A new method to estimate risk and return of non-traded assets from aggregate cash flows: The case of private equity funds*

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Abstract

We develop a new GMM-based methodology with good small sample properties to assess the abnormal performance and risk exposure of a non-traded asset from a cross-section of cash flow data. We apply this method to a sample of 958 mature private equity funds spanning 24 years. In contrast to existing work, our methodology mainly uses actual cash flow data and not intermediary self-reported net asset values. We find a beta for venture capital funds above 3 and a beta for buyout funds below 1. Venture capital funds have significantly negative abnormal performance while the abnormal performance of buyout funds is close to zero. Larger funds have higher returns due to higher risk exposures and not higher alphas. We also show that NAVs overstate fund market values for the subset of mature and inactive funds.

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We develop a new GMM-based methodology with good small sample properties to assess the abnormal performance and risk exposure of a non-traded asset from a cross-section of cash flow data. We apply this method to a sample of 958 mature private equity funds spanning 24 years. In contrast to existing work, our methodology mainly uses actual cash flow data and not intermediary self-reported net asset values. We find a beta for venture capital funds above 3 and a beta for buyout funds below 1. Venture capital funds have significantly negative abnormal performance while the abnormal performance of buyout funds is close to zero. Larger funds have higher returns due to higher risk exposures and not higher alphas. We also show that NAVs overstate fund market values for the subset of mature and inactive funds.

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1 Introduction

The estimation of risk (beta) and abnormal performance (alpha) is at the heart of financial economics. Since Jensen's (1968) time-series regression approach to determine the alpha and beta of a mutual fund, a large literature has been dedicated to refining measures of risk and return (see Cochrane, 2005a, for overview). A commonly observed situation that has not been investigated is that of a non-traded asset (or fund) for which we only observe the realized cash flows. For example, consider the problem of private equity fund investors. During the 10 years of the funds' life, investors give away cash at different points in time (fees and investments) and receive dividends at other points in time. In this paper, we propose a methodology to measure risk and abnormal return in such a context and use it on a sample of private equity funds.[1]

We show that observing a cross-section of asset cash flows can inform the econometrician about risk and return. The intuition is simple. When the partnership is terminated, we know that the market value at that date is zero. Hence, we have one equation which can be seen as a moment condition. If we have a sufficient number of moment conditions, the system is overidentified and we can find the parameters (alpha and beta) of the asset pricing model that best fit the observed cash flows using Generalized Method of Moments estimation (GMM).[2]

More specifically, the moment conditions state that the expected discounted value of all investments should equal the expected discounted value of all dividends paid out, where the discounting is done using the chosen asset pricing model. Effectively, our method searches for parameter values that bring the net present values of cash flows as close to zero as possible. Importantly, we do not need to make distributional assumptions for the factor returns and idiosyncratic shocks. This is a desirable feature because it is difficult to estimate

[1] Most private equity funds are structured as limited life non-traded private partnerships. Hence, investors only observe cash flows in and out of the partnership and some self reported Net Asset Values (NAVs). Several other investment vehicles are structured in the same way (e.g. some real estate funds or mezzanine debt funds). This situation is also frequently encountered for corporate investments. The CFO observes a stream of cash flows from a division/project but no market values. Our method can thus be applied to several relevant setups.

[2] This approach is related to the GMM estimation of standard Euler equations for the pricing kernel (see Cochrane (2005a) for an overview), but in contrast to the Euler equation approach our setup allows for direct estimation of abnormal performance and risk exposures.

the probability density function if market prices are not observable.

Our estimator is asymptotically consistent. In small samples, however, we show that there exists a bias and we investigate how to minimize it. We show that the bias is reduced when each moment condition corresponds to a portfolio of funds instead of a single fund. We also find that it is important to group funds into portfolio based on fund inception year. This enables a better identification of beta because portfolios with different inception years are subject to different market returns. Hence, the cross-section of inception-year portfolios provide better information on beta and other risk loadings. The alpha is essentially identified from the restriction that the expected NPV is zero for all portfolios. Finally, we show that a log-transformation of the moment condition further improves small sample estimates. We illustrate these findings with a Monte Carlo simulation. The simulation also shows that for reasonable calibration parameters our method produces a negligible bias in a small sample (less than three basis points per month for alpha and less than 0.01 for beta).

As mentioned above, we apply our new methodology to an economically significant asset class: private equity funds. Private equity funds are financial intermediaries that invest mainly in venture capital and leveraged buyouts. In 2005, a record high amount of $200 billion was invested in private equity funds for a total amount under management estimated to be above $1 trillion. Private equity funds are not publicly traded, investors observe only a stream of cash flows for about 10 years. Hence standard estimation techniques cannot be applied but we can use our new method.

Our dataset consists of 958 private equity funds with over 25,000 cash flow observations between 1980 and 2003. We include all funds that are more than 10 years old (it is the typical fund duration). For funds that are not reported as liquidated, we estimate a market value for on-going investments from an econometric model. We take all liquidated funds and compute their realized market value at different age as the present value of observed subsequent net cash flows. This market value is then regressed on the following fund characteristics: the Net-Asset-Value (NAV) reported, the time elapsed since the last dividend payment, time elapsed since the last NAV update, fund size and performance. This model is then used to predict the market value of non-liquidated funds. Note that the dependent variable is a function of the parameters of the asset pricing model (via the net present value calculation). Hence, we estimate this model for market values jointly with the abnormal return and risk exposures.

We find that venture capital funds have a significantly different risk profile than buyout funds. Venture capital funds have a market beta of 3.2, while buyout funds have a market beta of 0.3. Inspecting the data, we observe that many venture capital funds paid large dividends mainly in the late 1990s, precisely when the stock market had experienced large rate of returns. In 2000-2003, when stock markets have experienced lower returns, dividends from venture capital funds have been rare. This is this pattern that mainly explains why our beta for venture capital is so large. Korteweg and Sorensen (2008) also find a beta around 3 for venture capital even though they use a different dataset and approach. For buyout funds, in contrast, we do not observe such a strong dependence on market returns. The dividends of buyout funds have been quite steady throughout our time period. The underlying economic reason for a low beta may be that i) buyout funds hold companies that are typically in low beta industries and ii) buyout funds make many changes to the companies they purchase; if the success of these changes has little correlation with stock-market returns, then beta is dragged towards zero. On the other hand, buyout funds use leverage, which increases beta. Our results indicate that the first forces dominate.

When we add the Fama-French factors (SMB and HML), we find that venture capital fund returns resemble small growth stock returns, but the factor loadings are not statistically significant. Buyout fund returns resemble returns on large stocks and have no significant loading on HML. Turning to abnormal performance, we find a significant and large negative alpha of -125 basis points per month for venture capital funds. In line with these results, Moskowitz and Vissing-Jorgensen (2002) report low returns on entrepreneurial investments. For buyout funds alpha is 49 basis points per month but is not statistically significant.

Our model for market values predicts that the NAVs of non-officially liquidated funds are largely overstated. Specifically, non-liquidated funds that report a positive NAV beyond the typical liquidation age (10 years) have an estimated market value that is 30% of the self-reported NAV. In contrast, for funds that are liquidated, market values are close to NAVs.

This substantial discrepancy comes from the fact that the non-officially liquidated funds have not distributed dividends for a long time (more than 3 years) and have not updated their NAVs for a long time (more than 2 years), two characteristics that are significantly associated with poorer subsequent cash flows.

Our new GMM method estimates factor loadings even though the time series of the asset market values is not observable. It comes at the cost of assuming a common parametric structure for alpha and beta for the cross-section of funds. Note that we do not need to assume the same risk level, only the parametric structure needs to be the same. For example, we can make the alpha and beta a function of fund characteristics. To illustrate this point and to also shed light on the findings that large venture capital funds have higher total returns than small funds (Kaplan and Schoar, 2005, and Phalippou and Gottschalg, 2007), we let alpha and beta depend on fund size. We find that alpha is not related to size but beta is significantly and positively related to size. Higher returns of large venture capital funds are thus due to higher risk exposure and not higher abnormal performance.

We also conduct various robustness tests. We change sample selection criteria, weighting of moment conditions, and number of fund portfolios. We observe that results remain essentially unchanged for venture capital funds, while the results for buyout funds are somewhat less stable. Still, across all specifications and setups, buyout funds have low betas and zero abnormal performance.

In sum, this paper is the first to propose a method to consistently estimate risk and return faced by investors on their economically sizeable and fast growing allocations to private equity funds. Our results are relevant for performance evaluation and investment portfolio choice. Given the negative abnormal performance reported in this paper (net of fees), our results suggest a limited role for the average private equity fund in well-diversified portfolios. Finally, our method can be used for other applications such as evaluating risk and return of corporate investments.

The rest of the paper proceeds as follows. In Section 2, we discuss related literature. Section 3 contains a description of the GMM approach and presents a simulation study to assess the small-sample properties. Section 4 describes the private equity industry, our data, and the model for market values. Section 5 presents the empirical results and robustness checks. Section 6 discusses an NAV-based regression approach. Section 7 concludes.

2 Related Literature

Our work is related to Cochrane (2005b) who assesses the alpha and beta of US venture capital investments gross-of-fees with maximum likelihood and Korteweg and Sorensen (2008) who estimate an extended version of Cochrane's model using a Bayesian methodology. Both articles work with venture capital individual investment data (e.g. the return on Google realized by Sequoia from inception until the IPO date). The main issue they face given the nature of their data is that returns are observed mainly for investments that do well.[3] To correct for this sample selection bias, they need to estimate a selection equation and assume a distribution for idiosyncratic shocks.

We view the approach of Cochrane-Korteweg-Sorensen (CKS) and our approach as complementary. Using individual investment data (i) may lead to more precise estimates of risk and return since more information is used and (ii) allows for an analysis of risk and return as a function of project characteristics. However, this comes at the cost of assuming a probability distribution of project returns and assuming a specification for the selection equation. In contrast, our approach does not require distributional assumptions on the factor returns and idiosyncratic shocks. In addition, our paper provides three additional benefits. First, our data do not suffer from a project selection bias since the fund-level cash flows include all investments (both good and bad).[4] Second, we provide risk and return estimates for buyout funds as well. Third, the risk and return estimates CKS obtain are those faced by fund managers while our estimates are those faced by fund investors. As

[3] Venture Capital (VC) funds invest in distinct projects in so-called rounds. Hence, when a VC invests $1 in a project, the realized market value of this $1 is observed provided that there is a subsequent valuation round. In this case, the return on the project can be calculated in standard fashion. However, these subsequent rounds happen only if investments do well enough. Since only projects with subsequent rounds are observed, this creates a selection problem which is especially important for the estimation of abnormal performance as more than half of the investments do not make it to a new round.

[4] Sample selection issues are relatively low in our dataset as cash flows are reported by investors and for all the investments of a given fund, including bad ones. A small sample selection bias nonetheless exists in our dataset (Phalippou and Gottschalg, 2007). Investors that report fund cash flows to Venture Economics appear to have fund-picking abilities.

fees vary across funds, over time, and are non-linear in performance, they affect both alpha and beta. Also, funds keep a stake in companies after an IPO. Hence, investor returns may differ to what is found with the individual investment data of CKS.

In terms of empirical results, the beta for venture capital reported by Korteweg and Sorensen is close to our estimate, while Cochrane's estimate for beta is lower (1.9). The after-fee alpha, however, is significantly negative in our case, while both Cochrane (2005b) and Korteweg and Sorensen (2008) report large positive before-fee alphas for simple returns (30% and over 150% annual, respectively). The difference between these estimates for alpha is larger than what can be justified by fees.

Our paper is also related to that of Jones and Rhodes-Kropf (2004). They derive a theoretical model showing that idiosyncratic risk should be priced in private equity. To test it, they estimate the risk faced by private equity fund investors. They assume that the quarterly self-reported NAVs are stale but otherwise unbiased estimates of market values. They obtain alpha and betas by regressing NAV-based returns on both contemporaneous and lagged risk factors. Their approach is discussed below in section 6.[5]

Finally, our paper is related to Kaplan and Schoar (KS, 2005) and Phalippou and Gottschalg (PG, 2007) who benchmark private equity fund performance to that of the S&P 500, effectively assuming a CAPM with beta equals to one. Our results suggest that the natural benchmark for venture capital is much higher. According to the CAPM and using average past risk free rates and S&P 500 returns, our estimates imply a required return above 20% per year (instead of 10% per year with the S&P 500). In addition, our results suggest that most of the NAVs reported by mature funds (beyond their 10th anniversary) are too high, especially for inactive funds. KS assume that these final NAVs reflect market value while PG write them off. As NAVs are the main driver behind the discrepancy in the average performance results in KS and PG, our approach sheds light on this discrepancy.

[5] The literature also offers indirect estimates of fund risk. A first approach consists of assuming that each fund investment beta is the same as that of the publicly traded stocks in the same industry (e.g. Ljungqvist and Richardson (2003), and Phalippou and Gottschalg (2007)). An alternative approached, also used in Phalippou and Gottschalg (2007) infers the risk profile from post-IPO returns of buyout investments and venture capital investments. Finally, Bilo et al. (2006) make inference from publicly listed companies that are involved in private equity. We refer to Phalippou (2007) for a discussion of the literature.

3 A New Approach to Estimate Risk and Abnormal Return

Unlike common asset classes such as stocks and bonds, private equity funds are not publicly traded. Hence, alpha and factor-exposures (the betas) cannot be estimated directly via a time-series regression. Instead, we propose a GMM approach based on pricing restrictions for funds' cash flows. In this section, we first introduce our new approach with a simple example, and then describe our approach in details.

3.1 Example

Consider two private equity funds. They both live for 3 years and have two different inception years. We assume a CAPM economy with zero risk-free rate and no idiosyncratic risk. Then, the rate of return of fund value is by definition βR_m where R_m is the market return. The table below contains the cash flows from the funds to the investors and the stock market return in each period

Year	Market return	Fund 1: Cash flows (year end)	Fund 2: Cash flows (year end)
1	_	-100	0
2	20%	-50	-50
3	15%	159.25	-50
4	5%	65.84	0
5	-10%	0	101.66

Given these cash flow patterns and market returns, the unobserved market value $V_{i,j}$ of fund i at time j are given in the table below:

Year	Market return	Fund 1: Market value (year end)	Fund 2: Market value (year end)
1	_	$V_{1,1} = 100$	_
2	20%	$V_{1,2} = V_{1,1}(1 + \beta \cdot 0.20) + 50$	$V_{2,2} = 50$
3	15%	$V_{1,3} = V_{1,2}(1 + \beta \cdot 0.15) - 159.25$	$V_{2,3} = V_{2,2}(1 + \beta \cdot 0.15) + 50$
4	5%	$V_{1,4} = V_{1,3}(1 + \beta \cdot 0.05) - 65.84$	$V_{2,4} = V_{2,3}(1 + \beta \cdot 0.05)$
5	-10%	0	$V_{2,5} = V_{2,4}(1 + \beta \cdot -0.10) - 101.66$

Given that these funds are liquidated at year 4 and 5, respectively, both funds should have zero value at the liquidation date if the return-generating model is correctly specified, i.e. $V_{1,4} = 0$ and $V_{2,5} = 0$. In this case, both equations are solved by setting $\beta = 1.5$, since we chose the dividends exactly such that they were generated by βR_m with $\beta = 1.5$ for both funds.[6]

An important point is that of identification. Each equation above is a third order polynomial and thus may have multiple real solutions. In this example, for fund 1, the beta needs to be lower than -1.5 for multiple real solutions to exist, otherwise there is only 1 real solution. When beta is -2, for example, the other solutions are beta = -18 and beta = -14.2. For fund 2, one solution is, of course, -2 and the other solutions are -25.6 and 4.3. Hence, there is only one solution to the system of two equations. We do not have a formal proof for unicity of the solution but we never find any cases in the examples we took (and no local minimum in our real data application below). It appears that as long as the market returns that funds face are different, there is a unique solution. This indicates that it is important to consider funds that are active in different periods (and subject to different market returns) for the identification of beta. This makes intuitive sense. It is by observing cash flow amounts in different market environments that one can learn about systematic risk. In this example, the key assumption we make is that fund 1 and 2 have the same β.[7]

Note that, in the CKS approach, similar assumptions are needed to identify β although

[6] To be precise, we generated the distributions by assuming that each investment of fund 1 has a duration of 2 years and each investment of fund 2 lasts until fund liquidation. This is simply for illustration. Any cash flow timing can be assumed.

[7] In practice, we have an overidentified system and can allow for different β (and α) across funds as long as we can specify β as a function of some variables (e.g. fund size).

with their data they could theoretically assume that β is only constant across rounds of a given project.

Our example thus illustrates that we do not need to know the return generating process for the stock market return, nor which cash flow comes from which project, to estimate funds' risk profile. Nonetheless, the example above is stylized. In reality, we have more funds, more cash flows, and, importantly, private equity fund returns exhibit considerable idiosyncratic risk. This renders the task more difficult but the intuition provided by the above example still goes through. In the following subsection, we show how a GMM approach applied to a large cross-section of funds can be used to estimate abnormal returns and factor exposures in a more realistic setting.

3.2 GMM approach

We illustrate our approach with a one-factor market index model (including a constant mispricing parameter α). A generalization to multi-factor pricing models is trivial as long as the factors are traded assets in order to still measure abnormal performance with α.

As explained below, we work with portfolios of funds (i.e. fund-of-funds, FoF). Each FoF i invests an amount T_{ij} in n_i projects at date t_{ij}. The liquidation dividend of project j at date d_{ij} is stochastic since it depends on the realization of market returns and idiosyncratic shocks and is given by:[8]

$$D_{ij} = T_{ij} \prod_{t=t_{ij}+1}^{d_{ij}} (1 + r_{f,t} + \alpha + \beta r_{m,t} + \varepsilon_{ij,t}) \tag{1}$$

where $r_{f,t}$ is the risk-free rate, $r_{m,t}$ is the market excess return, and $\varepsilon_{ij,t}$ is an idiosyncratic shock with mean zero, independent from the market returns, and uncorrelated over time and across FoFs. These assumptions are the standard assumptions made for performance evaluation regressions (as typically applied to mutual funds for example). As described below, we use a bootstrap methodology to calculate standard errors, where we assume that

[8] We assume that each project has one investment and one distribution. This assumption is without loss of generality. If a project has more than one dividend or investment, one can view this as a combination of subprojects.

the idiosyncratic shocks are uncorrelated across funds but allow projects within a fund to have correlated idiosyncratic shocks.

If one would observe cash flows at the project level and each project would have only one investment and one dividend, one could estimate α and β by simply applying nonlinear regression techniques to equation (1). In practice, however, cash flows are only observed at the fund level and it is not known to which project a given cash flow belongs. Also, in practice, even if one has cash flow data at the project level, it is difficult to know when a project reached zero value (i.e. went bankrupt).

This implies that we have to discount or compound all cash flows to a date that is common to all projects in a FoF so that we can aggregate all the cash flows. Below, we discuss different ways to do this, leading to different estimation methods.

3.3 Method 1: GMM based on Net Present Values

We start by describing the most intuitive approach as it simply sets FoF Net Present Values to zero. Next, we show that this method generates small-sample biases and numerical problems when applied to aggregate data.

3.3.1 Derivation

This method discounts both takedowns (investments) and distributions (dividends) to the inception date t_{0i} of the fund-of-funds. Dividing equation (1) on both sides by $\prod_{s=t_{0i}+1}^{d_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})$ and taking expectations gives

$$E\left[\frac{D_{ij}}{\prod_{s=t_{0i}+1}^{d_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})}\right]=E\left[\frac{T_{ij}}{\prod_{s=t_{0i}+1}^{t_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})}\right] \tag{2}$$

This equation uses the assumption that the idiosyncratic shocks $\varepsilon_{ij,t}$ are uncorrelated over time and independent to the market returns, so that all expectations of the cross-products of the form $\varepsilon_{ij,t}\varepsilon_{ij,s}$ are equal to zero (as well as higher-order cross-products). It is important to note that we take the expectation with respect to the idiosyncratic shocks here. We treat the market returns as exogenous. In other words, we condition upon the realized market returns when constructing this moment condition.[9]

Next, we directly use the sample equivalent of the expectations in equation (2) by averaging across projects within a fund-of-funds. The LHS of (2) for fund-of-funds $i = 1,.., N$ is then estimated by

$$\overline{NPV}^{D_i}(\alpha, \beta) = \frac{1}{n_i} \sum_{j=1}^{n_i} \left[\frac{D_{ij}}{\prod_{s=t_{0i}+1}^{d_{ij}} (1 + r_{f,s} + \alpha + \beta r_{m,s})} \right], \quad i = 1, ..., N \tag{3}$$

and the RHS of (2) is estimated by

$$\overline{NPV}^{T_i}(\alpha, \beta) = \frac{1}{n_i} \sum_{j=1}^{n_i} \left[\frac{T_{ij}}{\prod_{s=t_{0i}+1}^{t_{ij}} (1 + r_{f,s} + \alpha + \beta r_{m,s})} \right], \quad i = 1, ..., N \tag{4}$$

As the number of projects (or funds) per FoF tends to infinity (so that $n_i \to \infty$), the average converges to the expectation asymptotically so that we obtain a consistent GMM estimator (under standard regularity conditions). Note that, by averaging projects to the FoF level, we only need FoF-level data to perform our estimation. Estimation can be done by applying GMM to all moment conditions across the fund-of-funds. For example, in case of equal weighting of moment conditions the first-step GMM estimator with identity weighting matrix can be written as

$$\min_{\alpha,\beta} \sum_{i=1}^{N} [\overline{NPV}^{D_i}(\alpha, \beta) - \overline{NPV}^{T_i}(\alpha, \beta)]^2 \tag{5}$$

Empirically, we weight the moment conditions in different ways.

[9] The expectation on the RHS of equation (2) is with respect to the takedowns T_{ij}. When considering asymptotics with respect to the number of projects within a FoF, we need the standard assumption that (i) the takedowns are drawn from a given probability distribution with finite variance and (ii) the takedowns are not 'very highly' correlated. This ensures that the sample average converges to the expectation. Economically, this means that no single fund dominates all other funds in terms of size of the takedowns.

3.3.2 Small-sample bias and numerical issues

Although intuitive, the NPV method suffers from a small-sample bias and numerical problems. To illustrate this analytically, we study in this subsection a simplified setup where within each fund-of-funds, all projects have a takedown equal to 1, a duration of one period and pay off at the same date. In this case, the average realized dividend of FoF i is

$$\frac{1}{n_i}\sum_{j=1}^{n_i} D_{ij} = \frac{1}{n_i}\sum_{j=1}^{n_i}\left[T_{ij}(1+r_f+\alpha+\beta r_{m,t(i)}+\varepsilon_{ij})\right] = (1+r_f+\alpha+\beta r_{m,t(i)}+\overline{\varepsilon_i}) \quad (6)$$

where $\overline{\varepsilon_i} = \frac{1}{n_i}\sum_{j=1}^{n_i}\varepsilon_{ij}$ and where $r_{m,t(i)}$ is the one-period market return that applies to all projects in FoF i. Note that asymptotically, as n_i tends to infinity, $\overline{\varepsilon_i}$ tends to zero ($\overline{\varepsilon_i} \to^P 0$).

The NPV approach discounts all cash flows to the starting date of the FoF. For illustration purposes, we assume here that the starting date is one period before the investments are made. This is to capture the fact that in reality projects start at different dates, so that some investments are discounted to an earlier date. We can then write the optimization in the simplified setup as follows (using equation (6))

$$\begin{aligned} &= \min_{\widetilde{\alpha},\widetilde{\beta}} \sum_{i=1}^{N}\Big[\frac{(1+r_f+\alpha+\beta r_{m,t(i)}+\overline{\varepsilon_i})}{(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)-1})(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)})} && (7)\\ &\quad -\frac{1}{(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)-1})}\Big]^2 && (8)\\ &= \min_{\widetilde{\alpha},\widetilde{\beta}} \sum_{i=1}^{N}\Big[\frac{(\alpha-\widetilde{\alpha})+(\beta-\widetilde{\beta})r_{m,t(i)}+\overline{\varepsilon_i}}{(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)-1})(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)})}\Big]^2 \end{aligned}$$

Since we divide by $(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)-1})(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)})$, we obtain an upward bias for (in particular) $\widetilde{\alpha}$, which is confirmed by unreported simulation results. Moreover, this estimator suffers from numerical problems: since the denominator in (8) has a cubic dependence on $\widetilde{\alpha}$, while the numerator depends linearly on $\widetilde{\alpha}$, the goal function in (8) tends to zero as $\widetilde{\alpha} \to \infty$. Hence, the goal function is not globally convex. Figure 1 illustrates this (this figure is based on the simulation setup in section 3.7). Hence, using this method is problematic in practice, especially if one does not have good starting values for the optimization algorithm.

Finally, instead of calculating net present values for the GMM estimation, one could consider compounding all cash flows to a final liquidation date, thus bringing net compounded values as close to zero as possible. In appendix 1, we show that such an approach also has small-sample biases and numerical problems. Specifically, in this case a local optimum is achieved by setting $\widetilde{\alpha}$ close to -1, so that the compounding term is essentially zero.

3.4 Solutions

Given the problems of the NPV approach, we consider three alternative ways of performing the GMM estimation. For all three alternative methods it is irrelevant whether one discounts or compounds the cash flows, so that these methods do not suffer from the numerical problems that arise due to the fact that the discounting or compounding term depends on the parameters over which we optimize.

3.4.1 Method 2: GMM based on Extended-PI

This method takes the ratio of the value of distributions and takedowns. The moment condition can thus be written as

$$E[\frac{D_{ij}}{\prod_{s=t_{0i}+1}^{d_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})}] \;/\; E[\frac{T_{ij}}{\prod_{s=t_{0i}+1}^{t_{ij}}(1+r_{f,s}+\alpha+\beta r_{m,s})}] = 1 \qquad (9)$$

In this case, it is irrelevant whether one discounts or compounds all cash flows since the discounting/compounding term affects the denominator and numerator in exactly the same way. Estimation is then performed as follows

$$\min_{\alpha,\beta}\sum_{i=1}^{N}[\frac{\overline{NPV}^{D_i}(\alpha,\beta)}{\overline{NPV}^{T_i}(\alpha,\beta)}-1]^2 \qquad (10)$$

Again, this estimator is asymptotically consistent.[10] It generates a small-sample bias for different reasons than method 1, as discussed below. We call this method 'Extended-PI'. Extending the profitability index (PI, also called public market equivalent, PME) to incorporate an α and β, this method essentially estimates α and β by bringing the 'extended profitability index' as close to 1 as possible.

3.4.2 Method 3: GMM based on Log-PI

Method 3 compares the log-values of distributions and takedowns. As we will see below, this method generates the smallest small-sample biases in our simulation study. Like method 2, it is irrelevant whether one discounts or compounds for this log-estimator. The moment condition underlying the estimation is $\ln(E[D_{ij} \,/\, \prod_{s=t_{0i}+1}^{d_{ij}}(1 + r_{f,s} + \alpha + \beta r_{m,s})]) = \ln(\ E[T_{ij} \prod_{t=t_{0i}+1}^{t_{ij}}(1 + r_{f,t} + \alpha + \beta r_{m,t})])$.

Estimation is performed as follows

$$\min_{\alpha,\beta} \sum_{i=1}^{N} [\ln(\overline{NPV}^{D_i}(\alpha, \beta)) - \ln(\overline{NPV}^{T_i}(\alpha, \beta))]^2 \tag{11}$$

Because the log of the average converges to the log of the expectation asymptotically, this estimator is consistent as the number of projects per fund or the number of funds per portfolio tends to infinity (see again appendix 2).

3.4.3 Method 4: Exact Identification (Method of Moments)

The final method is a simple method of moments where we set the number of funds-of-funds equal to the number of parameters ($N = 2$ in case of the market model). Setting $N = 2$ delivers a special case of all methods discussed above, since in this case all methods will give the same estimates. This is because in this setting the two moment conditions are matched exactly, so that it does not matter how the moment conditions are compounded or discounted. However, as discussed in the simple example in section 3.1, an important caveat is that with $N = 2$ we have in general multiple solutions for α and β because the

[10] Formally, this is not a GMM estimator, since it is based on a ratio of expectations. In appendix 2 we show however that our modification of the GMM estimator is still asymptotically normal.

compounding (or discounting) generates polynomials in α and β. Below we discuss other disadvantages of setting $N = 2$. Most importantly, we show that having more moment conditions leads to more precise estimates in small samples.

3.5 Small-sample biases for methods 2, 3 and 4

We now discuss the small-sample biases of these three methods. To get analytically tractable equations, we use the simplified setup in section 3.3.2 leading to equation (6).

3.5.1 Method 2: Extended-PI-GMM

In this case the estimation can be rewritten as

$$
\begin{aligned}
&= \min_{\widetilde{\alpha},\widetilde{\beta}} \sum_{i=1}^{N} [\frac{(1+r_f+\alpha+\beta r_{m,t(i)}+\overline{\varepsilon_i})/(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)-1})(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)})}{1/(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)-1})} - 1]^2 \\
&= \min_{\widetilde{\alpha},\widetilde{\beta}} \sum_{i=1}^{N} [\frac{(\alpha-\widetilde{\alpha})+(\beta-\widetilde{\beta})r_{m,t(i)}+\overline{\varepsilon_i}}{(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)})}]^2 \qquad (12)
\end{aligned}
$$

This method thus generates a 'discounting' bias since the pricing error is divided by $(1 + r_f + \widetilde{\alpha} + \widetilde{\beta} r_{m,t(i)})$, leading to an upward bias for $\widetilde{\alpha}$ in particular. However, relative to the NPV-estimator, this bias will be smaller because there is less discounting than in equation (7). Also, since both the denominator and numerator depend linearly on $\widetilde{\alpha}$, the goal function does not tend to zero as $\widetilde{\alpha} \to \infty$ so that there is a unique optimum for this estimator. This is confirmed by our simulation results in a more realistic setting.

3.5.2 Method 3: Log–PI-GMM

Next we turn to the bias of the Log-estimator. As with the extended PI estimator, this estimator is insensitive to compounding or discounting to a particular date. We can thus write the estimator as

$$\min_{\widetilde{\alpha},\widetilde{\beta}} \sum_{i=1}^{N} [\ln(1+r_f+\alpha+\beta r_{m,t(i)}+\overline{\varepsilon_i}) - \ln(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)})]^2 \qquad (13)$$

Intuitively, equation (13) generates a convexity bias because $E\left[\ln(1+r_f+\alpha+\beta r_{m,t(i)}+\overline{\varepsilon_i})\right] < E\left[\ln(1+r_f+\alpha+\beta r_{m,t(i)})\right]$. As discussed above, this bias disappears asymptotically since $\overline{\varepsilon_i}$ tends to zero as $n_i \to \infty$. In small samples, however, this will lead to a tendency to lower the term $(1+r_f+\widetilde{\alpha}+\widetilde{\beta}r_{m,t(i)})$, leading to (in particular) a downward bias for $\widetilde{\alpha}$. In general, the estimator $\widetilde{\beta}$ may be biased as well, but quantitatively the bias is smaller (see simulation results). Like the extended-PI method, this method does not suffer from numerical problems since this method is fully insensitive to the choice of discounting or compounding cash flows. This is confirmed by our simulations and empirical results. For example, we find a globally concave goal function in our simulations, as shown in figure 2.

3.5.3 Method 4: Exact Identification (MM)

In the simplified one-period setting, it is easy to show that solving the moment conditions in this case boils down to setting the pricing errors for the two FoFs to zero[11]

$$(\alpha-\widetilde{\alpha})+(\beta-\widetilde{\beta})r_{m,t(i)}+\overline{\varepsilon_i}=0,\ i=1,2 \tag{14}$$

Then, as long as $r_{m,t(1)} \neq r_{m,t(2)}$ the estimators $\widetilde{\alpha}$ and $\widetilde{\beta}$ are unbiased. However, in a more realistic setting where the duration of projects is larger than one period, the first-order conditions are not linear in $\widetilde{\alpha}$ and $\widetilde{\beta}$ anymore, because products of the form $(1+r_f+\widetilde{\alpha}+\widetilde{\beta}r_{m,t(i)})(1+r_f+\widetilde{\alpha}+\widetilde{\beta}r_{m,t(i)+1})$ will enter the pricing error formula. Hence, in general this method generates a small-sample nonlinearity bias and, as discussed in section 3.1, multiple solutions for $\widetilde{\alpha}$ and $\widetilde{\beta}$. Obviously, the other methods discussed above will also suffer from the nonlinearity bias, with the exception of the Log–PI-estimator[12].

3.6 Discussion and Implications

The analysis of the small-sample biases above has several implications. First, the first order condition in (14) shows that in the one-period case, β is identified if $r_{m,t(1)} \neq r_{m,t(2)}$, that is,

[11] If one uses the NPV estimator with $N=2$, the resulting MM estimator again has numerical problems as $\widetilde{\alpha}$ tends to ∞. However, using the extended-PI or Log-PI-estimator with $N=2$ generates unique and identical solutions.

[12] This is because the log of the compounded return can be rewritten as the sum of the log-returns.

if the different FoFs are subject to different market return shocks. It is useful to derive the first order conditions for a case where the two FoFs are formed such that each FoF has half of the projects in period $t(1)$ and half of the projects in period $t(2)$. In other words, both FoFs have projects in both periods with equal weights. In the simple one-period example, the first order conditions are

$$(\alpha - \widetilde{\alpha}) + \frac{1}{2}(\beta - \widetilde{\beta})(r_{m,t(1)} + r_{m,t(2)}) + \overline{\varepsilon_i} = 0,\ i = 1, 2 \tag{15}$$

In this case, α and β cannot be separately identified. As was already indicated by our simple example (section 3.1), this shows that it is important to create FoFs that have as little overlap in time as possible, to allow for cross-sectional identification of β. This is why we group funds by vintage year in our application below.

Second, equations (14) and (15) show that we need to consider asymptotics in terms of having more projects in a FoF in the *same* time period. If we add more projects over different time periods, it is easy to show that the first order conditions become

$$(\alpha - \widetilde{\alpha}) + (\beta - \widetilde{\beta})\overline{r_m(i)} + \overline{\varepsilon_i} = 0,\ i = 1, 2 \tag{16}$$

where $\overline{r_m(i)}$ is the average market return over all periods relevant for the FoF i. In this case, adding more projects from different periods implies that $\overline{r_m(i)} \to^P E[r_m]$, so that α and β can not be identified asymptotically. This again motivates our empirical strategy to group funds by vintage year.

Third, the bias derivations show that the bias is a direct function of the variance of $\overline{\varepsilon_i} = \frac{1}{n_i}\sum_{j=1}^{n_i} \varepsilon_{ij}$, the average of the idiosyncratic shocks across projects within a fund-of-funds. This suggests that it is important to group funds into funds-of-funds as much as possible. However, reducing the number of FoFs implies that the time-overlap between the FoFs becomes larger, and the simulation results show that this reduces the precision with which β is estimated. When choosing the number of funds-of-funds, we thus face a trade off in terms of precision and bias. The simulation results discussed below illustrate this in more detail. In any case, it is beneficial to have more moment conditions than parameters in order to generate (asymptotically) unique estimates for α and β.

To obtain standard errors, a bootstrap methodology is adopted. We first resample the funds with replacement within each fund-of-funds, and then re-estimate the alpha and the beta. Since we resample at the fund level, we thus assume that the idiosyncratic shocks of projects may be correlated within a given fund but not across funds. Repeating the process 1,000 times yields the bootstrap distributions of the alpha and the beta.

Although similar in spirit, our approach differs from the GMM estimation of the pricing kernel based on the Euler equation, which is frequently applied in asset pricing (see Cochrane, 2005a, for an overview). To illustrate this point, consider the standard CAPM pricing kernel $a + bR_{m,t}$. As shown in Cochrane (2005a, equation 8.3), the parameters a and b are fixed by imposing that the risk-free asset and market return are priced correctly. Then, one can write an Euler equation for the cash flows of the fund, and test whether this equation holds or not (at the fund or fund-of-funds level). However, this does not lead to direct estimates of the abnormal performance α or risk loading β. In principle, the monthly α could be inferred from the pricing error on the Euler equation, but this pricing error is a nonlinear and complicated function of α. The β is usually estimated in this standard setup by $Cov(R, R_m)/V(R_m)$. Given that we only observe irregular cash flows for private equity funds, estimating β in this way is not possible. In sum, in contrast to the Euler equation approach, our method renders direct estimates of risk exposures and abnormal performance.

The approach described in this section can readily be applied to a sample of private equity funds that are fully liquidated. For our empirical analysis, we will also include funds that are at the typical liquidation age but still reporting positive NAVs. In section 4.4, we describe how we incorporate these funds.

3.7 Small sample properties: A Monte Carlo Simulation

As discussed above, the GMM methodology generates asymptotically consistent estimates of α and β. To evaluate the small sample properties of the different methods, we run a Monte Carlo experiment. We only consider the methods 2,3 and 4, since the NPV-based method generates results that are numerically not stable.[13]

Our simulated economy works as follows. At the beginning of the year = 1980,...,1993, 50 funds are started. They all invest \$1 per project and start 3 projects per year for 5 years, so that a fund has 15 projects in total. Each project liquidates 5 years later. These numbers are chosen to match our venture capital dataset (see below). For this simulation exercise, we need to assume a true return generating process. For simplicity, we use that of Cochrane (2005b). That is, the quarterly growth in value of project j of fund i is assumed to be lognormally distributed:[14]

$$\ln(\frac{V_{ij,t+1}}{V_{ij,t}}) = \gamma + \ln R_t^f + \delta(\ln R_{t+1}^m - \ln R_t^f) + \eta_{ij,t+1}; \qquad \eta_{ij,t+1} \sim N(0, \sigma^2) \tag{17}$$

where $\eta_{ij,t}$ is i.i.d. normal across projects and over time.[15]

We group all the funds with the same vintage year into a fund-of-funds (FoF), which leads to 14 moment conditions, one for each vintage year. We assume a 4% annual risk-free rate and a 8% expected equity premium. Market index volatility is set at 15% annually (which reflects the US S&P index data over our sample period). As mentioned above, larger idiosyncratic shocks lower the precision of our estimator. We therefore show results for a low and high value for the idiosyncratic volatility. The low volatility level is 25% p.a., which is the estimate of Ang, Hodrick, Xing and Zhang (2006) for the highest idiosyncratic volatility quintile for US stocks.[16] For the high volatility level we use the estimate of idiosyncratic volatility of venture capital projects from Cochrane (86% p.a.). We show results for a

[13] Unreported simulation results show that, even if we carefully deal with the numerical issues of the NPV-based method by constraining the α to be in a range around 0 (the true value), the small-sample biases of this estimator are larger compared to the extended-PI and Log-PI methods.

[14] Consistency of our GMM method is obtained without specific distributional assumptions. We use lognormality here as an example to show the small-sample properties.

[15] As shown in Cochrane (2005b), the continuous limit of equation (17) can be used to obtain the α and β for the CAPM in simple returns which gives $\beta = \delta$ and

$$\alpha = \gamma + \frac{1}{2}\delta(\delta - 1)\sigma_m^2 + \frac{1}{2}\sigma^2 \tag{18}$$

where $\sigma_m^2 \equiv V(\ln(R^m))$.

[16] See Table 6 of Ang, Hodrick, Xing and Zhang (2006), which provides total volatility estimates across quintile portfolios. We correct these total volatilities for market volatility to obtain idiosyncratic volatility.

benchnmark parameter set with $\alpha = 0$ and $\beta = 1$.[17]

Using 1000 simulations, Panel B of Table 2 show the results for the extended-PI, Log-PI and MM methods discussed in section 3, in case of the low value for idiosyncratic volatility. In this case, idiosyncratic volatility across projects is averaged out to a large extent and all three estimators provide essentially unbiased estimates. Importantly, the Log–PI-estimator has the most precise estimates, while the MM estimator has the highest standard deviation for alpha and beta across simulations. As argued above, if moment conditions overlap too much in time, the FoFs are subject to mostly the same market returns and beta can not be estimated precisely, which is indeed what we find for the MM method which has only two FoFs.

If we set idiosyncratic volatility at a higher level (86% per year), the Log-PI estimator generates again the smallest small-sample bias for alpha and beta and provides the most precise estimates (Panel C, Table 2). The MM method is the noisest and most biased method for both alpha and beta. The Log-PI method generates a slightly negative bias for alpha (-0.03% per month). This is due to the convexity effect discussed in section 3.4.2. In line with the intuition provided in section 3.4.1, the E_PI method generates a slight upward bias for alpha (0.08% per month). The method of moments approach (method 5, $N = 2$), also generates an upward bias of 0.10% for alpha, which is due to the nonlinearity of the moment conditions.

For beta, Log-PI also generates the most precise estimate. It gets the estimate right on average and has a 0.22 standard deviation for alpha. E_PI also get the beta right on average but with more volatility (0.33 standard deviation).

For the best-performing method, the Log-estimator, we assess in more detail the sensitivity of the small-sample biases to the level of idiosyncratic volatility and the number of moment conditions. As shown by the results for the method of moments ($N = 2$), having few moment conditions does not generate satisfactory results because of the large overlap between moment conditions. We therefore consider increasing the number of moment conditions beyond the benchmark number ($N = 14$), by splitting each vintage year FoF into

[17] Results for other parameter values are similar.

subportfolios of equal size. Figure 3A and 3B show the mean estimates of alpha and beta (across 1000 simulations) for different values for N and the idiosyncratic volatility. These figures show first of all that the bias for alpha becomes more negative as we increase N. This is intuitive, since each moment conditions has less projects in this case so that the convexity effect discussed in section 3.4.2 is more relevant. Similarly, the alpha bias increases in volatility, since in this case there is less 'averaging' of idiosyncratic volatility (i.e., the variance of $\overline{\varepsilon_i}$ in equation (6) is larger). The mean beta is hardly affected by N or the level of idiosyncratic volatility.

In sum, this simulation study shows that the GMM method has satisfactory small-sample properties when the true model is log-normal even though the distribution is unknown to the econometrician.

4 Data

We apply the methodology developed above to the estimation of risk and abnormal return of venture capital funds and buyout funds. In this section, we provide key institutional details, describe our data source, the content of our dataset, and our treatment of non liquidated funds.

4.1 Institutional environment

The private equity funds that we study are organized as limited partnerships and have a finite life (typically 10 years). This structure is by far the most common in this industry. Investors, called Limited Partners (LPs), are principally institutional investors. LPs commit a certain amount of capital to private equity funds, which are run by General Partners (GPs). In general, when a GP identifies an investment opportunity, it "calls" money from its LPs up to the amount committed (undiscounted). Such "calls" mainly occur over the first 5 years of a fund's life. Each time an investment is (partially or fully) liquidated, the GP distributes the proceeds to its LPs. The timing of these cash flows is typically unknown ex ante. The year when a fund starts is called vintage year. Funds report quarterly their

Net Asset Values. NAVs typically equal the amount invested in the first years of funds' life.[18]

4.2 Data source

Data on private equity funds are from Thomson Venture Economics (TVE). TVE records the amount and date of all the cash flows. Cash flows are net of fees as they are what LPs have received and paid. TVE also provides quarterly NAVs from audited financial reports of the partnership.

Venture Economics offers the most comprehensive source of financial performance of both US and European private equity funds and has been used in previous studies (e.g., Kaplan and Schoar, 2005). It covers an estimated 66% of both venture capital funds and buyout funds (Phalippou and Gottschalg, 2007). TVE builds and maintains this dataset based on voluntarily reported information about cash flows between GPs and LPs. The main data providers are LPs and not GPs, which reduces sample selection bias concerns.

4.3 Sample selection

We consider all funds (with size over $5 million) raised between 1980 and 1993 as they have reached their normal liquidation age (10 years) at the end of our sample time period (2003). As discussed above, we construct venture capital fund-of-funds and buyout fund-of-funds based on vintage years. Hence, vintage years with less than 5 funds are excluded. This excludes buyout funds raised between 1980 and 1983 but does not affect venture capital funds.

Descriptive statistics are reported in Table 2. We have 958 funds, of which 686 have a Venture Capital (VC) objective and 272 have a buyout (BO) objective. In total, we have 25,800 cash flows. Our descriptive statistics are similar to what has been reported in the literature. Importantly, two thirds of the funds report a positive NAV at the end of our sample time period despite having passed their tenth anniversary. VCs have returned

[18] For further details on private equity fund contracts, see Axelson, Stromberg and Weisbach (2007), Gompers and Lerner (1999, 2000), Metrick and Yasuda (2007) and Phalippou (2007).

twice the capital invested and claim 20% extra of non-exited value (final NAV). VCs have returned 150% the capital invested and claim 25% extra of non-exited value.

4.4 Estimating Final Market Values

As mentioned above, about two thirds of the funds report a positive NAV at the end of our sample period. Existing work either treats these final NAVs as a final cash flow (Kaplan and Schoar, 2005) or writes them off (Phalippou and Gottschalg, 2007). One of the problems faced in the literature and which partly explains these simple choices is that the conversion of NAVs into a market value necessitates an estimate of risk.

We observe that a number of funds that are beyond their 10th anniversary have the same NAV every single quarter, all the way until the end of our sample time period while no cash flows are observed. The median non-liquidated fund reports the same NAV (and no cash flow) over the last 2.5 years. As much as a quarter of the funds report the same NAV (and no cash flow) for more than 6 years, i.e. the exact same NAV and no cash flow from January 1998 to December 2003. This fact could be either a deliberate action of funds in an attempt to "hide" bad performance (Phalippou and Gottschalg, 2007, report that these funds tend to have the lowest performance), or result from a data entry convention (if a fund liquidates but investors do not report that event to TVE then TVE keeps on repeating the latest NAV forever).[19]

In this paper, rather than making a judgement call, we estimate econometrically the relation between NAV and market value. Specifically, we take the fully liquidated funds at different ages, compute their market value (MV) as the net present value of subsequent cash flows. That is, for each age a=10,11,12, and 13, we estimate the following model:

$$\ln(1 + MV_{a,i}(\alpha, \beta)) = b_{a0} + b'_{a1} X_{a,i} + \varepsilon_{a,i} \tag{19}$$

The vector of explanatory variables $X_{a,i}$ includes log $(1 + NAV)$, the log of fund size (committed capital), the log of the time elapsed since the last dividend distribution, the log

[19] In theory, it is also possible that funds beyond their 10th anniversary hold some investments passively and keep them at the same value in their accounting each quarter. Unfortunately, we cannot obtain from TVE information on this issue.

of the time elapsed since the last NAV update, and fund's multiple excluding NAV (sum of capital distributed divided by sum of capital invested); where all variables are computed at age a.

Results from the regression (19) are shown in Table 3 - Panel A. We find that a 1% increase in NAV leads to slightly less than 1% increase in market value and that this elasticity decreases with age. It is 0.91% for NAVs reported at age 10 but only 0.72% for NAVs reported at age 13. Large funds and better performing funds tend to have higher market values, hence more conservative accounting valuations. Funds that have not paid a dividend for a long time have lower market values. Similarly, the NAV of funds that have not updated their NAVs for a long time is significantly exaggerated. This variable is the most significant of all explanatory variables besides NAV. The average time since last NAV update is 1.5 years. An increase to 2 years would decrease market value by about 10% everything else constant (taking the average coefficient across the 4 specifications).

Some descriptive statistics for fully-liquidated funds are shown at the bottom of Table 3 - Panel B. The ratio of (aggregate) market value to NAV is between 100% at age 10 to 113% at age 13. Hence, for the sample of liquidated funds, NAVs reported towards the end of funds' life (age 10 to 13) are close to market values. However, even though NAV and market values are similar on average, there are large cross sectional differences that are well captured by our set of explanatory variables (R-squared is between 64% and 71%).

The next step consists of predicting market values for the non-liquidated funds by applying the regression coefficients from equation (19) to the fund characteristics of the non-liquidated funds.[20]

Results of the extrapolation are shown in Table 3 - Panel B. The model predicts small market values compared to NAVs as the ratio of total market values to total reported

[20] Beyond the 13th anniversary (typically the maximum duration of a fund), we observe only very few funds that have a positive NAV and are subsequently liquidated. For funds older than 13 years old, we consider their NAV reported at the end of their 13th anniversary and use the coefficients from the age 13 regression. An alternative approach would be to consider their NAVs at the end of our sample and apply the coefficients from the age 13 regression. In this case we write-off their NAV almost entirely because some funds are 15 years old, have been inactive for 5 years and have been reporting the same NAV for 5 years. For them the model predicts worthless NAVs. So stopping at year 13 for all non-liquidated funds is a conservative treatment of NAVs. This choice also explains why most of the funds to be extrapolated are in the age 13 category (N=434). Note also that our extrapolation exercise is valid under the assumption that the model estimated with the liquidated fund sample is the same with the non-liquidated sample.

NAVs is between 21% (age 12) and 37% (age 10). Overall 70% of NAVs are written off according to the model. This is due to the different characteristics of non-liquidated funds. Non-liquidated funds have not paid any dividends for about 3.5 years while fully liquidated funds have on average paid a dividend 1 year ago. Similarly, non-liquidated funds have not updated their NAVs for 2.5 years while liquidated funds have updated their NAVs less than 6 months ago. As a consequence, the model predicts small market values. We thus provide evidence that NAVs of old and inactive funds largely overstate the true market value.

The results described above require a joint estimation setup. $MV_{a,i}$ depends on the discount rate, and is therefore a function of α and β. In turn, we need the estimate of $MV_{a,i}$ for the non-liquidated funds to apply our methodology and estimate α and β. Hence, we simultaneously estimate α and β from the GMM equation (11) and the regression coefficients in (19). The results described above are those obtained when estimating a one factor market model (which we call CAPM for convenience). However, we find that the coefficients are very similar across pricing models.

5 Risk and Return Estimates

In this section, we report private equity funds risk and abnormal return estimates using our methodology. We also analyze differences in α and β across funds by allowing these parameters to be a function of fund characteristics like size and experience. Finally, we present several robustness checks for the estimation setup and portfolio formation.

5.1 Benchmark Results

As discussed in section 3.7, we create one fund portfolio (fund-of-funds, FoF) per vintage year. This way, we reduce idiosyncratic risk, which is the main driver of the small-sample bias of our estimates. This comes at the cost of lowering the number of moment conditions. In the robustness section, we show that increasing the number of moment conditions actually changes results only marginally.

Note also that for each portfolio we add the cash flows across funds, hence, we value weight each fund. Our estimate of risk and abnormal performance is thus for $1 invested in a venture capital fund or a buyout fund.

The results for venture capital funds are shown in Table 4 - Panel A. The CAPM specification gives a beta of 3.21 and a significantly negative alpha of -1.25% per month. When the two Fama-French factors are added, alpha increases to -0.69% because venture capital funds overall are similar to small growth stocks, whose performance is lower than average. However, the loadings on SMB and HML are not significant. We also see that the precision of the estimates decreases when we add the Fama-French factors. Given that we perform a cross-sectional estimation, the parameter estimates are correlated to some extent and these parameter estimate correlations are higher for the 3-factor model. This makes it harder to precisely pin down the different risk exposures and alpha.

A beta of 3 for venture capital is high but plausible. The beta of the small growth portfolio of Fama-French (in the 10 by 10 dataset) is 1.67; this portfolio is probably the most closely related to that of venture capital investments. Cochrane (2005b) finds a beta of 1.90 at the project level (gross of fees) from 1987 to June 2000. Korteweg and Sorensen (2008) using a cleaner version of Cochrane's data and a similar model as Cochrane also find a beta around 3.

The results for buyout (BO) funds are shown in Table 4 - Panel B. Buyout funds have a much lower beta. It is 0.33 in the CAPM specification. Alpha is slightly positive but economically small and not statistically significant. Buyout funds load negatively on SMB which indicates that returns achieved by buyout funds are more related to the returns achieved by large caps than by small caps.

The beta for buyout is quite low. On the one hand, buyout funds purchase and sell companies that are similar to those publicly traded typically in low beta industries. In addition, BO funds make many changes to the companies they purchase. If the success of these changes has little correlation with stock-market returns, then beta is dragged towards zero. On the other hand, BO funds use leverage, which increases beta. Our results indicate that the first forces dominate.

To provide some further intuition for these results, Figure 4 shows dividend yields (sum of next 12 months dividends divided by fund size) averaged across funds, respectively, in their 4th, 5th,..., and 10th year. The time-series is shown both for venture capital and buyout funds. In addition, the cumulated stock market return over the previous 5 years is plotted (divided by 5 to show an annual number). It is apparent that the first pick of the stock-market in 1995 and the rally of 1998-1999 goes hand-in-hand with a huge spike in dividend yield for venture capital funds. When the stock market went down the following three years, so did the dividends. Our high estimate for the venture capital beta reflects these features of the data. Interestingly, the same figure shows that buyout fund dividends are smoother across years. This is consistent with our finding of a beta below unity for buyout funds. This figure is however only suggestive as these dividend yields are not monthly fund returns. Such returns are not readily computable given the nature of the data.

5.2 Return and fund characteristics

The literature has shown that some fund characteristics are related to returns and it is thus important to incorporate these regularities in our estimations to increase precision. At the same time, we shed light on the nature of these regularities. Kaplan and Schoar (2005) find that fund returns (measured by Public Market Equivalent or IRR) are positively related to the fund size. Our framework allows us to investigate whether this effect is due to higher abnormal performance or higher risk exposures. We make alpha and beta a function of these characteristics using a linear specification:

$$\alpha = a_0 + a_{size} * \ln(fund\ size) \tag{20}$$

$$\beta = b_0 + b_{size} * \ln(fund\ size) \tag{21}$$

Next, we form size-sorted portfolios (i.e. fund-of-funds) for each vintage year. This allows us to pin down the effect of size from the cross-section of moment conditions. If we would use the 14 vintage-year portfolios, size effects would only be identified to the extent that funds with different vintage year have different size. We thus form 2 portfolios per vintage year, sorted on size.

We show results in Table 4. We first include size in the alpha specification only. We find that the alpha of VC funds has a positive and significant loading on size. For buyouts it is also positive, but not significant. Next we also allow the beta to depend on size. The results change dramatically. For VC funds, the size effect in alpha becomes insignificant, while the beta depends positively on size (marginally significant). Hence, we conclude that the higher returns of large VC funds are mainly due to higher betas, and not higher alphas. For buyout funds, the size effect in alpha actually becomes negative once we control for size effects in beta. It is significant for the CAPM specification but not for the FF specification. The beta of buyout funds depends very strongly and significantly on fund size. In sum, we do not find evidence that larger buyout funds outperform. In contrast, there is weak evidence that their alphas are lower than those of small buyout funds.

We have also examined whether fund experience is related to alpha and/or beta. Kaplan and Schoar (2005) provide some evidence that more experienced fund have higher returns, using the fund sequence as a measure of experience. When we make the alpha and beta a function of experience (specifically, by including a dummy for first-time funds), unreported results from our method show no significant relation between experience and alpha or beta.

5.3 Robustness

In this section, we investigate the robustness of our results. We first show results with a different treatment of NAVs. Next, we show results for different samples and different methodological choices.

In Table 5, we estimate risk and abnormal return when i) treating the final NAVs of non-liquidated funds as fair market value (as in Kaplan and Schoar, 2005) and when ii) writing NAVs off (as in Phalippou and Gottschalg, 2007). Table 5 shows that the difference between the betas of treating the final NAV as fair market value and writing it off is relatively small. VC beta decreases from 3.45 and 2.98 under the CAPM framework when changing the final market value from reported NAV to zero. BO betas similarly decrease from 0.59 to 0.14. Spreads in the estimates are similar when we add the Fama-French factors.

In Table 6, we show estimates of alpha and beta (CAPM model) when the moment conditions are either equally weighted and when the moment conditions are value weighted. We start by changing the number of fund-of-funds (FoFs). For each vintage year, we group funds by size and create either 2, 3 or 4 portfolios. Irrespective of how many portfolios we create, we find similar beta estimates. We note that betas are basically unchanged when moment conditions are value weighted. When moment conditions are equally weighted, betas are still reasonably stable but less. There is a 0.56 maximum spread for venture capital betas and 0.29 maximum spread for buyout betas.

Given that the small-sample bias increase when we have more FoFs, these results are reassuring since they show that with our data, the small sample bias is negligible when we use the log_PI approach.

We note that when we value weight the moment conditions, betas increase by 0.4 for both VC and BO (except for VC in the default case). We also change the sample. First, we exclude funds raised in 1993 (the benchmark sample includes funds up to vintage year 1993). Second, we add funds with vintage year 1994. Third, we exclude non-US focus funds. Results are stable and we note that US venture capital funds have a higher beta but US buyout funds have a slightly lower beta.

Another robustness check concerns the way we weight funds within a FoF. So far, we have aggregated cash flows in dollars within a FoF, so that larger funds automatically have higher weights within the FoF. This could be efficient if larger funds have smaller idiosyncratic variance. As an alternative, we divide all cash flows of a given fund by the committed capital of this fund, thus equally-weighting funds within a FoF. For VC funds, this does not lead to a substantial change in the estimates. For buyout funds however, the estimates are somewhat different. This again shows that the VC sample is large and homogeneous, while the buyout sample is smaller and (perhaps) more heterogeneous.

Finally, we report the estimates from the extended-PI method. As discussed in section 3.7, our benchmark Log-PI estimator generates a small downward bias for the alpha, while the extended-PI method generates an upward bias. Asymptotically, these biases disappear. Hence, by comparing the Log-PI and extended-PI estimates, we can check to what extent small sample biases are at work. For the VC sample, the extended-PI method gives an estimate for alpha of -1.20% per month. In line with our derivations in section 3.5, the Log-PI method delivers a smaller alpha (-1.24%), but the difference is very small. The beta estimates are also very close. This shows, again, that the VC estimates are hardly affected by small-sample biases. For the buyout sample, we do obtain quite different estimates for alpha and beta.

In sum, the robustness checks show that the VC results are robust and not subject to small-sample biases. In contrast, the buyout sample is smaller and reliable inference is more difficult.

6 Alternative approach: NAV-Regression

In this section, we show results obtained with an alternative approach to infer the risk and return of private equity funds. This approach is the most common in practice. It consists of aggregating cash flows and NAVs across a portfolio of funds to obtain a time-series of quarterly returns.[21]

$$R_t = \frac{(AggNAV_{t+1} + AggDiv_t - AggInv_t)}{AggNAV_t} \tag{22}$$

where $AggNAV_t$ is the sum of NAVs across funds at the beginning of quarter t, $AggDiv_t$ is aggregate dividends and $AggInv_t$ is aggregate investments during quarter t. Betas are then estimated via a time-series OLS regression of returns on contemporaneous and lagged risk factors:

$$r_t = \alpha + \beta_0 r_{m,t} + \beta_{-1} r_{m,t-1} + ... + \beta_{-L} r_{m,t-L} + \varepsilon_t \tag{23}$$

where $r_t = R_t - r_{f,t-1}$ is the excess return over the T-bill rate and L is the number of lags. The idea is that the CAPM-beta can be estimated by simply summing up the betas on current and lagged factor returns as argued by Dimson (1979) in a similar context.

Applying this method to private equity data, however, is problematic. In order to better understand the issue, we briefly discuss the setup and derivation of Dimson (1979). Dimson

[21] See, for example, Woodward (2004) and Jones and Rhodes-Kropf (2004).

models stale prices by assuming that the observed price $\widehat{P}_t$ is equal to P_{t-i} with probability θ_i, $i = 0, .., n$, where P_t is the 'true' value at time t. The expected observed price at time t thus equals

$$E(\widehat{P}_t) = \sum_{i=0}^{n} \theta_i P_{t-i} \tag{24}$$

Turning to log-prices, it directly follows that

$$E(\ln \widehat{P}_t) = \sum_{i=0}^{n} \theta_i \ln P_{t-i} \tag{25}$$

It then follows that

$$E(\ln \widehat{P}_t - \ln \widehat{P}_{t-1}) = \sum_{i=0}^{n} \theta_i (\ln P_{t-i} - \ln P_{t-i-1}) \tag{26}$$

Then, arguing that the change in log-prices is close to simple returns, it follows that the sum of the coefficients on lagged market returns provides an accurate estimate of beta. Dimson's result is thus suitable for studies of daily returns for example (which was Dimson's focus). In private equity, however, the frequency is quarterly leading to potentially serious biases and making the estimator inconsistent.[22]

Furthermore, and more importantly, the NAV-regression approach relies on the assumption that NAVs are stale but unbiased estimates of market values. In practice, NAVs are typically kept at cost during the first 5 years. In addition, NAVs may be systematically biased as suggested by the results of our econometric model of final market values (equation (19)). Finally, they may offer asymmetric responses to changes in market values. E.g., Phalippou and Gottschalg (2007) show suggestive evidence that NAVs are rarely written down in bad times but are readily written up in good times.

When applying the NAV-regression approach to our data, we find low market beta and

[22] Note however that if one makes a specific distributional assumption such as log-normality, it is possible to explicitly correct for the bias in this approach. The main benefit of our approach is not to assume an arbitrary distribution for the true return process.

high alpha for both venture capital and buyout funds (Table 7 – Panel A).[23]

We note that when adding lagged market returns, lagged risk factors are significant but not at a decreasing rate. For venture capital, the loading on the 3rd and 4th lag is much higher than the loading on the 1st and 2nd lag (specification 3). Similarly, for buyout, the same specification shows that only lag 3 is significant. When Fama-French factors are added, this irregularity is exacerbated. For venture capital funds, lags 1 and 2 become insignificant, lags 0 and 3 are significant at a 10% level test, and lag 4 is significant (specification 4). For buyout funds, only lag 0 and 3 are significant. Moreover, the loadings on Fama-French factors show similar irregularities and switch signs at different lags (not tabulated). These results are counter-intuitive but not surprising given the problems associated with the NAV-regression approach, as discussed above.

Furthermore, results are sensitive to the choice of the starting date. The main reason is that NAVs equal capital invested at the beginning and no dividend happens, hence returns are close to zero, then little by little, as dividends starts to be paid, returns become more volatile (see figure 2). It is not clear how many quarters to skip if any. What is clear however is that this choice would change the estimates. In specification 5, we skip the first two years and the market beta increases by 20%. In Table 7 – Panel B, we change the sample to the same sample as in Jones and Rhodes-Kropf (2004), i.e. all funds raised from 1980 to 1999 and cash flows from 1980 to June 2002, and the estimates change again. Beta is 1.15 for venture capital funds and 0.48 for buyout funds.

7 Conclusion

We develop a new econometric methodology to estimate the risk and return using cash flow data, and apply this to the case of private equity funds. The GMM methodology is based on moment conditions that state that expected discounted dividends should equal expected discounted investments, where the discounting is done using a factor pricing model of which the parameters are to be estimated. This methodology does not use the stale and noisy accounting valuations (Net Asset Values), but instead uses data on fund investments and

[23] The high alpha may be partially due to the upward bias in NAVs identified in this paper.

dividends. It also allows us to leave the distribution of the factor returns and idiosyncratic shocks unspecified. This is an appealing feature in private equity as the return distribution is not directly observable for these non-publicly traded funds. The method is asymptotically consistent and we show how to optimize the small sample performance by constructing moment conditions and fund portfolios in the appropriate way. A simulation study shows that the small-sample performance is satisfactory.

We find that venture capital funds have a high CAPM-beta, while buyout funds have a much lower CAPM-beta. Venture capital funds have a significantly negative alpha. Buyout funds have a slightly positive alpha, but it is close to zero and statistically insignificant.

For funds that are inactive (no cash flows and no updating of NAVs), we show that the NAVs reported near the end of the typical fund life are highly upward biased estimates of the market value. Specifically, using a regression approach, we find that the final market values of inactive funds that are 10 to 13 years old are about 30% of their self-reported net asset values. We incorporate the results of this regression in our estimation of abnormal performance and risk exposure.

Finally, the flexibility of our GMM model enables us to study the interaction between the characteristics of the funds and their alpha and beta. We find that fund size has a positive influence on fund returns as documented in Kaplan and Schoar (2005). However, the higher performance is largely due to higher risk exposure. After allowing the beta to depend on size, we find that fund size does not have a significantly positive impact on the alpha.

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Appendix 1: GMM based on net compounded values instead of net present values

This appendix discusses a method that compounds all cash flows to the final liquidation date l_i of fund-of-funds i, instead of discounting as in the NPV-method. We show that, like the NPV-method, this method suffers from numerical problems and small-sample biases.

We multiply both sides of equation (1) by the compounding term $\prod_{s=d_{ij}+1}^{l_i}(1+r_{f,s}+\alpha+\beta r_{m,s})$ and obtain

$$\begin{aligned} & D_{ij}\prod_{s=d_{ij}+1}^{l_i}(1+r_{f,s}+\alpha+\beta r_{m,s}) \\ = \; & T_{ij}\prod_{t=t_{ij}+1}^{d_{ij}}(1+r_{f,t}+\alpha+\beta r_{m,t}+\varepsilon_{ij,t})\prod_{s=d_{ij}+1}^{l_i}(1+r_{f,s}+\alpha+\beta r_{m,s}) \end{aligned} \tag{27}$$

Taking expectations on both sides of equation (27), we get

$$E[D_{ij}\prod_{s=d_{ij}+1}^{l_i}(1+r_{f,s}+\alpha+\beta r_{m,s})] = E[T_{ij}\prod_{t=t_{ij}+1}^{l_i}(1+r_{f,t}+\alpha+\beta r_{m,t})] \tag{28}$$

Next, we directly use the sample equivalent of the expectations in equation (28) by averaging across projects within a fund-of-funds. The LHS of (28) for fund-of-funds $i = 1,..,N$ is then estimated by

$$\overline{NCV}^{D_i}(\alpha,\beta) = \frac{1}{n_i}\sum_{j=1}^{n_i}\left[D_{ij}\prod_{s=d_{ij}+1}^{l_i}(1+r_{f,s}+\alpha+\beta r_{m,s})\right], \quad i=1,...,N \tag{29}$$

and the RHS of (28) is estimated by

$$\overline{NCV}^{T_i}(\alpha,\beta) = \frac{1}{n_i}\sum_{j=1}^{n_i}\left[T_{ij}\prod_{t=t_{ij}+1}^{l_i}(1+r_{f,t}+\alpha+\beta r_{m,t})\right], \quad i=1,...,N \tag{30}$$

The first-step GMM estimator with identity weighting matrix can be written as

$$\min_{\alpha,\beta}\sum_{i=1}^{N}[\overline{NCV}^{D_i}(\alpha,\beta) - \overline{NCV}^{T_i}(\alpha,\beta)]^2 \tag{31}$$

We then use again the simplified setup leading to equation (6) to understand the problems of this method. To capture that the NCV-estimator compounds most projects beyond their dividend date, we assume in this simplified setup that the final liquidation date is $t(i)+1$, so that we compound one period beyond the date at which the projects pay out their dividend. Let α and β be the true parameter values, and $\widetilde{\alpha}$ and $\widetilde{\beta}$ be the parameters over which we optimize. In the simplified setup, we use equation (6) to write the NCV-based GMM estimation as follows

$$\min_{\widetilde{\alpha},\widetilde{\beta}} \sum_{i=1}^{N} [(1+r_f+\alpha+\beta r_{m,t(i)}+\overline{\varepsilon_i})(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)+1}) \tag{32}$$

$$-(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)})(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)+1})]^2 \tag{33}$$

$$= \min_{\widetilde{\alpha},\widetilde{\beta}} \sum_{i=1}^{N} [((\alpha-\widetilde{\alpha})+(\beta-\widetilde{\beta}) r_{m,t(i)}+\overline{\varepsilon_i})(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)+1})]^2$$

This expression shows that the 'pricing error' $(\alpha-\widetilde{\alpha})+(\beta-\widetilde{\beta})r_{m,t(i)}+\overline{\varepsilon_i}$ is multiplied with the compounding term $(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)+1})$. Asymptotically, the $\overline{\varepsilon_i}$'s tend to zero and $\widetilde{\alpha}$ and $\widetilde{\beta}$ are consistent estimators of α and β. In a small sample, however, minimizing these 'compounded pricing errors' generates a tendency to bring the (positive) term $(1+r_f+\widetilde{\alpha}+\widetilde{\beta} r_{m,t(i)+1})$ closer to zero. In particular, as shown in the simulations, this leads to a downward bias for $\widetilde{\alpha}$.[24] Moreover, the goal function in equation (32) is not globally convex because setting $\widetilde{\alpha}$ close to -1 gives a second local minimum (which may be lower than the 'correct' minimum in a small sample).

Appendix 2: GMM with nonlinear functions

This appendix provides a short derivation to show the asymptotic behavior of an extended-GMM estimator, where we take a function of expectations as moment condition. This situation applies to both the extended-PI estimator (method 3) and the log–PI-estimator (method 4). The derivation follows the standard way of deriving asymptotic normality of the GMM estimator. Using general notation, standard GMM has as moment

[24] Note that, if we would not have compounded by one 'extra' period, we would obtain unbiased estimators. This is however not possible in reality, since projects start and end at different dates.

condition

$$E\left(f(\theta_0, x_i)\right) = 0 \tag{34}$$

where $f(.,.)$ is a k-dimensional function. Let $g_N(\theta) = 1/N \sum_{i=1}^{N} f(\theta, x_i)$, then the GMM estimator is $\widehat{\theta}_N = \arg\min g_N(\theta)' W_N g_N(\theta)$, with W_N a weighting matrix. For the extended-PI and Log-PI estimator, we use as moment condition

$$h\left(E\left(f(\theta_0, x_i)\right)\right) = h(0) \tag{35}$$

Normalize $h(0) = 0$ without loss of generality and our estimator is $\widehat{\theta}_N = \arg\min h(g_N(\theta))' W_N h(g_N(\theta))$. Applying the mean value theorem we can write

$$h(g_N(\widehat{\theta}_N)) = h(g_N(\theta_0)) + \frac{\partial h(g_N(\widetilde{\theta}_N))'}{\partial \theta}(\widehat{\theta}_N - \theta_0) \tag{36}$$

where $\widetilde{\theta}_j$ is between $\theta_{0,j}$ and $\widehat{\theta}_{N,j}$. Then, using the first order condition $\frac{\partial h(g_N(\widehat{\theta}_N))}{\partial \theta'} W_N h(g_N(\widehat{\theta}_N)) = 0$, premultiplying the above equation by $\frac{\partial h(g_N(\widehat{\theta}_N))}{\partial \theta'} W_N$ gives

$$0 = \frac{\partial h(g_N(\widehat{\theta}_N))}{\partial \theta'} W_N h(g_N(\theta_0)) + \frac{\partial h(g_N(\widehat{\theta}_N))}{\partial \theta'} W_N \frac{\partial h(g_N(\widetilde{\theta}_N))'}{\partial \theta}(\widehat{\theta}_N - \theta_0) \tag{37}$$

which can be rewritten as

$$\sqrt{N}(\widehat{\theta}_N - \theta_0) = -\left(\frac{\partial h(g_N(\widehat{\theta}_N))}{\partial \theta'} W_N \frac{\partial h(g_N(\widetilde{\theta}_N))'}{\partial \theta}\right)^{-1} \frac{\partial h(g_N(\widehat{\theta}_N))}{\partial \theta'} W_N \sqrt{N} h(g_N(\theta_0)) \tag{38}$$

Then, under standard regularity conditions, applying the delta-method implies that $\sqrt{N} h(g_N(\theta_0))$ has an asymptotically normal distribution and the premultiplying weighting matrices converge to their probability limits, so that this 'extended' GMM estimator is consistent and asymptotically normal.

Table 1: Monte Carlo Simulations

This table shows results from a Monte Carlo simulation. Each year (from 1980 to 1993), a batch of 50 funds enter the economy. Each fund invests $1 per project and starts 3 projects per year for 5 years (thus 15 projects in total). Projects last for 5 years and their return follows a log-CAPM with zero alpha, beta of one, risk-free rate of 4%, equity risk premium of 8%, and (annual) market volatility of 15%. 1000 economies are simulated. For each economy, 3 estimation methods are executed (see section 3.3 for details). The mean, standard deviation, and inter-quartile range for the 1,000 estimated pair of parameters (alpha, beta; monthly frequency) are displayed. In panel A, the idiosyncratic volatility is set to 24% p.a. and in Panel B, it is set to 86% p.a.

Panel A: Low volatility

	GMM Extended-PI	GMM Log-PI	MM
Mean Alpha	0.00%	0.00%	0.00%
Std Alpha	0.05%	0.02%	0.21%
Inter-Quartile	[-0.04 0.04]	[-0.01 0.01]	[-0.02 0.02]
Mean Beta	1.00	1.00	0.99
Std Beta	0.02	0.02	0.11
Inter-Quartile	[0.98 1.02]	[0.98 1.02]	[0.96 1.04]

Panel B: High volatility

	GMM Extended-PI	GMM Log-PI	MM
Mean Alpha	0.08%	-0.03%	0.10%
Std Alpha	0.81%	0.44%	1.25%
Inter-Quartile	[-0.10 0.16]	[-0.15 0.07]	[-0.17 0.22]
Mean Beta	1.00	1.00	0.95
Std Beta	0.33	0.22	0.60
Inter-Quartile	[0.78 1.23]	[0.82 1.18]	[0.59 1.33]

Table 2: Descriptive Statistics

This table shows descriptive statistics for our sample. Venture Capital funds are raised between 1980 and 1993 and Buyout funds are raised between 1984 and 1993. We report: (i) the average and the median of the amount committed to funds in million of 2003 U.S. dollars; (ii) the total final NAV reported (December 2003), total capital distributed and total capital invested; (iii) the overall multiple (sum NAV + sum Distributed)/(sum Invested); (iv) the proportion of first time funds; (v) the proportion of non-US focused funds; (vi) the proportion of funds with positive final Net Asset Value; and (vii) the number of monthly cash flows and funds.

		All funds	Venture Capital	Buyout
Mean size	($ million)	170.40	90.86	371.02
Median size	($ million)	63.98	51.82	133.38
Sum NAV	($ billion)	27.93	8.08	19.87
Sum Distributed	($ billion)	209.69	81.40	128.69
Sum Invested	($ billion)	119.89	39.57	80.50
Multiple		1.98	2.26	1.85
First time funds		49%	46%	57%
Non-US funds		29%	22%	47%
Funds with positive final NAV		64%	63%	64%
Number of cash-flows		25,800	16,859	8,941
Number of funds		958	686	272

Table 3: Final Fund Market Value Estimates

Panels A shows the estimated relation between fund market value and fund characteristics for the sample of liquidated funds. Fund characteristics include reported NAV, fund size (total capital committed), time (months) elapsed since last dividend distribution and since last NAV change, and Profitability Index (present value of dividends over present value of takedowns). Market value at a given age is computed as the present value of the subsequent cash flows. *t*-statistics are reported below each coefficient in italics. The estimation is done separately for each age (10^{th} anniversary to 13^{th} anniversary). Panel B shows summary statistics of the liquidated sample and non-liquidated sample: mean size, mean LastDiv, mean LastNAV, mean Profitability Index and two ratios (NAV/size and MV/NAV for liquidated funds and extrapolated-MV/NAV for non-liquidated funds). Extrapolated-MV are computed from the model in Panel A.

Panel A: Market values as a function of fund characteristics – liquidated sample

	Dependent variable: ln (Market Value)			
	Age 10	Age 11	Age 12	Age 13
Constant	-0.06	-0.11	-0.42	-0.35
	-0.28	*-0.39*	*-1.55*	*-0.98*
ln(1+NAV)	0.89	0.84	0.83	0.73
	19.30	*16.50*	*16.32*	*11.68*
ln(Size)	0.09	0.12	0.13	0.13
	1.56	*1.80*	*2.10*	*1.79*
ln(LastDiv)	-0.09	-0.10	-0.03	-0.16
	-2.00	*-1.86*	*-0.56*	*-2.11*
ln(LastNAV)	-0.25	-0.22	-0.28	-0.18
	-5.88	*-4.26*	*-5.08*	*-2.55*
Profitability Index	-0.03	0.06	0.14	0.36
	-0.60	*0.60*	*1.25*	*2.65*
Adj. R-square	0.68	0.66	0.70	0.65
N-observations	280	226	182	136

Panel B: Summary Statistics

	Liquidated funds			
	Age 10	Age 11	Age 12	Age 13
NAV - Mean	35.32	30.96	21.15	19.76
Size - Mean	121.21	122.77	122.08	123.42
LastDiv - Mean	14.00	12.75	11.97	11.40
LastNAV - Mean	7.39	5.82	5.60	5.26
PI - Mean	0.90	0.92	0.94	0.99
NAV/Size	0.23	0.16	0.09	0.07
MV/NAV	1.00	1.00	1.06	1.13
	Non-liquidated funds			
	Age 10	Age 11	Age 12	Age 13+
NAV - Mean	78.47	48.55	56.09	55.89
Size - Mean	234.81	202.30	140.89	203.93
LastDiv - Mean	37.09	36.58	57.46	41.03
LastNAV - Mean	22.22	23.54	37.62	29.77
PI - Mean	0.82	0.94	0.59	0.81
NAV/Size	0.33	0.24	0.40	0.27
Extrapolated-MV/NAV	0.37	0.32	0.21	0.29
N_obs	79	50	50	434

Table 4: Risk and Abnormal Performance of Private Equity Funds

This table shows results from our 'log-PI-GMM' estimation approach. It reports the abnormal performance (Alpha) and risk loadings using either the CAPM or the three-factor Fama-French model. *t*-statistics are in italics and are obtained by bootstrapping. Panel A shows results for Venture Capital funds and Panel B shows results for Buyout funds. Monthly alpha is shown in spec 1 and spec 2. In spec 3, the underlying model is R-Rf = a_0+a_{size}*ln(Size) +Beta_market*(Rm-Rf).+e In spec 5, the model is R-Rf = a_0+a_{size}*ln(Size) + (b_0+b_{size}*ln(Size))*(Rm-Rf)+e. In spec 4 and 6, the model is the same as in spec 3 and 5 with SMB and HML as extra factors. For specs 3 to 6, Alpha and Beta_Market are evaluated at the first size-quartile (small) and third size-quartile (large) to obtain the alpha and beta of 'small funds' and 'large funds'.

Panel A: Venture Capital Funds

	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5	Spec 6
Alpha (%)	-1.25%	-0.69%				
	-23.25	*-1.97*				
a_0			-2.14	-2.08	-1.61	-1.05
			-9.73	*-5.71*	*-3.78*	*-1.34*
a_{size}			0.18	0.18	0.08	0.02
			3.69	*3.70*	*0.91*	*0.12*
Beta_Market	3.21	2.57	2.70	2.85		
	15.19	*5.00*	*8.95*	*5.35*		
b_0					0.15	-0.62
					0.11	*-0.31*
b_{size}					0.57	0.67
					1.85	*1.65*
Beta_SMB		0.99		-0.26		0.46
		1.21		*-0.43*		*0.82*
Beta_HML		-0.56		-0.29		-0.17
		-1.40		*-0.82*		*-0.47*
Number obs.	686	686	686	686	686	686
Alpha – Small funds	_	_	-1.53	-1.48	-1.34	-1.00
Alpha – Large funds	_	_	-1.30	-1.25	-1.24	-0.98
Beta – Small funds	_	_	_	_	2.04	1.60
Beta – Large funds	_	_	_	_	2.74	2.43

Panel B: Buyout Funds

	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5	Spec 6
Alpha (%)	0.49%	0.13%				
	1.31	*0.24*				
a_0			-0.08	-0.35	2.63	2.21
			-0.14	*-0.51*	*3.64*	*-1.44*
a_{size}			0.07	0.08	-0.39	-0.35
			0.93	*1.13*	*-3.56*	*-1.51*
Beta_Market	0.33	0.94	0.23	0.57		
	0.82	*1.90*	*0.60*	*1.07*		
b_0					-3.37	-3.16
					-4.84	*-1.74*
b_{size}					0.64	0.64
					5.51	*2.10*
Beta_SMB		-2.05		-1.78		-1.58
		-3.06		*-2.97*		*-2.58*
Beta_HML		-0.16		0.13		0.34
		-0.14		*0.13*		*0.36*
Number obs.	272	272	272	272	272	272
Alpha – Small funds	_	_	0.22	-0.03	1.02	0.77
Alpha – Large funds	_	_	0.36	0.11	0.32	0.15
Beta – Small funds	_	_	_	_	-0.74	-0.54
Beta – Large funds	_	_	_	_	0.42	0.60

Table 5: Impact of Final NAVs on GMM estimates

This table shows results from our 'log-PI-GMM' estimation approach when final NAVs are treated as either market value or are written-off. Monthly alpha is shown.

	Final NAV as Market Value				Final NAV written off			
	Venture Capital		Buyout		Venture Capital		Buyout	
	Spec 1	Spec 2	Spec 1	Spec 2	Spec 1	Spec 2	Spec 1	Spec 2
Alpha	-1.08%	-0.60%	0.37%	0.19%	-1.27%	-0.51%	0.62%	0.13%
	-17.45	*-2.46*	*1.14*	*0.43*	*-21.68*	*-1.35*	*1.64*	*0.11*
Beta	3.45	2.58	0.59	1.06	2.98	2.27	0.14	0.75
	18.16	*7.35*	*1.54*	*2.29*	*10.73*	*4.84*	*0.36*	*0.96*
SMB		1.35		-1.57		1.32		-2.56
		3.04		*-2.81*		*1.67*		*-2.52*
HML		-0.34		-0.08		-0.64		0.40
		-1.03		*-0.10*		*-1.49*		*0.24*

Table 6: Robustness Tests

This table shows robustness tests for the GMM estimation of the CAPM specification (section 3.2). Moment conditions are either weighted by the number of funds per FoF (Equally Weighted) or by the sum of the size of the funds entering the moment condition (Value Weighted Moments). Results are shown for different time periods, for different number of fund-of-funds, for the sub-sample of US focused funds. Results are also shown when cash flows are divided by capital invested for each fund, and for the extended-PI method.

	Equally Weighted Moments				Value Weighted Moments			
	VC		BO		VC		BO	
	Alpha	Beta	Alpha	Beta	Alpha	Beta	Alpha	Beta
Default (Table 4 for EW)	-1.24	3.21	0.49	0.33	-1.22	3.13	0.06	0.75
Number FoFs per vint. year								
Default + 1 (2 FoFs)	-1.38	2.88	0.25	0.31	-1.24	3.12	-0.02	0.82
Default + 2 (3 FoFs)	-1.40	2.76	0.11	0.28	-1.23	3.18	0.03	0.73
Default + 3 (4 FoFs)	-1.38	2.65	-0.10	0.57	-1.23	3.17	-0.02	0.80
Vintage cut (default is 1993)								
Default + 1 (1980-1994)	-1.24	3.22	0.63	0.16	-1.21	3.14	0.17	0.57
Default – 1 (1980-1992)	-1.26	3.15	0.47	0.38	-1.23	3.07	0.00	0.83
US focus only	-1.10	3.55	0.83	0.11	-1.11	3.50	0.37	0.50
Per dollar invested								
(cash flows/capital invested)	-1.28	3.06	0.17	0.43	-1.31	3.11	1.10	-0.19
Extended-PI method	-1.20	3.19	0.97	-0.02	-1.15	3.04	0.46	0.41

Table 7: NAV-based regression approach

This table shows results from NAV-based time-series regressions. The dependent variable is the quarterly excess return computed as ($\sum NAV_t + \sum Dividends_{(t\text{-}1 \text{ to } t)} - \sum Capital\_Invested_{(t\text{-}1 \text{ to } t)}$) / $\sum NAV_{t-1} - 1 - Rf$), where the sum is taken across all funds. Independent variables are contemporaneous and lagged quarterly excess S&P 500 returns, contemporaneous and lagged SMB returns and HML returns. In specification 5, the first two years are skipped. Robust t-statistics are reported in italics. Alpha is annualized and the sum of betas (contemporaneous plus lagged betas) is shown under each specification. In Panel A, returns are from the first quarter of 1981 to Q4 2003 for VCs and from Q1 1985 to Q4 2003 for BOs. Only funds raised between 1980 and 1993 (for VCs) and 1984 to 1993 (for BOs) are included (our working sample). In Panel B, all funds raised between 1980 and 1999 are included and returns are from Q1 1981 to Q2 2002 (Jones and Rhodes-Kropf, 2003) sample.

Panel A: Our working sample

	Venture Capital Funds					Buyout Funds				
	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5
Alpha (per quarter)	0.05	0.04	0.01	0.05	0.05	0.07	0.06	0.05	0.05	0.05
	2.84	*2.35*	*0.95*	*2.50*	*2.46*	*2.95*	*2.64*	*1.94*	*1.68*	*1.38*
Beta (t)	0.19	0.19	0.19	0.10	0.13	0.13	0.13	0.15	0.16	0.28
	3.37	*3.34*	*3.77*	*1.91*	*2.27*	*1.84*	*1.94*	*2.54*	*2.19*	*4.02*
Beta (t-1)		0.09	0.09	0.02	0.06		0.09	0.09	0.07	0.17
		1.64	*1.84*	*0.40*	*0.99*		*1.79*	*1.72*	*0.94*	*2.06*
Beta (t-2)			0.09	0.06	0.09			0.02	0.02	-0.02
			2.08	*1.20*	*1.54*			*0.25*	*0.15*	*-0.18*
Beta (t-3)			0.15	0.10	0.12			0.17	0.17	0.16
			3.98	*1.90*	*2.37*			*3.23*	*2.33*	*2.10*
Beta (t-4)			0.14	0.14	0.12			-0.03	-0.03	-0.08
			2.18	*2.21*	*1.89*			*-0.27*	*-0.25*	*-0.55*
β_{SMB} (t)				0.21	0.24				-0.07	-0.03
				2.66	*3.18*				*-0.69*	*-0.28*
β_{HML} (t)				-0.05	-0.08				0.02	0.05
				-0.56	*-0.80*				*0.21*	*0.50*
Lagged β_{SMB}, β_{HML}	no	no	no	yes	yes	no	no	no	yes	yes
∑ Beta	0.19	0.28	0.66	0.43	0.51	0.13	0.23	0.40	0.39	0.51
∑ β_{SMB}				0.18	0.27				-0.21	0.06
∑ β_{HML}				-0.41	-0.42				0.02	0.08
Adj. R^2	0.11	0.12	0.27	0.41	0.43	0.04	0.05	0.11	0.08	0.14
N_obs	88.00	88.00	88.00	88.00	80.00	80.00	80.00	80.00	80.00	64.00

Panel B: Jones and Rhodes-Kropf (2003) sample

	Venture Capital Funds					Buyout Funds				
	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5	Spec 1	Spec 2	Spec 3	Spec 4	Spec 5
Alpha (per quarter)	0.05	0.04	-0.01	0.10	0.08	0.08	0.08	0.06	0.06	0.05
	1.92	*1.38*	*-0.38*	*2.33*	*2.20*	*3.46*	*3.07*	*2.28*	*1.69*	*1.57*
Beta (t)	0.37	0.37	0.36	0.16	0.22	0.16	0.16	0.15	0.14	0.19
	2.95	*2.94*	*3.38*	*2.36*	*3.17*	*2.13*	*2.16*	*2.48*	*1.70*	*2.07*
Beta (t-1)		0.13	0.13	-0.02	0.06		0.05	0.05	0.06	0.11
		1.14	*1.20*	*-0.15*	*0.57*		*0.71*	*0.85*	*0.80*	*1.50*
Beta (t-2)			0.20	0.11	0.17			0.07	0.03	0.06
			2.50	*1.41*	*2.21*			*1.00*	*0.28*	*0.68*
Beta (t-3)			0.21	0.09	0.15			0.17	0.18	0.21
			2.89	*1.02*	*1.96*			*2.75*	*1.86*	*2.07*
Beta (t-4)			0.25	0.25	0.23			0.03	0.07	0.06
			1.50	*1.79*	*1.64*			*0.33*	*0.45*	*0.39*
β_{SMB} (t)				0.30	0.38				-0.03	0.02
				2.37	*3.17*				*-0.35*	*0.24*
β_{HML} (t)				-0.28	-0.33				-0.04	-0.03
				-1.57	*-1.89*				*-0.46*	*-0.35*
Lagged β_{SMB}, β_{HML}	no	no	no	yes	yes	no	no	no	yes	yes
$\sum$ Beta	0.37	0.50	1.15	0.59	0.83	0.16	0.21	0.48	0.47	0.64
$\sum \beta_{SMB}$				0.41	0.68				-0.16	0.11
$\sum \beta_{HML}$				-1.06	-1.08				0.01	0.07
Adj. R^2	0.13	0.13	0.26	0.49	0.56	0.06	0.05	0.11	0.07	0.09
N_obs	85.00	85.00	85.00	85.00	77.00	85.00	85.00	85.00	85.00	77.00

Figure 1: GMM goal function as a function of alpha and beta for the NPV method in a simulated economy.

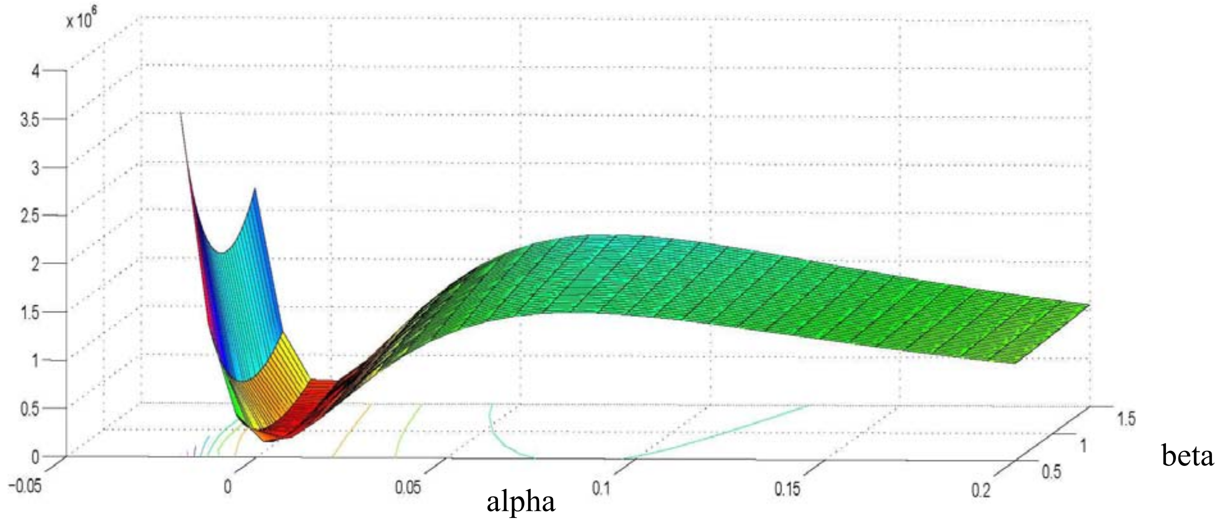


Figure 2: GMM goal function as a function of alpha and beta for the log-PI method in a simulated economy.

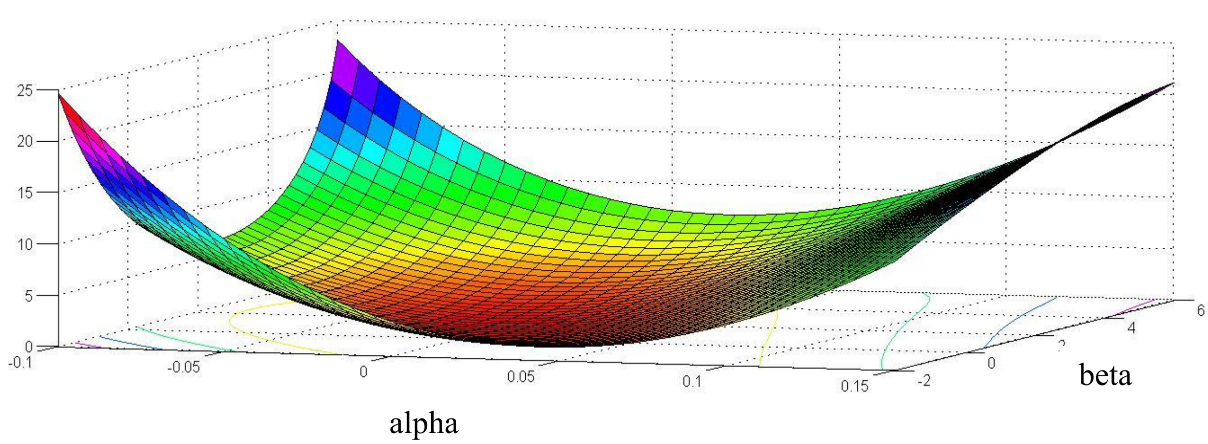
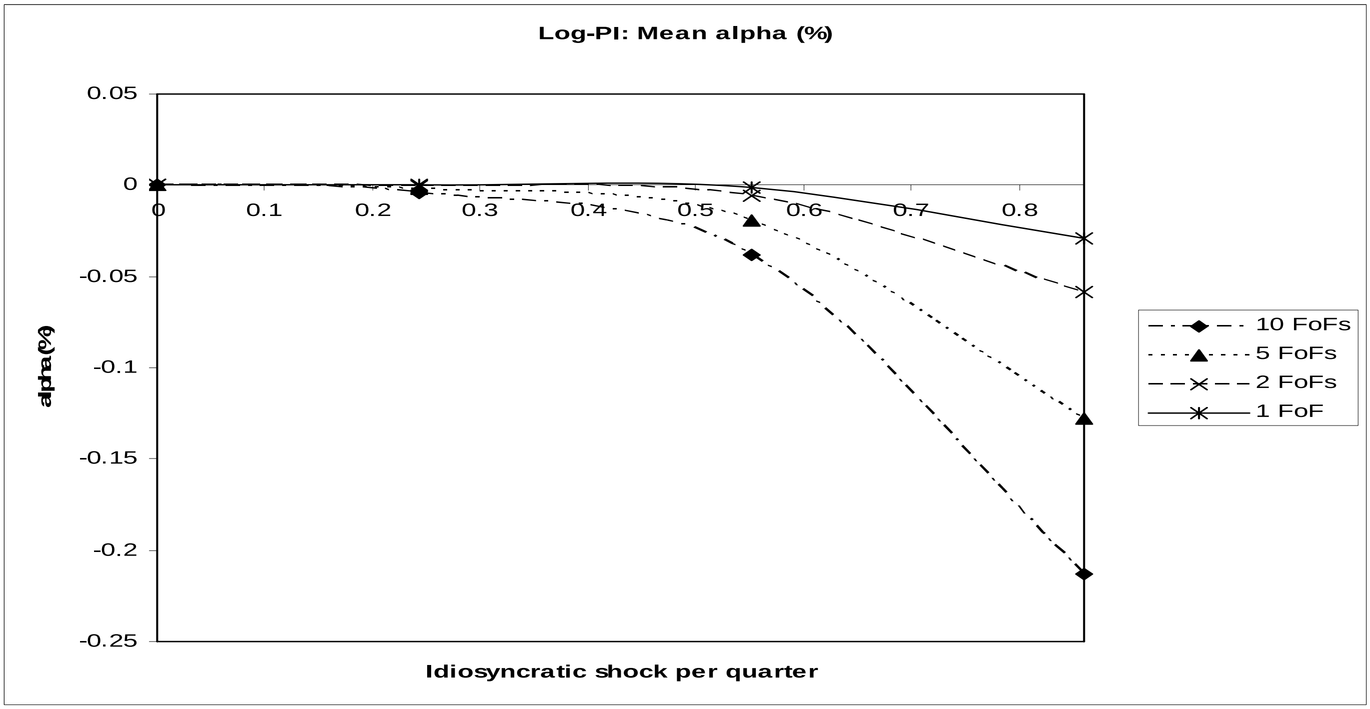


Figure 3: Mean estimated alpha (Panel A) and beta (Panel B) in the simulated economy (see Table 2) as a function of the idiosyncratic volatility. Method used is 'log-PI-GMM'.

Panel A: Mean estimated monthly alpha (%) with different number of FoFs



Panel B: Mean estimated beta with different number of FoFs

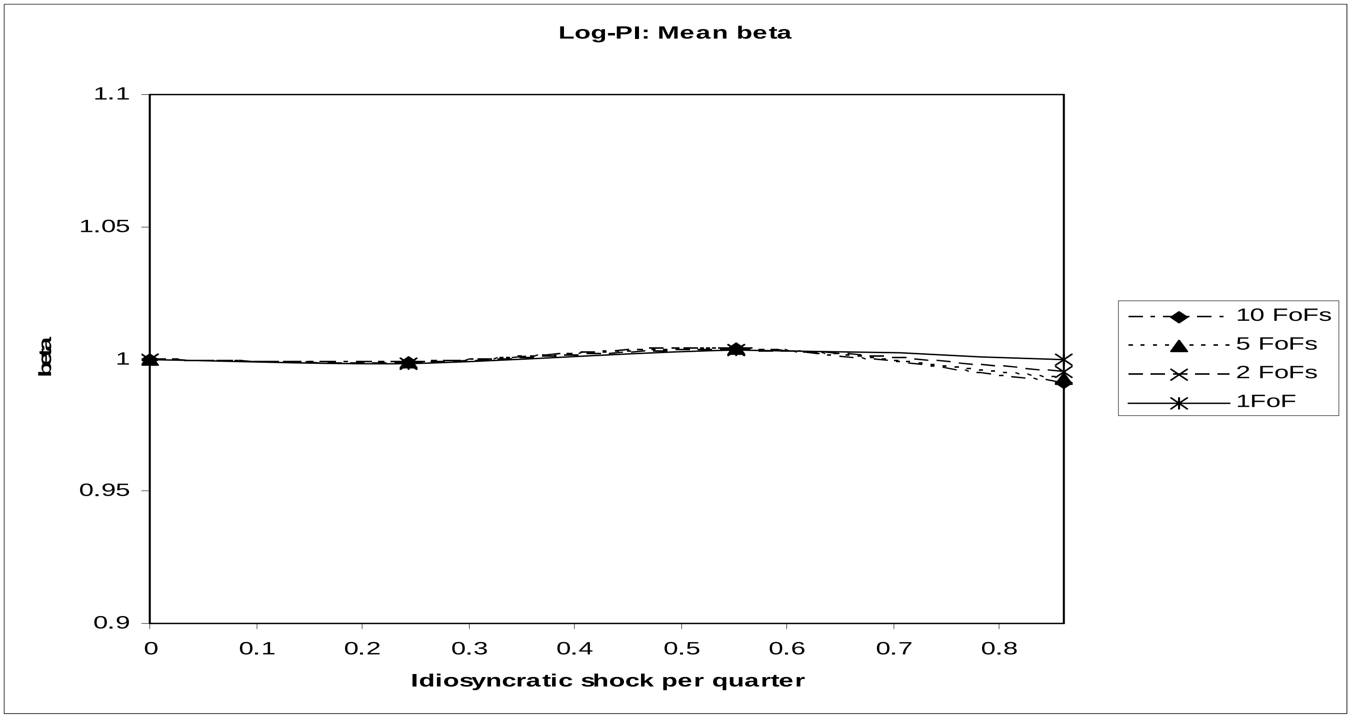


Figure 4: Fund dividend yields (average of the next 12 months dividend yields of funds in their 4th to 10th year). Dividend yield is the sum of the dividends paid divided by fund size. S&P 500 returns are the 5 years cumulated returns, divided by 5. Time spans 1990 to 2003.

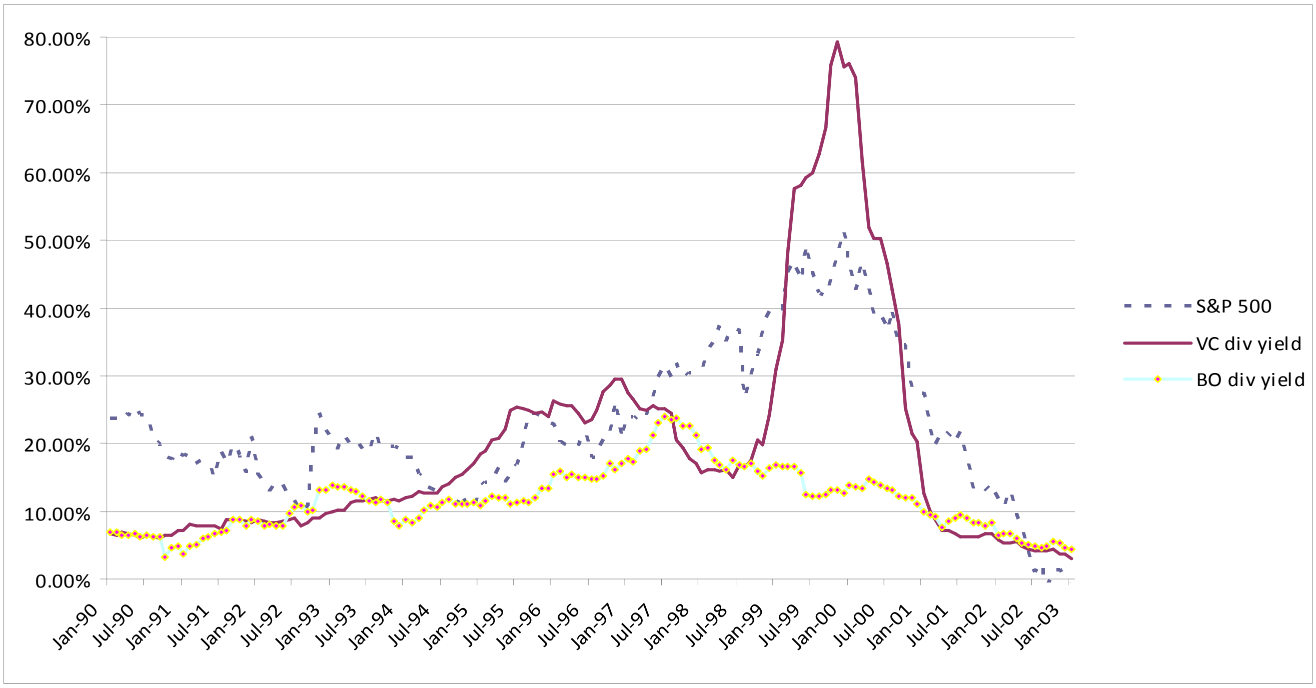


Figure 5: Quarterly returns based on NAVs for Venture Capital funds (plain line) and Buyout funds (dotted line).