Fertility and Income in the Cross Section: Theories and Evidence*

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Abstract

In this chapter we revisit the relationship between income and fertility. There is overwhelming empirical evidence that fertility is negatively related to income in most countries at most times. Several theories have been proposed in the literature to explain this somewhat puzzling fact. The most common one is based on the opportunity cost of time being higher for individuals with higher earnings. Alternatively, people might differ in their desire to procreate and accordingly some people invest more in children and less in market-specific human capital and thus have lower earnings. We revisit these and other possible explanations. We find that these theories are not as robust as is commonly believed. That is, several special assumptions are needed to generate the negative relationship. Not all assumptions are equally plausible. Such findings will be useful to distinguish alternative theories. We conclude that further research along these lines is needed.

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Contents

1 Introduction

Empirical studies find a clear negative relationship between income and fertility. This finding has been confirmed across time and for different countries. For example, Jones and Tertilt (2008) document a negative cross-sectional relationship between income and fertility in the United States and find that the relationship has been surprisingly stable over time. In particular, the paper shows a negative relationship for 30 birth cohorts between 1830 and 1960, and that the income elasticity of fertility has been roughly constant at about -0.30.[1]

This fact raises the question of why richer people have fewer children and what explains the relatively time-invariant nature of the relationship. The negative correlation is particularly puzzling if one thinks about children as a consumption good, unless one believes that children are an inferior good. An early discussion of this fact appears in the seminal article on fertility choice by Becker (1960). Indeed this puzzling correlation was one of the main impetus' to Becker's early work.[2] The ensuing literature can be roughly divided into two strands. One group of papers has focused on the idea that, properly interpreted, economic theory says that fertility *should* be negatively related to income.[3] The idea is that the price of children is largely time, and because of this, children are more expensive for parents with higher wages. Another argument that is often made is that higher wage people have a higher demand for child quality which makes quantity more costly and hence those same parents want fewer children.[4] The other strand of literature argues that the negative relationship is mainly a statistical

[1]We discuss the empirical evidence in more detail in Section 2.

[2]Quoting from Becker (1960), (p. 217): "Having set out the formal analysis and framework suggested by economic theory, we now investigate its usefulness in the study of fertility patterns. It suggests that a rise in income would increase both the quality and quantity of children desired; the increase in quality being large and the increase in quantity small. The difficulties in separating expenditures on children from general family expenditures notwithstanding, it is evident that wealthier families and countries spend much more per child than do poorer families and countries. The implication with respect to quantity is not so readily confirmed by the raw data. Indeed, most data tend to show a negative relationship between income and fertility." See also the discussion in Hotz, Klerman, and Willis (1993).

[3]The early seminal papers in this literature are Becker (1960), Becker and Lewis (1973), and Willis (1973).

[4]For example, Becker and Lewis (1973) emphasize the trade-off between quantity and quality of children.

fluke – i.e. due to a missing variables problem. This literature focuses on identifying those crucial missing variables.[5]

In this paper, we revisit these theories of the cross sectional relationship between income and fertility. We also add a new theory based on preference heterogeneity in the taste for children. For each of the theories, we catalogue whether they basically never work, work only with specific additional assumptions, or are relatively robust to changes in assumptions. For those theories that work sometimes, we try to be explicit about the assumptions that are needed to make them work. In particular, we give ideas about what kinds of sufficient conditions are needed (e.g., curvature and/or functional form restrictions) to generate a negative relationship between income and fertility. We also show what goes wrong by giving examples about how they fail. Finally, for those theories that work and seem pretty robust, we go on to ask for more. Can the theory also match the time series properties of fertility? If so, what exactly does it take? If not, why not? And finally, we want to know whether such a theory is consistent with a recursive formulation of dynastic altruism.

Our main findings are the following:

1. (Almost) all theories depend on the assumption that raising children takes time and that this time must be incurred by the parents.

2. Theories based on exogenous wage heterogeneity crucially depend on the assumption of a high elasticity of substitution between consumption and children.

3. The quantity-quality trade-off by itself does not generate a negative fertility-income relationship. It works only in conjunction with the same assumptions needed in (2).

4. Theories based on (child) preference heterogeneity are more robust.

5. Theories that explicitly distinguish between fathers and mothers are very similar to one-parent theories. However, to get fertility decreasing in men's

[5] See Hotz, Klerman, and Willis (1993) for a survey. An early literature review on fertility choice is Bagozzi and Van Loo (1978).

income, one needs to assume that there is positive assortative matching of spouses.

6. Several of the theories that match the cross-sectional patterns of fertility also match, at least loosely, some of the broad time series trends in fertility.

7. Trying to extend the models that are successful at matching the cross-sectional properties of fertility choice to fully dynamic models based on parental altruism is very challenging. Indeed, we were not successful in finding any examples that were (plausibly) capable of meeting this higher standard.

We believe that these findings will be useful in several different contexts.

First, there has been a recent increase in research relating the demographic transition and economic development among macroeconomists.[6] Similarly, several recent contributions try to understand why fertility is higher in poor countries than in rich.[7] There is also a recent literature that uses dynamic macro-style models to analyze the interplay between fertility, labor force participation, marriage, and inequality.[8] This literature includes studies that aim to understand the gender wage gap,[9] studies of the the baby boom after the second world war,[10] as well as several other applications. Often these models are used to analyze the impacts of various policy changes—parental leave policies, the impact of tax reform, welfare reform, social security, etc.[11]

These papers typically use an "off-the-shelf" fertility model as one of their building blocks and need to make an intelligent choice of which one to use. One thing that may help guide this choice is an informed understanding of the implications of the models for the fertility-income relationship in the cross section.

[6]See, for example, Becker, Murphy, and Tamura (1990), Galor and Weil (1996), Galor and Weil (1999), Galor and Weil (2000), Greenwood and Seshadri (2002), Hansen and Prescott (2002), Boldrin and Jones (2002), Doepke (2004, 2005), Greenwood, Seshadri, and Vandenbroucke (2005), Tertilt (2005), Jones and Schoonbroodt (2007b), Murtin (2007) and Bar and Leukhina (2007).

[7]See Manuelli and Seshadri (2007).

[8]E.g., see Alvarez (1999), Caucutt, Guner, and Knowles (2002), and Falcao and Soares (2007).

[9]For example, Erosa, Fuster, and Restuccia (2005b).

[10]For example, Greenwood, Seshadri, and Vandenbroucke (2005), Doepke, Hazan, and Maoz (2007), and Jones and Schoonbroodt (2007a).

[11]Recent contributions include Aiyagari, Greenwood, and Guner (2000), Erosa, Fuster, and Restuccia (2005a), Fernandez, Guner, and Knowles (2005), Greenwood, Guner, and Knowles (2003), Sylvester (2007), and Zhao (2008).

Because of this, it is natural to use successful models of the cross sectional properties of fertility as a way to inform that choice.

This is easier said than done, however. Even though economists have been developing and testing theories of fertility ever since Gary Becker, there is still no full consensus on the motivations behind fertility choices to this date. We hope that a clear catalogue of the theoretical possibilities including a systematic comparison of the properties of various theories of fertility choice, like what we try to provide here, will help in this endeavor.

This paper is organized as follows. In the next section, we summarize the empirical evidence on the fertility-income relationship. Section 3 describes a basic model and shows what assumptions are needed to get indeed a negative fertility-income relationship. Section 4 develops a new theory based on preference heterogeneity in the desire to have children. Section 5 adds quality to the basic model. In Section 6 we depart from the simplest framework and analyze more realistic theories with two parents. We investigate whether theories are robust to allowing people to hire nannies in Section 7, while Section 8 concludes. The Appendix analyzes in how much the results apply to a dynastic formulation of fertility.

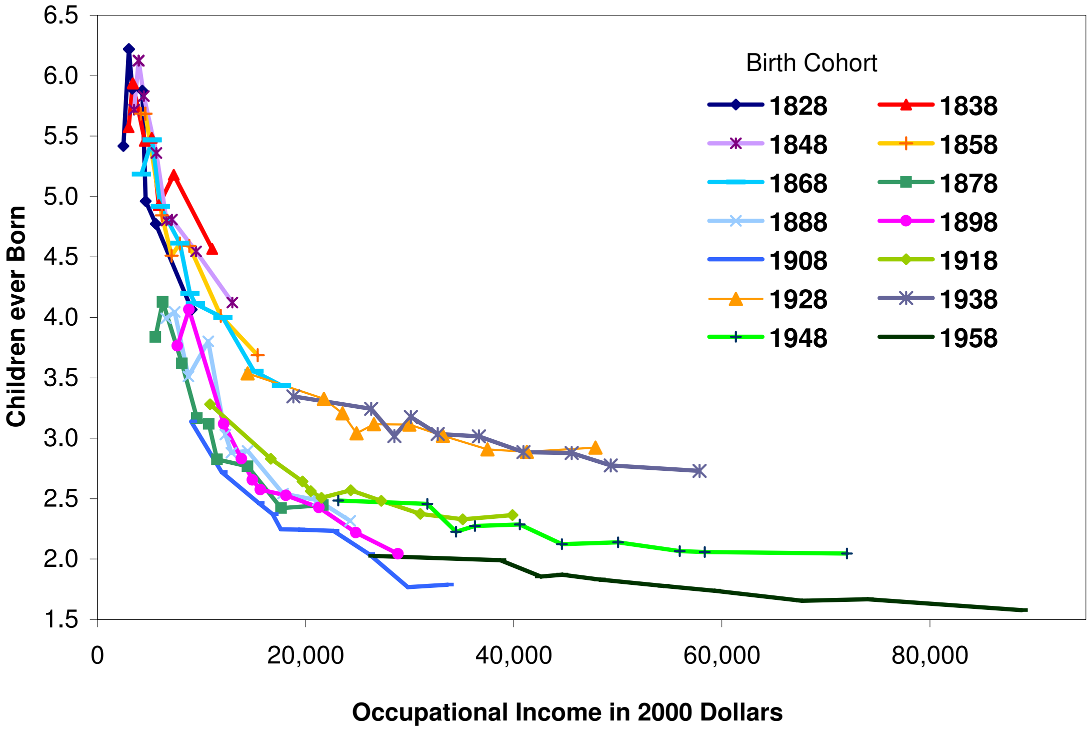
2 Data on Fertility and Income

A robust fact about fertility is that it is decreasing in income and this has been so for a long time. This fact has been documented from a time-series point of view, across countries, and in the cross section. An early discussion of this fact appears in the seminal article on fertility choice by Becker (1960). Indeed this puzzling correlation was one of the main impetus to Becker's early work.[12] Quoting from Becker (1960) (p. 217): "Indeed, most data tend to show a negative relationship between income and fertility. This is true of the Census data for 1910, 1940 and 1950, where income is represented by father's occupation, mother's education or monthly rental; the data from the Indianapolis survey, the data for nineteenth century Providence families, and several other studies as well."[13]

[12] See the discussion in Hotz, Klerman, and Willis (1993).

[13] The studies Becker is referring to are U.S. Census (1945), U.S. Census (1955), Whelpton and Kiser (1951), and Jaffe (1940).

Figure 1: Fertility by Occupational Income in 2000 Dollars
Source: Jones and Tertilt (2008)



In a recent study, Jones and Tertilt (2008) use U. S. Census Data on lifetime fertility and occupations to document this negative cross-sectional relationship in the United States.[14] They find a robust negative cross-sectional relationship between husband's income[15] and fertility for all cohorts for which data is available, that is for women born between 1826 and 1960.[16] Not only are the correlations always negative, but also they are surprisingly similar in magnitude over time. Figure 1, which is reproduced from their paper, shows this very clearly. While the relationship is not perfect, it seems that most of the fertility decline over time

[14]Income is based on the median annual income for a given occupation in 1950 and adjusted for TFP growth. A measure of income based on occupation is a better measure of life-time income than income in any particular year. See Ruggles, Sobek, Alexander, Fitch, Goeken, Hall, King, and Ronnander (2004) for a description of how occupational income scores (OIS) are constructed as well as its robustness as a proxy for income. See Jones and Tertilt (2008) for a description of how the OIS was converted into 2000 dollars.

[15]The focus on husband's income allows a consistent analysis over time. In particular, it allows the analysis of periods for which data on wife's income is practically non-existent.

[16]Fertility is measured as stated children ever born (CEB) to a given woman.

Birth Cohort	income elasticity	top/bottom fertility gap	CEB	occ. income in 2000 Dollar	Number of observations
1826-1830	-0.33	0.95	5.59	4,154	452
1836-1840	-0.20	0.74	5.49	5,064	1,960
1846-1850	-0.32	1.26	5.36	6,173	4,520
1856-1860	-0.35	1.24	4.90	7,525	7,241
1866-1870	-0.34	1.27	4.50	9,173	7,347
1876-1880	-0.42	1.06	3.25	11,182	3,203
1886-1890	-0.45	1.05	3.15	13,631	6,644
1896-1900	-0.50	0.93	2.82	16,616	8,462
1906-1910	-0.42	0.57	2.30	20,255	11,812
1916-1920	-0.25	0.34	2.59	24,690	46,908
1926-1930	-0.17	0.27	3.11	30,097	97,143
1936-1940	-0.19	0.31	3.01	36,688	44,428
1946-1950	-0.20	0.26	2.22	44,723	62,210
1956-1960	-0.22	0.23	1.80	54,517	71,517

Source: Jones and Tertilt (2008)

Table 1: Fertility-Income Relationship for 14 U.S. Cross Sections

can be "explained" by rising incomes alone, at least in a statistical sense.

To get a sense of the magnitudes, Table 1 reproduces some of the most interesting numbers from Jones and Tertilt (2008). For a selected number of birth cohorts, the table displays average husband's income and average fertility.[17] To quantify the fertility-income relationship, two different empirical measures were constructed: the income elasticity of fertility, and the fertility gap between the top and bottom 50 percent of the income distribution. The income elasticity roughly hovers around minus one third, meaning that for any 10% increase in income, the number of children decreases by about 3%. This is a large number. For example, for women born during the 19th century, those in the bottom half of the income distribution had easily one child more on average than those belonging to the top half of the income distribution. Today, the difference is substantially less in absolute numbers, with a fertility gap of roughly a quarter of a child, but, since fertility is significantly lower for all women, the income elasticity has only very mildly declined over time to about -0.20 for the most recent cohorts.

Many other studies have documented this kind of relationship in the data,

[17]The definitions of fertility and income in the table are identical to those used in Figure 1.

typically for a specific geographic area at a particular point in time. For example, Borg (1989) finds a negative relationship using panel data from South Korea in 1976 and Docquier (2004) documents a similar relationship for the U.S. using data from the PSID in 1994. Westoff (1954) finds a negative relationship between fertility and occupational status for the years 1900-1952 using U.S. Census data.

Part of the literature argues that a negative income-fertility relationship is primarily a statistical fluke – i.e., that it is due to a problem of missing variables. The idea is that once enough things are controlled for, one would actually find a positive income-fertility relation. Indeed, this was Becker's original view on the topic. He went into great detail focusing on knowledge of the proper use of contraceptives as the important missing variable. He showed that, in his sample, in those households that were actively engaged in family planning, fertility and income were positively related while the opposite was true for families not engaged in family planning.[18] Similarly, some authors have argued that a distinction between male and female income is crucial and that the relationship between male income and fertility is indeed (weakly) positive once one correctly controls for female income.[19] Authors of studies that find a positive relationship after controlling for women's wages, often interpret such finding as having resolved the "puzzle." This is, however, not necessarily the case.[20] In this paper we take a somewhat different approach: rather than controlling for important factors (such as wives' wages) in the data, we try to add such important factors into the model and then ask whether the augmented model delivers the same qualitative facts as the data does.

It is sometimes argued that early on in the development process, a positive relationship between income and fertility existed.[21] Most of the studies that docu-

[18]Other early papers along this line are cited by Becker in his original piece. They include Edin and Hutchinson (1935) and Banks (1955).

[19]Empirical studies distinguishing explicitly between husbands and wives include Cho (1968), Fleischer and Rhodes (1979), Freedman and Thorton (1982), Schultz (1986), Heckman and Walker (1990), Merrigan and Pierre (1998), Blau and van der Klaauw (2007), and Jones and Tertilt (2008). The findings are mixed.

[20]The reason is that even though the finding reconciles the conditional correlations in the data with the simplest model of fertility, the question remains of what kind of theories would explain the unconditional negative correlation of men's wages and fertility. At the very least it requires some assumptions about matching. We discuss this in detail in Section 6.

[21]A more recent version of such a positive relationship is that U.S. fertility is higher than most

ment such a positive relationship are set in agrarian economies, and often income is proxied by farm size. Examples include Simon (1977, chapter 16) who documents a positive relationship between farm size in hectors and the average numbers of children born for rural areas in Poland in 1948 and Clark and Hamilton (2006) who document a positive relationship between occupational status and the number of surviving children in England in the late 16th and early 17th century (see also Clark (2005) and Clark (2007)). Weir (1995) finds a weakly positive relationship between economic status and fertility in 18th century France, while Wrigley (1961) and Haines (1976) document higher fertility in the coal mining areas of France and Prussia than in surrounding agricultural areas during the end of the 19th century. Similarly, Lee (1987) document a similar finding using data from the U.S. and Canada.[22] This suggests that the fundamental forces determining the demand for children might be different in areas where agriculture is the primary economic activity.

Of course, there is no reason that the relationship should not change over time or be different in different cross sections. That is, it may be that in some subgroups of the population, fertility really is increasing in income once all other relevant correlates are controlled for, while in others the primary change across the income distribution is in the price of a child and it is because of this that fertility is lower at higher income levels. And in fact, it might be quite plausible that fertility and wealth were indeed positively related in early agrarian economies, but that this relationship reversed after industrialization.

In conclusion then, the fact that people with higher life-time earnings have fewer children seems very robust, at least during the last century and a half in the United States. Other countries and other episodes display a similar relationship. Inspired by these facts, this paper analyzes what theories of fertility are consistent with this relationship. We show how based on the simplest theory of fertility one would reach the opposite conclusion. We analyze what is needed for a theory to deliver a negative relationship.

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other countries in the OECD even though U.S. income is higher. This does not hold for a larger set of countries however. See Ahn and Mira (2002) and Manuelli and Seshadri (2007) for a discussion of related points.

[22] See also the papers cited in Lee (1987).

3 Basic Framework and Results

In this section we introduce notation and explore some basic models of fertility choice. The simple examples that we discuss here focus on the role played by the nature of the cost of children, the sources of family income and the formulation of preferences. We find that the simplest versions of these ideas do not generate a negative relationship between fertility and income. Special assumptions on the nature of costs of children, the utility function, the sources of income and/or the child quality production function are needed. This it not to say that these theories are wrong. Rather, we hope that making the assumptions behind the ideas explicit will guide and facilitate the testing of the theories and ultimately help improve our understanding of fertility decision-making.

To keep the analysis tractable, we focus on a static, monoparental set-up. The advantage of this is that it allows for closed form solutions and lets us focus on the basic mechanics behind the results. Obviously, there are many dynamic elements in real world fertility-decision making, for example, choices about the timing of births, etc. We see our basic examples as a way to gain insights into modeling ingredients of more complex dynamic models. Clearly, many important features are left out in the simplest example we start with. Some of these features are particularly important and we come back to those in later sections of this paper. One such element is that any child necessarily has a father and a mother. In fact, many authors have emphasized that it may be female time rather than male time that is important to generate the negative relationship between fertility and income. We get back to this in Section 6. In later sections of the paper we extend the model to include more dynamic elements including limited forms of human capital/child quality (Sections 4 and 5) and parental altruism (Appendix A).

3.1 The Basic Model

The general static model of fertility choice that we consider is as follows. People maximize utility subject to a budget constraint, a time constraint, and a child quality production function. People (potentially) derive utility from four different goods: consumption, c, number of children, n, the average quality of children,

q, and leisure, ℓ. Producing children takes b_0 units of goods and b_1 units of time (per child). We let l_w denote the time spent working and normalize the total time endowment to one. In addition to labor income, we also allow for non-labor income, y. Finally, child quality is a function of educational child inputs, s (we abstract from direct parental time inputs into child quality, except in Section 7). Thus, the choice problem is as follows:

$$\begin{aligned} \max_{c,n,q,e,l_w} \quad & U(c, n, q, \ell) \\ \text{s. t.} \quad & l_w + b_1 n + \ell \leq 1 \\ & c + (b_0 + s)n \leq y + w l_w \\ & q = f(e) \end{aligned} \tag{1}$$

In order to highlight the crucial ingredients for a negative income (or wage) to fertility relationship, we will distinguish between various combinations of utility specifications, concept of wealth/income/earnings used, costs of children and quality production functions. We now briefly discuss each of these components.

Utility: We focus on separable utilities. That is:

$$U(c, n, q, \ell) = u_c(c) + u_n(n) + u_q(q) + u_\ell(\ell)$$

We consider the CES utility case, $u_x(x) = \alpha_x \frac{x^{1-\sigma_x}-1}{1-\sigma_x}$ for values of $\sigma_x > 0$. We will often distinguish three cases: (i) $\sigma_x > 1$ (high curvature, low elasticity of substitution), (ii) $\sigma_x < 1$ (low curvature, high elasticity of substitution) and (iii) $\sigma_x = 1$ corresponding to log utility.[23]

Income/Wealth: We use the following (standard) language: w is the wage, $W = w + y$ is total wealth, and $I = wl_w$ is earned income (often also called labor earnings). In most of our examples, there are only two uses of time (working and child-rearing), in which case earned income is equal to $w(1 - b_1 n)$. An interesting special case is the case where all income is labor income, $y = 0$ and $W = w$.

[23]This utility function has the added advantage that, in some cases, it can be interpreted as the problem in Bellman's equation for a Barro-Becker style dynasty with parental altruism. There, the term $u_n(n)$ is the value function for continuations. This interpretation is only valid for certain choices of the α_n's however. See Appendix A for details.

In several examples, we focus on the fertility-earnings (rather than wage) relationship. In these examples, there is no wage heterogeneity. However, the logic underlying those examples can easily be generalized to (endogenous) wage heterogeneity. This is done in Section 4. In this context, the wage will be equal to human capital, H, and human capital is a function of schooling inputs. For simplicity, we will omit H and say that the wage w is a function of schooling inputs.

Costs of Children: We alow for both goods and time costs, denoted by b_0 and b_1, respectively. To get starker results, we sometimes shut down one of the two types of costs. It turns out that a time cost appears to be essential to almost all the theories and examples we present here. To see this note that with separable utility and no time cost ($b_1 = 0$), both n and q are normal goods, and hence, it follows that n is increasing in both y and w. Thus, we will typically require that $b_1 > 0$. While it seems fairly obvious that it takes time to raise a child, it is less clear whether this has to be the parent's time or could be someone else's time, e.g. a nanny or a day-care center. We analyze the implications of allowing for nannies in Section 7.

Quality Production Function: One important feature for the quantity-quality trade-off to generate the relationship we are after is the specification of the quality production function, $f(\cdot)$. We experiment with various specifications. Note that making special assumptions on $f(\cdot)$ is technically equivalent to making special assumptions on $u_q(\cdot)$. That is, let $v_q(\cdot) = u_q(f(\cdot))$ and make assumptions about this function. The interpretation, however, can be quite different. With homothetic preferences to start with, unless $f(e)$ is of the form $f(e) = e^{\kappa}$, this introduces non-homotheticity into the overall problem (1). We will analyze quality production functions in some detail in Section 5. The quality production function will also come up again in Section 7.

Leisure: For some of the examples in Sections 6 and 7, we need leisure as an alternative use of time in order to reproduce the negative fertility-income relation. For most examples, we will assume that $\alpha_\ell = 0$.

3.2 The Price of Time Theory

To highlight which of the various ingredients outlined above are necessary, we start by discussing a simple example that does not generate the desired negative relationship between fertility and income. We then show what the needed special assumptions are to obtain the desired result.

Starting from the general formulation (1), we assume log utility ($u_x(x) = \alpha_x \log(x)$), no utility from child quality ($\alpha_q = 0$) or leisure ($\alpha_\ell = 0$) and no non-labor income ($y = 0$). Then the problem reduces to

$$\begin{aligned} \max_{c,n} \quad & \alpha_c \log(c) + \alpha_n \log(n) \\ \text{s. t.} \quad & c + b_0 n \leq w(1 - b_1 n) \end{aligned} \tag{2}$$

The solution for fertility is:

$$n^* = \frac{\alpha_n w}{(\alpha_c + \alpha_n)(b_0 + w b_1)}$$

As is apparent from this example, as long as the goods cost of children is positive ($b_0 > 0$) higher-wage households (higher w) will have strictly *more* children in this set-up. This is the *opposite* prediction from what we observe in the data. Setting the goods cost to zero with just a time cost results in fertility choice being independent of w – still, not a negative relationship. Adding leisure or child quality (say, with $q = f(e) = e$) will not reverse this result (see Section 5).

To give the price of time theory a chance, it seems fairly obvious that a deviation from log utility is needed, i.e. a specification where income and substitution effects do not cancel out. We thus turn now to general CES utility functions. Also, since a time cost is essential here and a goods cost does not really add anything, we set $b_0 = 0$ and assume $b_1 > 0$. Thus, our next example takes the form

$$\begin{aligned} \max_{c,n} \quad & \alpha_c \frac{c^{1-\sigma} - 1}{1 - \sigma} + \alpha_n \frac{n^{1-\sigma} - 1}{1 - \sigma} \\ \text{s. t.} \quad & c \leq y + w(1 - b_1 n) \end{aligned} \tag{3}$$

It is easy to solve for a closed form solution of this specification. Optimal fertility is given by:

$$n^* = \frac{\frac{y}{w} + 1}{\left(\frac{\alpha_c b_1}{\alpha_n}\right)^{1/\sigma} w^{\frac{1-\sigma}{\sigma}} + b_1}$$

3.2.1 Wage Heterogeneity

Elasticity of substitution. In problem (3) wage heterogeneity leads indeed to a negative wage-fertility relationship if the right amount of curvature is assumed in the utility function. To see this, assume first that $y = 0$. If the only way in which individuals differ is in their wages, we can see that when $\sigma \geq 1$, fertility is either independent of or increasing in w. However, when $\sigma < 1$, it follows that $n^*(w)$ is decreasing.

The intuition here is simple, when the only cost of children is time, and that time must be the parents own time, higher wage families face a higher price of children. This induces the usual wealth and substitution effects familiar from demand theory. Certainly it implies that compensated demand for children is decreasing. This is not enough, however, to automatically imply that the demand for children is decreasing in income, since those families that face higher prices also have more wealth. Thus, it depends on which of these two forces is stronger. If the elasticity of substitution between children and consumption is high enough (low σ), the substitution effect dominates and $n^*(w)$ is decreasing, as in the data.

Moreover, it can be seen that this relationship is approximately isoelastic when y is small and w is large relative to b_1. It follows that in this case, the income elasticity of demand for children is $\frac{\sigma-1}{\sigma}$.

In sum, this theory works, but not without extra restrictions on preferences. This is not to say that it works perfectly however. For example, an additional requirement could be that the formulation be consistent with dynamic maximization in a setting with parental altruism. In Appendix A.1 we discuss the relationship between this static problem and a reinterpretation of it as the Bellman equation of a dynamic problem. The difficulty with the dynamic reinterpretation of the current example (i.e., with $\sigma < 1$) is that α_n becomes a function of the wage. It turns out that once this fact is taking into account properly, fertility is independent of the wage. Moreover, it has been shown however, that in this kind of models, $\sigma > 1$ is needed to generate the decreases in fertility observed over the past 200 years in response to increased productivity growth and decreased mortality (see Jones and Schoonbroodt (2007b)). Hence it seems that this theory is at an impasse to get both the cross-sectional and trend features of fertility at the same time. In Appendix A.1, we show that with preference heterogeneity, both the cross section as well as the trend observations can be generated.

Non-Labor Income. An alternative specification that also works is to assume log utility but reintroduced non-labor income.[24] Assume $\sigma = 1$ and $y > 0$, then the solution to (3) becomes

$$n^* = \frac{\alpha_n(\frac{y}{w} + 1)}{(\alpha_c + \alpha_n)b_1}$$

Note that for $y > 0$, fertility is indeed decreasing in the wage. Note that the slope of the relationship depends on the size of the non-labor income. That is, for small amounts of non-labor income fertility is decreasing in the wages only very mildly, and in the limit fertility is constant.

Note, however, that the only income that would really qualify as non-labor income here are gifts, lottery income, bequests and the like.[25] Since most families have no or very little such non-labor income, it is questionable whether this should be the main mechanism by which fertility and income are connected. Yet, variations of this formulation are used a lot in the literature. For example, the refinement that it is *female* time that determines the opportunity cost falls into this category. In particular, sometimes y is interpreted as the husband's income and w as the wife's wage. Then fertility is decreasing in the latter. We will turn our attention to two-parent fertility models in Section 6.

Non-Homothetic preferences. Another avenue to generate the desired relationship is to move away from homothetic utility. Assume for example that $\sigma_c = 0$. Then the problem to solve is

$$\begin{aligned} \max_{c,n} \quad & \alpha_c c + \alpha_n \frac{n^{1-\sigma} - 1}{1 - \sigma} \\ s.t. \quad & c \leq (1 - b_1 n)w \end{aligned} \tag{4}$$

[24] Adding non-labor income effectively changes the curvature of the utility function and hence the technical reason that makes it work is similar to the $\sigma < 1$ case in problem (3). The interpretation, of course, is very different.

[25] Any interest income from assets that are accumulated labor earnings would be proportional to labor income and hence would not generate the result outlined here.

And the solution is:

$$n^* = \left[\frac{\alpha_n}{\alpha_c b_1}\right]^{1/\sigma} w^{-1/\sigma}$$

which is clearly decreasing in w for any value of σ.[26] We are not emphasizing non-homothetic utilities any further. The reason for this choice is that one broader aim of a proposed research agenda here is to develop a theory that encompasses cross-sectional, trend, and cyclical features of fertility choice. Embedding this example into a fully dynamic growth model has the unfortunate property that income shares to consumption tend to one and because of this are of limited use.

4 Endogenous Wage Differences

In the previous section we focused on theories of the cross sectional relationship between fertility and wages in which the fundamental difference was exogenous variation in wages (ability). In this section, we explore an alternative view with an alternative causation. This is that the basic source of heterogeneity is in preferences for children—some people want large families and others don't. Given this basic difference, we examine two alternative paths through which fertility will be negatively correlated with economic well being. The first of these is straightforward. If children take time and some parents want large families, those parents will have less time available to work and hence will have lower earned income. The second has one extra degree of complexity. This is in the formation of human capital. Thus, parents that want large families will allocate less time to developing market based skills in anticipation of having many children. Because of this they have both lower wages, and lower earned income.

In this section, we explore different versions of this approach. To illustrate the basic mechanism, we start with a model with exogenous fertility heterogeneity. While this gets the basic relationship right, there are some problems with the timing of events. In particular, it works only if fertility is realized before any schooling decisions are made. We then move to a more general case that has a

[26] This specification (with $\sigma \to 1$) is used in Fernandez, Guner, and Knowles (2005), Erosa, Fuster, and Restuccia (2005a) and Erosa, Fuster, and Restuccia (2005b). Note that the income elasticity of demand for children here is $-1/\sigma$ which will be about right if $\sigma = 3.0$.

more plausible interpretation: deterministic heterogeneity in the preference for children.

4.1 Exogenous Fertility and Endogenous Wages

The simplest version illustrating the mechanism we want to focus on is one where fertility is exogenously different across people. Let $\bar{n}_i$ be the number of children that are attached to adult i. Each child requires b_1 units of parental time. The parent solves one life-time maximization problem choosing how much total lifetime time (net of child-rearing time) to allocate to schooling vs. earning wages. Even though we write this as a one-period problem, the decisions are best interpreted in a sequential fashion: time is first spent on schooling, l_s, which determines future human capital al_s. Normalizing the wage per unit of human capital to one, al_s is also the wage, so that total life-time income simply becomes $wl_w = al_s l_w$. The problem then is:

$$\begin{aligned} \max_{c,l_w,l_s} \quad & \alpha_c \frac{c^{1-\sigma}}{1-\sigma} + \alpha_n \frac{\bar{n}_i^{1-\sigma}}{1-\sigma} \\ \text{s. t.} \quad & l_s + l_w \leq 1 - b_1 \bar{n}_i \\ & w = al_s \\ & c \leq wl_w \end{aligned} \tag{5}$$

The solution is

$$l_s^i = l_w^i = \frac{1 - b_1 \bar{n}_i}{2}$$

It follows immediately that the wage is decreasing in fertility.

$$w^i = al_s^i = \frac{a}{2}(1 - b_1 \bar{n}_i)$$

Note that the derived negative relationship is quite robust, i.e. does not depend on specific functional forms or parameter restrictions. The only crucial assumption is that it takes time to raise children.

This example can be interpreted two different ways. On the one hand, one could think that people are ex-ante identical, but are exposed to stochastic fertility shocks (e.g. birth control failures). Then, ex-post, people will have different fertility realizations, which then leads them to optimally invest different amounts into human capital. However, for such shocks to be the main driving force of the negative fertility-income relationship, it would need to be the case that most people know their fertility realizations *before* they make their human capital accumulation decisions. While this seems implausible for schooling decisions, it is less obvious for human capital that is accumulated on the job through experience.[27] Furthermore, fertility shocks such as these may be important for some margins, such as drop out decisions for girls that become pregnant in high school.

4.2 Endogenous Fertility and Endogenous Wages

Next, we extend the basic intuition given above to allow for both the choice of fertility and the endogenous determination of wages. Assume now that parents differ in their preferences for children, i.e. some people value children more than others. To do this, we add a fertility choice to problem (5) and allow for preference heterogeneity. We also generalize the model along two other dimensions, which will turn out to be useful later on. First, following Ben-Porath (1976) and Heckman (1976) we allow for decreasing returns in the human capital accumulation process: $w = al_s^{\nu_s}$. Second, we allow for decreasing returns when working. That is, an individual working l_w units (hours/weeks/years) will earn a total income of $wl_w^{\nu_w}$. This may seem non-standard, as it is often assumed that income is linear in hours worked, yet, it does seem quite plausible because many jobs are salaried, for which income does not linearly increase in hours worked. Note also that setting $\nu_w = 1$ gives the standard model in which income is the product of

[27]This mechanism is indeed sometimes used in the literature. For example, Erosa, Fuster, and Restuccia (2005a) have stochastic fertility opportunities and stochastic values of children, together with learning-by-doing on the job, so that higher fertility translates into higher wages. (Although, this is not the only channel through which fertility and income are related in their model.) A similar mechanism is also at work in Erosa, Fuster, and Restuccia (2005b), Knowles (2007) and Buttet and Schoonbroodt (2006).

an hourly wage and hours worked. The modified problem then is

$$\begin{aligned}\max_{c,n,l_w,l_s} \quad & \alpha_c \frac{c^{1-\sigma}}{1-\sigma} + \alpha_n \frac{n^{1-\sigma}}{1-\sigma} \\ \text{s. t.} \quad & l_s + l_w \leq 1 - b_1 n \\ & w = a l_s^{\nu_s} \\ & c \leq w l_w^{\nu_w}\end{aligned} \tag{6}$$

The first order conditions are:

$$\begin{aligned} l_s: \quad & \alpha_c (a l_s^{\nu_s} l_w^{\nu_w})^{-\sigma} a \nu_s l_s^{\nu_s - 1} l_w^{\nu_w} = \alpha_n \left(\frac{1 - l_s - l_w}{b_1} \right)^{-\sigma} \frac{1}{b_1} \\ l_w: \quad & \alpha_c (a l_s^{\nu_s} l_w^{\nu_w})^{-\sigma} a \nu_w l_s^{\nu_s} l_w^{\nu_w - 1} = \alpha_n \left(\frac{1 - l_s - l_w}{b_1} \right)^{-\sigma} \frac{1}{b_1} \end{aligned}$$

It follows immediately that $l_s = \frac{\nu_s}{\nu_w} l_w$. Using this, the optimal amount of work solves the following equation

$$\alpha_c a^{1-\sigma} \nu_s \left(\frac{\nu_s}{\nu_w} \right)^{\nu_s - 1 - \nu_s \sigma} l_w^{-(\nu_s + \nu_w)\sigma + \nu_s + \nu_w - 1} = \alpha_n \left(\frac{1}{b_1} \right)^{1-\sigma} \left(1 - \frac{\nu_s + \nu_w}{\nu_w} l_w \right)^{-\sigma}$$

It is easy to derive closed form solutions for two special cases: (i) constant returns to scale ($\nu_w + \nu_s = 1$) and a general σ and (ii) general production function, but assuming log utility $\sigma = 1$.[28] The solution for case (ii) is

$$\begin{aligned} l_w^* &= \frac{\alpha_c \nu_w}{\alpha_n + (\nu_s + \nu_w)\alpha_c} \\ l_s^* &= \frac{\alpha_c \nu_s}{\alpha_n + (\nu_s + \nu_w)\alpha_c} \\ n^* &= \frac{1}{b_1} \left(\frac{\alpha_n}{\alpha_n + (\nu_s + \nu_w)\alpha_c} \right) \end{aligned}$$

Note that the wage rate is

$$w^* = a (l_s^*)^{\nu_s}$$

which increases monotonically in time spent at school. Taking derivatives with

[28] We analyze case (i) with dynastic altruism in Appendix A.2.

respect to the child preference parameters, α_n, gives

$$\begin{aligned}\frac{\partial n^*}{\partial \alpha_n} &= \frac{(\nu_s + \nu_w)\alpha_c}{b_1[\alpha_n + (\nu_s + \nu_w)\alpha_c]^2} > 0 \\ \frac{\partial l_s^*}{\partial \alpha_n} &= \frac{-\alpha_c \nu_s}{[\alpha_n + (\nu_s + \nu_w)\alpha_c]^2} < 0 \quad .\end{aligned}$$

So, clearly, people who have a higher preference for children will both have more children and also a lower wage.

As can be seen from these expressions, fertility is independent of the raw learning ability, a. That is, without differences in preferences, parents will all have the same fertility.

There are a couple of special cases where the implicit relationship between fertility and wages can be solved for explicitly.

Consider the special case $\nu_w = \nu_s = 1$: human capital is linear in years of schooling and total income is simply wage times time spent working. For this case, we can substitute out all preference parameters to derive an equilibrium relationship between wage and fertility that will hold across all consumers (i.e. independent of their individual α_n and α_c):

$$n^* = \frac{1}{b_1}(1 - \frac{2}{a}w^*).$$

In this case, it follows that fertility is linearly decreasing in wages.

A second case that admits a straightforward closed form solution is when $\nu_s = \nu_w$. Then, the relationship can be written as:

$$n^* = \frac{1}{b_1}\left(1 - 2\left(\frac{w^*}{a}\right)^{\frac{1}{\nu_s}}\right).$$

In this case the relationship between the wage and fertility is non-linear with its curvature determined by the parameter ν_s.

In sum, this direction of causation generates the negative income-fertility and wage-fertility relationships under fairly general assumptions. In Appendix A.2, we add parental altruism to this model where similar results go through.

4.3 An Aside on Wages vs. Income

Here we have focused on the cross sectional relationship between wages and fertility when the basic heterogeneity is differences, across people, in preferences for children, i.e. some people care a lot about children vis-à-vis consumption goods and others do not. To do this we needed a model in which wages themselves are endogenous. An alternative, weaker, version of a similar property can be derived without explicitly including human capital formation in the model. This involves the relationship between fertility and income. For simplicity, assume that all households have the same w. Recall the solution to Problem (3),

$$n^* = \frac{\frac{y}{w} + 1}{\left(\frac{\alpha_c b_1}{\alpha_n}\right)^{1/\sigma} w^{\frac{1-\sigma}{\sigma}} + b_1}$$

and consider two families which differ only in their values of α_n and/or α_c. As we can see from this, the family with the higher α_n will have more children for any value of σ and y. It also follows that they will have lower earned income, $I = [1 - b_1 n^*(\alpha_n, \alpha_c)]w$, simply because they will spend more time raising children and less time working. Thus, preference heterogeneity of this type will also generate a negative correlation between fertility and earned income without further assumptions on elasticities or the formation of human capital.

4.4 Outlook

The direction of causality from preference heterogeneity to a negative fertility/income correlation has not received much attention in the literature thus far. Rather, most authors assume that exogenous differences in income (or ability) cause fertility to vary systematically across the income distribution.[29] Because of this, we will address this channel in all subsequent sections. In particular, from the above it is clear that as long as child-rearing takes parental time, earned income will be lower for those who choose more children simply because there is less time left to earn income. For simplicity, we will use this shortcut version of the channel

[29]Some assume exogenous fertility variation in the population, which is a reduced form version of preference heterogeneity.

when we analyze preference heterogeneity in Sections 5 and 6. We reintroduce endogenous wages in Section 7 where we present an example where parental time is not essential and Appendix A.2 where we build the dynastic analog of Problem (6).

5 Quantity-Quality Theory

In this section, we revisit the idea that the demand for child quality naturally leads richer parents to want more quality and thus less quantity, what's often called the quantity-quality hypothesis. This idea turns out not to be a very robust theory of the negative fertility-income hypothesis.

In his seminal work, Becker (1960) argued that there was a trade-off between quantity and quality of children. However, originally, Becker did not propose the quantity-quality trade-off as an explanation for why fertility and income were negatively correlated. Indeed in the 1960 paper Becker argues, by analogy with other durable goods, that economic theory suggests that fertility and income should be positively related, but perhaps only weakly so, while quality of children and income should be strongly positively correlated. The intuition for Becker's argument is simple. While richer parents do spend more on their children (better schools, better clothes, higher bequests, etc.), richer people spend more on everything. They have higher quality houses and cars as well, yet no one would argue that we should expect rich people to have fewer houses than poor people. As a first cut, the same logic should apply to children, richer people would want more quality, but probably not less quality, the same way the also would not want better but fewer cars.

So what would makes children different? Hotz, Klerman, and Willis (1993), reviewing Becker's arguments, seem to emphasize that what might be the case is that not children per se are normal goods, but that expenditures on children are. We quote: "If children are normal goods in the sense that total expenditures on children are an increasing function of income, then the sum of the income elasticities of the number and quality of children must be positive [...], but it is still possible that the income elasticity of demand for the number of children is negative [...] if the income elasticity of quality is large enough."

This is not our reading of the paper. Our reading is that, by analogy, quantity should be slightly increasing in income and quality should be greatly increasing in income. Becker's argument is then that the observation of a negative relationship is a missing variables problem, namely knowledge about contraceptives. Becker and Lewis (1973) and Becker and Tomes (1976) were important follow ups on Becker (1960). Becker and Lewis (1973) argue that, once income is measured correctly, the true fertility-income elasticity is positive, even if the observed one is negative. Becker and Tomes (1976) argue that the quality production function has an endowment component which generates a decreasing correlation between fertility and income.

Below, we derive conditions under which simple examples including child quality can generate this negative correlation without making children inferior goods. We start with the simplest specification of the example in Section 3 with log utility and a linear quality production function. In this example, it becomes apparent that even with quality choice and ability heterogeneity, we need a positive time cost and zero goods costs for fertility to be non-increasing in income. Next, we derive the requirements on the quality production function for fertility to be strictly decreasing in wages. under ability and preference heterogeneity. One example that generates the desired relation is an affine production function with a positive constant as in Becker and Tomes (1976). Various interpretations of this specification can be used to accommodate the cross section of fertility with respect to income and the trend in fertility over time. Finally, under preference heterogeneity, none of these requirements on the quality production function are needed.

5.1 A Simple Example

First we show by example that including a quality choice of and by itself does not necessarily lead to a negative relationship between fertility and income. That is, including quality, by itself, does *not* lead richer people to want fewer children. They might want more quality and accordingly a smaller increase in number of children—as argued in Becker (1960)—but the relationship between fertility and income is still positive.

Suppose $U(c,n,q) = \alpha_c \log c + \alpha_n \log n + \alpha_q \log q$, $\alpha_q > 0$, $q = f(s) = s$ and $y = 0$. Then the problem from Section 3 is:

$$\begin{aligned} \max_{c,n,q,s,l_w} \quad & \alpha_c \log c + \alpha_n \log n + \alpha_q \log q \\ \text{s. t.} \quad & l_w + b_1 n \leq 1 \\ & c + (b_0 + s)n \leq w l_w \\ & q \leq s \end{aligned}$$

This is a version of the problem considered in Becker and Lewis (1973), while Becker (1960) assumed $b_0 = b_1 = 0$. The constraint set in this problem is not convex because of the term ne. We therefore rewrite the problem in terms of total quality, $Q = qn$. We also know that the constraints hold with equality. Using this, the problem becomes:

$$\begin{aligned} \max_{c,n,Q} \quad & \alpha_c \log c + (\alpha_n - \alpha_q) \log n + \alpha_q \log Q \\ \text{s. t.} \quad & c + b_0 n + Q \leq w(1 - b_1 n) \end{aligned}$$

This is now a standard problem under the assumption that $\alpha_n > \alpha_q$. The solution is given by:

$$\begin{aligned} n^* &= \frac{\alpha_n - \alpha_q}{(\alpha_c + \alpha_n)(b_0 + b_1 w)} w \\ q^* &= \frac{\alpha_q (b_0 + b_1 w)}{\alpha_n - \alpha_q} \\ c^* &= \frac{\alpha_c}{\alpha_c + \alpha_n} w \end{aligned}$$

Similar to what we found in the example in Section 3.2, as long as the goods cost is positive ($b_0 > 0$), fertility is strictly increasing in the wage (or ability), w.[30] On the other hand, if $b_0 = 0$, fertility is independent of w, while earned income

[30] Whether earned income, $I = (1 - b_1 n)w$, increases or decreases depends on the size of the increase in n in response to an increase in w. In the present example, we have::

$$\frac{dI}{dw} = (1 - b_1 n) - b_1 w \frac{dn}{dw} = \frac{(\alpha_c + \alpha_q)(b_0 + b_1 w)^2 + (\alpha_n - \alpha_q) b_0^2}{(\alpha_c + \alpha_n)(b_0 + b_1 w)^2} > 0$$

Thus, in this case, income and fertility are positively related.

is $I = w$. Again, this does not give a negative relationship between income and fertility. Instead, we get an extreme version of Becker's original argument. That is, if there is only a time cost of children, $b_0 = 0$, then we have high income elasticity of quality per child (q is strictly increasing in w and hence I) and low income elasticity of number of children (n is independent of w or I).[31]

There are two ways in which this "negative result" can be overturned. First, keeping wage heterogeneity, the quality production function can be generalized. Second, one can consider preference heterogeneity instead of ability heterogeneity in this simple example. We consider these two routes in turn below.[32]

5.2 The Quality Production Function.

Next, we analyze the properties of the quality production function that generate a negative fertility-income relationship under either ability or preference heterogeneity. The following example is similar to Becker and Tomes (1976) and was inspired by the analysis in Moav (2005). According to this paper, the relationship between fertility and parent's income depends on the curvature of the human capital production function of the child.[33] We make the same assumptions as above, except that we let $q = f(s)$ be unspecified for now. The maximization problem is given by:

$$\begin{aligned} \max_{c,n,q,s} \quad & \alpha_c \log c + \alpha_n \log n + \alpha_q \log q \\ \text{s. t.} \quad & c + b_0 n + sn \leq w(1 - b_1 n) \\ & q = f(s) \end{aligned} \tag{7}$$

[31]It is useful to note that the time intensity in the cost of children matters (the relative size of b_0 and b_1) for the size of these effects. Also, similarly to the cost of time theory, then, one could vary the elasticity of substitution in the utility function. We leave this part to the reader.

[32]We have also explored a third channel—non-separable preferences—to a limited degree (cf. Jones and Schoonbroodt (2007b)). For example:

$$\begin{aligned} \max_{\{c,n,q\}} \quad & \log c + \log\left[[\mu n^\rho + (1-\mu)(nq)^\rho]^{\frac{1}{\rho}} \right] \\ \text{s.t.} \quad & c + (b_0 + b_1 w)n + \phi n q \leq w \end{aligned}$$

In this case, if $\rho < 0$ then n and $Q = nq$ are complements in utility and fertility is decreasing in wages while the opposite is true if $\rho > 0$.

[33]See discussion on p. 98, right after equation (7).

The first order conditions give

$$\frac{sf'(s)}{f(s)} = \frac{\alpha_n}{\alpha_q}\left(\frac{\frac{s}{w}}{\frac{b_0}{w}+b_1+\frac{s}{w}}\right) \tag{8}$$

$$n^* = \left(\frac{\alpha_n}{\alpha_c+\alpha_n}\right)\frac{1}{\frac{b_0}{w}+b_1+\frac{s^*}{w}} \tag{9}$$

Let the elasticity on the left-hand side of equation (8) be $\eta(s) \equiv \frac{sf'(s)}{f(s)}$.[34]

5.2.1 Wage Heterogeneity

Suppose, that households differ in their abilities, w. In the case where $b_0 = 0$, we can see from equation (9) that for n^* to be a decreasing function in w, $\frac{s^*}{w}$ needs to be increasing in w. But the right-hand side of (8) is increasing in this ratio. Thus the left-hand side has to be increasing as well. Hence, we need that $\eta'(s) > 0$, which is purely a property of $f(s)$. The question is: which functional forms would satisfy these conditions? The desired conditions are satisfied in the production function first introduced by Becker and Tomes (1976):[35]

$$f(s) = d_0 + d_1 s, \quad d_0 > 0, d_1 > 0$$

In this case, the solution is:

$$s^* = \frac{\frac{\alpha_q}{\alpha_n}b_1 w - \frac{d_0}{d_1}}{(1-\frac{\alpha_q}{\alpha_n})}$$

which is well-defined as long as $\alpha_q < \alpha_n$ and d_0 is small enough, i.e. $d_0 < d_1\frac{\alpha_q}{\alpha_n}b_1 w$.[36] Solving for n^* gives

$$n^* = \frac{\frac{\alpha_n-\alpha_q}{\alpha_c+\alpha_n}}{b_1 - \frac{d_0}{wd_1}}$$

[34] Note that unless $f(s) = s^\lambda$ for some $\lambda > 0$, this formulation is very similar to the non-homothetic preference example given in Section 3 since we can rewrite the utility function as $\alpha_c \log c + \alpha_n \log n + \alpha_q \log f(s)$.

[35] Note that the following specifications do not work: $f(s) = s^a$ and $f(s) = as$ lead to a constant $\frac{s^*}{w}$, while $f(s) = \log(s)$ and $f(s) = \exp(as)$ lead to decreasing $\frac{s^*}{w}$.

[36] Otherwise $s = 0$ is the solution.

From this it is clear that $\frac{\partial n^*}{\partial w} < 0$.

Finally, notice that this example still requires a time cost. In fact, in the case with $b_0 > 0$, the solution is given by:

$$s^* = \frac{\frac{\alpha_q}{\alpha_n}(b_0 + b_1 w) - \frac{d_0}{d_1}}{(1 - \frac{\alpha_q}{\alpha_n})}$$

which is well-defined as long as

$$\alpha_q < \alpha_n \qquad \text{and} \qquad \frac{\alpha_q}{\alpha_n}(b_0 + b_1 w) > \frac{d_0}{d_1} \tag{10}$$

Solving for n^* gives

$$n^* = \frac{\frac{\alpha_n - \alpha_q}{\alpha_c + \alpha_n}}{b_1 + \frac{b_0}{w} - \frac{d_0}{wd_1}}$$

Hence, fertility is decreasing in w if and only if

$$\frac{d_0}{d_1} > b_0 \tag{11}$$

In the case where $b_1 = 0$, conditions (10) and (11) are mutually exclusive.

Interpretation and further predictions of the model. Becker and Tomes (1976) interpret d_0 as an endowment of child quality, or "innate ability". In this interpretation, one might want to take intergenerational persistence in ability into account. If the child's quality endowment and parent's ability, w, are perfectly positively correlated in the sense that $E(d_0) = w$, then fertility is, again, independent of w while quality is still increasing in w. An alternative would be that in those families in which parents have higher market wages, the marginal value of education is higher – d_1 is perfectly positively correlated with w. For example, assume that $d_1 = \kappa w$. Then even if innate ability, d_0, is perfectly correlated with w, fertility is still decreasing while education is increasing in w. This educational investment does not require time per se. Instead, for a given amount of goods, the high ability parent produces more quality.

An alternative interpretation of d_0 is publicly provided schooling. Since this has increased over time, we see that the predicted response is that fertility will increase, at least holding w fixed. In contrast, holding d_0 fixed, an increase in income over time would cause fertility to decrease. Hence, under this interpretation the example suggests that the increase in income was more important than the increase in publicly provided schooling.[37]

5.2.2 Preference Heterogeneity

Next, assume that w is the same for all households but suppose that people differ in their preference for the consumption good, α_c. In all the examples above, the more people like the consumption good, the fewer children they will have and, as long as $b_1 > 0$, the more income they will earn. However, the quality choice, q, is independent of α_c and hence income, I.

If, on the other hand, we consider heterogeneity in the preference for children, α_n, we see that the more people like children, n (relative to both consumption, c, and quality, q), the more they will have, the less income they will earn and the less quality investments they make per child. Thus, in this case, fertility and income are still negatively related, while quality per child will be positively related with income.

Note that this does not depend on any particular assumption about goods costs or the quality production function. As usual, however, a positive time cost is required so that earned income, I, is decreasing in number of children, n, which generates the negative correlation.

6 Married Couples and the Female Time Allocation Hypothesis

A refinement of the price of time theory of the negative relationship between fertility and income is to view the decision making unit as a married couple rather than a single decision maker. In this version, since it is typically the case that most childcare responsibility rests with the woman, it is the time of the wife that is critical to the problem.[38] In its simplest form the idea is that the price of children is

[37] See the conclusion for suggestive simulations of such changes over time.

[38] A related idea was first formalized in Willis (1973) who studied the time allocation problem for a couple in which the time of both the husband and wife are used in raising children while

higher for high productivity couples, even if only the husband is working.[39]

The aim of this section is threefold. First, we test how robust the results derived in Sections 3 (elasticity restrictions) and 4 (preference heterogeneity assumptions) are. Second, we study restrictions needed on the home production technology under log utility (in the spirit of Willis (1973)). Third, we show that specific patterns of assortive mating are needed to match the data. To this end, we compare the model implications for the following refined fertility-wage correlations to findings in the statistical literature (whenever possible):

(1) The theoretical correlation between fertility and wife's wage (or productivity), holding the husband's wage constant. Evidence suggests that this correlation is strongly negative.

(2) The theoretical correlation between fertility and husband's wage, holding the wife's wage constant. Evidence here is very mixed (e.g., Blau and van der Klaauw (2007) find it is strongly positive, Jones and Tertilt (2008) find it is negative and Schultz (1986) finds that it depends on the exact subgroup of the population one considers, see below).

(3) The theoretical partial correlation between fertility and husband's wage, under different assortative mating patterns. Evidence suggests that this correlation is strongly negative in the data which suggests positive assortative mating if (2) is weakly positive.

We show that simple examples imply that fertility should be decreasing in the productivity or wage of the wife (1) and (weakly) increasing in the wage of the

consumption is produced using the time of the wife and market purchased goods.

[39]In the words of Hotz, Klerman, and Willis (1993): "A second major reason for a negative relationship between income and fertility, in addition to quality-quantity interaction, is the hypothesis that higher income is associated with a higher cost of female time, either because of increased female wage rates or because higher household income raises the value of famale time in non-market activities. Given the assumption that childrearing is a relatively time intensive activity, especially for mothers, the opportunity cost of children tends to increase relative to other sources of satisfaction not related to children, leading to a substitution effect against children. As noted earlier, the cost of time hypothesis was first advanced by Mincer (1963) and, following Becker's (1965) development of the household production model, the relationship between fertility and female labor supply has become a standard feature of of models of household behavior."

husband (2). Because of this, from a purely statistical point of view then this approach takes the stand that the negative estimated correlation between income of the husband and fertility (3) is contaminated by a missing variables problem—the productivity of the wife. Since productivities or wages within couples are typically positively correlated, a downward bias (perhaps enough to change the sign) is induced on the true effect of husband's income on fertility. One might think that this effect is large enough, in theory, that any restrictions on the form of preferences, etc., are no longer necessary. This is not what we find in the examples below. Rather, we find that specific assumptions on elasticity, the home production function and assortive mating (either in terms of productivities or preferences) are still required to generate (1) and (3).[40]

Testing predictions (1) and (2) in the data is complicated because of the difficulty in obtaining direct measures of the value of the wife's time. Until recently many wives did not work and even now, those that do are a 'selected' sample. Hence, other proxies must be used such as inferred productivities based on a Mincer regression or education. The evidence on (1) and (3) are quite robust while evidence on (2) is mixed. Below is a summary of the findings of three recent studies.

Schultz (1986) estimates a reduced-form fertility equation given by

$$C_i = \beta_0 + \beta_1 \ln W_{wi} + \beta_2 W_{hi} + \beta V_i + e\epsilon_i$$

as suggested by his household demand framework.[41] This equation is estimated separately for different age and race groups. The data are from the 1967 Survey of Economic Opportunities, an augmented version of the Current population Survey. He finds that "in every age and race regression the wife's wage is negatively associated with fertility. The coefficient on the husband's predicted wages changes sign over the life cycle, adding to the number of children ever born for younger wives [...] but contributing to lower fertility among older wives. [...] For white wives over age 35 and for black wives aged 35-54, a higher predicted

[40] Given the mixed evidence on (2), we do not focus too much on the model prediction for (2).

[41] Schultz (1986, p. 91) also says: "Empirical studies of fertility that have sought to estimate the distinctive effects of the wage opportunities for men and women generally find β_1 to be negative, while β_2 tends to be negative in high-income urban populations and frequently positive in low-income agricultural populations Schultz (1981)".

husband's wage is significantly associated with lower completed fertility. The elasticities of fertility with respect to the wage rates of wives and husbands are of similar magnitude for blacks and whites, although for blacks the level of fertility is higher and wage levels are lower. [...] These estimates give credence to the hypothesis that children are time-intensive. In all age and race regressions the sum of the coefficients on the wife's and husband's wage rates is negative and increases generally for older age groups. [...] The hypothesis that children are more female than male time-intensive is also consistent with these estimates".(Table 1, pp. 93)

Using NLSY longitudinal data for women born between 1957 and 1964, Blau and van der Klaauw (2007) find that "a one standard deviation increase in the male wage rate is estimated to have some fairly large effects on white women, but none of the underlying coefficient estimates are significantly different from zero. Several of the black and Hispanic interactions are statistically significant, however, and the simulated effects are in some cases quite large. A higher male wage rate increases the number of children ever born to black women by .169. For Hispanic women, a higher male wage rate also increases fertility. Concerning female wage rates, a higher female wage rate generally has effects that are of the opposite sign from those of the male wage rate. As with the male wage rate, the effects are not significantly different from zero for whites, but for blacks and Hispanics a higher female wage rate has negative effects on fertility that are significantly different from zero. Children ever born decline by about 0.1 for blacks and Hispanics."

Jones and Tertilt (2008) also experiment with this hypothesis. Since very few women worked in the early cohorts, education is chosen as a measure of potential income. They find that CEB is declining in both the education level of the wife and the husband, and significantly so. Moreover, the coefficients of husband's and wife's education are similar in size (the wife's being slightly larger) and there is no systematic time trend.

It is convenient to break this variant of the story into two separate pieces, one when the woman does not work in the market and one where she can/does. Roughly, we can think of the first version as corresponding to a time in history when very few married women participated in the formal labor market. The second corresponds to more recent history. It is clear that the critical features necessary to reproduce the observations must be different in the two cases.

6.1 Full Specialization in the Household

In this example, the husband works in the market, l_m, earning wage, w_m, or enjoys leisure, ℓ_m while the wife works only in the home, l_{hf}, so that her tradeoff is between how much time to allocate to producing home goods versus raising children, $b_1 n$ or enjoy leisure, ℓ_f. Her productivity is denoted w_f. This is relevant to the early period in the data when (married) women's labor force participation was roughly zero.

The gender-specific utility function is given by

$$U_g = \alpha_{cg} \log(c_g) + \alpha_{ng} \log(n) + \alpha_{\ell g} \log(\ell_g) + \alpha_{hg} \log(c_{hg})$$

where $g = f, m$ indicates gender, c_g is market consumption, n is the number of children, ℓ is leisure and c_{hg} is the home good. Note that only husband's leisure is crucial in the results below. That is, $\alpha_{\ell f}$ could be zero but the husband needs an alternative use of time. Given our previous results, we assume that children cost only time, i.e., $b_0 = 0$

We assume that there is unitary decision making in the household. The family solves the problem:

$$\begin{aligned} \max_{\{c_m, c_f, c_{hm}, c_{hf}, n, \ell_m, \ell_f, l_m, l_f\}} \quad & \lambda_f U_f + \lambda_m U_m \\ \text{s. t.} \quad & c_f + c_m \leq w_m l_m \\ & l_m + \ell_m \leq 1 \\ & c_{hf} + c_{hm} \leq w_f l_{hf} \\ & l_{hf} + \ell_f + b_1 n \leq 1 \end{aligned}$$

Here ℓ_f and ℓ_m are leisure of the female and male respectively w_m is the wage of the man, w_f is the productivity of the woman in home production, c_{hf} and c_{hm} are consumption of home goods by the woman and the man respectively. Note that it is assumed that the wife spends b_1 hours for each child being raised (and the husband spends none). Assume that $\alpha_{xf} = \alpha_{xm}$ for $x = c, h, n, \ell$. Without loss of generality, assume $\lambda_f + \lambda_m = 1$ and $\alpha_c + \alpha_n + \alpha_\ell + \alpha_h = 1$.

This problem separates into two maximization problems, one concerning the allocation of the man's time and one concerning the allocation of the woman's time. The one for the man is straightforward and doesn't involve fertility. Notice however, that male earnings are increasing in α_c since leisure becomes less desirable relative to consumption. The problem for the woman is:

$$\max_{\{c_{hm}, c_{hf}, n, \ell_f\}} \quad \lambda_f \alpha_\ell \log(\ell_f) + \lambda_f a_h \log(c_{hf}) + \lambda_m \alpha_h \log(c_{hm}) + (\lambda_f + \lambda_m)\alpha_n \log(n)$$
$$\text{s. t.} \quad b_1 w_f n + c_{hf} + c_{hm} + \ell_f \leq w_f$$

The solution is:

$$b_1 w_f n^* = \frac{\alpha_n}{\lambda_f \alpha_\ell + \alpha_h + \alpha_n} w_f$$
$$n^* = \frac{\alpha_n}{\lambda_f \alpha_\ell + \alpha_h + \alpha_n} \frac{1}{b_1} \tag{12}$$

6.1.1 Wage Heterogeneity, elasticity and the home production function

Suppose households differ in their productivities, w_f and w_m. We see that n^* is independent of woman's productivity in the home. This productivity could be approximated by education as in Jones and Tertilt (2008) who find a strongly negative correlation, fact (1). Fertility is also independent of w_m, holding w_f fixed. Finally, even if the productivity of the husband and wife are positively correlated (or independent), fertility is independent of both productivities. Thus, fact (3) is not predicted here, either.[42]

As can be seen from the discussion above, since this problem splits into two separate maximization problems and the one for the time of the wife looks just like those discussed in section 3 above (additional goods permitting), the natural

[42]It can also be shown that if children have a nonmarket goods cost, $b_0 > 0$, n^* is increasing in w_f. It follows that if w_f is positively correlated with w_m (which is what we might expect), n^* and w_m will also be positively correlated.

next step is to look at a more general version in which utility is given by:

$$U_g = \alpha_c \frac{c_g^{1-\sigma}}{1-\sigma} + \alpha_n \frac{n^{1-\sigma}}{1-\sigma} + \alpha_\ell \frac{\ell_g^{1-\sigma}}{1-\sigma} + \alpha_h \frac{c_{hg}^{1-\sigma}}{1-\sigma}$$

With $\sigma < 1$, it follows that n^* will be decreasing in the productivity at home of the wife, w_f, fact (1). Holding the wife's productivity fixed, fertility is still independent of the husband's wage, "fact (2)". Thus, if w_f and w_m are positively correlated, and $\sigma < 1$, the partial correlation between n^* and w_m is negative as well, fact (3).

A second variation that also reproduces the negative correlation in the cross section can be obtained by making the home production technology slightly more complex. Assume that utility is given by

$$U_g = \alpha_n \log(n) + \alpha_h \log(c_{hg})$$

where the home good, c_{hg}, is produced using market goods, c, and time of the wife, l_{hf} with productivity w_f, i.e., $c_{hf} + c_{hm} = F(c, w_f l_{hf})$. Note that we have assumed that leisure is not valued, $\alpha_{\ell g} = 0$, to simplify the analysis. Thus the problem to solve is:

$$\begin{aligned} \max_{\{c_m, c_f, c_{hm}, c_{hf}, n, \ell_m, \ell_f\}} \quad & \lambda_f U_f + \lambda_m U_m \\ \text{s. t.} \quad & c \leq w_m \\ & b_1 n + l_{hf} \leq 1 \\ & c_{hf} + c_{hm} \leq F(c, w_f l_{hf}) \end{aligned}$$

The FOC's can be reduced to one equation involving the amount of time the wife spends making home goods which directly relates to fertility:

$$\begin{aligned} (1 - l_{hf}) &= \frac{\alpha_n}{\alpha_h} \frac{1}{w_f} \frac{F(w_m, w_f l_{hf})}{F_2(w_m, w_f l_{hf})} \\ n^* &= \frac{1 - l_{hf}}{b_1} \end{aligned}$$

That is, time spent in child rearing $(1 - l_{hf})$ is positively related to the relative desirability of children to consumption, $\frac{\alpha_n}{\alpha_h}$, and negatively related to the productivity of the wife, w_f, everything else equal. Thus, so is fertility, n^*. When F is assumed to be CES, $F(c, w_f l_{hf}) = [\delta c^\rho + (1-\delta)(w_f l_{hf})^\rho]^{\frac{1}{\rho}}$, this becomes:

$$n^* = \frac{1 - l_{hf}}{b_1} = \frac{\alpha_n}{\alpha_h}(b_1(1-\delta))^{-1}\left[\delta\left[\frac{w_m}{w_f}\right]^\rho l_{hf}^{1-\rho} + (1-\delta)l_{hf}\right] \tag{13}$$

We can see from the second equality that in the Cobb-Douglas case ($\rho \to 0$), $(1 - l_{hf})$ is independent of both w_m and w_f, but does depend on $\frac{\alpha_n}{\alpha_h}$. Thus, the same must be true of n^* (first equality).

We can also see that for any value of ρ, if w_f and w_m are proportional—$w_f = \phi\, w_m$, then l_{hf} is independent of w_m and w_f and hence the same is true for fertility. That is, under perfect assortive mating, fertility and the wage of the husband and the productivity of the wife are independent.

When this correlation is imperfect and $\rho \neq 0$, the analysis is more complicated. Let's assume another extreme—that w_m and w_f are independent—in what follows. When $\rho > 0$, market goods and female time are substitutes in the production of consumption, an increase in w_m holding w_f fixed causes h_f to fall and hence n^* rises in this case. That is, fertility is an increasing function of husband's wage if w_m and w_f are independent.

On the other hand, when $\rho < 0$, market goods and female time are complements in the production of consumption, an increase in w_m holding w_f fixed causes l_{hf} to rise and hence n^* falls in this case. That is, fertility is a decreasing function of husband's wage if time and goods are complements and wages of husbands and wives are independent.

Thus, assuming enough complementarity between time and goods in production, F, and enough independence between productivities of husbands and wives also gives a model that can reproduce the negative correlation between husbands income and fertility, fact (3).

In sum then, we see that fertility and wages/home productivities are uncorrelated without the same kinds of assumptions over utility function curvature that we have identified in earlier sections. As a substitute, we can generate the observed curvature, even with unitary elasticity in preferences if we move away from unitary elasticity in the home production technology, but this requires the right correlation between husband's wages and wife's productivity in the home.

6.1.2 Preference Heterogeneity

If instead, households differ in how much they like children, α_n, consumption, α_c, and/or the home good, α_h, one can show that in the simplest case in equation 12:

1. With heterogeneity in α_c alone, while (male) earnings are increasing in α_c, fertility is the same for all households.

2. With heterogeneity in α_n or α_h alone, (male) earnings are the same for all households while fertility is decreasing in α_h and increasing in α_n.

3. With simultaneous heterogeneity in α_c and α_h and a positive correlation of these preferences within households, fertility will be negatively correlated with husband's earnings, fact (3). This hinges on the husband having an alternative use of time to market work—leisure in this case.

Similar results can be derived in the examples with general elasticities or home production functions.

6.2 Partial Specialization

In this example, as before, the husband works in the market earning wage, w_m, or enjoys leisure, ℓ_m. Women, however, work only in the market, so that the tradeoff is between how much the woman works in the market and how many children to raise. This example is relevant to the later period in the data when women's labor force participation was positive.

The gender-specific utility function is given by

$$U_g = \alpha_{cg} \log(c_g) + \alpha_{ng} \log(n) + \alpha_{\ell g} \log(\ell_g)$$

Hence, the family solves the problem:

$$\begin{aligned} \max_{\{c_m, c_f, n, \ell_m, \ell_f\}} \quad & \lambda_f U_f + \lambda_m U_m \\ \text{s. t.} \quad & c_f + c_m \leq w_m(1 - \ell_m) + w_f(1 - \ell_f - b_1 n) \end{aligned}$$

where ℓ_f and ℓ_m are leisure of the female and male respectively and w_f and w_m are the respective wages. Note that it is assumed that the wife spends b_1 hours for each child being raised (and the husband spends none). Without loss of generality, assume that $\lambda_f + \lambda_m = 1$ and $\alpha_c + \alpha_n + \alpha_\ell = 1$. Let $W = w_f + w_m$.

We obtain the standard result that expenditure on each good is a constant fraction of wealth, given by preferences:

$$c_f = \lambda_f \alpha_c W;$$

$$c_m = \lambda_m \alpha_c W;$$

$$w_f \ell_f = \lambda_f \alpha_\ell W;$$

$$w_m \ell_m = \lambda_m \alpha_\ell W;$$

$$b w_f n = (\lambda_m + \lambda_f)\alpha_n W.$$

This immediately implies that:

$$n^* = \frac{(\lambda_m + \lambda_f)\alpha_n}{b_1}\left[1 + \frac{w_m}{w_f}\right] \tag{14}$$

6.2.1 Wage Heterogeneity

Suppose households differ in their market wages, w_f and w_m. We see that:

1. Fertility, n^*, is decreasing in the wife's wage, w_f, if the husband's wage, w_m, is held constant.

2. Fertility, n^*, is increasing in the husband's wage, w_m, if the wife's wage, w_f, is held constant.

Thus, this model is consistent with fact (1) and in line with some author's findings on "fact (2)" (e.g., Blau and van der Klaauw (2007)). What remains to be seen is under what conditions fact (3) can be accommodated as well. From equation 14, we also see that:

$$E\left[n|w_m\right] = \frac{(\lambda_m + \lambda_f)\alpha_n}{b_1}\left[1 + w_m E\left[\frac{1}{w_f}|w_m\right]\right]$$

Thus, the partial correlation between fertility and husbands income depends on $E\left[\frac{1}{w_f}|w_m\right]$. That is, it depends on the correlation between husband's and wife's market wages. We find that

1. **Perfectly (positively) correlated wages within couples**

   - If $w_f = \phi w_m$, then $E\left[\frac{1}{w_f}|w_m\right] = \frac{1}{\phi w_m}$ and so n^* is independent of w_m.
   - Similarly, if $w_f = \phi w_m^{\nu}$, $\nu < 1$, then $w_m E\left[\frac{1}{w_f}|w_m\right]$ is increasing in w_m if $\nu < 1$ and hence, so is n^*, and decreasing if $\nu > 1$. The restriction on curvature means that a 1% increase in the husband's wage is associated with a less (higher) than 1% increase in the productivity of his wife if $\nu < 1$ (> 1).

2. **Independent wages within couples** Then $E\left[\frac{1}{w_f}|w_m\right] = E[\frac{1}{w_f}]$ and so n^* is increasing as a function of w_m.

3. **Negatively correlated wages within couples.** Suppose that $w_f = D - \nu w_m$ (where $D > 0$ so that $w_f > 0$). In this case $w_m E\left[\frac{1}{w_f}|w_m\right] = \frac{w_m}{D-\nu w_m} = \frac{1}{D/w_m-\nu}$. Again this is increasing in w_m.

Thus, this version of the theory is consistent with the fact (1) that the regression coefficient on wife's wage is positive and the "debated fact (2)" that the regression coefficient on husband's income is positive (as in Blau and van der Klaauw (2007)). But it is not consistent with a negative partial correlation between husband's income and fertility (unless the correlation is positive with $\nu > 1$ which seems unlikely). Thus, this by itself does not get us away from having to make special assumptions about the curvature on utility as in the simpler examples above.

6.2.2 Preference Heterogeneity

From equation (14), we can also see the relationship between income and fertility when the basic source of heterogeneity is in preferences. Thus, for example, if couples differ in their values of α_c and assuming, both α_ℓ and α_n are lower so that $\alpha_c + \alpha_\ell + \alpha_n = 1$ for all households, those with higher desire for consumption choose lower leisure (both ℓ_f and ℓ_m), and also lower fertility, n^*. Because of this, those couples with higher α_c will have both higher incomes since they work more and lower fertility. Note that we have assumed that couples are perfectly matched in terms of their preferences.

7 Nannies

So far, the assumption that children take time has been an essential ingredient into deriving a negative wage-fertility relationship. It is easy to see that with a goods cost only, none of the examples above works. That is, with a goods cost only ($b_0 > 0, b_1 = 0$), the negative fertility-income relationships in any of the (working) examples of Sections 3, 4, 5 and 6 get reversed.

While it is fairly obvious that children are time-intensive, it is far less obvious that it is the parent's time that is essential. In fact, outsourcing of child care time is quite common, and has been throughout history. Examples include nannies, aupairs, relatives, wet nurses, and even orphanages.[43] In short, all these arrangements mean that even though children take time to raise, this time can, in principle, be hired, and hence, it is not clear why the price of children would be higher for high wage people.

In this section, we first show how, when buying nanny-time is an option, higher wage parents will choose to have more children. We then ask what additional assumptions would restore the negative wage-fertility relationship, *even when* hiring nannies is possible. We give one example where the specific functional form of the child quality production function gives the desired result. However, the interpretation is not straightforward.

[43]In the 19th century, many poor children were sent to orphanages, even when the parents where still alive, but too poor to feed the children. In 1853, Charles Loring Brace founded the *Children's Aid Society*, which rescued more than 150,000 abandoned, abused and orphaned children from the streets of New York City and took them by train to start new lives with families on farms across the country between 1853 and 1929.

7.1 A First Example

To see that the assumption of *parental* time is a crucial assumption, consider the following simple example.

$$\begin{aligned}\max_{c,n,l_p,l_n} \quad & \alpha_c u(c) + \alpha_n u(n) \\ s.t. \quad & b_1 n \leq l_n + l_p \\ & c + w_n l_n \leq w(1 - l_p)\end{aligned}$$

where $b_1 n$ is the total time requirement for raising n children, as before, but children can now be raised using either parental time, l_p, or nanny time, l_n. We denote the cost of a nanny by w_n per unit of time.

The optimal use of nannies in this example depends on the relative wage of nannies vs. parents. As long as $w < w_n$, it is never optimal to hire a nanny, and hence, this case is analogous to our previous analysis of examples in which children require parental time. On the other hand, when $w > w_n$, then parents prefer to hire a nanny, so that $l_p^* = 0$ and $l_n^* = b_1 n^*$. This case is equivalent to examples where children are a goods cost only, and there we have seen that $dn^*/dw \geq 0$. So while in this example $dn^*/dw < 0$ is possible, it occurs only in the region where nannies are irrelevant.

Thus, if some people have wages that are lower than the average nanny wage and others have higher wages, this model would imply a v-shaped fertility-income relationship. That is, fertility would be downward sloping in wages for people with wages below the nanny wage and upward sloping thereafter. This is *not* what Figure 1 shows.[44] So one might rephrase the question as follows: why is fertility decreasing in wages even for those people whose (after-tax) wages are higher than the hourly cost of day-care or nannies?

There are, of course, several plausible reasons for this, such as a moral hazard problem in the quality of child-care provided by a nanny. Or, perhaps parents

[44] Some authors have found that at the very top of the income distribution, the fertility-income relation might be positive. Due to top coding and small samples at the top of the income distributions, these estimates are often statistically insignificant. Also, if this theory was applied to such a v-shape, it would mean that nannies are so expensive (either due to high wages or high tax wedges) that only the top income group finds it worthwhile hiring nannies. This seems to be at odds with the evidence as well.

enjoy *spending time* with their children over and above the pure utility effect of having children. To the best of our knowledge, these ideas have not been seriously formalized, yet.[45] Yet a third possibility is that nanny quality enters the child quality production function. We study an example along these lines in the next subsection. We are able to recover the negative wage-fertility relationship in this example. However, it rests on very specific functional form assumptions. Rather than seeing this as a final answer to the question raised at the beginning of this section, we view this example as a starting point for discussion and further research.

7.2 An Example that works

The starting point for this example is the analysis of Section 4, i.e., the example in which parents make schooling choices for themselves, which in turn determine their wage. To keep it simple, assume $\nu_s = \nu_w = 1$. We add two additional goods to the utility function: child quality and leisure. As before, each child requires a time input b_1. However, now, this can be someone else's time, not necessarily the parent. There is a by-product of spending time with a child, however, namely that it also produces child quality, and child quality is strictly increasing in the human capital of the child-care provider. For simplicity, we assume that there are only two feasible child-care choices: either full time parental child-rearing or hiring a full-time nanny. Formally, we allow parents to choose an indicator function $I \in \{0, 1\}$, where $I = 1$ means the parent does the child-rearing and $I = 0$ means a nanny does. When a nanny is hired, the quality of the nanny (i.e., her human capital) is also a choice. That is, the parent faces a trade-off between hiring a more skilled nanny and paying more or hiring a cheaper lower-quality nanny. We denote human capital of the parent by H and nanny human capital by H^n. Note that in this simple set-up the wage is always equal to human capital, $w = H$ and $w^n = H^n$.

We also assume a rather special functional form for the child quality production function: $q = \exp\{\frac{H^{-d}}{-d}\}$, where H is either parent or nanny human capital

[45] Erosa, Fuster, and Restuccia (2005a) have an indirect way of modeling the idea that parents like to spend time with children. That is, the value of staying at home can only be enjoyed if the mother gave birth in the past but has not returned to work since.

and $d > 0$. The reason for using this unusual functional form is that with many of the more standard choices, the problem is either not well-defined or the desired result does not occur.[46] Even though this functional form is somewhat non-standard, it does have the usual properties – monotone increasing and strictly concave.

The choice problem is:

$$\begin{aligned}
\max_{c,n,q,\ell,l_s,l_w,I,H^n,w^n} \quad & \alpha_c \log(c) + \alpha_n \log(n) + \alpha_q \log(q) + \alpha_\ell \log(\ell) \\
\text{s. t.} \quad & l_s + l_w + \ell + b_1 n I \leq 1 \\
& H = a l_s \\
& c + (1-I) w^n b_1 n \leq w l_w \\
& q = \exp\{\frac{(IH + (1-I)H^n)^{-d}}{-d}\} \\
& w = H \quad \text{and} \quad w^n = H^n
\end{aligned}$$

Note that in this set-up hiring a nanny with one's own human capital or raising one's own children enters the problem exactly in the same way. Because of this we can rewrite the problem as one where all the child-rearing is done by nannies. In the special case where the optimal choice has the property that $H^n = H$, we can interpret the solution as either having the parent or a nanny raising the children. In all other cases, parents strictly prefer to hire a nanny. Then, the problem simplifies to

$$\begin{aligned}
\max_{c,n,q,\ell,l_s,l_w} \quad & \alpha_c \log(c) + \alpha_n \log(n) + \alpha_q \log(q) + \alpha_\ell \log(\ell) \\
\text{s. t.} \quad & l_s + l_w + \ell \leq 1 \\
& c + w^n b_1 n \leq a l_s l_w \\
& q = \exp\{\frac{(w^n)^{-d}}{-d}\}
\end{aligned}$$

[46] For example, if the child quality production function is of the form $q = BH^\alpha$, then only the product nH enters the problem and hence no separate solution for n^* and H^* can be obtained.

The solution is:

$$
\begin{aligned}
(w^n)^* &= \left(\frac{\alpha_q}{\alpha_n}\right)^{\frac{1}{d}} \\
l_s^* &= \frac{\alpha_c + \alpha_n}{\alpha_\ell + 2(\alpha_c + \alpha_n)} \\
n^* &= \frac{\alpha_n(\alpha_c + \alpha_n)a/b_1(\frac{\alpha_n}{\alpha_q})^{1/d}}{[\alpha_\ell + 2(\alpha_c + \alpha_n)]^2} \\
\ell^* &= \frac{\alpha_\ell}{\alpha_\ell + 2(\alpha_c + \alpha_n)}
\end{aligned}
$$

Suppose that people differ in their preference for the consumption good α_c. Then, wages are strictly increasing in α_c and fertility is strictly decreasing in α_c as long as leisure is not too important (the exact condition is: $2(\alpha_c + \alpha_n) > \alpha_\ell$). A further interesting feature of this examples is that the value of H^n chosen is identical across parents, i.e., higher wage parents will *not* hire a more qualified nanny.

The mechanism behind this example is similar to that in Section 4. People who put a higher weight on consumption goods will invest more in schooling, and hence have higher wages. At the same time, they care less about children and hence have fewer. Note that having leisure in this example is crucial. The reason is that once nannies are an option, then parents essentially allocate their time only between investing in (own) human capital and working. Given our functional forms, without leisure, the optimal allocation would always be $l_s^* = l_w^* = 0.5$. But then wages would no longer differ across people, since independent of the preference parameters, everyone would make the same schooling choice. Adding leisure gets around this "problem" as people who value consumption goods more choose more schooling and less leisure and therefore, have higher wages. These same people also have fewer children. This logic holds even when child-care time can be outsourced to nannies, since it is ultimately the dislike of children that drives the low fertility of high wage people, and not the high time cost of children. Because of this logic, heterogeneity in *preferences*, rather than exogenous ability, is essential to the result. Starting from exogenous ability heterogeneity would lead to very different conclusions, as is obvious from the solution above (and recalling $w = al_s^*$): higher a people have both higher wages *and* more children.

Of course, the mechanism in this example is probably not the only (or even the main) reason for why higher wage people choose lower fertility, even when nannies are an option. We merely mean to raise an important question and show that constructing an example with the desired properties is rather difficult – special assumptions are required. Note also, that adding nannies to any of the conventional mechanisms (from Section 3) would not work at all, in each case, people with higher wages would choose a higher fertility rate.

Another mechanism that comes to mind, is that there might be some moral hazard problem in child care work. Even though in principle, nannies can be hired, if there is some effort involved in raising a high quality child, then the incentives for a nanny might be different from a parent. If monitoring is costly, parents might optimally choose to do the child rearing themselves. In this case, the opportunity cost of a child is again increasing in income.

Finally, another possibility is that most models of fertility lack an important ingredient: it might not be actually the number of children that parents care about, but rather the time spent with their children that enters directly into the utility function. If people derive pleasure from, say, spending the weekend with their children, then nannies are a poor substitute for own child-rearing. While we believe these are interesting and potentially promising channels, they are well beyond the scope of this paper, and are left for future research.

8 Conclusion

Throughout most of this paper, we have focused on what kind of theories of fertility can match the downward sloping fertility-wage relationship observed in cross-sectional data. We have seen that this requires some special assumptions, such as a high elasticity of substitution between fertility and (parent's) consumption. One might want to ask more of such theories. For example, one might want to know under what conditions such models could also match the decline in average fertility over the last century and a half. In other words, which of these theories can also get the time series facts right, or how must they be modified to do so? Obviously our static examples are too stylized to take them to the data in any serious fashion. Yet, several stylized facts emerged from Section 2 and one way of getting at this question is to ask which of the theories can produce a picture that looks qualitatively like Figure 1. The stylized facts that emerge from this figure can be summarized as:

1. Fertility is very high at low wages (about 6).

2. Fertility is very low at high wages (about 2).

3. Fertility is decreasing (and convex) in wages for each cross section.

4. Fertility falls over time, as consecutive cross sections move to the right.

In terms of forcing variables, it is not obvious what exogenous changes over time one should consider when considering models and data. One obvious change over this time period are increases in wages driven by TFP growth. Another potential change that seems plausible is changes in education, both through technological change that made human capital production more efficient and changes in government policies through the (free) public provision of schooling. Sometimes it is argued that children have become more costly over time, and so we look at this change as well. It is not clear, however, what the interpretation of such a change should be. While we don't provide a clear answer to our own question, we still hope that these ideas will stimulate some further research and ultimately lead to a better understanding of fertility decision-making.

Below, we show four numerical examples, each based on a different theory that was analyzed in the text. Each graph displays four cross-sectional relationships between income and fertility. Depending on the example, the difference between people within a cross section (i.e., on one line) is either wages or preferences, while the difference between different cross sections (i.e. between the four different lines) is either wages, schooling technology, and/or child rearing costs.

The first two figures are based on two different examples from Section 3. Figure 2 is based on Problem (3) while Figure 3 is based on Problem (4), both variants of the simplest "price of time theory."[47] In each case, the only thing that differs

[47]The main qualitative difference between the two examples is that the income elasticity is constant in Figure 3, while it is increasing in absolute value in Figure 2. Recall also that the empirical elasticity appears slightly decreasing over this time horizon (as shown in Table 1).

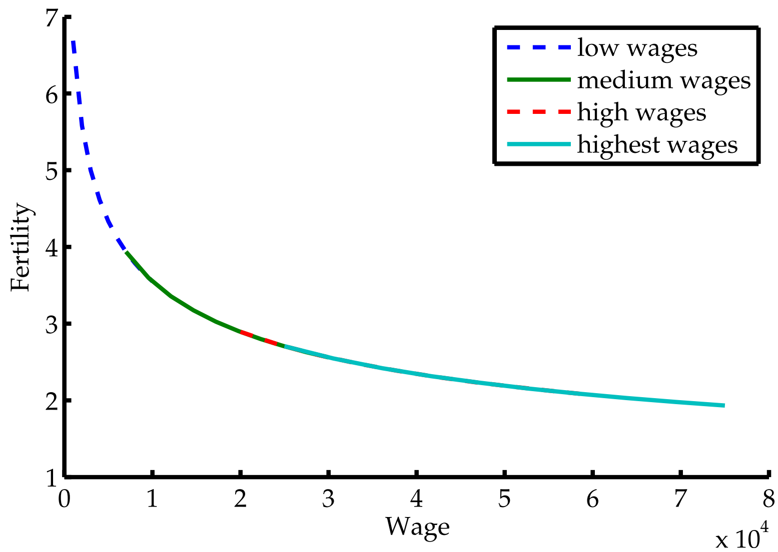

Figure 2: Time Series based on price of time example, $\sigma < 1$, increasing wages

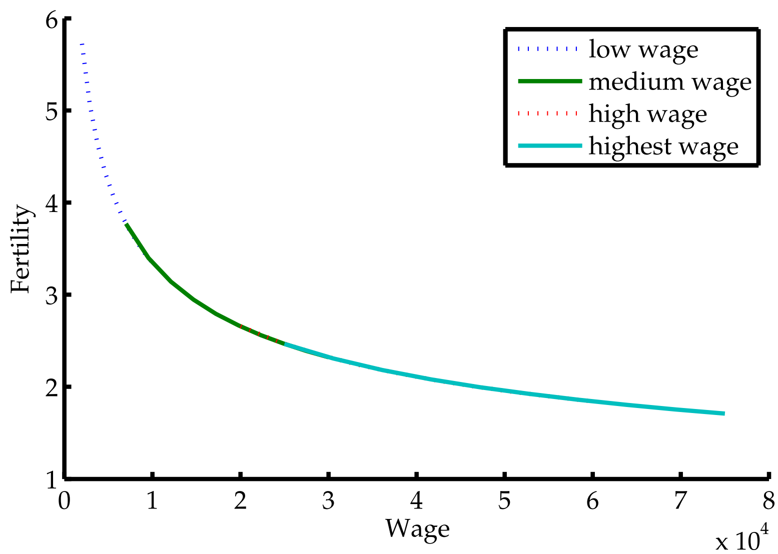

Figure 3: Time Series in example with non-homothetic utility, increasing wages

across people (both in the cross section and over time) is wages. Both examples match the stylized facts described above pretty well. So as long as one is willing to assume a high elasticity of substitution between parent's consumption and fertility, the basic theory seems to work well—at least in this simple formulation. Once one goes to a truly dynamic formulation, where parents have preferences over their children's utility, the same logic no longer goes through, as we discuss in Appendix A.1. The intuition is simple, when wages go up, this affects both parents's and children's wages. Thus, while the opportunity cost of having a child is higher for richer parents, the benefit of having a child also goes up (because the wage of a child of a rich parent is also high). Thus, even though these results seem like strong successes for the theory at first glance, there are other reasonable, but more stringent, requirements for which their success is more limited.

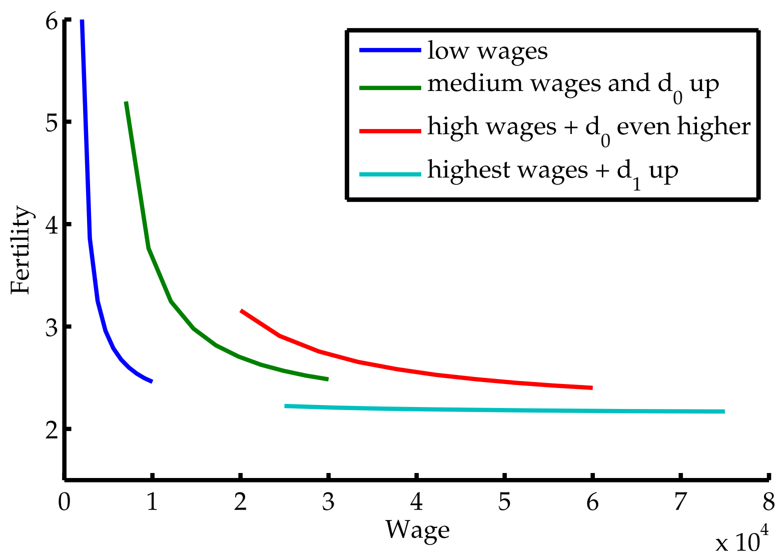

Figure 4: Time Series based on QQ example

Figure 4 considers the quantity-quality trade-off example from Problem (7) with $f(s) = d_0 + d_1 s$. Note that to distinguish this example from the first two pictures, this assumes log-utility, and all curvature comes in through the child quality production function only. In this example, fertility is essentially hyperbolic in wages and hence the shape of the curve does not match Figure 1 very well.[48] However, this example lends itself to think about potential changes in the education sector. Thus, in addition to increasing wages, consecutive cross sections in Figure 4 face different quality production functions. In particular, the second cross section has a higher d_0 which one could interpret as the introduction of elementary public education. The third cross section has an even higher d_0 which might stand for a further expansion of the public education system. The last cross section has a higher d_1, which is a parameter that determines the returns to parental education inputs. This could be interpreted as improvements in education technology. Alternatively, without this last change in the child quality production function, the last cross section would simply be a continuation of the third cross section, converging to 2.14 children (in this example) as wages go to infinity. So while this picture matches Figure 1 qualitatively, more work on the underlying changes in education technology (i.e., their historical analogues) would be required before one could call this theory a success.

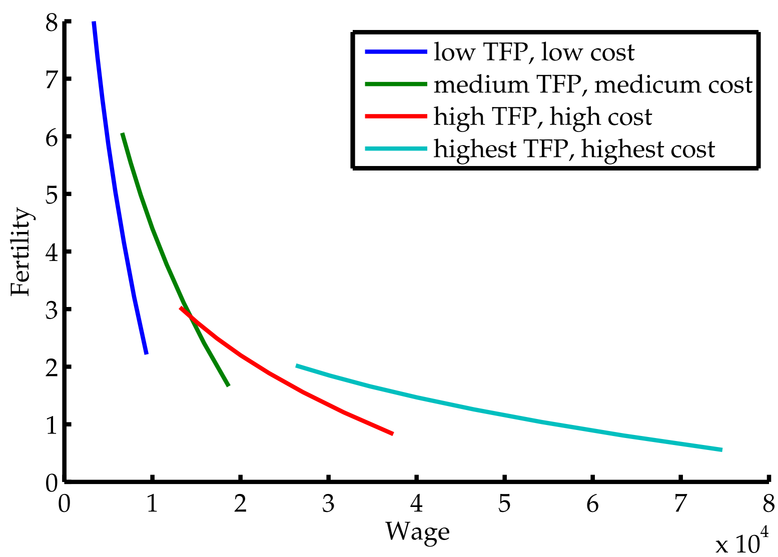

Figure 5: Time Series based on increasing TFP and increasing cost of children, cross section due to preference heterogeneity

[48]One way of stating the qualitative difference between Figure 4 and the data is that the income elasticity of fertility in the example converges to zero very fast as wages increase, while in the data, the elasticity is roughly constant.

Finally, Figure 5 is based on the preference heterogeneity example from Section 4. In this Figure the cross section and time series both slope downward but the mechanism behind the two is not the same. The cross section is based on preference heterogeneity. That is, those people who like children invest less in market-specific human capital and therefore have lower wages, while those who put a higher weight on consumption goods do the opposite and hence have higher wages. Over time, as in the examples above, we assume that average productivity, a, goes up. However, in this example, increases in productivity do not affect fertility decisions. Hence, without further bells and whistles (e.g. changing the curvature to the utility function), this example will not lead to falling fertility for consecutive cross sections. We have thus added a second channel to the time series in the figure: increases in child costs, i.e. the units of time required per child increases (exogenously) over time. This gives a picture that looks roughly like the data, but it is not clear how to interpret this, i.e. what is the real world analogue of an increase in child-rearing costs (measured in units of time)? [49]

These simple examples are only meant to be suggestive of the kinds of things that are possible with models like those that we have examined in this paper. Much more work in carefully calibrating/estimating the relevant parameters and documenting the needed changes in the forcing variables are necessary before any final conclusions can be drawn. This is a task left for future work.

[49] One rationale for this change may be the progressive introduction of child labor laws. That is, while the time cost remained the same, the time that children contribute to the household's income decreases. Hence, this would be equivalent to a net increase in the time cost.

A Appendix

A.1 Adding Parental Altruism

To this point, our focus has been on examining simple models of fertility choice that give rise to the observed pattern in the cross section with respect to income. As we have seen, there are several examples that are capable of this differing in their details. One could ask for more than this however. For example, one property that is missing from all of the examples in the main text is altruism from parents towards their children. That is, parents are made happy by those things that increase the utility of their children. This introduces an additional dynamic aspect to the fertility choice automatically. This is that parents, when choosing their own fertility levels, must forecast the utility levels of their own children. Following this logic, the utility of the children will depend on the utility levels of their own children – i.e., the grandchildren – and so forth. That is, the utility of the current period decision maker depends on the entire future evolution of the path of consumption and fertility, not just the levels chosen this period. Although this sounds complex, models of fertility choice based on parental altruism of this form have been worked out in detail in Becker and Barro (1988) and Barro and Becker (1989). Here we develop a simple version of the Barro-Becker model (B-B henceforth) and discuss its relationship with the examples developed in the main text. What we show is that the simple example discussed in Section 3 can be given an interpretation as the problem solved by the typical parent under a setting with dynastic altruism, but that this requires some extra assumptions and has some additional implications. In particular, the simple, static problem with homothetic preferences can be interpreted as the problem from the Bellman's equation for the fully dynamic model where the term relating to fertility choice corresponds to the value function for continuation payoffs. However, this interpretation has the additional implication that this value function also depends on the wage, and because of this has the property that families with different base wage rates all make the same fertility choices. Thus, although the high elasticity homothetic example has the correct cross sectional property in the static example, this property does not extend to the fully dynamic version of the model.

In the simplest version of the B-B model, the time t parent solves:

$$\max_{c_t, n_t} \qquad u(c_t) + \beta g(n_t) U_{t+1},$$

subject to: $\qquad c_t + \theta_t n_t \leq w_t,$

where c_t is current period consumption, n_t is the fertility choice, and U_{t+1} is the utility level of the typical child. Assuming that $g(n) = n^\eta$, $u(c) = c^{1-\sigma}/(1-\sigma)$, successively substituting and changing to aggregate variables for all of the descendants of a given time 0 household, the equilibrium sequence of choices can be represented as the solution to the following time 0 maximization problem:

$$\max_{\{C_t, N_t\}} \qquad \sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} C_t^{1-\sigma}/(1-\sigma)$$

Subject to:

$C_t + \theta_t N_{t+1} \leq w_t N_t,$

N_0 given,

where C_t is aggregate consumption in period t, N_t is the number of adults in period t, θ_t is the cost of producing a child and w_t is the wage rate. Implicit in this formulation is the assumption that each adult has the same level of consumption $\frac{C_t}{N_t} = c_t$ in any period.

For this problem to satisfy the typical monotonicity and concavity restrictions some restrictions on σ and η must be satisfied. There are two sets of parameter choices that satisfy these requirements. The first is the original assumption in Becker and Barro (1988) and Barro and Becker (1989): $0 \leq \eta + \sigma - 1 < 1$, $0 < 1 - \sigma < 1$ and $0 < \eta = \eta + \sigma - 1 + 1 - \sigma < 1$. In this case $U > 0$ for all $(N, C) \in R_+^2$. The second possibility is one which allows for intertemporal elasticities of substitution in line with the standard growth and business cycle literature: $\sigma > 1$, $\eta + \sigma - 1 \leq 0$. In this case, utility is negative and $\eta < 0$. When $\eta = 1 - \sigma$ (allowed under both configurations), utility becomes a function of aggregate consumption only.[50]

There are two types of situations under which this maximization problem becomes a stationary dynamic program (where the state variable is N). Both

[50]This formulation for the dynasty utility flow gives rise to some very useful simplifications that we will exploit below. One disadvantage of it however, is that it is not equivalent to logarithmic utility when $\sigma = 1$. However, when $\eta = 1 - \sigma$ and $\sigma \to 1$, the preferences, will converge to those given by the utility function $\sum \beta^t \log(C_t)$. See Bar and Leukhina (2007) for an explicit derivation of Barro-Becker preferences with an IES equal to one.

cases require constant growth in wages – $w_t = \gamma_w^t w_0$. The first is when the cost of children is in terms of goods, and this cost grows at the same rate as wages – $\theta_t = a\gamma_w^t$. The second case is when the cost of having a kid is in terms of time only, $\theta_t = b_{1t} w_t$ where b_{1t} is the amount of time it takes to raise one surviving child.

In either of these cases, the problem of the dynasty overall has a homogeneous of degree one constraint set and an objective function that is homogeneous of degree η. Because of this structure, it follows that the solution to the sequence problem has several useful properties that we will exploit below.

Following the discussion in Section 3, it follows that only the time cost case is capable of matching the facts from the cross section and hence, we will limit out attention to this case here.

Under the special case that $\eta = 1 - \sigma$, it follows that the value function for this problem, $V(N)$ is homogeneous of degree $1 - \sigma$ in N – $V(N) = V(1)N^{1-\sigma}$. Because of this fact, it follows that, after detrending, Bellman's equation for this problem can be written as:

$$\begin{aligned} V(N) = \sup_{\{C,N'\}} \quad & C^{1-\sigma}/(1-\sigma) + \hat{\beta}V(1)N'^{(1-\sigma)} \\ s.t. \quad & C + bwN' \leq wN \end{aligned}$$

where $\hat{\beta} = \beta\gamma_w^\eta$. $V(1)$ can be found explicitly. It is given by:

$$V(1) = \frac{(w + \theta(\pi - \gamma_N))^{1-\sigma}}{(1-\sigma)(1 - \beta\gamma_N^\eta \gamma^{1-\sigma})}$$

It follows that the solution to the dynastic problem has a representation in which each date t adult chooses his own consumption and fertility level so as to solve:

$$\begin{aligned} \max_{\{c,n\}} \quad & c_t^{1-\sigma}/(1-\sigma) + \hat{\beta}V(1)n_t^{1-\sigma} \\ s.t. \quad & c_t + bw_t n_t \leq w_t \end{aligned}$$

Note that this problem is similar to the CES utility function problem laid out in Section 3.2. However, there is one important difference. This is that the coefficient on fertility cannot be chosen freely. In particular, it is easy to see that $V(1)$

depends on the wage. Indeed, it follow directly that it is increasing in the wage. Because of this, it follows that the results from the comparative statics concerning the dependence of fertility on the wage are not necessarily valid – In the dynamic version of the problem both the objective function (i.e., Bellman's Equation) and the constraints depend on the wage.

In fact, it can be shown that the equilibrium choice of fertility is given by:

$$n_t = \frac{N_{t+1}}{N_t} = \gamma_N = \left(\beta\gamma_w^{1-\sigma}\left[\frac{w_0}{\theta_0} + \pi\right]\right)^{1/\sigma} = \left(\beta\gamma_w^{1-\sigma}\left[\frac{1}{b_1} + \pi\right]\right)^{1/\sigma} \tag{15}$$

It follows that fertility choices are independent of the level of wages of the family. Thus, although it seems like the time cost case can reproduce the cross sectional properties of fertility choice (when $\sigma < 1$ is assumed), this is not true once one restricts attention to static problems that have a dynamic rationalization.[51]

We can also use this framework to get some idea about the implications for differences in fertility across families when preferences for children are the basic source of heterogeneity. For example, we can see that if different families differ in their levels of patience – β – differences in the cross section are preserved in the time series. Thus, for example, if for two families, i and i', we have that $\beta_i > \beta_{i'}$, it follows that $n_{it} > n_{i't}$ for all t. Thus, the cross sectional variation in fertility choice is preserved in the time series.[52] It should be noted however, that this will also have the implication that families with higher fertility also have higher savings rates. This probably does not hold in the cross section.

[51]Here we have assumed that wage differences across families are permanent – i.e., if i and i' represent two distinct families then we are assuming that $\frac{w_{it+1}}{w_{i't+1}} = \frac{w_{it}}{w_{i't}} = \gamma_w$. An interesting question is whether this result will be overturned when one moves away from this assumption. Jones and Schoonbroodt (2007b) find that a high growth rate lowers fertility if $\sigma > 1$ and vice-versa (see also Equation 15). This suggests that with intergenerational mean reversion in income, poor households expect a high income growth rate and would have more children than rich ones as long as $\sigma < 1$. In this context, Zhao (2008) uses a model with filial altruism as in Boldrin and Jones (2002) where mean reversion is crucial, both in the cross section and over time (when social security crowds out fertility). We leave the analysis of intermediate cases (i.e. partially correlated dynastic incomes) to future research.

[52]As above, this assumes that the differences across families is permanent – $\beta_{it} > \beta_{i't}$ for all t.

A.2 A Dynamic Version of the Endogenous Wage Example

Next, we develop a version of the endogenous wage model in Section 4 that is consistent with parental altruism as in the B-B model.

Assume that the resource constraints are given by those of problem (6), but assume that $\nu_s + \nu_w = 1$. (To simplify notation, write $\nu_s = \nu$ and $\nu_w = 1 - \nu$.) Using capital letters to denote aggregate quantities (i.e. defining $L_t \equiv N_t l_t$ etc.), the planner's problem can be rewritten as:

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t N_t^{\eta+\sigma-1} C_t^{1-\sigma}/(1-\sigma) \\ s.t. \quad & L_{st} + L_{wt} + L_{nt} \leq N_t \\ & C_t \leq a L_{st}^{\nu} L_{wt}^{1-\nu} \\ & bN_{t+1} \leq L_{nt} \end{aligned} \tag{16}$$

As above, the constraint correspondence is homogeneous of degree 1 and the utility function is homogeneous of degree η in initial condition N_0. Assuming that $\eta = 1 - \sigma$ as above, the value function is of the form $V(N) = V(1)N^{1-\sigma}$. It follows that the Bellman Equation is:

$$\begin{aligned} V(N) \quad = \quad & \sup_{C,N'} C^{1-\sigma}/(1-\sigma) + \beta V(1) N'^{(1-\sigma)} \\ s.t. \quad & L_s + L_w + bN' \leq N \\ & C \leq a L_s^{\nu} L_w^{1-\nu} \end{aligned}$$

So for the appropriate choice of α_n and α_c, the solution to problem (6) can be interpreted as the solution to the dynamic problem (16) with $N_0 = 1$ in some cases. Here, normalizing $\alpha_c = 1$, it follows that $\alpha_n = \beta V(1)$.

It is not clear in this framework exactly what comparative statics exercise corresponds to the one in Section 4, in which α_n is increased. In principle, it could correspond either to an increase in β, or any increase that makes $V(1)$ larger. In what follows, we consider only the implications of increases, across dynasties, of increases in β's.

Using the first order conditions to the problem in sequence form and simplifying we obtain a characterization of the balanced growth path dynamics of the system which is completely determined by the division of time between schooling and working and the intertemporal choice of family size involving fertility. These are given by:

$\frac{L_{wt}}{L_{st}} = \frac{1-\nu}{\nu}$, and

$n_t^{\sigma} = \gamma_N^{\sigma} = \beta / b_1.$

That is, fertility is increasing in β. Because of this fact, it follows that both $\frac{L_{st}}{N_t}$ and $\frac{L_{wt}}{N_t}$ are decreasing in β, and hence, fertility and income (or wages) are negatively related as desired.

Thus, for the endogenous wage example, an explicit dynastic form can be provided that is still consistent with the cross sectional facts. There are still some issues about this however. Foremost among these is that it is common when discount factors differ across agents that strong forces for borrowing and lending are also present. The analysis here ignores these considerations. It is not clear that they will stand up to this extension.[53]

[53] Another issue not considered here is variants of intergenerational persistence in preferences.

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