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In or Out: Faculty Research and Consulting

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Abstract

We examine the empirical anomaly that 28% of the patents in a sample of patents listing university faculty as inventors are assigned to for-profit firms. This is an anomaly because, with rare exception, universities specify that inventions resulting from faculty research in the university belong to the university. We develop a model to show that this may not be nefarious, but instead is the result of consulting conducted outside the university. The model allows faculty inventors to conduct research inside the university with funding assistance from the federal government and/or a firm, and outside the university by selling their consulting services to a firm. Patents that result from research within the university are assigned to the university and licensed under the Bayh-Dole Act, while those from consulting are assigned to the firm. We analyze the equilibria of a two-stage game in which university funding levels are chosen in stage one, and consulting in stage two. We empirically test the model using data collected from 11 major US research institutions on the assignment of patents and the levels of funding from federal and industrial sources received by individual faculty members in engineering and sciences, as well as publication data, and citation data from publications and patents. In general, we find support for the theory, especially the fact that there are direct spillovers from government funding of university research to the firm's problem.

1 Introduction

This paper addresses the empirical anomaly that in a sample of 1767 US patents on which faculty from 11 major US universities are listed as inventors 28% are

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assigned to for-profit firms. This is an anomaly because, with rare exception, employment contracts in US universities specify that inventions resulting from a faculty member's research in the university belongs to the university.[1] Further, the Bayh Dole Act of 1980 allows universities to own the intellectual property (IP) resulting from federally-funded research and, in the case of industry sponsored research, US universities historically have insisted on ownership (Thursby et al.2001). The primary explanation given in Thursby *et al.*'s (2007) interviews with university technology licensing professionals and industry R&D executives was consulting conducted outside of university employment contracts.

In this paper, we argue that explaining the pattern of ownership of IP associated with faculty research requires an analysis, not only of faculty decisions to conduct research within the university or outside, but also the decisions of funding agents, both government and industrial, on the researchers they want to support. We develop a theoretical model in which a faculty researcher can conduct research inside the university with funding from the federal government and/or a firm, and can conduct research outside the university by selling her consulting services to a firm. Patents that result from research conducted within the university are assigned to the university, while those from consulting are assigned directly to the firm.

The model we construct has two stages. In the first stage, the government funding agency and the firm simultaneously choose funding levels for the researcher's university project. This stage is followed by another simultaneous-move game in which the firm chooses a unit consulting fee, and the researcher decides how much time to spend consulting for the firm. The model provides predictions for the amount of time spent consulting, the consulting fee, and the levels of government and firm funding for the faculty member's university research.

The model incorporates differences in the difficulty of research projects, both within the university and the firm, as well as the fact that faculty researchers vary in quality or academic reputation. An important element of the model is R&D spillovers in the sense that research conducted within the university by an inventor can enhance her probability of success in a consulting project. This allows us to relate consulting behavior, and therefore the assignment of patents to firms, to faculty quality, R&D spillovers, as well as the willingness of the firm and government to sponsor the faculty member's research. In the second stage, under plausible assumptions, we find that increases in the researcher's quality, the fraction of industrial funding that is equivalent to governmental funding, the researcher's share of license revenue from her university research, or the level of funding provided by the university all lead to a greater unit consulting fee, but less time spent in consulting. Conversely, increases in the extent to which her university research spills over into consulting, the research support provided by the firm in its lab, or the difficulty of the firm's research project results in a lower unit consulting fee and more time spent in consulting. Finally,

[1]In a survey of 65 major research universities only the University of Wisconsin gave ownership rights to faculty, but this was only for research not funded by the US government.

although the effects of changes in the levels of external funding are generally ambiguous, we find that an increase in governmental or industrial funding within the university must either decrease the unit consulting fee, decrease the time spent consulting, or both. Typically we would expect the consulting fee and time to both decrease.

In the first stage, under plausible assumptions, we show that an increase in the extent to which her university research spills over into consulting, the research support provided by the firm in its lab, or the difficulty of the firm's research project results in greater funding from the governmental agency and less funding from the firm for university research. The effect of a change in researcher quality is generally ambiguous, although an increase in quality must either increase university research funding from the government agency, from the firm, or both. In the typical case, we expect both to increase. Similarly, the effect of an increase in the difficulty of her university research project, research funding within the university, or the fraction of industrial funding that is equivalent to governmental funding are also ambiguous, although an increase in any of these must either decrease university research funding from the government agency, from the firm, or both.

We then consider a sample of faculty-invented patents assigned to either universities or to firms. Our data include publication and citation records for the inventors in the sample, as well as the levels of their research funding from the federal government and industry sources. We find that assignment to firms is negatively related to inventor quality as measured by publications and predicted by our theory. Further, firm assignment is positively associated with federal funding. In the context of our theoretical model, this result is possible only if there is a spillover from the faculty researcher's government sponsored research to the firm's research problem. Industry sponsored research is negatively associated with patent assignments to firms. Our theory does not provide a clear prediction for industry sponsored research.

This paper is one of only a few studies to examine consulting as a mechanism for university-industry technology transfer. While consulting is often cited as one of many such mechanisms, those which are more formal and easily tracked - such as publications, patents, licenses - have received more attention (Mowery et al., 2004). Notable exceptions include Mansfield (1995) and Cohen et al. (1998) which examine survey evidence from industrial R&D personnel. To our knowledge, the only theoretical analysis of consulting is Beath et al. (2003) which examines the potential for budget-constrained universities to relax the constraint by encouraging faculty to consult.

Our results also contribute to the growing literature on the impact of commercial opportunities on faculty research (Azoulay et al. 2006, 2007, Lach and Schankerman 2004; Thursby and Thursby 2007; and Thursby et al. 2007b). Much of this literature addresses the concern that commercial activity might divert faculty from research. We contribute by showing that the opportunity to earn license revenue is likely to increase time devoted to university research.

The work closest to ours is Thursby et al. (2007a) which examines a sample of 5811 patents on which faculty from 87 US universities are listed as inventors.

In their sample, 26% of the patents are assigned solely to firms rather than to the faculty member's university. Both their work and ours provide a more nuanced view of academic contributions to industrial patenting than that provided by citations to patents assigned to universities.[2] Although they focus on consulting, Thursby et al.(2007a) examine assignment as a function of patent characteristics and university policy rather than individual inventor characteristics or research funding. Consistent with our theoretical result on inventor share, they find that a higher inventor share increases the likelihood of university assignment as compared with assignment to a firm in which the inventor is a principal.

Finally, we contribute to the literature on the relationship between government and industry funding for research which has primarily focused on the complementarity or substitutability of public and private funding of R&D conducted by firms (David et al.2000). By contrast, we focus on government and industry funding for research in universities. Our combined theoretical and empirical results provide new insights into the ways in which firms benefit from spillovers from governmental funding for university research.

2 Environment

Our goal is to develop a theory to explain observed levels of governmental and industrial funding, consulting, publications, and assignment of patents among faculty researchers, and how they differ with the quality of the faculty. To this end, we employ Occam's razor and assume one faculty researcher, one firm interested in capitalizing on faculty expertise, and one government funding agency. There are many dimensions on which one can measure researcher quality, but for the purposes of this analysis, we assume it can be characterized by an observable variable q defined on the interval $[0, Q]$ such that higher values of q correspond to greater academic success.

2.1 Research Technology

Although there are also many dimensions on which one can categorize research, for the purposes of this analysis it is most useful to think in terms of the pure scientific component of a given research problem. Thus, we assume research problems can be characterized by a variable x, defined on the interval $[0, X]$, such that higher values of x correspond to research that has greater scientific merit and is inherently more difficult to solve.

Successfully solving a given research problem can generate multiple outputs of value to the researcher, university, government funding agency, or industrial sponsor. These can be generally thought of as those results of research that contribute to the scientific reputations and commercial payoffs associated with

[2] See, for example, Jaffe (1989), Jaffe et al.(1993), Henderson et al. (1998). For a similar point in a European context see Crespi et al.(2006), Geuna and Nesta (2006), and Saragossi and van Pottelsberghe de la Potterie (2003).

solving the problem, such as publications, citations, patents, and profits. The likelihood that a research project succeeds depends on a number of factors, including the nature of the problem to be solved (how fundamental or basic it is), the quality of the researcher, and the level of funding available.

For simplicity, we think of the researcher as working on a single research problem within the university, which has scientific merit x_I, with the possibility of also working outside the university as a consultant on a firm's research problem, which has scientific merit x_O, where $x_I > x_O$.[3] Assume that T is the total time available in the period, and that M is the (maximum) amount of time that she can spend consulting, $M < T$.[4] Then the timing of the problem is as follows. If t is the time she contracts to consult with the firm, $t \in (0, M]$, then she spends the first $T - t$ "months" working in the university on her own research project, and the last t months working on the firm's problem in its R&D lab. If she does not consult, $t = 0$, then she works all of the year on her own university research.[5]

Now consider her research funding. As a member of the faculty, she has at least minimal research support $K_I > 0$ from the university for her own project, which she can supplement with sponsored research funds from a government agency, G, and/or industrial firm, F. Her research on the firm's problem is conducted within the firm's own R&D lab, where $K_O > 0$ is the fixed level of research support provided by the firm in this lab. The faculty researcher is paid c per unit of time for consulting, so her consulting income is ct.

As is common, we model research as an uncertain production process in which the "production function" is a probability of success function. We assume that the probability of success in solving any specific research problem of scientific merit x undertaken by a researcher of quality q is $p(\tau, e; q, x)$, where τ represents the time the researcher devotes to the project, and e represents her effective funding on that project. From the production perspective, it is natural to assume that p is increasing and strictly concave in $(\tau,\ e,\ q)$, so these "inputs" have positive but diminishing marginal productivities. It is also natural to assume that these inputs are complements, so the second order cross-partial derivatives of p with respect to them are all positive. For example, the marginal effect of an additional hour of research on the probability a project will succeed should be greater for researchers with higher quality or greater levels of funding. Our assumption that it is more difficult to solve problems with greater scientific merit implies p is decreasing in x, and it also natural to assume that

[3] We make this assumption because it seems appropriate in the context of this problem. However, we emphasize that it is not necessary for any results that follow.

[4] Most funding agencies and universities will not allow researchers to sell more than 100% of their time, so a decision to consult for the firm in its research lab on its project clearly means that the researcher will not be spending all of her time on her university project. Indeed, if she chose to so this after accepting, for example, federal funding for the entire year, then the granting agency would undoubtedly adjust their level of funding for her to adjust for this.

[5] This implicitly assumes that our heroine is an obsessive-compulsive workaholic who prefers her own research to all forms of leisure activity. This interpretation is perhaps an over-simplification, but it highlights the stylized fact that most researchers view their own research as a consumption good.

this difficulty increases at a increasing rate, $\partial^2 p/\partial x^2 < 0$. We also assume that a more difficult project reduces the marginal effect on the probability of success of time, effective funding, and quality, so that the cross-partial derivatives with respect to x and each of the inputs are negative.

It is important to discuss in more detail what we mean by "effective" research funding. Essentially this concept has been developed to address two commonly observed stylized facts about research funding. First, funding sources typically differ in the types of constraints they place upon the uses of the funds they provide. It is generally conceded that funding from governmental agencies is "better" than that from industry, at least on average, because governmental agencies place fewer restrictions on the uses of those funds. Second, there are often spillovers between research projects, arising in this case because experience from basic research can affect the probability of success in applied projects, and vice versa (Mansfield 1995, Zucker et al. 1998).

Although there are several well-known and accepted methods for formalizing these stylized facts in models of R&D (see DeBondt (1997)), the approach we take is to define effective funding. Under the timing assumed, consulting occurs (if at all) at the end of a given period, so the only spillovers possible will be from the university project to the consulting project.[6] Thus, we define effective funding on the researcher's project in the university as

$$e_I = K_I + G + \alpha F \tag{1a}$$

where $\alpha \in (0, 1)$ represents the fraction of industrial funding that is equivalent to governmental funding. That is, if industrial funding had the same restrictions on its use as governmental funding, then we would have $\alpha = 1$. However, α decreases as the additional constraints imposed on industrial funding rise. Analogously, we define effective funding for the firm's project as

$$e_O = K_O + \beta G + F + ct, \tag{1b}$$

where $\beta \in [0, 1)$ represents the extent to which her university research experience can contribute to solving the firm's problem. It is worth noting that this structure assumes that industrial funding of basic research projects within universities can be justified not only by the possibility these projects might yield results with commercial application, but also by the possibility that this basic research experience might indirectly the firm's own internal research problems. (need cite).

2.2 Preferences and Payoffs

We assume that faculty utility depends on their academic reputation as well as income, $U(R, Y)$ where R is her reputation and Y her income. Let R_s

[6] We do not claim that there are no spillovers from applied to basic research, but rather that in this model any such spillovers would have to emanate from previous consulting projects not incorporated in this model. We abstract from these spillovers because they are not the focus of the analysis.

denote her scientific reputation if she successfully solves her university research problem in this period. We assume R_s is an increasing function of x_I, because successful solution of a research problem of greater scientific merit results in greater enhancement of her reputation.[7] Let R_f denote her reputation if she fails to solve the problem in this period. This is also her reputation at the beginning of the period, when the funding agency and firm make their funding decisions. Naturally we assume $R_s > R_f$.[8] Assume that S is her university salary, and that γ is her share of the license revenue paid to the university for a success, $L \geq 0$ (we allow $L = 0$, as this is the case for many university research projects). Therefore, the researcher's expected utility is

$$\begin{aligned} EU(G,F,t,c) &= p(T-t,e_I;q,x_I)U(R_s,S+\gamma L+ct) \\ &\quad +[1-p(T-t,e_I;q,x_I)]U(R_f,S+ct). \end{aligned} \tag{2}$$

The government funding agency is primarily interested in advancing basic scientific research, so its utility, U_g, depends upon the scientific reputation associated with the research it funds. Because there are alternative uses for its research budget, namely other researchers' projects, its net expected utility, EU_g, from funding this particular project is the expected utility of its reputation less the utility loss V from not funding alternative projects. Its net expected utility from devoting G to this project is then

$$\begin{aligned} EU_g(G,F,t,c) &= p(T-t,e_I;q,x_I)U_g(R_{gs}) \\ &\quad +[1-p(T-t,e_I;q,x_I)]U_g(R_{gf})-V(G) \end{aligned} \tag{3}$$

where R_{gs} is its reputation if she succeeds in her university project, and R_{gf} is its reputation if she does not. We also assume R_{gs} is an increasing function of x_I, because successful solution of a research problem of greater scientific merit results in greater enhancement of the reputation of the funding agency. However, the agency does not get reputational credit for her success if it does not fund her: $R_{gs} > R_{gf}$ if and only if $G > 0$. It is worth noting that an increase in consulting time by the researcher unambiguously decreases the funding agency's expected utility by reducing the probability of success in her university project.

Finally, the firm's expected profit can arise from either its own research problem or university research that it funds. We assume a firm does not fund a researcher's university project unless it obtains an option to license a success from that project. Let π_I denote firm profit from funding the researcher's university project if it succeeds, and π denote the profit from its own research project if it succeeds. Then its expected profit is

$$E\Pi(G,F,t,c) = p(T-t,e_I;q,x_I)(\pi_I - L) - F + p(t,e_O;q,x_O)\pi - ct. \tag{4}$$

[7] For notational convenience, we do not write this functional dependence explicitly except when necessary.

[8] As noted above, the empirical measures of quality and reputation could be the same, so it would not be unreasonable to assume that $R_f = q$.

Note that this form implicitly assumes that, if the firm is not interested in funding the research in exchange for a license option, then it would not be interested in a license from a success developed without its funding.

To save on notation, in the following we shall let p_I denote $p(T-t, e_I; q, x_I)$ and p_O denote $p(t, e_O; q, x_O)$ whenever we can do so without causing more confusion than usual.

3 The Funding Game

Our objective is to develop a game structure that conforms well to the stylized fact that faculty typically prefer their own research to consulting, and therefore they focus on trying to obtain funding for their research before making any agreements to consult, and they do not allow funding for their research to be tied to a consulting agreement. Thus, the game as we envision it has two stages. In the first stage, our heroine seeks support for her university research project from both the governmental funding agency and the firm. The agency and the firm then simultaneously choose funding levels for the researcher's university project. Then, after these decisions are made and revealed, that is followed immediately (i.e., before the success or failure of the university research project is observed) by another simultaneous-move game in which the firm chooses a unit consulting fee, and the researcher decides how much time to spend consulting for the firm.[9]

Two comments about this approach are in order. First, it assumes that the funding agency and firm must pre-commit to providing funds for the researcher's university project.[10] It also assumes that researchers cannot be treated as agents who must accept take-it-or-leave-it offers. That is, we are interested in modeling the behavior of those "star" scientists whose expertise gives them more "market power" than workers in a principal-agent model with a perfectly elastic supply of labor, an assumption which is unrealistic for star scientists.

3.1 Stage Two Equilibrium

As usual, we begin by considering the second stage game, in which the researcher chooses her consulting time t and the firm chooses its unit consulting fee c, given the values of funding for university research chosen in stage one, F and G. Firms that devote funds to R&D typically have some ability to adjust their budgets, at least in principle. However, generally such a firm allocates a fixed amount $B_f > 0$ to R&D, and does not make major adjustments until the next budget cycle. Therefore, we assume $c \in [0, B_f/M]$.

[9] This approach also conforms to the "standard" academic year of nine months in which faculty are paid by the university, followed by three summer months in which faculty are free to pursue external funding options.

[10] This approach is similar to that in Lacetera (2005), who assumes that firms commit to university research as a way of funding basic research.

Theorem 1 *Consider the strategic form game with the researcher and firm as the players, whose strategies are $t \in [0, M]$ and $c \in [0, B_f/M]$, and payoff functions are defined by (2) and (4). Also assume each player's payoff function is continuous and strictly quasi-concave in its own strategy, given any strategy choices by the other players. Then this game has a Nash equilibrium $(t^*(G,F), c^*(G,F))$.*[11]

As is well known, under the conditions of this theorem, choosing $t \in [0, M]$ to maximize $EU(G, F, t, c)$ yields a best a best reply function $\hat{t}(c)$ for the researcher[12], which gives the consulting time that maximizes her expected utility for any unit consulting fee chosen by the firm. Similarly, choosing $c \in [0, B_f/M]$ to maximize $E\Pi(G, F, t, c)$ yields a best a best reply function $\hat{c}(t)$ for the firm[13], which gives the unit consulting fee that maximizes its expected profit for any time in consulting chosen by the researcher. The possible equilibria of this game are more easily understood using diagrams of these best reply (or reaction) functions.

Because we are interested in deriving testable implications, we focus on the Nash equilibrium when it is interior, $t^* \in (0, M)$ and $c^* \in (0, B_f/M)$.[14] In this case it must satisfy

$$\frac{\partial EU(G, F, t^*, c^*)}{\partial t} = 0, \tag{5a}$$

and

$$\frac{\partial E\Pi(G, F, t^*, c^*)}{\partial c} = 0 \tag{5b}$$

where

$$\begin{aligned} \frac{\partial EU(G, F, t, c)}{\partial t} \quad = \quad & -\frac{\partial p_I}{\partial \tau}[U(R_s, S + \gamma L + ct) - U(R_f, S + ct)] \\ & +[p_I \frac{\partial U(R_s, S + \gamma L + ct)}{\partial Y} + (1 - p_I)\frac{\partial U(R_f, S + ct)}{\partial Y}]c \end{aligned} \tag{6a}$$

and

$$\frac{\partial E\Pi(G, F, t, c)}{\partial c} = \frac{\partial p_O}{\partial e_O}(t\pi) - t = (\frac{\partial p_O}{\partial e_O}\pi - 1)t. \tag{6b}$$

In this case, the best replies are implicitly defined by $(6a)$ and $(6b)$.

Examples of this equilibrium are depicted in Figures 1 and 2. To interpret the equilibrium conditions in (5), consider the expressions for marginal utility

[11] These equilibrium values are also functions of all the parameters of the model (α,β,q,x_I,K_I,x_O,K_O,S,L,γ). Although a minor abuse of notation, we omit these as arguemnts of the functions for clarity of exposition.

[12] We omit the parameters of the model as explicit arguemnts of this function for clarity of exposition.

[13] We omit the parameters of the model as explicit arguemnts of this function for clarity of exposition.

[14] Of course, these results provide some information about corner solutions as well. For example, a change that increases consulting time in an interior equilibrium is more likley to induce a researcher off the no-consulting corner and begin some consulting.

and marginal profit in (6). From (6b), an increase in the consulting fee increases effective funding e_O, and therefore increases the probability of success in the firm's project and its expected profit, so it increases this fee until the marginal increase in expected profit from the project is offset by this marginal consulting cost. The firm's best reply is, of course, also its inverse demand function for consulting. We therefore assume that

$$\frac{\partial^2 p_O}{\partial e_O \partial \tau} + \frac{\partial^2 p_O}{\partial e_O^2} c < 0 \tag{7}$$

to insure that this demand curve, and the firm's best reply function, are negatively sloped. Further note that, because effective funding e_O also depends on funding for the researcher's university project, (6b) shows how spillovers from basic university research can influence the firm's unit consulting fee, and so whether our heroine actually consults.

However, devoting more time to consulting has two conflicting effects on the researcher's expected utility, which are easily seen in (6a). First, for any fee, more time in consulting increases her income, whether either research project succeeds or not, as shown by the second term in. However, the first term in (6a) shows that diverting more time to consulting also decreases her expected utility by decreasing the probability of success in her university research, and thus the probability that she will enjoy the resulting reputational enhancement and license revenue. If the expected loss of utility from diverting any time to consulting is too high, then she will not do so. Otherwise, she increases time in consulting until the marginal gain in expected utility from consulting income is offset by this marginal expected loss in her university research. The slope of her best reply depends upon how changes in the unit consulting fee influence this trade-off between income and reputation.

Theorem 2 *When the researcher's best reply is interior, $\hat{t}(c) \in (0, M)$:*
(i) If she is risk-neutral, then her best reply function for consulting time is decreasing, increasing, or constant in the unit consulting fee if and only if her marginal utility of income increases, decreases, remains the same when her university research succeeds, $\frac{\partial U(R_s, S+\gamma L+ct)}{\partial Y} > (<, =) \frac{\partial U(R_f, S+ct)}{\partial Y}$.
(ii) If she is risk-averse, then her best reply function for consulting time is decreasing if her marginal utility of income does not decrease when her university research succeeds.
(iii) If she is risk-averse, then her best reply function for consulting time is increasing only if her marginal utility of income decreases sufficiently when her university research succeeds.

Proof. When her best reply is interior, its slope is $\frac{\partial \hat{t}(c)}{\partial c} = -(\frac{\partial^2 EU}{\partial t \partial c})/(\frac{\partial^2 EU}{\partial t^2})$, which has the sign of

$$\begin{aligned}\frac{\partial^2 EU}{\partial t \partial c} \quad = \quad & (-\frac{\partial p_I}{\partial \tau})[\frac{\partial U(R_s, S+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f, S+ct)}{\partial Y}]t + \\ & [p_I \frac{\partial^2 U(R_s, S+\gamma L+ct)}{\partial Y^2} + (1-p_I)\frac{\partial^2 U(R_f, S+ct)}{\partial Y^2}]ct,\end{aligned} \tag{8}$$

because expected utility is assumed strictly concave in t. Hence, if she is risk-neutral, so $\frac{\partial^2 U}{\partial Y^2} = 0$, then because $\frac{\partial p_I}{\partial \tau} > 0$, the sign of $\frac{\partial^2 EU}{\partial t \partial c}$ is given by the sign of $-[\frac{\partial U(R_s, S+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f, S+ct)}{\partial Y}]$. Statement (i) follows immediately. Statements (ii) and (iii) then follow from $\frac{\partial p_I}{\partial \tau} > 0$ and the fact that if she is risk-averse, then $\frac{\partial^2 U}{\partial Y^2} < 0$, and the second term in (8) is negative. ■

As a benchmark, suppose our heroine is risk-neutral and her utility is separable, $\frac{\partial^2 U}{\partial R \partial Y} = 0$. Then her best reply is a constant, the time she spends consulting for any fee. If she is risk-neutral but her utility is not separable, then her best reply is negatively (positively) sloped if her marginal utility of income decreases (increases) when her university research succeeds. In this case of risk-neutrality, this means her best reply is negatively (positively) sloped if income and academic reputation are complements (substitutes) in consumption, $\frac{\partial^2 U}{\partial Y \partial R} > 0$ ($\frac{\partial^2 U}{\partial Y \partial R} < 0$). If she is risk-averse, however, then the second term on the right-hand side of (8) is negative, and a sufficient condition for her best reply to be negatively sloped is that income and academic reputation are not substitutes in consumption, $\frac{\partial^2 U}{\partial Y \partial R} \geq 0$. In fact, in this case her best reply is negatively sloped even if income and academic reputation are substitutes in consumption, as long as this effect is not too large. Her best reply is positively sloped only if this substitution effect between income and reputation is large enough to offset the effects of risk-aversion.

The following result eliminates some of the ambiguity regarding the researcher's best reply by clarifying when it is interior.

Theorem 3 *The researcher's best reply function in consulting time is positive only if* $c_m = \min\{c : \frac{\partial EU(G,F,0,c)}{\partial t} = 0\}$ *exists and is finite. If so, then* $c_m > 0$ *and her best reply is positive and increasing,* $\hat{t}(c) > 0$ *and* $\hat{t}'(c) > 0$*, for all in a neighborhood above* c_m*.*

It is not surprising that the researcher does not consult for free. At $c = 0$, her expected marginal utility from consulting time is negative, because diverting time from her university project decreases the probability of success in it, and thus her expected utility, without providing any additional income in return. Therefore, she never consults unless her expected marginal utility is increasing in the consulting fee c for at least some values, so that as c increases her expected marginal utility, and her best reply, eventually become positive.

Therefore, two types of equilibria with consulting may occur. In one, the researcher's best reply is positively sloped when it intersects the firm's best reply, as shown in Figure 1. In the other, her best reply is positively sloped initially, but the effects of risk-aversion rapidly outweigh the substitutability of income and academic reputation as the fee increases, so her best reply reaches a maximum and begins to decline before it intersects the firm's best reply. This is depicted in Figure 2.[15] We analyze the comparative statics of each of these

[15] There is, of course, the possibility that her best reply not only intersects the firm's when it is increasing, but also turns down so sharply that it intersects the firm's again from above. In this case, however, the latter equilibrium is not locally stable, so we do not consider it.

equilibria in turn.

Theorem 4 *If the researcher's best reply function is positively sloped at equilibrium, then:*
(i) An increase in the extent to which her university research spills over into consulting, the research support provided by the firm in its lab, or the difficulty of the firm's research project results in less consulting time and a lower unit consulting fee, $\frac{\partial t^*}{\partial j} < 0$ *and* $\frac{\partial c^*}{\partial j} < 0$ *for* $j = \beta, K_O, x_O$.
(ii) If, in addition, she is risk-averse:
(a) An increase in her quality has an ambiguous effect on consulting time, but results in a higher unit consulting fee, $\frac{\partial c^*}{\partial q} > 0$.
(b) An increase in the fraction of industrial funding that is equivalent to governmental funding, the research funding provided by the university, her salary, license revenue, or her share of it results in less consulting time and a higher unit consulting fee, $\frac{\partial t^*}{\partial j} < 0$ *and* $\frac{\partial c^*}{\partial j} > 0$ *for* $j = \alpha, K_I, S, L, \gamma$.
(c) An increase in governmental or industrial funding within the university results in less consulting time, but has an ambiguous effect on the unit consulting fee, $\frac{\partial t^*}{\partial j} < 0$ *for* $j = G, F$.

This result is easily seen from Figure 1. An increase in β, K_O, or x_O has no effect on the researcher's best reply, but shifts the firm's best reply to the left. The effects of this change are determined completely by the positive slope of her best reply. The firm is willing to pay less per unit of time for her as a consultant, so c^* and t^* both decrease. Conversely, an increase in α, K_I, S, γ, or L has no effect on the firm's best reply, but shifts the researcher's best reply downward. The effects of these changes are determined completely by the negatively sloped firm best reply. She wants to spend less time consulting for any given fee, so t^* decreases and c^* increases. With an increase in q, both best reply functions shift but in opposite directions: the firm's best reply shifts out (to the right), but the researcher's shifts down. That is, the firm is willing to pay more per unit of time, but the researcher is willing to consult less for any given fee, so the fee c^* increases, but the effect on consulting time t^* is ambiguous, depending upon the relative magnitudes of these shifts. The effect of an increase in the scientific merit of her university research has an ambiguous effect on her best reply, and so on the equilibrium consulting time and fee. A more difficult project will enhance her reputation more, if she succeeds, but the probability of success is lower for such a project.

It is important to understand how changes in the levels of governmental and industrial funding chosen in stage one influence the consulting equilibrium in stage two. An increase in either G or F shifts the firm's best reply to the left, and shifts the researcher's best reply down. Therefore, consulting time t^* must decrease, but the effect on the fee c^* is ambiguous, depending upon the relative magnitudes of these shifts. It is most likely that total consulting expenditure by the firm decreases, which is not surprising given the spillovers from university research to the firm's project, which imply that an increase in the funding for university research essentially substitutes for consulting expenditures in the effective funding of the firm's project.

Finally, one aspect of these results that merits further discussion is that some of the effects on her best reply give a clearer indication of whether she consults. For example, an increase in her salary S shifts her best reply downward, making it more likely that she will not divert any time from her university research to consulting.

We now consider the comparative statics results when her best reply is negatively sloped at the equilibrium.

Theorem 5 *If the researcher's best reply function is negatively sloped at equilibrium, then:*
(i) An increase in the extent to which her university research spills over into consulting, the research support provided by the firm in its lab, or the difficulty of the firm's research project results in more consulting time and a lower unit consulting fee, $\frac{\partial t^*}{\partial j} > 0$ *and* $\frac{\partial c^*}{\partial j} < 0$ *for* $j = \beta, K_O, x_O$.
(ii) If, in addition, she is risk-averse, and her marginal utility of income does not increase if her university project succeeds, $\frac{\partial U(R_s, S+\gamma L+c^* t^*)}{\partial Y} \leq \frac{\partial U(R_f, S+c^* t^*)}{\partial Y}$, *then:*
(a) An increase in the quality of the researcher, the fraction of industrial funding that is equivalent to governmental funding, the research funding provided by the university, her salary, license revenue, or her share of it results in less consulting time and a higher unit consulting fee, $\frac{\partial t^*}{\partial j} < 0$ *and* $\frac{\partial c^*}{\partial j} > 0$ *for* $j = q, \alpha, K_I, S, L, \gamma$.
(b) An increase in governmental or industrial funding within the university must either decrease the equilibrium consulting time, decrease the equilibrium consulting fee, or both (for $j = G, F$, *either* $\frac{\partial t^*}{\partial j} < 0$, $\frac{\partial c^*}{\partial j} < 0$, *or both).*

Although the best replies shift in the same directions in this case, the difference is slope of the researcher's best reply causes some differences in the results. An increase in β, K_O, or x_O has no effect on the researcher's best reply, but shifts the firm's best reply to the left, so c^* decreases as before, but the effect on t^* is ambiguous. Similarly, an increase in α, K_I, S, γ, or L has no effect on the firm's best reply, but shifts the researcher's best reply downward. She wants to spend less time consulting for any given fee, so t^* decreases and c^* increases. Again, the efect of a change in x_I is ambiguous.

One important difference is that now an increase in q, which still shifts the firm's best reply to the right and the researcher's down, results in a decrease in consulting time t^* as well as an increase in the fee c^*. It is worth noting that this does not necessarily imply that higher quality researchers consult less, in general. Instead, it implies that, for any given consulting opportunity, a higher quality researcher will command a higher unit fee and spent less time consulting on that project.

Another important difference involves changes in the levels of governmental and industrial funding. An increase in either G or F again shifts the firm's best reply to the left, and the researcher's down, but now the ultimate change in both c^* and t^* depends on the relative magnitude of these shifts. When the best replies satisfy the local stability (relative-slope) condition, as shown in Figure 2, then both equilibrium values cannot increase, or even remain constant. Either c^* or t^* must decrease. If the shifts are of roughly equal magnitude, as might be viewed as typical, then an increase in funding for university research from the agency or the firm decreases both the time our heroine spends consulting and the unit fee she receives for it.

Nevertheless, it is important to note that, although there are some differences depending on the slope of the researcher's best reply, some comparative statics results do hold in either case, thus providing testable implications of the model.

Remark. Whatever the slope of the researcher's best reply function:
(i) An increase in the extent to which her university research spills over into consulting, the research support provided by the firm in its lab, or the difficulty of the firm's research project results in a lower consulting fee, $\frac{\partial c^*}{\partial j} < 0$ *for* $j = \beta, K_O, x_O$.
(ii) An increase in her quality results in a higher fee, $\frac{\partial c^*}{\partial q} > 0$.
(iii) An increase in the fraction of industrial funding that is equivalent to governmental funding, the research funding provided by the university, her salary, license revenue, or her share of it results in less consulting time and a higher unit consulting fee, $\frac{\partial t^*}{\partial j} < 0$ *and* $\frac{\partial c^*}{\partial j} > 0$ *for* $j = \alpha, K_I, S, L, \gamma$.
(iv) In the special case of $\beta = 0$, *an increase in governmental funding results in less consulting time,* $\frac{\partial t^*}{\partial G} < 0$.

Finally, it is also worth noting that this result has implications regarding the effects of the Bayh-Dole Act. Specifically, because this act gave rights from federally funded patents to universities and their researcher-inventors, its passage was equivalent to increase in license revenue L and the researcher's share of it γ. Our analysis shows that, whatever the slope of the researcher's best reply (e.g., consulting supply function), passage of this act had a tendency to reduce the time spent by researchers in consulting and increase their consulting fees. That is, our model predicts that the potential for income from their own university research led them to substitute time in university research for consulting. This seems important, as many have expressed concern that this act could lead to less fundamental research. Nonetheless empirical studies have failed to find such an effect (Azoulay *et al.* 2006, 2007; Thursby and Thursby 2007).

3.2 Stage One Equilibrium

In the first stage, the government funding agency and the firm simultaneously choose funding levels for the researcher's university project. As assumed above, the firm allocates a fixed amount $B_f > 0$ to R&D, and does not make major adjustments until the next budget cycle. Similarly, it is realistic to assume that the research budget of the governmental funding agency is also fixed at the level $B_g > 0$ during this period. To determine subgame perfect equilibria, we assume these funding choices are also made subject to equilibrium behavior in stage two, as detailed in the preceding subsection and embedded in the equilibrium functions $t^*(G,F)$ and $c^*(G,F)$. Substituting these into (3) and (4) gives the "reduced form" payoffs

$$P_g(G,F) = EU_g(G,F,t^*(G,F),c^*(G,F)) \tag{9}$$

and

$$P_f(G,F) = E\Pi(G,F,t^*(G,F),c^*(G,F)). \tag{10}$$

By construction, a Nash equilibrium (G^*,F^*) of the simultaneous-move game with these payoffs is a subgame perfect equilibrium of the two-stage funding game.

Theorem 6 *Consider the strategic form game with the government funding agency and firm as the players, whose strategies are $G \in [0,B_g]$ and $F \in [0,B_f]$, and payoff functions are defined by (9) and (10). Also assume each player's payoff function is continuous and strictly quasi-concave in its own strategy, given any strategy choices by the other players. Then this game has a Nash equilibrium (G^*,F^*), and $(G^*,F^*,t^*(G^*,F^*),c^*(G^*,F^*))$ is the subgame perfect equilibrium of the two-stage funding game.*[16]

Maximization of (9) by choosing $G \in [0,B_g]$ implicitly defines a best reply function $\hat{G}(F)$, giving the level of governmental funding for university research that maximizes the agency's expected utility for any choice of funding F by the firm. Similarly, maximization of (10) by choosing $F \in [0,B_f]$ implicitly defines a best reply function $\hat{F}(G)$, giving the level of industrial funding for university research that maximizes the firm's expected profit for any funding level chosen by the governmental agency.[17] Again, however, because we are interested in deriving testable implications, we focus on the interior equilibrium of this funding game.

If the Nash equilibrium is interior, $G^* \in (0,B_g)$ and $F^* \in (0,B_f)$, then it must satisfy

$$\frac{\partial P_g(G^*,F^*)}{\partial G} = 0, \tag{11a}$$

and

$$\frac{\partial P_f(G^*,F^*)}{\partial F} = 0 \tag{11b}$$

where

$$\frac{\partial P_g(G,F)}{\partial G} = \left(\frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial G}\right)[U_g(R_{gs}) - U_g(R_{gf})] - V'(G) \tag{12a}$$

and

$$\frac{\partial P_f(G,F)}{\partial F} = \left(\frac{\partial p_I}{\partial e_I}\alpha - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial F}\right)(\pi_I - L) + \left(\frac{\partial p_O}{\partial \tau}\frac{\partial t^*}{\partial F}\right)\pi. \tag{12b}$$

[16] Again, these equilibrium values are also functions of all the parameters of the model (α,β,q,x_I,K_I,x_O,K_O,S,L, γ).

[17] We omit the parameters of the model as explicit arguments in these best reply functions for clarity of exposition.

As expected, these conditions show both the initial marginal trade-offs between the benefits and costs of funding, and the effects of initial funding choices on the second stage equilibrium values. Notice the effect of these choices on the equilibrium consulting fee c^* does not directly enter the decision of either the firm or the governmental funding agency. The agency's payoff does not depend on c^*, and firm's second stage optimal choice of c^* eliminates its effect on the first stage funding choice (via a standard envelope theorem application). Examples of this equilibrium is depicted in Figures 3 and 4.

The conditions in ($11a$) and ($12a$) essentially show that increases in governmental funding directly increase effective funding e_I, and thus both the probability of success and expected utility, so the agency increases F until this marginal increase in expected utility from this project is offset by the marginal cost of reduced funding to other projects (embedded in V). Note that $\frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial t^*}{\partial G} > 0$ if $\frac{\partial t^*}{\partial G} < 0$, in which case it follows from ($12a$) that the agency's best reply is interior as long as the opportunity cost of funding our heroine is not too high. The conditions in ($11b$) and ($12b$) show that devoting more funds to our heroines's university research has conflicting effects for the firm. First, it increases the probability of success in university research, and expected licensing profit. However, if $\frac{\partial t^*}{\partial F} < 0$, this reduces time in consulting, and so the probability of success in and expected profit from the firm's project. Nevertheless, in this case, the firm's initial funding of university research does increase expected profit from licensing a university success, so it funds university research also as long as this outweighs the expected profit loss from its project.

Given the general ambiguity of $\frac{\partial t^*}{\partial G}$ and $\frac{\partial t^*}{\partial F}$, it is difficult to obtain unambiguous comparative statics results on equilibrium levels of funding for university research. Indeed, the signs of the slopes of the best reply funding functions are not obvious. Nevertheless, we can obtain results under some reasonable assumptions. First, because $\frac{\partial t^*}{\partial G}$ and $\frac{\partial t^*}{\partial F}$ are negative when the researcher's best reply is positively sloped, and because at least one of them must be negative when this best is negatively sloped, it is natural to assume $\frac{\partial t^*}{\partial G} < 0$ and $\frac{\partial t^*}{\partial F} < 0$. Next, the assumptions

$$-\frac{\partial^2 p_I}{\partial \tau^2}\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_I}{\partial \tau \partial e_I} < 0 \ for\ j = G, F, \tag{13a}$$

$$\frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial G} < 0, \tag{13b}$$

and

$$\alpha\frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial \tau \partial e_I}\frac{\partial t^*}{\partial F} < 0 \tag{13c}$$

essentially state that own-effects outweigh cross-effects for changes in the levels of funding for university research on the equilibrium probabilities of success. That is, (13) implies that an increase in the level of governmental (firm) funding cannot increase the equilibrium marginal probability of success in university research.

Theorem 7 *Assume that equilibrium consulting time is decreasing in both external funding levels,* $\frac{\partial t^*}{\partial G} < 0$ *and* $\frac{\partial t^*}{\partial F} < 0$*, that* (13) *holds, and that second-order effects on equilibrium consulting times are negligible,* $\frac{\partial^2 t^*}{\partial i \partial j} \approx 0$ *for all parameters* i *and* j*. Then:*
(i) The first-stage best reply function of the funding agency is negatively sloped.
(ii) The first-stage best reply function of the firm is negatively sloped if, in addition, an increase in governmental funding does not decrease the equilibrium effective funding for the firm's project, $\frac{\partial e_O^*}{\partial G} \geq 0$*.*

Under these circumstances, we can also identify some comparative statics results for the first stage.

Theorem 8 *Under the hypotheses of Theorem 7, if the first-stage equilibrium is locally stable, then:*
(i) The effect of a change in the extent to which her university research spills over into consulting is generally ambiguous, although an increase in spillovers must either increase equilibrium university research funding from the government agency, increase equilibrium university research funding from the firm, or both (either $\frac{\partial G^*}{\partial \beta} > 0$*,* $\frac{\partial F^*}{\partial \beta} > 0$*, or both) if, in addition, this does not increase equilibrium effective funding for the firm's project,* $\frac{\partial e_O^*}{\partial \beta} \leq 0$*.*
(ii) The effect of a change in research funding within the university, her salary, license revenue, or her share of it are also ambiguous, although an increase in any of these must either decrease equilibrium university research funding from the government agency, decrease equilibrium university research funding from the firm, or both (either $\frac{\partial G^*}{\partial j} < 0$*,* $\frac{\partial F^*}{\partial j} < 0$*, or both for* $j = K_I, S, L, \gamma$*); and*
(iii) An increase in research support provided by the firm in its lab decreases equilibrium university research funding from the government agency and increases equilibrium university research funding from the firm, $\frac{\partial G^*}{\partial j} < 0$ *and* $\frac{\partial F^*}{\partial j} > 0$*, if, in addition, this does not increase equilibrium effective funding for the firm's project,* $\frac{\partial e_O^*}{\partial K_O} \leq 0$*.*

When the best reply functions are negatively sloped, as depicted in Figures 3 and 4, we can draw interesting conclusions. First, an increase in spillovers to consulting shifts the agency's best reply up and the firm's best reply to the right, which must increase funding for her university research from either the government agency or firm, if not both. In "typical" situations we expect both will increase. Similarly, an increase in the level of university research support, her salary, license revenue, or her share of it shifts the agency's best reply down and the firm's best reply to the left, which must decrease funding for her university research from either the government agency or firm, if not both. Finally, all other changes are ambiguous in this case.

Finally, because the quality of the researcher and the difficulty of university research are important factors, we note the following limited results.

Corollary 9 *If the effects of first stage changes on second stage equilibrium values are sufficiently small, and the equilibrium is locally stable, then:*

(i) An increase in the quality of the researcher must either increase equilibrium university research funding from the government agency, increase equilibrium university research funding from the firm, or both (either $\frac{\partial G^*}{\partial q} > 0$*,* $\frac{\partial F^*}{\partial q} > 0$*, or both).*
(ii) An increase in the difficulty of the university research project must either decrease equilibrium university research funding from the government agency, decrease equilibrium university research funding from the firm, or both (either $\frac{\partial G^*}{\partial x_I} < 0$*,* $\frac{\partial F^*}{\partial x_I} < 0$*, or both).*

We emphasize caution in interpreting this result, because it is derived by minimizing the effects of changes in quality on the second stage equilibrium time spent consulting. Nevertheless, we do find this instructive, because it focuses on the "short-run" effects of changes on the marginal probability of success in the university research project. Because research quality and effective funding are "complements" in production, $\frac{\partial^2 p_I}{\partial e_I \partial q} > 0$, higher quality researchers are more likely to receive higher levels of external funding from either the government agency or industry, ceteris paribus. That is, both best reply functions shift outward, so the researcher definitely receives more funding. The ambiguity in the general case results from the fact that changes in quality and external funding have conflicting effects on time spent in consulting. Similarly, because research difficulty reduces the marginal productivity of all inputs, $\frac{\partial^2 p_I}{\partial e_I \partial x_I} > 0$, researcher pursuing more difficult projects are less likely to receive higher levels of external funding from either the government agency or industry, ceteris paribus. Both best reply functions shift inward, so the researcher definitely receives less funding. The ambiguity in this case stems from the ambiguity of the sign of $\frac{\partial^2 t^*}{\partial x_I}$.

3.3 Extensions

As noted in the introduction to this section, the game structure used above conforms well to the stylized fact that faculty typically prefer their own research, and therefore focus obtaining funds for it before making any consulting agreement. It is also often the case that faculty prefer governmental funding to industrial funding because the former has fewer ties on its use. We explicitly incorporated this notion in the definition of effective funding for university research. Thus, one might wonder whether it is more reasonable to consider a game structure in which our heroine seeks support from the governmental agency first, then (possibly) seeks support from the firm after learning how much the agency provides, followed again by the consulting game. We extend the analysis in this subsection to consider this sequence of events (assuming the researcher's best reply is negatively sloped). Because there is no change in the final stage, we can focus on the first stage equilibrium. However, rather than developing the formal machinery, we shall do this diagrammatically.

In this sequential move game, the governmental agency essentially acts as a Stackelberg leader, choosing the point on the firm's reaction function that gives it the greatest expected utility. Because the agency's payoff is increasing in the level of funding provided by the firm, and funding levels are strategic substitutes, it takes advantage of its leadership position to provide less funding (than when they move simultaneously), which induces the firm to provide more funding. That is, the equilibrium in this game is a point on the firm's best reply "southeast" of where it intersects the researcher's best reply (see Figure 4). Comparative statics results for this stage essentially follow from Theorem 7, because parametric changes shift the best replies as before, and we know the new equilibrium for this game will be southeast of the new simultaneous-move equilibrium point on the firm's best reply. In fact, the results of Theorem 7 hold for changes research funding within the university, and the fraction of industrial funding that is equivalent to governmental funding. Interestingly, the results for changes in the extent to which her university research spills over into consulting, the research support provided by the firm in its lab, or the difficulty of the firm's research project are now ambiguous. To see this, recall Figure 4, and note that an increase in any of these parameters shifts the firm's best reply to the left. Because the new equilibrium can be anywhere on the new best reply to the southeast of where it intersects the agency's best reply, both F^* and G^* can either increase or decrease (depending upon the shape of the iso-expected-utility curves for the agency).

4 Econometric Analysis

Thursby *et al* (2006) compare the names of over 34,000 science and engineering faculty at the 87 US Research I universities in 1993 with patents issued in various years from 1993 to 2004. After a series of filters to ensure that they had correctly matched faculty with patents they then examined the patent assignments. The filters eliminated common names and patent inventors who did not live close to the university listing the name as a faculty member. In addition, for patents applied for in years other than 1993 they checked that the faculty member was still employed at the university. In a sample of 5772 patents with university faculty as inventors 26% were assigned to (and therefore owned by) for-profit firms. Roughly 65% were assigned to not-for-profit entities (typically the inventor's university), and the remainder were either unassigned (therefore owned by the inventor) or were assigned to both for-profit and not-for-profit entities. In interviews, both university technology transfer professionals and industry R&D executives claimed that patents assigned to firms are typically the result of faculty/industry consulting arrangements.

Here we use assignment to firms as an indicator of consulting. Clearly, this is not a complete picture of consulting activity since it involves only inventions that are both patentable and patented. We restrict our analysis to the subset of faculty who were employed at Purdue, MIT, Stanford, Wisconsin, Georgia Tech, Cornell, Pennsylvania and Texas A&M universities from 1993 to 1999. For these faculty we have detailed information on publications, citations, and research funding over this period. While we have detailed information for all faculty for all years up to 1999, that information is not used unless a faculty member is known to have applied for a patent. Patents are restricted to those applied for starting in 1993 and granted by 1999. This yields 1767 patent/inventor pairs. The assignment of these pairs is found in Table 1. In what follows we only consider the 1687 pairs that are assigned either to one or more not-for-profits or to one or more firms. These 1687 pairs include 1527 patents and 600 faculty inventors. The distribution by application year is in Table 2.

We use a logit regression to explain the probability that a patent is assigned to a not-for-profit, rather than a firm, $P(Y_i = 1)$, where i refers to a patent/inventor pair.

According to our model consulting (and hence patent assignment) should be a function of the researchers quality and first stage federal and industry sponsored research funding. For quality we use the number of publications by the faculty member in the year of the patent application (PUBS) as well as the total number of citations to those publications received through 2003 (CITES). We have available for each faculty member the total US government sponsored research funds (FEDFND), industry sponsored research funds (INDFND) and sponsored research funds from other sources (OTHERFND) received in the year of the patent application. All funding is in millions of dollars. Demographic characteristics are the age of the inventor in the year of application (AGE) and their gender (MALE recorded as a 1 if the inventor is male). We also include university, year of application and field fixed effects. Fields are the major program areas engineering, physical sciences and biological sciences. Robust standard errors are reported. Summary statistics are in Table 3.

Results are in the first three columns of Table 4 and are presented as odds ratios. An odds ratio gives the effect of a unit change in a right hand side variable on the ratio of the probability of an assignment to a non/-profit divided by one minus that probability. Hence, an odds ratio of less than one implies that the right hand side variable has a negative effect on the probability of assignment to a non-profit.

Before discussing the results we consider a number of robustness checks. First, we drop the insignificant MALE, AGE and OTHERFND. Results are in columns 4-6 of Table 4. Note that the number of observations increases; this is to the fact that we do not have OTHERFND for Wisconsin. In addition, we do not have AGE and MALE for some observations. Results for the quality and funding variables do not change in any meaningful way. We then dropped all fixed effects. Results are in Table 4. CITES is no longer significant, but PUBS and the funding variables have similar results to those in Table 4. We have for each faculty member's publication the expected number of CITES based on the journal where the article appears. In Table 4 are results when we replace CITES with CITES minus the expected number of CITES (CITES-EXPECT). Again, results do not change. Our dependent variable is a patent/inventor pair. In a number of cases there are several faculty inventors on the same patent. When there are several faculty on the same patent we randomly dropped patent/inventor pairs so that a patent appears only once in the sample. The results, presented in Table 4, are similar to those in Table 4. Finally, we included indicator variables for the technology subcategory of which there are 26. As shown in Table 4 the results do not change.

Regardless of the slope of the researcher's best reply function, Theorems 4 and 5 predict that an increase in research quality will decrease consulting since an increase in quality shifts the researcher's best reply down and the firm's outward. For both of our measures of quality, the results support this theoretical prediction: higher quality faculty, as measured by publications and citations, are more likely to assign to the university.

Theorems 4 and 5 also predict that both best replies shift with an increase in either government or industry funding. In the case of government funding, the impact on consulting depends on the existence of spillovers. When $\beta > 0$, an increase in government funding shifts the researcher's best reply down and the firm's to the left, hence the ambigous theoretical results. When $\beta = 0$, only the researcher's best reply shifts with an increase in government funding, implying a decrease in consulting. In the econometric analysis, we find that an increase in government funding leads to an increase in consulting which suggests there are indeed direct spillovers from government funding to the firm's problem.

It is tempting to argue that federal funding is a signal to firms of inventor quality. That is, the peer review processes followed by federal agencies identifies, to some extent, the best researchers in a given field of inquiry. Thus federal funding is a signal of quality, hence, as one might argue, those with substantial amounts of federal funding are sought after to be consultants. However, higher quality faculty leads to less consulting in equilibrium. This is consistent with our empirical result that publications and citations are negatively related to firm assignment, but not the empirical result for federal funding which is the opposite.

5 Concluding Remarks

Several qualifiers to our work suggest directions for further research. First, one fourth of the patents in the sample assigned to for-profit firms are assignments to firms in which the inventor is a principal (founder, CEO, and/or scientific advisor). A role as scientific advisor is consistent with our interpretation of the faculty researcher choosing $t > 0$ and is consistent with most university policies as long at $t \leq M$. The patent may or may not be a follow-on patent to one from the faculty researcher's university research, in which case we would interpret the follow on project as x_o. Moreover, most conflict of interest policies prohibit faculty from receiving sponsored research from their start ups, so that this example would be the special case of our model in which $F = 0$. Of course, we do not differentiate between start ups and other types of firms in the analysis so we abstract from many of the nuances of faculty start ups.

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7 Appendix

I. Proof of Theorem 1.

Because the number of players is finite, their strategy sets are compact and nonempty, and their payoff functions are continuous and strictly quasi-concave, this follows directly from the well-known existence theorem for strategic form games with continuous strategy spaces (see, for example, Friedman 1977).

II. Proof of Theorem 3.

First observe from (6a) that $\frac{\partial EU(G,F,t,0)}{\partial t} = -\frac{\partial p_I}{\partial \tau}[U(R_s, S+\gamma L) - U(R_f, S)] < 0$ for all t because $R_s > R_f$, $\gamma L > 0$, and positive marginal utility imply that $U(R_s, S+\gamma L) > U(R_f, S)$, and $\frac{\partial p_I}{\partial \tau} > 0$. Hence, because t is constrained to be nonnegative, $\hat{t}(0) = 0$. That is, if we plotted $\frac{\partial EU(G,F,t,c)}{\partial t}$ as a function of c for fixed (G, F, t), then it would intersect the (vertical) utility axis at a negative value. Because $\frac{\partial^2 EU(G,F,t,c)}{\partial t^2} < 0$, if $\frac{\partial EU(G,F,0,c)}{\partial t} < 0$ for all $c \in [0, B_f/M]$, then $\frac{\partial EU(G,F,t,c)}{\partial t} < 0$ for all c, and consulting never occurs, $\hat{t}(c) = 0$ for all c. However, the slope of $\frac{\partial EU}{\partial t}$ with respect to c at $c = 0$ is $\frac{\partial^2 EU(G,F,t,0)}{\partial t \partial c} = (-\frac{\partial p_I}{\partial \tau})[\frac{\partial U(R_s,S+\gamma L)}{\partial Y} - \frac{\partial U(R_f,S)}{\partial Y}]t > 0$ if $\frac{\partial U(R_s,S+\gamma L)}{\partial Y} < \frac{\partial U(R_f,S)}{\partial Y}$. Thus, it is possible that $\frac{\partial EU(G,F,t,c)}{\partial t}$ increases (though perhaps not monotonically) as c increases, and eventually intersects the (horizontal) c axis. If so, there exists a positive, finite c_m defined as above. By continuity, $\frac{\partial^2 EU(G,F,0,c_m)}{\partial t \partial c} > 0$. Note that c_m is the fee at which the function $EU(G,F,t,c)$ takes on its unconstrained maximum at $t = 0$ (or the smallest fee if this occurs for more than one value). Therefore, for all fees in a neighborhood above c_m, $EU(G,F,t,c)$ takes on its unconstrained maximum at some $t > 0$, so $\hat{t}(c) > 0$. Moreover, $\hat{t}'(c) > 0$ in this neighborhood from the proof of Theorem 2 because $\frac{\partial^2 EU(G,F,0,c_m)}{\partial t \partial c} > 0$.

IV. Proof of Theorems 4 and 5.

Using standard comparative statics, $\frac{\partial t^*}{\partial j} = [\frac{\partial^2 EU}{\partial t \partial c}\frac{\partial^2 E\Pi}{\partial c \partial j} - \frac{\partial^2 E\Pi}{\partial c^2}\frac{\partial^2 EU}{\partial t \partial j}]/D_2$ and $\frac{\partial c^*}{\partial j} = [\frac{\partial^2 E\Pi}{\partial c \partial t}\frac{\partial^2 EU}{\partial t \partial j} - \frac{\partial^2 EU}{\partial t^2}\frac{\partial^2 E\Pi}{\partial c \partial j}]/D_2$, for $j = G,F,\alpha,\beta,q,x_I, K_I,x_O,K_O,S,L,\gamma$ where $D_2 = \frac{\partial^2 EU}{\partial t^2}\frac{\partial^2 E\Pi}{\partial c^2} - \frac{\partial^2 EU}{\partial t \partial c}\frac{\partial^2 E\Pi}{\partial c \partial t} > 0$ by the assumption that the equilibrium is locally stable. Differentiation yields $\frac{\partial^2 EU}{\partial t \partial G} = (-\frac{\partial^2 p_I}{\partial \tau \partial e_I})[U(R_s, S + \gamma L + ct) - U(R_f, S + ct)] + \frac{\partial p_I}{\partial e_I}[\frac{\partial U(R_s,S+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,S+ct)}{\partial Y}]c$, $\frac{\partial^2 EU}{\partial t \partial F} = \alpha \frac{\partial^2 EU}{\partial t \partial G}$, $\frac{\partial^2 EU}{\partial t \partial \alpha} = F\frac{\partial^2 EU}{\partial t \partial G}$, $\frac{\partial^2 EU}{\partial t \partial K_I} = \frac{\partial^2 EU}{\partial t \partial G}$, $\frac{\partial^2 EU}{\partial t \partial q} = (-\frac{\partial^2 p_I}{\partial \tau \partial q})[U(R_s, S+\gamma L+ct) - U(R_f, S+ct)] + \frac{\partial p_I}{\partial q}[\frac{\partial U(R_s,S+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,S+ct)}{\partial Y}]c$, $\frac{\partial^2 EU}{\partial t \partial x_I} = (-\frac{\partial^2 p_I}{\partial \tau \partial x_I})[U(R_s, S+\gamma L+ct) - U(R_f, S + ct)] + \frac{\partial p_I}{\partial x_I}[\frac{\partial U(R_s,S+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,S+ct)}{\partial Y}]c - \frac{\partial p_I}{\partial \tau}[\frac{\partial U(R_s,S+\gamma L+ct)}{\partial R}]R_s' + p_I[\frac{\partial^2 U(R_s,S+\gamma L+ct)}{\partial Y \partial R}]R_s'$, $\frac{\partial^2 EU}{\partial t \partial S} = (-\frac{\partial p_I}{\partial \tau})[\frac{\partial U(R_s,S+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,S+ct)}{\partial Y}] + [p_I\frac{\partial^2 U(R_s,S+\gamma L+ct)}{\partial Y^2} + (1-p_I)\frac{\partial^2 U(R_s,S+\gamma L+ct)}{\partial Y^2}]c = \frac{\partial^2 EU}{\partial t \partial c}/t$, and $\frac{\partial^2 EU}{\partial t \partial L} = (\frac{L}{\gamma})\frac{\partial^2 EU}{\partial t \partial \gamma} = [(-\frac{\partial p_I}{\partial \tau})\frac{\partial U(R_s,S+\gamma L+ct)}{\partial Y} + p_I\frac{\partial^2 U(R_s,S+\gamma L+ct)}{\partial Y^2}c]\gamma$, all of which are generally ambiguous in sign, whereas $\frac{\partial^2 EU}{\partial t \partial \beta} = \frac{\partial^2 EU}{\partial t \partial x_O} = \frac{\partial^2 EU}{\partial t \partial K_O} = 0$. Next note that $\frac{\partial^2 E\Pi}{\partial c \partial t} = [\frac{\partial^2 p_O}{\partial e_O \partial \tau} + \frac{\partial^2 p_O}{\partial e_O^2}(c)](t\pi) + (\frac{\partial p_O}{\partial e_O}\pi - 1) < 0$ at an interior solution to the firm's problem, $\frac{\partial^2 E\Pi}{\partial c \partial G} = \frac{\partial^2 p_O}{\partial e_O^2}\beta t\pi < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial F} = \frac{\partial^2 p_O}{\partial e_O^2}t\pi < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \alpha} = \frac{\partial^2 E\Pi}{\partial c \partial x_I} = \frac{\partial^2 E\Pi}{\partial c \partial K_I} = \frac{\partial^2 E\Pi}{\partial t \partial S} = \frac{\partial^2 EU}{\partial c \partial L} = 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \beta} = \frac{\partial^2 p_O}{\partial e_O^2}Gt\pi < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial q} = \frac{\partial^2 p_O}{\partial e_O \partial q}t\pi > 0$, $\frac{\partial^2 E\Pi}{\partial c \partial x_O} = \frac{\partial^2 p_O}{\partial e_O \partial x_O}t\pi < 0$, and $\frac{\partial^2 E\Pi}{\partial c \partial K_O} = \frac{\partial^2 p_O}{\partial e_O^2}t\pi < 0$.

From the proof of the Theorem 2, her best reply is positively sloped if and only if $\frac{\partial^2 EU}{\partial t \partial c} > 0$. Theorem 4(i) then follows from the definitions of $\frac{\partial t^*}{\partial j}$ and $\frac{\partial c^*}{\partial j}$ and $\frac{\partial^2 EU}{\partial t \partial \beta} = \frac{\partial^2 EU}{\partial t \partial x_O} = \frac{\partial^2 EU}{\partial t \partial K_O} = 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \beta} < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial x_O} < 0$, and $\frac{\partial^2 E\Pi}{\partial c \partial K_O} < 0$. As also shown in the proof of Theorem 2, if she is risk-averse, then her best reply is positively sloped only if $\frac{\partial U(R_s,S+\gamma L+ct)}{\partial Y} < \frac{\partial U(R_f,S+ct)}{\partial Y}$. From the expressions above, this implies that $\frac{\partial^2 EU}{\partial t \partial j} < 0$ for $j = G,F,\alpha,q,K_I,S,L,\gamma$, though the sign of $\frac{\partial^2 EU}{\partial t \partial x_I}$ is ambiguous. Theorem 4(ii) then follows from this, plus the signs of $\frac{\partial^2 E\Pi}{\partial c \partial j}$ above, and the definitions of $\frac{\partial t^*}{\partial j}$ and $\frac{\partial c^*}{\partial j}$.

Conversely, her best reply is negatively sloped if and only if $\frac{\partial^2 EU}{\partial t \partial c} < 0$. Theorem 5(i) then follows from the definitions of $\frac{\partial t^*}{\partial j}$ and $\frac{\partial c^*}{\partial j}$ and $\frac{\partial^2 EU}{\partial t \partial \beta} = \frac{\partial^2 EU}{\partial t \partial x_O} = \frac{\partial^2 EU}{\partial t \partial K_O} = 0$, $\frac{\partial^2 E\Pi}{\partial c \partial \beta} < 0$, $\frac{\partial^2 E\Pi}{\partial c \partial x_O} < 0$, and $\frac{\partial^2 E\Pi}{\partial c \partial K_O} < 0$. If, in addition, she is risk-averse and $\frac{\partial U(R_s,S+\gamma L+ct)}{\partial Y} \leq \frac{\partial U(R_f,S+ct)}{\partial Y}$, then $\frac{\partial^2 EU}{\partial t \partial G} = (-\frac{\partial^2 p_I}{\partial \tau \partial e_I})[U(R_s, S+\gamma L + ct) - U(R_f, S + ct)] + \frac{\partial p_I}{\partial e_I}[\frac{\partial U(R_s,S+\gamma L+ct)}{\partial Y} - \frac{\partial U(R_f,S+ct)}{\partial Y}]c < 0$ because

$\frac{\partial^2 p_I}{\partial\tau\partial e_I} > 0$, $U(R_s, S{+}\gamma L{+}ct) > U(R_f, S{+}ct)$, and $\frac{\partial^2 p_I}{\partial e_I} > 0$. Similarly, $\frac{\partial^2 EU}{\partial t\partial j} < 0$ for $j = F,\alpha,q,K_I,S,L,\gamma$, though $\frac{\partial^2 EU}{\partial t\partial x_I}$ remains ambiguous. Theorem5(ii)(a) then follows from the definitions of $\frac{\partial t^*}{\partial j}$ and $\frac{\partial c^*}{\partial j}$ and these facts.

Finally, because $\frac{\partial^2 EU}{\partial t\partial j} < 0$ and $\frac{\partial^2 E\Pi}{\partial c\partial j} < 0$, $\frac{\partial t^*}{\partial j} + \frac{\partial c^*}{\partial j} = \{\frac{\partial^2 EU}{\partial t\partial j}[\frac{\partial^2 E\Pi}{\partial c\partial t} - \frac{\partial^2 E\Pi}{\partial c^2}] + [\frac{\partial^2 EU}{\partial t\partial c} - \frac{\partial^2 EU}{\partial t^2}]\frac{\partial^2 E\Pi}{\partial c\partial j}\}/D_2 > 0$ if and only if both $\frac{\partial^2 E\Pi}{\partial c\partial t} - \frac{\partial^2 E\Pi}{\partial c^2} < 0$ and $\frac{\partial^2 EU}{\partial t\partial c} - \frac{\partial^2 EU}{\partial t^2} < 0$, which contradicts the local stability condition. This proves Theroem 5(ii)(b).

VI. The proof of Theorem 6 is analogous to that of Theorem 1.

VII. Proof of Theorem 7.

For notational convenience, set $A = -\frac{\partial^2 p_I}{\partial\tau^2}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial\tau\partial e_I}$, $B = \frac{\partial^2 p_I}{\partial e_I^2} - \frac{\partial^2 p_I}{\partial\tau\partial e_I}\frac{\partial t^*}{\partial G}$, $C = -\frac{\partial^2 p_I}{\partial\tau^2}\frac{\partial t^*}{\partial F} + \frac{\partial^2 p_I}{\partial\tau\partial e_I}\alpha$, $D = \frac{\partial^2 p_I}{\partial e_I^2}\alpha - \frac{\partial^2 p_I}{\partial\tau\partial e_I}\frac{\partial t^*}{\partial F}$, and $\Delta U_g = U_g(R_{gs}) - U_g(R_{gf}) > 0$. Recall $A < 0$, $B < 0$, and $D < 0$ by assumption, and note that $-\frac{\partial^2 p_I}{\partial\tau^2}\frac{\partial t^*}{\partial F} + \frac{\partial^2 p_I}{\partial\tau\partial e_I} < 0$ implies $C < 0$ because $\alpha \in [0,1]$ and $\frac{\partial^2 p_I}{\partial\tau\partial e_I} > 0$. Then $\frac{\partial^2 P_g}{\partial G\partial F} = [A(-\frac{\partial t^*}{\partial F}) + \frac{\partial p_I}{\partial\tau}(-\frac{\partial^2 t^*}{\partial G\partial F}) + B\alpha]\Delta U_g < 0$ because $\frac{\partial t^*}{\partial F} < 0$ and $\frac{\partial^2 t^*}{\partial F\partial G} \approx 0$ by assumption. Similarly,. $\frac{\partial^2 P_f}{\partial F\partial G} = [C(-\frac{\partial t^*}{\partial G}) - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial G\partial F} + D](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial\tau^2}(\frac{\partial t^*}{\partial G}) + \frac{\partial^2 p_O}{\partial\tau\partial e_O}(\frac{\partial e_O^*}{\partial G})]\frac{\partial t^*}{\partial F} + \frac{\partial^2 p_O}{\partial\tau\partial e_O}\frac{\partial^2 t^*}{\partial G\partial F}\}\pi < 0$ because $\frac{\partial t^*}{\partial G} < 0$, $\frac{\partial t^*}{\partial F} < 0$, $\frac{\partial^2 t^*}{\partial F\partial G} \approx 0$, and $\frac{\partial e_O^*}{\partial G} > 0$ by assumption.

VIII. Proof of Theorem 8 and Corollary 9.

In this case, we have $\frac{\partial G^*}{\partial j} = [\frac{\partial^2 P_g}{\partial G\partial F}\frac{\partial^2 P_f}{\partial F\partial j} - \frac{\partial^2 P_f}{\partial F^2}\frac{\partial^2 P_g}{\partial G\partial j}]/D_1$ and $\frac{\partial F^*}{\partial j} = [\frac{\partial^2 P_f}{\partial F\partial G}\frac{\partial^2 P_g}{\partial G\partial j} - \frac{\partial^2 P_g}{\partial G^2}\frac{\partial^2 P_f}{\partial F\partial j}]/D_1$, for $j = \alpha,\beta,q,x_I,K_I,x_O,K_O,\gamma,S,L$, where $D_1 = \frac{\partial^2 P_g}{\partial G^2}\frac{\partial^2 P_f}{\partial F^2} - \frac{\partial^2 P_g}{\partial G\partial F}\frac{\partial^2 P_f}{\partial F\partial G} > 0$ by the assumption that the equilibrium is locally stable, and where $\frac{\partial^2 P_g}{\partial G\partial F} < 0$ and $\frac{\partial^2 P_f}{\partial F\partial G} < 0$ from the preceding theorem. First, observe that $\frac{\partial^2 P_g}{\partial G\partial j} = [A(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial G\partial j}]\Delta U_g > 0$ for $j = \gamma,S,L$ because $\frac{\partial t^*}{\partial j} < 0$ for $j = \gamma,S,L$ and $\frac{\partial^2 t^*}{\partial i\partial j} \approx 0$, $\frac{\partial^2 P_g}{\partial G\partial j} = [A(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial G\partial j}]\Delta U_g < 0$ for $j = \beta,x_O,K_O$ because $\frac{\partial t^*}{\partial j} > 0$ for $j = \beta,x_O,K_O$, $\frac{\partial^2 P_g}{\partial G\partial j} = [A(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial G\partial j} + B]\Delta U_g < 0$ for $j = \alpha,K_I$ because $\frac{\partial t^*}{\partial j} < 0$ for $j = \alpha,K_I$, but $\frac{\partial^2 P_g}{\partial G\partial q} = [A(-\frac{\partial t^*}{\partial q}) - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial G\partial q} - \frac{\partial^2 p_I}{\partial\tau\partial q}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial e_I\partial q}]\Delta U_g$ and $\frac{\partial^2 P_g}{\partial G\partial x_I} = [A(-\frac{\partial t^*}{\partial x_I}) - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial G\partial x_I} - \frac{\partial^2 p_I}{\partial\tau\partial x_I}\frac{\partial t^*}{\partial G} + \frac{\partial^2 p_I}{\partial e_I\partial x_I}]\Delta U_g + AU_g' R_{sg}'$ are ambiguous. Similarly, $\frac{\partial^2 P_f}{\partial F\partial j} = [C(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial F\partial j}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial\tau^2} + \frac{\partial^2 p_O}{\partial\tau\partial e_O}c^*]\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_O}{\partial\tau\partial e_O}\frac{\partial c^*}{\partial j}\}\frac{\partial t^*}{\partial F}\pi + \frac{\partial p_O}{\partial\tau}\frac{\partial^2 t^*}{\partial F\partial j}\pi < 0$ for $j = \gamma,S,L$, because $\frac{\partial^2 p_O}{\partial\tau^2} + \frac{\partial^2 p_O}{\partial\tau\partial e_O}c^* < 0$ by (7) and $\frac{\partial t^*}{\partial j} < 0 < \frac{\partial c^*}{\partial j}$ for these j. Next note that $\frac{\partial^2 P_f}{\partial F\partial j} = [C(-\frac{\partial t^*}{\partial j}) - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial F\partial j}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial\tau^2}\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_O}{\partial\tau\partial e_O}\frac{\partial e_O^*}{\partial j}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial\tau}\frac{\partial^2 t^*}{\partial F\partial j}\}\pi < 0$ for $j = \beta, K_O$ because $\frac{\partial t^*}{\partial j} > 0 > \frac{\partial c^*}{\partial j}$ and $\frac{\partial e_O^*}{\partial j} \le 0$ is assumed for these j. Also, $\frac{\partial^2 P_f}{\partial F\partial K_I} = [C(-\frac{\partial t^*}{\partial K_I}) + D - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial F\partial K_I}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial\tau^2}\frac{\partial t^*}{\partial j} + \frac{\partial^2 p_O}{\partial\tau\partial e_O}\frac{\partial e_O^*}{\partial K_I}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial\tau}\frac{\partial^2 t^*}{\partial F\partial j}\}\pi < 0$. However, $\frac{\partial^2 P_f}{\partial F\partial q} = [C(-\frac{\partial t^*}{\partial q}) + \frac{\partial^2 p_I}{\partial e_I\partial q}\alpha - \frac{\partial^2 p_I}{\partial\tau\partial q}\frac{\partial t^*}{\partial F} - \frac{\partial p_I}{\partial\tau}\frac{\partial^2 t^*}{\partial F\partial q}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial\tau^2}\frac{\partial t^*}{\partial q} + \frac{\partial^2 p_O}{\partial\tau\partial e_O}\frac{\partial e_O^*}{\partial q} + \frac{\partial^2 p_O}{\partial\tau\partial q}]\frac{\partial t^*}{\partial F} +$

$\frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial q}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial x_I} = [C(-\frac{\partial t^*}{\partial x_I}) + \frac{\partial^2 p_I}{\partial e_I \partial x_I}\alpha - \frac{\partial^2 p_I}{\partial \tau \partial x_I}\frac{\partial t^*}{\partial F} - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_I}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial x_I} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial x_I}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_I}\}\pi$, $\frac{\partial^2 P_f}{\partial F \partial \alpha} = [C(-\frac{\partial t^*}{\partial \alpha}) + DF + \frac{\partial p_I}{\partial e_I} - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial \alpha}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial \alpha} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial \alpha}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial \alpha}\}\pi$, and $\frac{\partial^2 P_f}{\partial F \partial x_O} = [C(-\frac{\partial t^*}{\partial x_O}) - \frac{\partial p_I}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_O}](\pi_I - L) + \{[\frac{\partial^2 p_O}{\partial \tau^2}\frac{\partial t^*}{\partial x_O} + \frac{\partial^2 p_O}{\partial \tau \partial e_O}\frac{\partial e_O^*}{\partial x_O} + \frac{\partial^2 p_O}{\partial \tau \partial x_O}]\frac{\partial t^*}{\partial F} + \frac{\partial p_O}{\partial \tau}\frac{\partial^2 t^*}{\partial F \partial x_O}\}\pi$ are all ambiguous. The statements of the theorem then follow immediately from these results plus locally stability.

From the preceding, ignoring second stage effects, we have $\frac{\partial^2 P_g}{\partial G \partial q} = \frac{\partial^2 p_I}{\partial e_I \partial q}\Delta U_g > 0$ and $\frac{\partial^2 P_f}{\partial F \partial q} = \frac{\partial^2 p_I}{\partial e_I \partial q}\alpha(\pi_I - L) > 0$, so both best replies shift outward. However, $\frac{\partial^2 P_g}{\partial G \partial x_I} = \frac{\partial^2 p_I}{\partial e_I \partial x_I}\Delta U_g < 0$ and $\frac{\partial^2 P_f}{\partial F \partial x_I} = \frac{\partial^2 p_I}{\partial e_I \partial x_I}\alpha(\pi_I - L) < 0$, so in this case both best replies shift inward.

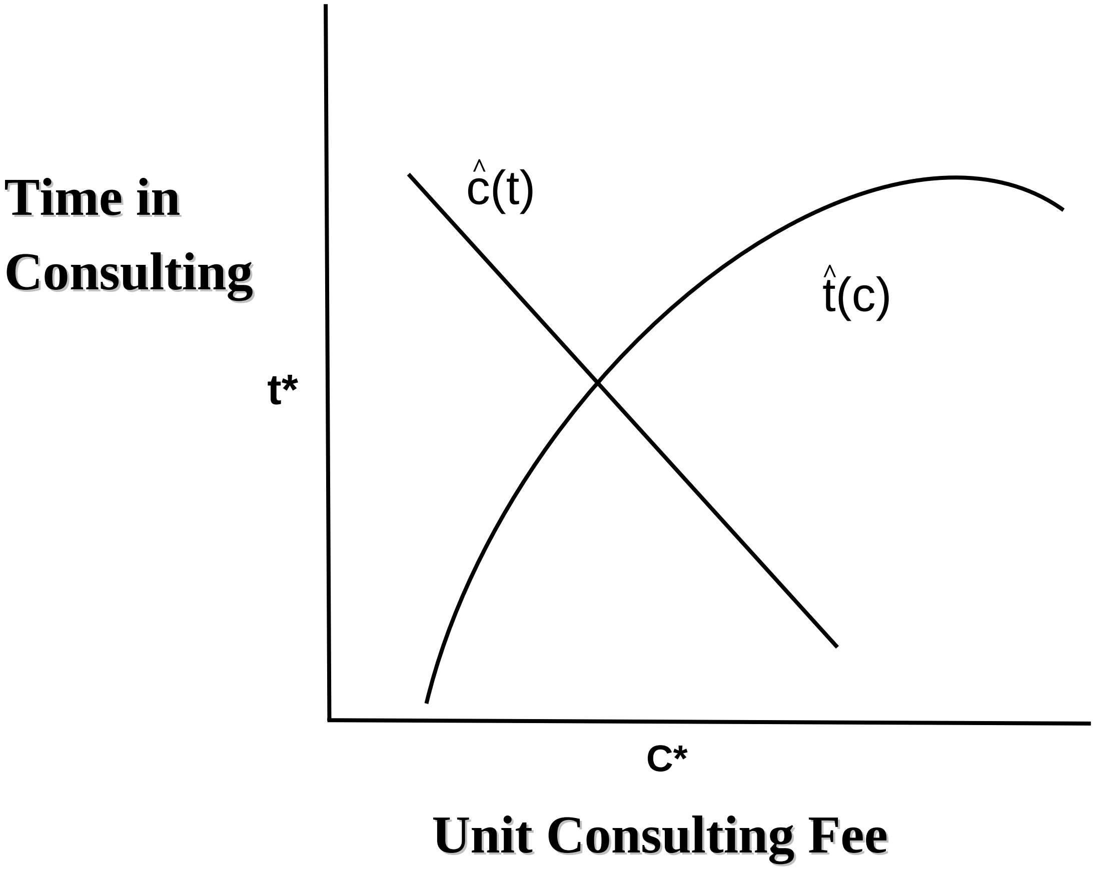

Figure 1

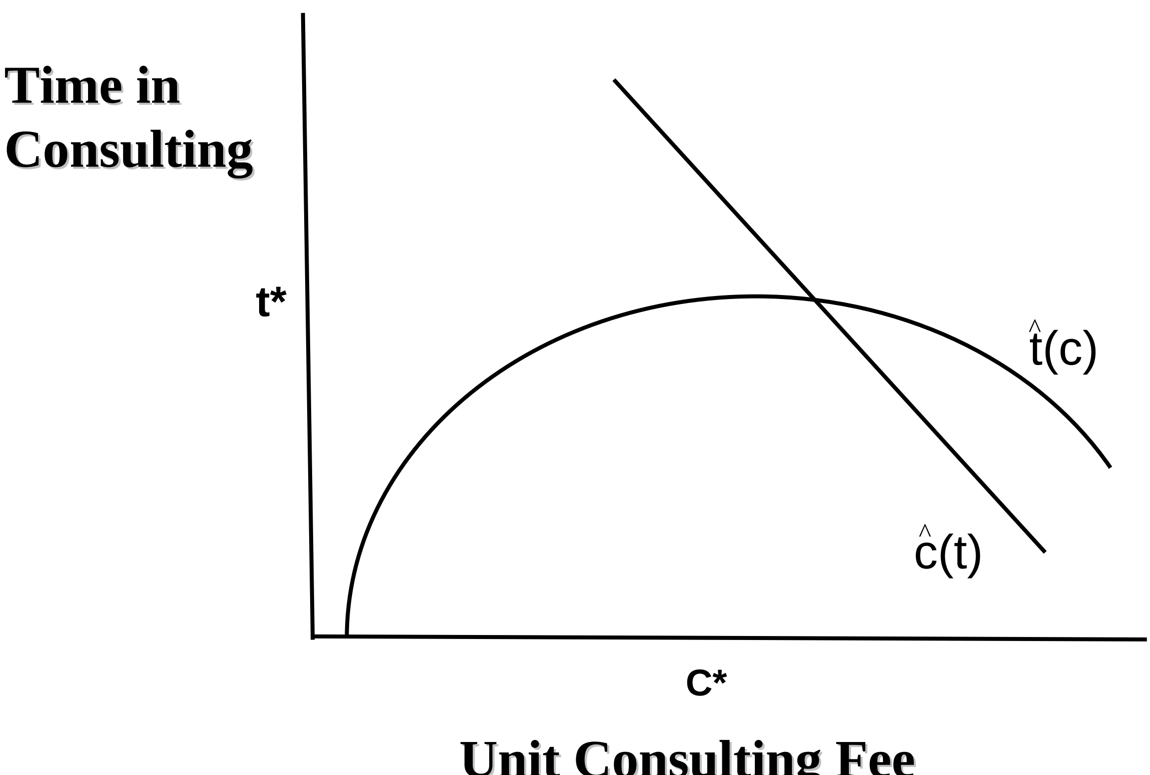

Figure 2

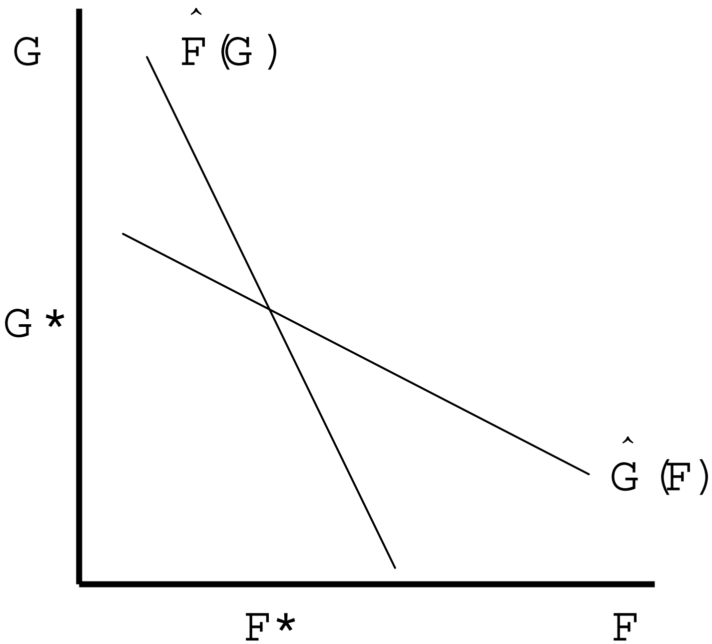

Figure 3

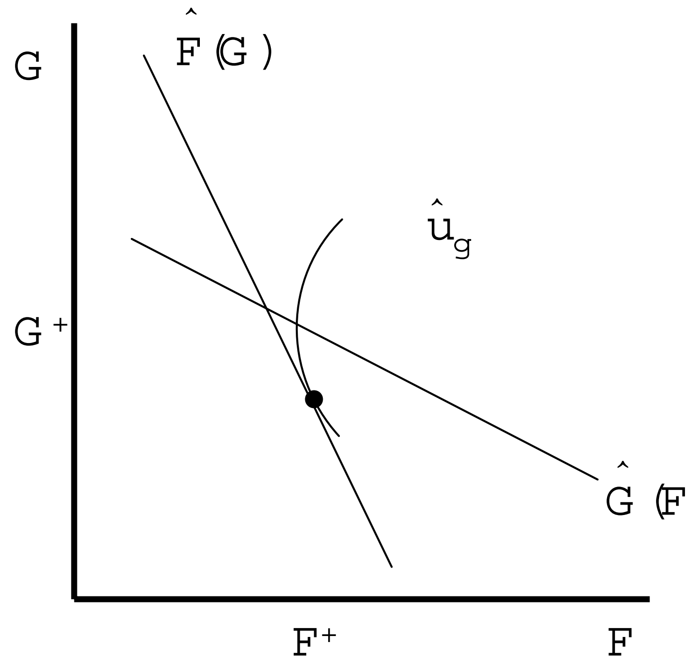

Figure 4

Table 1. Assignment

	Number	% of Sample
Firm	499	28.24
Not-For-Profit	1,188	67.23
Unassigned	34	1.92
US Gov't	15	0.85
Not-For-Profit & Firm	28	1.58
US Gov't & Not-For-Profit	3	0.17

Table 2. Application Year of Patents

	Number	% of Sample
Before 1993	243	14.40
1993	251	14.88
1994	334	19.80
1995	359	21.28
1996	255	15.12
1997	196	11.62
After 1998	49	2.90

Table 3. Summary Statistics

	No. Obs.	Mean	SE	Min	Max
PUBS	1687	7.279	8.498	0	51
CITES	1687	270.506	558.772	0	6557
FEDFND	1687	0.796	1.780	0	15.021
INDFND	1687	0.157	0.539	-0.08	4.18
OTHERFND	1368	0.077	0.215	-0.03	2.70
MALE	1656	0.950	0.218	0	1
AGE	1632	49.014	9.983	28	83

NOTES

Negative INDFND & OTHERFND are reimbursed overruns from a prior year.
OTHERFND not available for Wisconsin.

Table 4. Logistic Regression Results

	Odds Ratio	SE	p-Value	Odds Ratio	SE	p-Value	Odds Ratio	SE	p-Value
PUBS	1.068	0.016	0.000	1.047	0.013	0.000	1.064	0.015	0.000
CITES	1.001	0.000	0.015	1.001	0.000	0.008	1.000	0.000	0.113
FEDFND	0.890	0.028	0.000	0.899	0.027	0.000	0.792	0.024	0.000
INDFND	2.359	0.944	0.032	2.556	0.926	0.010	2.547	0.925	0.010
OTHERFND	1.344	0.501	0.427				1.518	0.831	0.446
MALE	1.300	0.527	0.518				1.201	0.415	0.595
AGE	1.000	0.007	0.995				0.988	0.006	0.051
University Fixed Effects	YES			YES			NO		
Year Fixed Effects	YES			YES			NO		
Field Fixed Effects	YES			YES			NO		
Tech Category Fixed Effects	NO			NO			NO		
No. Observations	1297			1687			1297		
Pseudo r-Square	0.174			0.121			0.093		

Table 4 (con"t). Logistic Regression Results

	Odds Ratio	SE	p-Value	Odds Ratio	SE	p-Value
PUBS	1.070	0.017	0.000	1.062	0.016	0.000
CITES-EXPECT	1.000	0.000	0.045	1.000	0.000	0.041
FEDFND	0.907	0.029	0.002	0.906	0.034	0.007
INDFND	2.468	1.014	0.028	2.312	0.972	0.046
OTHERFND	1.376	0.539	0.416	1.267	0.418	0.473
MALE	1.684	0.744	0.239	1.262	0.565	0.604
AGE	1.005	0.007	0.500	0.997	0.007	0.706
University Fixed Effects	YES			YES		
Year Fixed Effects	YES			YES		
Field Fixed Effects	YES			YES		
Tech Category Fixed Effects	NO			YES		
No. Observations	1181			1280		
Pseudo r-Square	0.171			0.223		