PESS
7/26/06
10:00 AM

Preliminary Draft
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Notional Defined Contribution Pension Systems in a Stochastic Context: Design and Stability

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July 2006

This research was supported by the U.S. Social Security Administration through grant #10-P-98363-1-02 to the National Bureau of Economic Research as part of the SSA Retirement Research Consortium. The findings and conclusions expressed are solely those of the authors and do not represent the views of SSA, any agency of the Federal Government, or the NBER. The authors gratefully acknowledge the excellent research assistance of Erin Metcalf, the contributions of Carl Boe in the development of the stochastic forecasting model, and support from Berkeley's NIA-funded Center for the Economics and Demography of Aging. The research funded here builds on basic research funded by NIA grant R37-AG11761.

Abstract

Around the world, Pay-As-You-Go (PAYGO) public pension programs face serious long-term fiscal problems due primarily to actual and projected population aging, and most appear unsustainable as currently structured. Some have proposed the replacement of such plans with systems of fully funded private or personal Defined Contribution (DC) accounts, but the difficulties of transition to funded systems have limited their implementation. Recently, a new variety of public pension program known as "Notional Defined Contribution" or "Non-financial Defined Contribution" (NDC) has been created, with the objectives of addressing the fiscal instability of traditional plans and mimicking the characteristics of funded DC plans while retaining PAYGO finance.

Using different versions of the system recently adopted in Sweden, calibrated to US demographic and economic parameters, we evaluate the success of the NDC approach in achieving fiscal stability. (In a companion paper, we will consider other aspects of the performance of NDC plans in comparison to traditional PAYGO pensions.) We find that, despite its built-in self-correction mechanisms, the basic NDC scheme is still subject to fiscal instability: there is a high probability that the system's debt-payroll ratio will explode over time. With adjustments, however, the NDC approach can be made considerably more stable.

Keywords: Social Security, Retirement, Deficits

JEL nos. H55, J11

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Introduction

Around the world, Pay-As-You-Go (PAYGO) public pension programs are facing serious long-term fiscal problems due primarily to actual and projected population aging, and most appear unsustainable as currently structured. All strict PAYGO programs (i.e., those that do not incorporate sizable trust fund accumulations) can feasibly pay an implicit rate of return equal to the growth rate of GDP (labor force growth plus productivity growth) once they are mature and in steady state. This rate of return is typically lower than the rate of return that can be earned in the market, either through low-risk bonds or through investment in equities. The programs' long-term fiscal problems relate to a misalignment between these low but feasible rates of return and promised rates of return that may once have been feasible but no longer are so. The traditional plans are mostly defined benefit, and have been criticized for creating strong incentives for early retirement. More generally there is a concern that the taxes that finance these programs distort labor supply incentives throughout life. Many also believe that these plans undermine motivations to save, and, because they are themselves unfunded, thereby reduce overall capital accumulation and consequently lead to lower labor productivity and slower growth.

Recently, a new variety of public pension program known as "Notional Defined Contribution" or "Non-financial Defined Contribution" (NDC) has been created and implemented by Sweden, with first payments in 2001. A number of other countries have introduced or are planning to introduce NDC plans, including Italy, Poland, Latvia, Mongolia and the Kyrgyz Republic, and proposed new plans for France and Germany have NDC aspects (Legros, 2003; Holtzmann and Palmer, 2005).

NDC programs differ in detail, but the basic principle is that they mimic Defined Contribution plans without actually setting aside assets as such plans do. Under an NDC program, a notional capital account is maintained for each participant. Balances in this account earn a rate of return that is declared by the pension plan each year; and notional payments into this account are made over the entire life history to mirror actual taxes or contributions. Together with the declared rate of return these notional contributions determine the value of the account at any point in time. After a designated age such as 62, a participant can choose to begin to draw benefits, which is done by using the account to purchase an annuity from the pension plan. The terms of the annuity will depend on mortality at the time the generation turns 65 (for example) and on a rate of return stipulated by the pension plan, which might be the same rate of return used in the pre-retirement accumulation phase.

NDC plans are seen as having many potential advantages over traditional PAYGO systems, but our focus in this paper is on just one of these potential advantages, stability. A plan of this sort appears structured to achieve a considerable degree of fiscal stability because the promised rates of return reflect the program's underlying PAYGO nature, rather than being market-based. Further, in the event that it begins to go off the tracks, a braking mechanism can be incorporated which automatically modifies the rate of return, to help restore the plan to financial health. Given the political difficulties of making frequent changes in PAYGO pension programs, the attractiveness of an inherently stable system is clear.

In this paper, we use a stochastic macro model for forecasting and simulating Social Security finances to examine the behavior of NDC-type public pension programs in the context of the US demography and economy. Given the structure and strategy of the stochastic model, we can study the probability distribution of outcomes (benefit flows and rates of return) for generations (birth cohorts) of plan participants for the NDC program, as well as the overall financial stability of the NDC system. The next section of the paper describes our stochastic forecasting model. In the following section, we describe in some detail the Swedish NDC program and our adaptation of it to US economic and demographic conditions. We then provide simulations of this basic US NDC plan, as well as variants incorporating modifications of two key attributes of the NDC plan, the method of determining rates of return, and the brake mechanism applied when the system appears headed for financial problems.

The Stochastic Forecasting/Simulation Model

The stochastic population model is based on a Lee-Carter (1992) mortality model and a somewhat similar fertility model (Lee, 1993; Lee and Tuljapurkar, 1994). Lee-Carter models the time series of a mortality index as a random walk with drift, estimated over US data from 1950 to 2003. This index then drives the evolution of age specific mortality rates and thereby survival and life expectancy. This kind of model has been extensively tested (Lee and Miller, 2003), and although we shall see that the probability intervals it produces for distant future life expectancy appear quite narrow, these intervals have performed well in within-sample retrospective testing.

In a similar way, a fertility index drives age specific fertility, but in this case it is necessary to prespecify a long term mean based on external information. We set the Total Fertility Rate equal to the 1.95 births per woman, as assumed by the Social Security Actuaries (Trustees Report, 2004, henceforth TR04). The estimated model then supplies the probability distribution for simulated outcomes. Because it is fitted on US data, the fertility model reflects the possibility of substantial baby boom and bust type swings.

Immigration is taken as given and deterministic, following the assumed level in TR04.

Following Lee and Tuljapurkar (1994), these stochastic processes can be used to generate population stochastic forecasts in which probability distributions can be derived for all quantities of interest. These stochastic population forecasts can be used as the core of stochastic forecasts of the finances of the Social Security system (Lee and Tuljapurkar, 1998a and b, and Lee, Tuljapurkar Anderson, 2003). Cross sectional age profiles of payroll tax payments and benefit receipts are estimated from administrative data. The tax profile is then shifted over time by a productivity growth factor which is itself modeled as a stochastic time series. The benefit age profile is shifted over time in more complicated ways based on the level of productivity at the time of retirement of each generation. The real rate of return on special issue Treasury Bonds is also modeled as a stochastic time series, and used to calculate the interest rate on the Trust Fund Balance. The long run mean values of the stochastic processes for productivity growth and rates of return are constrained to equal the central assumptions of TR04, but the actual stochastically generated outcomes will not exactly equal these central assumptions, of course, even when averaged over a 100 year horizon.

The probability distributions for the stochastic forecasts are constructed by using the frequency distributions for any variable of interest, or functions of variables of interest, from a large number of stochastically generated sample paths, say 1000 or 10,000, typically annually over a 100 year horizon. Essentially, this is a Monte Carlo procedure. The stochastic sample paths can equally well be viewed as stochastic simulations, and the set of sample paths can be viewed as describing the stochastic context within which any particular pension policy must operate.

The stochastic simulation model is not embedded in a macro-model, and therefore does not incorporate economic feedbacks, for example to saving rates and capital formation, and hence to wage rates and interest rates. For some purposes, this would be an important limitation. However, the model has given useful results for the uncertainty of Social Security finances, and it should also give useful results in the present context. Once the stochastic properties of different policy regimes have been studied in this manner, it may be appropriate to extend the analysis to incorporate more general economic feedbacks in future work.

A Stochastic Laboratory: Simulating Statistical Equilibrium

To date, the stochastic Social Security method just described has been used solely for projections or forecasts, based on the actual demography and Social Security finances of the United States. However, it can also be used as a stochastic laboratory to study how different pension systems would perform in a stochastic context divorced from the particularities of the actual US historical context with its baby boom, baby bust and other features. This is the main strategy we pursue in this paper, since we are hoping to find quite general properties of the NDC systems. We build on the important earlier work by Alho et al. (2005). This approach also enables us to avoid dealing initially with the problems of the transition from our current system to the new system. Instead we will analyze the performance of a mature and established system in stochastic steady state. In later work we hope to consider the transition and to account for the actual historical initial conditions such as the current age distribution as shaped by the baby boom.

The key feature of a stochastic equilibrium is that the mean or expected values of fertility, mortality, immigration, productivity growth, and interest rates have no trend, and the population age distribution is stochastically stable rather than reflecting peculiarities of the initial conditions. The basic idea is simple enough, but there are a number of points that require discussion, as follows.

1. Productivity growth and interest rates are already modeled as stationary stochastic processes with preset mean values, so these pose no particular problem.

2. Net immigration is set at a constant number per period, following the Social Security assumptions (Trustees Report, 2004, hereafter TR04). We treat immigration as deterministic and constant.
3. Fertility is also modeled as a stationary stochastic process with a long-term mean value of 1.95 births per woman, consistent with TR04. This is below replacement level, so absent positive net immigration, the simulated population would decline toward zero and go extinct, with the only possible equilibrium population being zero. But with immigration, there is some population size at which the natural decrease given a TFR of 1.95 will be exactly offset by the net immigrant inflow, and this will be the equilibrium population. The same principle applies in a stochastic context.
4. According to the fitted Lee-Carter mortality model, the mortality level evolves as a random walk with drift. First, we note that unless the drift term is set to 0, mortality will have a trend. So in constructing our stochastic equilibrium population, we will project mortality forward, with drift, until 2100 and then set the drift to zero thereafter. This sets equilibrium life expectancy at birth to be about 87 years. Second, we note that a random walk, even with zero drift, is not a stationary process. It has no tendency to return to an equilibrium level, but rather drifts around. Our strategy is simply to set the drift term to zero. This means that we cannot view the simulated process as truly achieving a statistical equilibrium, but this is unlikely to cause any practical problems. An alternative would be to alter the model to make it truly stationary by providing some weak equilibrating tendency, e.g. replacing the coefficient of unity on the previous level of mortality in the process by 0.99.
5. We also need to generate an appropriate initial state for our system. We begin by constructing a deterministic stable population corresponding to the mean values of fertility and mortality for the given inflow of immigrants. We then start our stochastic simulation from this initial population, but we throw out the first hundred years. We keep the next five hundred years of stochastic simulations as our experimental set. Figure 1 plots 15 stochastic sample paths for the old age dependency ratio defined as population 67+ divided by the population 21 to 66. Evidently the simulations often show very pronounced long term variations resulting from something like the baby boom and baby bust in the United States.

6. For our policy experiments, we have created a single set of 1000 sample paths or stochastic trajectories. We will examine the performance of different policies within the context of this single set of stochastic trajectories, which makes their performances more comparable.

NDC System Design

As is well-known, the feasible internal rate of return for a PAYGO system with stable population structure equals the rate of growth of the population (which equals the rate of growth of the labor force, in steady state) plus the rate of growth of output per worker. Alternatively, this implicit rate of return simply equals the growth rate of GDP, provided that covered wages are a constant share of GDP. NDC systems aim to mimic the structure of funded DC systems while maintaining fiscal stability by using such an internally consistent rate of return rather than a market-based rate of return.

As under any pension system, an individual goes through two phases under an NDC scheme, corresponding roughly to periods of work and retirement. During the work phase, the individual's payroll taxes (*T*) are credited to a virtual account typically referred to as the individual's "notional pension wealth" (*NPW*). Like the individual account under an actual DC plan, this account has a stated value that grows annually with contributions and the rate of return on prior balances; for an individual, this evolution is represented by:

$$NPW_{t+1} = NPW_t(1 + r_t^i) + T_t \tag{1}$$

where r_t^i is the rate of return credited to each individual's existing balances. Unlike the individual account balance under the DC plan, *NPW* is only a virtual balance and the rate of return is based on the system's internal growth rate. Once an individual retires, he or she receives an annuity based on the value of notional pension wealth at the time of retirement.

The Swedish system normally bases r^i on the contemporaneous rate of growth of the wages per worker, which we call *g*, rather than the total growth of wages, which would also account for the growth rate of the work force, which we label n.[1] The notional accounts of individuals in Sweden also receive an annual adjustment for so-called inheritance gains, representing a redistribution of the account balances of deceased cohort members. That is, the rate of return to the cohort as a whole, which we denote r, equals g, *where* $r=g<r^i$.[2]

Upon retirement in the Swedish system, the individual's *NPW* is converted into an annuity stream based on contemporaneous mortality probabilities and an assumed real rate of return of 1.6 percent. Letting the superscript t denote the generation that reaches retirement age in year t, the annuity in year t for an individual retiring that year, x_t^t, may be solved for implicitly from the formula,

$$(2) \qquad NPW_t^t = \sum_{s=t}^{T} (1.016)^{-(s-t+1)} P_{t,s}^t x_t^t$$

where NPW_t^t is the individual's notional pension wealth in the year of conversion, $P_{t,s}^t$ is the probability of survival from year t until year s, assessed in year t, and T is the maximum life span. In subsequent years, the individual's annuity level is increased or decreased according to whether average growth of wages per worker, denoted $r_t = g_t$ above, exceeds or falls short of 1.6 percent. If wage growth continues at 1.6 percent then the annuity level would remain constant throughout the individual's life. However, if the realized growth of wages per worker in year t were actually 1.3 percent, the annuity would be 0.3 percent lower in real terms in year t+1 than in year t. If r were 1.3 percent in every year, the annuity would fall in real terms at a rate of 0.3 percent per year.

This adjustment of pension benefits does not include any retrospective adjustments to account for previous pension benefits being too high or too low based on realized values of r. For example, if $x_{t+1}^t < x_t^t$ and the real value of benefits is adjusted downward, no further adjustment is made in calculating the benefits from year t+1 onward for the fact that too high a benefit was paid in year t. This means that if there are

[1] Our characterization of the Swedish system relies on several sources, including Palmer (2000) and Settergren (2001a, 2001b).

[2] Accounts in Sweden are also reduced annually to account for administrative costs. In our simulations, we ignore these adjustments and the underlying costs.

persistent downward revisions in the projected value of r (as in the example just given where r is 1.3 percent per year, rather than the assumed 1.6 percent), then the actual annuity stream, discounted at realized values of r, will exceed the value of NPW at retirement.

The Brake Mechanism

The system just described incorporates an adjustment mechanism aimed at keeping benefits in a range that can be supported by growth in the payroll tax base. However, the adjustment is not perfect. As just described, benefits are adjusted only prospectively. Also, benefits are not adjusted after retirement to reflect changes in mortality projections. Perhaps more importantly, the cohort rate of return r in the Swedish system is based on the growth rate of the average wage, g, rather than the growth rate of the covered payroll, $n+g$. Finally, as illustrated in the Appendix using a simplified version of the NDC plan, even an NDC plan without these problems is not assured of annual balance if $n+g$ varies over time. This is in line with the analysis of Valdes-Prieto (2000), who observed that, under certain conditions, an NDC plan might be stable in a steady state, but will not be so in the short run.

Although it was anticipated by its designers that the Swedish system would nevertheless be quite stable, they added to the system a "brake" that would slow the growth rate of notional pension wealth and reduce the level of pension benefits in the event of a threat to the system's financial stability, as measured by a "balance ratio" b based on the system's conditions,

$$(3) \qquad b = \frac{F + C}{NPW + P}$$

The numerator of the balance ratio is meant to account for the system's assets, and is the sum of two terms. The first term in the numerator (F) equals the financial assets of the system (negative if the system has financial debt); the second term in the numerator (C) is a so-called "contribution asset" equal to the product of a three-year moving average of tax revenues and a three-year moving average of "turnover duration," which is the average expected length of time between the payment of contributions and the payment of benefits, based on current patterns. If the economy were in a steady state, the contribution asset would provide a measure of the size of the pension liability that contributions could sustain. This balance measure can be calculated entirely from observed values and does not involve any projected values, reducing the risk of political manipulation.

The denominator is the pension system's liability, equal to the sum of two components. The first component of the denominator (*NPW*) is aggregate notional pension wealth for generations not yet retired; the second component (*P*) is an approximation of commitments to current retirees, equal to the sum over retired cohorts of current annual payments to each cohort multiplied by that cohort's life expectancy.

For a variety of reasons, the balance ratio is an imperfect measure of the system's financial health. For example, the contribution asset and the liability to current retirees are each based on current conditions, rather than on projected conditions. Also, the two components of the asset measure are based on inconsistent rate-of-return assumptions, the financial component being assumed to yield a market rate of return and the contribution asset being valued using the system's implicit rate of return. However, one would still expect a higher value of the balance ratio, in general, to be associated with a more viable system. For the Swedish system, the balance ratio is applied only when it is less than 1.0. Once this occurs, two things happen. First, cohort pension wealth accumulates not at a rate equal to g_t, but instead at a rate equal to $g_t b_t$, where b_t is the balance ratio. Second, the rate of growth used to adjust the pension benefits of retirees is also set equal to $g_t b_t$, meaning a greater likelihood of a real decline in pension benefits for any given cohort, since real benefits grow at a rate of $g_t b_t - .016$.

The Swedish brake is asymmetric in that it takes effect when the system is underfunded but not when it is overfunded. One could imagine a system with a symmetric brake, defined in the same manner but also in effect when the balance ratio exceeds 1 and thus raising benefits and pension accumulations. When examining the performance of NDC systems below, we will consider both asymmetric and symmetric brakes.

Adapting the NDC System to the US Context

We have already outlined the basic structure of the Swedish NDC system, but there are various details to be specified in adapting the system to the US context.

Contributions

What proportion of payroll is to be contributed? For comparability to our current Social Security system, we assume the OASI tax rate of 10.6 percent, applied to the fraction of total wages below the payroll tax earnings cap.

Rates of Return

Rate of return assumptions are required in two places in the NDC system, for use in accretions of Notional Pension Wealth and in conversion of Notional Pension Wealth into an annuity stream upon retirement. The Swedish plan sets the first of these rates equal to the growth rate of average wages, which should roughly equal the growth rate of productivity. It sets the second rate equal to 1.6 percent, taken to be the expected rate of productivity growth, and then adjusts annuities up or down in response to variations in the actual growth of the average wage. Sweden does not account for the growth rate of the labor force but in principle it should be included since it is a component of the rate of return to a PAYGO system. Note that even if the growth rate of the labor force is not included, demography will still influence the outcomes for generations through the back door, because if the system begins to go out of fiscal balance then the brake will be applied.

For the US system, we take the long-run annual productivity growth rate to be 1.1 percent, following the Social Security assumption (TR04)[3], as described below. We have implemented NDC in two ways for the United States, once with rate of return based only on wage growth (g), and once with the rate of return based on both wage growth and labor force growth (n+g). These will presumably distribute risk in different ways across the generations. In stochastic equilibrium population growth is near zero in any case, on average, but demographic change will certainly occur along simulated sample paths.

[3] We note that the growth rate of productivity (output per hour of labor) may overstate the growth rate of covered wages, as is explicitly taken into account by the US Social Security Administration. The growth rate of covered wages will be affected by changes in the supply of labor per member of the population of working age and by sex, both labor force participation and hours worked per participant, and by shifts in the population age distribution. It will also be affected by the proportion of compensation that is given in pretax fringe benefits.

Annuity Calculations

We assume that annuitization of *NPW* occurs at age 67, the normal retirement age to which the US system is currently in transition. We use the same rate of return for accumulations of *NPW* as in converting the account balance at retirement into the constant real annuitized income stream. That is, we use either the growth rate of wages (g) or the growth rate of wages plus labor force (n+g) in both cases. As in the Swedish system, we set the pattern of the annuity stream to be constant in real terms, based on the growth rate and mortality projections at the time of the original annuity computation. Unlike the Swedish system, we use the actual growth rate (either g or n+g) as of retirement, rather than an assumed long-run value (in the Swedish case, 1.6 percent).

The annuity calculation can either be set once at the time of retirement, or it might be updated during the benefit period to reflect changes in the implicit rate of return, as is done in Sweden. We have programmed both possibilities, referring to one as "updating" and the other as "no updating." What mortality schedule is used to compute the annuitized income stream? Once again, this can be based on conditions at the time a generation retires (as is done in Sweden), or it can be revised during the benefit period, an approach that Valdes-Prieto (2000) refers to as a CREF-style annuity. We have done it both ways, bundled into the "updating" and "no updating" programs. Because we wish to determine the extent to which the NDC plan can be made stable, we present the results for the "updating" version below. However, the difference between the two versions is minor in our simulations.

The Brake

As explained earlier, the Swedish program has a brake but no "accelerator," so that if surpluses begin to accumulate there is no mechanism to raise benefits or reduce taxes. In our NDC program, we have incorporated this asymmetric brake, but in another version we use a symmetric brake with an accelerator that raises the rate of return and raises current benefits when the fiscal ratio exceeds unity.

A second change we implement is in the design of the brake mechanism itself. As discussed above, the brake in the Swedish system multiplies the net return implied by wage growth by the "balance ratio" of system assets to system liabilities when this ratio is less than unity. That is, the adjusted rate of return, r_t^a, is give by

(4) $$r_t^a = r_t b_t$$

As we discuss below, this brake may prove inadequate to ensure financial stability of the NDC scheme. Even very low values of the balance ratio bring about relatively small reductions in the rate of growth of commitments. For example, a system with enormous liabilities (the denominator in expression 3) might have a balance ratio very close to zero, but even in this case pension accruals would continue and a positive rate of return would be used in the computation of annuities. That is, if the underlying rate of return, *r*, is positive, then the adjusted rate of return, r_t^a, will also be positive, no matter how large the denominator of the balance ratio. A second problem is that, when *r* is negative, as is certainly possible given its alternative definitions, a lower value of *b* and hence a system in greater need of adjustment results in a higher rate of pension accruals and annuity payments. A third problem is that the adjustment mechanism does not work properly when the balance ratio is negative, as it would be if the system already had accumulated financial debt in excess of its "contribution asset" (respectively, the terms *F* and *C* in expression 3). In this circumstance, a higher value of *r*, presumably a good outcome in terms of the system's financial health, would result in a more negative value of the adjusted rate of return credited to workers and pensioners, r_t^a.

In thinking about how the brake mechanism might be modified, it is useful to start with a characterization of what one would like the brake mechanism to accomplish. Letting $R^a = 1+r^a$ be the gross return that corresponds to the net adjusted return, r^a, the desired properties include:

(a) $dR^a/dr > 0$; the gross return actually credited should increase monotonically with the economy's "warranted" net return.

(b) $dR^a/db > 0$; the gross return should increase with the balance ratio, i.e., with the fiscal health of the system.

(c) $R^a > 0$; logically, it's not even clear what it means for *R* to be negative, and the system should certainly not be allowed to get this far out of fiscal equilibrium.

(d) $d^2R^a/dbdr > 0$; the sensitivity of the gross return to the balance ratio should increase with r.

The standard brake mechanism has problems satisfying some of these criteria:

(a.1) $dR^a/dr = b$; this is positive only if b is positive, leading to the third problem mentioned above.

(b.1) $dR^a/db = r$; this is positive only if r is positive, leading to the second problem mentioned above.

(c.1) $R^a > 0 \Leftrightarrow rb > -1$; this would not seem like a strong requirement, since the balance ratio would have to be hugely negative (if r is positive); but because the brake may have very little impact, the balance can get worse and worse.

(d.1) $d^2R^a/dbdr = 1 > 0$, so the criterion that this cross-derivative should be positive is satisfied.

Now, consider an alternative brake mechanism which is applied to the gross return ($1+r$) rather than the net return, r, scaling down the strength of the brake to reflect the fact that it's applied to the entire return, ($1+r$), and not just the net return r:

(5) $$R_t^a = (1+r_t)[1+A(b_t-1)] \quad \Rightarrow \quad r_t^a = (1+r_t)[1+A(b_t-1)]-1$$

where r and b are defined as before and A is a scaling factor, which should be on the order of $\rho/(1+\rho)$, where ρ is some "normal" net rate of return, perhaps the mean value of r. Consider the performance of this new mechanism with respect to the four criteria given above:

(a.2) $dR^a/dr = 1+A(b-1)$; this will be positive as long as $b > 1-1/A = -1/x$, which turns out to be the same requirement that $R > 0$ (see c.2 below). This is a much weaker condition to satisfy than that give in (a.1) above.

(b.2) $dR^a/db = A(1+r)$; this is positive as long as $r > -1$, which is a very weak requirement.

(c.2) $R^a > 0 \Leftrightarrow b > 1-1/A = -1/x$; this is a condition comparable to (c.1), but it is more likely to be satisfied here because the brake will work better.

(d.2) $d^2R^a/dbdr = A > 0$, so this criterion is satisfied.

Thus, a brake based on the gross return, (1+r), has many advantages over the brake based on the net return. A final advantage, related to the first problem cited above, is that, through the factor A in expression (5), it can be made to have much more powerful effects than a brake that simply scales back the *net* return.

We have experimented with various values of A, in each case also imposing a lower bound of zero on the adjusted gross return, R^a, should the balance ratio dictate a negative gross return in (5). The simulations presented below based on this gross brake mechanism are for A = .5. – a considerably larger value than the ratio $x/(1+x)$, where x is the mean value of r. This value was large enough to ensure that virtually none of the 500-year trajectories ever encountered the lower bound on R^a for NDC type systems with $r = g$ (only 2 of 1000 trajectories for the asymmetric brake case and 7 of 1000 for the symmetric brake case). Even for a much lower value of A = .2, the lower bound is basically irrelevant for trajectories with $r = n+g$ and binds along only relatively few trajectories for NDC systems with $r = g$ (15 for the asymmetric brake and 47 for the symmetric brake). On the other hand, the net brake mechanism modeled on the Swedish system resulted in at least one negative balance ratio among 251 – more than a quarter – of the 1000 simulated trajectories. In these instances where the balance ratio is negative, we set it to zero in expression (4) in computing the adjusted rate of return.

Initial Conditions

As discussed above, we start our simulations with a population structure based on a deterministic version of our demographic model, and then run the economy for a hundred-year "pre-sample" period to get a realistic distribution of demographic characteristics for the stochastic version of the model, which we then simulate over period of five hundred years.

We also use this initial hundred-year period to establish the initial conditions for the NDC system. As of the beginning of the actual simulation period, and for each trajectory, we calculate each working cohort's *NPW* based on its earnings during the pre-sample period and the relevant growth rates (g or $n+g$) used in compounding *NPW* accumulations. For each retired cohort, we calculate annuity values in the same manner. Finally we assume an initial stock of financial assets equal to 50 times the average primary deficit in the first year of the model based on g with no brake.

Simulation Results

We simulate seven versions of the NDC system, differing as to whether the rate of return is based on the productivity growth rate, g, or the growth rate of wages, $n+g$, and the type of brake used in attempting to achieve fiscal stability. There are two versions each (corresponding to the rates of return g and $n+g$) for the case of no brake, an asymmetric brake based on the gross return, as in (5), and a symmetric brake based on the gross return. The asymmetric brake applies whenever the balance ratio defined in (3) is less than 1.0, whereas the symmetric applies for all values of the balance ratio. Our seventh simulation is closest to the Swedish system; its rate of return is based on g and it incorporates an asymmetric brake based on expression (4), scaling the *net* rate of return.

In our projections, the mean rate of growth of the real covered wage is assumed to be 1.1 percent per year, following the assumptions of the Social Security Trustees. The long run growth rate of the projected population is close to 0 in the stochastic equilibrium we generate, so the growth rate of covered wages is also about 1.1 percent per year. The internal rates of return for individual cohorts along any given trajectory of our stochastic projections should therefore tend to fluctuate around this central value if the system maintains financial stability.[4] In addition to considering the internal rates of return (*IRR*s) under each NDC variant, we are also interested in the financial stability that each system provides. We measure financial or fiscal balance using the ratio of assets to payroll, where in the figures below a negative value indicates debt and a positive value indicates a positive fund balance. Note that the numerator of this expression includes only financial assets, not the "contribution asset" that is used in computing the balance ratio.

Consider first the performance of the NDC system based on $r = g$, roughly the Swedish approach without a brake mechanism. As shown in the first row of Table 1, this system provides a mean internal rate of return of 1.07 percent, in line with our expectations. The median *IRR* also equals 1.07 percent for this scheme. However, the need for a brake is quite evident from Figure 2, which shows the distribution of assets-payroll ratios for the system's first 100 years of operation. The median trajectory has

[4] The Social Security Trustees' assumption about GDP growth is 1.5 percent (this is from TR04, where it results from 1.6 percent productivity growth plus 0.2 percent growth in total employment plus the GDP deflator of 2.5 percent minus the CPI deflator of 2.8 percent). But this is not in stochastic equilibrium, the population growth rate is not near 0, and the ratio of covered payroll to GDP is changing over time.

essentially no accumulation of debt or assets. But, with no brake, some trajectories lead to accumulation of debt levels nearly 40 times payroll, clearly an unsustainable level. Indeed, a debt-payroll ratio of nearly 10 is present after 100 years in one-sixth of all trajectories, so this problem is not one limited to extreme draws from the distribution of outcomes. In addition, several trajectories involve substantial accumulation of assets relative to payroll.

Now consider the NDC system with the Swedish (net) asymmetric brake in place. As one would expect, imposing such a brake reduces the mean and median *IRR*s, as shown in the second row of Table 1. As Figure 3 shows, the lower tail of assets-payroll outcomes is raised, as also expected. But, the system still is not particularly stable, as the debt-payroll levels for the 2.5th percentile reach nearly 30 within 100 years. Thus, we consider the alternative (gross) brake mechanism introduced above, first for the asymmetric case.

This stronger brake mechanism results in somewhat higher mean and median *IRRs* than the Swedish-style brake (see the third row of Table 1). This version of the brake also produces a more acceptable range for the assets-payroll ratio. As one may observe in Figure 4, even the 2.5th percentile of the assets-payroll ratio has minimal debt-payroll ratios during the first 100 years of the program's operation. On the other hand, as Figures 2, 3, and 4 remind us, an asymmetric brake does nothing to reduce the substantial asset accumulation that can occur on some trajectories. Also, with its success in reducing the possibility of excessive debt accumulation, the asymmetric gross brake leads, on average, to the accumulation of assets, with both median and mean assets-payroll ratios positive after 100 years, and the upper 97.5th percentile at nearly 20.

To address this pattern, a symmetric brake mechanism is needed, to increase accumulations and annuity benefits when the system's fiscal health is assured. Implementing a symmetric version of the gross brake leads to more generous benefits for some trajectories, and hence higher mean and median *IRR*s, as the fourth row of Table 1 shows. The distribution of assets-payroll ratios is similar for the lower tail as under the asymmetric brake, but the upper tail has been pulled down by the brake's symmetry, with the 97.5th percentile assets-payroll ratio just below 1.5 after 100 years. Further, both the mean and median assets-payroll ratios stay very close to zero. Thus, the NDC system can be made to be quite stable financially with the introduction of two modifications of the Swedish brake, the use of a stronger, gross brake and the application of the brake not only when the balance ratio is too low but also when it is too high. This stability holds over the longer term as well, as shown in Figure 5, which exhibits the distribution of assets-payroll ratios over 500 years for the symmetric brake scheme.

Another potential modification of the NDC system involves the computation of the rate of return for *NPW* accumulations and annuity computations. Even if the average population growth rate is zero, this growth rate can fluctuate, and with this fluctuation the ability of the NDC system to cover benefits. Thus, building population growth into the rate of return should provide greater system stability, *ceteris paribus*. Figures 6, 7, and 8, and the last three rows of Table 1, present assets-payroll distributions and *IRR*s for NDC systems based on $r = n+g$ for the no-brake, asymmetric- and symmetric-brake variants.

The impact of this change in the method is most easily seen by comparing Figure 6, the trajectory of debt under the NDC system with no brake and with $r = n+g$, and Figure 2, the debt trajectory under the system with no brake and $r = g$. While the assets-payroll distribution still does not fully stabilize, its range is much smaller, especially in the lower tail. The (2.5, 97.5) range of outcomes is now (-2, +8) instead of (-35, +19). Still, a brake is needed to prevent eventual debt explosion along some paths, and a symmetric brake needed to stabilize the up side as well. Figure 8 shows the trajectory for the symmetric brake with $r=n+g$. As under the plan with $r=g$ pictured in Figure 5, the distribution of outcomes is quite acceptable over even 500 years. A comparison of the two figures indicates that using $n+g$ in calculating the rate of return is particularly effective at preventing debt accumulation, a result that was also evident in the earlier comparison of Figures 2 and 6.

Sources of Instability

As we have seen, the basic NDC system, even with Swedish-style net brake, is quite unstable financially. Even the NDC system based on setting the rate of return $r = n+g$ requires the application of a symmetric brake to head off substantial asset or debt accumulations on some trajectories. What is causing such instability? One can consider the impact of some sources of uncertainty by eliminating others from the simulations.

In our basic model, uncertainty arises from demographic and economic changes, the latter consisting of fluctuations in the interest rate and the rate of productivity growth.[5] These economic fluctuations, it turns out, are an important source of the NDC system's instability. Figure 2.a and 6.a present 100-year distributions of debt-payroll ratios for both versions of the NDC system (g and $n+g$) with no brake, corresponding to Figures 2 and 6 and differing from the systems depicted in those figures only in that productivity growth and the interest rate are held constant at their mean values. With only demographic fluctuations present, these new figures show, the distributions of asset-payroll ratios are substantially narrowed. Under the NDC(g) system, the (2.5,97.5) percentile range at 100 years shrinks from (-35,+19) to (-22,+8); under the NDC($n+g$) system, the same range shrinks from (-2, +8) to (-0.5,+2.5). Thus, even with the growth rate g incorporated in the rate of return used in the NDC system's calculations, this process does not come close to neutralizing fluctuations in that growth rate.

Conclusions

We have considered the financial stability of different variants of a system of Notional Defined Contribution accounts, using demographic and economic characteristics of the United States. In subsequent work, we will consider other aspects of NDC systems, notably their ability to smooth economic and demographic risks among different generations. Among our findings here are:

1. A system similar to that currently in use in Sweden, which bases net rates of return on the growth rate of average wages and utilizes a brake to adjust the net rate of return during periods of financial stress, does not ensure financial stability. For a large fraction of trajectories, the system accumulates an unsustainable level of debt within 100 years.
2. A brake mechanism that adjusts the *gross* rate of return provides greater flexibility and can be used to avoid these unsustainable outcomes.
3. Only the use of a symmetric brake, which raises rates of return during periods of financial strength, can avoid considerable accumulations of financial assets on some paths.

[5] The interest rate matters because the NDC approach is not a pure PAYGO system. With nonzero values

4. An NDC system in which rates of return are based on total rather than per capita economic growth is inherently more stable than the basic NDC system, without reference to the brake mechanism in use.
5. A considerable share of the volatility in the financial performance of NDC systems is attributable to economic, rather than demographic, uncertainty.

Evidently stochastic simulation of the system's finances can reveal aspects of its performance that are not otherwise obvious, and can assist in improving system design. This promises to be a valuable use for stochastic simulation models of pension systems.

Appendix: Benefits and Taxes under a Simple NDC Plan

This appendix illustrates the relationship between benefits and taxes at a given point in time under a simple version of the Notional Defined Contribution scheme in which the intrinsic rate of return is based on the growth rate of covered wages.

Consider the relationship between taxes and benefits at any given date *t* under a simplified version of an NDC system under which the rate of return used to accumulate notional pension wealth and to calculate annuities, r_t, is equal to the contemporaneous growth of covered wages.

Taxes at time *t* are:

$$T_t = \tau\left(W_t^{t+1} + W_t^{t+2} + \ldots + W_t^{t+L}\right) \tag{A1}$$

where W_t^{t+j} is covered wages in year *t* for the entire cohort that will retire *j* years hence and *L* is the number of years that individuals work.

For simplicity, assume that each retired cohort receives in benefits the annual real return on its notional pension wealth of r_t, so that the cohort's *NPW* will stay constant in real terms after retirement, and its annual payout is constant as well.[6] Then aggregate benefits at date *t* will equal:

of financial assets, the rate of return on these assets matters.

[6] This assumption implies that a cohort's benefits per capita grow as the cohort's population declines and indeed approaches infinity as the generation dies off, which is obviously unrealistic. We impose it here only for purposes of exposition

(A2) $$B_t = r_t\left(NPW_t^t + NPW_{t-1}^{t-1} + \ldots\right)$$

where NPW_{t-j}^{t-j} is the notional pension wealth in the year of retirement for cohort retiring in year *t-j*. The notional pension wealth at retirement for cohort *t-j* is:

(A3) $$NPW_{t-j}^{t-j} = \tau\left(W_{t-j-1}^{t-j} + W_{t-j-2}^{t-j}(1+r_{t-j-1}) + \ldots + W_{t-j-L}^{t-j}\prod_{l=1}^{L-1}(1+r_{t-j-l})\right)$$

Combining expressions (A2) and (A3) and comparing the resulting expression with expression (A1), we can see that a sufficient condition for taxes and benefits to be equal is that, for all *k* between 1 and *L*,

(A4) $$\tau W_t^{t+k} = \tau r_t \sum_{j=0}^{\infty} W_{t-j-k}^{t-j} \prod_{l=1}^{k-1}(1+r_{t-j-l})$$

or, that taxes paid by workers *k* years away from retirement equal benefits attributable to earnings at the same age for all retirees.

We have assumed that r_s equals the growth rate of covered wages between dates s-1 and s. If we assume in addition that this growth rate is shared by the entire age-wage distribution (i.e., that the relative age-distribution of covered wages remains fixed), then expression (A4) can be rewritten as:

$$W_t^{t+k} = r_t W_{t-k}^t \sum_{j=0}^{\infty}\prod_{m=1}^{j}(1+r_{t-m})^{-1}\prod_{l=1}^{k-1}(1+r_{t-j-l})$$

(A5) $$= r_t W_t^{t+k}\prod_{p=0}^{k-1}(1+r_{t-k+p})^{-1}\sum_{j=0}^{\infty}\prod_{m=1}^{j}(1+r_{t-m})^{-1}\prod_{l=1}^{k-1}(1+r_{t-j-l})$$

$$\Rightarrow r_t\prod_{q=0}^{k-1}(1+r_{t-1-q})^{-1}\sum_{j=0}^{\infty}\prod_{m=1}^{j}(1+r_{t-m})^{-1}\prod_{l=1}^{k-1}(1+r_{t-j-l}) = 1$$

The last line of expression (A5) is satisfied if *r* is constant over time, which reflects the underlying consistency of using the growth of covered wages as a rate of return for the

NDC system. If r varies over time, though, expression (A5) will generally not hold. For example, suppose $k = 1$, corresponding to wages in the year prior to retirement. Then the last line of (A5) reduces to:

$$\text{(A6)} \qquad r_t(1+r_{t-1})^{-1}\left(1+(1+r_{t-1})^{-1}+(1+r_{t-2})^{-1}+...\right)=1$$

From this, we can see that if the current growth rate used to compute annuities, r_t, is greater (less) than the growth rates of covered wages during the accumulation phase, then the expression on the left-side will be greater (less) than 1 and taxes on earnings for those in the year prior to retirement will be inadequate (more than adequate) to cover benefits for retirees based on earnings in the year prior to retirement. Although the results are more complicated for values of $k > 1$, the point is that variations in r over time can cause the NDC system to run deficits or surpluses, the variation being larger the larger is the variation in the growth rate of covered wages. This variation in deficits occurs even under the assumption of a fixed covered earnings age profile; relaxing this assumption adds yet another potential source of variation in the system's annual deficits.

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Table 1. Average Internal Rates of Return

Simulation	**Mean IRR**	**Median IRR**
NDC (*g*) No Brake	.0107	.0107
NDC (*g*) Asymmetric Brake (Net)	.0084	.0090
NDC (*g*) Asymmetric Brake (Gross)	.0093	.0105
NDC (*g*) Symmetric Brake (Gross)	.0106	.0130
NDC (*n*+*g*) No Brake	.0110	.0113
NDC (*n*+*g*) Asymmetric Brake (Gross)	.0109	.0112
NDC (*n*+*g*) Symmetric Brake (Gross)	.0133	.0134

Source: Calculated from stochastic simulations described in text.

Figure 1. Ratio of Retirees to Workers, 15 Sample Paths

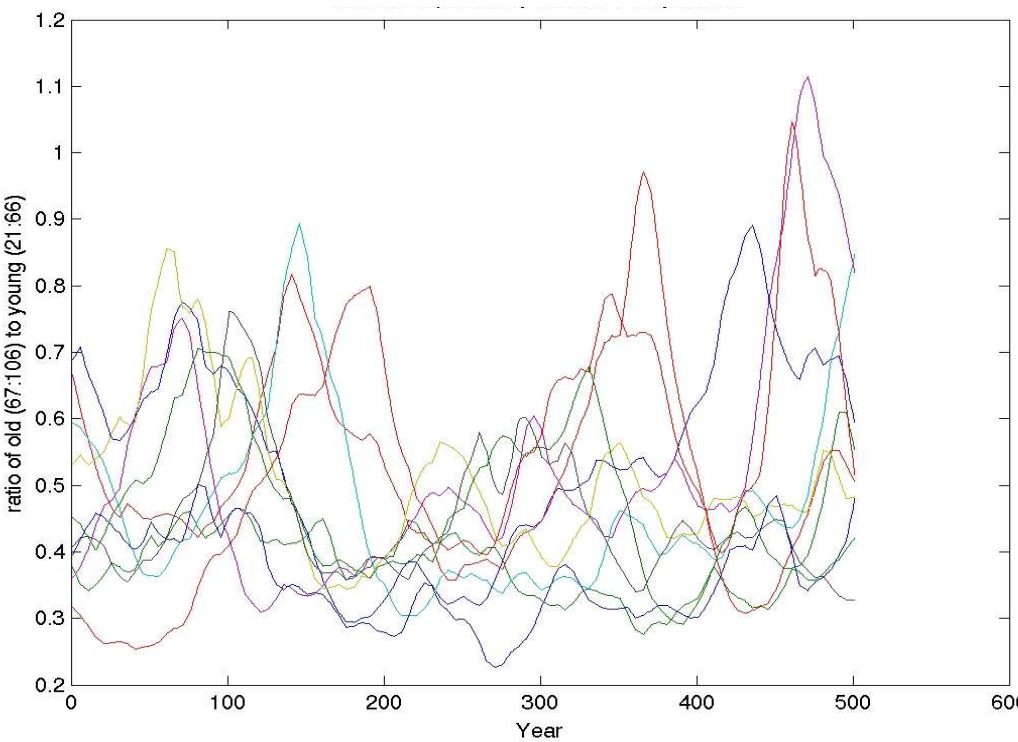

Figure 2. Assets/ Payroll (*r*=*g*, no brake)

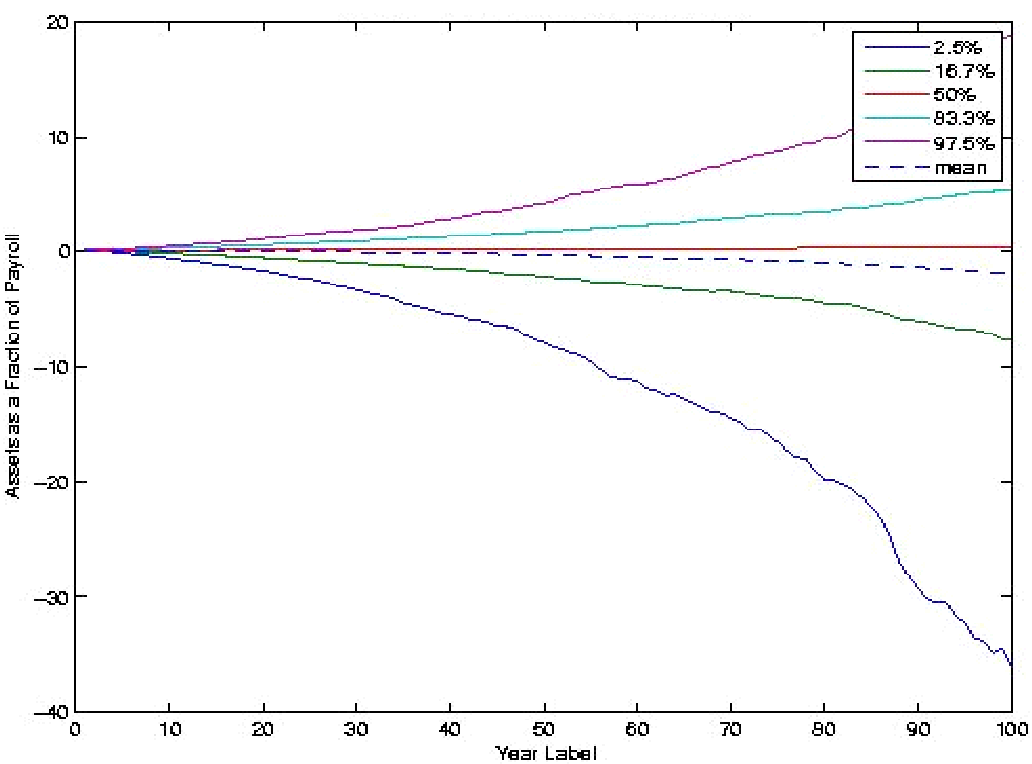

Figure 3. Assets/Payroll (*r=g*, asymmetric brake, net)

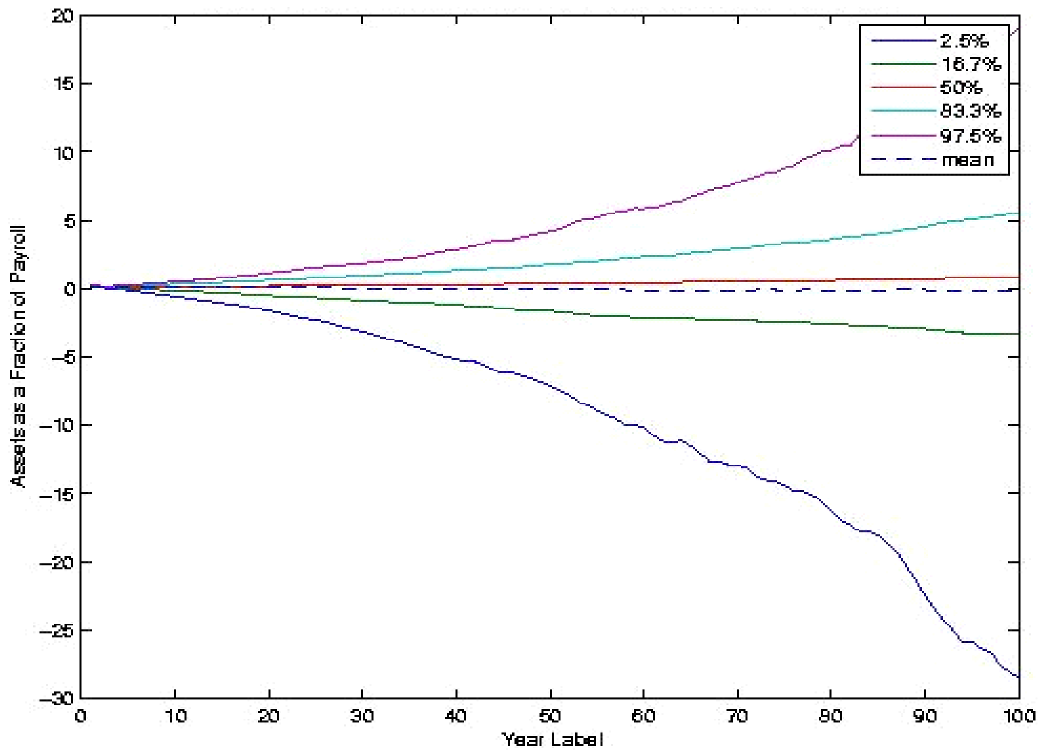

Figure 4. Assets/Payroll (*r=g*, asymmetric brake, gross: *A=.5*)

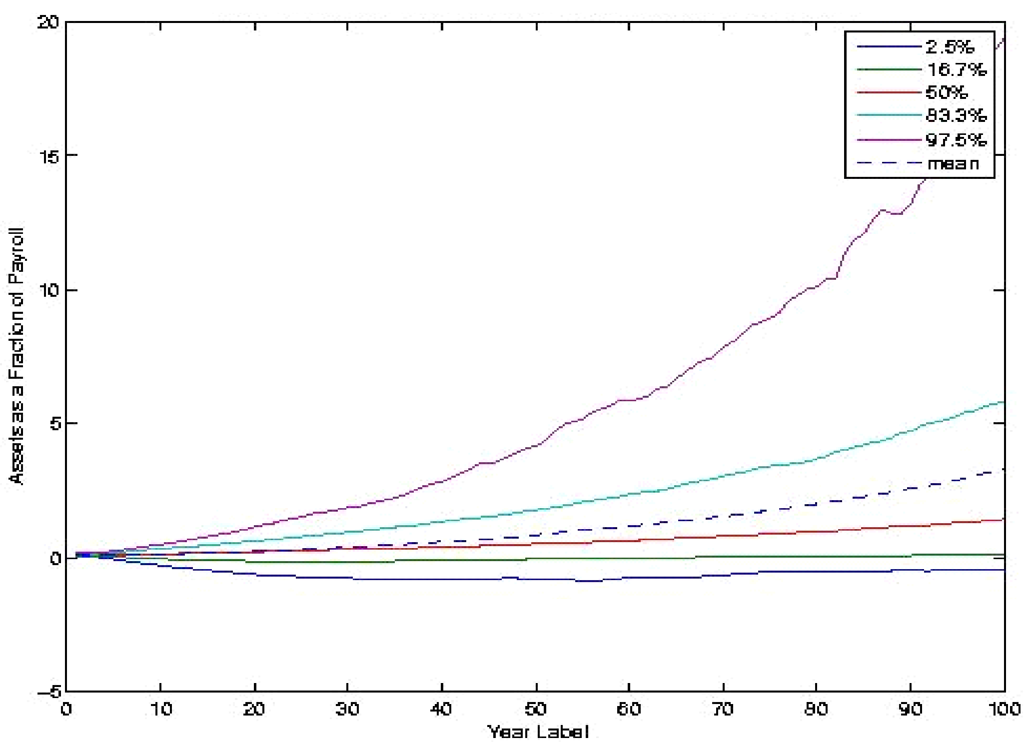

Figure 5. Assets/Payroll (*r*=*g*, symmetric brake, gross: *A*=*.5*)

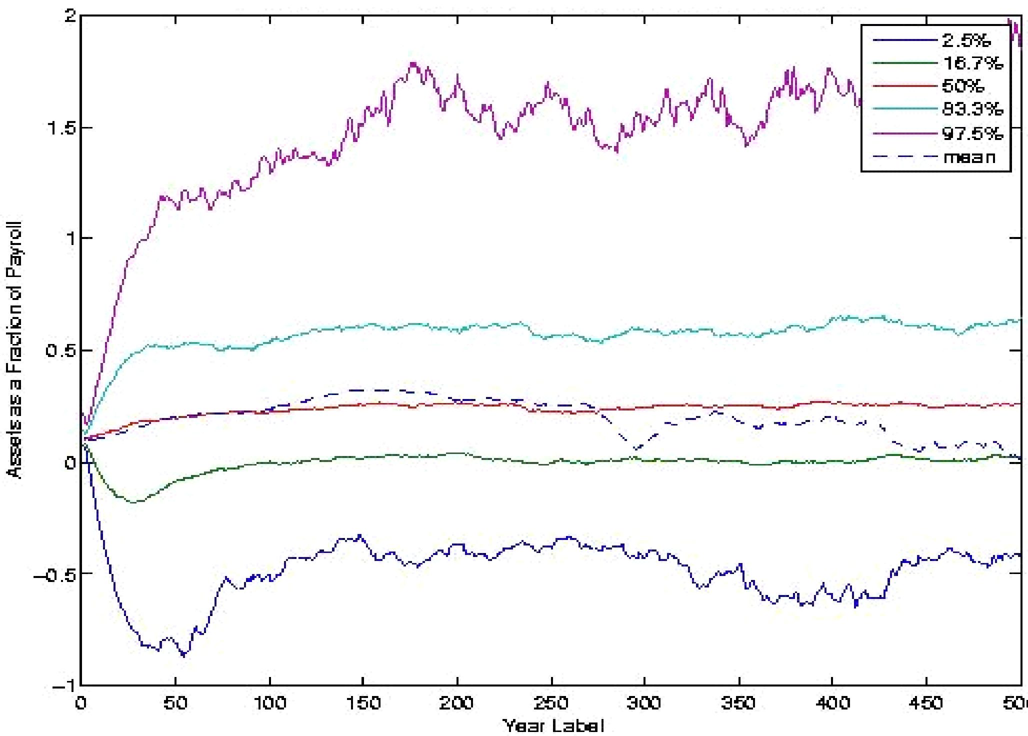

Figure 6. Assets/Payroll (*r*=*n*+*g*, no brake)

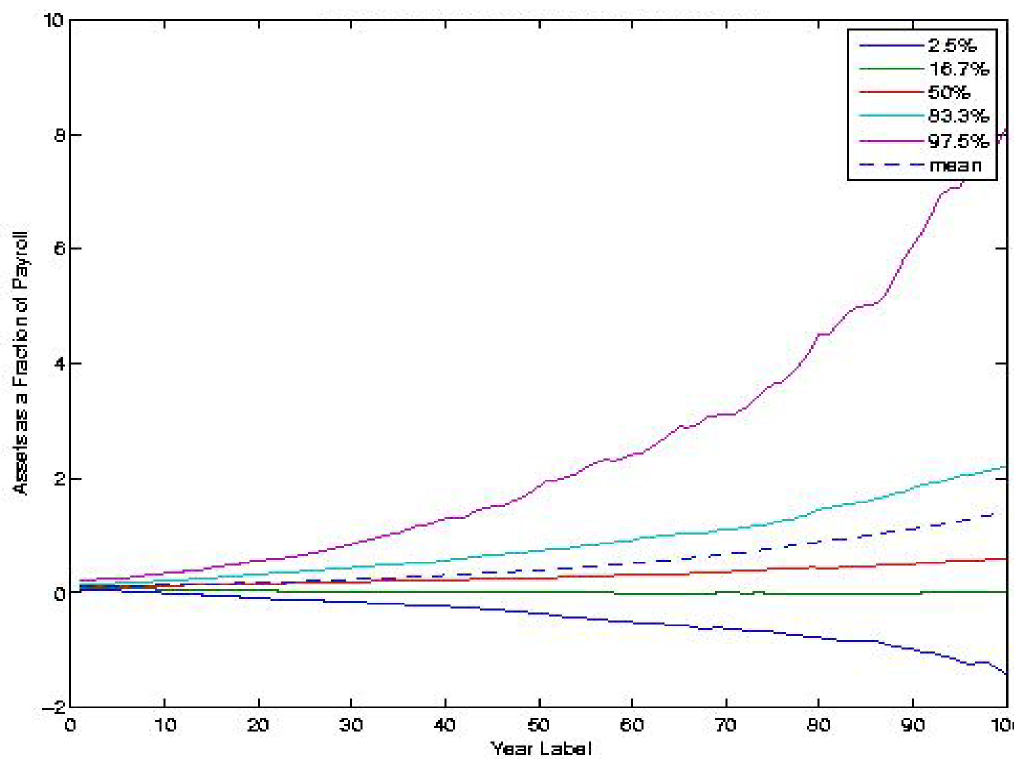

Figure 7. Assets/Payroll (*r=n+g*, asymmetric brake, gross: *A=.5*)

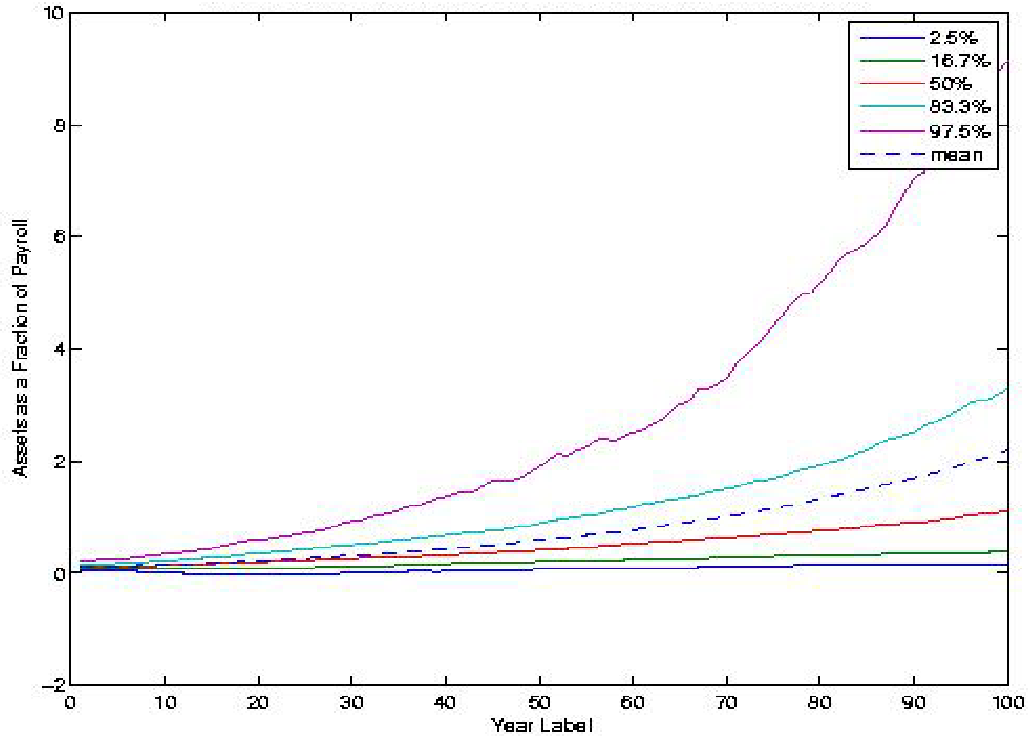

Figure 8. Assets/Payroll (*r=n+g*, symmetric brake, gross: *A=.5*)

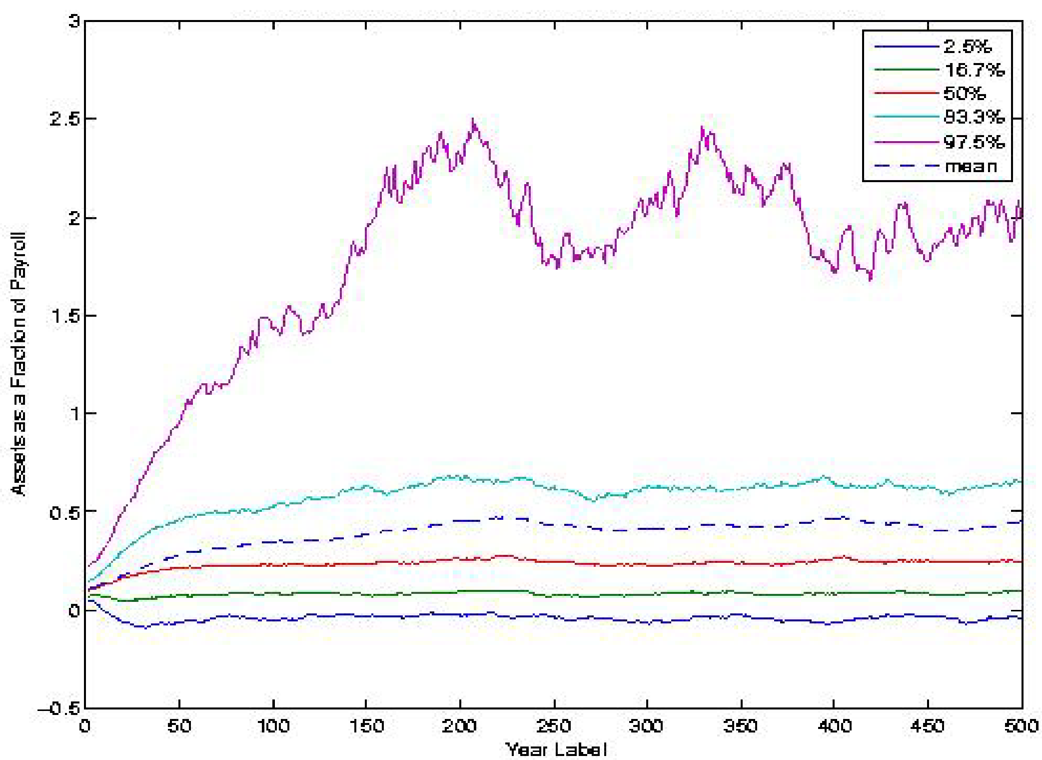

Figure 2.a. Assets/Payroll (*r*=*g*, no brake); constant interest, growth rates

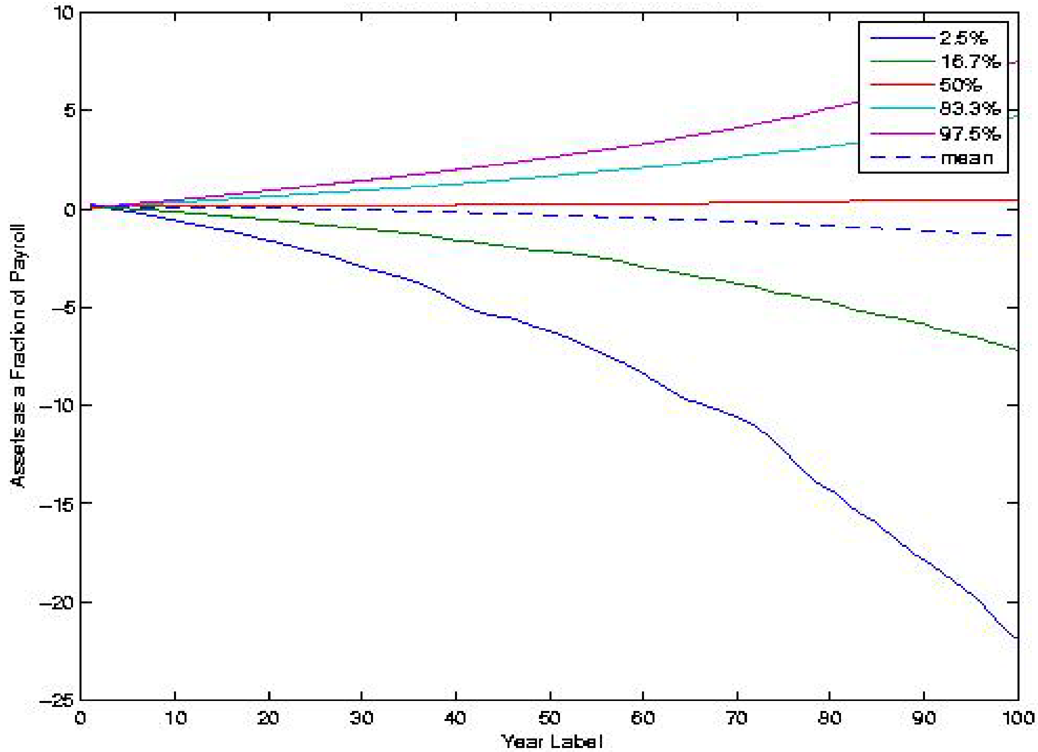

Figure 6.a. Assets/Payroll (*r*=*n*+*g*, no brake); constant interest, growth rates