Realization Utility

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February 2008*

Abstract

We study the possibility that, aside from standard sources of utility, investors also derive utility from *realizing* gains and losses on individual investments that they own. We propose a tractable model of this "realization utility," derive its predictions, and show that it can shed light on a number of puzzling facts. These include the poor trading performance of individual investors, the disposition effect, the greater turnover in rising markets, the negative premium to volatility in the cross-section, and the heavy trading of highly valued assets. Underlying some of these applications is one of our model's more novel predictions: that, even if the form of realization utility is linear or concave, investors can be risk-*seeking*.

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1 Introduction

When economists model the trading behavior of individual investors, they typically assume that these investors derive utility only from consumption or total wealth. In this paper, we study the possibility that investors also derive utility from another source, namely from *realized gains and losses* on risky assets that they own. Suppose, for example, that an investor buys a stock, and then, a few months later, sells his position. We analyze a model in which the investor gets a jolt of utility right then, at the moment of sale, and where the utility term depends on the size of the gain or loss realized – utility is positive if the stock is sold at a gain relative to purchase price, and negative otherwise. We label this source of utility "realization utility."

There are several reasons why an investor might derive utility from realizing a gain or loss. If he sells a stock at a gain, he can tell himself that he is a savvy investor, raising his self-esteem. The sale also gives him a piece of news to boast about to family and friends. While an investor can certainly also feel good about a stock trading at a *paper* gain, these feelings are likely to be more pronounced at the moment of sale. When an asset is sold, there is a sense that the transaction is "complete," making it easier to claim credit for a successful investment.[1]

Practitioners have long argued that realization utility plays an important role in individual decision making. For example, in a well-known manual for stock brokers, Gross (1982) discusses the pain associated with realizing a loss:

> *"Most clients, however, will never sell anything at a loss... Investors are reluctant to accept and realize losses because the very act of doing so proves that their first judgment was wrong... Investors who accept losses can no longer prattle to their loved ones, 'Honey, it's only a paper loss. Just wait. It will come back.' "*

Introspection and casual observation, then, suggest that realization utility may be a significant driver of trading behavior, and hence that it merits a more systematic analysis. In this paper, we do three things. First, we develop a tractable model of realization utility, one that is sophisticated enough to capture many features of actual trading, but also simple enough to allow for an analytical solution. Second, we lay out the model's predictions. And third, we link these predictions to a wide range of applications. We start with a partial equilibrium framework, but also show how realization utility can be embedded into a full equilibrium model. As a result, we are able to make predictions not only about trading behavior, but also about prices.

[1] A common terminology in behavioral economics is that, when an investor buys a stock, he opens a "mental account" for that asset, one that is closed only when the stock is fully sold (Thaler, 1999).

Our model is an infinite-horizon framework in which an investor switches back and forth between a stock and a risk-free asset. Whenever he liquidates his stock holdings, he receives a jolt of utility based on the size of the gain or loss realized, and pays a proportional transaction cost. He also faces random liquidity shocks: if such a shock occurs, he must sell any outstanding position in stock and exit the stock market. At each moment, the investor makes his allocation decision by maximizing the discounted sum of expected future realization utility flows. In our baseline model, we assume a linear functional form for realization utility, but also consider the case of piecewise-linear utility, under which the investor is more sensitive to realized losses than to realized gains.

We find that, in our model, the investor voluntarily sells his stock holdings only when the stock is trading at a sufficiently large gain relative to purchase price. We look at how this "liquidation point" – the percentage gain in price, relative to purchase price, at which the investor is willing to sell – depends on the stock's expected return, its standard deviation, the investor's time discount rate, the level of transaction costs, and the frequency of liquidity shocks. Our model also allows us to compute the probability that, within any given interval after first buying a stock, the investor sells it. We look at how this probability – a measure of trading frequency – depends on the aforementioned factors.

The model makes a number of interesting predictions. One of the more striking is that, even if realization utility has a linear or concave functional form, the investor can be risk-*seeking*: all else equal, his time 0 value function is increasing in the volatility of the stock available for trading. The intuition is straightforward. A highly volatile stock offers the chance of a large gain, which the investor can enjoy realizing. Of course, it may also experience a large drop in value; but in that case, the investor will simply postpone selling the stock until he is forced to by a liquidity shock. Any realized loss therefore lies in the distant, heavily discounted future and does not scare the investor very much. Overall, then, the investor prefers more volatility to less.

A related intuition underlies another of the model's predictions: that the investor is willing to buy a stock with a *negative* average excess return, so long as its volatility is sufficiently high. The model also predicts that more volatile stocks will be traded more frequently: roughly speaking, a more volatile stock reaches its liquidation point more rapidly.

We link our model to a wide range of financial phenomena. In particular, we argue that it offers a way of thinking about the subpar trading performance of individual investors (Odean, 1999; Barber and Odean, 2000), the disposition effect (Odean, 1998), the greater turnover in bull markets than in bear markets (Statman, Thorley, and Vorkink, 2006; Griffin, Nardari, and Stulz, 2007), the negative volatility premium (Ang et al., 2005), and the heavy trading associated with highly valued assets – as, for example, in the technology sector in the late 1990s (Hong and Stein, 2007).

To understand this last application, note that, in an economy where many investors care about realization utility, more volatile stocks will be both more heavily traded – such stocks reach their liquidation points faster – *and* more highly valued: since realization utility investors like volatility, they will collectively push the prices of volatile stocks up. Our model therefore predicts a coincidence of high valuations and heavy trading; and moreover, that this phenomenon will occur for assets whose fundamentals are particularly uncertain. Under this view, the late 1990s were years in which realization utility investors, attracted by the high uncertainty of technology stocks, bought these stocks, pushing their prices up; as (some of) these stocks rapidly reached their liquidation points, the realization utility investors sold them, and then immediately bought new ones.

Although we work mainly with exponential time discounting, we also consider the case of hyperbolic time discounting. While hyperbolic discounting has been linked to a number of economic phenomena, researchers have not, as yet, found many applications for it within the context of finance. We find that, in the presence of realization utility, hyperbolic discounting can play an interesting role: in particular, it can significantly amplify the effect of realization utility. As such, it may be relevant in exactly the applications that we link to realization utility.

As noted above, realization utility may offer a simple way of understanding a range of financial phenomena. In most cases, we did not foresee the link between realization utility and the particular application: the link emerged only after we had completed our analysis. Nonetheless, we are careful to not only offer explanations for known facts, but to also suggest new predictions. We noted one of these earlier: a realization utility investor will hold a more volatile stock for a shorter period of time, before selling it. Other predictions that we derive are that the more impatient the investor is, the more frequently he will trade; and that the more sensitive the investor is to realized losses as opposed to gains, the *less* frequently he will trade.

As mentioned earlier, practitioners have for decades noted the potential importance of realization utility. In the academic literature, an early discussion of this idea can be found in Shefrin and Statman (1985). They propose it, in combination with prospect theory, as a way of understanding the disposition effect, and present a two-period numerical example. More recently, Barberis and Xiong (2008) analyze a two-period model of realization utility, again in combination with prospect theory, and again with the disposition effect as the eventual application.

In this paper, we offer the first comprehensive analysis of realization utility. We move beyond the two-period setting and work in an infinite horizon framework. We allow for realistic features of trading, such as transaction costs and random liquidity shocks. We analyze the investor's trading strategy along several dimensions, including trading frequency. We present not only a partial equilibrium model, but also a full equilibrium. And we consider a wide range of applications, of which the disposition effect is just one.

In Section 2, we lay out our baseline model, one that assumes linear realization utility and exponential discounting. In Section 3, we consider alternative preference specifications, including piecewise-linear utility and hyperbolic discounting. In Section 4, we show how realization utility can be embedded in a full equilibrium framework. Section 5 considers a range of applications, and Section 6 concludes.

2 A Model of Realization Utility

We now present a model of trading behavior in which the investor cares about realization utility. We make two important modeling assumptions. First, we assume that the carriers of realization utility are *gains and losses* measured relative to purchase price. If realization utility is the idea that the investor derives pleasure from completing a successful investment in some asset, it is natural that how good he feels at the moment of sale is a function of the asset's change in value since purchase.

Second, we assume that realization utility is defined at the level of an individual asset. Again, if realization utility is the idea that the investor feels good when he completes a successful investment in some asset, it is natural that utility is defined at the asset level, even if the asset is just one of many in his portfolio. The idea that an investor might get utility from the outcome of one specific asset that he owns is sometimes known as "narrow framing." In short, then, realization utility leads naturally to narrow framing.

Taken together, these assumptions mean that the utility specification in our model differs from more traditional specifications in three ways: in that utility is defined over gains and losses rather than over absolute wealth levels; in that utility is defined at the level of individual assets; and in that the utility specification makes a distinction between realized and paper gain/losses, and defines utility only over *realized* gains and losses. To be clear, while there are now several papers that posit preferences defined over *paper* gains and losses, this paper is the first, to our knowledge, to present a comprehensive analysis of investor behavior when utility is defined over *realized* gains and losses.

We do not expect realization utility to be an important factor for all investors or in all circumstances. For example, we expect it to matter more for individual investors than for institutional investors, who, as trained professionals, are likely to ignore the emotions attached to realizing a gain or loss on an individual stock. Among individual investors, we expect realization utility to play a larger role when an asset's purchase price is particularly salient: it is easier to take pride in a successful investment when the success is easier to measure. Realization utility may therefore be more relevant to the trading of individual stocks or the sale of real estate than to the trading of mutual funds: the purchase price of a stock or of a house is typically more salient than that of a fund.

Another modeling choice concerns the functional form for realization utility. Since it is unclear what this form should be, we focus, in this section, on the simplest possibility, a linear functional form, and show that, even under this assumption, realization utility has a range of novel implications. Later in the paper, we consider some alternative specifications. For example, in Section 3.1, we consider a piecewise-linear specification. And in Section 3.2, we vary another dimension of preferences, and replace exponential time discounting with hyperbolic time discounting.[2]

We use a continuous-time framework because this allows us to solve the model analytically. We have also studied the discrete-time analog of our model. The results are similar, but can only be obtained numerically.

Consider an investor who starts at time 0 with wealth W_0. At each time $t \geq 0$, he has two investment options: a risk-free asset, which offers a net return of zero; and a risky asset whose price S_t follows

$$\frac{dS_t}{S_t} = \mu dt + \sigma dZ_t. \tag{1}$$

Perhaps the most natural application of our model is to understanding how individual investors trade stocks in their brokerage accounts. We therefore refer to the risky asset as a stock.

For simplicity, we assume that, at each time t, the investor either allocates all of his wealth to the risk-free asset or all of his wealth to the stock: no intermediate allocations are allowed. We also suppose that, if the investor sells his position in the stock at time t, he pays a proportional transaction cost, kW_t, $0 \leq k < 1$, where W_t is time t wealth. The investor's wealth therefore evolves according to

$$\frac{dW_t}{W_t} = \theta_t(\mu dt + \sigma dZ_t) - kI_{\{l_t=1\}}, \tag{2}$$

where θ_t takes the value 1 if he is holding the stock at time t, and 0 otherwise; and where l_t takes the value 1 if he sells the stock at time t, and 0 otherwise.

An important variable in our model is B_t. This variable, which is defined only if the investor is holding stock at time t, measures the cost basis of the stock position, in other

[2] One approach to specifying the functional form for realization utility is to adopt the specification proposed by Kahneman and Tversky (1979) in their prospect theory. We resist this approach. Prospect theory is intended to capture the way people evaluate wealth gambles. It is not clear that this makes it a good model of how people think about realized gains and losses. That said, the piecewise-linear specification we consider in Section 3.1 allows us to explore the best-known feature of prospect theory, namely the greater relative sensitivity to losses as opposed to gains.

words, the amount of money the investor put into the time t stock position at the time he bought it. Formally, if $\theta_t = 1$,

$$B_t = W_s, \text{ where } s = \max\{\tau \epsilon [0, t) : \theta_\tau = 0\}. \tag{3}$$

The key feature of our model is that the investor derives utility from realizing a gain or loss. Specifically, whenever he switches his wealth from the stock into cash, he receives a burst of utility given by

$$u((1 - k)W_t - B_t). \tag{4}$$

The argument of the utility term is the size of the realized gain or loss: the investor's wealth at the moment of sale, after the transaction cost, $(1-k)W_t$, minus the cost basis of the stock investment B_t. Throughout this section, we use a linear functional form,[3]

$$u(x) = x. \tag{5}$$

The investor also faces random liquidity shocks which arrive according to a Poisson process with parameter ρ. We can think of these shocks as occurring when the investor unexpectedly needs to draw on the funds in his brokerage account in order to finance his consumption. When a shock occurs, the investor liquidates his holdings and exits the stock market. Liquidity shocks serve an important purpose in our model: they ensure that the investor cares not only about realized gains and losses, but also about *paper* gains and losses. After all, even if a real-life investor cares about realization utility, he almost certainly also derives utility from paper gains and losses.

Suppose that, at time t, the investor's wealth is allocated to the stock. The investor's value function is a function of the current asset value, W_t, and of the asset's cost basis, B_t. We denote the value function as $V(W_t, B_t)$.

We further assume

$$V(W, W) > 0. \tag{6}$$

Note that $V(W, W)$ is the value function from investing in the stock *now*, so that the asset's current value and cost basis are both equal to current wealth W. Given a positive time discount rate, condition (6) implies two things. First, it implies that, at time 0, the investor allocates his wealth to the stock: since the risk-free asset generates no utility flows, he allocates to the stock as early as possible, in other words, at time 0. Second, condition (6) implies that, if, at any time $t > 0$, the investor sells a position in stock, he will then immediately re-establish the position. We verify the validity of condition (6) later.

[3] The investor only receives realization utility when he liquidates a position in stock and puts the proceeds into the risk-free asset, not when he sells the risk-free asset and puts the proceeds into stock. The reason is that, since the risk-free rate is zero, the realized gain or loss from selling the risk-free asset is always zero. We also assume that the investor does not incur a transaction cost when selling the risk-free asset.

We can now formulate the investor's decision problem. Let τ' be the random future time at which a liquidity shock occurs. Then, at time t, the investor solves

$$\begin{aligned} V\left(W_t, B_t\right) &= \max_{\tau \geq t} \; E_t\{e^{-\delta(\tau - t)}[u((1-k)W_\tau - B_\tau) + V\left((1-k)W_\tau, (1-k)W_\tau\right)]I_{\{\tau < \tau'\}} \\ &\qquad + e^{-\delta(\tau' - t)}u((1-k)W_{\tau'} - B_{\tau'})I_{\{\tau \geq \tau'\}}\}, \end{aligned} \tag{7}$$

subject to equation (3), (5), and

$$\frac{dW_s}{W_s} = \mu ds + \sigma dZ_s, \qquad t \leq s \leq \tau'. \tag{8}$$

The parameter δ is the time discount rate. To ensure that the investor does not hold his time 0 stock position forever, without selling it, we impose the following parameter restriction:

$$\max\left\{1, \frac{\delta}{\delta - \rho k}\right\}\mu < \rho + \delta. \tag{9}$$

To understand the formulation in (7), note that the investor's problem is to choose the optimal time τ, a random time in the future, at which to realize the gain or loss in his current position. Suppose first that $\tau < \tau'$, so that the investor voluntarily sells stock before a liquidity shock arrives. In this case, only the terms within the square parentheses are non-zero: when he liquidates his position at time τ, the investor receives a burst of utility $u((1-k)W_\tau - B_\tau)$ and a cash balance of $(1-k)W_\tau$ which he immediately reinvests in the stock. If $\tau \geq \tau'$, however, the investor is forced out of the stock market by a liquidity shock. In this case, only the final term is non-zero: the investor receives realization utility $u((1-k)W_{\tau'} - B_{\tau'})$ from the gain or loss at the moment of exit.

So far, we have interpreted the mathematical structure in (7) and (8) in terms of a *one*-stock model, so that, over time, the investor switches in and out of the same stock. However, the same mathematical structure also admits an alternative interpretation, which we prefer, and which we adopt going forward. Under this interpretation, the economy contains *many* stocks, all of which have the same return distribution in (1). At any time $t \geq 0$, the investor can either have all of his wealth in the risk-free asset, or all of his wealth in one of the stocks. And whenever the investor moves his wealth into a stock, the stock is a *new* stock, one that he has not previously owned.

Why is this a better interpretation? Under the one-stock interpretation, the objective function in (7) says that, if the investor liquidates his position in a stock and then immediately buys back the same stock, he nonetheless derives utility from the gain or loss realized at the moment of sale. This seems psychologically implausible. It is hard to imagine that the investor can derive realization utility from the sale of a stock if he then immediately buys the stock back: he can hardly claim credit for a successful "completed" transaction if he then immediately reopens the transaction. If, however, he sells a stock and then buys a *new* stock, it is more reasonable that he would derive utility from the gain or loss realized at the moment of sale: since the new stock is a different one, it is easier to think of the sale of the previous stock as a completed transaction.[4]

The proposition below summarizes the solution to the decision problem in (7). The variable

$$g_t = \frac{W_t}{B_t} \tag{10}$$

– in words, the percentage change in value, since purchase, of the risky asset the investor is holding at time t – plays an important role in the solution.

Proposition 1: Unless forced to exit the stock market by a liquidity shock, an investor with the decision problem in (7) will sell a position in stock once the gain $g_t = W_t/B_t$ reaches a liquidation point $g_* > 1$. The investor's value function is $V(W_t, B_t) = B_t U(g_t)$, where

$$U(g_t) = \begin{cases} a g_t^{\gamma_1} + \frac{\rho(1-k)}{\rho+\delta-\mu} g_t - \frac{\rho}{\rho+\delta} & \text{if} \quad g_t < g_* \\ (1-k) g_t (1 + U(1)) - 1 & \text{if} \quad g_t \geq g_* \end{cases}, \tag{11}$$

where

$$\gamma_1 = \frac{1}{\sigma^2}\left[\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2(\rho+\delta)\sigma^2} - \left(\mu - \frac{1}{2}\sigma^2\right)\right] > 0 \tag{12}$$

and

$$a = \frac{\delta}{g_*^{\gamma_1}(\gamma_1 - 1)(\rho+\delta)}. \tag{13}$$

The liquidation point g_* is the unique root, in the range $(1, \infty)$, of the following nonlinear equation:

$$(\gamma_1 - 1)\left(1 - \frac{\rho k(\rho+\delta)}{\delta(\rho+\delta-\mu)}\right) g_*^{\gamma_1} - \frac{\gamma_1}{1-k} g_*^{\gamma_1 - 1} + 1 = 0. \tag{14}$$

We prove the proposition in the Appendix. In brief, the proof proceeds by conjecturing that the investor sells his stock position once g_t exceeds some value g_*; by constructing the value function, first for the region below g_*, and then for the region above g_*; by requiring that the value function is continuous and continuously differentiable at g_*; and finally, by verifying that the constructed value function is indeed optimal.

[4]Even under the multi-stock interpretation, we still assume, for expositional simplicity, that the investor holds at most one stock at any time. However, the solution to (7) can also determine how the investor would trade in a setting where he holds *several* stocks concurrently. Suppose that he starts with wealth of mW_0 and spreads this wealth across m stocks, investing W_0 in each one. Suppose also, as is natural in the case of realization utility, that he derives utility *separately* from the realized gain or loss on each stock. The solution to (7) then describes how the investor trades each of his stocks in this multiple-concurrent-stock setting.

A corollary to Proposition 1 – one that holds even for the alternative preference specifications we consider in Section 3 – is that, under the most natural multiple-concurrent-stock extension of our model, the one described in footnote 4, the investor is indifferent to diversification. For example, he is indifferent between investing W_0 in just one stock at time 0 as compared to investing $W_0/2$ in each of two stocks at time 0: the value function for the first strategy, $W_0U(1)$, is the same as the value function for the second strategy, namely $W_0U(1)/2 + W_0U(1)/2$. If the investor's preferences included not only a realization utility term, but also a standard concave utility of consumption term, he would, for the usual reason, prefer to diversify.

Results

In this section, and again in Section 3, we draw out the implications of our model through two kinds of analysis. First, we look at the range of values of the model parameters for which the investor is willing to buy stock at time 0. Second, we look at how the liquidation point g_* and initial utility $U(1)$ depend on each of the model parameters. When assigning parameter values, we have in mind our model's most natural application, namely stock trading in brokerage accounts by individual investors.

The shaded area in the top graph in Figure 1 shows the range of values of the stock's expected return μ and standard deviation σ that satisfy $U(1) \geq 0$ – so that the investor is willing to buy the stock at time 0 – but also the restriction in (9), so that the investor is willing to sell the stock at a finite liquidation point. To create the graph, we need to assign values to the three remaining parameters, δ, k, and ρ. We set the time discount rate to $\delta = 0.08$ and the transaction cost to $k = 0.01$, which is of a similar order of magnitude to the transaction cost estimated by Barber and Odean (2000) for discount brokerage customers. Finally, we set $\rho = 0.1$. The probability of a liquidity shock over the course of a year is therefore $1 - e^{-0.1} \approx 0.1$.

The graph illustrates an interesting feature of our model, namely that the investor is willing to invest in a stock with a *negative* expected return, so long as its standard deviation σ is sufficiently high. The intuition is simple. So long as σ is sufficiently high, even a negative expected return stock has *some* chance of reaching the liquidation point g_*, at which time the investor can enjoy realizing the gain. Of course, more likely than not, the stock will lose value. However, since the investor does not voluntarily realize losses, this will only bring him disutility in the event of a liquidity shock. Any realized loss therefore lies in the distant, heavily discounted future and does not scare the investor very much. Overall, then, investing in stock, even if it has a negative expected return, is a better option than investing in the risk-free asset, which offers zero utility for sure.

Figures 2 and 3 show how the liquidation point g_* and time 0 utility $U(1)$ depend on the parameters μ, σ, δ, k, and ρ. The graphs on the left side of each figure correspond to the liquidation point, and those on the right side, to time 0 utility. For now, we focus on the solid lines; we discuss the dashed lines in Section 3.1.

To construct the graphs, we start with a set of benchmark parameters. We use the same benchmark parameters throughout the paper. We set the average excess return on stock to $\mu = 0.03$ and its standard deviation to $\sigma = 0.5$. We use a time discount rate of $\delta = 0.08$, a transaction cost of $k = 0.01$, and a liquidity shock intensity of $\rho = 0.1$. The graphs in Figures 2 and 3 show what happens as we vary each of μ , σ, δ, k, and ρ in turn, keeping the other parameters fixed at their benchmark levels.

The top graphs in Figure 2 show that, as we would expect, time 0 utility is increasing in the mean stock return μ. The liquidation point is also increasing in μ: if a stock that is trading at a gain has a high expected return, the investor is tempted to hold on to it, rather than sell it and incur a transaction cost.

The middle graphs illustrate one of the important predictions of our model: that, as stock return volatility goes up, the investor's time 0 utility also goes up. Put differently, even though the form of realization utility is linear, the investor is risk-*seeking*. While this is initially surprising, there is a simple intuition for it. A highly volatile stock offers the chance of a significant gain, which the investor can enjoy realizing. Of course, it also offers the chance of a significant loss. But the investor does not voluntarily realize losses, and so will only experience disutility in the event of a liquidity shock. Any realized loss therefore lies in the distant, heavily discounted future and does not scare the investor very much. Overall, then, more volatile stocks are more attractive. From a mathematical perspective, this prediction is a consequence of the fact that, while instantaneous utility is linear, the value function $U(g_t)$ in (11) is convex.[5]

The bottom-left graph in Figure 2 shows that, when the investor discounts the future more heavily, the liquidation point falls. An investor with a high discount rate is more impatient, and therefore cannot wait very long before realizing a gain.

The bottom-right graph shows that initial utility is relatively insensitive to the discount rate. There are two opposing forces at work here. On the one hand, a lower δ means that future utility flows are discounted at a lower rate, thereby raising initial utility. On the other hand, since an investor with a low discount rate sets a high liquidation point, he may have to endure the unpleasant scenario whereby the stock initially rises quite high, although not as high as the liquidation point, and then falls, generating a paper loss from which he is eventually forced to exit by a liquidity shock. By contrast, an investor with a high discount rate and hence a low liquidation point is less likely to experience such a scenario. These opposing effects lead to a relatively flat relationship between initial utility and the discount rate.

[5]The parameter restriction in (9) implies $\gamma_1 > 1$ and $a > 0$, which, in turn, implies the convexity of $U(\cdot)$.

The top graphs in Figure 3 show how the liquidation point and initial utility depend on the transaction cost k. As expected, a higher transaction cost lowers the investor's time 0 utility. It also increases the liquidation point: given that it is costly to liquidate a position, the investor waits longer before doing so.

What happens when there is no transaction cost? The top-left graph in Figure 3 suggests that, in this case, the liquidation point is $g_* = 1$. It is straightforward to confirm that, when $k = 0$, equation (14) is indeed satisfied by $g_* = 1$, so that the investor realizes all gains immediately. In other words, in our model, it is the transaction cost that stops the investor from realizing all gains as soon as they appear.

We suspect that, in reality, a transaction cost is not the only thing that deters an investor who cares about realization utility from realizing all gains immediately. For example, in reality, an investor may be reluctant to sell a stock until he has located *another* attractive stock to transfer the proceeds of a sale to. While our model assumes that an alternative investment is always available, it may, in reality, take the investor some time to locate such an investment. The harder it is to locate an alternative attractive investment, the more reluctant the investor will be to liquidate his current position.

To confirm this idea, we have analyzed an extension of our model in which, after selling his position in a stock, the investor is required to park the proceeds in the risk-free asset for T periods, during which time he searches for a new stock to buy: the higher T is, the harder it is to uncover a new investment opportunity. We find that the investor's optimal strategy in this model is, once again, to sell a position in stock only when the stock price reaches a sufficiently large premium to purchase price. In line with the intuition above, we find that the liquidation point is indeed increasing in T: the harder it is to locate a new investment opportunity, the more reluctant the investor is to liquidate his current position.[6]

The bottom graphs in Figure 3 show how the liquidation point g_* and initial utility $U(1)$ depend on the intensity of the liquidity shock ρ. The bottom-left graph shows that the liquidation point depends on ρ in a non-monotonic way. There are two factors at work here. As the liquidity shock intensity ρ goes up, the liquidation point initially falls. One reason the investor delays realizing a gain is the transaction cost that a sale entails. In the presence of liquidity shocks, however, the investor knows that he is likely to be forced out of the stock market at some point. The present value of the transaction costs he expects to pay is therefore lower than in the absence of liquidity shocks. As a result, he is willing to realize gains sooner.

At higher levels of ρ, however, there is a second factor which makes the investor more patient. If he is holding a stock with a gain, he is reluctant to exit the position, because he will then have to reinvest in another stock, which might do poorly, and from which he might

[6]For space reasons, we do not present the details of this analysis here. They are available upon request.

be forced to exit at a loss by a liquidity shock. This factor pushes the liquidation point back up.

The bottom-right graph shows that, as the liquidity shock intensity ρ rises, the agent's utility falls. Since a liquidity shock can force the investor to exit the stock market with a painful loss, it lowers his utility.

3 Other Preference Specifications

In Section 2, we took the functional form for realization utility $u(\cdot)$ to be linear, and assumed exponential time discounting. In Section 3.1, we consider an alternative specification – piecewise-linear utility – and show how it affects the results. In Section 3.2, we alter another dimension of preferences by replacing exponential discounting with hyperbolic discounting.

3.1 Piecewise linear utility

We used a linear functional form in Section 2 because of its simplicity, but also to show that we do not need strong assumptions about the form of utility in order to make interesting predictions. We now look at what happens when $u(\cdot)$ is piecewise-linear, rather than linear, so that the investor is more sensitive to realized losses than to realized gains:

$$u(x) = \begin{cases} x & \text{if} \quad x \geq 0 \\ \lambda x & \text{if} \quad x < 0 \end{cases}, \qquad \lambda > 1. \tag{15}$$

The parameter λ controls the relative sensitivity to realized losses, as opposed to realized gains.

It is not clear, ex-ante, whether a piecewise-linear form is more reasonable than a linear one. There is, of course, the well-known concept of "loss aversion" – but this is the idea that people are more sensitive to *wealth* losses than to wealth gains; in other words, more sensitive to *paper* losses than to paper gains. It is the premise of this paper that utility from realized gains and losses is distinct from utility from paper gains and losses, and that it involves different psychological factors. Even if people are more sensitive to paper losses as opposed to paper gains, we do not conclude that they are also more sensitive to realized losses, as opposed to realized gains.

The investor's decision problem is:

$$\begin{aligned} V(W_t, B_t) \;=\; \max_{\tau \geq t} \;\; & E_t\{e^{-\delta(\tau-t)}[u((1-k)W_\tau - B_\tau) + V((1-k)W_\tau, (1-k)W_\tau)]I_{\{\tau<\tau'\}} \\ & + e^{-\delta(\tau'-t)}u((1-k)W_{\tau'} - B_{\tau'})I_{\{\tau\geq\tau'\}}\}, \end{aligned} \tag{16}$$

subject to (3), (8), and (15). This is the same as decision problem (7) in Section 2, except that $u(\cdot)$ is no longer linear, but instead takes the form in (15).

In the Appendix, we prove:

Proposition 2: Unless forced to exit the stock market by a liquidity shock, an investor with the decision problem in (16) will sell a position in stock once the gain $g_t = W_t/B_t$ reaches a liquidation point $g_* > 1$. His value function is $V(W_t, B_t) = B_t U(g_t)$, where

$$U(g_t) = \begin{cases} b g_t^{\gamma_1} + \frac{\rho\lambda(1-k)}{\rho+\delta-\mu} g_t - \frac{\rho\lambda}{\rho+\delta} & \text{if} \quad g_t \in \left(0, \frac{1}{1-k}\right) \\ c_1 g_t^{\gamma_1} + c_2 g_t^{\gamma_2} + \frac{\rho(1-k)}{\rho+\delta-\mu} g_t - \frac{\rho}{\rho+\delta} & \text{if} \quad g_t \in \left(\frac{1}{1-k}, g_*\right) \\ (1-k) g_t (1 + U(1)) - 1 & \text{if} \quad g_t \in (g_*, \infty) \end{cases}, \tag{17}$$

where γ_1 is defined in equation (12), where

$$\gamma_2 = -\frac{1}{\sigma^2}\left[\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2(\rho+\delta)\sigma^2} + \left(\mu - \frac{1}{2}\sigma^2\right)\right] < 0, \tag{18}$$

and where b, c_1, c_2, and g_* are determined from

$$c_2 = \frac{(\lambda - 1)\rho(1-k)^{\gamma_2}(\mu\gamma_1 - \rho - \delta)}{(\gamma_1 - \gamma_2)(\rho + \delta - \mu)(\rho + \delta)} \tag{19}$$

$$(\gamma_1 - 1) c_1 g_*^{\gamma_1} + (\gamma_2 - 1) c_2 g_*^{\gamma_2} = \frac{\delta}{\rho + \delta} \tag{20}$$

$$c_1 \left(\frac{1}{1-k}\right)^{\gamma_1} + c_2 \left(\frac{1}{1-k}\right)^{\gamma_2} = b\left(\frac{1}{1-k}\right)^{\gamma_1} + \frac{(\lambda-1)\mu\rho}{(\rho+\delta-\mu)(\rho+\delta)} \tag{21}$$

$$c_1 g_*^{\gamma_1} + c_2 g_*^{\gamma_2} + \frac{(1-k)(\mu-\delta)}{\rho+\delta-\mu} g_* + \frac{\delta}{\rho+\delta} = (1-k) g_* \left(b + \frac{\rho\lambda(\mu - k\rho - k\delta)}{(\rho+\delta)(\rho+\delta-\mu)}\right). \tag{22}$$

Specifically, given values for μ, σ, δ, k, ρ, and λ, we first use equation (19) to find c_2; we then obtain c_1 from equation (20); we then use equation (21) to find b; finally, equation (22) allows us to solve for the liquidation point g_*.

Results

The shaded area in the lower graph in Figure 1 shows the range of values of μ and σ for which the investor is both willing to buy stock at time 0, so that $U(1)$, from (17), is positive, and also to sell it at a finite liquidation point. We set δ, k, and ρ to the benchmark values from before, namely 0.08, 0.01, and 0.1, respectively. We further assign λ the benchmark value of 1.5. Relative to the upper graph – the graph for the Section 2 model with *linear* realization utility – we see that the investor is now more reluctant to invest in a stock with a negative expected return. For a realization utility investor, the problem with a negative expected return stock is that it raises the chance that he will be forced, by a liquidity shock, to make a painful exit from a losing position. A high sensitivity to losses makes this prospect all the more unappealing. The investor will therefore only invest in a negative expected return stock if it is highly volatile, so that it at least offers a chance of a sizeable gain which he can enjoy realizing.

The graphs in Figure 4 show how the liquidation point g_* and initial utility $U(1)$ depend on the sensitivity to losses λ. Specifically, these graphs vary λ while maintaining

$$(\mu, \sigma, \delta, k, \rho) = (0.03, 0.5, 0.08, 0.01, 0.1). \tag{23}$$

In the top-left graph, we see that, the more sensitive the investor is to losses, the higher the liquidation point. The intuition is that, if the investor is holding a stock with a gain, he is reluctant to realize that gain, because if he does, he will have to invest the proceeds in a new stock, which might go down, and from which he might be forced to exit at a loss by a liquidity shock.

The top-right graph shows that, as the sensitivity to losses goes up, the investor's utility falls: a high λ raises the possibility that the investor may be forced, by a liquidity shock, to make a painful exit from a losing position.

The dashed lines in Figure 2 show how the liquidation point g_* and initial utility $U(1)$ depend on μ, σ, and δ when the investor is more sensitive to losses than to gains. Here, we vary each of μ , σ, and δ in turn, keeping the other parameters fixed at their benchmark values,

$$(\mu, \sigma, \delta, k, \rho, \lambda) = (0.03, 0.5, 0.08, 0.01, 0.1, 1.5). \tag{24}$$

Recall how the calculations for the solid lines in Figure 2 differ from those for the dashed lines: the solid lines correspond to linear realization utility, so that $\lambda = 1$; the dashed lines assume $\lambda = 1.5$. The dashed lines show that, for our benchmark parameter values, allowing for greater sensitivity to losses preserves the qualitative relationship between g_* and $U(1)$ on the one hand, and μ, σ, and δ on the other. As expected from Figure 4, increasing λ increases the liquidation point g_* and lowers utility $U(1)$.

The dashed line in the middle-right graph of Figure 2 deserves particular attention. It shows that, for the benchmark values in (24), the investor's initial utility $U(1)$ is still increasing in stock volatility σ. Put differently, even though the form of realization utility is now concave, the investor is still risk-seeking. If the sensitivity to losses λ or the intensity of liquidity shocks ρ rise significantly, however, this relationship will reverse, so that $U(1)$ becomes a decreasing function of σ.

3.2 Hyperbolic discounting

In the models presented so far, we assumed *exponential* time discounting. This is a standard assumption, and one that implies time-consistent behavior. Recently, however, there has been mounting evidence that time preferences are better captured by *hyperbolic* time discounting. Relative to the exponential case, hyperbolic discounting places more weight on the present, as opposed to the future: under hyperbolic discounting, immediate rewards are especially attractive, and immediate costs, especially repellent.

While hyperbolic discounting has been linked to a number of economic phenomena, researchers have not, as yet, found many applications for it within the specific context of finance. We now show that, as soon as we allow for realization utility, hyperbolic discounting can play an interesting role. Specifically, it leads an investor who cares about realization utility to realize gains *earlier* than suggested by exponential discounting: realizing a gain *now* provides an immediate reward, and, under hyperbolic discounting, this is highly valued. Hyperbolic discounting therefore amplifies the effect of realization utility, and, as such, it may play a role in the phenomena that we link to realization utility in Section 5.

Harris and Laibson (2004) and Grenadier and Wang (2007) show how hyperbolic discounting can be introduced into a continuous-time framework. We now use their approach to incorporate hyperbolic discounting into the model of Section 2.

Hyperbolic discounting is modeled by thinking of the investor as a sequence of different "selves," each of which exercises control at a different time. Specifically, from the vantage point of time 0, we divide the investor's horizon into two periods: a "present," which lasts until some random time $s > 0$; and a "future," which starts at time s. We think of the "present" as an interval during which control is exercised by the current self, and the "future" as an interval which is controlled by future selves. We assume that s follows a Poisson process with parameter ϕ, independent of other processes. At time s, a future self appears. That self's horizon can also be divided into a "present," which lasts until some random time, and a "future"; and so on.

Each self discounts utility which accrues within its "present" at $e^{-\delta t}$, and utility which accrues during its "future" at $\beta e^{-\delta t}$. The parameter $\beta < 1$ captures the idea that, under hyperbolic discounting, the present receives extra weight, relative to the future.

Hyperbolic discounting implies time-inconsistent behavior: the current self and future selves have different time preferences. The current self's beliefs about the actions of future selves are therefore important. Using the terminology of the literature, the current self can be "sophisticated," in that he correctly forecasts that future selves will use hyperbolic discounting; or he can be "naive," inaccurately believing that future selves will discount exponentially. For space reasons, we only analyze the naive case here. We expect the results for the sophisticated case to be qualitatively similar.

Consider a naive hyperbolic discounter who is holding stock at time t. Let τ' be the random future time at which a liquidity shock arrives, and let τ'' be the random future time at which the next self arrives. The current self's decision problem is then

$$\begin{aligned} & N\left(W_t, B_t\right) \quad (25) \\ = & \max_{\tau \geq t} \; E_t\{e^{-\delta(\tau-t)}\left[u((1-k)W_\tau - B_\tau) + N\left((1-k)W_\tau, (1-k)W_\tau\right)\right] I_{\{\tau<\min\{\tau',\tau''\}\}} \\ & \quad + e^{-\delta(\tau'-t)} u((1-k)W_{\tau'} - B_{\tau'}) I_{\{\tau'<\min\{\tau,\tau''\}\}} + e^{-\delta(\tau''-t)} \widehat{N}(W_{\tau''}, B_{\tau''}) I_{\{\tau''<\min\{\tau,\tau'\}\}}\}. \end{aligned}$$

To understand this, it is useful to distinguish between three possible scenarios, defined by which of the following three events occurs first: the investor sells his position voluntarily, a liquidity shock arrives, or the next self arrives.

Suppose first that the investor sells voluntarily before either the liquidity shock or the next self arrives, so that $\tau < \min\{\tau', \tau''\}$. In this case, only the first two terms on the right-hand side are non-zero: the investor receives realization utility of $u((1-k)W_\tau - B_\tau)$ and cash proceeds of $(1-k)W_\tau$ which he promptly reinvests. The second possibility is that the liquidity shock arrives before he has had a chance to sell, and before the next self arrives, so that $\tau' < \min\{\tau, \tau''\}$. In this case, only the third term on the right-hand side is non-zero: the investor liquidates his holdings and receives realization utility of $u((1-k)W_{\tau'} - B_{\tau'})$. The final possibility is that the next self arrives before the current self has had a chance to sell, and before a liquidity shock, so that $\tau'' < \min\{\tau, \tau'\}$. In this case, only the fourth term on the right-hand side is non-zero: the current self receives the value function that he *thinks* will result from the actions of the future self. We use $\widehat{N}(W_t, B_t)$ to denote this perceived value function.

An important step is to notice that

$$\widehat{N}(W_t, B_t) = \beta V(W_t, B_t). \quad (26)$$

Since the current self is a *naive* hyperbolic discounter, he thinks that future selves will use exponential discounting, and therefore that they will follow a strategy of selling once the gain reaches the liquidation point g_* derived in Proposition 1. The value function that the current self thinks will result from the actions of future selves is therefore $\beta V(W_t, B_t)$: the value function of an exponential discounter multiplied by β. The β factor appears because the current self discounts utility flows in the "future" period at $\beta e^{-\delta t}$ rather than at $e^{-\delta t}$.

In the Appendix, we prove:

Proposition 3: Unless forced to exit the stock market by a liquidity shock, an investor with the decision problem in (25) will sell a position in stock once the gain $g_t = W_t/B_t$ reaches a liquidation point $g_{**} > 1$. The value function is $N(W_t, B_t) = B_t n(g_t)$, where

$$n(g_t) = \begin{cases} d_1 g_t^{\gamma_1} + d_2 g_t^{\gamma_3} + d_3 g_t + d_4 & \text{if} \quad g_t < g_{**} \\ \\ (1-k) g_t (1 + n(1)) - 1 & \text{if} \quad g_t \geq g_{**} \end{cases}, \tag{27}$$

where γ_1 is given in (12),

$$\gamma_3 = \frac{1}{\sigma^2}\left[\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\left(\rho + \delta + \phi\right)\sigma^2} - \left(\mu - \frac{1}{2}\sigma^2\right)\right] > 0 \tag{28}$$

$$d_1 = \frac{a\phi\beta}{\rho + \delta + \phi - \mu\gamma_1 - \frac{\sigma^2}{2}\gamma_1(\gamma_1 - 1)} \tag{29}$$

$$d_3 = \frac{\rho(1-k)(1 + \frac{\beta\phi}{\rho+\delta-\mu})}{\rho + \delta + \phi - \mu} \tag{30}$$

$$d_4 = \frac{-\rho(1 + \frac{\beta\phi}{\rho+\delta})}{\rho + \delta + \phi}, \tag{31}$$

and where d_2 and g_{**} are jointly determined by

$$d_1 g_{**}^{\gamma_1} + d_2 g_{**}^{\gamma_3} + d_3 g_{**} + d_4 = (1-k) g_{**}(1 + d_1 + d_2 + d_3 + d_4) - 1 \tag{32}$$

$$d_1 \gamma_1 g_{**}^{\gamma_1 - 1} + d_2 \gamma_3 g_{**}^{\gamma_3 - 1} + d_3 = (1-k)(1 + d_1 + d_2 + d_3 + d_4). \tag{33}$$

The parameter a is the same as in Proposition 1, and is determined from equations (13) and (14).

Results

The top graphs in Figure 5 show how the liquidation point and initial utility $n(1)$ depend on the hyperbolic discounting parameter β. Here, we vary β while maintaining

$$(\mu, \sigma, \delta, k, \rho, \phi) = (0.03, 0.5, 0.08, 0.01, 0.1, 3). \tag{34}$$

In particular, we set the arrival intensity of new selves to $\phi = 3$. The key finding is that, as we predicted earlier, hyperbolic discounting makes the investor more impatient to realize gains: as β falls, the liquidation point g_{**} also falls. Initial utility is relatively insensitive to β, a finding that parallels the flat relationship between initial utility and the discount rate δ in the bottom-right graph of Figure 2.

The solid lines in the middle and bottom panels of Figure 5 show how the liquidation point g_{**} and initial utility $n(1)$ depend on the stock's expected return μ and standard deviation σ. When we vary μ or σ, we keep the remaining parameters fixed at their benchmark values, namely

$$(\mu, \sigma, \delta, k, \rho, \phi, \beta) = (0.03, 0.5, 0.08, 0.01, 0.1, 3, 0.8). \tag{35}$$

Note that the benchmark value of β is 0.8.

The dashed lines in these graphs show what happens in the exponential discounting case, in other words, when $\beta = 1$ and $\phi = 0$. By comparing the solid and dashed lines, we see that hyperbolic discounting preserves the qualitative relationship between the liquidation point and μ and σ; the effect is simply to shift the liquidation point down. It also preserves the qualitative relationship between initial utility and μ and σ; it simply shifts the utility level down.

4 Asset Pricing

In Sections 2 and 3, we studied realization utility in a partial equilibrium context. In this section, we show how realization utility can be embedded in a full equilibrium model. This, in turn, will allow us to understand its implications for asset prices. Of course, for realization utility to affect prices, *many* investors must care about it. Ex-ante, it is hard to know whether this is the case. Perhaps the best way to find out is to derive the pricing implications of realization utility and to see if this sheds light on puzzling facts, or if it leads to new predictions which can be tested and confirmed.

We study the simplest possible equilibrium model, one with *homogeneous* realization utility investors. Specifically, consider an economy with a risk-free asset and N risky stocks, indexed by $i \in \{1, \ldots, N\}$. The risk-free asset is in perfectly elastic supply and earns a net return of zero. The risky stocks are in limited supply and the price process for stock i is

$$\frac{dS_i}{S_i} = \mu_i dt + \sigma_i dZ_{i,t}, \tag{36}$$

where σ_i is constant over time. We assume, for now, that μ_i is also constant over time, and confirm this assumption later.

The economy contains a continuum of realization utility investors. At each time $t \geq 0$, each investor must either allocate all of his wealth to the risk-free asset, or all of his wealth to one of the stocks. We allow for transaction costs, liquidity shocks, and piecewise-linear utility. As noted above, investors are homogeneous, so that δ, ρ, and λ are the same for all investors. Transaction costs, however, can differ across stocks. The transaction cost for stock i is k_i.

In this economy, a condition for equilibrium is

$$V(W, W) = 0. \tag{37}$$

In words, this means that an investor who is buying a stock is indifferent between allocating his wealth to that stock or to the risk-free asset. Under this condition, we can clear markets at time 0 by assigning some investors to each stock and the rest to the risk-free asset. If, at any point in the future, some investors sell their holdings of stock i – whether because of a liquidity shock or because, for these investors, the stock has reached its liquidation point – condition (37) means that we can reassign some investors from the risk-free asset to the stock, thereby again clearing markets. Condition (37) will allow us to determine the equilibrium expected return of each stock.

Formally, the decision problem for an investor holding stock i at time t is

$$V\left(W_t, B_t\right) = \max_{\tau \geq t} \quad E_t\{e^{-\delta(\tau-t)}u((1-k_i)W_\tau - B_\tau)I_{\{\tau<\tau'\}} + e^{-\delta(\tau'-t)}u((1-k_i)W_{\tau'} - B_{\tau'})I_{\{\tau\geq\tau'\}}\}, \tag{38}$$

where τ' is the random future time at which a liquidity shock arrives. This differs from the decision problem in (16) in that it imposes the market clearing condition (37): after selling his stock holdings at time τ, the investor's future value function is zero. We summarize the solution to the decision problem in (38) in the following proposition. The proof is in the Appendix.

Proposition 4: Unless forced to exit the stock market by a liquidity shock, an investor in the economy described above will sell a position in stock once the gain $g_t = W_t/B_t$ reaches a liquidation point $g_* > 1$. His value function when holding stock i at time t is $V(W_t, B_t) = B_t U(g_t)$, where

$$U(g_t) = \begin{cases} b g_t^{\gamma_1} + \frac{\rho\lambda(1-k_i)}{\rho+\delta-\mu_i} g_t - \frac{\rho\lambda}{\rho+\delta} & \text{if} \quad g_t \,\epsilon\, (0, \frac{1}{1-k_i}) \\ c_1 g_t^{\gamma_1} + c_2 g_t^{\gamma_2} + \frac{\rho(1-k_i)}{\rho+\delta-\mu_i} g_t - \frac{\rho}{\rho+\delta} & \text{if} \quad g_t \,\epsilon\, (\frac{1}{1-k_i}, g_*) \\ (1-k_i) g_t - 1 & \text{if} \quad g_t \,\epsilon\, (g_*, \infty) \end{cases}, \tag{39}$$

where γ_1 and γ_2 are given by

$$\gamma_1 = \frac{1}{\sigma_i^2}\left[\sqrt{\left(\mu_i - \frac{1}{2}\sigma_i^2\right)^2 + 2\left(\rho+\delta\right)\sigma_i^2} - \left(\mu_i - \frac{1}{2}\sigma_i^2\right)\right] > 0 \tag{40}$$

$$\gamma_2 = -\frac{1}{\sigma_i^2}\left[\sqrt{\left(\mu_i - \frac{1}{2}\sigma_i^2\right)^2 + 2\left(\rho+\delta\right)\sigma_i^2} + \left(\mu_i - \frac{1}{2}\sigma_i^2\right)\right] < 0, \tag{41}$$

and where b, c_1, c_2, and g_* are determined from

$$c_2 = \frac{\left(\lambda-1\right)\rho(1-k_i)^{\gamma_2}\left(\mu_i\gamma_1 - \rho - \delta\right)}{\left(\gamma_1-\gamma_2\right)\left(\rho+\delta-\mu_i\right)\left(\rho+\delta\right)} \tag{42}$$

$$\left(\gamma_1 - 1\right) c_1 g_*^{\gamma_1} + \left(\gamma_2 - 1\right) c_2 g_*^{\gamma_2} = \frac{\delta}{\rho+\delta} \tag{43}$$

$$c_1\left(\frac{1}{1-k_i}\right)^{\gamma_1} + c_2\left(\frac{1}{1-k_i}\right)^{\gamma_2} = b\left(\frac{1}{1-k_i}\right)^{\gamma_1} + \frac{(\lambda-1)\mu_i\rho}{(\rho+\delta-\mu_i)(\rho+\delta)} \tag{44}$$

$$c_1 g_*^{\gamma_1} + c_2 g_*^{\gamma_2} + (1-k_i) g_* \frac{\mu_i - \delta}{\rho+\delta-\mu_i} = -\frac{\delta}{\rho+\delta}. \tag{45}$$

To determine μ_i, the equilibrium expected return of stock i, we require that the value function satisfy condition (37), namely $V(W, W) = 0$, or equivalently, $U(1) = 0$. The parameter μ_i therefore satisfies

$$b + \frac{\rho\lambda(1 - k_i)}{\rho + \delta - \mu_i} - \frac{\rho\lambda}{\rho + \delta} = 0. \quad (46)$$

Since the parameters δ, ρ, and λ are constant across investors, μ_i is constant over time, as assumed earlier.

5 Applications

Our model may be helpful for thinking about a wide range of financial phenomena. We now discuss some of these potential applications. We divide the applications into those that relate to investor trading behavior (Section 5.1); and those that relate to asset pricing (Section 5.2). In Section 5.3, we discuss some of our model's testable predictions.

5.1 Investor trading behavior

The disposition effect

The disposition effect is the finding that individual investors have a greater propensity to sell stocks that have gone *up* in value since purchase, rather than stocks that have gone down (Odean, 1998). This fact has turned out to be something of a puzzle, in that the most obvious explanations fail to explain important features of the data. Consider, for example, the most obvious explanation of all, the "informed trading" hypothesis. Under this view, investors sell stocks that have gone up in value because they have private information that these stocks will subsequently fall, and they hold on to stocks that have gone down in value because they have private information that these stocks will subsequently rebound. The difficulty with this explanation, as Odean (1998) points out, is that the prior winners people sell subsequently do *better*, on average, than the prior losers they hold on to. Odean (1998) also considers other potential explanations based on taxes, rebalancing, and transaction costs, but argues that all of them fail to capture important aspects of the data.

Our analysis shows that a model that combines realization utility with a positive time discount rate predicts a strong disposition effect. In fact, unless forced to sell by a liquidity shock, the investor in our model *only* sells stocks trading at a gain, never a stock trading at a loss.

In simple two-period settings, Shefrin and Statman (1985) and Barberis and Xiong (2008) show that realization utility, with no time discounting but with a prospect theory functional form for utility, can predict a disposition effect. This paper proposes a related, but distinct view of the disposition effect, namely that it arises from realization utility coupled with a *linear* functional form for utility and a *positive* time discount rate.

We emphasize that realization utility does not, on its own, predict a disposition effect. In other words, it is not enough to assume that the investor derives pleasure from realizing a gain and pain from realizing a loss. We need an extra ingredient in order to explain why the investor would want to realize a gain *today*, rather than hold out for the chance of realizing an even bigger gain tomorrow. Shefrin and Statman (1985) and Barberis and Xiong (2008) point out one possible extra ingredient: a prospect theory functional form for utility, and, in particular, a utility function that is concave over gains and convex over losses. Such a functional form indeed explains the expediting of gains and the postponement of losses. Here, we propose an alternative extra ingredient: a sufficiently positive time discount rate.

Our model is also well-suited for thinking about the disposition-type effects that have been uncovered in other settings. Genesove and Mayer (2001), for example, find that homeowners are reluctant to sell their houses at prices below the original purchase price; and Heath, Huddart, and Lang (1999) find that executives are more likely to exercise stock options when the underlying stock price exceeds a reference point – the stock's highest price over the previous year – than when it falls below that reference point. Our analysis shows that a model that combines linear realization utility with a positive time discount rate can capture this evidence very easily.

Of all the applications we consider in Section 5, the disposition effect is perhaps the most obvious, in the sense that the distance between assumption and conclusion is relatively small. We emphasize, however, that realization utility is in no sense a "relabelling" of the disposition effect. To the contrary, it is just one of a number of possible theories of the disposition effect, and can be distinguished from these other theories through carefully constructed tests.

An example of a clever test that distinguishes various theories of the disposition effect can be found in Weber and Camerer (1995). In particular, these authors test the realization utility view of the disposition effect against the alternative view that it is driven by an irrational belief in mean-reversion. In a laboratory setting, they ask subjects to trade six stocks over a number of periods. In each period, each stock can either go up or down. The six stocks have different probabilities of going up in any period, ranging from 0.35 to 0.65, but subjects are not told which stock is associated with each possible up-move probability.

Weber and Camerer (1995) find that, just as in field data, their subjects exhibit a disposition effect: they have a greater propensity to sell stocks trading at a gain relative to purchase price, rather than stocks trading at a loss. To try to understand the source of the effect, the authors consider an additional experimental condition in which the experimenter liquidates subjects' holdings, and then tells them that they are free to reinvest the proceeds in any way they like. If subjects were holding on to their losing stocks because they thought that these stocks would rebound, we would expect them to re-establish their positions in these losing stocks. In fact, subjects do *not* re-establish these positions. This casts doubt on the mean-reversion view of the disposition effect, and lends support to the realization utility view, namely that subjects were refusing to sell their losers simply because it would have been painful to do so. Under this view, subjects were relieved when the experimenter intervened and did it for them.

Excessive trading

Using a database of trading activity at a large discount brokerage firm, Barber and Odean (2000) show that, before transaction costs, the average return of the individual investors in their sample is on par with the returns on a range of benchmarks; but that, after transaction costs, it falls *below* the benchmark returns. This last finding is puzzling: Why do people trade so much, when their trading activity hurts their performance? Barber and Odean (2000) consider a number of potential explanations, including taxes, rebalancing, and liquidity needs, but conclude that none of them can fully explain the patterns they observe.

Our model offers a simple explanation for this post-transaction-cost underperformance. From the perspective of investors, the underperformance is compensated by the occasional bursts of positive utility they experience when they realize gains.

It is straightforward to compute the probability that, over any interval after he first establishes a position in stock, the investor in our model trades at least once. This is not the same thing as a turnover rate, but it is related, and can therefore help us compare the trading frequency predicted by our model with that observed in actual brokerage accounts. When the investor first establishes a position in stock, $g_0 = 1$. When g_t reaches an upper barrier $g_* > 1$ or when a liquidity shock arrives, he sells the stock. To compute the probability that, after establishing a position, the investor trades at least once in the s periods thereafter, we therefore need to compute the probability that g_t passes g_* in $(0, s)$ or that a liquidity shock arrives during the same interval. The next proposition, which we prove in the Appendix, reports the result of this calculation.

Proposition 5: The probability that at least one trade occurs in $(0, s)$ is:

$$\begin{aligned} G(s) &= 1 - e^{-\rho s} \\ &\quad + e^{-\rho s}\left[N\left(\frac{-\ln g_* + \left(\mu - \frac{\sigma^2}{2}\right)s}{\sigma\sqrt{s}}\right) + e^{\left(\frac{2\mu}{\sigma^2}-1\right)\ln g_*} N\left(\frac{-\ln g_* - \left(\mu - \frac{\sigma^2}{2}\right)s}{\sigma\sqrt{s}}\right)\right] \end{aligned} \quad (47)$$

The expression within the square parentheses in (47) is the probability that g_t reaches g_* in the interval $(0, s)$. Equation (47) therefore has a simple interpretation. The investor trades during the interval $(0, s)$ if one of two mutually exclusive events occurs: if there is a liquidity shock in $(0, s)$; or if there is no liquidity shock in $(0, s)$ but g_t reaches g_* within $(0, s)$. The probability of a trade in $(0, s)$ is therefore the probability of a liquidity shock in $(0, s)$, namely $1 - e^{-\rho s}$, plus the probability of no liquidity shock, namely $e^{-\rho s}$, multiplied by the probability that g_t reaches g_*.

Figure 6 shows how the probability of at least one trade in a stock over the year after it is bought, $G(1)$, depends on the model parameters. For the first five graphs in Figure 6 – for the graphs that correspond to μ, σ, δ, k , and λ – we use the model of Section 3.1, which allows for a transaction cost, a liquidity shock, and piecewise-linear utility. For any given parameter values, we compute the liquidation point g_* from equations (19)-(22) and substitute the result into the expression for $G(1)$ in Proposition 5. The graphs vary each of μ, σ, δ, k, and λ in turn, while keeping the remaining parameters fixed at their benchmark values

$$(\mu, \sigma, \delta, k, \rho, \lambda) = (0.03, 0.5, 0.08, 0.01, 0.1, 1.5).$$

For the sixth graph in Figure 6 – the graph that corresponds to β – we use the hyperbolic discounting model of Section 3.2. For any given parameter values, we compute g_* from equations (32)-(33) and substitute the result into the expression for $G(1)$ in Proposition 5. The graph varies β while keeping the remaining parameters fixed at the values:

$$(\mu, \sigma, \delta, k, \rho, \phi) = (0.03, 0.5, 0.08, 0.01, 0.1, 3).$$

Some of the results in Figure 6 are not surprising. As the investor becomes more impatient – as the exponential discount rate δ goes up, or as the hyperbolic discount rate β goes down – the probability of a trade rises. And as transaction costs fall, the probability of a trade again rises.

The graphs with μ and σ on the horizontal axis are less predictable. In both cases, there are two factors at work. On the one hand, for any fixed liquidation point g_*, a higher μ or σ raises the likelihood that g_* will be reached. However, as we saw in Figure 2, the liquidation point g_* itself goes up as μ and σ go up, thereby lowering the chance that g_* will be reached. Without computing $G(1)$ explicitly, we cannot tell which factor will dominate.

Figure 6 shows that, interestingly, a different factor dominates in each of the two cases. As μ rises, the probability of a trade *falls*. Roughly speaking, as μ rises, the liquidation point rises more quickly than the stock's ability to catch it. As σ rises, however, the probability of a trade goes *up*: in this case, the liquidation point rises *less* quickly than the stock's ability to catch it.

The graph that corresponds to the parameter λ shows that trading frequency declines as the investor's sensitivity to losses rises. The intuition is that, if λ is high, the investor is very reluctant to sell a stock trading at a gain because if he does, he will have to buy a new stock, which might go down, and from which he might be forced, by a liquidity shock, to make a painful exit.

Barber and Odean (2000) find that, in their sample of households with brokerage accounts, the mean annual turnover rate is 75%, and the median annual turnover rate, 30%. Figure 6 shows that, for the benchmark parameter values, our model predicts a trading frequency that is of a similar order of magnitude. When $\sigma = 50\%$, for example, the probability that an investor trades a specific stock in his portfolio within a year of purchase is about 50%.

Underperformance ***before*** transaction costs

Some studies find that individual investors underperform benchmarks even before transaction costs (Odean, 1999). Our model may be able to shed light on this. The key insight is that, in our model, the investor *is* willing to buy a stock with a negative return premium, so long as the stock's volatility is sufficiently high. The reason is that, if the stock is volatile enough, it offers the chance of a sizeable gain, which the investor can enjoy realizing. Of course, a negative expected return stock can also fall in value. But the investor does not voluntarily realize losses, so this outcome only brings him disutility in the event of a liquidity shock. So long as the intensity of liquidity shocks and the investor's sensitivity to losses are not too high, the investor is willing to invest in a negative expected return stock if its standard deviation is sufficiently large.

Trading in rising and falling markets

Our model suggests a reason for the high overall level of trading activity, but also for why there is more trading in rising markets than in falling markets (Statman, Thorley, and Vorkink, 2006; Griffin, Nardari, and Stulz, 2007). First, our investor has a much greater propensity to *sell* stocks in a rising market: a rising market gives him more opportunities to realize gains, which is something he enjoys doing. This immediately implies that he will also have a much greater propensity to *buy* in a rising market. In order to buy a stock, our investor needs capital. To free up capital, he needs to sell his holdings of other stocks. But he is far more willing to do this in a rising market than in a falling market.

5.2 Asset pricing

Our model may also be helpful for understanding certain asset pricing patterns. We now discuss two such applications.

The negative volatility premium

Ang et al. (2005) show that, in the cross-section, and after controlling for well-known predictors of cross-sectional returns, a stock's daily return volatility over the previous month *negatively* predicts its return in the following month: highly volatile stocks subsequently earn low average returns.

This is a puzzling finding. Even if we allow ourselves to think of a stock's own volatility as risk, the result is the opposite of what we would expect: it says that "riskier" stocks have *lower* average returns. Nor can the result be fully explained using a model that combines differences of opinion with short-sale constraints: the effect persists even after controlling for differences of opinion using dispersion in analyst forecasts.

Our model offers a novel explanation. The key insight comes from the middle-right graph in Figure 2: the finding that, holding other parameters constant, initial utility is increasing in a stock's volatility. This result suggests that highly volatile stocks may experience heavy buying pressure from investors who care about realization utility. These stocks may therefore become overpriced, and, as a result, may earn low average returns.

We can check this intuition using the simple equilibrium model of Section 4. We assign all investors the same benchmark parameters

$$(\delta, \rho, \lambda) = (0.08, 0.1, 1.5), \tag{48}$$

and suppose that the transaction cost parameter is the same for all stocks, namely $k = 0.01$. For values of σ ranging from 0.01 to 0.9, we use the equilibrium condition in (46) to compute the expected return that a stock with any particular standard deviation must earn.

The top-left graph in Figure 7 plots the resulting relationship between expected return and standard deviation. The graph confirms our prediction: more volatile stocks earn lower average returns; in this sense, they are overpriced.

A counterfactual prediction of the top-left graph is that the aggregate equity premium is negative. One way to obtain a negative relationship between expected return and volatility in the cross-section in conjunction with a *positive* equity premium is to suppose that investors apply different decision rules to different components of their wealth. In particular, suppose that investors use a standard concave utility function to allocate most of their wealth between a risk-free asset and a stock market index; but that, for the remainder of their wealth – the "play" money in their brokerage accounts which they allocate across individual stocks – realization utility preferences apply. The combination of a concave utility function on the one hand and realization utility on the other may be able to reconcile a high aggregate equity premium with a negative cross-sectional relationship between expected return and standard deviation.

Heavy trading of overvalued assets

A robust empirical finding is that assets that are highly valued, and possibly overvalued, are also heavily traded (Hong and Stein, 2007). Growth stocks, for example, are more heavily traded than value stocks; the highly-priced internet stocks of the late 1990s changed hands at a rapid pace; and shares at the center of famous bubble episodes, such as those of the East India Company at the time of the South Sea bubble, also experienced heavy trading.

Our model may be able to explain this coincidence of high prices and heavy trading. Moreover, it predicts that this phenomenon should occur when the value of the underlying asset is especially uncertain.

Suppose that the uncertainty about an asset's value goes up, pushing up its standard deviation σ. As noted earlier, investors who care about realization utility will now find the asset more attractive. If there are many such investors in the economy, the asset's price may be pushed up.

At the same time, the top-right graph in Figure 6 shows that, as σ goes up, the probability that the investor will trade the asset *also* goes up: simply put, a more volatile stock will reach its liquidation point more rapidly. In this sense, the overvaluation will coincide with higher turnover, and this will occur when uncertainty about the underlying asset value is especially high. Under this view, the late 1990s were years where realization utility investors, attracted by the high uncertainty of technology stocks, bought these stocks, pushing their prices up; as (some of) these stocks rapidly reached their liquidation points, the realization utility investors sold them, and then immediately bought new ones.

We can illustrate this result using the simple equilibrium framework of Section 4. As in our discussion of the negative volatility premium, we assign all investors the benchmark parameters in (48) and assume that the transaction cost parameter is the same for all stocks, namely $k = 0.01$. For values of σ ranging from 0.01 to 0.9, we compute, as before, the equilibrium expected return the stock must earn, but also, as a guide to the intensity of trading, the probability of trade in (47).

The top-right graph in Figure 7 plots the resulting relationship between expected return and trade probability. It confirms that stocks with lower expected returns – stocks that are more "overpriced" – do indeed experience more turnover.

5.3 Testable predictions

In Sections 5.1 and 5.2, we argued that realization utility offers a simple way of understanding a range of financial phenomena. In most cases, we did not foresee the link between realization utility and the particular application: the link emerged only after we had completed our analysis. Nonetheless, we are careful to not only offer explanations for known facts, but to also suggest new predictions. The most natural predictions come from Figure 6, which shows how the probability of trade depends on various parameters.

One of these predictions is not especially surprising: the investor trades more frequently when transaction costs are lower. Four other predictions, however, are more novel: The investor holds stocks with a higher average return for longer, before selling them. Stocks with higher volatility, however, are sold more quickly. The more impatient the investor is, the more often he trades. And the more sensitive he is to losses, the less he trades.

The prediction relating how long a stock is held to its average return is difficult to test because the average return perceived by individual investors may differ from the actual average return. Growth stocks, for example, have low average returns, but it is likely that some individual investors *perceive* them to have high average returns.

The prediction relating how long a stock is held to its volatility is easier to test. Indeed, after making this prediction, we found that the answer is already available in the literature. Zuckerman (2006) reports that the individual investors in the Barber and Odean (2000) database *do* hold more volatile stocks for shorter periods of time before selling them.

Our predictions relating trading frequency to investor impatience and investor sensitivity to losses are harder to test, but by no means impossible. The difficulty here is obtaining estimates of impatience and sensitivity to losses. In recent years, however, researchers have pioneered clever techniques for extracting information about investors' psychological profiles. Grinblatt and Keloharju (2006), for example, use military test scores from Finland to estimate overconfidence. This success raises the possibility that a test of the link between impatience and loss sensitivity on the one hand, and trading frequency on the other, can also be implemented.

6 Conclusion

We study the possibility that, aside from standard sources of utility, investors also derive utility from *realizing* gains and losses on individual investments that they own. We propose a tractable model of this "realization utility," derive its predictions, and show that it can shed light on a number of puzzling facts. These include the poor trading performance of individual investors, the disposition effect, the greater turnover in rising markets, the negative premium to volatility in the cross-section, and the heavy trading of highly valued assets. Underlying some of these applications is one of our model's more novel predictions: that, even if the form of realization utility is linear or concave, investors can be risk-*seeking*.

7 Appendix

Proof of Proposition 1: At time t, the investor can either liquidate his position, or hold it for an infinitesimal period dt. We therefore have:

$$\begin{aligned}
& V(W_t, B_t) \\
= \; & \max\left\{u\left((1-k)W_t - B_t\right) + V\left((1-k)W_t, (1-k)W_t\right),\right. \\
& \qquad \left.(1-\rho dt)E_t[e^{-\delta dt}V(W_{t+dt}, B_{t+dt})] + \rho dt\left[u\left((1-k)W_t - B_t\right)\right]\right\} \qquad (49) \\
= \; & \max\{u\left((1-k)W_t - B_t\right) + V\left((1-k)W_t, (1-k)W_t\right), \\
& \qquad E_t\left[e^{-\delta dt}V(W_{t+dt}, B_{t+dt})\right] + \rho dt\left[u\left((1-k)W_t - B_t\right) - V(W_t, B_t)\right]\}. \qquad (50)
\end{aligned}$$

The first expression on the right-hand side of (49) shows what happens if the investor liquidates his position at time t: he receives realization utility of $u((1-k)W_t - B_t)$ and cash proceeds of $(1-k)W_t$ which he promptly reinvests. The second expression on the right-hand side shows what happens if the investor instead holds his position for an infinitesimal period dt. With probability $e^{-\rho dt} \approx 1 - \rho dt$, there is no liquidity shock during this interval, and the investor's value function is simply the expected future value function, discounted back. With probability $1 - e^{-\rho dt} \approx \rho dt$, there *is* a liquidity shock, and the investor sells his holdings and exits. This entails realization utility of $u((1-k)W_t - B_t)$.

We conjecture that the value function takes the form

$$V(W_t, B_t) = B_t U(g_t).$$

Substituting this into (50), cancelling the B_t factor from both sides, and applying Ito's lemma gives

$$\begin{aligned}
U(g_t) \; = \; & \max\{u((1-k)g_t - 1) + (1-k)g_t U(1), \\
& U(g_t) + \left[\frac{1}{2}\sigma^2 g_t^2 U''(g_t) + \mu g_t U'(g_t) - (\rho+\delta)U(g_t) + \rho u\left((1-k)g_t - 1\right)\right]dt\} \qquad (51)
\end{aligned}$$

Equation (51) implies that any solution to (7) must satisfy

$$U(g_t) \geq u((1-k)g_t - 1) + (1-k)g_t U(1) \qquad (52)$$

and

$$\frac{1}{2}\sigma^2 g_t^2 U''(g_t) + \mu g_t U'(g_t) - (\rho+\delta)U(g_t) + \rho u\left((1-k)g_t - 1\right) \leq 0. \qquad (53)$$

Formally speaking, the decision problem in (7) is an optimal stopping problem. To solve it, we first construct a function $U(g_t)$ that satisfies conditions (52) and (53) and that is also continuously differentiable – this last condition is sometimes known as the "smooth pasting" condition. We then verify that $U(g_t)$ does indeed solve problem (7).

We construct $U(g_t)$ in the following way. If g_t is low – specifically, if $g_t \epsilon (0, g_*)$ – we suppose that the investor continues to hold his current position. In this "continuation" region, then, equation (51) is maximized by the second term within the curly brackets, so that condition (53) holds with equality. If g_t is sufficiently high – specifically, if $g_t \epsilon (g_*, \infty)$ – we suppose that the investor liquidates his position. In this "liquidation" region, equation (51) is maximized by the first term within the curly brackets, so that condition (52) holds with equality. As in the statement of the proposition, we refer to g_* as the liquidation point.

Since $u(\cdot)$ is linear, the value function $U(\cdot)$ in the **continuation** region satisfies

$$\frac{1}{2}\sigma^2 g_t^2 U''(g_t) + \mu g_t U'(g_t) - (\rho+\delta) U(g_t) + \rho((1-k)g_t - 1) = 0.$$

The solution to this equation is

$$U(g_t) = a g_t^{\gamma_1} + \frac{\rho(1-k)}{\rho+\delta-\mu} g_t - \frac{\rho}{\rho+\delta} \quad \text{for} \quad g_t \epsilon (0, g_*), \tag{54}$$

where γ_1 is given in equation (12) and where a is determined below.

In the **liquidation** region, we have

$$U(g_t) = (1-k)g_t(1+U(1)) - 1. \tag{55}$$

Note that the liquidation point g_* satisfies $g_* \geq 1$. For if $g_* < 1$, then $g_t = 1$ would fall into the liquidation region, which, from (55), would imply

$$U(1) = (1-k)U(1) - k.$$

For $k > 0$ and $U(1) > 0$, this is a contradiction. Since $g_* \geq 1$, then, we infer from (54) that

$$U(1) = a + \frac{\rho(1-k)}{\rho+\delta-\mu} - \frac{\rho}{\rho+\delta}. \tag{56}$$

The value function must be continuous and smooth around the liquidation point g_*. This implies

$$\begin{aligned} a g_*^{\gamma_1} + \frac{\rho(1-k)}{\rho+\delta-\mu} g_* - \frac{\rho}{\rho+\delta} &= (1-k)g_*(1+U(1)) - 1 \\ a\gamma_1 g_*^{\gamma_1-1} + \frac{\rho(1-k)}{\rho+\delta-\mu} &= (1-k)(1+U(1)). \end{aligned}$$

Solving these two equations, we obtain the expression for a in (13) and the following nonlinear equation for g_*:

$$(\gamma_1 - 1)\left(1 - \frac{\rho k(\rho+\delta)}{\delta(\rho+\delta-\mu)}\right) g_*^{\gamma_1} - \frac{\gamma_1}{1-k} g_*^{\gamma_1-1} + 1 = 0. \tag{57}$$

Equation (57) has a unique solution in the range $(1,\infty)$. To see this, define

$$f(g) \equiv (\gamma_1 - 1)\left(1 - \frac{\rho k(\rho+\delta)}{\delta(\rho+\delta-\mu)}\right) g^{\gamma_1} - \frac{\gamma_1}{1-k} g^{\gamma_1 - 1} + 1.$$

The parameter restriction in (9) implies that $\gamma_1 > 1$ and that $1 > \frac{\rho k(\rho+\delta)}{\delta(\rho+\delta-\mu)}$. It is then straightforward to see that

$$f(1) < 0 \text{ and } f(\infty) > 0.$$

As a result, $f(g)$ has at least one root above 1. We now rule out the possibility that $f(g)$ has more than one root above 1. Suppose instead that $f(g)$ does have more than one root above 1. Then, it must have a local maximum $g_m > 1$ which satisfies $f'(g_m) = 0$ and $f''(g_m) < 0$. The condition $f'(g_m) = 0$ implies

$$g_m = \frac{1}{(1-k)\left(1 - \frac{\rho k(\rho+\delta)}{\delta(\rho+\delta-\mu)}\right)}$$

and

$$f''(g_m) = \gamma_1 (\gamma_1 - 1) g_m^{\gamma_1 - 3} \frac{1}{1-k} > 0.$$

The last inequality contradicts the initial assumption that g_m is a local maximum. The function $f(g)$ therefore has a unique root above 1.

We now verify that the constructed value function is indeed optimal. Substituting $V(W_t, B_t) = B_t U(g_t)$ into (7) and cancelling the B_t factor reduces the stopping problem to

$$\begin{aligned} U(g_t) &= \max_{\tau \geq t} \; E_t\{e^{-\delta(\tau-t)}[u((1-k)g_\tau - 1) + (1-k)g_\tau U(1)] I_{\{\tau<\tau'\}} \\ &\qquad + e^{-\delta(\tau'-t)} u((1-k)g_{\tau'} - 1) I_{\{\tau \geq \tau'\}}\}. \end{aligned} \tag{58}$$

We first verify that the function $U(g_t)$ summarized in equation (11) satisfies conditions (52) and (53). Define

$$f_1(g) \equiv (1-k)(1+U(1))g - 1.$$

Note that, by construction, $f_1(g)$ is a straight line which coincides with $U(g)$ for $g \geq g_*$. Since $\gamma_1 > 1$, $U(g)$ in equation (11) is a convex function. It must therefore lie above the straight line $f_1(g)$ for all $g < g_*$. Condition (52) is therefore satisfied.

We now check that condition (53) holds. Define

$$H(g) \equiv \frac{1}{2}\sigma^2 g^2 U''(g) + \mu g U'(g) - (\rho+\delta) U(g) + \rho((1-k)g - 1).$$

Note that for $g < g_*$, $H(g) = 0$ by construction. For $g \geq g_*$, $U(g) = f_1(g)$, so that

$$H(g) = -(1-k)[(\rho+\delta-\mu)(1+U(1)) - \rho] g + \delta.$$

Substituting (56) and (13) into this expression, we obtain

$$
\begin{aligned}
H(g) &= -(1-k)\,g\left\{\frac{\delta(\rho+\delta-\mu)}{\rho+\delta}\left[1+\frac{1}{(\gamma_1-1)\,g_*^{\gamma_1}}\right]-\rho k-\frac{\delta}{(1-k)\,g}\right\} \\
&\le -(1-k)\,g\left\{\frac{\delta(\rho+\delta-\mu)}{\rho+\delta}\left[1+\frac{1}{(\gamma_1-1)\,g_*^{\gamma_1}}\right]-\rho k-\frac{\delta}{(1-k)\,g_*}\right\} \\
&= -\frac{g}{g_*}\frac{\delta}{(\rho+\delta)(\gamma_1-1)}(\rho+\delta-\mu\gamma_1)\,.
\end{aligned}
$$

The last equality follows by applying equation (14). Using (12), it is straightforward to show that if $\mu<\rho+\delta$, as assumed in parameter restriction (9), then $\rho+\delta-\mu\gamma_1>0$. Therefore, $H(g)<0$ for $g\ge g_*$. We have therefore confirmed that condition (53) holds for all $g_t\ \epsilon\ (0,\infty)$.

Now note that $U(g)$ has an increasing derivative in $(0,g_*)$ and a derivative of $(1-k)(1+U(1))$ in (g_*,∞). $U'(g)$ is therefore bounded. Define the stopping time

$$
\iota\equiv\min(\tau,\tau')\,,
$$

where τ is *any* selling strategy and τ' is the time at which a liquidity shock arrives. Ito's lemma for twice-differentiable functions with absolutely continuous first derivatives – see, for example, Revuz and Yor (1999), Chapter 6 – implies

$$
\begin{aligned}
e^{-\delta(\iota-t)}U(g_\iota) &= U(g_t)+\int_t^{\iota}\sigma g_s U'(g_s)\,dZ_s \\
&\quad+\int_t^{\iota}\left[\frac{1}{2}\sigma^2 g_s^2 U''(g_s)+\mu g_s U'(g_s)-(\rho+\delta)\,U(g_s)+\rho u\left((1-k)g_s-1\right)\right]ds.
\end{aligned}
$$

The bound on $U'(g)$ implies that the first integral is a martingale, while condition (53) implies that the second integral is non-positive. We therefore have

$$
U(g_t)\ge E_t\left[e^{-\delta(\iota-t)}U(g_\iota)\right]. \tag{59}
$$

Note also, from condition (52), that

$$
E_t\left[e^{-\delta(\iota-t)}U(g_\iota)\right]\ge E_t\left\{e^{-\delta(\iota-t)}\left[((1-k)g_\iota-1)+(1-k)g_\iota U(1)\right]\right\}. \tag{60}
$$

Now consider the expression in the expectation operator of (58). If $\tau\le\tau'$, then $\iota=\tau$ and

$$
\begin{aligned}
&e^{-\delta(\tau-t)}[u((1-k)g_\tau-1)+(1-k)g_\tau U(1)]I_{\{\tau<\tau'\}}+e^{-\delta(\tau'-t)}u((1-k)g_{\tau'}-1)I_{\{\tau\ge\tau'\}} \\
=\ &e^{-\delta(\iota-t)}\left[((1-k)g_\iota-1)+(1-k)g_\iota U(1)\right]. \qquad (61)
\end{aligned}
$$

If $\tau > \tau'$, so that $\iota = \tau'$, the expression satisfies

$$\begin{aligned}
& e^{-\delta(\tau-t)}[u((1-k)g_\tau - 1) + (1-k)g_\tau U(1)]I_{\{\tau<\tau'\}} + e^{-\delta(\tau'-t)}u((1-k)g_{\tau'} - 1)I_{\{\tau\geq\tau'\}} \\
\leq \; & e^{-\delta(\tau-t)}[u((1-k)g_\tau - 1) + (1-k)g_\tau U(1)]I_{\{\tau<\tau'\}} \\
& \quad + e^{-\delta(\tau'-t)}\left[u((1-k)g_{\tau'} - 1) + (1-k)g_{\tau'}U(1)\right] I_{\{\tau\geq\tau'\}} \\
= \; & e^{-\delta(\iota-t)}\left[((1-k)g_\iota - 1) + (1-k)g_\iota U(1)\right], \qquad (62)
\end{aligned}$$

where the inequality follows from $U(1) \geq 0$. For any stopping time τ, we therefore have

$$\begin{aligned}
& E_t\{e^{-\delta(\tau-t)}[u((1-k)g_\tau - 1) + (1-k)g_\tau U(1)]I_{\{\tau<\tau'\}} + e^{-\delta(\tau'-t)}u((1-k)g_{\tau'} - 1)I_{\{\tau\geq\tau'\}}\} \\
\leq \; & E_t\left\{e^{-\delta(\iota-t)}\left[((1-k)g_\iota - 1) + (1-k)g_\iota U(1)\right]\right\} \\
\leq \; & E_t\left[e^{-\delta(\iota-t)}U(g_\iota)\right] \\
\leq \; & U(g_t),
\end{aligned}$$

where the first inequality follows from (61) and (62), the second from (60), and the third from (59). The constructed value function $U(g_t)$ is therefore at least as good as the value function generated by any alternative selling strategy. This completes the proof.

Proof of Proposition 2: We conjecture that the value function takes the form

$$V(W_t, B_t) = B_t U(g_t).$$

Following the same logic as in the proof of Proposition 1, we find that $U(\cdot)$ again satisfies equation (51) and inequalities (52) and (53). The only difference is that $u(\cdot)$ now has the piecewise-linear form in (15).

As before, we conjecture two regions: a continuation region, $g_t \epsilon (0, g_*)$, and a liquidation region, $g_t \epsilon (g_*, \infty)$. In the **continuation** region, $U(\cdot)$ satisfies

$$\frac{1}{2}\sigma^2 g_t^2 U''(g_t) + \mu g_t U'(g_t) - (\rho+\delta)U(g_t) + \rho u((1-k)g_t - 1) = 0. \qquad (63)$$

The form of the $u(\cdot)$ term depends on whether its argument, $(1-k)g_t - 1$, is greater or less than zero. Note that the cross-over point, $g_t = \frac{1}{1-k}$, is below g_*, so that $g_* \geq \frac{1}{1-k}$. For if $g_* < \frac{1}{1-k}$, then $g_t = \frac{1}{1-k}$ would be in the liquidation region, which, from (17), would imply

$$U\left(\frac{1}{1-k}\right) = U(1),$$

contradicting the plausible restriction that $U(g_t)$ be increasing in g_t. Since $g_* \geq \frac{1}{1-k}$, we further subdivide the continuation region $(0, g_*)$ into two subregions, $\left(0, \frac{1}{1-k}\right)$, and $\left(\frac{1}{1-k}, g_*\right)$.

For $g_t \epsilon \left(0, \frac{1}{1-k}\right)$, equation (63) becomes

$$\frac{1}{2}\sigma^2 g_t^2 U''(g_t) + \mu g_t U'(g_t) - (\rho+\delta)U(g_t) + \rho\lambda((1-k)g_t - 1) = 0.$$

The solution to this equation is

$$U\left(g_t\right) = b g_t^{\gamma_1} + \frac{\rho\lambda(1-k)}{\rho+\delta-\mu} g_t - \frac{\rho\lambda}{\rho+\delta} \quad \text{for} \quad g_t \in \left(0, \frac{1}{1-k}\right), \tag{64}$$

where γ_1 is defined in equation (12), and where b is determined below.

For $g_t \epsilon \left(\frac{1}{1-k}, g_*\right)$, equation (63) becomes

$$\frac{1}{2}\sigma^2 g_t^2 U''\left(g_t\right) + \mu g_t U'\left(g_t\right) - (\rho+\delta) U\left(g_t\right) + \rho\left((1-k)g_t - 1\right) = 0.$$

The solution to this equation is

$$U\left(g_t\right) = c_1 g_t^{\gamma_1} + c_2 g_t^{\gamma_2} + \frac{\rho(1-k)}{\rho+\delta-\mu} g_t - \frac{\rho}{\rho+\delta} \quad \text{for} \quad g_t \,\epsilon\, \left(\frac{1}{1-k}, g_*\right),$$

where

$$\gamma_2 = -\frac{1}{\sigma^2}\left[\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2(\rho+\delta)\sigma^2} + \left(\mu - \frac{1}{2}\sigma^2\right)\right] < 0,$$

and where c_1 and c_2 are determined below.

The value function must be continuous and smooth around $g_t = \frac{1}{1-k}$. We therefore have

$$b\left(\frac{1}{1-k}\right)^{\gamma_1} = c_1\left(\frac{1}{1-k}\right)^{\gamma_1} + c_2\left(\frac{1}{1-k}\right)^{\gamma_2} - \frac{(\lambda-1)\mu\rho}{(\rho+\delta-\mu)(\rho+\delta)},$$

which is equation (21), and

$$b\gamma_1\left(\frac{1}{1-k}\right)^{\gamma_1-1} = c_1\gamma_1\left(\frac{1}{1-k}\right)^{\gamma_1-1} + c_2\gamma_2\left(\frac{1}{1-k}\right)^{\gamma_2-1} - \frac{(\lambda-1)(1-k)\rho}{\rho+\delta-\mu}.$$

Together, these equations imply equation (19), namely

$$c_2 = \frac{(\lambda-1)\,\rho(1-k)^{\gamma_2}\left(\mu\gamma_1 - \rho - \delta\right)}{(\gamma_1-\gamma_2)(\rho+\delta-\mu)(\rho+\delta)}.$$

In the **liquidation** region, $g_t \,\epsilon\, (g_*, \infty)$, using the fact that $g_* \geq 1$, we have

$$U\left(g_t\right) = (1-k)g_t(1 + U(1)) - 1.$$

The value function must be continuous and smooth around the liquidation point, so that

$$\begin{aligned} c_1 g_*^{\gamma_1} + c_2 g_*^{\gamma_2} + \frac{\rho(1-k)}{\rho+\delta-\mu} g_* &= (1-k)g_*(1+U(1)) - \frac{\delta}{\rho+\delta} \\ c_1\gamma_1 g_*^{\gamma_1-1} + c_2\gamma_2 g_*^{\gamma_2-1} + \frac{\rho(1-k)}{\rho+\delta-\mu} &= (1-k)(1+U(1)). \end{aligned}$$

Since, from equation (64),

$$U(1) = b + \frac{\rho\lambda(\mu - k\rho - k\delta)}{(\rho+\delta)(\rho+\delta-\mu)},$$

we obtain equation (22),

$$c_1 g_*^{\gamma_1} + c_2 g_*^{\gamma_2} + (1-k)g_* \frac{\mu-\delta}{\rho+\delta-\mu} + \frac{\delta}{\rho+\delta} = (1-k)g_* \left(b + \frac{\rho\lambda(\mu - k\rho - k\delta)}{(\rho+\delta)(\rho+\delta-\mu)} \right),$$

and equation (20),

$$(\gamma_1 - 1)\, c_1 g_*^{\gamma_1} + (\gamma_2 - 1)\, c_2 g_*^{\gamma_2} = \frac{\delta}{\rho+\delta}.$$

All that remains is to verify the optimality of the constructed value function. This part of the derivation is similar to the final part of the proof of Proposition 1. For space reasons, we do not repeat it here.

Proof of Proposition 3: At time t, the investor can either liquidate his position, or hold it for an infinitesimal period dt. We therefore have:

$$\begin{aligned} N(W_t, B_t) \;=\; & \max\{(1-k)W_t - B_t + N((1-k)W_t, (1-k)W_t), \\ & (1-e^{-\rho dt})u((1-k)W_t - B_t) + e^{-\rho dt}e^{-\phi dt}E_t[e^{-\delta dt}N(W_{t+dt}, B_{t+dt})] \\ & + e^{-\rho dt}(1-e^{-\phi dt})E_t[e^{-\delta dt}\widehat{N}(W_{t+dt}, B_{t+dt})]\}. \end{aligned} \tag{65}$$

If the current self sells stock now, he receives realization utility of $(1-k)W_t - B_t$ and a cash balance of $(1-k)W_t$ which he promptly reinvests. Alternatively, he may continue to hold his stock position for an infinitesimal period dt. With probability $1 - e^{-\rho dt}$, there is a liquidity shock during this interval. In this case, the investor exits the stock market and receives realization utility of $u((1-k)W_t - B_t)$. With probability $e^{-\rho dt}e^{-\phi dt}$, neither a liquidity shock nor a new self arrives during the interval. In this case, the current self receives the discounted expected value function $E_t[e^{-\delta dt}N(W_{t+dt}, B_{t+dt})]$. Finally, with probability $e^{-\rho dt}(1-e^{-\phi dt})$, a liquidity shock does not arrive during the interval, but a new self *does*, in which case the current self receives the discounted expected value function that he *thinks* will result from the actions of the future self.

We conjecture that

$$N(W_t, B_t) = B_t n(g_t).$$

Substituting this and $\widehat{N}(W_t, B_t) = \beta V(W_t, B_t)$ into (65) and cancelling the B_t factor leads to

$$\begin{aligned} n(g_t) \;=\; \max\; & \{(1-k)g_t - 1 + (1-k)g_t n(1), \;\; (1-e^{-\rho dt})((1-k)g_t - 1) \\ & + e^{-(\rho+\delta+\phi)dt}E_t(n(g_{t+dt})) + (1-e^{-\phi dt})e^{-(\rho+\delta)dt}\beta E_t(U(g_t))\}. \end{aligned}$$

Applying Ito's lemma, we obtain

$$\begin{aligned} n(g_t) &= \max\{(1-k)g_t - 1 + (1-k)g_t n(1), \quad n(g_t) \\ &+ \left[\frac{1}{2}\sigma^2 g_t^2 n''(g_t) + \mu g_t n'(g_t) - (\rho+\delta+\phi)n(g_t) + \rho((1-k)g_t - 1) + \phi\beta U(g_t)\right] dt\}. \end{aligned}$$

As before, we conjecture that there are two regions: a continuation region, $g_t \, \epsilon \, (0, g_{**})$, where the current self keeps holding the stock, and a liquidation region, $g_t \, \epsilon \, (g_{**}, \infty)$, where the current self sells his position.

In the **continuation** region,

$$\frac{1}{2}\sigma^2 g_t^2 n''(g_t) + \mu g_t n'(g_t) - (\rho+\delta+\phi)n(g_t) + \rho((1-k)g_t - 1) + \phi\beta U(g_t) = 0. \tag{66}$$

We conjecture that $g_{**} < g_*$, where g_* is the liquidation point in the *exponential* discounting model of Section 2. From (11), this means that

$$U(g_t) = a g_t^{\gamma_1} + \frac{\rho(1-k)}{\rho+\delta-\mu} g_t - \frac{\rho}{\rho+\delta} \qquad \text{if } g_t \, \epsilon \, (0, g_{**}),$$

where γ_1 is given in (12) and where a is determined from (13) and (14). The solution to (66) is

$$n(g_t) = d_1 g_t^{\gamma_1} + d_2 g_t^{\gamma_3} + d_3 g_t + d_4, \tag{67}$$

where

$$\begin{aligned} \gamma_3 &= \frac{1}{\sigma^2}\left[\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\left(\rho+\delta+\phi\right)\sigma^2} - \left(\mu - \frac{1}{2}\sigma^2\right)\right] > 0 \\ d_1 &= \frac{a\phi\beta}{\rho+\delta+\phi-\mu\gamma_1 - \frac{\sigma^2}{2}\gamma_1(\gamma_1 - 1)} \\ d_3 &= \frac{\rho(1-k)(1+\frac{\beta\phi}{\rho+\delta-\mu})}{\rho+\delta+\phi-\mu} \\ d_4 &= \frac{-\rho(1+\frac{\beta\phi}{\rho+\delta})}{\rho+\delta+\phi}, \end{aligned}$$

and where d_2 is determined below.

In the **liquidation** region, $g_t \, \epsilon \, (g_{**}, \infty)$, we have

$$n(g_t) = (1-k)g_t(1+n(1)) - 1.$$

By the usual argument, $g_{**} \geq 1$, so that, from (67), $n(1) = d_1 + d_2 + d_3 + d_4$. The value function must be continuous and smooth around the liquidation point, so that

$$\begin{aligned} d_1 g_{**}^{\gamma_1} + d_2 g_{**}^{\gamma_3} + d_3 g_{**} + d_4 &= (1-k)g_{**}(1 + d_1 + d_2 + d_3 + d_4) - 1 \\ d_1\gamma_1 g_{**}^{\gamma_1 - 1} + d_2\gamma_3 g_{**}^{\gamma_3 - 1} + d_3 &= (1-k)(1 + d_1 + d_2 + d_3 + d_4). \end{aligned}$$

Proof of Proposition 4: We solve the decision problem in (38) using a procedure very similar to the one we employed in the proofs of Propositions 1 and 2. In particular, we replace μ, σ, and k in (51) with μ_i, σ_i, and k_i, the expected return, standard deviation, and transaction cost of stock i. We also note that $U(1) = 0$ in equilibrium. It is then straightforward to obtain the results in Proposition 4.

Proof of Proposition 5: Define

$$x_t \equiv \ln(g_t) \quad \text{and} \quad x_* \equiv \ln(g_*).$$

Then,

$$dx_t = \mu_x dt + \sigma dZ_t, \qquad \mu_x = \mu - \frac{\sigma^2}{2}.$$

If the investor has not yet traded, what is the probability that he trades at least once in the following s periods? Note that he will trade if the stock price level rises sufficiently high so that the process x_t hits the barrier x_*; or if there is a liquidity shock. The probability is therefore a function of x_t and of the length of the period s. We denote it by $p(x, s)$.

Since a probability process is a martingale, its drift is zero, so that

$$-p_s + \mu_x p_x + \frac{1}{2}\sigma^2 p_{xx} + \rho(1 - p) = 0.$$

The last term on the left hand side is generated by the liquidity shock: if a liquidity shock arrives, the probability of a trade jumps from p to 1. The probability function must also satisfy two boundary conditions. First, if the process x_t is already at the barrier x_*, there is a trade for sure:

$$p(x_*, s) = 1, \qquad \forall s \geq 0.$$

Second, if the length of the remaining time period is zero and the price level is such that $x < x_*$, there can be no trade:

$$p(x, 0) = 0, \qquad \forall x < x_*.$$

The solution to the differential equation, subject to the boundary conditions, is

$$p(x, s) = 1 - e^{-\rho s} + e^{-\rho s}\left[N\left(\frac{x - x_* + \mu_x s}{\sigma\sqrt{s}}\right) + e^{-\frac{2\mu_x}{\sigma^2}(x - x_*)} N\left(\frac{x - x_* - \mu_x s}{\sigma\sqrt{s}}\right)\right].$$

Substituting $x = 0$, $x_* = \ln g_*$, and $\mu_x = \mu - \frac{\sigma^2}{2}$ into this expression, we obtain the result in Proposition 5.

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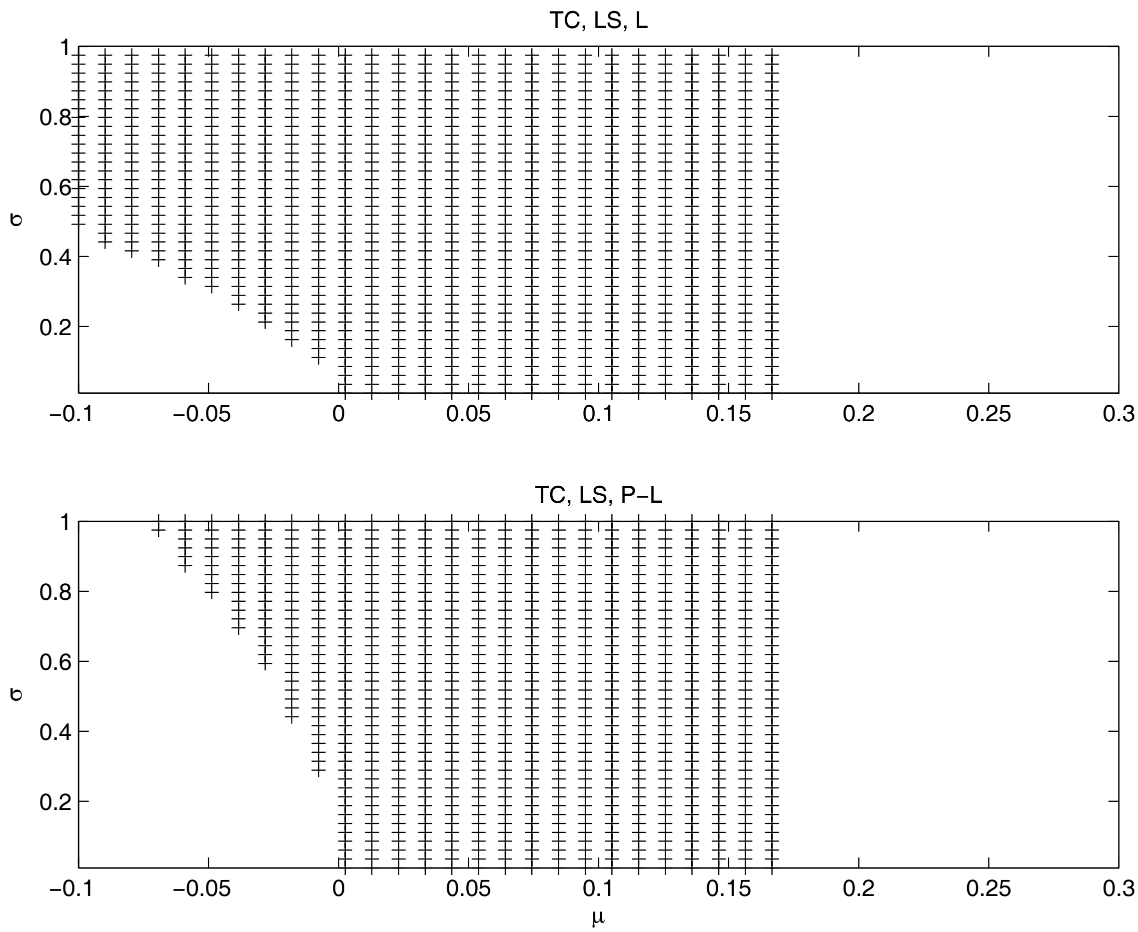

Figure 1. The graphs show, for an investor who derives utility from realized gains and losses, the range of values of a stock's expected return μ and standard deviation σ for which the investor is willing both to buy the stock and to sell it once its price reaches a sufficiently high liquidation point. The top graph corresponds to a model that allows for a transaction cost (TC) and an exogeneous liquidity shock (LS), and in which realization utility has a linear form (L). The bottom graph corresponds to a model that also allows for a transaction cost and an exogeneous liquidity shock, but in which realization utility has a piecewise-linear form (P-L), so that the investor is 1.5 times as sensitive to realized losses as to realized gains.

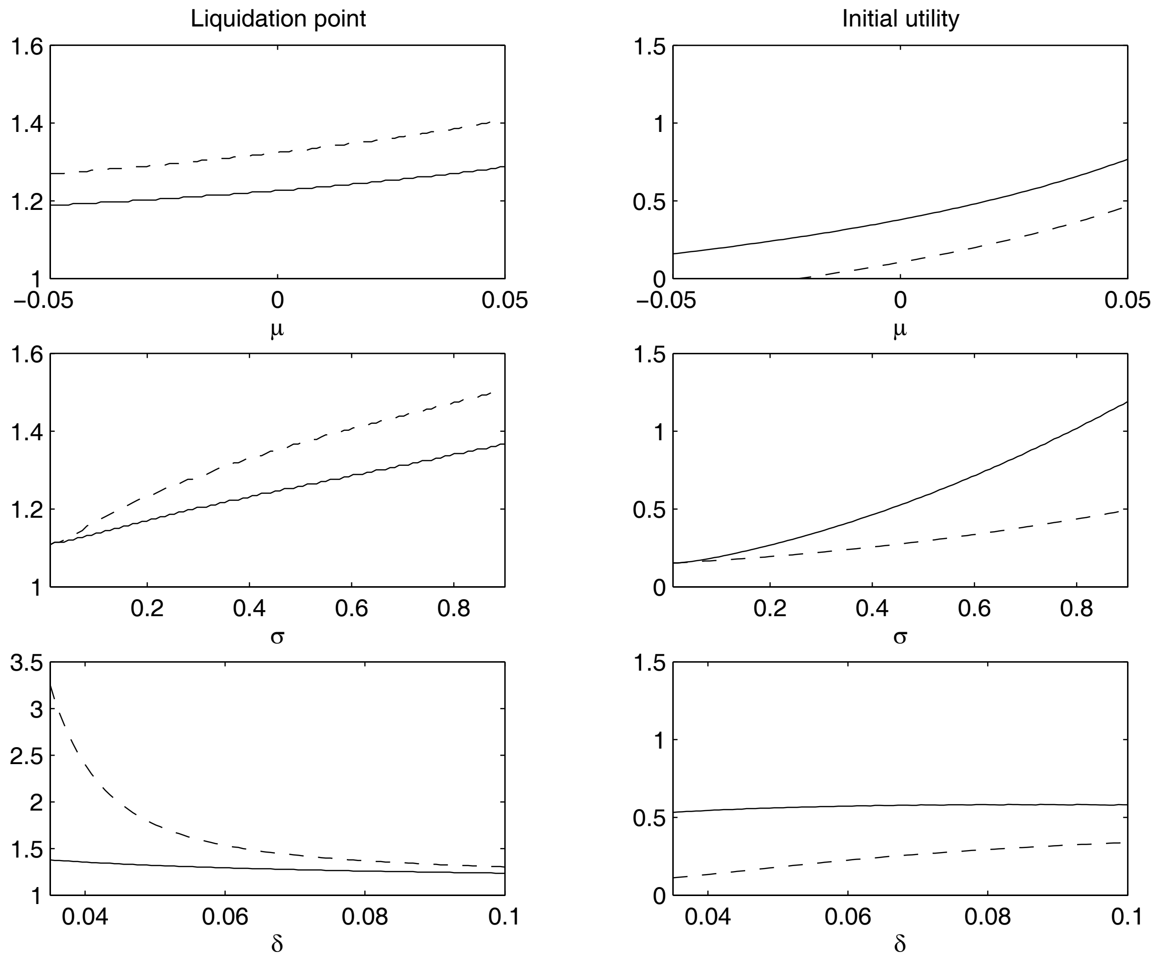

Figure 2. The graphs show, for an investor who derives utility from realized gains and losses, how the liquidation point at which he sells a stock and the initial utility from buying it depend on the stock's expected return μ, its standard deviation σ, and the investor's time discount rate δ. The solid lines correspond to a model that allows for a transaction cost and an exogeneous liquidity shock, and in which realization utility has a linear form. The dashed lines correspond to a model that also allows for a transaction cost and an exogeneous liquidity shock, but in which realization utility has a piecewise-linear form, so that the investor is 1.5 times as sensitive to realized losses as to realized gains.

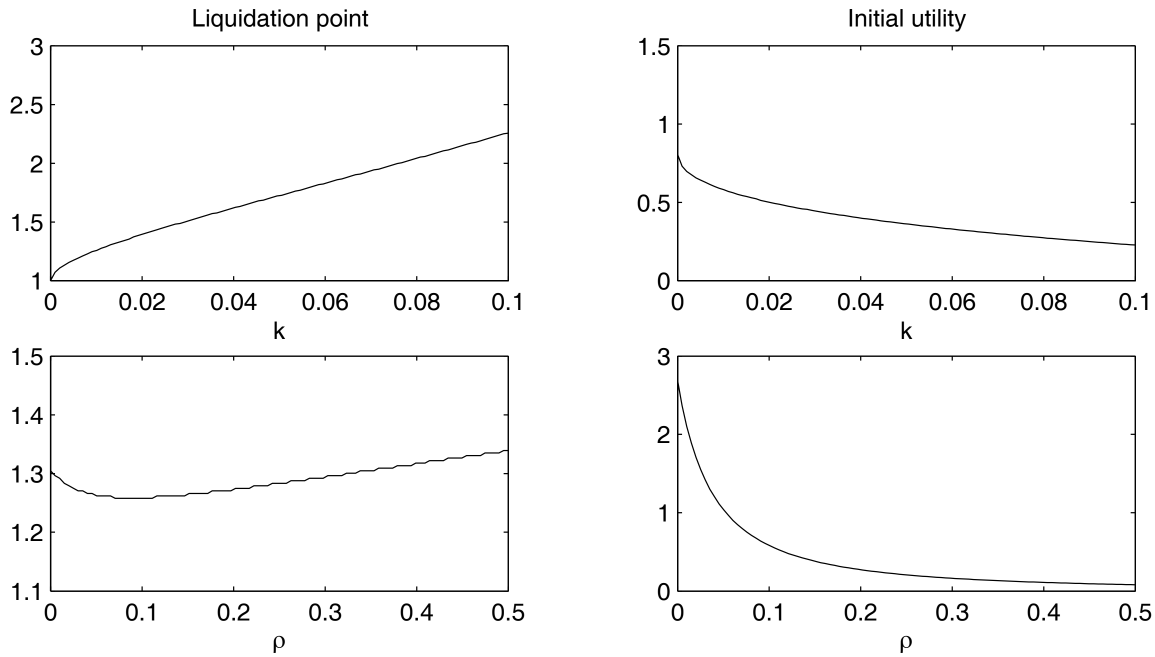

Figure 3. The graphs show, for an investor who derives utility from realized gains and losses, how the liquidation point at which he sells a stock and the initial utility from buying it depend on the transaction cost k and the arrival rate ρ of an exogeneous liquidity shock. In these computations, realization utility has a linear form.

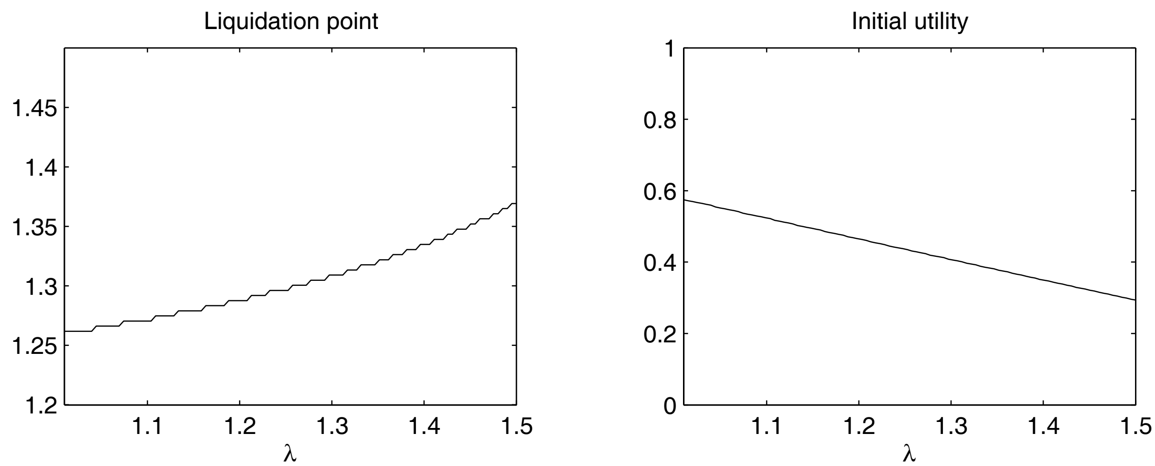

Figure 4. The graphs show, for an investor who derives utility from realized gains and losses, how the liquidation point at which he sells a stock and the initial utility from buying it depend on λ, his relative sensitivity to realized losses as opposed to realized gains. The computations are based on a model that allows for a transaction cost and an exogeneous liquidity shock.

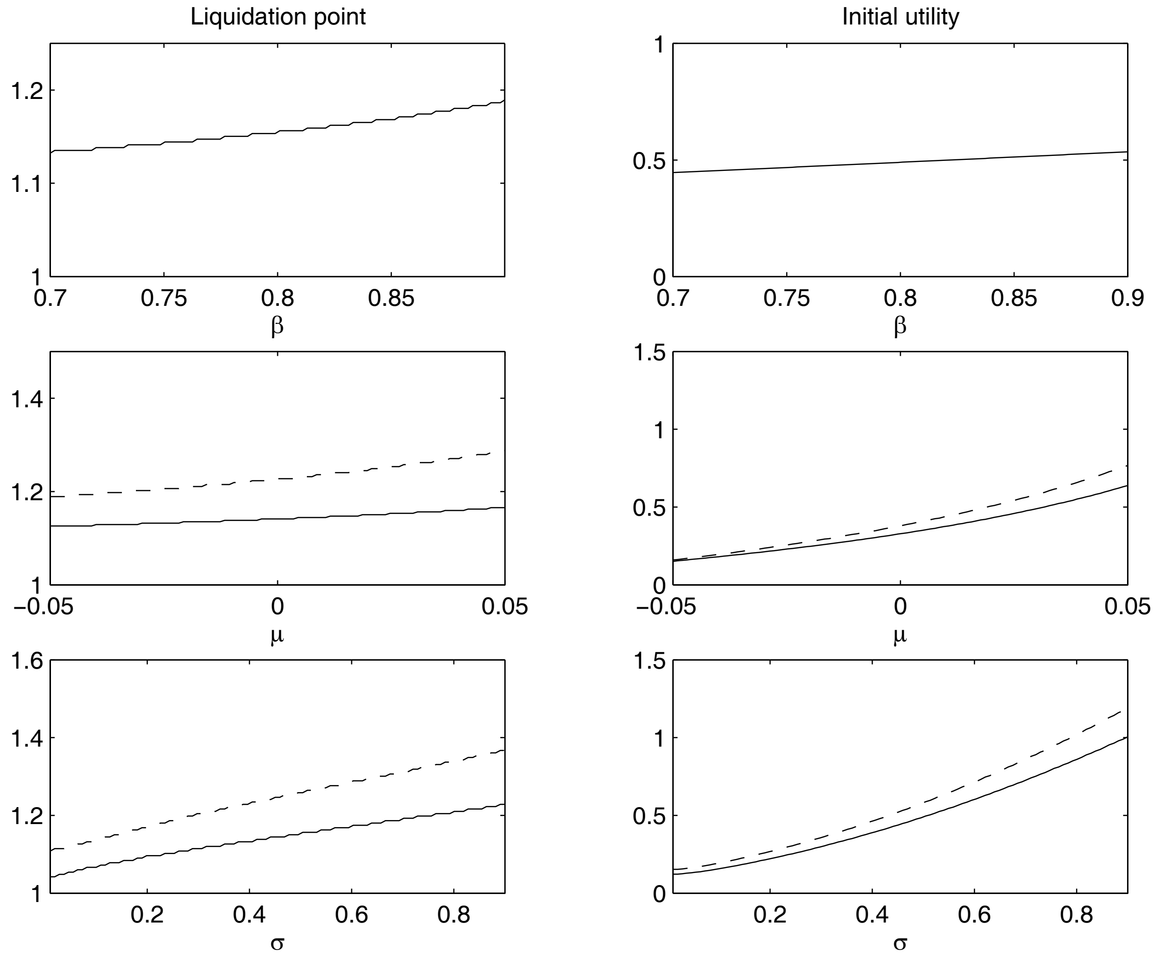

Figure 5. The solid lines in the graphs show, for an investor who derives utility from realized gains and losses and who exhibits hyperbolic time discounting, how the liquidation point at which he sells a stock and the initial utility from buying it depend on the hyperbolic discounting parameter β, the stock's expected return μ, and its standard deviation σ. (The lower the value of β, the more heavily the investor weighs the present, as opposed to the future). The dashed lines in the middle and bottom panels correspond to a model with standard exponential time discounting. The computations for both the solid and dashed lines allow for a transaction cost and an exogeneous liquidity shock, and use a linear functional form for realization utility.

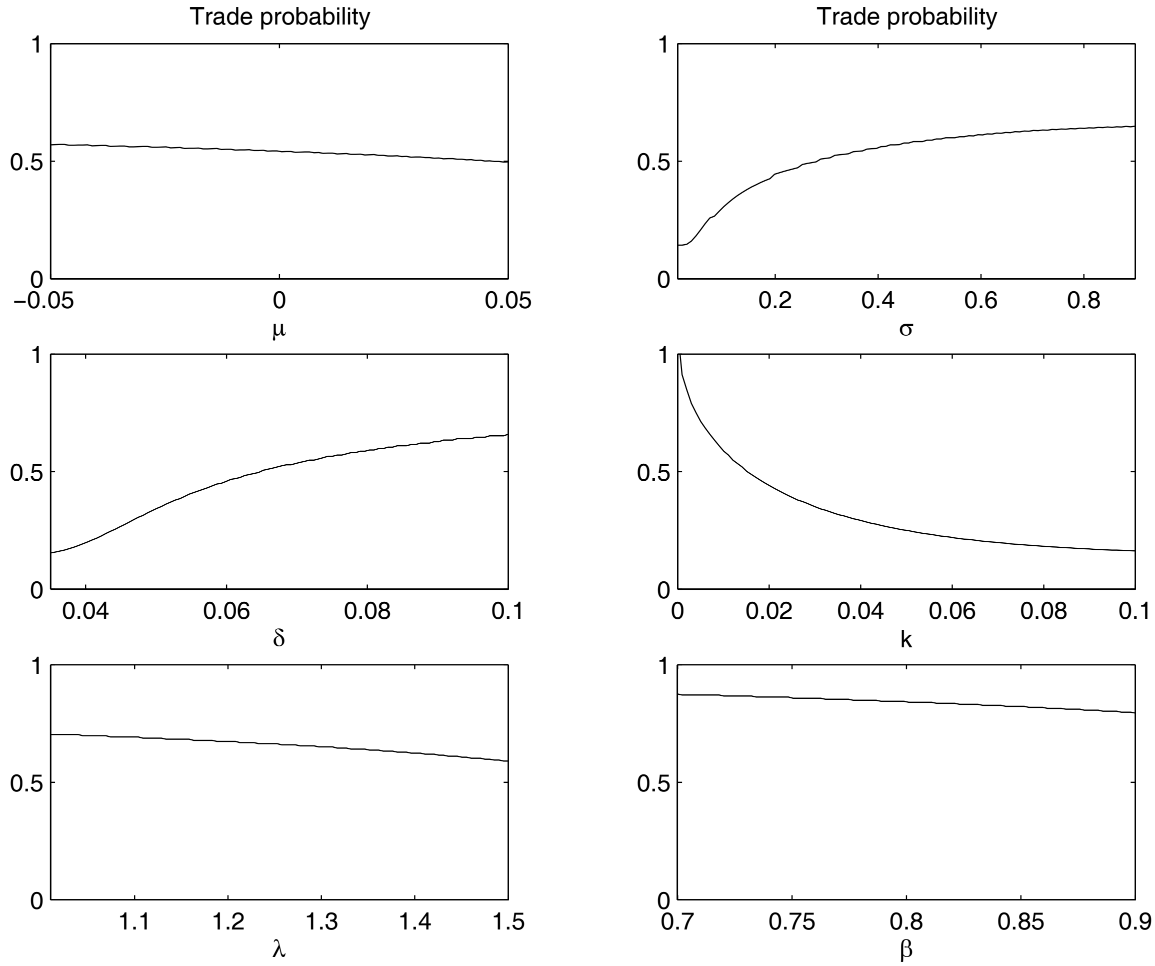

Figure 6. The graphs show, for an investor who derives utility from realized gains and losses, how the probability that the investor will sell a specific stock within a year of buying it depends on the stock's expected return μ, its standard deviation σ, the exponential time discount rate δ, the transaction cost k, the investor's relative sensitivity to realized losses as opposed to realized gains λ, and the hyperbolic time discount rate β.

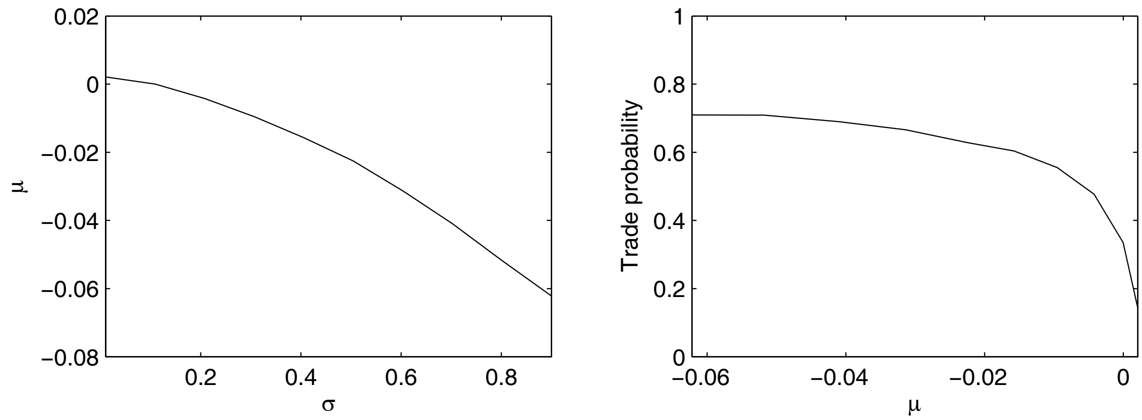

Figure 7. The top-left graph shows, for an economy populated by investors who derive utility from realized gains and losses, the equilibrium relationship between expected return and standard deviation in a cross-section of stocks. The top-right graph shows, for the same cross-section, the equilibrium relationship between expected return and trading intensity. The computations are based on a model that allows for a transaction cost and an exogeneous liquidity shock, and in which realization utility has a piecewise-linear functional form, so that the investor is 1.5 times more sensitive to realized losses as opposed to realized gains.