Financing Risk Transfer under Governance Problems: Mutual versus Stock Insurers*

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Abstract

Mutual insurance companies and stock insurance companies are different forms of organized risk sharing: policyholders and owners are two distinct groups in a stock insurer, while they are one and the same in a mutual. This distinction is relevant to raising capital and selling policies under governance problems. In the presence of an owner-manager conflict, capital is costly. Free-rider and commitment problems limit the degree of capitalization that a stock insurer can obtain. The mutual form, by tying sales of policies to the provision of capital, can overcome these problems at the cost of less diversified owners.

Key Words ownership structure, insurance, owner-manager conflict, capital, default

JEL Classification G22, G32

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1 Introduction

An insurance corporation "organizes" risk sharing between individuals, and in principle, there are two different ways to do this. First, risks can be shared within the pool of policyholders. A prominent example is a mutual insurer where policyholders are also the owners of the insurance corporation. In this case, policyholders have participating contracts as all participate in the insurer's surplus. Second, risks can be transferred from policyholders to another group of individuals (investors). Stock insurers, for example, transfer risks from policyholders to shareholders, i.e. the capital market. This transfer is achieved through the separation of rights to profits and rights to indemnity claims, thereby decoupling owners and customers (policyholders).

A large body of the insurance literature assumes that the level of aggregate future claims payment is certain. In this case, an actuarially fair premium equals the expected indemnity payment, and no capital beyond premiums is needed. Moreover, there is no insolvency and, with full insurance, the actuarially fair premium equals the expected loss. This implies that there is no difference between a mutual insurer and a stock insurer with respect to the required capital, the fair premium, and the distribution of risk. In this context, it is irrelevant whether rights to profits and rights to indemnity claims or, equivalently, owners and customers, are separated or not.

A different picture emerges if the level of total claims payment is uncertain. Then, the average policyholder's claim can be higher or lower than the insurer's total capital, which includes risk capital (equity) and premiums. If the policyholders' claims are higher than the total capital, the company is insolvent and total capital is distributed among policyholders. If claims are lower, the excess funds accrue to the owners of the insurer. Risk capital is important to reallocate funds from states where total claims are lower than total premiums to states where the reverse holds. The way that risk capital is raised within these two organizational forms is markedly different. A stock insurer first raises risk capital from investors (shareholders) and then sells insurance policies; while a mutual insurer raises risk capital through premiums (raising risk capital is tied to selling insurance contracts). We show that these different forms of raising capital have different consequences in the presence of governance problems.

The governance problem in our paper stems from a conflict between the manager and owners, and this conflict exists for both stock and mutual insurers. The manager is able to expropriate a fraction of the surplus, and the firm returns only the remaining surplus to owners (shareholders or policyholders). Thus, it costs the same to provide a given amount of capital either through issuing shares or through premiums. The main difference between the stock and the mutual forms is that it is possible under the mutual form to restrict sales of policies to those who also provide capital. If a stock insurer raises capital before selling policies then competition in the insurance market restricts the return to shareholders. Without market frictions the timing is not crucial. In contrast, with frictional costs of capital, the insurer incurs the cost of capital before premiums are determined. This imposes an upper bound on the level of reimbursement for the (sunk) frictional cost of capital and thereby on the capital the stock insurer can raise. For policyholders it would be optimal to collectively provide additional capital, despite the frictional cost, to improve risk sharing. Thus, the increase in utility from improved risk sharing compensates policyholders for a negative (unfair) return on the invested capital. However, if buying insurance policies and providing capital are separated, each policyholder has an incentive to free-ride on the capital provided by the others and will not provide capital. For a mutual, the sales of policies and provision of capital are linked, and the free-rider problem is overcome. This benefit of the mutual comes at the cost of less diversified owners.

The model provides a set of interesting predictions related to this trade-off. The advantage of the mutual form arises when insurers have to raise capital, e.g. after large catastrophes or when a company is founded, while the advantage of the stock form in spreading risks is important if risks are large, e.g. due to high variance or correlation of losses. The trade-off is also influenced by regulation that differentially strengthens either shareholders' or policyholders' rights and by regulation related to capital requirements. Strengthening policyholders' rights and high initial capital requirements increases the problem of raising sufficient capital under the stock form. In contrast, strengthening shareholders' rights and regulation aimed at maintaining a minimum level of capital reduces the disadvantage of the stock insurer by reducing the governance problem and the need to raise capital.

Our paper contributes to an understanding of the differences between stock and mutual insurers. The existing literature discusses two main differences: differences in risk bearing (participating versus non-participating contracts) and differences in governance (reducing the owner-customer conflict versus reducing the owner-manager conflict). Smith and Stutzer (1990, 1995) focus on the different contractual structures of insurance contracts offered by mutual insurers (participating contract) and stock insurers (indemnity payment with a flat fee). They argue that undiversifiable risk drives participating contracts and that these contracts reduce problems of adverse selection and moral hazard. Mayers and Smith (1981, 2005) focus on governance issues and argue that different organizational forms have different advantages in dealing with different types of agency problems. The stock form is better suited to reducing the owner-manager conflict through the market for corporate control, whereas the mutual form internalizes the owner-customer problem at the expense of a higher owner-manager conflict. In our model, both organizational forms face an identical owner-manager conflict: the manager can extract a fraction of the surplus. The distinction that is central to our discussion is how the separation and non-separation of owners and customers differentially impacts the raising of capital and selling of policies in the presence of this agency problem. Thus, while the focus of Mayers and Smith is on how different organizational forms can influence (increase or decrease) different types of agency problems, we take the agency problem as given for both forms and analyze the effects that it has on raising capital and providing insurance. Assuming that the owner-manager problem is lower for a stock insurer than for a mutual insurer, as suggested by Mayers and Smith, does not affect our qualitative results.

Doherty and Dionne (1993) argue that when external capital is costly, consumers will substitute by bearing risk themselves. Zanjani (2004) combines the arguments of Mayers and Smith (1981) and Doherty and Dionne (1993): when external capital is costly, the level of external capital is low so that the owner-manager conflict becomes less important and the mutual form is chosen to reduce the owner-customer conflict. In our setting, the cost of capital includes agency costs that apply equally to both capital provided by shareholders and premiums paid by policyholders. Therefore, external capital is not more costly than internal capital. Nevertheless, it may be optimal for policyholders to provide the capital and thus bear the risk themselves. The benefit of capital provided by policyholders is that the free-rider and commitment problems can be overcome when raising capital is tied to selling policies. A direct consequence of tying ownership rights to policies is that policyholders, instead of investors, have to bear the insurer's surplus risk.

We also contribute to the literature that analyzes the role of financial distress in insurance markets. Doherty and Schlesinger (1990) examine the demand for insurance under financial distress, and Mahul and Wright (2004a, 2004b) explore the optimal structure of insurance contracts for a mutual insurer with limited capital. This literature does not analyze the relation among capital, premiums, and financial distress. Instead, the authors fix the level of pre-paid premiums either directly or indirectly by fixing the probability of financial distress. Cagle and Harrington (1995) and Cummins and Danzon (1997) analyze insurance supply with capacity constraints and endogenous insolvency risk. Their approach is quite different from ours, as the authors focus on the effect of loss shocks on capitalization and premiums in insurance markets. Shareholders supply insurance based on maximizing the expected value of net cash flows under the assumption that capital is costly. The demand side is given by some exogenously specified demand curve, which is assumed to be negatively related to the price of insurance. In contrast to this literature, our paper focuses on the trade-off between price and quality of insurance, which is crucial for the distinction between a mutual and stock insurer in terms of the relation among capital, premiums, and risk sharing.

The pricing of insurance policies in the presence of insolvency risk is discussed by Doherty and Garven (1986) and Gründl and Schmeiser (2002). Doherty and Garven were the first to propose a contingent-claims approach to deal with insolvency risk in pricing insurance contracts. They focus on price regulation in property-liability insurance. Gründl and Schmeiser compare different approaches for pricing double-trigger reinsurance contracts that are subject to default and also discuss the relation between risk capital and financial distress.

The paper is structured as follows. We present the model in Section 2 and examine the role of capital and premium under the two different organizational forms in Section 3. In Section 4, we discuss the role of the corporate form in raising capital and sharing risk under governance problems. In Sections 5 and 6, we discuss several extensions and the empirical predictions of our model. We conclude in Section 7.

2 The Model

There are n identical, risk-averse individuals with increasing and concave utility function u. Each individual is endowed with initial wealth w_0 and faces a loss of random size X_i, $i = 1, ..., n$. We assume that losses are independent and identically distributed according to a continuous distribution function F^1 with $F^1(0) = 0$ and density function f^1. The aggregate loss in the economy, $\sum_{i=1}^{n} X_i$, is then distributed according to the n-fold convolution $F^n = \left(F^1\right)^{*(n)}$ with density function f^n.

Risk sharing is organized through an insurance company, which can either be a stock insurer or a mutual insurer. The insurer is run by a risk-neutral manager and the stock insurer is owned by risk-neutral shareholders.

Stock insurer. We consider the following three stages of setting up a stock insurance company. First, the company raises risk capital C from shareholders. Second, the manager sells insurance policies offering full coverage to the n individuals at a premium P per policy. Last, losses are realized and the total capital, $nP + C$, is distributed to policyholders and shareholders.

Mutual insurer. A mutual insurance company is owned by its policyholders, who own the right to the insurer's surplus. Since the company does not raise capital from shareholders there is no first stage. Rather, at the second stage, the company raises capital through selling full coverage policies for a premium P^m per policy, i.e. the mutual has no capital other than the collected insurance premiums, nP^m. At the third stage, losses are realized and the company distributes the total capital, nP^m, amongst policyholders.

The distinctions that are central to our discussion are the tying versus non-tying of ownership rights to insurance policies as well as the sequential versus simultaneous way of raising capital and selling policies in the presence of an owner-manager conflict.

At the last stage, indemnity and ownership claims are settled according to the following rules.

Indemnity claims. Let TC denote the insurer's total capital, which is $nP + C$ for a stock insurer and nP^m for a mutual. The insurer is solvent if $\sum_{i=1}^{n} X_i \leq TC$, and insolvent otherwise. If the insurer is solvent, then policyholders are fully indemnified. If the insurer is insolvent, then the company declares bankruptcy and the total capital is split amongst policyholders according to some pre-specified bankruptcy rule, $I_i(X_1, ..., X_n)$, with $\sum_{i=1}^{n} I_i(X_1, ..., X_n) = TC$ and $E[I_i(X_1, ..., X_n)] = TC/n$ for all $i = 1, ..., n$. A pro-rata rule would be defined by[1]

$$I_i(X_1, ..., X_n) = \frac{X_i}{\sum_{i=1}^{n} X_i} \cdot TC.$$

[1]Such a sharing rule, where policyholders receive a share of the insurer's assets that is proportional to their claim, is assumed in much of the literature (see, e.g., Cummins and Danzon, 1997, who confirm that "this liquidation rule is consistent with the way insurance bankruptcies are handled in practice," footnote 22).

Ownership claims. If the insurer is insolvent, owners are protected by limited liability and their payoff is zero. If the insurer is solvent, owners have a claim to the excess funds, i.e. total capital net of total claims payments, $TC - \sum_{i=1}^{n} X_i$. We refer to these excess funds as surplus.

If raising capital from shareholders is costless, it is optimal to raise infinite capital and sell full insurance policies at a premium equal to the expected loss, $P = E[X_1]$. The stock insurer is always solvent and dominates the mutual insurer since risk sharing is more efficiently organized through risk-neutral shareholders. In our paper, raising capital is costly owing to an owner-manager conflict. We assume that the owner-manager conflict exists for both corporate forms and is of the following form.

Owner-manager conflict. We assume that management derives a private benefit that is increasing in the resources under its control (Stein, 1997, and Hellwig, 2000, 2001). As a consequence, management maximizes the value of resources as long as it retains control over them. This gives rise to a conflict of interest between owners and management. Instead of returning resources to owners, management will rather carry out new projects even if they have a negative net present value, fight owners' decision to pay out funds, and hide or expropriate capital. As a consequence, owners are only able to extract the fraction $(1 - \alpha)$ of any capital that they want the firm to return. The parameter α is a measure for how severe the conflict between owners and management is when it comes to a redistribution of funds: it can be interpreted as a measure for the power of management relative to owners and verification costs that are proportional to the surplus.

This frictional cost of capital does not represent an exogenous bias for or against providing capital through either corporate form. While differences in the governance mechanisms certainly can result in shifts of the relative importance of different types of agency conflicts, the purpose of our paper is to highlight differences in organizational forms that are above and beyond direct differences in the relative importance of agency problems. Therefore, we assume that α is independent of the organizational form.

Debt versus equity financing. Of course, owners may consider using debt to reduce the problems stemming from management's unwillingness to return funds. If the debt is subordinate, the claims of policyholders are not affected. Debt, in contrast to equity, is a hard claim on the firm's surplus funds (i.e. capital net of policyholders' claims). If faced by a hard claim, management will be less successful in expropriating funds. Indeed, to avoid financial distress, management is likely to repay the debt holders' claim. This suggests that debt may be the optimal source of financing to overcome the owner-manager conflict. However, there are limits to using debt as increasing amounts of debt also increase the likelihood of financial distress, which is also associated with potentially high costs (e.g., verification costs, legal expenses, incentive problems). To make the equity claim arbitrarily small, the debt repayment obligation has to equal the insurer's total capital and the probability of distress approaches one. In this case, the cost of financial distress is likely to exceed the frictional cost of equity. Thus, there is a trade-off between using debt and equity. The ability of insurers to use debt financing is also constrained by regulation. E.g., in Germany insurers are not allowed to finance the underwriting business with debt. But even in the US, minimum capital requirements put constraints on the type and quantity of debt.

While some subordinate debt may optimally be used, explicitly taking into account subordinate debt does not change our qualitative results regarding equity financing, which is still needed. For ease of exposition, we assume zero debt.

3 Optimal Premium and Financial Distress

In this section, we examine the role of capital provided by owners under the two organizational forms. We show that it can be optimal for policyholders in a stock insurer to raise additional funds through a loading in order to improve risk sharing. To highlight this effect, in this section we assume that there is no frictional cost of capital under either corporate form, i.e. $\alpha = 0$, and that the amount of capital provided by shareholders is exogenously fixed.

Actuarially Fair Premium. In the absence of financial distress, the actuarially fair premium equals the expected indemnity payment to the policyholder which, under full insurance, equals the expected loss. In contrast, if there is a positive probability that the company can become insolvent, the expected indemnity payment under full insurance is lower than the policyholder's expected loss. The actuarially fair premium therefore depends on the company's insolvency probability, which, in turn, depends on the level of the premium, as collected premiums are available for claim payments. Another important determinant of the insolvency risk and level of the actuarially fair premium is the amount of shareholders' capital in the company, C.

To clarify the interconnection between the actuarially fair premium and insolvency risk of a stock corporation, we examine the two extreme scenarios: zero capital and unlimited financial capital held by the company. With unlimited capital the company never goes bankrupt and policyholders are always fully indemnified. This implies that the actuarially fair premium equals the expected loss to the insured. If the stock corporation has no capital, i.e. $C = 0$, there is still a strictly positive probability that the company remains solvent and that the collected premiums exceed the aggregate claims by policyholders. In this case, the remaining funds are paid out to shareholders, and the expected payout to policyholders is lower than the premium, which contradicts the definition of an actuarially fair premium. Without capital the only actuarially fair premium is therefore zero.

Under a mutual organization, policyholders are also the owners of the firm and all premiums collected are redistributed. Each policyholder receives $X_i + P^m - \frac{1}{n}\sum_{i=1}^{n} X_i$ in case of solvency and $I_i(X_1, ..., X_n)$ in case of insolvency. The premium therefore comprises the expected indemnity payment and the value of the ownership right. This implies that any premium P^m provided by the policyholder of a mutual insurer is actuarially fair.

In the following proposition, we formalize these arguments and show that for stock insurers the actuarially fair premium is increasing in the amount of capital provided by shareholders.

Proposition 1 *For a stock insurer, there exists a unique actuarially fair premium for each fixed level of capital provided by shareholders. Furthermore, the actuarially fair premium is increasing in the amount of capital, from zero without capital to the level of the expected loss with unlimited capital. For a mutual insurer, any premium provided by policyholders is actuarially fair.*

Proof. See Appendix A.1. ■

Proposition 1 highlights an important distinction between stock and mutual insurers. A sufficiently high level of capital C is required to offer a substantial amount of insurance at an actuarially fair premium in the case of a stock insurer. This capital has to be provided by shareholders who benefit from states in which total premiums exceed total claims. The role of capital is thus to reallocate funds from states where shareholders make a profit to states where total premiums are lower than the policyholder's losses.

Optimal Loading. Since the actuarially fair premium, and thereby the likelihood that the company stays solvent, is increasing in the amount of capital C provided by shareholders, policyholders' level of expected utility is also increasing in C. If providing risk capital is costless, i.e. if capital markets are perfect, then it would be optimal to have unlimited risk capital available. In this case, the insurer is always solvent, and full insurance is achieved at a fair premium equal to the expected loss, as shown in Proposition 1. Policyholders are fully indemnified and thus not willing to pay a loading in excess of the fair premium.

Suppose now that a stock insurer's risk capital is limited. If the company is insolvent, then policyholders will not be fully indemnified. Their marginal utility is therefore higher in states in which the company is insolvent compared to states in which funds are sufficient to receive full coverage. As policyholders are risk-averse, they wish to transfer wealth from solvency-states to insolvency-states and in particular to those insolvency-states with relatively high claims. In the following proposition, we show that this wealth transfer can be achieved by paying a loading on top of the actuarially fair premium.

Proposition 2 *For a stock insurer, if capital provided by shareholders is small (great), then it is optimal (not optimal) for policyholders to pay a loading in excess of the actuarially fair premium. If policyholders' preferences exhibit constant absolute risk aversion (CARA), then the optimal loading is decreasing in the level of capital provided by shareholders.*

Proof. See Appendix A.2. ■

This proposition implies that there exist two critical thresholds of capital such that it is optimal for policyholders to collectively pay a loading for all levels of capital below the one threshold and not to pay a loading for all levels above the other threshold. In the case of CARA, these two thresholds coincide.

By collectively paying a loading in excess of the actuarially fair premium, policyholders reduce their wealth in solvency-states to the benefit of shareholders. At the same time, more funds are available to be distributed to policyholders in insolvency-states. Reasonable bankruptcy rules may therefore create a form of coinsurance amongst policyholders if these additional funds accrue to those policyholders with relatively high claims.[2] Policyholders thus trade off higher premiums for additional insurance against the possibility that these funds are not used to pay claims and instead accrue to shareholders. In addition to this trade-off, the loading also reduces the probability of insolvency. On the one hand, this is beneficial to policyholders, as they are more likely to be fully indemnified. On the other hand, a reduction of the insolvency probability has a negative effect on the trade-off described above. It is now more likely that the additional funds accrue to shareholders. The proposition shows that creating this form of coinsurance in insolvency-states is particularly beneficial if little capital is provided by shareholders.

If increasing shareholders' capital is not an option, paying a loading is akin to "back door" capital. In the extreme scenario with no capital, we have shown in Proposition 1 that no insurance can be offered at an actuarially fair premium. By paying a loading, policyholders would in fact initiate risk sharing. Providing these additional funds, however, is costly for policyholders and generates strictly positive rents for existing shareholders. Alternatively, policyholders could form a mutual in which they have a claim on the excess funds.

4 Governance Problems and the Role of the Corporate Form for Raising Capital and Risk Sharing

4.1 Raising Capital and Corporate Form

In this section, we investigate the optimal level of total capital that the insurance company raises in the presence of the owner-manager-conflict.

4.1.1 Stock Insurer

The optimal combination of premium P and capital C for policyholders has to satisfy the shareholder's participation constraint. A general assumption in the insurance literature is that policyholders pay an actuarially fair premium that equals the expected indemnity payment. Underlying this assumption are (i) competition in the insurance market where shareholders earn zero expected profit and (ii) a frictionless capital market where shareholders can recall their capital at zero cost so

[2] Mahul and Wright (2004b) show that the Pareto-optimal mutual risk sharing contract with limited capital includes a deductible which is adjusted *ex-post* depending on realized losses to meet the capital constraint.

that they would not sell insurance if premiums fall below the actuarially fair premium. The second argument is important in our setting. In practice, a stock insurer raises capital before selling insurance policies. Therefore, premiums are determined in competition after capital has been raised. Without market frictions the timing is not crucial. In contrast, with frictional costs of capital, the insurer incurs the cost of capital before premiums are determined.

We follow the insurance literature in assuming that the shareholders' participation constraint is binding (zero expected profit condition). Thus, the amount of capital that shareholders provide equals the expected repayment to shareholders:

$$C = (1-\alpha)\, E\left[\left(C + nP - \sum\nolimits_{i=1}^{n} X_i\right)^{+}\right]. \tag{1}$$

Stock insurer with commitment. As a benchmark case, we first examine the optimal solution for a stock insurer if premium and capital are chosen simultaneously. Thus, the benchmark case corresponds to a situation where policyholders commit to a premium P when capital is raised from shareholders. With commitment, the optimal capital C^* and premium P^* are determined by the following optimization problem

$$(C^*, P^*) = \arg\max_{(C,P)} E\left[u\left(W_1^s\left(C,P\right)\right)\right] \tag{2}$$

subject to (1) where each policyholder's final wealth under the stock form is given by

$$W_1^s\left(C,P\right) = \begin{cases} w_0 - P & \text{if } \sum_{i=1}^{n} X_i \le C + nP \\ w_0 - P - X_1 + \dfrac{X_1}{\sum_{i=1}^{n} X_i}\left(C + nP\right) & \text{if } \sum_{i=1}^{n} X_i > C + nP \end{cases}. \tag{3}$$

Lemma 1 *The zero expected profit condition for shareholders* (1) *provides a one-to-one, increasing mapping between capital and premium. Furthermore, the optimal premium P^* paid by policyholders includes a loading that compensates shareholders for the frictional cost of capital.*

Proof. See Appendix A.3. ■

The optimal premium P^* can be interpreted as the fair premium that assures that shareholders earn a fair return on their invested capital. Therefore, shareholders earn a (quasi) rent that compensates them for the frictional cost of capital.

Stock insurer without commitment. Now suppose that policyholders cannot commit to a premium P that allows shareholders to earn a quasi rent.[3] After capital has been raised, the frictional cost of capital is sunk and the insurer's interim participation constraint is given by

$$(1-\alpha)\,E\left[\left(C+nP-\sum\nolimits_{i=1}^{n}X_i\right)^+\right]\geq(1-\alpha)\,C. \tag{4}$$

We note that the interim participation constraint is equivalent to the participation constraint in a frictionless capital market where $\alpha=0$. After capital has been raised, shareholders are willing to accept a lower premium compared to the one implied by the ex-ante participation constraint (1). However, shareholders will foresee that in this case their ex-ante participation constraint will be violated.

Therefore, the governance problem results in an additional constraint: for a given level of capital, C, the premium, P, for which the zero expected profit condition for shareholders (1) is binding, must also be optimal for policyholders after capital has been raised. Put differently, it must be optimal for policyholders to pay a voluntary loading (see Proposition 2), which compensates shareholders for the frictional cost of capital.

Without commitment, the optimal level of capital $\underline{C}$ (and premium $\underline{P}$) is thus determined by the following optimization problem

$$(\underline{C},\underline{P})=\arg\max_{(C,P)}E\left[u\left(W_1^s\left(C,P\right)\right)\right] \tag{5}$$

subject to

$$\underline{P}=\arg\max_{P}E\left[u\left(W_1^s\left(\underline{C},P\right)\right)\right] \tag{6}$$

and (1) where W_1^s is given by (3). Constraint (6) is the policyholders' incentive-compatibility constraint that assures that the premium $\underline{P}$ is still optimal after capital has been raised.

Proposition 3 *For* $\alpha>0$*, a solution* $(\underline{C},\underline{P})$ *to optimization problem* (5) *exists and satisfies* $0<\underline{C}<C^*$ *and* $0<\underline{P}<P^*$*.*

[3] A commitment problem is also the reason why policies cannot be sold before raising capital. In this case, initial owners have an incentive to not raise additional capital. This problem is directly related to the debt-overhang problem discussed by Myers and Majluf (1984).

Proof. See Appendix A.4. ■

The important implication of the incentive-compatibility constraint (6) is that the amount of capital that a stock insurer can raise is lower than with commitment. As in the case with commitment, a necessary condition for a positive level of capital is that the premium P exceeds the actuarially fair premium by a loading to compensate shareholders for the frictional cost of capital. The main difference is that with commitment it is possible to commit to the level of premium loading before capital is raised while without commitment no such ex ante specification of premium loading is possible. In this case, it may be optimal for policyholders to collectively provide additional capital to improve risk sharing. The increase in utility from improved risk sharing compensates policyholders for a negative (unfair) return of their stock. However, as buying insurance policies and buying stock are separated, each policyholder has an incentive to free-ride on the capital provided by the others and will not buy stock.

We note that replacing the shareholders' interim participation constraint by "management's interim participation constraint" does not relax the commitment problem. For example, if management derives a private benefit that is linear in the insurer's surplus, "management's interim participation constraint" is equivalent to shareholder's interim participation constraint (4). However, it is also conceivable that the management may accept any premium just to continue. In both cases, the incentive-compatibility constraint (6) is the relevant constraint.

4.1.2 Mutual Insurer

For a mutual insurer capital is raised through the premium when selling insurance policies. This is an important difference between a stock insurer and a mutual, which results from the policyholders also being the owners.

In the case of a mutual, the optimal premium P^{m*} for policyholders is determined by the optimization problem

$$P^{m*} = \arg\max_{P^m} E\left[u\left(W_1^m\left(P^m\right)\right)\right] \tag{7}$$

where W_1^m is given by

$$W_1^m(P^m) = \begin{cases} w_0 - P^m + \frac{1}{n}(1-\alpha)\left(nP^m - \sum_{i=1}^n X_i\right) & \text{if } \sum_{i=1}^n X_i \leq nP^m \\ w_0 - P^m - X_1 + \dfrac{X_1}{\sum_{i=1}^n X_i} nP^m & \text{if } \sum_{i=1}^n X_i > nP^m \end{cases}. \quad (8)$$

The premium P^m is a joint payment for insurance policy and ownership rights. A mutual thereby overcomes the commitment and free-rider problem by tying selling insurance policies and raising capital. This results in a higher and more efficient level of total capital than the stock insurer can provide under the commitment problem.

4.2 Risk Sharing and Corporate Form

The benefit of a mutual discussed above has to be traded off against the cost of lower diversification since risk is shared within the pool of risk-averse policyholders only. To carve out this difference we first focus on the sharing of surplus risk by comparing equally capitalized insurers. Then we introduce the benefit of the mutual to provide a higher level of total capital which is beneficial for the sharing of insolvency risk.

Surplus risk.

Proposition 4 *If the stock and mutual insurer are equally capitalized then the stock insurer dominates the mutual insurer.*

Proof. See Appendix A.5. ■

The intuition for the proposition is that it is optimal for risk-averse policyholders to transfer the insurer's surplus risk to the risk-neutral capital market. Since policyholders are risk-averse their marginal utility in insolvency states is higher than in solvency states. Changing from the mutual to the stock form improves welfare as it allows to transfer wealth from solvency to insolvency states at a fair rate. This dominance extends to the situation when both the mutual and the stock insurer with commitment choose their optimal level of total capital.

Corollary 1 *The stock insurer with commitment dominates the mutual insurer.*

Proof. See Appendix A.6. ■

Insolvency risk. As shown in Proposition 3 the stock insurer is undercapitalized due to the commitment problem, i.e. $n\underline{P}+\underline{C} < nP^*+C^*$. A mutual insurer with equivalent total capital, i.e. $n\underline{P}^m = n\underline{P}+\underline{C}$, implies the following distribution of final wealth

$$W_1^m\left(\underline{P}^m\right) = \begin{cases} w_0 - \underline{P}^m + \frac{1}{n}(1-\alpha)\left(n\underline{P}^m - \sum_{i=1}^n X_i\right) & \text{if } \sum_{i=1}^n X_i \leq n\underline{P}^m \\ w_0 - \underline{P}^m - X_1 + \dfrac{X_1}{\sum_{i=1}^n X_i} n\underline{P}^m & \text{if } \sum_{i=1}^n X_i > n\underline{P}^m \end{cases}.$$

This mutual insurer is dominated by the stock insurer (see Proposition 4) since it transfers the surplus risk to the capital market. However, by bundling insurance and capital, the mutual can raise a level of total capital, nP^{m*}, which exceeds the overall capital that can be raised by a stock insurer in the presence of commitment problems, i.e. $nP^{m*} > n\underline{P}+\underline{C}$ (see Proposition 3). Let Δ be the additional capital per policy that the mutual insurer can raise, i.e. $\Delta = P^{m*} - \underline{P}^m$. The distribution of final wealth under the mutual form with the optimal amount of capital can be expressed as

$$W_1^m\left(P^{m*}\right) = \begin{cases} w_0 - \underline{P}^m + \frac{1}{n}(1-\alpha)\left(n\underline{P}^m - \sum_{i=1}^n X_i\right) - \alpha\Delta & \text{if } \sum_{i=1}^n X_i \leq n\left(\underline{P}^m + \Delta\right) \\ w_0 - \underline{P}^m - X_1 + \dfrac{X_1}{\sum_{i=1}^n X_i} n\underline{P}^m + \left(\dfrac{nX_1}{\sum_{i=1}^n X_i} - 1\right)\Delta & \text{if } \sum_{i=1}^n X_i > n\left(\underline{P}^m + \Delta\right) \end{cases}.$$

Increasing the level of premium from $\underline{P}^m$ to $P^{m*} = \underline{P}^m + \Delta$ implies three effects:

1. the probability of insolvency decreases
2. the level of wealth in solvency states is reduced by $\alpha\Delta$
3. the zero-mean lottery $\left(\dfrac{nX_1}{\sum_{i=1}^n X_i} - 1\right)\Delta$ is added to the level of wealth in insolvency states

The first effect is beneficial to risk-averse policyholders. The second and third effect imply a wealth transfer where at a cost of $\alpha\Delta$ in solvency states (second effect) a zero mean lottery is added to insolvency states (third effect). This zero mean lottery is beneficial for policyholders since it provides additional insurance within insolvency states: wealth is transferred from states with lower marginal utility, where $X_1 < \frac{1}{n}\sum_{i=1}^n X_i$, to states with higher marginal utility, where $X_1 > \frac{1}{n}\sum_{i=1}^n X_i$.

To overall compare the mutual insurer and the stock insurer without commitment, these effects of a higher level of overall capital under the mutual form have to be traded off against the stock insurer's benefit of transferring risk to the capital market. This trade-off is determined by the severity of the governance problem, α, the number of policyholders, n, policyholders' degree of risk aversion, and the standard deviation of the policyholders' losses.

4.3 Numerical Comparison of Corporate Forms

To compare the corporate forms for different parameters, we numerically solve each optimization program: program (2) for the stock insurer with commitment, program (5) for the stock insurer without commitment, and program (7) for the mutual insurer. We assume that policyholders' preferences exhibit constant absolute risk aversion (CARA) with coefficient γ, i.e. $u(w) = -\exp(-\gamma w)$. Each individual loss is distributed according to a Gamma distribution $\Gamma(k, \theta)$ with shape parameter k and scale parameter θ. The first two moments are $E[X_1] = k\theta$ and $\sigma^2(X_1) = k\theta^2$. Since the class of Gamma distributions is closed under convolution, aggregate losses are also distributed according to a Gamma distribution $\Gamma(nk, \theta)$ with shape parameter nk and scale parameter θ. In each figure, we plot, for each of the three settings, the maximized expected utility as a function of the governance problem's severity, α.

Figure 1 plots the base scenario with the following parameters: $n = 250$, $\gamma = 1$, $w_0 = 50$, $E[X_1] = 1$, and $\sigma(X_1) = 3$ (that is, $k = 1/9$ and $\theta = 9$).

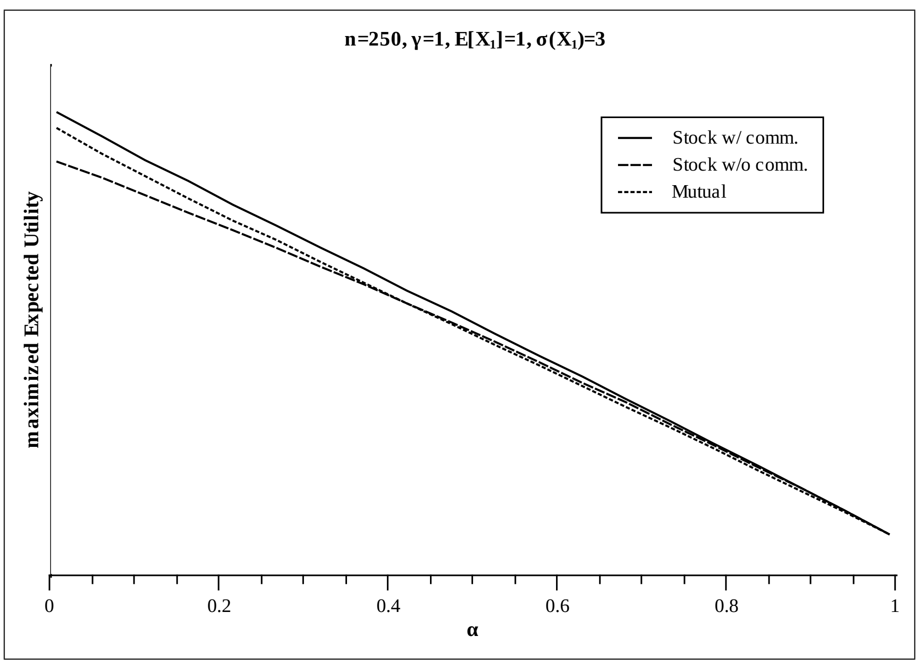

Figure 1

As shown in Corollary 1 the stock insurer with commitment dominates both the mutual insurer and the stock insurer without commitment. We furthermore observe that the stock insurer with commitment dominates the mutual insurer for large values of α. This is due to the second effect described above where α is a measure of the cost involved in providing additional risk sharing in insolvency states under the mutual form. For high levels of α, providing such additional risk sharing is costly and its benefit does not outweigh the cost of less diversified owners under the mutual form. In contrast, for low levels of α, the benefit of a mutual in providing such additional risk sharing dominates the cost of less diversified owners.

In the following, we focus on the effects of the number of policyholders, n, policyholders' degree of risk aversion, γ, and the standard deviation of the policyholders' losses, $\sigma(X_1)$.

Figures 2 provides the plots for $n = 50$, $n = 500$, $n = 750$, and $n = 1000$.

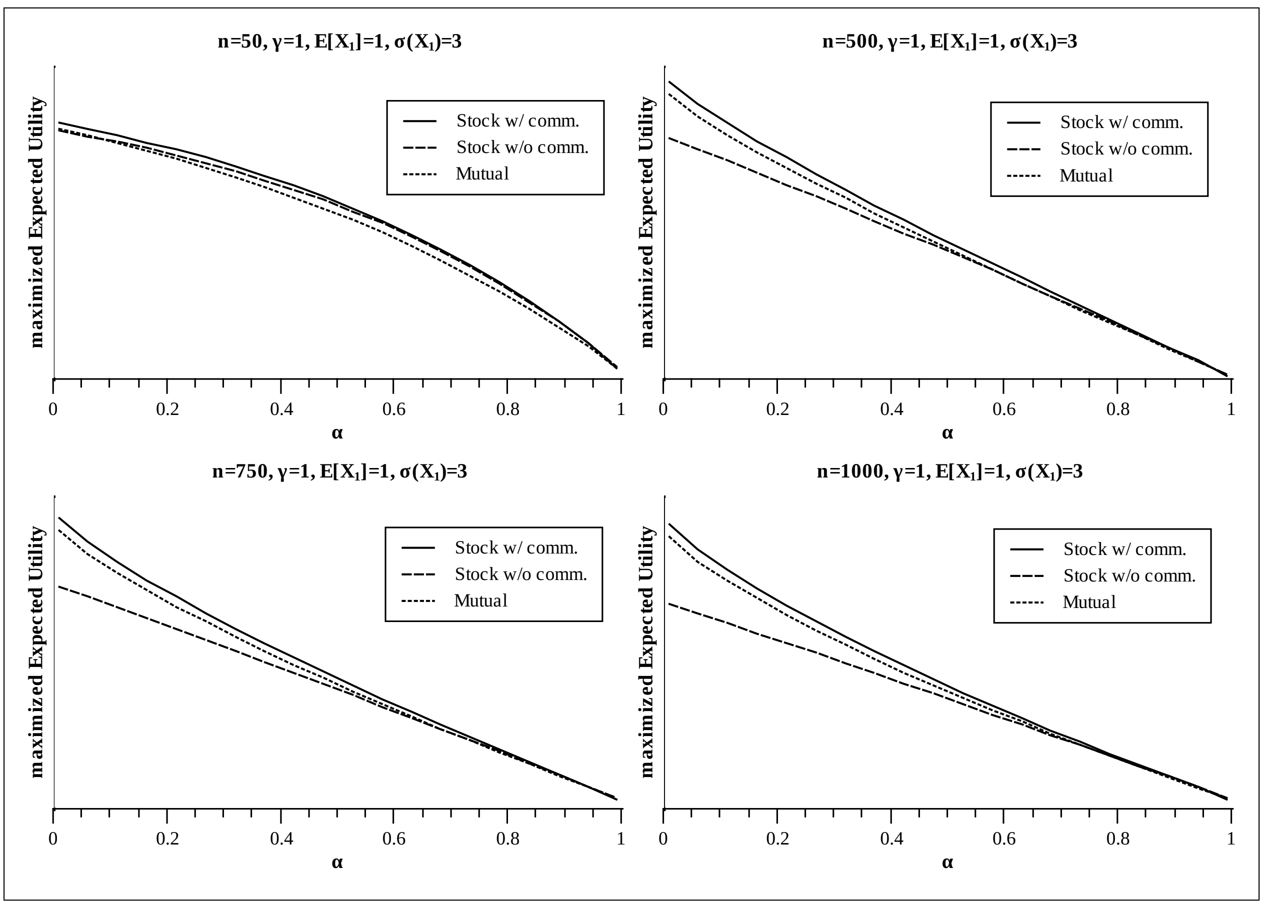

Figure 2

The larger the number of policyholders, n, the relatively less important is the benefit of transferring risk to the capital market. Therefore the benefit of overcoming the commitment problem through the mutual insurer is more likely to dominate.

Figure 3 provides the plots for $\gamma = 0.5$ and $\gamma = 2$.

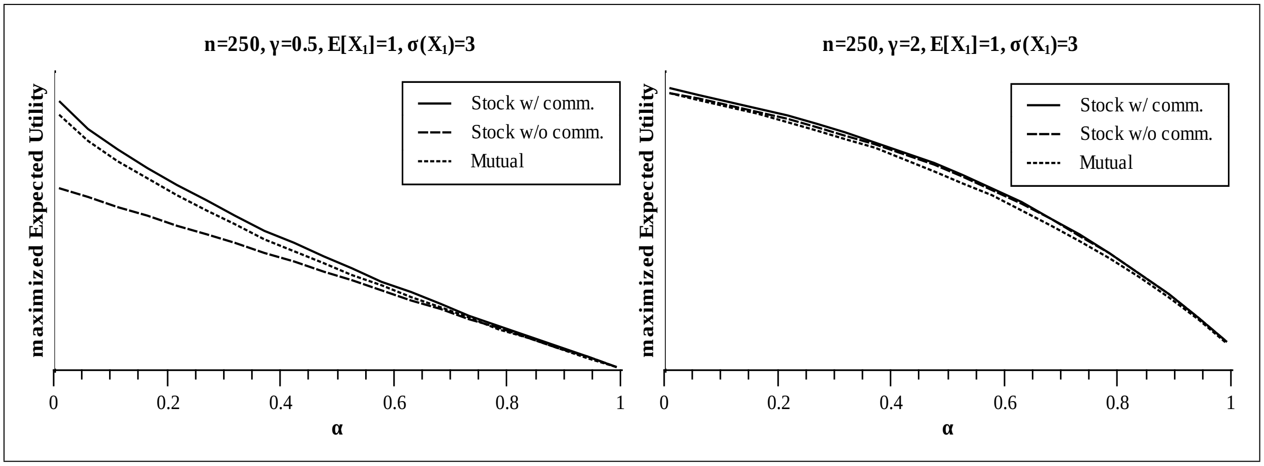

Figure 3

If policyholders are more risk-averse the stock insurer without commitment is more likely to dominate the mutual form since the benefit of transferring risk to the capital market is relatively more important.

Figure 4 provides the plots for $\sigma(X_1) = 1$ and $\sigma(X_1) = 5$.

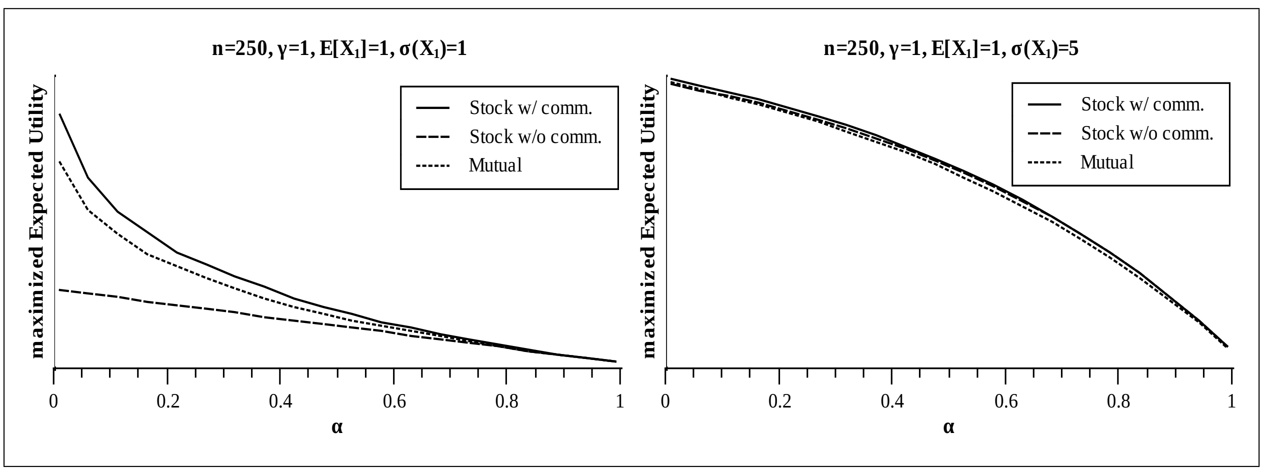

Figure 4

Analogously to the discussion above, a higher standard deviation puts a higher weight on the benefit of transferring risk to the capital market.

We conclude from Figures 3, 4, and 5 that the stock insurer without commitment is more likely to dominate the mutual insurer when risk and thereby its sharing becomes more important, that is for a

- lower number of policyholders, n, i.e., lower diversification at the level of the insurer,
- higher degree of policyholders' risk aversion, γ, and
- higher standard deviation of losses, $\sigma(X_1)$.

5 Discussion and Extensions

5.1 Differences in Governance Problems

We assumed that the stock insurer and the mutual insurer face an identical owner-manager conflict. This allowed us to point out differences between the two organizational forms above and beyond an exogenous bias against the mutual form in dealing with this conflict. Mayers and Smith (1981, 2005) and Zanjani (2004) argue that the stock form is better suited in dealing with owner-manager conflicts than the mutual form due to improved governance of stock corporations. The implication for our model is that α may be lower for a stock insurer than for a mutual insurer. A lower governance problem would be an additional advantage of a stock insurer that may help to overcome the commitment problem. We can use the figures above to compare the benefit of a stock insurer and a mutual insurer for different α. Holding α fixed for the mutual form and reducing it for the stock insurer without commitment, increases the benefit of the stock form relative to the mutual form. As a consequence, the stock form may now dominate the mutual even for parameter constellations for which the mutual dominates if the governance problem is the same for both corporate forms. The effect is stronger the more important the potential benefits of risk sharing are: higher risk aversion of policyholders, higher variance of losses, lower number of policyholders.

5.2 Correlated Losses

Correlation among losses plays an important role for the comparison between the two corporate forms. To model correlation, we assume that policyholders fall into k subgroups with n/k policyholders in each. Within each subgroup policyholders' losses are perfectly positively correlated whereas between different subgroups they are independent and identically distributed. Aggregate losses in each subgroup are therefore $\frac{n}{k}X_i$ and aggregate losses over all policyholders can be written as

$$\sum\nolimits_{i=1}^{n} X_i \sim^d \frac{n}{k}\sum\nolimits_{j=1}^{k} X_j$$

where $(X_j)_{j=1,\ldots,k}$ are independent and identically distributed and $\sim^d$ denotes identical in distribution. The degree of correlation is therefore inversely related to the number of subgroups, k. For $k = 1$, all losses are perfectly positively correlated, whereas $k = n$ describes the model above where all losses are independent and identically distributed.

Proposition 5 *Under both corporate forms, the effect of correlation between losses is equivalent to the effect of reducing the number of policyholders from n to k.*

Proof. See Appendix A.7. ■

Thus, the insight derived from the number of policyholders n also holds for correlation: a higher correlation of losses makes it more likely that the stock insurer dominates the mutual insurer.

5.3 Multiple Periods

In this section, we extend our one-period model to multiple periods. We consider only one-period insurance policies. In each period, policyholders therefore only consider a one-period problem. Since shareholders' participation constraint is binding in each period, there is no dynamic effect stemming from future periods. For that reason, the reaming question is how capital is accumulated in the stock insurer over multiple periods.

In each period, the company can be solvent or insolvent. If the stock insurer is insolvent at the end of a period, then the launching a new company is described by our model above. In case that the company is solvent at the end of the period, let $\bar{C}$ denote the insurer's surplus from the previous period. In the one-period model the insurer is liquidated and the surplus is returned to shareholders who receive $\alpha\bar{C}$. This is still an option even with multiple periods. But with multiple periods, the insurer can also continue to do business. Let C the amount of capital retained in the insurer. If part of the surplus is returned to owners, i.e. $C < \bar{C}$, then owners receive $(1-\alpha)\left(\bar{C} - C\right)$ in addition to the expected future surplus from continuation given capital C. For every choice of C, shareholders' ex ante participation constraint has to be satisfied:

$$(1-\alpha)\, E\left[\left(C+nP-\sum\nolimits_{i=1}^{n} X_i\right)^{+}\right]+(1-\alpha)\left(\bar{C}-C\right)\geq(1-\alpha)\,\bar{C},$$

which is equivalent to

$$E\left[\left(C+nP-\sum\nolimits_{i=1}^{n} X_i\right)^{+}\right]\geq C. \tag{9}$$

The participation constraint for the level of surplus retained in the insurer is equivalent to the case without market frictions, $\alpha = 0$. As unlimited capital is optimal for policyholders without market frictions, it immediately follows that no funds are paid out. We note that constraint (9) is equivalent to the interim participation constraint. Thus, the commitment problem does not arise for capital that is already in the firm. The reason is that the capital is already exposed to the governance problem. Put differently, paying out funds is subject to the same cost, independent of when they are paid out. Policyholders therefore have no hold-up power over funds retained in the insurer. This implies that the benefit of the stock insurer is increasing in the amount of surplus retained in the company, since the commitment problem becomes less severe or even diminishes.

However, if the level of retained surplus $\bar{C}$ is low, then it may be optimal to raise capital in addition to the surplus, i.e. $C \geq \bar{C}$. For the newly raised capital, $C-\bar{C}$, the commitment problem arises again.

5.4 Managerial Incentives to Expand

The two different organizational forms also provide contrasting managerial incentives to expand the number of customers. Suppose that a manager has to exert privately costly effort, $c(q)$, to sell q policies, which is increasing in q. Furthermore, the manager derives a private benefit of δ that is increasing in the surplus.[4] If q policies are sold, the manager's utility under the two organizational forms are

$$U^{s}(q)=\delta E\left[\left(C+qP-\sum\nolimits_{i=1}^{q} X_i\right)^{+}\right]-c(q)$$

[4] Note that we allow the benefit to the manager to differ from the cost to owners. Moreover, similar results obtain if the manager incurs a disutility from the company's insolvency.

in case of a stock insurer and

$$U^m(q) = \delta E\left[\left(qP^m - \sum\nolimits_{i=1}^{q} X_i\right)^+\right] - c(q)$$

in case of a mutual insurer. We assume that the objective is to sell n insurance contracts and analyze how the manager's utility changes when the number of policies increases from q to n under the two organizational forms.

Proposition 6 *Suppose that the number of policyholders is large such that the Central Limit Theorem can be applied. The managerial incentives to expand the number of customers are higher under the mutual form than under the stock form if* $P^{m*} > E[X_1]$ *and* $qP^m > C + qP$.

Proof. See Appendix A.8. ■

The difference in incentives stems from the difference in how capital is raised. Stock insurers raise a fixed amount of capital first, then sell policies. Increasing the number of policies then reduces the average capital available for each policy. This effect dominates the diversification benefit. Mutual insurers raise capital while selling insurance policies. Since the total premium and capital for ownership rights are constant, the benefit of reducing the variance of the average claim dominates. This implies that if the mutual and the stock insurer are equally capitalized, i.e. $qP^m = C + qP$, the managerial incentives to expand are higher under the mutual form. If the mutual company is better capitalized, i.e. $qP^m > C + qP$, we show in the proof of Proposition 6 that the managerial incentives to expand under a mutual form are increasing in the amount of capital, as long as $P^{m*} > E[X_1]$, and thereby are also higher than under the stock form.

The ex ante available capital in a stock insurer provides a cushion for the manager—the fraction δ of which he is even able to consume if the firm is solvent—while the manager of a mutual has to "earn" this cushion by selling insurance contracts.

6 Empirical Predictions and Evidence

Our model and its extensions provides a set of empirical predictions about the relative dominance of one corporate form over the other. The first set of predictions relates to their ability to raise capital and spread risk.

1. The potential advantage of a mutual insurer arises when insurers have to raise large amounts of capital, e.g., after large shock to capital due to a catastrophic event or when an insurance company is founded.

2. The benefit of a stock insurer comes to force in its ability to better spread risk if the involved risk is high, e.g., because of a high variance of losses and a high correlation of losses.

3. The potential disadvantage of a stock insurer decreases (i) if the owner-manager conflict is higher for a mutual than for a stock insurer and (ii) if the stock insurer accumulated capital after periods of low or moderate losses.

These predictions give rise to a development of corporate form that is consistent with empirical evidence.

First, mutual formation was associated with times of insurance market crises.[5] For example, the New York Fire of 1835 wiped out most stock insurers and stimulated the formation of mutual insurers (Smith and Stutzer, 1995). Zanjani (2004, 2007) finds that mutual insurers were used more often in times of financial crises. Financial crises have the potential to deplete the value of an insurer's capital, making it necessary to raise new capital. Consistent with our paper, he argues that the reason is that mutuals may substitute for (external) capital in production.

Second, mutuals are less risky than stock insurers, which is consistent with the evidence found by Lamm-Tennant and Starks (1993).

Third, capital market regulation aimed at increasing shareholders' rights may reduce governance problems for a stock insurer, but not for a mutual insurer. As a consequence, the relative benefit of stock insurers increases. Moreover, if a mutual insurer accumulates capital in times when losses are low and the asset returns are high, the mutual's benefit of raising new capital becomes less important relative to the disadvantage of not transferring risk to the capital market. This is consistent with observed demutualization in countries with highly developed stock markets (Viswanathan and Cummins, 2003).

[5] Even though insurance market crisis are also associated with high premiums, stock insurers cannot just raise capital. Due to the commitment problem, shareholders of a stock insurer must fear that, after a large increase in capacity if several insurers raise capital after a shock, insurance premiums will fall quickly, leaving them uncompensated for the frictional cost of capital. This is in line with Winter (1988) and Gron (1994) who argue that after a sudden loss, insurers respond by increasing premiums rather than raising external capital.

A second set of predictions relates to the implications of how regulation affects the benefit of one organizational form over the other.

4. Regulation that aims at increasing policyholders' rights may harm stock insurers more than mutual insurers.

5. Initial minimum capital requirements imply that capital has to be raised before policies are sold. If the required capital is very high for a stock insurer, the commitment problem combined with the frictional cost of capital may be prohibitive.

6. A regulatory requirement of maintaining a minimum level of capital to support insurance operations might reduce the relative disadvantage of stock insurers.

The rationale underlying prediction 4 is that stock insurers rely on quasi rents to compensate shareholders for the frictional cost of capital. Increasing policyholders' rights reduces the insurers rent, which makes it more difficult for a stock insurer to raise capital. This is consistent with findings by Fletcher (1966) who argues that the 1905 Armstrong Investigation's negative impact on the profit making opportunities of insurers played an important role in motivating mutualizations of three major stock life insurance companies in New York.

High initial capital requirements may pose a problem for a stock insurer due to the commitment problem of raising funds through policies to reimburse shareholders for high frictional cost of capital. Zanjani (2007) finds that initial capital requirements were a major determinant of the choice of mutuals between 1900 and 1949. Mutuals were formed in states with low initial capital requirements for mutuals and differentially high initial capital requirements for stock corporations. The economic rationale for this finding is not immediately clear. High capital improves risk sharing and may therefore be even beneficial. However, as we argue, it can be difficult to raise capital ex ante in the presence of frictional cost of capital.

In contrast, selling policies first and then being forced to raise some capital can reduce the problem that premiums may not cover the (sunk) frictional cost of capital if capital is raised first. Regulatory capital requirements can thus serve as a commitment device to the extent that the insurer can raise the required capital after policies have been sold.

7 Conclusion

In this paper, we emphasize the distinction between mutual and stock insurers in organizing risk sharing in the presence of governance problems that may exist under both corporate forms. In a stock corporation, the efficiency of risk sharing is inherently linked to the level of capital provided by shareholders and the degree to which shareholders are diversified. In a mutual corporation, risks are shared among policyholders only, and the efficiency of risk sharing therefore depends on the size of the pool of policyholders. In an efficient capital market without frictions and where shareholding is dispersed, risk sharing can optimally be organized through a stock insurer. However, in the presence of an owner-manager conflict where the manager expropriates a fraction of the insurer's surplus, the insurance premium has to compensate shareholders for this expropriation of funds. When insurance policies are sold, shareholders have already exposed their capital and competition in the insurance market may result in a premium that does not provide a sufficiently high (quasi) rent to cover the loss from expropriation. When governance problems are large, the level of capital and risk sharing in a stock insurer may be low. A mutual links the provision of capital and premium. Thus, policyholders directly bear the cost of providing capital. Moreover, policyholders cannot free-ride on others to provide capital at unfair terms.

A Appendix: Proofs

A.1 Proof of Proposition 1

For a stock insurer, the actuarially fair premium $P_{fair}(C)$ as a function of capital C is implicitly defined by

$$P_{fair}(C) = E\left[X_i \cdot 1_{\{\Sigma_i X_i \leq nP_{fair}(C)+C\}} + I_i(X_1, ..., X_n) \cdot 1_{\{\Sigma_i X_i > nP_{fair}(C)+C\}}\right].$$

Summing over all policies yields

$$\begin{aligned} nP_{fair}(C) &= E\left[\sum_{i=1}^{n} X_i \cdot 1_{\{\Sigma_i X_i \leq nP_{fair}(C)+C\}} + (nP_{fair}(C)+C) \cdot 1_{\{\Sigma_i X_i > nP_{fair}(C)+C\}}\right] \\ &= \int_0^{nP_{fair}(C)+C} x dF^n(x) + (nP_{fair}(C)+C)(1 - F^n(nP_{fair}(C)+C)), \end{aligned} \tag{10}$$

where F^n is the n-fold convolution of F^1 and thus the distribution function of the aggregate loss $\sum_{i=1}^{n} X_i$. For $C = 0$ we have

$$nP_{fair}(0) = \int_0^{nP_{fair}(0)} x dF^n(x) + nP_{fair}(0)(1 - F^n(nP_{fair}(0)))$$

which is satisfied for $P_{fair}(0) = 0$. For any $P_{fair}(0) > 0$ we deduce

$$\int_0^{nP_{fair}(0)} x dF^n(x) + nP_{fair}(0)(1 - F^n(nP_{fair}(0))) < nP_{fair}(0).$$

$P_{fair}(0) = 0$ is therefore the unique solution to (10).
For $C = +\infty$, the company is never insolvent and the actuarially fair premium is given by $P_{fair}(\infty) = E[X_1]$.
For any $0 < C < \infty$, define the f as

$$f(P) = nP - \int_0^{nP+C} x dF^n(x) - (nP+C)(1 - F^n(nP+C)).$$

The actuarially fair premium $P_{fair}(C)$ is determined by $f(P_{fair}(C)) = 0$. We have

$$f(0) = -\int_0^{C} x dF^n(x) - C(1 - F^n(C)) < 0$$

and $f(\infty) = \infty$ for all $0 < C < \infty$. Furthermore

$$f'(P) = n - n(1 - F^n(nP+C)) = nF^n(nP+C) > 0.$$

As f is a continuous function in P the intermediate value theorem implies that there exists a unique solution $P_{fair}(C) > 0$ for $f(P_{fair}(C)) = 0$.
Implicitly differentiating (10) with respect to C yields

$$nP'_{fair}(C) = \left(nP'_{fair}(C) + 1\right)(1 - F^n(nP_{fair}(C)+C))$$

which implies

$$P'_{fair}(C) = \frac{1}{n}\frac{1 - F^n(nP_{fair}(C)+C)}{F^n(nP_{fair}(C)+C)} > 0$$

for all $C > 0$. The actuarially fair premium is thus strictly increasing in the amount of risk capital.

For a mutual insurer, suppose policyholders provide a premium P^m. This premium is actuarially fair if and only if

$$P^m = E\left[\left(X_i + P^m - \frac{1}{n}\sum_{i=1}^{n} X_i\right) \cdot 1_{\{\Sigma_i X_i \leq nP^m\}} + I_i(X_1, ..., X_n) \cdot 1_{\{\Sigma_i X_i > nP^m\}}\right]$$

with $\sum_{i=1}^{n} I_i(X_1,...,X_n) = nP^m$ and $E[I_i(X_1,...,X_n)] = P^m$ for all $i = 1,...,n$. Summing over all policies yields that total premiums provided, nP^m, are actuarially fair. Since all policies have the same expected value of payout, each single premium provided is actuarially fair.

A.2 Proof of Proposition 2

Suppose that the insolvency rule specifies a pro-rata rule, i.e.

$$I_i(X_1,...,X_n) = \frac{X_i}{\sum_{i=1}^{n} X_i} \cdot TC,$$

and let Δ denote the loading in excess of the actuarially fair premium.[6] The final level of wealth of a policyholder is then given by

$$W_1^s(C,\Delta) = \begin{cases} w_S(C,\Delta) = w_0 - P_{fair}(C) - \Delta \\ w_{IS}(C,\Delta) = w_0 - P_{fair}(C) - \Delta - X_1 + \dfrac{X_1}{\sum_{i=1}^{n} X_i}\left(n\left(P_{fair}(C)+\Delta\right)+C\right) \end{cases}$$

where $w_S(C,\Delta)$ and $w_{IS}(C,\Delta)$ are the levels of final wealth in solvency and insolvency states, i.e. if $\sum_{i=1}^{n} X_i \leq n(P_{fair}(C)+\Delta)+C$ and $\sum_{i=1}^{n} X_i > n(P_{fair}(C)+\Delta)+C$, respectively. The actuarially fair premium $P_{fair}(C)$ is implicitly defined by (10). The policyholder's expected utility of final wealth is given by

$$\begin{aligned} &E[u(W_1^s(C,\Delta))] \\ &= u(w_S(C,\Delta))\,F^n(n(P_{fair}(C)+\Delta)+C) + \int_{n(P_{fair}(C)+\Delta)+C}^{\infty} u(w_{IS}(C,\Delta))\,dF^n\left(\textstyle\sum_{i=1}^{n} x_i\right) \\ &= u(w_S(C,\Delta))\,F^n(n(P_{fair}(C)+\Delta)+C) + \int_0^{\infty}\int_{n(P_{fair}(C)+\Delta)+C-x_1}^{\infty} u(w_{IS}(C,\Delta))\,dF^{n-1}(x_{-1})\,dF^1(x_1) \end{aligned}$$

where $x_{-1} = \sum_{i=2}^{n} x_i$. Differentiating expected utility with respect to Δ yields

$$\begin{aligned} \frac{\partial E[u(W_1^s(C,\Delta))]}{\partial \Delta} &= -u'(w_S(C,\Delta))\,F^n(n(P_{fair}(C)+\Delta)+C) \\ &\quad + \int_0^{\infty}\int_{n(P_{fair}(C)+\Delta)+C-x_1}^{\infty} \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) u'(w_{IS}(C,\Delta))\,dF^{n-1}(x_{-1})\,dF^1(x_1). \end{aligned}$$

[6] For expositional purposes, we focus on the pro-rata rule as bankruptcy rule. The results, however, are robust to any "reasonable" bankruptcy rule that allow to create the form of coinsurance decsribed above. More precisely, the bankruptcy rule must be such that the marginal benefit of an extra dollar under bankruptcy is increasing in the realized size of the loss.

The second derivative is given by

$$\begin{aligned}
&\frac{\partial^2 E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial \Delta^2} \\
&= u''\left(w_S\left(C,\Delta\right)\right) F^n\left(n\left(P_{fair}\left(C\right)+\Delta\right)+C\right) - nu'\left(w_S\left(C,\Delta\right)\right) f^n\left(n\left(P_{fair}\left(C\right)+\Delta\right)+C\right) \\
&+ \int_0^\infty \int_{n(P_{fair}(C)+\Delta)+C-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right)^2 u''\left(w_{IS}\left(C,\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
&- nu'\left(w_S\left(C,\Delta\right)\right) \int_0^\infty \left(-1+\frac{nx_1}{n\left(P_{fair}\left(C\right)+\Delta\right)+C}\right) f^{n-1}\left(n\left(P_{fair}\left(C\right)+\Delta\right)+C-x_1\right) dF^1\left(x_1\right) \\
&= u''\left(w_S\left(C,\Delta\right)\right) F^n\left(n\left(P_{fair}\left(C\right)+\Delta\right)+C\right) \\
&- \frac{n^2}{n\left(P_{fair}\left(C\right)+\Delta\right)+C} u'\left(w_S\left(C,\Delta\right)\right) \int_0^\infty x_1 f^{n-1}\left(n\left(P_{fair}\left(C\right)+\Delta\right)+C-x_1\right) dF^1\left(x_1\right) \\
&+ \int_0^\infty \int_{n(P_{fair}(C)+\Delta)+C-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right)^2 u''\left(w_{IS}\left(C,\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
&< 0.
\end{aligned}$$

Expected utility is globally concave in Δ and any inner solution $\Delta^*\left(C\right)$ to the FOC

$$\frac{\partial E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial \Delta}|_{\Delta=\Delta^*(C)} = 0 \tag{11}$$

is the unique global maximum. For $C=\infty$, we have $P_{fair}\left(\infty\right)=E\left[X_1\right]$ and the first derivative is given by

$$\frac{\partial E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial \Delta}|_{C=\infty} = -u'\left(w_0-E\left[X_1\right]-\Delta\right) < 0.$$

As expected utility is decreasing in Δ, we get the corner solution $\Delta^*\left(\infty\right)=0$.[7] As $\Delta^*\left(C\right)$ is continuous in C, $\Delta^*\left(C\right)=0$ for large values of C. For $C=0$, we have $P_{fair}\left(0\right)=0$ (see Proposition A.1) and the first derivative is given by

$$\begin{aligned}
&\frac{\partial E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial \Delta}|_{C=0} \\
&= -u'\left(w_0-\Delta\right) F^n\left(n\Delta\right) \\
&+ \int_0^\infty \int_{n\Delta-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) u'\left(w_0-\Delta-x_1\left(1-\frac{1}{x_1+x_{-1}}\left(n\Delta\right)\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).
\end{aligned}$$

Evaluating this derivative at $\Delta=0$ yields

$$\begin{aligned}
\frac{\partial E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial \Delta}|_{C=0,\Delta=0} &= \int_0^\infty \int_0^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) u'\left(w_0-x_1\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
&= E\left[u'\left(w_0-X_1\right)\left(-1+\frac{nX_1}{X_1+X_{-1}}\right)\right] \\
&= Cov\left(u'\left(w_0-X_1\right), \frac{nX_1}{X_1+X_{-1}}\right) + E\left[u'\left(w_0-X_1\right)\right] E\left[-1+\frac{nX_1}{X_1+X_{-1}}\right].
\end{aligned}$$

[7] The participation constraint for risk-neutral shareholders providing capital imposes $\Delta \geq 0$.

We have

$$
\begin{aligned}
E\left[-1+\frac{nX_1}{X_1+X_{-1}}\right] &= -1+nE\left[\frac{X_1}{\Sigma_{i=1}^n X_i}\right] \\
&= -1+\textstyle\sum_{i=1}^n E\left[\frac{X_i}{\Sigma_{i=1}^n X_i}\right] \\
&= 0
\end{aligned}
$$

and therefore

$$
\frac{\partial E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial \Delta}|_{C=0,\Delta=0} = Cov\left(u'\left(w_0-X_1\right), \frac{nX_1}{X_1+X_{-1}}\right) > 0.
$$

This implies $\Delta^*(0) > 0$. Again, as $\Delta^*(C)$ is continuous in C, $\Delta^*(C) > 0$ for small values of C. Total differentiation of the FOC (10) with respect to C and Δ implies

$$
\frac{d\Delta^*(C)}{dC} = -\frac{\frac{\partial^2 E\left[u\left(W_1^s(C,\Delta)\right)\right]}{\partial\Delta\partial C}|_{\Delta=\Delta^*(C)}}{\frac{\partial^2 E\left[u\left(W_1^s(C,\Delta)\right)\right]}{\partial^2\Delta}|_{\Delta=\Delta^*(C)}}.
$$

As expected utility is globally concave in Δ we derive

$$
sign\left(\frac{d\Delta^*(C)}{dC}\right) = sign\left(\frac{\partial^2 E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial\Delta\partial C}|_{\Delta=\Delta^*(C)}\right). \tag{12}
$$

The cross-derivative is then given by

$$
\begin{aligned}
&\frac{\partial^2 E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial\Delta\partial C} \\
&= P'_{fair}(C)\, u''\left(w_S(C,\Delta)\right) F^n\left(n\left(P_{fair}(C)+\Delta\right)+C\right) \\
&- \left(nP'_{fair}(C)+1\right) u'\left(w_S(C,\Delta)\right) f^n\left(n\left(P_{fair}(C)+\Delta\right)+C\right) \\
&+ \int_0^\infty \int_{n(P_{fair}(C)+\Delta)\ +C-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\
&\qquad \cdot \left(-P'_{fair}(C)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}(C)+1\right)\right) u''\left(w_{IS}(C,\Delta)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
&- \left(nP'_{fair}(C)+1\right) u'\left(w_S(C,\Delta)\right) \\
&\qquad \cdot \int_0^\infty \left(-1+\frac{nx_1}{n\left(P_{fair}(C)+\Delta\right)+C}\right) f^{n-1}\left(n\left(P_{fair}(C)+\Delta\right)+C-x_1\right) dF^1\left(x_1\right) \\
&= P'_{fair}(C)\, u''\left(w_S(C,\Delta)\right) F^n\left(n\left(P_{fair}(C)+\Delta\right)+C\right) \\
&+ \int_0^\infty \int_{n(P_{fair}(C)+\Delta)\ +C-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\
&\qquad \cdot \left(-P'_{fair}(C)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}(C)+1\right)\right) u''\left(w_{IS}(C,\Delta)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
&- \frac{n\left(nP'_{fair}(C)+1\right)}{n\left(P_{fair}(C)+\Delta\right)+C} u'\left(w_S(C,\Delta)\right) \int_0^\infty x_1 f^{n-1}\left(n\left(P_{fair}(C)+\Delta\right)+C-x_1\right) dF^1\left(x_1\right).
\end{aligned}
$$

In Proposition A.1, we have shown that $P'_{fair}(C) > 0$ which implies

$$
\begin{aligned}
\frac{\partial^2 E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial\Delta\partial C} &< \int_0^\infty \int_{n(P_{fair}(C)+\Delta)\ +C-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\
&\cdot \left(-P'_{fair}(C)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}(C)+1\right)\right) u''\left(w_{IS}(C,\Delta)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).
\end{aligned}
$$

Introducing the constant coefficient of absolute risk aversion $R_a = -\frac{u''(w)}{u'(w)}$ yields

$$\frac{\partial^2 E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial\Delta\partial C} < -R_a \int_0^\infty \int_{n(P_{fair}(C)+\Delta)+C-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \cdot \left(-P'_{fair}\left(C\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(C\right)+1\right)\right) u'\left(w_{IS}\left(C,\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).$$

For $-1+\frac{nx_1}{x_1+x_{-1}} > 0$ we have

$$-P'_{fair}\left(C\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(C\right)+1\right) > \frac{1}{n}$$

and thus

$$-\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)\left(-P'_{fair}\left(C\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(C\right)+1\right)\right) < -\frac{1}{n}\left(-1+\frac{nx_1}{x_1+x_{-1}}\right).$$

For $-1+\frac{nx_1}{x_1+x_{-1}} < 0$ we have

$$-P'_{fair}\left(C\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(C\right)+1\right) < \frac{1}{n}$$

and thus

$$-\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)\left(-P'_{fair}\left(C\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(C\right)+1\right)\right) < -\frac{1}{n}\left(-1+\frac{nx_1}{x_1+x_{-1}}\right).$$

This implies

$$\frac{\partial^2 E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial\Delta\partial C} < -R_a\frac{1}{n}\int_0^\infty \int_{n(P_{fair}(C)+\Delta)+C-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) u'\left(w_{IS}\left(C,\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).$$

The FOC (11) for $\Delta^*\left(C\right)$ implies

$$\int_0^\infty \int_{n(P+\Delta^*(C))+C-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) u'\left(w_{IS}\left(C,\Delta^*\left(C\right)\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) = u'\left(w_S\left(C,\Delta^*\left(C\right)\right)\right) F^n\left(n\left(P+\Delta^*\left(C\right)\right)+C\right)$$

and therefore

$$\frac{\partial^2 E\left[u\left(W_1^s\left(C,\Delta\right)\right)\right]}{\partial\Delta\partial C}\Big|_{\Delta=\Delta^*(C)} < -R_a\frac{1}{n}u'\left(w_S\left(C,\Delta^*\left(C\right)\right)\right) F^n\left(n\left(P_{fair}\left(C\right)+\Delta^*\left(C\right)\right)+C\right) < 0.$$

Finally, (12) implies $\frac{d\Delta^*(C)}{dC} < 0$.

A.3 Proof of Lemma 1

We prove that the condition

$$C = \left(1-\alpha\right) E\left[\left(C+nP-\sum_{i=1}^n X_i\right)^+\right] \tag{13}$$

is a one-to-one, increasing mapping between C and P. Let P be given and define the function f by

$$\begin{aligned} f(C) &= C - (1-\alpha) E\left[\left(C + nP - \sum\nolimits_{i=1}^{n} X_i\right)^{+}\right] \\ &= C - (1-\alpha)\left((C+nP) F^n (C+nP) - \int_0^{C+nP} x dF^n(x)\right). \end{aligned}$$

We have

$$\begin{aligned} f(0) &= -(1-\alpha)\left(nPF^n(nP) - \int_0^{nP} x dF^n(x)\right) < 0 \\ f(\infty) &= \infty \end{aligned}$$

and

$$f'(C) = 1 - (1-\alpha) F^n (C+nP) > 0.$$

This shows that for all P there exists a unique $C = C(P)$ such that (13) is satisfied.
Implicitly differentiating (13) with respect to P yields

$$C'(P) = \frac{(1-\alpha) nF^n (C(P) + nP)}{1 - (1-\alpha) F^n (C(P) + nP)}.$$

Thus $C'(P) > 0$ for $0 \leq \alpha < 1$ and $C'(P) = 0$ for $\alpha = 1$.

The solution P^* can be decomposed in the actuarially fair premium

$$P^*_{fair} = E\left[X_i \cdot 1_{\{\Sigma_i X_i \leq C^* + nP^*\}} + \frac{X_i}{\sum_{i=1}^{n} X_i}(C^* + nP^*) \cdot 1_{\{\Sigma_i X_i > C^* + nP^*\}}\right]$$

and a premium loading $P^*_{load} = P^* - P^*_{fair}$. Summing over all policies yields

$$nP^*_{fair} = E\left[\min\left(\sum\nolimits_{i=1}^{n} X_i, C^* + nP^*\right)\right].$$

Combining this equation with (13) implies

$$nP^*_{load} = \frac{\alpha}{1-\alpha} C^* = \alpha E\left[\left(C^* + nP^* - \sum\nolimits_{i=1}^{n} X_i\right)^{+}\right].$$

A.4 Proof of Proposition 3

First we show that there exists a strictly positive solution $(\underline{C}, \underline{P})$ of the optimization problem. From Lemma 1 we know that for each level of capital C there exists a unique $P = P(C)$ such that the ex-ante zero expected profit condition

$$C = (1-\alpha) E\left[\left(C + nP(C) - \sum\nolimits_{i=1}^{n} X_i\right)^{+}\right]$$

is satisfied. As argued in the text, the premium $\underline{P}(C)$ that results from the maximization of policyholders' expected utility given a level of capital C must include a strictly positive, optimal loading, $\Delta^*(C)$, that compensates shareholders for the frictional cost of capital, i.e.

$$n\Delta^*(C) = \alpha E\left[\left(C + n\left(P_{fair}(C) + \Delta^*(C)\right) - \sum\nolimits_{i=1}^{n} X_i\right)^{+}\right].$$

Define the function f by

$$f(C) = n\Delta^*(C) - \alpha E\left[\left(C + n\left(P_{fair}(C) + \Delta^*(C)\right) - \sum\nolimits_{i=1}^{n} X_i\right)^{+}\right].$$

We note that

$$f(0) = n\Delta^*(0) - \alpha E\left[\left(n\Delta^*(0) - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] > 0$$

as $\Delta^*(0) > 0$ (see Proposition 2). We have also shown in Proposition 2 that there exists a critical threshold level of capital $\hat{C} > 0$ above which the optimal loading is strictly zero, i.e. $\Delta^*(C) = 0$ for all $C \geq \hat{C}$. Evaluating the function f at $C = \hat{C}$ yields

$$f\left(\hat{C}\right) = -\alpha E\left[\left(\hat{C} + nP_{fair}\left(\hat{C}\right) - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] < 0.$$

The intermediate value theorem implies that there exists a solution $\underline{C}$ with $0 < \underline{C} < \hat{C}$ such that $f(\underline{C}) = 0$. The optimal premium $\underline{P}$ is then given by $\underline{P} = \underline{P}(C)$ and satisfies $\underline{P} > 0$.

To show that $\underline{C} < C^*$ and $\underline{P} < P^*$, we first show that $\underline{C} \leq C^*$ and $\underline{P} \leq P^*$ and then $\underline{C} < C^*$ and $\underline{P} < P^*$. (C^*, P^*) is defined by

$$(C^*, P^*) = \arg\max_{C,P} E\left[u\left(W_1^s(C, P)\right)\right]$$

subject to the participation constraint. The incentive-compatibility constraint for the stock insurer without commitment

$$\underline{P}(C) = \arg\max_{P} E\left[u\left(W_1^s(C, P)\right)\right] \tag{14}$$

implies that $\underline{P}(C^*) \leq P^*$. Otherwise, if $\underline{P}(C^*) > P^*$ then $(C(\underline{P}(C^*)), \underline{P}(C^*))$, where $C(\underline{P}(C^*)) > C^*$ is defined by the participation constraint, dominates (C^*, P^*). This contradicts the optimality of (C^*, P^*). Now suppose $\underline{C} > C^*$. Then $\underline{P}(\underline{C}) > P^* \geq \underline{P}(C^*)$. Since $\underline{P}(C)$ is strictly decreasing in C (this can be shown by implicitly differentiating the FOC of (14)) $\underline{P}(\underline{C}) > \underline{P}(C^*)$ implies $\underline{C} < C^*$ which is a contradiction to the assumption. Therefore, $\underline{C} \leq C^*$. This, in turn, implies $\underline{P} \leq P^*$ since the participation constraint provides an increasing mapping (see Lemma 1).

Last, we show that $\underline{C} < C^*$ and $\underline{P} < P^*$. For a stock insurer facing the commitment problem, the optimal loading $\Delta^*(\underline{C})$ provided by policyholders satisfies the following FOC (see Proof A.2 of Proposition 2)

$$\begin{aligned}
&\frac{\partial E\left[u\left(W_1\left(C, \Delta^*(\underline{C})\right)\right)\right]}{\partial \Delta} \\
&= -u'\left(w_S\left(C, \Delta^*(\underline{C})\right)\right) F^n\left(n\left(P_{fair}(\underline{C}) + \Delta^*(\underline{C})\right) + \underline{C}\right) \\
&+ \int_0^\infty \int_{n(P_{fair}(\underline{C})+\Delta^*(\underline{C}))+\underline{C}-x_1}^\infty \left(-1 + \frac{nx_1}{x_1 + x_{-1}}\right) u'\left(w_{IS}\left(C, \Delta^*(\underline{C})\right)\right) dF^{n-1}(x_{-1})\, dF^1(x_1) \\
&= 0.
\end{aligned}$$

For a stock insurer not facing the commitment problem, policyholders' expected utility of final wealth is given by

$$\begin{aligned}
&E\left[u\left(W_1\left(C(P), P\right)\right)\right] \\
&= u\left(w_S\left(C(P), P\right)\right) F^n\left(nP + C(P)\right) + \int_0^\infty \int_{nP+C(P)-x_1}^\infty u\left(w_{IS}\left(C(P), P\right)\right) dF^{n-1}(x_{-1})\, dF^1(x_1),
\end{aligned}$$

where

$$W_1\left(C(P), P\right) = \begin{cases} w_S(P) = w_0 - P & \text{if } \sum_{i=1}^n X_i \leq nP + C(P) \\ w_{IS}(P) = w_0 - P - X_1 + \dfrac{X_1}{\sum_{i=1}^n X_i}\left(nP + C(P)\right) & \text{if } \sum_{i=1}^n X_i > nP + C(P) \end{cases}.$$

and $C(P)$ is the unique level of capital that satisfies the zero expected profit condition (see Lemma 1).

Differentiating expected utility with respect to P yields

$$\frac{\partial E\left[u\left(W_1\left(C\left(P\right),P\right)\right)\right]}{\partial P}$$
$$= -u'\left(w_S\left(C\left(P\right),P\right)\right)F^n\left(nP+C\left(P\right)\right)$$
$$+\int_0^\infty\int_{nP+C(P)-x_1}^\infty\left(-1+\left(n+C'\left(P\right)\right)\frac{x_1}{x_1+x_{-1}}\right)u'\left(w_{IS}\left(C\left(P\right),P\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right).$$

Evaluating this derivative at the optimal premium level of the stock insurer facing the commitment problem, i.e. at $P=\underline{P}\left(\underline{C}\right)=P_{fair}\left(\underline{C}\right)+\Delta^*\left(\underline{C}\right)$, yields

$$\frac{\partial E\left[u\left(W_1\left(C\left(P\right),P\right)\right)\right]}{\partial P}|_{P=\underline{P}(\underline{C})}$$
$$= C'\left(\underline{P}\left(\underline{C}\right)\right)\int_0^\infty\int_{n\underline{P}(\underline{C})+\underline{C}-x_1}^\infty\frac{x_1}{x_1+x_{-1}}u'\left(w_{IS}\left(\underline{C},\underline{P}\left(\underline{C}\right)\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right).$$

In Lemma 1 we have shown $C'\left(P\right)>0$ and thus $\frac{\partial E[u(W_1(C(P),P))]}{\partial P}|_{P=\underline{P}(\underline{C})}>0$. This implies $P^*>\underline{P}\left(\underline{C}\right)=\underline{P}$.[8] As the participation constraint provides an increasing mapping between capital and premium (see Lemma 1) we conclude $C^*>\underline{C}$.

A.5 Proof of Proposition 4

Since $nP+C=nP^m$ the premium P under the stock form can be expressed as

$$P=P^m-\frac{1}{n}C\left(P\right)=P^m-\frac{1}{n}\left(1-\alpha\right)E\left[\left(nP^m-\sum\nolimits_{i=1}^n X_i\right)^+\right]$$

where $C\left(P\right)$ is defined by the shareholders' participation constraint (1). The policyholder's final wealth distribution (3) under the stock form can then be written as

$$W_1^s\left(C,P\right)=W_1^s\left(P^m\right)$$
$$=\begin{cases} w_0-P^m-\frac{1}{n}\left(1-\alpha\right)E\left[\left(nP^m-\sum_{i=1}^n X_i\right)^+\right] & \text{if } \sum_{i=1}^n X_i\le nP^m \\ w_0-P^m-X_1+\dfrac{X_1}{\sum_{i=1}^n X_i}nP^m-\frac{1}{n}\left(1-\alpha\right)E\left[\left(nP^m-\sum_{i=1}^n X_i\right)^+\right] & \text{if } \sum_{i=1}^n X_i>nP^m \end{cases}.$$

Subtracting the policyholder's final wealth distribution (8) under the mutual form yields

$$W_1^s\left(C,P\right)-W_1^m\left(P^m\right)=\frac{1}{n}\left(1-\alpha\right)E\left[\left(nP^m-\sum\nolimits_{i=1}^n X_i\right)^+\right]-\frac{1}{n}(1-\alpha)\left(nP^m-\sum\nolimits_{i=1}^n X_i\right)\cdot 1_{\{\Sigma_i X_i\le nP^m\}}.$$

Since $E\left[W_1^s\left(C,P\right)-W_1^m\left(P^m\right)\right]=0$ and since policyholders are risk-averse they prefer the wealth distribution under the stock form over the wealth distribution under the mutual form, i.e. $E\left[u\left(W_1^s\left(C,P\right)\right)\right]>E\left[u\left(W_1^m\left(P^m\right)\right)\right]$.

A.6 Proof of Corollary 1

Let P^{m*} denote the solution to the optimization problem (7) of the mutual insurer. Define the premium P' of an stock insurer such that it is equally capitalized to the mutual insurer, i.e. P' satisfies

$$nP'+C\left(P'\right)=nP^{m*}$$

[8] Note that, for this conclusion, we do not need the concavity of expected utility in P, since we have shown above that $P^*\ge\underline{P}$.

where $C(P')$ is uniquely defined by the shareholders' participation constraint (1).[9] Proposition 4 implies that the stock insurer dominates the mutual insurer, i.e. $E[u(W_1^s(C(P'),P'))] > E[u(W_1^m(P^{m*}))]$. For the optimal level of total capital, $nP^* + C(P^*)$, of the stock insurer we have $E[u(W_1^s(C(P^*),P^*))] > E[u(W_1^s(C(P'),P'))]$ and therefore $E[u(W_1^s(C(P^*),P^*))] > E[u(W_1^m(P^{m*}))]$.

A.7 Proof of Proposition 5

Stock insurer with and without commitment. The optimization problem (2) with k subgroups is

$$(C^*, P^*) = \arg\max_{(C,P)} E[u(W_1^s(C,P))]$$

subject to

$$C = (1-\alpha) E\left[\left(C + nP - \frac{n}{k}\sum\nolimits_{j=1}^{k} X_j\right)^+\right].$$

with

$$W_1^s(C,P) = \begin{cases} w_0 - P & \text{if } \frac{n}{k}\sum_{j=1}^{k} X_j \leq C + nP \\ w_0 - P - X_1 + \dfrac{X_1}{\frac{n}{k}\sum_{j=1}^{k} X_j}(C + nP) & \text{if } \frac{n}{k}\sum_{j=1}^{k} X_j > C + nP \end{cases}.$$

By defining $c = \frac{1}{n}C$ as shareholders' capital per policyholder we can rewrite this problem as

$$(c^*, P^*) = \arg\max_{(c,P)} E[u(W_1^s(c,P))]$$

subject to

$$kc = (1-\alpha) E\left[\left(k(c+P) - \sum\nolimits_{j=1}^{k} X_j\right)^+\right].$$

with

$$W_1^s(c,P) = \begin{cases} w_0 - P & \text{if } \sum_{j=1}^{k} X_j \leq k(c+P) \\ w_0 - P - X_1 + \dfrac{X_1}{\sum_{j=1}^{k} X_j} k(c+P) & \text{if } \sum_{j=1}^{k} X_j > k(c+P) \end{cases}.$$

This optimization problem is equivalent to the one with k policyholders facing independent and identically distributed losses. The same comparison holds for the optimization problem of the stock insurer without commitment.

Mutual insurer. For the mutual, the optimization problem with k subgroups is

$$P^{m*} = \arg\max_{P^m} E[u(W_1^m(P^m))]$$

with

$$\begin{aligned} W_1^m(P^m) &= \begin{cases} w_0 - P^m + \frac{1}{n}(1-\alpha)\left(nP^m - \frac{n}{k}\sum_{j=1}^{k} X_j\right) & \text{if } \frac{n}{k}\sum_{j=1}^{k} X_j \leq nP^m \\ w_0 - P^m - X_1 + \dfrac{X_1}{\frac{n}{k}\sum_{j=1}^{k} X_j} nP^m & \text{if } \frac{n}{k}\sum_{j=1}^{k} X_j > nP^m \end{cases} \\ &= \begin{cases} w_0 - P^m + \frac{1}{k}(1-\alpha)\left(kP^m - \sum_{j=1}^{k} X_j\right) & \text{if } \sum_{j=1}^{k} X_j \leq kP^m \\ w_0 - P^m - X_1 + \dfrac{X_1}{\sum_{j=1}^{k} X_j} kP^m & \text{if } \sum_{j=1}^{k} X_j > kP^m \end{cases}. \end{aligned}$$

Again, the optimization problem of the mutual insurer is exactly identical to the one with k policyholders facing independent and identically distributed losses.

[9] Note that P' exists and is unique since $nP + C(P)$ is increasing in P from 0 to $+\infty$ accoridng to Lemma (1).

A.8 Proof of Proposition 6

Incentives stemming from the effect on the utility are higher under the mutual form if

$$U^m(n) - U^m(q) > U^s(n) - U^s(q). \tag{15}$$

For $\beta = \beta^*$, $P^{m*} = P(\beta^*) + C(\beta^*)/n$ and $U^s(n) = U^m(n)$. The inequality holds if $U^s(q) > U^m(q)$. For $P^{m*} = P(\beta^*) + C(\beta^*)/n$, we obtain

$$\begin{aligned} U^s(q) &= \delta E\left[\left(C(\beta^*) + qP(\beta^*) - \sum\nolimits_{i=1}^{q} X_i\right)^+\right] \\ &> \delta E\left[\left(\frac{q}{n}C(\beta^*) + qP(\beta^*) - \sum\nolimits_{i=1}^{q} X_i\right)^+\right] \\ &= U^m(q) \end{aligned}$$

and condition (15) holds.

For $\beta < \beta^*$, the mutual insurer is more capitalized than the stock insurer, i.e. $P^{m*} > P(\beta) + C(\beta)/n$. We prove (15) by showing that the managerial incentives to expand a mutual insurer are increasing in the capital of the company. We thus have to show that

$$U^m(n|P^{m*}) - U^m(q|P^{m*}) > U^m(n|\gamma P^{m*}) - U^m(q|\gamma P^{m*})$$

for $0 \leq \gamma < 1$ and

$$U^m(q|P^{m*}) = \delta E\left[\left(qP^{m*} - \sum\nolimits_{i=1}^{q} X_i\right)^+\right] - c(q).$$

This inequality is equivalent to

$$\begin{aligned} & E\left[\left(nP^{m*} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] - E\left[\left(n\gamma P^{m*} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] \\ & > E\left[\left(qP^{m*} - \sum\nolimits_{i=1}^{q} X_i\right)^+\right] - E\left[\left(q\gamma P^{m*} - \sum\nolimits_{i=1}^{q} X_i\right)^+\right]. \end{aligned}$$

We will prove the inequality by using the Central Limit Theorem (CLT) to show that $E\left[\left(nP^{m*} - \sum_{i=1}^{n} X_i\right)^+\right] - E\left[\left(n\gamma P^{m*} - \sum_{i=1}^{n} X_i\right)^+\right]$ is increasing in n. The CLT implies

$$\begin{aligned} & E\left[\left(nP^{m*} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] - E\left[\left(n\gamma P^{m*} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] \\ & = \frac{1}{\sqrt{2\pi}} \frac{n\sqrt{n}}{\sigma(X_1)} \left(\int_0^{z_2(n)} (z_2(n) - z)\, e^{-\frac{1}{2}z^2} dz - \int_0^{z_1(n)} (z_1(n) - z)\, e^{-\frac{1}{2}z^2} dz\right) \end{aligned} \tag{16}$$

where $z_1(n) = \frac{\gamma P^{m*} - E[X_1]}{\sigma(X_1)/\sqrt{n}}$ and $z_2(n) = \frac{P^{m*} - E[X_1]}{\sigma(X_1)/\sqrt{n}}$. The first term of the product is increasing in n. Differentiating the second term with respect to n yields

$$\begin{aligned} & \frac{\partial}{\partial n}\left(\int_0^{z_2(n)} (z_2(n) - z)\, e^{-\frac{1}{2}z^2} dz - \int_0^{z_1(n)} (z_1(n) - z)\, e^{-\frac{1}{2}z^2} dz\right) \\ & = z_2'(n) \int_0^{z_2(n)} e^{-\frac{1}{2}z^2} dz - z_1'(n) \int_0^{z_1(n)} e^{-\frac{1}{2}z^2} dz \\ & = \frac{1}{2\sigma(X_1)\sqrt{n}} \left((P^{m*} - E[X_1]) \int_0^{z_2(n)} e^{-\frac{1}{2}z^2} dz - (\gamma P^{m*} - E[X_1]) \int_0^{z_1(n)} e^{-\frac{1}{2}z^2} dz\right). \end{aligned}$$

Define the function f by

$$f(\gamma) = \left(P^{m*} - E[X_1]\right) \int_0^{z_2(n)} e^{-\frac{1}{2}z^2} dz - \left(\gamma P^{m*} - E[X_1]\right) \int_0^{z_1(n)} e^{-\frac{1}{2}z^2} dz.$$

If $\gamma P^{m*} - E[X_1] \leq 0$ then $f(\gamma) > 0$ (as $P^{m*} > E[X_1]$) and (16) is increasing in n. Suppose $\gamma P^{m*} - E[X_1] > 0$. Then $f(1) = 0$ and

$$f'(\gamma) = -P^{m*} \int_0^{z_1(n)} e^{-\frac{1}{2}z^2} dz - \left(\gamma P^{m*} - E[X_1]\right) \frac{P^{m*}}{\sigma(X_1)/\sqrt{n}} e^{-\frac{1}{2}z_1(n)^2} < 0.$$

Since f is strictly decreasing and $f(1) = 0$, we derive that $f(\gamma) > 0$ for all $0 \leq \gamma < 1$. (16) is therefore increasing in n.

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