Capital, Contracts and the Cross Section of Stock Returns*

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Abstract

We present a tractable, static, general equilibrium model with multiple sectors in which firms offer workers incentive contracts and simultaneously raise capital in stock markets. Workers optimally invest in the stock market and at the same time hedge labor income risk. Firms rationally take agents' portfolio decisions into account. In equilibrium, the cost of capital of each sector is endogenous. We compare the first-best, in which workers' effort levels are observable, to an economy in which workers' effort is observed with noise. In the presence of moral hazard, the CAPM fails because firms, by choosing optimal incentive contracts, transfer risk both through wages and through the stock market. This leads to several cross-sectional asset pricing "anomalies," such as size and value effects. As we characterize optimal contracts, we present empirical predictions relating workers' compensation, firm productivity, firm size and financial market abnormal returns. We also demonstrate some general equilibrium implications of endogenous contracts; for example the ex ante value of human capital can be higher in an economy with moral hazard.

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1 Introduction

In the United States, labor accounts for about two thirds of national income and there is a great deal of heterogeneity in the source of this income. Each worker is typically tied to one industry, be it a hedge fund in New York or almond production in California, and all workers differ in the marginal productivity of their labor. Further, households with labor income are three times more likely to own financial assets than those without.[1] Given that workers supply human and investment capital to firms, how do firms optimally compensate workers and what effect does this have on production?

Understanding the cross-sectional implications of labor supply is important both to understand capital market equilibrium and labor market equilibrium. First, there is empirical evidence that human capital can help explain returns both in the time series and in the cross–section.[2] However, there are few theoretical predictions on how the cross section of returns should relate to labor productivity. Second, few labor papers explicitly take into account the fact that workers can trade in financial markets. Given that workers are also investors, how do workers in different industries optimally invest and how is capital and labor priced?

To answer these questions we present a two period general equilibrium model with a continuum of workers and many firms that are organized into different sectors. In the first period, workers endowed with wealth accept employment contracts offered by firms and their effort is used as an input into production. Firms in different sectors have different labor productivity and therefore elicit different effort levels from their employees. Firms' production is uncertain in that each sector is subject to a common revenue shock. Workers and firms simultaneously trade in the stock market: Workers with CARA utility functions trade to lay off wage income risk and to transfer wealth across time, while firms raise investment capital.

A distinctive feature of our paper is that we induce heterogeneity across investors by explicitly modeling the relationship between firms and workers. Specifically, firms offer workers an optimal wage contract. We first consider complete markets in which effort is observable and the optimal incentive contracts are risk free. We then consider the case in which firms observe the employees' effort with noise that is both systematic and idiosyncratic. We demonstrate an optimal incentive contract in the presence of moral hazard that includes some systematic risk. (Relative performance evaluation is irrelevant because workers have access to capital markets and therefore firms and employees have the same valuation of systematic risk.)

We fully characterize equilibrium in both cases. At the firm level, the endogenous variables are the wage contract (and attendant employee effort) and the initial investment level (which is just firm size). Economy–wide, we solve for the equilibrium size of each sector and its cost of capital. We also determine the endogenous effort–free wage: the minimum wage required to induce a worker to join a firm. This is the equilibrium, ex ante value of human capital.

If there are no frictions in the labor market then the CAPM holds. However, if there is moral hazard then stock market returns do not reflect the simple CAPM. This is because

[1]According to the 2004 survey of consumer finances, approximately 60% of households with labor income own financial assets, while approximately 20% of households with no labor income own financial assets.

[2]The seminal time series paper is due to Jagannathan and Wang (1996), while using micro data, Malloy, Moskowitz and Vissing-Jorgenson (2005) find evidence that labor income risk (through a firing decision) can explain the value effect. Further use of human capital proxies have improved the performance of the conditional CAPM, as in, for example, Palacios-Huerta (2005).

labor compensation and dividends are two channels through which a firm can distribute its total value. If less risk is paid out in the stock market, then the price of the traded claims reflects this.

The same real variables drive both returns and wage contracts. In equilibrium, firms with different characteristics pay out a different percentage of their factor risk in the wage bill. This has implications for well known cross-sectional asset pricing anomalies. For example, a size effect naturally occurs in our model (Fama and French 1993). Further, the characteristics of firms' wage bills should be correlated with abnormal returns in the data. Indeed, we provide explicit predictions on the relationship between compensation (either wage or profit sharing), and asset market returns.

We embed an optimal contract in general equilibrium. However, the novel aspect is not the contract per se, but rather the properties of the minimum wage required to entice a worker to join a firm (in addition to the cross–sectional differences in the contracts). Even under first best the contracts are not unique: a continuum of contracts elicit first best behavior. Under moral hazard, if effort is unobservable, workers retain some systematic risk in their contract. However, workers can trade in financial markets and therefore lay off any untoward systematic risk but retain idiosyncratic risk. Frictionless access to capital markets renders the equilibrium tractable, but the contract not unique. However, irrespective of the form of the optimal contract, the minimum wage is unique (the workers are all driven down to this level) and we examine its properties. For example, one can construct economies in which the ex ante value of human capital is higher in the presence of moral hazard.

We interpret our results in light of the Roll (1977) critique of the CAPM. Roll's critique, that the only test of the CAPM is whether the market portfolio is efficient, follow from the fact that the true market portfolio is unobservable. In our model, risky labor income is the main source of the discrepancy between the observed and true market portfolio, and drives the failure of the CAPM to price assets correctly. The consumption CAPM (Breeden 1979), on the other hand, does hold in our model. In the context of these models, our analysis suggests that a supply-side proxy for the true market portfolio is given by stock returns together with firms' risky labor expenses. Empirically, while we do not frequently observe labor contracts, we do observe the returns to human capital. These reflect the risk paid out to workers in the form of wages. An empirical literature has tried to proxy for the investments that agents make through their human capital. For example, Jagannathan and Wang (1996) finds that a labor factor improves predictability of expected returns.

Following the insights of Telmer (1993), and Heaton and Lucas (1996), idiosyncratic labor income and incomplete markets do not seem to fully resolve asset pricing anomalies in the time series. We take a different approach, namely that labor income is related to the real economy and therefore might help with cross–sectional predictions. Specifically, in our model, labor income is tied to a particular sector of the economy and therefore generates a particular hedging demand which affects firms' cost of capital. A literature has developed to analyze the effect of labor income in explaining cross sectional asset prices. Most recently, Santos and Vieronesi (2006) demonstrates how including stochastic labor income in a representative agent economy generates return predictability. They find that the labor to consumption ratio is predictor of long run returns. Danthine and Donaldson (2002) demonstrates in a dynamic model that the implicit leverage implied by wage payments combined with uninsurable labor income risk generate realistic equity premia.

Our model is simple compared with their work in that we model the productive sector in a static (two-period) economy. This allows us to study, cross-sectionally, the close link between wage compensation and returns in capital markets. In this respect, we are closer to Bodie, Merton and Samuelson (1993), who assume perfect correlation between human capital and stock return in a one asset portfolio choice model, and also to Qin (2003) who introduces a similar model. However, compared with these two papers, our analysis goes further, by endogenizing firms' labor compensation decisions and the industry cost of capital.

Ou-Yang(2005) presents an equilibrium model of asset pricing and moral hazard. Our work is similar in that we both exploit the tractability of the normal–exponential framework pioneered by Holmstrom and Milgrom (1987). A key difference between our frameworks is that in our model, workers are also the investors and can trade on any systematic risk in their compensation package. This obviates the need for relative performance evaluation. It further ensures that the objective function of the firm is well defined (i.e., all investors agree on profit maximization). We also fully endogenize agents' participation wages.

These differences also distinguish our work from Zame (2007) who considers a general equilibrium model in which firms, firm organization and the prices of inputs and outputs for all the consumption goods are endogenous. In his model, state contingent profit–sharing plans are part of the description of the firm, the prices of which are determined in equilibrium. In other words, intra–firm transfers replace an external asset market. By contrast, to distinguish between wage payments and asset market returns, we restrict attention to the particular utility function for which the objective function of the firm is well defined, i.e., all shareholders agree on the "positive NPV" rule. Further, by explicitly allowing all agents to trade in securities markets we can relate asset market returns to our endogenous wage contract.

We establish the general framework in Section 2 and analyze the case with observable effort levels in the following, Section 3. We completely characterize the equilibrium in this framework. In Section 4, we introduce unobservable effort levels and develop the equilibrium theory for this case and also compare the two equilibria. We present our predictions and implications in Section 5: these demonstrate how prices and allocations change in the economy as a result of a productivity shock (Subsection 5.1), show how asset pricing "anomalies" can arise (Subsection 5.2); and present cross-sectional labor market predictions (Subsection 5.3). After a brief conclusion, all proofs are presented in the first appendix. In the second appendix we establish the utility equivalence of contracts under moral hazard.

2 Model

Consider the following two period economy populated by firms and workers. At time $t = 1$, workers sign employment contracts with firms and trade in financial markets. Simultaneously, firms raise money in the financial markets and make investment decisions. At $t = 2$, firms' cash flows are realized and all claims pay out. We describe the decisions of agents who consume and work, the real production sector, and the financial sector in more detail below. Throughout the paper we adhere to the convention that a boldface letter presents a vector, the superscript T denotes a transpose and the operator $()_i$ selects the ith element of a vector. For an arbitrary vector, $\mathbf{a}$, we use the notation $diag(\mathbf{a})$ to denote the diagonal matrix with $\mathbf{a}$ on its diagonal.

The economy is populated by a continuum of ex ante identical workers, each indexed by

m with total mass $M > 0$. Each agent has a CARA utility function

$$U_m = U(W_m, e_m) = -e^{-\rho(W_m - \frac{k e_m^2}{2})},$$

over $t = 2$ wealth denoted by W_m and effort, e_m, expended by the agent between $t = 1$ and $t = 2$. The cost per unit effort expended by a worker is $k > 0$. We interpret the continuous effort variable $e_m \geq 0$ as an investment in firm–specific human capital. The $t = 2$ wealth of agent m, W_m, is made up of his wage income, w_m, and the payoff of his investment portfolio, which we describe below. At $t = 1$, all agents have initial wealth W.

Firms transform human capital and investment capital into wealth with industry specific production risk and labor productivity. A continuum of firms, each indexed by ℓ, is divided into $n = 1, \ldots, N$ sectors (also called industries). Each firm in the same industry is identical. Thus there are many workers within each firm and many firms within each industry. This will allow us to apply the law of large numbers both to diversify human capital specific risk within a firm and firm specific risk within an industry.

In each industry, one unit of investment requires one unit of workers. Thus, if firm ℓ in industry n chooses a nonnegative level of investment, $I^{n,\ell}$, it must hire a mass $I^{n,\ell}$ of workers. It offers to each worker a wage contract, $w_m^{n,\ell}$, and workers decide how much effort to expend.

All contracts are written on a performance report generated by the worker's effort. The report of agent m employed in sector n in firm ℓ is denoted $\eta_m^{n,\ell}$. We consider two types of contracting regimes that differ in the information content of the report and specify them in more detail below. First, a frictionless economy in which workers and firms write wage contracts contingent on the effort level (observable effort); and second, an economy impaired by a moral hazard problem in which workers and firms contract on a noisy signal of effort (unobservable effort). However, in both cases the effort that each worker exerts is endogenous, rendering the effective labor per unit capital endogenous.

If each worker, m, chooses an effort level e_m, the revenue of firm ℓ in industry n is

$$\widetilde{R}^{n,\ell} = \alpha^n \int_0^{I^{n,\ell}} e_m^{n,\ell} dm + \tilde{x}^n I^{n,\ell}.$$

The productivity of workers differs across industries. In sector n each unit of human capital increases revenues by $\alpha^n > 0$. Without loss of generality, we assume that $\alpha^1 \leq \alpha^2 \leq \cdots \leq \alpha^N$.

Production is also uncertain; each industry is subject to a shock to productive capital, $\tilde{x}^n$, where $\tilde{x}^n$, $n = 1, \ldots, N$, are jointly normally distributed, $\tilde{x}^n \sim N(1, \sigma_x^2)$. Economy wide, these multivariate real risk factors are the $N \times 1$ vector, $\tilde{\mathbf{x}} = (\tilde{x}^1, \ldots, \tilde{x}^N)^T$, where $\tilde{\mathbf{x}} \sim N(\mathbf{1}, \Sigma)$, and $\mathbf{1}$ is an $N \times 1$ vector of ones.[3] In what follows, we assume that the $N \times N$ covariance matrix, Σ, is nonsingular; throughout most of the paper we will allow for arbitrary covariance matrices.

Capital in this economy does not depreciate, however, firms face a convex investment cost, $\kappa + \gamma\alpha^n (I^{n,l})^2$, where $\kappa > 0$ and $\gamma > 0$ are constants.[4] These costs capture the fact that physical capital and financial capital are not perfectly fungible. We specifically interpret the costs as payments for research and development. This functional form is motivated by two stylized facts. First, marginal investment costs are increasing in investment level. Second,

[3]It is straightforward to generalize the model to arbitrary means, $\tilde{\mathbf{x}} \sim N(\bar{\mathbf{x}}, \Sigma)$.

[4]The model is easy to generalize to industry specific κ's and γ's.

R&D costs per unit of investment are higher in high productivity industries than in low productivity ones.[5] Mechanically, the R&D cost renders the production function concave.

If the risk-adjusted cost of capital in industry n is r^n, then the economic profits of a firm are:

$$\tilde{\pi}^{n,\ell} = \underbrace{\alpha^n \left(\int_0^{I^{n,\ell}} e_m^{n,\ell} dm \right) + \tilde{x}^n I^{n,\ell}}_{\text{Revenues}} - \underbrace{\int_0^{I^{n,\ell}} w_m^{n,\ell} dm}_{\text{Cost of wages}} - \underbrace{(\kappa + \gamma \alpha^n (I^{n,l})^2)}_{\text{Production costs}} - \underbrace{r^n I^{n,\ell}}_{\text{Cost of capital}} \quad (1)$$

We note in passing that because all agents can trade in financial markets the objective of the firm is well defined and all shareholders agree on economic profit maximization.

Prices in the financial market are determined by the interaction of workers who hedge consumption risk and firms who raise capital. The financial market opens at time $t = 1$. Agents may trade securities which are claims to the firms' profits. As risk within each industry is perfectly correlated, without loss of generality, we can assume that there are N representative stocks. Stock n has price $S^n(t)$ at $t = 1, 2$. The return of stock n is denoted $\tilde{\mu}^n$, i.e., $S^n(2) = (1 + \tilde{\mu}^n)S^n(1)$. The random market returns can then be summarized by $\tilde{\boldsymbol{\mu}} \sim N(\bar{\boldsymbol{\mu}}, \Sigma_\mu)$, where $\tilde{\boldsymbol{\mu}}$ is an $N \times 1$ vector of returns with typical element μ^n, and Σ_μ is the $N \times N$ covariance matrix. We assume that Σ_μ is invertible.[6] We define $\boldsymbol{\sigma}_{\mu,n} = \mathrm{cov}(\tilde{\boldsymbol{\mu}}, \tilde{x}^n)$. There is also a risk-free asset in perfectly elastic supply, with excess return normalized to zero.

Each investor working in industry n chooses a portfolio of dollar amounts in each industry denoted by $\boldsymbol{q}^n$, which is an $N \times 1$ vector describing his investment in each industry. At $t = 2$ his portfolio has value $\tilde{\theta}^n = \tilde{\boldsymbol{\mu}}^T \boldsymbol{q}^n$. For later convenience, we introduce the matrix $\boldsymbol{Q} = [\boldsymbol{q}^1, \ldots, \boldsymbol{q}^N]$.

In what follows, we focus on symmetric outcomes, so that all firms within the same industry offer the same contract to their employees and all employees in a particular industry invest in the same way. The exogenous parameters in the frictionless economy are given by the tuple $\mathcal{E}_0 = (M, \kappa, \gamma, \Sigma, k, \rho, \boldsymbol{\alpha})$. Here, economy–wide human capital productivity is characterized by the vector $\boldsymbol{\alpha} = (\alpha^1, \ldots, \alpha^N)^T$. An equilibrium is characterized by the tuple of endogenous quantities: $\mathcal{X}_0 = (w_0, \boldsymbol{L}, \boldsymbol{I}, \boldsymbol{e}, \bar{\boldsymbol{\mu}}, \boldsymbol{Q})$. The first four elements of $\mathcal{X}_0$ constitute the real part of the economy.

The variable w_0 in an economy is the ex ante value of a worker's human capital. Specifically, it is the difference between the certainty equivalent of earned labor income and the disutility of working. As we demonstrate below, in the optimal contract, the worker retains no surplus and therefore w_0 can be thought of as the wage a worker would earn at a hypothetical firm that requires no effort from its workers. We will also refer to w_0 as the *effort-free wage* or the *participation wage*.[7]

[5]Growth firms — which are usually in high-productivity industries — tend to be R&D intensive even when adjusted for size, see e.g., Chan, Lakonishok and Sougiannis (2000), hence the α^n-term in $\gamma\alpha^n(I^{N,l})^2$.

[6]It will become clear that the equilibrium distribution of returns is normal, and that in equilibrium, invertibility of Σ is equivalent to invertibility of Σ_μ.

[7]In constructing equilibrium, workers are indifferent between remaining in their sector and going elsewhere including this effort free alternative.

The vector $\boldsymbol{L} = (L^1, \ldots, L^N)^T$, summarizes the distribution of firms across all sectors. The initial size of each firm, or alternatively investment in physical capital is given by $\boldsymbol{I} = (I^1, \ldots, I^N)^T$, and the workers' supply of human capital by $\mathbf{e} = (e^1, \ldots, e^N)^T$. For a given technology and exogenous structure of risk, the amount of risk generated will depend on these production choices and is therefore endogenous. The last two elements in $\mathcal{X}_0$ are the equilibrium financial variables: the expected return of each asset ($\bar{\boldsymbol{\mu}}$) and agents' portfolio choices ($\boldsymbol{Q}$). We restrict attention to equilibria that are feasible: $I^n \geq 0$, $L^n \geq 0$, $e^n \geq 0$ for all $n = 1, \ldots, N$. Further, we focus on equilibria that are interior: those that are feasible and for which all the previous inequalities are strict.

Definition 1 *General equilibrium of the frictionless economy $\mathcal{E}_0$ is characterized by $\mathcal{X}_0$ in which:*
(i) *Each firm optimally chooses an investment level and a wage contract to maximize expected profits leading to $\boldsymbol{I}$.*
(ii) *Given a wage contract, each worker optimally chooses his effort level and stock-market investment to maximize expected utility, leading to $\mathbf{e}$ and $\boldsymbol{q}$.*
(iii) *Asset markets clear: $M\boldsymbol{q} = diag(\boldsymbol{I})\boldsymbol{L}$.*
(iv) *Labor markets clear: $M = \boldsymbol{I}^T\boldsymbol{L}$.*
(v) *For each sector $n = 1, \ldots, N$, the expected return on financial assets equals the cost of capital in the real economy, so that $\bar{\mu}^n = r^n$.*

Part *(iv)*, the labor market clearing condition, reflects our assumption that each worker is paired with a unit of capital, irrespective of the amount of effort he exerts. Thus, while the "effective labor" supplied by each worker is endogenous, this market clearing condition on bodies reduces the dimensionality of the general equilibrium problem to permit a solution.

An implication of the last condition *(v)*, that the expected return in each sector of the financial market equals the cost of capital of that sector in the real economy, is that firms earn zero economic profits. In this two-period world, a return higher than the cost of capital is equivalent to positive profits. Such rents are incompatible with general equilibrium as firms would enter into industries in which there are positive profits, driving rents to the fixed entry costs (κ) and thus generating zero economic profits.

Notice that there are no frictions to prevent agents from trading. In particular, an agent trades at time $t = 1$ against his labor income which will be realized at time $t = 2$. Also, each agent may trade in the securities issued by his industry. To summarize, the sequence of events is presented in Figure 1.

3 Equilibrium in the Frictionless Economy

Suppose that an employee m in firm ℓ in industry n generates a performance report of the form:

$$\tilde{\eta}_m^{n,\ell} \quad = \quad \tilde{R}^{n,\ell}/I^{n,l} + \alpha^n(e_m - e^n).$$

This is increasing in the average revenue of the firm and also in the amount of effort the employee exerts. In equilibrium, the firm knows the effort it elicits from all the other employees (e^n) and (ex post) how much revenue it has garnered. Therefore, it effectively observes the

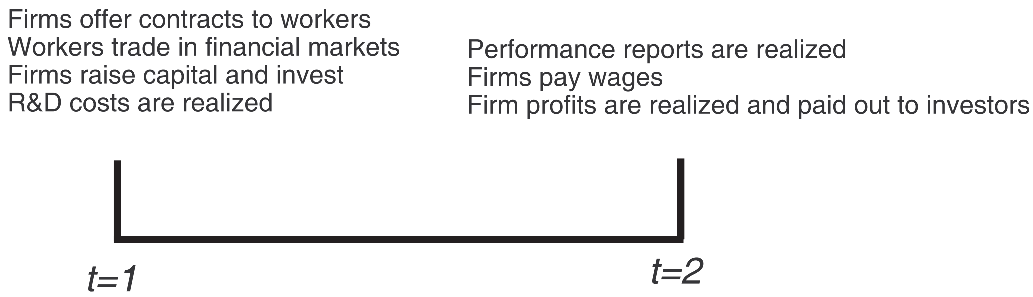

Figure 1: ***Summary of events with observable effort level.***

actions of employee m. If effort is observable and contractible, the optimal wage contract is simply a take–it or leave–it offer that makes the worker indifferent between accepting the firm's offer and rejecting it for the next best outside employment alternative. While all firms offer the same type of contract, as firms are characterized by different labor productivity, they elicit different effort levels. To do so, they compensate workers for the cost of exerting higher effort. However, in any equilibrium, workers must be indifferent between remaining in their current employment or moving to another industry, else the firm is overpaying.

Recall, that $w_0 \geq 0$ is the equilibrium effort–free wage. The firm that wishes to induce a worker to exert effort e, offers a contract $w(e)$ that recompenses the agent for the effort he undertakes and makes him indifferent to the outside opportunity.

Lemma 1 *If the effort levels are observable and contractible, then a firm wishing to elicit effort level $\widetilde{e}$ offers a worker*

$$\begin{array}{rl} w_0 + k\widetilde{e}^2/2 & \text{if } e = \widetilde{e} \\ 0 & \text{otherwise.} \end{array}$$

We note in passing that this contract is not unique. In particular, any promised payment off the equilibrium path (i.e., if the worker shirks) less than w_0 is optimal and induces an effort level of $\widetilde{e}$. Irrespective of the "shirking" payment, the wage for satisfactory performance is paid ex post to the worker, conditional on the effort that he puts into the firm. Therefore, the firm incurs no wage bill until after the production process has been completed. In the next section, in which we consider the case in which effort is not contractible, firms optimally offer part of the wage contract ex ante to workers (before the production risk is realized).

Given the wage contract, a representative, price–taking firm in industry n chooses e^n and I^n to solve:

$$\max_{e^n, I^n} E\left[\widetilde{\pi}^n\right] = \alpha^n e^n I^n + I^n - (w_0 + k(e^n)^2/2)I^n - (\kappa + \gamma\alpha^n (I^n)^2) - r^n I^n. \tag{2}$$

Therefore, necessary conditions satisfied by the firm's optimal choice of e^n and I^n are:

$$\alpha^n e^n + 1 - (w_0 + \frac{k(e^n)^2}{2}) - 2\gamma\alpha^n I^n - r^n = 0 \tag{3}$$

$$\alpha^n - ke^n = 0. \tag{4}$$

As the wage is deterministic and there are no wealth effects, the decision to provide labor and how the agent invests in the stock market are independent. In other words, agents make effort and stock investment decisions separately.

Lemma 2 *Every agent in the benchmark economy chooses an identical portfolio investment,* $\boldsymbol{q} = \Sigma_\mu^{-1}\frac{\bar{\boldsymbol{\mu}}}{\rho}$. *The certainty equivalent of this stock market participation is* $A/2\rho$, *where* A *is the squared Sharpe ratio of the market, so that* $A \stackrel{\text{def}}{=} \bar{\boldsymbol{\mu}}^T \Sigma_\mu^{-1} \bar{\boldsymbol{\mu}}$.

To ensure the existence of general equilibrium, we impose two technical conditions. Let

$$v^n = 1 + (\alpha^n)^2/2k - 2\sqrt{\kappa\gamma\alpha^n}, \qquad n = 1, \ldots, N$$

and $\mathbf{v} = (v^1, \ldots, v^N)^T$. We assume that $v^n > 0$, $n = 1, \ldots N$. After we have characterized equilibrium we provide a natural economic interpretation of v^n.

Assumption 1 (i) *The risk aversion of investors is bounded above. Specifically,*

$$\rho < \mathbf{1}^T \Sigma^{-1} \mathbf{v},$$

(ii) *The risk aversion of investors is bounded below. Specifically,*

$$\rho \Sigma^{-1}\mathbf{1} > (\mathbf{1}^T\Sigma^{-1}\mathbf{v})\Sigma^{-1}\mathbf{1} - (\mathbf{1}^T\Sigma^{-1}\mathbf{1})\Sigma^{-1}\mathbf{v}. \tag{5}$$

The first part of Assumption 1 ensures that all workers are employed in equilibrium. Or, equivalently, that the endogenous participation constraint is nonnegative, $w_0 \geq 0$. If this condition does not hold, then investors are too risk-averse to want to absorb the risk inherent in a full-employment equilibrium. Thus, they are better off at lower production and concomitant risk levels. The condition is always satisfied if there is at least one sector with very low risk. In this case, workers can always contribute to total surplus by working in a low-risk sector.

The second part of the assumption guarantees that agents are sufficiently risk averse so that they will not wish to hold negative positions in any of the assets in equilibrium. Equivalently, in general equilibrium this assumption ensures that investment in all firms is strictly positive. If this condition were violated (suppose that all agents were risk neutral) each would optimally invest arbitrary large amounts in the asset with the highest mean return and short all assets with lower returns. In this case, the markets for physical investment would not clear.

Using Lemma 2 and firms' optimal investment choices, we can characterize equilibrium in the real economy. From Equation (1), that specifies the objective function of the firm, variability in the profits comes from $\tilde{x}I^n$. Therefore, $\tilde{\boldsymbol{\mu}}$ is also normally distributed, and $\Sigma_\mu = \Sigma$, so invertibility of Σ carries over to Σ_μ. Also, recall, $v^n = 1 + (\alpha^n)^2/2k - 2\sqrt{\kappa\gamma\alpha^n}$, $n = 1, \ldots, N$ and $\mathbf{v} = (v^1, \ldots, v^N)^T$.

Proposition 1 *In an economy that satisfies Assumption 1, there is a unique general equilibrium which is feasible, interior, and Pareto efficient. The real economy has the following properties:*
(i) *Investment in human capital is* $e^n = \alpha^n/k$,

(ii) *Investment in physical capital is* $I^n = \sqrt{\frac{\kappa}{\gamma\alpha^n}}$*,*

(iii) *The effort-free wage is* $w_0 = \frac{\mathbf{1}^T\Sigma^{-1}\mathbf{v}-\rho}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}}$*,*

(iv) *The size of each sector is such that* $L^n = M(\boldsymbol{q})_n/I^n$*,*

(v) *Agents' certainty equivalent is* $W + w_0 + (\mathbf{v} - w_0\mathbf{1})^T\Sigma^{-1}(\mathbf{v} - w_0\mathbf{1})/2\rho$*.*

Not surprisingly, the equilibrium human capital investment is increasing in a firm's productivity as evinced by *(i)*. Firms and sectors that are more productive will elicit higher effort levels from their workers. However, from *(ii)* physical investment is *decreasing* in productivity. Alternatively, as the investment determines the size of the firm in this two period economy, more productive firms are smaller. This is a general equilibrium effect.

To see this, consider the optimal investment decision of a firm in partial equilibrium, holding fixed workers' participation constraints (w_0) and the industry cost of capital (r^n). Specifically, partial differentiation of the first order condition of the firm from Equation (2) yields

$$\frac{\partial I^n}{\partial \alpha^n} = \frac{1}{2k\gamma} + \frac{w_0 + r^n - 1}{(\alpha^n)^2\gamma}.$$

Hence, $\frac{\partial I^n}{\partial \alpha^n} > 0$ if $\frac{1}{2k} > \frac{1-r^n-w_0}{(\alpha^n)^2}$. Therefore, in partial equilibrium, a firm with an unexpected increase in productivity ($\partial\alpha^n$) could either increase or decrease its size, depending on the minimum wage, cost of capital, and the productivity level. For low productivity sectors, a shock may lead to smaller firms, whereas it leads to larger firms in high-productivity industries. It also always leads to larger firms in economies where the participation constraint is high enough, or alternatively, where the cost of capital is sufficiently low i.e., when $w_0 > 1 - \min_n r^n$.

By contrast, in general equilibrium (from *(ii)* above), higher productivity always leads to smaller firms. This is because higher productivity sparks competition and the subsequent entry of new firms. Total investment in that sector typically increases, leading to a higher industry cost of capital (as risk averse investors value this investment less). Further, due to the concave production function, firm sizes are smaller.

That the equilibrium size of each industry (L^n) in our economy is also endogenous is readily seen from *(iv)* above, in which total investment in an industry is equal to the total demand by investors. The dollar amount that each identical investor is willing to put into an industry (recall that $(\boldsymbol{q})_n$ selects the nth element of the vector) depends on how investors value the risk in each industry relative to the whole economy. Risk averse investors are willing to supply more capital to an industry that has a low correlation with other sectors. It is their willingness to invest that drives industry size.

In this economy the CAPM holds; not surprising as there are no frictions.

Proposition 2 *In an economy that satisfies Assumption 1, expected returns are* $\bar{\mu}^n = v^n - w_0$ *and are described by the Capital Asset Pricing Model:*

$$\bar{\mu}^n = \beta^n E[\tilde{\mu}_{market}], \qquad \textit{where } \beta^n = \frac{cov(\tilde{\mu}^n, \tilde{\mu}_{market})}{var(\tilde{\mu}_{market})}, \ \textit{and } \tilde{\mu}_{market} = M^{-1}\sum_{n=1}^{N} I^n L^n \tilde{\mu}^n.$$

The expected returns (or equivalently, the cost of capital) are just the excess of v^n over the participation wage, w_0. The characterization of the real economy in Proposition 1, combined with the equilibrium cost of capital admits an economic interpretation of v^n. By inspection of the first order condition for investment capital (Equation 3) evaluated at the equilibrium effort level, the equilibrium value of the marginal product of investment capital is $v^n - w_0$. Increasing investment has a direct effect on profits measured by v^n, however in addition the firm has to hire an extra worker which decreases profits by the minimum participation wage, w_0. Thus stock market returns reflect the productivity of a marginal dollar invested in an industry. Notice that v^n is the value of the marginal product of capital plus the participation wage. In other words, it is the equilibrium *marginal total productivity* — the value generated to all stakeholders (shareholders and workers) of an extra unit of investment capital.

Another important variable determined in general equilibrium is the effort free wage or endogenous participation constraint, w_0, described in part *(iii)* of Proposition 1. This is set so that agents are indifferent between remaining in their sector or moving to another one. It is also the ex ante value of human capital with which each worker is endowed. The participation wage is affected by the aggregate risk structure. As workers both invest and work, an increase in the size of a particular sector changes the distribution of risk in the economy and therefore how much workers value such investment. To see this, consider the capital market clearing condition. If another worker is added to the economy he will invest in a financial portfolio so that $\boldsymbol{q} = \Sigma^{-1}\frac{\bar{\boldsymbol{\mu}}}{\rho}$. As we have demonstrated, in equilibrium, expected returns are related to the marginal productivity of investment, less the participation wage. Therefore, $\boldsymbol{q} = \frac{\Sigma^{-1}(\mathbf{v}-w_0\mathbf{1})}{\rho}$. In full employment equilibrium, each worker is paired with one unit of investment capital. Therefore, as capital markets clear, the sum across all investments must be 1, or $\mathbf{1}^T\boldsymbol{q} = 1$. We stress that this reflects our simplifying assumption that each worker is associated with one unit of productive investment capital.

Therefore, capital market clearing yields

$$1 = \frac{\mathbf{1}^T\Sigma^{-1}(\mathbf{v} - w_0\mathbf{1})}{\rho},$$

which pins down the effort free wage. If this is too high, then the right hand size is too low, and firms wish to disinvest. Whereas, if it is too low, then firms demand more labor.

The certainty equivalent,*(v)* in Proposition 1, will allow us to Pareto rank equilibria. It can be decomposed into two parts: one due to initial wealth and an increase due to stock and human capital investments. Specifically the equilibrium certainty equivalent can be rewritten as $W + \Delta W$, where $\Delta W = w_0 + A/2\rho$. The total welfare increase, ΔW, thus has two components: the welfare gained in the labor market, w_0, and the welfare gained in the stock market, $A/2\rho$ (Recall, that A is the squared market Sharpe ratio.)

Finally, note that all marginal investments in this economy are evaluated by a "positive NPV" rule. In an economy in which the risk-free rate is normalized to zero, only loadings on $\tilde{x}$ risks generate a positive cost of capital. Specifically, under standard noarbitrage assumptions, the linear pricing rule for investments q^0 in bonds and q^n in stock n is:

$$\text{Price}_{t=1}\left(\sum_{n=1}^{N} q^n S^n(2) + q^0\right) = \text{Price}_{t=1}(q^0) + \sum_{n=1}^{N} q^n \text{Price}_{t=1}(S^n(2))$$

$$= \quad q^0 + \sum_{n=1}^{N} q^n S^n(1).$$

This implies a linear pricing rule for $\tilde{x}^n$ risk, i.e.,

$$\begin{aligned} \text{Price}_{t=1}\left(q^0 + \sum_{n=1}^{N} q^n \tilde{x}^n\right) &= q^0 + \sum_{n=1}^{N} q^n \text{Price}_{t=1}(\tilde{x}^n) \\ &= q^0 + \sum_{n=1}^{N} \frac{q^n}{1+z^n}. \end{aligned}$$

where z^n is the required excess return per unit of $\tilde{x}^n$ risk. A firm's cost of capital thus depends on how much $\tilde{x}$ risk it represents. This observation will be useful as we turn to an economy with frictions.

4 An Economy with Moral Hazard

In reality, effort is not observable and contracts are typically tied to the cash flows of the firm. We therefore extend the model of the previous section to the case of unobservable effort levels. Specifically, we assume that firms cannot perfectly observe the effort expended by each agent; rather they receive a garbled performance report. A worker supplying effort level e_m, for firm ℓ in industry n generates a performance report of

$$\tilde{\eta}_m \quad = \quad R^{n,\ell}/I^{n,l} + \alpha^n(e_m - e^n) + \tilde{\epsilon}_m^{n,\ell},$$

where $\tilde{\epsilon}_m^{n,\ell}$ is independently drawn for each worker from $N(0, \sigma_{\epsilon,n}^2)$. Thus, the firm observes a worker's true productivity with noise. In our formulation, workers in "more productive" industries produce higher performance reports for the same effort level, and all performance reports are noisy. Of course if $\sigma_{\epsilon,n}^2 = 0$, then effort is contractible as in the frictionless case. We assume that the performance report variances across all industries are summarized by the vector $\boldsymbol{\sigma}_\epsilon = (\sigma_{\epsilon,1}, \ldots, \sigma_{\epsilon,N})^T$, however for the most part we take them to be constant across industries.

The workers' situation under moral hazard differs in an important way that affects the equilibrium: The structure of the incentive contract offered to him by the firm differs. Under the assumption of CARA utility, and normal performance reports, the optimal contract is well-known to be linear. In particular, it is made up of a fixed wage that satisfies agents' participation constraint plus a component that is sensitive to each agent's performance. The idiosyncratic risk inherent in the garbling of the performance report (σ_ϵ) cannot be laid off in the market and therefore is borne by the worker. As a risk averse agent, he deplores this uncertainty and requires an ex ante compensation from the firm to participate in the production process. As a consequence of these different contracts, and in particular because the optimal incentive contract includes some systematic risk, an agent's optimal portfolio now depends on the industry in which he is employed.

If the agent is risk averse, the firm is risk neutral, and agents are constrained in asset trades then as Holmstrom (1982) observed, the principal should design a relative performance contract.[8] In our framework, by the law of large numbers, the firm could eliminate the systematic risk by comparing any individual's report with all the others'. However, in equilibrium, it is *not* inefficient for the worker to bear systematic risk. Agents may short-sell the stock of the company at which they are employed, and the firm and each agent has the same valuation for each unit of systematic risk that is included in wage. (We demonstrate this rigorously in the appendix).[9]

Given the structure of the optimal contract, an industry-n contract will take the form

$$\widetilde{w}_m^n = s^n + b^n \times \tilde{\eta}_m.$$

The contract can be interpreted as one with a fixed wage that recompenses the worker for idiosyncratic risk, and a revenue sharing part. A natural assumption is that the fixed part of the wage is paid out in the first time period, whereas the bonus part can not be paid out until revenues have been realized, as described in Figure 2. When we discuss empirical results and in our predictions we distinguish between wages (s^n), incentives (b^n) and total compensation w^n.

As the firm hires a continuum of workers, all the idiosyncratic worker risk $\epsilon_m^{n,\ell}$ cancels out by the law of large number and the average performance report only depends on revenue risk.[10] Given that the contract offered in industry n is characterized by the pair (b^n, s^n), which induces a mean effort level of e^n, the firm's expected profit function is:

$$E\widetilde{\pi}^n = \underbrace{I^n(\alpha^n e^n + \tilde{x}^n)}_{\text{Revenue}}(1 - b^n) - I^n s^n - r^n I^n - \left(\kappa + \gamma\alpha^n (I^n)^2\right). \tag{6}$$

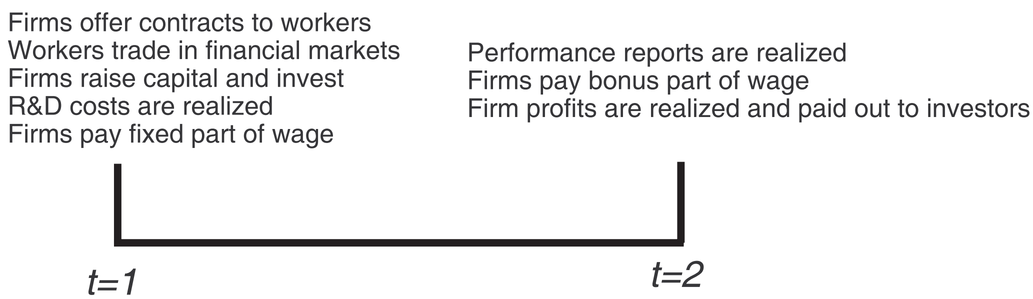

Figure 2: ***Summary of events with unobservable effort level***.

[8]The empirical evidence on this is that relative performance contracts are not pervasive as summarized by Prendergast(1999). Garvey and Milbourn (2003) demonstrates that relative performance evaluation is more likely in executives who are more constrained.

[9]We found qualitatively similar numerical results in a version in which workers are not allowed to short-sell their own sector. Alternatively, if workers cannot short sell stock in their own company, but can short sell industry risk, they could obtain hedge $\tilde{x}$ risk, but not $\tilde{\eta}$ risk. If workers coordinate their effort level, perhaps due to peer pressure (see, e.g., Kandel and Lazear 1992), similar results obtain.

[10]By the law of large numbers for a continuum of random variables (see e.g., Judd 1985), the average performance report of workers in industry n is $\bar{\eta}^n = \frac{1}{I^n L^n} \int \widetilde{\eta}_m dm = \tilde{R}^n / I^n$.

Observe that a portion of the firm's revenue is paid out to workers. The wage bill acts as "leverage," on the revenue of the firm. In other words, part of the risks in the real economy are transferred to the workers through their incentive contracts. This is captured by the $(1-b^n)$-term.

4.1 Agents' Optimal Portfolios and Effort levels

Agents choose their portfolios and effort levels given the incentive contracts offered by the firms. To avoid possible confusion, we use a hat to distinguish variables that obtain in the moral hazard economy. Because the performance part of the contract includes systematic risk, the worker optimally chooses a portfolio that is correlated with the firm's output. Faced with a wage contract, the agent chooses a portfolio $\hat{\tilde{\theta}}^n$ (represented by the dollar portfolio $\hat{\boldsymbol{q}}^n$) and an effort level $\hat{e}^n$ to maximize the certainty equivalent of the increase in her utility:

$$\Delta\widehat{W}^n = \max_{\hat{\boldsymbol{q}}^n, \hat{e}^n} \left[E\left(\widetilde{w}^n + \tilde{\theta}^n\right) - \frac{k}{2}(e^n)^2 - \frac{\rho}{2}\mathrm{var}(\widetilde{w}^n + \tilde{\theta}^n)\right]. \tag{7}$$

Lemma 3 *If firm n offers a linear contract (s^n, b^n) then:*
(i) *The worker supplies effort $\hat{e}^n = \frac{b^n\alpha^n}{k}$,*
(ii) *The worker holds a portfolio position $\hat{\boldsymbol{q}}^n = \Sigma_{\hat{\mu}}^{-1}\left(\frac{\hat{\boldsymbol{\mu}}}{\rho} - b^n\boldsymbol{\sigma}_{\hat{\mu},n}\right)$.*

It is immediate that if $b^n \neq 1$, then workers will not exert the same effort level as in Section 3. Feasibility (firms cannot pay out more than the revenues of the firm) dictates that $b < 1$ and therefore they will exert lower effort.

Workers hedge the risk that they are exposed to through the incentive contract: That is, they also seek to eliminate exposure to $\tilde{x}$ risk by shorting claims correlated with the industry in which they work (part (*(ii)*). This mirrors the argument made in Bodie, Merton and Samuelson (1993) that even though human capital is not tradable, human capital risk may be partly hedgeable in the market. If $b^n = 0$, so that the firm does not offer an incentive component, all workers are faced with idiosyncratic risk and hold the same portfolio, $\hat{\boldsymbol{q}}^n = \Sigma_{\hat{\mu}}^{-1}\frac{\hat{\boldsymbol{\mu}}}{\rho}$, as in the case where effort is observable. Thus, the term $b^n\Sigma_{\hat{\mu}}^{-1}\boldsymbol{\sigma}_{\hat{\mu},n}$ in part (*ii*) of Lemma 3 is the distortion in portfolio holdings that comes about because the firm transfers wealth to the agent through labor income.[11] The only risk that the agent is unable to trade is the idiosyncratic risk that arises because he cannot perfectly communicate his effort level to the firm.

As in the observable effort case, agents will not work for a firm unless they are at least as well off as they would be if they worked in another industry or did not work at all. Therefore they must be recompensed both for their effort and the idiosyncratic risk inherent in the performance report. The former will depend on the performance report, the latter will be the fixed wage. A firm offering an optimal contract ensures that each worker's participation constraint is binding or that it appropriates any surplus, i.e., $E[U_m] = -e^{-(W+\Delta W)}$ for all workers. Notice that the firms take into account the utility workers get from optimally trading

[11] That investments in such hedging portfolios arise when labor income risk is present was observed in Mayers (1973) and in several subsequent papers.

in financial markets, by offering contracts that drive workers down to their participation constraint, *including* their portfolio holdings.

Lemma 4 *A firm in sector n will offer the following fixed wage*

$$s^n(b^n, \hat{I}^n) = (b^n)^2 \left(-\frac{(\alpha^n)^2}{2k} - \frac{\rho}{2}(C^n - \sigma^2_{x,n}) + \frac{\rho\sigma^2_{\epsilon,n}}{2} \right) + b^n(B^n - 1) + \Delta\widehat{W} - \frac{\hat{A}}{2\rho},$$

where

$$\hat{A} = \hat{\boldsymbol{\mu}}^T \Sigma_{\hat{\mu}}^{-1} \hat{\boldsymbol{\mu}}, \qquad B^n = \hat{\boldsymbol{\mu}}^T \Sigma_{\hat{\mu}}^{-1} \boldsymbol{\sigma}_{\hat{\mu},n}, \qquad C^n = \boldsymbol{\sigma}'_{\hat{\mu},n} \Sigma_{\hat{\mu}}^{-1} \boldsymbol{\sigma}_{\hat{\mu},n}.$$

We define the vectors $\boldsymbol{b} = (b^1, \dots, b^N)$ and $\boldsymbol{s} = (s^1, \dots, s^n)$.

4.2 A Firm's Partial Equilibrium Investment in Physical and Human Capital

In the economy with moral hazard, the cost of capital is reduced by the risk that will be paid out to the workers. The risky cash flows are the part of the revenue that is not paid out to the employees in compensation: $\widetilde{R}^n(1 - b^n)$. This implies a slightly different formulation of the cost of capital. Suppose that in equilibrium, one unit of $\widetilde{x}^n$ risk commands a required rate of return, z^n, then a unit investment in industry n generates a unit of $\widetilde{x}$ risk of which only $(1 - b^n)$ accrue to the shareholders. Therefore, the cost of capital in this industry is

$$\hat{r}^n = (1 - b^n) z^n.$$

Notice, that if firms pay out all their risky cash flows to workers, the firm is risk free and the cost of capital falls to zero, the risk free rate. If the firm pays out no incentive bonuses, then it retains the industry $\widetilde{x}$ risk and the cost of capital is maximal. As individual firms take z^n as given, they choose physical investment and labor investment to maximize risk-adjusted expected profits:

$$\begin{aligned} \max_{\hat{I}^n, b^n} E[\widetilde{\pi}^n] &= \max_{\hat{I}^n, b^n} I^n \Big((\alpha^n e^n + 1)(1 - b^n) - s^n(b^n, I^n) - (1 - b^n) z^n - \alpha^n \gamma I^n \Big) - \kappa \\ &= \max_{I^n, b^n} I^n \left(\Big(\frac{b^n (\alpha^n)^2}{k} + 1 \Big)(1 - b^n) - s^n(b^n, I^n) - (1 - b^n) z^n - \alpha^n \gamma I^n \right) - \kappa. \end{aligned} \tag{8}$$

The first order condition for physical capital:

$$\frac{\partial E[\widetilde{\pi}^n]}{\partial I^n} = \underbrace{\Big(\frac{b^n (\alpha^n)^2}{k} + 1 \Big)(1 - b^n) - 2\alpha^n \gamma \hat{I}^n}_{\text{value of the marginal product of capital}} - \underbrace{\left(\hat{I}^n \frac{\partial s^n(b^n, \hat{I}^n)}{\partial I^n} + s^n(b^n, \hat{I}^n) + (1 - b^n) z^n \right)}_{\text{change in cost}} = 0,$$

suggests that the firm equates the value of the marginal revenue product of capital (the first term), with the change in costs it incurs if it increases capital (the second term). The change in cost consists of three terms. The first is the effect on the wage bill of increasing investment. If the firm increases investment then, ceteris paribus, the firm has to recompense existing workers for the fact that the value of the workers' shares is now lower. Secondly, an increase in investment induces an increase in the labor force and therefore the firm pays out an extra fixed wage. Finally, for every increase in investment, the firm pays the direct cost of $\widetilde{x}$ risk in the capital markets.

Solving the two first order conditions yields the firm's optimal decision:

Lemma 5 *Firms in sector n will choose*

$$\begin{aligned} b^n &= \frac{(\alpha^n)^2/k + z^n - B^n}{(\alpha^n)^2/k + \rho(\sigma^2_{x,n} - C^n + \sigma^2_{\epsilon,n})}, \\ \hat{I}^n &= \frac{1}{4\alpha^n\gamma^n}\left(\frac{((\alpha^n)^2/k + z^n - B^n)^2}{2[(\alpha^n)^2/k + \rho(\sigma^2_{x,n} - C^n + \sigma^2_{\epsilon,n})]} + \frac{\hat{A}}{2\rho} - \Delta\widehat{W} + 1 - z^n\right), \end{aligned}$$

if $\hat{I}^n > 0$ *and* $0 < b^n < 1$.

Recall, that the optimal effort level is increasing in b^n, but by a factor of $\frac{\alpha^n}{k}$. Thus, Lemma 5 can also be viewed as a firm's choice of human capital and physical capital. Notice also that an increase in physical investment increases the aggregate $\widetilde{x}$ risk in the economy.

4.3 General Equilibrium with Moral Hazard

We proceed along the same lines as in the case of observable effort level in Section 3. However, in addition to the exogenous variables in the frictionless case we also include the noise inherent in each agent's performance report. Recall, the vector $\boldsymbol{\sigma}_\epsilon$ captures the extent of the moral hazard problem industry by industry. We let $\mathcal{E}_1$, denote the parameters of the economy that comprises $\mathcal{E}_0$ with the vector of "garbles." The endogenous variables comprise all the ones of the frictionless economy. However, in this case the optimal wage contract is a two part payment. Finally, observe that, just as the cost of capital reflects the actual risks borne by the shareholders of the firm, the covariance structure of returns also respects this. Thus,

$$[\Sigma_{\hat{\mu}}]_{i,j} = (1 - b^i)[\Sigma]_{i,j}(1 - b^j), \quad \text{and} \quad (\boldsymbol{\sigma}_{\hat{\mu},i})_j = [\Sigma]_{i,j}(1 - b^j). \tag{9}$$

This follows from the linear pricing rule implied by noarbitrage. As the traded securities are claims to the underlying firm, the real risk retained by the firm must be reflected in the asset market. As in the observable case, multivariate normality of $\tilde{\boldsymbol{x}}$ implies multivariate normality of $\hat{\tilde{\boldsymbol{\mu}}}$, and as long as $b^n < 1$, $n = 1, \ldots, N$, invertibility of Σ is equivalent to invertibility of $\Sigma_{\hat{\mu}}$. In the presence of moral hazard, total marginal productivity of capital is

$$\hat{v}^n = 1 + b^n(\alpha^n)^2/2k - 2\sqrt{\kappa\gamma\alpha^n}, \qquad n = 1, \ldots, N.$$

This is identical to the definition in the observable case, except for the factor b^n that enters before the $(\alpha^n)^2/2k$ term. The incentive problems under moral hazard reduce $\hat{v}^n$ compared to the observable case. We define the vector $\hat{\mathbf{v}} = (\hat{v}^1, \ldots, \hat{v}^N)^T$.

Finally, we need slightly different parameter restrictions to ensure the existence of an interior equilibrium:

Assumption 2 (i) *The risk aversion of investors is bounded above. Specifically,*

$$\rho < \mathbf{1}^T \Sigma^{-1} \hat{\mathbf{v}},$$

(ii) *The risk aversion of investors is bounded below. Specifically,*

$$\rho \Sigma^{-1} \mathbf{1} > (\mathbf{1}^T \Sigma^{-1} \hat{\mathbf{v}}) \Sigma^{-1} \mathbf{1} - (\mathbf{1}^T \Sigma^{-1} \mathbf{1}) \Sigma^{-1} \hat{\mathbf{v}}. \tag{10}$$

With these caveats and following a similar line of argument to that which established Proposition 1, we can establish:

Proposition 3 *In an economy, $\mathcal{E}_1$, that satisfies Assumption 2, there is a unique equilibrium. The equilibrium is feasible, interior, and strictly Pareto dominated by the equilibrium in the observable economy $\mathcal{E}_0$, and*
(i) *Effort levels are $\hat{e}^n = b^n \alpha^n / k$,*
(ii) *Investment is $\hat{I}^n = \sqrt{\frac{\kappa}{\gamma \alpha^n}}$,*
(iii) *The optimal incentive part of the wage contract is $b^n = \frac{1}{1 + \frac{k \rho \sigma^2_{\epsilon,n}}{(\alpha^n)^2}}$,*
(iv) *The fixed part of the wage contract is $s^n = (1 - b^n) \hat{w}_0 + \frac{(b^n)^2 \rho \sigma^2_{\epsilon,n}}{2} - 2b^n \sqrt{\kappa \gamma \alpha^n}$,*
(v) *The effort-free wage is $\hat{w}_0 = \frac{\mathbf{1}^T \Sigma^{-1} \hat{\mathbf{v}} - \rho}{\mathbf{1}^T \Sigma^{-1} \mathbf{1}}$,*
(vi) *The number of firms in each sector is $\hat{\boldsymbol{L}} = M \rho^{-1} \Lambda_{\hat{I}}^{-1} \Sigma^{-1} (\hat{\mathbf{v}} - \hat{w}_0 \mathbf{1})$,*
(vii) *Agents' certainty equivalent is $W + \hat{w}_0 + (\hat{\mathbf{v}} - \hat{w}_0 \mathbf{1})^T \Sigma^{-1} (\hat{\mathbf{v}} - \hat{w}_0 \mathbf{1}) / 2\rho$.*

By inspection of Proposition 3, the larger the moral hazard problem (as measured by σ_ϵ) the smaller the equilibrium value of b. This is quite natural as it becomes more expensive for a firm to provide incentives to a risk averse agent and, therefore, firms in equilibrium provide fewer. The fact that less effort is induced in economies with a moral hazard problem directly leads to the welfare loss relative to the frictionless economy as evinced by the lower certainty equivalent.

The contract in industry n is described by the pair (s^n, b^n). As $\partial b / \partial \alpha > 0$ (from (*iii*)), higher productivity industries always offer steeper incentive contracts. The effects of different productivity on the fixed part of the contract, s, (through (*vii*)) is, however, not as clear. For industries with low α^n, $s \approx (1 - b^n) \hat{w}_0$ and is decreasing and approaches $\hat{w}_0$ as α approaches zero. Indeed, as long as $b^n < \frac{\hat{w}_0}{\rho \sigma^2_{\epsilon,n}}$, s is decreasing in α. Similarly, for high α (i.e., $\alpha^n > \frac{\rho^2 \sigma^4_{\epsilon,n}}{4 \kappa \gamma}$), s is decreasing in α. Thus, for low productivity and high productivity industries, the fixed wage contract is decreasing in productivity. For intermediate values of α, however, the sign of $ds/d\alpha$ is ambiguous.

Firms with higher productivity, ceteris paribus, elicit higher levels of effort therefore, given the optimal contract, offer higher levels of total compensation. Specific micro wage data is very difficult to obtain, but Abowd, Kramarz and Margolis (1999) use a French longitudinal sample and find that firms with higher (total) wages are more productive.[12]

Other comparisons can be made between economies with and without moral hazard, that satisfy assumptions 1 and 2. Notice that a sector's equilibrium marginal total productivity, $\hat{v}^n$ is always lower in the economy with moral hazard than in the observable economy. In fact, $\hat{v}^n$ is strictly decreasing in $\sigma_{\epsilon,n}$. However, the investment in physical capital in each sector is identical to the frictionless case. Therefore, the size of firms within sector are the same and are unaffected by moral hazard.

Finally, we note that as the moral hazard problem vanishes, $\boldsymbol{\sigma}_{\epsilon,n} \to \mathbf{0}$, the equilibrium in real economy $\mathcal{E}_1$ converges to the equilibrium in the economy without moral hazard, $\mathcal{E}_0$.

In our economy, agents differ in the industry in which they work, and therefore agents choose different optimal portfolios. Thus, the CAPM (based on observations of the stock market) does not hold. Instead, the following modification of CAPM explains cross-sectional asset returns.

Proposition 4 *In an economy that satisfies the assumptions of Proposition 3, expected returns are* $\hat{\bar{\mu}}^n = (1-b^n)(\hat{v}^n - \hat{w}_0)$*. Moreover, assume that the value-weighted market portfolio is* $\hat{\boldsymbol{q}}$*. Define the diagonal matrix* $\Lambda = diag(\alpha^1/\sigma_{\epsilon,1}, \ldots, \alpha^N/\sigma_{\epsilon,N})$*. Then,*

$$\hat{\bar{\boldsymbol{\mu}}} = \hat{\boldsymbol{\beta}}\hat{\nu}, \tag{11}$$

where

$$\hat{\boldsymbol{\beta}} = \frac{\Sigma_{\hat{\mu}}(\bar{\boldsymbol{I}} + \frac{1}{k\rho}\Lambda^2)\hat{\boldsymbol{q}}}{\hat{\boldsymbol{q}}^T(\bar{\boldsymbol{I}} + \frac{1}{k\rho}\Lambda^2)\Sigma_{\hat{\mu}}(\bar{\boldsymbol{I}} + \frac{1}{k\rho}\Lambda^2)\hat{\boldsymbol{q}}}, \quad \textit{and } \hat{\nu} = \hat{\boldsymbol{q}}^T(\bar{\mathbf{I}} + \frac{1}{k\rho}\Lambda^2)\hat{\bar{\boldsymbol{\mu}}}, \tag{12}$$

and $\bar{\boldsymbol{I}}$ *is the* $N \times N$ *identity matrix.*

The first part of the proposition shows that the expected return in the case with moral hazard is modified in three ways compared to the observable case. First, the factor b^n enters in the definition of $\hat{v}^n$, as workers' effort levels are lower, leading to lower output. Second, the factor $1 - b^n$ enters, as the risk in capital markets per unit of capital investment, as some of the risk is paid out through wages. Both these effects are firm-specific, i.e., they only depend on a firm's characteristics. Third, the participation constraint, $\hat{w}_0$ differs from the frictionless case, which will influence all sectors in general equilibrium.

The second part of the proposition has a CAPM interpretation. Equation (11) shows that expected excess returns are given by the product of a market return and a "beta factor." However, the market portfolio and beta's are defined with respect to a modified portfolio,

$$\tilde{\nu} = \frac{\hat{\boldsymbol{q}}^T(\bar{\mathbf{I}} + \frac{1}{k\rho}\Lambda^2)\hat{\bar{\boldsymbol{\mu}}}}{\hat{\mathbf{q}}^T(\bar{\boldsymbol{I}} + \frac{1}{k\rho}\Lambda^2)\mathbf{1}}. \tag{13}$$

This portfolio measures the true risk in the economy, taking into account the risk that is paid out through wages. The return on the market portfolio is $\hat{\bar{\mu}}_{market} = M^{-1}\sum_{n=1}^N \hat{I}^n\hat{L}^n\hat{\bar{\mu}}^n$.

[12] The proportion of Executive compensation from high productivity firms is found to be higher than in low productivity firms. (See, for example Gaver and Gaver (1993,1995), Bizjak, Brickley and Coles (1993), Smith and Watts (1992)). We observe, however, that our workers are not necessarily executives.

In line with the argument in Roll (1977), we will see deviations from the CAPM when we measure each industry's expected return with respect to $\hat{\bar{\mu}}_{market}$. Proposition 3 can be rewritten in the CAPM-like form

$$\hat{\bar{\mu}}^n = \hat{\beta}^n E[\tilde{\nu}], \qquad \text{where } \hat{\beta}^n = \frac{cov(\hat{\bar{\mu}}^n, \tilde{\nu})}{var(\tilde{\nu})}. \tag{14}$$

5 Comparative Statics and Predictions

Our framework yields three types of results: comparative statics, cross–sectional asset pricing and cross–sectional labor implications. In addition, we can measure the effect of moral hazard by comparing the equilibria of the two economies. We observe that the assumptions required on risk aversion to establish equilibrium are somewhat different in the economies with and without moral hazard. In the specific case of a symmetric one-factor economy (defined precisely below), both the bounds are lower under moral hazard. However, the intervals do overlap. We therefore consider comparative statics under levels of risk aversion that satisfy both Assumptions 1 and 2.

5.1 Productivity Shocks

A standard intuition is that if a sector is (ceteris paribus) larger; in equilibrium, its expected return must be higher to recompense risk averse investors for the increased undiversifiable risk. Such an argument is the basis for the wealth puzzle documented in Bansal, Fang and Yaron (2006). In our framework, as firm size, industry size and returns are endogenous, we can also examine this intuition.

Sector sizes differ in our model because of different technological parameters. We thus consider the effect of a productivity shock ($\partial\alpha^n$) on industry size and returns. For simplicity, we consider a simple one factor structure on risk. Specifically, if risks have a symmetric one-factor structure, then the real risks in the economy can be expressed as $\Sigma = c_0\bar{\boldsymbol{I}} + c_1\mathbf{1}\mathbf{1}^T$, $c_0 > 0$, $c_1 > 0$ and $\bar{\boldsymbol{I}}$ is the identity matrix, while $\mathbf{1}$ is a vector of ones. In such a world, covariances are positive and equal to c_1, while variances are simply $c_0 + c_1$.

Simple derivation yields that the intuition of a Lucas tree economy goes through in our frictionless benchmark.

Corollary 1 *In a one factor economy with no moral hazard, productivity shocks in sector n that increase the cost of capital in sector n, decrease the cost of capital in sector $p \neq n$. Further, a productivity shock that increases the cost of capital in sector n also increases the size of sector n.*

By contrast in the presence of moral hazard, the results are ambiguous. There are two channels through which a productivity shock affects the cost of capital. First, directly, through an effect on total productivity ($\hat{v}$), secondly through an effect on the effort free wage, w_0. Through this channel, in general equilibrium, a change in the productivity of one sector will also affect the cost of capital in other sectors. This is intuitive: If the ex ante value of human capital is higher, this affects firms of different productivity to different degrees, and therefore the distribution of risk in the economy. A risk averse agent's valuation for any risk factor depends on the others to which he is exposed.

To see this, consider the effect on the sector p's cost of capital of a change in the productivity of sector n.

Corollary 2 *In a one factor model with moral hazard, a productivity shock in sector n affects the cost of capital in sector n*

$$\frac{d\hat{r}^n}{d\alpha^n} = (1-b^n)\left(1-\frac{1}{N}\right)\times\frac{\partial\hat{v}^n}{\partial\alpha^n}-\frac{\partial b^n}{\partial\alpha^n}\left(\hat{v}^n-\hat{w}_0\right), \tag{15}$$

and the cost of capital in sector $p \neq n$

$$\frac{d\hat{r}^p}{d\alpha^n} = -\frac{1-b}{N}\times\frac{\partial\hat{v}^n}{\partial\alpha^n}\ \ p\neq n. \tag{16}$$

Notice that the sign of $d\hat{r}^p/d\alpha^n$ is always opposite to that of $\partial\hat{v}^n/\partial\alpha^n$. By contrast, the effect of a productivity shock on the same sectors' cost of capital ($d\hat{r}^n/d\alpha^n$) is ambiguous. Indeed, if $\frac{1-b^n}{\partial b^n/\partial\alpha^n} < \frac{1-1/N}{\hat{v}^n-\hat{w}_0}$, then $\partial\hat{r}^n/\partial\alpha^n$ has the same sign as $\partial\hat{v}^n/\partial\alpha^n$, otherwise it has the opposite sign.[13]

A productivity shock affects the equilibrium total productivity of a firm positively ($\partial\hat{v}^n/\partial\alpha^n > 0$) if $\kappa\gamma < \frac{(b^n)^2(\alpha^n)^3}{k^2}$. This condition demands that the industry cost of entry be low relative to productivity. Changes in relative productivity change the size of each sector as capital flows to more productive sectors in the economy.

Corollary 3 *In a one factor economy with moral hazard, the effect on own industry size of a productivity shock is:*

$$\frac{d(\hat{L}^n\hat{I}^n)}{d\alpha^n} = \left(1-\frac{1}{N}\right)\times\frac{M}{\rho c_0}\times\frac{\partial\hat{v}^n}{\partial\alpha^n},$$

and on other industries is

$$\frac{d(\hat{L}^p\hat{I}^p)}{d\alpha^n} = -\frac{1}{N}\times\frac{M}{\rho c_0}\times\frac{\partial\hat{v}^n}{\partial\alpha^n}.$$

We show in the proof of Corollary 1 that the comparative statics for industry size in the observable case are identical to Corollary 3, except that $\boldsymbol{v}$ replaces $\hat{\boldsymbol{v}}$.

Comparing Corollaries 2 and 3, situations can arise in which $\frac{\partial\hat{r}^n}{\partial\alpha} > 0$ (because $\frac{\partial\hat{v}^n}{\partial\alpha^n} < 0$), in which case

$$\frac{d(\hat{L}^n\hat{I}^n)/d\alpha^n}{d\hat{r}^n/d\alpha^n} < 0.$$

Thus, in the GE economy with moral hazard the intuition of the standard Lucas tree model, (that if one sector increases its return, its size must increase), may not hold. The intuition for this is clear: A productivity increase leads to a larger sector, but more risk is then paid out through the wage channel, which *decreases* the cost of capital observed in the capital market.

[13] The same argument can be made for Sharpe ratios, which is the focus of Bansal, Fang and Yaron (2006).

5.1.1 Welfare Implications of Productivity Shocks

The frictionless economy Pareto dominates that with moral hazard. However, a natural question is whether agents are better off if there is a productivity shock in one sector. Once again, we perform comparative statics under the assumption of a symmetric one factor structure. In both economies (with and without moral hazard), agents' equilibrium certainty equivalent is the sum of the effort free wage plus the value of investing in the stock market. Of course, the value of both differs across the two economies. In the absence of moral hazard, agents are better off if there is a productivity shock, as long as $\partial v/\partial \alpha > 0$. Even though all workers are driven down to their endogenous participation constraint, they are all better off in aggregate because employment is more productive; implicitly this increases the ex ante value of human capital.

To show this, we use the equilibrium identity: $\Delta W = w_0 + \frac{A}{2\rho}$. Therefore,

$$\begin{aligned} \Delta W &= w_0 + \frac{A}{2\rho} \\ &= w_0 + \frac{\bar{\boldsymbol{\mu}}^T \Sigma_\mu^{-1} \bar{\boldsymbol{\mu}}}{2\rho}. \end{aligned}$$

Hence,

$$\frac{d(\Delta W)}{d\alpha^n} = \tfrac{dw_0}{d\alpha^n} + \left(\mathbf{q}^T \tfrac{d\boldsymbol{\mu}}{d\alpha^n}\right).$$

where the last line follows from the equilibrium identity that $\boldsymbol{q} = \Sigma_\mu^{-1} \frac{\bar{\mu}}{\rho}$.[14] Observing that in equilibrium, $\bar{\boldsymbol{\mu}} = \mathbf{v} - \mathbf{1} w_0$, allows us to write

$$\begin{aligned} \frac{d(\Delta W)}{d\alpha^n} &= \left(\frac{1}{N} + (\boldsymbol{q})_n - \frac{\boldsymbol{q}^T \mathbf{1}}{N}\right) \times \frac{\partial v^n}{\partial \alpha^n} \\ &= (\boldsymbol{q})_n \frac{\partial v^n}{\partial \alpha^n}. \end{aligned}$$

The $1/N$ factor enters through the effect an industry shock indirectly has on the participation wage of workers. Notice also that $\boldsymbol{q}^T \mathbf{1} = 1$. That is, the sum of all dollar positions is one. (This stems from our assumption that each firm requires a unit of investment capital for each worker.)

On inspection, $\frac{d(\Delta W)}{d\alpha^n}$ is strictly positive if $\frac{\partial v^n}{\partial \alpha^n} > 0$. In the frictionless economy, the condition is $\kappa\gamma < \frac{(\alpha^n)^2}{k^2}$. This is intuitive: If the cost of entering the market is low relative to the productivity, then social welfare increases if there is a productivity shock as capital flows easily into the newly more productive industries.

In the presence of moral hazard, the general form of the change in the certainty equivalent is similar, however, equilibrium values of the market returns and the effort free wage differ. Specifically, as above we obtain

[14] $d(\Delta W)/d\alpha^n = dw_0/d\alpha^n + (2\rho)^{-1} d(\bar{\boldsymbol{\mu}}^T \Sigma_\mu^{-1} \bar{\boldsymbol{\mu}})/d\alpha^n = dw_0/d\alpha^n + 2(2\rho)^{-1} \bar{\boldsymbol{\mu}}^T \Sigma_\mu^{-1} (d\bar{\boldsymbol{\mu}}/d\alpha^n) = dw_0/d\alpha^n + \boldsymbol{q}^T d\bar{\boldsymbol{\mu}}/d\alpha^n$.

$$\frac{d\Delta\widehat{W}}{d\alpha^n} \quad = \quad (\hat{\boldsymbol{q}})_n \frac{\partial \hat{v}^n}{\partial \alpha^n},$$

where $\hat{\boldsymbol{q}} = \Sigma^{-1}(\hat{\mathbf{v}} - \hat{w}_0 \mathbf{1})/\rho$ is the portfolio of a hypothetical worker, working in a risk-free industry. The difference between the two expressions is the adjustment for moral hazard, i.e., $\hat{v}^n$, rather than v^n. Recall that a productivity shock increases equilibrium total productivity under moral hazard ($\partial \hat{v}^n/\partial \alpha^n > 0$) if $\kappa\gamma < \frac{(b^n)^2(\alpha^n)^3}{k^2}$. Therefore, consider an industry with entry costs so that

$$\kappa\gamma \in \left(\frac{\alpha^n}{k}\right)^2 \left[(b^n)^2\alpha^n,\ 1\right].$$

For these industries, a productivity shock decreases welfare, whereas in the frictionless case a shock would increase welfare. Further, as b^n is decreasing in the degree of moral hazard measured by σ_ϵ, the larger the moral hazard problem the more likely a productivity shock is to lead to a decrease in welfare. Also, for a fixed level of moral hazard, because high productivity industries have higher incentive pay (b^n is higher), aggregate welfare is more likely to be lower with a positive productivity shock amongst these firms.

Surprisingly, the ex ante value of human capital or effort-free wage in the economy with moral hazard may be *higher* than that in the economy with observable effort levels. From Propositions 1 and 2 it is clear that $\hat{w}_0 - w_0 > 0$ is equivalent to $\mathbf{1}^T\Sigma^{-1}(\hat{\mathbf{v}} - \mathbf{v}) > 0$. Consider an economy in which the noisiness is very low in all industries but one, $\sigma_{\epsilon,i} > 0$, $\sigma_{\epsilon,j} \approx 0$, $j \neq i$. In this economy $[(\hat{\mathbf{v}} - \mathbf{v})]_i < 0$, and $[(\hat{\mathbf{v}} - \mathbf{v})]_j \approx 0$, for $j \neq i$. We may interpret this as an economy without moral hazard, $\mathcal{X}_0$, that is struck by a "noise shock" in sector i, leading to a new equilibrium, $\mathcal{X}_1$. For example, a workforce deregulation in an industry may lead to uncertainty about worker input. It is easy to see that

$$\sum_j [\Sigma^{-1}]_{i,j} < 0 \tag{17}$$

is a necessary and sufficient condition for $\hat{w}_0 - w_0 > 0$ in this economy.[15]

The productivity in the noisy sector is lower than in the observable case, which — all else equal — leads to lower expected returns in that sector. For a sector in which condition (17) is satisfied, however, lower expected returns lead to higher total dollar demand in the stock market.[16] In the "noisy" economy, demand for stocks is now higher than supply after the shock, which leads firms to compete for labor, driving up wages. The wage increases further decrease the return in sector i, which has a multiplicative effect on stock demand. However, the wage increase also decreases expected returns in all other sectors, $j \neq i$, which decreases the demand pressure on stocks and more than offsets the effects in sector i. A new equilibrium is reached, in which $\hat{w}_0$ is higher than before the noise shock.

[15] A three-sector example, satisfying the condition is $[\Sigma]_{11} = 3$, $[\Sigma]_{21} = 5$, $[\Sigma]_{31} = 4$, $[\Sigma]_{22} = 9$, $[\Sigma]_{32} = 7$, $[\Sigma]_{33} = 6$, for which (17) is satisfied for the second sector.

[16] The total dollar demand from an investor is $\hat{D} = \mathbf{1}^T\hat{\boldsymbol{q}} = \mathbf{1}^T\Sigma_{\hat{\mu}}^{-1}\hat{\boldsymbol{\mu}}$, which increases if there is a negative shock in $\hat{\mu}^i$, since $\partial\hat{D}/\partial\hat{\mu}^i = \sum_j [\Sigma_{\hat{\mu}}^{-1}]_{i,j} < 0$.

It may seem counterintuitive that the labor market actually provides more wealth in the Pareto dominated economy than in the Pareto efficient observable economy. The total wealth increase in the economy, however, also contains the wealth provided from the stock market, $\Delta W = w_0 + A/2\rho$. As Propositions 1 and 2 show, the total wealth increase in the observable economy is always higher than in the economy with moral hazard. Thus, in the cases where $\hat{w}_0 - w_0 > 0$, the negative effects of the less efficient capital market more than outweighs the wage increase in the labor market.

5.2 Moral hazard and Financial "Anomalies"

Our model also provides implications for the cross-section of expected returns. The key driver behind the discrepancies between the CAPM and our model are the productivity parameters, α^n, and the unobservability of effort levels, measured by σ_ϵ^n. High-productivity firms pay out a higher fraction of $\tilde{x}$-risk through wages, as they gain more from workers exerting high effort levels. In other words, the industries of such firms look "too small", and the firms earn excess returns compared with CAPM. Specifically,

Corollary 4 *Compared to the CAPM predictions:*
i) Industries with high α will seem to be "too small,"
ii) Industries with low α will seem to be "too big,"
iii) Firms with high α have positive abnormal returns,
iv) Firms with low α have negative abnormal returns.

We have proved *i-ii* for general economies, whereas we have only proved *iii-iv* for the symmetric one-factor economy in which $\partial \hat{v}^n / \partial \alpha > 0$ in all industries (see appendix).

As high-productivity firms are small (Proposition 3:*iii*), this immediately implies the following size-effect related result:[17]

Corollary 5 Size effect*:*
i) Small firms have higher expected returns than those predicted by the CAPM.
ii) Large firms have lower expected returns than those predicted by the CAPM.

A value-like effect can also arise within the model. The book-to-market ratio in an industry is

$$1 - \frac{\kappa + \gamma \alpha^n (\hat{I}^n)^2 + s^n \hat{I}^n}{\hat{I}^n}.$$

This is the remaining capital in the firm at $t = 1$, divided by the market value of the firm at that point (see Figure 2). The crucial condition that ensures a value effect for all firms is that

Condition 1 *For all* $\alpha \in [\alpha^1, \alpha^N]$, $\frac{ds}{d\alpha} < -\sqrt{\frac{\kappa\gamma}{\alpha}}$.

[17] In the model, we make specific assumptions on firms' production and cost functions. The corollary, however, does not depend on the specific functional form, as long as the cost is an increasing function of productivity. Any cost function of the form $\kappa + \gamma(\alpha) I^2$, where γ is increasing in α, will lead to a size anomaly.

Intuitively, this condition ensures that firms with higher productivity decrease the fixed-wage part of workers' compensation more than enough to offset the increase in spending on R&D (which will have an offsetting effect). Obviously, this condition is stronger than $\frac{ds}{d\alpha}$ just being negative, which we discussed in Section 4.3. However, for large α, the R.H.S. of Condition 1 is small so the condition is only slightly more restrictive than the condition that $ds/d\alpha < 0$. In this case, high α firms have high book-to-market ratios and we immediately get

Corollary 6 Value effect: *If Condition 1 is satisfied, then*

i) Firms with high book-to-market ratios have expected returns greater than those predicted by the CAPM.

ii) Firm with low book-to-market ratios have lower expected returns than those predicted by the CAPM.

In Figure 3 below, we show an example with ten industries. Expected return as a function of beta is shown as a solid line. It is convex, driven by the cross sectional differences of wage risk. A regression of market beta versus expected return (shown by dotted line with circles) will therefore not capture expected returns well. In this world, size (dotted line with pluses) and book-to-market (dotted line with crosses) do a better job at capturing cross sectional differences in expected returns.

5.3 Moral Hazard and the Labor Market

Our model links firm productivity, investments in R&D, labor compensation and stock returns and is consistent with several observed phenomena in the literature on executive compensation. Recall that s is a proxy for fixed wage compensation, whereas b is a proxy for variable compensation, e.g., stock and stock options or profit sharing. Mehran (1995) and later Frye (2004) find that firms with higher relative investments in R&D (measured as R&D/Sales) have higher employee equity compensation. Furthermore, Mehran reports a negative relationship between R&D/Sales and fixed wage compensation. Kedia and Mozundar (2002) find that small firms grant more stock options than large firms and that firms that grant more stock options earn abnormal returns. All these results are in line with our model. Kedia and Mozundar (2002) also report that low book-to-market companies grant more stock options, which is not consistent with our model.

If the model presented in this paper provides an accurate description, the type and size of labor compensation that a firm offers is important to understand its stock returns. Our predictions are based on the equilibrium result that compensation and firm size are driven by the same parameter – α. Recall that b is increasing in α, but firm size is decreasing in α. Therefore firms that are more productive offer a higher bonus component and are smaller.

Corollary 7 *The smaller the firm, the larger the profit sharing.*

Our model suggests that the total profit sharing portion of the wage bill (ex ante) should be $b^n \hat{I}^n$, therefore b^n measures the profit sharing normalized by firm size. This is consistent with Kruse (1992) who finds that profit sharing and Employee Stock Ownership Plans (ESOPs) Granger cause labor productivity.

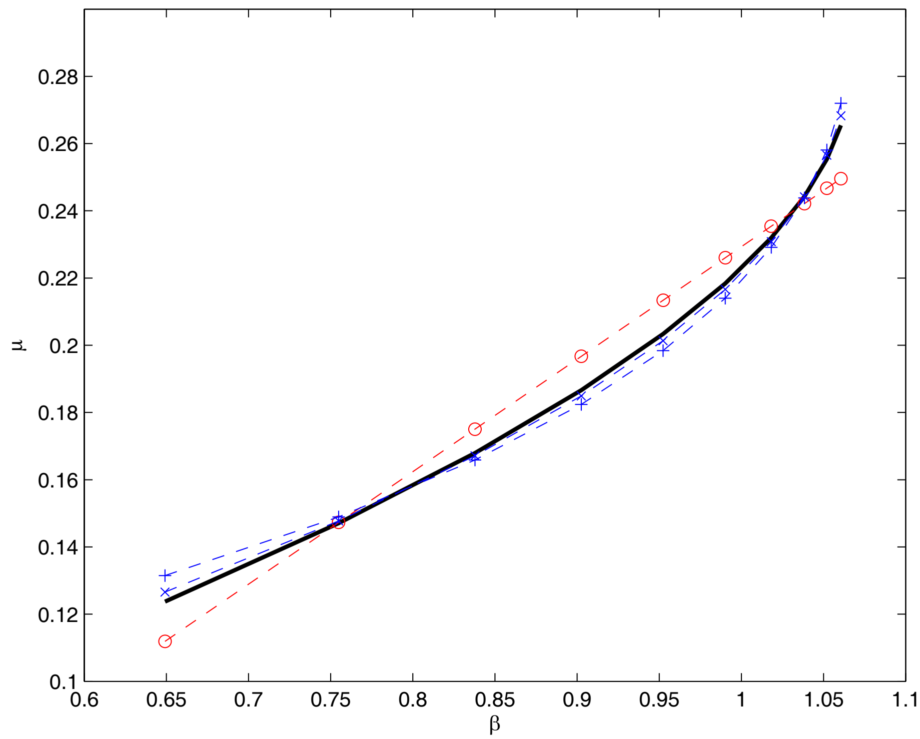

Figure 3: ***Expected returns. True expected returns are represented by solid line. Regressed returns on market beta (dotted line with circles), on size (dotted line with pluses) and on book-to-market ratio (dotted line with crosses) are also shown.***

Under the optimal contract we exhibit, employees are also paid a fixed wage; the total wage bill is $s^n \hat{I}^n$. Smaller firms pay out most of their compensation in profit sharing and therefore we expect[18] that

Corollary 8 *Small firms will have lower wage per employee than large firms.*

In the previous section, we demonstrated that cross–sectional asset pricing anomalies follow from the existence of incentive contracts. Using the same logic that generated the value effect in the previous section, since small firms offer wages with a larger incentive component than large firms,

Corollary 9 *Firms with low wages per employee will earn abnormal returns.*

5.4 Conclusions

We have characterized a tractable general equilibrium model of production in which workers, remunerated by firms, hedge labor income in financial markets. The CARA/Normal framework admits both simple, optimal incentive contracts and closed form solutions for capital market equilibrium.

More broadly, as investors are also workers, firm characteristics should help explain the cross-section of returns. Specifically, asset pricing, firm balance sheet characteristics and returns to human capital are jointly determined in equilibrium.

[18] Although, the effect may not be monotone for all α, in line with our discussion of $ds/d\alpha$.

6 Appendix I

Proof of Lemma 2

As a worker's effort and investment decisions are separate, the solution is the standard mean variance optimum, see, e.g., Ingersoll 1987, leading to the expression for $\boldsymbol{q}$ and the certainty equivalent.

To see that A is the squared Sharpe ratio, S^2, we note that

$$S^2 = (\boldsymbol{q}^T \bar{\boldsymbol{\mu}})^2/(\boldsymbol{q}^T \Sigma_\mu \boldsymbol{q}) = (\bar{\boldsymbol{\mu}}^T \Sigma_\mu^{-1} \bar{\boldsymbol{\mu}})^2/(\bar{\boldsymbol{\mu}}^T \Sigma_\mu^{-1} \Sigma_\mu \Sigma_\mu^{-1} \bar{\boldsymbol{\mu}}) = A.$$

■

Proof of Proposition 1

Throughout the appendix, we prove more general versions of the proposition, namely with general $\kappa^n > 0$, $\gamma^n > 0$ and $\bar{x}^n > 0$. The Propositions in the body of the text are obtained by choosing $\kappa = \kappa^1 = \cdots = \kappa^N$, $\gamma = \gamma^1 = \cdots = \gamma^N$, $\bar{x}^1 = \cdots = \bar{x}^N = 1$.

The first condition, *(i)*, follows from the firms' first order conditions (4).

For *(ii)*, we use that *(i)* together with (2) implies that

$$\bar{\pi}^n = E[\tilde{\pi}^n] = I^n \left((\alpha^n)^2/2k + \bar{x}^n - w_0 - r^n\right) - \alpha^n \gamma^n (I^n)^2 - \kappa^n. \tag{18}$$

The zero economic profit condition and firm optimization condition implies that $\bar{\pi}^n = \partial \bar{\pi}^n/\partial I^n = 0$. For a general quadratic equation: $f(x) = ax^2 + bx + c$, a necessary condition for $f(x) = f'(x) = 0$ is that $x = \pm\sqrt{c/a}$, which in this case implies *(ii)*,

$$I^n = \sqrt{\frac{\kappa^n}{\gamma^n \alpha^n}}.$$

Clearly (18) implies that

$$2\alpha^n \gamma^n I^n = (\alpha^n)^2/2k + \bar{x}^n - w_0 - r^n,$$

which by the equilibrium condition on I^n and the definition of v^n implies that

$$v^i = r^i + w_0, \tag{19}$$

i.e., as $r^i = \bar{\mu}^i$,

$$\bar{\boldsymbol{\mu}} = \mathbf{v} - w_0 \mathbf{1}.$$

To prove *(iii)*, we observe that the stock market clearing condition is

$$M\boldsymbol{q} = M\Sigma_\mu^{-1}\bar{\boldsymbol{\mu}}/\rho = \Lambda_I \boldsymbol{L} \Rightarrow \boldsymbol{L} = M\Lambda_I^{-1}\Sigma_\mu^{-1}\bar{\boldsymbol{\mu}}/\rho, \tag{20}$$

i.e., *(iv)*. By premultiplying with $\mathbf{1}^T$, this also leads to

$$M\mathbf{1}^T\Sigma_\mu^{-1}\bar{\boldsymbol{\mu}}/\rho = \mathbf{1}^T\Lambda_I \boldsymbol{L} = M.$$

This in turns means that

$$\mathbf{1}^T\Sigma_\mu^{-1}(\mathbf{v} - w_0\mathbf{1}) = \rho \Rightarrow \mathbf{1}^T\Sigma_\mu^{-1}\mathbf{v} - \rho = w_0\mathbf{1}^T\Sigma_\mu^{-1}\mathbf{1},$$

leading to *(iii)*.

(v) follows from (19) and the fact that the certainty equivalent of the increase in expected utility is the sum of the wage surplus w_0 and the value-add from stock market participation, $A/2\rho$. The solution thus constitutes an equilibrium.

Under Assumption 1:*i*, *(iii)* implies that w_0 is strictly positive, and Assumption 1:*ii* together with (19) implies that $L^n > 0$, $n = 1, \ldots, N$. Thus, all variables are strictly positive so the equilibrium is interior. Moreover, all equations in the derivation are unique, so the equilibrium is unique.

Finally, by solving the social planner's problem, one easily checks that the equilibrium is indeed the unique Pareto optimal solution, up to pure wealth distributions, i.e., any Pareto optimal equilibrium will have the same value of $\mathbf{e}$, $\boldsymbol{I}$, $\boldsymbol{L}$ and $\boldsymbol{q}$.

■

Proof of Proposition 2

The condition $\bar{\mu}^n = v^n - w_0$ is shown in (19). Returns are normal and the stock market is the only source of risk, so the CAPM holds with respect to the market portfolio (see, e.g., Ingersoll 1987). Moreover, by Proposition 1, the return of the market portfolio is $M^{-1}\sum_{n=1}^{N} I^n L^n \tilde{\mu}^n$. ■

Proof of Lemma 3

The agent's optimization problem (7) takes the form

$$\Delta\widehat{W}^n = \max_{\hat{e}^n, \hat{\boldsymbol{q}}^n} V^n = \max_{\hat{e}^n, \hat{\boldsymbol{q}}^n} \Big[s^n + b^n\alpha^n e^n + \hat{\boldsymbol{\mu}}^T \boldsymbol{q}^n - \frac{k}{2}(e^n)^2 - \\ -\frac{\rho}{2}\left((b^n)^2\sigma_{x,n}^2 + (b^n)^2\sigma_{\epsilon,n}^2 + (\boldsymbol{q}^n)^T\Sigma_{\hat{\mu}}(\boldsymbol{q}^n) + 2b^n\boldsymbol{\sigma}_{\hat{\mu},n}^T\boldsymbol{q}^n\right)\Big].$$

The first order conditions are therefore

$$\frac{\partial V^n}{\partial e^n}: \qquad b^n\alpha^n - k\hat{e}^n = 0 \;\Rightarrow\; \hat{e}^n = \tfrac{\alpha^n b^n}{k}, \tag{21}$$

$$\frac{\partial V^n}{\partial \boldsymbol{q}^n}: \qquad \hat{\boldsymbol{\mu}} - \rho\left(\Sigma_{\hat{\mu}}\boldsymbol{q}^n + b^n\boldsymbol{\sigma}_{\hat{\mu},n}\right) = \boldsymbol{0} \;\Rightarrow\; \hat{\boldsymbol{q}}^n = \Sigma_{\hat{\mu}}^{-1}\left(\tfrac{\hat{\boldsymbol{\mu}}}{\rho} - b^n\boldsymbol{\sigma}_{\hat{\mu},n}\right). \tag{22}$$

As the decision variables are separated over e^n and $\boldsymbol{q}^n$, and as the highest order terms in e^n and $\boldsymbol{q}^n$ are strictly negative definite quadratic forms, a solution to the f.o.c. is also global maximum. ■

Proof of Lemma 4

Equation (22) implies the following values:

$$\begin{aligned} \hat{\boldsymbol{\mu}}^T\hat{\boldsymbol{q}}^n &= \frac{\hat{A}}{\rho} - b^n B, \\ (\hat{\boldsymbol{q}}^n)^T\Sigma_{\hat{\mu}}(\hat{\boldsymbol{q}}^n) &= \frac{\hat{A}}{\rho^2} + (b^n)^2 C^n - \frac{2b^n B^n}{\rho}, \\ \boldsymbol{\sigma}_{\hat{\mu},n}^T\hat{\boldsymbol{q}}^n &= \frac{B^n}{\rho} - b^n C^n. \end{aligned}$$

A worker in sector n, at his participation constraint will therefore have

$$\begin{aligned} \Delta\widehat{W} = {} & s^n + b^n\bar{x}^n + b^n\alpha^n\frac{\alpha^n b^n}{k} + \left(\frac{\hat{A}}{\rho} - b^n B^n\right) - \frac{(\alpha^n b^n)^2}{2k} - \\ & -\frac{\rho}{2}\left((b^n)^2\sigma_{x,n}^2 + (b^n)^2\sigma_{\epsilon,n}^2 + \frac{\hat{A}}{\rho^2} + (b^n)^2C^n - \frac{2b^nB^n}{\rho} + 2b^n(\frac{B^n}{\rho} - b^nC^n)\right) \\ = {} & s^n + (b^n)^2\left(\frac{(\alpha^n)^2}{2k} + \frac{\rho}{2}(C^n - \sigma_{x,n}^2) - \frac{\rho}{2}\sigma_{\epsilon,n}^2\right) - b^n(B^n - \bar{x}^n) + \frac{\hat{A}}{2\rho}. \end{aligned}$$

This leads to

$$s^n(b^n, \hat{I}^n) = (b^n)^2\left(-\frac{(\alpha^n)^2}{2k} - \frac{\rho}{2}(C^n - \sigma_{x,n}^2) + \frac{\rho}{2}\sigma_{\epsilon,n}^2\right) + b^n(B^n - \bar{x}^n) + \Delta\widehat{W} - \frac{\hat{A}}{2\rho}.$$

■

Proof of Lemma 5

From (6) a firm in sector n solves

$$\max_{\hat{I}^n, b^n} \bar{\pi}^n = E[\widetilde{\pi}^n] = I^n\Big((\alpha^n e^n + \bar{x}^n)(1-b^n) - s^n(b^n, I^n) - (1-b^n)z^n - \gamma^n I^n\Big) - \kappa^n =$$

$$\max_{\hat{I}^n, b^n} \; I^n\left(\Big(\frac{b^n(\alpha^n)^2}{k} + \bar{x}^n\Big)(1-b^n) - s^n(b^n, I^n) - (1-b^n)z^n - \gamma^n I^n\right) - \kappa^n =$$

$$\max_{\hat{I}^n, b^n} \; I^n\Big(\Big(\frac{b^n(\alpha^n)^2}{k} + \bar{x}^n\Big)(1-b^n) - \Big((b^n)^2\Big(-\frac{(\alpha^n)^2}{2k} - \frac{\rho}{2}(C^n - \sigma^2_{x,n})$$

$$+\frac{\rho}{2}\sigma^2_{\epsilon,n}\Big) + b^n(B^n - \bar{x}^n) + \Delta\widehat{W} - \frac{\hat{A}}{2\rho}\Big) - (1-b^n)z^n - \gamma^n I^n\Big) - \kappa^n =$$

$$\max_{\hat{I}^n, b^n} \; I^n\Big((b^n)^2\Big(-\frac{(\alpha^n)^2}{2k} + \frac{\rho}{2}(C^n - \sigma^2_{x,n}) - \frac{\rho}{2}\sigma^2_{\epsilon,n}\Big) +$$

$$+b^n\Big(\frac{(\alpha^n)^2}{k} + z^n - B^n\Big) + \frac{\hat{A}}{2\rho} - \Delta\widehat{W} + \bar{x}^n - z^n\Big) - \alpha^n\gamma^n(I^n)^2 - \kappa^n$$

$$\stackrel{\text{def}}{=} I^n(M_2^n(b^n)^2 + M_1^n b^n + M_0^n) - \alpha^n\gamma^n(I^n)^2 - \kappa^n.$$

The first order conditions in b^n is:

$$\frac{\partial\bar{\pi}^n}{\partial b^n}: \qquad 2M_2^n b^n + M_1^n = 0 \Rightarrow$$

$$b^n = -\frac{M_1^n}{2M_2^n} = \frac{(\alpha^n)^2/k + z^n - B^n}{(\alpha^n)^2/k + \rho(\sigma^2_{x,n} - C^n + \sigma^2_{\epsilon,n})}. \tag{23}$$

Cauchy-Schwarz' inequality, $Cov(\tilde{x}^n, \tilde{y}) \le \sqrt{Var(\tilde{x}^n)Var(\tilde{y})}$, ensures that $C^n \le \sigma^2_{x,n}$: For the choice $\tilde{y} = \mathbf{a}^T\hat{\boldsymbol{\mu}}$ with $\mathbf{a} = \Sigma_{\hat{\mu}}^{-1}\boldsymbol{\sigma}_{\hat{\mu},n}$ the inequality leads to $\boldsymbol{\sigma}^T_{\hat{\mu},n}\Sigma^{-1}_{\hat{\mu}}\boldsymbol{\sigma}_{\hat{\mu},n} \le \sqrt{\boldsymbol{\sigma}^T_{\hat{\mu}n}\Sigma^{-1}_{\hat{\mu}}\boldsymbol{\sigma}_{\hat{\mu},n} \times \sigma^2_{x,n}}$, i.e., $C^n \le \sqrt{C^n} \times \sqrt{\sigma^2_{x,n}}$. This immediately implies that M_2 is strictly negative, and in fact, $M_2 < -(\alpha^n)^2/k$.

The first-order conditions for $\hat{I}^n$ is:

$$\frac{\partial\bar{\pi}}{\partial I}: \qquad M_2^n(b^n)^2 + M_1^n b^n + M_0^n - 2\gamma^n\hat{I}^n = 0 \implies \hat{I}^n = \frac{M_2^n(b^n)^2 + M_1^n b^n + M_0^n}{2\alpha^n\gamma^n}. \tag{24}$$

From the f.o.c. on b^n, (23), this is equivalent to:

$$\hat{I}^n = \frac{1}{2\alpha^n\gamma^n}\left(-\frac{(M_1^n)^2}{4M_2^n} + M_0^n\right) \stackrel{\text{def}}{=} \frac{T^n}{2\alpha^n\gamma^n}. \tag{25}$$

We note that solution, $(b^n, \hat{I}^n)$, to the first order condition is unique. We now prove that it is a global maximum, as long as $\hat{I}^n > 0$ and $0 < b^n < 1$: Plugging the solutions to the f.o.c. into the profit function leads to

$$\bar{\pi} = \frac{(T^n)^2}{4\alpha^n\gamma^n} - \kappa^n =$$

$$\frac{1}{4\alpha^n\gamma^n}\left(\frac{((\alpha^n)^2/k + z^n - B^n)^2}{(\alpha^n)^2/k + \rho(\sigma^2_{x,n} - C^n + \sigma^2_{\epsilon,n})} + \frac{\hat{A}}{2\rho} - \Delta\widehat{W} + \bar{x}^n - z^n\right)^2 - \kappa^n \ge -\kappa^n,$$

so any strictly better strategy must lead to a profit greater than $-\kappa^n$.

Clearly, the optimization problem is smooth, so an optimum will either be at a boundary (including the extended boundary $\hat{I}^n = \infty$), or satisfy the first order conditions. As the solution to the f.o.c. is unique, we check the boundaries: 1. $\hat{I}^n = \infty$: The Hessian of this optimization is on the form $H = [2M_2^n I^n, M_1^n; M_1^n, -2\gamma^n]$, with characteristic equation $(\lambda - 2M_2^n I^n)(\lambda + 2\gamma^n) - (M_1^n)^2 = 0$, i.e., $\lambda^2 + 2(\gamma^n - \hat{I}^n M_2^n)\lambda - 4\gamma^n I^n M_2^n - (M_1^n)^2 \stackrel{\text{def}}{=} \lambda^2 + a_1\lambda + a_0 = 0$. Clearly, $a_1 > 0$, $a_0 < a_1$ and, for large enough $\hat{I}^n$, $a_0 > 0$, which implies that, for large enough $\hat{I}^n$, both characteristic roots are negative. Thus, there is always an $\hat{I}^n$, such that $\bar{\pi}$ is decreasing regardless of b^n and the optimum can not be reached at the (extended) boundary $\hat{I}^n = \infty$.

Moreover, any $b^n \geq 1$ will lead to $\bar{\pi} \leq -\kappa$, so no interesting solution can have $b^n \geq 1$, and the boundary $\hat{I}^n = 0$ will lead to $\bar{\pi} = -\kappa$. Thus, any noninterior optimum must lie on the boundary $b^n = 0$. On this boundary, the optimal investment level is $\hat{I}^n = M_0^n/2\gamma^n$, which is feasible if $M_0^n \geq 0$ (as otherwise $I^n < 0$), in this case leading to $\bar{\pi} = M_0^2/4\gamma^n - \kappa^n$. However, if $M_0^n \geq 0$, then this boundary solution is obviously dominated by $\bar{\pi} = (-(M_1^n)^2/4M_2^n + M_0)^2/4\gamma^n - \kappa^n$ (as $M_2^n < 0$), so no solution on the boundary $b^n = 0$ can dominate the interior solution. ∎

Proof of Proposition 3

We construct an equilibrium satisfying *(i)-(vii)* and then show that it is unique. We define $\Lambda_I = diag(\hat{I}^1, \ldots, \hat{I}^N)$, $\Lambda_\alpha = diag(\alpha^1, \ldots, \alpha^N)$, $\Lambda_b = diag(b^1, \ldots, b^N)$ and $\Lambda_{1-b} = diag(1-b^1, \ldots, 1-b^N)$.

First, from Lemma 3, an equilibrium with optimizing workers will satisfy *(i)*.

We note that (9) implies that in equilibrium, $C^n = [\Sigma_x]_{n,n} = \sigma_{x,n}^2$ and that $B^n = \hat{\bar{\mu}}^n/(1-b^n) = z^n$, which in turn, through the relation $\hat{r}^n = \hat{\bar{\mu}}^n$, implies *(iii)*:

$$b^n = \frac{1}{1 + \frac{k\rho\sigma_{\epsilon,n}^2}{(\alpha^n)^2}} \in (0,1).$$

Also, $M_1^n = (\alpha^n)^2/k$ and $M_2^n = -((\alpha^n)^2/k + \rho\sigma_{\epsilon,n}^2)/2$, where M_1^n and M_2^n were defined in the proof of Lemma 5.

Moreover, (8) together with $E[\tilde{\pi}^n] = 0$ implies *(ii)*:

$$\hat{I}^n = \sqrt{\frac{\kappa^n}{\gamma^n \alpha^n}},$$

which is strictly positive. However, we also have through (25):

$$2\sqrt{\gamma^n\kappa^n\alpha^n} + \frac{(M_1^n)^2}{4M_2^n} = \frac{\hat{A}}{2\rho} - \Delta\widehat{W} - z^n, \tag{26}$$

which through the relations $\Delta\widehat{W} = \hat{w}_0 + \hat{A}/2\rho$ and $\hat{\mu}^n/(1-b) = z^n$ leads to

$$\hat{\bar{\mu}}^n = (1-b^n)\left(2\sqrt{\gamma^n\kappa^n\alpha^n} + \frac{(M_1^n)^2}{4M_2^n} - \hat{w}_0\right). \tag{27}$$

The market clearing condition in the stock market now gives us

$$\hat{\boldsymbol{Q}}\Lambda_I\hat{\boldsymbol{L}} = \Lambda_I\hat{\boldsymbol{L}},$$

which implies that

$$(\bar{\boldsymbol{I}} - \hat{\boldsymbol{Q}})\Lambda_I\hat{\boldsymbol{L}} = \boldsymbol{0}, \tag{28}$$
$$\tag{29}$$

(where $\bar{\boldsymbol{I}}$ is the identity matrix) i.e., 1 must be an eigenvalue to $\hat{\boldsymbol{Q}}$, with eigenvector $\boldsymbol{\lambda}$. Given such a $\boldsymbol{\lambda}$, the vector of firm mass $\hat{\boldsymbol{L}} = \frac{M}{\mathbf{1}^T\boldsymbol{\lambda}}\Lambda_I^{-1}\boldsymbol{\lambda}$ will be a solution. Moreover, the labor market condition, $M = \hat{\boldsymbol{I}}^T\hat{\boldsymbol{L}}$, can be rewritten as

$$\mathbf{1}^T\Lambda_I\hat{\boldsymbol{L}} = M.$$

We have

$$\hat{\boldsymbol{Q}} = \Sigma_{\hat{\mu}}^{-1}\left(\frac{\hat{\boldsymbol{\mu}}\mathbf{1}^T}{\rho} - \Lambda_b\Sigma_{\hat{\mu},x}\right), \tag{30}$$

where $\Sigma_{\hat{\mu},x} = [\boldsymbol{\sigma}_{\hat{\mu},1},\ldots,\boldsymbol{\sigma}_{\hat{\mu},n}]$. Since from (9), we know that $\Sigma_{\hat{\mu}} = (\bar{\boldsymbol{I}} - \Lambda_b)\Sigma(\bar{\boldsymbol{I}} - \Lambda_b)$ and $\Sigma_{\hat{\mu},x} = \Sigma(\bar{\boldsymbol{I}} - \Lambda_b)$, (30) can be rewritten as

$$\hat{\boldsymbol{Q}} = (\bar{\boldsymbol{I}} - \Lambda_b)^{-1}\Sigma^{-1}(\bar{\boldsymbol{I}} - \Lambda_b)^{-1}\left(\frac{\hat{\boldsymbol{\mu}}\mathbf{1}^T}{\rho} - \Lambda_b\Sigma(\bar{\boldsymbol{I}} - \Lambda_b)\right).$$

This in turn, using the definitions of $\hat{\mathbf{v}}$, and (27) means that (28) can be rewritten as:

$$M\hat{w}_0\rho\left(\Sigma_{\hat{\mu}}^{-1}(\Lambda_{1-b}\hat{\mathbf{v}}\mathbf{1}^T/\rho - \Lambda_{1-b}\Sigma_{\hat{\mu},x}) - \bar{\boldsymbol{I}}\right)^{-1}\Sigma_{\hat{\mu}}^{-1}\Lambda_{1-b}\mathbf{1} = \boldsymbol{\lambda} = M\hat{w}_0\rho\boldsymbol{Z}\mathbf{1}, \tag{31}$$

where $\boldsymbol{Z} = \left(\Sigma_{\hat{\mu}}^{-1}(\Lambda_{1-b}\hat{\mathbf{v}}\mathbf{1}^T/\rho - \Lambda_{1-b}\Sigma_{\hat{\mu},x}) - \bar{\boldsymbol{I}}\right)^{-1}\Sigma_{\hat{\mu}}^{-1}\Lambda_{1-b}$. Since $\mathbf{1}^T\lambda = M$, we have

$$\hat{w}_0 = \frac{1}{\rho\mathbf{1}^T\boldsymbol{Z}\mathbf{1}}.$$

Now, expanding $\boldsymbol{Z}$ through the definitions of $\Sigma_{\hat{\mu}}$ and $\Sigma_{\hat{\mu},x}$ leads to

$$\boldsymbol{Z} = \left(\frac{\hat{\mathbf{v}}\mathbf{1}^T}{\rho} - \Sigma\right)^{-1},$$

which through the Sherman-Morrison-Woodberg formula,

$$(\boldsymbol{A} + \boldsymbol{U}\boldsymbol{V}^T)^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{U}(I + \boldsymbol{V}^T\boldsymbol{A}^{-1}\boldsymbol{U})^{-1}\boldsymbol{V}^T\boldsymbol{A}^{-1},$$

leads to *(iv)*: $\hat{w}_0 = 1/(\rho\mathbf{1}^T\boldsymbol{Z}\mathbf{1}) = \frac{\mathbf{1}^T\Sigma^{-1}\hat{\mathbf{v}}-\rho}{\mathbf{1}^T\Sigma_{\hat{\mu}}^{-1}\mathbf{1}}$.

From (31), and the relation $\hat{\boldsymbol{L}} = \frac{M}{\mathbf{1}^T\boldsymbol{\lambda}}\Lambda_I^{-1}\boldsymbol{\lambda}$ it follows that

$$\hat{\boldsymbol{L}} = M\hat{w}_0\rho\Lambda_I^{-1}\mathbf{Z}\mathbf{1},$$

and another application of the Sherman-Morrison-Woodberg formula finally leads to *(v)*.

Condition *(vi)*, $W + \hat{w}_0 + (\hat{\mathbf{v}} - \hat{w}_0\mathbf{1})^T\Sigma^{-1}(\hat{\mathbf{v}} - \hat{w}_0\mathbf{1})/2\rho$, follows from the relation $\Delta\widehat{W} = \hat{w}_0 + \hat{A}/2\rho$ and (27).

Condition *(vii)* follows immediately from plugging the derived values of variables into (8) of Lemma (4).

A similar argument as in the proof of Proposition 1 shows that the equilibrium is unique and interior if Assumption 2 is satisfied.

Finally, we observe that the resource allocation $\hat{\boldsymbol{L}} = M\rho^{-1}\Lambda_I^{-1}\Sigma^{-1}(\hat{\mathbf{v}} - \hat{w}_0\mathbf{1})$ in economy $\mathcal{E}_0$ (with observable effort level) is strictly Pareto dominated by the observable economy equilibrium outcome, $\mathcal{X}_0$ (as the equilibrium is the unique Pareto optimal outcome from Proposition 1). This outcome, in turn, strictly Pareto dominates the equilibrium outcome in $\mathcal{E}_1$, $\mathcal{X}_1$, as each worker will get additional disutility from the noisiness of the performance report, and the value creation will decrease with the factor $(1-b)\alpha^2/(2k)$. Therefore $\mathcal{X}_0$ strictly Pareto dominates $\mathcal{X}_1$.

■

Proof of Proposition 4

The first part follows immediately from (27) and $M_1^n = (\alpha^n)^2/k$, $M_2^n = -((\alpha^n)^2/k + \rho\sigma^2_{\epsilon,n})/2$.

The value-weighted market portfolio is $\hat{\boldsymbol{q}} = diag(\hat{\boldsymbol{I}})\hat{\boldsymbol{L}}/(\hat{\boldsymbol{I}}^T\hat{\boldsymbol{L}}) = \Sigma^{-1}(\hat{\mathbf{v}} - \hat{w}_0\mathbf{1})/\mathbf{1}^T\Sigma^{-1}(\hat{\mathbf{v}} - \hat{w}_0\mathbf{1})$. However, the mean-variance efficient portfolio in the financial market is $\boldsymbol{q}^* = \Sigma_{\hat{\mu}}^{-1}\hat{\boldsymbol{\mu}}/\mathbf{1}^T\Sigma_{\hat{\mu}}^{-1}\hat{\boldsymbol{\mu}} = (\bar{\boldsymbol{I}} - \Lambda_b)^{-1}\Sigma^{-1}(\hat{\mathbf{v}} - \hat{w}_0\mathbf{1})/[\mathbf{1}^T(\bar{\boldsymbol{I}} - \Lambda_b)^{-1}\Sigma^{-1}(\hat{\mathbf{v}} - \hat{w}_0\mathbf{1})]$. The CAPM will hold with respect to this portfolio (see, e.g., Ingersoll 1987), and obviously to any scaled version of this portfolio, for example

$$\boldsymbol{\nu} = (\bar{\boldsymbol{I}} - \Lambda_b)^{-1}\hat{\boldsymbol{q}}.$$

Now,

$$b^n = \frac{1}{1 + \frac{k\rho\sigma^2_{\epsilon,n}}{(\alpha^n)^2}} \in (0,1),$$

and since

$$(\bar{\boldsymbol{I}} - \Lambda_b)^{-1} = diag(1/(1-b^1), \ldots, 1/(1-b^N)),$$

and also

$$\frac{1}{1-b^n} = 1 + \frac{\alpha^n}{k\rho\sigma^2_{\epsilon,n}},$$

this leads to

$$\boldsymbol{\nu} = \left(I + k^{-1}\rho^{-1}\Lambda^2\right)\hat{\boldsymbol{q}}.$$

Since the CAPM holds with respect to $\boldsymbol{\nu}$, we have $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{\beta}}[\boldsymbol{\nu}^T\hat{\boldsymbol{\mu}}]$, where $\hat{\boldsymbol{\beta}} = \Sigma_{\hat{\mu}}\boldsymbol{\nu}/\boldsymbol{\nu}^T\Sigma_{\hat{\mu}}\boldsymbol{\nu}$, which, plugging in the definition of $\boldsymbol{\nu}$, leads to (11-12). ∎

Proof of Corollary 1

From Proposition 1, it follows that $\partial r^n/\partial\alpha^p = \partial v^n/\partial\alpha^p - \partial w_0/\partial\alpha^p$. Since $\Sigma = c_0\bar{\boldsymbol{I}} + c_1\mathbf{1}\mathbf{1}^T$, it follows that

$$\Sigma^{-1} = \frac{1}{c_0}\left(\bar{\mathbf{I}} - \frac{c_1}{c_0 + c_1 N}\mathbf{1}\mathbf{1}'\right),$$

which implies that

$$w_0 = \frac{\mathbf{1}^T\mathbf{v}}{N} - \rho \times \left(\frac{c_0}{N} + c_1\right).$$

Therefore, $\partial r^n/\partial\alpha^n = (1 - 1/N)\partial v^n/\partial\alpha^n$, whereas $\partial r^n/\partial\alpha^p = -N^{-1}\partial v^p/\partial\alpha^p$, $n \neq p$. Thus,

$$\frac{\partial r^n/\partial\alpha^n}{\partial r^p/\partial\alpha^n} = -(N-1) < 0, \qquad n \neq p$$

which proves the first part of the corollary.

For the second part, we note that

$$\frac{d(L^n I^n)}{d\alpha^p} = \frac{M}{\rho}\left[\Sigma^{-1}\frac{\partial(\mathbf{v} - w_0\mathbf{1})}{\partial\alpha^p}\right]_n,$$

which equals $\frac{M}{\rho c_0}(1 - 1/N)(\partial v^n/\partial\alpha^n)$ for $n = p$, and $\frac{M}{\rho c_0}(-1/N)(\partial v^p/\partial\alpha^p)$, for $n \neq p$. This implies that

$$\frac{d(L^n I^n)/d\alpha^p}{dr^n/dr^p} = \frac{M}{\rho c_0} > 0, \qquad \forall n, p.$$

∎

Proof of Corollary 2

Similar to the proof of Corollary 1, using the results of Proposition 3. ■

Proof of Corollary 3

Similar to the proof of Corollary 1, using the results of Proposition 3. ■

Proof of Corollary 4

We first show *i-ii*, that, for an arbitrary economy, the value-weighted market portfolio will seem to be under-weighted on high-productivity firms and over-weighted on low-productivity firms. We define $\mathbf{z} = \hat{\mathbf{v}} - \hat{w}_0\mathbf{1}$. The value-weighted market portfolio is then $\boldsymbol{q} = \Sigma^{-1}\mathbf{z}/(\mathbf{1}^T\Sigma^{-1}\mathbf{z})$. The portfolio that the CAPM would predict, on the other hand, is $\boldsymbol{q}^* = \Sigma_\mu^{-1}\hat{\boldsymbol{\mu}}/(\mathbf{1}^T\Sigma_\mu^{-1}\hat{\boldsymbol{\mu}}) = \Lambda_{1-b}^{-1}\Sigma^{-1}\mathbf{z}/(\mathbf{1}^T\Lambda_{1-b}^{-1}\Sigma^{-1}\mathbf{z})$.

We therefore have

$$\frac{[\boldsymbol{q}]_i}{[\boldsymbol{q}^*]_i} = c \times (1 - b_i), \qquad \text{where} \qquad c = \frac{\mathbf{1}^T\Lambda_{1-b}^{-1}\Sigma^{-1}\mathbf{z}}{\mathbf{1}^T\Sigma^{-1}\mathbf{z}} > 0,$$

(where the strict positivity of c is guaranteed by the strict positivity of all the elements of $\boldsymbol{L}$, since $\Sigma^{-1}\mathbf{z} = M^{-1}\rho\Lambda_I\boldsymbol{L}$ as showed by Proposition 3:*vi*) which is a decreasing functions of b, and therefore, as b is an increasing function of α, a decreasing function of α.

Further, as $\mathbf{1}^T\boldsymbol{q}^* = \mathbf{1}^T\boldsymbol{q} = 1$, and the portfolio weights are all nonnegative, it is clear that, as long as there is dispersion of productivity, $\alpha^i \neq \alpha^j$ for some i, j, there is a $\bar{\alpha} \in (\alpha^1, \alpha^N)$, such that for all industries, n, in which $\alpha^n < \bar{\alpha}$, $[\boldsymbol{q}^*]_i < [\boldsymbol{q}]_i$, and for all industries in which $\alpha^n > \bar{\alpha}$, $[\boldsymbol{q}^*]_i > [\boldsymbol{q}]_i$ ($\boldsymbol{q} \neq \boldsymbol{q}^*$ (W.l.o.g., assume that there is a i such that $[\boldsymbol{q}]_i < [\boldsymbol{q}^*]_i$. Then, there has to be a j such that $[\boldsymbol{q}]_j > [\boldsymbol{q}^*]_j$, as $\mathbf{1}^T\boldsymbol{q}^* = \mathbf{1}^T\boldsymbol{q} = 1$). Thus, low productivity industries will indeed look "too big," whereas high-productivity industries will look "too small." This general result is true regardless of the parameters of the economy. We note that, as $c/(1 - b_i) > 1$ for some i, $c < 1$.

We now turn to *iii-iv*. We prove the result for the case where $\Sigma = c_0\bar{\boldsymbol{I}} + c_1\mathbf{1}\mathbf{1}^T$, $c_0 > 0$, $c_1 > 0$ and $\bar{\boldsymbol{I}}$ is the identity matrix, while $\mathbf{1}$ is a vector of ones.

It is straightforward to show that the CAPM, based on the value-weighted market portfolio, predicts a vector of expected returns of

$$\boldsymbol{z}^* = d \times \Sigma\Lambda_{1-b}\Sigma^{-1}\mathbf{z}, \qquad \text{where} \qquad d = \frac{\mathbf{z}^T\Lambda_{1-b}\Sigma^{-1}\mathbf{z}}{\mathbf{z}^T\Sigma^{-1}\Lambda_{1-b}\Sigma\Lambda_{1-b}\Sigma^{-1}\mathbf{z}},$$

whereas the true expected returns (per unit of $\tilde{\mathbf{x}}$ risk) are $\mathbf{z}$. However, using the definitions of $\boldsymbol{q}$ and $\boldsymbol{q}^*$, this is equivalent to

$$r_i \stackrel{\text{def}}{=} \frac{[\mathbf{z}^*]_i}{[\mathbf{z}]_i} = d \times c \times \frac{[\Sigma\boldsymbol{q}^*]_i}{[\Sigma\boldsymbol{q}]_i}.$$

Since $\Sigma = c_0\bar{\boldsymbol{I}} + c_1\mathbf{1}\mathbf{1}^T$, this implies that

$$r_i = d \times \frac{[\boldsymbol{q}]_i/(1 - b_i) + c \times c_1/c_0}{[\boldsymbol{q}]_i + c_1/c_0}.$$

Now, since $c < 1$, we have $r_i < d/(1 - b_i)$ (This can for example be seen by defining $R(q) = \frac{q + c\times(1-b_i))c_1/c_0}{q + c_1/c_0}$, noting that $R(0) = c(1 - b_i) < (1 - b_i) < 1$, and $dR/dq = (q + c \times c_1/c_0(1 - b_i))^{-1}(1 - R) > 0$ iff $R > 0$, and $\lim_{q\to\infty} R(q) = 1$, so $R(q) < 1$ regardless of q, and, because $r_i = d \times R([\boldsymbol{q}]_i)/(1 - b_i)$, the result follows).

For arbitrary i, j, such that $b_i < b_j$, we define the functions

$$Q(b) \stackrel{\text{def}}{=} [\Sigma\boldsymbol{q}]_i + \frac{[\Sigma\boldsymbol{q}]_j - \Sigma[\mathbf{q}]_\mathbf{i}}{b_j - b_i}(b - b_i),$$

and

$$Z(b) \stackrel{\text{def}}{=} \frac{Q(b)/(1-b) + c \times c_1/c_0}{Q + c_1/c_0}.$$

Clearly, $Q(b) > 0$, and $Z(b) > 0$. We first show that $[\Sigma \boldsymbol{q}]_i < [\Sigma \boldsymbol{q}]_j$. From Proposition 3, b is strictly increasing in α. We have

$$\Sigma \boldsymbol{q} = (c_0 \bar{\boldsymbol{I}} + c_1 \mathbf{1}\mathbf{1}^T)\mathbf{z} = c_0 \mathbf{z} + c_1(\mathbf{1}^T \mathbf{z})\mathbf{1},$$

so

$$[\Sigma \boldsymbol{q}]_j - [\Sigma \boldsymbol{q}]_i = c_0([\mathbf{z}]_j - [\mathbf{z}]_i) > 0,$$

as $\hat{v}^j > \hat{v}^i$ when $\alpha^j > \alpha^i$ and $[\mathbf{z}]_j - [\mathbf{z}]_i = \hat{v}^j - \hat{v}^i$.

We then have $r_j - r_i = Z(b_j) - Z(b_i)$, so showing that $Z'(b) > 0$, for $b_i < b < b_j$ is enough to ensure that $r_j > r_i$. It is easy to check that

$$Z'(b) = \frac{1}{(1-b)[\boldsymbol{q}^*]_i + c_1/(c_0 \times c)} \left(Q'(b)(1 - c(1-b)Z(b)) + Q(b)Z(b) \right).$$

Moreover, since $Z(b) > 0$ and $(1-b)Z(b) < d$ (which is clearly the case since $(1-b)Z(b) = R(Q(b))$, and $d(R(Q(b))/db = R'Q' > 0$, so $(1-b)Z(b)$ reaches its maximum at b_j, at which point it is $r_j(1-b_i) < d$), if $cd < 1$, then $Z'(b) > 0$. A similar argument to the one used in showing that $c < 1$, indeed confirms that $d < 1$, so $cd < 1$, and indeed $Z'(b) > 0$, and $r_j > r_i$. Thus, r_n — the rate of CAPM-predicted expected returns to true expected returns — is an increasing function of n. A similar argument as that made when proving *i*-*ii* shows that for i's such that $\alpha_i < \bar{\alpha}$, $r_i < 1$, whereas for i's such that $\alpha_i > \bar{\alpha}$, $r_i > 1$. This concludes the proof of Corollary 4. ■

7 Appendix II: Utility Equivalence of Contracts

As noted in the main text, there are a continuum of optimal contracts. We demonstrate that all choices lead to utility equivalent equilibria. This is not a priori clear, as the participation constraint is endogenous in our GE framework. We also demonstrate that irrespective of the conditioning variables that all equilibria are also "productively," equivalent, specifically that the firm will elicit the same investment in human capital from all employees. The intuition for this result stems from the ease with which workers can access the capital markets. It is irrelevant to the contract if the firm offers systematic risk in the incentive contract. The agent can lay off any unwanted risk in the stock market. Exposure to this risk has a price in equilibrium, as agents can trade the risk. Therefore, the firm has to "repay," the worker by offering a larger fixed fee component. In general equilibrium, for any compensation contract that includes a wage written on the systematic risk generated by the firm, there is an equivalent economy in which it is not.

Proposition 5 *Assume that firms can offer contracts of the following generalized form:*

$$w_m^{n,\ell} = s_m^{n,\ell} + b_m^{n,p}\tilde{\eta}_m + v_m^{n,p}\tilde{x}^{n,p} - \gamma^n I^{n,p}, \tag{32}$$

where $0 \le v_m^{n,\ell} < 1$*. Then, in equilibrium, the choices of* $v_m^{n,\ell}$ *are irrelevant for the real part of the realization of the economy, i.e., regardless of choices, there is an equilibrium with the same* ΔW, $\boldsymbol{L}$, $\boldsymbol{I}$, *and* $\mathbf{e}$ *as the equilibrium described in Proposition 3.*

Proof: Intuitively, the Proposition should hold, as $b^n < 1$, so $\tilde{x}^n$-risk is a tradable asset that both the firm and the worker will agree upon the price for. The new contract,

$$y_m^n = s^n + b^n\tilde{\eta}_m + v^n\tilde{x}^n - \gamma^n I^n,$$

can be rewritten as the contract analyzed in the text plus an additional $\tilde{x}^n$-risk term,

$$w_m^n + \phi^n\tilde{x}^n,$$

where $\phi^n = v^n - b^n$. By showing that expected excess profit (8) does not depend on ϕ^n, the irrelevance result follows. The same derivation as in the proof of Lemma 4 shows that the new reservation wage, s^n_{new} is

$$s^n_{\text{new}} = s^n + (\phi^n - b)^2\frac{\rho}{2}(\sigma_{x,n} - C) + (\phi^n - b^n)B,$$

which in equilibrium reduces to

$$s^n_{\text{new}} = s^n + (\phi^n - b^n)B. \tag{33}$$

Plugging this expression into (8), in exactly the same way as in the proof of Lemma 5, makes the $(\phi^n - b^n)B^n$-terms cancel out against the changed opportunity cost which introduces an extra $-(\phi^n - \beta^n)z^n$ term, as $B^n = z^n$ in equilibrium. Thus, expected excess profit does not depend on ϕ, so firms are indifferent in their choice of ϕ. A similar argument shows that each agent is indifferent too, and her effort level does not depend on ϕ so all real variables $(\Delta W, \boldsymbol{L}, \boldsymbol{I}, \mathbf{e})$ are the same regardless of ϕ. ■

In equilibrium, the price of traded assets are the same if they appear in the incentive contract or in the stock markets. Therefore, the only source of risk that the firm can use to elicit effort is the idiosyncratic risk which is non–traded. Notice however, that the amount of risk which agents receive through wage contracts will affect equilibrium in the capital market. Suppose that no $\widetilde{x}$ risk is paid out to the employees. In this case, all employees hold the same portfolio position and the amount of $\widetilde{x}$ risk in the economy is determined by the productivity of firms (α). Now suppose, that a large amount of $\widetilde{x}$ risk is paid out to employees. In this case, agents in firms will lay off the risk in the market, however the total amount of x risk and the total effort level elicited by firms will be the same, although economies that differ in the amount of $\widetilde{x}$ risk paid out to workers will have different asset pricing implications, since different ϕ's lead to different degrees of systematic risk being paid out through the labor channel.

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