Advance Information and Asset Prices*

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Abstract

This paper provides an explanation for momentum and reversal in stock returns within a rational expectations equilibrium framework in which investors are heterogeneous in their information and investment opportunities. We assume that informed investors privately receive advance information about company earnings that materializes into the future. This information is immediately incorporated into prices, and thus stock prices may move in ways unrelated to current fundamentals. Investors' speculative and rebalancing trades in response to advance information generate short-run momentum, mimicking an underreaction pattern. When this information materializes, the stock price reverts back to its long-run mean, mimicking an overreaction pattern.

Key words: advance information, momentum and reversal effects, underreaction, overreaction, rational expectations equilibrium

JEL Classification: G11, G12, G14.

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1. Introduction

Many empirical studies have documented evidence of the momentum and reversal effects in the aggregate and cross-sectional stock returns. The momentum effect refers to the phenomena that stock returns tend to exhibit unconditional positive serial correlation in the short to medium run. A related phenomenon is that conditional on observable public events, stocks tend to experience post-event drift in the same direction as the initial event impact. The reversal effect refers to the phenomena that stock returns are negatively correlated in the long run, and that stock returns are positively related to price-scaled variables such as the book-to-market ratio.[1]

The momentum and reversal effects provide a serious challenge to the efficient markets hypothesis and standard risk-based models. Many researchers have recently shifted attention to behavioral models, where investors are boundedly rational and arbitrage is limited. A common behavioral interpretation is that investors underreact to current news and overreact to consistent patterns of news pointing in the same direction.

In this paper, we provide a rational, heterogeneous-investor model with asymmetric information that can deliver both the momentum and reversal effects. We start with a benchmark model which is a simplified version of Wang (1994). In this benchmark model, two types of investors, informed and uninformed, trade in the financial market. Informed investors can invest in both publicly traded assets (a stock and a risk-free bond) and in a private investment opportunity. Informed investors have private information about the persistent and transitory components of dividends or earnings as well as the return on the private investment. Uninformed investors can invest in the stock and bond only. Their information consists of past earnings and stock price realizations. Based on this public information, they infer informed investors' private information. As discussed in Wang (1994), informed investors trade for speculative and rebalancing reasons. Uninformed investors trade for noninformational reasons only, because they cannot distinguish between the informed investors' trading motives. Uninformed investors trade only to accommodate what they perceive to be rebalancing trades by informed investors.

When informed investors trade for rebalancing reasons due to an increase in the current expected return on the private investment, they sell the stock to invest in the private opportu-

[1] See Cutler et al. (1991), Jegadeesh and Titman (1993), Chan et al. (1997) and Rouwenhorst (1998) for time-series and cross-sectional evidence on short term momentum, and Bernard (1992) for evidence on price continuation after public news events. For evidence on long term return reversals see DeBondt and Thaler (1985), Fama and French (1988), Poterba and Summers (1988). For evidence on the negative association between stock returns and past price-scaled variables see DeBondt and Thaler (1987), Fama and French (1992) and La Porta et al. (1997).

nity. The current stock price falls to induce uninformed investors to buy, who in turn expect high stock returns in the next period. This trading pattern decreases the current stock return and raises next period's stock return if the private investment returns are independent over time. Thus, stock returns generated by rebalancing trades tend to reverse themselves. If the private investment returns are persistent, future private investment returns will change when the current private investment return changes, and thus future stock prices will also change due to future rebalancing trades. This effect may dominate, and thus may make excess stock returns continue themselves forever.

When informed investors trade for speculative reasons, the stock price changes to reflect these investors' expectations of the stock's future payoffs. These expectations are fulfilled later on as private information becomes public. Thus, a price change generated by a speculative trade implies future returns of the same sign, and stock returns generated by speculative trades tend to continue themselves. We show that, in our benchmark model, the effect of speculative trading is always dominated by the effect of rebalancing trading. Thus, the benchmark model cannot generate short-run momentum and long-run reversals simultaneously.

In order to explain these phenomena in a unified way, we extend our benchmark model by introducing a new mechanism that may generate momentum from both speculative and rebalancing trades. Our novel model ingredient is to assume that informed investors possess private advance information about a firm's future performance, such as future shocks to earnings, in addition to their other private information. In this case, the stock price immediately partly incorporates this information. In addition, informed investors' speculative trading may be generated by private advance information, reinforcing the return continuation effect discussed earlier.

In contrast to the benchmark model, rebalancing trades in the presence of advance information may also help generate return continuation, even though the persistence of private investment returns is low. We explain the intuition by considering the effects of a good piece of one-period-ahead advance information. In response to this information, the current stock price rises because it immediately incorporates this good information. Thus, the current excess stock return also rises. When the stock and the private investment opportunity are substitute assets, good advance information about a firm's future performance also constitutes good news about the private investment return. Therefore, as informed investors buy into the private investment, they sell the stock for rebalancing reasons, even though they expect high future returns. Uninformed investors buy the stock, expecting high returns in the next period. Thus, informed investors follow contrarian strategies and uninformed investors are trend chasers. This trading behavior generates short-run momentum.

In addition to generating short-run momentum in excess stock returns, private advance information may also generate long-run excess stock return reversals. This is because stock prices may move with advance information, even though fundamentals (earnings) do not change. Once the advance information is materialized, the stock price reverts to its long-run mean giving rise to what appears to be an overreaction effect: Long-run returns display negative serial correlation and also high prices relative to earnings can forecast low future stock returns.

We show that with a single piece of advance information, the short-run momentum and long-run reversal effects can occur when this information is about next period's earnings innovations. An undesirable prediction of this case is that momentum lasts only for a few periods, which seems inconsistent with empirical evidence. When the single piece of advance information is about many-period-ahead earnings innovations, the model may generate counterfactual cyclic return dynamics. We thus extend this model by assuming that informed investors receive increasingly precise signals about earnings innovations as these are closer to materialize. In this case, stale information is useful for forecasting and informed investors trade on this information.[2] As a result, the effects of advance information described above can last for a long period, causing long-lived momentum. Moreover, after a sustained streak of good news, the stock price appears to have overshoot its fundamental value and ultimately must revert itself.

To the best of our knowledge, there is no rational model in the extent literature that can explain the momentum and reversal effects simultaneously in a unified way. First, some models have the mechanisms to explain momentum but not reversals. Berk, Green and Naik (1999) show that a rich variety of return patterns, including momentum effects, result from the variation of risk exposures over the life-cycle of a firm's endogenously chosen projects.[3] Johnson (2002) provides a standard model of firm cash flows discounted by an ordinary pricing kernel, that can deliver the momentum effect. His key idea is that expected dividend growth rates vary over time and growth rate risk varies with the growth rates. Both models, however, cannot deliver the long-horizon reversal effect. Second, some models have the mechanisms to explain reversals but not momentum. Lewellen and Shanken (2002) predict return reversals due to correction of past forecast errors, but not momentum. Fama and French (1993, 1996) show that many of the long-horizon results–such as return reversals, the book-to-market effect, and the earnings to price ratio effect–can be largely subsumed within their three-factor model. However, Fama and French (1996) point out that the momentum result of Jegadeesh and

[2] See Bernhardt and Miao (2005) for a strategic model of stock prices with stale information. See Tetlock (2007) for empirical evidence of the importance of stale information.

[3] See Gomes, Kogan and Zhang (2003) for an extension in a general equilibrium framework.

Titman (1993) constitutes the "main embarrassment" for their three-factor model. Finally, some models can predict either momentum or reversals but not both (see Wang (1993) and our benchmark model discussed above).

A common behavioral interpretation of the momentum and reversal effects is based on under- and over-reaction to news. Daniel et al. (1998) present a model based on investor overconfidence and self-attribution. In their model, investors are overconfident about the precision of their private signals. In addition, they update their confidence in a biased manner as they observe the outcomes of their actions. Thus, they overreact to private information and underreact to public information. The Barberis et al. (1998) model is based on different psychology phenomena, i.e., conservatism and the representativeness heuristic. Both of the preceding models study a representative agent framework. Hong and Stein (1999) analyze a model with two types of investors–news watchers and momentum traders. Each news watcher observes some private information, but fails to extract other newswatchers' information from prices. Momentum traders can profit by trend chasing, but they use simple strategies, leading to overreaction at long horizons.

Daniel and Titman (2006) dispute both the behavioral and risk-based interpretations that the reversal and book-to-market effects are a result of high expected returns on stocks of distressed firms with poor past performance. They decompose individual firm returns into two components, one that is associated with past performance – based on a set of accounting performance measures – and one that is orthogonal to past performance. They show that future returns are unrelated to the accounting measures of past performance, which they call tangible information, but are strongly negatively related to the component of news about future performance, which is unrelated to past performance. They refer to this last component as intangible information. We may interpret the advance information in our model as intangible information. Our advance information is also unrelated to past performance, but impacts prices. Using a rational-expectations, risk-based model with asymmetric information, we show that the presence of this intangible information is important to generate the momentum and reversal effects.

We organize the rest of the paper as follows. Section 2 presents the model. Section 3 studies a benchmark model without advance information. Section 4 analyzes the equilibrium when informed investors possess a single piece of advance information about future earnings. Section 5 extends this model to incorporate multiple-pieces of advance information. Section 6 concludes. Proofs are relegated to appendices.

2. The Model

Time is discrete and indexed by $t = 1, 2, ...$ Consider an economy with a single good that can be either consumed or invested. There are two types of infinitely-lived investors in the economy, informed and uninformed. Informed and uninformed investors differ in their information structure and investment opportunities. The fraction of informed investors is $\lambda \in (0, 1)$ and the fraction of uninformed investors is $1 - \lambda$.

2.1. Preferences

All investors have expected exponential utility with an identical constant absolute risk aversion parameter γ. We assume that investors are myopic and derive utility from next period's accumulated wealth. Thus, time t preferences are represented by

$$E_t \left\{ -e^{-\gamma W_{t+1}} \right\}, \tag{1}$$

where E_t is the expectation operator conditional on an investor's information at time t and W_{t+1} is the wealth level at time $t+1$. The assumption of myopic preferences rules out dynamic hedging demands and simplifies our analysis significantly. Introducing a dynamic hedging demand, however, does not change our key insights.[4]

2.2. Investment opportunities

Investors can trade publicly a riskless bond and a risky stock in the economy. The riskless bond is assumed to have an infinitely elastic supply at a positive constant interest rate r. Let $R = 1 + r$ denote the gross interest rate in the economy.

The stock generates earnings D_t at time t. The earnings process is described by,

$$D_t = F_t + \varepsilon_t^D, \tag{2}$$

where F_t follows an AR(1) process,

$$F_t = a_F F_{t-1} + \varepsilon_t^F, \ 0 < a_F < 1. \tag{3}$$

Earnings have both a persistent component F_t, with persistence given by a_F, and a temporary component ε_t^D. We assume that shocks to both components, ε_t^D and ε_t^F, are independently and identically distributed (i.i.d.) normal random variables with means of zero and variances σ_D^2

[4] Other papers also adopt myopic preferences for tractability (see, e.g., Bacchetta and van Wincoop (2004, 2006), Campbell et al. (1993), and LIorente et al. (2002)).

and σ_F^2, respectively. The firm is assumed to distribute 100 percent of its earnings as dividends. We therefore use the terms earnings and dividends interchangeably. The stock is in unit supply.

In addition to the publicly traded assets, there is a risky investment opportunity that is available only to the informed investors. This investment opportunity has constant returns to scale. Its return between period t and $t+1$ is $R + q_{t+1}$, where the excess return q_{t+1} satisfies

$$q_{t+1} = Z_t + \varepsilon_{t+1}^q. \tag{4}$$

Here, Z_t is the expected excess return to the private investment opportunity and follows an AR(1) process:

$$Z_t = a_Z Z_{t-1} + \varepsilon_t^Z,\ 0 < a_Z < 1, \tag{5}$$

The expected return to the private investment opportunity has a persistent component given by Z_t, with persistence given by a_Z, and a transitory component given by ε_t^q. We assume that shocks to both components, ε_t^q and ε_t^Z, are i.i.d. normal random variables with means of zero and variances σ_q^2 and σ_Z^2, respectively. We also assume that all shocks are uncorrelated except for ε_t^D and ε_t^q. Specifically, $E\left[\varepsilon_t^D \varepsilon_t^q\right] = \sigma_{Dq} > 0$ so that the stock and the private investment are substitutes.

2.3. Information structure

All investors observe the past and current realizations of earnings and prices of the stock. Informed investors have private information about the persistent component F_t of the stock and the expected return Z_t on the private investment, while uninformed investors do not. In addition to these pieces of private information, informed investors receive news about future earnings announcements which we label advance information. The advance information is modeled as a noisy signal of future earnings innovations. In the analysis below, we consider in turn two information structures:

1. *Single piece of advance information.* At time t, informed investors receive a single signal

   $$S_t = \varepsilon_{t+k}^D + \varepsilon_t^S, \tag{6}$$

   about time $t+k$ earnings innovations, where ε_t^S is an i.i.d. normal random variable with mean zero and variance σ_S^2. The shock ε_t^S is assumed independent of all other shocks. The information set of informed investors is given by:

   $$\mathcal{F}_t^i = \{D_s, F_s, P_s, Z_s, S_s : s \le t\}. \tag{7}$$

The information set of uninformed investors is given by:

$$\mathcal{F}_t^u = \{D_s, P_s : s \leq t\}. \tag{8}$$

2. *Multiple pieces of advance information.* At time t, informed investors receive a vector of signals $\left(S_t^k, ..., S_t^1\right)$ about earnings innovations at $t+1$ through $t+k$:

$$S_t^k = \varepsilon_{t+k}^D + \varepsilon_t^{S_k}, ..., S_t^1 = \varepsilon_{t+1}^D + \varepsilon_t^{S_1}. \tag{9}$$

Each $\varepsilon_t^{S_n}$ is assumed to be an i.i.d. normal random variable with mean zero, variance $\sigma_{S_n}^2$, and independent of any other shock. The informed investors' information set is given by:

$$\mathcal{F}_t^i = \left\{D_s, F_s, P_s, Z_s, (S_s^n)_{n=1,...,k} : s \leq t\right\}. \tag{10}$$

The uninformed investors' information set is given by (8).

The modeling of advance information in (6) or in (9) is stylized in many respects and can be generalized to deliver better quantitative predictions. However, we think that the qualitative mechanisms we describe survive these generalizations. First, advance information news at time t refers to a known date into the future $t+k$. This need not be the case and in fact the relevant future date associated with advance information is usually uncertain, perhaps one that can be influenced by the firm. Second, investors need not obtain advance information in every period. While this is a critical abstraction for stationarity of the model and tractability, it gains more realism when the period considered is 1 quarter or 1 year as opposed to 1 day. Finally, the informativeness of the advance information need not be constant and may vary over time or with the news themselves. We discuss some of the implications of this possibility in Section 5 in the context of specification (9).

2.4. Equilibrium

A rational expectations equilibrium is defined in the usual way. We focus on a stationary equilibrium in which the stock price is a stationary function of state variables. The key step to define an equilibrium is to formulate the investors' portfolio choice problems. We start with an informed investor's decision problem. We use the superscript i to index a variable associated with an informed investor. Let P_t denote the time t stock price and $Q_t = P_t + D_t - RP_{t-1}$ the stock's excess return at time t. An informed investor solves the following problem:

$$\max -E_t^i\left[\exp\left(-\gamma W_{t+1}^i\right)\right], \tag{11}$$

subject to the budget constraint:

$$\begin{aligned} W^i_{t+1} &= \theta^i_t \left(P_{t+1} + D_{t+1}\right) + \alpha^i_t \left(R + q_{t+1}\right) + \left(W^i_t - \left(\theta^i_t P_t + \alpha^i_t\right)\right) R \\ &= \theta^i_t Q_{t+1} + \alpha^i_t q_{t+1} + W^i_t R, \end{aligned} \tag{12}$$

where E^i_t denotes the conditional expectation operator given the information set $\mathcal{F}^i_t$, W^i_t denotes the wealth level, θ^i_t denotes the fraction of the stock held by i, and α^i_t denotes the invested amount in the private investment opportunity. Similarly, we write an uninformed investor's decision problem as follows:

$$\max -E^u_t \left[\exp\left(-\gamma W^u_{t+1}\right)\right], \tag{13}$$

subject to

$$W^u_{t+1} = \theta^u_t Q_{t+1} + R W^u_t, \tag{14}$$

where E^u_t denotes the conditional expectation operator given the information set $\mathcal{F}^u_t$, W^u_t denotes the wealth level, and θ^u_t denotes the fraction of the stock held by u. Note that an uninformed investor does not have any private investment opportunities.

The market clearing condition is given by:

$$\lambda \theta^i_t + (1-\lambda)\,\theta^u_t = 1. \tag{15}$$

3. A benchmark model without advance information

As a benchmark, we start with a model in which investors do not have advance information. This model is similar to Wang (1994) with two differences. First, we assume that investors are myopic and ignore dynamic hedging demands. Second, we assume that uninformed investors do not have any information about the persistent component of earnings beyond what they can infer from earnings and price realizations. Our simplified model allows us to make our analysis more transparent when deriving new results regarding momentum and reversal effects absent from Wang (1994).

3.1. Equilibrium stock price

We follow a similar solution method to that in Wang (1994). Our approach is more general and can be easily adapted to the models with advance information analyzed later. The key step is to determine the stock price function. To do so, we first define the state variable as $\mathbf{x}_t = (F_t, Z_t)^\intercal$ and the unforecastable shock vector as $\boldsymbol{\varepsilon}_t = \left(\varepsilon^D_t, \varepsilon^F_t, \varepsilon^Z_t, \varepsilon^q_t\right)^\intercal$. Note that $\boldsymbol{\varepsilon}_t \sim N\left(0, \Sigma\right)$, where Σ is the covariance matrix. We then conjecture that the price function takes the following form:

$$P_t = -p_0 + \mathbf{p}_i \mathbf{x}_t + \mathbf{p}_u \hat{\mathbf{x}}^u_t, \tag{16}$$

where $\hat{\mathbf{x}}_t^u = E_t^u[\mathbf{x}_t]$, p_0 is a constant, and $\mathbf{p}_i = [p_{i1}, p_{i2}]$ and $\mathbf{p}_u = [p_{u1}, p_{u2}]$ with $p_{u2} = 0$. In general, one may include $\hat{Z}_t^u$ in the price function in that $p_{u2} \neq 0$. However, from the current price, P_t, the uninformed investors can infer the following sum:

$$P_t + p_0 - p_{u1}\hat{F}_t^u = p_{i1}F_t + p_{i2}Z_t \equiv \Pi_t, \tag{17}$$

since $\hat{F}_t^u$ is observable by the uninformed investors. Thus, $p_{i1}F_t + p_{i2}Z_t$ represents the information content of the equilibrium price. This implies that $p_{i1}F_t + p_{i2}Z_t = p_{i1}\hat{F}_t^u + p_{i2}\hat{Z}_t^u$. We then obtain

$$\hat{Z}_t^u = Z_t + \frac{p_{i1}}{p_{i2}}\left(F_t - \hat{F}_t^u\right). \tag{18}$$

Using this equation, we can eliminate $\hat{Z}_t^u$ in (16), and thus set $p_{u2} = 0$.

Proposition 1 *Consider the benchmark model without advance information. If there is a solution to the system of equations given in Appendix A, then the economy has a stationary rational expectations equilibrium in which the equilibrium stock price is given by*

$$P_t = -p_0 + \frac{a_F}{R - a_F}F_t + p_{i2}Z_t - p_{u1}\left(F_t - \hat{F}_t^u\right), \tag{19}$$

where $p_0, p_{u1} > 0, p_{i2} < 0$. *Also,* $p_{i1} > 0$ *with*

$$p_{i1} + p_{u1} = a_F / (R - a_F). \tag{20}$$

This result is similar to Wang (1994).[5] The properties of the price function are intuitive. The negative constant (or $p_0 > 0$) in the price function reflects the discount on the price to compensate for the risk in future earnings. The second term in the price function reflects the present discounted value of dividends conditional on knowing F_t. The third term in the price function reveals that the stock and the private investment opportunity are substitutes because $p_{i2} < 0$. When the expected return on the private investment opportunity is high (i.e., Z_t is high), informed investors invest in the private investment and rebalance their portfolios by selling the stock. This causes the stock price to drop. These three effects would be present in a full information setup. The fourth and final term in the price function is what gives rise to speculative trading by informed investors. When uninformed investors underestimate the persistence component of dividends and $F_t - \hat{F}_t^u > 0$, the stock price does not immediately reflect the high value of expected discounted future dividends because $p_{u1} > 0$. This causes the stock price to be lower than what it would be under full information.

[5] Wang (1994) gives the same restrictions on the coefficients in the price function without a proof.

Equation (19) demonstrates that the equilibrium price does not reveal the informed investors' private information. That is, uninformed investors cannot distinguish between persistent shocks to earnings and persistent shocks to expected returns to the private investment: Good news about future earnings (high F_t) or bad private investment opportunities (low Z_t) can both cause informed investors to buy the stock and its price to rise. In other words, both a high F_t and a low Z_t lead to a high observed Π_t (see (17)). Observing price and earnings is thus insufficient for uninformed investors to identify the two shocks. This implies that information asymmetry persists in the equilibrium.

The fact that the stock and the private investment opportunity are substitutes, i.e., $p_{i2} < 0$, also has implications for the forecast errors that uninformed investors make: Uninformed investors' forecast errors on the persistent components F_t and Z_t are positively correlated (see (18)). That is, if an uninformed investor underestimates the level of F_t it will also underestimate the level of Z_t.

3.2. Uninformed investors' forecast problem

Uninformed investors do not observe the persistent component of earnings, F_t, nor the persistent component of private investment returns, Z_t. They forecast these variables using available information as described by $\mathcal{F}_t^u$ in (8).

Proposition 2 *Consider the benchmark model without advance information. Given* $\mathcal{F}_t^u = \{D_s, P_s : s \leq t\}$, *the conditional expectations* $\hat{F}_t^u$ *and* $\hat{Z}_t^u$ *are given by the following steady-state Kalman filtering equation:*

$$\left[\begin{array}{c} \hat{F}_t^u \\ \hat{Z}_t^u \end{array}\right] = \left[\begin{array}{c} a_F \hat{F}_{t-1}^u \\ a_Z \hat{Z}_{t-1}^u \end{array}\right] + \mathbf{K} \left[\begin{array}{c} D_t - E_{t-1}^u [D_t] \\ \Pi_t - E_{t-1}^u [\Pi_t] \end{array}\right], \tag{21}$$

where $\mathbf{K}$ *is a* 2×2 *matrix with elements* $k_{11}, k_{12}, k_{21} > 0$ *and* $k_{22} < 0$.

The intuition behind the filtering equation (21) is as follows. The first term on the right-hand side gives the expectation based on information prior to period t. The second term gives the update in expectations based on new information from unexpected fluctuations in earnings and the stock price.

The sign restrictions on the elements of the Kalman gain matrix $\mathbf{K}$ reveal several properties.[6] First, unexpected high earnings (i.e., $D_t - E_{t-1}^u [D_t] > 0$) can be attributed to (i) a high transitory shock ε_t^D, (ii) a high persistent shock ε_t^F, or (iii) a high past forecasting error $F_{t-1} - \hat{F}_{t-1}^u$. Because uninformed investors cannot tell these shocks apart, they increase their

[6] Wang (1994) gives the same restrictions on $\mathbf{K}$ without a proof.

estimate of F_t whenever they observe high unexpected earnings (i.e., $k_{11} > 0$). Because an unexpected positive earnings surprise may arise from a high past forecasting error $F_{t-1} - \hat{F}^u_{t-1}$ and because forecast errors of Z_t and F_t are positively correlated as shown in (18), uninformed investors also revise upwards their expectation of Z_t. Hence, $k_{21} > 0$.

Second, an unexpected increase in $\Pi_t = p_{i1}F_t + p_{i2}Z_t$ may indicate an increase in F_t or a decrease in Z_t. Uninformed investors do not observe these two components separately, and thus raise their estimate of F_t and decrease their estimate of Z_t accordingly. This explains why $k_{12} > 0$ and $k_{22} < 0$.

3.3. Excess stock returns

Using Propositions 1-2, we can derive per-share, excess stock returns and investors' estimates of these returns. Formally, in Appendix A, we show that

$$Q_{t+1} = e_0 + e_{i2}Z_t + e_{i1}\left(F_t - \hat{F}^u_t\right) + \mathbf{b}_Q\boldsymbol{\varepsilon}_{t+1}, \tag{22}$$

where $e_0 = rp_0$ is the unconditional mean excess returns, $e_{i1} > 0$ and $e_{i2} > 0$ are constants, and $\mathbf{b}_Q$ is a constant vector, all given in Appendix A. This equation reveals that changes in the expected private investment return Z_t or changes in the uninformed investors' estimation error $\left(F_t - \hat{F}^u_t\right)$ affect the excess stock return. In addition, exogenous shocks $\boldsymbol{\varepsilon}_{t+1}$ to dividends and private investment returns also affect the excess stock return.

The informed investors' estimate of excess stock returns depends on Z_t and $\left(F_t - \hat{F}^u_t\right)$:

$$E^i_t\left[Q_{t+1}\right] = e_0 + e_{i2}Z_t + e_{i1}\left(F_t - \hat{F}^u_t\right). \tag{23}$$

An increase in Z_t makes informed investors want to substitute stocks for private investments and thus lowers the stock price P_t and raises the future excess stock return Q_{t+1}. If uninformed investors underestimate F_t, in the sense that $F_t > \hat{F}^u_t$, they are likely to revise their estimates upward in the next period as more information on F_t is revealed. As a result, the stock price is expected to rise in period $t+1$. Informed investors observe the size of uninformed investors' underestimation and thus expect the excess stock return to rise.

The uninformed investors' estimate of the excess stock return depends on their estimates of the expected return on the private investment Z_t:

$$E^u_t\left[Q_{t+1}\right] = e_0 + e_{i2}\hat{Z}^u_t. \tag{24}$$

Therefore, uninformed investors expect high stock returns when they expect a lot of rebalancing-motivated selling by informed investors. Using (18), we can rewrite (24) as

$$E^u_t\left[Q_{t+1}\right] = e_0 + e_{i2}Z_t + e_{i2}\frac{p_{i1}}{p_{i2}}\left(F_t - \hat{F}^u_t\right),$$

while noting that uninformed investors do not observe Z_t and $F_t - \hat{F}_t^u$ separately. Changes in Z_t change the excess stock return in the same way it did previously for informed investors. However, in contrast with informed investors, a positive forecast error, i.e., $F_t - \hat{F}_t^u > 0$, leads uninformed investors to expect a low stock return ($e_{i2}\frac{p_{i1}}{p_{i2}} < 0$): When informed investors are buying for speculative reasons, uninformed investors are bound to loose money.

3.4. Optimal portfolios

We now solve the investors' optimal portfolio choice problems. It is straightforward to derive the informed investors' optimal portfolio:

$$\theta_t^i = \frac{E_t^i\left[Q_{t+1}\right]}{\gamma\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)} - \frac{\rho_{Qq}^i E_t^i\left[q_{t+1}\right]}{\gamma\sigma_Q^i\sigma_q^i\left(1-\left(\rho_{Qq}^i\right)^2\right)}, \tag{25}$$

where $\sigma_Q^i = \sqrt{Var_t^i\left(Q_{t+1}\right)}$, $\sigma_q^i = \sqrt{Var_t^i\left(q_{t+1}\right)}$, and

$$\rho_{Qq}^i = Cov_t^i\left(Q_{t+1}, q_{t+1}\right)/\sqrt{Var_t^i\left(Q_{t+1}\right)Var_t^i\left(q_{t+1}\right)}.$$

Note that the preceding conditional variances and covariance are independent of time t due to the property of normal random variables. Similarly, the optimal portfolio for an uninformed investor is given by

$$\theta_t^u = \frac{1}{\gamma}\frac{E_t^u\left[Q_{t+1}\right]}{Var_t^u\left(Q_{t+1}\right)}. \tag{26}$$

Equations (25)-(26) reveal that the optimal portfolios are mean-variance efficient reflecting the trade-off between expected return and risk. Since all investors are myopic and maximize utility from terminal wealth, there is no dynamic hedging demand. However, informed investors have a static hedging demand (last term on the right hand side of (25)) arising from the positive correlation between the stock return and the private investment return. Using the conditional expectations of excess stock returns derived in the previous subsection, we can provide a sharper characterization of the optimal portfolios.

Proposition 3 *Consider the benchmark model without advance information. The equilibrium trading strategies satisfy*

$$\theta_t^i = f_0^i + f_Z^i Z_t + f_F^i\left(F_t - \hat{F}_t^u\right), \tag{27}$$

$$\theta_t^u = f_0^u + f_Z^u \hat{Z}_t^u, \tag{28}$$

where f_0^i, f_0^u, f_F^i, $f_Z^u > 0$ *and* $f_Z^i < 0$ *are constants.*

Equation (27) shows that informed investors trade in the stock when Z_t or $\left(F_t - \hat{F}_t^u\right)$ change. An increase in Z_t leads informed investors to sell the stock, giving rise to their rebalancing trades. The forecast error of uninformed investors $\left(F_t - \hat{F}_t^u\right)$ gives rise to informed investors' speculative trading in the stock market. When uninformed investors underestimate F_t, in the sense that $F_t > \hat{F}_t^u$, informed investors' speculative trading induces them to buy stocks. Informed investors expect to receive a high return in the future as high dividends are realized.

Equation (28) reveals that the uninformed investors' optimal stockholding changes only when their expectation of the private investment return Z_t changes. Using (18) to rewrite (28) we get

$$\theta_t^u = f_0^u + f_Z^u Z_t + f_Z^u \frac{p_{i1}}{p_{i2}} \left(F_t - \hat{F}_t^u\right).$$

This expression demonstrates that trading by uninformed investors is subject to adverse selection. When they trade because informed investors rebalance their portfolios in response to changes in the private investment return as given by Z_t, uninformed investors trade at favorable prices and earn abnormal returns. However, uninformed investors cannot tell these trades from the speculative trades of informed investors. In the later, they trade at unfavorable prices and earn a negative return: Uninformed investors are sellers at times when they underestimate persistent shocks to earnings, i.e. $f_Z^u \left(F_t - \hat{F}_t^u\right) p_{i1}/p_{i2} < 0$ when $F_t - \hat{F}_t^u > 0$.

3.5. Momentum and reversal effects

While Wang (1994) uses a similar model to analyze properties of trading volume, we instead focus on the serial correlation properties of excess stock returns. Empirical evidence documents short-run momentum and long-run reversal effects in returns. We now examine these effects in the benchmark model without advance information. We focus on excess stock returns, while some empirical studies use stock returns.

Proposition 4 *Consider the benchmark model without advance information. For any $n \geq 1$, we have*

$$E\left[Q_{t+n}|Q_t\right] = e_0 + e_{i2} a_Z^{n-1} \frac{Cov\left(Z_t, Q_t\right)}{Var\left(Q_t\right)} Q_t = e_0 + \left(1 - Ra_Z\right) a_Z^{n-1} f_Q Q_t, \tag{29}$$

where $e_0, e_{i2} > 0$ and $f_Q < 0$ are constants.

This proposition shows how current excess returns Q_t can forecast n-period-ahead, single-period excess returns. Empirical studies usually use cumulative n-period returns. Let $Q_{t,t+n} =$

$\sum_{j=1}^{n} Q_{t+j}$ denote the cumulative n-period excess return. We can then derive:

$$E\left[Q_{t,t+n}|Q_t\right] = ne_0 + \left(1 - Ra_Z\right) f_{Qn} Q_t,$$

where $f_{Qn} = \left(1 + a_Z + ... + a_Z^{n-1}\right) f_Q$. Thus, the properties of momentum and reversals follow from the properties of $E\left[Q_{t+n}|Q_t\right]$ given in equation (29). This equation demonstrates that the sign of the correlation between Q_{t+n} and Q_t is the same as the sign of $(Ra_Z - 1)$ for all $n \geq 1$. This means that the model cannot predict both short-run momentum and long-run reversals in excess returns simultaneously. To see this, note that short-run momentum requires that $Cov\left(Q_{t+1}, Q_t\right) > 0$ or $1 - Ra_Z < 0$. However, this condition also implies that $Cov\left(Q_{t+n}, Q_t\right) > 0$ for all $n \geq 1$. As a result, we cannot obtain the long-run return reversal effect documented in empirical studies. In short, excess stock returns are negatively serially correlated at every horizon if $Ra_Z < 1$; otherwise they are positively serially correlated at every horizon.[7]

We now turn to the intuition behind Proposition 4. As we show in Section 3.3 (see equation (22)), next period excess returns can change because either one of the following three components changes: (i) the expected private investment return, (ii) uninformed investors' errors in estimating next-period's persistent component of earnings, and (iii) cash-flow related shocks. Since the latter two components are not correlated with the current excess returns, the serial correlation of excess returns are determined by the correlation between the first component and the current excess return. To derive the sign of this correlation, we start with $n = 1$ and consider the effects of a positive shock to private investment returns, Z_t. There are two effects generated by rebalancing and speculative trading motives, respectively. First, for rebalancing reasons, informed investors sell the stock and invest in the private technology, causing the current stock price P_t and excess stock returns Q_t to fall, *ceteris paribus*. Uninformed investors buy the stock in expectation of high excess stock returns Q_{t+1} in the next period. On the other hand, a high Z_t tends to follow a high Z_{t-1}, because this process is persistent. A high Z_{t-1} causes period $t-1$ stock price P_{t-1} to fall due to rebalancing reasons. This price drop raises the current excess stock returns, Q_t, *ceteris paribus*. This effect dominates when the persistence of expected private investment returns is high enough in that $Ra_Z > 1$, causing positive correlation between Z_t and Q_t, and hence continuation in excess stock returns. Otherwise, excess stock returns tend to reverse themselves.

Consider next the effect of speculative trading. Uninformed investors do not observe the actual increase in Z_t, but can estimate this increase by observing the falling stock prices.

[7] A similar condition is suggested in other papers with different models (see Wang (1993)). We note that the same result applies when $\sigma_{Dq} < 0$. Its proof is available upon request.

Because the information content of the price is given by $\Pi_t = p_{i1}F_t + p_{i2}Z_t$, which decreases with Z_t, observing a low price or a low Π_t leads to an increase in the estimate of Z_t ($\hat{Z}_t^u$) and a decrease in the estimate of F_t ($\hat{F}_t^u$) (see equation (21)). The increase in $\hat{Z}_t^u$ induces uninformed investors to accommodate the rebalancing trades of informed investors. The decrease in $\hat{F}_t^u$ leads to speculative buy trades by informed investors because uninformed investors underestimate F_t. Proposition 4 shows that this speculative trade effect is always dominated by the previous rebalancing trade effect.

We now consider the case with $n > 1$. Because changes in the return on the private investment are persistent, the effect of rebalancing trades is also persistent, causing excess stock returns in period $t + n$ to be correlated with excess stock returns in period t. This correlation follows the same sign as the correlation between successive period returns. Its impact decays at the rate a_Z as n increases.

[Insert Figure 1 Here.]

Figure 1 illustrates the impulse response to a one-time positive shock to the private investment return Z_t at time 0 when the economy is initially at the deterministic steady state.[8] The parameter values are such that $Ra_Z < 1$. The top right panel of Figure 1 reveals that the stock price falls at time 0 and then gradually reverts back to its long-run mean as the shock to Z_t decays over time. The excess return also falls at time 0, but rises at time 1 and then gradually reverts back to the long run mean. The speed of reversion depends on the persistence of Z_t. In our numerical illustration, the persistence is not high and hence the excess return reverts fast back to its long-run mean. The bottom left panel illustrates the estimation errors that generate speculative trades. The bottom right panel shows the trading strategies. In response to an increase in the private investment return, the informed investors sell the stock and uninformed investors buy the stock. As the shock to Z_t decays over time, both informed and uninformed investors revert their trading positions.

3.6. Earnings announcement drift

The empirical literature on earnings announcements documents a persistent price drift after earnings surprises. In our model with heterogeneous information, we assume that earnings surprises are computed with respect to all available public information. Thus, we define earnings surprises as $D_t - E_{t-1}^u(D_t)$. We show the following result:

[8] To produce better visual effects, means are subtracted from all variables in all figures in the paper.

Proposition 5 *Consider the benchmark model without advance information. For any $n \geq 1$, we have*

$$E\left[Q_{t+n}|D_t - E_{t-1}^u(D_t)\right] = e_0 + e_{i2}a_Z^{n-1}E\left[Z_t|D_t - E_{t-1}^u(D_t)\right] = e_0 + a_Z^{n-1}d_1\left[D_t - E_{t-1}^u(D_t)\right], \tag{30}$$

where e_0 and $e_{i2}, d_1 > 0$ are constants.

Empirical studies of price drift after earnings announcements consider the cumulative stock return after an earnings surprise. The cumulative return is characterized by the following equation:

$$E\left[Q_{t,t+n}|D_t - E_{t-1}^u(D_t)\right] = ne_0 + d_{1n}\left[D_t - E_{t-1}^u(D_t)\right],$$

where $d_{1n} = \left(1 + a_Z + ... + a_Z^{n-1}\right)d_1$. This equation reveals that positive surprises lead to positive returns because $d_{1n} > 0$. In addition, cumulative stock returns increase with the investment horizon since $\partial(d_{1n})/\partial n > 0$, but at a decreasing rate since $\partial^2(d_{1n})/\partial n < 0$.

To understand Proposition 5, we start with the case with perfect and symmetric information. In this case, an earnings surprise in the current period is observed by all investors, as is all other information, and thus is immediately included in the current stock price. From the next period on, the surprise is already realized and thus does not affect the stock price. This implies that the price drift cannot occur under symmetric information.[9] In our model with asymmetric information, uninformed investors may interpret a positive earnings surprise as an increase in the persistent component of earnings, and thus raise their estimate of this component $\left(\hat{F}_t^u\right)$ and their estimate of the private investment return $\left(\hat{Z}_t^u\right)$ by Proposition 2. As uninformed investors overestimate the persistent component of earnings, informed investors take speculative sale positions. Uninformed investors do not know informed investors' trading motives and may think they sell for rebalancing reasons because uninformed investors believe that the private investment return goes up. Thus, uninformed investors buy the stock to accommodate informed investors' speculative trades, in anticipation of a high return in the next period. This causes a positive correlation between excess return and earnings announcements. Because both $\hat{F}_t^u$ and $\hat{Z}_t^u$ are persistent, the above trading patterns are also persistent, leading to a persistent return drift.

Note that in the model, the phenomenon of price earnings announcement drift does not depend on whether there is momentum. While earnings price drift arises from uninformed investors' estimation errors, we have shown in the previous subsection that informed investor's

[9] Banerjee et al. (2008) provide a model in which differences in higher-order beliefs may lead to price drift.

speculative trades based on these estimation errors are not sufficient to overcome the strength of the rebalancing trades in driving the serial correlation in returns.

4. Equilibrium with a single piece of advance information

In the next two sections, we analyze models with advance information. We start with the case where informed investors receive a single piece of advance information each period. In particular, we assume that the informed and uninformed investors' information sets are given by (7) and (8), respectively.[10]

In the presence of advance information, solving an equilibrium is nontrivial. The key is to construct suitable state variables. We define the state vector as

$$\mathbf{x}_t = \left(F_t, Z_t, \varepsilon^D_{t+k}, ..., \varepsilon^D_t, \varepsilon^q_{t+k}, ..., \varepsilon^q_{t+1}\right)^{\intercal}. \tag{31}$$

The state vector is conceivably much bigger than in the previous section because it includes all future realizations of the transitory shocks to earnings and returns on private investment up to $t+k$. This is because informed investors can use their private information to forecast these values. Note that we also include ε^D_t as part of the state vector for the technical reason to make the forecasting problem easy to solve. We show below that it is not priced in equilibrium, and that it does not appear in the asset demand functions, because ε^D_t has already been paid out in the form of earnings at time t and has no value for forecasting future earnings.

4.1. Stock price

We conjecture that the equilibrium stock price function takes the following form:

$$P_t = -p_0 + \mathbf{p}_i \hat{\mathbf{x}}^i_t + \mathbf{p}_u \hat{\mathbf{x}}^u_t, \tag{32}$$

where $\hat{\mathbf{x}}^i_t = E^i_t[\mathbf{x}_t]$, $\hat{\mathbf{x}}^u_t = E^u_t[\mathbf{x}_t]$, p_0 is a constant, and $\mathbf{p}_i$ and $\mathbf{p}_u$ are row vectors of constants to be determined in equilibrium. In addition, we set $p_{u2} = 0$ as in the benchmark model in Section 3. The reason is that the equality $\mathbf{p}_i \hat{\mathbf{x}}^i_t = \mathbf{p}_i \hat{\mathbf{x}}^u_t$ holds in equilibrium because $\mathbf{p}_i \hat{\mathbf{x}}^i_t$ is in the information set of uninformed investors. Therefore,

$$\hat{Z}^u_t = \frac{1}{p_{i2}} \mathbf{p}_i \hat{\mathbf{x}}^i_t - \frac{\mathbf{p}_i \mathbf{I}_{-2}}{p_{i2}} \hat{\mathbf{x}}^u_t, \tag{33}$$

where $\mathbf{I}_{-2}$ conforms with the state vector and denotes the matrix that is the same as the identity matrix except that the $(2,2)$ element equals zero. Thus, we can eliminate $\hat{Z}^u_t$ in the

[10] Baccheta and van Wincoop (2004, 2006) construct models of investor heterogeneity with advance information to study the role of higher order expectations. However, they do not address the questions we study here.

price function. Also, like equation (18), equation (33) indicates that uninformed investors' forecast errors are perfectly linearly correlated.

Proposition 6 *Consider the model with a single piece of advance information. If there is a solution to the system of equations given in Appendix B, then the economy has a stationary rational expectations equilibrium in which the equilibrium stock price is given by*

$$\begin{aligned} P_t &= -p_0 + \frac{a_F}{R-a_F}F_t + p_{i2}Z_t - p_{u1}\left(F_t - \hat{F}_t^u\right) \\ &\quad + \sum_{j=1}^{k}\left\{\frac{1}{R^j}E_t^i\left[\varepsilon_{t+j}^D\right] - p_{u,k+3-j}\left(E_t^i\left[\varepsilon_{t+j}^D\right] - E_t^u\left[\varepsilon_{t+j}^D\right]\right)\right\} \\ &\quad + \sum_{j=1}^{k}\left\{-\frac{e_{i2}}{R^j}E_t^i\left[\varepsilon_{t+j}^q\right] - p_{u,2k+4-j}\left(E_t^i\left[\varepsilon_{t+j}^q\right] - E_t^u\left[\varepsilon_{t+j}^q\right]\right)\right\}, \end{aligned} \tag{34}$$

where $p_0 > 0$, $p_{u,k+3} = p_{i,k+3} = 0$, and $e_{i2} > 0$ iff $Cov_t^i\left(Q_{t+1}, q_{t+1}\right) > 0$.

The interpretation of the first line of equation (34) is similar to that of (19) in the benchmark model. Unlike the benchmark model, we are unable to prove $p_{i2} < 0$ because we are unable to show $Cov_t^i\left(Q_{t+1}, q_{t+1}\right) > 0$ analytically. This positive covariance is intuitive, however. It reflects the fact that the stock and the private investment are substitutes because unexpected earnings and the unexpected private investment return are positively correlated, i.e., $\sigma_{Dq} > 0$. We verify this result numerically in all our examples below.

The second and third lines of equation (34) reflect the effects of advance information. In the presence of advance information about earnings innovations in the future, informed investors forecast these innovations and their forecasts are incorporated into the stock price. Specifically, earnings expected next period are discounted at rate R, earnings expected two periods later are discounted at rate R^2, and so on up to $t+k$. Since date-t advance information is not informative about earnings after date $t+k$, we have $E_t^i\left[\varepsilon_{t+s}^D\right] = 0$, for $s > k$. Uninformed investors do not have any advance information, and forecast it based on current and past stock price and earnings data. Thus, they make forecast errors relative to informed investors' information. These forecast errors lead to speculative trades, and hence also move prices. This discussion explains the second line of (34).

The terms on the price function that appear on the third line of (34) arise because earnings innovations are correlated with innovations to the private investment return. As informed investors learn about ε_{t+j}^D based on date $t+j-k$ advance information for $1 \le j \le k$, they also improve their forecast about ε_{t+j}^q and can better forecast private investment returns. This information may generate rebalancing trades of informed investors. For example, when the private investment and the stock are substitutes, $Cov_t^i\left(Q_{t+1}, q_{t+1}\right) > 0$, a good piece of advance information at date t raises $E_t^i\left[\varepsilon_{t+k}^q\right]$, which induces informed investors to sell the stock at time $t+k-1$, causing the current stock price to decline. As before, the stock price also incorporates the uninformed investors' errors in estimating ε_{t+j}^q, which generate speculative trades.

Unlike in Wang (1994), in our model with advance information both the informed and uninformed investors must solve forecasting problems.[11] We next turn to these problems and solve the informed investor's forecasting problem first.

4.2. Informed investors' forecast

Informed investors use their private signals on k-period-ahead earnings innovations to learn about the growth potential in both the stock and private investment. Their information processing problem is greatly simplified for two reasons. First, their information set includes knowledge of all past values of the persistent components of earnings and private investment returns. Second, their information set includes that of uninformed investors, which means that informed investors do not learn from the price level.

Proposition 7 *Consider the model with a single piece of advance information. Informed investors' conditional expectations are given by:*

$$\begin{aligned} E_t^i\left[\varepsilon_{t+k-j}^D\right] &= \frac{\sigma_D^2}{\sigma_S^2+\sigma_D^2} S_{t-j}, \qquad 0 \leq j \leq k-1 \\ E_t^i\left[\varepsilon_{t+k-j}^q\right] &= \frac{\sigma_{Dq}}{\sigma_S^2+\sigma_D^2} S_{t-j}, \qquad 0 \leq j \leq k-1. \end{aligned}$$

The signals S_j, $j \leq t$, do not help forecast any other variable.

The information content of a signal S_t about earnings ε_{t+k}^D is given by $\sigma_D^2/\left(\sigma_S^2+\sigma_D^2\right)$. If the signal is infinitely precise and $\sigma_S^2=0$, then $\sigma_D^2/\left(\sigma_S^2+\sigma_D^2\right)=1$ and S_t reveals the true innovation value. If the signal is worthless and $\sigma_S^2=\infty$ then $\sigma_D^2/\left(\sigma_S^2+\sigma_D^2\right)=0$ and the informed investors do not use this information to update their expectation of future earnings. The signal S_t is also informative about shocks to the private investment return ε_{t+k}^q because $E\left[\varepsilon_t^D \varepsilon_t^q\right]=\sigma_{Dq}>0$. The information content of the signal is given by $\sigma_{Dq}/\left(\sigma_S^2+\sigma_D^2\right)$.

With i.i.d. signals, a signal S_t observed at t about earnings ε_{t+k}^D contains no new information about other innovations ε_{t+k-j}^D for $0 \leq j \leq k-1$. Likewise, S_t contains no new information

[11] Albuquerque et al. (2007) generalize Wang (1994) to an international economy and also require both informed and uninformed agents to solve forecasting problems.

about innovations ε^q_{t+k-j} for $0 \leq j \leq k-1$. Finally, because ε^D_{t+k} is uncorrelated with every other shock, the signal S_t cannot be used to forecast any other future variable.

It is useful to write the forecasting problem of informed investors as a filtering problem in terms of the state-space system representation. We write

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t, \tag{35}$$

where the matrices $\mathbf{A}_x$ and $\mathbf{B}_x$ are defined in Appendix B. We also construct the unforecastable shock vector (based on period $t-1$ information) as

$$\boldsymbol{\varepsilon}_t = \left(\varepsilon^D_{t+k}, \varepsilon^F_t, \varepsilon^Z_t, \varepsilon^q_{t+k}, \varepsilon^S_t\right)^\intercal.$$

This vector is normally distributed with mean zero and covariance matrix $\Sigma = E\left[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^\intercal\right]$, where the only nonzero covariance is $\sigma_{Dq} = E\left[\varepsilon^D_{t+k}\varepsilon^q_{t+k}\right] > 0$.

The informed investors' observable signals are summarized in the vector $y^i_t = (D_t, F_t, Z_t, S_t)^\intercal$. This vector satisfies:

$$\mathbf{y}^i_t = \mathbf{A}_{yi}\mathbf{x}_t + \mathbf{B}_{yi}\boldsymbol{\varepsilon}_t.$$

where $\mathbf{A}_{yi}$ and $\mathbf{B}_{yi}$ are given in Appendix B. Then we have the steady-state Kalman filter:

$$\hat{\mathbf{x}}^i_t = \mathbf{A}_x \hat{\mathbf{x}}^i_{t-1} + \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}^i_t, \tag{36}$$

where the innovation

$$\hat{\boldsymbol{\varepsilon}}^i_t \equiv \mathbf{y}^i_t - E^i_{t-1}\left[\mathbf{y}^i_t\right], \tag{37}$$

is normally distributed with mean zero and covariance matrix $\boldsymbol{\Sigma}_i = E\left[\hat{\boldsymbol{\varepsilon}}^i_t \left(\hat{\boldsymbol{\varepsilon}}^i_t\right)^\intercal\right]$. The expressions of the Kalman gain matrix $\mathbf{K}_i$ and the matrix $\boldsymbol{\Sigma}_i$ are given in Appendix B. We note that the first two components of $\hat{\mathbf{x}}^i_t$ are given by F_t and Z_t since they are observable. Also, since D_t and F_t are observable, $\hat{\mathbf{x}}^i_t$ contains ε^D_t. The other components of $\hat{\mathbf{x}}^i_t$ can be obtained via proposition 7.

4.3. Uninformed investors' forecast

We next consider an uninformed investor's forecasting problem. Because informed investors know more than uninformed investors, the most that uninformed investors can hope to learn is what informed investors know. Therefore, it is sufficient for uninformed investors to track the dynamics of the state vector (36). The uninformed investors' observation is summarized by the vector $\mathbf{y}^u_t = \left(D_t, \mathbf{p}_i \hat{\mathbf{x}}^i_t\right)^\intercal$. We write the observation system as

$$\mathbf{y}^u_t = \mathbf{A}_{yu}\hat{\mathbf{x}}^i_t,$$

where $\mathbf{A}_{yu} = \left[\mathbf{c}_1^\intercal + \mathbf{c}_{k+3}^\intercal, \mathbf{p}_i^\intercal\right]^\intercal$. By the Kalman filtering theory, we have:

Proposition 8 *Consider the model with a single piece of advance information. Uninformed investors' conditional forecast of the state vector is given by the steady-state Kalman filters:*

$$\hat{\mathbf{x}}_t^u = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \mathbf{K}_u \hat{\boldsymbol{\varepsilon}}_t^u, \tag{38}$$

and

$$\hat{\boldsymbol{\varepsilon}}_t^u \equiv \mathbf{y}_t^u - E_{t-1}^u \left[\mathbf{y}_t^u \right], \tag{39}$$

where $\hat{\boldsymbol{\varepsilon}}_t^u$ is normally distributed with mean zero and covariance matrix

$$\boldsymbol{\Sigma}_u = \mathbf{A}_{yu} \mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\intercal \mathbf{A}_{yu}^\intercal + \mathbf{A}_{yu} \mathbf{K}_i \boldsymbol{\Sigma}_i \mathbf{K}_i^\intercal \mathbf{A}_{yu}^\intercal.$$

Moreover, the covariance matrix $\boldsymbol{\Omega}_u = E_t^u \left[\left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u \right) \left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u \right)^\intercal \right]$ and the Kalman gain matrix $\mathbf{K}_u$ satisfy:

$$\boldsymbol{\Omega}_u = \left(\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) - \mathbf{K}_u \mathbf{A}_{yu} \left(\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right), \tag{40}$$

where

$$\mathbf{K}_u = \left(\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) \mathbf{A}_{yu}^\intercal \left[\mathbf{A}_{yu} \left(\mathbf{A}_x \boldsymbol{\Omega}_u \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) \mathbf{A}_{yu}^\intercal \right]^{-1}, \tag{41}$$

and $\boldsymbol{\Sigma}_{xx} = \mathbf{K}_i \boldsymbol{\Sigma}_i \mathbf{K}_i^\intercal$.

We are now ready to solve for the estimates of excess stock returns and optimal portfolios of informed and uninformed investors.

4.4. Excess stock returns

In Appendix B, we use Propositions 6-8 to derive the excess stock return

$$\begin{aligned} Q_{t+1} \quad = \quad & e_0 + e_{i2} \left(Z_t + E_t^i \left[\varepsilon_{t+1}^q \right] \right) + e_{i1} \left(F_t - \hat{F}_t^u \right) + \mathbf{b}_Q \boldsymbol{\varepsilon}_{t+1} \\ & + \sum_{j=1}^{k} \left\{ e_{i,k+3-j} \left(E_t^i \left[\varepsilon_{t+j}^D \right] - E_t^u \left[\varepsilon_{t+j}^D \right] \right) - e_{u,2k+4-j} \left(E_t^i \left[\varepsilon_{t+j}^q \right] - E_t^u \left[\varepsilon_{t+j}^q \right] \right) \right\}, \end{aligned} \tag{42}$$

where $\mathbf{b}_Q$ is a constant vector, e_0, $e_{i1}, ..., e_{i,k+2,}$ $e_{u,k+4}, ...,$ and $e_{u,2k+3}$ are constants given in Appendix B. The first line of equation (42) is similar to equation (22) for the benchmark model with one difference. Informed investors' forecast of the innovation ε_{t+1}^q to the private investment return also affects the $t+1$ excess stock return. This reflects informed investors' increased ability to forecast private returns using the time $t-k+1$ advance information about ε_{t+1}^D. The informed investors' estimates of ε_{t+1}^q change the time t expected private investment return $Z_t + E_t^i \left[\varepsilon_{t+1}^q \right]$ and generate rebalancing trades which affect excess stock returns at time $t+1$.

The second line of equation (42) reveals that the uninformed investors' error in estimating future earning innovations ε_{t+1}^{D} and private return innovations ε_{t+1}^{q} relative to the informed investors' estimates influences excess stock returns. The reason is that the informed investors can profit from this estimation error in their speculative trading.

Using equation (42), we can easily derive the informed and uninformed investors' estimates of excess returns:

$$\begin{aligned} E_t^i\left[Q_{t+1}\right] &= e_0+e_{i2}\left(Z_t+E_t^i\left[\varepsilon_{t+1}^q\right]\right)+e_{i1}\left(F_t-\hat{F}_t^u\right) \\ &+\sum_{j=1}^{k}\left\{e_{i,k+3-j}\left(E_t^i\left[\varepsilon_{t+j}^D\right]-E_t^u\left[\varepsilon_{t+j}^D\right]\right)-e_{u,2k+4-j}\left(E_t^i\left[\varepsilon_{t+j}^q\right]-E_t^u\left[\varepsilon_{t+j}^q\right]\right)\right\}, \end{aligned} \tag{43}$$

and

$$E_t^u\left[Q_{t+1}\right]=e_0+e_{i2}\left(\hat{Z}_t^u+E_t^u\left[\varepsilon_{t+1}^q\right]\right). \tag{44}$$

The error term $\mathbf{b}_Q\boldsymbol{\varepsilon}_{t+1}$ is unforecastable by both informed and uninformed investors and thus does not change either investors' estimates of excess stock returns.

4.5. Optimal portfolios

As in Section 3.2, optimal portfolios are given by equations (25) and (26). We use the previously derived conditional expectations of excess stock returns to derive the following:

Proposition 9 *Consider the model with a single piece of advance information. The equilibrium trading strategies satisfy*

$$\begin{aligned} \theta_t^i &= f_0^i+f_Z^i\left(Z_t+E_t^i\left[\varepsilon_{t+1}^q\right]\right)+f_F^i\left(F_t-\hat{F}_t^u\right) \\ &+\sum_{j=1}^{k}\left\{f_{Dj}^i\left(E_t^i\left[\varepsilon_{t+j}^D\right]-E_t^u\left[\varepsilon_{t+j}^D\right]\right)+f_{qj}^i\left(E_t^i\left[\varepsilon_{t+j}^q\right]-E_t^u\left[\varepsilon_{t+j}^q\right]\right)\right\}, \end{aligned} \tag{45}$$

and

$$\theta_t^u=f_0^u+f_Z^u\left(\hat{Z}_t^u+E_t^u\left[\varepsilon_{t+1}^q\right]\right), \tag{46}$$

where f_0^i, $f_0^u>0$, f_Z^i, f_F^i, f_Z^u, f_{Dj}^i *and* f_{qj}^i *are constants. Also,* $f_Z^i<0$ *and* $f_Z^u>0$ *iff* $Cov_t^i\left(Q_{t+1},q_{t+1}\right)>0$.

This proposition shows that the informed investors trade for both speculative and rebalancing reasons as in the benchmark model. The expected return on the private investment determines their rebalancing trades. Unlike the benchmark model, with advance information on earnings innovations, informed investors' forecast of ε_{t+1}^{q} is not zero and hence is included in the expected private investment return. When the assets are substitutes and $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$ a high expected return on the private investment leads to rebalancing selling of the stock.

Speculative trading by informed investors arises from their knowledge of the forecast errors made by uninformed investors about F_t, $E_t^i\left[\varepsilon_{t+j}^D\right]$ and $E_t^i\left[\varepsilon_{t+j}^q\right]$. The forecasting errors $E_t^i\left[\varepsilon_{t+j}^D\right] - E_t^u\left[\varepsilon_{t+j}^D\right]$ and $E_t^i\left[\varepsilon_{t+j}^q\right] - E_t^u\left[\varepsilon_{t+j}^q\right]$ are new to the model with advance information, and, together with $F_t - \hat{F}_t^u$, constitute the private information of informed investors. This private information allows them to take speculative positions against expected future corrections of the uninformed investors' expectations.

Uninformed investors trade for noninformational reasons. They are willing to trade only when they perceive to be accommodating informed investors' rebalancing trades because these trades always occur at favorable prices (i.e. $p_{i2} < 0$ whereas $f_Z^i < 0$ and $f_Z^u > 0$). However, as in the benchmark model, their trading is subject to adverse selection because they do not know whether informed investors are trading in response to a change in $Z_t + E_t^i\left[\varepsilon_{t+1}^q\right]$ or to superior private information.

4.6. Momentum and reversal effects

Because of advance information the properties of momentum and reversal in stock returns differ substantially from the benchmark model. As in Section 3.3, in order to study the properties of cumulative returns, $E\left[Q_{t,t+n}|Q_t\right]$, commonly used to measure momentum and reversal effects in the empirical studies, we only need to focus on single-period returns, $E\left[Q_{t+n}|Q_t\right]$.

Proposition 10 *Consider the model with a single piece of advance information. For any $n \geq 1$, we have*

$$E\left[Q_{t+n}|Q_t\right] = e_0 + e_{i2}\frac{a_Z^{n-1}Cov\left(Z_t, Q_t\right) + \mathbf{1}_{\{n\leq k\}}Cov\left(E_t^i\left[\varepsilon_{t+n}^q\right], Q_t\right)}{Var\left(Q_t\right)}Q_t, \tag{47}$$

where $e_0 > 0$ and $e_{i2} > 0$ if and only if $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$. The indicator function $\mathbf{1}_{\{n\leq k\}}$ equals 1 if $n \leq k$ and 0 otherwise.

This proposition decomposes the correlation between Q_{t+n} and Q_t into two parts. The first part is determined largely by the covariance between the stock return and the expected return on the private investment, $Cov\left(Z_t, Q_t\right)$. Its sign depends on the persistence of the expected private investment return, much like in the benchmark model. In particular, if the persistence is sufficiently high, then the covariance is positive. Otherwise, it is negative.

The second part reflects the effect of advance information. This part is represented by the term $Cov\left(E_t^i\left[\varepsilon_{t+n}^q\right], Q_t\right)$ which plays a role if and only if $n \leq k$. After period $t+k$, all advance information up to date t loses its value and the correlation between Q_{t+n} $(n > k)$ and Q_t is determined as in the benchmark model without advance information. From the analysis in Section 3, we deduce that we must have two conditions for our model to generate short-run momentum and long-run reversals simultaneously. First, the persistence a_Z must be sufficiently small. If it is too large, we cannot generate long-run reversals. Second, given a small value of a_Z, we must have $Cov\left(E_t^i\left[\varepsilon_{t+n}^q\right], Q_t\right) > 0$. Otherwise, we cannot generate short-run momentum. Therefore, in the discussion below we assume that a_Z is sufficiently small.

[Insert Figure 2 Here.]

To describe the intuition behind the mechanism that generates momentum, consider first the case of one-period-ahead advance information, i.e., $k = 1$. We suppose the economy is in the deterministic steady state. Suppose informed investors receive a good signal about future earnings ε_{t+1}^D at time $t = 0$. We assume that this signal materializes fully at time $t+1$ in the form of high earnings; but what happens at $t+1$ is not known by any investor at time t. The effects of this good signal are described in Figure 2. The good news leads to an increase in price P_t and in the excess stock return Q_t by the amount $E_t^i\left[\varepsilon_{t+1}^D\right]/R$. In addition, there are two main effects that partly offset the effect of the good news on the stock price. First, uninformed investors underestimate the private signal and $E_t^i\left[\varepsilon_{t+1}^D\right] - E_t^u\left[\varepsilon_{t+1}^D\right] > 0$ as shown in the bottom left panel of Figure 2. This gives rise to an increase in informed investors' speculative demand for the stock. Second, a good signal about ε_{t+1}^D helps informed investors forecast a high return in their private investments as well, $E_t^i\left[\varepsilon_{t+1}^q\right] > 0$ because ε_{t+1}^D and ε_{t+1}^q are positively correlated. This gives rise to rebalancing stock sales by informed investors.

The two effects that keep the price from fully incorporating the value of the advance information are also responsible for a high return next period and for momentum in stock returns. First, at time $t+1$ earnings are realized to be high, which surprises uninformed investors. Second, informed investors undo their rebalancing trades which forces prices up as well.

The bottom right panel of Figure 2 shows that uninformed investors buy the stock in expectation of high returns in the next period. Because the price is going up in the next period, they are perceived as trend-chasers. Informed investors sell because of the previously discussed rebalancing trades. They are thus viewed as contrarian investors. Therefore, momentum occurs as contrarian investors sell in spite of the fact that they expect high future stock returns.[12] That

[12] This type of behavior is consistent with the trading pattern that seems to arise when mutual funds sell large amounts of stock for what appears to be liquidity reasons. The returns for investors that buy are positive and significant (see Coval and Stafford (2007)).

is, in order for momentum to occur, while all investors must be expecting return continuation, a subset of them must be willing to take the opposite side of trading positions.

We next consider the case of advance information about ε^D_{t+k} with $k > 1$. We argue that serial correlation in one-period returns may display a cyclical pattern.[13] Intuitively, the effect of rebalancing trades due to advance information shows up only in period $t + k - 1$. Up to period $t + k - 1$, there is return reversal provided a_Z is sufficiently small, as in the benchmark model. To understand the intuition, consider the effects of a one time good signal about $k = 2$ period ahead earnings at time $t = 0$. This signal is realized in period $t + k$. The effects are described in Figure 3. We find that the stock price and the stock return respond immediately by increasing up to $E^i_t \left[\varepsilon^D_{t+k}\right] / R^k$. In addition, the stock price and return also immediately reflect the discounted value of uninformed investors' forecast error and of the expected high private investment return. Thus, informed investors buy the stock, speculating on uninformed investors' forecast errors, as shown in the bottom right panel of Figure 3. These speculative trades should lead to return continuation, but as in the benchmark model they may not be strong enough to overturn the return reversal phenomenon when a_Z is small: As revealed in the top right panel, the excess return Q_{t+1} is positive but very small, because the news about two-period-ahead earnings is not realized at $t + 1$.

[Insert Figure 3 Here.]

Informed investors' rebalancing trades in response to the good advance information only occur at time $t + k - 1$: Only then do they wish to invest more in the private technology in expectation of a high subsequent return, $Z_{t+k-1} + E^i_{t+k-1}\left[\varepsilon^q_{t+k}\right]$ (see the second term in (45)). Therefore, as shown in the bottom right panel of Figure 3, at time $t + k - 1$ informed investors act as contrarian investors and sell after a price run-up, but expecting high future returns when the good advance information materializes next period. Uninformed investors act as return chasers, and are willing to buy because they expect high returns due to rebalancing by informed investors. Therefore we obtain $Cov\left(Q_{t+k}, Q_t\right) > 0$. After period $t + k$, the date t advance information has no effect on future returns and stock returns revert to the long-run mean in that $Cov\left(Q_{t+n}, Q_t\right) < 0$ for $n > k$.

We now conduct some numerical experiments to illustrate our previous intuition. Table 1 shows the slope coefficients on the forecast of single-period returns Q_{t+n} conditional on Q_t as well as the slope coefficient of the forecast of cumulative returns $Q_{t,t+n}$ conditional on Q_t. We

[13] Interestingly, Jegadeesh and Titman (1993) find that the strategy of buying winners and selling loosers based on the last six months of trading produces negative returns for a one-month holding period.

find that when $k = 1$, our model generates two-period momentum followed by a stock return reversal. The two-period cumulative return is positive because the first period positive return compensates for the second period negative return. We also find that when $k = 3$, cumulative returns for all horizons are negative and there is no momentum effect.

[Insert Table 1 Here.]

4.7. Earnings announcement drift

Advance private information about future transitory earnings also alters the nature of the earnings announcement price drift. As in Section 3.4, we only need to focus on $E\left[Q_{t+n}|D_t - E^u_{t-1}(D_t)\right]$. We have the following result:

Proposition 11 *Consider the model with a single piece of advance information. For any $n \geq 1$, we have*

$$E\left[Q_{t+n}|D_t - E^u_{t-1}(D_t)\right] = e_0 + e_{i2}a_Z^{n-1}E\left[Z_t|D_t - E^u_{t-1}(D_t)\right] + e_{i2}d_1\frac{\mathbf{1}_{\{n\leq k-1\}}Cov\left(E^u_t\left[\varepsilon^D_{t+n}\right], D_t - E^u_{t-1}(D_t)\right)}{Var\left(D_t - E^u_{t-1}(D_t)\right)}\left[D_t - E^u_{t-1}(D_t)\right], \quad (48)$$

where e_0 is a constant, $d_1 > 0$, and $e_{i2} > 0$ if and only if $Cov^i_t(Q_{t+1}, q_{t+1}) > 0$. The indicator function $\mathbf{1}_{\{n\leq k-1\}}$ equals 1 if $n \leq k-1$ and 0 otherwise.

With advance information, we have not been able to show that earnings price drift always occurs. There are two main effects. One is common to the benchmark model. A positive earnings surprise calls for an upward revision of uninformed investors' estimates of F_t and Z_t. Thus, informed investors take speculate sale positions. Uninformed investors buy the stock because they believe informed investors may trade for rebalancing reasons. This causes excess returns to rise.

With advance information there is a second effect. A positive earnings surprise at time t may also occur because uninformed investors underestimate F_{t-1}. Observing past prices, but not advance information signals, they may believe informed investors receive good advance information about $\varepsilon^D_{t+1}, ...\varepsilon^D_{t+k-1}$, which materializes at any time from $t+1$ to $t+k-1$.[14] In response to a positive earnings surprise at date t, uninformed investors raise their estimate of F_t and revise the forecasts of $E^u_t\left[\varepsilon^D_{t+n}\right]$ down, for $n \leq k-1$. As a result, low expected future dividends are then associated with low expected returns.

[14] Note that advance information about ε^D_{t+k} was just released which explains why the seocnd term only appears for $n \leq k-1$.

Despite this dampening effect, Table 2 illustrates numerically that our model with advance information can still generate the earnings announcement drift phenomenon. The table displays the slope coefficients of the forecast of single-period returns Q_{t+n} conditional on the earnings announcement $D_t - E^u_{t-1}[D_t]$ as well as the slope coefficients of cumulative returns $Q_{t,t+n}$ conditional on $D_t - E^u_{t-1}[D_t]$ for various values of n and k.

[Insert Table 2 Here.]

5. Equilibrium with multiple pieces of advance information

The analysis thus far has shown that the benchmark model without advance information in Section 3 cannot generate momentum and reversal effects simultaneously. In addition, we have shown in Section 4 that the model with one-period-ahead advance information can generate short-run momentum followed by reversals in stock returns. However, momentum lasts only for very few periods. We also show that when informed investors receive a single piece of long-horizon advance information, the model generates a counterfactual cyclic behavior of serial correlation in one-period excess stock returns.

In order to generate long-lived momentum followed by long-run reversals, we extend the model in Section 4 to incorporate multiple pieces of advance information. Specifically, we assume that at each period t informed investors receive signals about earnings in each period from $t+1$ to $t+k$. At time t, they will have received k correlated signals about period $t+1$ earnings. Thus, the informed and uninformed investors' information sets are given by (10) and (8), respectively. This assumption is quite natural as new information, say about end-of-quarter earnings, is likely to arrive at intermediate periods as the quarter nears its end. In addition, past stale information is still useful for forecasting and thus affects stock prices.

The intuition behind this modeling device is that the successive advance information news about the same future earnings can generate long-lived, large speculative trading effects and momentum. An important modeling issue is how to specify the quality of signals. Because up to period t informed investors will have received $k-1$ signals on ε^D_{t+1} already, the stock price increasingly reveals ε^D_{t+1} to the uninformed investors, reducing the motive for speculative trading by informed investors. It is therefore possible that, with too much information in previous periods, only the rebalancing trade motive is at work, generating negative serial correlation in returns. In order to obtain long-lived momentum when $k > 1$ it is thus needed that the advance information increases in quality as we approach the earnings realization, i.e., $\sigma^2_{S_k} > ... > \sigma^2_{S_1}$. In this case, return reversals occur after at least k periods as the advance information effect dissipates and the stock price overshoots its long-run mean. This gives rise to the same overreaction patterns as described in behavioral theories.

In appendix C, we show that after casting the problem into vector and matrix forms, we can use the previous solution method to show that the equilibrium in this section displays the same form as in Section 4. The only difference is that the informed investors' forecasting problem is different because they now have multiple pieces of advance information. We omit the detailed derivation here and turn to a numerical analysis.

For ease of exposition, we focus on the case with $k = 2$. We first discuss two limiting results. First, when the signal about two-period-ahead earnings is completely uninformative (i.e., $\sigma_{S_2} = \infty$), the model becomes that in Section 4 with $k = 1$. Consequently, our previous results in Section 4 apply here. Second, when the signal about the two-period-ahead earnings innovation is extremely precise (i.e., $\sigma_{S_2} \to 0$), we find numerically that asymmetric information increases so much that there is no trading in equilibrium. As a result, stock returns are serially uncorrelated. The intuition is as follows. The stock price incorporates the information about the persistent and transitory components of earnings as well as the expected return on the private investment. Uninformed investors use the earnings realizations and the stock price to infer the value of these variables, but may attribute changes in earnings innovations to changes in the various components of earnings or to changes in the private investment return. When informed investors receive very precise information about earnings innovations, they trade on this information more aggressively and uninformed investors believe that informed investors' trading is generated by a speculative motivate and not by a rebalancing motive. Because uninformed investors know that they will lose if they trade with informed investors when their trades are solely motived by speculative reasons, they refrain from trading.

We now turn to intermediate values of σ_{S_2}. Table 3 displays the slope coefficients of the forecast of single-period returns Q_{t+n} conditional on Q_t as well as the slope coefficients of the forecast of cumulative returns $Q_{t,t+n}$ conditional on Q_t for $\sigma_{S_2} = 1$ and various values of σ_{S_1}. This table reveals that our model with advance information about earnings innovations over two successive periods can generate momentum and reversal effects simultaneously. In addition, the duration of momentum depends on the precision of the advance information signals. In particular, when the signal about one-period-ahead earnings innovations becomes more precise relative to the signal about two-period-ahead earnings innovations, the momentum effect lasts longer. On the other hand, when this signal is sufficiently imprecise, the momentum effect disappears.

[Insert Table 3 Here.]

Table 4 displays the slope coefficients of the forecast of single-period returns Q_{t+n} conditional on the earnings announcement $D_t - E^u_{t-1}[D_t]$ as well as the slope coefficients of the forecast of cumulative returns $Q_{t,t+n}$ conditional on $D_t - E^u_{t-1}[D_t]$, for $\sigma_{S_2} = 1$ and various values of σ_{S_1}. The table shows that the earnings announcement drift effect is stronger as the signal about one-period-ahead earnings innovations becomes less precise relative to the signal about two-period-ahead earnings innovations.

[Insert Table 4 Here.]

6. Conclusion

In this paper, we present a heterogeneous-investor rational expectations equilibrium model with asymmetric information that can deliver the momentum and reversal effects in a unified way. Our key insight is to assume that informed investors possess advance information about future earnings which is correlated with other (private) investment opportunities of these investors. The stock price underreacts to this information and uninformed investors can profit by following trend-chasing strategies. Advance information also makes prices move in ways that are unrelated to current fundamentals. These price movements, which Daniel and Titman (2006) call the intangible component of returns, can predict future return reversals. After a sustained streak of good news, the advance information materializes and the price starts reverting back to its long-run mean, giving the appearance of having overshoot its fundamental value. Thus, our model provides a rational account of the underreaction and overreaction phenomena.

We can extend our model in a number of directions. First, for tractability, we have assumed myopic preferences and ignored hedging demands. While it is not difficult to introduce hedging demand, we believe that this extension does not change the main predictions and insights of our model. Second, our model focuses on the implications of advance information for momentum and reversal effects. It would be interesting to study the implications for trading volume, as in Wang (1994) and LIorente et al. (2002). Third, we follow Wang (1994) and assume a hierarchical information structure. It would be interesting to consider the case where information is symmetrically dispersed (see, e.g., Bacchetta and van Wincoop (2004, 2006)). In this case, higher order expectations play an important role. Finally, we assume that advance information is private to informed investors only. In work in progress, we consider the case where advance information is public to all investors in the economy.

Appendices

A Proofs for the benchmark model

We first prove Proposition 2 since we need to solve the investors' forecasting problems before we derive an equilibrium.

Proof of Proposition 2: An uninformed investor's forecasting problem is the classical Kalman filtering problem. To solve this problem, we use the state-space system representation. Let

$$\mathbf{x}_t = \mathbf{A}_x \mathbf{x}_{t-1} + \mathbf{B}_x \boldsymbol{\varepsilon}_t. \tag{A.1}$$

where

$$\mathbf{A}_x = \begin{bmatrix} a_F & 0 \\ 0 & a_Z \end{bmatrix}, \ \mathbf{B}_x = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{A.2}$$

The uninformed investor has signals $\mathbf{y}_t = (D_t, \Pi_t)^\intercal$, which satisfies

$$\mathbf{y}_t = \mathbf{A}_y \mathbf{x}_t + \mathbf{B}_y \boldsymbol{\varepsilon}_t, \tag{A.3}$$

where

$$\mathbf{A}_y = \begin{bmatrix} 1 & 0 \\ p_{i1} & p_{i2} \end{bmatrix}, \ \mathbf{B}_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.4}$$

Define

$$\boldsymbol{\Sigma}_{xx} = \mathbf{B}_x \boldsymbol{\Sigma} \mathbf{B}_x^\intercal = \begin{bmatrix} \sigma_F^2 & 0 \\ 0 & \sigma_Z^2 \end{bmatrix}, \ \boldsymbol{\Sigma}_{yy} = \mathbf{B}_y \boldsymbol{\Sigma} \mathbf{B}_y^\intercal = \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & 0 \end{bmatrix} \tag{A.5}$$

and $\boldsymbol{\Omega}_t = E_t^u \left[(\mathbf{x}_t - \hat{\mathbf{x}}_t^u) (\mathbf{x}_t - \hat{\mathbf{x}}_t^u)^\intercal \right]$. Then by Kalman filtering

$$\hat{\mathbf{x}}_{t+1}^u = \mathbf{A}_x \hat{\mathbf{x}}_t^u + \mathbf{K}_t \left[\mathbf{y}_{t+1} - E_t^u \left[\mathbf{y}_{t+1} \right] \right] \tag{A.6}$$

$$\boldsymbol{\Omega}_t = \left(\mathbf{A}_x \boldsymbol{\Omega}_{t-1} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) - \mathbf{K}_t \mathbf{A}_y \left(\mathbf{A}_x \boldsymbol{\Omega}_{t-1} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) \tag{A.7}$$

where

$$\mathbf{K}_t = \left(\mathbf{A}_x \boldsymbol{\Omega}_{t-1} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) \mathbf{A}_y^\intercal \left[\mathbf{A}_y \left(\mathbf{A}_x \boldsymbol{\Omega}_{t-1} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) \mathbf{A}_y^\intercal + \boldsymbol{\Sigma}_{yy} \right]^{-1} \tag{A.8}$$

As in Wang (1994), we focus on the steady-state Kalman filtering. Let $\boldsymbol{\Omega}$ be the solution to the Riccati equation

$$\boldsymbol{\Omega} = \left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) - \mathbf{K} \mathbf{A}_y \left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right), \tag{A.9}$$

where

$$\mathbf{K} = \left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) \mathbf{A}_y^\intercal \left[\mathbf{A}_y \left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx} \right) \mathbf{A}_y^\intercal + \boldsymbol{\Sigma}_{yy} \right]^{-1}. \tag{A.10}$$

We then obtain the following the steady-state filters:

$$\hat{\mathbf{x}}_t^u = \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \mathbf{K}\hat{\boldsymbol{\varepsilon}}_t^u, \tag{A.11}$$

and

$$\mathbf{y}_t = \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u + \hat{\boldsymbol{\varepsilon}}_t^u. \tag{A.12}$$

where $\hat{\boldsymbol{\varepsilon}}_t^u = \mathbf{y}_t - E_{t-1}^u [\mathbf{y}_t]$ is the is the innovation, which is normally distributed with mean of zero and covariance matrix

$$\begin{aligned}
E\left[\hat{\boldsymbol{\varepsilon}}_t^u \hat{\boldsymbol{\varepsilon}}_t^{u\intercal}\right] &= E\left[\left(\mathbf{y}_t - \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u\right)\left(\mathbf{y}_t - \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u\right)^\intercal\right] \\
&= E\left[\left(\mathbf{A}_y \mathbf{x}_t + \mathbf{B}_y \boldsymbol{\varepsilon}_t - \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u\right)\left(\mathbf{A}_y \mathbf{x}_t + \mathbf{B}_y \boldsymbol{\varepsilon}_t - \mathbf{A}_y \mathbf{A}_x \hat{\mathbf{x}}_{t-1}^u\right)^\intercal\right] \\
&= E[\left(\mathbf{A}_y \mathbf{A}_x \left(\mathbf{x}_{t-1} - \hat{\mathbf{x}}_{t-1}^u\right) + \left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y\right)\boldsymbol{\varepsilon}_t\right) \\
&\quad \left(\left(\mathbf{x}_{t-1} - \hat{\mathbf{x}}_{t-1}^u\right)^\intercal \mathbf{A}_x^\intercal \mathbf{A}_y^\intercal + \boldsymbol{\varepsilon}_t^\intercal \left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y\right)^\intercal\right)] \\
&= \mathbf{A}_y \mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal \mathbf{A}_y^\intercal + \left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y\right)\boldsymbol{\Sigma}\left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y\right)^\intercal,
\end{aligned}$$

because by construction

$$E\left[\left(\mathbf{x}_{t-1} - \hat{\mathbf{x}}_{t-1}^u\right)\boldsymbol{\varepsilon}_t^\intercal\right] = E\left[E_{t-1}\left(\left(\mathbf{x}_{t-1} - \hat{\mathbf{x}}_{t-1}^u\right)\boldsymbol{\varepsilon}_t^\intercal\right)\right] = 0.$$

Thus, we have

$$Var\left(\hat{\boldsymbol{\varepsilon}}_t^u\right) = \mathbf{A}_y \mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal \mathbf{A}_y^\intercal + \left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y\right)\boldsymbol{\Sigma}\left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y\right)^\intercal. \tag{A.13}$$

Post-multiplying both sides of (A.9) by $\mathbf{A}_y^\intercal$ and subtracting $\mathbf{K}\boldsymbol{\Sigma}_{yy}$ from both sides yields

$$\boldsymbol{\Omega}\mathbf{A}_y^\intercal - \mathbf{K}\boldsymbol{\Sigma}_{yy} = \left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_y^\intercal - \mathbf{K}\left[\mathbf{A}_y\left(\mathbf{A}_x \boldsymbol{\Omega} \mathbf{A}_x^\intercal + \boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_y^\intercal + \boldsymbol{\Sigma}_{yy}\right] = 0.$$

This equality can be written as

$$\begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} 1 & p_{i1} \\ 0 & p_{i2} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & 0 \end{bmatrix},$$

where $\Omega = (\omega_{ij})$ with $\omega_{12} = \omega_{21}$. Therefore, we get the following 4 equations

$$\begin{bmatrix} \omega_{11} & \omega_{11}p_{i1} + \omega_{12}p_{i2} \\ \omega_{21} & \omega_{21}p_{i1} + \omega_{22}p_{i2} \end{bmatrix} = \sigma_D^2 \begin{bmatrix} k_{11} & 0 \\ k_{21} & 0 \end{bmatrix}.$$

Solving yields

$$\begin{aligned}
k_{11} &= \omega_{11}/\sigma_D^2 > 0,\ k_{21} = \omega_{21}/\sigma_D^2 > 0, && \text{(A.14)} \\
\omega_{12} &= -p_{i1}/p_{i2}\omega_{11} > 0,\ \omega_{21} = -p_{i2}/p_{i1}\omega_{22} > 0, && \text{(A.15)}
\end{aligned}$$

where we have used the sign restrictions in Proposition 1.

Now post-multiply both sides of (A.9) by $\left(\mathbf{A}_x\mathbf{\Omega}\mathbf{A}_x^{\intercal}+\mathbf{\Sigma}_{xx}\right)^{-1}$ to get

$$\mathbf{\Omega}\left(\mathbf{A}_x\mathbf{\Omega}\mathbf{A}_x^{\intercal}+\mathbf{\Sigma}_{xx}\right)^{-1}=\mathbf{I}-\mathbf{K}\mathbf{A}_y$$

or

$$\begin{aligned}&\omega_{11}\begin{bmatrix}1 & -\frac{p_{i1}}{p_{i2}}\\ -\frac{p_{i1}}{p_{i2}} & \left(\frac{p_{i1}}{p_{i2}}\right)^2\end{bmatrix}\begin{bmatrix}a_F^2\omega_{11}+\sigma_F^2 & -a_Z a_F\frac{p_{i1}}{p_{i2}}\omega_{11}\\ -a_F a_Z\frac{p_{i1}}{p_{i2}}\omega_{11} & a_Z^2\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2\end{bmatrix}^{-1}\\ &=\begin{bmatrix}1-k_{11}-k_{12}p_{i1} & -k_{12}p_{i2}\\ -k_{21}-k_{22}p_{i1} & 1-k_{22}p_{i2}\end{bmatrix}\end{aligned}$$

Simplify this equation to obtain:

$$\begin{aligned}&\frac{\omega_{11}}{\Delta}\begin{bmatrix}a_Z\left(a_Z-a_F\right)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2 & \frac{p_{i1}}{p_{i2}}\left[a_F\left(a_Z-a_F\right)\omega_{11}-\sigma_F^2\right]\\ -\frac{p_{i1}}{p_{i2}}\left[a_Z\left(a_Z-a_F\right)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2\right] & -\left(\frac{p_{i1}}{p_{i2}}\right)^2\left[a_F\left(a_Z-a_F\right)\omega_{11}-\sigma_F^2\right]\end{bmatrix}\\ &=\begin{bmatrix}1-k_{11}-k_{12}p_{i1} & -k_{12}p_{i2}\\ -k_{21}-k_{22}p_{i1} & 1-k_{22}p_{i2}\end{bmatrix},\end{aligned}\tag{A.16}$$

with

$$\begin{aligned}\Delta &= \left(a_F^2\omega_{11}+\sigma_F^2\right)\left(a_Z^2\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2\right)-\left(a_F a_Z\frac{p_{i1}}{p_{i2}}\omega_{11}\right)^2\\ &= \sigma_F^2 a_Z^2\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+a_F^2\sigma_Z^2\omega_{11}+\sigma_F^2\sigma_Z^2>0\end{aligned}\tag{A.17}$$

Now using the top right hand corner equation:

$$\frac{\omega_{11}}{\Delta}\frac{p_{i1}}{p_{i2}^2}\left[\sigma_F^2-\left(1-a_F^2\right)\omega_{11}+\left(1-a_F a_Z\right)\omega_{11}\right]=k_{12},\tag{A.18}$$

with $\sigma_F^2-\left(1-a_F^2\right)\omega_{11}>0$ and $a_F,a_Z\in(0,1)$, we obtain $k_{12}>0$. Similarly, we have

$$-\frac{\omega_{11}}{\Delta}\frac{p_{i1}}{p_{i2}}\left[a_Z\left(a_Z-a_F\right)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2\right]=-k_{21}-k_{22}p_{i1}>0.\tag{A.19}$$

Since

$$a_Z\left(a_Z-a_F\right)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2=\sigma_Z^2-\left(1-a_Z^2\right)\omega_{22}+\left(1-a_F a_Z\right)\omega_{22}>0,$$

we obtain

$$-p_{i1}k_{22}=k_{21}-\frac{\omega_{11}}{\Delta}\frac{p_{i1}}{p_{i2}}\left[a_Z\left(a_Z-a_F\right)\left(\frac{p_{i1}}{p_{i2}}\right)^2\omega_{11}+\sigma_Z^2\right]>0\tag{A.20}$$

implying $k_{22} < 0$. Note using the top left hand corner equation in (A.16),

$$1 - k_{11} - k_{12} p_{i1} = \frac{\omega_{11}}{\Delta} \left[a_Z \left(a_Z - a_F \right) \left(\frac{p_{i1}}{p_{i2}} \right)^2 \omega_{11} + \sigma_Z^2 \right] > 0.$$

Having shown that the right hand side of this expression is positive and that under our guess that $p_{i1} > 0$, we obtain $k_{11} < 1$. Q.E.D.

Proof of Proposition 1: We will use the market-clearing condition (15) to verify that the equilibrium price function takes the form in (16) and that the coefficients satisfy the sign restrictions. We start with the investors' optimal portfolios. We first derive the conditional expectations, variances, and covariance of the excess returns. We use the conjectured pricing function (16) and Kalman filters (A.11)-(A.12) to rewrite the excess return as

$$\begin{aligned} Q_{t+1} &= P_{t+1} + D_{t+1} - RP_t \\ &= -p_0 + \mathbf{p}_i \mathbf{x}_{t+1} + p_{u1} \mathbf{c}_1 \hat{\mathbf{x}}_{t+1}^u + F_{t+1} + \varepsilon_{t+1}^D - R \left(-p_0 + \mathbf{p}_i \mathbf{x}_t + p_{u1} \mathbf{c}_1 \hat{\mathbf{x}}_t^u \right) \\ &= rp_0 + \left(\left(\mathbf{p}_i + \mathbf{c}_1 \right) \mathbf{A}_x - R \mathbf{p}_i \right) \mathbf{x}_t + p_{u1} \mathbf{c}_1 \left(\mathbf{A}_x - R \mathbf{I} \right) \hat{\mathbf{x}}_t^u \\ &\quad + \left(\left(\mathbf{p}_i + \mathbf{c}_1 \right) \mathbf{B}_x + \mathbf{c}_1 \right) \boldsymbol{\varepsilon}_{t+1} + p_{u1} \mathbf{c}_1 \mathbf{K} \left(\mathbf{y}_{t+1} - E_t^u \left[\mathbf{y}_{t+1} \right] \right) \\ &= e_0 + \mathbf{e}_i \mathbf{x}_t + \mathbf{e}_u \hat{\mathbf{x}}_t^u + \mathbf{b}_Q \boldsymbol{\varepsilon}_{t+1}. \end{aligned} \tag{A.21}$$

Here the coefficients are defined as

$$e_0 = rp_0, \tag{A.22}$$

$$\mathbf{e}_i = \left(\mathbf{p}_i + \mathbf{c}_1 \right) \mathbf{A}_x - R \mathbf{p}_i + p_{u1} \mathbf{c}_1 \mathbf{K} \mathbf{A}_y \mathbf{A}_x \left[\begin{array}{c} 1 \\ -p_{i1}/p_{i2} \end{array} \right] \mathbf{c}_1, \tag{A.23}$$

$$\mathbf{e}_u = p_{u1} \mathbf{c}_1 \left[\mathbf{A}_x - R \mathbf{I} - \mathbf{K} \mathbf{A}_y \mathbf{A}_x \left[\begin{array}{c} 1 \\ -p_{i1}/p_{i2} \end{array} \right] \mathbf{c}_1 \right], \tag{A.24}$$

$$\mathbf{b}_Q = \left(\mathbf{p}_i + \mathbf{c}_1 \right) \mathbf{B}_x + \mathbf{c}_1 + p_{u1} \mathbf{c}_1 \mathbf{K} \left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y \right), \tag{A.25}$$

where $\mathbf{c}_j$ is the standard row vector with the j^{th} element being 1 and the rest being zero. Note that in deriving equation (A.21), we have used the following:

$$\begin{aligned} \hat{\boldsymbol{\varepsilon}}_{t+1}^u &= \mathbf{y}_{t+1} - E_t^u \left[\mathbf{y}_{t+1} \right] \\ &= \mathbf{A}_y \mathbf{x}_{t+1} + \mathbf{B}_y \boldsymbol{\varepsilon}_{t+1} - E_t^u \left[\mathbf{A}_y \mathbf{x}_{t+1} + \mathbf{B}_y \boldsymbol{\varepsilon}_{t+1} \right] \\ &= \mathbf{A}_y \mathbf{A}_x \left(\mathbf{x}_t - \hat{\mathbf{x}}_t^u \right) + \left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y \right) \boldsymbol{\varepsilon}_{t+1} \\ &= \mathbf{A}_y \mathbf{A}_x \left[\begin{array}{c} 1 \\ -p_{i1}/p_{i2} \end{array} \right] \left(F_t - \hat{F}_t^u \right) + \left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y \right) \boldsymbol{\varepsilon}_{t+1} \\ &= \mathbf{A}_y \mathbf{A}_x \left[\begin{array}{c} 1 \\ -p_{i1}/p_{i2} \end{array} \right] \mathbf{c}_1 \mathbf{x}_t - \mathbf{A}_y \mathbf{A}_x \left[\begin{array}{c} 1 \\ -p_{i1}/p_{i2} \end{array} \right] \mathbf{c}_1 \hat{\mathbf{x}}_t^u + \left(\mathbf{A}_y \mathbf{B}_x + \mathbf{B}_y \right) \boldsymbol{\varepsilon}_{t+1} \end{aligned} \tag{A.26}$$

where the next-to-last step uses (18) again. Note that from (A.24) it must true that $e_{u2} = 0$.

We next use (A.21) to derive the conditional variances and covariance of excess returns:

$$\left(\sigma_Q^i\right)^2 = Var_t^i\left(Q_{t+1}\right) = \mathbf{b}_Q \Sigma \mathbf{b}_Q^{\intercal}, \tag{A.27}$$

$$\left(\sigma_q^i\right)^2 = Var_t^i\left(q_{t+1}\right) = \sigma_q^2, \tag{A.28}$$

$$Cov_t^i\left(Q_{t+1}, q_{t+1}\right) = \mathbf{b}_Q E_t^i\left[\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}_{t+1}^{\intercal}\right]\mathbf{c}_4^{\intercal} = \mathbf{b}_Q \Sigma \mathbf{c}_4^{\intercal} = \left(1 + p_{u1}k_{11}\right)\sigma_{Dq} \tag{A.29}$$

which is positive by (A.14) and the guess that $p_{u1} > 0$. In addition,

$$\begin{aligned} \left(\sigma_Q^u\right)^2 &= Var_t^u\left(Q_{t+1}\right) = E_t^u\left[\left(\mathbf{e}_i\left(\mathbf{x}_t - \hat{\mathbf{x}}_t^u\right) + \mathbf{b}_Q\boldsymbol{\varepsilon}_{t+1}\right)\left(\mathbf{e}_i\left(\mathbf{x}_t - \hat{\mathbf{x}}_t^u\right) + \mathbf{b}_Q\boldsymbol{\varepsilon}_{t+1}\right)^{\intercal}\right] \\ &= E_t^u[\mathbf{e}_i\left(\mathbf{x}_t - \hat{\mathbf{x}}_t^u\right)\left(\mathbf{x}_t - \hat{\mathbf{x}}_t^u\right)^{\intercal}\mathbf{e}_i^{\intercal} + \mathbf{e}_i\left(\mathbf{x}_t - \hat{\mathbf{x}}_t^u\right)\boldsymbol{\varepsilon}_{t+1}^{\intercal}\mathbf{b}_Q^{\intercal} \\ &\quad + \mathbf{b}_Q\boldsymbol{\varepsilon}_{t+1}\left(\mathbf{x}_t - \hat{\mathbf{x}}_t\right)^{\intercal}\mathbf{e}_i^{\intercal} + \mathbf{b}_Q\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}_{t+1}^{\intercal}\mathbf{b}_Q^{\intercal}] \\ &= \mathbf{e}_i\boldsymbol{\Omega}\mathbf{e}_i^{\intercal} + \mathbf{b}_Q\boldsymbol{\Sigma}\mathbf{b}_Q^{\intercal}, \end{aligned} \tag{A.30}$$

where we have used $E\left[\left(\mathbf{x}_t - \hat{\mathbf{x}}_t^u\right)\boldsymbol{\varepsilon}_{t+1}^{\intercal}\right] = 0$ to derive that last equality.

We now use equation (A.21) to derive the informed and uninformed investors' conditional expectations of excess returns:

$$E_t^i\left[Q_{t+1}\right] = e_0 + \mathbf{e}_i\mathbf{x}_t + \mathbf{e}_u\hat{\mathbf{x}}_t^u, \tag{A.31}$$

$$E_t^u\left[Q_{t+1}\right] = e_0 + \left(\mathbf{e}_i + \mathbf{e}_u\right)\hat{\mathbf{x}}_t^u, \tag{A.32}$$

and

$$E_t^i\left[q_{t+1}\right] = Z_t = \mathbf{c}_2\mathbf{x}_t. \tag{A.33}$$

We rewrite (18) as

$$\hat{Z}_t^u = \frac{1}{p_{i2}}\mathbf{p}_i\mathbf{x}_t - \frac{p_{i1}}{p_{i2}}\hat{F}_t^u.$$

Substituting this equation into (A.31)-(A.32) yields

$$E_t^i\left[Q_{t+1}\right] = e_0 + \mathbf{e}_i\mathbf{x}_t + e_{u1}\hat{F}_t^u \tag{A.34}$$

$$\begin{aligned} E_t^u\left[Q_{t+1}\right] &= e_0 + e_{i1}\hat{F}_t^u + e_{i2}\hat{Z}_t^u + e_{u1}\hat{F}_t^u \\ &= e_0 + e_{i1}\hat{F}_t^u + e_{i2}\left[\frac{1}{p_{i2}}\mathbf{p}_i\mathbf{x}_t - \frac{p_{i1}}{p_{i2}}\hat{F}_t^u\right] + e_{u1}\hat{F}_t^u \\ &= e_0 + \frac{e_{i2}}{p_{i2}}\mathbf{p}_i\mathbf{x}_t + \left(e_{i1} - e_{i2}\frac{p_{i1}}{p_{i2}} + e_{u1}\right)\hat{F}_t^u. \end{aligned} \tag{A.35}$$

Substituting the preceding equations into (25) and (26), we obtain

$$\theta_t^i = \frac{e_0 + \mathbf{e}_i\mathbf{x}_t + e_{u1}\hat{F}_t^u}{\gamma\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)} - \frac{\rho_{Qq}^i\mathbf{c}_2\mathbf{x}_t}{\gamma\sigma_Q^i\sigma_q^i\left(1 - \left(\rho_{Qq}^i\right)^2\right)}, \tag{A.36}$$

$$\theta_t^u = \frac{e_0 + \frac{e_{i2}}{p_{i2}}\mathbf{p}_i\mathbf{x}_t + \left(e_{i1} - e_{i2}\frac{p_{i1}}{p_{i2}} + e_{u1}\right)\hat{F}_t^u}{\gamma\left(\sigma_Q^u\right)^2}. \tag{A.37}$$

We now use the market-clearing condition (15) to determine the coefficients p_0, $\mathbf{p}_i$ and $\mathbf{p}_u$. Using this condition and equations (A.36)-(A.37), we have

$$\begin{aligned} 1 &= \frac{\lambda}{\gamma}\left[\frac{e_0 + \mathbf{e}_i\mathbf{x}_t + e_{u1}\hat{F}_t^u}{\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)} - \frac{\rho_{Qq}^i\mathbf{c}_2\mathbf{x}_t}{\sigma_Q^i\sigma_q^i\left(1 - \left(\rho_{Qq}^i\right)^2\right)}\right] \\ &\quad + (1-\lambda)\frac{e_0 + \frac{e_{i2}}{p_{i2}}\mathbf{p}_i\mathbf{x}_t + \left(e_{i1} - e_{i2}\frac{p_{i1}}{p_{i2}} + e_{u1}\right)\hat{F}_t^u}{\gamma\left(\sigma_Q^u\right)^2}. \end{aligned}$$

Matching coefficients yields the following four equilibrium restrictions:

$$e_0 = \gamma\frac{\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)\left(\sigma_Q^u\right)^2}{\lambda\left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)}, \tag{A.38}$$

$$0 = \lambda\frac{e_{i1}}{\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)} + (1-\lambda)\frac{e_{i2}\frac{p_{i1}}{p_{i2}}}{\left(\sigma_Q^u\right)^2}, \tag{A.39}$$

$$0 = \lambda\left[\frac{e_{i2}}{\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)} - \frac{\rho_{Qq}^i}{\sigma_Q^i\sigma_q^i\left(1 - \left(\rho_{Qq}^i\right)^2\right)}\right] + (1-\lambda)\frac{e_{i2}}{\left(\sigma_Q^u\right)^2}, \tag{A.40}$$

and

$$0 = \lambda\frac{e_{u1}}{\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)} + (1-\lambda)\frac{e_{i1} - e_{i2}\frac{p_{i1}}{p_{i2}} + e_{u1}}{\left(\sigma_Q^u\right)^2}. \tag{A.41}$$

Equation (A.40) gives the solution for e_{i2} :

$$e_{i2} = \frac{\lambda\left(\sigma_Q^u\right)^2}{\lambda\left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)}\frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2}. \tag{A.42}$$

Equation (A.39) gives the solution for e_{i1} :

$$e_{i1} = -\frac{p_{i1}}{p_{i2}}\frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)}{\lambda\left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2\left(1 - \left(\rho_{Qq}^i\right)^2\right)}\frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2}. \tag{A.43}$$

Equation (A.41) gives the solution for e_{u1} :

$$e_{u1} = \frac{p_{i1}}{p_{i2}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)} \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2}. \tag{A.44}$$

Having solved for e_{i1}, e_{i2} and e_{u1} as a function of the price coefficients we can use (A.22)-(A.24) to determine these coefficients. First, equation (A.22) implies

$$p_0 = r^{-1} e_0 > 0,$$

verifying our conjectured sign in Proposition 1. The rest gives us a system of 3 equations in 3 unknowns p_{i1}, p_{i2}, and p_{u1}. As noted before, we have $e_{u2} = 0$ since we have substituted our $\hat{Z}_t^u$ in (A.21). We can verify that equation (A.24) implies that this is always true. To explicitly derive those 3 equations, we first use (A.24) to derive

$$e_{u1} = p_{u1}\left[a_F\left(1-k_{11}-p_{i1}k_{12}\right)-R+a_Z p_{i1} k_{12}\right]. \tag{A.45}$$

We then add equations (A.23) and (A.24) to derive

$$\mathbf{e}_i + \mathbf{e}_u = \left(\mathbf{p}_i + \mathbf{c}_1\right)\mathbf{A}_x - R\mathbf{p}_i + p_{u1}\mathbf{c}_1\left(\mathbf{A}_x - R\mathbf{I}\right).$$

That is,

$$e_{i1} + e_{u1} = -\left(p_{i1}+p_{u1}\right)\left(R-a_F\right)+a_F, \tag{A.46}$$

$$e_{i2} = -p_{i2}\left(R-a_Z\right). \tag{A.47}$$

Note that the system of three equations (A.45)-(A.47) along with (A.42)-(A.44) do not admit an analytical solution for p_{i1}, p_{i2} and p_{u1}. An equilibrium exists if this system has a solution. A simple iterative numerical procedure can be used to solve this system. We should emphasize that when solving this system we must substitute the equations for conditional variances and covariance (A.27)-(A.37) and the equations for the Kalman gain matrix (A.9)-(A.10).

Even though we cannot solve the equilibrium explicitly, we can derive restrictions on the coefficients in the price function. First, adding equations (A.43) and (A.44) yields $e_{i1}+e_{u1}=0$. Equation (A.46) then implies equation (20) in Proposition 1. We then take the ratio of (A.44) and (A.42), and substitute for e_{i2} using (A.47) to derive

$$p_{i1} = -\left(R-a_Z\right)e_{u1}\frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}{\lambda\left(\sigma_Q^u\right)^2}. \tag{A.48}$$

We conclude that $p_{i1} > 0$. Suppose to the contrary $p_{i1} < 0$. Then equation (A.48) implies that $e_{u1} > 0$. We next use (A.45) to show that $p_{u1} < 0$. To this end, we use the two equations implied in the top row of (A.16) to substitute for $1 - k_{11} - p_{i1}k_{12}$ and $p_{i1}k_{12}$, respectively. We then obtain

$$\begin{aligned}
& a_F\left(1 - k_{11} - p_{i1}k_{12}\right) - R + a_Z p_{i1} k_{12} \\
= \; & \frac{\omega_{11}}{\Delta} a_F \left[a_Z \left(a_Z - a_F\right) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} + \sigma_Z^2 \right] - R \\
& - \frac{\omega_{11}}{\Delta} a_Z \left(\frac{p_{i1}}{p_{i2}}\right)^2 \left[a_F\left(a_Z - a_F\right)\omega_{11} - \sigma_F^2\right] \\
= \; & \frac{\omega_{11}}{\Delta} \left(a_F \sigma_Z^2 + a_Z \left(\frac{p_{i1}}{p_{i2}}\right)^2 \sigma_F^2 \right) - R. \\
= \; & \frac{a_Z \left(\frac{p_{i1}}{p_{i2}}\right)^2 \sigma_F^2 \omega_{11} + a_F \sigma_Z^2 \omega_{11}}{a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \sigma_F^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2} - R,
\end{aligned}$$

We now show this expression is negative and thus deduce $p_{u1} < 0$. It suffices to show that

$$\frac{\omega_{11}}{\Delta} \left(a_F \sigma_Z^2 + a_Z \left(\frac{p_{i1}}{p_{i2}}\right)^2 \sigma_F^2 \right) = \frac{a_Z \left(\frac{p_{i1}}{p_{i2}}\right)^2 \sigma_F^2 \omega_{11} + a_F \sigma_Z^2 \omega_{11}}{a_Z^2 \left(\frac{p_{i1}}{p_{i2}}\right)^2 \sigma_F^2 \omega_{11} + a_F^2 \sigma_Z^2 \omega_{11} + \sigma_F^2 \sigma_Z^2} < 1,$$

where we have substituted equation (A.17). This inequality is equivalent to

$$a_Z\left(1 - a_Z\right) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \sigma_F^2 \omega_{11} + \left(1 - a_F\right) a_F \sigma_Z^2 \omega_{11} < \sigma_F^2 \sigma_Z^2.$$

This inequality is true since we can show that

$$\begin{aligned}
& a_Z\left(1 - a_Z\right) \left(\frac{p_{i1}}{p_{i2}}\right)^2 \sigma_F^2 \omega_{11} + \left(1 - a_F\right) a_F \sigma_Z^2 \omega_{11} \\
= \; & a_Z\left(1 - a_Z\right) \sigma_F^2 \omega_{22} + \left(1 - a_F\right) a_F \sigma_Z^2 \omega_{11} \\
< \; & a_Z\left(1 - a_Z\right) \frac{\sigma_F^2 \sigma_Z^2}{\left(1 - a_Z^2\right)} + \left(1 - a_F\right) a_F \frac{\sigma_F^2 \sigma_Z^2}{\left(1 - a_F^2\right)} \\
= \; & \frac{a_Z}{1 + a_Z} \sigma_F^2 \sigma_Z^2 + \frac{a_F}{1 + a_Z} \sigma_F^2 \sigma_Z^2 \\
< \; & \sigma_F^2 \sigma_Z^2,
\end{aligned}$$

where we have used (A.15) and the definition of Ω to derive

$$\begin{aligned}
\left(\frac{p_{i1}}{p_{i2}}\right)^2 \omega_{11} &= \omega_{22} = Var\left(Z_t - \hat{Z}_t\right) = \frac{\sigma_Z^2}{1 - a_Z^2} - Var\left(\hat{Z}_t\right) \\
\omega_{11} &= Var\left(F_t - \hat{F}_t\right) = \frac{\sigma_F^2}{1 - a_F^2} - Var\left(\hat{F}_t\right),
\end{aligned}$$

and we note

$$a_Z, a_F \in (0,1) \text{ and } \frac{a_Z}{1+a_Z} + \frac{a_F}{1+a_Z} < 1.$$

Therefore, equation (A.45) and $e_{u1} > 0$ imply that $p_{u1} < 0$, which contradicts with (20), because equation (20) implies that $p_{i1} + p_{u1}$ must be positive.

Thus, we must have $p_{i1} > 0$. We then use (A.48) to deduce that $e_{u1} < 0$. Since we have shown that the expression in the square bracket in equation (A.45) is negative, we conclude that $p_{u1} > 0$. It follows from (A.29) that $Cov_t^i(Q_{t+1}, q_{t+1}) > 0$. Use this result and equation (A.42) to obtain $e_{i2} > 0$. By this result, equation (A.47), and $a_Z < R$, we obtain $p_{i2} < 0$. Q.E.D.

Proof of Proposition 3: We only need to consider conditional expectations of future returns in (25) and (26) since their coefficients are constant. Equation (A.33) gives $E_t^i[q_{t+1}] = Z_t$. Since $e_{i1} + e_{u1} = 0$, we can rewrite equations (A.34) and (A.35) as

$$E_t^i[Q_{t+1}] = e_0 + e_{i2}Z_t + e_{i1}\left(F_t - \hat{F}_t^u\right), \tag{A.49}$$

$$E_t^u[Q_{t+1}] = e_0 + e_{i2}\hat{Z}_t^u. \tag{A.50}$$

Plugging the preceding conditional expectations in equations (25) and (26) yields the desired result. Q.E.D.

Proof of Proposition 4: We use equation (A.50) to compute:

$$\begin{aligned} E[Q_{t+1}|Q_t] &= E[E_t^u[Q_{t+1}]|Q_t] = e_0 + e_{i2}E[E_t^u[Z_t]|Q_t] && \text{(A.51)} \\ &= e_0 + e_{i2}E[Z_t|Q_t] = e_0 + e_{i2}\frac{Cov(Z_t, Q_t)}{Var(Q_t)}Q_t, && \text{(A.52)} \end{aligned}$$

where e_0 and $e_{i2} > 0$ are constants as shown in the proof of Proposition 1.

Now we use (16) to compute the unconditional covariance $Cov(Z_t, Q_t)$:

$$\begin{aligned} Cov(Z_t, Q_t) &= E[Z_t(p_{i1}F_t + p_{u1}\hat{F}_t^u + p_{i2}Z_t + F_t + \varepsilon_t^D \\ &\quad -R(p_{i1}F_{t-1} + p_{u1}\hat{F}_{t-1}^u + p_{i2}Z_{t-1}))] \\ &= E\left[p_{i2}Z_t^2 - p_{i2}RZ_tZ_{t-1} + p_{u1}Z_t\hat{F}_t^u - p_{u1}a_ZRZ_{t-1}\hat{F}_{t-1}^u\right] \\ &= p_{i2}(1 - Ra_Z)Var(Z_t) + p_{u1}(1 - Ra_Z)E\left(Z_t\hat{F}_t^u\right), \end{aligned}$$

where we use the fact that $E[Z_tF_t] = E[Z_tF_{t-1}] = E\left[Z_t\varepsilon_t^D\right] = 0$.

Multiplying by Z_t in both sides of (18) and taking expectations, we obtain:

$$E\left[Z_t\hat{Z}_t^u\right] = Var(Z_t) - \frac{p_{i1}}{p_{i2}}E\left[Z_t\hat{F}_t^u\right].$$

We then derive:

$$\frac{p_{i1}}{p_{i2}} E\left[Z_t \hat{F}_t^u\right] = Var\left(Z_t\right) - Var\left(\hat{Z}_t^u\right)$$

using the fact that

$$E\left(Z_t \hat{Z}_t^u\right) = E\left[E_t^u\left(Z_t \hat{Z}_t^u\right)\right] = E\left[\hat{Z}_t^u \hat{Z}_t^u\right] = Var\left(\hat{Z}_t^u\right).$$

Finally we obtain:

$$Cov\left(Z_t, Q_t\right) = \left(1 - Ra_Z\right) p_{i2} \left\{ Var\left(Z_t\right) + \frac{p_{u1}}{p_{i1}} \left[Var\left(Z_t\right) - Var\left(\hat{Z}_t^u\right)\right] \right\}$$

Define

$$f_Q = p_{i2} e_{i2} \frac{Var\left(Z_t\right) + \frac{p_{u1}}{p_{i1}} \left[Var\left(Z_t\right) - Var\left(\hat{Z}_t^u\right)\right]}{Var\left(Q_t\right)}.$$

To show this expression is negative, we only need to prove $Var\left(Z_t\right) > Var\left(\hat{Z}_t^u\right)$ because Proposition 1 shows that $p_{i2} < 0$, and $p_{i1}, p_{u1} > 0$. This fact follows from the decomposion of variance. Finally, for $n > 1:$

$$\begin{aligned} E\left[Q_{t+n} | Q_t\right] &= E\left[E_t^u\left[Q_{t+n}\right] | Q_t\right] = e_0 + e_{i2} E\left[E_t^u\left[Z_{t+n-1}\right] | Q_t\right] \\ &= e_0 + a_Z^{n-1} e_{i2} E\left[Z_t | Q_t\right] = e_0 + a_Z^{n-1} e_{i2} \frac{Cov\left(Z_t, Q_t\right)}{Var\left(Q_t\right)} Q_t. \end{aligned}$$

We can then use the previous analysis to obtain the desired result. Q.E.D.

Proof of Proposition 5: Noting that the earnings surprise is in uninformed investors' information set at time t:

$$\begin{aligned} & E\left[Q_{t+1} | D_t - E_{t-1}^u\left(D_t\right)\right] = E\left[E_t^u\left(Q_{t+1}\right) | D_t - E_{t-1}^u\left(D_t\right)\right] = e_0 + e_{i2} E\left[Z_t | D_t - E_{t-1}^u\left(D_t\right)\right] \\ = \; & e_0 + e_{i2} \frac{E\left[Z_t\left(\varepsilon_t^D + \varepsilon_t^F + a_F\left(F_{t-1} - \hat{F}_{t-1}^u\right)\right)\right]}{Var\left(D_t - E_{t-1}^u\left(D_t\right)\right)} \left[D_t - E_{t-1}^u\left(D_t\right)\right] \\ = \; & e_0 + e_{i2} a_Z \frac{a_F E\left[Z_{t-1}\left(F_{t-1} - \hat{F}_{t-1}^u\right)\right]}{Var\left(D_t - E_{t-1}^u\left(D_t\right)\right)} \left[D_t - E_{t-1}^u\left(D_t\right)\right] \\ = \; & e_0 + e_{i2} a_Z \frac{a_F E\left[\left(Z_{t-1} - \hat{Z}_{t-1}^u\right)\left(F_{t-1} - \hat{F}_{t-1}^u\right)\right]}{Var\left(D_t - E_{t-1}^u\left(D_t\right)\right)} \left[D_t - E_{t-1}^u\left(D_t\right)\right], \end{aligned}$$

where the second line follows from equation (22) and the last line follows from

$$E\left[\hat{Z}_{t-1}^u\left(F_{t-1} - \hat{F}_{t-1}^u\right)\right] = E\left[\hat{Z}_{t-1}^u E_{t-1}^u\left(F_{t-1} - \hat{F}_{t-1}^u\right)\right] = 0.$$

Finally, noting that (18) implies that

$$E\left[\left(Z_{t-1}-\hat{Z}^u_{t-1}\right)\left(F_{t-1}-\hat{F}^u_{t-1}\right)\right]>0,$$

we get that

$$d_1=a_F\frac{E\left[\left(Z_{t-1}-\hat{Z}^u_{t-1}\right)\left(F_{t-1}-\hat{F}^u_{t-1}\right)\right]}{Var\left(D_t-E^u_{t-1}\left(D_t\right)\right)}>0.$$

Similarly, we can show that for all $n\geq 2$,

$$\begin{aligned}E\left[Q_{t+n}|D_t-E^u_{t-1}\left(D_t\right)\right] &= E\left[E^u_t\left[Q_{t+n}\right]|D_t-E^u_{t-1}\left(D_t\right)\right]\\ &= e_0+e_{i2}E\left[E^u_t\left[Z_{t+n-1}\right]|D_t-E^u_{t-1}\left(D_t\right)\right]\\ &= e_0+e_{i2}a_Z^{n-1}E\left[Z_t|D_t-E^u_{t-1}\left(D_t\right)\right]\\ &= e_0+a_Z^{n-1}d_1\left[D_t-E^u_{t-1}\left(D_t\right)\right],\end{aligned}$$

as desired. Q.E.D.

B Proofs for the model with a single piece of advance information

As in the benchmark model, we first prove Propositions 7-8 and then prove Proposition 6.

Proof of Proposition 7: We use the following state-space system representation

$$\mathbf{x}_t=\mathbf{A}_x\mathbf{x}_{t-1}+\mathbf{B}_x\boldsymbol{\varepsilon}_t,$$

where we write $\mathbf{A}_x$ and $\mathbf{B}_x$ as:

$$\mathbf{A}_x=\begin{bmatrix} a_F & & & & & \\ & a_Z & & & & \\ & & 0 & & & \\ & & I_k & & & \\ & & & 0 & & \\ & & & & I_{k-1} & 0\end{bmatrix}_{2k+3},\quad \mathbf{B}_x=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\1&0&0&0&0\\ & & \cdots & 0 & \\ 0&0&0&1&0\\ 0& &\cdots & &0\end{bmatrix}_{[2k+3]\times 5}$$

Note that $\mathbf{A}_x$ has two columns with zeros only, column $k+3$ associated with ε^D_t and column $2k+3$ associated with $2k+3$. The informed investors' observable signals are summarized in the vector $y^i_t=(D_t,F_t,Z_t,S_t)^\intercal$. This vector satisfies:

$$\mathbf{y}^i_t=\mathbf{A}_{yi}\mathbf{x}_t+\mathbf{B}_{yi}\boldsymbol{\varepsilon}_t,$$

where we write $\mathbf{A}_{yi}$ as:

$$\mathbf{A}_{yi}=\begin{bmatrix} 1 & 0 & ... & 1_{\text{at}}[k+3] & 0 & ... & 0 \\ 1 & 0 & & & & & \\ 0 & 1 & 0 & & & & \\ 0 & 0 & 1 & 0 & ... & & 0 \end{bmatrix}_{4\times[2k+3]}, \quad \mathbf{B}_{yi}=\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{c}_5 \end{bmatrix}_{4\times 5}.$$

Note that the first two components of $\hat{\mathbf{x}}_t^i$ are given by F_t and Z_t since they are observable. Also, since D_t and F_t are observable, $\hat{\mathbf{x}}_t^i$ contains ε_t^D.

We can now derive the steady-state Kalman filters as in Section 3.1:

$$\hat{\mathbf{x}}_t^i=\mathbf{A}_x\hat{\mathbf{x}}_{t-1}^i+\mathbf{K}_i\hat{\boldsymbol{\varepsilon}}_t^i \tag{B.1}$$

and

$$\mathbf{y}_t^i=\mathbf{A}_{yi}\mathbf{A}_x\hat{\mathbf{x}}_{t-1}^i+\hat{\boldsymbol{\varepsilon}}_t^i, \tag{B.2}$$

where the innovation $\hat{\boldsymbol{\varepsilon}}_t^i=\mathbf{y}_t^i-E_{t-1}^i\left[\mathbf{y}_t^i\right]$ is normally distributed with mean zero and variance

$$\boldsymbol{\Sigma}_i=E\left[\hat{\boldsymbol{\varepsilon}}_t^i\left(\hat{\boldsymbol{\varepsilon}}_t^i\right)^\intercal\right]=\mathbf{A}_{yi}\mathbf{A}_x\boldsymbol{\Omega}_i\mathbf{A}_x^\intercal\mathbf{A}_{yi}^\intercal+\left(\mathbf{A}_{yi}\mathbf{B}_x+\mathbf{B}_{yi}\right)\boldsymbol{\Sigma}\left(\mathbf{A}_{yi}\mathbf{B}_x+\mathbf{B}_{yi}\right)^\intercal. \tag{B.3}$$

The covariance matrix $\boldsymbol{\Omega}_i=E_t^i\left[\left(\mathbf{x}_t-\hat{\mathbf{x}}_t^i\right)\left(\mathbf{x}_t-\hat{\mathbf{x}}_t^i\right)^\intercal\right]$ and the Kalman gain matrix $\mathbf{K}_i$ satisfy:

$$\boldsymbol{\Omega}_i=\left(\mathbf{A}_x\boldsymbol{\Omega}_i\mathbf{A}_x^\intercal+\boldsymbol{\Sigma}_{xx}\right)-\mathbf{K}_i\mathbf{A}_{yi}\left(\mathbf{A}_x\boldsymbol{\Omega}_i\mathbf{A}_x^\intercal+\boldsymbol{\Sigma}_{xx}\right), \tag{B.4}$$

and

$$\mathbf{K}_i=\left(\mathbf{A}_x\boldsymbol{\Omega}_i\mathbf{A}_x^\intercal+\boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_{yi}^\intercal\left[\mathbf{A}_{yi}\left(\mathbf{A}_x\boldsymbol{\Omega}_i\mathbf{A}_x^\intercal+\boldsymbol{\Sigma}_{xx}\right)\mathbf{A}_{yi}^\intercal+\boldsymbol{\Sigma}_{yy}\right]^{-1}, \tag{B.5}$$

where we define

$$\boldsymbol{\Sigma}_{xx}=\mathbf{B}_x\boldsymbol{\Sigma}\mathbf{B}_x^\intercal,\ \boldsymbol{\Sigma}_{yy}=\mathbf{B}_{yi}\boldsymbol{\Sigma}\mathbf{B}_{yi}^\intercal. \tag{B.6}$$

As in Proposition 2, we have $\boldsymbol{\Omega}_i\mathbf{A}_{yi}^\intercal=\mathbf{K}_i\boldsymbol{\Sigma}_{yy}$ and $\boldsymbol{\Omega}_i\left(\mathbf{A}_x\boldsymbol{\Omega}_i\mathbf{A}_x^\intercal+\boldsymbol{\Sigma}_{xx}\right)^{-1}=\mathbf{I}-\mathbf{K}_i\mathbf{A}_{yi}$. We will use these two equations to simplify the informed investors' forecast problem. For expositional convenience, we consider the case with $k=1$. The solution for general $k>1$ is similar. Let $\boldsymbol{\Omega}_i=\left(\omega_{ij}\right)_{5\times 5}$. Using the first equation yields

$$\begin{bmatrix} \omega_{11}+\omega_{14} & \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21}+\omega_{24} & \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31}+\omega_{34} & \omega_{31} & \omega_{32} & \omega_{33} \\ \omega_{41}+\omega_{44} & \omega_{41} & \omega_{42} & \omega_{43} \\ \omega_{51}+\omega_{54} & \omega_{51} & \omega_{52} & \omega_{53} \end{bmatrix}=\sigma_S^2\begin{bmatrix} 0 & 0 & 0 & k_{14} \\ 0 & 0 & 0 & k_{24} \\ 0 & 0 & 0 & k_{34} \\ 0 & 0 & 0 & k_{44} \\ 0 & 0 & 0 & k_{54} \end{bmatrix}.$$

We thus obtain:

$$\begin{aligned}
\omega_{1j} &= \omega_{j1} = 0, \\
\omega_{2j} &= \omega_{j2} = 0, \\
\omega_{4j} &= \omega_{j4} = 0, \\
\sigma_S^2 k_{j4} &= \omega_{j3}.
\end{aligned}$$

That is,

$$\mathbf{\Omega}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_S^2 k_{34} & 0 & \sigma_S^2 k_{54} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_S^2 k_{54} & 0 & \omega_{55} \end{bmatrix}.$$

Using the second equation, we can derive

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{54}\sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2\sigma_D^2} - k_{34}\sigma_S^2 \frac{\sigma_q^2}{\sigma_{qD}^2 - \sigma_q^2\sigma_D^2} & 0 & k_{34}\sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2\sigma_D^2} - k_{54}\sigma_S^2 \frac{\sigma_D^2}{\sigma_{qD}^2 - \sigma_q^2\sigma_D^2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_{55} \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2\sigma_D^2} - k_{54}\sigma_S^2 \frac{\sigma_q^2}{\sigma_{qD}^2 - \sigma_q^2\sigma_D^2} & 0 & -\omega_{55} \frac{\sigma_D^2}{\sigma_{qD}^2 - \sigma_q^2\sigma_D^2} + k_{54}\sigma_S^2 \frac{\sigma_{qD}}{\sigma_{qD}^2 - \sigma_q^2\sigma_D^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1-(k_{11}+k_{12}) & -k_{13} & -k_{14} & -k_{11} & 0 \\ -(k_{21}+k_{22}) & 1-k_{23} & -k_{24} & -k_{21} & 0 \\ -(k_{31}+k_{32}) & -k_{33} & 1-k_{34} & -k_{31} & 0 \\ -(k_{41}+k_{42}) & -k_{43} & -k_{44} & 1-k_{41} & 0 \\ -(k_{51}+k_{52}) & -k_{53} & -k_{54} & -k_{51} & 1 \end{bmatrix}.$$

Equating terms gives

$$\begin{aligned}
k_{34} &= \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2}, k_{54} = \frac{\sigma_{qD}}{\sigma_S^2 + \sigma_D^2}, \\
\omega_{55} &= \sigma_q^2 \left(1 - \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} \rho_{qD}^2\right).
\end{aligned}$$

And generalizing to $k \geq 1$ yields:

$$\mathbf{K}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{\sigma_D^2}{\sigma_S^2 + \sigma_D^2} \\ \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sigma_{qD}}{\sigma_S^2 + \sigma_D^2} \\ \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} & \mathbf{0}_{k-1} \end{bmatrix}.$$

Substituting this equation into equation (B.1) yields the desired result. Q.E.D.

Proof of Proposition 8: It follows from the standard filtering theory (see, e.g., Wang (1994)). So we omit the detailed proof. Q.E.D.

Proof of Proposition 6: We first use the conjectured price function (32) to derive

$$\begin{aligned} Q_{t+1} &= P_{t+1} + D_{t+1} - RP_t \\ &= -p_0 + \mathbf{p}_i \hat{\mathbf{x}}_{t+1}^i + \mathbf{p}_u \mathbf{I}_{-2} \hat{\mathbf{x}}_{t+1}^u + F_{t+1} + \varepsilon_{t+1}^D - R\left(-p_0 + \mathbf{p}_i \hat{\mathbf{x}}_t^i + \mathbf{p}_u \mathbf{I}_{-2} \hat{\mathbf{x}}_t^u\right) \\ &= e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u + \mathbf{b}_Q \hat{\boldsymbol{\varepsilon}}_{t+1}^i, \end{aligned} \tag{B.7}$$

where

$$e_0 = rp_0, \tag{B.8}$$

$$\mathbf{e}_i = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3})\, \mathbf{A}_x - R\mathbf{p}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\right), \tag{B.9}$$

$$\mathbf{e}_u = \mathbf{p}_u \mathbf{I}_{-2} \mathbf{A}_x - R\mathbf{p}_u \mathbf{I}_{-2} - \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\right), \tag{B.10}$$

$$\mathbf{b}_Q = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3})\, \mathbf{K}_i + \mathbf{p}_u \mathbf{I}_{-2} \mathbf{K}_u \mathbf{A}_{yu} \mathbf{K}_i. \tag{B.11}$$

and where we have used

$$\begin{aligned} \hat{\boldsymbol{\varepsilon}}_{t+1}^u &= \mathbf{y}_{t+1}^u - \mathbf{A}_{yu} \mathbf{A}_x \hat{\mathbf{x}}_t^u \\ &= \mathbf{A}_{yu} \hat{\mathbf{x}}_{t+1}^i - \mathbf{A}_{yu} \mathbf{A}_x \hat{\mathbf{x}}_t^u \\ &= \mathbf{A}_{yu} \mathbf{A}_x \left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) + \mathbf{A}_{yu} \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}_{t+1}^i \\ &= \mathbf{A}_{yu} \mathbf{A}_x \left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\right) \left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) + \mathbf{A}_{yu} \mathbf{K}_i \hat{\boldsymbol{\varepsilon}}_{t+1}^i, \end{aligned}$$

noting that ε_{t+1}^D –and not its expectation– is in $\hat{\mathbf{x}}_{t+1}^i$ at time $t+1$. The last equality follows from (33):

$$\hat{Z}_t^i - \hat{Z}_t^u = \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \left(\hat{\mathbf{x}}_t^u - \hat{\mathbf{x}}_t^i\right),$$

and the fact that

$$\begin{aligned} \hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u &= \mathbf{I}_{-2} \left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) + \mathbf{c}_2^\intercal \left(\hat{Z}_t^i - \hat{Z}_t^u\right) \\ &= \mathbf{I}_{-2} \left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) - \mathbf{c}_2^\intercal \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2} \left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right) \\ &= \left(\mathbf{I}_{-2} - \mathbf{c}_2^\intercal \frac{1}{p_{i2}} \mathbf{p}_i \mathbf{I}_{-2}\right) \left(\hat{\mathbf{x}}_t^i - \hat{\mathbf{x}}_t^u\right). \end{aligned}$$

We next derive the conditional expectations:

$$E_t^i \left[Q_{t+1}\right] = e_0 + \mathbf{e}_i \hat{\mathbf{x}}_t^i + \mathbf{e}_u \hat{\mathbf{x}}_t^u, \tag{B.12}$$

and

$$E_t^u\left[Q_{t+1}\right]=e_0+\left(\mathbf{e}_i+\mathbf{e}_u\right)\hat{\mathbf{x}}_t^u, \tag{B.13}$$

We use (33) to substitute out $\hat{Z}_t^u$ in $E_t^u\left[Q_{t+1}\right]$ (because likely $e_{i2}\neq 0$) and derive:

$$\begin{aligned}
E_t^u\left[Q_{t+1}\right] &= e_0+\left(\mathbf{e}_i+\mathbf{e}_u\right)\hat{\mathbf{x}}_t^u \\
&= e_0+\mathbf{e}_i\left[\mathbf{I}_{-2}\hat{\mathbf{x}}_t^u+\mathbf{c}_2^\intercal\hat{Z}_t^u\right]+\mathbf{e}_u\hat{\mathbf{x}}_t^u \\
&= e_0+\mathbf{e}_i\left[\mathbf{I}_{-2}\hat{\mathbf{x}}_t^u+\mathbf{c}_2^\intercal\hat{Z}_t^i-\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2}\left(\hat{\mathbf{x}}_t^u-\hat{\mathbf{x}}_t^i\right)\right]+\mathbf{e}_u\hat{\mathbf{x}}_t^u \\
&= e_0+\mathbf{e}_i\left(\mathbf{c}_2^\intercal\mathbf{c}_2+\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2}\right)\hat{\mathbf{x}}_t^i+\left[\mathbf{e}_i\left(\mathbf{I}_{-2}-\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2}\right)+\mathbf{e}_u\right]\hat{\mathbf{x}}_t^u \\
&= e_0+\tilde{\mathbf{e}}_i\hat{\mathbf{x}}_t^i+\tilde{\mathbf{e}}_u\hat{\mathbf{x}}_t^u.
\end{aligned}$$

It is easy to derive

$$E_t^i\left[q_{t+1}\right]=E_t^i\left[Z_t+\varepsilon_{t+1}^q\right]=\left(\mathbf{c}_2+\mathbf{c}_{2k+3}\right)\hat{\mathbf{x}}_t^i.$$

We can also derive the conditional variances:

$$Var_t^i\left(Q_{t+1}\right)=\mathbf{b}_Q\mathbf{\Sigma}_i\mathbf{b}_Q^\intercal,$$

$$Var_t^i\left(q_{t+1}\right)=\sigma_q^2,$$

$$\begin{aligned}
Cov_t^i\left(Q_{t+1},q_{t+1}\right) &= \mathbf{b}_QE_t^i\left[\hat{\boldsymbol{\varepsilon}}_{t+1}^i\varepsilon_{t+1}^q\right] \\
&= \mathbf{b}_QE_t^i\left[\hat{\boldsymbol{\varepsilon}}_{t+1}^i\mathbf{x}_t^\intercal\right]\mathbf{c}_{2k+3}^\intercal \\
&= \mathbf{b}_Q\mathbf{A}_{yi}\mathbf{A}_x\mathbf{\Omega}_i\mathbf{c}_{2k+3}^\intercal,
\end{aligned}$$

where we have used the following result to derive the last equation,

$$\begin{aligned}
E_t^i\left[\hat{\boldsymbol{\varepsilon}}_{t+1}^i\mathbf{x}_t^\intercal\right] &= E_t^i\left[\mathbf{A}_{yi}\mathbf{A}_x\left(\mathbf{x}_t-\hat{\mathbf{x}}_t^i\right)\mathbf{x}_t^\intercal+\left(\mathbf{A}_{yi}\mathbf{B}_x+\mathbf{B}_{yi}\right)\boldsymbol{\varepsilon}_{t+1}\mathbf{x}_t^\intercal\right] \\
&= E_t^i\left[\mathbf{A}_{yi}\mathbf{A}_x\left(\mathbf{x}_t-\hat{\mathbf{x}}_t^i\right)\mathbf{x}_t^\intercal\right] \\
&= E_t^i\left[\mathbf{A}_{yi}\mathbf{A}_x\left(\mathbf{x}_t-\hat{\mathbf{x}}_t^i\right)\left(\mathbf{x}_t-\hat{\mathbf{x}}_t^i\right)^\intercal\right] \\
&= \mathbf{A}_{yi}\mathbf{A}_x\mathbf{\Omega}_i.
\end{aligned}$$

We can also derive the uninformed investors' conditional variance:

$$\begin{aligned}
Var_t^u\left(Q_{t+1}\right) &= E_t^u\left[\left(\mathbf{e}_i\left(\hat{\mathbf{x}}_t^i-\hat{\mathbf{x}}_t^u\right)+\mathbf{b}_Q\hat{\boldsymbol{\varepsilon}}_{t+1}^i\right)\left(\mathbf{e}_i\left(\hat{\mathbf{x}}_t^i-\hat{\mathbf{x}}_t^u\right)+\mathbf{b}_Q\hat{\boldsymbol{\varepsilon}}_{t+1}^i\right)^\intercal\right] \\
&= \mathbf{e}_i\mathbf{\Omega}_u\mathbf{e}_i^\intercal+\mathbf{b}_Q\mathbf{\Sigma}_i\mathbf{b}_Q^\intercal.
\end{aligned}$$

where we have used the following result:

$$E_t^u\left[\left(\hat{\mathbf{x}}_t^i-\hat{\mathbf{x}}_t^u\right)\hat{\varepsilon}_{t+1}^{i\intercal}\right]=E_t^u\left[E_t^i\left[\left(\hat{\mathbf{x}}_t^i-\hat{\mathbf{x}}_t^u\right)\hat{\varepsilon}_{t+1}^{i\intercal}\right]\right]=E_t^u\left[\left(\hat{\mathbf{x}}_t^i-\hat{\mathbf{x}}_t^u\right)E_t^i\left[\hat{\varepsilon}_{t+1}^{i\intercal}\right]\right]=0.$$

Now we use (25) and (26) to show that optimal stock holdings are linear functions of $\hat{\mathbf{x}}_t^i$ and $\hat{\mathbf{x}}_t^u$ by substituting the preceding conditional expectations:

$$\theta_t^i=\frac{e_0+\mathbf{e}_i\hat{\mathbf{x}}_t^i+\mathbf{e}_u\hat{\mathbf{x}}_t^u}{\gamma\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}-\frac{\rho_{Qq}^i\left(\mathbf{c}_2+\mathbf{c}_{2k+3}\right)\hat{\mathbf{x}}_t^i}{\gamma\sigma_Q^i\sigma_q^i\left(1-\left(\rho_{Qq}^i\right)^2\right)},\tag{B.14}$$

and

$$\theta_t^u=\frac{e_0+\tilde{\mathbf{e}}_i\hat{\mathbf{x}}_t^i+\tilde{\mathbf{e}}_u\hat{\mathbf{x}}_t^u}{\gamma\left(\sigma_Q^u\right)^2}.\tag{B.15}$$

We use the market clearing condition (15) to determine the coefficients in the price function. Substituting (B.14) and (B.15) into (15) yields:

$$1=\frac{\lambda}{\gamma}\left[\frac{e_0+\mathbf{e}_i\hat{\mathbf{x}}_t^i+\mathbf{e}_u\hat{\mathbf{x}}_t^u}{\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}-\frac{\rho_{Qq}\left(\mathbf{c}_2+\mathbf{c}_{2k+3}\right)\hat{\mathbf{x}}_t^i}{\sigma_Q^i\sigma_q^i\left(1-\rho_{Qq}^2\right)}\right]+(1-\lambda)\frac{e_0+\tilde{\mathbf{e}}_i\hat{\mathbf{x}}_t^i+\tilde{\mathbf{e}}_u\hat{\mathbf{x}}_t^u}{\gamma\left(\sigma_Q^u\right)^2}$$

Matching coefficients on constant, $\hat{\mathbf{x}}_t^i$ and $\hat{\mathbf{x}}_t^u$, we obtain

$$e_0=\gamma\frac{\left(\sigma_Q^i\right)^2\left(\sigma_Q^u\right)^2\left(1-\rho_{Qq}^2\right)}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}>0,\tag{B.16}$$

$$\frac{\lambda}{\gamma}\left[\frac{\mathbf{e}_i}{\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}-\frac{\rho_{Qq}^i\left(\mathbf{c}_2+\mathbf{c}_{2k+3}\right)}{\sigma_Q^i\sigma_q^i\left(1-\left(\rho_{Qq}^i\right)^2\right)}\right]+\frac{1-\lambda}{\gamma}\frac{\mathbf{e}_i\left(\mathbf{c}_2^\intercal\mathbf{c}_2+\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2}\right)}{\left(\sigma_Q^u\right)^2}=0,\tag{B.17}$$

$$0=\frac{\lambda}{\gamma}\frac{\mathbf{e}_u}{\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}+(1-\lambda)\frac{\mathbf{e}_i\left(\mathbf{I}_{-2}-\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2}\right)+\mathbf{e}_u}{\gamma\left(\sigma_Q^u\right)^2}.\tag{B.18}$$

Equations (B.8) and (B.16) imply that $p_0>0$. Using

$$\mathbf{e}_i\left(\mathbf{c}_2^\intercal\mathbf{c}_2+\frac{1}{p_{i2}}\mathbf{c}_2^\intercal\mathbf{p}_i\mathbf{I}_{-2}\right)=e_{i2}\left[\begin{array}{ccccc}\frac{p_{i1}}{p_{i2}} & 1 & \frac{p_{i3}}{p_{i2}} & \ldots & \frac{p_{i,2k+3}}{p_{i2}}\end{array}\right]$$

together with (B.17), we have

$$
\begin{aligned}
\frac{\lambda}{\gamma}\frac{e_{i1}}{\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}+\frac{1-\lambda}{\gamma}\frac{e_{i2}\frac{p_{i1}}{p_{i2}}}{\left(\sigma_Q^u\right)^2} &= 0,\\
\frac{\lambda}{\gamma}\left[\frac{e_{i2}}{\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}-\frac{\rho_{Qq}^i}{\sigma_Q^i\sigma_q^i\left(1-\left(\rho_{Qq}^i\right)^2\right)}\right]+\frac{1-\lambda}{\gamma}\frac{e_{i2}}{\left(\sigma_Q^u\right)^2} &= 0,\\
\frac{\lambda}{\gamma}\frac{e_{i3}}{\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}+\frac{1-\lambda}{\gamma}\frac{e_{i2}\frac{p_{i3}}{p_{i2}}}{\left(\sigma_Q^u\right)^2} &= 0,\\
&\ldots\\
\frac{\lambda}{\gamma}\left[\frac{e_{i,2k+3}}{\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}-\frac{\rho_{Qq}^i}{\sigma_Q^i\sigma_q^i\left(1-\left(\rho_{Qq}^i\right)^2\right)}\right]+\frac{1-\lambda}{\gamma}\frac{e_{i2}\frac{p_{i,2k+3}}{p_{i2}}}{\left(\sigma_Q^u\right)^2} &= 0.
\end{aligned}
$$

Solving these equation gives

$$
\begin{aligned}
e_{i2} &= \frac{\lambda\left(\sigma_Q^u\right)^2}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}\frac{Cov_t^i\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma_q^i\right)^2}, \qquad \text{(B.19)}\\
e_{i1} &= -\frac{p_{i1}}{p_{i2}}\frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}\frac{Cov_t^i\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma_q^i\right)^2},\\
e_{i3} &= \frac{p_{i3}}{p_{i1}}e_{i1},\\
&\ldots\\
e_{i,2k+2} &= \frac{p_{i,2k+2}}{p_{i1}}e_{i1},\\
e_{i,2k+3} &= \left[1-\frac{p_{i,2k+3}}{p_{i2}}\frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}{\lambda\left(\sigma_Q^u\right)^2+(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\left(\rho_{Qq}^i\right)^2\right)}\right]\frac{Cov_t^i\left(Q_{t+1},q_{t+1}\right)}{\left(\sigma_q^i\right)^2}.
\end{aligned}
$$

Turning now to (B.18), we use

$$
\mathbf{e}_i\left(\mathbf{I}_{-2}-\frac{1}{p_{i2}}\mathbf{c}_2^{\intercal}\mathbf{p}_i\mathbf{I}_{-2}\right)=\left[\begin{array}{ccccc} e_{i1}-e_{i2}\frac{p_{i1}}{p_{i2}} & 0 & e_{i3}-e_{i2}\frac{p_{i3}}{p_{i2}} & \ldots & e_{i,2k+3}-e_{i2}\frac{p_{i,2k+3}}{p_{i2}}\end{array}\right]
$$

to derive:

$$
\begin{aligned}
0 &= \frac{\lambda}{\gamma} \frac{e_{u1}}{\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} + (1-\lambda) \frac{e_{i1} - e_{i2}\frac{p_{i1}}{p_{i2}} + e_{u1}}{\gamma \left(\sigma_Q^u\right)^2}, \\
0 &= \frac{\lambda}{\gamma} \frac{e_{u2}}{\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} + (1-\lambda) \frac{e_{u2}}{\gamma \left(\sigma_Q^u\right)^2}, \\
0 &= \frac{\lambda}{\gamma} \frac{e_{u3}}{\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} + (1-\lambda) \frac{e_{i3} - e_{i2}\frac{p_{i3}}{p_{i2}} + e_{u3}}{\gamma \left(\sigma_Q^u\right)^2}, \\
& \ldots \\
0 &= \frac{\lambda}{\gamma} \frac{e_{u,2k+3}}{\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} + (1-\lambda) \frac{e_{i,2k+3} - e_{i2}\frac{p_{i,2k+3}}{p_{i2}} + e_{u,2k+3}}{\gamma \left(\sigma_Q^u\right)^2}.
\end{aligned}
$$

Solving these equations yields:

$$
\begin{aligned}
e_{u1} &= \frac{p_{i1}}{p_{i2}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2}, \\
e_{u2} &= 0, \\
e_{u3} &= \frac{p_{i3}}{p_{i2}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2}, \\
& \ldots \\
e_{u,2k+3} &= \frac{p_{i,2k+3} - p_{i2}}{p_{i2}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda \left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2}
\end{aligned} \tag{B.20}
$$

To solve an equilibrium, we need to determine $1+(2k+3)+(2k+2)$ price coefficients $p_0, \mathbf{p}_i$, and $\mathbf{p}_u$ (note that $p_{u2} = 0$). Equations (B.8) and (B.16) determine p_0. Equating equations (B.9) with equations in (B.19), we obtain $(2k+3)$ equations. Equating equations (B.10) with equations in (B.20), we obtain $(2k+3)$ equations. Note that the second equation $e_{u2} = 0$ is an redundant identity. Therefore, we essentially have $(2k+2)$ equations. In summary, we obtain $(2k+3)+(2k+2)$ equations to solve for $(2k+3)+(2k+2)$ unknowns of $\mathbf{p}_i$, and $\mathbf{p}_u$. When solving these equations, we need to substitute in the preceding variances and covariances. If there is a solution, then we obtain a stationary equilibrium.

Now we derive restrictions on the coefficients in the price function. Adding equations in (B.19) and (B.20) yields:

$$e_{i,j} + e_{u,j} = 0, \text{ for all } j \neq 2, 2k+3, \tag{B.21}$$

$$e_{i2} + e_{u2} = e_{i2}, \tag{B.22}$$

and

$$e_{i,2k+3} + e_{u,2k+3} = e_{i2}. \tag{B.23}$$

Adding equations (B.9)-(B.10), we get

$$\mathbf{e}_i + \mathbf{e}_u = (\mathbf{p}_i + \mathbf{c}_1 + \mathbf{c}_{k+3})\,\mathbf{A}_x - R\mathbf{p}_i + \mathbf{p}_u\mathbf{I}_{-2}\mathbf{A}_x - R\mathbf{p}_u\mathbf{I}_{-2}. \tag{B.24}$$

Simplifying yields

$$\begin{aligned}
p_{i1} + p_{u1} &= a_F / (R - a_F)\,, && \text{(B.25)}\\
p_{i2} &= \frac{-e_{i2}}{R - a_Z}, && \text{(B.26)}\\
p_{ij} + p_{uj} &= \frac{1}{R^{3+k-j}},\ \text{for } 3 \leq j \leq k+2, && \text{(B.27)}\\
p_{ij} + p_{uj} &= -\frac{e_{i2}}{R^{2k+3-(j-1)}},\ \text{for } j \geq k+4, && \text{(B.28)}
\end{aligned}$$

and

$$p_{i,k+3} + p_{u,k+3} = 0. \tag{B.29}$$

We next show that $p_{u,k+3} = p_{i,k+3} = 0$. Note that from equation (B.10) we get

$$e_{u,k+3} = -Rp_{u,k+3} + \mathbf{p}_u\mathbf{K}_{u,.2}a_Z p_{i,k+3},$$

where $\mathbf{K}_{u,.2}$ is the second column of matrix $\mathbf{K}_u$. Substituting for the equation for $e_{u,k+3}$ in (B.19) and $p_{i,k+3} + p_{u,k+3} = 0$ yields:

$$\frac{p_{i,k+3}}{p_{i2}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}{\lambda\left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2} = \left(R + \mathbf{p}_u\mathbf{K}_{u,.2}a_Z\right)p_{i,k+3}.$$

Suppose $p_{i,k+3} \neq 0$. Then we write the above expression as

$$\frac{1}{p_{i2}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}{\lambda\left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2} = R + \mathbf{p}_u\mathbf{K}_{u,.2}a_Z. \tag{B.30}$$

Now use the last equation (for $2k+3$) in (B.10) to derive

$$e_{u,2k+3} = -Rp_{u,2k+3} + \mathbf{p}_u\mathbf{K}_{u,.2}a_Z p_{i,2k+3}.$$

We substitute the equation for $e_{u,2k+3}$ in (B.20) and $p_{i,2k+3} + p_{u,2k+3} = -\frac{e_{i2}}{R}$ to derive

$$\frac{p_{i,2k+3} - p_{i2}}{p_{i2}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)}{\lambda\left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)} \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2} - e_{i2} = p_{i,2k+3}\left(R + \mathbf{p}_u\mathbf{K}_{u,.2}a_Z\right).$$

Substituting the equation for e_{i2} in (B.19) yields:

$$\left(\frac{p_{i,2k+3}}{p_{i2}} \frac{(1-\lambda)\left(\sigma_Q^i\right)^2 \left(1-\rho_{Qq}^2\right)}{\lambda\left(\sigma_Q^u\right)^2 + (1-\lambda)\left(\sigma_Q^i\right)^2\left(1-\rho_{Qq}^2\right)} - 1 \right) \frac{Cov_t^i\left(Q_{t+1}, q_{t+1}\right)}{\left(\sigma_q^i\right)^2} = p_{i,2k+3}\left(R + \mathbf{p}_u \mathbf{K}_{u,.2} a_Z\right).$$

Replacing the value of $R + \mathbf{p}_u \mathbf{K}_{u,.2} a_Z$ from (B.30), we get an impossibility. Therefore $p_{i,k+3} = 0 = p_{u,k+3}$. Using (B.20) again we obtain $e_{u,k+3} = e_{i,k+3} = 0$. Q.E.D.

Proof of Proposition 9: As in the proof of Proposition 3, we focus on conditional expectations of future returns in (25) and (26). Using equations (B.12) and (B.21), we can derive the expression in (45). Using equations (B.13) and (B.21)-(B.23), we can derive

$$E_t^u\left[Q_{t+1}\right] = e_0 + e_{i2} E_t^u\left[Z_t + \varepsilon_{t+1}^q\right], \tag{B.31}$$

and (46). Q.E.D.

Proof of Proposition 10: We proceed as in Proposition 4. Using (B.31) and the law of iterated expectations, we compute

$$\begin{aligned} E\left[Q_{t+1}|Q_t\right] &= E\left[E_t^u\left[Q_{t+1}\right]|Q_t\right] = E\left[E_t^i\left[Q_{t+1}\right]|Q_t\right] \\ &= e_0 + e_{i2} E\left[E_t^i\left(Z_t + \varepsilon_{t+1}^q\right)|Q_t\right] \\ &= e_0 + e_{i2}\frac{Cov\left(Z_t, Q_t\right) + Cov\left(E_t^i\left[\varepsilon_{t+1}^q\right], Q_t\right)}{Var\left(Q_t\right)} Q_t. \end{aligned}$$

We also have for general $n \geq 1$:

$$\begin{aligned} E\left[Q_{t+n}|Q_t\right] &= e_0 + e_{i2} E\left[Z_{t+n-1} + \varepsilon_{t+n}^q|Q_t\right] \\ &= e_0 + e_{i2}\frac{\left(\mathbf{c}_2 + \mathbf{c}_{2k+3}\right)\mathbf{A}_x^{n-1} Cov\left(\hat{\mathbf{x}}_t^i, Q_t\right)}{Var\left(Q_t\right)} Q_t \\ &= e_0 + e_{i2}\frac{a_Z^{n-1} Cov\left(Z_t, Q_t\right) + \mathbf{1}_{\{n\leq k\}} Cov\left(E_t^i\left[\varepsilon_{t+n}^q\right], Q_t\right)}{Var\left(Q_t\right)} Q_t \end{aligned}$$

noting that $\left(\mathbf{c}_2 + \mathbf{c}_{2k+3}\right)\mathbf{A}_x^{n-1} = a_Z^{n-1}\mathbf{c}_2 + \mathbf{1}_{\{n\leq k\}}\mathbf{c}_{2k+3-(n-1)}$, where $\mathbf{1}_{\{n\leq k\}}$ is an indicator function equal to 1 if $n \leq k$ and 0 otherwise. Q.E.D.

Proof of Proposition 11: We prove the case with $n = 1$ and $k = 2$. The proof for general n and k is similar. By the law of iterated expectations, we have:

$$\begin{aligned}
& E\left[Q_{t+1}|D_t - E_{t-1}^u(D_t)\right] \\
= \; & E\left[E_t^u\left[Q_{t+1}\right]|D_t - E_{t-1}^u(D_t)\right] = e_0 + e_{i2}E\left[E_t^u\left[Z_t + \varepsilon_{t+1}^q\right]|D_t - E_{t-1}^u(D_t)\right] \\
= \; & e_0 + e_{i2}E\left[E_t^i\left[Z_t + \varepsilon_{t+1}^q\right]|D_t - E_{t-1}^u(D_t)\right] \\
= \; & e_0 + e_{i2}E\left[Z_t|D_t - E_{t-1}^u(D_t)\right] + e_{i2}E\left[E_t^i\left[\varepsilon_{t+1}^q\right]|D_t - E_{t-1}^u(D_t)\right] \\
= \; & e_0 + e_{i2}E\left[Z_t|D_t - E_{t-1}^u(D_t)\right] + e_{i2}\frac{\sigma_{Dq}}{\sigma_D^2}E\left[E_t^i\left[\varepsilon_{t+1}^D\right]|D_t - E_{t-1}^u(D_t)\right] \\
= \; & e_0 + e_{i2}E\left[Z_t|D_t - E_{t-1}^u(D_t)\right] + e_{i2}\frac{\sigma_{Dq}}{\sigma_D^2}E\left[E_t^i\left[\varepsilon_{t+1}^D\right]|D_t - E_{t-1}^u(D_t)\right] \\
= \; & e_0 + e_{i2}E\left[Z_t|D_t - E_{t-1}^u(D_t)\right] + e_{i2}d_1\frac{Cov\left(E_t^i\left[\varepsilon_{t+1}^D\right], D_t - E_{t-1}^u(D_t)\right)}{Var\left(D_t - E_{t-1}^u(D_t)\right)}\left[D_t - E_{t-1}^u(D_t)\right]
\end{aligned}$$

where we have used Porposition 7 and defined $d_1 = \sigma_{Dq}/\sigma_D^2 > 0$. Q.E.D.

C Solution for the model with multiple pieces of advance information

To solve the model with multiple pieces of advance information, we define the state vector in equation (31), as in Section 4. Unlike Section 4, we define the unforecastable error term based on the period $t-1$ information as

$$\boldsymbol{\varepsilon}_t = \left(\varepsilon_{t+k}^D, \varepsilon_t^F, \varepsilon_t^Z, \varepsilon_{t+k}^q, \varepsilon_t^{S_k}, ..., \varepsilon_t^{S_1}\right)^{\intercal}.$$

It is normally distributed with mean zero and covariance matrix $\Sigma = E\left[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t^{\intercal}\right]$. The only positive covariance that shows up is between $E\left(\varepsilon_{t+k}^D\varepsilon_{t+k}^q\right) = \sigma_{Dq} > 0$. We can apply the same method in Section 4 to solve the model. The only modification lies in the inference problem of informed investors. This problem results in conditional expectations given by

$$\begin{aligned}
E_t^i\left[\varepsilon_{t+k}^D\right] &= \rho_k S_t^k, \\
E_t^i\left[\varepsilon_{t+j}^D\right] &= E_{t-1}^i\left[\varepsilon_{t+j}^D\right] + \rho_j S_t^j, \; 1 \leq j \leq k-1,
\end{aligned}$$

and

$$\begin{aligned}
E_t^i\left[\varepsilon_{t+k}^q\right] &= \frac{\sigma_{Dq}}{\sigma_D^2}\rho_k S_t^k, \\
E_t^i\left[\varepsilon_{t+j}^q\right] &= E_{t-1}^i\left[\varepsilon_{t+j}^q\right] + \frac{\sigma_{Dq}}{\sigma_D^2}\rho_j S_t^j, \; 1 \leq j \leq k-1,
\end{aligned}$$

where

$$\rho_k = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_{S_k}^2},\ \rho_{k-1} = \frac{(1-\rho_k)\,\sigma_D^2}{(1-\rho_k)\,\sigma_D^2 + \sigma_{S_{k-1}}^2}, ..., \rho_1 = \frac{(1-\rho_k)\,...\,(1-\rho_2)\,\sigma_D^2}{(1-\rho_k)\,...\,(1-\rho_2)\,\sigma_D^2 + \sigma_{S_1}^2}.$$

Because the signals $S_{t+j-k}^k, ..., S_{t+j-1}^1$ arrive sequentially and are correlated, each of them contributes to lowering the conditional variance of earnings innovations ε_{t+j}^D for $0 < j < k$, but incrementally, the precision of each new signal changes with $\sigma_{S_j}^2$. Using these expressions we can construct a new Kalman gain matrix $\mathbf{K}_i$. Additional details are available upon request. Q.E.D.

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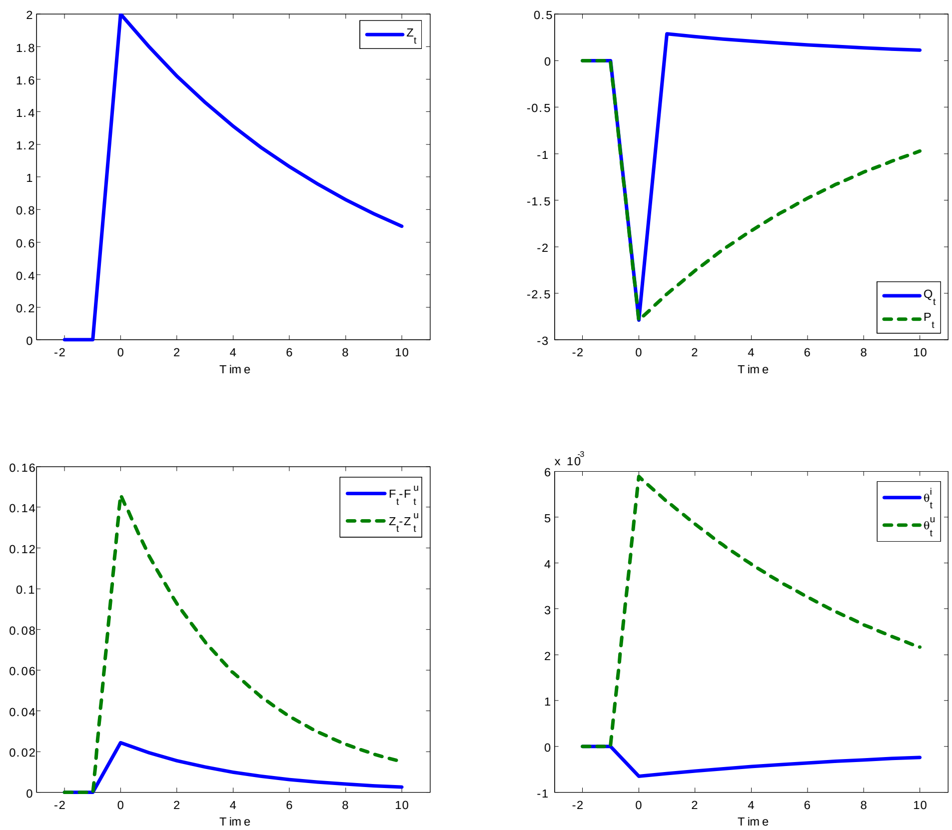

Figure 1: **Impulse response function for a shock to Z_t in the benchmark model without advance information**. We subtract means from all variables in the figure. Shock equals one standard deviation of ε_t^Z and occurs at $t = 0$. Parameters are: $\sigma_D = .5, \sigma_{Dq} = 1.35$, $\sigma_F = .1, \sigma_Z = 2, \sigma_q = 3$, $a_F = a_Z = .9$, $\gamma = 5$, $\lambda = .9$, and $r = .0025$.

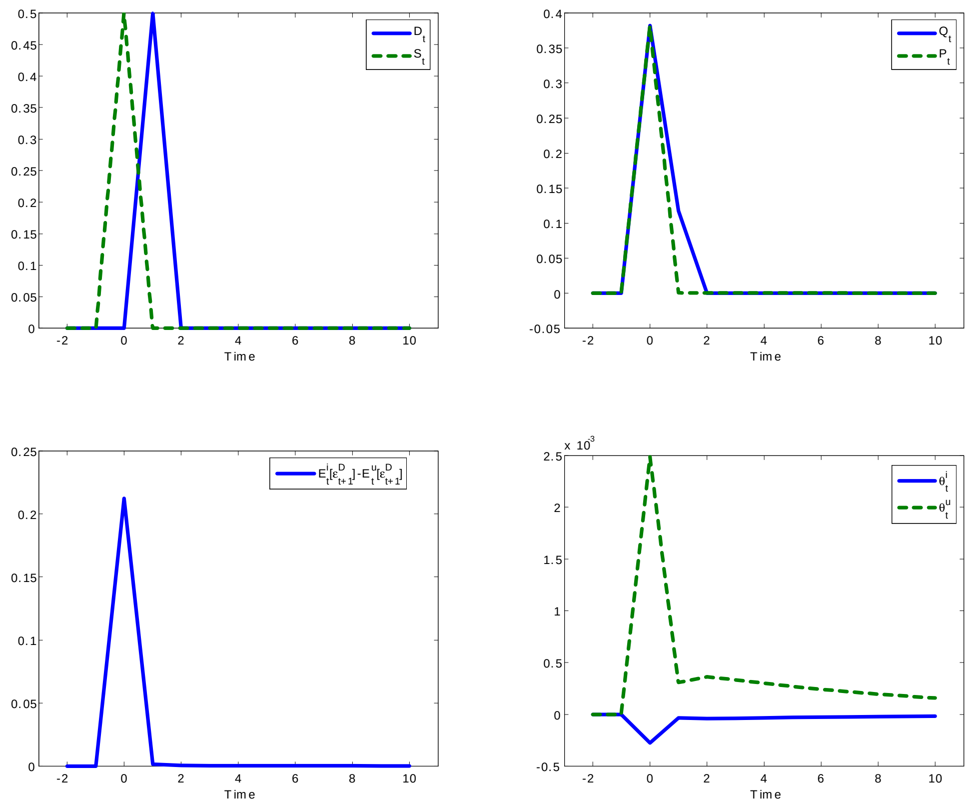

Figure 2: **Impulse response functions for a shock to ε_1^D that is first observed through a signal at S_0.** Shock to ε_1^D equals one standard deviation of ε_t^D. Parameters are: $k = 1$, $\sigma_D = .5, \sigma_F = .01, \sigma_Z = 2, \sigma_q = 3, \sigma_S = .2$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = .9$, $\gamma = 5$, $\lambda = .9$, and $r = .0025$.

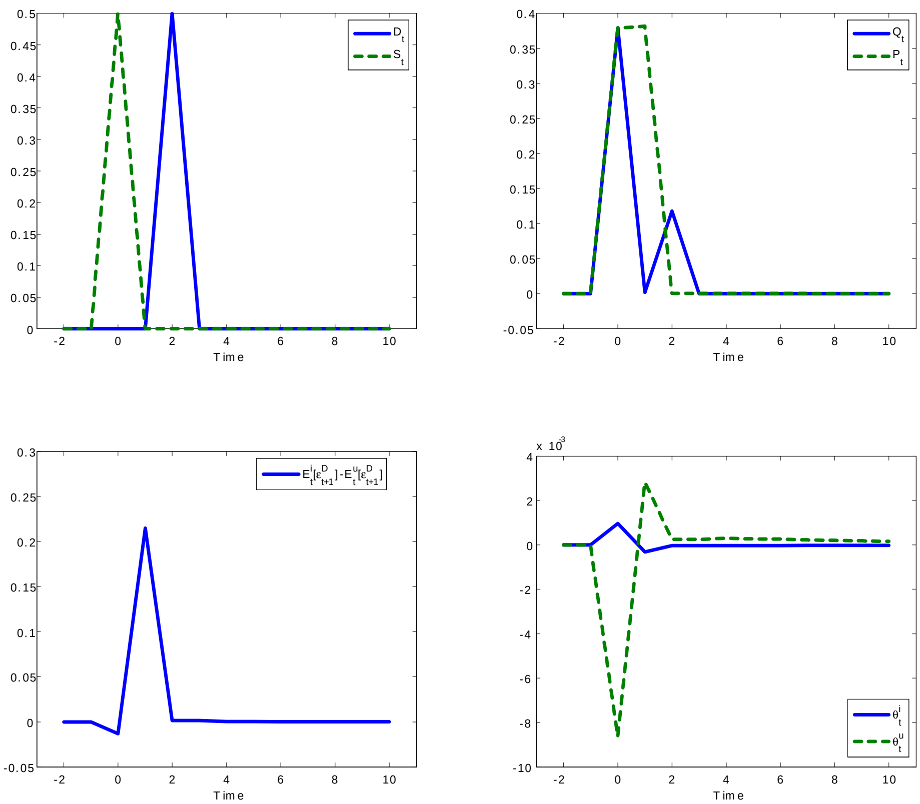

Figure 3: **Impulse response functions for a shock to ε_2^D that is first observed through a signal at S_0.** Shock to ε_2^D equals one standard deviation of ε_t^D. Parameters are: $k = 2$, $\sigma_D = .5, \sigma_F = .01, \sigma_Z = 2, \sigma_q = 3, \sigma_S = .2$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = .9$, $\gamma = 5$, $\lambda = .9$, and $r = .0025$.

Table 1.
MOMENTUM AND REVERSAL
IN THE MODEL WITH A SINGLE PIECE OF ADVANCE INFORMATION

$n \setminus k$	1		2		3	
	One period, b_n	Cumulative	One period, b_n	Cumulative	One period, b_n	Cumulative
1	0.0337	0.0337	-0.0197	-0.0197	-0.0198	-0.0198
2	-0.0178	0.0160	0.0353	0.0156	-0.0178	-0.0377
3	-0.0160	-0.0000	-0.0161	-0.0005	0.0366	-0.0010
4	-0.0144	-0.0144	-0.0145	-0.0150	-0.0146	-0.0157
5	-0.0130	-0.0274	-0.0131	-0.0281	-0.0132	-0.0288
6	-0.0117	-0.0391	-0.0118	-0.0399	-0.0118	-0.0407
7	-0.0105	-0.0495	-0.0106	-0.0504	-0.0107	-0.0513
8	-0.0094	-0.0590	-0.0095	-0.0600	-0.0096	-0.0609
9	-0.0085	-0.0675	-0.0086	-0.0685	-0.0086	-0.0696
10	-0.0077	-0.0752	-0.0077	-0.0762	-0.0078	-0.0773

The columns labeled "One period" display the slope coefficients of regressing single period returns, Q_{t+n}, on current returns:

$$Q_{t+n} = a_n + b_n Q_t + \varepsilon_{t,n}.$$

Likewise, the columns labeled "Cumulative" display the slope coefficients of regressing cumulative excess returns, $Q_{t,t+n}$, on current returns, Q_t. n is the holding period and k is the number of advance periods in the advance information. We set $\sigma_D = 0.5$, $\sigma_F = 0.01$, $\sigma_Z = 2$, $\sigma_q = 3$, $\sigma_S = 0.2$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.0025$.

Table 2.

Price earnings announcement drift
in the model with a single piece of advance information

$n \setminus k$	1		2		3	
	One period, d_n	Cumulative	One period, d_n	Cumulative	One period, d_n	Cumulative
1	0.0593	0.0593	0.0505	0.0505	0.0469	0.0469
2	0.0533	0.1126	0.0501	0.1006	0.0421	0.0891
3	0.0480	0.1606	0.0451	0.1457	0.0423	0.1314
4	0.0432	0.2038	0.0406	0.1863	0.0380	0.1694
5	0.0389	0.2427	0.0365	0.2228	0.0342	0.2036
6	0.0350	0.2776	0.0329	0.2557	0.0308	0.2345
7	0.0315	0.3091	0.0296	0.2852	0.0277	0.2622
8	0.0283	0.3375	0.0266	0.3119	0.0250	0.2872
9	0.0255	0.3630	0.0240	0.3358	0.0225	0.3096
10	0.0230	0.3859	0.0216	0.3574	0.0202	0.3298

The columns labeled "One period" display the slope coefficients of regressing single-period returns, Q_{t+n}, on the earnings announcement surprise, $D_t - E^u_{t-1}[D_t]$:

$$Q_{t+n} = a_n + d_n \left[D_t - E^u_{t-1}(D_t)\right] + \varepsilon_{t,n}.$$

Likewise, the columns labeled "Cumulative" display the slope coefficients of regressing cumulative excess returns, $Q_{t,t+n}$, on the earnings surprise, $D_t - E^u_{t-1}[D_t]$. n is the holding period and k is the number of advance periods in the advance information. We set $\sigma_D = 0.5$, $\sigma_F = 0.01$, $\sigma_Z = 2$, $\sigma_q = 3$, $\sigma_S = 0.2$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.0025$.

Table 3.

MOMENTUM AND REVERSAL

IN THE MODEL WITH MULTIPLE PIECES OF ADVANCE INFORMATION

$n \setminus \sigma_{S_1}$	0.1		0.15		0.2		0.30	
	One Period	Cumu-lative	One Period	Cumu-lative	One Period	Cumu-lative	One Period	Cumu-lative
1	0.0180	0.0180	0.0267	0.0267	0.0222	0.0222	-0.0046	-0.0046
2	0.0033	0.0213	0.0018	0.0285	-0.0048	0.0173	-0.0204	-0.0250
3	-0.0018	0.0195	-0.0071	0.0213	-0.0153	0.0020	-0.0281	-0.0530
4	-0.0016	0.0179	-0.0064	0.0149	-0.0138	-0.0118	-0.0252	-0.0783
5	-0.0014	0.0164	-0.0058	0.0091	-0.0124	-0.0242	-0.0227	-0.1010
6	-0.0013	0.0151	-0.0052	0.0039	-0.0112	-0.0354	-0.0204	-0.1214
7	-0.0012	0.0140	-0.0047	-0.0008	-0.0101	-0.0454	-0.0184	-0.1398
8	-0.0011	0.0129	-0.0042	-0.0050	-0.0091	-0.0545	-0.0166	-0.1564
9	-0.0009	0.0120	-0.0038	-0.0088	-0.0081	-0.0626	-0.0149	-0.1713
10	-0.0009	0.0111	-0.0034	-0.0122	-0.0073	-0.0700	-0.0134	-0.1847

The columns labeled "One period" display the slope coefficients of regressing single period returns, Q_{t+n}, on current returns:

$$Q_{t+n} = a_n + b_n Q_t + \varepsilon_{t,n}.$$

Likewise, the columns labeled "Cumulative" display the slope coefficients of regressing cumulative excess returns, $Q_{t,t+n}$, on current returns, Q_t. n is the holding period and σ_{S_1} is the variance of advance information about one-period ahead earnings. We set $k = 2$, $\sigma_D = 0.5$, $\sigma_F = 0.01$, $\sigma_Z = 2$, $\sigma_q = 3$, $\sigma_{S2} = 1$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.0025$.

Table 4.

PRICE EARNINGS ANNOUNCEMENT DRIFT

IN THE MODEL WITH MULTIPLE PIECES OF ADVANCE INFORMATION

$n \setminus \sigma_{S_1}$	0.1		0.15		0.2		0.30	
	One period	Cumu-lative	One period	Cumu-lative	One period	Cumu-lative	One period	Cumu-lative
1	0.0100	0.0100	0.0289	0.0289	0.0407	0.0407	0.0443	0.0443
2	0.0209	0.0309	0.0396	0.0685	0.0478	0.0885	0.0455	0.0898
3	0.0188	0.0497	0.0357	0.1042	0.0430	0.1315	0.0409	0.1307
4	0.0169	0.0666	0.0321	0.1363	0.0387	0.1701	0.0368	0.1676
5	0.0152	0.0819	0.0289	0.1652	0.0348	0.2050	0.0332	0.2007
6	0.0137	0.0956	0.0260	0.1912	0.0313	0.2363	0.0298	0.2306
7	0.0123	0.1079	0.0234	0.2146	0.0282	0.2645	0.0269	0.2574
8	0.0111	0.1191	0.0211	0.2356	0.0254	0.2899	0.0242	0.2816
9	0.0100	0.1291	0.0190	0.2546	0.0228	0.3128	0.0218	0.3034
10	0.0090	0.1381	0.0171	0.2717	0.0206	0.3333	0.0196	0.3229

The columns labeled "One period" display the slope coefficients of regressing single-period returns, Q_{t+n}, on the earnings announcement surprise, $D_t - E^u_{t-1}[D_t]$:

$$Q_{t+n} = a_n + d_n \left[D_t - E^u_{t-1}(D_t)\right] + \varepsilon_{t,n}.$$

Likewise, the columns labeled "Cumulative" display the slope coefficients of regressing cumulative excess returns, $Q_{t,t+n}$, on the earnings surprise, $D_t - E^u_{t-1}[D_t]$. n is the holding period and σ_{S_1} is the variance of advance information about one-period ahead earnings. We set $k = 2$, $\sigma_D = 0.5$, $\sigma_F = 0.01$, $\sigma_Z = 2$, $\sigma_q = 3$, $\sigma_{S_2} = 1$, $\sigma_{Dq} = 1.35$, $a_F = a_Z = 0.9$, $\lambda = 0.9$, $\gamma = 5$, and $r = 0.0025$.