GUY CARPENTER

John A. Major, ASA, MAAA

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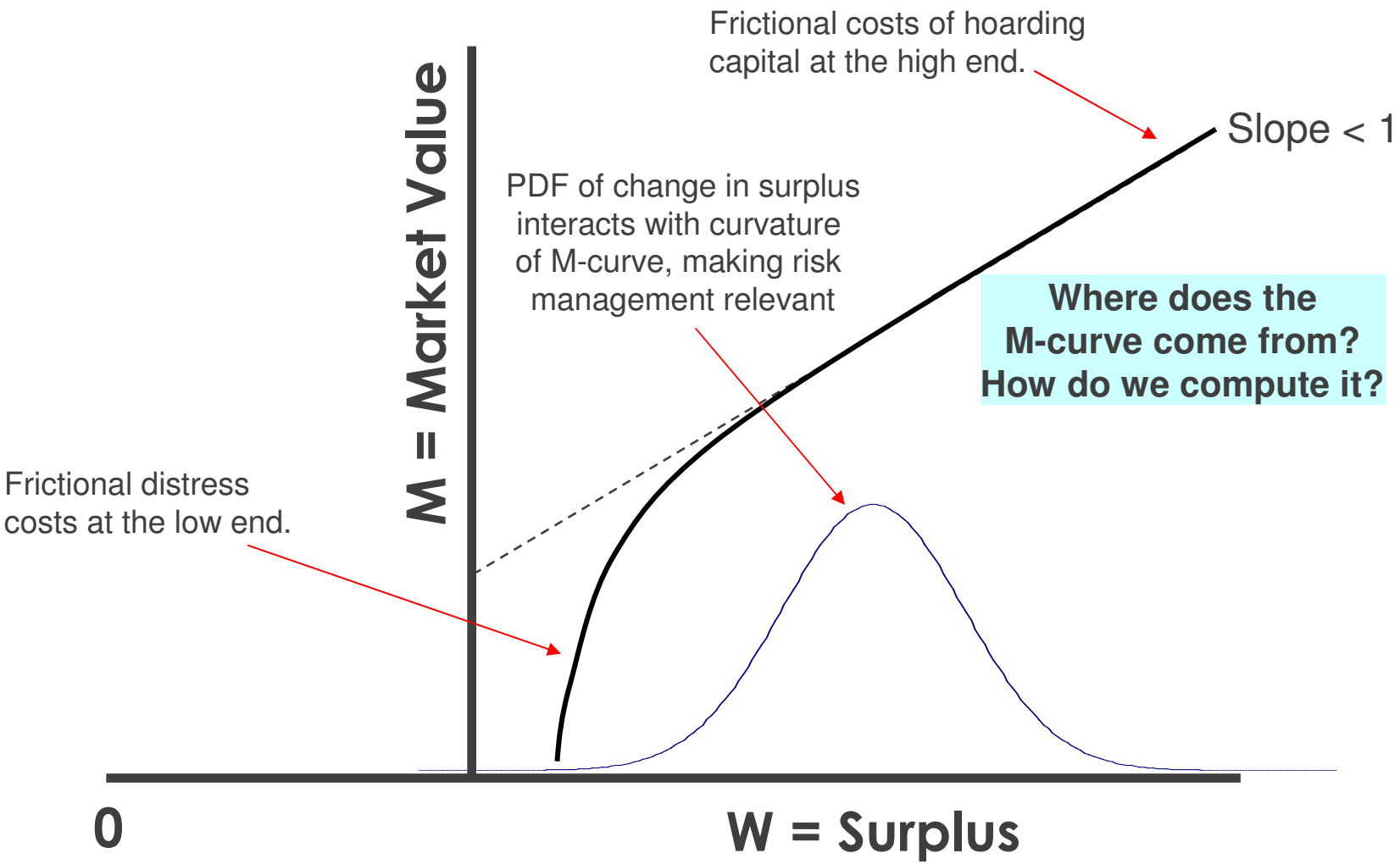
On a connection between Froot-Stein and optimal dividends models

john.major@guycarp.com

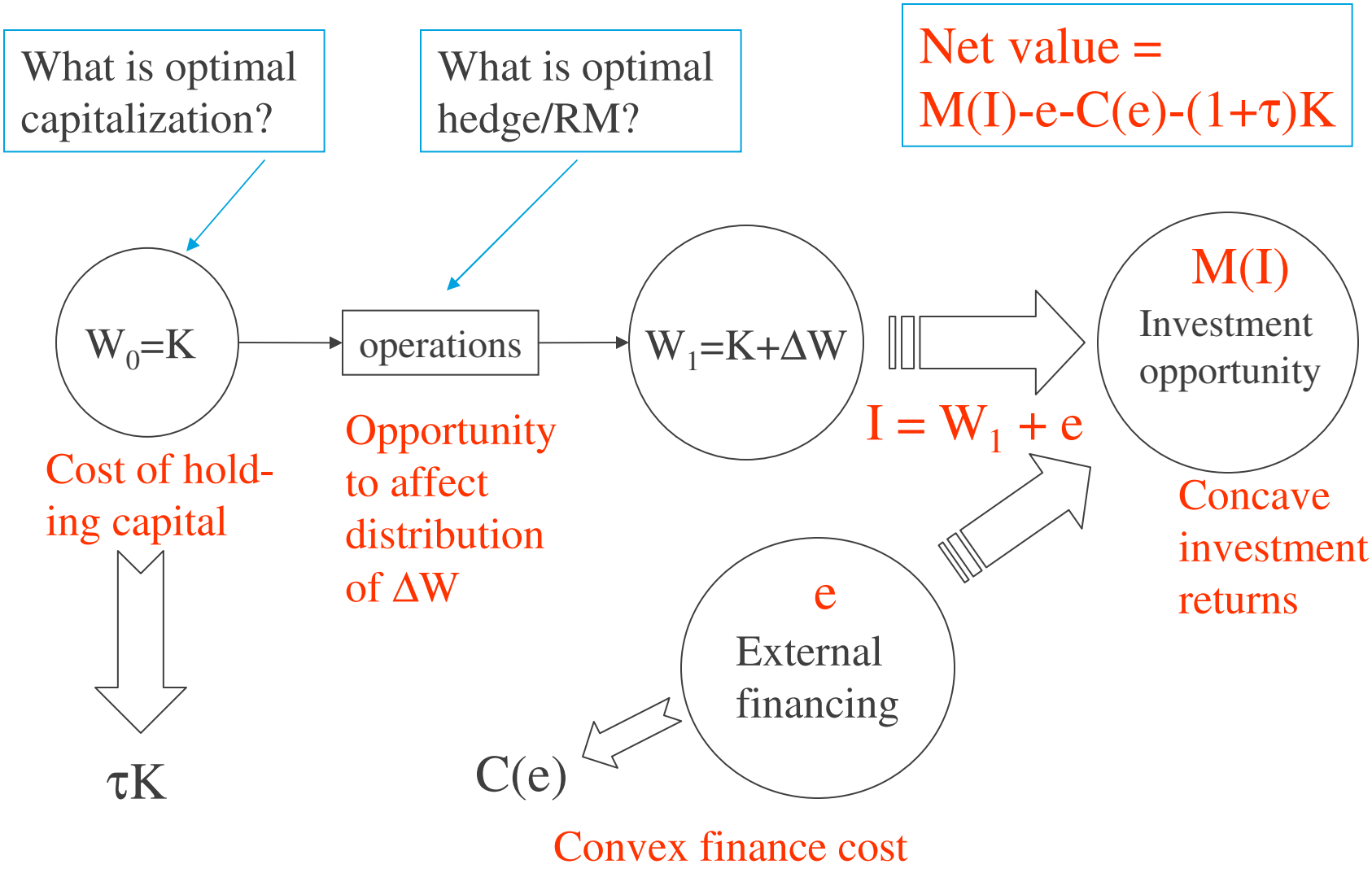
Objective

- “A credible, recognized capability to estimate and assess the shareholder value of risk in an insurance company.”
- Collapse 2-D (risk,reward) to 1-D (reward)
- The “M-curve”

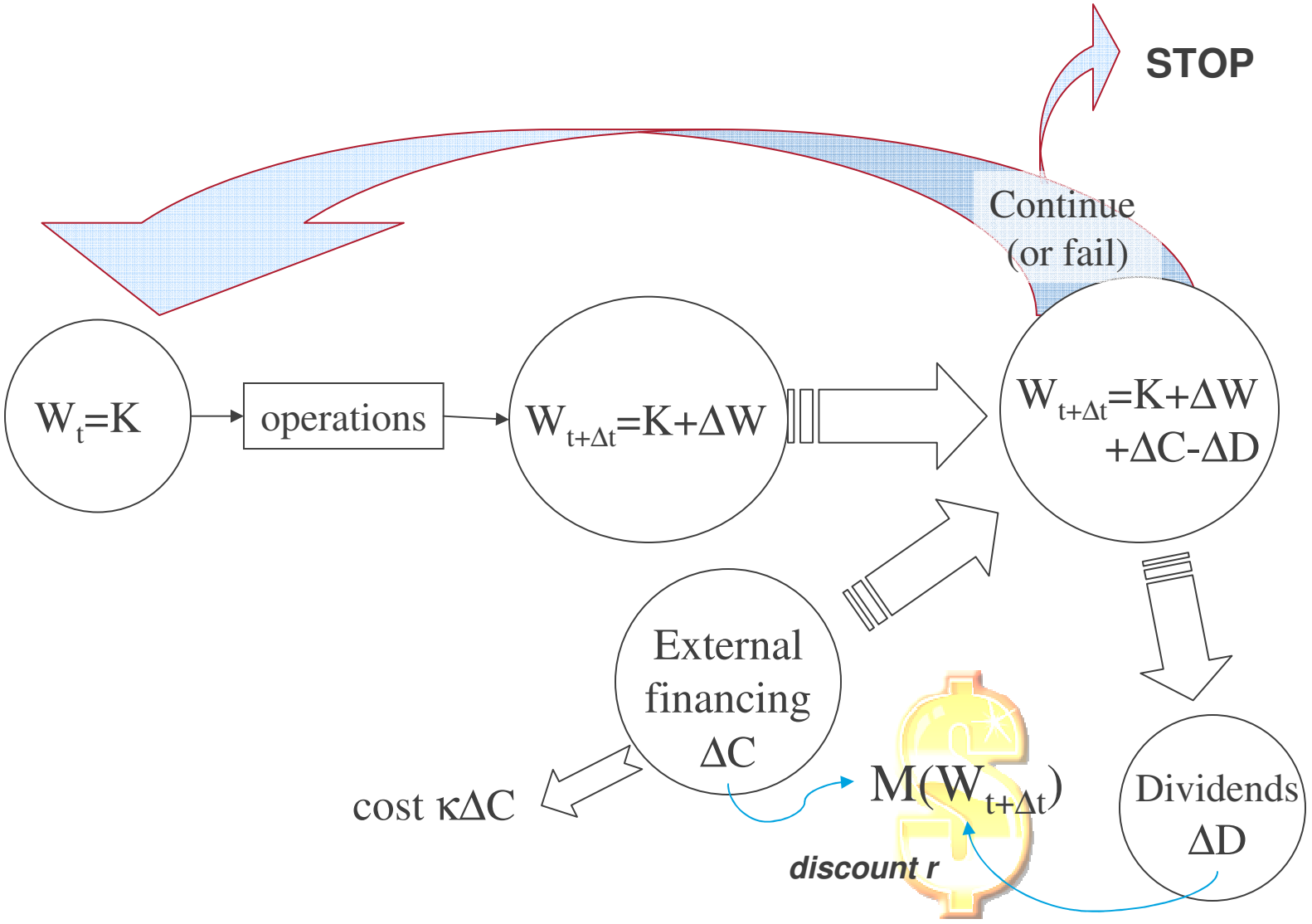
The M-curve (Froot, Venter, Major 2005)
Shareholder value of the firm M as function of wealth W



Froot et al. model



Closing the loop in Froot et al.



Recursive definition of M-curve (Bellman Equation)

take money out

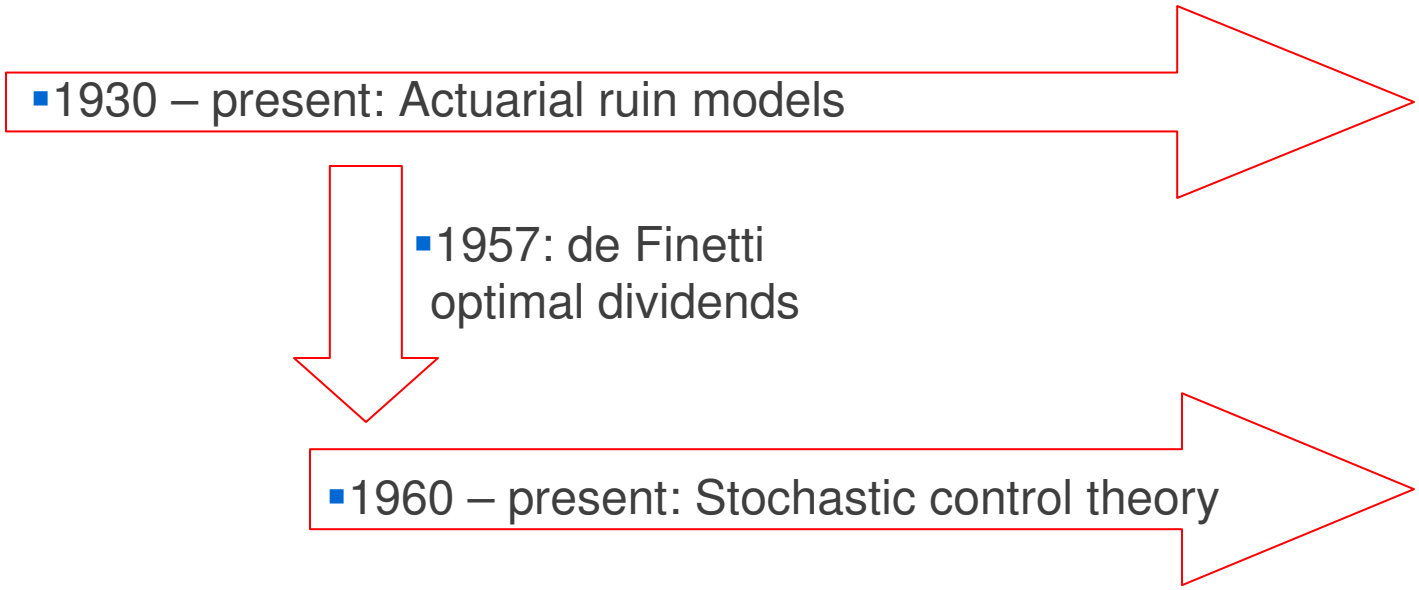
or put money in

$$M(W_t) = \max_{U} \begin{Bmatrix} M(W_t - \Delta D) + \Delta D, \\ M(W_t + \Delta C) - (1+\kappa)\cdot \Delta C, \\ e^{-r\cdot\Delta t}\cdot E\left[M(W_{t+\Delta t}) \mid W_t, U\right] \end{Bmatrix}$$

or let it ride

with your best RM and capital strategy

The optimal dividends literature

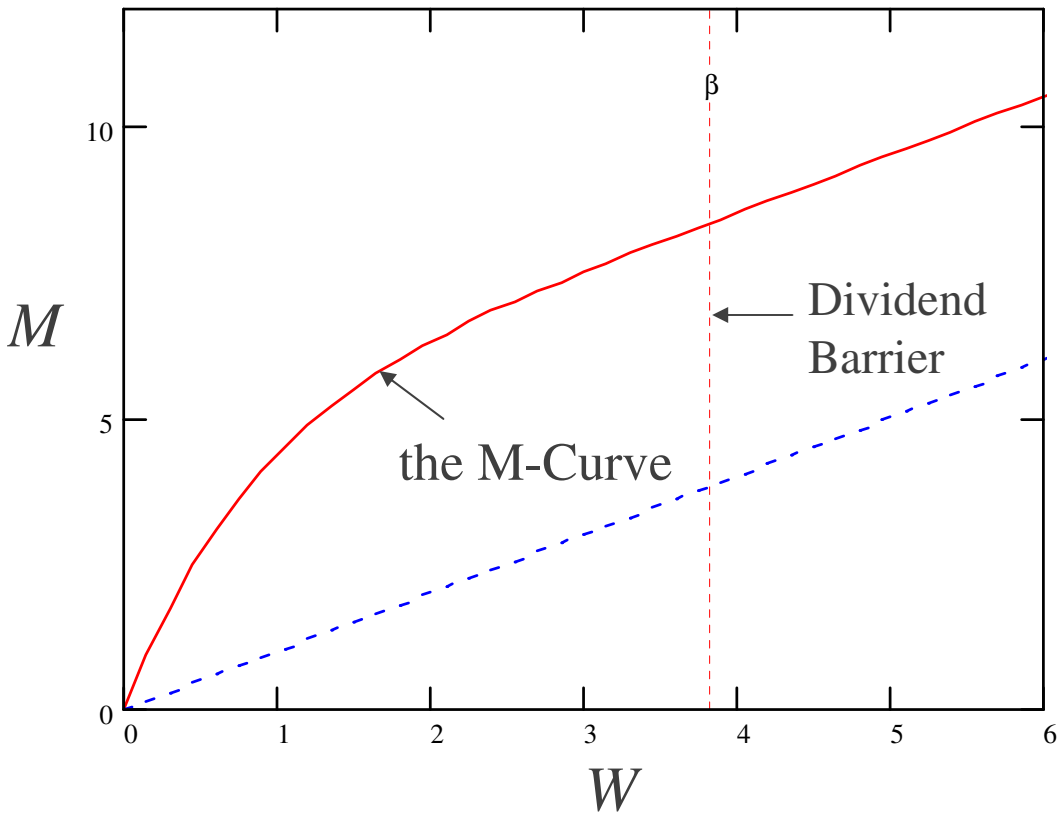


- Still focused on analytical solutions (not ready for application)
- ***NO*** attempt to justify w.r.t. financial economics
  - ***Little*** explanation of relationship to M&M (just invoke DDM)
  - ***NO*** explanation of relationship to frictions, etc.

Simplest continuous-time example (circa 1960)

$$dW_t = \mu \cdot dt - \sigma \cdot dB_t - dD_t$$

Brownian Motion



My contribution

- Represent evolution of W suitable for insurance
  - Risk as jump-diffusion process
  - Include XOL as risk management

- Develop software for numerical solution
  - Fast enough, accurate enough

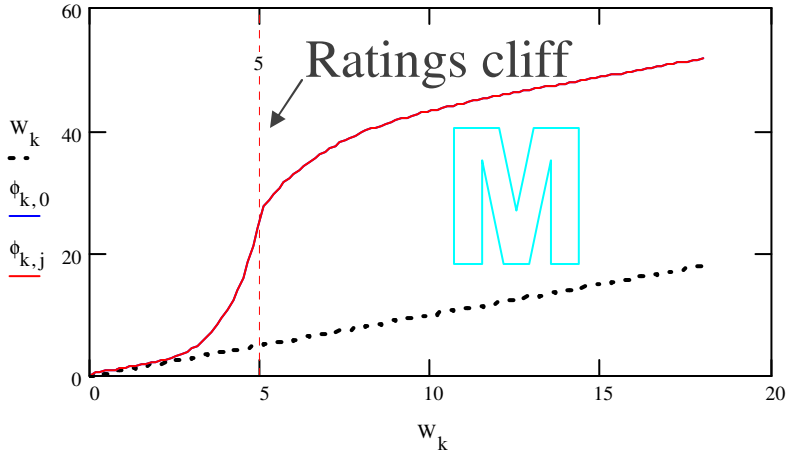
- Establish credibility by publishing in a *finance* journal

More sophisticated example (numerical solution only)

- Risk
  - Compound Poisson representing cats
  - Ratings “cliff”
    - Expected profit above
    - Expected loss below (but c/b lucky)
- Risk management
  - Available reinsurance: 1-XS-3 XOL layer
  - Premium 4.4x actuarial value
  - Can buy and pay portion 0-100% (decision variable “U”)
- Questions
  - Market value?
  - Optimal capital level?
  - When, how much to use reinsurance?
- Solve at various levels of external capital cost factor κ

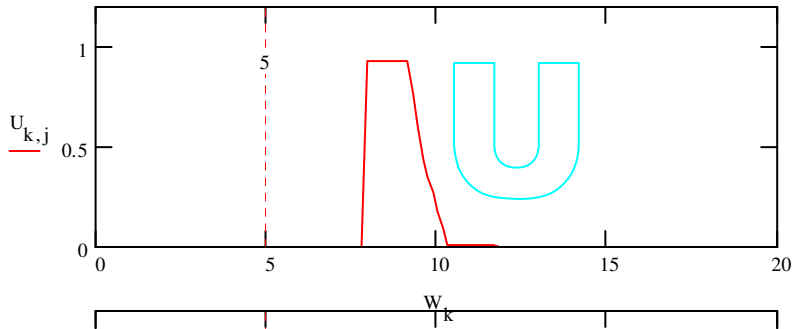
Solution (no ext. capital)

Value of the firm



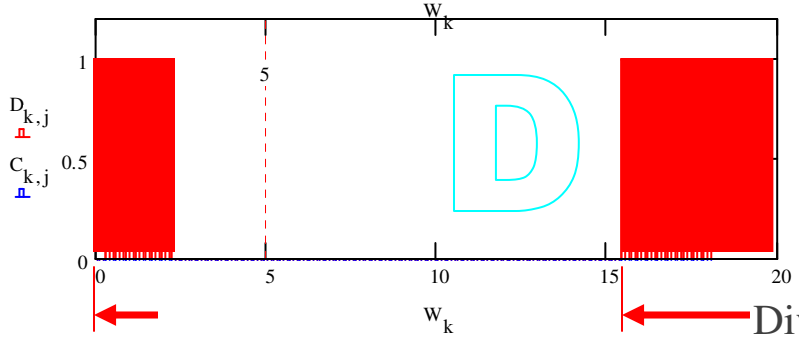
Franchise value M-W climbs rapidly around cliff, then levels off. Constant above W=15.4

Rein-surance strategy



Purchase reinsurance when W between 8, 10. Above, not worth it; below, not effective enough

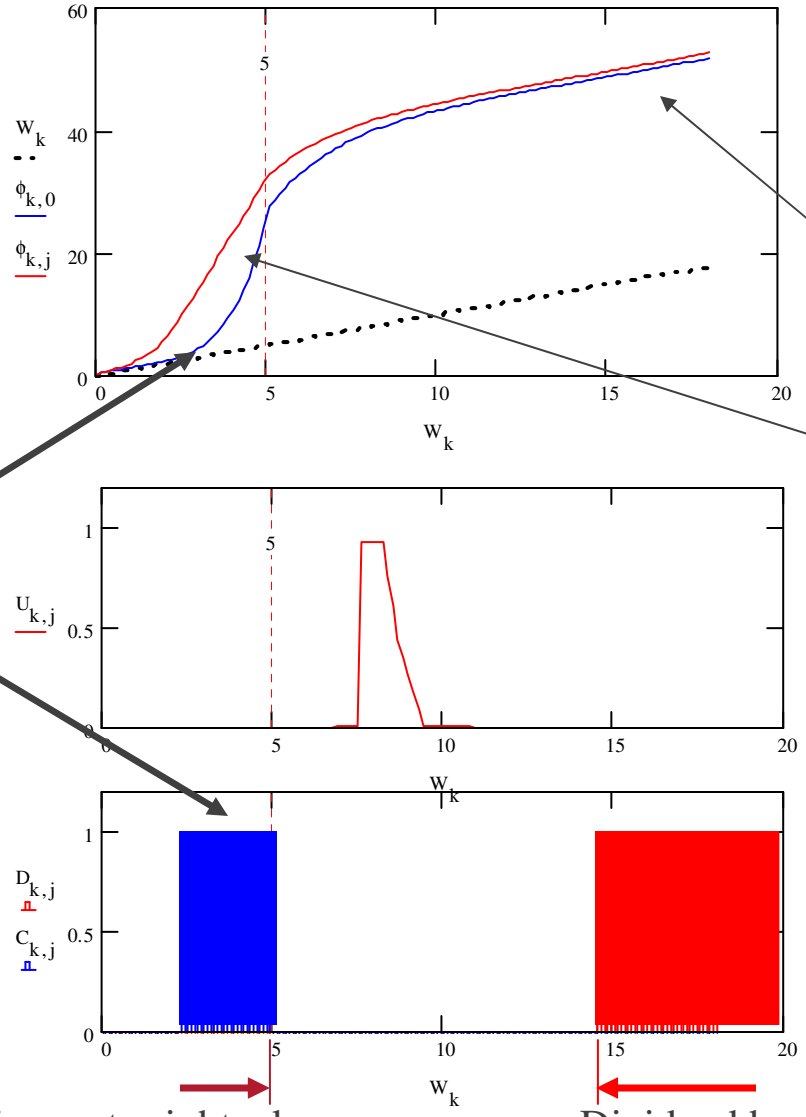
Dividend strategy



Optimal capital = 15.4; dividend above that, retain earnings below. If W<2, go out of business.

Dividend back to left edge

Solution: $\kappa = 8$



M-curve has sufficient slope to allow giving up 8x capital gain when just under the ratings cliff

Still pretty costly (anything over 30 is effectively infinite) so the boost to high-W franchise value is relatively small.

Significant in low-W (distress) however.

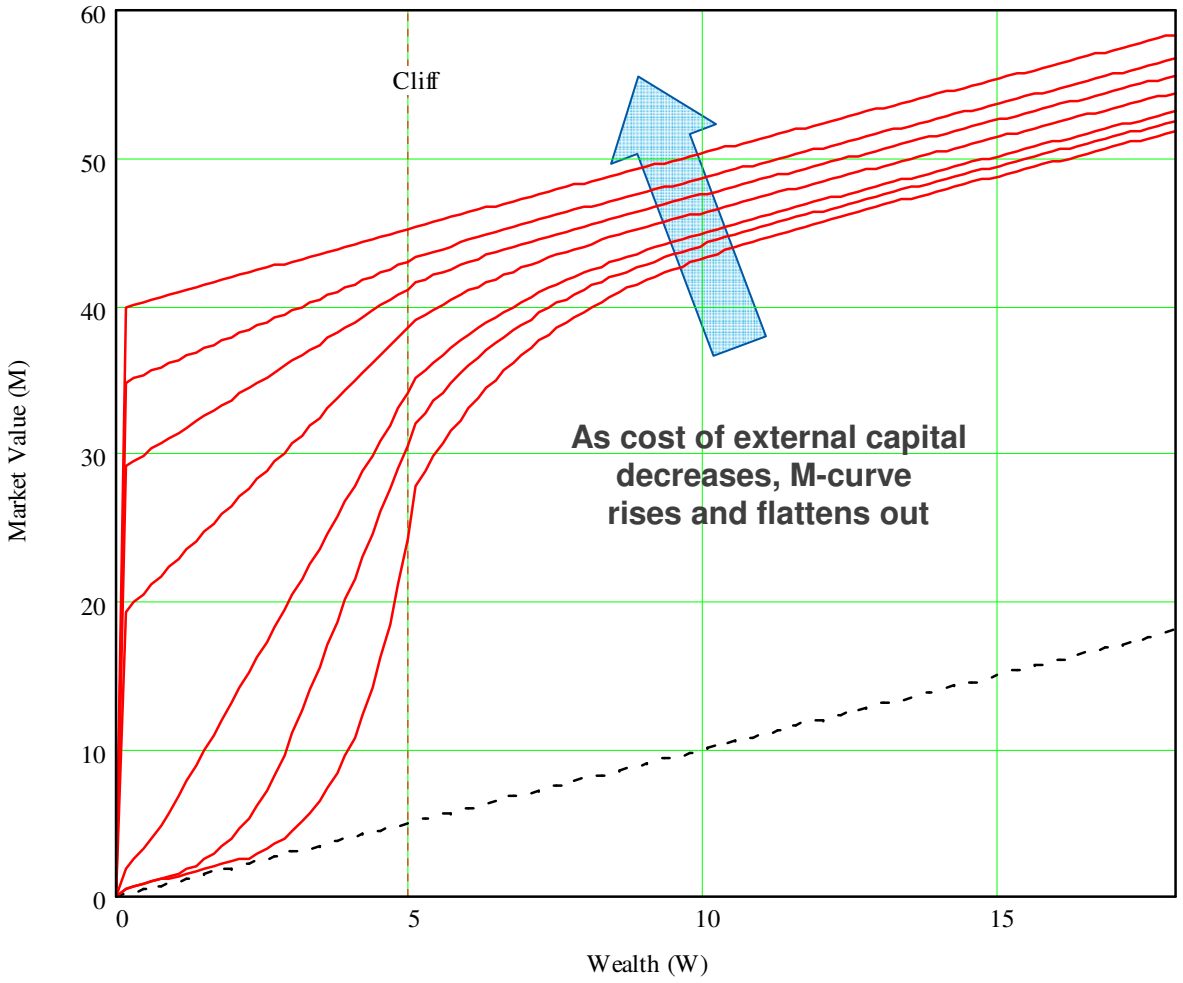
XOL is used a bit less than with no-recap solution

Optimal capital level has shifted down to below 15

Recapitalize up to right edge

Dividend back to left edge

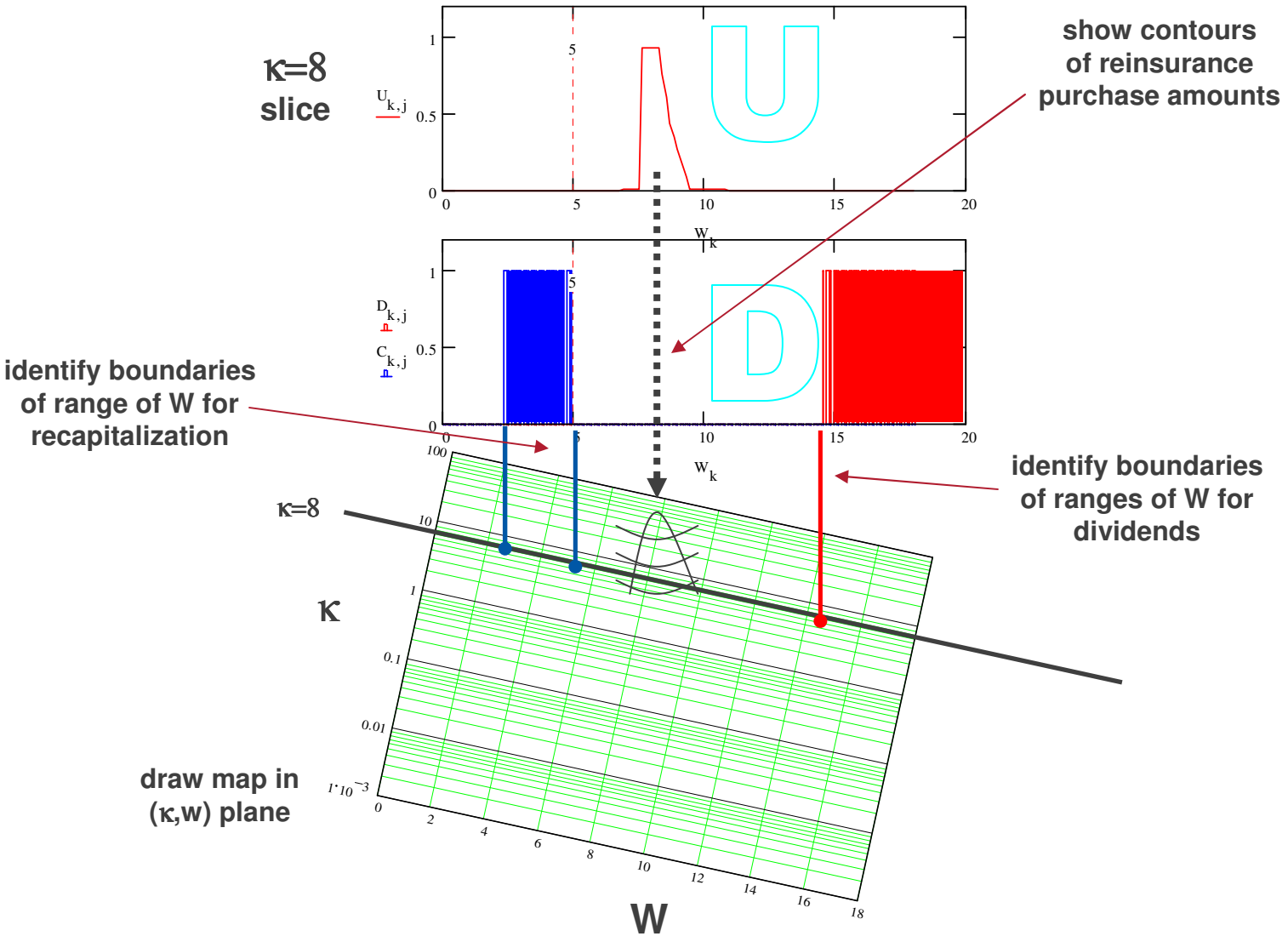
Big picture: stacking all the M-Curves together



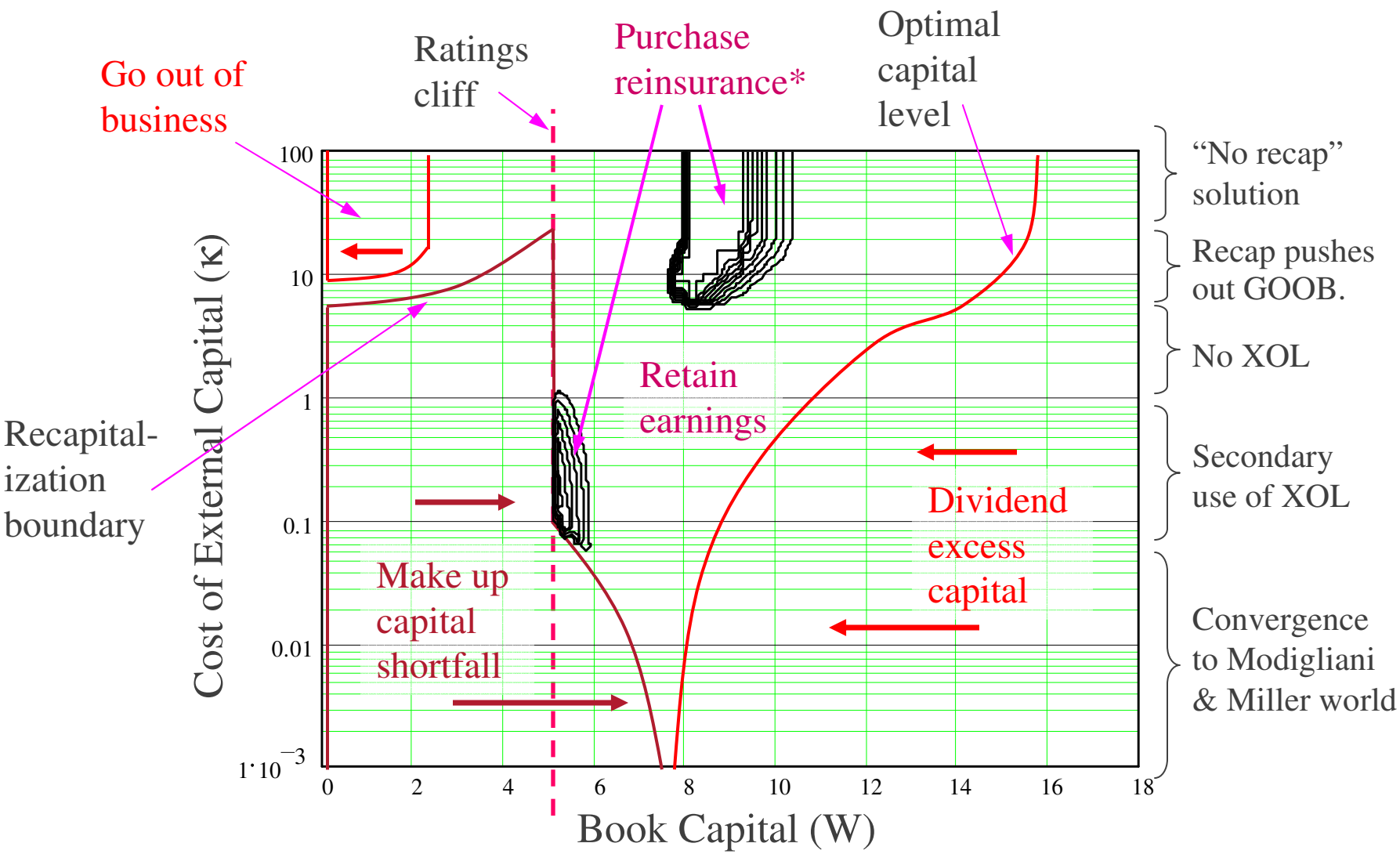
From top to bottom, κ =

0.1
0.7
1.5
3
6
9
30 (=∞)

Big picture: stacking the capital and reinsurance strategies



Big Picture: Capital and Reinsurance Strategy



* Reinsurance purchase contours start at 0.1 on outside and increase by 0.1 at each inward step.

Experience to date

- 10s of models, 1000s of solutions
- Submitted to finance journals
  - *J. Finance*, rejected in 5 days
  - *J. Risk & Insurance*, sent back for revisions
- Sophisticated, paying customer
  - *correct treatment of valuation rate (r) very important*