Consumption, Retirement and Social Security:
Evaluating the Efficiency of Reform that Encourages Longer Careers*

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Abstract

This paper analyzes the effect on individuals' retirement and consumption choices of a potential reform to the U.S. Social Security system. We first estimate the parameters of a life–cycle model. We assume intratemporally nonseparable preference orderings and the possibility of disability. The specification predicts a change in consumption at retirement. We use the empirical magnitude of the change, together with the model's predicted retirement age, to identify key parameters, including the curvature of the utility function. We then qualitatively and quantitatively study the possible long–run effect of a Social Security reform in which individuals no longer face the old–age and survivors insurance payroll tax after some specified age, and their subsequent earnings have no bearing on their Social Security benefits. Simulations indicate that retirement ages could rise by as much as one year, equivalent variations could average $6000 (1984 dollars) per household or more, and the reform could generate as much as $3000 of additional income tax revenue per household.

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1. Introduction

This paper analyzes the effect of a potential reform to the U.S. Social Security system on individuals' retirement and consumption choices, and, in particular, on economic efficiency. Our goal is to describe and evaluate a simple reform that will address potentially important labor–supply distortions inherent in the current program. The proposed reform is as follows: after a long vesting period (say, 35-40 years of contributions), the Social Security Administration (SSA) would determine a worker's benefits for various prospective retirement ages using the current formula. Subsequently, a worker would no longer face the old–age and survivors insurance (OASI) payroll tax and his/her benefits would cease to depend on his/her actions. Individuals who continue to work after "vesting" would receive a 10.6 percent payroll tax reduction.[1] To maintain revenue neutrality within the system, there would be a slight increase in the payroll tax during the vesting period. Following the tradition of Auerbach and Kotlikoff [1987] and others, this paper evaluates its reform in the context of a certainty equivalent life–cycle model. In contrast to the tradition, this paper estimates the parameters of its model using microeconomic data on earnings, consumption, and retirement. We employ what we think is a novel estimation strategy that delivers quite precise estimates of key parameters. The strategy makes use of recently available panel data from the Health and Retirement Study (HRS), including linked lifetime Social Security annual earning records, and pseudo panel consumption expenditure data from the Consumer Expenditure Survey (CEX). Our simulations show that the proposed reform could raise retirement ages by a year, on average; equivalent variations from the reform could average $6,000 per household (1984 dollars, present value age 50) or more; and, society's additional income tax revenues could average $3,000 per household.

The logic of the proposed reform becomes evident when we integrate the structure the current Social Security system with a life–cycle model (e.g., Diamond [1965], Tobin [1967], Auerbach and Kotlikoff [1987], Modigliani [1986], Hubbard *et al.* [1995], Altig *et al.* [2001], and many others) in which jobs require full-time work and retirement is irreversible. Under the current Social Security system, a household's benefits, on average, have smaller lifetime present value than its contributions. This generates an income effect that presumably tends to prolong careers. We treat this income effect, stemming from the legacy costs of the system, as unavoidable.[2] There is, on the other hand, also a substitution effect. For many households with long earnings histories, the present value of Social Security benefits is

[1] Similar reforms have been proposed elsewhere, both in an earlier version of this paper (ASSA annual meeting [2006]), and in work by others (Shah *et al.* [2006], Burtless and Quinn [2002]). Our work is, as far as we know, the first to evaluate the effects of this reform with a model with estimated parameters — see below.

[2] Others have suggested alleviating the legacy costs, and the distortions of labor supply

quite insensitive to marginal earnings. Thus, Social Security taxes tend to lead households to substitute leisure, in the form of earlier retirement, for work, while benefits, which are often effectively lump sum, fail to generate countervailing incentives. Our proposed reform dismantles the payroll tax late in careers — but before many households' optimal retirement. We can hope, therefore, to diminish the Social Security system's substitution effect, which distorts private retirement decisions.

To quantify the effects of our reform, we develop a life–cycle model in which households choose their retirement age as well as their lifetime consumption/saving profile, jobs require full-time work, and retirement is permanent. The benefit to a household of later retirement is greater lifetime earnings; the cost is forgone leisure. A household derives a flow of services from its consumption expenditure and leisure — as in Auerbach and Kotlikoff, for example. The service flow, in turn, yields utility through a conventional isoelastic utility function. Although our "basic model" ignores health considerations, we present a second formulation with a stochastic, but insurable, chance of disability.

Our model predicts a discontinuous change in consumption expenditure at a household's retirement, due to the abrupt change in leisure and the intratemporal non–separability of consumption and leisure in the preference ordering. In fact, a number of recent empirical studies have described a drop in household consumption expenditure at the time of retirement (e.g., Banks *et al.* [1998], Bernheim *et al.* [2001], Hurd and Rohwedder [2003], Haider and Stephens [2005], Aguiar and Hurst [2005], Blau [2006], and others) — sometimes referred to as the "retirement consumption puzzle." We use the magnitude of the drop, which this paper measures from CEX data, as well as age of retirement, measured from the HRS, to identify the model's key parameters.[3]

Our analysis predicts that stopping the Social Security payroll tax after a vesting period of 34 years of contributions could lead households to postpone their retirement by about one year, on average. This suggests that the social security system has important effects on labor supply. The analysis also suggests that potential efficiency gains from the proposed reform are substantial. In particular, consumers, on average, would pay

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that they generate, with (partial) privitization schemes combined with consumption taxes. Recent analysis (Nishiyama and Smetters [2005]) indicates that such a reform would, in fact, likely generate efficiency losses due in large part to the lost income insurance that the system provides. Our proposed reform would not affect this social insurance value of the system.

[3] We first described our strategy in Laitner and Silverman [2005]. Hall [2006] develops a similar method for estimating the curvature of the utility function. In a similar vein, Chetty [2006] shows how to estimate risk aversion, in part, from the change in consumption associated with a random change in labor supply.

approximately \$6,000 (1984 dollars, in present value at age 50) for the preceding reform. When we account for the social gain from income taxes on longer careers, the total benefit could increase to \$9,000 per household.

Certain assumptions of our model, such as jobs requiring full-time work and permanent retirement, may amplify the behavioral changes and the efficiency gains of the reform. (If, for example, we allowed adjustments to labor supply during the vesting period, reactions to the small increase in the payroll tax early in life might partly counteract advances in efficiency from removing the tax late in life.) The estimated magnitudes of the efficiency gains in our model, nevertheless, indicate, we believe, that the reform is worth further consideration. It is also important to note that the efficiency gains from the reform do not represent Pareto improvements — the reform causes some agents who, absent the reform, would have retired before the vesting age, to suffer welfare losses. Our panel data enables us to study both average potential benefits from reform and the corresponding full distribution of underlying gains and losses.

This paper joins a large literature aimed at evaluating the effects of Social Security on labor supply. See Feldstein and Liebman [2002] for a review. By applying an explicit life–cycle model, we differ from much of this literature, which seeks reduced form estimates. Implementing a structural model allows us, most importantly, to evaluate the *life–cycle* effects on retirement and consumption of both the existing social security system and counterfactual reforms. By estimating the parameters of a fully-specified model, our paper also joins a smaller literature that provides structural estimates of life-cycle models of retirement (see, for example, Gustman and Steinmeier [1986], Rust and Phelan [1997], Bound *et al.* [2005], French [2005], and van der Klaauw and Wolpin [2005]). Our work is distinguished from this literature by its emphasis on a particular reform and by its use of both earnings and consumption data. Our estimation differs from many recent structural models of retirement in its certainty equivalent approach. On the other hand, policy simulations frequently employ such a framework, and we believe that it provides a rich yet tractable formulation — permitting analytic as well as numerical insights. In fact, our paper is an effort to bridge structural econometric and policy–oriented literatures.

The organization of this paper is as follows. Section 2 describes our basic model and its formulation with stochastic disability. Section 3 discusses our pseudo–panel data on consumption expenditure, our HRS data on lifetime earnings and retirement ages, and our parameter estimates. Section 4 qualitatively and quantitatively analyzes the Social Security reform outlined above. Section 5 concludes.

2. Model

This section presents our basic model. Then it elaborates the framework to include stochastic disability.

2.1 Basic Model. We have a partial equilibrium overlapping generations model. This paper restricts its analysis to couples. A household begins when its male reaches age S. He marries at age S_0 and has children $k = 1, ..., K$ at age S_k. Males die at age T^M; females at age T^F. Set $T \equiv \max\{T^M, T^F\}$.

A key feature of our model is intratemporally nonseparable preferences. A household's current utility depends on its current service flow from market consumption and leisure (cf. Auerbach and Kotlikoff [1987]). This paper assumes the service flow is a Cobb–Douglas function of household consumption, c, and leisure, ℓ, per capita:[4]

$$f(c, \ell) \equiv [c]^{\alpha} \cdot [\ell]^{1-\alpha}, \quad \alpha \in (0,1).$$

For couples, the man and woman work full time until retirement and retire when the male is age R. We normalize $\ell = 1$ post retirement; prior to retirement

$$\ell = \bar{\ell} \in (0, 1).$$

A household's utility flow is an isoelastic function of its service flow:

$$\frac{[f(c, \ell)]^{\gamma}}{\gamma}, \quad \gamma < 1.$$

This paper's treatment of life–cycle changes in family composition follows Tobin [1967]. For household i at age t define

$$\chi^S(i, t) \equiv \begin{cases} 1, & \text{if age–}t\text{ household includes a spouse,} \\ 0, & \text{otherwise.} \end{cases}$$

If household i at age t has K "kids" of ages 0-17, define

$$\chi^K(i, t) \equiv K.$$

The number of "equivalent adults" in the household when it is age t is

$$n_{it} \equiv 1 + \chi^S(i, t) \cdot \xi^S + \chi^K(i, t) \cdot \xi^K, \tag{1}$$

where ξ^S and ξ^K are positive parameters. Economies of scale in household operation and/or the public goods of household consumption might leave $\xi^S < 1$ and $\xi^K < 1$. The utility flow of household i at age t is

[4] See section 2.2 for a discussion of this and other assumptions of the model.

$$u(c_{it}) = \frac{1}{\gamma} \cdot n_{it} \cdot [\frac{c_{it}}{n_{it}}]^{\alpha\cdot\gamma} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\,, \qquad \text{for } t \in [S\,,\,R)\,,$$
$$v(c_{it}) = \frac{1}{\gamma} \cdot n_{it} \cdot [\frac{c_{it}}{n_{it}}]^{\alpha\cdot\gamma}\,, \qquad \text{for } t \in [R\,,\,T]\,. \tag{2}$$

In other words, flow utility depends upon consumption per equivalent adult and leisure per adult, weighted by number of equivalent adults.

Household i solves the following life–cycle problem:

$$\max_{R_i\,,\,c_{it}} \int_{S_i}^{R_i} e^{-\rho\cdot t} \cdot u(c_{it})\,dt + \varphi\big(a_{i,R_i} + B_i(R_i)\cdot e^{r\cdot R_i}\,,\,R_i\big) \tag{3}$$

$$\text{subject to:} \quad \dot{a}_{it} = r\cdot a_{it} + y_{it} - c_{it}\,,$$

$$a_{iS} = 0\,,$$

where ρ is the subjective discount rate; the household's adult male supplies e^M_{it} "effective hours" in the labor market per hour of work time; the adult female supplies e^F_{it} "effective hours;" the wage rate per effective hour is w; the income–tax rate is τ; the Social Security and Hospital Insurance tax rate is τ^{ss}; household net worth is a_{it}; and,

$$y_{it} \equiv \begin{cases} (1-\bar{\ell})\cdot[e^M_{it} + e^F_{it}]\cdot w\cdot(1-\tau-\tau^{ss})\,, & \text{for } S_i \leq t < R_i, \\ 0\,, & \text{otherwise.} \end{cases}$$

"Effective hours" change with age, reflecting an individual's cumulative experience and economywide technological progress. The function $\varphi(.)$ is

$$\varphi\big(A + B_i(R_i)\cdot e^{r\cdot R_i}\,,\,R_i\big) \equiv \max_{c_{it}} \int_{R_i}^{T} e^{-\rho\cdot t}\cdot v(c_{it})\,dt \tag{4}$$

$$\text{subject to:} \quad \dot{a}_{it} = r\cdot a_{it} - c_{it}\,,$$

$$a_{iR_i} = A + B_i(R_i)\cdot e^{r\cdot R_i} \quad \text{and} \quad a_{iT} \geq 0\,,$$

where the age–0 present value of capitalized Social Security, Medicare, and private defined–benefit pension benefits is $B_i(R_i)$. A household takes r, w, τ, τ^{ss}, e^M_{it}, e^F_{it}, and $B(.)$ as given. Social Security benefits only begin at age $\max\{R_i\,,\,62\}$; Medicare benefits begin at age 65. Social Security and private–pension benefits depend upon retirement age. Social Security benefits are taxed at rate $\tau/2$, private–pension benefits at rate τ, and Medicare benefits are not taxed.

There may be inducements to retire at particular ages implicit in some defined benefit pension plans (or, indeed, in some employer–provided health insurance packages) — c.f., Ippolito [1997]. One hypothesis is that both employers and workers are heterogeneous in their preferences about retirement ages and that workers choose employers whose preferences match their own. A second hypothesis is that employers have unified preferences and that these affect some workers' retirement choices. This paper follows the first hypothesis, leaving our $B(.)$ to reflect Social Security alone. (We hope, however, that our data resources will allow us to contribute to the literature on the second hypothesis in the future.)

The criteria and asset constraints of (3)-(4) are only piecewise continuously differentiable; nevertheless, conventional optimality conditions remain valid and, importantly, costate variables have continuous time paths — see Lemma 1 in Appendix I.

Our empirical analysis rests on two features of solutions to (3)-(4). The sign and magnitude of changes in c_{it} at retirement are the first (recall the introduction to this paper). We have

Proposition 1: *Let household i choose to retire at age $R = R_i$. Suppose that discontinuities in n_{it} and labor supply at retirement make the criterion and right–hand side of the asset equation discontinuous at monotone increasing ages t_j, $j = 1, ..., J$. Let $t_0 \equiv S = S_i$ and $t_{J+1} \equiv T$. Then a solution of (3)-(4) has*

$$\frac{\dot{c}_{it}}{c_{it}} = \frac{r-\rho}{1-\alpha\cdot\gamma}\,, \tag{5}$$

$$c_{it+} = M_{ij}\cdot c_{it-}\,, \quad M_{ij} \equiv \frac{n_{it+}}{n_{it-}}\,, \quad t = t_j\,,\ j = 1, ..., J,\ \textit{but } t \neq R\,, \tag{6}$$

$$c_{iR+} = M_{ij}\cdot c_{iR-}\,, \quad M_{ij} \equiv [\bar{\ell}]^{-\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}}\,, \quad t = t_j = R\,. \tag{7}$$

Letting $M_{i0} = 1$, initial household consumption is

$$c_{iS} = \psi(i\,,\,R_i) \equiv \frac{\int_S^{R_i} e^{-r\cdot t}\cdot y_{it}\,dt + e^{-r\cdot R_i}\cdot B_i(R_i)}{\sum_{j=0}^{J}\left[\prod_{k=0}^{j} M_{ik}\right]\cdot\int_{t_j}^{t_{j+1}} e^{-r\cdot t}\cdot e^{\frac{r-\rho}{1-\alpha\cdot\gamma}\cdot t}\,dt}\,. \tag{8}$$

Proof: See Appendix I.

The result of greatest interest here is (7), which describes a discontinuous change in consumption at retirement. The intuition is as follows. Inputs to a bivariate neoclassical production function are complementary in the sense that more of one raises the other's marginal product. If $u(.)$ were linear, a household would desire to raise its consumption at retirement to take advantage of this complementarity. If $u(.)$ departs from linearity, a second, competing force arises: a household desires to "smooth" its service flow intertemporally and may, therefore, want to decrease c_{it} upon the cessation of work to offset increases in leisure. Condition (7) shows that complementarity predominates for $\gamma \in (0\,,\,1)$, but proclivities to smooth service flows win out for $\gamma < 0$.

The second foundation of our empirical analysis is households' choice of R_i. To develop intuition on the choice, we have

Proposition 2: *Given a solution to (3)-(4), at* $R = R_i \in (S_i\,,\,T)$ *one has*

$$\big[\alpha \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot [c_{iR-}]^{\alpha\cdot\gamma-1} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\big] \cdot [y_{iR-} - c_{iR-} + c_{iR+} + B_i'(R) \cdot e^{r\cdot R}] =$$
$$\frac{1}{\gamma} \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot \big[[c_{iR+}]^{\alpha\cdot\gamma} - [c_{iR-}]^{\alpha\cdot\gamma} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\big]\,. \tag{9}$$

Proof: See Appendix I.

The idea of (9) is as follows. The left–hand side registers the advantage of a marginal increase in retirement age R: y_{iR-} measures additional earnings contingent upon postponing retirement; $c_{iR+} - c_{iR-}$ registers the fact that if desired consumption declines after retirement, household resources may stretch farther; and, $B_i'(R) \cdot e^{r\cdot R}$ measures incremental pension–benefit gains. The left side of (9) multiplies the sum of these dollar advantages by the marginal utility of consumption, thus converting it to units of utility. The right–hand side of (9) captures the disadvantage of postponing retirement, namely, the difference between the flow of utility after and before retirement.[5]

[5] The right–hand side of (9) is positive as follows. Using (7), the sign on the right–hand side is

$$\text{sgn}(\frac{1}{\gamma}) \cdot \text{sgn}([\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma\cdot\alpha\cdot\gamma}{1-\alpha\cdot\gamma}} - [\bar{\ell}]^{(1-\alpha)\cdot\gamma}) \cdot \text{sgn}([c_{iR-}]^{\alpha\cdot\gamma}) =$$
$$\text{sgn}(\frac{1}{\gamma}) \cdot \text{sgn}(1 - [\bar{\ell}]^{\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}}) \cdot \text{sgn}([\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma\cdot\alpha\cdot\gamma}{1-\alpha\cdot\gamma}}) =$$
$$\text{sgn}(\frac{1}{\gamma}) \cdot \text{sgn}(1 - [\bar{\ell}]^{\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}})\,.$$

A second interpretation is also illuminating. Divide both sides of (9) by $[n_{iR}]^{1-\alpha\cdot\gamma} \cdot [c_{iR-}]^{\alpha\cdot\gamma}$, and notice that c_{iR+}/c_{iR-} is a constant. Then the left side depends only on

$$\frac{y_{iR-} + B_i'(R) \cdot e^{r\cdot R}}{c_{iR-}}, \tag{10}$$

while the right side is constant. If (10) falls with age, retirement occurs when the two sides become equal. So, reductions in the growth rates of earnings and pension accumulation in old age encourage retirement. Increases in consumption — provided the latter's growth rate from (5) is positive — have the same effect because higher consumption expenditures raise the value of leisure. (In fact, we might surmise that differences in households' lifetime earnings and pension–accumulation patterns tend to promote heterogeneity of retirement ages, whereas the common lifetime rate of increase in consumption (see (5)) promotes homogeneity.)

2.2 Discussion. Two especially important features of our model are discrete labor supply options and intratemporally non-separable preferences.

In our framework, households must either work full time or retire. While in practice employers do offer part–time jobs, the rate of pay may be lower than that for full–time work, possibilities of advancement more limited, etc.[6] As Rust and Phelan [1997] write,

> The finding that most workers make discontinuous transitions from full–time work to not working, and the finding that the majority of the relatively small number of 'gradual retirees' reduce their annual hours of work by taking on a sequence of lower wage partial retirement 'bridge jobs' rather than gradually reducing hours of work at their full-time pre-retirement 'career job' suggests the existence of explicit or implicit constraints on the individual's choice of hours of work. [p.786]

An indivisible work day seems consistent with the fact that U.S. data show little trend in male work hours or participation rates after 1940, except for a trend toward earlier retirement 1940-80 (e.g., Pencavel [1986], Blundell and MaCurdy [1999], and Burkhauser *et al.* [1999]).

Although intratemporal additivity is perhaps the most common specification for utility in the life–cycle literature, both Blau [2006] and Haider and Stephens [2005] indicated

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Recall that $\bar{\ell} \in (0,1)$ and $\alpha \in (0,1)$. If $\gamma \in (0,1)$, the sign of both terms in the last product is positive. If $\gamma < 0$, both are negative.

[6] Reasons for the wage penalty for part-time work include daily fixed costs of startup and shutdown, scheduling and coordination problems, employer concern for timely return on training investments, and the fixed–cost nature of some employee benefits (e.g., Hurd [1996]).

that, even with uncertainty, such a formulation leaves much of the observed drop in consumption expenditure at retirement unexplained. Our nonseparable specification is similar to a number of papers (e.g., Auerbach and Kotlikoff [1987], King *et al.* [1988], Hurd and Rohwedder [2003], and Cooley and Prescott [1995]). With discrete retirement, nonseparability provides, as we have seen, an explanation of the "retirement consumption puzzle." Auerbach and Kotlikoff employ a CES aggregator for service flows, assuming an elasticity of substitution for $f(.)$ of 0.75 in their "base case;" Cooley and Prescott [1995], for example, use the same functional forms as we do. (Allowing greater flexibility with a CES aggregator would complicate (5) and (7), exacerbating the shortcomings of our consumption data. Lower substitution possibilities would, on the other hand, have potentially interesting implications for retirement patterns in the long run; hence, less restrictive specifications remain a topic for future research.)

As the introduction suggests, our assumptions that jobs require full-time work and that retirement is permanent may function to amplify the behavioral changes and efficiency gains of the reform proposed in this paper. If predicted consequences from the reform were small in this context, one might conjecture that aspects of a richer model might cancel them altogether. As we will see below, however, our estimated effects from reform are substantial. Moreover, we find that augmenting our basic model to accommodate the potentially dampening effects of disability on career length has only a modest influence on the model's predictions.

2.3 Disability. This section augments the basic model to include a stochastic chance of disability. This paper considers only the case with actuarially fair disability insurance, it assumes that exogenous factors cause disability, and it assumes that one's health status is objectively verifiable.

Assume that a household's health status is either "not disabled" or "disabled," and that a disabled household can never again work. Once a household becomes disabled, it remains disabled until it reaches its (previously) chosen retirement age R, at which point we reclassify it as retired. Let $p(t)$ be the probability that a household becomes disabled at age t. Let $P(s)$ be the probability of becoming disabled after age s:

$$P(s) \equiv 1 - \int_S^s p(t)\,dt = \int_S^T p(t)\,dt - \int_S^s p(t)\,dt = \int_s^T p(t)\,dt\,. \tag{11}$$

At age $t < R$, a nondisabled household purchasing term disability insurance during the interval $[t\,,\,t+dt)$ would pay an insurance premium with annual rate $p(t)/P(t)$ per dollar of benefits. In other words, a household would pay total premiums $p(t)\cdot X_{it}\,dt/P(t)$, to receive (current–dollar) lump–sum benefit X_{it} in the event of disability. Whether disabled or not, household i receives capitalized sum $B_i(R_i)\cdot e^{r\cdot R_i}$, in current dollars, at its chosen retirement age R_i; thus, retirement benefits implicitly include a disability–insurance component in our framework, and disability insurance need only tide a household over until its retirement age.

Disabled households benefit from full-time leisure; disability may lower their utility as well. We could allow for the latter with an additively separable term in the flow utility function. Such a term does not affect household behavior; so, for simplicity, our analysis omits it.[7]

Behavior after retirement is the same as before; hence, problem (4) remains as above. If household i happens to become disabled at age $D = D_i < R_i = R$ and has insurance payout X_{iD}, its cumulative utility for ages $t \in [D\,,\,T]$ is

$$\bar{\varphi}\big(A + X_{iD}\,,\,D\,,\,R\big) \equiv \max_{\bar{c}_{it}} \int_D^R e^{-\rho \cdot t} \cdot v(\bar{c}_{it})\,dt + \varphi\big(\bar{a}_{iR-} + B_i(R) \cdot e^{r \cdot R}\,,\,R\big) \tag{12}$$

$$\text{subject to:} \quad \dot{\bar{a}}_{it} = r \cdot \bar{a}_{it} - \bar{c}_{it}\,,$$

$$\bar{a}_{iD+} = A + X_{iD}\,,$$

$$\bar{a}_{iR} \geq 0\,.$$

We are now ready to set out a household's complete life–cycle problem for the environment with disability. Continue to let $D = D_i$ and $R = R_i$. At its inception, household i solves

$$\max_{R\,,\,c_{it}\,,\,X_{it}} \int_S^R p(D) \cdot [\int_S^D e^{-\rho \cdot t} \cdot u(c_{it})\,dt + \bar{\varphi}(a_{iD-} + X_{iD}\,,\,D\,,\,R)]\,dD +$$
$$[1 - \int_S^R p(t)\,dt] \cdot [\int_S^R e^{-\rho \cdot t} \cdot u(c_{it})\,dt + \varphi\big(a_{iR+} + B_i(R) \cdot e^{r \cdot R}\,,\,R\big)] \tag{13}$$

$$\text{subject to:} \quad \dot{a}_{it} = r \cdot a_{it} + y_{it} - c_{it} - \frac{p(t) \cdot X_{it}}{P(t)} \quad \text{for} \quad t \leq D\,,\,R\,,$$

[7] Our model would have different predictions for the change in consumption upon becoming disabled if poor health affected either the technology of household service production or the utility flow from the same level of service. We leave treatment of these effects to later work.

$$a_{iS} = 0\,.$$

The criterion's first term captures lifetime utility if the household becomes disabled at age $D < R$; the second component captures lifetime utility if the household reaches its chosen retirement age R without first becoming disabled.

The analog to Proposition 1 is

Proposition 3: *Consider the model with disability. Let household i choose to retire at age $R = R_i$. Suppose that discontinuities in n_{it} and labor supply at retirement or disability make the criterion and right–hand side of the asset equation discontinuous at ages t_j, $j = 1, ..., J$. Let $t_0 \equiv S = S_i$ and $t_{J+1} \equiv T$. If $D = D_i \geq R$, a solution of (12)-(13) has*

$$\frac{\dot{c}_{it}}{c_{it}} = \frac{r - \rho}{1 - \alpha \cdot \gamma}\,, \tag{14}$$

$$c_{it+} = \frac{n_{it+}}{n_{it-}} \cdot c_{it-}\,, \quad t = t_j\,, \quad j = 1, ..., J, \quad \textit{but} \quad t \neq R\,, \tag{15}$$

$$c_{iR+} = [\bar{\ell}]^{-\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}} \cdot c_{iR-}\,. \tag{16}$$

If the household becomes disabled at age $D < R$, we replace (16) with

$$c_{iD+} = [\bar{\ell}]^{-\frac{(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}} \cdot c_{iD-} \quad \textit{and} \quad c_{iR+} = c_{iR-}\,. \tag{17}$$

Let M_{ij} be the consumption jump from (15)-(17) at t_j, $j = 1, ..., J$, and let $M_{i0} = 1$. Since one breakpoint occurs at age $\min\{D\,,\, R\}$, write $t_j = t_j(D)$ when $D < R$ and $t_j = t_j(R)$ otherwise. Then the initial consumption of household i is

$$c_{iS} = \bar{\psi}(i\,,\, R) \equiv$$
$$\frac{\int_S^R p(D) \cdot [\int_S^D e^{-r\cdot t} \cdot y_{it}\, dt]\, dD + P(R) \cdot \int_S^R e^{-r\cdot t} \cdot y_{it}\, dt + e^{-r\cdot R} \cdot B_i(R)}{DEN}\,, \tag{18}$$

$$DEN \equiv \int_S^R p(D) \cdot [\sum_{j=0}^{J} [\prod_{k=0}^{j} M_{ik}] \cdot \int_{t_j(D)}^{t_{j+1}(D)} e^{-r\cdot t} \cdot e^{\frac{r-\rho}{1-\alpha\cdot\gamma}\cdot t}\, dt]\, dD +$$
$$P(R) \cdot \sum_{j=0}^{J} [\prod_{k=0}^{j} M_{ik}] \cdot \int_{t_j(R)}^{t_{j+1}(R)} e^{-r\cdot t} \cdot e^{\frac{r-\rho}{1-\alpha\cdot\gamma}\cdot t}\, dt\,.$$

Proof: See Appendix I.

The new feature of Proposition 3 is the change in consumption upon pre-retirement disability, namely, condition (17). The intuition for (17) is as follows. Although the possibility of disability reduces lifetime resources (c.f. (8) and (18)), households adopt full insurance. The need to pay insurance premiums causes lifetime consumption to be lower, but, given insurance, the advent of disability causes a household no further financial hardship. The latter fact implies that a household chooses the same consumption change after becoming disabled as at the arrival of its planned retirement age in other circumstances.

The analog to Proposition 2 provides a first–order condition for each household's utility–maximizing retirement age:

Proposition 4: *Given a solution to (4) and (12)-(13), at $R = R_i \in (S\,,\,T)$ one has*

$$\big[\alpha \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot [c_{iR-}]^{\alpha\cdot\gamma-1} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\big] \cdot [y_{R-} - c_{iR-} + c_{iR+} + B_i'(R) \cdot e^{r\cdot R} - \frac{p(R)\cdot X_{iR}}{P(R)}] =$$
$$\frac{1}{\gamma} \cdot [n_{iR}]^{1-\alpha\cdot\gamma} \cdot \big[[c_{iR+}]^{\alpha\cdot\gamma} - [c_{iR-}]^{\alpha\cdot\gamma} \cdot [\bar{\ell}]^{(1-\alpha)\cdot\gamma}\big] \tag{19}$$

when $R_i \le D_i$. Furthermore,

$$X_{iR} = y_{iR-} - c_{iR-} + c_{iR+}\,. \tag{20}$$

Proof: See Appendix I.

As in Proposition 2, (19) balances retirement–induced losses of wages and retirement benefits against utility gains from more leisure. What is new is that only earnings net of disability–insurance cost constitute an advantage for postponing retirement.

<u>2.4 Estimation equations</u>. This section derives the two equations on which our estimation depends.

The first equation comes from Proposition 3. When household i is age s, it has experienced a set of ages, say, $\kappa(s\,,\,i)$, with breakpoints from family composition changes and retirement or disability. Let the consumption–level adjustment factors corresponding to breakpoints be M_{ik} as in Proposition 3. Then Proposition 3 shows that

$$c_{is} = \bar{\psi}(i\,,\,R_i) \cdot [\prod_{k\in\kappa(s\,,\,i)} M_{ik}] \cdot e^{\frac{r-\rho}{1-\alpha\cdot\gamma}\cdot(s-S_i)}\,.$$

Let D_i be the household's age of disability. Define

$$\chi^{RD}(i\,,\,t) \equiv \begin{cases} 1\,, & \text{if } t \ge \min\{R_i\,,\,D_i\}, \\ 0\,, & \text{otherwise.} \end{cases}$$

Noting that $\kappa(s\,,\,i) \subseteq \kappa(s+1\,,\,i)$, and using (1), we then have

$$\ln(c_{i,s+1}) - \ln(c_{i,s}) = \frac{r-\rho}{1-\alpha\cdot\gamma} + \sum_{k\in\kappa(s+1,i)-\kappa(s,i)} \ln\bigl(M_{ik}\bigr) \approx$$
$$\frac{r-\rho}{1-\alpha\cdot\gamma} + \xi^S\cdot[\chi^S(i\,,\,s+1) - \chi^S(i\,,\,s)] + \xi^K\cdot[\chi^K(i\,,\,s+1) - \chi^K(i\,,\,s)]+$$
$$\frac{(1-\alpha)\cdot\gamma\cdot\ln(\bar{\ell})}{\alpha\cdot\gamma-1}\cdot[\chi^{RD}(i\,,\,s+1) - \chi^{RD}(i\,,\,s)]\,, \tag{21}$$

where the approximation comes from a first–order Taylor series.

Consumption is difficult to measure in practice. Our consumption data, taken from the CEX and described below, provide a pseudo panel of average consumption expenditures, $\bar{c}_{st}$, for households of age s at time t. If i indexes individual households and ω_{ist} gives household weights,

$$\bar{c}_{st} = \sum_i \omega_{ist}\cdot c_{ist}\,.$$

Since the distribution of earnings in practice is roughly lognormal and our life–cycle preferences are homothetic, think of household consumption in each age–year cell as lognormally distributed: $\ln(c_{ist}) \sim N(\mu_{st}\,,\,\sigma^2_{st})$. Then

$$\sum_i \omega_{ist}\cdot\ln(c_{ist}) \approx E[\ln(c_{ist})] = \mu_{st}\,,$$

$$\ln(\sum_i \omega_{ist}\cdot c_{ist}) \approx \ln(E[c_{ist}]) = \mu_{st} + \frac{\sigma^2_{st}}{2}\,.$$

Assuming $\sigma_{s+1,t+1} \approx \sigma_{st}$ and letting υ_{st} register consumption measurement error, our first equation for estimation follows from (21) and the two preceding expressions:

$$\ln(\bar{c}_{s+1,t+1}) - \ln(\bar{c}_{st}) =$$
$$\frac{r-\rho}{1-\alpha\cdot\gamma} + \xi^S\cdot[\sum_i \omega_{i,s+1,t+1}\cdot\chi^S(i\,,\,s+1\,,\,t+1) - \sum_j \omega_{jst}\cdot\chi^S(j\,,\,s\,,\,t)]+$$
$$\xi^K\cdot[\sum_i \omega_{i,s+1,t+1}\cdot\chi^K(i\,,\,s+1\,,\,t+1) - \sum_j \omega_{jst}\cdot\chi^K(j\,,\,s\,,\,t)]+$$
$$\frac{(1-\alpha)\cdot\gamma\cdot\ln(\bar{\ell})}{\alpha\cdot\gamma-1}\cdot[\sum_i \omega_{i,s+1,t+1}\cdot\chi^{RD}(i\,,\,s+1\,,\,t+1) - \sum_j \omega_{jst}\cdot\chi^{RD}(j\,,\,s\,,\,t)]+$$
$$\upsilon_{s+1,t+1} - \upsilon_{st}\,. \tag{22}$$

Hall [1978, 1988] would include another error, say, ζ_{st}, to capture the effect of information newly available to households during each period t. Shocks to individual households are beyond the scope of our CEX, averaged data; however, our actual regressions include annual time–dummy variables that can capture aggregative shocks. (See also the discussion below of columns 5-6 of Table 4.)

Our second estimation equation comes from the retirement–age choices of individual households. Given the consumption path from Proposition 3, we maximize (12)-(13) with respect to R_i. We have HRS data, described below, on the lifetime earnings, retirement age, and demographics for individual households. Letting R_i be the actual retirement age of household i and R_i^* our model's prediction, we estimate

$$R_i = R_i^* + \epsilon_i \, , \tag{23}$$

where ϵ_i captures factors independent of our model, such as a household's responsiveness to its particular employer's wishes, etc. Proposition 4 makes no reference to second–order conditions; consequently, we generate R_i^* from a global maximization algorithm described below.

3. Data and Estimation

As the introduction previewed, this paper estimates (22) from CEX data and (23) from the HRS. After discussing the data sources, this section presents our parameter estimates.

3.1 CEX Data. Our primary data source for estimating (22) is the U.S. Consumer Expenditure Survey (CEX). It provides comprehensive consumption data. The CEX obtains diary information on small purchases from one set of households; with a second set of households, it conducts quarterly interviews that catalog major purchases. The survey also collects demographic data and self-reports on the value of the respondent's house. At any given time, the sample consists of approximately 5,000 (7,000 after 1999) households, which each remain in the survey for at most 5 quarters. The survey was conducted at multi–year intervals prior to 1984, and annually thereafter. This paper uses the surveys for 1984-2001.[8]

Our earlier work (Laitner and Silverman [2005]) compared CEX annual consumption totals with the National Income and Product Accounts. Assuming that the NIPA numbers are accurate, that item–nonresponse and other measurement errors of the survey typically

[8] The web site http://stats.bls.gov.gov/csxhome.htm presents aggregative tables, codebooks, etc., for the CEX. This paper uses raw CEX data from the ICPSR archive, and we gratefully acknowledge the assistance of the BLS in providing "stub files" of changing category definitions.

make CEX totals too low, and that the relative magnitude of survey errors does not systematically vary with age, for each year we scale CEX consumption categories, uniformly across ages, to match NIPA amounts. Appendix II lists our categories and describes in detail three additional adjustments concerning the treatment of housing services, health care, and personal business expenditures. This paper abstracts from the empirical difference between consumption and expenditure (e.g., Aguiar and Hurst [2005]) and, except in the case of housing, draws no distinction between consumer durable stocks and flows.

Deflating with the NIPA personal consumption deflator, and using survey weights, we derive an adjusted average consumption amount, our $\bar{c}_{st}$, for each age s and year t. Due to the construction of the CEX from separate interview and diary surveys, and annual aggregation from quarterly, rotating–sample data, we do not have consumption figures for individual households. We organize the CEX data so that a household's age is the age of the wife for a married couple (and the single household head in other cases).

The CEX provides information on whether the household is married. Although the CEX also reports number of children age 0-17, we construct our own measure of children per household to gain more flexibility: using Census data on births per woman at age $s \in \{15, ..., 49\}$ in year $t \in \{1920, ..., 2001\}$, we simulate the number of children of each age for women of each age 1984,...,2001.

CEX data on retirement is unsatisfactory because the CEX questionnaire only asks whether the respondent is "retired" when he or she had zero weeks of work in the prior twelve months; therefore, we turn to the March Current Population Survey (CPS) 1984-2001 for our $\chi_s(R_i)$ variable.[9] We consider a CPS household retired if the head is over 50 years old and answers that he or she is out of the labor force at the time of the March survey for reasons other than unemployment or, in the case of a male, is not "unemployed" but reports less than 30 hours per week of work. This paper focuses on male retirement because males were more attached to the labor force in the HRS cohorts and because our analysis abstracts from a detailed model of decision making within dual–earner households.

3.2 Retirement Data. The HRS is our data source for estimating equation (23), though we calibrate some parts of our life–cycle framework.

Consider the calibrations first. We assume a constant gross–of–income tax real interest rate of 5%/yr.[10] We disregard government transfer payments other than Social Security.

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[9] Indeed, the average median retirement age 1984-2001 in the CEX data is 64-65, whereas it is about 62 in the CPS.

[10] Our real interest rate comes from a ratio of factor payments to capital over the market value of private net worth. For the numerator, NIPA Table 1.13 gives corporate business income, indirect taxes, and total labor compensation. The first less the other two is our measure of corporate profits; the ratio of profits to profits plus labor remuneration is

Our income tax rate τ comes from government spending on goods and services less indirect taxes (already removed from profits, and implicitly absent from wages and salaries below). Dividing by national income, the average over 1952–2003 is 14.28%/year.[11]

In the calculations below, the Social Security benefit formula, including the ceiling on taxable annual earnings, follows the history of the U.S. system. One–half of Social Security benefits are subject to the income tax.

Our theoretical model assumes that adults work 40 hours per week until retirement and 0 hours per week thereafter. With 16×7 waking hours per week, we set[12]

$$\bar{\ell} = \frac{16 \times 7 - 40}{16 \times 7} = .6429\,.$$

Turning to the HRS data, we derive earnings profiles and retirement ages from the original HRS survey cohort, consisting of households in which the respondent was age 51-61 in 1992. A majority of participant households signed a permission waiver allowing the HRS to link to their Social Security Administration (SSA) earnings history. Each history runs 1951-1991; our HRS survey data covers 1992, 1994, 1996, 1998, 2000, and 2002. We restrict attention to once–married couples with both spouses alive in 1992, with the husband having linked SSA earnings and remaining in the labor force until at least age 51, and with the wife having linked SSA data or reporting no market work prior to 1992. Men and women must have 8-24 years of education. They become adults at the age equaling years of education plus 6, and we drop those reaching this age before 1951. Men and women live independently until marriage. We set our age of marriage at the minimum of the reported age and age at first birth. We assume that the children of HRS households leave home at age 22. We assume that men die at the close of age 74 and women at the close of 80. We exclude couples with more than 10 years age difference. Omitting households with incomplete data leaves 1121 couples.

"profits share." We multiply the latter times corporate, noncorporate, and nonprofit–institution income less indirect taxes. We add household–sector income (NIPA Table 1.13) less indirect taxes and labor remumeration. Finally, we subtract personal business expenses (brokerage fees, etc. from NIPA Table 2.5.5, rows 61–64). The denominator is U.S. Flow of Funds household and non–profit institution net worth (Table B.100, row 19), less government liabilities (Table L106c, row 20). We average beginning and end of year figures. For 1952–2003, the average is .0504. For comparison, Auerbach and Kotlikoff [1987] use 6.7%/year, Altig *et al.* [2001] 8.3%/yr., Cooley and Prescott [1995] 7.2%/yr., and Gokhale *et al.* [2001] use post–tax rates of 4%/yr. and 6%/yr.

[11] Auerbach and Kotlikoff [1987], for example, use 15%/year.

[12] See also Cooley and Prescott [1995] — who, on the basis of time–use studies, determine that households devote 1/3 of waking hours to work.

As stated, we assume that a household retires when its male adult does. The HRS twice asks if each adult is retired and when retirement took place. Prior to 1992, a male is retired if he reports that status on either question. After 1992, a male who reports being retired and works less than 1500 hours per year, or who works less than 1500 hours and never again more than 1500 hours per year, is "retired." We exclude households that pass our criterion for retirement in one survey wave but fail to do so in a subsequent wave, or that pass age 70 without retiring, reducing our sample to 1025.

For men, we estimate a so–called earnings dynamics model of earnings, dividing the total HRS sample into 4 education groups, and regressing log constant–dollar earnings on a quartic in age and dummy variables for time. The regression error has an individual effect as well as a random term. The data are right censored at the Social Security tax cap prior to 1980; at $125,000 for earnings 125,000-250,000, at $250,000 for earnings 250,000-500,000, and at $500,000 for earnings 500,000+ for 1981-1991. Our likelihood function takes the censoring into account. Laitner and Silverman [2005] present details. After 1991, survey data is available every other year. As a protection against coding errors, we exclude survey earnings greater than twice, or less than half, the earnings dynamics equation prediction for the same age. This paper assesses late–in–life earnings as follows. Using quadratic programming, we fit a convex quadratic function to each male's available earnings figures from 1986 onward, constraining the function to match 1986 earnings and have a non–positive slope at the last available work age. We interpolate missing data and extrapolate prospective earnings through age 69. (In our model, a household has zero earnings after its male retires; however, our global maximization algorithm for R^* — see below — utilizes the extrapolated figures.)

Although we use similar steps for female earnings, there are several differences. A woman who never works remains in our sample. As stated above, we assume a woman retires when her spouse does. We extrapolate non–zero late–in–life earnings only for women who supply market hours in the survey in the last year that their husband works. We are much more concerned than for men that women might have part–time earnings. Prior to 1992, therefore, a woman's earnings are her SSA earnings unless the latter are censored, in which case we impute from female earnings dynamics equations. For the latter, see Laitner and Silverman [2005]. The HRS provides information in 1996 on whether a woman had non–FICA earnings prior to 1992 (i.e., earnings not covered in the Social Security system). If a woman had non–FICA jobs and provided beginning and end dates, we impute her earnings from our earnings–dynamics regressions; if she provided only the span of non–FICA employment, we subtract non–FICA employment years 1980-91, which are evident from the data, and probabilistically impute remaining years using our earnings–dynamics regressions; if a woman said she had non–FICA employment but provided no information on when or how long, we drop the couple from the sample on the basis of incomplete data.

Since HRS earnings are net of employer benefits (including health insurance, pension contributions, and employer Social Security tax), we multiply household earnings for each year by the ratio of NIPA total compensation to NIPA wages and salaries.

We derive Social Security benefits after retirement from the statutory benefit formula for 2000. We also incorporate a stream of Medicare benefits after age 65, less participant SMI cost. To do this, for each adult 65 and older, we add to household resources Medicare benefits equaling the SMI annual premium for 2000 (i.e., $546) multiplied by the ratio of HI and SMI total expenditures to SMI premiums for 2000 (i.e., 10.7282 less 1).

This paper considers two possible measures of disability. In a sequence of questions about work status, the HRS asks respondents whether they are disabled and, if so, the year of onset. Table 1, column 1, presents the cumulative fraction of men who characterize themselves in this way as disabled and are retired. As stated above, our sample is limited to men who retire after age 50. In terms of Section 2, the cumulative probability corresponds to $1 - P(t)$ for each age t. Column 2 considers a less stringent measure. In a sequence of questions about health status, the HRS queries respondents on whether they have any health problems that "limit their ability to perform work." Column 2 presents cumulative fractions of men who are retired and who characterize themselves as disabled or who say they have a work–limiting health condition.

Table 1. Cumulative Probability of Male Disability: HRS Couples 1992-2002

Age	Retired and Disabled	Retired and Disabled or Work-Limitation	Age	Retired and Disabled	Retired and Disabled or Work-Limitation
51	.0008	.0027	61	.0743	.1701
52	.0016	.0036	62	.0935	.2334
53	.0106	.0162	63	.1164	.3132
54	.0140	.0235	64	.1339	.3604
55	.0198	.0373	65	.1532	.4175
56	.0249	.0511	66	.1720	.4726
57	.0326	.0662	67	.2032	.5422
58	.0489	.0907	68	.2370	.6004
59	.0545	.1088	69	.2969	.6980
60	.0640	.1355	70	.3619	.7829

Source: see text. HRS household weights.

Tables 2-3 present summary statistics on other aspects of our HRS sample. Table 2 calculates the present value at age 50 of after–tax lifetime earnings (1984 dollars) for men, $YM50$, and women, $YF50$. (In these figures, earnings end at retirement or the respondent's last survey wave.) We can see that for this cohort, females' earnings average only 20 percent of males'. Table 2 also computes the present value of household Social Security benefits at male age 50. Table 3 shows, importantly, that many men do not reach retirement age in our sample.

Table 2. Statistics for HRS Couples

Variable	Mean	Min	Median	Max	Coef. Var.
Age Male Last Works in Sample	61.3084	50	61	69	.0628
Age Diff. Couple: Male Age - Female	2.7934	-8	3	10	1.0700
Male Age Marriage	24.0276	14	23	56	.1743
Child per Couple	2.7689	0	3	10	.5197
YM50 (thou.)	1,569	458	1,400	12,278	.5683
YF50 (thou.)	316	0	243	2,499	.9719
B50 (thou.)	95	53	93	145	.1473

Source: see text. HRS household weights. Note: ages integer variables this paper.

Table 3. HRS Couples by Male Retirement Status

Category	Stringent Definition Disability	Broad Definition Disability$^{a\,,\,b}$
Retired/not disabled	631	429
Never retires in sample	287	279
Retires after disability	74	197
Dies prior to retirement	33	32
Total sample	1025	947

Source: see text. (a) "Disability" includes work limitations – - see text.
(b) Omits non-respondents work-limitations question.

<u>3.3 Estimation.</u> Our estimation uses a method of moments approach. Letting $\vec{\beta}$ be the vector of parameters to be estimated, rewrite (22)-(23) as

$$q^1_{st}(\vec{\beta}) = \upsilon_{s+1,t+1} - \upsilon_{st}\,, \tag{24}$$

$$q^2_i(\vec{\beta}) = \epsilon_i\,. \tag{25}$$

Assume that the υ_{st} and ϵ_i random variables are iid and mean 0. Standard steps yield a matrix A diagonalizing the covariance matrix for (24):

$$q^{1*}_{st}(\vec{\beta}) \equiv A \cdot q^1_{st}(\vec{\beta}) = \Upsilon_{st} \tag{26}$$

with Υ_{st} iid and mean 0. Let

$$\vec{\beta} \equiv (\alpha\,,\,\gamma\,,\,\rho\,,\,\xi^S\,,\,\xi^K\,,\,\sigma_\Upsilon\,,\,\sigma_\epsilon)\,. \tag{27}$$

Then our CEX moment conditions are[13]

$$\sum_{st} q^{1*}_{st}(\vec{\beta}) \cdot [A \cdot V^1_j(st)] = 0\,, \quad j = 1, ..., 4, \tag{28}$$

where

$$V^1_1(st) = 1\,, \quad V^1_2(st) = \sum_i \omega_{i,s+1,t+1} \cdot \chi^S(i\,,\,s+1\,,\,t+1) - \sum_j \omega_{jst} \cdot \chi^S(j\,,\,s\,,\,t)\,,$$

$$V^1_3(st) = \sum_i \omega_{i,s+1,t+1} \cdot \chi^K(i\,,\,s+1\,,\,t+1) - \sum_j \omega_{jst} \cdot \chi^K(j\,,\,s\,,\,t)\,,$$

$$V^1_4(st) = \sum_i \omega_{i,s+1,t+1} \cdot \chi^{RD}(i\,,\,s+1\,,\,t+1) - \sum_j \omega_{jst} \cdot \chi^{RD}(j\,,\,s\,,\,t)\,.$$

Our estimation of (28) also includes annual dummy variables for 1984-1999.

[13] Parenthetically, solving these moment conditions is equivalent to estimating the following vector of parameter composites from (26) with FGLS:

$$\left(\frac{r-\rho}{1-\alpha\cdot\gamma}\,, \quad \xi^S\,, \quad \xi^K\,, \quad \frac{(1-\alpha)\cdot\gamma\cdot\ln(\bar{\ell})}{\alpha\cdot\gamma-1}\right).$$

Turning to $q_i^2(\vec{\beta}) = R_i - R_i^*$, for each household i we solve (12)-(13) for integer values $R \in \{51, ..., 70\}$, setting the path of each household's consumption as in Proposition 3; we fit a quadratic to lifetime utility at the integer with the highest utility and its two closest neighbors; we determine R_i^* from the quadratic's peak. In comparison to Proposition 4, this procedure has the advantage of ensuring that sufficiency conditions hold.

To address the censoring problems evident in Table 3, assume $\epsilon_i \sim N(0\,,\,\sigma_\epsilon^2)$ and, letting $\phi(.\,,\,\sigma_\epsilon^2)$ be the normal density, define

$$q_i^{2*}(\vec{\beta}) \equiv E[\epsilon_i \,|\, \text{data}\,,\,\vec{\beta}] = \begin{cases} q_i^2(\vec{\beta}) & \text{if voluntarily retires in sample} \\ \dfrac{\int_{q_i^2(\vec{\beta})}^{\infty} e\cdot\phi(e\,,\,\sigma_\epsilon^2)\,de}{\int_{q_i^2(\vec{\beta})}^{\infty} \phi(e\,,\,\sigma_\epsilon^2)\,de} & \text{otherwise,} \end{cases}$$

$$q_i^{3*}(\vec{\beta}) \equiv E[(\epsilon_i)^2 \,|\, \text{data}\,,\,\vec{\beta}] = \begin{cases} [q_i^2(\vec{\beta})]^2 & \text{if voluntarily retires in sample} \\ \dfrac{\int_{q_i^2(\vec{\beta})}^{\infty} e^2\cdot\phi(e\,,\,\sigma_\epsilon^2)\,de}{\int_{q_i^2(\vec{\beta})}^{\infty} \phi(e\,,\,\sigma_\epsilon^2)\,de} & \text{otherwise.} \end{cases}$$

Then our HRS moment conditions are

$$\sum_i q_i^{2*}(\vec{\beta}) \cdot 1 = 0 \quad \text{and} \quad \sum_i q_i^{3*}(\vec{\beta}) \cdot 1 = \sigma_\epsilon^2\,. \tag{29}$$

Table 4 presents our estimates. Columns 1-2 implement (28)-(29), with different definitions of disability. We call this our basic model. We employ pseudo–panel CEX data for s=30,...,80, t=1984,...,2001; after differencing, this yields 850 observations. (Starting the CEX data at very youthful ages introduces a selection problem because college graduates, for example, only join the sample after age 22. With initial age 20, our estimates would change; for starting ages 25-35, they are stable.) HRS sample sizes are as in Table 3.

Parameter estimates are similar in columns 1-2 — though σ_ϵ seems noticeably larger in column 2, perhaps suggesting that the stringent definition of disability is more consistent with agents' actual behavior.

Columns 3-4 consider a substitute for (29). Column 1's methodology has the advantages of satisfying second–order conditions; it has the liability of requiring extrapolations of households' earning profiles past actual retirement — see, for example, French [2005]. (In fairness, however, such extrapolations are necessary for the policy simulations of Section 4 in any case.) Columns 3-4 report results from an estimator that relies on optimality condition (19) and has the opposite strengths and weaknesses. In particular, define a

Table 4. Estimated Coefficients Equations (28)-(29):[a]
Estimated Parameter (Std. Error/T Stat.)

Par.[b]	Basic Model (see text)		Alt. Estimation Using Prop. 4	
	Stringent Def. Male Disability[c]	Broad Def. Male Disability[c]	Stringent Def. Male Disability	Broad Def. Male Disability
α	0.3289 (0.0050/64.1428)	0.3629 (0.0053/68.2478)	0.3469 (0.0054/63.8677)	0.4032 (0.0076/53.1299)
γ	-0.8518 (0.1825/-4.6679)	-0.9400 (0.2110/-4.4545)	-.8963 (0.1981/-4.5252)	-1.0717 (0.2681/-3.9978
ρ	0.0080 (0.0029/2.7530)	0.0063 (0.0034/1.8701)	0.0071 (0.0032/2.2582)	0.0038 (0.0044/0.8752)
ξ^S	0.3979 (0.0541/7.3486)	0.3979 (0.0541/7.3486)	0.3979 (0.0541/7.3486)	0.3979 (0.0541/7.3486)
ξ^C	0.1469 (0.0105/14.0131)	0.1469 (0.0105/14.0131)	0.1469 (0.0105/14.0131)	0.1469 (0.0105/14.0131)
σ_ϵ	5.8998 (0.0179/328.7092)	6.5652 (0.0109/601.1751)		
σ_η			0.0116 (0.0076/1.5223)	0.0040 (0.0042/0.9465)
Calculated Parameters:[d]				
$\alpha\cdot\gamma$	-0.2801 (0.0596/-4.6985)	-0.3411 (0.0426/17.4673)	-0.3109 (0.0762/-4.4752)	-0.4322 (0.1112/-3.8857)
$\frac{r-\rho}{1-\alpha\cdot\gamma}$	0.0273 (0.0013/21.5044)	0.0273 (0.0013/21.5044)	0.0273 (0.0013/21.5044)	0.0273 (0.0013/21.5044)
$\frac{(1-\alpha)\cdot\gamma\cdot\ln(\bar{\ell})}{\alpha\cdot\gamma-1}$	-0.1973 (0.0334/-5.9147)	-0.1973 (0.0334/-5.9147)	-0.1973 (0.0334/-5.9147)	-0.1973 (0.0334/-5.9147)
Observations Eq. (28)/Eq. (29):[e]				
	850/1025	850/947	850/1025	850/947

Table 4 (cont). Estimated Coefficients Equations (28)-(29):[a] Estimated Parameter (Std. Error/T Stat.)		
Par.[b]	Basic Model; Time–Aggregated CEX Data	
	Stringent Def. Male Disability[c]	Broad Def. Male Disability[c]
α	0.3282 (0.0166/19.776)	0.3642 (0.0174/20.8473)
γ	-0.6496 (0.4962/-1.3093)	-0.7092 (0.5603/-1.2658)
ρ	0.0121 (0.0062/1.9622)	0.0110 (0.0072/1.5370)
ξ^S	0.4318 (0.2261/1.9101)	0.4318 (0.2261/1.9101)
ξ^C	0.1298 (0.0265/4.9030)	0.1298 (0.0265/4.9030)
σ_ϵ	5.9138 (0.0403/146.9143)	6.5908 (0.0120/549.4285)
	Calculated Parameters:[d]	
$\alpha \cdot \gamma$	-0.2132 (0.1588/-1.3421)	-0.2570 (0.1986/-1.2940)
$\frac{r-\rho}{1-\alpha\cdot\gamma}$	0.0253 (0.0027/9.2428)	0.0253 (0.0027/9.2428)
$\frac{(1-\alpha)\cdot\gamma\cdot\ln(\bar{\ell})}{\alpha\cdot\gamma-1}$	-0.1589 (0.1023/-1.5537)	-0.1589 (0.1023/-1.5537)
	Observations Eq. (28)/Eq. (29):[e]	
	50/1025	50/947

a. Annual time dummies 1984-99 for eq. (28) not reported.
b. Note: column 1-2 estimates of ξ^S, ξ^C, and second and third "calculated parameters" identical because of exact identification — recall fn 13.
c. "Stringent case male disability" refers to table 1, column 1; "broad case" refers to table 1, column 2.
d. Standard error from the so–called delta method first row below; from GLS on (28) next two rows (see fn 13).
e. For sample size changes, see text.

replacement $Q_i^2(\vec{\beta})$ for $q_i^{2*}(\vec{\beta})$ from the left–hand side of marginal condition (19) minus the right–hand side, evaluating the difference at each household's actual retirement age R_i. Set $Q_i^2(\vec{\beta}) = \eta_i$, and assume $\eta_i \sim N(0\,,\, \sigma_\eta^2)$. One can interpret η_i as an idiosyncratic (across households), additive preference for leisure — i.e., substitute $v(c_{it}) + \eta_i$ in (2) for $v(c_{it})$. Proposition 4 shows that $Q_i^2(\vec{\beta}) = 0$ at the desired retirement age in the absence of such heterogeneity. Observations from households that never retire in sample, that die before retiring, or that retire with disabilities, provide upper bounds for η_i, which straightforwardly generate an analog $Q_i^3(\vec{\beta})$ to $q_i^{3*}(\vec{\beta})$. Equation (28) remains as before.

The results in Table 4 do not point to a difficult choice between alternative estimation methods; the parameter estimates are very similar. (This is despite the fact that violations of second–order conditions do, nevertheless, arise for a number of cases in when we implement the first–order condition method.)

As stated, the analysis of columns 1-4 incorporates time dummies in (28). This specification assumes that aggregative shocks affect households of different ages in the same way. Since disturbances might, in practice, influence households of different ages differently, we also consider an alternative specification: to attenuate the effect of yearly shocks altogether, columns 5-6 average, for each age, consumption expenditure changes through all sample years. Standard errors rise because the number of consumption observations shrinks to 50; nevertheless, point estimates of parameter values change only very modestly.

Table 4's estimates of γ vary from -.65 to -1.07; the estimates of α vary from .33 to .40; and, the estimates of ρ vary from 0.00 to 0.01. These correspond to estimates of an intertemporal elasticity of substitution (IES) for services, $1/(1-\gamma)$, of 0.48 to 0.61, and an IES for pre–retirement consumption itself, $1/(1-\alpha\cdot\gamma)$, of 0.70 to 0.83. All estimates of γ, α, and $\alpha\cdot\gamma$ in columns 1-4 are statistically different from zero at the 5 percent significance level.

Our estimates of γ, α, and ρ are similar to a number of calibrations in the literature. For example, Auerbach and Kotlikoff's [1987] favorite calibration has $\gamma = -3$, α (roughly) $= .4$, and $\rho = .015$; Altig *et al.* [2001] use $\gamma = -3$, α (roughly) $= .5$, and $\rho = .004$; and, Cooley and Prescott [1995] set $\gamma = 0$, $\alpha = .36$, and $\rho = .053$.

Our results may also be compared with estimates that have identified the IES from expected changes interest rates. Using aggregate consumption data Hall [1988], Cambell and Mankiw [1989], and Patterson and Pesaran [1992], for example, estimate the IES for consumption to be at most half the size of our esimates. Micro studies tend to estimate somewhat larger intertemporal elasticities. Banks *et al.* [1998], for instance, estimate the average IES for consumption to be approximately 0.5. In another example, Attanasio and Weber [1993] estimate an IES for consumption of approximately 0.75 from micro data.[14]

[14] Barsky *et al.* [1997] use hypothetical questions to estimate an IES distribution for

Although our calculations rely a very different source of variation to estimate the IES, Table 4's outcomes are similar to, if on the larger end of, those obtained in micro studies from the change in consumption growth with expected changes in interest rates.

Table 4's second "calculated parameter" provides an estimate of the average lifetime growth rate for households' per capita consumption (see Prop. 3) of 2.5-2.7%/yr. This suggests that between, say, ages 25 and 62, in the absence of retirement a household's consumption per equivalent adult rises by a factor of about 2.62. In Auerbach and Kotlikoff [1987] the corresponding factor is about 1.54; in Gokhale *et al.* [2001], it is 1.74; in Tobin [1967], it is 13.33. For an infinite–lived representative agent model (e.g., Cooley and Prescott [1995]), the growth rate of consumption in a steady–state equilibrium would, of course, match the growth rate of GDP.

Our estimate of ξ^S suggests that the addition of a spouse raises household consumption by 39-44 percent. This agrees fairly closely with the U.S. Social Security System's award to retired households of 50 percent extra benefits for a spouse. It is consistent with substantial returns to scale for larger households.

Estimates of our third "calculated parameter" suggests a 19-20 percent drop in consumption at retirement. This is consistent with, though at the smaller end of, estimates in Bernheim *et al.* [2001], Banks *et al.* [1998], Hurd and Rohwedder [2003], and the retirement brochures cited in Laitner [2001].

Our estimate of ξ^C suggests an increase in household consumption of 13-15 percent for each child age 0-22. Since two parents correspond to 1.4 "equivalent adults," a child adds about 20 percent as much as each parent. Mariger [1986] estimates that children consume 30 percent as much as adults; Attanasio and Browning [1995, p. 1122] suggest 58 percent; Gokhale *et al.* [2001] assume 40 percent; most of the analysis in Auerbach and Kotlikoff [1987] implicitly weights children at zero; and, Tobin [1967] assumes teens consume 80 percent as much as adults, and minor children 60 percent. Our estimate would be consistent with parents who spend a great deal on their children but reduce expenditures on themselves at the same time — perhaps vicariously enjoying their children's consumption.

4. Social Security Reform

This section investigates the consequences of Social Security reforms in which the OASI tax, and benefit adjustments based on new earnings, cease at a specific age or following

their sample. They find an average IES of 0.2, with less than 20% of respondents having an IES greater than 0.3. Others who have attempted to estimate a distribution of intertemporal elasticities of substitution find evidence that the IES is increasing with wealth (e.g., Blundell *et al.* [1994]).

a specific span of career years. Individuals who avoid disability could retire at any age; however, those who continue working after the Social Security vesting age/period would enjoy a 10.6 percent increase in their aftertax wage. (As with the present system, individuals could start collecting Social Security benefits at age 62 or later, with an actuarially fair adjustment for postponed receipt.)

Table 5 presents simulation outcomes for different reforms and different parameter estimates. The table compares behavior of our HRS couples under the existing Social Security System to the same sample if it had lived its life under the specified reform. Our analysis is not general equilibrium in nature — wages and interest rates are exogenous — nor does this section study transitions after reforms announced in a household's midlife. Because we want to examine prospective reforms in an environment that is revenue neutral from the standpoint of the Social Security System, each of the table's simulations introduces a constant adjustment to historical Social Security taxes that equates the sample–average present value (at age 50) of Social Security taxes less benefits before and after reform.[15]

In row 1 of Table 5, for example, under the reform couples realize that their Social Security vesting ends at age 54. Subsequent to male (female) age 54, aftertax male (female) wages rise 10.6 percent. If we disregard disability–shock realizations, the reform lengthens careers by 1.08 years on average. In practice, the onset of disability (or death) can limit one's ability to adjust labor supply. Taking that into account, the average actual change in career length according to the simulation is 0.97 years. If we ask households *ex ante* how much they would pay to participate in the reformed Social Security System, column 3 shows they would offer, on average, \$6193 (in 1984 dollars, present value at male age 50), which amounts to 0.3 percent of their aftertax earnings.

To gain further insight into Table 5's results, recall Proposition 4. As in the discussion of Table 4, let $Q_i^2(R)$ equal the left–hand side of (19) minus the right–hand side. As in our basic formulations, assume homogeneity of tastes; hence, at desired retirement age R_i^*, we have $Q_i^2(R_i^*) = 0$. If $Q_i^2(R)$ is positive (negative), household i should retire later (earlier). Using (17) and (20), and dropping the subscript i for convenience,

$$\text{sign}\big(Q^2(R)\big) = \text{sign}\Big(\big(\frac{y_{R-}}{c_{R-}}\big) \cdot \big(1 - \frac{p(R)}{P(R)}\big) - \big[[\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma}{1-\alpha\cdot\gamma}} - 1\big] \cdot \big(1 - \frac{p(R)}{P(R)}\big) + \frac{B'(R)\cdot e^{r\cdot R}}{c_{R-}} - \frac{1}{\alpha\cdot\gamma} \cdot \big[1 - [\bar{\ell}]^{\frac{\gamma\cdot(1-\alpha)}{1-\alpha\cdot\gamma}}\big] \cdot [\bar{\ell}]^{\frac{-(1-\alpha)\cdot\gamma\cdot\alpha\cdot\gamma}{1-\alpha\cdot\gamma}}\Big).$$

Assuming that $p(R)/P(R)$ changes slowly with R, one can, therefore, limit one's attention to

[15] As this is an aggregative condition, the present value calculation employs the gross–of–tax real interest rate.

Table 5. Simulations with Vesting by Age or by Span of Career: Point Estimate [95% Confidence Interval][a] (Estimated Parameters as in Table 4; 1984 Dollars; NIPA PCE Deflator)

Vesting Age or Vesting Span (Years)	Average Change Actual Career Years	Average Change Desired Career Years	Average Equivalent Variation (PV Age 50)	Average [Equivalent Variation ÷ Lifetime Earnings]	Average Per Household Additional Income Tax Revenue (PV Age 50)
Age	Stringent Definition Disability; Vesting by Age[b]				
54	0.9733 [0.9559 , 1.0306]	1.0832 [1.0614 , 1.1559]	6193 [6105 , 6352]	0.0030 [0.0030 , 0.0031]	2997 [2943 , 3193]
58	0.7820 [0.7620 , 0.8237]	0.8904 [0.8651 , 0.9435]	4376 [4284 , 4510]	0.0022 [0.0022 , 0.0023]	2341 [2268 , 2495]
62	0.4281 [0.4235 , 0.4858]	0.5174 [0.5127 , 0.5924]	2349 [2314 , 2487]	0.0012 [0.0012 , 0.0013]	1150 [1146 , 1383]
66	0.1803 [0.1585 , 0.2025]	0.2336 [0.2067 , 0.2614]	904 [870 , 956]	0.0005 [0.0005 , 0.0005]	451 [380 , 523]
Age	Broad Definition Disability; Vesting by Age[c]				
54	0.9472 [0.9125 , 0.9575]	1.2661 [1.2101 , 1.2814]	7196 [7087 , 7303]	0.0036 [0.0035 , 0.0036]	2431 [2344 , 2470]
58	0.7736 [0.7311 , 0.7941]	1.0834 [1.0145 , 1.1080]	5449 [5366 , 5511]	0.0028 [0.0028 , 0.0029]	1903 [1816 , 1969]
62	0.4716 [0.4491 , 0.4896]	0.7231 [0.6841 , 0.7503]	3327 [3268 , 3366]	0.0018 [0.0017 , 0.0018]	1070 [1023 , 1122]
66	0.2141 [0.1866 , 0.2173]	0.3412 [0.2931 , 0.3466]	1425 [1368 , 1427]	0.0008 [0.0007 , 0.0008]	470 [409 , 487]

Table 5 (cont.). Simulations with Vesting by Age or by Span of Career: Point Estimate [95% Confidence Interval] [a] (Estimated Parameters as in Table 4; 1984 Dollars; NIPA PCE Deflator)					
Vesting Age or Vesting Span (Years)	Average Change Actual Career Years	Average Change Desired Career Years	Average Equivalent Variation (PV Age 50)	Average [Equivalent Variation ÷ Lifetime Earnings]	Average Per Household Additional Income Tax Revenue (PV Age 50)
Span	Stringent Definition Disability; Vesting by Career Span[b]				
34	0.8743 [0.8591 , 0.9192]	0.9702 [0.9514 , 1.0240]	5171 [5055 , 5355]	0.0026 [0.0025 , 0.0026]	2725 [2667 , 2877]
38	0.7579 [0.7374 , 0.8043]	0.8543 [0.8276 , 0.9083]	4135 [4036 , 4320]	0.0023 [0.0022 , 0.0024]	2261 [2174 , 2445]
42	0.5046 [0.4879 , 0.5346]	0.5886 [0.5691 , 0.6287]	2508 [2429 , 2604]	0.0015 [0.0014 , 0.0015]	1300 [1241 , 1415]
46	0.2071 [0.1916 , 0.2360]	0.2578 [0.2356 , 0.2912]	933 [890 , 1009]	0.0006 [0.0006 , 0.0006]	443 [400 , 535]
Span	Broad Definition Disability; Vesting by Career Span[c]				
34	0.8006 [0.7700 , 0.8128]	1.0521 [1.0002 , 1.0672]	5847 [5746 , 5966]	0.0030 [0.0029 , 0.0030]	2058 [1990 , 2103]
38	0.7484 [0.6997 , 0.7534]	0.9974 [0.9260 , 1.0043]	4944 [4807 , 4994]	0.0028 [0.0027 , 0.0028]	1876 [1744 , 1904]
42	0.5131 [0.4854 , 0.5312]	0.7286 [0.6817 , 0.7544]	3207 [3135 , 3264]	0.0019 [0.0019 , 0.0020]	1172 [1112 , 1229]
46	0.2261 [0.1931 , 0.2331]	0.3608 [0.3029 , 0.3707]	1299 [1244 , 1312]	0.0009 [0.0009 , 0.0009]	449 [352 , 468]

a. Based on 1000 random parameter vector draws from the multivariate normal distribution determined by our estimated parameter vector and its covariance matrix, recalculating values, as in columns 1-5, at each draw.

b. Average lifetime earnings (PV age 50) $1,888,478. Cases 994. In contrast to Table 4, we eliminate households retiring before age 52 or after 68 — allowing our existing solution algorithm (see text) a minimum of two years of latitude for post–reform retirement–age changes.

c. 918 cases (see preceding note). Average lifetime earnings (PV age 50) $1,884,320.

$$(\frac{y_{R-}}{c_{R-}}) \cdot (1 - \frac{p(R)}{P(R)}) \quad \text{and} \quad \frac{B'(R) \cdot e^{r \cdot R}}{c_{R-}} . \tag{30}$$

Under the current Social Security System, with the stringent definition of disability, average retirement–age values for households in our sample who are not forced to retire early because of disability or death are $y_{R-} \approx \$35000$, $c_{R-} \approx \$40000$, $p(R)/P(R) \approx 0.26$, and $B'(R) \cdot e^{r \cdot R} \approx \$1125$. In other words, in terms of Social Security cumulative benefits, the advantage of working one more year is relatively small; thus, (30) shows that the major determinants of retirement, on average, seem to be forgone earnings relative to consumption together with the value of leisure.

To understand better the distribution of effects of our reform, it is helpful to characterize different types of households. Suppose the reform institutes full vesting at age 54. Then there are four types of household. (i) Some households are disabled. Following a reform to Social Security, they cannot work longer. (ii) For a typical unconstrained couple who would otherwise retire at $R > 54$, subsequent to reform y_{R-} rises about \$3600 and $B'(R) \cdot e^{r \cdot R}$ declines to 0. Thus, Proposition 4 shows that such a couple should choose to work longer. Net of disability insurance, an extra year's work yields about \$2700. Table 4 shows consumption rises steadily with age. Each extra planned year's work raises lifetime resources and, hence, consumption, as well. With one additional year's work, the rise in the denominator of the right–hand term of (30) and the disappearance of the right–hand term would offset nearly all the gain in aftertax earnings from reform. In other words, we expect an average career extension of about one year. (iii) A couple who had previously chosen to retire before age 54 would have no marginal incentive to work longer after reform. On the contrary, y_{R-} for $R < 54$ would fall from the (slight) increase in Social Security taxes prior to age 54. Since we focus on permanent reforms and ignore transitions, c_{R-} would fall as lifetime resources diminish from the tax increase. Nevertheless, because the Social Security benefit component of household resources remains fixed, the drop in consumption would be less in percentage terms than the drop in earnings. Thus, the left–hand term of (30) would fall, causing R^* to fall (slightly). (iv) Some couples who chose to retire before age 54 in the absence of reform might make a non–marginal response to reform, choosing to work beyond age 54. Couples in this category could show labor supply increases of more, perhaps much more, than 1 year.

Chart 1 displays the simulated distribution of retirement–age changes for vesting at age 58. One can see evidence of behavior from all four groups. Among all households, 54% strictly increase the length of their careers, and 25% increase it by more than one year — including a substantial fraction (12%) that works at least two additional years. Another large fraction (28%) makes no adjustment to their labor supply. The remainder (18%) shorten their careers very slightly, with the average decrease among this group amounting

Chart 1: Distribution of Change in Retirement Age after Reform: Vesting Age 58

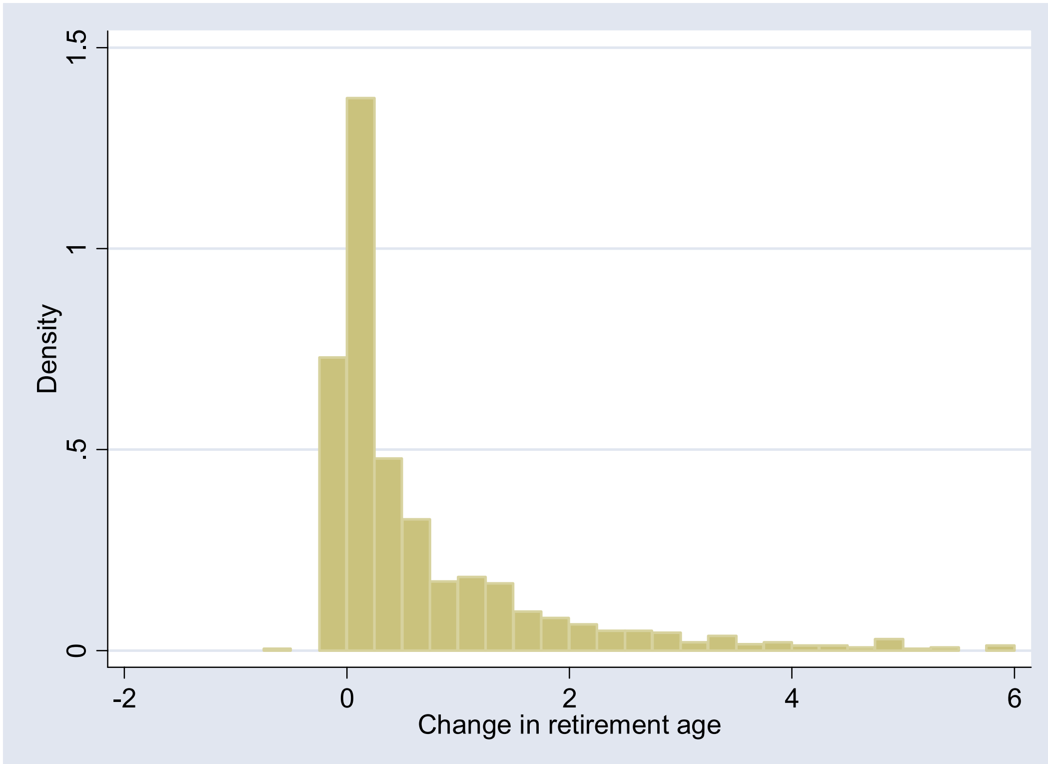
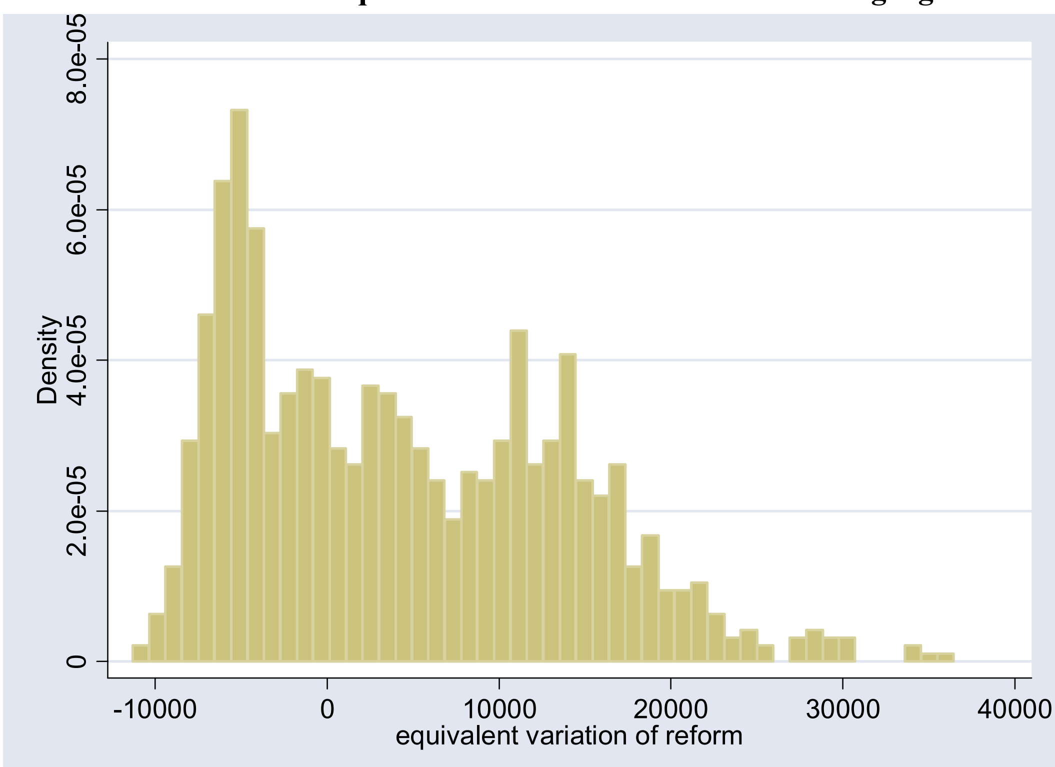


Chart 2: Distribution of Equivalent Variation of Reform: Vesting Age 58



to just one week less work. Overall, households increase their careers by an average of 0.8 years.

Table 5 also shows that later vesting leads to smaller labor supply increases on average. This is expected: if a reform vests later in life, groups (i) and (iii) expand. Indeed, differences are substantial. When, for example, the vesting age is 54, households are predicted to work an average of 0.95–0.97 additional years depending on which definition of disability we use. If instead the vesting age is set to 62, the average change in work career length drops to 0.43–0.47 years.

We turn next to the welfare consequences of the reform and, in particular, equivalent variations. It is important to remember that our definition of revenue neutrality demands an increase in the Social Security tax following reform to make up for lost payroll tax revenue from those who, absent the reform, would have worked beyond the vesting age. For example, the reform summarized in line 1 of Table 5 requires a tax increase in the years before vesting of about 0.5 percent. Households in groups (i) and (iii) pay the higher tax over their entire work lives but receive no benefit from reform. So, their equivalent variation can be quite negative. Groups (ii) and (iv), in contrast, can show positive equivalent variations. To the extent that members of group (ii) originally choose to work beyond the vesting age, they achieve redistributive gains at the expense of group (iii). Efficiency gains, gathered through longer working lives, add to the equivalent variations of groups (ii) and (iv). In our partial equilibrium framework, redistributive gains and losses cancel one another out. In each version of the reform, however, Table 5 shows that average gains are positive. Hence, overall efficiency gains are achieved by every version of the reform.

Chart 2 shows the distribution of gains (and losses) for the version of the reform summarized in row 2. The average equivalent variation is $4,444, (1984 dollars), with 60% of households willing to pay a positive amount to live under the reformed system. Approximately 42% of households would be willing to pay more than $7,000 to live under the reform, and 15% would pay more than $15,000. Some of these welfare gains come at the expense of substantial losses to groups (i) and (iii). Nearly 7% of households would pay more than $7,000 (1984 dollars) to avoid the reform, and 20% would pay at least $5,000. In the end, while the reform generates efficiency gains, there are important winners and losers.

The welfare gains are, themselves, a function of the vesting age. If one contemplates reforms with a later vesting age, group (iii) should expand in relative size. Thus, Table 5's decline with vesting age in the average equivalent variation is not surprising.

In sum, any of the variants of our proposed reform yield positive average equivalent variations. On the other hand, by no means do all couples benefit, and *ex post* losses for some households are quite large.

Equivalent variations do not, however, capture the entirety of the welfare gains from this reform. To the extent that the reform increases years of work, income tax revenues will rise. The latter generate social gains — assuming that households consume the services of government whether they work or not. Table 5's last column assesses these social bonuses, measured on a per household basis exactly commensurate in units to column 3. We see that social gains augment column 3's personal gains by about 50 percent. If these additional income tax revenues were redirected into the Social Security system, they would represent a gain of up to 3% of the average lifetime value of Social Security benefits.

5. Conclusion

Many recent proposals for U.S. Social Security reform do not focus on the potential inefficiencies that the system creates. This paper describes and analyzes an alternative, simple reform aimed at alleviating labor–supply distortions from the current program. After a long vesting period (say, 35-40 years of contributions), the reformed policy would determine a worker's benefits using the current formula for all prospective retirement dates; beyond the vesting period, the worker would no longer face the OASI payroll tax. For those who continued to work after vesting, wages would, in partial equilibrium, increase 10.6 percent. Lost revenues to the system would be made up by a small increase in the payroll tax during the vesting period. In a life-cycle model where the only margin of choice in labor supply is the timing of retirement, this reform eliminates the distortions of the Social Security system for those whose optimal retirement occurs after the vesting period.

We find that the proposed reform could have substantial effects on both behavior and welfare. This paper's simulations indicate that retirement ages could rise by nearly a year on average, that a typical household might willingly pay as much as $6,000 (1984 dollars, present value age 50) to participate in the reformed system, and that additional gains accruing to society from extra income taxes due to longer careers could average another $3000 per household.

The heterogeneity of welfare consequences evident in Charts 1-2 suggests that the reform that this paper studies would not be unanimously embraced, and aspects of our model may tend to amplify the predicted behavioral and welfare consequences of the reform. Nevertheless, magnitudes of the estimated increases in the average retirement age and the average welfare gain suggest to us that the policy is worthy of continued consideration. Increases in longevity in recent decades have, somewhat puzzlingly, failed to induce men to work longer on average, and the reform analyzed here might mitigate one of the impediments to their deciding to do so in the future.

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Appendix I: Proofs

Throughout this appendix, we omit the subscript i for expositional convenience.

Lemma 1: *Suppose that discontinuities in n_t and labor supply at retirement make criterion (3)-(4) and the right–hand side of the asset equation discontinuous at ages t_j. Define a present–value Hamiltonian with costate variable λ:*

$$\mathcal{H} \equiv \begin{cases} e^{-\rho \cdot t} \cdot u(c_t) + \lambda_t \cdot [r \cdot a_t + y_t - c_t]\,, & \text{for } t \in [S\,,\, R)\,, \\ e^{-\rho \cdot t} \cdot v(c_t) + \lambda_t \cdot [r \cdot a_t - c_t]\,, & \text{for } t \in [R\,,\, T]\,. \end{cases}$$

Then for a given R, the following conditions are necessary and sufficient for an optimum:

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \quad all \quad t\,, \tag{i}$$

$$\dot{\lambda}_t = -\frac{\partial \mathcal{H}}{\partial a} \quad all \quad t\,, \tag{ii}$$

$$\dot{a}_t = r \cdot a_t + y_t - c_t \quad all \quad t \neq R\,, \tag{iii}$$

$$a_S = 0\,, \quad a_{R+} = a_{R-} + B(R) \cdot e^{r \cdot R}\,, \quad and \quad a_T = 0\,. \tag{iv}$$

Proof of Lemma 1: Let R be given. Begin with problem (4). Suppose it has one breakpoint, $t_1 \in (R\,,\, T)$. Solving the subproblem for $t \geq t_1$ — which is standard — we have (i)-(iv). Call the subproblem's maximized criterion $\Phi(a_{t_1}\,,\, t_1)$. Next, solve

$$\max_{c_t} \int_R^{t_1} e^{-\rho \cdot t} \cdot u(c_t, t, R)\, dt + \Phi(a_{t_1}\,,\, t_1)$$

with the same constraints as (4). This is a standard problem: we have (i)-(iii) and

$$\lambda_{t_1 -} = \frac{\partial \Phi(a_{t_1}\,,\, t_1)}{\partial a}\,. \tag{v}$$

(See, for example, Kamien and Schwartz [1981].) Since a_i is continuous by nature, it only remains to show that λ_t is continuous at t_1. But, the envelope theorem shows

$$\frac{\partial \Phi(a_{t_1}\,,\, t_1)}{\partial a} = \lambda_{t_1 +}\,. \tag{vi}$$

Equations (v)-(vi) establish the costate's continuity at t_1. Induction on the number of breakpoints, say, J', in (4) establishes continuity for any J'. The logic of (v)-(vi), with

$\Phi(.) = \varphi(.)$, establishes continuity of the costate at $t = R$. The same arguments apply for $t < R$. ▮

Proof of Proposition 1: Suppose we have a solution of (3)- (4). Fix the R. The optimal consumption path must solve (3)-(4) conditional on this R. Follow Lemma 1. From (ii), $\dot{\lambda}_t = -r \cdot \lambda_t$. Then for $t \in (t_j\,,\, t_{j+1})$, we have

$$e^{-\rho \cdot t} \cdot [n_t]^{1-\alpha \cdot \gamma} \cdot [c_t]^{\alpha \cdot \gamma - 1} \cdot [\ell_t]^{(1-\alpha) \cdot \gamma} = \lambda_t \qquad \text{from Lemma 1, (i)}$$

$$\Longleftrightarrow (\alpha \cdot \gamma - 1) \cdot \frac{\dot{c}_t}{c_t} = \rho - r\,, \qquad \text{since } t \in (t_j\,,\, t_{j+1})$$

establishing (5). For $t = t_j$, $j = 1, ..., J$, Lemma 1 shows λ_t is continuous; so,

$$e^{-\rho \cdot t} \cdot [n_{t-}]^{1-\alpha \cdot \gamma} \cdot [c_{t-}]^{\alpha \cdot \gamma - 1} \cdot [\ell_t]^{(1-\alpha) \cdot \gamma} = \lambda_t = e^{-\rho \cdot t} \cdot [n_{t+}]^{1-\alpha \cdot \gamma} \cdot [c_{t+}]^{\alpha \cdot \gamma - 1} \cdot [\ell_t]^{(1-\alpha) \cdot \gamma}\,,$$

establishing (6). For $t = R$, by the same logic, since $\ell_{t+} = 1$,

$$e^{-\rho \cdot t} \cdot [n_t]^{1-\alpha \cdot \gamma} \cdot [c_{t-}]^{\alpha \cdot \gamma - 1} \cdot [\ell_{t-}]^{(1-\alpha) \cdot \gamma} = \lambda_t = e^{-\rho \cdot t} \cdot [n_t]^{1-\alpha \cdot \gamma} \cdot [c_{t+}]^{\alpha \cdot \gamma - 1}\,,$$

establishing (7). Integrating budget constraint (iii) from $t = S$ to T gives (8). ▮

Proof of Proposition 2: For any $R = R_i$, define a Hamiltonian as in Lemma 1. Let

$$V(R\,,\, A) \equiv \max_{c_s} \int_S^R e^{-\rho \cdot t} \cdot u(c_t)\, dt \qquad (vii)$$

$$\text{subject to:} \quad \dot{a}_t = r \cdot a_t + y_t - c_t \quad \text{and} \quad a_S = 0\,.$$

Solving (3)-(4) requires solving

$$\max_{R\,,\, A} \{V(R\,,\, A) + \varphi\big(A + B(R) \cdot e^{r \cdot R}\,,\, R\big)\}\,.$$

First–order conditions for the latter are

$$\frac{\partial V}{\partial R} + \frac{\partial \varphi}{\partial R} = 0 \quad \text{and} \quad \frac{\partial V}{\partial A} + \frac{\partial \varphi}{\partial A} = 0\,. \qquad (viii)$$

Kamien and Schwartz [1981, Sect.7] shows

$$\mathcal{H}\big|_{R-} + \frac{\partial \varphi}{\partial R}\big|_R = 0 \,. \tag{ix}$$

The first part of (viii) then shows

$$\frac{\partial V}{\partial R} = \mathcal{H}\big|_{R-} \,. \tag{x}$$

Given (x), the analogy between (vii) and (4) yields that at an optimum,

$$\varphi_2\big|_R = -\mathcal{H}\big|_{R+} \,. \tag{xi}$$

Lemma shows $\lambda_{R-} = \lambda_{R+}$ Using (ix) and (v), we have

$$\begin{aligned}
&\mathcal{H}\big|_{R-} + \lambda_R \cdot [B'(R) \cdot e^{r \cdot R} + r \cdot B(R) \cdot e^{r \cdot R}] + \varphi_2\big|_R = 0 && \text{from (ix), (v)} \\
&\Longrightarrow \mathcal{H}\big|_{R-} + \lambda_R \cdot [B'(R) \cdot e^{r \cdot R} + r \cdot B(R) \cdot e^{r \cdot R}] - \mathcal{H}\big|_{R+} = 0 && \text{from (xi)} \\
&\Longrightarrow e^{-\rho \cdot R} \cdot u(c_{R-}) + \lambda_R \cdot [r \cdot a_{R-} + y_{R-} - c_{R-}] + \\
&\lambda_R \cdot [B'(R) \cdot e^{r \cdot R} + r \cdot B(R) \cdot e^{r \cdot R}] - e^{-\rho \cdot R} \cdot v(c_{R+}) - \lambda_R \cdot [r \cdot a_{R+} - c_{R+}] = 0 \,.
\end{aligned}$$

Because $a_{R+} = a_{R-} + B(R) \cdot e^{r \cdot R}$, the latter yields

$$\lambda_R \cdot [y_R - c_{R-} + c_{R+} + B'(R) \cdot e^{r \cdot R}] = e^{-\rho \cdot R} \cdot [v(c_{R+}) - u(c_{R-})] \,.$$

Proposition 2 then follows from Lemma 1,(i). ▮

Proof of Proposition 3: First, looking at the first component of the first integral in (13), one has

$$\begin{aligned}
&\int_S^R p(D) \cdot \int_S^D e^{-\rho \cdot t} \cdot u(c_t)\, dt\, dD \\
&= \int_S^R \int_t^R p(D) \cdot e^{-\rho \cdot t} \cdot u(c_t)\, dD\, dt && \text{from Fubini's theorem} \\
&= \int_S^R [P(t) - P(R)] \cdot e^{-\rho \cdot t} \cdot u(c_t)\, dt \,.
\end{aligned}$$

This logic enables us to rewrite the criterion as

$$\int_S^R [P(t) \cdot e^{-\rho \cdot t} \cdot u(c_t) + p(t) \cdot \bar{\varphi}(a_{t-} + X_t\,,\, t\,,\, R)]\, dt +$$
$$P(R) \cdot \varphi\big(a_{R-} + B(R) \cdot e^{r \cdot R}\,,\, R\big)\,.$$

Now, fix R for the remainder of this proof. Set up Hamiltonians for, respectively, disability problem (12), retirement problem (4), and lifetime problem (13):

$$\mathcal{D} \equiv e^{-\rho \cdot t} \cdot v(\bar{c}_t) + \bar{\Lambda}_t \cdot [r \cdot \bar{a}_t - \bar{c}_t]\,, \quad t \geq D\,,$$

$$\mathcal{R} \equiv e^{-\rho \cdot t} \cdot v(c_t) + \Lambda_t \cdot [r \cdot a_t - c_t]\,, \quad t \geq R\,,$$

$$\mathcal{H} \equiv P(t) \cdot e^{-\rho \cdot t} \cdot u(c_t) + p(t) \cdot \bar{\varphi}(a_{t-} + X_t\,,\, t\,,\, R) +$$
$$\lambda_t \cdot [r \cdot a_t + y_t - c_t - \frac{p(t) \cdot X_t}{P(t)}]\,, \quad t < R\,.$$

The costate variables are $\bar{\Lambda}_t$, Λ_t, and λ_t, respectively.

Step 1. At demographic breakpoints, the analysis follows the proof of Proposition 1.

Step 2. The next four equations together establish (16):

$$\frac{\partial \mathcal{R}}{\partial c} = 0 \Rightarrow e^{\rho \cdot R} \cdot \frac{\partial v(c_R)}{\partial c} = \Lambda_R\,, \qquad \text{F.O.C. for (4)}$$

$$\Lambda_R = \frac{\partial \varphi(a_R + B(R) \cdot e^{r \cdot R}\,,\, R)}{\partial a_R}\,, \qquad \text{envelope theorem}$$

$$\lambda_R = P(R) \cdot \frac{\partial \varphi(a_R + B(R) \cdot e^{r \cdot R}\,,\, R)}{\partial a_R}\,, \qquad \text{F.O.C. for (13)}$$

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \Rightarrow P(R) \cdot e^{-\rho \cdot R} \cdot \frac{\partial u(c_R)}{\partial c} = \lambda_R\,. \qquad \text{F.O.C. for (13)}$$

Step 3. The next four equations together establish (17):

$$\frac{\partial \mathcal{H}}{\partial X} = 0 \Longrightarrow p(D) \cdot \frac{\partial \bar{\varphi}(a_{D-} + X_D \, , \, D \, , \, R)}{\partial X} = \lambda_D \cdot \frac{p(D)}{P(D)}$$

$$\Longrightarrow P(D) \cdot \frac{\partial \bar{\varphi}(a_{D-} + X_D \, , \, D \, , \, R)}{\partial X} = \lambda_D \, ,$$

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \Longrightarrow P(D) \cdot e^{-\rho \cdot D} \cdot \frac{\partial u(c_D)}{\partial c} = \lambda_D \, ,$$

$$\frac{\partial \mathcal{D}}{\partial \bar{c}} = 0 \Rightarrow e^{-\rho \cdot D} \cdot \frac{\partial v(\bar{c}_D)}{\partial \bar{c}} = \bar{\Lambda}_D \, ,$$

$$\frac{\partial \bar{\varphi}(a_{D-} + X_D \, , \, D \, , \, R)}{\partial X} = \bar{\Lambda}_D \, . \qquad \text{envelope theorem}$$

Step 4. The numerator of (18) is the expected present value of the household's lifetime earnings and retirement benefits. (One could subtract disability–insurance premiums and add expected disability–insurance benefits, but they would exactly balance.) The denominator times c_S is the expected present value of lifetime consumption. ▮

Proof of Proposition 4: Use the notation from the proof of Proposition 3. Analogous to the proof of Proposition 2, we have

$$\begin{aligned} &\mathcal{H}\big|_{R-} + \frac{\partial [P(R) \cdot \varphi\big(a_R + B(R) \cdot e^{r \cdot R} \, , \, R\big)]}{\partial R} \\ &= \mathcal{H}\big|_{R-} + P(R) \cdot \frac{\partial \varphi}{\partial R}\big|_R - p(R) \cdot \varphi\big|_R = 0 \, , \end{aligned} \qquad (xii)$$

$$\frac{\partial \varphi\big(a_R + B(R) \cdot e^{r \cdot R} \, , \, R\big)}{\partial R} = \Lambda_R \cdot [B'(R) \cdot e^{r \cdot R} + r \cdot B(R) \cdot e^{r \cdot R}] - \mathcal{R}\big|_{R+} \, . \qquad (xiii)$$

Combining (xii)-(xiii),

$$\begin{aligned} &P(R) \cdot e^{-\rho \cdot R} \cdot u(c_{R-}) + p(R) \cdot \bar{\varphi}(a_R + X_R \, , \, R \, , \, R) + \lambda_R \cdot [r \cdot a_{R-} + y_R - c_{R-} - \frac{p(R) \cdot X_R}{P(R)}] + \\ &P(R) \cdot \Lambda_R \cdot [B'(R) \cdot e^{r \cdot R} + r \cdot B(R) \cdot e^{r \cdot R}] - \\ &P(R) \cdot \big[e^{-\rho \cdot R} \cdot v(c_{R+}) + \Lambda_R \cdot [r \cdot a_{R-} + r \cdot B(R) \cdot e^{r \cdot R} - c_{R+}]\big] - \\ &p(R) \cdot \varphi\big(a_{R-} + B(R) \cdot e^{r \cdot R} \, , \, R\big) = 0 \, . \end{aligned} \qquad (xiv)$$

The proof of Proposition 3 shows

$$\lambda_R = P(R) \cdot \Lambda_R \,.$$

By construction,

$$\bar{\varphi}(a_{R-} + X_R \,,\, R \,,\, R) = \varphi\big(a_{R-} + B(R) \cdot e^{r \cdot R} \,,\, R\big) \,.$$

First–order conditions for (13) imply

$$P(R) \cdot e^{-\rho \cdot R} \cdot u'(c_R) = \lambda_R \,.$$

So, (xiv) simplifies to

$$P(R) \cdot e^{-\rho \cdot R} \cdot u'(c_R) \cdot [y_{R-} - c_{R-} + c_{R+} - \frac{p(R) \cdot X_R}{P(R)} + B'(R) \cdot e^{r \cdot R}] =$$
$$P(R) \cdot e^{-\rho \cdot R} \cdot [v(c_{R+}) - u(c_{R-})] \,,$$

which establishes (19).

Proposition 3 shows that term disability insurance for $[t \,,\, t + dt)$, where the interval ends with retirement, should cover lost earnings, corrected for changing consumption needs in the disabled state; hence,

$$\frac{p(R)}{P(R)} \cdot X_R \, dt = \frac{p(R)}{P(R)} \cdot [y_{R-} - c_{R-} + c_{R+}] \, dt \,,$$

which completes the proof. ▮

Appendix II: Adjustments of the CEX Data

We divide the NIPA and CEX data into 11 categories: food, apparel, personal care, shelter, household operation, transportation, medical care, recreation, education, personal business, and miscellaneous. Detailed adjustments include the following.

(1) We subdivide "shelter" into "services from own house" and "other." We scale the latter as we do other categories, but we drop the CEX "services from own house" and impute a substitute that allocates the annual NIPA total service flow from residential houses to the CEX in proportion to CEX reported house values.

(2) CEX medical expenditures omit employer contributions to health insurance and services that Medicare covers. We annually, proportionately, and for every age adjust CEX expenditures on private health insurance to match the Department of Health and Human

Services total for all premiums for private health insurance; and, we adjust out–of–pocket health spending from the CEX to match annual DHHS totals.[16] Turning to Medicare, funding for the benefits comes from a hospital insurance (HI) tax on wages and salaries, monthly premiums for supplementary medical insurance (SMI) from people currently eligible for benefits, and contributions from general tax revenues to SMI. The CEX registers only SMI premiums from participants; so, we allocate the yearly total of Medicare benefits (both HI and all SMI expenditure) to the CEX sample in proportion to SMI premium payments (principally for people over 65).[17]

(3) The NIPA "personal business" category includes bank and brokerage fees, many of which are hidden in the form of low interest on saving accounts, etc., and hence absent from expenditures that CEX households perceive. We assume that bank and brokerage fees make their way into the life–cycle model in the form of lower–than–otherwise interest rates on saving; therefore, we normalize annual personal business expenditures measured in the CEX to match the corresponding NIPA amount less bank and brokerage fees, and omit bank and brokerage fees from our measure of consumption.

[16] See http://www.cms.hhs.gov/statistics/burden-of-health-care-costs/table01.asp. The annual figures cover 1987-2000. We extrapolate to 1984-86 and 2001 using the growth rate of NIPA total medical consumption.

[17] For HI expenditures, see *Social Security Bulletin, Annual Statistical Supplement 2001*, table 8.A1; for SMI receipts and receipts from participant premiums, see table 8.A2.