EFACR
7/18/07
9:00 AM

Information Heterogeneity in the Macroeconomy*

Ponpoje Porapokkarm†
Department of Economics
University of Virginia

Eric R. Young‡
Department of Economics
University of Virginia

June 20, 2007

Abstract

This paper considers the role that information heterogeneity can play in generating wealth inequality. We compute a model of uninsurable idiosyncratic risk and aggregate shocks to TFP under different assumptions about the information set of the households. Information has two effects in our economy. The first effect is direct: information changes the distribution of future states an agent expects tomorrow; this effect is standard in models of asymmetric information. But an additional indirect effect arises in our model: information heterogeneity alters the shape of the value function, leading to heterogeneity in the marginal value of wealth. A better-informed agent receives more utility from an additional unit of saving. In our calibrated economy we find that the second effect is far more important and is a potential mechanism to induce wealth concentration; for our particular calibration it turns out to be insufficiently strong quantitatively to dramatically increase the Gini coefficient on wealth. We find that the assumption of information heterogeneity has a non-trivial effect on the cost of aggregate fluctuations; the cost of business cycles is 4.5 times as large for poorly-informed agents and marginal gains from smoothing the cycle are large and decreasing for the poorly-informed.

Keywords: Heterogeneity, Information, Wealth Distribution, Costs of Business Cycles

JEL Classification Codes: E21, E25, E32

---

* We would like to thank Dave DeJong, Per Krusell, Rob Martin, and Tom Tallarini for helpful comments, without implicating them in any errors, and seminar participants at the New York Fed who endured a very early version of this paper, and participants at the 2007 Society for Computational Economics Meetings. We also thank the Bankard Fund for Political Economy for financial support.

† PO Box 400182, Charlottesville, VA 22904. Email: pp9s@virginia.edu.

‡ Corresponding author. PO Box 400182, Charlottesville, VA 22904. Email: ey2d@virginia.edu.

1. Introduction

Inequality is a fact of life in modern economies, but its origins have been difficult to uncover. The extreme wealth inequality observed in US cross-sectional data has been particularly difficult to understand, despite a number of papers dedicated to this issue. The literature on inequality in general equilibrium has typically found that measured shocks to labor earnings are quantitatively insufficient to generate wealth concentration anywhere close to what we observe in the data; for example, the Gini coefficient for wealth in the US is around 0.8, while Aiyagari (1994) finds that measured labor earnings shocks only produce a Gini coefficient of 0.4. Three responses have emerged as a result. One response, argued in Castañeda, Díaz-Giménez, and Ríos-Rull (2004), is to note that labor earnings measurements are fraught with measurement error; the very rich do not appear in the surveys used to estimate the dynamics of earnings, but they do appear in the cross-sectional surveys used to measure wealth inequality.[1] The authors therefore assume that labor earnings shocks are whatever is needed to match wealth inequality with the context of a particular model; the direct measurement of earnings is abandoned. A second response is to modify preferences, generating classes of agents with strong desires to save – for example, Krusell and Smith (1998) show that discount factor heterogeneity can generate large inequities in wealth. A third approach models the differential return across occupations, noting that much of the wealth is held by individuals who are self-employed (Cagetti and De Nardi 2006); wealth concentration is then generated by high average returns to entrepreneurs.[2]

This paper takes a different approach. What we are interested in studying is the role of information in a dynamic economy, particularly related to its contribution to inequality. We consider agents who observe only prices and individual state variables (everything that appears in their budget sets) but not aggregates. Some agents know the structural equations of the model; we show these agents can infer the relevant aggregate states, so we refer to them as 'fully-informed' or 'FI' agents. Other agents do not know the structural equations of the model. Since they cannot evaluate the laws of motion for aggregate states, their forecasts of future prices must be computed using only current prices; we assume that this forecasting takes the form of an unrestricted VAR run

[1] The Panel Study of Income Dynamics (PSID) is commonly used to estimate earnings dynamics and the Survey of Consumer Finances (SCF) to estimate the distribution of wealth. The SCF oversamples the very wealthy and therefore gets a more complete picture of the economy, while the PSID misses important fractions of income and wealth by failing to have the wealthy in the sample.

[2] Guvenen (2005) shows how the combination of heterogeneity in the intertemporal elasticity of substitution and asymmetric access to asset markets can generate wealth concentration, combining some aspects of the latter two modifications.

on a large sample of past prices.[3] Because these agents have less information than the FI agents, we label them 'partially-informed' or 'PI.' We do not model the reasons why some households know more than others; we simply want to know whether it matters.[4]

We ask four questions of our model. First, we want to know whether economies populated only by PI agents behave similarly to those populated only by FI agents. The information assumptions implicit in dynamic general equilibrium models are often quite strong, requiring the individual households to know more about the structure of the economy than we economists do and to observe aggregate states (like capital) we ourselves do not.[5] Our model in particular has a state variable – the distribution of wealth – that would seem completely unavailable to real households over the frequency needed for it to guide consumption-savings decisions, making it reasonable to ask how agents would do without knowledge of this object. Thus, while we study both types of agents here, we believe that the 'real world' is populated by PI agents; the FI agents are used as a benchmark because their behavior is well-understood. We find that the aggregate behavior of the model economy is invariant to the measure of PI agents present in the economy in terms of the usual second moments studied in the business cycle literature; the one difference that emerges is that the total capital stock is a little larger in an economy populated entirely by FI agents than in one populated by PI agents.

Our second question asks how the different information sets affect individual behavior. We simulate the behavior of one FI and one PI agent who live together in an economy where the measures of both agents are equal. Each individual receives the same sequence of idiosyncratic shocks and the same initial wealth, but their accumulated wealth diverges over time due to differential savings rates. Specifically, the FI agent saves more than the PI agent. Our results show that the saving divergence is not primarily the result of different perceptions about tomorrow's states of the economy; instead, a more important indirect effect is that different information sets induce a different shape of indirect utility. In our setting FI agents have a higher marginal value

[3]The VAR is not implied by the structure of the model, which is nonlinear; one could call these agents 'rule-of-thumb' forecasters. Our agents differ from those in Bomfim (2001a) in that ours derive optimal forecasts given the information they observe, while Bomfim (2001a) endows his rule-of-thumb agents with biased forecasts. Our results are similar to his in the absence of strategic complementarity; we have serious reservations about the assumption that the goods produced by FI and PI agents are not perfect substitutes.

[4]Our approach differs from many models of asymmetric information in that some of our agents do not observe the model, while typically it is assumed only that some agents do not observe relevant state variables. The one paper that has a similar approach is Bomfim (2001a), which we discussed in the previous footnote.

[5]At least, not without a significant amount of error. An (2006) compares the empirical capital stock series to one constructed using a DSGE model that treats the capital stock as an unobserved state. The two series diverge, despite the model fitting the observed series quite well.

over wealth, leading to higher asset levels. This effect seems not to have been emphasized in the current literature. We point out here that more savings by FI agents is not a trivial implication because their information advantage would seem to reduce their demand for precautionary savings; since all savings in our model is precautionary (the average return is smaller than the time rate of preference), FI agents could have been poorer.[6]

Our third question asks whether economies populated by nontrivial fractions of PI and FI agents feature more wealth inequality than standard models (all FI agents) predict. We find only a small increase in wealth Gini coefficients when the measure of PI agents is 0.5, with only minor differences across types (the FI group has a slightly lower Gini coefficient). But there is a split in the wealth distribution between the PI and FI populations: wealthy agents are disproportionately FI agents, a consequence of the differential savings behavior noted above. In our calibrated model we find that 60 percent of the bottom 10 percent of the wealth distribution are PI agents while 72 percent of the top 1 percent are FI agents; if information heterogeneity did not play a role in inequality we would see these fractions equal to 50 percent. Information heterogeneity is a mechanism for generating inequality, although some features of our model limit its quantitative significance; in particular, we believe that the introduction of a risk-free asset would exacerbate the differences for reasons we discuss in the conclusion.

Given our finding that the economy's aggregate behavior is robust to the information assumption, the last question we ask is how much the cost of aggregate fluctuations would change under different assumptions about information. The existing literature on the costs of fluctuations is very large, but to our knowledge no one has studied an economy where households have only limited information about the state of economy. Our model shows that the cost of fluctuations in an economy populated with PI agents is 4.5 times larger than the equivalent one populated by FI agents, although it is still small. Modifications that raise the average welfare cost of cycles – such as the alternative preference structures used in Dolmas (1998) or Tallarini (2000) – could lead to nontrivial costs for uninformed agents, weakening the case that stabilization is a suboptimal policy goal. Furthermore, the marginal gain from reducing fluctuations is very large and declining for PI agents but essentially constant for FI agents; thus, policies that eliminate only a small fraction of the business cycle may be welfare-enhancing in a PI world.

[6] Chamberlain and Wilson (2000) prove that the average return is smaller than the homogeneous time rate of preference for this class of models. Aiyagari (1994) proves the analogous result for economies without aggregate shocks.

2. Model

The model economy is populated by a continuum of households and a continuum of firms, both with unit measure. The production sector is represented by a stand-in firm that operates a Cobb-Douglas production technology,

$$Y_t = \exp(z_t) K_t^{\alpha} H_t^{1-\alpha}, \tag{2.1}$$

where K_t and H_t are aggregate capital and labor inputs in the economy and $\alpha \in (0,1)$ is capital's share of income. The aggregate shock in the economy is the technology shock z_t, which evolves as

$$z_{t+1} = \rho_z z_t + e_{t+1}; \quad e_t \sim \text{iid } N\left(0, \sigma_e^2\right); \tag{2.2}$$

we assume $|\rho_z| < 1$. With competitive factor markets the factor prices would satisfy

$$\begin{aligned} \log(r_t + \delta) &= \log(\alpha) + (1-\alpha)\log(H_t) + z_t + (\alpha - 1)\log(K_t) \\ \log(w_t) &= \log(1-\alpha) - \alpha \log(H_t) + z_t + \alpha \log(K_t); \end{aligned} \tag{2.3}$$

these expressions are simply the logarithms of the marginal products of capital and labor, respectively. $\delta \in [0,1]$ is a fixed depreciation rate.

The other sector of the economy is represented by a continuum of infinitely-lived households with total measure 1. These agents are heterogeneous *ex post* along three dimensions: their uninsurable idiosyncratic shock ϵ_t^i, their accumulated cash on hand m_t^i, and their information sets Ω_t^i. ϵ_t^i evolves according to an exogenous AR(1) process

$$\begin{gathered} \epsilon_{t+1}^i = \rho_\epsilon \epsilon_t^i + \nu_{t+1}^i; \quad \nu_t^i \sim \text{iid } N\left(0, \sigma_\nu^2\right) \\ E\left(\nu_t^i e_\tau\right) = 0 \quad \forall\, t \text{ and } \tau. \end{gathered} \tag{2.4}$$

We assume that $|\rho_\epsilon| < 1$. Note that we have assumed the distribution of the idiosyncratic shock is independent of the aggregate shock.[7] Since we will also assume inelastic labor supply by households, H_t will be a constant (denoted $\overline{H}$).[8]

[7] This assumption is for computational reasons, as it simplifies the estimation procedure of the PI agents. We intend to relax it in the future.

[8] We experimented with a model that permitted elastic labor supply and found the results were similar. For clarity of presentation we concentrate on the inelastic labor case. We also looked at versions of the model with shocks to depreciation because these shocks reduce the excessive correlation between r and w; our empirical measure of these shocks turned out to be small. Even when we made the depreciation shocks large they did not affect our answers.

2.1. Information structure

The study focuses on an economy populated by two types of households: *fully-informed agents (FI)* and *partially-informed agents (PI)*. These two types of agents are identical except their information sets; in particular, we assume that they face the same process for the idiosyncratic shock.

2.1.1. FI agent

An FI agent knows the model structure and observes all the relevant state variables. We therefore define their information set as

$$\Omega_t^{FI} \equiv \left\{ m_t^i, \epsilon_t^i, \Gamma_t\left(m, \epsilon, \theta\right), z_t, r_t, w_t, \mathcal{Q} \right\},$$

m_t^i is the sum of total wealth and current income and Γ is the distribution over cash on hand m, individual shock ϵ, and type of agents $\theta \in \{\text{'PI'},\text{'FI'}\}$. Denote $\Lambda = \int_{\theta^i = PI} \Gamma_t\left(m, \epsilon, \theta\right)$, the exogenous proportion of PI agents in the economy.[9] An FI agent also knows equations (2.2), (2.3) and the values of the parameters; we denote this information by $\mathcal{Q}$. The FI agent's recursive problem is

$$V^{FI}\left(m, \epsilon, \Gamma, z\right) = \max_{k' \in [0,m]} \left\{ u\left(m - k'\right) + \beta E\left[V^{FI}\left(m', \epsilon', \Gamma', z'\right) | \Omega^{FI} \right] \right\} \tag{2.5}$$

subject to the budget constraint and law of motion for Γ

$$\begin{aligned} m' &= k'\left(1 + r'\right) + w' \exp\left(\epsilon'\right) \overline{h} \\ \Gamma' &= F\left(\Gamma, z, z'\right) \end{aligned}$$

and the shock processes (2.2), (2.4). k' is individual savings in capital. $E\left[\cdot|\Omega_{FI}\right]$ is the expectation operator conditioned on information set Ω_{FI}. The last equation is the law of motion for the distribution. Following the approximate aggregation results in Krusell and Smith (1998) and Young (2006) the only relevant aggregate variables are K_t and z_t; other moments of Γ_t do not contribute to forecasting future prices.[10] It is obvious that any FI agent who knows (r_t, w_t) can compute (K_t, z_t) by using (2.3); thus prices fully reveal the relevant state variables in our setting. It

[9] In Krusell and Smith (1998) every household is fully-informed; their equilibrium can be approximated by allowing households to use only information in current period. We also show here that in an economy where not all households are fully-informed, knowledge of current period values is sufficient for an FI agent to accurately forecast the evolution of aggregate capital.

[10] This result is due to the near-linearity of the optimal saving function k' with respect to m combined with the fact that changes in the aggregate states linearly displace the savings function.

is also true that K_{t+1} is Ω_t^{FI}-measurable (since it is a deterministic function of individuals' current saving). Following Krusell and Smith (1998), we parameterize the law of motion for K_{t+1} as

$$\log\left(K_{t+1}\right) = a_0 + a_1 z_t + a_2 \log\left(K_t\right); \tag{2.6}$$

this assumption is based on results in Young (2006) that show more flexible functional forms do not change the implied law of motion.

To make comparisons across agents simple and relatively free of numerical error, we rewrite the FI agent problem using (r, w) as state variables rather than (K, z). The recursive problem of an FI agent is therefore

$$V^{FI}\left(m, \epsilon, r, w\right) = \max_{k' \in [0,m]} \left\{ u\left(m - k'\right) + \beta \sum_{\epsilon'|\epsilon} \pi\left(\epsilon'|\epsilon\right) \left(\int_{r',w'} V^{FI}\left(m', \epsilon', r', w'\right) dF\left(r', w'|r, w\right) \right) \right\} \tag{2.7}$$

subject to

$$\begin{aligned} m' &= k'\left(1 + r'\right) + w' \exp\left(\epsilon'\right) \overline{h} \qquad (2.8) \\ \log(r' + \delta) &= A_0 + A_1 \log\left(r + \delta\right) + A_2 \log\left(w\right) + e' \\ \log(w') &= A_3 + A_4 \log\left(r + \delta\right) + A_5 \log\left(w\right) + e'. \end{aligned}$$

Appendix A shows that dynamic equations of r and w shown above can be derived from equations (2.3), (2.2), and (2.6), where the coefficients $\{A_0, A_1, A_2, A_3, A_4, A_5\}$ will be determined endogenously in equilibrium. Independence between ϵ_t and (r_t, w_t) comes from independence between ϵ_t and e_t; note that the error in (2.8) is the innovation in the technology shock process.

2.1.2. PI agent

PI agent i's information set in period t is defined by

$$\Omega_t^{PI} = \left\{m_t^i, \epsilon_t^i, r_t, w_t\right\} \subset \Omega_t^{FI}.$$

We restrict Ω_t^{PI} to contain only current variables; our focus is on Markov recursive equilibria consistent with this restriction. By observing ϵ_t^j a PI agent can infer the true individual shock process (2.4). However, PI agents do not know the structure of the model economy; specifically equations (2.2) and (2.3) are not known. PI agents also do not observe the aggregate variables (K_t, z_t).[11] As a result, it is impossible for a PI agent to derive the true dynamic equation for r_t and w_t that the FI agent obtains.[12] In this case, the dynamic problem of PI agent can be written as

$$V^{PI}(m,\epsilon,r,w) = \max_{k'\in[0,m]} \left\{ u(m-k') + \beta \sum_{\epsilon'|\epsilon} \pi(\epsilon'|\epsilon) \left(\int_{r',w'} V^{PI}(m',\epsilon',r',w')\, dG(r',w'|r,w) \right) \right\} \tag{2.9}$$

subject to the budget constraint and a stochastic price process $G(r,w)$ defined as

$$\begin{aligned} \log(r') &= b_0 + b_1 \log(r) + b_2 \log(w) + \varepsilon'_r & (2.10) \\ \log(w') &= b_3 + b_4 \log(r) + b_5 \log(w) + \varepsilon'_w \\ \varepsilon &= \begin{bmatrix} \varepsilon_r \\ \varepsilon_w \end{bmatrix} \sim \text{iid } N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \sigma_r^2 & \sigma_{rw}^2 \\ \sigma_{rw}^2 & \sigma_w^2 \end{bmatrix} \right). \end{aligned}$$

PI agents use VAR(1) representations to forecast price movements. We believe that this restriction is reasonable, given that VARs are standard tools in the applied forecasting literature and the PI agent does not know anything about the structure of the model that would suggest an alternative. The assumption that only one lag is used in the VAR may be restrictive, but adding more lagged terms drastically increase the computational cost.[13] Note that the PI agent's forecast is based on statistical estimation while the FI agent's forecast is derived directly from the model structure; thus the FI agent has no estimation error. PI agents know their forecasts have error – they take ε' into account and integrate over it when forming expectations.[14] Finally, the distribution of ε is endogenous; we verify that it is approximately iid normal in the simulations, consistent with the endowed beliefs.[15]

[11] Although irrelevant, we also assume they do not observe any other moments of $\Gamma_t(m,\epsilon,\theta)$.

[12] Note that the PI agent only sees the net return $r_t = MPK - \delta$; MPK and δ cannot be identified separately from the mean of r_t under the information assumptions. Thus not knowing MPK, a PI agent cannot solve for K and z.

[13] Technically, we should permit PI agents to use their individual cash-on-hand m to forecast future prices as well, since it is correlated with them. In such a world, beliefs about future prices would be naturally heterogeneous; agents with high m would infer something different about the direction of the economy than would agents with low m. We found that this extension was computationally very burdensome and the resulting coefficients were quantitatively unimportant. We will comment more on this extension in the conclusion.

[14] Here the PI agents discard the coefficients' estimation errors. Since in our equilibrium PI agents use a very long time series of observations in their estimation, these errors are negligible.

[15] The approximation does not hold in the tails of the distribution, but these events are too rare to matter quantitatively.

The sequential form of the PI agents' problem is nonstandard. Information for the PI agent does not evolve as a filtration because we do not permit them to condition forecasts on information dated $t-1$ or earlier. Thus, the recursive problem cannot be iterated to obtain the standard representation

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\middle|\,\Omega_0\right]. \tag{2.11}$$

Instead, the sequential problem takes the form

$$\sum_{t=0}^{\infty}\beta^t \widetilde{E}_0\left[\left.u\left(c_t\right)\right|\Omega_0\right] \tag{2.12}$$

where

$$\begin{aligned}
\widetilde{E}_t\left(x_{t+1}\right) &= E_t\left[x_{t+1}|\Omega_t\right] \\
\widetilde{E}_t\left(x_{t+2}\right) &= E_t\left[E_{t+1}\left[x_{t+2}|\Omega_{t+1}\right]|\Omega_t\right]
\end{aligned}$$

and so on. The law of iterated expectations does not apply to the PI agent so these expressions do not collapse; that is, the conditional distribution of $\{r_\tau\}_{\tau\geq t+2}$ is not Ω_t^{PI}-measurable. However, it may be true that the expressions are approximately equal if past information is not independently useful for the forecasts of future prices, so that a VAR(p) does not forecast quantitatively better than the VAR(1). As noted above, the burden of even a VAR(2) is tremendous, so we leave this extension for future work.

We present the formal definition of the recursive equilibrium for the economy in Appendix D. PI agents are not learning in our model; because we assume that this VAR is estimated over the infinite past sequence of prices, any learning that could be done must already have taken place. Obviously this assumption limits the extent to which information can play a role in generating inequality, so we view our results as establishing a lower bound. Computational considerations do not permit us to consider a full Bayesian procedure in which households update their estimates of the coefficients in the VAR period-by-period, unfortunately.[16]

[16]There would be two nontrivial problems associated with that model. First, the state space would include an economic model, so each parameter would be a state variable; the size of this space would be very large. Second, the Bayesian estimation of nonlinear models is very time-consuming, making computational time very long.

2.2. Calibration

We assume one model period corresponds to one quarter. The felicity function $u(c)$ is chosen to be $\log(c)$, so that relative risk aversion equals one. The chosen parameters of the model are $\beta = 0.99$, $\alpha = 0.36$, $\delta = 0.0217$, and $\overline{h} = 0.3271$; these values yield aggregate outcomes generally consistent with US data on capital/output and investment/output ratios and capital's share of income. The log of technology shock z_t is estimated from the annual series of GDP and capital stock from the National Income and Product Accounts and then converted into a quarterly process, yielding $\rho_z = 0.96429$ and $\sigma_e^2 = 0.0071^2$. We approximate the AR(1) process for ϵ_t^i with a Markov chain with 7 states.[17]

Appendix C presents an extensive discussion of our solution method, which is based on Young (2006). We use Gauss-Hermite quadrature and product rule methods to compute the integrals over the continuous random variables and cubic spline interpolation combined with linear interpolation to evaluate the value functions.[18]

3. Results

In the first subsection we compare two extreme economies at the aggregate level. The first economy is populated by a full measure of FI agents, called the FI economy ($\Lambda = 0.0$). The second economy is populated by a full measure of PI agents, called the PI economy ($\Lambda = 1.0$). In the second subsection we examine the behavior of one FI agent and one PI agent who live in a mixed economy ($\Lambda = 0.5$); the results in this subsection also extend to other economies with different mixtures of PI and FI agents.[19] In our simulation the two agents receive the same idiosyncratic shock each period. Since they make different decisions, their endogenous states diverge over time, but this divergence is entirely driven by information heterogeneity and not luck. In the third subsection, we use the mixed economy to explore the consequences of information heterogeneity for inequality. In the final subsection we estimate the cost of aggregate fluctuations in the PI economy relative to the FI economy.

[17] See Appendix B.

[18] Fortran code to solve the model is available upon request. Typically the code takes several days to converge and may be unstable for poor initial guesses.

[19] The choice of $\Lambda = 0.5$ helps us to see the effect of information heterogeneity on the wealth distribution. Specifically, if information plays no role, the wealth distributions of both types of agents would be identical.

3.1. Aggregate Fluctuations

Table 1 presents the standard set of aggregate first and second moments used to assess business cycle models; it is clear that the two economies display essentially the same fluctuations, since none of the statistics are quantitatively different. This similarity also holds for the mixed economy where $\Lambda = 0.5$. Krusell and Smith (1998) demonstrates that the implications of representative agent models regarding the response of the economy to technology shocks were robust to the introduction of idiosyncratic risk. The results in their paper extend to our models in which agents are not fully informed about the economic model they inhabit; information heterogeneity (at least as modelled here) does not undermine the lessons learned from the vast real business cycle literature.

To see this more clearly, Table 2 reports the forecast rules of FI and PI agents in the two extreme economies. It shows that K_t and z_t are sufficient statistics for K_{t+1} in both economies. The evolution equation of K is deterministic; in both economies, the R^2 of this equation is virtually one. This result strengthens the approximate aggregation results in Krusell and Smith (1998) and Young (2006); approximate aggregation obtains in economies where some measure of agents observe only equilibrium prices. This near-equivalence may be of interest to researchers studying more elaborate extensions of Krusell and Smith (1998), particularly those where aggregate prices are unknown functions of the state of the world. If accurate representation of those economies is difficult using constituent moments from the distribution, one may be able to approximate the aggregate dynamics simply using prices instead; if there are not many prices this approach will be less computationally demanding.[20] Because the behavior of individuals in these economies can be different, whether the approximation is good will depend critically on the question being explored.

Figure F.1 compares the time-series of aggregate variables in the two economies. Panel (a) shows that aggregate capital in the FI economy is a little higher than in the PI economy. We discuss in the next subsection why FI agents tend to save more than PI agents.

3.2. Individual Savings Behavior

In Figure F.2, panels (a) and (b) compare simulated individual savings of an FI agent and a PI agent in the mixed economy ($\Lambda = 0.5$). The FI agent accumulates more wealth than the PI agent. Table 3 shows some statistics at individual level. The FI agent's average wealth is almost 24 percent

[20] We are not the first researchers to use prices as state variables. Ríos-Rull and Sanchez-Marcos (2006) use prices as state variables in a model of house price fluctuations. Telmer and Zin (2002) show that the pricing kernel in a particular incomplete market model can be approximated accurately using only prices.

higher than PI agent's. Consequently, the aggregate capital accumulated by the FI population is on average 26.5 percent higher than the aggregate capital of PI population. The FI agent also has higher average consumption, although the difference is much smaller. This resulting pattern is robust to other economies with different measures of PI and FI agents. For example, in the PI economy the FI agent's average wealth is 26.8 percent higher while in the FI economy the FI agent's average wealth is 21.5 percent higher.[21]

Panels (a) and (b) of Figure F.3 show that the saving functions of both agents are nearly linear, with the exception of very low levels of m where they become noticeably convex. Panel (c) of Figure F.3 and panels (a)-(d) of Figure F.4 compare the saving functions of PI and FI agents for various different values of (r, w, ϵ), confirming that FI agents' savings rates are higher than PI agents'.[22] Though FI agents' current-period savings is only a little higher than PI agents, accumulation over time generates large wealth gaps as shown in Figure F.2. To see why the saving rates of the two agents are different, we analyze the agent's problem as a standard two-period saving problem:

$$V^i(m,\epsilon) = \max_{k' \in [0,m]} \left\{ u\left(m - k'\right) + \beta E\left[\mathcal{V}^i\left(\left(1+r'\right)k' + w' \exp\left(\epsilon'\right)\overline{h}, \epsilon'\right) | \Omega^i\right]\right\}. \tag{3.1}$$

Since both agents have the same (r, w), we omit these state variables to simplify the expressions.[23] Today's return function is the underlying preference over consumption while the second-period return function is the indirect utility function over m. Since the FI and PI agents are different only in their forecast functions, due to different information sets, this difference can affect the savings decision through two channels. First, the different forecast directly causes the two agents perceive tomorrow's returns and risks differently: the conditional distributions of (r', w') are different.[24] The second effect is indirect: the different forecasts induce different shapes in the indirect utility functions $\mathcal{V}^i$. We refer to the first effect as 'forecast heterogeneity' and the second as 'value heterogeneity'; we are interested in assessing the quantitative importance of each factor.

[21] In the FI (PI) economies we introduce one PI (FI) agent; since this agent is measure zero the equilibrium is not affected. We can therefore simulate the behavior of this single agent along the equilibrium path of an economy populated entirely by the other type.

[22] In panel b of Figure F.4, there is a small range of low values for m where a PI agent saves more than an FI agent. This reversal occurs when r is very low while w is very high. Since r and w have a positive correlation, the savings reversal rarely occurs. In our simulation of $99,000$ periods, we find a positive measure of PI agents who save more than FI agents in only 47 periods. Conditioned on these periods, the average measure of PI and FI agents who exhibit this saving reversal is only 0.015. All of them are among the very poor; a model which captures better the distribution of wealth and income may produce more such reversed agents.

[23] $V^i(\cdot)$ is the same as $\mathcal{V}^i(\cdot)$ for an infinitely-lived agent.

[24] The conditional distribution of ϵ' is the same by assumption, both across agents and over time.

3.2.1. Direct effect: Forecast Heterogeneity

Figure F.5 shows the distribution of forecast errors of the mixed economy; the same picture emerges for the PI and FI economy. A normal density fits the distribution of forecast errors very closely. Table 2 also shows that the autocorrelation of the VAR error is almost zero. As a result, our assumption that the PI agent believes the error in his forecast rule is white noise normal is consistent with the equilibrium outcome.

Table 2 reports R^2 of both agents' forecasts; they are practically the same, implying that the VAR(1) does a very good job producing accurate one-period forecasts. However, the PI agent's VAR(1) forecasts are just a close approximation of the true price processes, namely the FI agent's forecasts. Since these forecasts are just approximations, both agents do not have to agree on the distribution of the future prices; while both agents observe the same time series of prices, their inferences are different. Figure F.6 compares the conditional expectation and standard deviation of both agents' one-period forecasts.[25] In panels (a) and (b), the disagreement in the expected r' is less than 0.25 percent while the disagreement in the expected w' is less than 0.05 percent. Note that there is no systematic mistake in PI agent's forecast; the average of this disagreement is zero. Panels (c) and (d) show that the PI agent does not always perceive higher variance in r but always perceives higher variance in w.[26]

To gauge the effect of forecast heterogeneity on the savings decision, we ask how much the PI agent would save if endowed with FI's forecast for the current period only. The difference between the original PI agent's problem and this one is that integration over (r', w') is based on FI's conditional distribution. More explicitly, in this experiment and in Subsection (3.2.2), we substitute the equilibrium value functions obtained earlier in the RHS of the two-period problem (3.1) and solve for the policy functions associated with the new one-period forecast rule. Figure

[25] Conditional expectation and variance are calculated from

$$E_t^{PI} r_{t+1} = \exp\left(\mu_r + \frac{\sigma_r^2}{2}\right);\ V_t^{PI}\left(r_{t+1}\right) = \left(E_t^{PI} r_{t+1}\right)^2 \left(\exp(\sigma_r^2) - 1\right),$$

$$E_t^{PI} w_{t+1} = \exp\left(\mu_w + \frac{\sigma_w^2}{2}\right);\ V_t^{PI}\left(w_{t+1}\right) = \left(E_t^{PI} w_{t+1}\right)^2 \left(\exp(\sigma_w^2) - 1\right),$$

$$E_t^{FI} r_{t+1} = \exp\left(\widehat{\mu}_r + \frac{\sigma_e^2}{2}\right) - \delta;\ V_t^{FI}\left(r_{t+1}\right) = \left(E_t^{FI} r_{t+1} + \delta\right)^2 \left(\exp(\sigma_e^2) - 1\right),$$

$$E_t^{FI} w_{t+1} = \exp\left(\widehat{\mu}_w + \frac{\sigma_e^2}{2}\right);\ V_t^{PI}\left(w_{t+1}\right) = \left(E_t^{FI} w_{t+1}\right)^2 \left(\exp(\sigma_e^2) - 1\right),$$

where $\left(\widehat{\mu}_r, \widehat{\mu}_w, \sigma_e^2\right)$ and $\left(\mu_r, \mu_w, \sigma_r^2, \sigma_w^2\right)$ are calculated from (2.8) and (2.10) respectively.

[26] Because prices follow a lognormal distribution, the disagreement in expectation translates into a difference in variance.

F.7 plots the simulated saving of a PI agent who is endowed with FI agent's forecast, together with the original PI and FI agents. The lower panel shows that the new 'informed' PI agent does not change his optimal saving by a significant amount; that is, forecast heterogeneity is not the main contributor to savings heterogeneity in this model. We get the same result in the PI and FI economy.

3.2.2. Indirect effect: Value Heterogeneity

Since different forecasts cannot account for the large savings divergence, it must come from differences in the shape of the indirect utility functions. Figures F.8 and F.9 plot the value functions of PI and FI agents together with their differences at various r and w. The value functions of both agents are concave (panels (a) and (b) of Figure F.8). Notice that despite information inferiority, PI agents are not always worse off. Figure F.6 shows that the PI agent is too 'optimistic' for some combinations of r and w; his forecasts have higher expectation or lower variance. Consequently, his *ex ante* indirect utility is higher than an FI agent's, conditional on these states of the world. However, these states are relatively rare, leading to overall lower utility for the PI agents.

We want to examine what characteristic of their indirect utility accounts for the savings divergence. The difference in the shape of $\mathcal{V}^i$ will result in different responses to risks. For the two-period model (3.1) there are two risks: return risk from saving and background risk from income, and these random variables are positively correlated. First we control for the risks by endowing both agents with the same one-period forecast, say FI agent's forecast.[27] To quantify the response to the saving risk we endow both agents with w' and ϵ' before they make their savings decision; these agents do not face background risk. Given $\mathcal{V}^i$ as the value function for a given type, we solve the PI and FI agents' new problems to get their savings decision in the absence of background risk. Then we use the new policy function to simulate a time series of savings to compare with the equilibrium decisions. We conduct similar experiments by eliminating the saving risk (agents know r' but not w' or ϵ'), and no risk (agents know r', w', and ϵ'). We will discuss the no-risk case explicitly since it turns out that the other experiments are qualitatively similar but of smaller magnitudes.

Figure F.10 plots the time series in the case of no risk. Panel (a) shows that even though both agents know with certainty the value of (r', w', ϵ') before making their decision, the divergence in

[27] The following results are not different when the PI agent's forecast is used instead.

saving is still large. Panel (b) shows the difference between the no-risk case and the equilibrium behavior: risks do not play much of a role in the saving divergence.[28] Thus the divergence has to be from heterogeneity in the marginal value of wealth. Under certainty, the first-order condition of problem (3.1) is

$$-u'\left(m-k'\right)+\beta\frac{\partial\mathcal{V}}{\partial m}\left(k'\left(1+r'\right)+w'\exp\left(\epsilon'\right)\overline{h}\right)=0. \tag{3.2}$$

If the FI agent saves more for a given m, his marginal value over wealth has to be higher. Figure F.11 compares the marginal value of the PI and the FI agent for various r and w values; the FI agent's marginal value over wealth is greater than the PI agent's. The pattern of the difference is similar to the pattern observed in savings shown in Figure F.3. Thus the saving heterogeneity is mainly due to heterogeneity in the marginal value of wealth. Our quantitative model shows that better information increases the marginal value over wealth, leading to the FI agent saving more.

Before ending this subsection, we present a discussion of the indirect effect of information on savings. Specifically, we show how different information sets induce a wedge in the marginal value of wealth. Our argument can be developed using a simple three-period model. We add another period to (3.1):

$$\begin{aligned} V^i(m_0) &= \max_{s_1^i\in[0,m_0]}\left\{u\left(m_0-s_1^i\right)+\beta\int\limits_{r_1,w_1}\mathcal{V}^i\left(s_1^i\left(1+r_1\right)+w_1\right)d\mathcal{F}^i\left(r_1,w_1\right)\right\} \\ \mathcal{V}^i\left(m_1\right) &= \max_{s_2^i\in[0,m_1]}\left\{u\left(m_1-s_2^i\right)+\beta\int\limits_{r_2,w_2}u\left(s_2^i\left(1+r_2\right)+w_2\right)d\mathcal{F}^i\left(r_2,w_2\right)\right\}. \end{aligned} \tag{3.3}$$

Since the role of ϵ is the same as w, generating background risk, it is removed to simplify the expression. For expository purposes, assume that $\left\{r_1^i,r_2^i,w_1^i,w_2^i\right\}$ are iid. Both PI and FI agents have the same period utility function $u\left(c\right)$, an increasing concave function. By applying the envelope theorem to the second period problem, the marginal value over wealth in the second period can be written as

$$\frac{\partial\mathcal{V}^i\left(m_1\right)}{\partial m}=u'\left(m_1-s_2^*\left(m_1\right)\right),$$

where $s_2^*\left(m_1\right)$ is the optimal savings function; the factors that determine the size of the wedge are

[28] The small role that risk plays here could be due to two features: risks are relatively small or the value function $\mathcal{V}^i$ is close to linear. Since the background risk is not small – in particular, ϵ has a large standard deviation relative to aggregate income – the near-linearity of the indirect utility $\mathcal{V}$ is likely to be the culprit.

therefore the slope of the period utility function and factors that affect the shape of the savings function $s_2^*(m_1)$. If $s_2^{FI}(m_1) > s_2^{PI}(m_1)$, we have $\frac{\partial \mathcal{V}^{FI}(m_1)}{\partial m} > \frac{\partial \mathcal{V}^{PI}(m_1)}{\partial m}$; the FI agent's marginal utility of future wealth is higher than the PI agent's. This effect feeds into the first-period problem and magnifies the wedge in s_1^*, consequently increasing the wedge in the first-period marginal value over wealth. For a long-lived agent, this feedback will continue to accumulate, generating a potentially large difference in current savings. Note that this effect happens even when the agent perceives no risk in (r_1, w_1), as in the no-risk case in the previous section.

The propositions below illustrate how the distribution of $\left(r_2^i, w_2^i\right)$ affects $s_2^i(m_1)$.[29] In the propositions, we consider only the case when $s_2^i(m_1) > 0$ for $i \in \{PI, FI\}$.

Proposition 1. *If $r_2^{FI} \succ_{FOSD} r_2^{PI}$ and $u(c) = \log(c)$, then $s_2^{FI}(m_1) > s_2^{PI}(m_1)$.*

Proposition 2. *If $r_2^{FI} \succ_{SOSD} r_2^{PI}$ and $u(c) = \log(c)$, then $s_2^{FI}(m_1) > s_2^{PI}(m_1)$.*

Proposition 3. *If $w_2^{FI} \succ_{FOSD} w_2^{PI}$, then $s_2^{FI}(m_1) > s_2^{PI}(m_1)$.*

Proposition 4. *If $w_2^{FI} \succ_{SOSD} w_2^{PI}$ and $u(c)$ is CRRA, then $s_2^{FI}(m_1) > s_2^{PI}(m_1)$.*

The proofs of the above propositions can be found in Appendix E; interpretation of these propositions is straightforward and can be found in Gollier (2001). If the information sets of the two agents imply one of the conditions in the above propositions, the FI agent's marginal value over wealth will be higher. Unfortunately, our quantitative model does not always satisfy these conditions; for example, FI and PI agents disagree about both the conditional expectation and the variance of (r', w'), but the direction of the disagreement is not constant over time. As a result, these propositions serve to highlight mechanisms at work in our model only; ultimately, the quantitative results resolve the issue in favor of higher savings by FI agents. Note that while this result may seem obvious, it need not hold in general because better information could translate into a decreased demand for precautionary savings. In our model, the proof in Chamberlain and Wilson (2000) can be used to show that

$$\overline{r} < \frac{1-\beta}{\beta}$$

[29] $x \succ_{FOSD} y$ denotes x first-order stochastically dominates y. Likewise, $x \succ_{SOSD} y$ denotes x second-order stochastically dominates y. Our definitions of FOSD and SOSD are taken from Hadar and Russell (1969).

where $\bar{r}$ is the average return on savings, so that in the absence of risk households would all have zero assets.[30] Thus, in a sense all savings is precautionary, so anything that reduces the demand for precautionary savings will lead to less asset accumulation; that is, the FI agents could have been poorer.

It is important to note that it is the effect of permanent information that significantly changes the saving rate. This can be seen in Figure F.7, where the PI agent is endowed with the FI forecasts before making their savings decision. Since information in this experiment is temporary, the savings decision is a trade-off between the marginal utility of consumption today and the marginal value of saving without this information in the future. The effect of temporary information on savings is very small compared with the effect of permanent information (that is, the difference between knowing what an FI agent knows today and actually being an FI agent). This result suggests that the PI agent might not gain much from temporary information. In other words, the PI agent is willing to pay a little to get rid of risks in the near future but much more to get rid of risks permanently. The implication of this result is that the welfare cost of business cycles could be larger in an economy populated by PI agents, since elimination of business cycle risk renders all agents equally informed. We examine the size of this effect below.

3.3. Information and Inequality

As mentioned in the introduction, the existing literature has resorted to several different mechanisms to account for the extreme wealth concentration evident in the data. One mechanism in particular generates inequality by differential access to high-return assets, such as risky stocks (Guvenen 2005) or business capital (Cagetti and De Nardi 2006). In our model all households can freely access to the same asset markets but they are differently informed about the return distribution. In the previous section, we showed that FI agent has an incentive to save more, so in an economy where both FI and PI agents live together, we would expect to see more FI agents in the top deciles of the wealth distribution and more PI agents in the bottom. We are interested in assessing the quantitative strength of this effect: does information heterogeneity play an important role in producing inequality?

Figure F.13 displays the wealth distribution of the mixed economy ($\Lambda = 0.5$). The plot shows the wealth distribution of each three groups: the entire population, the PI agents only, and the

[30] In a partial equilibrium sense; obviously if all agents hold zero assets the rental rate of capital would rise until positive amounts of capital are held.

FI agents only. Table 4 shows that almost 60 percent of the bottom 10 percent of the wealth distribution are PI agents while 72 percent of the top 1 percent are FI agents; PI agents are concentrated among the poor and FI agents are concentrated among the rich. If information heterogeneity played no role in determining the wealth distribution, the share of households would be 50 percent independent of the wealth percentage.[31]

Figure F.14 shows the Lorenz curves for the total population, the PI fraction only, and the FI fraction only. While FI agents are generally wealthier than PI agents, the distribution of wealth within each group is not significantly different from the population as a whole. In addition, the wealth distribution is not highly concentrated as observed in the US data. The Gini coefficient of wealth distribution in the U.S. is around 0.8, while the Gini coefficients for the total population is 0.43 and those of the PI and FI populations are 0.437 and 0.418, respectively.[32] For comparison, we note that the extreme economies do not produce very different results from the mixed economy: the Gini coefficient in the FI economy is 0.424 while in the PI economy it is 0.431. Our model suggests only a modest role for information heterogeneity in the generation of inequality; we will have some comments regarding this result in the conclusion.

We examine the underlying mechanisms in our model by studying another mixed economy ($\Lambda = 0.5$) where PI agents use the same forecast function as FI agents but their forecasts are exogenously different. We compute four cases: i) $E_t^{PI} r_{t+1}$ is biased downward; ii) $V_t^{PI} r_{t+1}$ is arbitrarily larger; iii) $E_t^{PI} w_{t+1}$ is biased downward; and iv) $V_t^{PI} w_{t+1}$ is arbitrarily larger.[33] The purpose of these exercises is to isolate the effect of each mechanism on wealth distribution, particularly the disagreement in expectations and variances. Figures F.15 and F.16 report the results. In summary, lower $E_t^{PI} r_{t+1}$ or higher $V_t^{PI} r_{t+1}$ causes PI agents to receive less utility value from saving, generating low wealth for this group. On the contrary, lower $E_t^{PI} w_{t+1}$ or higher $V_t^{PI} w_{t+1}$ induces PI agents to receive more value from saving due to an increased precautionary motive, thus saving more and becoming relatively wealthy.

Finally, to demonstrate the potential role of information as a source of wealth concentration we compute an economy where $\Lambda = 0.8$ and PI agents have a downward biased $E_t^{PI} r_{t+1}$. In

[31] Our economy with homogeneous information satisfies the mixing conditions in Aiyagari and Alvarez (2001), so all agents have the same long-run probability of entering any interval $[m_1, m_2]$. Since they also face the same process for the idiosyncratic shock, all agents have the same unconditional distribution of individual states. Any deviation from these independence is due to the small sample properties of z, the aggregate shock, which we minimize by exploiting a very long data series.

[32] The empirical value is taken from Budría Rodríguez *et al.* (2001).

[33] Appendix F explains how we formulate the forecasts of PI agents.

this experiment, we assume that $E_t^{PI} r_{t+1} \simeq 0.952 E_t^{FI} r_{t+1}$, or $E_t^{PI} r_{t+1}$ is around 4.8 percent lower than $E_t^{FI} r_{t+1}$.[34] Table 5 reports the results comparing with the US data. In terms of wealth concentration, there is a dramatic improvement when we assume downward biased $E_t^{PI} r_{t+1}$. The wealth Gini coefficient increases from 0.42 in the FI economy to 0.73 in the biased economy. The top 20 percent richest holds almost 82 percent of total wealth (all of them FI agents), while the bottom 40 percent are mostly PI agents and own only 3.38 percent; these numbers are very close to US data.[35] However, the model still fails to generate very rich households: the top one percent richest in the model owns only 9.5 percent of total wealth while in the data they own 32 percent.[36] Figures F.17 and F.18 show the wealth distribution and the Lorenz curve.[37]

We can explain the large effect of a small bias in $E_t^{PI} r_{t+1}$ using Figure 1 from Aiyagari and McGrattan (1998). In the class of models we consider here, average asset supply is upward-sloping in the long run, ranging from the borrowing constraint to ∞ as r approaches its upper bound $\beta^{-1} - 1$. In a complete market economy $r = \beta^{-1} - 1$, so the deviation between the average interest rate and the time rate of preference measures the extent to which efficient risk-sharing fails; it is known that this class of models one asset does a good job of providing self-insurance, leading to a small gap. Thus the economy's average interest rate lies on the very elastic portion of the asset supply curve, leading to large changes in holdings when expected returns vary. It is interesting that the bias generated endogenously in our model is so small that agents with different information endowments still end up close together.

3.4. Information and the Costs of Fluctuations

To examine the effect of information on the cost of business cycles, we compute another economy where there is only an idiosyncratic shock, as in Aiyagari (1994).[38] Then we compare the ag-

[34] Note that this bias is much larger than the one observed in the equilibrium with endogenous PI forecasts, which was just under 0.25 percent.

[35] Biased expectations function similarly to heterogeneity in discount factors. Tsyrennikov (2006) shows an exact correspondence in a complete market economy.

[36] Because we calibrate the earnings process using the PSID, it is not surprising that we fail to generate very wealthy households – these households are not present in the sample, as we noted in the Introduction.

[37] We also examined the effect of bias on the wealth distribution when both FI and PI agents are more risk averse. For the case that $V_t^{PI} r_{t+1}$ is arbitrarily larger (ii) and the coefficient of relative risk aversion is 2, the Gini coefficient falls to 0.394, compared with 0.425 under logarithmic utility. The fact that higher risk aversion tends to lead to less wealth inequality has been noted before; see Díaz, Pijoan-Mas, and Ríos-Rull (2003) for a discussion.

[38] These calculations are 'behind the veil of ignorance' as they do not condition on current states. Our calculation therefore ignores the welfare cost during the transition, which can be significant. In addition, since in our model z_t is independent of ϵ_t, removing business cycles does not affect the process generating ϵ_t. Krusell and Smith (2002) discusses the issue of how to remove aggregate risk when idiosyncratic risk is correlated with it.

gregate welfare averaged over business cycles of the two extreme economies to average welfare in the economy without aggregate shocks. Not surprisingly, the aggregate welfare when there is no aggregate risk is highest at -6.87640, followed by the average welfare in FI economy at -6.92037. The average welfare in PI economy is lowest at -7.07462.

To meaningfully measure the cost of fluctuations, we compute the fraction of consumption x agents are willing to give up in each period if allowed to live in the new economy where there is no aggregate shock. $\overline{V}^i$ is the average welfare of agents living in PI (FI) economy and $\overline{W}$ is the aggregate welfare of agents living in the economy without aggregate shocks. We can solve for x as

$$x = 1 - \exp\left((1-\beta)\left(\overline{V}^i - \overline{W}\right)\right).$$

The fraction of consumption agents in the PI economy are willing to give up is 0.1982 percent while the fraction in the FI economy is 0.044 percent. While both numbers are small, the PI cost is 450 percent of the FI cost.[39] Above we noted that there are states of the world where PI agents are too 'optimistic;' it turns out that their expected lifetime utility is higher than FI agents' lifetime utility conditioned on being in one of those states currently. But these states are extremely rare, so that integrating over the stationary distribution does not assign them much weight. Of course, with a different information structure the results could be different, although we do not find it likely that the cost of fluctuations would ever be higher for FI agents than PI agents.[40]

Despite the small numbers obtained here, we think our results regarding the relative cost of fluctuations has some importance. Alternative preference structures can generate much larger welfare costs for cycles – Dolmas (1998) and Tallarini (2000) are two prominent examples. In our economy, these preferences could generate very large costs for PI agents; one example in Dolmas (1998) suggests a cost on the order of 1 percent of consumption for an FI agent, meaning that 4 percent of consumption would be in the ballpark for PI agents. The small aggregate numbers do not tell the entire story, either. If we look at 'marginal gains' for business cycles – meaning the gain in consumption equivalents generated by reducing the variance of the aggregate shock incrementally to zero – we find additional heterogeneity across the two types. As seen in Table 6,

[39] Small aggregate costs of fluctuations in this class of models are also found in Krusell and Smith (1999,2002) and Mukoyama and Şahin (2006). We stress that none of these models are calibrated to match the behavior of the cross-sectional distribution of consumption, so small costs may be an indication of excessively-smooth consumption, a common problem in business cycle models. Since those papers concentrate on the relative cost of cycles across the wealth distribution this criticism does not diminish their contributions.

[40] Because the allocation is not constrained-efficient, PI agents could have higher welfare than FI agents despite their information disadvantage.

the marginal gain for FI agents is essentially constant.[41] For PI agents, the marginal gain function is decreasing: removing one quarter of the variance in the aggregate shock generates a welfare gain of nearly 0.08 percent, 40 percent of the total gain of 0.1982 percent. Thus, even small reductions in business cycle volatility could have important welfare consequences; we think is premature to dismiss the potential of stabilization policy given our results. The explicit study of policies that reduce business cycles is a topic for future work, however, as this paper is already lengthy.

4. Conclusion

Our paper is a study of the role of information in wealth inequality. While we find only modest implications in the calibrated model, we have demonstrated the potential role of information to generate wealth concentration. Our next step is to construct a more elaborate information structure which allows information to have more impact on agents' decisions. We would like to introduce more sophisticated behavior by PI agents by permitting them to fit more elaborate time series models to the artificial data; ideally, we would define the observable aggregates and let the households learn as much as they can about the underlying economy from these observables using filtering methods. It also seems important to permit learning – agents estimate models and update them period-by-period – since information would play a more important role in its presence. While the computational burden of that economy exceeds anything feasible currently, it may not in the future.

Another feature which we plan to relax is the assumption of only one asset. The results in Krusell and Smith (1997) show that extreme portfolios typically result in an FI economy: the poor hold bonds and the rich hold stocks. Poor agents have little incentive to hold stocks because the return is highly correlated with their background risk, making it a poor vehicle for insurance; as a result they tend save in the form of bonds. In contrast, rich agents feel 'well-insured' and therefore hold stocks to claim the equity premium. Our mixed economy has the potential to increase the incentive of the rich FI agents to take extreme portfolio positions as they become wealthier (in a relative sense) in the presence of PI agents. Conversely, the poor PI agents will do the opposite; even a small equity premium might eventually generate significant wealth concentration among

[41] Alvarez and Jermann (2004) discuss the theoretical calculation of the marginal cost function for business cycle uncertainty. The total cost number is the value of reducing the business cycle volatility from σ_e to the number in the first column, while the marginal cost is the value of reducing it to the number in the first column from the next higher value.

the informed.[42] Furthermore, the increased sensitivity to business cycles displayed by the PI agents would act to drive up the equity premium. A similar effect may play an important role in generating the home bias puzzle observed in international financial markets.[43]

We also believe that information plays a role in the nature of the idiosyncratic risk faced by households – the FI and PI agents would choose to face different idiosyncratic risk. In particular, information heterogeneity may play an important role in determining the occupational choices of households; for example, given the high risk faced by entrepreneurial activities (some of which is aggregate risk, as shown by the high volatility of proprietor's share of income), FI agents would seem to be more likely to engage in them. Furthermore, this environment may help explain the absence of a significant private equity premium. It would also play a role in the decision to participate in costly labor market or product market search; poorly-informed agents may be at a serious disadvantage and end up with lower wages or higher costs, magnifying the wealth gap.

Another line of extension we have planned is to use the model to study an equilibrium with heterogeneous beliefs by introducing m into the forecasting equation for (r, w). While m is idiosyncratic it is correlated with K; during periods in which aggregate capital is high, individual capital will tend to be high as well. As a result, PI agents will gain forecasting accuracy by conditioning on their individual cash-on-hand level. The upshot of that change is that 'beliefs' – probability distributions over future events – are now heterogeneous; each individual will have a distinct distribution of future returns. As mentioned above, because the wealth distribution and K are not cyclically volatile we found that this change made little difference in our current model; in models with state variables that fluctuate more over time, heterogeneity of beliefs may play a more important role.[44] A similar effect could be achieved by allowing correlation between ϵ and z, although in that case the heterogeneity of beliefs would be exogenous; this method would be considerably less computationally burdensome and we are currently pursuing it.

Finally, our study also offers another perspective on policies related to business cycles. Our result shows that the cost of aggregate fluctuations is larger when the population is poorly informed. If the policies that completely remove business cycles are too costly, a cheaper alternative may be a policy that moves the partially-informed economy towards the fully-informed one; this policy

[42] Guvenen (2005) generates a wide disparity in wealth levels by proscribing stockholding for some households.

[43] Ahearne, Griever, and Warnock (2004) find evidence that information plays an important role in determining the share of foreign assets in domestic portfolios.

[44] Heterogeneity in beliefs could help account for the puzzlingly-high volume of trade in risky asset markets documented by DeJong and Espino (2006), particularly at high frequencies.

emphasizes the important role of institutions that provide timely public information and reflects the relatively large gain experienced by PI agents from the partial elimination of cycles.[45] We show that there is only a small gain if agents do not believe that the institution can always provide short-term accurate forecasts on a timely schedule; that situation would approximate the case of PI agents who are endowed with FI agents' one-period forecasts. Larger effects are generated if the PI agent actually becomes fully-informed (that is, actually has the information set of the FI agent forever), meaning that the institution is reliably providing information. A model with endogenous and costly acquisition of information, such as Veldkamp (2006), would be needed to develop this idea further.

[45] Two papers that investigate information provision and business cycles are Bomfim (2001b) and Aruoba (2004).

References

[1] Ahearne, Alan G., William L. Griever, and Francis E. Warnock (2004), "Information Costs and Home Bias: An Analysis of US Holdings of Foreign Equities," *Journal of International Economics* **62(2)**, pp. 313-36.

[2] Aiyagari, S. Rao (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* **109**, pp. 659-84.

[3] Aiyagari, S. Rao and Fernando Alvarez (2001), "Efficient Dynamic Monitoring of Unemployment Insurance Claims," unpublished manuscript, University of Chicago.

[4] Aiyagari, S. Rao and Ellen R. McGrattan (1998), "The Optimum Quantity of Debt," *Journal of Monetary Economics* **42(3)**, pp. 447-69.

[5] Alvarez, Fernando and Urban J. Jermann (2004), "Using Asset Prices to Measure the Cost of Business Cycles," *Journal of Political Economy* **112(6)**, pp. 1223-56.

[6] An, Sungbae (2006), "On Accommodation of Capital Stock Series in the Estimation of Dynamic Economy," unpublished manuscript, University of Pennsylvania.

[7] Aruoba, S. Boragan (2004), "Data Uncertainty in General Equilibrium," unpublished manuscript, University of Maryland.

[8] Bomfim, Antulio N. (2001a), "Heterogeneous Forecasts and Aggregate Dynamics," *Journal of Monetary Economics* **47(1)**, pp. 145-61.

[9] Bomfim, Antulio N. (2001b), "Measurement Error in General Equilibrium: The Aggregate Effects of Noisy Economic Indicators," *Journal of Monetary Economics* **48(3)**, pp. 585-603.

[10] Budría Rodríguez, Santiago, Javier Díaz-Giménez, Vincenzo Quadrini, and José-Víctor Ríos-Rull (2001), "Updated Facts on the U.S. Distributions of Earnings, Income, and Wealth," Federal Reserve Bank of Minneapolis *Quarterly Review* **26(3)**, pp. 2-35.

[11] Cagetti, Marco and MariaCristina De Nardi (2006), "Entrepreneurship, Frictions, and Wealth," forthcoming, *Journal of Political Economy.*

[12] Castañeda, Ana, Javier Díaz-Giménez, and José-Víctor Ríos-Rull (2003), "Accounting for Wealth and Income Inequality," *Journal of Political Economy* **111(4)**, pp. 818-57.

[13] Chamberlain, Gary and Charles A. Wilson (2000), "Optimal Intertemporal Consumption Under Uncertainty," *Review of Economic Dynamics* **3(3)**, pp. 365-95.

[14] DeJong, David N. and Emilio Espino (2006), "The Cyclical Behavior of Equity Turnover in Financial Markets," unpublished manuscript, University of Pittsburgh.

[15] Díaz, Antonia, Josep Pijoan-Mas, and José-Victor Ríos-Rull (2003), "Precautionary Savings and Wealth Distribution Under Habit Formation Preferences," *Journal of Monetary Economics* **50(6)**, pp. 1257-91.

[16] Dolmas, Jim (1998), "Risk Preferences and the Welfare Cost of Business Cycles," *Review of Economic Dynamics* **1(3)**, pp. 646-76.

[17] Gollier, Christian (2001), *Economics of Risk and Time*, MIT Press.

[18] Guvenen, M. Fatih (2005), "A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-Sectional Heterogeneity?" working paper, University of Texas-Austin.

[19] Hadar, Josef, and William R. Russell (1969), "Rules for Ordering Uncertain Prospects," *American Economic Review* **59**, pp. 25-33.

[20] Judd, Kenneth L. (1998), *Numerical Methods in Economics*, MIT Press.

[21] Krusell, Per and Anthony A. Smith, Jr. (1997), "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics* **1(2)**, pp. 387-422.

[22] Krusell, Per and Anthony A. Smith, Jr. (1998), "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy* **106(6)**, pp. 867-96.

[23] Krusell, Per and Anthony A. Smith, Jr. (1999), "On the Welfare Effects of Eliminating Business Cycles," *Review of Economic Dynamics* **2(1)**, pp. 245-72.

[24] Krusell, Per and Anthony A. Smith, Jr. (2002), "Revisiting the Welfare Effects of Eliminating Business Cycles," unpublished manuscript, University of Rochester.

[25] Mukoyama, Toshihiko and Ayşegül Şahin (2006), "Costs of Business Cycles for Unskilled Workers," *Journal of Monetary Economics* **53(8)**, pp. 2179-93.

[26] Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron (2004), "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy* **112(3)**, pp. 695-717.

[27] Tallarini, Thomas D., Jr. (2000), "Risk-Sensitive Real Business Cycles," *Journal of Monetary Economics* **45(3)**, pp. 507-32.

[28] Telmer, Chris I. and Stanley E. Zin (2002), "Prices as Factors: Approximate Aggregation with Incomplete Markets," *Journal of Economic Dynamics and Control* **26(7-8)**, pp. 1127-57.

[29] Tsyrennikov, Viktor (2006), "Heterogeneous Information, Wealth Distribution, and Asset Prices," unpublished manuscript, New York University.

[30] Veldkamp, Laura (2006), "Information Markets and the Comovement of Asset Prices," *Review of Economic Studies* **73**, pp. 823-45.

[31] Young, Eric R. (2006), "Approximate Aggregation," unpublished manuscript, University of Virginia.

A. Appendix A

In this appendix we show how to derive the price process for r and w used by an FI agent. Given competitive factor markets we can solve for z and $\log(K)$ by equating factor prices to marginal products:

$$\begin{aligned} r+\delta &= \alpha \exp(z) K^{\alpha-1}\overline{H}^{1-\alpha} \\ w &= (1-\alpha)\exp(z) K^{\alpha}\overline{H}^{-\alpha} \end{aligned}$$

implies

$$\begin{aligned} \log\left(\frac{K}{\overline{\overline{H}}}\right) &= \log(w) - \log(r+\delta) + \log\left(\frac{\alpha}{1-\alpha}\right) \\ z &= \log(r+\delta) - \log(\alpha) + (1-\alpha)\log\left(\frac{K}{\overline{\overline{H}}}\right). \end{aligned}$$

Now substitute these expressions into the dynamic equation (2.6) to obtain

$$\log\left(\frac{K'}{\overline{\overline{H}}}\right) = a_0 + a_1 \log\left(\frac{\alpha}{1-\alpha}\right) + (a_1-1)\log\left(\overline{H}\right) + a_1\log(w) - a_1\log(r+\delta) + a_2 z.$$

Now we substitute the above result and the technology shock process (2.2) into the expressions for r' and w' to obtain

$$\begin{aligned} \log\left(r'+\delta\right) &= \log(\alpha) + (\alpha-1)\left(a_0 + a_1\log\left(\frac{\alpha}{1-\alpha}\right) + (a_1-1)\log\left(\overline{H}\right)\right) + (\alpha-1)a_1\log(w) - \\ &\quad (\alpha-1)a_1\log(r+\delta) + ((\alpha-1)a_2+\rho)z + e' \\ \log\left(w'\right) &= \log(1-\alpha) + \alpha\left(a_0 + a_1\log\left(\frac{\alpha}{1-\alpha}\right) + (a_1-1)\log\left(\overline{H}\right)\right) - \alpha a_1\log(r+\delta) + \\ &\quad \alpha a_1\log(w) + (\rho+\alpha a_2)z + e'. \end{aligned}$$

Then substitute out the expressions for $\log(K)$ and z to obtain (r', w') as functions of (r, w):

$$\begin{aligned}
\log\left(r'+\delta\right) &= \log(\alpha) + (\alpha-1)\left(a_0 + a_1 \log\left(\frac{\alpha}{1-\alpha}\right) + (a_1-1)\log\left(\overline{H}\right)\right) + (\alpha-1)\,a_1 \log(w) - \\
&\quad (\alpha-1)\,a_1 \log(r+\delta) + ((\alpha-1)\,a_2+\rho) + \\
&\quad (1-\alpha)\left(\log(w) - \log(r+\delta) + \log\left(\frac{\alpha}{1-\alpha}\right)\right) \\
\log\left(w'\right) &= \log(1-\alpha) + \alpha\left(a_0 + a_1\log\left(\frac{\alpha}{1-\alpha}\right) + (a_1-1)\log\left(\overline{H}\right)\right) - \alpha a_1 \log(r+\delta) + \\
&\quad \alpha a_1 \log(w) + (\rho + \alpha a_2) \\
&\quad (1-\alpha)\left(\log(w) - \log(r+\delta) + \log\left(\frac{\alpha}{1-\alpha}\right)\right).
\end{aligned}$$

Simplifying this expression yields

$$\begin{aligned}
\log\left(r'+\delta\right) &= A_0 + A_1 \log(r+\delta) + A_2 \log(w) + \varepsilon_r' \\
\log\left(w'\right) &= A_3 + A_4 \log(r+\delta) + A_5 \log(w) + \varepsilon_w'
\end{aligned}$$

where

$$\begin{aligned}
A_0 &= \log(\alpha) + (\alpha-1)\left(a_0 + a_1\log\left(\frac{\alpha}{1-\alpha}\right) + (a_1-1)\log\left(\overline{H}\right)\right) - \\
&\quad ((\alpha-1)\,a_2+\rho)\,(\alpha\log(\alpha) + (1-\alpha)\log(1-\alpha)) \\
A_1 &= (1-\alpha)\,a_1 + \alpha\,((\alpha-1)\,a_2+\rho) \\
A_2 &= (\alpha-1)\,a_1 + (1-\alpha)\,((\alpha-1)\,a_2+\rho) \\
A_3 &= \log(1-\alpha) + \alpha\left(a_0 + a_1\log\left(\frac{\alpha}{1-\alpha}\right) + (a_1-1)\log\left(\overline{H}\right)\right) - \\
&\quad (\alpha a_2+\rho)\,(\alpha\log(\alpha) + (1-\alpha)\log(1-\alpha)) \\
A_4 &= -\alpha a_1 + \alpha\,(\alpha a_2+\rho) \\
A_5 &= \alpha a_1 + (1-\alpha)\,(\alpha a_2+\rho)\,.
\end{aligned}$$

B. Appendix B

In this appendix we first discuss our estimation of the process for z and then the discretized approximation for ϵ. For the technology shock, we use aggregate output, fixed assets and inventories, and average weekly hours from BEA (1964-2000). Since the data of aggregate capital is annual, we approximate the annual process first, then find a quarterly representation of this annual process using an Indirect Inference approach.

Given $\alpha = 0.36$, we construct the annual series of z_t^a. In addition, we also construct an annual series of depreciation rate δ_t^a. Our estimated AR(1) process of these annual series is

$$\begin{aligned} z_{t+1}^a &= \rho_z^a z_t^a + \eta_{t+1} \\ \delta_{t+1}^a &= \mu_\delta^a + \rho_\delta^a \delta_t^a + \upsilon_{t+1} \end{aligned} \tag{B.1}$$

with $\rho_z^a = 0.8586$, $\sigma_\eta^2 = 0.0134^2$, $\mu_\delta^a = 0.0075$, $\rho_\delta^a = 0.7854$, $\sigma_\upsilon^2 = 0.0004^2$, and the correlation coefficient between η_t and υ_t is 0.4163. In the above expression we remove the linear time trends and normalize mean of z to zero. We then approximate a quarterly process from this annual process as follows. We can write the relationship between the quarter series and annual series as

$$\begin{aligned} z_t^a &= z_{t,4}^q \\ 1 - \delta_t^a &= (1 - \delta_{t.1}^q)(1 - \delta_{t.2}^q)(1 - \delta_{t.3}^q)(1 - \delta_{t.4}^q), \end{aligned} \tag{B.2}$$

where $x_{t.j}^q$ is a quarterly data in quarter j of year t. Our objective is to find a stationary quarterly process that can generate an annual process close to the one represented by (B.1). We restrict the class of approximated quarterly processes to be an AR(1).[46]

$$\begin{aligned} z_{t+1}^q &= \rho_z^q z_t^q + e_{t+1} \\ \delta_{t+1}^q &= \mu_\delta^q + \rho_\delta^q \delta_t^q + \varsigma_{t+1} \\ \begin{bmatrix} e_t \\ \varsigma_t \end{bmatrix} &\sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_e^2 & \sigma_{e\varsigma}^2 \\ \sigma_{e\varsigma}^2 & \sigma_\varsigma^2 \end{bmatrix} \right). \end{aligned} \tag{B.3}$$

To find the coefficients of this quarterly process, we perform the following steps.

1. Simulate two independent series of normal random variables of length 10^6.

2. Guess the parameters in the quarterly process (B.3).

3. Use the normal random variables from step 1 to simulate the quarterly series for (z_t^q, δ_t^q).

4. Use the relationship (B.2) to get the simulated annual series and estimate an AR(1) process on this series.

[46] Notice that the relationship between the annual series and quarterly series is nonlinear. Even though the annual process is AR(1), the quarterly process may not be.

5. Calculate the weighted sum of differences between the estimated coefficients and the ones in (B.1).[47]

6. Use a numerical minimization routine to find the coefficients of quarterly process.[48]

The result from our estimation is following: $\rho_z^q = 0.964$, $\sigma_e^2 = 0.0071^2$, $\mu_\delta^q = 0.00087$, $\rho_\delta^q = 0.9298$, $\sigma_\varsigma^2 = 0.000081^2$, and the correlation coefficient between e and ς is 0.5326. Since the variance of depreciation shock is very small, in our model we assume it to be a constant and have only one aggregate shock z_t.

For the idiosyncratic productivity shock, we use the AR(1) earning process from Storesletten, Telmer, and Yaron (2004), yielding $\rho_\epsilon = 0.9412$ and $\sigma_\nu^2 = 0.0958^2$. Then we approximate this continuous process with a Markov chain that has 7 states; the transition matrix is given by

$$\pi\left(\epsilon'|\epsilon\right) = \begin{bmatrix} 0.8133 & 0.1866 & 0.0001 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0366 & 0.8213 & 0.1421 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0535 & 0.8410 & 0.1055 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.00005 & 0.0761 & 0.8477 & 0.0761 & 0.00005 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.1055 & 0.8410 & 0.0535 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.1421 & 0.8213 & 0.0366 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0001 & 0.1866 & 0.8133 \end{bmatrix}$$

and the vector of realizations for $\exp(\epsilon)$ is $\{0.447, 0.589, 0.749, 0.942, 1.185, 1.508, 1.986\}$.

C. Appendix C

Assume $\Omega_i^{FI} = \left\{m^i, \epsilon^i, \Gamma, z, \mathcal{Q}\right\}$ where $\mathcal{Q}$ is the collection of structural equations for the model, z is the aggregate shock, and Γ is the distribution over (m, ϵ, θ) where θ denotes 'type'.[49] Also, $\Omega_i^{PI} = \left\{m^i, \epsilon^i, r, w\right\}$, where r is the current realization of the rental rate and w is the current realization of the wage rate; the PI agent does not know $(z, \Gamma, \mathcal{Q})$. Note that $\Omega^{PI} \subset \Omega^{FI}$. Define $f(\mathcal{X})$ as the space of distributions over elements of the space $\mathcal{X}$. Denote $(\mathcal{E}, \mathcal{T}, \mathcal{Z})$ as the spaces of (ϵ, Γ, z) respectively. Also note that (m, r, w) is an element of $\mathcal{R}_+^3$.

[47] An alternative is to minimize the difference between the spectrum of the annual process (B.1) and the simulated process.

[48] In another estimation where we assume the quarterly process is ARMA(1,1), the simulated annual process fits the annual process (B.1) better. Because an ARMA(1,1) increases the state space of the model, the computational cost is too large to consider currently.

[49] If agents know z and Γ, they can derive (r, w). Thus we drop (r, w) from Ω^{FI}.

A recursive competitive equilibrium for the economy is a value function $V^{FI}:\mathcal{R}_+\times\mathcal{E}\times\mathcal{T}\times\mathcal{Z}\to\mathcal{R}$, a value function $V^{PI}:\mathcal{R}_+\times\mathcal{E}\times\mathcal{R}^2_{++}\to\mathcal{R}$, decision rules $\left(g_k^{FI},g_c^{FI}\right):\mathcal{R}_+\times\mathcal{E}\times\mathcal{T}\times\mathcal{Z}\to\mathcal{R}^2_+$, decision rules $\left(g_k^{PI},g_c^{PI}\right):\mathcal{R}_+\times\mathcal{E}\times\mathcal{R}^2_{++}\to\mathcal{R}^2_+$, pricing functions $(R,W):\mathcal{T}\times\mathcal{Z}\to\mathcal{R}^2_{++}$, a transition function $F:\mathcal{T}\times\mathcal{Z}\times\mathcal{Z}\to\mathcal{T}$, and a stochastic transition function $G:\mathcal{R}^2_{++}\to f\left(\mathcal{R}^2_{++}\right)$ such that

Definition 1. *1. Given (R,W,F), $\left(V^{FI},g_k^{FI},g_c^{FI}\right)$ solve the FI agent problem (2.5) for all (m,ϵ,Γ,z);*

2. Given G, $\left(V^{PI},g_k^{PI},g_c^{PI}\right)$ solve the PI agent problem (2.9) for all (m,ϵ,r,w);

3. $r=R(K,z)=MPK-\delta$ and $w=W(K,z)=MPH$ for all (K,z);

4. Markets clear:

$$\begin{aligned}\overline{H} &= \sum_{\epsilon}\sum_{\theta='PI','FI'}\int \epsilon d\Gamma(m,\epsilon,\theta)\\ K &= \sum_{\epsilon}\sum_{\theta='PI','FI'}\int k d\Gamma(m,\epsilon,\theta)\\ \exp(z)K^{\alpha}\overline{H}^{1-\alpha} &= \sum_{\epsilon}\sum_{\theta='PI','FI'}\int c d\Gamma(m,\epsilon,\theta)+\sum_{\epsilon}\sum_{\theta='PI','FI'}\int k' d\Gamma(m,\epsilon,\theta)-(1-\delta)K;\end{aligned}$$

5. F is consistent with aggregation of $\left(g_k^{FI},g_c^{FI},g_k^{PI},g_c^{PI}\right)$, individual shock process (2.4), and the pricing functions (R,W);

6. G is consistent with (R,W,F) and technology shock process (2.2).

The last two conditions require clarification. Condition (*5*) states that given z', the evolution of Γ is obtained by aggregating decision rules and the individual shock process (2.4). For example, F must be consistent with

$$K'=\sum_{\epsilon}\sum_{\theta='PI','FI'}\int k' d\Gamma(m,\epsilon,\theta).$$

All other statistics must also be consistent. Condition (*6*) requires that the distribution of (r',w') given (r,w) is consistent with the equilibrium pricing functions applied to $F(\Gamma,z,z')$ and technology shock (2.2); that is, $r'=R(K',z')$ and $w'=W(K',z')$.

D. Appendix D

This appendix explains the algorithm to compute the equilibrium of the model. Our algorithm for solving the FI agent's problem is modified from Krusell and Smith (1998) and Young (2006). The objective of the algorithm is to approximate the value functions $\left(V^{FI}, V^{PI}\right)$, the law of motion for aggregate capital (2.6), and the price process (2.10). We divide the algorithm into three main parts. In summary, the first part is to solve for the value functions $\left(V^{FI}, V^{PI}\right)$ over a finite grid of (m, r, w), given a law of motion (2.6) and a price process (2.10). The second part is to solve for the policy functions k' over a much finer grid of (m, r, w) using V^{FI} and V^{PI} from the first part. The third part is to simulate the time series of (K_t, r_t, w_t) using the policy function from the second part and update the law of motion (2.6) and the price process (2.10). Then iterate from the first part using the updated law of motion and price process until their coefficients converge. The following subsections explain the algorithm in detail.

D.1. Part 1: Solving for $V^{FI}(m, \epsilon, r, w)$ and $V^{PI}(m, \epsilon, r, w)$

1. Discretize the space of m and denote this grid $\mathbf{m1}$. Since the value functions have more curvature where m is close to 0, we concentrate our grid points at low values of m. The total number of points we use is 135.

2. Guess $\{a_j\}_{j=0}^{2}$ in (2.6) and $\{b_j\}_{j=0}^{5}$ and $\left\{\sigma_r^2, \sigma_w^2, \sigma_{rw}^2\right\}$ in (2.10). Then compute $\{A_j\}_{j=0}^{5}$ as shown in Appendix A.

3. Discretize the space of (r, w) and denote these grids $(\mathbf{r1}, \mathbf{w1})$. To keep the range of $(\mathbf{r1}, \mathbf{w1})$ tight and consistent with the price process (2.10), we define the minimum and maximum grid points of $\mathbf{r1}$ and $\mathbf{w1}$ as five times their unconditional standard deviation computed from (2.10). The number of grid points in $\mathbf{r1}$ and $\mathbf{w1}$ is 4. In our model there is a little curvature along the r and w dimensions, so we evenly-space our points.

4. Guess initial value function V_0^{FI} and V_0^{PI} on the discretized grids of $(\mathbf{m1}, \boldsymbol{\epsilon}, \mathbf{r1}, \mathbf{w1})$.

5. Given $\{A_j\}_{j=0}^{5}$ and V_0^{FI}, solve the FI agent's problem (2.7) to get the policy function k' and use it to get V_1^{FI}. Iterate until V_n^{FI} converges. Given $\{b_j\}_{j=0}^{5}$, $\left\{\sigma_r^2, \sigma_w^2, \sigma_{rw}^2\right\}$, and V_0^{PI}, do the same in PI agent's problem.

D.2. Part 2: Solving for the policy functions $k' = g_k^i(m, \epsilon, r, w)$

1. Define finer grids $(\mathbf{m2}, \mathbf{r2}, \mathbf{w2})$. We use 270 points for **m2** and put more points near zero as before. For **r2** and **w2**, we use 15 points that are again equally-spaced.

2. Use the resulting value function from the first part in the RHS of problem (2.7) and (2.9) and solve for the policy functions k' for all points in the grids $(\mathbf{m2}, \mathbf{r2}, \mathbf{w2})$.

D.3. Part 3: Update law of motion and price process

1. Discretize the distribution $\Gamma(m, \epsilon, \theta)$ by defining a much finer grid along the m dimension, called **m3**. We use 2000 points in **m3**.

2. Simulate a long time series of $\{z_t\}_{t=1}^T$. We simulate $T = 30,000$ periods.

3. Guess the initial distribution Γ_1. Given z_1 and Γ_1, calculate $\{K_1, r_1, w_1\}$.[50]

4. Use policy function k' from the second part, together with z_2, to calculate the next period distribution Γ_2. Repeat the same calculation for $\{z_t\}_{t=3}^T$. Note that since $\mathbf{m}_3$ and (r_t, w_t) will typically not lie on the grids $(\mathbf{m}_2, \mathbf{r}_2, \mathbf{w}_2)$, we use three-dimensional linear interpolation to approximate the policy function k'. We find that k' is nearly linear along the m dimension except when m is close to zero and there is little curvature along the r and w dimensions. In addition, since the next period m' will typically not lie on the grid **m3**, for each period we use a weighted method to reallocate the mass of agents back to the grids (see Young 2006 for a complete description); because the grid points are densely packed this randomness does not significantly alter the properties of the distribution.

5. Use OLS on the equilibrium time series of $\{K_t, r_t, w_t\}_{t=T_0}^T$ to get a new value of $\{a_j\}_{j=0}^2$ and $\{b_j\}_{j=0}^5$.

6. Update these coefficients using the updating rule: $x_{update} = \lambda x_{new} + (1-\lambda) x_{old}$ and repeat from step 3 in part 1 till all the coefficients $\{a_j\}_{j=0}^2$, $\{b_j\}_{j=0}^5$, and $\{\sigma_r^2, \sigma_w^2, \sigma_{rw}^2\}$ converge. (More sophisticated methods of solving the implicit nonlinear equation for $x = (a, b, \sigma)$, such as Broyden's method, are not stable because the numerical derivatives are poorly behaved).

[50] Given $\int m d\Gamma_0 = K_0(1 + r_0) + w_0 \bar{\epsilon} \overline{H}$ and (2.3) and z_0, we can solve for the unique K_0 that solves this equation using Brent's method.

In the remainder of this section we will discuss how we solve the recursive problems (2.7) and (2.9) in step 5 of Part 1 and step 2 of Part 2. To compute the maximization on the RHS of the Bellman equation we solve

$$\begin{aligned} 0 &\geq -u'\left(m-k'\right)+\beta\sum_{\epsilon}\pi\left(\epsilon'|\epsilon\right)\int_{(r',w')}\frac{\partial V}{\partial m}\left(m',\epsilon',r',w'\right)\left(1+r'\right)d\mathcal{F}\left(r',w'|r,w\right), \\ m' &= k'\left(1+r'\right)+w'\epsilon'\overline{h}, \end{aligned}$$

where $\mathcal{F}(\cdot)$ is the distribution according to the corresponding forecast rule. We use bisection and Newton-Raphson procedures to solve for the optimal k', depending on whether we are close to the borrowing limit. To calculate the above integral we use Gauss-Hermite quadrature, since the errors are normal. Since (r',w') have errors which are perfectly correlated in the FI agent's problem, this integral is only one-dimensional, which we calculate using 20 nodes. For the PI agent's problem, we use Gauss-Hermite quadrature and the product rule to integrate over (r',w').[51] In this case, the number of nodes for each dimension is 13. Judd (1998) contains references for the algorithm that produces the nodes and weights for the quadrature.

The last issue is how to approximate V and V_m. Since (m',r',w') will be off the grids, we use the following steps in the interpolation:

1. For every grid point of $\mathbf{m1}$ (or $\mathbf{m2}$), we use two-dimensional linear interpolation over the $(\mathbf{r},\mathbf{w})$ dimensions to obtain $V(\mathbf{m1},\epsilon,r',w')$.[52]

2. We then construct a cubic spline over the $\mathbf{m1}$ dimension to obtain $V(m',\epsilon',r',w')$. We can then evaluate $\frac{\partial V}{\partial m}(m',\epsilon',r',w')$ from the cubic spline; $\frac{\partial V}{\partial m}$ is continuous and smooth in m'.

With a good initial guess V_0^{FI}, V_0^{PI}, $\{a_j\}_{j=0}^{2}$,$\{b_j\}_{j=0}^{5}$, we find that the value functions converge monotonically and all coefficients converge nicely (although not always monotonically). The typical

[51] For any bivariate normally-distributed variable, $\mathbf{y}\sim N\left(\overline{\mathbf{y}},\mathbf{\Sigma}\right)$, we can approximate its integral by

$$\begin{aligned} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f\left(y_1,y_2\right)d\Phi\left(y_1,y_2\right) &\simeq \pi^{-1}\left|\mathbf{\Sigma}\right|^{-1/2}\left|L\right|\sum_{j=1}^{N_2}\sum_{i=1}^{N_1}f\left(y_{1,i}^{*},y_{2,j}^{*}\right)\omega_{1,i}^{*}\omega_{2,j}^{*}; \\ \begin{bmatrix} y_{1,i}^{*} \\ y_{2.j}^{*}\end{bmatrix} &= \sqrt{2}L\begin{bmatrix} x_{1,i}^{*} \\ x_{1,j}^{*}\end{bmatrix}+\begin{bmatrix}\overline{y}_1 \\ \overline{y}_2\end{bmatrix}, \end{aligned}$$

where $\left(x_{1,i}^{*},x_{2,j}^{*}\right)$ and $\left(\omega_{1,i}^{*},\omega_{2,j}^{*}\right)$ are nodes and weights for Gauss-Hermite quadrature and L is from LU decomposition of $\mathbf{\Sigma}$. For a discussion of Gauss-Hermite quadrature as an approximation for the expectation of functions of normal variables, see Judd (1998).

[52] ϵ' is always on the grid, so no interpolation is needed along this dimension.

runtime is several days. We doubt that the algorithm would work well with arbitrary initial guesses; our initial guess for the PI agent coefficients is taken from the FI economy.

E. Appendix E

This appendix proves Propositions (1)-(4). Consider the second period problem in (3.3). Omitting the time subscript, the FOCs of the PI and FI agents are

$$-u'\left(m-s^i\right)+\beta\int u'\left(s^i\left(1+r^i\right)+w^i\right)\left(1+r^i\right)d\mathcal{F}^i\left(r^i,w^i\right)=0. \tag{E.1}$$

We consider only the case when $s^i>0$. Using the FOC of an FI agent, define

$$\Phi\left(s\right)=-u'\left(m-s\right)+\beta\int u'\left(s\left(1+r^{FI}\right)+w^{FI}\right)\left(1+r^{FI}\right)d\mathcal{F}^{FI}\left(r^{FI},w^{FI}\right).$$

Note that $\Phi\left(s\right)$ is strictly decreasing. If $\Phi\left(s^{PI}\right)>0$, then we can conclude that $s^{FI}>s^{PI}$. Subtracting the FOC of the PI agent from $\Phi\left(s^{PI}\right)$, we have $\Phi\left(s^{PI}\right)>0$ if and only if

$$\begin{aligned}&\int u'\left(s^{PI}\left(1+r^{FI}\right)+w^{FI}\right)\left(1+r^{FI}\right)d\mathcal{F}^{FI}\left(r^{FI},w^{FI}\right)\\ >\ &\int u'\left(s^{PI}\left(1+r^{PI}\right)+w^{PI}\right)\left(1+r^{PI}\right)d\mathcal{F}^{PI}\left(r^{PI},w^{PI}\right).\end{aligned} \tag{E.2}$$

Now we show that condition (E.2) is satisfied in each proposition.

Proposition 1. *Given $u\left(c\right)=\log\left(c\right)$ and $r^{FI}\succ_{FOSD}r^{PI}$, then $s^{FI}>s^{PI}$.*

In this case the distribution of w^{FI} and w^{PI} are the same. Under the iid assumption, condition (E.2) can be written as

$$\begin{aligned}&\int E_w\left[u'\left(s^{PI}\left(1+r^{FI}\right)+w\right)\right]\left(1+r^{FI}\right)d\mathcal{F}^{FI}\left(r^{FI}\right)\\ >\ &\int E_w\left[u'\left(s^{PI}\left(1+r^{PI}\right)+w\right)\right]\left(1+r^{PI}\right)d\mathcal{F}^{PI}\left(r^{PI}\right).\end{aligned} \tag{E.3}$$

$E_w[\cdot]$ is conditional expectation over w. Let $x = S^{PI}(1+r)$ and define $g(r)$ and write out its first derivative as following

$$\begin{aligned} g(r) &= E_w\left[u'(x+w)\right](1+r) \\ g'(r) &= E_w\left[u'(x+w) + u''(x+w)x\right] \\ &= E_w\left[u'(x+w)\left(1 + \frac{u''(x+w)(x+w)}{u'(x+w)}\frac{x}{(x+w)}\right)\right]. \end{aligned}$$

Notice that if $u(c)$ is logarithmic, $g(r)$ is strictly monotone-increasing since w is always greater than zero. Thus if $r^{FI} \succ_{FOSD} r^{PI}$, condition (E.3) is satisfied.[53]

Proposition 2. *Given $u(c) = \log(c)$ and $r^{FI} \succ_{SOSD} r^{PI}$, then $s^{FI} > s^{PI}$.*

Following the same steps in the proof of Proposition (1), we will get

$$\begin{aligned} g''(r) &= s^{PI}E_w\left[2u''(x+w) + u'''(x+w)x\right] \\ &= s^{PI}E_w\left[u''(x+w)\left(2 + \frac{u'''(x+w)(x+w)}{u''(x+w)}\frac{x}{x+w}\right)\right]. \end{aligned}$$

Notice that if $u(c)$ is logarithmic, $g(r)$ is a strictly concave function. Thus if $r^{FI} \succ_{SOSD} r^{PI}$, condition (E.3) is satisfied.[54]

Proposition 3. *Given $w^{PI} \succ_{FOSD} w^{FI}$, then $s^{FI} > s^{PI}$.*

In this case the distribution of r^{FI} and r^{PI} are the same. Under the iid assumption, condition (E.2) can be written as

$$\begin{aligned} &\int E_r\left[u'\left(s^{PI}(1+r) + w^{FI}\right)(1+r)\right] d\mathcal{F}^{FI}\left(w^{FI}\right) \\ > &\int E_r\left[u'\left(s^{PI}(1+r) + w^{PI}\right)(1+r)\right] d\mathcal{F}^{PI}\left(w^{PI}\right). \end{aligned} \tag{E.4}$$

Let $x = S^{PI}(1+r)$ and define $g(w)$ and write out its first derivative as

$$\begin{aligned} g(w) &= E_r\left[u'(x+w)(1+r)\right], \\ g'(w) &= E_r\left[u''(x+w)(1+r)\right]. \end{aligned}$$

[53] By definition, if $\widetilde{x}_1 \succ_{FOSD} \widetilde{x}_2$, $Eg(\widetilde{x}_1) > Eg(\widetilde{x}_2)$ for all strictly increasing functions $g(x)$.

[54] By definition, if $\widetilde{x}_1 \succ_{SOSD} \widetilde{x}_2$, $Eg(\widetilde{x}_1) > Eg(\widetilde{x}_2)$ for all strictly concave function $g(x)$.

If $u(c)$ is a concave function, $g(w)$ is monotone-decreasing. Thus if $w^{PI} \succ_{FOSD} w^{FI}$, condition (E.4) is satisfied.

Proposition 4. *Given $u(c)$ is CRRA and $w^{PI} \succ_{SOSD} w^{FI}$, then $s^{FI} > s^{PI}$.*

Following the same steps in the proof of Proposition 3, we will get

$$g''(w) = E_r\left[u'''(x+w)(1+r)\right].$$

If $u(c)$ is a CRRA utility function, $g(w)$ is a strictly convex function. Thus if $w^{PI} \succ_{SOSD} w^{FI}$, condition (E.3) is satisfied.

F. Appendix F

This appendix explains how we set the PI agent's forecast equations in the four cases: *i)* $E_t^{PI} r_{t+1}$ is downward biased, *ii)* $V_t^{PI} r_{t+1}$ is arbitrarily larger, *iii)* $E_t^{PI} w_{t+1}$ is downward biased, and *iv)* $V_t^{PI} w_{t+1}$ is arbitrarily larger. Let $x = r_{t+1|t}^{FI} + \delta$ and $y = w_{t+1|t}^{FI}$. Then we have

$$x \sim \log N\left(\mu_r, \sigma_e^2\right); \quad y \sim \log N\left(\mu_w, \sigma_e^2\right),$$

where μ_r, μ_w and σ_e^2 are calculated according to FI agent's forecast function (2.8). Assume that the PI agent's forecast functions have the form as (2.8) but with different coefficients. Let $\widehat{x} = r_{t+1|t}^{PI} + \delta$ and $\widehat{y} = w_{t+1|t}^{PI}$, where

$$\widehat{x} \sim \log N\left(\widehat{\mu}_r, \sigma_x^2\right); \quad \widehat{y} \sim \log N\left(\widehat{\mu}_w, \sigma_y^2\right)$$

Our objective is to find μ_r, μ_w, σ_x^2, and σ_y^2 for each of the four cases.

Case *i)* $E_t^{PI} r_{t+1}$ **is downward biased.** In this case $E_t^{PI} r_{t+1} < E_t^{FI} r_{t+1}$ while $V_t^{PI} r_{t+1} = V_t^{FI} r_{t+1}$. Let $E\widehat{x} = \gamma E x$ for $0 < \gamma < 1$. Using the expressions for the expectation and variance of a log normal random variable we obtain

$$\begin{aligned} \exp\left(\widehat{\mu}_r + \frac{\sigma_x^2}{2}\right) &= \gamma \exp\left(\mu_r + \frac{\sigma_e^2}{2}\right), \\ (\gamma E x)^2 \left(\exp\left(\sigma_x^2\right) - 1\right) &= (E x)^2 \left(\exp\left(\sigma_e^2\right) - 1\right). \end{aligned}$$

Solve for σ_x^2 and $\widehat{\mu}_r$:

$$\begin{aligned}\sigma_x^2 &= \log\left(\frac{\exp\left(\sigma_e^2\right) - 1 + \gamma^2}{\gamma^2}\right), \\ \widehat{\mu}_r &= \mu_r + \frac{\sigma_e^2 - \sigma_x^2}{2} + \log\gamma.\end{aligned}$$

Then given γ, we use the above formulas to transform an FI agent's forecast into a PI agent's biased forecast. In our experiment, we set $\gamma = 0.9975$, so $E_t^{PI}\left(r_{t+1} + \delta\right) = 0.9975 E_t^{FI}\left(r_{t+1} + \delta\right)$; approximately $\frac{E_t^{FI} r_{t+1}}{E_t^{PI} r_{t+1}} \simeq 0.992$.

Case $ii)$ $V_t^{PI} r_{t+1}$ **is arbitrarily larger.** In this case $V_t^{PI} r_{t+1} > V_t^{FI} r_{t+1}$ but $E_t^{PI} r_{t+1} = E_t^{FI} r_{t+1}$. Let $\sigma_x = \gamma\sigma_e$ with $\gamma > 1$. In this case we have

$$\exp\left(\widehat{\mu}_r + \frac{\gamma^2\sigma_e^2}{2}\right) = \exp\left(\mu_r + \frac{\sigma_e^2}{2}\right).$$

Solve for $\widehat{\mu}_r$ as

$$\widehat{\mu}_r = \mu_r + \frac{\sigma_e^2}{2}\left(1 - \gamma^2\right).$$

In our experiment, we set $\gamma = 6.0$, so $\frac{V_t^{PI} r_{t+1}}{V_t^{FI} r_{t+1}} = \frac{\exp\left(\gamma^2\sigma_e^2 - 1\right)}{\exp\left(\sigma_e^2 - 1\right)} = 1.00177$.

Case $iii)$ $E_t^{PI} w_{t+1}$ **is downward biased.** In this case $E_t^{PI} w_{t+1} < E_t^{FI} w_{t+1}$ while $V_t^{PI} w_{t+1} = V_t^{FI} w_{t+1}$. Following the same steps in case $i)$, we can get the same formulas for σ_y^2 and $\widehat{\mu}_w$. In our experiment, we set $\gamma = 0.9975$ so that $\frac{E_t^{FI} w_{t+1}}{E_t^{PI} w_{t+1}} = 0.9975$.

Case $ii)$ $V_t^{PI} w_{t+1}$ **is arbitrarily larger.** In this case $V_t^{PI} w_{t+1} > V_t^{FI} w_{t+1}$ but $E_t^{PI} w_{t+1} = E_t^{FI} w_{t+1}$. Following the same steps in case $ii)$, we can get the same formulas for $\widehat{\mu}_w$. In our experiment, we set $\gamma = 3.0$ so that $\frac{V_t^{PI} w_{t+1}}{V_t^{FI} w_{t+1}} = 1.0004$.

Table 1

Aggregate Implications

	z	$\log(Y)$	$\log(K)$	$\log(C)$	$\log(I)$	r	w
PI economy							
Mean	0.000	0.242	2.692	-0.047	-1.140	0.0094	2.540
SD	0.026	0.035	0.035	0.028	0.068	0.0006	0.089
Relative SD	0.730	1.000	0.998	0.804	1.936	0.018	2.533
Corr with GDP	0.962	1.000	0.829	0.936	0.900	0.295	0.999
Autocorrelation	0.961	0.980	0.999	0.996	0.950	0.940	0.980
FI economy							
Mean	0.000	0.245	2.701	-0.046	-1.131	0.0092	2.549
SD	0.026	0.035	0.035	0.028	0.067	0.0006	0.089
Relative SD	0.734	1.000	0.986	0.807	1.913	0.018	2.541
Corr with GDP	0.962	1.000	0.828	0.938	0.901	0.315	0.999
Autocorrelation	0.961	0.980	0.999	0.996	0.950	0.936	0.980

Table 2

Forecast Rules[55]

PI agent $(\Lambda = 0.5)$	Constant	$\log(r)$	$\log(w)$	σ_ε	$\rho(\varepsilon_t, \varepsilon_{t-1})$	R^2
$\log(r')$	-0.1195	0.9532	-0.1066	0.02385	0.0037	0.8795
$\log(w')$	0.0311	0.0023	0.9780	0.00712	0.0038	0.9590
FI agent $(\Lambda = 0.5)$						
$\log(r' + \delta)$	-0.1382	0.9524	-0.0292	0.00710	0.0	0.8796
$\log(w)$	0.0413	0.0067	0.9807	0.00710	0.0	0.9590

[55] For an FI agent we compute a comparable R^2 from $1 - \frac{\sigma_e^2}{\sigma_y^2}$, where σ_y^2 are variances of the dependent variable.

Table 3

Individual Statistics

	m_i	k_i'	c_i
Mixed economy (FI,PI)			
Mean	$(16.42, 13.27)$	$(15.46, 12.34)$	$(0.960, 0.931)$
SD	$(13.06, 11.37)$	$(12.89, 11.21)$	$(0.184, 0.178)$
PI Economy (FI,PI)			
Mean	$(18.68, 14.73)$	$(17.70, 13.79)$	$(0.980, 0.944)$
SD	$(14.34, 12.34)$	$(14.16, 12.17)$	$(0.190, 0.182)$
FI Economy (FI,PI)			
Mean	$(14.93, 12.28)$	$(13.98, 11.35)$	$(0.946, 0.922)$
SD	$(12.19, 10.70)$	$(12.02, 10.54)$	$(0.179, 0.175)$

Table 4

Wealth Distribution ($\Lambda = 0.5$)

Wealth Percentile	% of Population	Fraction of PI	Fraction of FI
$< 1\%$	0.0099	0.5857	0.4143
$1\% - 10\%$	0.0898	0.5823	0.4177
$10\% - 20\%$	0.0961	0.5661	0.4339
$50\% - 60\%$	0.0990	0.5041	0.4959
$95\% - 99\%$	0.0400	0.3498	0.6502
$99\% - 100\%$	0.0101	0.2828	0.7172

Table 5

Wealth By Deciles

	Wealth	Wealth Deciles				
	Gini	Top 1%	Top 5%	Top 10%	Top 20%	Bottom 40%
US data[56]	0.806	32.27%	57.45%	69.65%	82.62%	1.11%
FI economy ($\Lambda = 0.0$)	0.424	3.93%	15.47%	26.83%	45.15%	12.66%
Model with biased $E_t^{PI} r_{t+1}$	0.733	9.46%	35.9%	58.7%	81.60%	3.38%
Fraction of PI agents		0.0%	0.0%	0.0%	6.97%	99.60%

[56]Data is from the 2001 wave of the Survey of Consumer Finances.

Table 6

Welfare Gains

	FI Economy		PI Economy		
Volatility	Total Gain	Marginal Gain	Total Gain	Marginal Gain	Ratio
0	0.0122%	0.0107%	0.0199%	0.0199%	1.639
$\frac{1}{4}\sigma_e$	0.0229%	0.0096%	0.0592%	0.0393%	2.586
$\frac{1}{2}\sigma_e$	0.0325%	0.0115%	0.1181%	0.0590%	3.635
$\frac{3}{4}\sigma_e$	0.0440%	0.0122%	0.1980%	0.0799%	4.505

Table 2 Supplement

Forecast Rules[57]

PI agent (PI economy)	Constant	$\log(r)$	$\log(w)$	σ_ε	$\rho(\varepsilon_t, \varepsilon_{t-1})$	R^2
$\log(r')$	-0.1196	0.9531	-0.1067	0.02367	0.0037	0.8803
$\log(w')$	0.0310	0.0023	0.9781	0.00712	0.0038	0.9593
FI agent (PI economy)	Constant	$\log(r+\delta)$	$\log(w)$	σ_e	$\rho(e_t, e_{t-1})$	R^2
$\log(r'+\delta)$	-0.1381	0.9522	-0.0294	0.00710	0.0	0.8804
$\log(w)$	0.0412	0.0067	0.9808	0.00710	0.0	0.9593
PI agent (FI economy)	Constant	$\log(r)$	$\log(w)$	σ_ε	$\rho(\varepsilon_t, \varepsilon_{t-1})$	R^2
$\log(r')$	-0.1193	0.9533	-0.1066	0.02400	0.0037	0.8789
$\log(w')$	0.0311	0.0022	0.9779	0.00712	0.0038	0.9589
FI agent (FI economy)	Constant	$\log(r+\delta)$	$\log(w)$	σ_e	$\rho(e_t, e_{t-1})$	R^2
$\log(r'+\delta)$	-0.1381	0.9524	-0.0290	0.00710	0.0	0.8790
$\log(w)$	0.0412	0.0066	0.9806	0.00710	0.0	0.9589
PI agent (mixed economy)	Constant	$\log(r)$	$\log(w)$	σ_ε	$\rho(\varepsilon_t, \varepsilon_{t-1})$	R^2
$\log(r')$	-0.1195	0.9532	-0.1066	0.02385	0.0037	0.8795
$\log(w')$	0.0311	0.0023	0.9780	0.00712	0.0038	0.9590
FI agent (mixed economy)	Constant	$\log(r+\delta)$	$\log(w)$	σ_e	$\rho(e_t, e_{t-1})$	R^2
$\log(r'+\delta)$	-0.1382	0.9524	-0.0292	0.00710	0.0	0.8796
$\log(w)$	0.0413	0.0067	0.9807	0.00710	0.0	0.9590

[57] For an FI agent we compute a comparable R^2 from $1 - \frac{\sigma_e^2}{\sigma_y^2}$, where σ_y^2 are variances of the dependent variable.

Figure F.1: Comparison of Aggregates, PI and FI economies

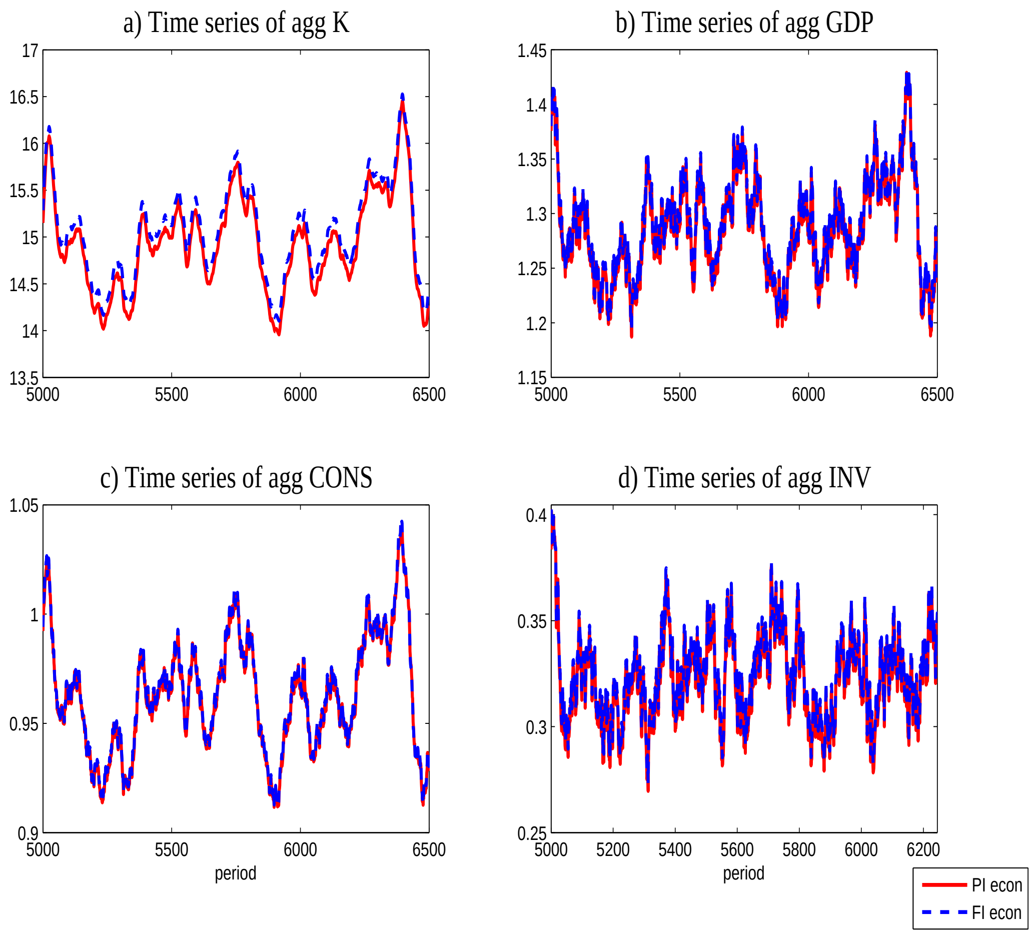

Figure F.2: Individual Saving (Mixed Economy)

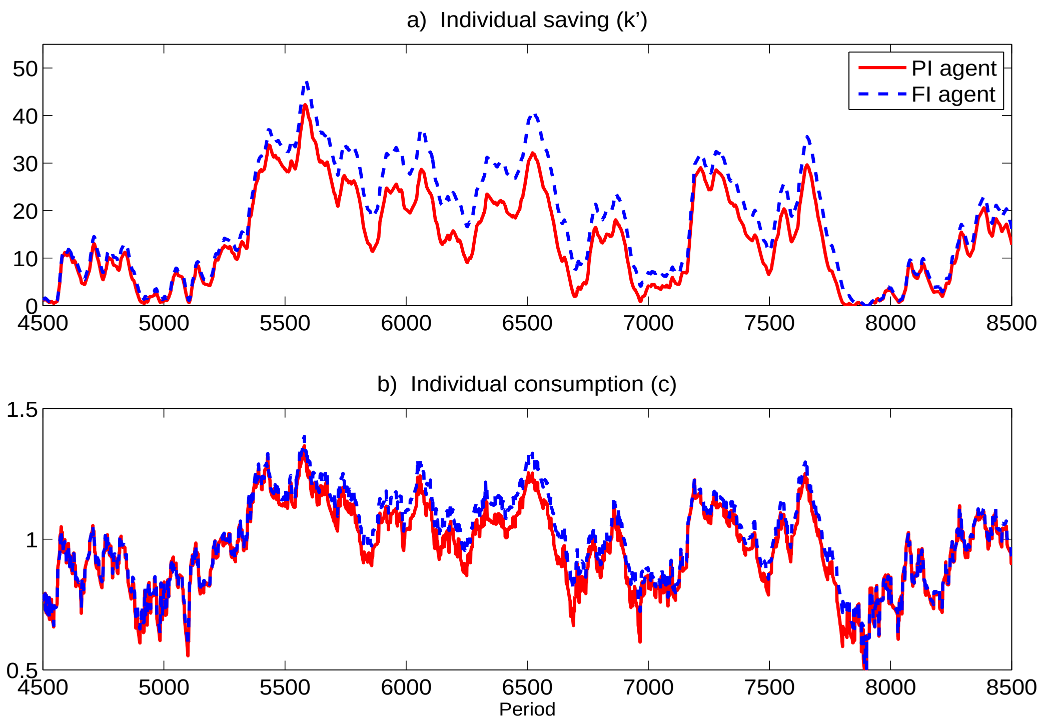

Figure F.3: Savings Functions (Mixed Economy)

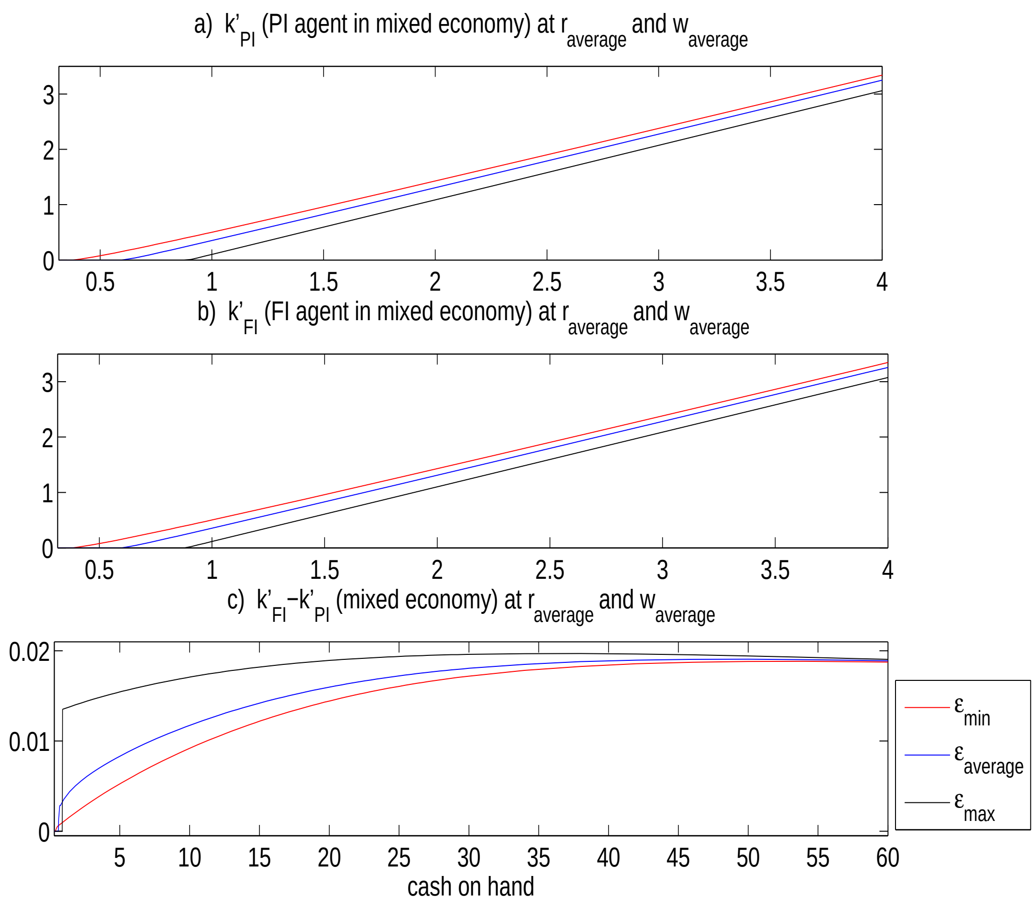

Figure F.4: Comparison of Savings Functions (Mixed Economy)

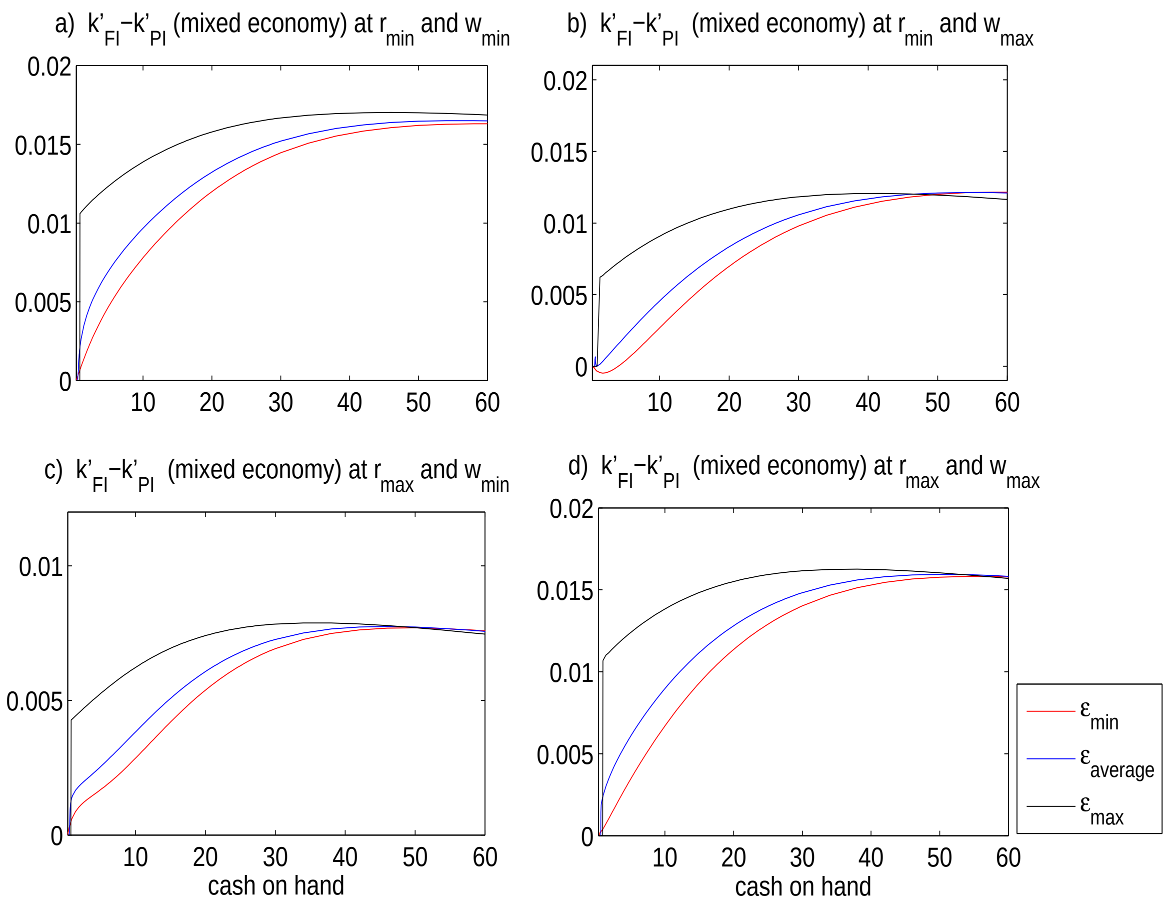

Figure F.5: One period Forecast Error (Mixed Economy)

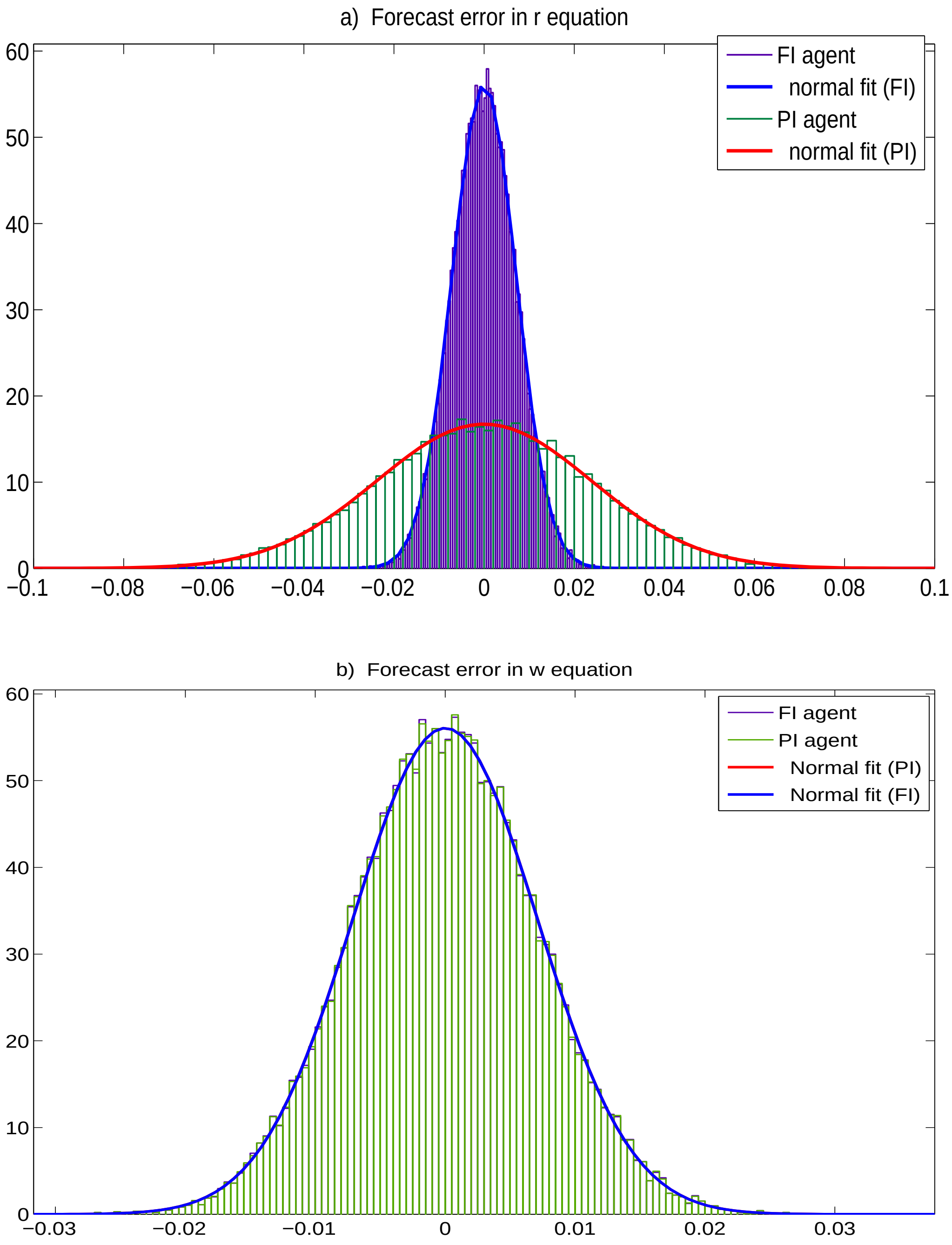

Figure F.6: Expected r' and w' (Mixed Economy)

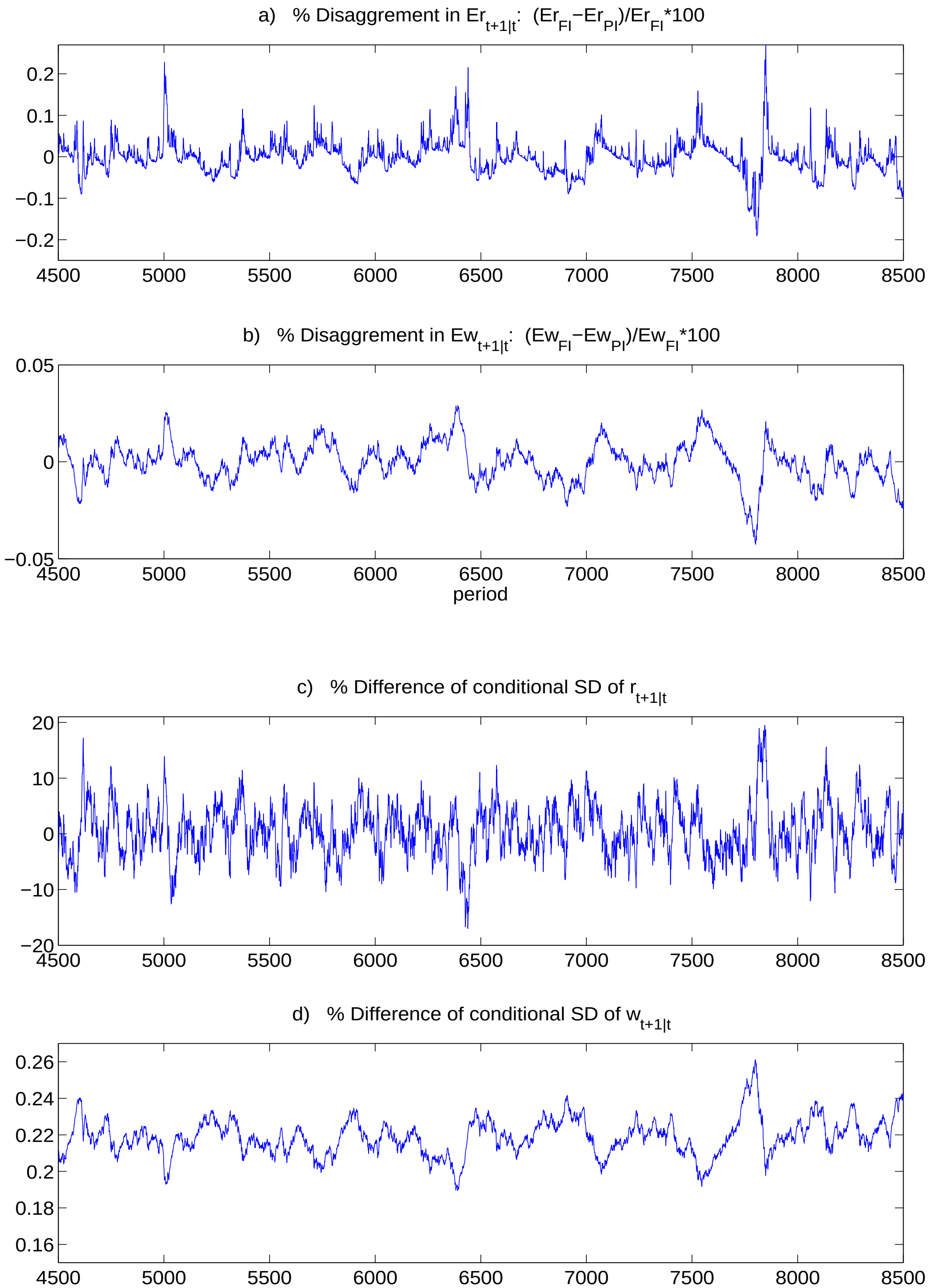

Figure F.7: PI agent Endowed with FI Forecast (Mixed Economy)

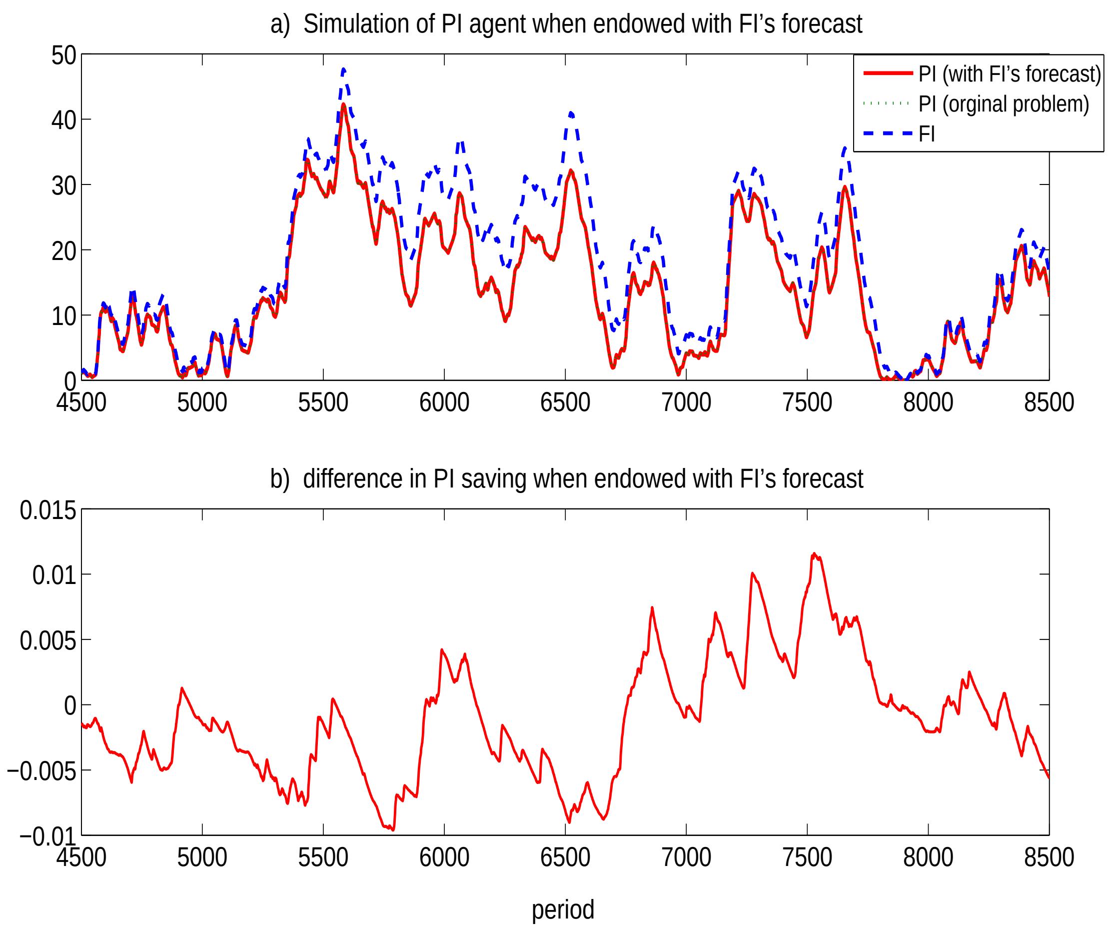

Figure F.8: Value Functions (Mixed Economy)

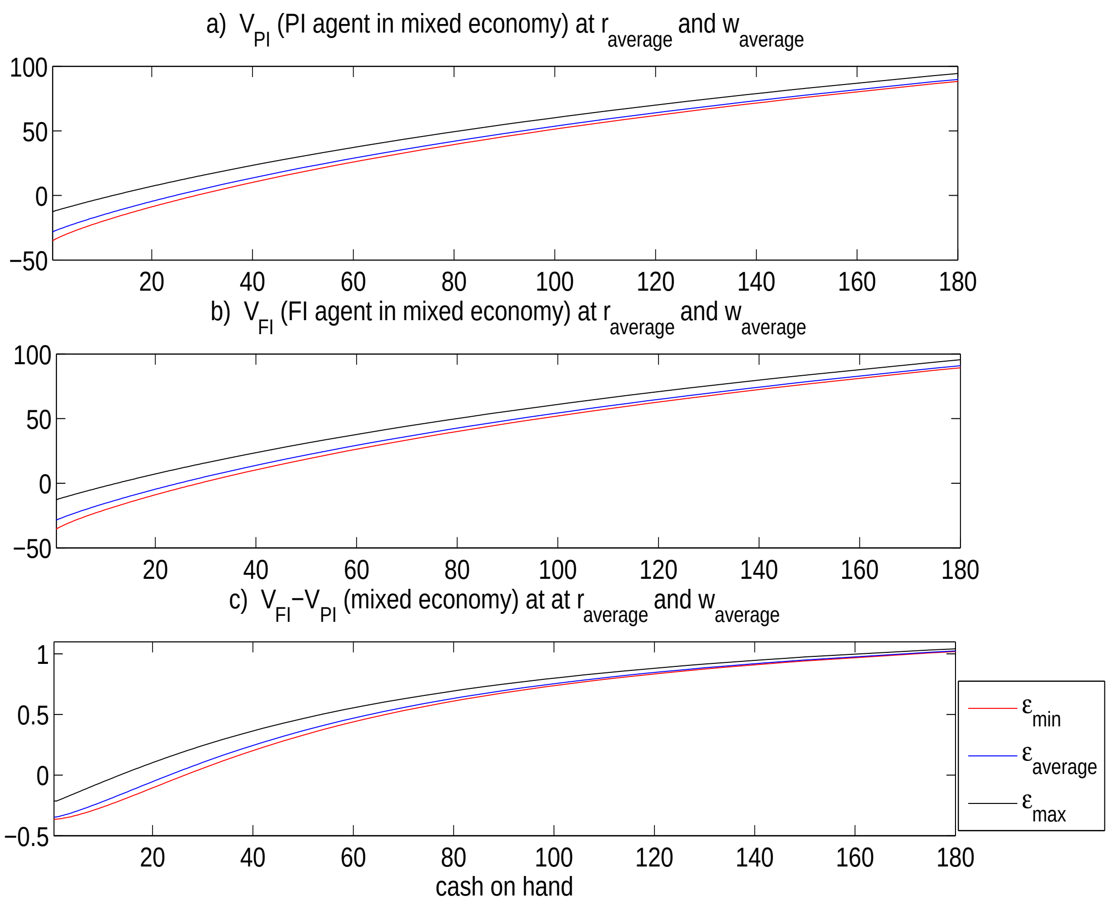

Figure F.9: Comparison of Value Functions (Mixed Economy)

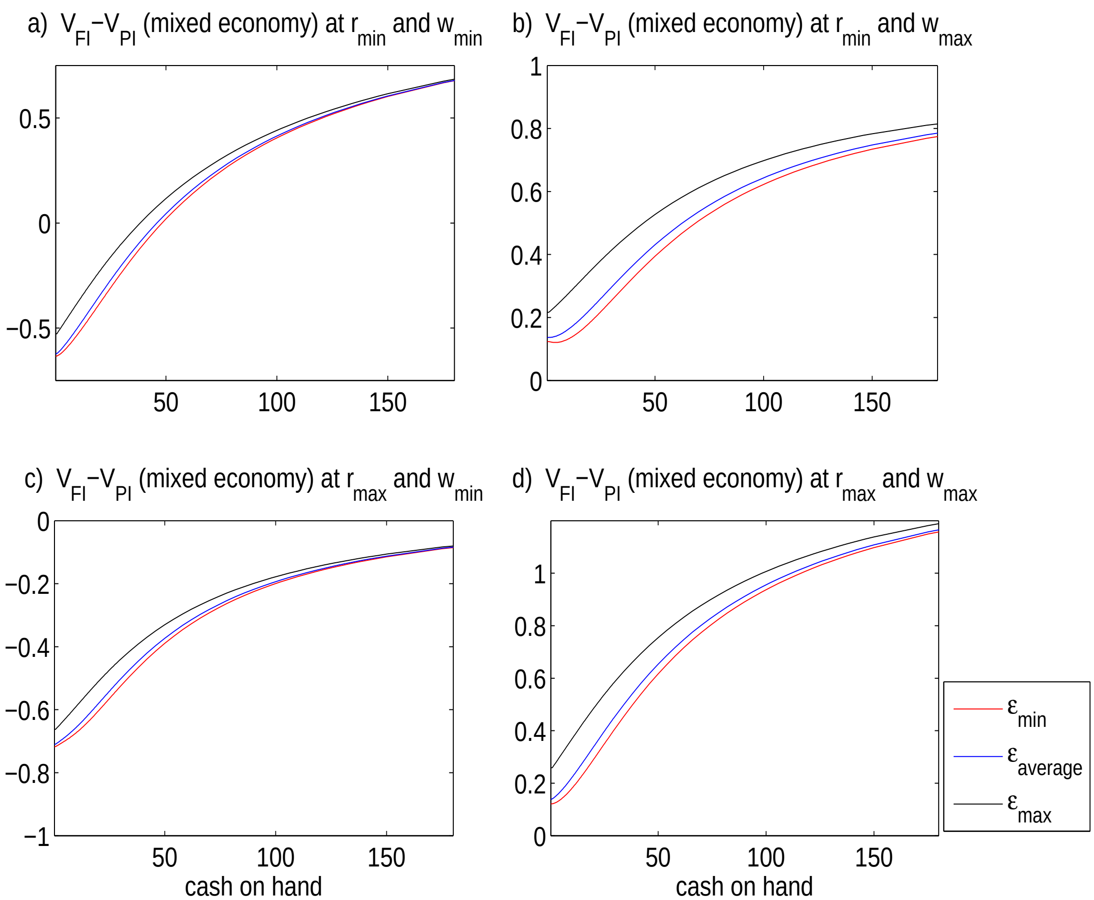

Figure F.10: PI and FI agents, No One-Period Risk (Mixed Economy)

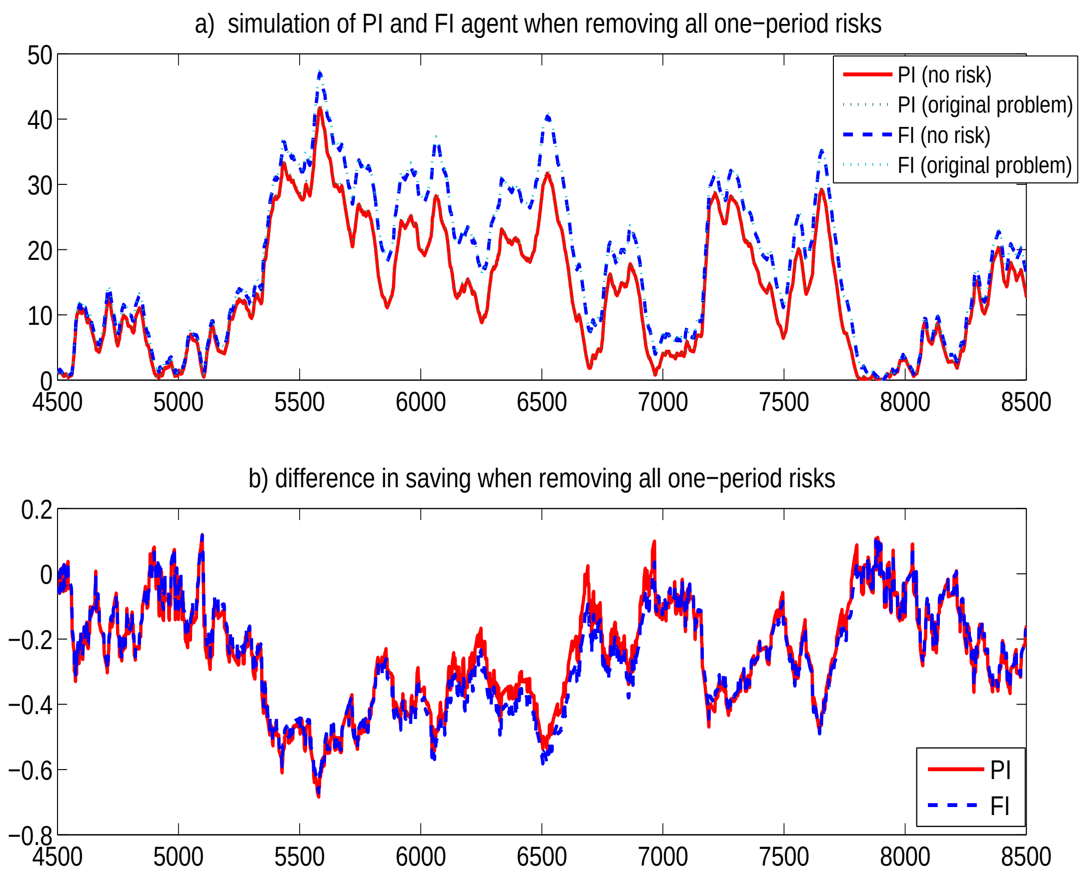

Figure F.11: Marginal Value of m (Mixed Economy)

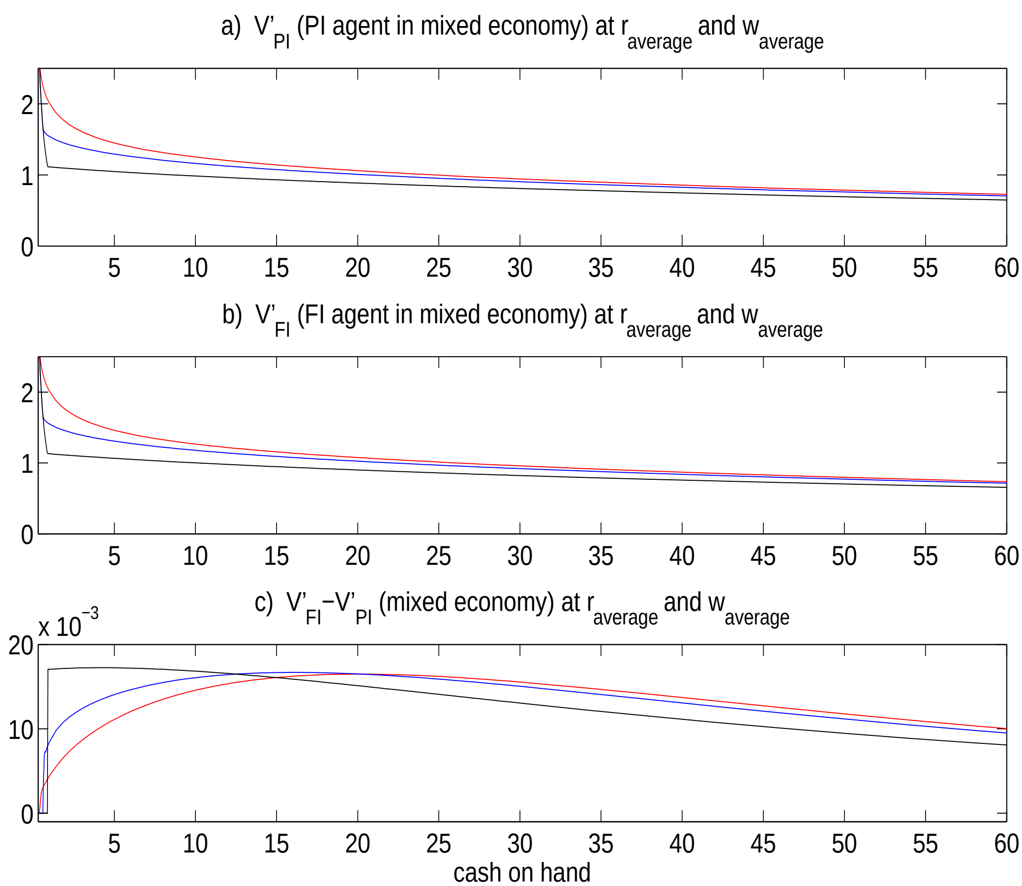

Figure F.12: Comparison of Marginal Value of m (Mixed Economy)

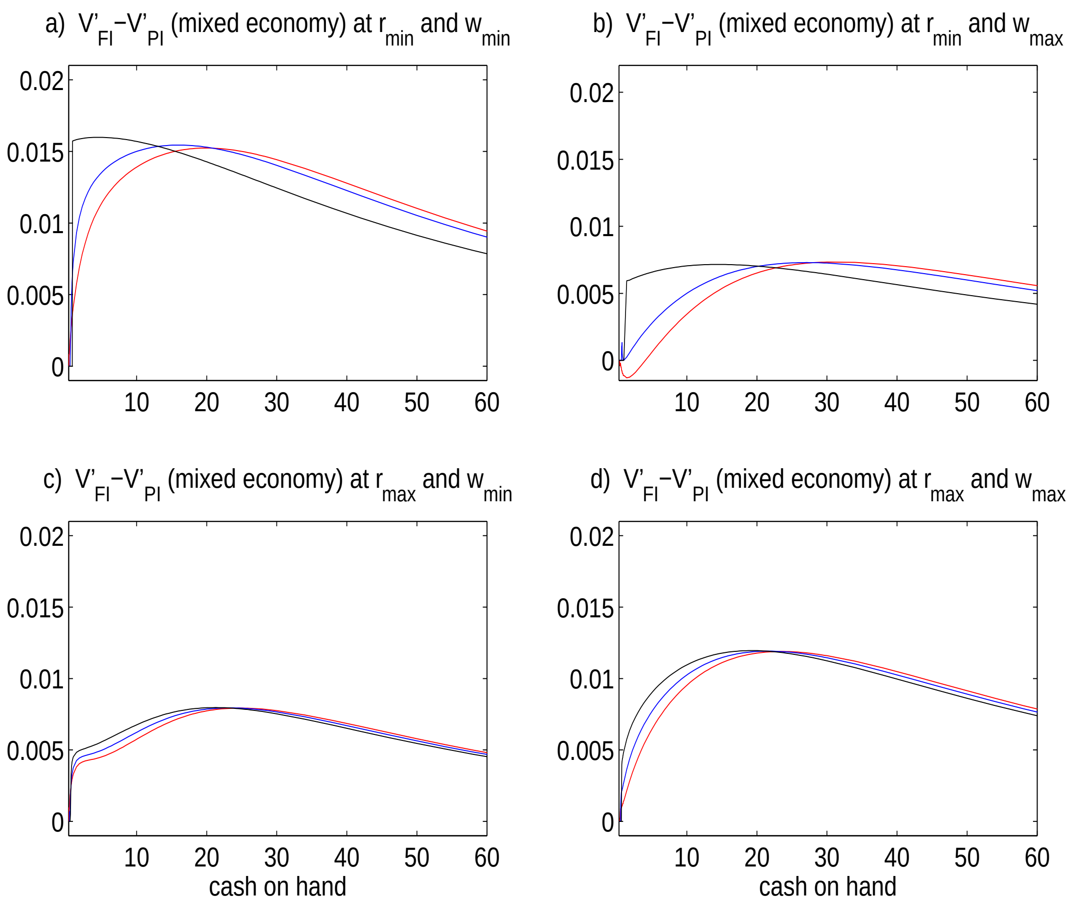

Figure F.13: Wealth Distribution (Mixed Economy)

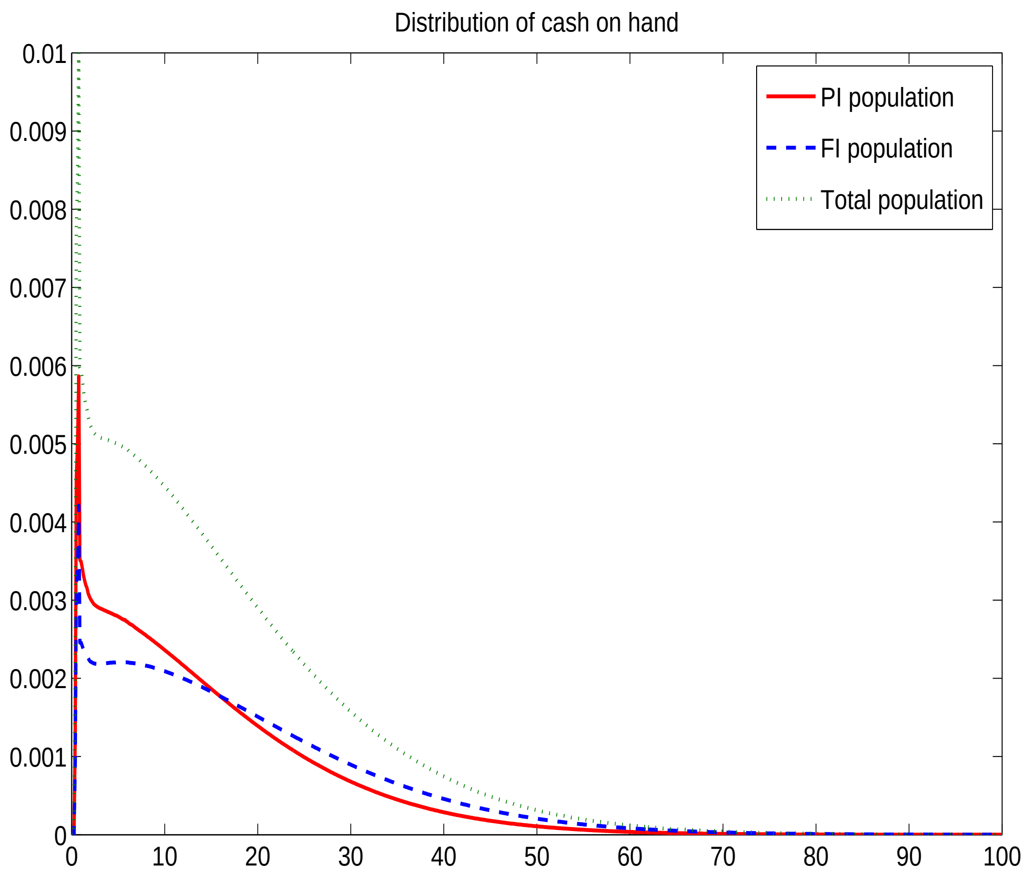

Figure F.14: Wealth Concentration (Mixed Economy)

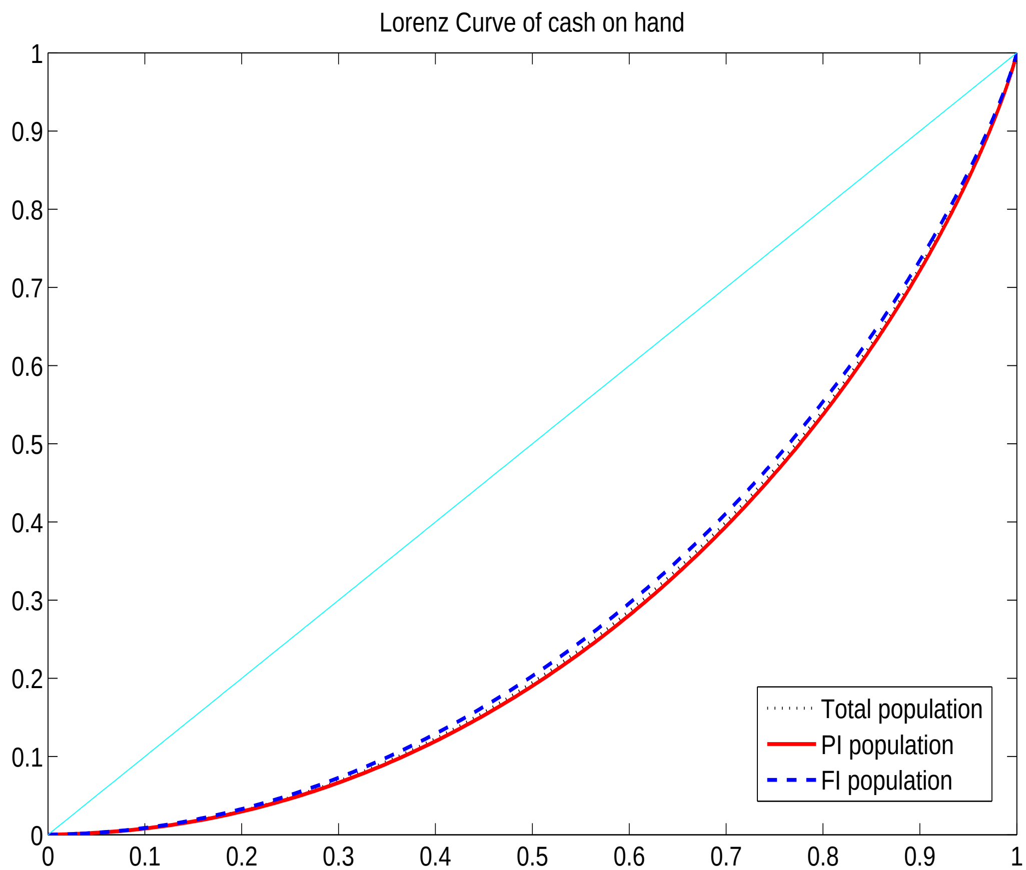

Figure F.15: **Case 1**: $E_t^{PI} r_{t+1} \simeq 0.992 E_t^{FI} r_{t+1}$ and **Case 2**: $\frac{V^{PI}(r_{t+1})}{V^{FI}(r_{t+1})} = 1.001766$

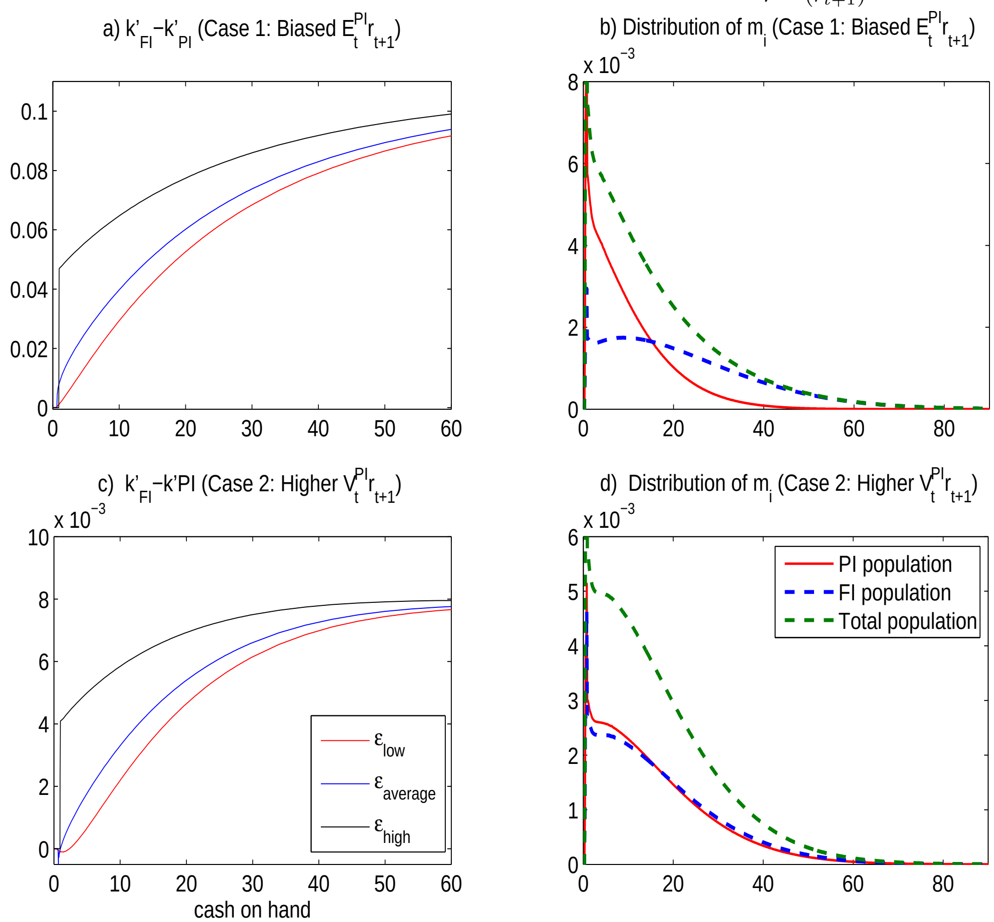

Figure F.16: **Case 3**: $E_t^{PI} w_{t+1} = 0.9975 E_t^{FI} w_{t+1}$ and **Case 4**: $\frac{V^{PI}(w_{t+1})}{V^{FI}(w_{t+1})} = 1.000403$

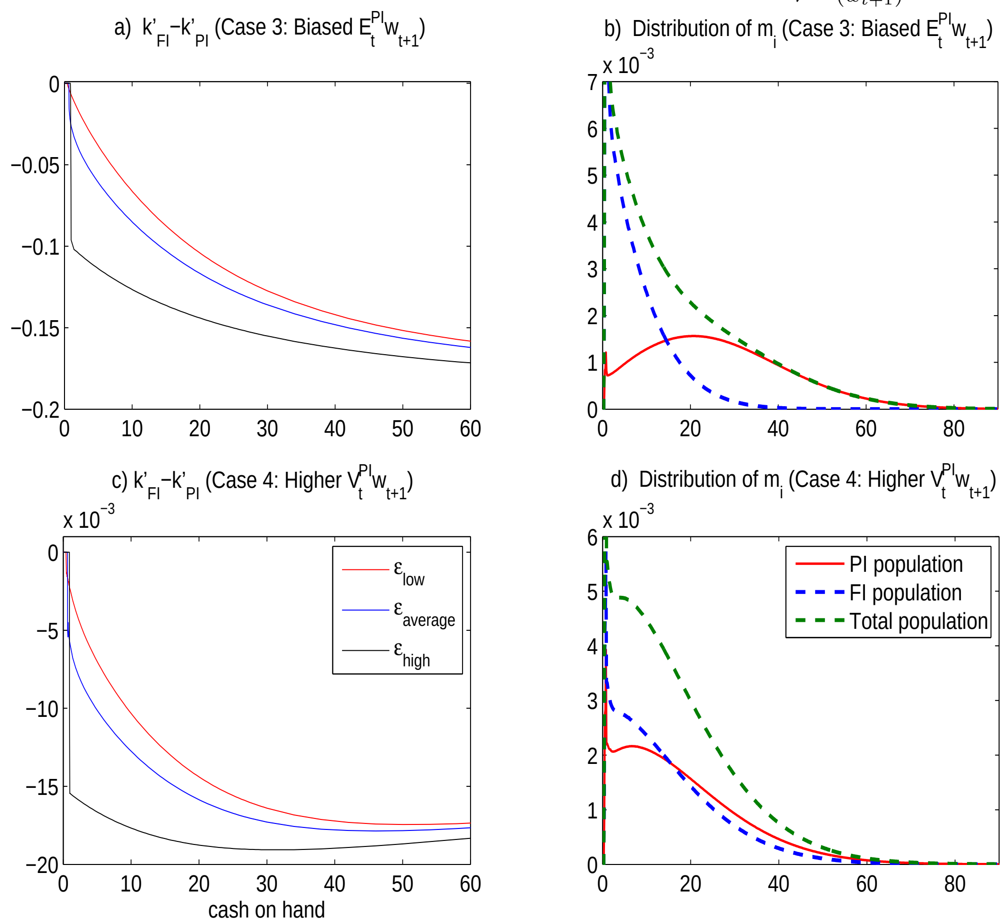

Figure F.17: Wealth Distribution with $E_t^{PI} r_{t+1} \simeq 0.952 E_t^{FI} r_{t+1}$ $(\Lambda = 0.8)$

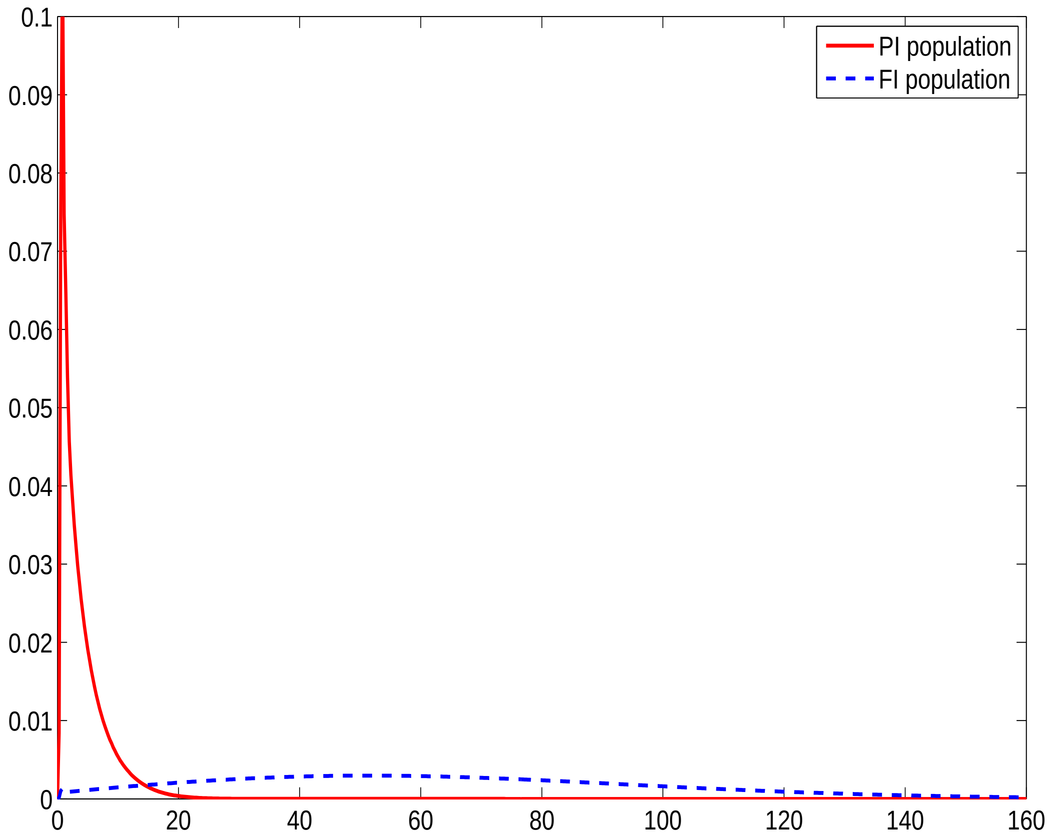

Figure F.18: Wealth Concentration with $E_t^{PI} r_{t+1} \simeq 0.952 E_t^{FI} r_{t+1}$ ($\Lambda = 0.8$)