heathcote & Perri (2004) report an international correlation of investment equal to zero.

Table 5: International Prices: Theory versus Data<sup>a</sup>

Statistic	Data <sup>b</sup>	Results		Robustness	
		Benchmark	BKK	Benchmark Fin. Aut.	BKK with Adj. Cost
<i>A. Correlation</i>					
$p_x, p_m$	0.75	0.98	-1.00	1.00	0.62
$p_x, x$	0.46	0.99	-1.00	1.00	0.71
$p_m, x$	0.69	1.00	1.00	1.00	0.43
$p, x$	0.61	0.95	1.00	0.99	-0.46
<i>B. Volatility relative to<sup>e</sup> <math>x</math></i>					
$p_x$	0.37	0.37	0.16	0.37	0.90
$p_m$	0.61	0.62	1.16	0.63	0.75
$p$ (no fuels <sup>d</sup> )	0.27	0.26	1.32	0.26	0.75
$p_x/p_d$	n/a <sup>f</sup>	0.19	0.00	0.65	0.20
<i>C. Standard deviation of <math>x</math></i>					
	3.60	0.43	0.49	1.65	0.20
<i>D. Correlation of <math>c/c^*</math> with <math>x</math></i>					
	-0.71	1.0	1.0	-0.60	1.0
<i>E. Price elasticities of trade</i>					
Short-Run	0.7	0.7	0.7	0.7	7.9
Long-Run	7.9	7.9	0.7	7.9	7.9

<sup>a</sup>Statistics based on logged and H-P-filtered time-series with smoothing parameter  $\lambda = 1600$ .

<sup>b</sup>Data column refers to U.S. data for the period 1980 : 1 – 2004 : 1.

<sup>c</sup>This setting of  $\gamma$  is consistent with model implied volatility ratio of 0.7.

<sup>d</sup>Refers to terms of trade series cleaned from the influence of fuels (SITC 3); relative volatility of the overall terms of trade is about 0.41 for U.S.

<sup>e</sup>Ratio of corresponding standard deviation to the standard deviation of the real exchange rate  $x$ .

<sup>f</sup>We do not report a number for the U.S. because data on producer price index includes exported goods, and thus is not a good measure of the domestic prices. In disaggregated Japanese data the median is 0.43.

successful on the quantity dimension.

An additional prediction of our richer framework pertains to the behavior of marketing expenditures over the business cycle. The evidence on the behavior of marketing expenditures over the business cycle is scant. However, annual aggregate figures for advertising expenditures on the national level are readily available from the *Statistical Abstract of the United States* published by the U.S. Census Bureau. These figures reveal that advertising expenditures are a highly pro-cyclical series; in particular, the share of advertising expenditures in GDP is highly pro-cyclical. This observation is qualitatively consistent with the predictions of our model.

Table 6: Quantities: Theory versus Data<sup>a</sup>

Statistic	Data <sup>b</sup>	Results		Robustness	
		Benchmark	BKK	Benchmark Fin. Aut.	BKK with Adj. Cost
<i>A. Correlations</i>					
<i>domestic with foreign</i>					
TFP (actual <sup>c</sup> )	0.30	0.30	0.30	0.30	0.30
GDP	0.40	0.35	0.36	0.37	0.22
Consumption	0.25	0.23	0.32	0.34	0.62
Employment	0.21	0.32	0.48	0.27	0.07
Investment	0.23	0.03	0.16	0.35	0.11
<i>GDP with</i>					
Consumption	0.83	0.93	0.94	0.90	0.92
Employment	0.85	0.80	0.98	0.68	0.99
Investment	0.93	0.83	0.66	0.94	0.68
Net exports	-0.49	-0.56	-0.77	n/a	-0.11
<i>Terms of trade with</i>					
Net exports	-0.17	-0.89	-0.81	n/a	-0.85
<i>B. Volatility</i>					
<i>relative to GDP<sup>d</sup></i>					
Consumption	0.74	0.32	0.31	0.38	0.28
Investment	2.79	3.67	3.36	3.07	3.20
Employment	0.81	0.69	0.48	0.80	0.51
Net exports	0.29	0.21	0.13	0.00	0.04

<sup>a</sup>Statistics based on logged and H-P-filtered time-series with smoothing parameter  $\lambda = 1600$ .

<sup>b</sup>Data column refers to U.S. data for the time period 1980:1-2004:1.

<sup>c</sup>This setting of  $\gamma$  is consistent with model implied volatility ratio of 0.7.

<sup>d</sup>Ratio of corresponding standard deviation to the standard deviation of GDP.

<sup>e</sup>Calculated using actual national accounting procedures; see technical appendix.

## V. Mechanics Behind the Results

Compared to the standard theory, our model brings the aggregate price statistics closer to the data in the following dimensions: (i) the real export and import prices are *both* positively correlated with the real exchange rate, (ii) terms of trade is *less* volatile than the real exchange rate, and (iii) producers price-to-market. The goal of this section is to provide an intuitive understanding of these implications of the model.

We start by analyzing the critical features that give rise to the above patterns. These features are: (i) *bargaining* and (ii) *market expansion friction*. We then proceed to analyze the sources of the real exchange rate fluctuations.

For expositional purposes, we study the impulse response functions to a one-time, one percent positive productivity shock in the domestic country. Panels A and B of Figure 1 and

2 present the response of prices in the benchmark model and in the standard BKK model, respectively. In the benchmark model when the real exchange rate depreciates following the shock (panel A), the real export price  $p_x$  goes up. At the same time, the price of the same good sold at home  $p_d$  actually falls (panel B). In contrast, in the standard model these two prices are always equal by the law of one price and following the shock both fall.<sup>37</sup> This feature of our model, labeled in the literature as pricing-to-market, is the major difference between the two environments. Below, we discuss intuitively the key forces that give rise to pricing-to-market in our environment.

**Bargaining** Bargaining sets the stage for pricing-to-market to occur by explicitly linking export and import prices to the valuation of the good by the local retailers. From the bargaining equations,

$$\begin{aligned} p_x(s^t) &= \theta x(s^t) P_d^*(s^t) + (1 - \theta)v(s^t), \\ p_d(s^t) &= \theta P_d(s^t) + (1 - \theta)v(s^t), \end{aligned} \tag{40}$$

we can observe that the wholesale prices of the domestic good not only depend on the marginal cost,  $v$ , but also on the valuation of the goods by the retailers,  $xP_d^*$  and  $P_d$ . This contrasts with the standard model, in which by the law of one price both prices are tied to domestic marginal cost.

**Market expansion friction** Bargaining alone, however, is not enough to generate the observed behavior of prices. Without certain dynamic properties of the valuations of the retailers, export and import would still correlate the wrong way in our model. The reason why this is not the case is because producers face the *market expansion friction*.

Mechanically, this friction makes the endogenous list of customers respond sluggishly to shocks. As a result, the relative scarcity of domestic and foreign goods remains relatively stable over the business cycle and is also sluggish. This connection can readily be seen from

<sup>37</sup>An immediate consequence of such behavior of export and import prices is that the terms of trade, which can be expressed as the ratio of export to import prices, is no longer more volatile than the real exchange rate.

the feasibility condition pertaining to the export market:

$$\frac{d^*}{f^*} = \frac{H_d^*}{H_f^*} = \frac{(1 - \delta_h)H_{d,-1}^* + \frac{\bar{m}_d^*}{\bar{m}_f^* + \bar{m}_d^*} h^*}{(1 - \delta_h)H_{f,-1}^* + \frac{\bar{m}_f^*}{\bar{m}_f^* + \bar{m}_d^*} h^*}. \quad (41)$$

From this formula, observe that the adjustment of the scarcity ratio  $\frac{d^*}{f^*}$  is hardwired to the adjustment of the relative marketing capital,  $\frac{\bar{m}_f^*}{\bar{m}_f^* + \bar{m}_d^*}$ , which, in turn, is subject to the market expansion friction by (21).

**Pricing-to-market** The implication of the market expansion friction described above matters for pricing-to-market because it crucially affects the dynamics of retail prices, and thereby the valuation of the good by local retailers. To understand the connection between the market expansion friction and retail prices, consider the implication of (14) for the price of the domestic good sold in the foreign market:

$$P_d^* = \omega \left[ \omega + (1 - \omega) \left( \frac{d^*}{f^*} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{1}{\gamma-1}}. \quad (42)$$

The above formula reveals two key features. First, retail prices respond only to the change in the scarcity ratio  $d^*/f^*$ , and second, the higher the elasticity of substitution  $\gamma$  between foreign and domestic goods, the less sensitive retail prices are to the scarcity ratio. Because the elasticity of substitution is set to a high value in our model, and the scarcity ratio moves sluggishly in response to the shocks, the retail prices measured in local consumption units remain almost constant over the business cycle.

Panels C and D of Figure 1 document this property of the model. Comparing with similar plots for the standard BKK model included in panels C and D of Figure 2, we see that even though the scarcity ratio moves about as much in our model as in the standard model, these movements translate to almost negligible movements of the retail prices.

Panel C of Figure 1 illustrates the consequence of retail price sluggishness in their respective local consumption units. Following the shock the foreign retail price of the domestic good expressed in the *domestic* consumption units  $xP_d^*$  increases almost one-to-one with the real exchange rate  $x$ .

This increase in  $xP_d^*$  creates extra surplus from trade within each existing match, as

it is the foreign retailer’s valuation of the exported good entering the bargaining problem (40). If the bargaining power of the producer  $1 - \theta$  is positive, this extra surplus partially goes to the domestic producer and results in increased markups on the exported goods. This increase in markups leads to an increase in the export price  $p_x$ , despite the fall of the price of the same good sold at home. This effect is illustrated in Panels A and B of Figure 1.

**Sources of incomplete arbitrage** The above analysis leads to the natural question about the source of incomplete arbitrage in our model. The price differential between the home and the export market visible in panel B of Figure 1 encourages domestic producers to relocate sales from the less profitable home market to the more profitable export market.

What precludes them from taking advantage of this price difference is the fact that the producers first need to match with buyers and expand their customer lists. Following the shock, this process is more costly abroad than at home due to market expansion friction, which allows the export price  $p_x$  to persistently depart from the home price  $p_d$ . The variation in the shadow cost of marketing is what drives time-varying markups in our economy and gives bargaining some bite. It is the combination of the two frictions that makes prices move the right way, with the intermediate value of bargaining being essential for the results.<sup>38</sup>

**Real exchange rate movements and the law of one price** In the benchmark model the real exchange rate responds to shocks similarly to the standard model (Panel A of Figure 1 and 2). However, the mechanics of these movements can be almost entirely attributed to the deviations from the law of one price, unlike in the standard model. Below, we first explain the forces behind the real exchange rate fluctuations in our model, and then show how they are related to the deviations from the law of one price.

In the calibrated benchmark model the market shares of the producers are biased towards the local good, i.e.,  $\pi > 1 - \pi$  and  $\pi^* > 1 - \pi^*$ , respectively. This asymmetry in market shares, combined with their sluggishness is critical to give rise to real exchange rate fluctuations.<sup>39</sup>

To illustrate the mechanism at work, consider a positive productivity shock in the

<sup>38</sup>When bargaining power is too high, short-run price elasticity of trade flows is hardwired to elasticity  $\gamma$  just as in the standard model. Bargaining power that is too low shuts down pricing-to-market.

<sup>39</sup>Without the home-bias, i.e., when  $\omega = \frac{1}{2}$ , the real exchange rate does not move over the business cycle either in our model or in the standard model.



domestic country. Such shock makes good  $d$  more abundant, and the additional supply of good  $d$  can be shipped to the households in each country via two channels. The retailers can search more intensively ( $h$  and  $h^*$  go up) or, alternatively, the market shares at home and abroad can adjust towards the more abundant domestic good ( $\pi$  and  $1 - \pi^*$  go up). This link can be established from the following feasibility condition,

$$h\pi + h^*(1 - \pi^*) - \delta_H(H_d + H_d^*) = \Delta, \quad (43)$$

where  $\Delta$  denotes the extra supply of  $d$  goods to be distributed from producers to consumers relative to the previous period. The left hand side is the net increase in the number of matches with  $d$  producers.

In the benchmark model, the market expansion friction impairs the adjustment through  $\pi$  and  $\pi^*$ , and the asymmetry implied by home-bias ( $\pi > 1 - \pi^*$ ) makes search by retailers relatively more efficient in finding abundant domestic goods in the domestic country than in the foreign country. As a result, following the shock, the domestic retailers are willing to search more insensitively than foreign retailers (at the same prices), and thus in consistency with (16), the price of domestic consumption must fall relative to the price of the foreign consumption (real exchange rate depreciates), with  $c$  increasing relative to  $c^*$  to soak up the extra supply of  $d$ -goods.

Next, we proceed to show that the real exchange rate fluctuations in the benchmark model can be linked to the deviations from the law of one price on the commodity level. Using the ideal CPIs and the bargaining equations together with (28), by definition of the real exchange rate as the ratio of CPI's measured in common unit, we have

$$x \equiv \frac{CPI^*}{CPI} = \frac{((P_f + \frac{1}{\theta}(x\Lambda_f^* - x\Lambda_f))^{1-\gamma}\omega^\gamma + (P_d + \frac{1}{\theta}(\Lambda_d^* - \Lambda_d))^{1-\gamma}(1-\omega)^\gamma)^{\frac{1}{1-\gamma}}}{(P_d^{1-\gamma}\omega^\gamma + P_f^{1-\gamma}(1-\omega)^\gamma)^{\frac{1}{1-\gamma}}}, \quad (44)$$

where  $\Lambda_k, \Lambda_k^*$  are the shadow costs of marketing for a producer of good  $k$  in the domestic and foreign markets, respectively.<sup>40</sup>

The above formula shows that the movements of the real exchange rate in the benchmark model can be attributed to two sources. First, they can be driven by the relative price movements of the price of the domestic good relative to the foreign good  $\frac{P_f}{P_d}$ —just like in

<sup>40</sup>By definition,  $\Lambda_k$  is the difference between the wholesale price and the marginal cost of production.

the standard model. Second, they may additionally come from the shadow cost differences between the domestic and the foreign market (deviations from LOP),  $x\Lambda_f^* - x\Lambda_f$  and  $\Lambda_d^* - \Lambda_d$ , respectively. The comparison of the behavior of a *hypothetical* real exchange rate without the shadow price terms,

$$\hat{x} \equiv \frac{(P_f^{1-\gamma}\omega^\gamma + P_d^{1-\gamma}(1-\omega)^\gamma)^{\frac{1}{1-\gamma}}}{(P_d^{1-\gamma}\omega^\gamma + P_f^{1-\gamma}(1-\omega)^\gamma)^{\frac{1}{1-\gamma}}}, \quad (45)$$

and the actual real exchange given by (44) reveals the critical role of the shadow terms and thus the dominant role of the deviations from the law of one price. In our calibrated economy, the ratio of the standard deviation of  $\hat{x}$  to the standard deviation of  $x$  is equal to 0.038, meaning that almost all movements can be attributed to deviations from the law of one price. This prediction is broadly consistent with the evidence documented in Goldberg & Campa (2008), showing that the retail prices of imported goods carry much less volatility than the real exchange rates.

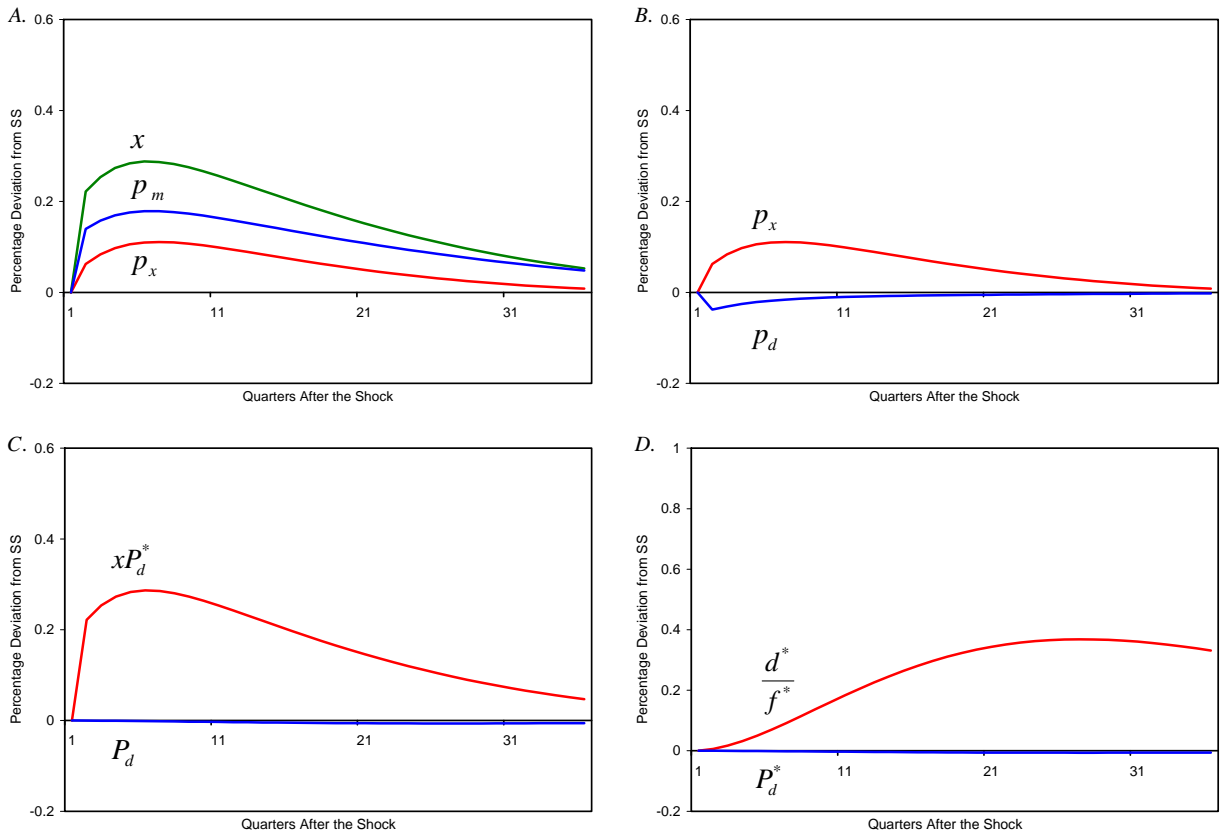


Figure 1: Benchmark model: Impulse response to a positive productivity shock in the domestic country.

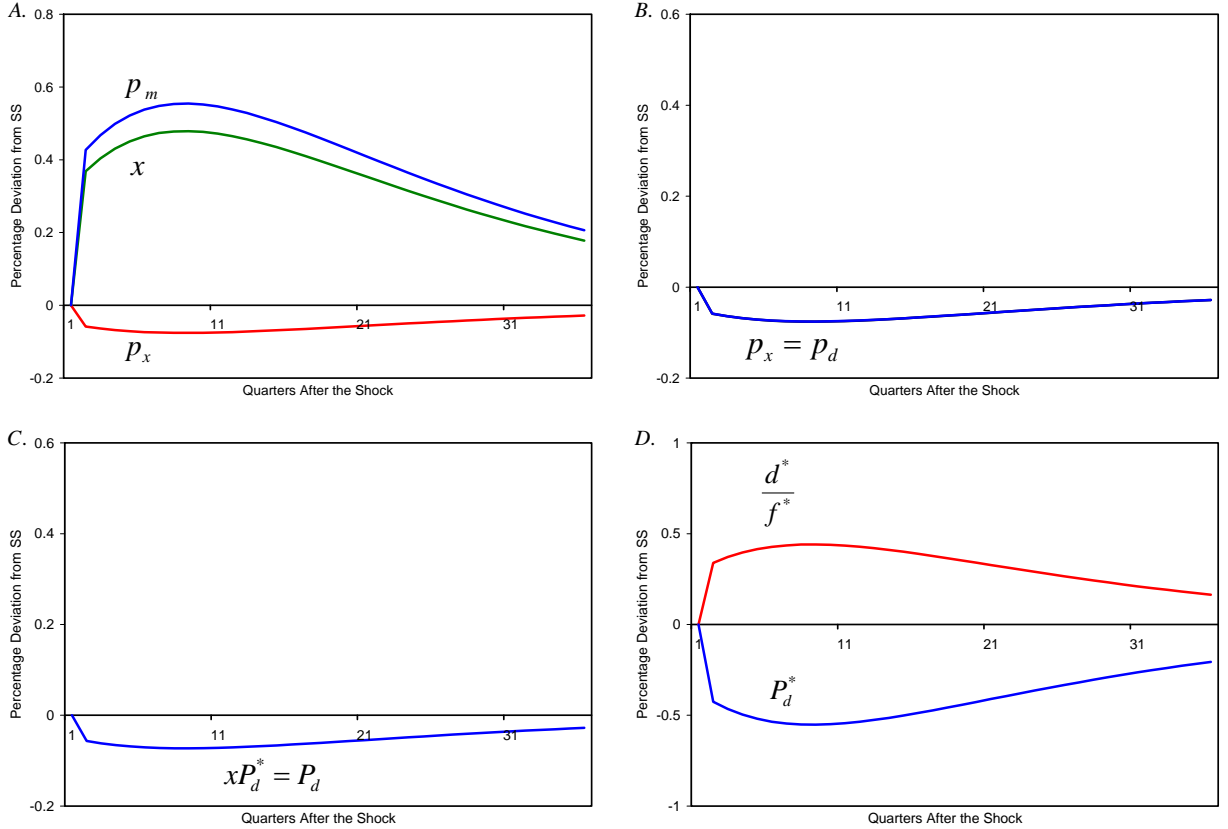


Figure 2: Standard model ( $\gamma = 0.7$ ): Impulse response to a positive productivity shock in the domestic country.

## VI. Robustness and Sensitivity

In this section, we examine the robustness of our results. We report two exercises. In the first exercise, we show that the sources of dynamics of the real exchange rate do not affect the pricing-to-market predictions of our model, and thus neither the volatility puzzle nor the Backus-Smith puzzle affect the key mechanism of our model. To boost the volatility of the real exchange rate, we consider a variant of our economy in which we assume financial autarky. The second exercise answers the question of whether a simple adjustment cost, as explicitly suggested by Krugman (1986), can generate the same behavior of prices as our marketing friction. We find that it can account for some observations, but it fails to account for the positive correlation of terms of trade and real exchange rate.<sup>41</sup>

<sup>41</sup>There are two additional exercises that we conducted, in which we are interested in the impact of the assumed value of match destruction rate  $\delta_h$ , which we set arbitrarily equal to  $\delta_h = 0.1$  in the benchmark parameterization, and the share of marketing expenditures in GDP, for which we lack good data. We show that possible disturbance to the value of  $\delta_h$  or the share of marketing expenditures to GDP has little impact

We report the results of these exercises, called *Benchmark under Financial Autarky* and *BKK with Adjustment Cost*, in Tables 5 and 6. Parameters are reported in Table 4.

**Financial autarky** In this exercise, we demonstrate that the price dynamics generated by our model *relative to the real exchange rate* do not depend on the driving forces behind exchange rate movements. For this purpose, we assume that countries are in financial autarky, which increases the volatility of the real exchange rate to the levels observed in the data. In particular, we impose the condition that the current account be zero at each date and state

$$x(s^t) p_d^*(s^t) d^*(s^t) + v_d(s^t) a_f(s^t) = p_f(s^t) f(s^t) + x(s^t) v_f(s^t) a_d^*(s^t).$$

The rest of the parameters are chosen to match the same targets as in the benchmark case. We can see in Table 5 that for the price statistics, changing the real exchange rate dynamics does not affect the relative price dynamics in our model. In particular, the model still matches the import and export price comovement, as well as the volatilities of these prices and the terms of trade relative to the real exchange rate.

**Adjustment cost in BKK** This exercise answers the question of whether a simple adjustment cost suggested by Krugman (1986) could generate quantitatively similar behavior of prices as our micro-founded frictions. Krugman (1986) argued that a convex trade cost would induce producers to price-to-market and potentially account for the observed behavior of prices. In this spirit, we introduce a quadratic adjustment cost directly on the quantity sold by producers into the standard BKK model. Formally, the domestic producers solve:

$$\max \sum \int_{s^t} Q(s^t) [P_d(s^t)d(s^t) + x(s^t)P_d^*(s^t)d^*(s^t) - w(s^t)l(s^t) - r(s^t)k(s^t)] \quad (46)$$

subject to

$$d(s^t) + d^*(s^t) = f(s^t)(k(s^t), l(s^t)) - v_1 \left( \frac{d(s^t)}{d(s^{t-1})} - 1 \right)^2 - v_2 \left( \frac{d^*(s^t)}{d^*(s^{t-1})} - 1 \right)^2, \quad (47)$$

on the overall results. These results are available in the working paper version of the paper.

where  $v_1$  and  $v_2$  are the adjustment costs for changing the sales in the domestic and foreign market, respectively.

First of all, we should point out that this model no longer accounts for the elasticity puzzle, and therefore cannot be quantitatively disciplined in a similar fashion as the benchmark model. To understand this point, note that here, unlike in the benchmark model, producer prices are equal to retail prices and therefore are tightly linked to the scarcity ratios  $\frac{f}{d}$  and  $\frac{f^*}{d^*}$  through the consumer first order conditions (14). These conditions fix the volatility of the price ratio  $\frac{p_d}{p_f}$  relative to the quantity ratio  $\frac{f}{d}$ , and consequently both the short-run and the long-run price elasticity of trade flows are equal to  $\gamma$ . In contrast, in the benchmark model bargaining disconnects wholesale and retail prices, and gives potential to account for the elasticity puzzle.

Given that we cannot follow the same approach of disciplining the parameter values governing market share sluggishness, we will select the values of  $\gamma$  and  $v_1, v_2$  that are most favorable for the overall fit of the model, and conduct extensive sensitivity. The most promising case turns out to be a high elasticity ( $\gamma = 7.9$ ) case (similar to the benchmark parameterization), with an asymmetric adjustment cost  $v_1 = 0, v_2 = 50$ . In this case, we find that the model still falls short relative to the benchmark model. Except failing to account for the long-run versus short-run elasticity puzzle, it counterfactually predicts that the export price is more volatile than the import price, and that the terms of trade is strongly negatively correlated with the real exchange rate. Other combinations of the parameters yield a strictly worse fit with the data. We conclude that our micro-founded model allows to discipline the quantitative exercise in the first place, and matches the statistics strictly better.

## VII. Conclusions

In this paper, we have demonstrated that dynamic frictions of building market shares have the potential to account for pricing-to-market, and the discrepancy between the short-run and the long-run price elasticity of trade flows. Given the anecdotal evidence about the importance of switching costs and the long-lasting nature of producer-supplier relations in international trade, we believe that the mechanism proposed by us is an important step toward a better understanding of the fundamental reasons behind the deviations from the law of one price.

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