Levered Returns

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Abstract

In this paper we revisit the theoretical relation between financial leverage and stock returns in a dynamic world where both the corporate investment and finance decisions are endogenous. We find that the link between leverage and stock returns is more complex than the static textbook examples suggest and will usually depend on the investment opportunities available to the firm. In the presence of financial market imperfections leverage and investment are generally correlated so that highly levered firms are also mature firms with relatively more (safe) book assets and fewer (risky) growth opportunities. We use a quantitative version of our model to generate empirical predictions concerning the empirical relationship between leverage and returns. We test these implications in actual data and find support for them.

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1 Introduction

Standard finance textbooks propose a relatively straightforward link between capital structure and the expected returns on equity: increases in financial leverage directly increase the risk of the cash flows to equity holders and thus raise the required rate of return on equity. This remarkably simple idea has proved extremely powerful and has been used by countless researchers and practicioners to examine returns across and within firms with varying capital structures.

Unfortunately, despite, or perhaps because of, its extreme clarity, this relation between leverage and returns has met with only mixed success empirically. Notable early papers (Bhandari (1988) and Fama and French (1992)) were somewhat inconclusive and a negative relation between stock returns and various measures of financial leverage is documented in several more recent studies (George and Hwang (2007), Penman, Richardson, Tuna (2007), Korteweg (2004)).

This paper suggests that the link between financial leverage and stock returns is generally complex and depends crucially on how debt is used and on its impact on the firm's investment opportunities. Extant literature generally assumes that debt will be used to fund changes in equity, a tradition that is rooted both in the static trade off view of optimal leverage (Miller (1977)) and the Modigliani-Miller theorem decoupling the firm's investment and financing strategies.

Our analysis focuses instead on the effects of debt on the left side of the balance sheet as firms use debt to finance capital spending. Since this expansion naturally increases the value of assets in place to growth options it may reduce the underlying (total) risk of the firm and thus its equity risk as well. While these effects can be dismissed in the benchmark Modigliani-Miller setting, they become of paramount importance in the presence of financial frictions, when investment and financing strategies must be examined jointly.

Our theoretical results can be used to interpret the contradictory empirical evidence about the role of leverage in determining expected returns. In a world of financial market imperfections leverage and investment are often strongly correlated. This, in turn, implies that highly levered firms are also more mature firms with (relatively safe) book assets and fewer (risky) growth opportunities. As a result, cross-sectional studies that fail to control for the interdependence of leverage and investment decisions are unlikely to be very informative.

Clearly real life decisions by corporations will reflect both the existing textbook analysis and our new view. Nevertheless this subtle new link between leverage and expected equity return raises some doubts about the usefulness of the standard textbook formulas in real world applications. This is particularly true when changes in the asset side of the balance sheet are important such as when making cross-sectional comparisons across firms, or when constructing the cost of capital for new projects within a firm.

We begin by constructing a very simple continuous time real options model that formalizes our basic intuition and delivers closed form expressions linking expected returns and corporate decisions on investment and financing. Although stylized, the only key assumptions in this example are that debt and investment decisions are linked and, that growth options are relatively less important for large mature firms. If both assumptions are satisfied then highly levered firms will face less underlying (asset) risk and, possibly, also less equity risk as well.

This simple example is very useful to develop intuition for our key insights, but it is necessarily far too stylized. Accordingly we then proceed to construct a more detailed quantitative model that inherits the key properties of our simple example, but also introduces additional features such as endogenous borrowing constraints, investment costs, and equity issues. We then use this model to show more generally how the link between expected return and leverage arises endogenously as a result of optimal investment and financing policies of the firm and is, in general, more complex than the simple textbook formula implies.

Our quantitative model it is also suitable to develop a number of empirical predictions. To accomplish this we simulate artificial panels of firms and use them as our laboratory. To test the quantitative model predictions for leverage and returns, we provide our own empirical evidence using data from the widely used CRSP/Compustat dataset. Specifically we show that simulated data from the model can successfully replicate the empirical relationships between leverage and returns, even after one controls for variables such as size and book-to-market. Interestingly we find that, book-to-market does a very good job of capturing the effects of leverage on returns, while size is a more appropriate indicator of growth options.

Our work is at the center of several converging lines of research. First, it builds on the growing theoretical literature that attempts to link corporate decisions to the behavior of asset returns (a partial list includes Berk, Green and Naik (1999), Gomes, Kogan and Zhang (2003), Carlson, Fisher and Gianmarino (2004), Cooper (2005) and Zhang (2005)). From this point of view the novelty in our work is the fact that we explicitly allow for deviations of the Modigliani-Miller theorem so that corporate financing decisions will affect investment and thus asset prices.

Our paper also adds to the recent literature on dynamic models of the capital structure that attempt to link the corporate investment and leverage policies of firms (a partial list includes Hennessy and Whited (2005, 2007), Miao (2005) and Sundaresan and Wang (2006)). Here the key novelty of our work is allowing for exposure to systematic risk and our specific focus on the asset pricing implications of these models.

Our work is most closely related to a growing literature on dynamic quantitative models investigating the implications of firms' financing decisions on asset returns. Some recent papers along these lines include Garlappi and Yan (2007), Livdan, Sapriza Zhang (2006), Li, Livdan and Zhang (2007), Li (2007) and Obreja (2007). Livdan, Sapriza and Zhang study the quantitative implications of firms' financing constraints and leverage in a model without default or taxes while Li, Livdan and Zhang (2007) look at the asset pricing implications of firms' equity issuance decisions. Allowing for deviations from the Modigliani-Miller assumptions, Li (2007) focuses on the link between investment, leverage and corporate governance issues while Garlappi and Yan (2007) examine the link between distress risk and asset returns, allowing for deviations from the absolute priority rule.

Like us Obreja (2007) also investigates the link between leverage and returns but focuses instead on the role of leverage in generating the observed size and book-to-market factors in cross-sectional equity regressions. By contrast our work seeks to first understand how the interaction of corporate investment and leverage decisions lead to different patterns in equity returns.

This paper is organized as follows. Section 2 provides a simple example where we can derive in closed form the effects of endogenous leverage on expected returns. Section 3 builds on this intuition to develop our argument in a more general model where firms make joint decisions about investment, debt, and equity issues in the presence of adjustment costs to capital and leverage. Section 4 examines some of the model's quantitative implications for the cross-section returns and compares them with the empirical evidence. Finally section 5 offers a few concluding remarks.

2 Leverage, Investment, and Returns: A Simple Example

In this section we construct a simple continuous time real options model that formalizes our basic insights and delivers closed form expressions linking expected returns and corporate decisions on investment and financing. These ideas are then integrated in the more general model developed in the next section.

2.1 Profits and Dividends

We consider the problem of value maximizing firms, indexed by the subscript i, that operate in a perfectly competitive environment. The instantaneous flow of (after tax) operating profits , Π_i, for each firm i is completely described by the expression

$$\Pi_i = (1 - \tau) X_t K_i^{\alpha}, 0 < \alpha < 1$$

where K_i is the productive capacity of the firm, τ is the corporate tax rate, and the variable X is an exogenous state variable that captures the state of aggregate demand (or productivity).

As usual we think of this profit function as that resulting from the determination of the optimal choices for all other (static) inputs such as labor and raw materials for example. This combination of perfect competition with decreasing returns to scale can be shown to be equivalent to that of a monopolist facing a downward sloping demand curve for its output, so that our assumptions are not too restrictive.

The state variable, X_t, is assumed to follow the stochastic process

$$dX_t/X_t = \mu dt + \sigma d\varepsilon_t$$

where we assume for simplicity that ε_t is a standard Brownian motion under a risk-neutral measure.[1]

2.2 Investment and Financing

A typical firm is endowed with an initial capacity K_0 and one option to expand this capacity to K_1 by purchasing additional capital in the amount $I = K_1 - K_0 > 0$. We assume that the relative price of capital goods is one and that there are no adjustment costs to this investment. In what follows we will say that the firm is "young" if it has not yet exercised this growth option and "mature" if this option has already been exercised.

For this example, we assume that to finance this investment opportunity a firm needs to raise debt in the amount of I. Formally this requires us to make two simplifying assumptions. First, we need to assume that a young firm will distribute its entire earnings in every period. Second we also rule out new equity issues at the time of investment by assuming that the costs of doing so for a young firm are prohibitive.

While these are convenient assumptions for the purposes of our illustration, neither of them is really essential and they will both be relaxed in the more general model below. Our basic insights will survive as long as at least some of the investment is financed with debt. Given the tax benefits of debt this will always be the case.

Given our simplifying assumptions debt then will have a face value of I. We assume also that this debt takes the form of a consol bond that pays a

[1] As is well known this measure may or may not be unique depending on whether financial markets are assumed to be complete or not. At this stage however we only require the existence of one such measure.

fixed coupon c. Young firms have no debt outstanding, so that c represents the total flow of interest commitments per period for a mature firm.

2.2.1 The Problem for Mature Firms

Given our assumptions it follows that the instantaneous dividends for the equity holders of a mature firm are equal to

$$(1-\tau)\left(X_t K_1^{\alpha} - c\right).$$

Given debt, I, and its associated coupon payment, c, the value of a mature firm, $V_1(X;c)$, satisfies the following Bellman equation

$$V_1(X;c) = (1-\tau)\left(X_t K_1^{\alpha} - c\right)dt + (1+rdt)^{-1}\,E[V_1(X+dX;c)] \quad (1)$$

Here our choice of notation, $V_1(X;c)$, emphasizes the dependence of equity value on the firm's leverage.

Equation (1) holds only as long as the firm meets its obligations to the debt holders. However it is reasonable to assume that equity-holders will choose to close the firm and default on their debt repayments if the prospects for the firm are sufficiently bad. If equity holders have no outside options this (optimal) default occurs whenever $V_1(X;c)$ reaches zero. Alternatively, default occurs as soon as the value of X reaches some (endogenous) default threshold X_D. The optimal default threshold is determined by imposing the usual value matching and smooth pasting condition, requiring that at X_D the derivative of the equity value function be zero.

$$V_1(X_D;c) = 0 \quad (2)$$

$$V_1'(X_D;c) = 0 \quad (3)$$

2.2.2 The Problem for Young Firms

Young firms have no leverage, but they have an option to expand their productive capacity and become mature firms. For young firms the flow of operating profits (and dividends) per unit of time is then given by the expression

$$(1-\tau)XK_0^{\alpha}.$$

This yields the following Bellman equation for equity value, $V_0(X)$:

$$V_0(X) = \max\left\{V_1(X;c), (1-\tau)XK_0^{\alpha}dt + (1+rdt)^{-1}E[V_0(X+dX)]\right\} \quad (4)$$

The maximum in equation (4) now reflects the existence of an investment opportunity for the young firm. If demand grows sufficiently, so that X is above an investment threshold X_I, the firm will choose to expand its productive capacity to K_1. At this investment threshold firm value must obey the usual boundary conditions:

$$V_1(X_I;c) + B(X_I;c) - I = V_0(X_I) \quad (5)$$

$$V_1'(X_I;c) + B'(X_I;c) = V_0'(X_I) \quad (6)$$

where $B(X_I;c)$ denotes the value of debt issues at the time of investment. Given our assumption that all investment is financed through debt issuance $B(X_I;c) = I$ and the value matching condition (5) collapses to

$$V_1(X_I;c) = V_0(X_I).$$

2.2.3 Debt Value and Coupon Payments

Before computing the value of each firm explicitly it is helpful to construct the market value of the debt outstanding and the instantaneous coupon payments, since both of these values are linked to the firm's decision to investment.

The possibility of default will naturally induce a deviation between the market, B, and the book value of debt, I, at any point in time. As in Leland (1994), as long as the firm does not default this market value satisfies the Bellman equation

$$B(X;c) = cdt + (1 + rdt)^{-1} E[B(X + dX;c)]$$

Bankruptcy costs are assume large enough so that the firm is liquidated and no value is left for bondholders. Again this is extreme but not really important. Formally this implies that, at default, $B(X_D;c) = 0$. Given this boundary condition at default we can easily construct the expression for the market value of debt, $B(X;c)$. This is given by

$$B(X;c) = \frac{c}{r}\left(1 - \left(\frac{X}{X_D}\right)^{v_1}\right) \tag{7}$$

where $v_1 < 0$ so that the market value converges to c/r as X approaches infinity.

To determine the value of the periodic coupon payment, c, we use the fact that the initial debt issue must be enough to finance investment, so that $B(X_I;c) = I$. Replacing in the expression for the market value of debt, (7), we obtain

$$c = \frac{r}{1 - \left(\frac{X_I}{X_D}\right)^{v_1}} I \tag{8}$$

Hence the value of the coupon payment depends both on the face value of debt as well as default and investment thresholds X_D and X_I, respectively. The impact of the former is fairly standard and is due to the fact that the possibility of future default raises the required coupon payments. Holding the face value of debt, I, fixed, the effect of the investment threshold is related to its impact of the probability of future default. The larger the threshold the less likely the firm is to default. As we will see below this is something that a young firm will take into account when making investment decisions.

2.3 Valuation

We are now ready to compute the value of equity for both young and mature firms. To compute the value of a mature firm, given a pre-determined coupon payment, c, we use Ito's Lemma in equation (1) and impose default when $X = X_D$ to solve the associated second order differential equation.

This procedure implies that the value of a mature firm satisfies the expression

$$V_1(X;c) = \frac{(1-\tau)XK_1^{\alpha}}{r-\mu} - \frac{(1-\tau)c}{r} + A_1 X^{v_1} \tag{9}$$

where $v_1 < 0$, and the value for the constant $A_1 > 0$ can be obtained using the relevant boundary conditions at the default threshold.[2]

The first term in equation (9) is the present value of the future cash flows generated by existing assets, K_1. From this value we must then deduct the present value of all future debt obligations, which is captured by the term $\frac{(1-\tau)c}{r}$. Finally, the last term shows the impact of default on the value of the firm to its shareholders.

In the case of a young firm we apply Ito's Lemma to the Bellman equation (4) and solve the associated differential equation to obtain the expression

$$V_0(X) = \frac{(1-\tau)XK_0^{\alpha}}{r-\mu} + A_0 X^{v_0} \tag{10}$$

where $v_0 > 1$, and $A_0 > 0$ is determined by imposing the boundary conditions at X_I.

The first term in equation (10) for the equity value of young firms, is the present value of the future cash flows generated by existing assets and is

[2] In this case we obtain that

$$A_1 = -\left(\frac{(1-\tau)X_D K_1^{\alpha}}{r-\mu} - \frac{(1-\tau)c}{r}\right)\left(\frac{1}{X_D^{v_1}}\right).$$

essentially the same as that in the equation for the value of mature firms in (9).

More importantly equation (10) shows that the value of young firms, V_0, differs from that of mature firms, V_1, in a number of ways. First, the equity value of young firms will depend on the (positive) value of future growth options, here captured by the term $A_0 X^{v_0}$. In this simple example this piece is entirely missing from the expression for the value of mature firms. While this is clearly too extreme, it is nevertheless plausible to expect that the value of growth options to be relatively more important for young firms.

Second, mature firms are larger ($K_1 > K_0$) and precisely for that reason they are also more levered. These additional effects of debt are captured in the last two terms in equation (9).

2.4 Leverage and Risk

Expected returns can be recovered by looking directly at the equity betas implied by the the valuation expressions (9) and (10).[3] In our simple example these conditional betas can be computed in closed form by examining the elasticities of the value functions with respect to X_t.

We will express conditional equity betas β_{it}, for any firm, young ($i = 0$) or old ($i = 1$), in a general form as

$$\beta_{it} = 1 + \frac{(1-\tau)c}{rV_{it}} + \frac{V_{it}^D}{V_{it}}(v_1 - 1) + \frac{V_{it}^G}{V_{it}}(v_0 - 1), \quad i = 0, 1 \tag{11}$$

Here we use $V_{it}^G = A_0 X^{v_0}$ to denote the value of the young firm's growth options and $V_{it}^D = A_1 X^{v_1}$ is the value of the default option for the mature

[3]The corresponding conditional one-factor asset pricing model is derived as follows. Assuming a constant factor risk premium λ, the conditional expected return on equity is obtained as

$$E_t[R_{it+1}] = r + \beta_{it}\sigma\lambda$$

where $\beta_{it} = \frac{d \log V_{it}}{d \log X_t}$

firm.

The first term in this expression is common to both young and old firms and is simply the firm's revenue beta, which captures the (unlevered) riskiness of assets in place. Since operating profits are linear in the aggregate state of demand, this term is here effectively normalized to 1.

The next two components of equity risk are directly tied to leverage and, in our simple example are only relevant for mature firms. Together they capture the traditional effects of leverage on returns so often emphasized in the static literature. The second term, $\frac{(1-\tau)c}{rV_{it}}$, shows the effects of levering up equity cash flows on expected returns, even in the absence of any default risk.[4] The third term on the other hand reflects the impact of default on equity risk. Together these two terms imply the usual positive relation between leverage and expected equity returns that is described in most finance textbooks.[5]

The novelty however is the last term in equation (11). This term reflects the effect of growth options and depends on the relative importance of these options to the equity value of the firm. In our simple example this term will add to the underlying risk of the young firm, since mature firms no longer have any growth options, since $v_0 > 1$.

Thus, our expression for equity betas, (11), illustrates the potential pitfalls of searching for simple mappings between leverage and equity risk. This equation implies that, all else constant, financial leverage clearly increases equity risk. However this simple rule only holds in a static world when leverage is already pre-determined.

In a richer dynamic setting leverage is itself endogenous and generally

[4]Note that $\frac{(1-\tau)c}{r}$ is simply the value of a riskless perpetuity.

[5]Here the endogenous nature of default limits the firms downside risk ($A_1 > 0$). This may change however if we allow for more sophisticated default mechanisms in which the firm may be liquidated sub-optimally due to covenant violations.

related to investment decisions of varying degrees of risk. And because leverage tends to be generally higher for mature, low growth, firms which are otherwise less risky, simple correlations between discount rates and leverage are unlikely to produce meaningful results (see Barclay, Morellec and Smith (2006) for example). More precisely, equation (11) suggests that accounting for the importance of growth (and default) options is crucial when examining this relation between leverage and returns.

2.5 Numerical Illustration

Our key insights can now be developed with a numerical example. Since our focus is no longer on obtaining closed form solutions we can also begin to relax some of our more restrictive assumptions about the environment. The most significant change is that we now allow the firm to finance investment with both debt and newly issued equity. Hence a firm is now simultaneously choosing optimal investment and financing policies at the investment threshold X_I. Mathematically this implies that the boundary condition $B(X_I; c) = I$ is no longer required.

A less important but nevertheless realistic change concerns the assumed recovery rate on assets. We now assume that debt holders will be able to recover a fraction, $\phi > 0$, of the asset value of the firm upon default. Formally we now impose the boundary condition on debt

$$B(X_D; c) = \phi \frac{(1-\tau) X K_1^{\alpha}}{r - \mu}.$$

Effectively this assumes that, after accounting for some transaction costs, debt holders will take over the firm and will be entitled to the entirety of its future cash flows.

2.5.1 Leverage

Figure 1 shows the betas for several hypothetical mature firms as a function of alternative levels of (book) leverage as measured by their periodic coupon payment – the dashed line.[6] Because these firms differ only in their leverage the curve is upward sloping, conforming with the static view that, if all else is constant, higher leverage will raise expected equity returns.

The figure also shows the betas of unlevered young firms – the solid line. However since young firms are not levered the beta is just a constant here. Because of the role of growth options however this beta will be relatively high particularly when compared with moderately levered firms. In this case it is quite possible that unlevered young firms will have higher expected returns than levered mature firms. These two lines then provide an effective graphical illustration of the basic intuition from equation (11) and the limitations of the usual textbook intuition.

2.5.2 Business Cycle Effects

Figure 2 provides additional insights into the role of leverage in determining equity risk. This figure plots equity betas for both (optimally) levered and unlevered firms as a function of the state of demand, X. By optimally levered we refer to the situation where firms issue debt to maximize firm value when investing. Therefore, investment is partly debt and partly equity financed.

As before the dashed line shows the beta for mature firms, while the solid line shows the beta for the young firms. Not surprisingly we see that expected returns rise with X for the young firms because this increases the relative importance of their growth options in total firm value. Also intuitive is the pattern for mature firms. Here risk increases as demand conditions,

[6] Here we hold the value of the state variable X fixed.

X, worsen since this makes it more likely that the firms will find itself in default.

Figure 2 also confirms our findings that expected returns will not in general be monotonic in leverage. Depending on demand conditions it is possible for unlevered firms to be either more or less risky (as measured by expected equity returns).

Another implication of this result is that it suggests that the relationship between leverage and returns is conditional in nature: In bad times the contribution of default and cash flow risk is greater, while in good times the investment channel dominates. Thus when default risk is rather small, the figures suggest that expected returns are decreasing at least in book leverage, a finding that seems consistent with the recent empirical literature.

Finally this cyclical pattern of equity risk across firms is also interesting because it shows how financial leverage can generate endogenously the kind of variation in equity returns that is often required replicate the value premium (See for example Carlson, Fisher and Gianmarino (2004), Cooper (2005), or Zhang (2005)). Unlike the existing literature however, our mechanism does not rely on exogenous technological assumptions but is instead linked to the capital structure of the firm.

3 The General Model

The simple example in the section 2 provides much of the intuition for our findings although at the cost of some loss of generality. The model is also too stylized to allow for a more serious quantitative investigation of its key predictions.

In this section we embed the key ideas from our example in a more general environment that allows for more complex investment and financing strategies. Specifically, we now let firms have access to multiple investment options, while also relaxing the assumption that investment and financing must be perfectly coordinated. Firms can now issue debt (and equity) at any point in time and in any amount, subject to the natural financing constraints.

In addition we now allow for additional cross sectional firm heterogeneity in the form of firm specific shocks to both current profitability and the value of growth options. Moreover, aggregate shocks to the state of demand now impact both firm profitability and the discount rates as we no longer conduct our analysis under risk-neutral valuation.

Although this more general environment contains several additional ingredients its basic features are very similar, and our notation is, when possible, identical to that in the section 2.

3.1 Firm Problem

3.1.1 Profits and Investment

As before we begin by considering the problem of a typical value maximizing firm in a perfectly competitive environment. Time is now discrete. The flow of after tax operating profits per unit of time for each firm i is described by the expression

$$\Pi_{it} = (1-\tau)(Z_{it} X_t K_{it}^{\alpha} - f), \quad 0 < \alpha < 1 \tag{12}$$

where Z_i captures a firm specific component of profits and the variables X_t and K_{it} denote, as before, the aggregate state of productivity and the book value of the firm's asset. We use $f \geq 0$ to denote a (per-period) fixed cost of production.

Both X and Z are assumed to be lognormal and obey the following laws of motion

$$\begin{aligned}\log(X_t) &= \rho_x \log(X_{t-1}) + \sigma_x \varepsilon_t \\ \log(Z_{it}) &= \rho_z \log(Z_{it-1}) + \sigma_z \eta_{it}\end{aligned}$$

and both η_i and ε are (standard) normal variables.[7] The assumption that Z_{it} is firm specific requires that

$$\begin{aligned}E\varepsilon_t \eta_{it} &= 0 \\ E\eta_{jt}\eta_{it} &= 0, \text{ for } i \neq j\end{aligned}$$

The firm is now allowed to scale operations by picking between any level of productive capacity in the set $[K_0, K_N]$. This can be accomplished through (irreversible) investment, I_{it}, which is linked to capacity by the standard capital accumulation equation

$$I_{it} = K_{it+1} - (1-\delta)K_{it} \geq 0 \tag{13}$$

where $\delta > 0$ denotes the depreciation rate of capital per unit of time.

3.1.2 Financing

Corporate investment as well as any distributions, can be financed with either the internal funds generated by operating profits or net new issues which can take the form of new debt (net of repayments) or new equity.

As before we assume that debt B can take the form of a consol bond that pays a periodic coupon c per unit of time. However each firm is now allowed to renegotiate the terms of any outstanding issue at any point in time. This is then equivalent to letting the firm refinance the entire value

[7] This is slightly incorrect. To ensure the existence of a solution to the firm's problem the shocks must be finite. We accomplish this by imposing a (very large) upper bound on the ε and η.

of outstanding liabilities in every period. Formally, letting B_{it} denote the book value of outstanding liabilities for firm i at the beginning of period t we define the value of net new issues as

$$B_{it+1} - (1 + c_{it})B_{it}.$$

where c_{it} is again the coupon payment on B_{it} which will in general depend on a number of firm and aggregate variables. Note that now both debt and coupon payments will exhibit potentially significant time variation.

The firm can also raise external finance by means of seasoned equity offerings. For added realism however, we now assume that these issues entail additional costs so that firms will never find it optimal to simultaneously pay dividends and issue equity. Following the existing literature we assume that these costs include both fixed and variable components, which we denote by λ_0 and λ_1, respectively.[8] Thus, letting E_{it} denote the net payout to equity holders, total issuance costs are given by the function:

$$\Lambda(E_{it}) = (\lambda_0 - \lambda_1 \times E_{it})\, \mathbb{I}_{\{E_{it}<0\}}$$

where the indicator function implies that these costs apply only in the region where the firm is raising new equity finance so that net payout, E_{it}, is negative.

Investment, equity payout, and financing decisions must meet the following identity between uses and sources of funds

$$E_{it} + I_{it} = \Pi_{it} + \tau\delta K_{it} + B_{it+1} - (1 + (1-\tau)c_{it})B_{it} \tag{14}$$

where again E_{it} denotes the equity payout. Note that the resource constraint (14) recognizes the tax shielding effects of both depreciated capital and interest expenditures. Distributions to shareholders are then given as equity

[8] See Gomes (2001) and Hennessy and Whited (2007).

payout net of issuance costs. That is, we have

$$D_{it} = E_{it} - \Lambda(E_{it})$$

3.1.3 Valuation

The equity value of the firm, V, is defined as the discounted sum of all future equity distributions. Here again we assume that equity-holders will choose to close the firm and default on their debt repayments if the prospects for the firm are sufficiently bad, i.e., whenever V reaches zero.

To discount future cash flows we directly parameterize the discount factor applied to future cash flows as a stochastic process given by the expression

$$\log M_{t,t+1} = \log \beta - \gamma \log(X_{t+1}/X_t)$$

with $\gamma > 0$. Although this pricing kernel is exogenous its basic properties seem plausible, most notably, the idea that the risk premium is directly related to aggregate growth in cash flows.[9]

The complexity of the problem is reflected in the dimensionality of the state space necessary to construct the equity value of the firm. This includes both aggregate and idiosyncratic components of demand, productive capacity, and total debt commitments, defined as

$$\hat{B}_{it} \equiv (1 + (1-\tau)c_{it})B_{it}$$

To save on notation we henceforth use the $S_{it} = \{K_{it}, \hat{B}_{it}, X_t, Z_{it}\}$ to summarize our state space.

We can now characterize the problem facing equity holders, taking coupon payments as given. These payments will be determined endogenously below. Shareholders jointly choose investment (the next period capital stock) and

[9] See Berk et all (1999) and Zhang (2005) for similar applications and in-depth explorations of this assumption.

financing (next period total debt commitments) strategies to maximize the equity value of each firm, which accordingly can then be computed as the solution to the following dynamic program

$$\begin{aligned} V(S_{it}) &= \max\{0, \max_{K_{it+1},\hat{B}_{it+1}} \{D(S_{it}) + E\left[M_{t,t+1}V(S_{it+1})\right]\}\} \\ \text{s.t. } K_{it+1} &\geq (1-\delta)K_{it} \end{aligned} \tag{15}$$

where the expectation in the left hand side is taken by integrating over the conditional distributions of X and Z, Note that the first maximum captures the possibility of default at the beginning of the current period, in which case the shareholders will get nothing.[10] Finally, aside from the budget constraint, the only significant constraint on this problem is the requirement that investment is irreversible.

3.1.4 Default and Bond Pricing

We now turn to the determination of the required coupon payments, taking into account the possibility of default by equity holders. Assuming debt is issued at par, the market value of new issues must satisfy the following condition

$$B_{it+1} = E\left[M_{t,t+1}((1+c_{it+1})B_{t+1}\mathbb{I}_{\{V_{it+1}>0\}} + R_{it+1}(1-\mathbb{I}_{\{V_{it+1}>0\}}))\right]$$

where R_{it+1} denotes the recovery on a bond in default and $\mathbb{I}_{\{V_{it+1}>0\}}$ is an indicator function that takes the value of 1 if the firm remains active and 0 when equity chooses to default.

Finally, we follow Hennessy and Whited (2007) and specify the deadweight losses at default to consist of a fixed and a proportional component.

[10] In practice, there can be violations of the absolute priority rule, implying that shareholders in default still recover value. Garlappi and Yan (2007) analyze the asset pricing implications of such violations.

Thus, creditors are assumed to recover a fraction of the firm's current assets and profits net of fixed liquidation costs. Formally the default payoff is equal to:

$$R_{it} = \Pi_{it} + \tau\delta K_{it} + \xi_1(1-\delta)K_{it} - \xi_0$$

Since the equity value V_{it+1} is endogenous and itself a function of the firms' debt commitments this equation cannot be solved explicitly to determine the value of the coupon payments, c_{it}. However, using the definition of $\hat{B}$, we can rewrite the bond pricing equation as

$$\begin{aligned} B_{it+1} &= \frac{E\left[M_{t,t+1}(\frac{1}{1-\tau}\hat{B}_{it+1}\mathbb{I}_{\{V_{it+1}>0\}} + R_{it+1}(1-\mathbb{I}_{\{V_{it+1}>0\}}))\right]}{1+\frac{\tau}{1-\tau}(E\left[M_{t+1}\mathbb{I}_{\{V_{it+1}>0\}}\right])} \\ &= \mathbf{B}(K_{it+1}, \hat{B}_{it+1}, X_t, Z_{it}) \end{aligned}$$

Given this expression and the definition of $\hat{B}$ we can easily deduce the implied coupon payment as

$$c_{it+1} = \frac{1}{1-\tau}(\frac{\hat{B}_{it+1}}{B_{it+1}} - 1)$$

Defining $\hat{B}$ as a state variable and constructing the bond pricing schedule $\mathbf{B}(\cdot)$ offers an important computational advantage. Because equity and debt values are mutually dependent (since the default condition affects the bond pricing equation) we would normally need jointly solve for both the interest rate schedule (or bond prices) and equity values. Instead our approach requires only a simple function evaluation during the value function iteration. This automatically nests the debt market equilibrium in the calculation of equity values and greatly reduces computational complexity.

3.2 Optimal Firm Behavior

Given our assumptions, the dynamic programming problem (15) has a unique solution, that can be characterized efficiently by the optimal distribution, financing, and investment, policies.[11] We now investigate some of the properties of optimal strategies implied by the solution to the firm's problem. Since this cannot be solved in closed form we must resort to numerical methods, which are detailed in Appendix A.

3.2.1 Investment and Financing

Figure 3 illustrates the optimal financing and investment policies of the firm as well as their implications for equity values conditional on the aggregate level of demand. The dashed line corresponds to a high realization for the aggregate state of demand (an economic boom), the dotted line corresponds to the long-run mean of aggregate demand, while the solid line shows the results when demand is relatively weak (a recession). In all cases se set the idiosyncratic profitability shock, Z_{it}, to its mean.

The top panels, labeled "new capital stock" depicts the optimal choice of next period capital, K_{it+1}, as a function of the underlying variables. These panels neatly illustrate the interaction of financing and investment decisions, particularly for small firms. With unlimited access to external funds, the optimal choice of capacity would be independent of this period capital stock, at least for low values of K as the irreversibility constraint only binds on disinvestment. Here however this is not the case. This is because an increase in existing K_{it} generates both higher internal cash flows and more collateral, thus alleviating the effect of financing constraints. As the picture shows this effect is particularly important for small firms. The plot also illustrates the constraints imposed by irreversibility: Independent of the aggregate level of demand, disinvestment is infeasible for high levels of the capital stock.

Equally interesting is the fact that the optimal capacity choice is declining

[11] It is straightforward but cumbersome to show this formally. The interest reader is referred to Gomes (2001) and Hennessy and Whited (2006) for similar proofs.

in current liabilities, B_{it}. Although reminiscent of the popular debt overhang effect this finding result is worthy of note since we explicitly allows firms to renegotiate the terms of their debt in every period.

The "new debt" panels show the optimal choice for new debt outstanding, B_{it+1}. A notable feature is the strong positive relation between current and lagged leverage, a phenomenon sometimes dubbed as 'hysteresis', and that suggests that our model is also consistent with the well documented finding that financial leverage is extremely persistent. Note that this result arises even in the absence of any of the usual suspects such as: market timing or costly debt issues. Here, persistence in leverage is due almost exclusively to the nature of investment decisions of the firm since as we have seen above investment and financing are closely linked.

For completeness Figure 3 also shows the behavior or the equity value of the firm. Not surprisingly these values are increasing in current assets and profitability and declining in the amount of outstanding debt.

3.2.2 Risk and Returns

Figure 4 investigates the implications of these firm decisions on various measures of risk and returns. As before the dashed line corresponds to a high realization for the aggregate state of demand (an economic boom) while the solid line shows the results when demand is relatively weak (a recession).

The top four panels show the effects on default probabilities and credit spreads, measured as the difference between the yield on the debt outstanding and the risk-free rate. Both credit spreads and default probabilities are countercyclical, in the sense that they are declining in the state of aggregate demand (X_t).

More interestingly we note that the model can give rise to a sizable credit spread. The intuition here is straightforward: What matters for credit spreads are not so much the actual default probabilities depicted but much more the risk-adjusted pricing kernel weighted default probabilities. Hence the time variation in both the pricing kernel and default probabilities ensures that risk adjusted default probabilities are much higher than the historical probabilities in recessions thus creating significant credit spreads.

From a cross-section point of view that credit risk rises substantially when the firm is very small and leverage is high, since this scenario leads to a dramatic increase in the probability of default.

Finally the bottom panels, labeled "Beta", show the induced variation in expected equity returns. The first panel shows that controlling for both current assets and profitability, leverage increases the systematic risk to equity holders. This is precisely the result identified in traditional static models and discussed in section 2.

As before however we also find that in our more general model equity risk declines fairly quickly in firm size and is significant smaller for larger firms. The intuition is precisely the same that we identified in section 2: with decreasing returns to scale, large firms also have fewer growth options which reduces their risk.[12]

Thus to the extent that leverage and investment policies are jointly determined, the link between expected returns and leverage is likely to be more subtle than what is traditionally suggested in the literature. In fact if decreasing returns are sufficiently strong it is actually possible that the relation between returns and debt could be fairly flat or even negative.

[12]Note that the presence of decreasing returns to scale effectively ties the value of growth options to current size since it ensures that the marginal value of new additions to productive capacity is always lower for large firms.

4 Cross-Sectional Implications: Theory and Evidence

In this section we investigate some of the empirical implications of our general model in section 3. We then compare our theoretical findings with our empirical work on leverage and equity returns and find evidence for the model implications in the data.

4.1 Basic Methodology and Definitions

To accomplish this we first construct an artificial cross-section of firms by simulating the investment and leverage rules implied by the model. The numerical procedure used is described in more detail in Appendix A.

We then construct theoretical counterparts to the empirical measures of returns, beta, book-to-market, and leverage in the widely used CRSP/Compustat dataset. Specifically, taking into account that in the model a firm's equity value is cum-dividend while in the data one typically uses the ex-dividend value, equity returns between t and $t+k$ are defined in a straightforward fashion by the identity

$$r_{t,t+k} = \frac{V_{t+k}}{V_t - D_t}$$

In our model the book value of assets is simply given K, while the book value of equity is $BE = K - B$. To facilitate comparisons with these studies we will henceforth use the notation $ME = V$ to denote the market value of equity. Book leverage is then measured by the ratio B/K, while book-to-market equity is defined as BE/ME.

To assess the model implications, we use data from CRSP/Compustat dataset and perform the same exercise as in the simulated data on real data. Specifically, we use panel data from CRSP/Compustat Merged database between 1963 and 2006 to perform the empirical tests. We construct our empirical measures of returns, book-to-market, and leverage following carefully the procedures given in Fama and French (1992, 1993). A more detailed description of our dataset and our empirical procedure can be found in appendix C.

4.2 Leverage and Returns: Unconditional Moments

Table 3 is constructed by creating five value-weighted portfolios that are ranked by either book or market leverage. These portfolios are then held for 12 months following its formation and their returns are computed. The table reports the average monthly return associated with this buy-and-hold strategy, both in actual and simulated data.

The rows labeled "book-leverage" show the results of constructing portfolios that are sorted according to the book leverage of the firm, while the portfolios labeled "market-leverage" show the results of sorting on market leverage.

Although the magnitude of our numbers is a little high we see that in both cases our model conforms with the broad pattern in the data. Specifically, we find that equity returns seem positively related to market leverage, but essentially flat on book leverage. Moreover the quantitative spread in returns induced by sorting on market leverage is also very similar to that obtained in the data. The result that book leverage is essentially unrelated to cross-sectional dispersion in returns is consistent with the inconclusive results in the empirical literature. On the other hand, market leverage, containing market capitalization in the denominator, is mechanically positively related to returns.

4.3 Size and Book-to-Market

The evidence in Table 3 is useful, but it offers little more than a crude test of the model. More interesting is to look at the role of our leverage measures when interacted with other variables. A natural benchmark is to focus on the usual suspects of firm size and the book-to-market ratio.

Table 4 looks at the relationship between market leverage and returns controlling first for either size (panels on the left) or book to market (panels on the right). The bottom row in all of these panels (labeled "All") shows the average pattern of returns across the various portfolios and is a good way of thinking about the *conditional* relation between leverage and returns.

Thus comparing this row with the *unconditional* result obtained in the one-way sorts in Table 3 provides an effective summary of the role of either size or book to market in capturing the effects of leverage on returns.

In general we find that once again our model performs very well. Looking first at the left panels of Table 4 we see that leverage and returns retain a clear positive relation even after controlling for firm size. This is true both in the model and in the data. It is also true both on average and across all of the size portfolios.

However, the two book-to-market panels on the right suggest a different story. Controlling for book to market yields only a very mild link between leverage and returns. While this is more pronounced in the model, it is also significantly smaller than the unconditional link in Table 3.

We find these results informative in a number of important ways. First, they seem to connect back in the context of a dynamic model of leverage and returns to the common intuition, put forward first by Fama and French (1992), that book-to-market is related to a financial distress factor, as it captures much of the impact of leverage in returns.

Secondly both sets of results confirm our model's intuition that the book to market ratio is not a very useful measure of growth options. Although not ideal, market size is a much more useful proxy for these options. In particular, consistent with the intuition developed in our simple example, both in the model and in the data the link between leverage and returns remains apparent even after controlling for firm size.

For completeness we also include Table 5 which shows the relation between book leverage and returns controlling for either size or book-to-market. This Table confirms our earlier view that book leverage is much less informative about expected returns even after we control for size and book-to-market. Both in the data and in our model there is, at best, a very small positive link between this measure of leverage and equity returns.

5 Conclusion

In this paper we revisit the theoretical relation between financial leverage and stock returns in a dynamic world where both the corporate investment and finance decisions are endogenous. We find that in general the link between leverage and stock returns is more complex than the static textbook examples suggest and will generally depend on the investment opportunities available to the firm. In the presence of financial market imperfections leverage and investment are generally correlated so that highly levered firms are also mature firms with relatively more (safe) book assets and fewer (risky) growth opportunities. We first develop the underlying intuition qualitatively in a simple real options model, which delivers closed form expressions for firms' equity betas as functions of firm characteristics. We then construct a quantitative model incorporating the same economic mechanisms to analyze the empirical implications of our framework and test them on actual data.

Our results help interpreting recent puzzling empirical evidence concerning leverage and returns and provide new insights in the economic determinants of size and book-to-market factors in equity returns. In particular, we show that the quantitative version of our model can successfully replicate the empirical relationships between leverage and returns, even after one controls for variables such as size and book to market.

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A Appendix: Computational Details

We use a standard value function iteration algorithm on a discretized state space to solve the model. The major advantage of this approach, in spite of being relatively time-consuming, is its robustness and precision.

To that end we discretize all variables in the model to lie on finite grids. The capital stock K is constrained to lie on a equally-spaced grid with n_k elements. Similarly, the face value of debt B lies on a grid with n_b elements. The lower and upper bounds of the grids are chosen to ensure that they never bind.

The state variables X and Z are defined on continuous state spaces and need to be transformed into discrete state spaces as well. We use the Tauchen procedure to transform the autoregressive processes into finite Markov Chains. Specifically, we use n_x points for the aggregate shock x and n_z points for the idiosyncratic shocks Z. Due to the high persistence of the shocks in the monthly calibration, we need a relatively large number of points here to achieve a satisfactory precision.

On this state space with $n_k \times n_b \times n_x \times n_z$ elements we guess an initial value function at every point. We then iterate until convergence on the Bellman equation to find the value function and the optimal investment and financing policies. To do so, we restrict the control variables K' and B' to lie on equally-spaced grids. Since the value function is defined on a smaller grid, we use linear interpolation extensively to find values on non-grid points.

B Appendix: Parameter Choices

Our choice of parameter values, summarized in Table 1, follows closely the existing literature (e.g. Gomes (2001), Cooley and Quadrini (2001), Hennessy and Whited (2005), Zhang (2005)). These values are picked so that the model matches key unconditional moments of investment, returns, and cash flows both in the cross-section and at the aggregate level.

The persistence, ρ_x, and conditional volatility, σ_x, of aggregate productivity, are set equal to 0.983 and 0.0023 which is close to the corresponding values reported in Cooley and Prescott (1995). For the persistence, ρ_z, and conditional volatility, σ_z, of firm-specific productivity, we choose values close to the corresponding ones constructed by Gomes (2001) to match the cross-sectional properties of firm investment and valuation ratios.

The depreciation rate of capital, δ, is set equal to 0.01 which provides a good approximation to the average monthly rate of investment found in both macro and firm level studies. For the degree of decreasing returns to scale we use 0.65. Although probably low this number is almost identical to the estimates in Cooper and Ejarque (2003) as well as several other recent micro studies.

We set ξ_1 which is one minus the proportional cost of bankruptcy equal to 0.75, which is in line with recent empirical estimates in Hennessy and Whited (2006) as well as consistent with values traditionally used in the macroeconomics. Additionally, under the assumption that close to default the asset value of the unlevered firm is close to its book value, the number is consistent with the traditional estimates of the direct costs of bankruptcy obtained in the empirical corporate finance literature. We then choose ξ_0, the fixed cost of bankruptcy, such that we match average market-to-book values in the economy.

The costs of equity issuance λ_0 and λ_1 are chosen similarly as in Gomes (2001). Later empirical studies (Hennessy and Whited, 2004) have confirmed that these values are good estimates.

Following Zhang (2005) we choose the pure time discount factor β and the pricing kernel parameter γ such that the model approximately matches two key moments of asset markets, namely the mean risk free rate and the equity premium. This pins down β at 0.995 and we set γ equal to 15. We note that this parameterization pins down aggregate risk characteristics, whereas our emphasis is on cross-sectional risk characteristics.

To assess the fit of our calibration, we report in table 2 the implied moments generated by our parameterization for some key statistics. Our calibration seems to perform rather well along some dimensions crucial for the model. The simulated data match some key statistics related to asset market data and firms' investment and financing decisions reasonably well.

C Appendix: Data Description

Our empirical results are based on the merged CRSP and Compustat database, specifically on the Industrial Annual Data from CRSP/Compustat Merged data base). Our dataset goes from 1963 to 2006.

To construct our measures of book-to-market, size, book and market leverage we proceed following Fama and French (1992, 1993) as follows. Total assets is item 6, book value of common equity is defined as the Compustat book value of stockholders' equity, plus balance-sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock, which is estimated using redemption, liquidation or par value (items 216, 35, 56, 10, 130). Size is price times shares outstanding. Book-to-market is then book value divided by size, market leverage is (total asset minus book value) divided by (total asset minus book value plus market value) and book leverage is (total asset minus book value) divided by total assets.

Portfolios are formed on June 30th every year (t) and run through June

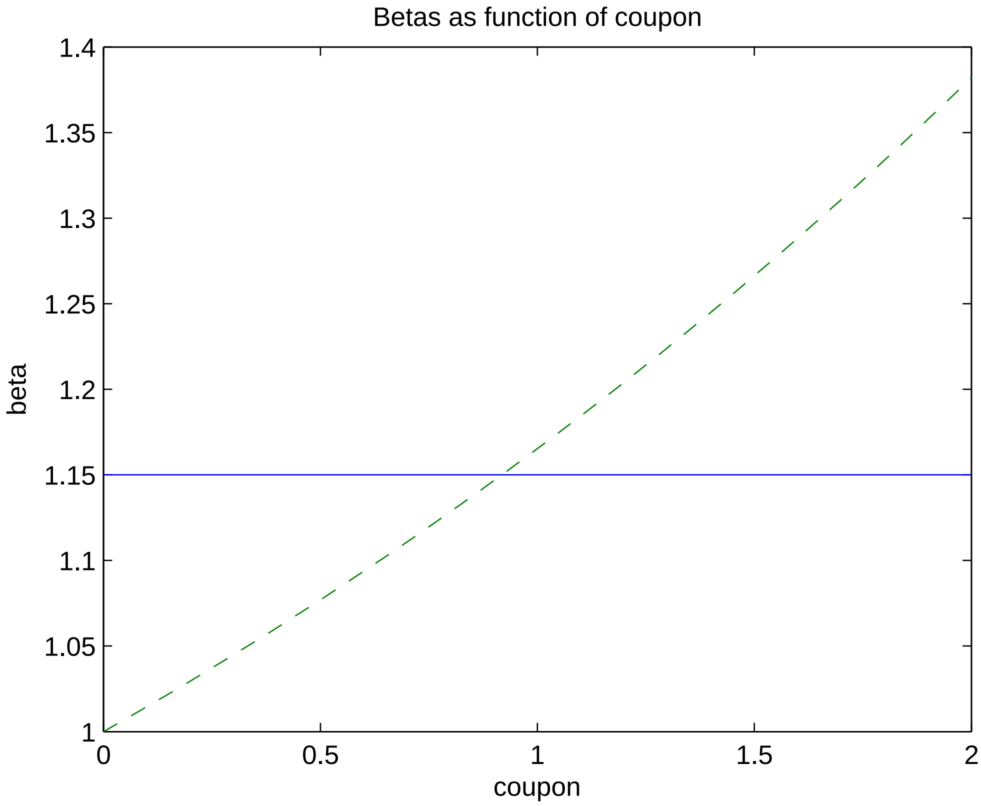
30th of the next year (t+1) based on Compustat and CRSP data for each firm as of December of the previous year ($t - 1$). Size bins are created by sorting on NYSE stocks only and then using the break points for all NYSE, Amex and NASDAQ stocks. All other bins are created equal sized. We drop all observations with negative book values. To correct for survival bias we only include stocks which are in Compustat for more than two years and restrict our sample to common stocks. For portfolio formation only firms with asset, book and size as of December of $t - 1$ are included in portfolios. We use monthly value weighted excess returns (over 30 day T-bill) that are averaged over all months and years. We included a bias correction for delisted firms suggested by Shumway (1997) and Shumway and Warther (1999).

Table 1: **: Parameter Values**

Parameter	**Values**
α	0.65
β	0.995
δ	0.01
γ	15
τ	0.2
λ_0	0.01
λ_1	0.025
ξ_0	0.1
ξ_1	0.75
f	0.01
ρ_x	0.983
σ_x	0.0023
ρ_z	0.92
σ_z	0.15

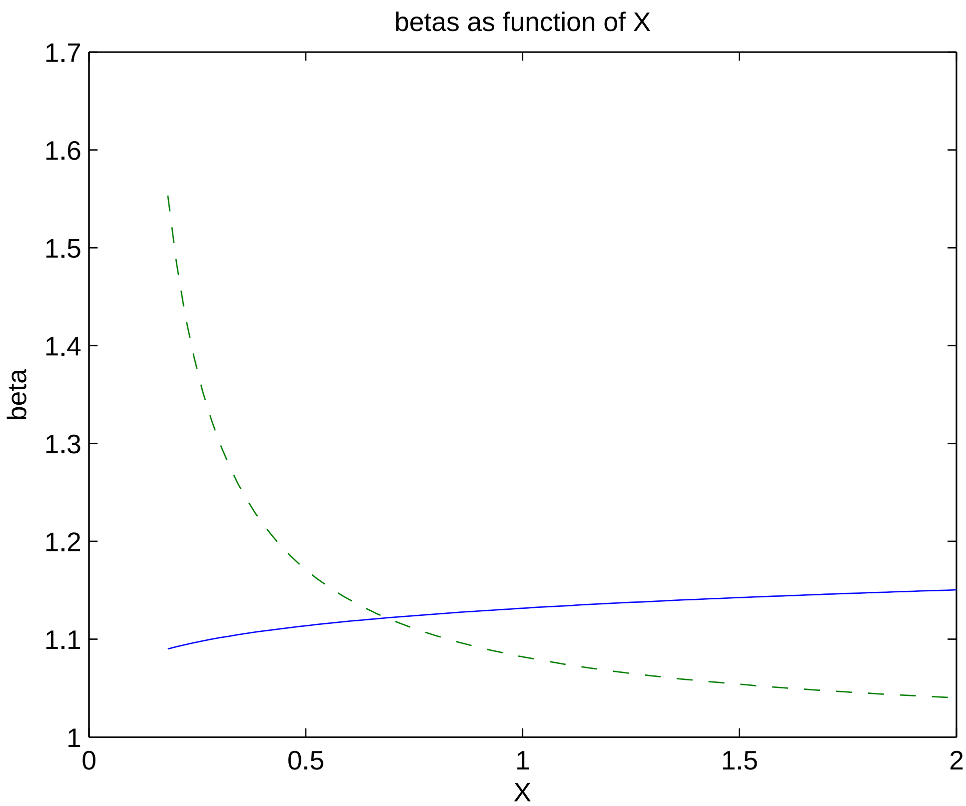
This table reports parameter choices for our general model. The model is calibrated to match annual data both at the macro level and in the cross-section. The persistence, ρ_x, and conditional volatility, σ_x, of aggregate productivity, are set close to the corresponding values reported in Cooley and Prescott (1995). The persistence, ρ_z, and conditional volatility, σ_z, of firm-specific productivity, are close to the corresponding ones constructed by Gomes (2001) to match the cross-sectional properties of firm investment and valuation ratios. The parameter δ is equal to the depreciation rate of capital and is set to approximate the average monthly investment rate. For the degree of decreasing returns to scale we use 0.65 which is the value in Cooper and Ejarque (2003). Finally the pricing kernel parameter γ is chosen as in Zhang (2005) to match average asset market data.

Figure 1: **: Beta and Leverage**



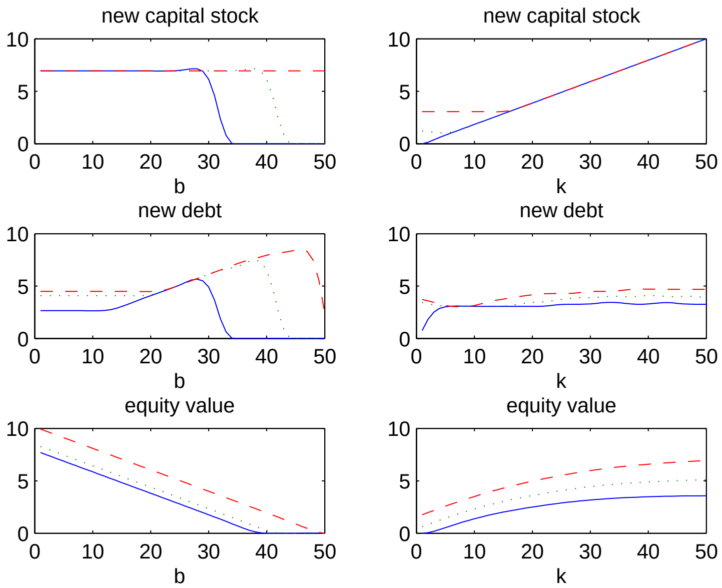
This figure presents betas for young and mature firms as a function of an exogenously chosen coupon c. The beta for young firms is represented by the solid line, while the beta for mature firms is represented by the dashed line. Parameter values in the example are $r = 0.05, \mu = 0.03, \tau = 0.35, \sigma = 0.2$, recovery rate on debt $\phi = 0.9, I = 10, \alpha = 0.3, K_0 = 1, K_1 = 11$. The value of the shock X is chosen such that it is below the investment trigger for the young firm for every choice of the coupon.

Figure 2: **: Beta and Business Cycles**



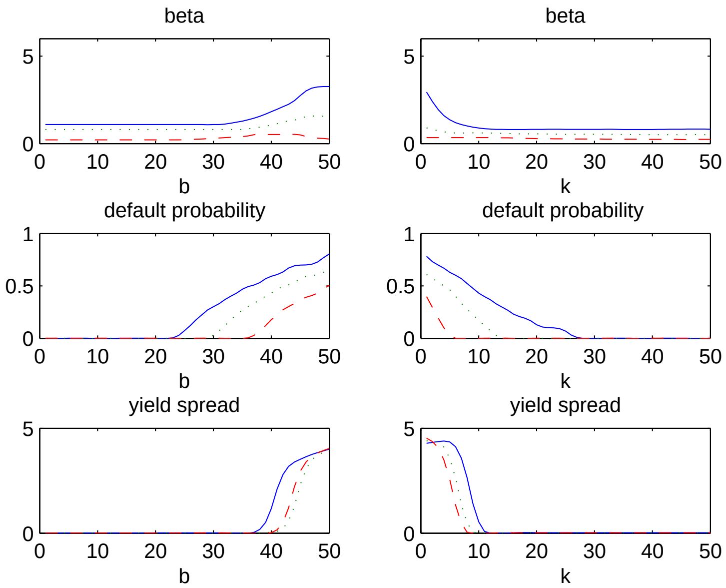
This figure presents betas for young and mature firms as a function of the shock X for an optimally chosen coupon. The beta for young firms is represented by the solid line, while the beta for mature firms is represented by the dashed line. Parameter values in the example are $r = 0.05, \mu = 0.03, \tau = 0.35, \sigma = 0.2$, recovery rate on debt $\phi = 0.9, I = 10, \alpha = 0.3, K_0 = 1, K_1 = 11$. This gives an investment trigger $x_I = 1.55$ for the young firm. The dashed line represents the mature firm.

Figure 3: **: Optimal Policies**



This figure summarizes the optimal investment and financing policies as a function of existing debt (B) and firm size (K). The bottom pictures show the resulting value of the firm to equity holders. The dashed line refers to a realization of the aggregate shock, X, that is one standard deviation above its mean, the dotted line holds the aggregate shock fixed at its long-run mean, while the solid line refers to a realization of the aggregate shock that is one standard deviation below its mean.

Figure 4: **: Returns**



This figure shows the spread on corporate bonds, implied default probabilities and the equity betas implied by the corporate strategies of the firm for each possible level of current assets (K) and debt B). The dashed line refers to a realization of the aggregate shock, X, that is one standard deviation above its mean, the dotted line holds the aggregate shock fixed at its long-run mean, while the solid line refers to a realization of the aggregate shock that is one standard deviation below its mean.

Table 2: **: Sample Moments**

Variable	Data	Model
Annual risk-free rate	0.018	0.025
Annual volatility of risk-free rate	0.030	0.019
Annual Equity Premium	6.00	7.81
Investment-to-asset ratio	0.14	0.17
Market leverage	0.29	0.35
Frequency of Equity Issuance	0.09	0.15
Default rate	0.02	0.02

This table reports unconditional sample moments generated from the simulated data of some key variables of the model. Data moments on asset returns come from Campbell, Lo, and McKinlay (1997). The data moments on the investment-to-asset ratio and the market-to-book ratio are taken from Gomes (2001) and Zhang (2005) respectively. Leverage and aggregate default rate are taken from Covas and Den Haan (2006). All data are annualized

Table 3: **: Univariate Portfolio Sorts**

Variable	**Mean Monthly Returns**				
	Actual Data				
	Low	2	3	4	High
Book Leverage	0.48	0.49	0.52	0.52	0.50
Market Leverage	0.38	0.46	0.54	0.60	0.73
	Simulated Data				
	Low	2	3	4	High
Book Leverage	0.62	0.69	0.63	0.67	0.66
Market Leverage	0.57	0.68	0.72	0.77	0.82

This table reports average monthly realized returns of portfolios sorted first by either book leverage (top row) or by market leverage (bottom row). The top panel reports the empirical results for the CRSP/Compustat data set using the procedure and data definitions in Fama and French (1992). The bottom panel reports the results for our artificial dataset generated by simulating the model with 2000 firms over 1500 periods and dropping the first 1000 periods. This procedure is repeated 50 times and the average results reported in the Table. Book leverage is defined as the ratio between book debt and the book value of equity plus book debt. Market leverage is the ratio between book debt and the market value of equity plus book debt.

Table 4: **: Market Leverage Sorts**

Mean Monthly Returns															
Actual Data															
		Market leverage								Market leverage					
		Low	2	3	4	High	All			Low	2	3	4	High	All
	Small	0.39	0.88	0.93	1.03	1.12	0.86		Low	0.32	0.26	0.31	0.38	0.29	0.31
	2	0.54	0.67	0.95	0.83	0.98	0.80		2	0.66	0.50	0.43	0.59	0.51	0.52
	3	0.54	0.51	0.70	0.74	0.83	0.67	Book to	3	0.57	0.68	0.64	0.56	0.75	0.64
Size	4	0.62	0.58	0.59	0.62	0.76	0.64	Market	4	0.80	0.63	0.73	0.78	0.76	0.72
	Large	0.41	0.30	0.46	0.51	0.51	0.41		High	1.05	0.81	0.86	1.14	1.19	0.89
	All	**0.41**	**0.35**	**0.52**	**0.57**	**0.63**			**All**	**0.46**	**0.47**	**0.45**	**0.52**	**0.51**	
Simulated Data															
		Market leverage								Market leverage					
		Low	2	3	4	High	All			Low	2	3	4	High	All
	Small	0.71	0.85	0.87	0.91	0.96	0.81		Low	0.48	0.49	0.51	0.55	0.57	0.53
	2	0.63	0.77	0.82	0.84	0.92	0.74		2	0.56	0.57	0.62	0.69	0.70	0.60
	3	0.56	0.68	0.72	0.79	0.81	0.63	Book to	3	0.62	0.66	0.71	0.73	0.75	0.77
Size	4	0.52	0.60	0.64	0.74	0.72	0.61	Market	4	0.71	0.73	0.77	0.82	0.85	0.82
	Large	0.47	0.57	0.63	0.69	0.70	0.57		High	0.81	0.83	0.82	0.89	0.91	0.84
	All	**0.55**	**0.62**	**0.67**	**0.72**	**0.74**			**All**	**0.65**	**0.64**	**0.67**	**0.72**	**0.71**	

This table reports average monthly realized returns of portfolios sorted first by size and then market leverage (left panels) or first by book-to-market and then market leverage (right panels). The top panels report the empirical results for the CRSP/Compustat data set using the procedure and data definitions in Fama and French (1992). The bottom panels report the results for our artificial dataset generated by simulating the model with 2000 firms over 1500 periods and dropping the first 1000 periods. This procedure is repeated 50 times and the average results reported in the Table. Market leverage is defined as the ratio between book debt and the market value of equity plus book debt.

Table 5: **: Book Leverage Sorts**

Mean Monthly Returns															
Actual Data															
		Book leverage								Book leverage					
		Low	2	3	4	High	All			Low	2	3	4	High	All
	Small	0.68	0.96	0.90	0.82	0.89	0.86		Low	0.45	0.31	0.37	0.34	0.21	0.31
	2	0.78	0.78	0.89	0.73	0.80	0.80		2	0.72	0.54	0.35	0.56	0.56	0.52
	3	0.67	0.61	0.78	0.65	0.64	0.67	Book to	3	0.59	0.68	0.67	0.54	0.71	0.64
Size	4	0.60	0.67	0.73	0.56	0.62	0.64	Market	4	0.90	0.59	0.70	0.78	0.80	0.72
	Large	0.39	0.42	0.48	0.41	0.39	0.41		High	1.16	0.82	0.85	0.91	1.28	0.89
	All	**0.42**	**0.46**	**0.56**	**0.48**	**0.47**			**All**	**0.52**	**0.49**	**0.45**	**0.47**	**0.52**	
Simulated Data															
		Book leverage								Book leverage					
		Low	2	3	4	High	All			Low	2	3	4	High	All
	Small	0.76	0.78	0.75	0.74	0.81	0.78		Low	0.52	0.53	0.50	0.51	0.50	0.51
	2	0.64	0.68	0.73	0.72	0.72	0.70		2	0.64	0.66	0.66	0.67	0.65	0.66
	3	0.63	0.67	0.70	0.71	0.70	0.68	Book to	3	0.71	0.73	0.76	0.74	0.73	0.73
Size	4	0.58	0.61	0.60	0.64	0.65	0.62	Market	4	0.79	0.81	0.80	0.81	0.77	0.79
	Large	0.57	0.56	0.59	0.60	0.58	0.58		High	0.88	0.90	0.92	0.87	0.86	0.89
	All	**0.62**	**0.64**	**0.67**	**0.65**	**0.67**			**All**	**0.68**	**0.69**	**0.71**	**0.72**	**0.70**	

This table reports average monthly realized returns of portfolios sorted first by size and then book leverage (left panels) or first by book-to-market and then book leverage (right panels). The top panels report the empirical results for the CRSP/Compustat data set using the procedure and data definitions in Fama and French (1992). The bottom panels report the results for our artificial dataset generated by simulating the model with 2000 firms over 1500 periods and dropping the first 1000 periods. This procedure is repeated 50 times and the average results reported in the Table. Book leverage is defined as the ratio between book debt and the book value of equity plus book debt.