# THE FINAL INEQUALITY: VARIANCE IN AGE AT DEATH

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## Introduction

Demography and economics shape many aspects of individual lives and social policy. The most dramatic effects of demography result from changes over time in patterns of vital events – birth, marriage, death, migration, and so on. Some changes, such as the recent decline of fertility in many industrialized countries (e.g., in southern Europe and Japan), have occurred over one human generation (about 25 yrs), others, such as the expansion of human life span, have been ongoing for over a century. Increasing life expectancy coupled with fertilities near replacement levels result in population aging, which in turn drives increases in the estimated costs of retirement and health care. Life expectancy and mortality are also of direct interest as indices of population health. Analyses of mortality change and of the determinants of mortality often use life expectancy at birth as a key statistic summarizing the lifetime effects of mortality and health. Many studies have examined trends and forecasts of aggregate life expectancy, while others have examined the effect on health of inequalities in wealth, income, or education by studying differences in life expectancy between groups that differ in these characteristics.

This paper takes a different look at mortality change and its determinants, by examining uncertainty in the age of adult death. Infant and child mortality are now at historically low levels in most industrialized countries, and are falling rapidly in the largest developing countries including India and

China. The probability of dying below age 10 or 20 years has fallen steadily over time; for example in the US, about 98% of deaths in period lifetables from 1980 onwards occur at ages over 10 yrs (see Figure 1). Uncertainty in the age at death is now primarily in the age of adult death, where I use the term "adult" somewhat loosely as all ages over 10 or 20 yrs. The relationships between mortality and economic issues, such as lifetime consumption and income or the cost of retirement, and between mortality and health, are now mainly driven by the pattern of adult death. An accurate description of the distribution of adult deaths is also the principal target of efforts to forecast mortality. Thus variation between individuals in the age of adult death can be thought of as a central inequality that shapes many important outcomes.

I begin by discussing measures of dispersion in age at death and defining the variance in age at adult death. Next I discuss historical trends in this variance and the effect of variance in adult death on simple economic measures in an overlapping generations setting. I then turn to the relationship between the pattern of adult death and inequalities rooted in education, income, and race in the US. Finally I consider international trends, first with a comparison of industrialized countries and then with a comparison of changes in India with historical changes in the industrialized countries. This paper draws on joint work with Ryan Edwards, including Edwards and Tuljapurkar (2005), Tuljapurkar and Edwards (2008), on Qi Li's 2005 PhD thesis at Stanford, and calculations that we provided to the OECD for their

Social indicators report (2006); I also present some more recent work on India with Anant Sudarshan and Debarun Bhattacharya.

# Variance in Age at Adult Death

## Measuring Dispersion

The age pattern of mortality is described by an age-specific mortality rate $\mu(a)$ and a corresponding survivorship $l(a)$ which is the probability of living to at least age $a$. The probability that an individual dies at age $a$ is described by the density $\phi(a) = \mu(a)\, l(a)$. Fig. 2 shows this distribution using 1999 data from the US when the period life expectancy was 77 yrs. Infant deaths are in the peak near zero, the probability of death then falls to a low until age nearly 20 yrs, then rises slowly till age 60 yrs, and the mass of $\phi$ is mostly centered around a mode at about 85 yrs. Just over 85% of all deaths occur between ages 60 and 99 yrs. It is the variation in this age range that describes the bulk of variation in "adult" death. However the variance of the entire distribution $\phi$ will always be strongly affected by the infant mortality peak even when infant mortality is as small as it is in Fig. 2. If we choose an age $A$ near the minimum of $\phi$, say in the range 10 to 20 yrs, then deaths above such an $A$ are what we call "adult" deaths. Following Edwards and Tuljapurkar (2005), we describe the variance in the age of adult death by the conditional variance of the distribution of age at death given that death occurs past age $A$. Within the above range, the value of $A$ used is largely a matter of convenience and different choices do not materially alter the results.

## Historical Dispersion in Adult Death

Fig. 3 shows the variance in age at death for Sweden from 1893 to 2003, using deaths at ages over 10 (dashed line) and 20 (solid line). The curves follow similar trajectories overall – there is a substantial numerical difference before 1950, when child deaths before age 20 were still relatively common. After 1960 the two curves are very close. Note the historical pattern, with very little change before 1920, rapid and steady decline from 1920 to 1960, and then a modest decline till 2003. Fig. 4 compares the variance of deaths at age over 10 yrs with the total variance in age at death – the latter is strongly influenced by infant mortality and shows a decline starting much earlier, a more rapid decline overall, and a continuing decline in recent decades. Clearly the rate of change of the dispersion of adult mortality has slowed substantially in recent years.

From here on I will describe dispersion in adult death by the standard deviation $S(A)$ of the age at death conditional on deaths occurring at ages past $A$. Typically I will use $S(10)$ or $S(20)$. Thinking about $S(20)$ allows us to ask whether mortality improvement means that both the modal age and the variance in adult age at death change together. In other words, are we compressing inequality in age at adult death while also delaying death? Fig. 5 plots $S(20)$ versus life expectancy $e_0$ for Sweden. Time turns out to run from left to right across the plot. In the 19th century, there was a negative correlation between $S(10)$ and $e_0$ but a lot of variation. In about 1920, $e_0$ approached 60 yrs, and for the next 50 years $S(10)$ fell and $e_0$ rose in concert, until about 1960 when $e_0$ was 73 yrs. In the years after 1975, the pattern appears to have changed significantly, as $e_0$ continues to increase but $S(20)$ is relatively unchanged. Before 1975, mortality declines in the 20th century clearly acted as a "rising tide" that reduced inequality in age at adult death across the population as a whole. The pattern shown here for Sweden is repeated in all countries for which I have found reasonably long historical series.

## International Trends and the Future

The slowdown in decline of $S(20)$ in Sweden since about 1960, seen in Fig. 5, is partially mirrored across the industrialized world. Fig. 6, originally drawn for an OECD comparison, shows $S(10)$ since 1960 for a subset of the OECD countries. Note that modest declines have continued in some but not all countries: Japan's $S(10)$ declined until 1990 and Sweden, Canada, and Australia resumed declines after 1980. The US has the highest $S(10)$ across these countries (and indeed across the industrialized world) over the entire period, and the difference between the US and Canada after 1980 is striking.

Bongaarts (2007) recently proposed an interesting model of mortality change to be used in making forecasts. He proposed that life expectancy simply increases at some steady rate per year and that the shape of the distribution of deaths, our $\phi(a)$, does not change with time for deaths over age 25 yrs. All that happens is that the mass of deaths, as in our Fig. 1, translates to later ages at some steady rate, but with the variance in adult death held constant. He arrived at his model using rather different arguments about the nature of senescence and so our historical analysis provides a direct test of his assumption. It is clear from Fig. 6 that his approximation is a plausible approximation to trends in the US since 1960; it may also be plausible for some other but not all countries in recent decades. His model would clearly not be correct as a description of historical change prior to 1960.

## Economics and Variance in Adult Death

Our variance $S(10)$ is simply the dispersion of the random age at death, call it $T$, across adult individuals in a population. We can approximate the distribution of adult deaths by a normal distribution around the modal age at death, call it $\mu$, with a standard deviation $\sigma = S(10)$. This approximation undershoots the true left-skewed distribution at ages below $\mu$ and overshoots the true distribution at ages much over $\mu$, but it is quite reasonable for seeing how variance in $T$ affects lifetime income, consumption and utility. Suppose that wages are fixed at some value $W$ and an individual works starting at some age $a_s$ (upon leaving school or college, say) until the earlier of death or retirement at age $a_r$. For a given interest rate $r$, lifetime earnings are

$$I = W \int_0^{(T \wedge a_r)} ds\, e^{-rs} = \left(\frac{W}{r}\right) [e^{-r\, a_s} - \mathcal{E} e^{-r\,(T \wedge a_r)}].$$

Here $\mathcal{E}$ indicates an expectation over the distribution of age at death $T$, which we take to be a normal distribution as above. The exact expressions here are messy but they are closely approximated by

$$I = \left(\frac{W}{r}\right) \left\{e^{-r\,a_s} - l(a_r)e^{-r\,a_r} - [1 - l(a_r)]e^{-r\mu+(1/2)r^2\sigma^2}\right\}.$$

This is sensible: when retirement occurs at an age well below the modal age at death $\mu$, uncertainty in death has little effect on lifetime income. As age at retirement increases towards $\mu$, the dispersion $\sigma$ in $T$ translates into dispersion in lifetime income. There is a tradeoff between $\mu$ and $\sigma$, in that

$$\frac{\partial I}{\partial \sigma} = -r\,\sigma\,\frac{\partial I}{\partial \mu}.$$

For interest rate   0.03, and $\sigma$ 14, which is typical of industrialized countries, the multiplier is 0.42; in developing countries with $\sigma$ 25, the multiplier is 1. So the effect of increasing $\mu$ by a year is about the same as decreasing $\sigma$ by half a year in industrialized countries and by a year in developing countries.

Lifetime consumption also depends on $T$. In simple overlapping generations models (Blanchard-Fischer 1989) with constant relative risk aversion (CRRA) utility, the optimal consumption at age $x$ is a function

$$c(x) = c_0\, e^{kx}, \text{ where } k = (r-\theta)/\gamma,$$

where $r$ is interest rate, $\theta$ is the discount rate, and $\gamma$ is the coefficient of risk aversion. Lifetime consumption then depends on $e^{kT}$ and we have

$$\mathcal{E}e^{kT} = e^{k\mu+(1/2)k^2\sigma^2}.$$

So inequality in $T$ translates into inequality in lifetime consumption. This fact suggests that it would be useful to incorporate uncertainty in $T$ into analyses of the benefits of increasing lifespan.

Lifetime utility depends on consumption in these settings, and in the CRRA model, utility at age $x$ is proportional to $c(x)^{(1-\gamma)}/(1-\gamma)$. Expected lifetime utility averages over the variation in $T$ and thus also depends on $\sigma$. The effect of $\sigma$ on lifetime consumption depends on the factor $k$ but the effect on lifetime utility depends on the product $k(1-\gamma)$, being modified by the level of risk aversion. Qi Li (2005) has explored these connections in more detail by studying the equilibrium of a simple closed economy model with adult deaths distributed normally as above.

# The Sources of Variance in Adult Death

I turn now to a different question: what causes differences in mortality between groups that are distinguished by characteristics such as income, education, race or other factors that we expect to influence mortality risk? This question has become particularly important in recent discussions about the relationships between inequality measured in various ways and mortality. Typically, attention has focused on the effect of a particular risk factor and life expectancy, though the epidemiological literature tends to focus on the effect on mortality rates. Controlling for differences in other likely risk factors, a successful analysis detects a difference in the $e_0$ corresponding to differences in the factor in question. Such studies measure what I call the variance *between* groups that are distinguished by particular explanatory factors. We can look at these differences in a different and informative way by asking how an explanatory factor affects the variance of adult age at death both *between* groups and *within* groups.

I consider three examples. The first two use data from the National Longitudinal Mortality Study using data from 1981. Fig. 7 looks at the distribution of adult death among people with less than a high school education and those who have graduated high school. Note that the difference in modal ages at death is significant, about 5 yrs. But note also that the variance within each group is large, with each $\sigma$ $4yrs$. If we think in terms of variance, the variance between groups is about 6.25 and that within groups is about 15. Clearly education matters, but it matters rather less to dispersion, i.e., to the predictive power of education about the age of death. A similar result is found when looking at age at death as a function of household income: Fig. 8 compares death distributions for the highest and lowest income quantiles. I argue that an examination of variances adds substantially to the information that we get from studying how possible causal factors affect lifespans within groups.

A different decomposition that is of particular interest in the US is by race. Fig. 9 uses 1981 data to compare adult death distributions between African-Americans and whites in the US. Again we find that variances within groups are substantially larger than variance between groups. In work with Ryan Edwards, we find that the variances within and between these racial groups have been strikingly stable over the past 5 decades in the US, leading to the overall stability of the aggregate $S(10)$ seen in Fig. 6. The conclusion is that we need to explore factors that explain the variances within these groups, whether by education, income or race. More generally, we could search for the collection of risk factors that best separates groups, i.e., that maximizes the ratio of between-roup variance to within-group variance in adult age at death.

Figs. 7 through 9 also point to interesting differences in mortality by age between the groups considered. In every case, the probability of death is more sharply skewed to younger ages in the group that has a lower modal age at death. This fact suggests that it would also be useful to explore risk factors that appear to result in higher mortality at ages much below the modal age at death.

A final point about looking at $S(10)$ over historical periods is that it is an aggregate measure of variance. Say that we decompose the population into groups by risk factors. A decline in $S(10)$ over any period means that the sum of the variances within and between these groups has declined. Given that variances within groups tend to dominate these decompositions, it is plausible that much of the effect of mortality decline has been to reduce the within-group variances for all groups, rather than to reduce the variances within particular groups.

## The Tides of History

I close by taking a long view at the correlated changes in variance in age at adult death and life expectancy and also a look at what has happened in India over the past 4 decades. Here I will use $S(16)$ – this may seem odd, but 16 yrs is an appropriate choice for the cutoff of "adult" ages for the Indian data. These data are from the Sample Registration System and were provided to us by Mari Bhat. The other data we use are from the Human Mortality Database (HMD) that is supported at Berkeley and Rostock.

Fig. 10 plots $S(16)$ against $e_0$ for the nineteenth century. That was still the age of epidemic disease and there was little systematic improvement in mortality over much of the period. The trajectories of change seen in the figure are correspondingly chaotic, with values dispersed around $S(16)$ of 31 yrs (rage 28 to 33) and $e_0$ about 35 yrs (range 20 to 50). Fig. 11 takes us to the age of rapid mortality decline in the first half of the twentieth century, through 1960. Now there is a dramatic structural change from the chaos of Fig. 10, as trajectories mostly congeal around a clear negative correlation, with $S(16)$ falling to 18 (with a range 18 to 33) and $e_0$ rising to 65 or more. Finally, we move to a period of slow but persistent change in Fig. 11, post-

1960, as $S(16)$ falls to around 15 and $e_0$ marches towards 75-80 yrs.

It has often been suggested that as countries march towards industrialization they will follow roughly similar trajectories of change in aspects of human and economic development. Fig. 13 plots values of $S(16)$ and $e_0$ for the states of India (we used 22 that had the same identity over the period), shown in red. The figure also shows values of $S(16)$ and $e_0$ over the period for the HMD countries. The trajectories of change in India are somewhat similar to those for the HMD but shifted to lower $e_0$ and higher $S(16)$. The values of $S(16)$ and $e_0$ for India over the period are in the range shown by the HMD countries nearly fifty years previously. But the Indian trajectory of change is consistent in its pattern and has about the same slope as the HMD trajectories. There is also an interesting "floor" to the Indian trajectories – there appear to be more deviations above than below the main trend line. This suggests that development is improving conditions everywhere but that few states are doing much better than the majority. To the extent that all states are benefiting from rapid economic growth, these results suggest that growth is certainly good for the mortality conditions of all states, including those with high levels of poverty.

# Conclusion

I have attempted to show that the variance in age at adult death is a useful and important dimension of mortality change. Trends in this variance are informative about the speed and the age-pattern of mortality change. The decomposition of this variance with respect to risk factors provides useful insights into the explanatory power of different factors that are correlated with mortality. Historical and economic analyses can benefit from an examination of variance in age at death in addition to the traditionally important studies of life expectancy.

# Literature Cited

Blanchard,O.J.,Fischer,S.,1989. Lectures on macroeconomics. The MIT Press.

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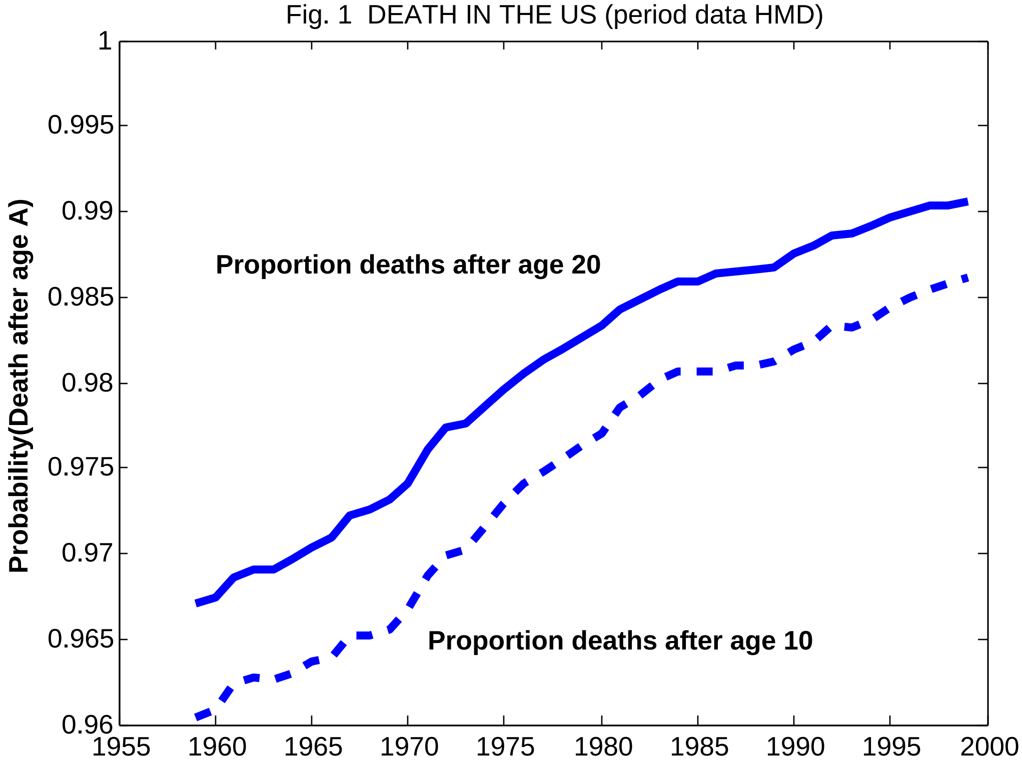
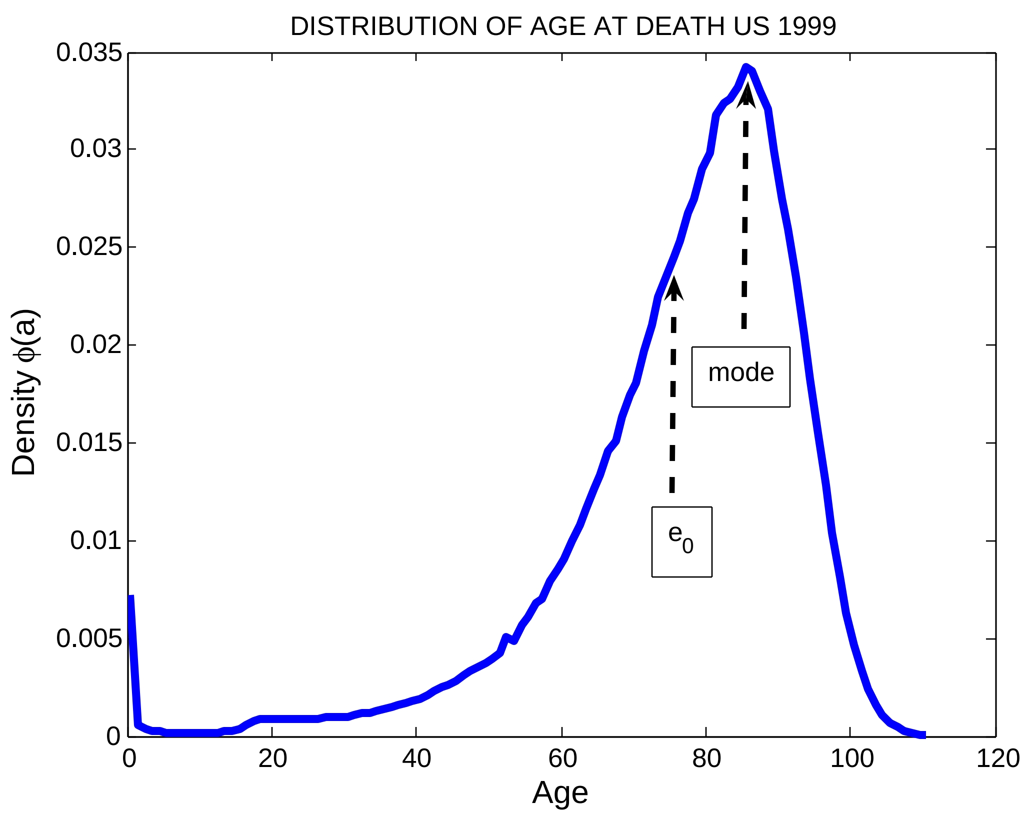
Fig. 1  DEATH IN THE US (period data HMD)





DISTRIBUTION OF AGE AT DEATH US 1999
Density φ(a)
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
20
40
60
80
100
120
Age
mode
e0

**Fig. 3 Sweden Female Var(10+) [dashes], Var(20+) [solid]**

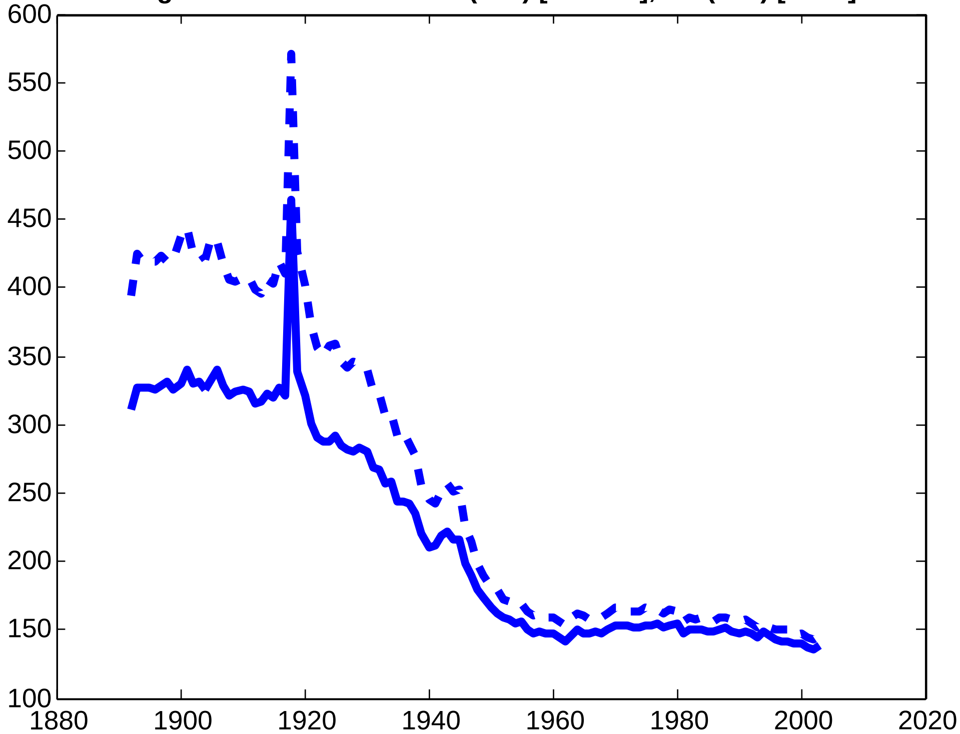


Fig. 4 Sweden Female Var(10+) [solid], Total Var [dashes]

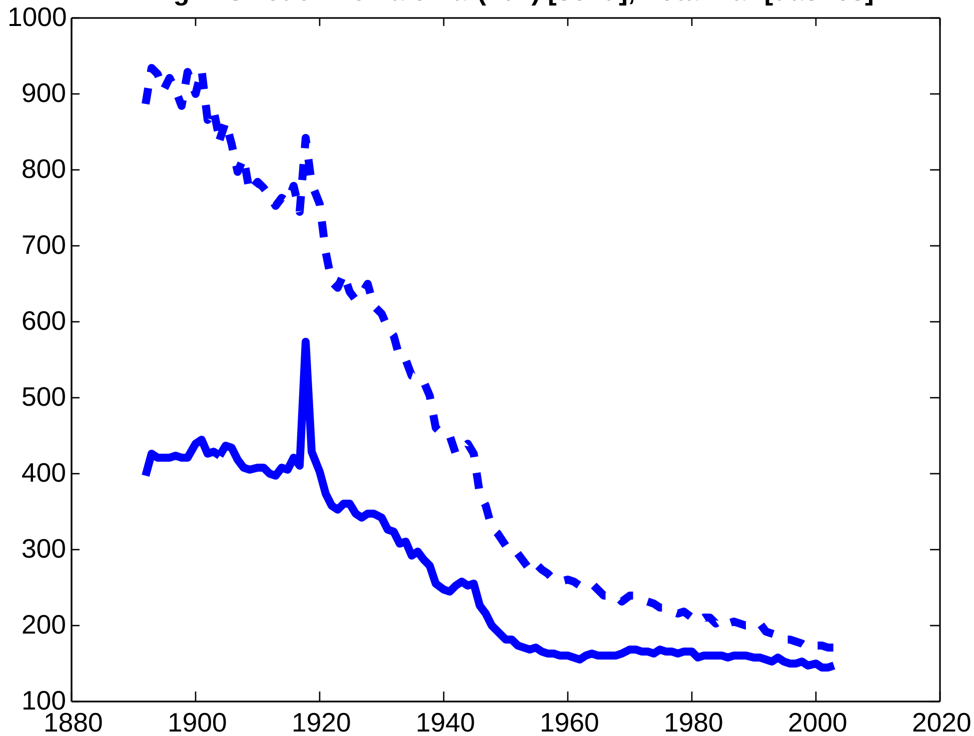


**Fig. 5 Sweden Female S(20) VS $e_0$. 1891−1953**

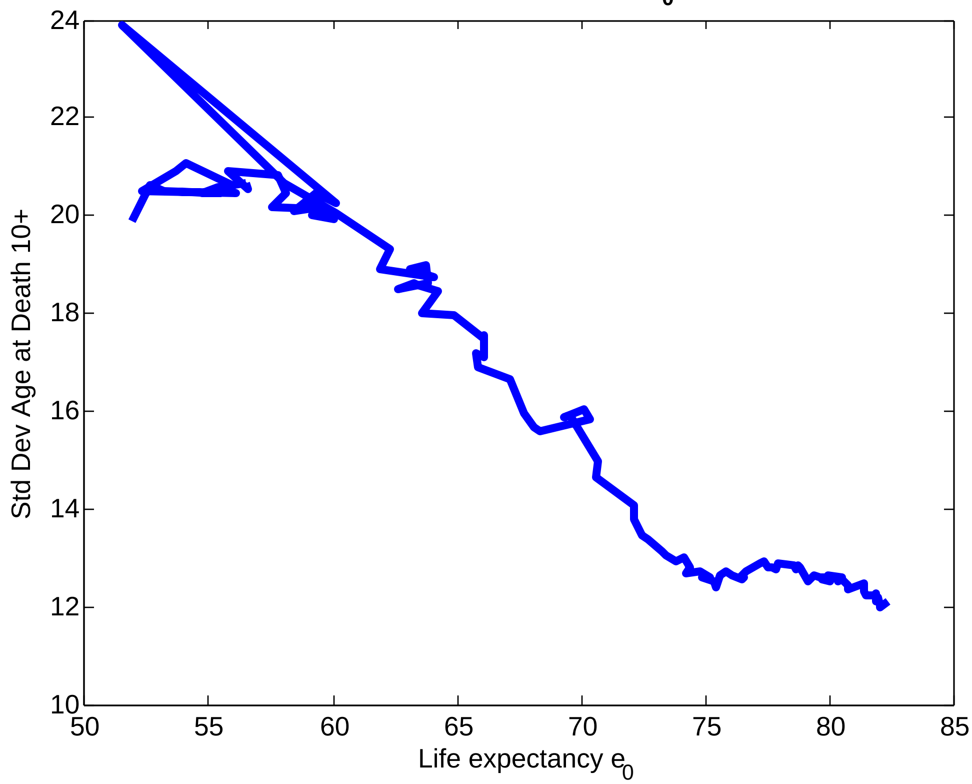
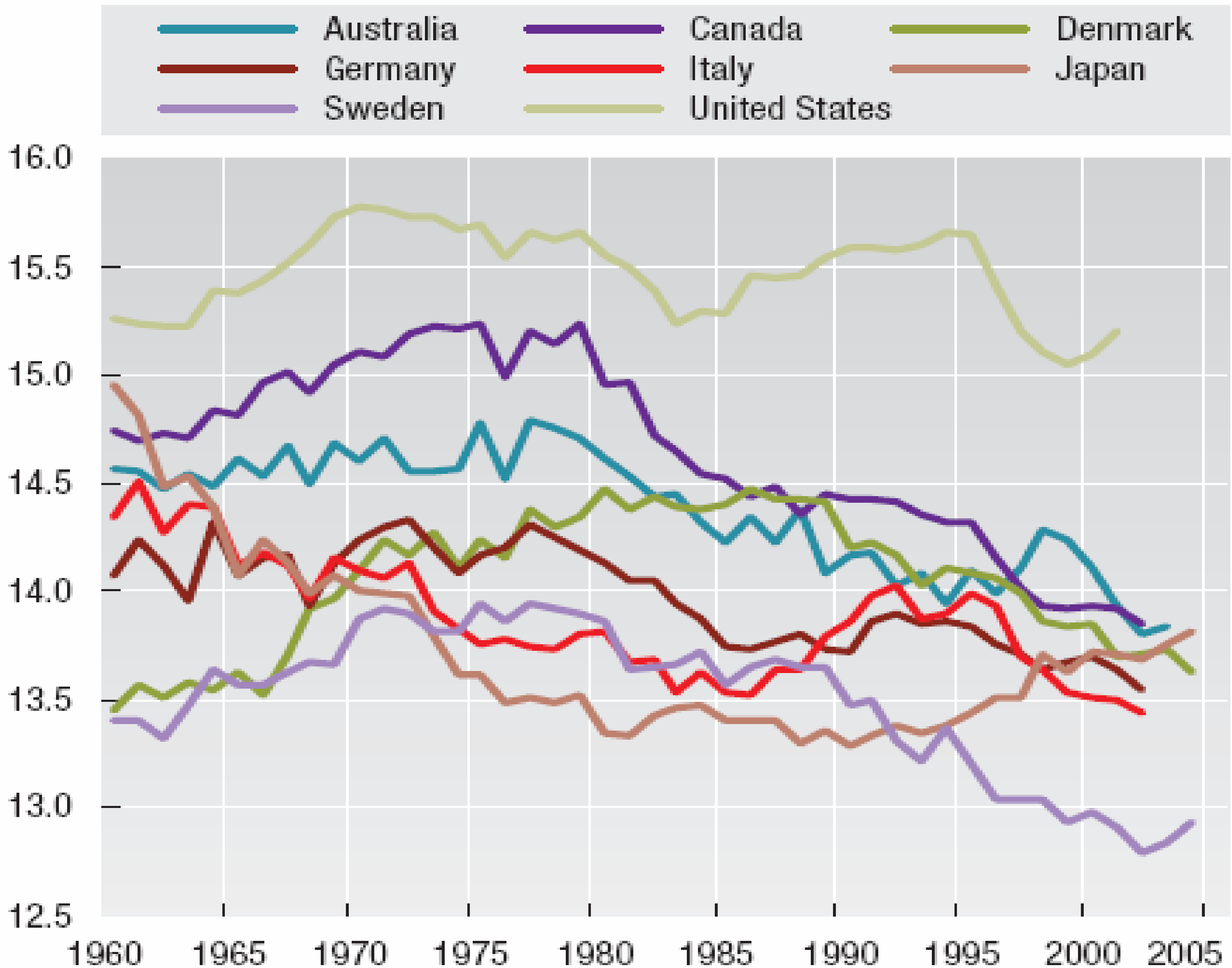


Fig. 6



Australia
Canada
Denmark
Germany
Italy
Japan
Sweden
United States
16.0
15.5
15.0
14.5
14.0
13.5
13.0
12.5
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005

# Fig. 7 Education and age at death using the NLMS

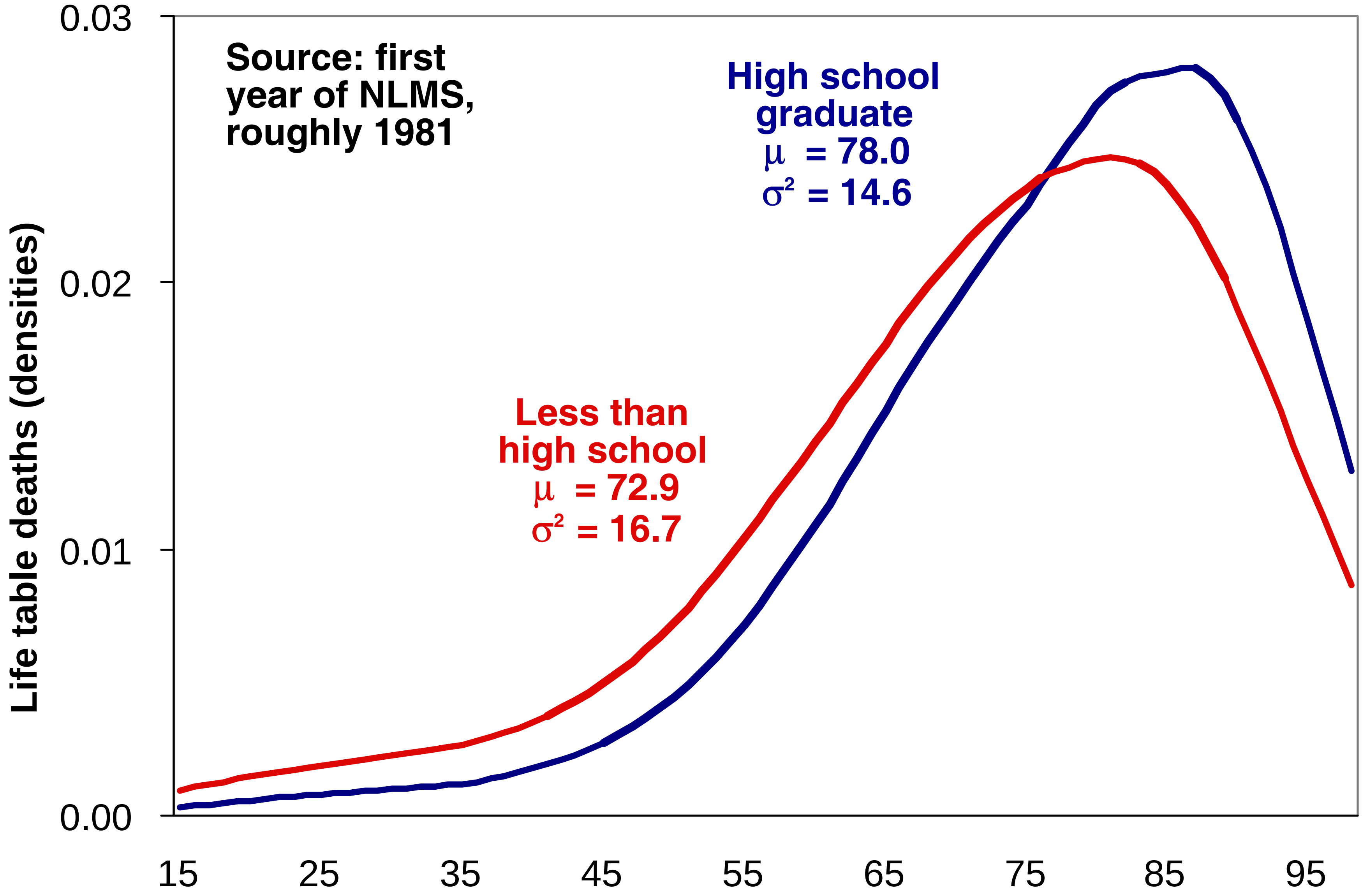
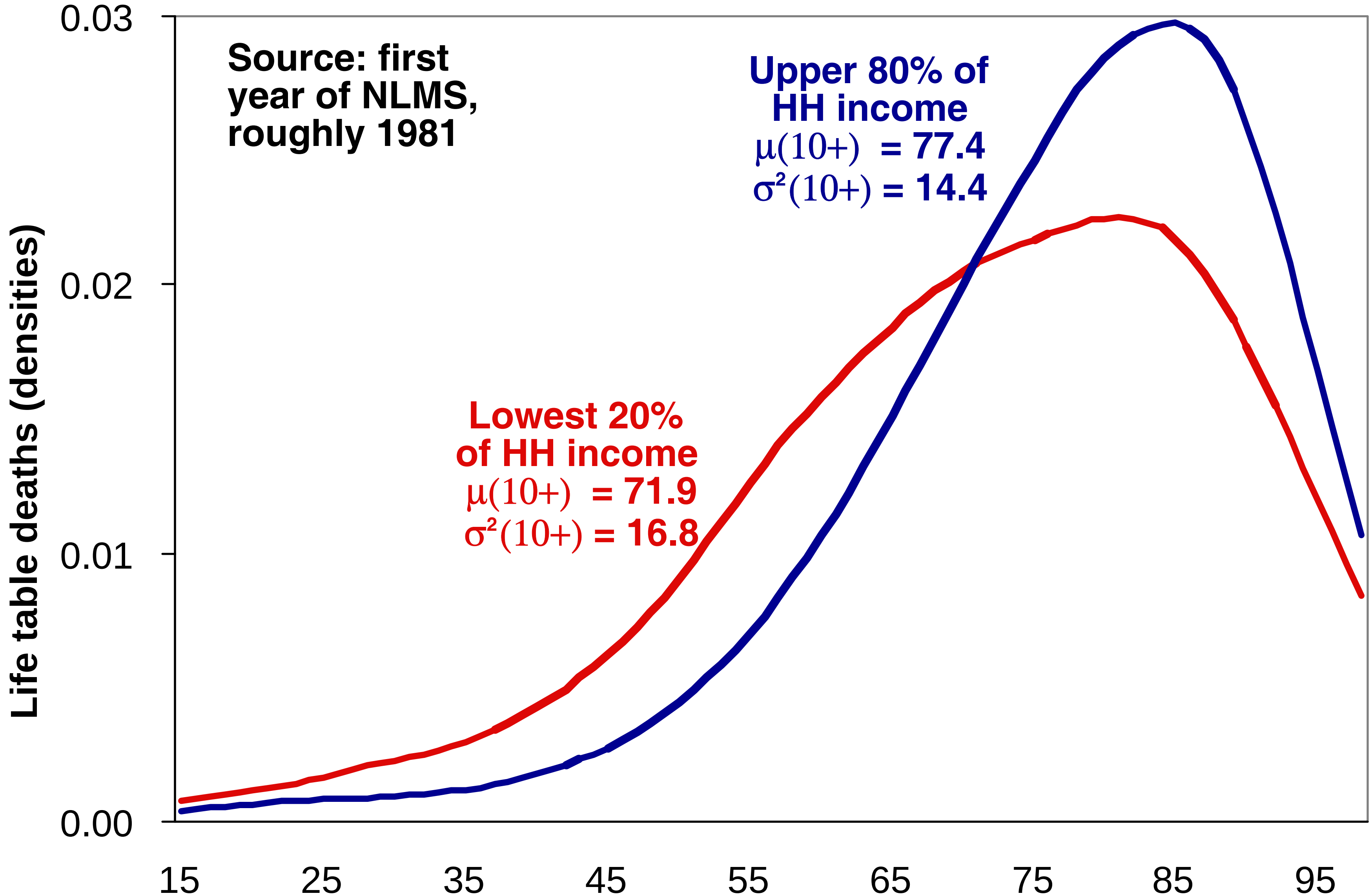
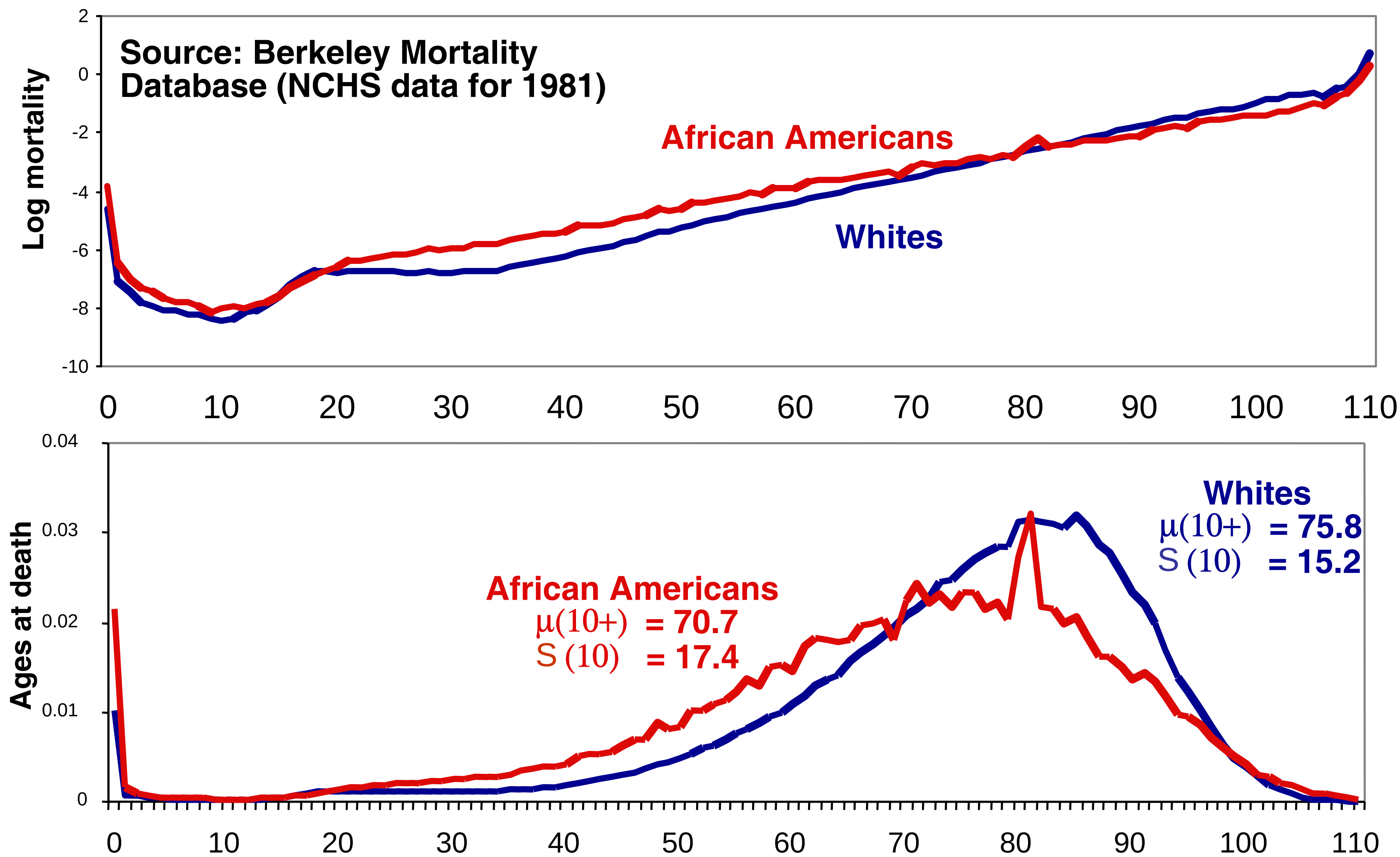


**Fig. 8 HH income and age at death using the NLMS**



# Fig. 9 African Americans & Whites



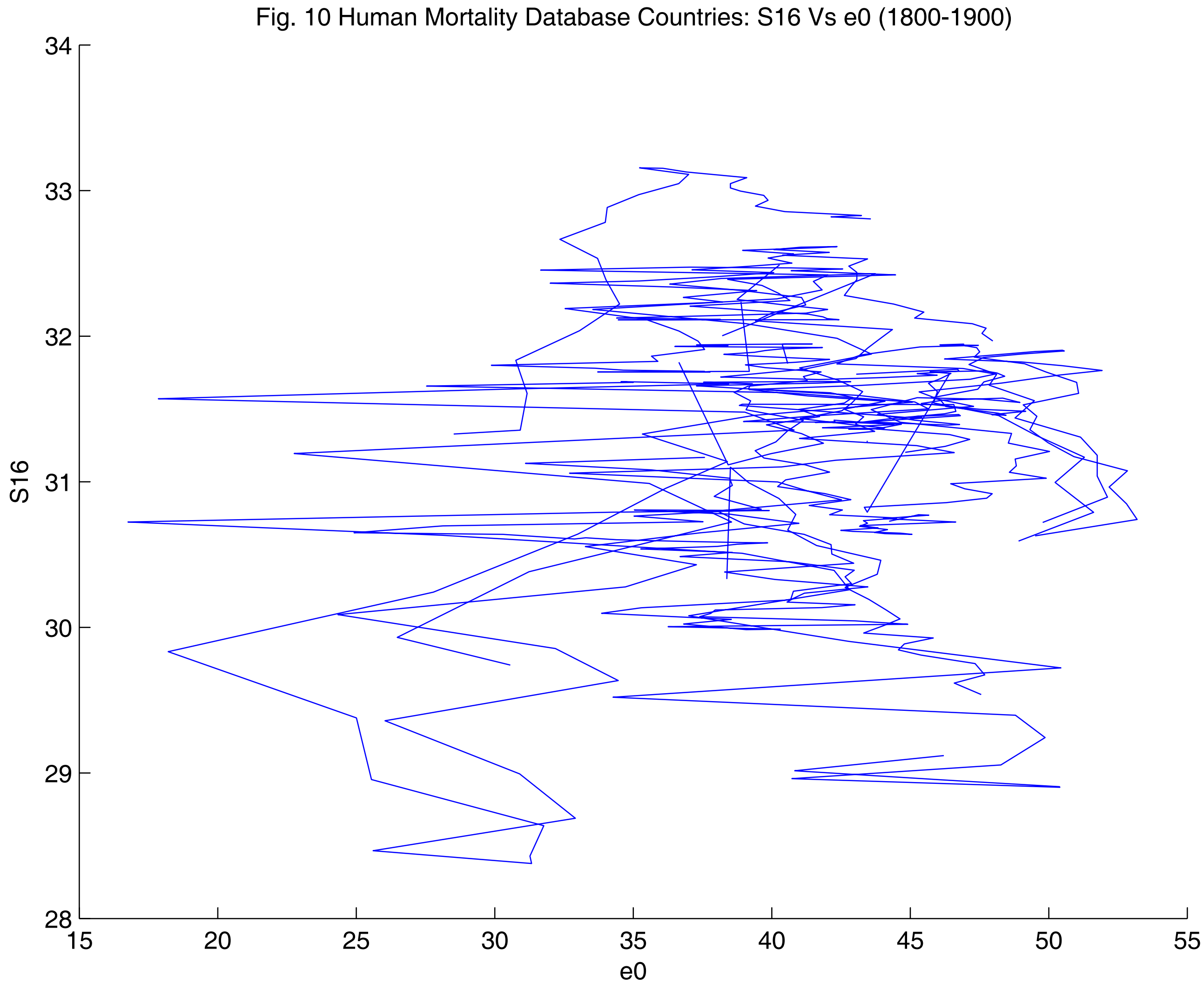


Fig. 10 Human Mortality Database Countries: S16 Vs e0 (1800-1900)
S16
e0
34
33
32
31
30
29
28
15
20
25
30
35
40
45
50
55

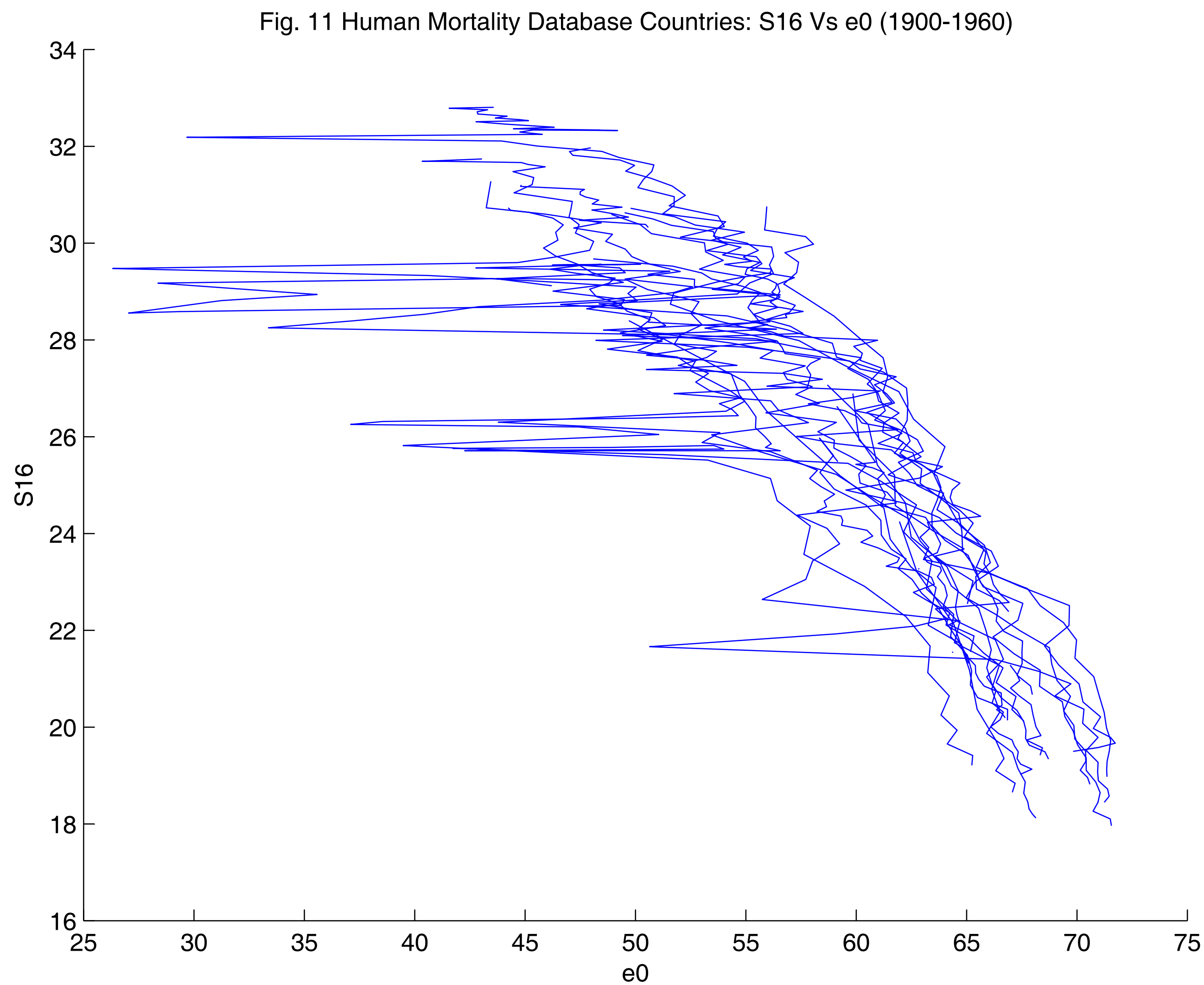


Fig. 11 Human Mortality Database Countries: S16 Vs e0 (1900-1960)
S16
e0
34
32
30
28
26
24
22
20
18
16
25
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35
40
45
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60
65
70
75

Fig. 12 Human Mortality Database Countries: S16 Vs e0 (1960-2007)

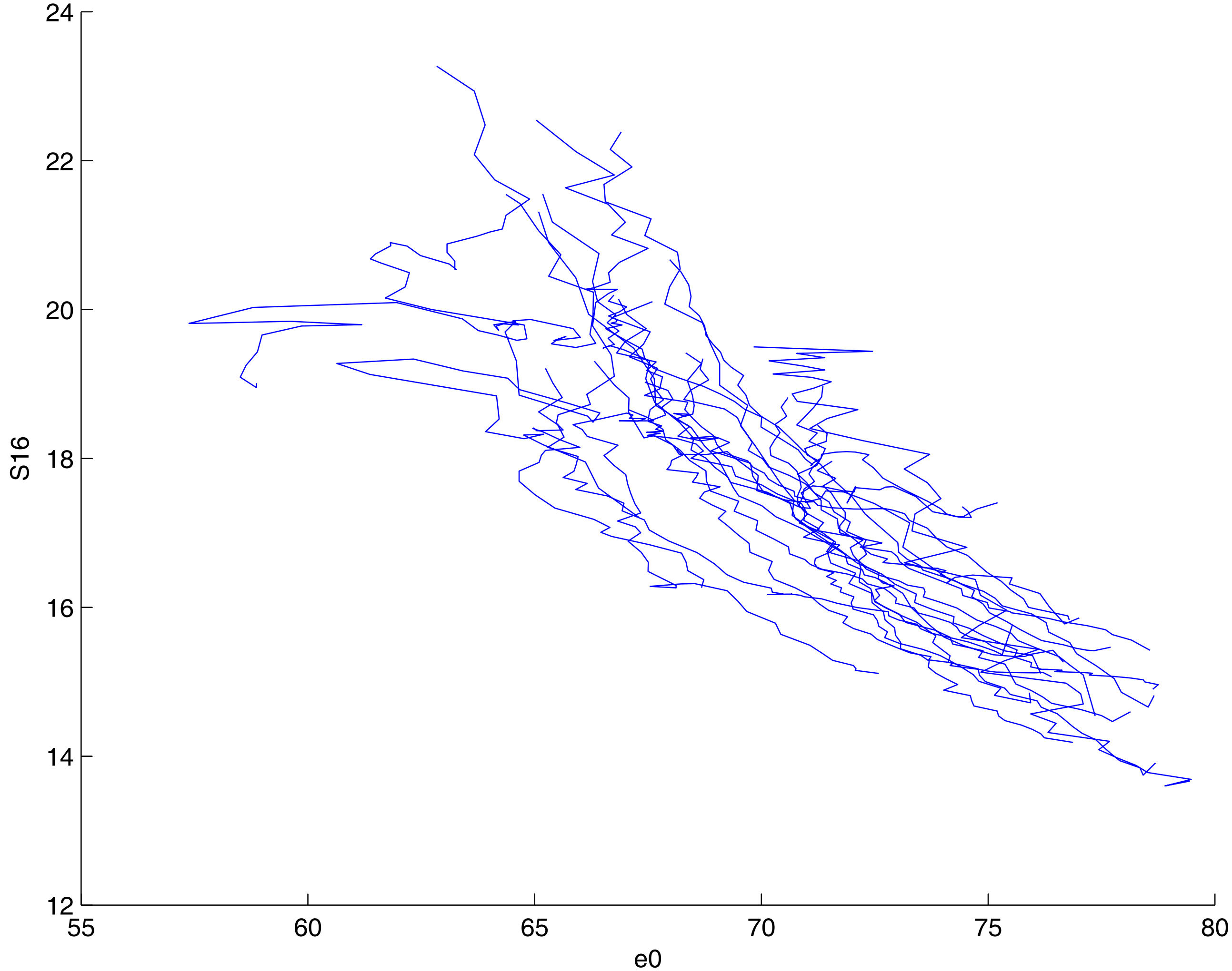

Fig. 13  Comparison of India and HMD Countries S16 versus e0 (1960-2007)