Financial Distress and the Cross Section of Equity Returns[*]

Lorenzo Garlappi[†]
University of Texas at Austin

Hong Yan[‡]
University of South Carolina

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[†]Department of Finance, McCombs School of Business, University of Texas at Austin, Austin, TX 78712. Email: `lorenzo.garlappi@mccombs.utexas.edu`

[‡]Department of Finance, Moore School of Business, University of South Carolina, Columbia, SC 29208. Email: `yanh@moore.sc.edu`.

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Abstract

In this paper, we propose a new perspective for understanding cross-sectional properties of equity returns. We explicitly introduce financial leverage in a simple equity valuation model and consider the likelihood of a firm defaulting on its debt obligations as well as potential deviations from the absolute priority rule (APR) upon the resolution of financial distress. We show that financial leverage amplifies the magnitude of the book-to-market effect and hence provide an explanation for the empirical evidence that value premia are larger among firms with higher likelihood of financial distress. By further allowing for APR violations, our model generates two novel predictions about the cross section of equity returns: (i) the value premium (computed as the difference between expected returns on mature and growth firms), is hump-shaped with respect to default probability, and (ii) firms with a higher likelihood of deviation from the APR upon financial distress generate stronger momentum profits. Both predictions are confirmed in our empirical tests. These results emphasize the unique role of financial distress—and the ensuing nonlinear relationship between expected return and risk—in understanding cross-sectional properties of equity returns.

1 Introduction

The cross section of stock returns has been a focus of research efforts in asset pricing for the last two decades. As summarized by Fama and French (1996), with the exception of the momentum effect documented in Jegadeesh and Titman (1993), much of the empirically observed regularities can be accounted for by the size effect and the value premium associated with the book-to-market ratio (Fama and French (1992)). Because these relationships between stock returns and firm characteristics cannot be reconciled within the context of the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), they are usually referred to as pricing "anomalies". The lack of a unified risk-based framework to explain these cross-sectional features of returns has spawned a spirited debate on market efficiency and stimulated competing interpretations of these anomalies.

Recently, a series of empirical studies indicate that financial distress seems to play an essential role in the cross section of stock returns. Griffin and Lemmon (2002) show that the value premium is most significant among firms with high probabilities of financial distress, and Vassalou and Xing (2004) demonstrate that both the size and the book-to-market effects are concentrated in high default risk firms. Furthermore, Avramov, Chordia, Jostova, and Philipov (2006a) document that momentum profits are mainly associated with firms with low credit ratings. These empirical findings are consistent with the conjecture of Fama and French (1996) that cross-sectional patterns in stock returns may reflect distress risk. However, details of the underlying economic mechanism remain elusive.

In this paper, we develop a simple theoretical framework that can *simultaneously* account for the value premia and the momentum effect in the cross section of equity returns. Specifically, we demonstrate that the likelihood of financial distress and the potential violation of the absolute priority rule (APR) upon its resolution are important determinants of these empirical regularities. We also provide empirical evidence supporting the unique predictions of our risk-based theory.

Our work builds on the growing literature, stemming from Berk, Green, and Naik (1999), that provides rational explanations for *either* value premia *or* momentum profits based on optimal firm-level investment decisions. Carlson, Fisher, and Giammarino (2004), for example, consider operating leverage and finite growth opportunities in a dynamic investment model to show that

asset betas contain time-varying size and book-to-market components, reflecting the changing risk of assets-in-place and growth opportunities.[1] Sagi and Seasholes (2006), on the other hand, demonstrate how one can derive momentum profits within this type of investment-based models. In a setting with both growth and mature firms, they show that rising operating profits not only increase a growth firm's stock price, but also the relative importance of growth options as a fraction of total assets. Because growth options are riskier, momentum emerges from the fact that higher realized returns are then associated with greater firm risk, and hence higher expected returns in the future.

Although these models provide intuitive economic explanations for understanding the relationship between expected returns and firm characteristics, none of them explicitly models financial leverage. Because their theoretical predictions are more suitable for *asset* returns, these all-equity real-options models face two important challenges in explaining equity returns. First, while the value premium is significant in equity returns, it is not in asset returns, as documented by Hecht (2000), who reconstructs asset values by combining equity and debt. Second, ignoring financial leverage makes these models less suited to understand the recent empirical evidence on the relationship between cross-sectional return anomalies and financial distress.

We explicitly introduce financial leverage into a partial equilibrium, real-option valuation framework and consider the role of *potential* deviation from the absolute priority rule (APR) in the event of financial distress. In our model, APR violations refer not only to the result of bankruptcy proceedings, but also, more generally, to the expected outcome of common workout procedures among different claim-holders without formal bankruptcy filings. Garlappi, Shu, and Yan (2006) show that a similar mechanism can explain the counter-intuitive relationship between default probability and stock returns, originally documented in Dichev (1998) and more recently confirmed in Campbell, Hilscher, and Szilagyi (2006). In this paper, we demonstrate that potential deviations from the APR rule are an *essential* mechanism to establish a theoretical connection between financial distress and the empirically observed cross-sectional patterns of stock returns.

Using this modeling framework, we show that, similar to Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2004), the book-to-market effect is embedded into equity

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[1]The related literature also includes Gomes, Kogan, and Zhang (2003), Zhang (2005), Cooper (2006), Gala (2006). Section 2 provides a more detailed review of the literature.

beta which, for growth firms, also contains a size effect. More importantly, we demonstrate that, in the presence of financial leverage, the magnitude of the book-to-market effect increases with the likelihood of financial distress. Intuitively, this happens because equity is *de facto* a call option on the firm's assets, and hence its beta is equal to the product of *asset* beta and the elasticity of equity price with respect to asset value. This elasticity is in turn increasing in the probability of default.[2] As leverage increases, *ceteris paribus*, the likelihood of default increases, and thus the equity beta is amplified. While the basic economic mechanism for the value and size effects in our model remains the same as in Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2004), we show that the value premium is exacerbated as default probability increases and, in our numerical analysis, reaches magnitudes that are comparable to empirical estimates. This also explains why the observed book-to-market effect tends to be concentrated in low-credit-quality firms.

By further allowing for violations of the APR during the resolution of financial distress, our model predicts a *hump-shaped* relationship between the value premium and default probability. This happens because at high levels of default probability, the potential payoff to equity holders upon default counter-balances the debt burden. This reduces the risk of assets-in-place, and hence the expected return to equity holders.

Using *Moody's KMV* Estimated Default Frequency (EDF) as a measure of default probability, we verify that value premia indeed exhibit this hump shape in the full sample which includes low-priced, high-default-probability stocks, thus providing confirming evidence for the unique role of the potential APR violation in the cross section of equity returns.

The hump-shaped relation between default probability and equity beta in the presence of potential APR violations upon financial distress also has interesting implications for momentum in stock returns. All else being equal, as a firm's profitability and stock price decline, its probability of default increases. Because the hump-shape implies that at high levels of default likelihood equity beta is decreasing in default probability, low (high) realized return are followed by low (high) expected returns. In other words, our model predicts that return-continuation (momentum) is more likely to be concentrated within the group of firms with high default probabilities. This finding is consistent with the recent evidence in Avramov, Chordia, Jostova, and

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[2]Default refers to financial distress, which includes instances of missed payments, modified terms and structure of debt in private workouts, and ultimately bankruptcy filings. In this paper, we use the terms "default" and "financial distress" interchangeably.

Philipov (2006a). It is important to note that our model is capable of generating momentum in equity returns *without* assuming predictability (e.g., mean reversion) of the underlying fundamental process of revenues. Moreover, our mechanism generates momentum through potential violations of the APR and hence is different from that proposed in Sagi and Seasholes (2006), which relies on growth options.[3] A unique prediction of our theory is that highly levered firms with larger possible deviations from the APR tend to generate stronger momentum profits. Our empirical analysis corroborates this prediction and thus further validates the role of financial distress and associated APR violations in explaining the cross-sectional patterns of stock returns.

The main contribution of our paper is to provide a unified valuation framework to *qualitatively* explain the major cross-sectional properties of stock returns simultaneously. In particular, by accounting for financial leverage and potential APR violations, we demonstrate the economic impact of financial distress on value premium and momentum in equity returns. Our results imply that these anomalies may be different manifestations of a *nonlinear* relationship between time-varying risk and expected equity returns. While the value premium is a direct consequence of this relationship, momentum profits are linked to the connection between the *changes* in these variables. Therefore, the risk of financial distress leaves its footprints in the cross-sectional return patterns by affecting the role of the book-to-market and momentum factors, usually invoked in multi-factor asset pricing models of equity returns.

There are, of course, several other explanations for the patterns observed in the cross section of stock returns. Promising alternatives include explanations based on information dissemination, institutional ownership, and individual trading behavior.[4] Our work does not preclude these alternatives and our findings are consistent with these explanations because the evidence suggests that the information environment is poor and institutional ownership is low for stocks with high default probabilities. Assessing the relative merits of these explanations is an important empirical question that is beyond the scope of this paper. Our aim for this paper is to present a valuation framework, based on fundamental asset pricing principles, that can account for the main features observed in the cross section of equity returns without explicitly considering trading motivations.

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[3] We note that the mechanism proposed in Sagi and Seasholes (2006) may be more applicable to the momentum in high-tech stocks which do not usually have significant financial leverage. Our mechanism applies more suitably to firms with substantial leverage, similar to the subset of stocks studied in Avramov, Chordia, Jostova, and Philipov (2006a).

[4] See, for example, Lakonishok, Shleifer, and Vishny (1992, 1994), Hong, Lim, and Stein (2000), Grinblatt and Han (2005) and Han and Wang (2005).

The paper proceeds as follows. We review the related literature in the next section. Our modeling framework is presented in Section 3 with a discussion of empirical implications and predictions. Empirical results are provided in Section 4. Section 5 concludes. All proofs are collected in Appendix A and the numerical procedure and parameter choice are described in Appendix B.

2   Related literature

The literature on the cross section of stock returns is vast and varied. Cumulative empirical evidence, culminated in Fama and French (1992), identifies the size and the book-to-market effects in the cross section of stock returns. Jegadeesh and Titman (1993) document a momentum phenomenon in stock returns in which past winners outperform past losers for up to one year. While a debate ensues regarding whether the cross section of stock returns is based on risk factors (Fama and French (1993)) or determined by characteristics alone (Daniel and Titman (1997)), the traditional capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) is widely considered a failure in accounting for the observed patterns of returns.[5]

Recently, a growing literature has explored the role of investment options in explaining the cross section of stock returns. Berk, Green, and Naik (1999) consider a firm as a portfolio of past projects, which may become obsolete at some random dates, and future opportunities with heterogenous risk profiles. They show that the relative weight of growth options versus assets-in-place captures the size effect in expected returns while the systematic risk in assets-in-place is linked to the book-to-market effect. This intuition retains in the general equilibrium extension of Gomes, Kogan, and Zhang (2003) which highlights the connection between expected returns and firm characteristics through beta and potential measurement problems in empirical work.

Cooper (2006) and Zhang (2005) argue that due to adjustment costs in investments, value firms with excessive capital capacity may benefit during economic expansion and suffer during economic downturn, leading to a higher exposure to systematic risk and hence a risk premium. Similarly, in a general equilibrium model, Gala (2006) shows that growth firms have lower

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[5]This failure has also brought serious challenges to the notion of market efficiency and lead to behavior-based theories for either the value premium or momentum. For a summary of the debate on the implications of "anomalies" for market efficiency, see, e.g., Fama (1998), Schwert (2003) and Shleifer (2000). For behavioral explanations of value premium and momentum, see Lakonishok, Shleifer, and Vishny (1994), Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (1999), Daniel, Hirshleifer, and Subrahmanyam (2001) and Grinblatt and Han (2005).

expected returns than value firms because their investment flexibility allows growth firms to weather adverse shocks better than value firms, hence providing "consumption insurance" to investors in economic downturns.[6]

Instead of linking the value premium to the business cycle, Carlson, Fisher, and Giammarino (2004) study a monopolistic firm making investment decision at different growth stages (juvenile or adolescent) or producing at full capacity (mature). They show that the growth opportunity and the operating leverage related to production costs are two important factors for connecting firm characteristics with time-varying beta. The intuition for the size and the book-to-market effects is similar to that in Berk, Green, and Naik (1999), except that the book-to-market effect captures the risk of assets-in-place through the operating leverage.

Working within a similar framework, Sagi and Seasholes (2006) argue that firms with growth options and mean-reverting revenues will exhibit momentum in returns. This happens because as revenues, and hence the firm value, increase, the likelihood of these growth options being exercised rises, so does their weight in the total firm value. Since growth options are riskier than assets-in-place, this implies that the firm risk and expected return increase when the firm value increases, leading to "rational" momentum. This mechanism provides an economic interpretation of the notion of log-convexity discussed in Johnson (2002).

While these models develop intuitive economic arguments to explain various aspects of the cross section of returns, none of them explicitly considers financial leverage and the effect of financial distress. Therefore, *de facto*, these models describe the cross section of *asset*, not equity, returns. In an interesting paper, Hecht (2000) illustrates that most of the cross-sectional features found in equity returns become insignificant for asset returns.[7] A related issue concerns the fact that, in order to match empirically observed levels of value premia, some of these investment-based models require unusually high equity premia.

A more serious challenge for this class of models comes from the recent empirical evidence that indicates that cross-sectional features of stock returns, such as the size, book-to-market

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[6]There is an extensive literature on consumption-based asset pricing models with implications for the cross section of equity returns. An excellent survey is Cochrane (2006). While this literature provides some intuitions on the link between the macroeconomy and asset returns, little has been written about the impact of financial distress at the firm level on the cross section of stock returns.

[7]Following a different approach,Ferguson and Shockley (2003) argue that the use of an equity-only proxy for the market portfolio biases downward the estimation of equity beta and this estimation error is increasing with leverage and relative distress. Their empirical analysis shows that portfolios based in debt-to-equity ratio and Altman's Z scores of financial distress subsume the role of the SMB (size) and HML (book-to-market) portfolios in the Fama-French three-factor model.

and momentum effects are much stronger for firms with high financial leverage and hence low credit quality. For instance, Griffin and Lemmon (2002) show that the value premium is most significant among firms with high probabilities of financial distress, and Vassalou and Xing (2004) demonstrate that both the size and the book-to-market effects are concentrated in high default risk firms.[8] Avramov, Chordia, Jostova, and Philipov (2006a) document that momentum profits are mainly associated with firms of low credit ratings. While this body of evidence seems to lend credence to the suggestion of Fama and French (1996) that a distress factor may affect the cross section of stock returns, the specific economic mechanism underlining this argument is still elusive.[9]

In the next section, we build on the literature linking investment and returns and introduce financial leverage to endogenously determine the likelihood of financial distress. We demonstrate that the consideration of financial distress and the outcome of its resolution are essential in simultaneously accounting for the cross-sectional features of stock returns and their relation to default probability.

3 A model of the cross section of equity returns

In order to understand the economic intuition underlying size, book-to-market and momentum effects in the cross section of equity returns, we develop a simple equity valuation model in which firms have both operating and financial leverage and can expand their operating scale by exercising an investment growth option.

Our valuation framework is a partial equilibrium one, similar to Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004) and Sagi and Seasholes (2006). Therefore, we consider the cross-sectional return patterns in an unconditional sense without analyzing their time-series properties. We consider two types of firms: a "mature" firm, which can not change its scale and will produce at capacity for as long as the firm is alive, and a "growth" firm, which

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[8]These papers originated from the desire to understand the relation between default risk and stock returns first documented by Dichev (1998), which is also recently studied by Campbell, Hilscher, and Szilagyi (2006) and George and Hwang (2006). Garlappi, Shu, and Yan (2006) provide an explanation based on strategic bargaining between debt- and equity-holders upon default. Empirically, Avramov, Chordia, Jostova, and Philipov (2006b) argue that this inverse relation is linked to the momentum in low-credit-quality firms.

[9]Gomes, Yaron, and Zhang (2003, 2006) and Lidvan, Sapriza, and Zhang (2006) examine the impact of financing frictions on stock returns within a production-based asset pricing model. They find that financing constraints are an important determinant of cross sectional returns and that the shadow price for external funds is pro-cyclical. While these papers provide useful insights into the importance of financing costs for equity returns, they do not address explicitly the mechanism that links the likelihood of financial distress to the value and momentum anomalies.

has the option to make an investment to expand its scale. We assume that the price P of the output produced by each firm follows a geometric Brownian motion

$$dP = \mu P dt + \sigma P dW, \tag{1}$$

where μ is the growth rate of the product price, σ its volatility and dW denotes the increment of a standard Brownian motion.[10] This price process may be thought of as a revenue stream for a standard productive unit. It is the sole source of risk at the firm level in our model and captures the systematic risk exposure of a firm. We further assume that the risk premium associated with the price P is equal to a positive constant λ. Hence, under the risk-neutral measure, the risk-adjusted revenue stream from production obeys the following process:

$$dP = (\mu - \lambda) P dt + \sigma P d\widehat{W}, \tag{2}$$

where $d\widehat{W}$ is a standard Brownian motion under the risk neutral measure. For ease of notation we will denote by δ the difference between the quantity $r + \lambda$ and the growth rate μ, i.e.,

$$\delta \equiv r + \lambda - \mu, \tag{3}$$

where r is the constant risk-free rate. In the cross section, firms differ in their cost structure, financial leverage, operating scale and growth opportunity. To highlight the intuition in a cleaner way, we abstract from optimal capital structure decisions and take the cross-sectional distribution of leverage as given. While endogenizing this decision is an important and challenging issue, our simplifying assumptions allow a first step towards fully understanding the effect of investment and financing decisions on equity returns while preserving a certain level of tractability.

Our main focus is to examine how, in general, firms' characteristics affect equity expected returns and momentum and, in particular, the effects of leverage and default probability on these relations. The product price P represents the state variable in our model, and we denote by $E(P)$ the market value of equity. Equity exhibits positive return autocorrelation, i.e.,

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[10]In the presence of competition in the product market, it is common to assume that the price follows a mean-reverting process. The geometric Brownian motion assumption in (1) can be seen as a limiting case in which the speed of mean reversion is set to zero. This limit coincides with the monopolistic setting studied in Carlson, Fisher, and Giammarino (2004). While under this assumption the product price is non-stationary, the analytical tractability allows us to better highlight the economic mechanism underlying the cross-sectional properties of equity returns.

momentum, if its expected return increases with stock price. The following lemma provides a formal characterization of expected return and return autocorrelation in our framework.

Lemma 1 *The sensitivity β of equity return to the state variable P is given by*

$$\beta = \frac{\partial \ln E(P)}{\partial \ln P}, \tag{4}$$

and the expected return on equity is given by

$$\text{Expected Return} = r + \beta\lambda. \tag{5}$$

The instantaneous return autocorrelation is

$$\text{AutoCorr} = \frac{\lambda}{\beta}\frac{\partial \beta}{\partial \ln P} = \lambda\frac{P}{\beta}\frac{\partial \beta}{\partial P}. \tag{6}$$

The lemma shows that, in our model, cross-sectional differences in equity expected returns are completely characterized by β. This quantity measures the equity exposure to the risk inherent in P, which is compensated by the positive risk premium λ. Note that β in expression (5) is *not* the CAPM beta. Following Duffie and Zame (1989), the CAPM beta can be derived from the covariance of the P process and the pricing kernel in the economy. Hence the expected return on equity may be further expressed as

$$\text{Expected Return} = r + \beta \cdot SR \cdot \rho \cdot \sigma, \tag{7}$$

where SR is the maximal Sharpe ratio attainable in the economy and ρ is the correlation of the price process P with the price kernel in the economy. This implies that the risk premium λ associated with the output price P is

$$\lambda = SR \cdot \rho \cdot \sigma. \tag{8}$$

As emphasized by Gomes, Kogan, and Zhang (2003), measurement errors in equity β create a role for the empirically observed importance of "omitted" variables related to firm characteristics, even if returns conform to a single factor structure, as in our model.

Because momentum in equity returns manifests itself through return continuation, or positive autocorrelation, following Sagi and Seasholes (2006) we use the autocorrelation defined in

(6) as a measure of momentum. An intuitive way of understanding this measure is to focus on the first equality in equation (6). If $\beta > 0$, the expression implies that positive autocorrelation occurs whenever a change in expected returns (captured by $\partial\beta$) are positively associated with a change in *realized* returns (captured by $\partial \ln P$).[11] In the rest of this section we derive analytical expressions for the equity value of mature and growth firms and illustrate cross-sectional properties of equity expected returns.

3.1 Mature firms

Mature firms have no access to growth options. Their value derives from the revenue stream generated by production for as long as the firm is alive. Producing one unit of goods requires an operating cost of c per unit of time. We assume that the firm operates at a fixed scale ξ. Hence the net profit from operation per each unit of time is equal to $\xi(P_t - c)$. The capital structure of the firm is characterized by a single issue of of perpetual debt with a continuous and constant coupon payment of l. The profit after interest service is thus $\xi(P_t - c) - l$. We ignore tax considerations.

As long as the firm is operating, equity holders enjoy the stream of profits. When the firm encounters financial distress and defaults on its debt, we assume that equity holders can recover a fraction η of the book value of assets. This assumption captures the deviation from the absolute priority rule (APR) documented empirically (e.g., Franks and Torous (1989), Eberhart, Moore, and Roenfeldt (1990), Weiss (1991) and Betker (1995)). Fan and Sundaresan (2000), among others, argue that strategic interaction between equity holders and bond holders can lead to APR violation as an optimal outcome.[12] Following Carlson, Fisher, and Giammarino (2004), we define the book value of the firm as the capitalized operating costs, $\xi c/r$. The book value of equity is hence $(\xi c - l)/r$, obtained by subtracting the book value of debt (l/r) from the firm's book value of assets.

The value of equity of a mature firm can therefore be expressed as follows

$$E^m(P_t) = \mathbb{E}\left[\int_0^{\tau_L}(\xi(P_{t+s} - c) - l)e^{-rs}ds + \eta V_A^m(\underline{P}^m)e^{-r\tau_L}\right] \tag{9}$$

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[11]Because we use the instantaneous autocorrelation to describe momentum in stock returns, we are silent in our discussion on possible reversal over a longer horizon. As it will become clear later, however, our model can be consistent with the presence of such reversals.

[12]Although we take η as a constant that may vary across firms, it may be viewed as a stochastic variable following a distribution. However, since adding this layer of complexity does not alter the basic intuition, we keep η deterministic in our exposition for simplicity.

where $\tau_L = \inf\{t : P_t = \underline{P}^m\}$ denotes the first time price P hits the threshold $\underline{P}^m$, at which point the firm defaults. The threshold, $\underline{P}^m$, is determined endogenously as it is chosen optimally by shareholders.[13] The expectation $\mathbb{E}$ is taken under the risk-neutral probability measure. The integrand in equation (9) represents the stream of profits received by equity holders until default. The last term represents the salvage value of equity upon default, which is a fraction η of the book value of assets, $V_A^m(\underline{P}^m) = \xi c/r$. The following proposition characterizes the equity value of a mature firm and its endogenous default boundary.

Proposition 1 *The equity value of a mature firm is given by*

$$E^m(P) = \begin{cases} \xi\left(\frac{P}{\delta} - \frac{c}{r}\right) - \frac{l}{r} + A_1 P^{\gamma_1}, & \text{if } P > \underline{P}^m \\ \eta\frac{\xi c}{r}, & \text{if } P = \underline{P}^m \end{cases}, \tag{10}$$

where

$$\gamma_1 = \left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0, \tag{11}$$

$$A_1 = -\frac{\xi}{\delta\gamma_1}(\underline{P}^m)^{1-\gamma_1} > 0, \quad and \quad \underline{P}^m = \frac{(l + \xi c(1+\eta))}{r}\frac{\gamma_1\delta}{(\gamma_1 - 1)\xi}. \tag{12}$$

Notice that the endogenous default boundary $\underline{P}^m$ is monotonically increasing in η. This is intuitive: if shareholders are getting more in default, they will choose to default earlier, all else being equal. Substituting the expression of $\underline{P}^m$ in (10) we obtain

$$E^m(P) = \xi\left(\frac{P}{\delta} - \frac{c}{r}\right) - \frac{l}{r} - \pi\left(\frac{\xi\underline{P}^m}{\delta\gamma_1}\right), \tag{13}$$

where

$$\pi \equiv \mathbb{E}\left[e^{-r\tau_L}\right] = \left(\frac{P}{\underline{P}^m}\right)^{\gamma_1} \tag{14}$$

is the risk-neutral probability of default. The above expression explicitly links equity value to financial leverage l and to a measure of default probability π. While for our theoretical derivations we will refer to π as the "probability of default", in our numerical analysis we adhere to the industry practice and adopt a definition derived under the real probability measure, provided in Lemma 2 of Appendix A (equation (A20)). Notice, however, that the use of the

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[13]The endogenous choice of default boundary by shareholders is a common feature in theoretical models (see, e.g., Black and Cox (1976) and Leland (1994)). Empirically, Brown, Ciochetti, and Riddiough (2006) show that default decisions are endogenous responses to the anticipated restructuring outcomes.

risk-neutral probability of default π does not alter any of the properties we derive in this section since the two quantities are monotonically related.

Using the result in Lemma 1 we obtain the following characterization of the β of a mature firm.

Corollary 1 *The β of a mature firm is*

$$\beta \;=\; 1 + \left(\frac{1}{rE^m}\right)(\xi c + l - \pi((1+\eta)\xi c + l)) \tag{15}$$

$$=\; 1 + \left(\frac{(\xi c - l)/r}{E^m}\right)\left(\frac{\xi c + l}{\xi c - l}\right)\left(1 - \pi\frac{(1+\eta)\xi c + l}{\xi c + l}\right), \tag{16}$$

where $\frac{(\xi c - l)/r}{E^m}$ represents the equity book-to-market ratio of the mature firm.

The corollary highlights two important aspects of the effect of the book-to-market ratio and leverage on the cross section of equity returns. First, for mature firms, the cross-sectional variation in equity returns is completely determined by the book-to-market effect, which, by the definition of a mature firm, captures only the risk in assets-in-place. This intuition of the book-to-market effect is similar to the one in Berk, Green, and Naik (1999) and Carlson, Fisher, and Giammarino (2004), which, in equation (16), corresponds to the case of no debt, i.e., when $l = 0$. In this case, we have

$$\beta = 1 + \left(\frac{\xi c/r}{A^m}\right)(1 - \pi), \tag{17}$$

where A^m denotes the market value of assets, and $1 - \pi$ represents the (risk-neutral) probability of a firm remaining operationally viable.[14]

Second, Corollary 1 shows that financial leverage and the associated probability of default play an important role in the book-to-market effect on equity returns. This happens through two channels: (i) a *direct* channel, operating through the fact that in the expression of β in (16) the book-to-market effect is explicitly dependent on leverage l and default probability π, and (ii) an *indirect* channel, operating through the fact that the equity value E^m is also affected by financial leverage and default probability.

To illustrate this result, we generate a cross section of firms differing by operating costs c, leverage l, scale of operation ξ, volatility of profits σ, degree of correlation ρ between the price

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[14]In Carlson, Fisher, and Giammarino (2004), a firm's exit is not considered endogenously. This is equivalent to setting π to be an exogenous quantity.

process and the pricing kernel in the economy, severity of the APR violation η and investment costs I. Appendix B.1 contains details of the parameter choice used to generate cross-sectional data from our model.

In Figure 1 we report the relationships of expected returns (solid line, left axis) and equity beta (dash-dotted line, right axis) with respect to default probability, based on the simulated cross-sectional data. Expected returns are computed according to (7) and β is given in equation (16). Firms are ranked in deciles based on their default probability computed according to equation (A20) in Appendix A, which refers to the likelihood of the firm defaulting within a year. Within each default probability decile, we obtain the average expected return and average β by equally weighting each firm in the decile. Panel A of Figure 1 shows that, when $\eta = 0$, i.e., when there is no deviation from the APR upon default, the expected return is monotonically increasing in default probability, so is the risk of equity associated with the book-to-market effect, as measured by β. The dramatic increase in the magnitude of the book-to-market effect in the expected return highlights the crucial role of financial leverage.

Panel B of Figure 1 shows that when $\eta > 0$, even if the expected recovery by equity holders upon default is set at a modest level of only 5% of the asset value, both the expected return and β exhibit a hump shape with respect to default probability. Financial leverage increases the risk of equity until the default probability reaches a relatively high level. At this point, the magnitude of the book-to-market effect is several times stronger than that at the lower end of the default probability spectrum. Beyond this point, the prospect of recovering a fraction of the assets with lower risk outweighs the leverage effect in determining the risk of equity, which is further reduced as the firm inches closer to the point of default. This is the argument used in Garlappi, Shu, and Yan (2006) to interpret the cross-sectional pattern linking stock returns and default probability, and it applies here to our analysis of the dependence of the book-to-market effect of mature firms on financial leverage and the associated default probability.

The results discussed above and illustrated in Figure 1 can be summarized more generally in the following corollary.
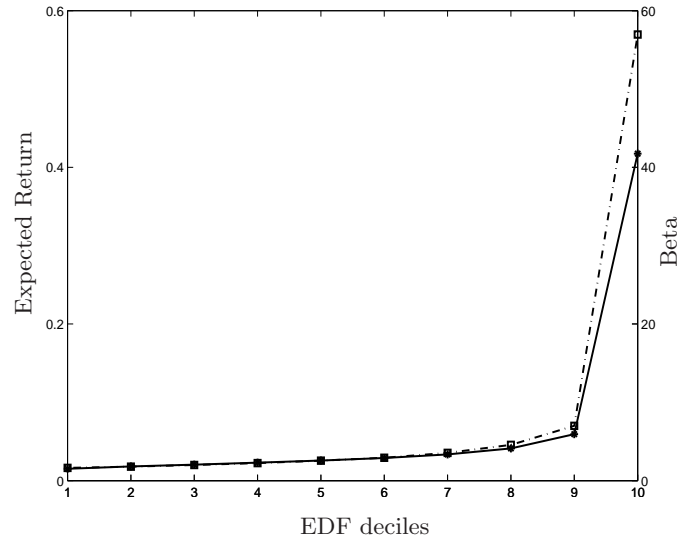
Corollary 2 *In the cross section,*

1. *If $\eta = 0$, equity betas and expected returns of mature firms are increasing in default probability π, with $\lim_{\pi \to 1} \beta = \infty$.*

Figure 1: Mature firms equity return and β versus default probability

The figure reports the expected return (solid line, left axis) and β (dash-dotted line, right axis) of mature firms as a function of default probability, with β described in equation (16) and the expected return defined in (7). The graphs are obtained from a cross section of firms by varying firm-level characteristics as described in Appendix B. Firms are ranked in deciles based on their default probability computed. according to equation (A20) in Lemma 2 of Appendix A and refers to the likelihood of the firm defaulting within a year. Panel A refers to the case of no violation of APR, $\eta = 0$ while Panel B refers to the case in which $\eta = 5\%$

Panel A: No violations of APR $(\eta = 0)$

Panel B: With violations of APR $(\eta = 5\%)$
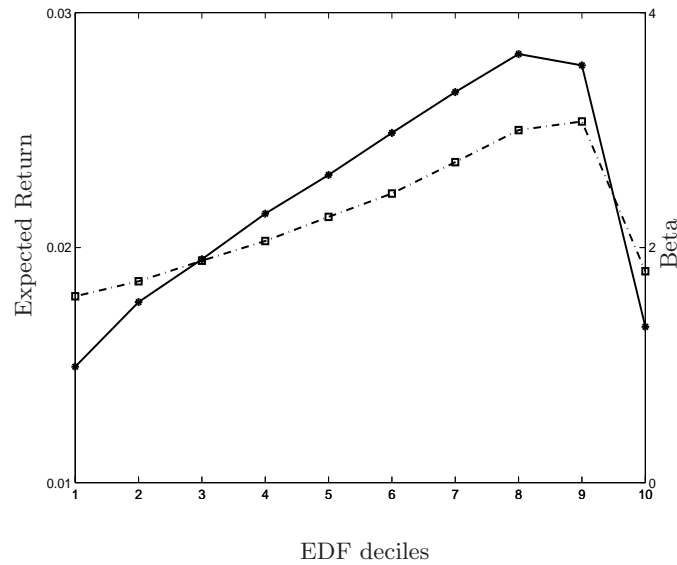
2. *If $\eta > 0$, equity betas and expected returns of mature firms are increasing in default probability π for low levels of π, and decreasing in default probability for high levels of π, with $\lim_{\pi \to 1} \beta = 0$.*

The mechanism highlighted above also plays an important role in the understanding of momentum in the cross section of equity returns of mature firms. Based on the results in Lemma 1, the return autocorrelation in (6) takes the following form for mature firms:

$$\text{AutoCorr} = \lambda \left[1 - \beta - \left(\frac{\gamma_1 \pi}{\beta E^m} \right) \left(\frac{l + \xi c(1 + \eta)}{r} \right) \right], \tag{18}$$

where λ is the risk premium associated with the output price P and defined in (8). The next proposition provides a formal link between default probability and momentum.

Proposition 2 *If $\eta > 0$, the return on equity of a mature firm exhibits positive autocorrelation, i.e., AutoCorr > 0, only for high level of default probability, and, ceteris paribus, autocorrelation increases in η. If $\eta = 0$ or default probability is low, there is no momentum in equity returns, i.e., AutoCorr < 0.*

This proposition highlights the crucial role of financial distress for leveraged equity—and the ensuing potential deviation from the absolute priority rule—in the determination of momentum in equity returns. The intuition behind this result is that, because of the potential APR violation, the prospect of default may actually *reduce* the risk of the leveraged equity. This happens because, as a consequence of the APR violation, equity holders are relieved from the debt burden and receive either a payout (as in our setting) or a stake in the restructured firm (as in Fan and Sundaresan (2000) and Garlappi, Shu, and Yan (2006)) with much lower risk. Therefore, for these firms, as the firm edges towards default with a declining stock price, the ex-ante risk level of equity *decreases* too, and so does the expected return for the future period, as depicted in Panel B of Figure 1. Similarly, as a firm moves away from the brink of bankruptcy, its stock price rises, but the risk of its equity *increases* because of the debt burden and so does the expected return in the future period. Both scenarios depict a return pattern that exhibits momentum. Notice that this mechanism applies *only* to firms with high default probability. For this reason, the risk dynamic we highlighted can explain the recent empirical finding of Avramov, Chordia,

Jostova, and Philipov (2006a) who document that the momentum effect in stock returns is driven primarily by firms with low credit ratings.[15]

There are two final points worth noting about the analysis of mature firms. First, our model produces momentum in equity returns even though the fundamental process of the revenue of the firm is not predictable, since P follows a geometric Brownian motion. In contrast, the existing models of rational momentum in Johnson (2002) and Sagi and Seasholes (2006) rely on a fundamental process that is itself mean-reverting and hence with a positive instantaneous autocorrelation. Second, our model is able to generate momentum for mature firms which have *no growth options*. In Sagi and Seasholes (2006), growth options are instrumental for inducing return momentum. To examine how growth options may affect our results on the book-to-market effect and the momentum effect, we turn to growth firms in the next subsection.

3.2 Growth firms

We define a growth firm as a firm which currently produces one unit of product but has a perpetual option to expand its operating scale to ξ (> 1) units of product upon making a one-time investment of I.[16] In other words, a growth firm is the predecessor, in the life-cycle of firms, to the mature firm discussed in the previous subsection. In our current framework, we abstract away from the endogenous financing decision and assume that the investment is financed by new equity. Consistent with the case of mature firms, the growth firm has an existing level of leverage that is represented by a console bond paying a continuous coupon of l.

A growth firm maintains its status until it either defaults or exercises its growth option and becomes a mature firm. As for the mature firm case, we allow for possible APR violations upon default that enable equity holders to receive a fraction η of the book value of assets. Given this setup, the equity value can be written as follows

$$
\begin{aligned}
E^g(P_t) &= \mathbb{E}\left[\int_0^{\tau_L \wedge \tau_G}(P_{t+s} - c - l)e^{-rs}ds\right] \\
&\quad + \eta V_A^g(\underline{P}^g)\mathbb{E}[e^{-r\tau_L}\mathbf{I}_{\{\tau_L < \tau_G\}}] + (E^m(\overline{P}) - I)\mathbb{E}\left[e^{-r\tau_G}\mathbf{I}_{\{\tau_G < \tau_L\}}\right],
\end{aligned} \tag{19}
$$

---

[15]This mechanism is also consistent with the reversal in the momentum in stock returns. As the fortune of a low-credit-quality firm improves, its default probability is reduced and its expected return may shift over the hump in Panel B of Figure 1. When this happens, its autocorrelation turns negative and the momentum in stock returns is reversed.

[16]The assumption of one unit of current production scale does not make a material difference in the intuition of our results.

where $\underline{P}^g$ and $\overline{P}$ are the prices at which the growth firm defaults or expands, respectively; and τ_L and τ_G the times at which these two events take place; $V_A^g(\underline{P}^g) = c/r$ is the book value of assets, and $E^m(\overline{P})$ is the equity value of the corresponding mature firm, derived in (13). Equation (19) states that the equity value of a growth firm is equal to the present value of its stream of profits, net of operating and interest costs, until the firm is no longer operative as a growth firm, i.e., until the arrival of the smaller of the two stopping times τ_L and τ_G. If the firm defaults before it expands ($\tau_L < \tau_G$), equity holders receive a fraction η of book value of assets. If, on the other hand, the firm expands before it defaults ($\tau_L > \tau_G$), equity holders pay I and receive the equity value of the mature firm it transforms into. The boundaries $\underline{P}^g$ and $\overline{P}$ are chosen optimally by shareholders. The following proposition characterizes the equity value of such a growth firm.

Proposition 3 *The equity value of a growth firms is given by*

$$E^g(P) = \frac{P}{\delta} - \frac{c+l}{r} + AP^{\gamma_1} + BP^{\gamma_2} + \eta \left(\frac{c}{r}\right) f(P) + (E^m(\overline{P}) - I)g(P), \tag{20}$$

where $f(P) = \mathbb{E}\left[e^{-r\tau_L}\mathbf{I}_{\{\tau_L < \tau_G\}}\right]$ is the price of a perpetual barrier option that pays off one dollar if the price P reaches the default boundary before the growth option is exercised, and $g(P) = \mathbb{E}\left[e^{-r\tau_G}\mathbf{I}_{\{\tau_G < \tau_L\}}\right]$ is the price of a perpetual barrier option that pays off one dollar if the price P reaches the expansion boundary before the firm defaults. Their expressions are given, respectively, in equations (A44) and (A44) of Appendix A. The four unknowns A, B, $\underline{P}^g$, $\overline{P}$ are obtained from the value-matching and smooth-pasting conditions (A46)–(A47) derived in Appendix A.

Based on the above proposition, we can characterize the β of a growth firm, as summarized in the following corollary.

Corollary 3 *The β of a growth firm is given by*

$$\begin{aligned}
\beta &= 1 + \frac{1}{E^g(P)}\left[\frac{l+c}{r} + \left((\gamma_1 - 1)(\underline{P}^g)^{\gamma_1}A' + (\gamma_2 - 1)\overline{P}^{\gamma_1}B'\right)f(P)\right] \\
&\quad + \frac{1}{E^g(P)}\left[\left((\gamma_1 - 1)(\underline{P}^g)^{\gamma_2}A' + (\gamma_2 - 1)\overline{P}^{\gamma_2}B'\right)g(P)\right].
\end{aligned} \tag{21}$$

where A' and B' are constants defined in equations (A49) and (A50) of Appendix A.

The structure of β is represented in an intuitive form. The first part of the β expression depends upon the "limited liability option" captured by the term containing $f(P)$ while the second part depends upon the "growth option" captured by the term containing $g(P)$. The term included in the first square bracket of equation (21) represents the book-to-market effect in the same fashion as for a mature firm. The illustration of the relationship between this component of β and default probability looks very similar to that in Figure 1, indicating that the risk of assets-in-place to equity holders increases with default probability for the most part, i.e., the book-to-market effect in equity returns is stronger for firms with lower credit worthiness. The effect of η is similar for growth firms, except that the relative magnitude may be reduced due to the difference in operating scales ξ.

The term included in the second square bracket of (21) represents a unique component of the β of a growth firm that is directly related to the likelihood of exercising the growth option. As this likelihood, $g(P)$, increases, the weight of the growth option in the equity value gets larger, and hence the equity risk increases. This component is ascribed to capturing the size effect by Berk, Green, and Naik (1999) and others, and it is only present for growth firms.

As for the case of mature firms, we rely on the result in Lemma 1 to obtain a measure of momentum for the growth firm, as the following corollary illustrates.

Corollary 4 *The autocorrelation in equity returns of a growth firm is given by*

$$AutoCorr = \lambda \left[1 - \beta - \left(\frac{1}{\beta E^m} \right) (\gamma_1(\gamma_1 - 1)A'P^{\gamma_1} + \gamma_2(\gamma_2 - 1)B'P^{\gamma_2}) \right], \qquad (22)$$

where λ is the risk-premium associated with the output revenues P and A' and B' are constants defined in equations (A49) and (A50) of Appendix A.

Since the expression for the equity value of growth firms is not available in fully closed form, we cannot perform more analytical characterization of momentum for these firms. In the next subsection we conduct a numerical analysis to show that the momentum measure is positive only in the range of high default probability with $\eta > 0$, similar to the pattern for mature firms. Interestingly, we find that possible violations of APR are *necessary* for obtaining this result and that, contrary to the models that rely on a mean-reverting price process, momentum *cannot* be generated by simply considering growth options when the underlying revenue process follows a geometric Brownian motion.

3.3 Discussion and empirical implications

The framework we have presented above combines the essential features of Carlson, Fisher, and Giammarino (2004), Sagi and Seasholes (2006) and Garlappi, Shu, and Yan (2006). While it generates the same book-to-market and size effects in cross-sectional returns as in Carlson, Fisher, and Giammarino (2004), the financial leverage incorporated in our model distinguishes equity from assets and establishes a clear link between the book-to-market effect and default probability. This is significant because it helps explain several puzzling pieces of empirical evidence in the literature regarding the book-to-market effect. First, because of the elasticity effect of leverage on the equity beta, for most firms, the risk of assets-in-place to equity holders is higher with higher levels of financial leverage, and hence the magnitude of the book-to-market effect is stronger for more heavily levered firms. This is consistent with the evidence that the value premium is most significant for firms with high default probability (see, e.g., Griffin and Lemmon (2002), Vassalou and Xing (2004), and Chen (2006)).

Second, our model provides a perspective for understanding the results of Hecht (2000) that firm-level asset returns do not exhibit strong cross-sectional patterns, such as the book-to-market and momentum effects, because these patterns are generally enhanced by leverage in equity returns, and their magnitude may be too small in asset returns to be statistically detectable. This insight may help resolve the problem of unusually high risk premia required to match empirically observed levels of value premia in Carlson, Fisher, and Giammarino (2004) and Zhang (2005). These models are based on the dynamics of a firm's asset returns and yet calibrated to equity returns. A high risk premium is hence required to reconcile the discrepancy between the theoretical model, which ignores financial leverage, and the empirical data, which include such leverage.

Moreover, our framework can also accommodate the findings of Chung, Johnson, and Schill (2006) who show that once higher-order co-moments are taken into account, the Fama-French factors lose their cross-sectional pricing power. Given the option feature of equity, accounting for the higher-order co-moments is akin to accounting for the effect of leverage, and the residual effect of the book-to-market ratio then becomes insignificant in magnitude. This is also consistent with the results of Ferguson and Shockley (2003) who argue that the SMB and HML factors in the Fama-French three-factor model are instruments for measurement errors in equity beta due to leverage and financial distress.

Potential violations of the absolute priority rule upon default alter the risk structure of equity. The risk of equity is in fact reduced as the firm edges to default, since this event presents an opportunity for restructuring that can bring shareholders relief from the debt burden. While this intuition is used by Garlappi, Shu, and Yan (2006) to explain the empirical association between equity return and default probability, our analysis demonstrates that this mechanism is more general and has implications for both the book-to-market and momentum effects on equity returns.

As illustrated in Figure 1, for mature firms, without APR violations (i.e., $\eta = 0$), the magnitude of the book-to-market effect is increasing in default probability. However, when APR violations are present (i.e., $\eta > 0$), the equity risk associated with the book-to-market effect, and hence the expected return, exhibits a hump shape as default probability increases. Similar pattern can be obtained for growth firms (not shown). Even though in our model the book-to-market effect itself does not rely on the existence of growth opportunities, it is common in the literature to associate growth firms with firms having a low book-to-market ratio and mature (or value) firms with firms having a high book-to-market ratio. To understand the link between book-to-market ratio and growth opportunities in our model, we report in Figure 2 the book-to-market-equity ratio of mature and growth firms as a function of default probability. Specifically, following the same methodology described earlier, we sort firms in our cross section by default probability and then compute the average book-to-market ratio within each default probability decile.[17]

As Figure 2 illustrates, the book-to-market-equity ratio is always larger for mature firms than for growth firms with similar levels of default probability. This happens because, for a given scale of operation, the equity value of a growth firm contains growth options that are not available to mature firms. The figure suggests that, within our model, ranking firms by the book-to-market-equity ratio is capturing the difference in the growth potential of firms. It is important to realize that the presence of firms at different stages of their life-cycle allows us to establish this link between the book-to-market effect and the difference in mature and growth firms. The "value premium", which is empirically defined as the difference between high and low book-to-market-equity ratios, may then be represented as the difference in expected returns of mature and growth firms. This is the representation of the value premium that we will use

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[17]The book-to-market ratio for mature firms is computed as $(\xi c - l)/(rE^m)$ and that for growth firms is $(c - l)/E^g$, where E^m and E^g represent equity values of mature and growth firms, respectively.

Figure 2: Book-to-market ratio of mature and growth firms

The figure reports the book-to-market equity ratio of mature and growth firms. The solid line refers to the case of mature firms and the dash-dotted line represents the case of growth firms. The figure is drawn assuming a value of $\eta = 5\%$
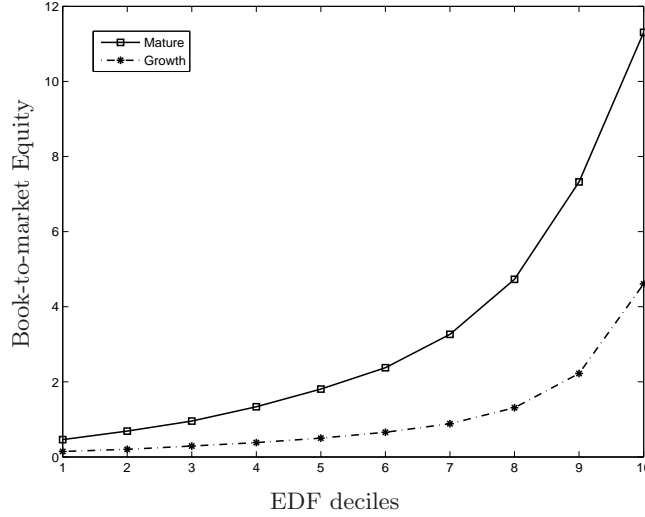

in our subsequent analysis, with the understanding that such a spread corresponds to what is empirically referred to as the premium on the HML portfolio.

Figure 3 reports the average value premium across default probability deciles. The figure illustrate a similar pattern to that observed for expected returns of mature firms in Figure 1. Value premia are increasing with default probability when $\eta = 0$ and humped with respect to default probability when $\eta > 0$. The hump shape reflects the fact that, for a given η, because of the difference in the operating scale, shareholders of growth firms receive lower payoffs than shareholders of mature firms during the resolution of financial distress. This makes growth firms with high default probabilities to have higher expected returns than those of mature firms with similar default probabilities. The result with $\eta > 0$ may seem to contradict the existing empirical evidence that the value premium is higher for low credit-quality firms. However, it is important to realize that large part of these earlier studies usually exclude stocks with prices lower than $5 per share, exactly those associated with high default probabilities. Our theoretical model suggests then the following testable empirical implication, based on the role played by APR violations upon default.

Figure 3: Value premium and default probability

The figure reports the difference in the expected returns of mature and growth firms as a function of default probability. The graphs are obtained from a cross section of firms by varying firm-level characteristics as described in Appendix B. Firms are ranked based on their default probability computed according to equation (A20) in Lemma 2 of Appendix A and refers to the likelihood of the firm defaulting within a year. Panel A refers to the case of no violation of APR, $\eta = 0$ while Panel B refers to the case in which $\eta = 5\%$

Panel A: No violations of APR $(\eta = 0)$
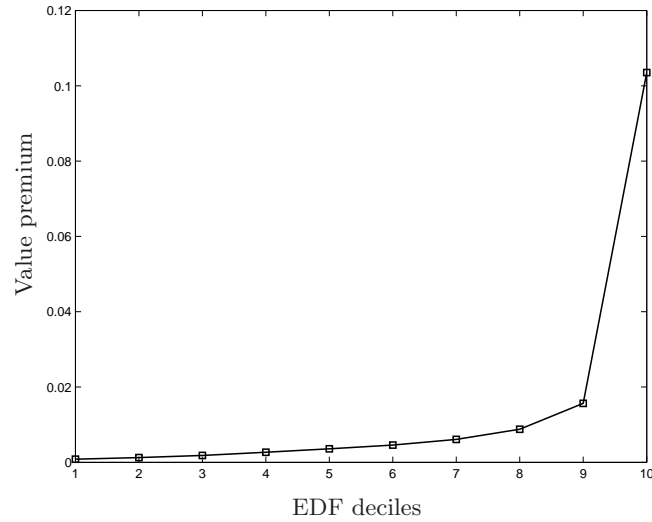
Panel B: With violations of APR $(\eta = 5\%)$
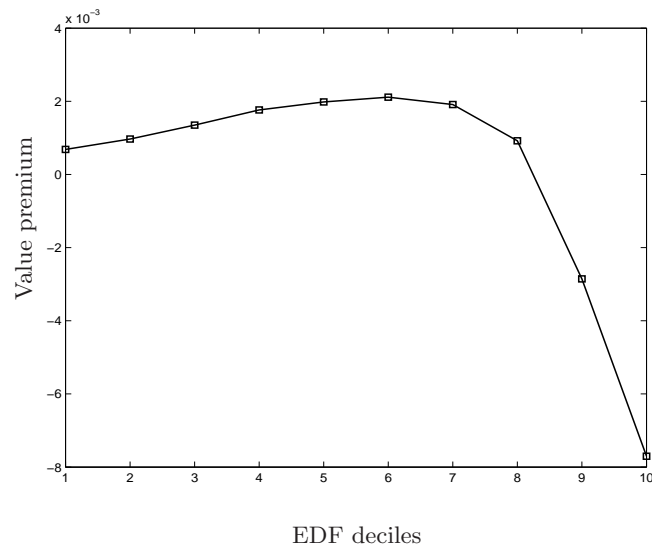
Prediction 1 *In the presence of potential violations of the APR upon the resolution of financial distress, the value premium is hump-shaped with respect to default probability. All else being equal, the value premium is more likely to be positively related to default probability for firms with higher stock prices.*

As discussed in Proposition 2, possible deviations from APR upon default also play an instrumental role in generating momentum in equity returns. To illustrate this result, we use the cross section of firms generated in our numerical analysis and compute the average return correlation within each default probability decile. The results are reported in Figure 4. As shown in Panel A, when $\eta = 0$, the autocorrelation in equity returns is negative across the spectrum of default probability. In this case, the debt burden drives the equity value to zero when the default boundary is approached. That is, as the equity price decreases with default probability, the expected return increases due to the higher debt burden, hence the negative autocorrelation in returns.

However, when $\eta > 0$ and default probability is large, the expected return is decreasing in default probability. This implies that as stock prices decrease with default probability, future expected returns also decrease. This is the signature of momentum. As illustrated in Panel B of Figure 4, this feature is present *only* with high levels of default probability and with $\eta > 0$.

To verify that the result regarding the autocorrelation in returns is robust in producing momentum profits similar to those in Jegadeesh and Titman (1993), we extend our numerical exercise by adding a time-series dimension to the already-generated cross section of firms. Specifically, we simulate quarterly returns for each firm in the cross section and, after sorting by default probability, we construct portfolios of winners and losers based on realized returns. We then calculate the differences in average expected returns of winner and loser portfolios. The details of the calculation are in Appendix B.2 and the results are presented in Figure 5. The patterns are similar to those in Figure 4. Namely, when $\eta = 0$, there are no detectable momentum profits. At very high default probabilities, past winners have much *lower* expected returns than past losers, contrary to what is needed to generate momentum profits. However, when $\eta > 0$, at high levels of default probability past winners do have higher expected returns on average than past losers.

While our results for $\eta > 0$ are consistent with the empirical evidence of Avramov, Chordia, Jostova, and Philipov (2006a) that momentum profits are mainly contributed by firms with low

Figure 4: Return autocorrelation and default probability

The figure reports the momentum measure for mature and growth firms [equations (18) and (22)] as a function of default probability. The graphs are obtained from a cross section of firms by varying firm-level characteristics as described in Appendix B. Firms are ranked based on their default probability computed according to equation (A20) in Lemma 2 of Appendix A and refers to the likelihood of the firm defaulting within a year. Panel A refers to the case of no violation of APR, $\eta = 0$ while Panel B refers to the case in which $\eta = 5\%$

Panel A: No violations of APR $(\eta = 0)$
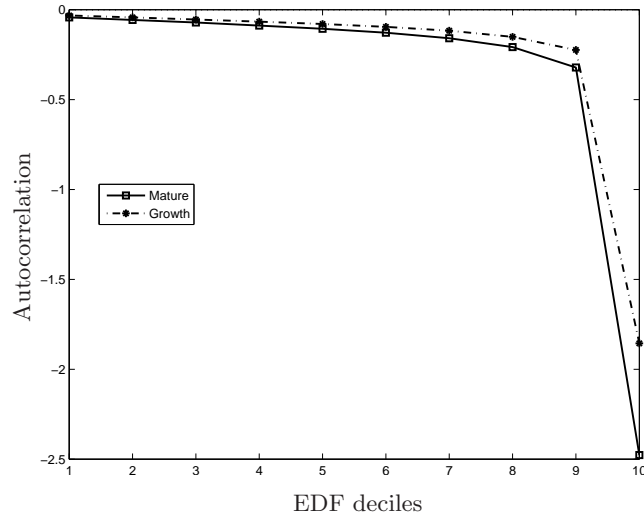
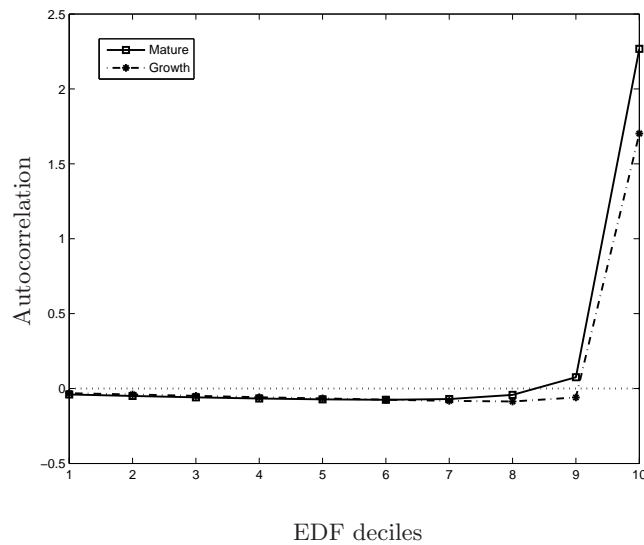Panel B: With violations of APR $(\eta = 5\%)$

Figure 5: Momentum profits and default probability

The figure reports the momentum profits from long the winner-portfolio and short the loser-portfolio as a function of default probability. The graphs are obtained from a cross section of firms by varying firm-level characteristics as described in Appendix B.2. Firms are ranked based on their default probability computed according to equation (A20) in Lemma 2 of Appendix A and refers to the likelihood of the firm defaulting within a year. There are equal numbers of growth and mature firms. Panel A refers to the case of no violation of APR, $\eta = 0$ while Panel B refers to the case in which $\eta = 5\%$

Panel A: No violations of APR ($\eta = 0$)
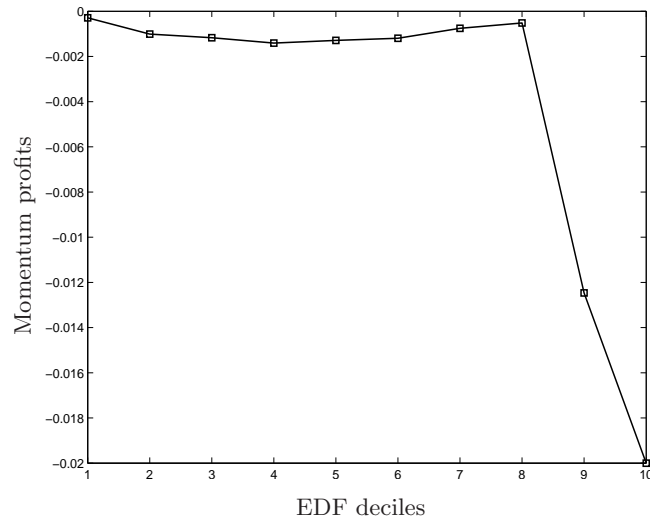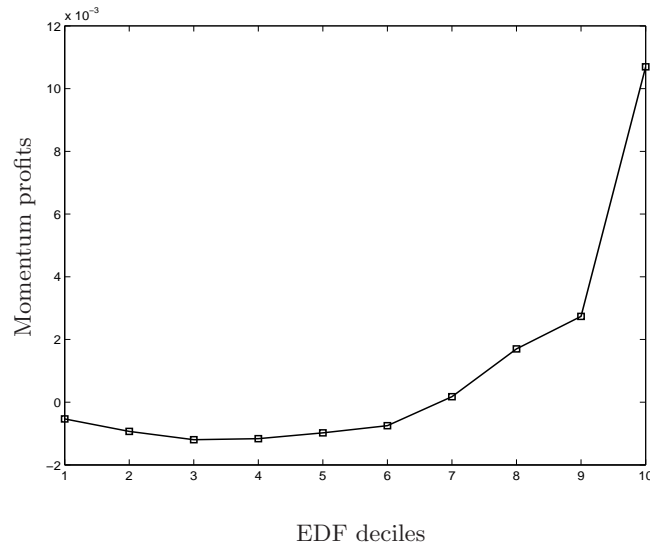

Panel B: With violations of APR ($\eta = 5\%$)

credit ratings, the importance of APR violations for generating momentum profits in our model leads to the following new and testable empirical prediction.

Prediction 2 *Among firms with high default probabilities, those with higher likelihood of APR violations upon default should exhibit stronger momentum in stock returns.*

In summary, the likelihood of default and the possible APR violation upon default constitute a simple mechanism that can account for major cross-sectional patterns in stock returns. In addition to its impact on the cross-sectional variation in the value premium, this mechanism can also be responsible for the concentration of momentum profits in low credit quality firms, as demonstrated above, and for the negative relation between expected return and default probability, as discussed in Garlappi, Shu, and Yan (2006). The empirical evidence in Avramov, Chordia, Jostova, and Philipov (2006b) seems to be supportive of this interpretation.

In the next section, we empirically verify the unique predictions of our model regarding the relation between value premium and default probability and the impact of possible APR violations on the relation between momentum profits and default probability.

4 Empirical evidence

4.1 Data and summary statistics

In our empirical investigation, we use a market-based measure of default probability, the *Expected Default Frequency* (EDF), obtained directly from *Moody's KMV* (MKMV hereafter). This measure is constructed from the Vasicek-Kealhofer model (Kealhofer (2003a,b)) which adapts the Black and Scholes (1973) and Merton (1974) framework and is mapped with a comprehensive database of historical default experiences.[18]

We match the EDF database with the CRSP and COMPUSTAT databases, i.e., a stock needs to have data in all three databases to be included in our analysis. Specifically, for a given month, we require a firm to have an EDF measure and an implied asset value in the MKMV dataset, stock price, shares outstanding and returns data from CRSP, and accounting numbers from COMPUSTAT for firm-level characteristics. We limit our sample to non-financial US firms.[19] We also drop from our sample stocks with a negative book-to-market ratio. Our

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[18]See Crosbie and Bohn (2003) for details on how MKMV implements the Vasicek-Kealhofer model to construct the EDF measure.

[19]Financial firms are identified as firms whose industrial code (SIC) are between 6000 and 6999.

baseline sample contains 1,430,713 firm-month observations and spans from January 1969 to December 2003.[20] This is the same data sample used in Garlappi, Shu, and Yan (2006).

Summary statistics for the EDF measure are reported in Table 2. The average EDF measure in our sample is 3.44% with a median of 1.19%. The table shows that there are time-series variations in the average as well as in the distribution of the EDF measure, and that the majority of the firms in our dataset have an EDF score below 4%. One caveat is that MKMV assigns an EDF score of 20% to all firms with an EDF measure larger than 20%. Around 5% of the firms are in this group at any given time.

Since the EDF measure is based on market prices, in order to mitigate the effect of noisy stock prices on the default score, we use an exponentially smoothed version of the EDF measure, based on a time-weighted average. Specifically, for default probability in month t, we use

$$\overline{EDF_t} = \frac{\sum_{s=0}^{5} e^{-s\nu} EDF_{t-s}}{\sum_{s=0}^{5} e^{-s\nu}}, \tag{23}$$

where ν is chosen to satisfy $e^{-5\nu} = 1/2$, such that the five-month lagged EDF measure receives half the weight of the current EDF measure. The empirical results are reported based on $\overline{EDF_t}$, which we will still refer to as EDF for notational convenience.

4.2 Value premium and default probability

We first examine how value premium changes with default probability. We sort *all* stocks in our sample each month into ten deciles according to their EDF scores and, *independently*, into three terciles according to their book-to-market ratios. We then record both value-weighted and equal-weighted returns of each portfolio in the second month after portfolio formation to avoid possible market microstructure effects. Panel A of Table 3 presents the results which are averaged over time.

The results show that in the full sample value premium initially rises with default probability and then starts to decline at high levels of default probability. For value-weighted returns, the value premium rises with EDF scores until the seventh EDF decile and then turns and drops to a lower level in the last decile. This hump-shaped pattern is particularly pronounced with

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[20]We follow Shumway (1997) to deal with the problem of delisted firms. Specifically, whenever available, we use the delisted return reported in the CRSP datafile for stocks that are delisted in a particular month. If the delisting return is missing but the CRSP datafile reports a performance-related delisting code (500, 520-584), then we impute a delisted return of -30% in the delisting month.

equal-weighted returns, with a clear decline starting from the seventh decile all the way to the highest level of EDF scores. The more pronounced pattern is due to the fact that stocks with higher EDF scores, which usually have lower market capitalizations, within each EDF decile take more weight in equal-weighted portfolio returns. This hump shape in the relationship between value premium and default probability is precisely the prediction of our model as a consequence of potential APR violations upon default.

The results presented here seem to contradict the existing evidence in the literature that the value premium is larger for firms with higher default probability (e.g., Griffin and Lemmon (2002)). Note that in these empirical studies, a customary sample filtering rule is to exclude stocks with price per share less than $5 to avoid market microstructure issues. As we discussed earlier, filtering out these stocks exactly excludes the stocks with very high levels of default probability. Therefore, it is likely that the extant empirical evidence reflects the variation of the value premium over a limited range of default likelihood where the value premium increases in default probability, as indicated in Figure 3.

To test this notion, we restrict our sample of stocks to those with stock prices larger than $5 per share or with market capitalization larger than the breakpoint of the lowest size decile of NYSE stocks, and redo the same portfolio formation and return recording as for the full sample. We again report only the second-month portfolio returns to mitigate liquidity and market microstructure concerns. The results, presented in Panel B of Table 3, confirm that for this subset of stocks, the value premium is indeed monotonically increasing in EDF scores, consistent with the existing evidence in the literature. Taken together, results in Table 3 provide a solid confirmation of the prediction of our model for the relationship between value premium and default probability, and hence validate the importance of potential APR violations for the cross section of stock returns.

4.3 Financial distress, APR violations, and momentum profits

Our model also predicts that momentum in stock returns is strongest for firms with high levels of default probability. This is consistent with the evidence in Avramov, Chordia, Jostova, and Philipov (2006a). Furthermore, our model yields an additional unique prediction regarding how the expected outcome of APR violations, as represented by η in our model, can affect the cross-sectional pattern of momentum. A verification of this prediction will be a strong

piece of supporting evidence that our model provides a valid mechanism for understanding the momentum phenomenon.

In order to test the model prediction, we need to have proxies for the role of η. We use three proxies: asset size, R&D expenditure, and industry concentration measured by the Herfindahl index of sales. The justification of these proxies is discussed in detail in Garlappi, Shu, and Yan (2006). Basically, as documented by Franks and Torous (1994) and Betker (1995), firm size is a persistent determinant of the deviation from the APR. Opler and Titman (1994) show that firms with high costs of R&D suffer the most in financial distress and may subject to liquidity shortage that diminishes the bargaining power of shareholders in financial distress (e.g., Fan and Sundaresan (2000)). Therefore, firms with smaller asset bases or higher R&D expenditures are likely to have a smaller η. Moreover, Shleifer and Vishny (1992) argue that specificity of a firm's assets increases liquidity costs when the firm is in financial distress. High liquidity costs can motivate creditors to negotiate with shareholders in the resolution of financial distress and therefore increase the chance of APR violations. Firms in a more concentrated industry is likely to have more specific assets, and hence a larger η.

To test the prediction that stocks with high η and high default probability have stronger momentum, we sort all stocks in each month independently into terciles of a proxy for η, terciles of EDF scores and quintiles of losers and winners based on past six-month returns. We then record the equal-weighted portfolio returns over the six-month period, starting from the second month after portfolio formation. We report in Table 4 the results of monthly momentum returns, averaged over the sample period, for portfolios in the top tercile of EDF scores.

Panel A of Table 4 shows that for firms with high EDF scores and large asset bases, winners outperform losers by 1.14% per month over the next six-month period. This is compared with those high EDF firms with small asset bases, among whom past winners do not outperform past losers, i.e., there is no momentum for this group of firms. This result is consistent with our prediction and is significant with a t-statistic of 3.92. Panel B demonstrates that firms with high EDF scores and low R&D expenditures experience strong momentum in stock returns, but firms with high R&D expenditures with similar credit profiles do not. Moreover, high EDF firms in a more concentrated industry are more likely to have momentum in stock returns than similar firms in a more competitive industry. All of these results directly confirm the prediction

of our model regarding the role of APR violations for financially distressed firms in inducing momentum in equity returns.

5   Conclusion

Recent empirical evidence strongly suggests that financial distress is instrumental in explaining the cross section of stock returns. While this seems to confirm the conjecture of Fama and French (1996) that the book-to-market effect be related to financial distress, efforts of finding a distress risk factor have not been successful.

In this paper, we propose a new perspective for understanding the empirical regularities in the cross section of equity returns. We explicitly introduce financial leverage in a simple equity valuation model and consider the likelihood of a firm defaulting on its debt obligations as well as the possible ensuing deviation from the absolute priority rule upon the resolution of financial distress. In this simple modeling framework, we derive two important insights. First, since financial leverage distinguishes equity from firm assets, we show that the option feature of equity amplifies the cross-sectional patterns in stock returns. Therefore, introducing financial leverage validates the intuition of investment-based models for explaining cross-sectional returns by enhancing the magnitude of these effects and reconciling the seemingly contradictory evidence regarding asset returns vs stock returns.

The second insight of our work is about the importance of APR violations in effecting the cross-sectional patterns in returns. We show that while the value premium does increase with default probability, it declines at very high levels of default likelihood. Hence the value premium exhibits a hump shape with respect to default probability. This new prediction, verified empirically, is a consequence of the role of possible APR violations. Moreover, we illustrate that this role of APR violations in financial distress is also a rational mechanism to explain the concentration of momentum profits in low credit quality firms. While this is consistent with the existing empirical evidence in the literature, our additional empirical tests further confirm the unique role of APR violations for inducing momentum in returns.

It is important to note that our simple framework is capable of accounting for the main features of cross-sectional stock returns simultaneously. Yet our model is driven by a single source of risk and does not require an additional risk factor of financial distress to price the cross section

of returns. This implies that the cross-sectional variation of returns is substantially driven by the difference in cash flows (e.g., Vuolteenaho (2002)) and characterized by the nonlinear relationship between returns and cash flows, analogous to returns on options. Our work also reiterates the importance of time-varying risk associated with changes in cash flows for understanding the cross section of returns.

We should also note that while our model provides a rational explanation of the cross section of stock returns based on fundamental characteristics of a firm, it is not inconsistent with other explanations based on information flows or institutional and individual trading behaviors. This is because firms with high default probabilities are usually associated with opaque information environment and/or low levels of institutional holdings. Although the relative importance of different mechanisms in accounting for the regularities in the data is an empirical question, it is essential to be able to provide an explanation, such as ours, that is based on fundamental asset pricing principles.

The simplicity of our framework allows us to distill the basic intuition more clearly, and also suggests a number of possible generalizations to account for richer features in stock returns. For instance, endogenizing the financing choice for investments may enhance our understanding of the effect of optimal capital structure decisions on stock returns. A general equilibrium extension may also enable us to investigate both the time-series and cross-sectional features of stock returns and examine the link between macroeconomic conditions, corporate investments and asset prices. These are exciting directions left for our future research.

A   Appendix: Proofs

Proof of Lemma 1

The total rate of return on equity is given by

$$\text{Expected reurn} \cdot dt = \frac{\mathbb{E}^P[dE] + D \cdot dt}{E} \tag{A1}$$

where $\mathbb{E}^P$ is the expectation under the true probability measure and D denotes generically cash flows received by equity in each period. Using Ito's lemma (A1) can be written as

$$\text{Expected reurn} \cdot dt = \frac{1}{E}\left[\frac{1}{2}\frac{\partial^2 E}{\partial P^2}P^2\sigma^2 + \frac{\partial E}{\partial P}P\mu + D\right] \cdot dt \tag{A2}$$

The fundamental valuation equation under the risk-neutral probability measure implies that

$$E_t = e^{-rdt}\mathbb{E}\left[E_{t+dt} + D_{t+dt} \cdot dt\right], \tag{A3}$$

which, after applying Ito's lemma and simplifying the terms in dt yields

$$\frac{1}{2}\frac{\partial E}{\partial P^2}P^2\sigma^2 + \frac{\partial E}{\partial P}P(\mu - \lambda) - rE + D = 0 \tag{A4}$$

Using (A4) in (A2) and simplify delivers

$$\text{Expected reurn} = r + \lambda\beta \tag{A5}$$

where $\beta = \frac{P}{E}\frac{\partial E}{\partial P} = \frac{\partial \ln(E)}{\partial \ln(P)}$.

As in Sagi and Seasholes (2006), the autocorrelation is defined as the ratio of the covariance between changes in expected returns and changes in equity value, i.e., $\text{cov}\left(\Delta\left(\lambda\frac{\partial \ln(E)}{\partial \ln(P)}\right), \Delta \ln(E)\right)$, and the variance of the changes in equity value, i.e., $\text{var}(\Delta \ln(E))$. Application of Ito's lemma yields (6). ∎

Proof of Proposition 1

The problem can be solved via dynamic programming as follows:

$$E^m(P_t) = e^{-rdt}\mathbb{E}\left[\int_0^{\tau_L}(\xi(P_{t+dt+s} - c) - l)e^{-rs}ds + \eta V_A^m(\underline{P}^m)e^{-r\tau_L}\right] \tag{A6}$$

$$= e^{-rdt}\mathbb{E}\left[\int_0^{\tau_L}(\xi(P_{t+s}+dP-c)-l)e^{-rs}ds + \eta V_A^m(\underline{P}^m)e^{-r\tau_L}\right] \tag{A7}$$

$$= e^{-rdt}\mathbb{E}\left[E^m(P_t+dP)\right] \tag{A8}$$

Using Ito's lemma in the equality $E^m(P_t) = e^{-rdt}\mathbb{E}\left[E^m(P_t+dP)\right]$ we obtain the following ODE

$$\frac{1}{2}\sigma^2 P^2\frac{\partial^2 E^m}{\partial P^2} + (r-\delta)P\frac{\partial E^m}{\partial P} - rE^m + \xi(P-c) - l = 0 \tag{A9}$$

The solution of the homogeneous part of (A9) is

$$E_{homo}^m(P) = A_1 P^{\gamma_1} + A_2 P^{\gamma_2}, \quad \gamma_1 < 0, \gamma_2 > 0, \tag{A10}$$

where $\gamma_1 < 0$ and $\gamma_2 > 1$ are the two roots of the characteristic equation

$$\frac{1}{2}\sigma^2\gamma(\gamma-1) + (r-\delta)\gamma - r = 0. \tag{A11}$$

The particular solution of (A9) is

$$E_{part}^m(P) = \xi\left(\frac{P}{\delta} - \frac{c}{r}\right) - \frac{l}{r}. \tag{A12}$$

Hence

$$E^m(P) = E_{part}^m(P) + E_{homo}^m(P) = \xi\left(\frac{P}{\delta} - \frac{c}{r}\right) - \frac{l}{r} + A_1 P^{\gamma_1} + A_2 P^{\gamma_2}. \tag{A13}$$

As $P \to \infty$, the probability of the firm not meeting the cost/coupon requirement is nil and so the boundary condition is the no-transversality (or no bubble) condition:

$$\lim_{P\to\infty} E^m(P) = \xi\left(\frac{P}{\delta} - \frac{c}{r}\right) - \frac{l}{r}, \tag{A14}$$

which means that $A_2 = 0$ in (A13). This yields the final value of the firm with operating and financial leverage as

$$E^m(P) = \begin{cases} \xi\left(\frac{P}{\delta} - \frac{c}{r}\right) - \frac{l}{r} + A_1 P^{\gamma_1}, & \text{if } P \geq \underline{P}^m \\ \eta\frac{\xi c}{r}, & \text{if } P < \underline{P}^m. \end{cases} \tag{A15}$$

The numbers of perpetual limited liability (put) options A_1, and the default threshold $\underline{P}$ are determined by the following value-matching and smooth-pasting conditions.

$$E^m(\underline{P}^m) = \eta V_A^m(\underline{P}^m) \equiv \eta \frac{\xi c}{r} \tag{A16}$$

$$\left.\frac{\partial E^m}{\partial P}\right|_{P=\underline{P}^m} = 0. \tag{A17}$$

Solving these two conditions yields

$$A_1 = -\frac{\xi}{\delta \gamma_1}(\underline{P}^m)^{1-\gamma_1} > 0 \tag{A18}$$

$$\underline{P}^m = \frac{(l + \xi c(1+\eta))}{r}\frac{\gamma_1 \delta}{(\gamma_1 - 1)\xi} \tag{A19}$$

∎

Definition of default probability

Lemma 2 *Let P_0 be the current product price, evolving according to the process described in (1), and $\underline{P}$ the endogenous determined default trigger. The time 0 cumulative real default probability* $\Pr_{(0,T]}$ *over the time period $(0,T]$ is given by*

$$\Pr_{(0,T]}(P_0) = \mathcal{N}\left(h(T)\right) + \left(\frac{P_0}{\underline{P}}\right)^{-\frac{2\gamma}{\sigma^2}} \mathcal{N}\left(h(T) + \frac{2\gamma T}{\sigma\sqrt{T}}\right), \tag{A20}$$

with $\gamma = \mu - \frac{1}{2}\sigma^2 > 0$, $h(T) = \frac{\log(\underline{P}/P_0) - \gamma T}{\sigma\sqrt{T}}$ and $\mathcal{N}(\cdot)$ the cumulative standard normal function.

Proof: Direct application of the property of hitting time distribution of a geometric Brownian motion, see, e.g., (Harrison, 1985, equation (11), p. 14). ∎

Proof of Corollary 1

The β of a mature firm is

$$\beta = \frac{\partial \ln E^m(P)}{\partial \ln P} = \frac{P}{E^m}\frac{\xi}{\delta}\left(1 - \pi\frac{\underline{P}^m}{P}\right) \tag{A21}$$

$$= \frac{1}{E^m}\left[\xi\left(\frac{P}{\delta} - \frac{c}{r}\right) - \frac{l}{r} + A_1 P^{\gamma_1} + \frac{\xi c + l}{r} - A_1 P^{\gamma_1} - \pi\frac{\xi}{\delta}\underline{P}^m\right], \tag{A22}$$

$$= 1 + \frac{1}{E^m} \left[\frac{l + \xi c}{r} - \pi \left(1 - \frac{1}{\gamma_1} \right) \frac{\xi}{\delta} \underline{P}^m \right] \tag{A23}$$

where the second equality follows by using the definition of risk-neutral probability (14), the third equality simply re-writes (A21) by isolating the expression of E^m in (13) and the last equality follows from using the expression of A_1 in (12). After substituting the expression of $\underline{P}^m$ in (12) and rearranging, we obtain

$$\beta = 1 + \left[\frac{1}{E^m} \left(\frac{\xi c - l}{r} \right) \right] \left(\frac{\xi c + l}{\xi c - l} \right) \left(1 - \pi \frac{(1+\eta)\xi c + l}{\xi c + l} \right). \tag{A24}$$

∎

Proof of Corollary 2

Let us consider separately the cases of small and large default probabilities.

i. For low levels of default probability, $\pi \approx 0$, or alternatively $P \gg \underline{P}^m$. In this case the equity value $E^m \approx \xi(P/\delta) - (\xi c + l)/r > 0$ and the equity beta is approximated by

$$\beta|_{\pi \approx 0} \approx 1 + \frac{1}{E^m} \left(\frac{\xi c + l}{r} \right) = 1 + \frac{1}{\frac{\xi P/\delta}{(\xi c + l)/r} - 1} \tag{A25}$$

It is immediate to see that, at a very low level of default probability π, equity beta is increasing in the leverage l and cost c, and decreasing in P. By (12) the default boundary $\underline{P}^m$ is increasing in c and l, and, by (14), the probability of default π is increasing in the default boundary $\underline{P}^m$ and decreasing in P. Hence, for low levels of π equity betas are increasing in the default probability, independently of the value of η.

ii. For very high levels of default probability, $\pi \approx 1$, or alternatively $P \approx \underline{P}^m$. We consider the effect of a change in the default threshold $\underline{P}^m$ induced by a change in either c or l on the default probability π and on equity value E^m. Using a Taylor approximation around $\pi = 1$ and assuming leverage decreases by an infinitesimal amount x we can locally express the probability of default (14) as

$$\pi|_{\pi \approx 1^-} \approx 1 + \frac{\partial \pi}{\partial l}\bigg|_{\pi=1} (-x) + \frac{1}{2} \frac{\partial^2 \pi}{\partial l^2}\bigg|_{\pi=1} x^2 + o(x^3) \tag{A26}$$

$$= 1 + \frac{\gamma_1}{\xi c(1+\eta)+l}x + \frac{\gamma_1(\gamma_1+1)}{2(\xi c(1+\eta)+l)^2}x^2 + o(x^3), \tag{A27}$$

where we used the expression of $\underline{P}^m$ in (12). The equity value for $\pi \approx 1$ can be approximated via Taylor expansion of (13) for small changes x in leverage:

$$E^m|_{\pi\approx1^-} \approx E^m|_{\pi=1} + \frac{\partial E}{\partial l}\bigg|_{\pi=1}(-x) + \frac{1}{2}\frac{\partial^2 E}{\partial l^2}\bigg|_{\pi=1}x^2 + o(x^3) \tag{A28}$$

$$= \eta\left(\frac{\xi c}{r}\right) - \frac{1}{2}\frac{\gamma_1}{r(\xi c(1+\eta)+l)}x^2 + o(x^3). \tag{A29}$$

To approximate the equity beta, we use the expansion of (A29) for the denominator in the expression (15) and the expansion of (A27) for the numerator. Hence,

$$\beta|_{\pi\approx1^-} \approx 1 + \frac{-\eta\xi c - \gamma_1 x + o(x^2)}{\eta\xi c - \frac{1}{2}\frac{\gamma_1}{(\xi c(1+\eta)+l)}x^2 + o(x^3)}. \tag{A30}$$

When $\eta = 0$,

$$\beta|_{\pi\approx1^-} \approx 1 + \frac{2(\xi c(1+\eta)+l)}{x}. \tag{A31}$$

Hence, $\beta \to \infty$ as $\pi \to 1$, i.e., as $x \to 0^+$. Moreover, locally, β is decreasing in x, or, equivalently, increasing in leverage l.

When $\eta > 0$, from (A30), as $\pi \to 1$, i.e., as $x \to 0^+$, $\beta \to 0^+$. Moreover, it can be shown that

$$\frac{\partial \beta}{\partial x}\bigg|_{\pi\approx1^-} = -\frac{\gamma_1}{\eta\xi c} > 0. \tag{A32}$$

Hence, for high levels of default probability, if $\eta > 0$, β is increasing in x, or equivalently, *decreasing* in leverage l. ∎

Proof of Proposition 2

Because the risk premium λ associated with the price process is positive and constant, according to (6), positive autocorrelation in returns exists if the quantity $\theta \equiv \frac{P}{\beta}\frac{\partial\beta}{\partial P} > 0$. Using the fact that $\beta = \frac{P}{E^m}\frac{\partial E^m}{\partial P}$ together with expression (10) of E^m, we obtain:

$$\theta = \frac{P}{\beta}\frac{\partial}{\partial P}\left(\frac{P}{E}\frac{\partial E}{\partial P}\right) \tag{A33}$$

$$= \frac{1}{\beta}\left(\frac{P}{E^m}\frac{\partial E^m}{\partial P} - \left(\frac{P}{E^m}\frac{\partial E^m}{\partial P}\right)^2 + \frac{P^2}{E^m}\frac{\partial^2 E^m}{\partial P^2}\right) \tag{A34}$$

$$= \frac{1}{\beta}\left(\beta - \beta^2 + \frac{A_1\gamma_1(\gamma_1 - 1)P^{\gamma_1}}{E^m}\right) \tag{A35}$$

$$= 1 - \beta - \frac{1}{\beta E^m}\frac{\gamma_1(l + \xi c(1 + \eta))}{r}\pi, \tag{A36}$$

where the second equality follows from the definition of β in (4), the third equality relies on the definition of E^m in (10) and the last equality uses the definitions of A_1, $\underline{P}^m$ in (12) and π in (14). When $\pi \to 0^+$, $\theta < 0$ because $\beta > 1$, independently of η. When $\pi \to 1^-$, as in the proof of Corollary 2, we use a Taylor approximation around $\pi = 1$ and consider an infinitesimal reduction x in the level of leverage. This allows us to locally approximate the values of E^m and β as in (A29) and (A30) and obtain, as $x \to 0^+$, i.e., as $\pi \to 1^-$

$$\beta E^m|_{\pi\approx 1} = \left(\frac{-\gamma_1 x}{\eta\xi c}\right)\left(\frac{\eta\xi c}{r}\right) + o(x^2) = -\frac{\gamma_1}{r}x + o(x^2). \tag{A37}$$

When $\eta = 0$, using (A31) and (A37) we can rewrite (A36) as

$$\theta|_{\pi\approx 1^-} \approx -\frac{\xi c + l}{x}. \tag{A38}$$

Hence, as $\pi \to 1^-$, i.e., as $x \to 0^+$, $\theta \to -\infty$. When $\eta > 0$, from Corollary 2, β decreases to zero as $\pi \to 1^-$. Substituting (A37) into (A36), and setting $\beta \approx 0$, yields

$$\theta|_{\pi\approx 1^-} \approx 1 + \frac{\xi c(1 + \eta) + l}{x}. \tag{A39}$$

Hence, as $\pi \to 1^-$, i.e., $x \to 0^+$, $\theta \to \infty$. Note finally that, θ is increasing in η. ∎

Proof of Proposition 3

To solve (19), let us define the following expectations

$$f(P) = \mathbb{E}\left[\int_0^{\tau_L \wedge \tau_G}[P_{t+s} - c - l]e^{-rs}ds\right] \tag{A40}$$

$$g(P) = \mathbb{E}\left[e^{-r\tau_L}\mathbf{I}_{\{\tau_L < \tau_G\}}\right] \tag{A41}$$

$$h(P) = \mathbb{E}\left[e^{-r\tau_G}\mathbf{I}_{\{\tau_G < \tau_L\}}\right] \tag{A42}$$

By the analysis above, $f(P)$ can be solved via dynamic programming and yields

$$f(P) = \frac{P}{\delta} - \frac{c+l}{r} + AP^{\gamma_1} + BP^{\gamma_2} \tag{A43}$$

with $\gamma_1 < 0$ and $\gamma_2 > 1$ solutions of (A11) and A and B arbitrary constants. For *given* $\underline{P}^g$ and $\overline{P}$, the solutions of (A41) and (A42), as obtained in Geman and Yor (1996), are:

$$g(P) = \frac{P^{\gamma_1}\overline{P}^{\gamma_2} - P^{\gamma_2}\overline{P}^{\gamma_1}}{\overline{P}^{\gamma_2}(\underline{P}^g)^{\gamma_1} - \overline{P}^{\gamma_1}(\underline{P}^g)^{\gamma_2}}, \quad h(P) = \frac{P^{\gamma_2}(\underline{P}^g)^{\gamma_1} - P^{\gamma_1}(\underline{P}^g)^{\gamma_2}}{\overline{P}^{\gamma_2}(\underline{P}^g)^{\gamma_1} - \overline{P}^{\gamma_1}(\underline{P}^g)^{\gamma_2}}. \tag{A44}$$

Notice that, as it should be, $g(\underline{P}^g) = 1$ and $h(\underline{P}^g) = 0$. Similarly, $g(\overline{P}) = 0$ and $h(\overline{P}) = 1$. Using the fact that the salvage value $V_A(\underline{P}^g) = \eta(c/r)$ and combining (A43), (A44) and (A44), the value of a growing firm can be expressed as

$$\begin{aligned}
E^g(P) &= \frac{P}{\delta} - \frac{c+l}{r} + AP^{\gamma_1} + BP^{\gamma_2} \\
&+ \left(\frac{\eta c}{r}\right)\frac{P^{\gamma_1}\overline{P}^{\gamma_2} - P^{\gamma_2}\overline{P}^{\gamma_1}}{\overline{P}^{\gamma_2}(\underline{P}^g)^{\gamma_1} - \overline{P}^{\gamma_1}(\underline{P}^g)^{\gamma_2}} + (E^m(\overline{P}) - I)\frac{P^{\gamma_2}(\underline{P}^g)^{\gamma_1} - P^{\gamma_1}(\underline{P}^g)^{\gamma_2}}{\overline{P}^{\gamma_2}(\underline{P}^g)^{\gamma_1} - \overline{P}^{\gamma_1}(\underline{P}^g)^{\gamma_2}}
\end{aligned} \tag{A45}$$

The above expression contains four unknown A, B, $\underline{P}^g$, $\overline{P}$. These can be obtained by imposing the following two pairs of value-matching and smooth-pasting conditions

$$E^g(\underline{P}^g) = \frac{\eta c}{r}, \qquad \left.\frac{\partial E^g(P)}{\partial P}\right|_{P=\underline{P}^g} = 0, \tag{A46}$$

$$E^g(\overline{P}) = E^m(\overline{P}) - I, \qquad \left.\frac{\partial E^g(P)}{\partial P}\right|_{P=\overline{P}} = \left.\frac{\partial E^m(P)}{\partial P}\right|_{P=\overline{P}}, \tag{A47}$$

where $\left.\frac{\partial E^m(P)}{\partial P}\right|_{P=\overline{P}} = 1/\delta + A_1\gamma_1\overline{P}^{\gamma_1-1}$, by (10) and (12). ∎

Proof of Corollary 3

To make expressions in the beta calculation simpler, we write the equity value in the following form

$$E^g(P) = \frac{P}{\delta} - \frac{c+l}{r} + A'P^{\gamma_1} + B'P^{\gamma_2} \tag{A48}$$

where

$$A' = A + \eta\left(\frac{c}{r}\right)\frac{\overline{P}^{\gamma_2}}{a} - (E^m(\overline{P}) - I)\frac{(\underline{P}^g)^{\gamma_2}}{a} \tag{A49}$$

$$B' = B - \eta \left(\frac{c}{r}\right)\frac{\overline{P}^{\gamma_1}}{a} + (E^m(\overline{P}) - I)\frac{(\underline{P}^g)^{\gamma_1}}{a} \tag{A50}$$

where A and B are obtained from the solution of the value-matching and smooth pasting conditions (A46)–(A47) in Proposition 3, and $a = \overline{P}^{\gamma_2}(\underline{P}^g)^{\gamma_1} - \overline{P}^{\gamma_1}(\underline{P}^g)^{\gamma_2}$. The β of a growth firm is

$$\beta = \frac{\partial \ln E^g(P)}{\partial \ln P} \tag{A51}$$

$$= \frac{1}{E^g(P)}\left(\frac{P}{\delta} + \gamma_1 A' P^{\gamma_1} + \gamma_2 B' P^{\gamma_2}\right) \tag{A52}$$

$$= 1 + \frac{1}{E^g(P)}\left[\frac{l + c}{r} + (\gamma_1 - 1)A' P^{\gamma_1} + (\gamma_2 - 1)B' P^{\gamma_2}\right]. \tag{A53}$$

¿From equations (A44), we have

$$P^{\gamma_1} = (\underline{P}^g)^{\gamma_1} g(P) + \overline{P}^{\gamma_1} h(P), \quad P^{\gamma_2} = (\underline{P}^g)^{\gamma_2} g(P) + \overline{P}^{\gamma_2} h(P). \tag{A54}$$

Substitution in the expression for β yields

$$\beta = 1 + \frac{1}{E^g(P)}\left[\frac{l + c}{r} + \left((\gamma_1 - 1)A'(\underline{P}^g)^{\gamma_1} + (\gamma_2 - 1)B'\overline{P}^{\gamma_2}\right)g(P)\right] \tag{A55}$$

$$+ \frac{1}{E^g(P)}\left[\left((\gamma_1 - 1)A'\overline{P}^{\gamma_1} + (\gamma_2 - 1)B'\overline{P}^{\gamma_2}\right)h(P)\right]. \tag{A56}$$

∎

Proof of Corollary 4

Direct application of the definition of autocorrelation (6) from Lemma 1. ∎

B   Appendix: Numerical analysis

B.1   Cross-sectional model data

There are a total of twelve parameters in our model, out of which two are common for the overall economy (i.e., the risk-free rate r and the maximal Sharpe ratio SR); five refer to the firm's output price process and can be thought of as industry-specific (i.e., the growth rate in the price process μ, the parameter δ, the volatility σ, the correlation with the pricing kernel in the economy ρ, and the initial output price P_0); and five are firm specific (i.e., the operating cost c, the financial leverage l, the scale of operation of mature firms ξ, the investment cost I, and the degree of APR violation η). To construct the cross section we need to select a set of parameter combination characterizing each firm and a set of initial values for the price of the firm's output. Below we provide a description of our choice of parameters.

1. *Economy-wide parameters (r and SR).* We select the risk-free rate r to be 3% per annum to roughly match empirical estimates of the short rate. The qualitative nature of the results is unaffected by this choice. We choose the maximal Sharpe ratio SR attainable in the economy to be 0.5, in line with other studies (e.g., Campbell (2003)).

2. *Price-process parameters (μ, δ, σ, ρ, P_0).* Given that our price process (1) is non-stationary, we cannot rely on long-run properties to determine the growth rate μ. From (2), this quantity is equal to $\mu = r - \delta + \lambda$, with λ being the risk premium associated with the price process. As in Sagi and Seasholes (2006), we rely on a single-factor model to express the risk premium as $\lambda = \rho SR \sigma$, where SR is the maximal Sharpe ratio attainable in the economy and $-\rho$ is the correlation of the price process with the pricing kernel (see Duffie and Zame (1989)). We choose a benchmark value for ρ of 0.7 consistent with Sagi and Seasholes (2006) and allow 3 values of ρ ranging from 0.5 to 0.9 in constructing the cross section. We choose a benchmark value of the volatility of output price (σ) to be 0.3, based on Sagi and Seasholes (2006) estimates of the annual volatility of revenues, and vary it from 0.2 to 0.4 when generating the cross section. δ is a "free" parameter which has to be less than the risk-free rate in order to insure that the growth option is ever exercised. There are not further restriction that we can impose based on actual data and, as the risk-free rate, the role of this parameter is to act as a scaling factor without affecting the qualitative results. We set it to be 1%, i.e., one-third of the risk-free rate. Hence we have

$9 = 3 \times 3$ different values for the growth rate $\mu = r - \delta + \rho S R \sigma$. Finally, we choose 21 different level of the initial output P_0, ranging from 0.1 to 0.3. The different initial values of P_0 represent differences across industries due to idiosyncratic shocks. The magnitude of P_0 does not matter as it serves to scale other variables accordingly.

3. *Firm-specific parameters* (c, l, ξ, I, η). To choose the level of operating expenses c we rely on the functional form of equity value in Propositions 1 and 3. Absent leverage and the limited liability/growth options, the net value of equity would be zero if $P/\delta = c/r$. We use this as a reference point for the range of values of c to consider. Given the range of initial prices and the selected values of r and δ, the implied range of c is $r/\delta \times [0.1, 0.3] = [0.3, 0.9]$. We choose 7 different values of c in this range. This choice guarantees that at least for some firms in the cross section the limited liability option is valuable. The financial leverage is chosen as a fraction of the operating cost to guarantee that the book-equity $(c - l)/r$ is not negative. We choose 6 different levels of financial leverage, ranging from 40% to 90% of operating costs. We select three values for the scale of operation for mature firms: $\xi = 1.5, 2$ and 2.5, indicating that in our population we allow for growth firms that, upon exercising their investment options, can grow from 50% to 150% of their pre-growth asset size. The investment cost I is linked to the size of growth and is chosen to be equal to the increase in the scale of operation $(\xi - 1)$ times the capitalized value of operating costs c/r, a proxy for the book value of assets. Moreover, we select three different values for the expected deviation from the APR upon financial distress at $\eta = 0, 2.5\%$ and 5% of asset value. $\eta = 0$ represents no APR violations while the positive values for η selected are consistent with the empirical evidence on the unconditional average amount recovered by shareholders in bankruptcy proceedings (e.g., Eberhart, Moore, and Roenfeldt (1990)).

4. *EDF horizon*. To match our empirical data we choose a horizon of one year to compute default probability according to equation (A20).

A summary of our parameter choice is reported in Table 1. In total, for any given value of η, our cross section of firms at time 0 consists of 40,824 firms equally split between growth and mature firms: (2 types of firms)$\times$(21 initial prices)$\times$(9 levels of μ)$\times$(6 levels of c)$\times$(6 levels of l)$\times$(3 levels of ξ).

B.2 Momentum portfolios

To construct momentum portfolios we need to generate also a time series of realized returns that will determine winners and losers in each period. Instead of simulating an entire time-series, we follow a methodology in Sagi and Seasholes (2006) and draw shocks from a discretized version of a steady-state distribution. Precisely, given an initial price P_0, we assume that the shock dW in (1) is governed by a mixture of systematic and idiosyncratic components so that, over the next Δt interval, the output price is given by

$$P_1 = P_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}\,\tilde{x}}, \tag{B57}$$

where

$$\tilde{x} = \rho\tilde{\varepsilon}_z + \sqrt{1 - \rho^2}\tilde{\varepsilon}_p \tag{B58}$$

where the systematic shock $\tilde{\varepsilon}_z \in \{-1, +1\}$ with equal probability and the idiosyncratic shock $\tilde{\varepsilon}_p \in \left\{-\sqrt{\frac{3}{2}}, 0, \sqrt{\frac{3}{2}}\right\}$ with equal probability. This guarantees that both systematic and idiosyncratic shocks have zero mean and unit volatility. The realized return over the period Δt is computed as

$$R_1 = \exp\left\{(r + \beta(P_0)\lambda)\Delta t\right\} \frac{\exp\left\{\beta(P_0)\sigma\tilde{x}\sqrt{\Delta t}\right\}}{E\left[\exp\left\{\beta(P_0)\sigma\tilde{x}\sqrt{\Delta t}\right\}\right]}, \tag{B59}$$

where $\beta(P_0)$ is the equity beta with respect to the process P and computed at P_0 and $\lambda = \rho SR\sigma$ the risk premium for the price process. The normalization by the expected value of $\exp\left\{\beta(P_0)\sigma\tilde{x}\sqrt{\Delta t}\right\}$ insures that the first term in (B59) corresponds indeed to expected returns. In our implementation we take $\Delta t = 0.25$, i.e., a quarter. Conditional on the realization of the systematic shock, we first sort firms by their default probability and then by their realized returns due to the random draw of idiosyncratic shocks. Within each one of these bins we compute the equally weighted average of expected returns from time 1 to 2 obtained by using the information of equity beta at time t, i.e.,

$$E_1[R_2] = \exp\left\{(\beta(P_1)\lambda + r)\Delta t\right\}. \tag{B60}$$

Finally, we average the results over 100 different draws of idiosyncratic shocks and two draws of systematic shocks.

Table 1: Parameter values used in numerical analysis

Parameter	Values
r	3%
SR	0.5%
δ	1%
σ	$20\%, 30\%, 40\%$
ρ	$0.5, 0.7, 0.9$
μ	$r - \delta + \rho SR \sigma$
P_0	$[0.1, 0.3]$, 21 values equally spaced
c	$0.3, 0.5, 0.7, 0.9$
l	$[0.4c \text{ to } 0.9c]$
ξ	$1.5, 2, 2.5$
η	$0, 2.5\%, 5\%$
I	$\frac{(\xi-1)c}{r}$

Table 2: Summary statistics of the EDF measure

Our sample period spans from January 1969 to December 2003. At the beginning of every three-year interval (starting from January 1970), the table reports the number of firms in our sample, the mean, standard deviation, median, first and third quartile of the EDF distribution. EDF quantities are expressed in percent units.

Month	# Firm	Mean	Std.	Median	Quart 1	Quart 3
Jan-70	1,455	1.19	1.76	0.56	0.17	1.50
Jan-73	1,894	2.00	3.20	0.83	0.23	2.25
Jan-76	2,945	3.87	4.77	2.06	0.88	4.58
Jan-79	3,149	2.57	4.21	0.97	0.31	2.56
Jan-82	3,116	3.19	4.60	1.42	0.59	3.40
Jan-85	3,566	3.21	5.17	0.98	0.34	3.18
Jan-88	3,745	4.25	5.83	1.68	0.48	5.02
Jan-91	3,627	5.48	7.11	1.80	0.37	8.08
Jan-94	3,916	2.73	4.56	0.85	0.22	2.82
Jan-97	4,541	2.72	4.61	0.78	0.18	2.82
Jan-00	4,246	3.68	5.11	1.53	0.52	4.26
Jan-03	3,572	5.23	6.52	2.03	0.59	7.39
Full Sample	1,430,713	3.44	5.22	1.19	0.35	3.75

Table 3: Relationship between value premium and default probability

Each month, stocks are sorted independently into terciles of book-to-market ratios (BM) and deciles of MKMV's EDF scores (EDF). The value-weighted (VW) and equal-weighted (EW) returns of each portfolio in the second month after portfolio formation are recorded and averaged over time. Panel A includes all stocks in our sample, while Panel B excludes those with stock prices less than $5 per share or with a market capitalization smaller than the NYSE bottom size decile breakpoint.

| | Low EDF | | | | | | | | | High EDF | | |
	1	2	3	4	5	6	7	8	9	10	High-Low	t-value
					Panel A: Full sample							
					VW Returns							
B/M												
Low	0.96	1.02	0.71	0.72	0.63	0.63	0.05	-0.01	-0.20	0.19	-0.77	*-1.50*
Medium	1.05	1.11	1.19	1.16	1.23	1.10	1.11	1.32	0.81	0.48	-0.57	*-1.10*
High	0.97	1.13	1.30	1.41	1.33	1.27	1.74	1.36	1.55	1.42	0.45	*1.02*
High - Low	0.00	0.11	0.59	0.69	0.71	0.64	1.69	1.37	1.74	1.23		
t-value	*0.01*	*0.48*	**2.27**	**2.49**	**2.59**	**2.38**	**5.68**	**3.95**	**4.55**	**3.16**		
B/M					*EW Returns*							
Low	1.13	1.01	0.86	0.79	0.62	0.72	0.33	0.55	0.66	1.76	0.63	*1.21*
Medium	1.15	1.27	1.34	1.34	1.46	1.35	1.38	1.34	1.35	1.79	0.63	*1.43*
High	1.32	1.34	1.42	1.66	1.58	1.65	1.78	1.79	1.87	2.13	0.81	**2.11**
High - Low	0.19	0.33	0.56	0.87	0.96	0.93	1.45	1.24	1.21	0.37		
t-value	*1.01*	*1.55*	**2.55**	**3.75**	**4.15**	**4.13**	**6.15**	**4.42**	**4.26**	*1.22*		
			Panel B: Subsample of stocks with per-share price greater than $5									
					VW Returns							
B/M												
Low	0.95	1.06	1.07	0.50	0.73	0.76	0.59	0.74	0.27	-0.44	-1.40	**-3.61**
Medium	0.99	1.04	1.10	1.14	1.11	1.10	1.22	1.25	0.96	0.92	-0.07	*-0.18*
High	1.03	1.04	1.23	1.19	1.26	1.38	1.23	1.38	1.25	1.52	0.48	*1.45*
High - Low	0.08	-0.02	0.16	0.69	0.53	0.62	0.64	0.63	0.98	1.96		
t-value	*0.34*	*-0.09*	*0.64*	**2.69**	**1.82**	**2.05**	**2.15**	**2.19**	**3.51**	**6.15**		
B/M					*EW Returns*							
Low	1.15	1.14	0.98	0.86	0.77	0.76	0.67	0.73	0.23	-0.02	-1.18	**-3.73**
Medium	1.08	1.14	1.30	1.28	1.33	1.30	1.25	1.45	1.19	1.10	0.02	*0.06*
High	1.19	1.26	1.35	1.43	1.42	1.56	1.48	1.59	1.44	1.43	0.24	*0.90*
High - Low	0.03	0.11	0.36	0.57	0.65	0.80	0.81	0.86	1.22	1.45		
t-value	**0.16**	**0.52**	**1.57**	**2.38**	**2.72**	**3.26**	**3.34**	**3.55**	**5.29**	**6.14**		

Table 4: Effect of APR violations on momentum profits

Each month, all stocks are sorted independently into terciles of EDF scores, terciles of a proxy for likelihood of APR violations and quintiles of winners/losers according to past six-month returns. The returns of each portfolio for the next six-month period are recorded and averaged through time. Only portfolios in the top terciles are reported in the table. The proxies for the likelihood of APR violations are: asset size (AVL), R&D expenditure-asset ratio (R&D), and Herfindahl index of sales (SalesHfdl).

Panel A: Momentum profits across AVL groups

AVL	Loser	2	3	4	Winner	W-L	t-stat
Low	2.09	1.90	1.93	2.03	2.03	-0.06	-0.27
Med	0.65	1.18	1.34	1.45	1.70	1.05	**3.82**
High	0.36	1.01	1.29	1.25	1.53	1.14	**3.07**
High-Low						1.20	**3.92**

Panel B: Momentum profits across R&D groups

R&D	Loser	2	3	4	Winner	W-L	t-stat
Low	1.05	1.36	1.54	1.57	1.68	0.63	**2.43**
Med	1.98	1.85	2.07	1.94	1.96	0.04	0.14
High	2.73	2.53	2.30	2.39	2.52	-0.34	-1.23
Low-High						0.97	**3.73**

Panel C: Momentum profits across SalesHfdl groups

SalesHfdl	Loser	2	3	4	Winner	W-L	t-stat
Low	1.71	1.65	1.76	1.81	1.81	0.10	0.43
Med	1.68	1.69	1.75	1.97	2.07	0.39	1.49
High	1.34	1.52	1.60	1.61	1.81	0.47	**1.74**
High-Low						0.37	**1.65**

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