Contracts as Reference Points*

by

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July 2006

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* An early version of this paper was entitled “Partial Contracts.” We would like to thank Philippe Aghion, Dan Benjamin, Richard Craswell, Florian Englmaier, Ben Hermalin, Henrik Lando, Steve Leider, Sendhil Mullainathan, Andrei Shleifer, and Birger Wernerfelt for helpful suggestions. We are grateful for comments from audiences at the Max Planck Institute for Research on Collective Goods in Bonn, the Harvard-MIT Organizational Economics Seminar, the University of Zurich, the 2006 Columbia University Conference on The Law and Economics of Contracts, Cornell University (Center for the Study of Economy & Society), Yale University Law School (where part of the text formed the first author’s Raben Lecture), Edinburgh University, London School of Economics, the American Law and Economics Association Meetings (where part of the text formed the first author’s Presidential address), Copenhagen Business School, the University of Stockholm, and Stockholm School of Economics. We acknowledge financial support from the U.S. National Science Foundation through the National Bureau of Economic Research, and the U.K. Economic and Social Research Council.
Abstract

We argue that a contract provides a reference point for a trading relationship: more precisely, for parties’ feelings of entitlement. A party’s ex post performance depends on whether he gets what he is entitled to relative to outcomes permitted by the contract. A party who feels shortchanged shades on performance. A flexible contract allows parties to adjust their outcome to uncertainty, but causes inefficient shading. Our analysis provides a basis for long-term contracts in the absence of noncontractible investments, and elucidates why “employment” contracts, which fix wage in advance and allow the employer to choose the task, are optimal.
1. Introduction

What is a contract? Why do people write (long-term) contracts? The classical view held by economists and lawyers is that a contract provides parties with a set of rights and obligations, and that these rights and obligations are useful, among other things, to encourage long-term investments. In this paper we present an alternative, and complementary, view. We argue that a contract provides a reference point for the parties’ trading relationship: more precisely for their feelings of entitlement. We develop a model in which a party’s ex post performance depends on whether the party feels that he (or she) is getting what he’s entitled to relative to the outcomes permitted by the contract. A party who feels shortchanged shades on his performance, which causes a deadweight loss. One way the parties can reduce this deadweight loss is for them to write an ex ante contract that pins down future outcomes very precisely, and that therefore leaves little room for disagreement and aggrievement. The drawback of such a contract is that it does not allow the parties to adjust the outcome to the state of the world. We study the trade-off between rigidity and flexibility. Our analysis provides a basis for long-term contracts in the absence of noncontractible relationship-specific investments, and throws light on why simple “employment” contracts are sometimes optimal.

To motivate our work, it is useful to relate it to the literature on incomplete contracts. A typical model in that literature goes as follows. A buyer and seller meet initially. Since the future is hard to anticipate, they write an incomplete contract. As time passes and uncertainty is resolved, the parties can and do renegotiate their contract, in a Coasian fashion, to generate an ex post efficient outcome. However, as a consequence of this renegotiation, each party shares some of the benefits of prior (noncontractible) relationship-specific investments with the other party.

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1 For up-to-date syntheses of the classical view, see Bolton and Dewatripont (2005) and Shavell (2004).
Recognizing this, each party underinvests ex ante. The literature studies how the allocation of asset ownership and formal control rights can reduce this underinvestment.  

While the above literature has generated some useful insights about firm boundaries, it has some shortcomings. Two that seem particularly important to us are the following. First, the emphasis on noncontractible ex ante investments seems overplayed: although such investments are surely important, it is hard to believe that they are the sole drivers of organizational form. Second, and related, the approach is ill-suited for studying the internal organization of firms, a topic of great interest and importance. The reason is that the Coasian renegotiation perspective suggests that the relevant parties will sit down together ex post and bargain to an efficient outcome using side payments: given this, it is hard to see why authority, hierarchy, delegation, or indeed anything apart from asset ownership matters.

We believe that in order to develop more general and compelling theories of organizational form it is essential to depart from a world in which Coasian renegotiation always leads to ex post efficiency. The purpose of our paper is to move in this direction. To achieve this goal we depart from the existing literature in two key ways. First, we drop the assumption made in almost all of the literature that ex post trade is perfectly contractible. Instead we suppose that trade is only partially contractible. Specifically, we distinguish between perfunctory performance and consummate performance, or performance within the letter of the contract and performance within the spirit of the contract. Perfunctory performance can be

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3 For a discussion, see Holmstrom (1999).
4 One obvious possibility is to introduce asymmetric information. However, to date such an approach has not been very fruitful in the theory of the firm.
5 We do not go as far as some of the recent incomplete contracting literature that supposes that ex post trade is not contractible at all (see, e.g., Baker et al. (2006)).
6 The perfunctory and consummate language is taken from Williamson (1975, p. 69). See also Goldberg and Erickson (1987, p. 388).
judicially enforced, while consummate performance cannot. Second, we introduce an important behavioral element. We suppose that a party is happy to provide consummate performance if he feels that he is getting what he is entitled to, where entitlement is measured relative to the contract, but will withhold some part of consummate performance if he feels shortchanged—we refer to this as shading. A final element of the story is that there is no reason why parties’ feelings of entitlement should be consistent. In other words, it may well be the case that the sum of the parties’ entitlements exceeds the total amount available.

These ingredients yield the above-described trade-off between flexibility and rigidity. A flexible contract has the advantage that parties can adjust the outcome to the state of the world, but the disadvantage that any outcome selected will cause at least one party to feel aggrieved and shortchanged, which leads to a loss of surplus from shading. An optimal contract trades off these two effects.

The paper is organized as follows. Section 2 presents the model and discusses the two key assumptions of partial contractibility and shading. In Section 3 we analyze a case of the model where there is uncertainty about value and cost but not about the type of good to be traded. In Section 4 we consider a second case where there is uncertainty about the nature of the good. This section also shows why a simple employment contract may be optimal. In Section 5 we show that the model can be used to throw light on vertical integration and on why parties deliberately write incomplete contracts, and we also discuss renegotiation. Finally, Section 6 concludes. The Appendix considers a more general class of contracts than those studied in the text and includes proofs of theorems.
2. The Model

We consider a buyer B and a seller S who are engaged in a long-term relationship. The parties meet at date 0 and can trade at date 1. We assume a perfectly competitive market for buyers and sellers at date 0, but that competition is much reduced at date 1: in fact, for the most part we suppose that B and S face bilateral monopoly at date 1. In other words, there is a fundamental transformation in the sense of Williamson (1985).

We do not model why this fundamental transformation occurs. It could be because the parties make relationship-specific investments, but there may be other more prosaic reasons. For example, imagine that B is organizing a wedding for his daughter. S might be a caterer. Six months before the wedding, say, there may be many caterers that B can approach and many weddings that S can cater. But it may be very hard for B or S to find alternative partners a week before the wedding. While there are no very obvious relationship-specific investments here, the fundamental transformation seems realistic, and the model applies.

It would be easy to fit relationship-specific investments explicitly into the analysis, but we would then suppose that these investments are contractible. That is, an important feature of our model is that it does not rely on noncontractible investments.

We make some standard assumptions. Any uncertainty at date 0 is resolved at date 1. There is symmetric information throughout, and the parties are risk neutral and face no wealth constraints.

We now come to the two assumptions that represent significant departures from the literature. The first is that ex post trade is only partially contractible. Specifically, while the
broad outlines of ex post trade are contractible, the finer points are not. As noted in the 
Introduction, we distinguish between perfunctory and consummate performance, or performance 
within the letter of the contract and performance within the spirit of the contract. Perfunctory 
performance is enforceable by a court while consummate performance can never be judicially 
enforced.

For instance, in the wedding example, a judge can determine whether food was provided, 
but not the quality of the cake or whether the catering staff was friendly to the guests.

Before we describe our second assumption, it is useful to provide a time line; see Figure 
1. The parties meet and contract at date 0. At this stage there may be uncertainty and so the 
parties typically choose to write a flexible contract that admits several outcomes. At date 1 the 
uncertainty is resolved and the parties refine the contract, that is, they decide which outcome to 
pick. After this, trade occurs and the degree of consummate performance is determined.⁷

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Figure 1

What determines the degree of consummate performance? This is where our second key 
(behavioral) assumption comes in. We suppose that consummate performance doesn’t cost 

⁷ Since a court can determine whether trade took place, any payments that B has promised S conditional on trade 
must be made: if not, S would sue for breach of contract. (In other words, payments are part of perfunctory 
performance.)
much relative to perfunctory performance, and that if the circumstances are “right” a party is willing to provide consummate performance – he may even be happy to do so. Moreover, the circumstances are right if the party feels that he is getting what he is entitled to relative to the outcomes permitted by the contract.

Note the key idea that entitlements are defined relative to the contract. That is, the contract acts as a reference point in the sense that it governs or anchors the parties’ entitlements or expectations. We discuss this further below.

We take the view that there is no particular reason why the parties’ feelings of entitlement should be consistent. To make things as simple as possible, we consider the extreme case where each party feels entitled to 100% of the residual surplus, i.e., to the best outcome permitted by the contract.

Getting less than what you feel entitled to causes aggrievement and leads to retaliation. Specifically, for each dollar that a party feels shortchanged he shades his performance so that the other party’s payoff falls by $\theta$, where $0 \leq \theta \leq 1$. We take $\theta$ to be exogenous – it represents both the desire and opportunity for retaliation – but in future work it would be interesting to endogenize it. Note that we do not suppose that shading confers any (significant) direct benefit on the party who shades: it simply hurts the other party.

Shading decisions are made simultaneously by B and S, and are not observable to an outsider. Hence they are not contractible (recall our assumption that consummate performance can never be enforced judicially). They are, however, anticipated by both parties, i.e., the parties have rational expectations. Finally, we suppose that shading is infeasible if the parties do not trade at date 1 (trade can be shaded but no trade cannot be shaded).
The assumption that B and S can shade symmetrically and face the same parameter $\theta$ may seem strong. In the wedding example it is easy to think of ways in which S can shade, but perhaps harder to think of ways in which B can. However, buyers can hurt sellers, e.g., by refusing to make minor concessions or to be cooperative, or by quibbling about details of performance. We view the case of symmetry as a reasonable starting point for our analysis. In Section 6 we discuss alternative interpretations and extensions of the model for the case where shading is asymmetric.

The behavioral assumptions we have made are strong and deserve greater discussion. Before we provide this, it is useful to illustrate the model with a simple example.

Suppose that B requires one unit of a standard good – a widget – from S at date 1. Assume that it is known at date 0 that B’s value is 100 and S’s cost is zero: there is no uncertainty. What is the optimal contract?

The answer found in the standard literature is that, in this setting without noncontractible investments, no ex ante contract is necessary: the parties can wait until date 1 to contract. To review the argument, imagine that the parties do wait until date 1. Assume that Nash bargaining occurs and they divide the surplus 50 : 50, i.e., the price $p = 50$. Of course, a 50 : 50 division may not represent the competitive conditions at date 0. For simplicity, suppose that there is one buyer and many sellers at date 0, so that in competitive equilibrium B receives all the surplus. Then S will make a lump-sum payment of 50 to B at date 0: in effect S pays B up front for the privilege of being able to hold B up once the parties are in a situation of bilateral monopoly.

This “no contract” solution, combined with a lump-sum payment, no longer works in our context. To see why, note that, if the parties do not write a contract at date 0, this is equivalent to
saying that trade can occur at any price between zero and 100. (Prices above 100 are irrelevant since B will reject the widget and prices below zero are irrelevant since S will refuse to supply.) But this means that when the parties reach date 1 there is much to argue about. The best deal possible for B is a zero price and he will feel entitled to it; and the best deal possible for S is a price of 100 and she will feel entitled to it. Our assumption is that, in spite of these conflicting feelings of entitlement, the parties will settle on some price p between 0 and 100, and trade will occur. However, each party will feel aggrieved and will shade. Since B feels shortchanged by p, he shades by $\theta p$, and since S feels shortchanged by $(100 – p)$, she shades by $\theta(100 – p)$. Thus the final payoffs are

\begin{align}
(2.1) \quad U_B &= 100 – p – \theta(100 – p) = (1 – \theta)(100 – p), \\
(2.2) \quad U_S &= p – \theta p = (1 – \theta)p,
\end{align}

and total surplus is given by

\begin{align}
(2.3) \quad W &= (1 – \theta)100.
\end{align}

We see that, independent of p, there is a loss of $100 \theta$.

What can be done to eliminate this loss? The first point to note is that ex post Coasian renegotiation does not do the job. The reason is that shading is not contractible, and thus a contract not to shade is not enforceable. To put it another way, if B offers to pay S more not to shade, then while this will indeed reduce S’s shading since S will feel less aggrieved, it will
increase B’s shading because B will feel more aggrieved! In fact, it is clear from (2.3) that changes in p do not affect aggregate shading, which is given by 100θ.

Note that the conclusion that the loss from shading equals 100θ remains true even if the parties replace renegotiation at date 1 by a mechanism. For example, suppose B and S agree at date 0 that B will make a take-it-or-leave-it offer to S at date 1. The best offer for B to make is p = 0. However, S will feel that B could and should have offered p = 100 since S is entitled to this. Thus S will be aggrieved by 100, and will shade by 100θ. Hence the loss from shading is again 100θ.

Although these approaches do not work, there is a very simple solution to the shading problem. The parties can write a contract at date 0 that fixes p at some level between 0 and 100, e.g., if there are many sellers and only one buyer at date 0, then it would be natural to set p = 0. Then there is nothing to argue about at date 1. Neither party will feel aggrieved or shortchanged since each receives exactly what he or she bargained for and expected. Hence no shading occurs and total surplus equals 100.8

As we have noted earlier, a contract that fixes price works because it anchors the parties’ expectations and feelings of entitlement: the contract is a reference point.9 An obvious question to ask is, what changes between dates 0 and 1? Why does a date 0 contract that fixes p = 0 avoid aggrievement by S, whereas a date 1 contract that fixes p = 0 does not? Our view is that the ex

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8 Note that we are ignoring “efficiency wage” considerations in our analysis. Regardless of date 0 market conditions, B might well feel that it makes sense to offer S a price in excess of cost in order to encourage better performance. However, note that efficiency wage ideas are not inconsistent with our approach. Our view is that, whatever the level of the price, it still makes sense for B and S to fix price in advance in order to avoid argument about the right price later.

9 The notion of a reference point has played an important role in the recent behavioral economics literature, including that concerned with contractual relationships. Kahneman et al. (1986) provide evidence that for transactions between firms and consumers customers use past prices as a reference point for judging the fairness of a transaction. See also Okun (1981), Falk et al. (2006), Frey (1997, Chapter 2), and Gneezy and Rustichini (2000) for related ideas. Benjamin (2005) analyzes the implications of reference points for optimal incentive schemes, and Carmichael and MacLeod (2003) for the hold-up problem. Our paper owes a lot to the above literature, but differs from it in supposing that a contract governing a transaction is a reference point for the transaction itself.
ante market plays a crucial role here. Since the date 0 market is much more competitive than the
date 1 market – for simplicity we have supposed that the date 0 market is perfectly competitive
while the date 1 market is perfectly noncompetitive – it provides an external, i.e., objective,
measure of what B and S bring to the relationship. Our assumption is that B and S continue to
accept these external measures, now embodied in the contract, once their relationship is
underway. In contrast, if B and S pass up the opportunity to write a contract at date 0, then by
the time date 1 arrives there are no external measures to control entitlements, and the result will
be argument, aggrievement, and shading.\textsuperscript{10}

At this point it makes sense to discuss a little further the retaliation and entitlement
assumptions underlying our analysis. Start with retaliation. One view is that consummate
performance costs only a little more than perfunctory performance, and that a party will willingly
bear this cost if he feels “well-treated,” given that the benefit to the other party is large, i.e.,
parties are naturally at least slightly altruistic. However, if the first party feels shortchanged,
then he will no longer be willing to help the other party and will shade. A second view is that
consummate performance is actually more pleasurable to provide than perfunctory performance,
but that a party will forego this pleasure to punish a partner who he feels did not treat him
generously.

Both views find support in the large behavioral economics literature, which has examined
altruism, reciprocity, and retaliation.\textsuperscript{11} The easiest way to connect our model to this literature is

\textsuperscript{10} To emphasize this point, suppose that B and S are in a situation of bilateral monopoly already at date 0, i.e., the
date 0 market is no more competitive than the date 1 market. Then, if they agree on $p = 50$, say, at date 0, our view
is that each will be aggrieved by 50 and this aggrievement will carry over to date 1 and lead to shading.

\textsuperscript{11} For example, in the ultimatum game (see, e.g., Guth et al. (1982)), a suggested split of surplus by the proposer that
is seen as “greedy” will often elicit retaliation in the form of rejection by the responder, even though this is costly
for the responder. See Camerer and Thaler (1995) for a discussion, and Andreoni et al. (2003) for experimental
evidence for the case where the responder can scale back the level of trade rather than rejecting trade entirely
(scaling back is like “shading” in our model). Other important works on reciprocity and retaliation include Akerlof
(1982), Rabin (1993), Fehr et al. (1997), and Bewley (1999); for surveys see Fehr and Gachter (2000) and Sobel
to think of the choice of the outcome at date 1 – the refinement process in Figure 1 – as determining how well-treated a party feels, and shading as each party’s response to/retaliation for this treatment.\textsuperscript{12}

Let’s turn next to our assumption about entitlements, particularly the idea that each party feels entitled to 100% of the residual surplus. This is obviously a very different view from that found in the fairness part of the behavioral literature, e.g., according to the fairness view, the parties might be expected to agree on and be comfortable with a 50 : 50 split of the surplus: \( p = 50 \) in the example.\textsuperscript{13} Our view is closer to the literature that emphasizes self-serving biases.\textsuperscript{14} According to that literature, each party can tell a story that justifies an outcome that is very favorable to him. For instance, in our example, if no contract is written at date 0, S can argue to herself that the widget is really worth 200 to B at date 1 because S produces fabulous widgets, even though B does not realize this (i.e., B is not willing to pay more than 100). Hence according to S the fair price \( p = 100 \). Similarly, B can argue to himself that S’s costs are actually extremely low (-100) or that S receives some other benefit (100) from the transaction with B, even though again S does not realize this. Hence according to B the fair price \( p = 0 \). In both cases the parties are constrained by the reality that B is not willing to pay more than 100 and S is not willing to receive less than zero (the IR constraints). However, within this reality, each party can tell a story that justifies his most favorable outcome as being fair.\textsuperscript{15}

\textsuperscript{12} Note that the experimental evidence of Falk et al. (2003) is consistent with the idea that the level of aggrievement/retaliation/shading will depend not only on the outcome that occurs but also on what other outcomes were available (see also Camerer and Thaler (1995)).

\textsuperscript{13} On fairness, see, e.g., Fehr and Schmidt (1999).

\textsuperscript{14} See, e.g., Hastorf and Cantril (1954), Messick and Sentis (1979), Ross and Sicoly (1979), and Babcock et al. (1995). For a discussion see Babcock and Loewenstein (1997).

\textsuperscript{15} See Rabin (1995) for a general discussion of self-serving biases. Conflicting notions of entitlement may also arise because of differences in information about the total surplus available. See Ellingsen and Johannesson (2005). As we have noted, the assumption that each party feels entitled to 100% of the residual surplus is extreme. It is enough
All of this, of course, requires further examination, both theoretically and empirically/experimentally. For the moment, we view the behavioral literature as providing some loose support for the approach taken here.

The example analyzed in this section is very special because a date 0 contract that fixes price achieves the first-best. The first-best is no longer achievable if either (a) \( v, c \) are uncertain; or (b) the nature of the good (the widget) is uncertain. We study case (a) in Section 3 and case (b) in Section 4.

3. The Case Where Value and Cost Are Uncertain

In this section we consider the case where \( B \) wants one unit of a standard good – a widget – from \( S \) at date 1 but there is uncertainty about \( B \)’s value \( v \) and \( S \)’s cost \( c \). This uncertainty is resolved at date 1. There is symmetric information throughout, so that \( v, c \) are observable to both parties. However, \( v, c \) are not verifiable, and so state-contingent contracts cannot be written.

We make an important simplifying assumption in both this and the next section. We suppose that trade occurs at date 1 if and only if both parties want it, i.e., trade is voluntary. To put it another way, if no trade occurs an outsider (e.g., a judge) cannot tell whether this is because the seller refused to supply the widget or the buyer refused to accept it.\(^{16}\) As a result, a party cannot be punished for breach of contract. We are confident that the main ideas of the

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\(^{16}\) This assumption is taken from Hart and Moore (1988).
paper generalize to the case where specific performance is possible, but the details become more complicated.

In this setting the simplest kind of contract consists of a no-trade price $p_0$ and a trade price $p_1$. Given such a contract, trade will occur ($q = 1$) if and only if

\[(3.1) \quad v \geq p_1 - p_0 \geq c.\]

From (3.1) it is clear that only the difference between $p_1$ and $p_0$ matters, and so, given the existence of lump-sum transfers, we can normalize $p_0$ to be zero.

It is worth comparing (3.1) to the first-best trading rule, given by

\[(3.2) \quad q = 1 \Leftrightarrow v \geq c.\]

Since we want to allow for contractual flexibility we shall wish to generalize beyond simple contracts. One way to introduce flexibility is to suppose that the contract specifies a no-trade price $p_0$ and an interval of trading prices $[p, \bar{p}]$. Suppose for simplicity that $B$ chooses the trade price at date 1. Then

\[(3.3) \quad q = 1 \Leftrightarrow \exists \quad p \leq p_1 \leq \bar{p} \quad \text{s.t.} \quad v \geq p_1 - p_0 \geq c.\]
In other words, trade occurs if and only if B can find a price in the range \([p, \bar{p}]\) so that the parties want to trade (B will choose the lowest such price). Actually, it is clear that the same trading rule (3.3) holds if S chooses \(p_1\) (S will choose the highest price in the range \([p, \bar{p}]\) that guarantees trade). This feature – that the mechanism for choosing the outcome doesn’t matter – is special to the model of this section: it will importantly not hold in the model of Section 4.

It follows from (3.3) that, again, only the difference between \(p_1\) and \(p_0\) matters, and so we can normalize \(p_0 \equiv 0\) and rewrite (3.3) as

\[
(3.4) \quad q = 1 \iff \exists \quad p \leq p \leq \bar{p} \text{ s.t. } v \geq p \geq c
\]

\[
\iff v \geq c, v \geq p, c \leq \bar{p}.
\]

More general contracts than \(p_0 \equiv 0, p \in [p, \bar{p}]\) are in fact possible. For example, a contract could permit \(p\) not to be in an interval, or it could allow \(p_0\) and \(p_1\) both to vary. In the Appendix we show that our main ideas extend to the case where a contract consists of an arbitrary set of \((p_0, p_1)\) pairs and a mechanism – a game – for choosing among them. In order to carry out this generalization, we need to slightly refine our assumptions about the determinants of a party’s aggrievement level.

We need to deal with one further issue before we proceed. After the uncertainty about \(v, c\) is resolved, suppose \(v > c\) but either \(v < p\) or \(c > \bar{p}\). At this stage, the parties might want to renegotiate their contract. Renegotiation does not fundamentally change our results and so, for the moment, we ignore it; we return to it in Section 5.

Given a contract \([p, \bar{p}]\) (that is, \(p_0 \equiv 0, p \in [p, \bar{p}]\)), what determines aggrievement? Recall from Section 2 that each party feels entitled to the best outcome possible subject to the other
party’s individual rationality constraint being satisfied. Given our voluntary trade assumption, this means that S feels entitled to \( p = \min(v, \bar{p}) \) and S feels entitled to \( p = \max(c, \underline{p}) \). Thus aggregate aggrievement equals \( \{ \min(v, \bar{p}) - \max(c, \underline{p}) \} \). An optimal contract maximizes expected surplus net of shading costs. (Lump-sum transfers are used to reallocate surplus.) Thus an optimal contract solves:

\[
\text{(3.5)} \quad \max_{p, \bar{p}} \int_{v \geq c} [v - c - \theta(\min(v, p) - \max(c, p))] dF(v, c),
\]

where \( F \) is the distribution function of \((v, c)\).

The trade-off is clear. A large interval \([p, \bar{p}]\) makes it more likely that trade will occur if \( v \geq c \). (If \( p = -\infty, \bar{p} = \infty \), the trading rule becomes the first-best one: \( q = 1 \Leftrightarrow v \geq c \).) However, it also increases expected shading costs.

We refer to a contract where \( p = \bar{p} \) as a **simple** contract, and a contract where \( p < \bar{p} \) as a **non-simple** contract. We start off with some cases where the first-best is achievable with a simple contract.

**Proposition 3.1.** A simple contract achieves the first-best if (i) only \( \bar{v} \) varies; (ii) only \( \bar{c} \) varies; (iii) the smallest element of the support of \( \bar{v} \) is at least as great as the largest element of the support of \( \bar{c} \).
The proof of Proposition 3.1 is immediate. If only \( \tilde{v} \) varies, choose a simple contract with \( p = c \). If only \( \tilde{c} \) varies, choose a simple contract with \( p = v \). If (iii) holds, choose a simple contract with \( p \) between the smallest \( v \) and largest \( c \).

In some cases one needs a non-simple contract to achieve the first-best.

**Example 3.1**

Suppose that there are two states of the world. In \( s_1 \), \( v = 9 \), \( c = 0 \). In \( s_2 \), \( v = 20 \), \( c = 10 \). In other words, either \( v \) and \( c \) are both low or they are both high.

\[
\begin{array}{ccc}
\text{s1} & \text{s2} \\
v & 9 & 20 \\
c & 0 & 10
\end{array}
\]

Obviously, one cannot get the first-best with a simple contract since there is no price \( p \) that lies both between 0 and 9 and between 10 and 20. However, a contract that specifies an interval of trading prices \([9,10]\) (\( p = 9 \), \( \bar{p} = 10 \)) does achieve the first-best. To see why, note that in \( s_1 \) B will choose \( p = 9 \) since this is the lowest available price. S will not be aggrieved since, even if S could choose the price, she would not pick a price above 9 given that this would cause B not to trade. In \( s_2 \) B picks \( p = 10 \) since this is the lowest price consistent with S being willing to trade. S is again not aggrieved since she couldn’t hope for a higher price than 10 given that 10 is the highest available price. Thus, the contract \( p = 9 \), \( \bar{p} = 10 \) achieves trade in both states without any shading.
Note that in this example the optimal contract is unique. Any price range smaller than [9,10] would fail to generate trade in one of the states, and any price range larger than [9,10] would cause aggrievement in at least one of the states. (If \( p < 9 \), the parties would argue about where \( p \) should be in the range \([p,9]\) in \( s_1 \), and if \( \overline{p} > 10 \), the parties would argue about where \( p \) should be in the range \([10,\overline{p}]\) in \( s_2 \).)

We now turn to an example where the first-best cannot be achieved even with a non-simple contract.

Example 3.2

The example is the same as the previous one except that there is a third state, \( s_3 \), where \( v \) is high and \( c \) is low.

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The first-best cannot be achieved because, in order to ensure trade in \( s_1, s_2 \), we need \( p \leq 9, \overline{p} \geq 10 \). But such a price range leads to aggrievement and shading in \( s_3 \).

There are three possible candidates for a second-best optimal contract:

(a) \( p = \overline{p} = 9 \).
This contract yields trade in s1 and s3 but not in s2. Since there is nothing to argue about – the price is fixed at 9 -- there is no shading. Total surplus is given by

\[ W_a = 9 \pi_1 + 20 \pi_3, \]

where \( \pi_1, \pi_3 \) are the probabilities of s1, s3, respectively.

(b) \( p = \bar{p} = 10. \)

This contract yields trade in s2 and s3 but not in s1. Since there is nothing to argue about – the price is fixed at 10 -- there is no shading. Total surplus is given by

\[ W_b = 10 \pi_2 + 20 \pi_3, \]

where \( \pi_2 \) is the probability of s2.

(c) \( p = 9, \bar{p} = 10. \)

This contract yields trade in all three states, but there is aggregate aggrievement of 1 in s3. Total surplus is given by

\[ W_c = 9 \pi_1 + 10 \pi_2 + (20 - \theta) \pi_3. \]
Obviously, which of these contracts is optimal depends on the probabilities \(\pi_1, \pi_2, \pi_3\) and \(\theta\). Contract (a) is optimal if \(\pi_2\) is small, contract (b) is optimal if \(\pi_1\) is small, and contract (c) is optimal if \(\pi_3\) or \(\theta\) is small.

Two observations can be made about this example. First, if third parties are permitted, the first-best can be achieved. Consider a contract that fixes the trade price and makes both B and S pay a large amount to a third party in the absence of trade (i.e., the no-trade price is large and positive for B and large and negative for S). This leads to trade in all states, and no aggrievement, since the consequences of not trading are dire. However, this arrangement works only because trade is efficient in every state. In a more general example where trade is efficient in some states but not others, third parties do not guarantee the first-best. In what follows we ignore third parties.

Second, the reader may wonder whether “Maskin mechanisms” could improve matters.\(^{17}\) Maskin mechanisms are a way of making observable information verifiable. Note that if the state were verifiable, it would be easy to achieve the first-best. For example, a contract that specifies \(p = 9\) in \(s_1\) and \(s_3\), and \(p = 10\) in \(s_2\) would do the job. Call this contract (d).

However, Maskin mechanisms do not work in our situation. A Maskin mechanism is a subtle version of the following. Each party reports the state of the world. If they agree the price is as in contract (d), say. If they disagree they pay a large penalty (e.g., to a third party). The problem is that in \(s_3\) S would like B to play the Maskin mechanism as if it were \(s_2\) and will be aggrieved by \(1\) if B refuses to go along with this. On the other hand B will be aggrieved by \(1\) if S refuses to play the mechanism as if it were \(s_3\). Either way aggregate aggrievement in \(s_3\) is \(1\), which yields total surplus equal to \(W_c\), as in contract (c).

\(^{17}\) See Maskin (1999).
This brings to a close our discussion of the case where parties induce flexibility by specifying a price range. It is not clear how realistic this case is. One reason is that in practice the parties may be able to ensure trade when \( v \) and \( c \) vary through specific performance. In any event, we see the model of this section as something of a five-finger exercise (although it will be useful when we study vertical integration in Section 5). In the next section we consider a model where the uncertainty concerns the nature of the good to be provided. We will show that this model can shed light on the employment relationship.

4. The Case Where the Nature of the Good is Uncertain

In this section we consider the case where there is uncertainty about the nature of the good or service \( B \) requires from \( S \). For example, \( S \) might provide secretarial services for \( B \), and \( B \) may not know in advance whether he wants \( S \) to type letters or file papers. We will actually use a more colorful example. We will suppose that \( B \) is arranging an evening with friends and wants \( S \) to perform music. The nature of the music may depend on eventualities that will occur between dates 0 and 1, e.g., who is coming to the evening, what other performances \( S \) is involved in, etc.

To make matters as simple as possible, we will assume that there are two types of music/composers that it might be efficient for \( S \) to play: Bach and Shostakovich. In the Appendix we also allow for convex combinations of Bach and Shostakovich, but in the text we will not need to do this.

We assume the simplest possible stochastic structure. Each composer can take on one of two value-cost combinations, given by \((v, c)\) and \((v - \Delta, c - \delta)\), respectively, where \( v > v - \Delta > c > c - \delta \). (Everything is measured in money terms.) In other words a composer can be “high
value-high cost” or “low value-low cost.” We do not insist on stochastic independence of the
two value-cost combinations, but we do impose symmetry, i.e., the probability that Bach is “high
value-high cost” and Shostakovich is “low value-low cost” is the same as the probability of the
reverse. Thus, there are four states of the world:

\[
\begin{align*}
\text{s1 (Prob } \pi_1) & \quad \text{s2 (Prob } (1 - \pi_1 - \pi_4)/2) & \quad \text{s3 (Prob } (1 - \pi_1 - \pi_4)/2) & \quad \text{s4 (Prob } \pi_4) \\
\text{Bach} & \quad (v,c) & \quad (v,c) & \quad (v-\Delta,c-\delta) & \quad (v-\Delta,c-\delta) \\
\text{Shostakovich} & \quad (v,c) & \quad (v-\Delta,c-\delta) & \quad (v,c) & \quad (v-\Delta,c-\delta)
\end{align*}
\]

Figure 2

We start with the case $\Delta > \delta$. This implies that the high value-high cost composer yields
more surplus than the low value-low cost composer and should be chosen whenever available.
Thus the first-best has any music in states s1 and s4, Bach in s2, and Shostakovich in s3. Total
surplus is $W = v - c - \pi_4 (\Delta - \delta)$.

What is the optimal second-best contract given that the state is observable but not
verifiable?

We proceed heuristically in the text, leaving the formal argument to the Appendix. We
continue to assume voluntary trade and set $p_0 \equiv 0$. We also focus on contracts that deliver
symmetric outcomes, i.e., whatever composer occurs in s2, the “mirror image” composer occurs
in s3, and the prices are the same in the two states.

Suppose first that s1 is the only state, i.e., it occurs with probability 1. Then we are in a
situation similar to that of the model of Section 3 with no uncertainty (see also the example in
Section 2). We know that an optimal contract fixes $p$ somewhere between $c$ and $v$. The choice
of composer doesn’t matter in s1 since both parties are completely indifferent. So the contract
could fix the composer (at Bach, say) or let either B or S choose the composer, given the fixed price p.

Exactly the same argument applies if s4 is the only state, with the one difference being that p should lie between c – δ and v – Δ.

If we put s1 and s4 together, i.e., allow both states to occur with positive probability, then an optimal contract is to fix c ≤ p ≤ v – Δ, and either fix the composer or let B or S choose it.

Now turn to the more interesting states s2, s3. We consider which of the contracts that are optimal for s1 and s4 also work well in s2 and s3.

Fixing c ≤ p ≤ v – Δ works well in s2, s3 since both parties are willing to trade at this price. What about the choice of composer? One possibility is to fix the composer, at Bach say. This yields the efficient choice of composer half the time (Bach is efficient in s2, but inefficient in s3), and so provides average surplus in s2, s3 equal to

$$W' = \frac{1}{2} (v - c) + \frac{1}{2} (v - Δ - c + δ) = v - c - \frac{1}{2} Δ + \frac{1}{2} δ.$$  

(There is no deadweight loss of shading since, with price and composer fixed, there is nothing to argue about.)

Another possibility is to let B choose the composer. Given the fixed price p, B will always choose the highest value composer, i.e., Bach in s2 and Shostakovich in s3, which is efficient. However, S will be aggrieved since S feels entitled to the composer that is best for her, i.e., the low-cost composer. The level of S’s aggrievement equals δ, the difference between her payoff if the low-cost composer were selected, p – (c – δ), and her payoff given that the high-
cost composer is selected, \( p - c \). Hence the deadweight loss from shading = \( \theta \delta \), and average surplus in \( s_2, s_3 \) equals

\[
(4.2) \quad W'' = v - c - \theta \delta.
\]

Finally the contract could specify that \( S \) chooses the composer. If \( \theta \) is small, \( S \) will choose the low-cost, inefficient composer in each state. \( B \) will be aggrieved by \( v - p - (v - \Delta - p) = \Delta \), and will shade by \( \theta \Delta \). Thus average surplus in \( s_2 \) and \( s_3 \) equals \( v - c - \theta \Delta \), which is less than \( W'' \) in (4.2). If \( \theta \) is large, the cost to \( S \) of \( B \)'s shading exceeds the gain \( S \) receives from choosing the low-cost composer (\( \theta \Delta > \delta \)), and so \( S \) chooses what \( B \) wants anyway: the high-cost composer. We see that when \( \Delta > \delta \) a contract where \( S \) chooses the composer is weakly dominated by one where \( B \) chooses the composer.

Thus we have two candidates for an optimal contract when \( \Delta > \delta \): fix the price and the composer or fix the price and let \( B \) choose the composer. To determine which is better, one just compares (4.1) and (4.2). In the Appendix we establish formally:

**Proposition 4.1** Assume \( v > v - \Delta > c > c - \delta \), and \( \Delta > \delta \). Suppose in addition \( \frac{\pi_4}{1 - \pi_1} \geq \frac{m}{(v - c - \Delta + \delta + m)} \), where \( m = \min \{ \theta \delta, \frac{1}{2}(\Delta - \delta) \} \), and \( \frac{\pi_1}{1 - \pi_4} \geq \frac{\delta}{\Delta + \delta} \). Then the optimal second-best contract fixes \( c \leq p \leq v - \Delta \). In addition, if \( \Delta > (1 + 2\theta)\delta \), \( B \) is given the right to choose the composer, while, if \( \Delta < (1 + 2\theta)\delta \), the parties fix the composer, at Bach say.\(^{18}\)

\(^{18}\) Before we go on, it is worth reviewing our assumptions about aggrievement. Take the contract where \( B \) chooses the composer, and suppose \( s_2 \) or \( s_3 \) is realized. We assume that \( S \) feels entitled to the low-cost composer and shades by \( \theta \delta \) when \( B \) chooses the high-cost composer. The reader may wonder whether such behavior makes sense. After
Proposition 4.1 illuminates the different roles played by price and music in the model of this section. Price has no allocative role – its choice is a zero sum game – and so, in order to avoid aggrievement, it is better to fix it in advance. Music does serve an allocative role and so, if $\Delta/\delta$ is large or $\theta$ is small, it makes sense to leave it open. Moreover, when $\Delta > \delta$, B should choose the composer since B will make an efficient choice, and, given that S cares relatively little, aggrievement will be low.

Note that there are two implicit assumptions underlying Proposition 4.1. First, aggregate uncertainty is small ($v - \Delta > c$), and so price does not have to vary across the four states in order to ensure that both parties wish to trade, as it did in Section 3. Second, there is no systematic relationship between composer and cost. In contrast, if Shostakovich, say, was on average costlier for S to play than Bach, then it would be optimal to have a higher price for Shostakovich than Bach, in order to reduce S’s aggrievement in states where Shostakovich is chosen.

Let’s now turn to the case where $\Delta < \delta$, i.e., the low value-low cost composer yields more surplus than the high value-high cost composer. The argument goes through as above except that now it is never optimal for B to choose the composer but it may be optimal for S to choose the composer. We have

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all, the parties have rational expectations and so doesn’t S anticipate that she will never get the composer she wants? Why is she annoyed by B’s behavior and why is she motivated to retaliate? We believe that our assumptions about S’s behavior can be justified using the notion of self-serving biases described in Section 2. Suppose the state is s2. S can tell herself the following. It’s true that B thinks that Bach is worth $\Delta$ more to him than Shostakovich, while the incremental cost to me, S, is only $\delta$, and B therefore feels justified in choosing Bach. However, I, S, think that even though B doesn’t realize it, the true value of Bach is close to $v - \Delta$. (Who knows why S thinks this, but there are undoubtedly many ways in which she can convince herself that it is true.) Given this, I do not think that B’s request to me to play Bach is reasonable. Hence I, S, will find some way to get back at B if he insists on Bach, e.g., by not putting my heart into the performance, quibbling about details, etc.

24
Proposition 4.2 Assume \( v > v - \Delta > c > c - \delta \), and \( \Delta < \delta \). Suppose in addition

\[
\frac{\pi_1}{1 - \pi_4} \geq \frac{m'}{(v - c + m')}, \text{ where } m' = \min \{\theta \Delta, \frac{1}{2}(\delta - \Delta)\}, \text{ and } \frac{\pi_4}{1 - \pi_1} \geq \frac{\Delta}{\delta + \Delta}.
\]

Then the optimal second-best contract fixes \( c \leq p \leq v - \Delta \). In addition, if \( \delta > (1 + 2\theta) \Delta \), S is given the right to choose the composer, while, if \( \delta < (1 + 2\theta) \Delta \), the parties fix the composer, at Bach say.

We believe that Propositions 4.1 and 4.2 can throw light on a classic question: the nature of the employment relationship and the difference between an employee and an independent contractor. In early work, Coase (1937) and Simon (1951) argued that a key feature of the employment relationship is that wage is fixed and an employer tells an employee what to do. This view was challenged by Alchian and Demsetz (1972), and the more recent literature has emphasized asset ownership as the distinguishing aspect of these relationships (see Grossman and Hart (1986) and Hart and Moore (1990)).\(^{19}\) The current model allows us to return to the ideas of Coase and Simon. We interpret the case where B chooses the composer as an employment relationship and the case where S chooses the composer as independent contracting.\(^{20}\) That is, if B hires S’s musical services for the evening, with the understanding that B will tell S what to do, then S is working for B. In contrast, if B engages S to provide an evening of music, with the details of exactly how this is to be done left up to S, then S is an independent contractor.

Propositions 4.1 and 4.2 tell us that when \( \theta \) is small, if B cares more about the composer, that is, \( \Delta > \delta \), then employment is better; while if S cares more about the composer, that is, \( \Delta < \delta \),

\(^{19}\) But see Wernerfelt (1997).

\(^{20}\) This notion of independent contracting differs from Simon’s. Simon views independent contracting as corresponding to the case where the price and composer are both fixed.
then independent contracting is better. In both cases it is optimal for the parties to fix the price in advance.

While these results are in the spirit of Coase and Simon, they differ from Simon’s formal argument in important ways. Simon would also argue that B should choose the composer if B cares more about the composer than S. However, in Simon’s model it is not clear why an ex ante contract is needed at all. Since there is no aggrievement (not surprisingly!) and no noncontractible investments, the parties can rely on Coasian bargaining at date 1. Also, a contract that achieves the first-best in Simon’s model is one where B has the right to make a take-it-or-leave-it offer to S; i.e., B proposes a price-composer pair, and S can accept or reject it. In other words, in Simon’s model there are many optimal contracts (a continuum, in fact), of which the employment contract is just one.

We saw in Section 2 that this is not true in our model. For example, consider the contract in which B offers a price-composer pair. In s1 B will suggest any composer at price c, in s2 Bach at price c, in s3 Shostakovich at price c, and in s4 any composer at price c – δ. There will be aggrievement and shading in all states, since S will feel entitled to a higher price (v in s1, s2, s3, and v – Δ in s4). Thus this contract performs strictly worse than the employment contract.

In other words a virtue of our model is that it can explain why the employment contract is uniquely optimal when Δ > δ and θ is small; why independent contracting is uniquely optimal when Δ < δ and θ is small; and why in all the cases considered in this section it makes sense for the parties to fix price ex ante, i.e., to take price off the table.

5. An Application and an Extension
In this section we apply the model to vertical integration and discuss renegotiation.

5.1 Vertical Integration

Consider the model of Section 3, but now interpret $c$ as an opportunity cost. That is, suppose that $S$ can supply the widget to another buyer and receive an amount $r = c$ for it. For simplicity focus on simple contracts, i.e., a contract consists of a trading price $p$. Then the trading rule is, as in Section 3,

\[(5.1) \quad q = 1 \Leftrightarrow v \geq p \geq r.\]

We want to compare this arrangement, which we think of as nonintegration, with an alternative arrangement: vertical integration. We interpret (vertical) integration to mean that $B$ owns $S$’s assets and therefore in particular $B$ owns, i.e., can seize, $S$’s widget at date 1. However, $B$ can sell it back to $S$, to whom it is worth $r$, if both parties agree. We continue to assume that only $S$ has access to the outside opportunity $r$; for example, this outside opportunity might be embodied in $S$’s human capital.

Under integration a contract again consists of a trading price $p'$. The status quo is now that $B$ consumes the widget, i.e., $q = 1$, and receives $v$. However, if both parties are better off, the widget will be transferred to $S$ ($q = 0$), and $S$ will receive $r$. In other words, the new trading rule is

\[(5.2) \quad q = 0 \Leftrightarrow v \leq p' \leq r.\]
It is easy to compare the two arrangements. The first-best has $q = 1 \Leftrightarrow v \geq r$. Under nonintegration, (5.1) implies that we have $q = 1$ too little of the time since $v \geq r$ is a necessary but not a sufficient condition for $v \geq p \geq r$. Under integration it’s the other way around. It follows from (5.2) that $q = 0$ too little of the time since the first-best has $q = 0 \Leftrightarrow v \leq r$, and $v \leq r$ is a necessary but not a sufficient condition for $v \leq p' \leq r$.

Thus nonintegration and integration are both inefficient, but in opposite ways. Under nonintegration B consumes the widget too little of the time. Under integration B consumes the widget too much of the time. Consider Example 3.2 in Section 3. In that example $v \geq r = c$ in all states. Thus $q = 1$ is always efficient, and integration guarantees this outcome. In contrast we saw that under nonintegration the first-best cannot be achieved in Example 3.2. However, if we considered an example where $v \leq r$ with probability 1, then our conclusion would be reversed: nonintegration would dominate integration.

In our model, vertical integration is a substitute for specific performance: another way for B and S to achieve the first-best in Example 3.2 would be for them to write a specific performance contract, $q = 1$. However, under our voluntary trade assumption, a specific performance contract is not feasible. That is, the only way to ensure $q = 1$ is to have a range of trading prices, but this leads to aggrievement in s3. We believe that the model of Section 3 can be extended to allow for specific performance, and that the distinction between nonintegration and integration will continue to matter. However, we leave this for future work.

5.2 Renegotiation
So far we have ignored renegotiation. We now consider what happens if renegotiation is allowed. Our discussion will also throw light on why parties deliberately write incomplete contracts.

Start with the model of Section 3. Suppose B and S write a contract consisting of the price range \([p, \bar{p}]\). Then after the uncertainty about \(v\), \(c\) is resolved at date 1 it is possible that \(v > c\) and yet either \(v < p\) or \(c > \bar{p}\). In other words, trade is efficient but won’t occur under the contract. What happens?

One possibility is to suppose that B and S write a new contract. Assume they do this as if no contract ever existed. Then B will feel entitled to \(p = c\) and S to \(p = v\). Total shading is \(\theta (v – c)\), and net surplus is \((1 – \theta) (v – c)\). Note that renegotiation does not achieve the first-best whenever \(\theta > 0\).

How does this affect the analysis? Take Example 3.2. Contract (c) is unchanged since no renegotiation occurs. However, the surplus in contracts (a), (b) rises. Under (a) renegotiation will take place if \(s_2\) occurs. Under (b) renegotiation will take place if \(s_1\) occurs. Thus total surplus under (a), (b) is now

\[
W_a' = 9 \pi_1 + 10(1 - \theta) \pi_2 + 20 \pi_3,
\]
\[
W_b' = 9(1 - \theta) \pi_1 + 10 \pi_2 + 20 \pi_3.
\]

Contracts (a) or (b) might now beat contract (c) even if, in the absence of renegotiation, they did not.

In our opinion this view of renegotiation is too rosy. It seems optimistic to suppose that the possibility of changing price in one state will not affect parties’ feelings of entitlement in
other states. For example, if under contract (a), the price is raised to 10 in s2 as a result of renegotiation, why wouldn’t S feel entitled to a price of 10 in s3? Of course, if S does think this way, then it is as if the contract specified that the price could be in the [9, 10] range in the first place, and we are back to contract (c).

In our opinion an intellectually more coherent position is that renegotiation of the trading price is impossible once it has been specified: any flexibility in the trading price must be built into the initial contract. That is, one can set \( p = 9 \) or \( p = 10 \) or \( p \in [9, 10] \), but one cannot set \( p = 9 \) and then change it to \( p = 10 \).

Moreover, as we have discussed elsewhere, we believe that this position is consistent with legal practice and social custom.\(^{21}\) The courts regard contract renegotiations with some suspicion and may overturn them if they believe that opportunism or duress has played a role. (Social attitudes and norms often mirror the law.) To this end, the courts require that renegotiation must be in “good faith,” but, since this is difficult to monitor, they will often substitute the requirement that the renegotiation can be justified objectively, e.g., the price increases because the seller is supplying an additional service and her costs have risen.\(^{22}\) In our model, no extra service is provided, and so there is no objective justification for a price change, say from \( p = 9 \) to \( p = 10 \).\(^{23}\)

It is important to emphasize that the above argument does not rule out all renegotiations. Suppose that the parties have the opportunity to trade a second widget. Imagine that they specified the price of the first widget but never mentioned a price for an additional widget. Then

\(^{22}\) See Restatement (Second) of Contracts, Section 84(a)(1979); Farnsworth (1999, pp. 276-95); Jolls (1997, pp. 228-301); Muris (1981, particularly p. 530); and Shavell (2005).
\(^{23}\) In fact, without some constraint on price changes, a long-term contract would have little meaning. Almost every contract is incomplete in the sense that some ex ante noncontracted-for cooperation is required ex post for the contract to succeed. If each party can demand a large sidepayment for that cooperation that is completely unrelated to costs – you want a glass of water that will cost you $1,000 – the initial contract will be vitiated.
a price for the second widget that is in some way related to the price of the first widget might pass muster with the courts and with society. (Of course, if some measure of cost is available, a price for the second widget based on S’s incremental cost would also be justifiable.) In other words the parties have more flexibility in renegotiating or setting the price of something that has never been priced before than in changing the price of something that has.

There is a further important element to the story. Given that a second widget was never mentioned in the initial contract, it seems plausible that neither party will feel entitled to a second widget, i.e., the possibility of a second widget will not affect parties’ aggrievement levels or shading behavior in other states of the world. This is the assumption we will make: outcomes excluded from a contract do not affect feelings of entitlement.24

This perspective yields an interesting trade-off between writing a more or less complete contract. Putting an extra outcome into the contract, e.g., trade of a second widget, has an advantage and a disadvantage. The advantage is that the parties can choose any price they like for this outcome. The disadvantage is that once such an outcome is in the contract it affects parties’ feelings of entitlement in all states of the world. Similarly, not putting an extra outcome into the contract has an advantage and a disadvantage. The advantage is that entitlements are not affected. The disadvantage is that if the outcome is added later it can be done so only at a price that is subject to some constraints.

It is useful to apply these ideas to the model of Section 4. To do so we modify slightly the example in Figure 2. Suppose that in s3 Bach has the value-cost pair \( (v - \Delta, c + \delta) \) instead of \( (v - \Delta, c - \delta) \). In other words, in s3 both parties prefer Shostakovich to Bach, given a constant price. All other states and value-cost pairs stay the same. Consider two contracts. The first

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24 This assumption is related to ideas in Gneezy and Rustichini (2000) and Frey (1997).
fixes \( c \leq p \leq v - \Delta \), and lets B choose the composer. The second fixes both the price, \( c \leq p \leq v - \Delta \), and the composer, at Bach.

Under the first contract, the logic is as in Section 3. B chooses Bach in s2 and Shostakovich in s3. S is aggrieved by \( \delta \) in s2 and is not aggrieved in s3 (S feels entitled to Shostakovich and gets it). Thus there is a welfare loss of \( \theta \delta \) in s2. (There is no aggrievement or welfare loss in s1 or s4.)

Consider the second contract. In s2 Bach is efficient and, since the contract specifies Bach, Bach will be the outcome. S will not be aggrieved since she does not feel entitled to anything but Bach given that the contract mentions no other composer. Suppose s3 occurs. Now both parties prefer Shostakovich to Bach at the same price. Will they renegotiate their contract? Our view is that they will. Changing composers without changing price is likely to pass muster with society since there is no objective change in S’s costs (or B’s value): one composer looks very much like another to outsiders. Thus, a zero-price change is justified.

Hence the second contract achieves the first-best and dominates the first contract. Of course, the example is very special. Nonetheless it shows that it may make sense for parties deliberately to write an incomplete contract, that is, leave out an outcome – and renegotiate the contract, that is, add the outcome, later.25

6. Summary and Conclusions

In this paper we have developed a theory of contracts based on the view that a contract provides a reference point for a parties’ trading relationship. The idea is that a contract written

25 Ours is, of course, not the only explanation for why parties write incomplete contracts. See, e.g., Bernheim and Whinston (1998).
early on when an external measure of the parties’ contributions to the relationship is provided by competitive markets can continue to govern the parties’ feelings of entitlement later when they become locked in to each other. The anchoring of entitlements in turn limits disagreement, aggrievement, and the deadweight losses from shading. We have shown that our theory yields a trade-off between contractual rigidity and flexibility, provides a basis for long-term contracts in the absence of noncontractible investments, and throws light on the nature of the employment relationship.

Our theory is based on strong behavioral assumptions. While we believe that these have merit, the reader may wonder whether a trade-off between rigidity and flexibility could be obtained in more traditional ways. While we are sympathetic to this possibility, it is not clear to us how to carry it out.\(^2^6\) Take rent-seeking, for example.\(^2^7\) Rent-seeking arguments suggest that a flexible contract that “leaves money on the table” will generate inefficiency ex post as the parties fight over the surplus. However, rent-seeking theories suffer from two shortcomings. First, it is not clear why parties cannot negotiate around the rent-seeking. Second, rent-seeking theories do not explain why mechanisms don’t work. For example, if one party, B say, is given the right to make a take-it-or-leave-it offer to the other party, S, flexibility can be achieved without any obvious rent-seeking costs.

A second possibility is to invoke influence-cost arguments.\(^2^8\) These can explain why mechanisms are costly – in the above example S will waste resources trying to influence B’s decision. However, it is again not clear why the parties cannot negotiate around this. Also,

\(^{26}\) But see Bajari and Tadelis (2001). Bajari and Tadelis consider a model where rigidity, in the form of a fixed price, is good because it encourages efficient cost reduction by the seller S, but bad because it impedes ex post adjustment. Our model has somewhat similar ex post characteristics to Bajari and Tadelis’ s, but ignores ex ante incentives for cost minimization.

\(^{27}\) See, e.g., Tullock (1967).

\(^{28}\) See, e.g., Milgrom (1988).
influence-cost models typically do not yield a simple formula for welfare losses in the way that our model does.

We believe that some of our behavioral assumptions can be relaxed. Consider first our assumption that a contract perfectly controls entitlements. In some situations this is too strong. Consider an employee who agrees to work for her employer at some wage. Six months later she finds that one of her colleagues is being paid 25% more than she is. She is likely to feel aggrieved and may engage in shading. This example shows that events external to the contract can affect entitlements and shading. It would be interesting to generalize the model to allow for this possibility. We believe that our theory will survive such a generalization: it is enough for the theory that a contract has a role in controlling entitlements; it is not necessary that it has the only role.

Consider next our assumption that shading by B and S is symmetric. As mentioned in the text, it is easier to think of examples of S’s shading than of B’s shading. If we supposed no B shading, the model as it stands would collapse: the first-best could be achieved by giving S the right to make a take-it-or-leave-it offer to B. S would choose the efficient price-output or price-quality combination, thereby extracting all the surplus, and, while B does not like this, there is nothing he can do about it. One way to avoid this conclusion is to suppose that B must make an ex ante noncontractible investment, so that complete hold-up by S is undesirable. Another way is to suppose that S is wealth constrained and so cannot compensate B in advance for the 100% of the surplus that S will obtain ex post. A third possibility, which relies on even stronger behavioral assumptions than we have made, is to suppose that B dislikes feeling “taken advantage of” or “exploited” by S, that is, a contract that allows S to make a take-it-or-leave-it offer to B will cause a psychic loss for B if S chooses not to be generous. For instance, in the
wedding example, B obtains disutility from being gouged ex post by the caterer. This psychic loss is akin to the deadweight loss from shading: the difference is that B internalizes it (“eats it”) rather than being able to retaliate and shift it to S. (One could even suppose that S incurs a psychic loss from the fact that S’s offer will be perceived as ungenerous by B.)

There is a third assumption that could be relaxed. We have supposed that the parties’ feelings of entitlement are maximally inconsistent: each party feels entitled to 100% of the residual surplus. As noted earlier, we believe that our results would go through as long as the sum of the parties’ entitlements exceeds the total amount available. In fact, one could even embellish the model so that, with some probability, the parties’ entitlements sum to less than the total available; under these conditions a grateful party who receives more than his entitlement might shade negatively, i.e., provide more than consummate performance!29

In the Introduction we motivated the paper by pointing out some limitations of existing models of the firm. We believe that the model developed here can help to overcome some of those limitations and can be applied to organizational and contract economics more generally. We have used the model to understand the nature of the employment relationship, some aspects of vertical integration, and why parties deliberately write incomplete contracts. We believe that there are many other possibilities. To mention two: First, the model may throw light on the role of the courts in filling in the gaps of contracts deliberately left incomplete, e.g., on how the courts should assess damages for breach of contract. Second, the model of Section 4, extended from two to many people, may help us to understand how authority should be allocated, i.e., who, out of a group of individuals, should be boss. We believe that this second application may

29 It is worth noting that there is some experimental evidence suggesting that negative shading is unlikely to outweigh positive shading, that is, the increase in performance from receiving a dollar more than one’s entitlement is likely to be less than the decrease in performance from receiving a dollar less. See, e.g., Charness and Rabin (2002) and Offerman (2002).
be a useful step in allowing incomplete contracting ideas to be applied to the very interesting and important topic of the internal organization of firms.
Appendix A

In this appendix we use the framework of Section 3, first, to refine our assumption about what determines a party’s level of aggrievement; and, second, to prove a result giving circumstances where we can restrict attention to contracts in which the no-trade price $p_0$ is zero and one party unilaterally chooses the terms of trade.

To see why a refinement is required, consider a slight variant of the three-state example from Section 3:

<table>
<thead>
<tr>
<th></th>
<th>s1</th>
<th>s2</th>
<th>s3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>9</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>$c$</td>
<td>0</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

The only change is that in state $s_3$ the $(v,c)$ pair equals $(10,9)$ rather than $(20,0)$. All the analysis of contracts (a) – (c) still pertains, except that now they yield expected total surplus of

\[
W_a = 9\pi_1 + \pi_3,
\]

\[
W_b = 10\pi_2 + \pi_3,
\]

\[
W_c = 9\pi_1 + 10\pi_2 + (1 - \theta)\pi_3,
\]

respectively. As before, in general none of these contracts achieves first-best.

Now consider contract (e), in which $p_0 \equiv 0$ and $B$ chooses $p_1$ from a set of three discrete prices, $\{8\frac{1}{2}, 9\frac{1}{2}, 10\frac{1}{2}\}$. If we think of a contract where $p_1$ must lie in an interval $[\underline{p}, \overline{p}]$ as a
“standard” contact (and \( p = \bar{p} \) as a “simple” contract), then, unlike contracts (a) – (c), contract (e) is “non-standard”.

On the face of it, contract (e) achieves first-best because in each state only one \( p_1 \) out of the three allowable prices delivers trade (remember trade is voluntary, so each party has to be better off than not trading at \( p_0 = 0 \)). Surely, this means that there is no source of aggrievement?

We take the view that this is the wrong answer: contract (e) will generate aggrievement and hence shading. The reason is that in state s3, when B chooses \( p_1 = 9\frac{1}{2} \), S will feel aggrieved that B didn’t choose a 50:50 lottery between \( p_1 = 9\frac{1}{2} \) and \( p_1 = 10\frac{1}{2} \), with B committing to trade whatever the outcome of the lottery. The 50:50 odds are such that before the lottery, B would be no worse off than not trading, given that he values the widget at 10. At a price of \( 9\frac{1}{2} \), then, S feels aggrieved by \( \frac{1}{2} \). Equally, B feels aggrieved by \( \frac{1}{2} \) too! Even though he has the contractual right to choose \( p_1 \), and chooses \( 9\frac{1}{2} \), he would prefer to choose a 50:50 lottery between \( p_1 = 8\frac{1}{2} \) and \( p_1 = 9\frac{1}{2} \), with S committing to trade whatever the outcome: before the lottery, S would be no worse off than not trading, given that the widget costs her 9. In aggregate, shading in state s3 amounts to \( (\frac{1}{2} + \frac{1}{2})\theta \), which is the same as under contract (c). Worse, in state s1, when B chooses \( p_1 = 8\frac{1}{2} \), S feels aggrieved that B didn’t choose a 50:50 lottery between \( p_1 = 8\frac{1}{2} \) and \( p_1 = 9\frac{1}{2} \), with B committing to trade whatever the outcome, so that in this state there is shading. Similarly, there is shading in state s2. All in all, contract (e) is strictly dominated by contract (c).

We are adopting the position that when he or she thinks how aggrieved to feel, a party (A1) conjures with mixed strategies in the contractual mechanism (and with correlated strategies if the mechanism has simultaneous moves);

(A2) imagines a commitment to trade on the part of the other party, whatever the outcome of the randomization.
However, no-one believes they can force the other party beyond his or her participation constraint. Specifically, no-one thinks they can push the other party’s expected payoff below what he or she could get from simply walking away prior to the randomization, refusing to trade.

If the no-trade price $p_0$ varies with the strategies, and cannot simply be normalized to zero, then it is a little less obvious how these participation constraints should be factored into the parties’ thinking.

Consider a contract (f) in which B chooses $(p_1, p_0)$ from a set of three pairs: $(9, 1), (9\frac{1}{2}, \frac{1}{2})$ or $(10, 0)$. If there is no shading, then in state s1 B will choose $(p_1, p_0) = (9, 1)$, leading to trade at a price of 9 – actually, he is indifferent between this outcome and choosing $(p_1, p_0) = (10, 0)$, leading to no trade at $p_0 = 0$. In state s3 B will choose $(p_1, p_0) = (9\frac{1}{2}, \frac{1}{2})$, leading to trade at a price of $9\frac{1}{2}$ – if he chose $(p_1, p_0) = (9, 1)$ then S would not trade. In state s2 B will choose $(p_1, p_0) = (10, 0)$, the only price pair at which S will trade.

Again, on the face of it, if we do not invoke (A1) or (A2), contract (f) looks attractive. It could be argued that, since B is choosing the pair $(p_1, p_0)$ he is never aggrieved. And the contract has been cleverly designed so that in state s3 S is aggrieved by only $\frac{1}{2}$. She would increase her payoff by $\frac{1}{2}$ if, instead of B choosing $(p_1, p_0) = (9\frac{1}{2}, \frac{1}{2})$ leading to trade at $p_1 = 9\frac{1}{2}$, B were to choose $(p_1, p_0) = (9, 1)$ leading to no trade at $p_0 = 1$; equally, S would increase her payoff by $\frac{1}{2}$ if B were to choose $(p_1, p_0) = (10, 0)$ leading to trade at $p_1 = 10$. Arguably, then, shading in state s3 amounts to $\frac{1}{2}\theta$ – half that under contract (c). Likewise, in state s1 there is shading of $\frac{1}{2}\theta$ because S would prefer B to choose $(p_1, p_0) = (9\frac{1}{2}, \frac{1}{2})$ leading to trade at $p_1 = 9\frac{1}{2}$; and in state s2 there is shading of $\theta$ because S would prefer B to choose $(p_1, p_0) = (9, 1)$ leading to no trade at $p_0 = 1$.

Overall, it therefore could be argued that expected total surplus from contract (f) is
\[(9 - \frac{1}{2}\theta) \pi_1 + (10 - \theta) \pi_2 + (1 - \frac{1}{2}\theta) \pi_3,\]

which means that if, for example, \(\pi_1 + 2\pi_2 < \pi_3\) and \(\theta = 1\), contract (f) would dominate contracts (a), (b) and (c).

Once (A1) and, in particular, (A2) are invoked, however, this conclusion changes. To see why, we first need to check what would happen under contract (f) if either party walked away and refused to trade. B would minimize \(p_0\) by choosing \((p_1, p_0) = (10, 0)\). So, in reckoning how low they can push the other’s payoff, each party thinks in terms of a default no-trade price of \(p_0^* = 0\). Precisely, \(p_0^*\) is the right-hand side (RHS) of S’s participation constraint; and \((-p_0^*)\) is the RHS of B’s participation constraint.

Knowing this, we can invoke (A2) to calculate aggrievement levels in each state. In particular, in state s3, when B chooses \((p_1, p_0) = (9\frac{1}{2}, \frac{1}{2})\), B is actually aggrieved by \(\frac{1}{2}\) because he would prefer to choose \((p_1, p_0) = (9, 1)\), with S committing to trade – from (A2), we see that this would not violate S’s participation constraint given \(p_0^* = 0\). S is also aggrieved by \(\frac{1}{2}\) because she would prefer B to choose \((p_1, p_0) = (10, 0)\) and commit to trade. In aggregate, shading in state s3 amounts to \((\frac{1}{2} + \frac{1}{2})\theta\) – the same as under contract (c). One can in fact show that the two contracts, (f) and (c), deliver the same expected total surplus.

All this begs the question: assuming (A1) and (A2), can non-zero values of \(p_0\) ever help? If not, to find an optimal contract, can we narrow down the class that we need to consider, e.g., restrict attention to contracts in which one party unilaterally chooses the terms of trade?

A general contract C can be viewed as a stochastic mechanism mapping from a pair of messages \(\beta\) and \(\sigma\), reported by B and S respectively, onto either a pair of (trade, no-trade) prices, \((p_1, p_0)\), or onto simply a no-trade price, \(p_0\). In other words, following the report of messages \(\beta\)
and $\sigma$, there is an exogenous lottery to determine (i) if trade is allowed or not; (ii) the terms of trade/no-trade. If trade is allowed then it occurs if and only if both parties want it (at price $p_1$). Otherwise, there is no trade (at price $p_0$). (Remember we are assuming no renegotiation, so if the mechanism specifies no trade then that outcome is final.)

In effect, then, a mechanism allows for probabilistic trade – a surrogate for fractional trade (our widget is assumed to be indivisible).

Under contract $C$, $p^*$ (the default no-trade price used to determine the RHS’s of the parties’ participation constraints) is the value of the zero-sum game over the $p_0$’s specified in the mechanism – as if one or other party “quits”, i.e. always chooses to veto trade, so that the specified $p_1$’s are irrelevant. Let $(\tilde{\beta}^*, \tilde{\sigma}^*)$ be the (possibly mixed) equilibrium strategies of this zero-sum message game.

We consider, for each state $(v, c)$, a (possibly mixed strategy) subgame perfect equilibrium of the game induced by contract $C$. Let $q(v, c)$ be the probability of trade in equilibrium, and $p(v, c)$ the expected payment from B to S. Since trade is voluntary, $q(v, c) > 0$ only if $v \geq c$. If there is more than one equilibrium, we pick the one that maximizes $q(v, c)$.

No party is worse off than if he or she quit:

Lemma $vq(v, c) - p(v, c) \geq -p_0^*$,

$p(v, c) - cq(v, c) \geq p_0^*$.

Proof Consider B. In state $(v, c)$ he could deviate from his equilibrium message-reporting-cum-trading strategy to report $\tilde{\beta}^*$ and always refuse to trade. Were he to do so, his payoff would drop from $vq(v, c) - p(v, c)$ to, say, $(-\hat{p}_0)$, where $\hat{p}_0$ is the ensuing (expected) no-
trade price. But \( \hat{p}_0 \) cannot be any less than \( p_0^* \), because \( \hat{\sigma}^* \) is a best reply to \( \hat{\beta}^* \) for S in the zero-sum game over the \( p_0 \)’s. This proves the first inequality in the Lemma. The second follows symmetrically – reversing the roles of B and S.

QED

For future reference, let \( H \subseteq [0,1] \times R \) denote the convex hull of (0, 0) and all pairs

\[ \{q(v, c), p(v, c) - p_0^*\}, \]

where, notice, we are netting prices by subtracting \( p_0^* \). In the space of quantity \( q \) and net price \( p - p_0^* \), the set \( H \) might look as follows:

Invoking (A1) and (A2), in state \( (v, c) \) we define B’s [resp. S’s] “aspiration level” \( b(v, c) \) [resp. \( s(v, c) \)] to be his [resp. her] maximum payoff across all correlated message pairs and trading rules subject to the constraint that S [resp. B] gets no less than \( p_0^* \) [resp. \( -(p_0^*) \)].

Figure 2
In particular, B and S each imagine that they could jointly precommit to the (mixed) message-cum-trading equilibrium strategies pertaining to some other state, or some convex combination thereof.

Hence

\[(i) \quad b(v, c) \geq \max_{q, p} \{vq - p \mid (q, p - p_0^*) \in H \text{ and } p - cq \geq p_0^* \}, \]
\[(ii) \quad s(v, c) \geq \max_{q, p} \{p - cq \mid (q, p - p_0^*) \in H \text{ and } vq - p \geq -p_0^* \}. \]

Note that, thanks to the Lemma,

\[b(v, c) \geq vq(v, c) - p(v, c), \]
\[s(v, c) \geq p(v, c) - cq(v, c), \]
i.e., aspiration levels are at least as high as equilibrium payoffs.

In each state \((v, c)\), once equilibrium play is over, B shades by reducing S’s payoff down to

\[p(v, c) - cq(v, c) - \theta[b(v, c) - [vq(v, c) - p(v, c)]] .\]

And S shades by reducing B’s payoff down to

\[vq(v, c) - p(v, c) - \theta[s(v, c) - [p(v, c) - cq(v, c)]] .\]
Hence, in the special case $\theta = 1$ (the case considered in the Proposition below), total surplus in this equilibrium equals

\[(iii) \quad 2(v - c)q(v, c) - b(v, c) - s(v, c)\].

Now consider contract $\hat{C}$, in which $p_0 \equiv 0$ and one party (say $B$ - it doesn’t matter who) chooses from a set of exogenous lotteries, each corresponding to a different point $(q, p) \in H$:

\[
\begin{cases}
\text{with probability } q, \text{ trade is allowed at } p_1 = \frac{p}{q}, \\
\text{with probability } 1-q, \text{ trade is not allowed.}
\end{cases}
\]

At first sight contract $\hat{C}$ may look a little strange, but that is because it is dealing with probabilistic trade. Note that for $q = 1$ – corresponding to the right-hand edge of the set $H$ in Figure 2 – contract $\hat{C}$ is nothing more than our “standard” contract in which $B$ chooses a trading price $p_1$ from an interval $[p, \overline{p}]$. To put this another way, if the upper and lower edges of the set $H$ in Figure 2 were linear rather than piecewise linear, $H$ would correspond to a standard contract.

**Proposition** Suppose $\theta = 1$. Then contract $\hat{C}$ yields at least as much total surplus in each state as does contract $C$.

**Proof** Under contact $\hat{C}$, in state $(v, c)$, $B$’s aspiration level is
\[ \hat{b}(v, c) = \max_{q, p} \{ vq - p \mid (q, p) \in H \quad \text{and} \quad p - cq \geq 0 \} \]

(iv) \[ \leq p_0^* + b(v, c) \]

by (i). And S’s aspiration level is

\[ \hat{s}(v, c) = \max_{q, p} \{ p - cq \mid (q, p) \in H \quad \text{and} \quad vq - p \geq 0 \} \]

(v) \[ \leq -p_0^* + s(v, c) \]

by (ii). In each state \((v, c)\), B chooses the lottery corresponding to a point \((q, p) \in H\) to maximise his net payoff (i.e. net of S’s shading), taking into account that S may not be willing to trade at \(p_1 = \frac{P}{q}\) for \(q \neq 0\). That is, given \(\theta = 1\), B chooses \((q, p) \in H\) to maximise

\[ vq - p - \{ \hat{s}(v, c) - [p - cq] \} \quad \text{subject to} \quad p - cq \geq 0. \]

If \(v > c\), in effect B will maximise the probability of trade, \(q\), subject to \((q, p) \in H\) for some \(p \geq cq\). Call this maximum \(\hat{q}(v, c)\). But from the definition of the set \(H\),

\[ (q(v, c), p(v, c) - p_0^*) \in H; \]

and from the second inequality in the Lemma,
\[ p(v, c) - p_0^* \geq cq(v, c). \]

Hence \( \hat{q}(v, c) \) is at least \( q(v, c) \) whenever \( v > c \).

If \( v < c \), B will choose \((q, p) = (0, 0)\); in this case set \( \hat{q}(v, c) = 0 \).

Combining these two cases, we have

\[(vi) \quad (v - c)[\hat{q}(v, c) - q(v, c)] \geq 0 \text{ in all states } (v, c).\]

Just as total surplus in state \((v, c)\) was given by expression (iii) under contract C, so too under contract \( \hat{C} \) it is given by

\[(vii) \quad 2(v - c)\hat{q}(v, c) - \hat{b}(v, c) - \hat{s}(v, c).\]

But, making use of inequalities (iv) – (vi), we see that the expression in (vii) is no less than that in (iii).

QED

In words, the Proposition states that, without loss of generality, \( p_0 \) can be normalized to zero and one party (B, say) can be given control over the terms of trade. The subset \( H \) of \([0,1] \times R \) is the “design variable”. It is this set that the contract specifies, taking any shape (along
the lines of that in Figure 2) – but it must be convex and, for \( q = 0 \), come to a point at the origin (i.e., when \( q = 0, p = p_0^* = 0 \)).

Of course, the weakness of this result is that it applies only to the limit case \( \theta = 1 \).

However the Proposition is suggestive of other more general results. For example, it may be quite general that (A1) and (A2) are enough to allow us to ignore the possibility of non-zero values of \( p_0 \). The Proposition as it stands may apply if \( \theta \) is close enough to 1. And for lower values of \( \theta \), somewhat more complex allocations of control rights over the terms of trade (not merely giving unilateral control to either B or S) may turn out to be optimal. All this awaits further research.
Appendix B

In this appendix we prove Proposition 4.1.

Think of composers as lying in the \([0, 1]\) interval, with \(\lambda = 0\) corresponding to Bach and \(\lambda = 1\) corresponding to Shostakovich. In state \(s_2\), the value of composer \(\lambda\) to B is \(v - \lambda \Delta\), and the cost to S is \(c - \lambda \delta\). (That is, \(\lambda\) is equivalent to a convex combination of Bach and Shostakovich.) In state \(s_3\), the value of \(\lambda\) to B is \(v - (1 - \lambda) \Delta\), and the cost to S is \(c - (1 - \lambda) \delta\).

As a preliminary, we should observe that if no music is played in state \(s_4\) then in the other three states the first-best could be achieved using a contract that fixes the price at \(v\) and has B choose the composer. In state \(s_2\), B would choose \(\lambda = 0\) (as first-best requires) but there would be no aggrievement on the part of S since at price \(v\) no other composer would satisfy B’s participation constraint. Likewise in state \(s_3\), B would choose \(\lambda = 1\) and there would be no aggrievement. In state \(s_1\), there would be no aggrievement either (B could choose any composer). At this high price B would be unwilling to trade in state \(s_4\). Overall, expected total surplus would be \((1 - \pi_4)(v - c)\). For small enough \(\pi_4\), this would be the optimal contract. But we see this as a peculiar case, which we can later confirm is ruled out if \(\pi_4/(1 - \pi_1)\) is above the lower bound in Proposition 4.1. (Incidentally, this is the role of state \(s_4\) in the model when \(\Delta > \delta\). State \(s_1\) plays an analogous role when \(\Delta < \delta\).)

From now on, we suppose that music is played in state \(s_4\), at price \(p_4\), which must lie below B’s value, \(v - \Delta\). It is straightforward to confirm that in states \(s_2\), \(s_3\) and \(s_4\) music is also played under an optimal contract. Let the price be \(p_1\) in state \(s_1\). In state \(s_2\), suppose composer \(\lambda_2\) is played at price \(p_2\). Since we are restricting attention to symmetric contracts, in state \(s_3\) composer \(1 - \lambda_2\) is played, also at price \(p_2\).
The method we will use to characterize an optimal contract is to include in our mathematical programme only those inequality constraints that are critical. At the end, we will need to confirm that the (many) missing constraints are satisfied. In particular, at this point we shall ignore the question of who controls the choice of composer and price.

Let $a_2$ be the total level of aggrievement (B’s plus S’s) in state $s_2$ when $\lambda_2$ is played at price $p_2$. (By symmetry, $a_2$ is also the total level of aggrievement in state $s_3$.) Now S may prefer to switch from composer $\lambda_2$ to composer $1 - \lambda_2$ at the same price $p_2$ (which is admissible in the contract, since that is what occurs in state $s_3$), to reduce her costs from $c - \lambda_2 \delta$ to $c - (1 - \lambda_2) \delta$ – unless this switch would violate B’s participation constraint, $v - (1 - \lambda_2) \Delta - p_2 \geq 0$, in which case the best S could wish for is to switch to composer $\frac{v - p_2}{\Delta}$ at price $p_2$ and reduce her costs from $c - \lambda_2 \delta$ to $c - \frac{v - p_2}{\Delta} \delta$. Thus

(i) \[ a_2 \geq \delta \min \{1 - 2\lambda_2, \frac{v - p_2}{\Delta} - \lambda_2\}. \]

Let $a_1$ be the total level of aggrievement in state $s_1$. Given that $p_4 \leq v - \Delta$, $a_1$ must be at least $p_2 - v + \Delta$, irrespective of the price $p_1$ (if $p_1$ does not lie between $v - \Delta$ and $p_2$ then $a_1$ will be higher still):

(ii) \[ a_1 \geq p_2 - v + \Delta. \]
If $a_4$ is the total level of aggrievement in state $s_4$, consider the relaxed programme:

Choose $\lambda_2$, $p_2$, $a_1$, $a_2$ and $a_4$ to maximize

$$
(iii) \quad W \equiv \pi_1 [v - c - \theta a_1] + (1 - \pi_1 - \pi_4)[v - c - \lambda_2(\Delta - \delta) - \theta a_2] + \pi_4 [v - c - \Delta + \delta - \theta a_4]
$$

subject to (i), (ii) and the constraint that total aggrievement is always nonnegative:

$$
(iv) \quad a_1 \geq 0, \ a_2 \geq 0, \ a_4 \geq 0.
$$

In a solution to this relaxed programme, the tighter of the lower bound constraints on $a_1$ will bind. Likewise for $a_2$. And $a_4 = 0$.

Now consider the role of $p_2$, which affects $W$ in (iii) only via $a_1$ and $a_2$. Via $a_1$, the slope of $W$ w.r.t. $p_2$ is $(-\pi_1 \theta)$ if $p_2 \geq v - \Delta$, and is zero otherwise. Via $a_2$, the slope of $W$ w.r.t. $p_2$ is at most $(1 - \pi_1 - \pi_4)\theta \delta / \Delta$, and is zero if $p_2 < v - (1 - \lambda_2)\Delta$. Hence, from the lower bound on $\pi_i / (1 - \pi_4)$ in Proposition 4.1, $p_2$ should be as small as possible. However, there is no value in pushing $p_2$ below $v - \Delta$ since this would not affect $W$.

Next, consider the role of $\lambda_2 \in [0, 1]$, given $p_2 = v - \Delta$. $\lambda_2$ affects $W$ only through the middle term in (iii), and via $a_2$. Values of $\lambda_2$ above $\frac{1}{2}$ are clearly not optimal. If $\Delta > (1 + 20)\delta$, 

50
\( \lambda_2 \) should be zero; whereas if \( \Delta < (1 + 2\theta)\delta \), \( \lambda_2 \) should equal \( \frac{1}{2} \). These are two conditions that appear in Proposition 4.1.

Where does this leave us? On the one hand, if \( \Delta > (1 + 2\theta)\delta \) we can implement the above solution to the relaxed programme (viz., \( p_2 = v - \Delta \), \( \lambda_2 = 0 \), \( a_1 = a_4 = 0 \) and \( a_2 = \delta \)) using a contract in which the price is fixed at \( v - \Delta \) and B chooses the composer. (Actually, any fixed price between \( c \) and \( v - \Delta \) would yield the same \( W \).) On the other hand, if \( \Delta < (1 + 2\theta)\delta \) we can implement the solution (viz. \( p_2 = v - \Delta \), \( \lambda_2 = \frac{1}{2} \) and \( a_1 = a_2 = a_4 = 0 \)) using a contract in which the price is again fixed at \( v - \Delta \), but so too is the composer, at \( \lambda_2 = \frac{1}{2} \). (Actually in this latter case, it doesn’t matter which composer is fixed; it could instead be Bach, \( \lambda_2 = 0 \).) The fact that in all cases the solution to the relaxed programme can be implemented using some contract vindicates our earlier decision to omit the other inequality constraints.

Proposition 4.1 is proved.

QED
References


RESTATEMENT (SECOND) OF CONTRACTS, § 89(a) (1979).


