Work-Consumption Preferences and Employment Volatility*

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Abstract

Preferences about work and consumption play a key role in employment volatility. If potential workers respond sensitively to earnings, small changes in productivity result in large changes in employment and unemployment. Adjustments relating to aggregate fluctuations are mostly on the extensive margin of employment against non-employment. The class of models launched by Mortensen and Pissarides provides a coherent view of labor-market equilibrium, but its characterization of preferences is stylized—non-workers enjoy a constant flow benefit from not working, measured in consumption units. I calibrate a parametric representation of preferences to provide a framework to measure the flow benefit and describe its relation to fundamentals. With full personal insurance against idiosyncratic labor-market shocks, the flow value of non-work is the same for all people sharing the same preferences, irrespective of their histories. Absent such insurance, people self-insure using the ability to save. I show that the full-insurance model is a reasonable approximation to the more realistic case of limited or no insurance. The main results of the paper relate a central property of preferences—the Frisch elasticity of labor supply—to the response of unemployment to productivity. At the calibrated elasticity, the response is too low to provide much of an explanation of observed volatility, a point made forcefully by Shimer. I consider extensions of the Mortensen-Pissarides model that raise the response, but not enough to close the gap by much at the calibrated elasticity. On the other hand, at an elasticity at the upper end of plausibility, the enhanced response puts the model into a range where productivity fluctuations are a major driving force of employment and unemployment volatility.

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1 Introduction

Preferences over consumption and work determine the net payoff to work, counting the earnings positively and the time requirement of work negatively. The theory of labor supply gives a precise meaning to the payoff from work. Although the theory of unemployment has made important advances in recent years, its connection with labor supply has remained relatively primitive. In the Mortensen-Pissarides class of models, the subject of intense development recently, non-workers enjoy an exogenous flow value from not working. They sacrifice this value in favor of the market wage if they take jobs. Roughly speaking, if labor supply is highly elastic, the flow value of non-work is close to the wage, while if inelastic, the flow value is well below the wage. An interesting controversy has broken out recently over the flow value of non-work. Only a calibrated model of labor supply can resolve that controversy. One of the objectives of this paper is to provide that calibrated model.

The current controversy parallels similar controversies in macroeconomics for the past four decades. In the first round, Lucas and Rapping (1969) invoked elastic labor supply directly to explain employment volatility. The second involved models of employment volatility based on contracts between risk-neutral employers and risk-averse workers, where employment was made sensitive to the marginal product of labor. In the third round, Kydland and Prescott (1982) specified preferences in the real business cycle model that implied near-indifference between working and not working. Rogerson (1988) followed up with a model in which workers are literally indifferent. In the most recent round, Hagedorn and Manovskii (2006) specify a flow value of non-work in the Mortensen and Pissarides (1994) (MP) model that implies virtual indifference between work and non-work. In all models with near-indifference, employment is correspondingly sensitive to driving forces.

Throughout the paper, I make the key distinction between the flow utility that individuals enjoy from employment or non-employment, on the one hand, and the flow value they derive. Under all but the most extreme conditions, people derive more utility from not working, because they can use their time for more enjoyable purposes and they continue to consume almost as much as they would while working. The near-indifference that helps explain employment volatility is in value, not utility. I use measures of flow value derived from the Frisch theory of labor supply and consumption demand that fit the concept in the MP model exactly. These measures also correspond to flow values in the theory of insurance. The value of non-employment converts utility to consumption units and deducts the amount
of consumption. The value of employment is similar, but adds the amount earned from working.

I focus on the discrete choice between non-work and full-time work as embodied in the MP model, because this is the more important dimension of aggregate fluctuations in total labor input. I also generally focus on the relationship between productivity and unemployment fluctuations. I consider three factors that shift the relationship in a way that matters for employment volatility. The first is non-labor income. If workers enjoy a positive flow of non-labor income, the flow value of non-work is closer to the wage. Consequently, the response of unemployment to productivity is higher, possibly substantially higher.

The second factor is heterogeneity in jobs. Job-seekers facing a range of potential wages will accept a larger fraction if a rise in aggregate productivity shifts the distribution upward. This influence, absent from the standard model, makes the job-taking rate more sensitive to aggregate productivity. I quantify the resulting increase in the sensitivity of unemployment to productivity.

The third factor is variations in job-seeking effort that occur in response to changes in the payoff to finding work. A model incorporating variable search effort amplifies productivity fluctuations as well.

Most of the paper considers the case where actuarially fair insurance insulates individuals against idiosyncratic shocks in the labor market. With this insurance, all individuals with the same preferences place the same value on non-work, irrespective of their personal histories. The assumption of full insurance provides an exact rationalization of the stylized representation of preferences in the MP model—it delivers a constant flow value of non-work.

Under the more realistic assumption of partial insurance provided by public programs and by family members and friends, each worker has a personal state variable, wealth, and the model is no longer discrete Markoff in the available activities. I show that the ability to save even modest amounts supports substantial self-insurance against unemployment risk, so the model with full insurance is a reasonable approximation to the much more complicated model with partial insurance.

2 Preferences

As in most research on choices over time, I assume that preferences are time-separable, though I am mindful of Browning, Deaton and Irish (1985)’s admonition that “the fact
that additivity is an almost universal assumption in work on intertemporal choice does not suggest that it is innocuous.” In particular, additivity fails in the case of habit.

My approach to calibration will consider choices made with linear budget constraints. Taking the Arrow-Debreu time-0 price of consumption in the period under examination to be \( p \) and the wage to be \( w \), I let \( c(\lambda p, \lambda w) \) be the Frisch consumption demand and \( h(\lambda p, \lambda w) \) be the Frisch labor supply. See Browning et al. (1985) for a complete discussion of Frisch systems in general. The consumption demand and labor supply satisfy

\[
uc (c(\lambda p, \lambda w), h(\lambda p, \lambda w)) = \lambda p
\]

and

\[
uh (c(\lambda p, \lambda w), h(\lambda p, \lambda w)) = -\lambda w
\]

Frisch demands are symmetric: \( \partial c/\partial (\lambda w) = -\partial h/\partial (\lambda p) \). They have three basic first-order or slope properties:

- **Intertemporal substitution in consumption**, \( \partial c/\partial (\lambda p) \), the response of consumption to changes in its time-0 price
- **Frisch labor-supply response**, \( \partial h/\partial (\lambda w) \), the response of hours to changes in the wage
- **Consumption-hours cross effect**, \( \partial c/\partial (\lambda w) \), the response of consumption to changes in the wage (and the negative of the response of hours to the consumption price). The expected property is that the cross effect is positive, implying substitutability between consumption and hours of non-work or complementarity between consumption and hours of work.

Each of these responses has generated a body of literature, which I will draw upon to calibrate the utility kernel, \( u(c, h) \). In addition, in the presence of uncertainty, the curvature of \( u \) also controls risk aversion, the subject of another literature.

Chetty (2006) considers the issues surrounding this calibration. He shows that the value of the coefficient of relative risk aversion (or, though he does not pursue the point, the inverse of the intertemporal elasticity of substitution in consumption) is implied by a set of other measures. He solves for the consumption curvature parameter by drawing estimates of responses from the literature on labor supply. One is the third item on the list above, consumption-hours complementarity. The others are the compensated wage elasticity of
static labor supply and the elasticity of static labor supply with respect to unearned income, \( \bar{y} \). The static labor-supply function \( \bar{h}(p, w, \bar{y}) \) maximizes \( u(c, h) \) subject to the constraint \( pc = wh + \bar{y} \). The derivatives of \( \bar{h}(p, w, \bar{y}) \) are functions of the derivatives listed above, so information about static labor supply does not add anything that those derivatives miss. In principle, as long as the mapping has adequate rank, one could take any set of measures of behavior and solve for the slopes of the Frisch functions or any other representation of preferences. My procedure links the empirical measures more directly to the underlying basic properties of preferences. I do, however, study the implications of my calibration for static labor supply. My calibration lies within the space of values that Chetty extracts from a wide variety of studies of static labor supply.

Basu and Kimball (2000) pursue an idea related to Chetty’s. They calibrate preferences to an outside estimate of the intertemporal elasticity of substitution in consumption and to zero uncompensated elasticity of static labor supply with respect to the wage. They constrain the complementarity of consumption and hours to have the multiplicative form of King, Plosser and Rebelo (1988).

2.1 Functional form

I posit that individuals order consumption-hours pairs according to the period utility of Malin (2006),

\[
u(c, h) = \frac{1}{1 - \delta} \left[ \frac{c^{-(1/\sigma - 1)} - c^{-(1/\sigma - 1)}}{1/\sigma - 1} - \frac{\gamma}{1/\psi + 1} h^{1/\psi + 1} \right]^{1-\delta}.
\]

The kernel inside the brackets governs the marginal rate of substitution between consumption and work within a month and state of the world. A key feature of the specification is to take a concave transformation of the kernel before adding across months or states. The parameter \( \delta \) controls the concavity—positive values of \( \delta \) correspond to complementarity of hours and work, with \( u_{c,h} > 0 \).

Absent complementarity, with \( \delta = 0 \), a widely used specification, the Frisch consumption demand is

\[
u_c(c, h) = c^{-\frac{1}{\sigma}} = \lambda \text{ or } c = \lambda^{-\sigma},
\]

can be lower...
in states or at times when people are not working. But that finding is consistent with the impression only in the special case where $u_{c,h} = 0$. More recent work—such as Attanasio and Davis (1996) and Jappelli and Pistaferri (2005)—considers complementarity and does not diagnose the failure of insurance from the positive correlation of consumption and hours of work.

Consumption and hours are Frisch complements if consumption rises when the wage rises (work rises and non-work falls)—see Browning et al. (1985) for a discussion of the relation between Frisch substitution and Slutsky-Hicks substitution. In the specification with $\delta = 0$, consumption and work are at the boundary. If $\delta$ is positive, consumption and work are unambiguously Frisch complements. People consume more when wages are high because they work more and consume less leisure. Nothing can be deduced about limited insurance or consumption-smoothing from the reaction of consumption to changes in work opportunities, without studying the related movements of wages or, alternatively, of hours.

2.2 Calibration

The Frisch consumption demand and labor supply are:

$$
c^{-1/\sigma} \left[ \frac{c^{-1/\sigma - 1} - c^{-1/\sigma - 1}}{1/\sigma - 1} - \frac{\gamma}{1/\psi + 1} h^{1/\psi + 1} \right]^{-\delta} = \lambda p \tag{5}$$

$$
\gamma h^{1/\psi} \left[ \frac{c^{-1/\sigma - 1} - c^{-1/\sigma - 1}}{1/\sigma - 1} - \frac{\gamma}{1/\psi + 1} h^{1/\psi + 1} \right]^{-\delta} = \lambda w. \tag{6}
$$

I calibrate at the point where consumption $c$, hours $h$, the consumption price $p$, and the wage $w$ are all 1. By taking the ratio of equations (5) and (6), I infer that $\gamma = 1$ under this normalization, so I will presume that value in what follows.

The four remaining parameters of preferences are the intercept in the kernel, $c$, the complementarity parameter, $\delta$, the curvature parameter for consumption, $\sigma$ and the curvature parameter for work, $\psi$.

The intercept $c$ controls the point of disaster for consumers with $\delta > 0$ (it is irrelevant for those with $\delta = 0$). Disaster occurs when the term in brackets reaches zero and marginal utility rises to infinity. If $\sigma < 1$, the range of values I find appropriate, disaster occurs at consumption levels above the level of zero that drives $c^{-1/\sigma}$ to infinity. Consumers with Malin preferences who are not working ($h = 0$) reach the disaster point when $c = c$, provided $\sigma < 1$. Disasters are only likely for people who are not working, because earnings are generally above the disaster level. I take the disaster level of consumption to be $c = 0.2$. 
I calibrate the three curvature parameters to the three basic properties of consumer-worker behavior listed earlier. In all cases, I will draw primarily upon research at the household rather than the aggregate level. The first property is risk aversion and intertemporal substitution in consumption. With additively separable preferences across states and time periods, the coefficient of relative risk aversion (CRRA) and the intertemporal elasticity of substitution are reciprocals of one another. But there is no widely accepted definition of measure of substitution between pairs of commodities when there are more than two of them. Chetty (2006) discusses two natural measures of risk aversion when hours of work are also included in preferences. In one, hours are held constant, while in the other, hours adjust when the random state becomes known. He notes that risk aversion is always greater by the first measure than the second. The measures are the same when consumption and hours are neither complements nor substitutes. In this case, the CRRA is $\frac{u_{cc}}{cu_c}$.

For the purposes of this calibration, I assume that research on intertemporal substitution/risk aversion measures the Frisch elasticity. I believe this is a reasonable approximation for two reasons: First, the evidence suggests that complementarity is not strong. When it is absent, all measures of the elasticity agree. Second, my results later in the paper on the relevance of the fully insured model even when, in truth, insurance is only partial, imply that the research actually reveals the Frisch elasticity. When confronted with a change in relative prices from one period to the next, the fully insured consumer responds along the Frisch demand function—the Borch-Arrow insurance condition has the same content as the Frisch demand.

### 2.3 Risk aversion

Research on the value of the CRRA falls into several broad categories. In finance, a consistent finding within the framework of the consumption capital-asset pricing model is that the CRRA has high values, in the range from 10 to 100 or more. Mehra and Prescott (1985) began this line of research. A key step in its development was Hansen and Jagannathan (1991)’s demonstration that the marginal rate of substitution—the universal stochastic discounter in the consumption CAPM—must have extreme volatility to rationalize the equity premium. Models such as Campbell and Cochrane (1999) generate a highly volatile marginal rate of substitution from the observed low volatility of consumption by the trick of subtracting an amount almost equal to consumption before measuring the MRS. This trick does not seem
A second body of research considers experimental and actual behavior in the face of small risks and generally finds high values of risk aversion. For example, Cohen and Einav (2005) find that the majority of car insurance purchasers behave as if they were essentially risk neutral in choosing the size of their deductible, but a minority are highly risk-averse, so the average coefficient of relative risk aversion is about 80. But any research that examines small risks, such as having to pay the amount of the deductible or choosing among the gambles that an experimenter can offer in the laboratory, faces a basic obstacle: Because the stakes are small, almost any departure from risk-neutrality, when inflated to its implication for the CRRA, implies a gigantic CRRA. The CRRA is the ratio of the percentage price discount off the actuarial value of a lottery to the percentage effect of the lottery on consumption. For example, consider a lottery with a $20 effect on wealth. At a marginal propensity to consume out of wealth of 0.05 per year and a consumption level of $20,000 per year, winning the lottery results in consumption that is 0.005 percent higher than losing. So if an experimental subject reports that the the value of the lottery is one percent—say 10 cents—lower than its actuarial value, the experiment concludes that the subject’s CRRA is 200! 

Remarkably little research has investigated the CRRA implied by choices over large risky outcomes. One important contribution is Barsky, Juster, Kimball and Shapiro (1997). This paper finds that almost two-thirds of respondents would reject a new job with a 50 percent chance of doubling income and a 50 percent chance of cutting income by 20 percent. The cutoff level of the CRRA corresponding to rejecting the hypothetical new job is 3.8. Only a quarter of respondents would accept other jobs corresponding to CRRA of 2 or less. The authors conclude that most people are highly risk averse. The reliability of this kind of survey research based on hypothetical choices is an open question, though hypothetical choices have been shown to give reliable results when tied to more specific and less global choices, say, among different new products.

2.4 Intertemporal substitution

Attanasio, Banks, Meghir and Weber (1999), Attanasio and Weber (1993), and Attanasio and Weber (1995) are leading contributions to the literature on intertemporal substitution in consumption at the household level. These papers examine data on total consumption
(not food consumption, as in some other work). They all estimate the relation between consumption growth and expected real returns from saving, using measures of returns available to ordinary households. All of these studies find that the elasticity of intertemporal substitution is around 0.7.

Barsky et al. (1997) asked a subset of their respondents about choices of the slope of consumption under different interest rates. They found evidence of quite low elasticities, around 0.2.

Guvenen (2006) tackles the conflict between the behavior of securities markets and evidence from households on intertemporal substitution. With low substitution, interest rates would be much higher than are observed. The interest rate is bounded from below by the rate of consumption growth divided by the intertemporal elasticity of substitution. Guvenen’s resolution is in heterogeneity of the elasticity and highly unequal distribution of wealth. Most wealth is in the hands of those with elasticity around one, whereas most consumption occurs among those with lower elasticity.

Finally, Carroll (2001) and Attanasio and Low (2004) have examined estimation issues in Euler equations using similar approaches. Both create data from the exact solution to the consumer’s problem and then calculate the estimated intertemporal elasticity from the standard procedure, instrumental-variables estimation of the slope of the consumption growth-interest rate relation. Carroll’s consumers face permanent differences in interest rates. When the interest rate is high relative to the rate of impatience, households accumulate more savings and are relieved of the tendency that occurs when the interest rate is lower to defer consumption for precautionary reasons. Permanent differences in interest rates result in small differences in permanent consumption growth and thus estimation of the intertemporal elasticity in Carroll’s setup has a downward bias. Attanasio and Low solve a different problem, where the interest rate is a mean-reverting stochastic time series. The standard approach works reasonably well in that setting. They conclude that studies based on fairly long time-series data for the interest rate are not seriously biased. My conclusion favors studies with that character, accordingly.

I calibrate to a Frisch elasticity of consumption demand of -0.4. Again, I associate the evidence described here about the intertemporal elasticity of substitution as revealing the Frisch elasticity, even though many of the studies do not consider complementarity of consumption and hours explicitly.
2.5 Frisch elasticity of labor supply

The second property is the Frisch elasticity of labor supply. Pistaferri (2003) is a leading recent contribution to estimation of this parameter. This paper makes use of data on workers’ personal expectations of wage change, rather than relying on econometric inferences, as has been standard in other research on intertemporal substitution. Pistaferri finds the elasticity to be 0.70 with a standard error of 0.09. This figure is somewhat higher than most earlier work in the Frisch or elasticity of substitution framework (here, too, I proceed on the assumption that the two are the same as a practical matter). Kimball and Shapiro (2003) survey the earlier work.

Mulligan (1998) challenges the general consensus among labor economists about the Frisch elasticity of labor supply with results showing elasticities well above one. My discussion of the paper, published in the same volume, gives reasons to be skeptical of the finding, as it appears to flow from an implausible identifying assumption.

Kimball and Shapiro (2003) estimate the Frisch elasticity from the decline in hours of work among lottery winners, based on the assumption that the uncompensated elasticity of labor supply is zero. They find the elasticity to be about one. But this finding is only as strong as the identifying condition.

I calibrate to a Frisch elasticity of labor supply of 0.7.

2.6 Consumption-hours complementarity

The third property is the relation between hours of work and consumption. A substantial body of work has examined what happens to consumption when a person stops working, either because of unemployment following job loss or because of retirement, which may be the result of job loss.

Browning and Crossley (2001) appears to be the most useful study of consumption declines during periods of unemployment. Unlike most earlier research in this area, it measures total consumption, not just food consumption. They find a 14 percent decline on the average from levels just before unemployment began.

A larger body of research deals with the “retirement consumption puzzle”—the decline in consumption thought to occur upon retirement. Most of this research considers food consumption. Aguiar and Hurst (2005) show that, upon retirement, people spend more time preparing food at home. The change in food consumption is thus not a reasonable guide to
the change in total consumption.

Banks, Blundell and Tanner (1998) use a large British survey of annual cross sections to study the relation between retirement and nondurables consumption. They compare annual consumption changes in 4-year wide cohorts, finding a coefficient of -0.26 on a dummy for households where the head left the labor market between the two surveys. They use earlier data as instruments, so they interpret the finding as measuring the planned reduction in consumption upon retirement.

Miniaci, Monfardini and Weber (2003) fit a detailed model to Italian cohort data on non-durable consumption, in a specification of the level of consumption that distinguishes age effects from retirement effects. The latter are broken down by age of the household head. The pure retirement reductions range from 4 to 20 percent. This study also finds pure unemployment reductions in the range discussed above.

Fisher, Johnson, Marchand, Smeeding and Terrey (2005) study total consumption changes in the Consumer Expenditure Survey, using cohort analysis. They find small declines in total consumption associated with rising retirement among the members of a cohort. Because retirement in a cohort is a gradual process and because retirement effects are combined with time effects on a cohort analysis, it is difficult to pin down the effect.

Based on this research, my calibration assumes that people consume 15 percent less when their work falls to zero, compared to their consumption when working a normal amount. I incorporate this property in the calibration by requiring

\[
0.85^{1/\sigma} \left[ \frac{c^{-(1/\sigma-1)} - 0.85^{-1/\sigma-1}}{1/\sigma - 1} \right]^{-\delta} = \lambda \tag{7}
\]

Notice that this calibration does not require me to take a stand on whether people who are not working have chosen that condition voluntarily, against other available choices. Equation (7) holds whether the choice is voluntary or involuntary. Some of the research on the effects of unemployment and retirement on consumption has interpreted the decline as the result of frictions in capital and insurance markets. I make the hypothesis in this part of the paper that the decline arises mainly from the Frisch substitutability of work and consumption, not from failures of insurance and capital markets. A number of the studies cited above support this hypothesis.
2.7 Parameter values

The calibration takes a Frisch elasticity of consumption demand of \(-0.4\) and of labor supply of 0.7 and a consumption decline from 1.00 to 0.85 if hours fall from 1.0 to 0. The corresponding values of the parameters of preferences are \(\sigma = 0.48\), \(\psi = 1.27\), and \(\delta = 3.48\), obtained by numerical solution of equations (5), (6), and (7). The uncompensated elasticity of static labor supply is \(-0.38\), so supply is backward-bending, consistent with many studies of static labor supply—see Blundell and MaCurdy (1999). The compensated elasticity is 0.35, also in line with these studies.

3 Models with Homogeneous Job Opportunities

Mortensen and Pissarides (1994) developed a model that has proven remarkably instructive about unemployment. In its simplest version, workers and jobs are homogeneous. Matching frictions delay the process of finding a new job after an earlier job has ended. Individuals face a transition matrix \(\pi_{i,i'}\) from state \(i\) (0 if unemployed; 1 if employed) to state \(i'\). The job-finding rate is \(\pi_{0,1}\) and the job-separation rate is \(\pi_{1,0}\). Employed workers always have the same hours, normalized at one: \(h_1 = 1\). Naturally \(h_0 = 0\). Unemployed individuals receive benefits \(y_0 = b\). Workers’ earnings are \(y_1 = w\).

Suppose, first, that workers have access to a market providing actuarially fair insurance against the personal risk of the labor market. Their choices obey the Borch-Arrow condition, equating the marginal utility of consumption across states of the world and across time periods to the common value \(\lambda\). The value of earnings in utility units is \(\lambda\) as well. Consider an individual in activity \(i\), who chooses consumption to maximize

\[
u(c, h_i) - \lambda c.
\]

The maximizing \(c\) is the Frisch consumption demand. At that value of \(c\), I let

\[
v_i = u(c, h_i) - \lambda(c - y_i),
\]

the flow value of activity \(i\) in utility units. Individuals place asset values on activities according to the Bellman equation,

\[
\bar{V}_i = v_i + \beta \sum_{i'} \pi_{i,i'} \bar{V}_{i'}.
\]
I also let
\[ v_i = \frac{1}{\lambda} \tilde{v}_i \]  
and
\[ V_i = \frac{1}{\lambda} \tilde{V}_i, \]
the flow and asset values in consumption units.

In summary, with full insurance, individuals choose an unemployment consumption level \( c_0 \) and an employment consumption level \( c_1 \) that depend only on the current activity and not on history. Their choices impute a constant shadow value \( \lambda \) to consumption.

### 3.1 Solving the model in the case of full insurance

Suppose that a person’s distribution across activities is the stationary distribution. The person’s budget constraint, given actuarially fair insurance, is
\[
\sum_i p_i (c_i(\lambda) - y_i) = 0. \tag{13}
\]
Solving the model requires finding the root, \( \lambda \), of this equation. From the root, one can calculate the flow values \( v_i \) and then the asset values \( V_i \) from
\[
V = (I - \beta \pi)^{-1} v. \tag{14}
\]

### 3.2 Implications for the Mortensen-Pissarides model

The Mortensen-Pissarides model embeds preferences about consumption and work neatly in the case of full insurance. I will discuss the integration of preferences in a totally standard MP model taken almost directly from Shimer (2005). I focus on the stationary equilibrium of a model with constant productivity. In the model, the exogenous monthly separation rate is 3 percent and the equilibrium job-finding rate is 52 percent per month, so the equilibrium unemployment rate is 5.5 percent. The job-finding rate varies with the square root of the vacancy-unemployment rate. A symmetric Nash bargain sets the wage. I calibrate the vacancy cost and the constant in the matching function to yield the 52 percent job-finding rate and a vacancy/unemployment ratio of 0.5.

The model comprises equation (14) (which assigns Bellman values for the worker while unemployed and employed), equations expressing equal division of the surplus, a Bellman
equation for the value the employer assigns to the employment relationship, and the zero-profit condition for job creation, and an equation expressing the flow value of not working, \( z \), including both the benefits \( b \) and the value of leisure or home production.

To avoid duplication of well-known equations, I will reference equations appearing later in the paper for a variant of the MP model. Equation (25) gives the Bellman value for unemployment, with \( G(R) = 1 \). Equations (14) and (27) give the Bellman values for employment and for a filled job. Equation (28) states the symmetric Nash wage bargain. Finally, equation (29) states the free-entry condition that the net value of a vacancy is zero, again with \( G(R) = 1 \).

To accommodate the normalization in the MP model that equates the flow value of working to earnings, I will add a constant \( \alpha \) to the flow utility. Equations (9) and (11) imply, for employed workers consuming \( c_1 \) and earning \( y_1 = 1 \),

\[
\frac{\alpha + u(c_1, 1)}{\lambda} - c_1 + w = w. \tag{15}
\]

The \( w \) on the right-hand side corresponds to the MP normalization that the flow value of employment is earnings. Similarly, the flow value of not working and receiving benefits of \( b \) is

\[
\frac{\alpha + u(c_0, 0)}{\lambda} - c_0 + b = z. \tag{16}
\]

The solution is

\[
z = b + c_1 - c_0 + \frac{u(c_0, 0) - u(c_1, 1)}{\lambda}. \tag{17}
\]

I solve equation (17) along with the standard equations of the MP model to find the stationary state. I solve for all the endogenous variables except the unemployment rate and I also solve for the flow cost of a vacancy needed to match the prescribed unemployment rate of 5.5 percent. I assume unemployment benefits equal to 30 percent of productivity (\( b = 0.3 \)). I find that consumption levels are \( c_0 = 0.82 \) and \( c_1 = 0.97 \), corresponding to utility levels of \( u(c_0, 0)/\lambda = -1.88 \) and \( u(c_1, 1)/\lambda = -2.17 \). The preference component is \( c_1 - c_0 + \frac{u(c_0, 0) - u(c_1, 1)}{\lambda} = 0.43 \) so the total flow value, from benefits and the preference for not working, is \( z = 0.73 \).

These findings shed some light on Hagedorn and Manovskii (2006)’s estimate of \( z \) obtained from a radically different calibration strategy. They require their version of the MP model to match the derivative of the wage with respect to productivity and they calibrate to an outside estimate of the cost of posting a vacancy. They show that the criteria imply that
the flow value of non-work has the high value of 0.955 (in units of productivity) and that
the bargaining power of labor is weak, with only 5.2 percent of the surplus accruing to the
worker. Unemployment is highly sensitive to productivity—vibrations in technology alone
fully explain the observed volatility of unemployment.

To investigate Hagedorn and Manovskii’s calibration, I repeat my earlier calibration to
the two Frisch elasticities and the level of consumption among non-workers. I keep the Frisch
elasticity of consumption demand at -0.4 and non-worker consumption at 0.85. I recompute
the values of $\sigma$, $\psi$, and $\delta$ for a range of values of the Frisch elasticity of labor supply. The
values of $\sigma$ and $\delta$ are not sensitive to the labor-supply elasticity, while $\psi$ varies in approximate
proportion to the elasticity. I keep the worker’s share of the surplus at one-half to separate
the effect of higher $z$ from the effect of the other major difference in their calibration, the
low bargaining power of workers. I solve for a lower flow cost of a vacancy in this case, so
that I again match the unemployment rate of 5.5 percent.

Figure 1 illustrates the importance of the Frisch elasticity of labor supply. The horizontal
axis is the Frisch elasticity of labor supply. Evidence from household data finds a value of
about 0.7, as I discussed earlier. The upper curve shows the flow value of work, $z$, calculated
according to equation (17). The value of the elasticity that corresponds to a $z$ of 0.955
is about 1.5. The vertical lines mark the two calibrations. The lower curve measures the
amplification of unemployment fluctuations, measured as $-\frac{du}{dA}$, where $A$ is the level of
productivity, taken to be one at the calibration point.

Figure 1 confirms Shimer (2005)’s finding of low amplification, even for values of $z$
well above Shimer’s value of 0.4. At my calibration point, a decrease of one percent in
productivity—typical of the decline in a recession—causes only a 0.1 percentage point in-
crease in unemployment, far below the increase of two or three percentage points that usually
occurs. The figure also shows much more amplification in Hagedorn and Manovskii’s case of
$z = 0.955$. Unemployment rises by 0.6 percentage points. However, to match what appears
to be the actual amplification with symmetric bargaining power, the Frisch elasticity would
have to be much higher than the 1.5 implicit in their calibration, with $z$ extremely close to 1.

The estimates reviewed earlier on the Frisch elasticity of labor supply may have a down-
ward bias—see Domeij and Floden (2006) for simulation results suggesting that the true
value of the elasticity may be double the estimated value as a result of omitting considera-
tion of borrowing constraints. This much bias would not put the value of $z$ high enough to generate realistic volatility of unemployment from productivity fluctuations in a model with symmetric bargaining power.

Hagedorn and Manovskii’s calibration actually generates much more amplification than shown in Figure 1. Most of their amplification comes from the low response of wages to productivity they infer from the data. For all the equilibria shown in the figure, the derivative of the wage with respect to productivity is 0.97. Symmetry of the wage bargain is an important determinant of the near-unitary response of wages. Hagedorn and Manovskii calibrate to a response of 0.449. Their MP model accommodates the low response by assigning very low bargaining power to workers. In effect, their calibration takes as given a relatively large amount of wage rigidity and finds a version of the MP model that delivers that amount. Their model is a close cousin of others that rationalize wage rigidity by dropping Nash wage bargaining.

4 Comparing Models with and without Insurance

In this section I investigate the complications arising from absent insurance markets. Without insurance, workers make use of their opportunity to save and thus achieve some but not
all of the benefits of insurance. A specific question of interest is how close workers can come to the fully insured allocation.

Suppose that workers cannot borrow against future income, so they face the constraint $c \leq W + y$. Let $\tilde{V}_i(W)$ be the worker’s expected present value having just chosen job $i$ with non-human wealth $W$, measured prior to this period’s consumption and earnings. The worker’s Bellman equation is:

$$\tilde{V}_i(W) = \max_c \left( u(c, h_i) + \beta \sum_{i'} \pi_{i,i'} \tilde{V}_{i'} \left( \frac{W - c + y_i}{\beta} \right) \right). \tag{18}$$

Let $W' = W - c + y_i$ be the wealth carried forward to the next period and define the flow value of activity $i$ to be

$$\tilde{v}_i(W') = u(c, h_i) - \left( \tilde{V}_i(W' + c - y_i) - \tilde{V}_i(W') \right). \tag{19}$$

The Bellman equation becomes

$$\tilde{V}_i(W') = \tilde{v}_i(W') + \beta \sum_{i'} \pi_{i,i'} \tilde{V}_{i'} \left( \frac{W'}{\beta} \right). \tag{20}$$

Notice that if $\tilde{V}_i$ is linear, with

$$\tilde{V}_i(W') = \alpha_i + \lambda W',$$ \tag{21}

the flow and asset values are as given earlier for the case of full insurance.

The worker is either not working with zero hours of work and cash earnings of $y_0 = 0.3$ or working with $h_1 = 1$ and $y_1 = 1$. Time is in quarters and the transition probabilities are $\pi_{0,1} = 1$ and $\pi_{1,0} = 0.082$. The implied unemployment rate is 5.5 percent. In addition to the binary state variable showing which job the worker currently occupies, wealth $W$ is a continuous state variable.

Following the dictates of Judd (1998), Chapter 12, I represent the value function as piecewise linear, with 40 nodes. Starting with $V_{i,T} = 0$, I iterate backwards, solving for the 40 nodal values of $V_{i,t}$ from the Bellman equation. I take $T = 160$ for a working life of 40 years.

To calculate the distribution of wealth at mid-career, $T/2$, I proceed as follows: From the value function at $T/2$, I compute a grid of 300 values of $W$ satisfying the recursion,

$$W_n = (1 + r)(W_{n-1} - c_1 + y_1), \tag{22}$$
where $c_1$ is the consumption chosen by a worker in state 1, employed. The sequence $W_n$ describes the growth of wealth of a worker who stays employed indefinitely; its limit is an upper bound on $W$.

Define a worker’s overall discretized state as $s = 2n + i$, where $n$ is the bin containing the worker’s wealth, $W$: $W_{n-1} < W \leq W_n$. I take the initial state of the worker to be unemployed at minimum wealth, so the initial distribution across states is $p = 0$ except $p_1 = 1$. I calculate $s_{i,i',n}$, the new state of a worker previously in job $i$ with wealth $W_n$ who has drawn new job $i'$. By construction, $s_{1,1,n} = 2(n+1) + 1$, at mid-career. For each quarter, I construct the 600 X 600 transition matrix from $s_{i,i',n}$ and the original transition matrix, $\pi_{i,i'}$. I then multiply $p$ by the transition matrix to form the next distribution across states. The vector $p$ is the joint distribution of the job $i$ and wealth $W$, which I consider at mid-career. Finally, I calculate the marginal distribution of $W$.

Figure 2 shows the slope of the value function and the marginal distribution of wealth at mid-career. Because the utility discount and interest rate offset each other exactly, the only reason that people accumulate wealth is precautionary—absent the probability of occasional unemployment, workers would hold no wealth. Typical wealth holdings are fairly small, about 1.2 quarters of earnings. The interquartile range of wealth is from 0.8 to 1.5 quarters of earnings. Over this range, the marginal value of wealth, $V'$, falls from 1.57 to 1.46. In the case of full insurance, the marginal value is constant at 1.49.
Figure 3. Cumulative Distributions of Consumption While Unemployed, with and without Insurance

Figure 3 studies the difference between the insured and uninsured cases in terms of the distribution of consumption while unemployed. With insurance, unemployment consumption is always 77 percent of employment earnings. Without insurance, unemployment consumption is less than that amount if unemployment hits when wealth is low and greater if wealth is high. The interquartile range of unemployment consumption is from 75 percent of earnings to 78 percent. The ability to save (but not borrow) gives people a powerful tool to stabilize consumption when cash income drops to 30 percent of its normal level. In the presence of insurance at 50 percent of its full level, the distribution of unemployment consumption is even tighter—the interquartile range is from 75.9 percent of earnings to 77.1 percent.

Figure 4 compares the full-insurance and no-insurance cases in terms of the distributions of the flow value of unemployment, \( z \). I calculate \( z \) in the no-insurance case from equation (9), substituting the state-dependent marginal value of wealth, \( V_0'(W) \) for the constant marginal value \( \lambda \). Again, the distribution centers fairly tightly around \( \lambda \), so using a flow value \( z \) based on a constant value of wealth is probably a good approximation.

I conclude that the full-insurance model is sufficiently close to the more realistic no-insurance model and vastly easier to handle. The rest of the paper further explores the full-insurance version.
5 The Desire to Find Work

Suppose that an individual in the setting described in Section 3 contemplated remaining out of work for the indefinite future. Let the value, in consumption units, of that strategy be $Z$, the present value of the non-work flow value, $z$. Let the value associated with search be $U = V_0$ and the value associated with working be $W = V_1$ (note that $W$ is not the present value of the earnings $w$ in the course of a job, but is rather the present value of the rest of a worker’s career, spanning all jobs and all periods of unemployment separating the jobs).

The payoff to finding work has two components. First, the value of choosing the strategy of looking for work over the strategy of not working at all is $U - Z$. Second, the further increment to finding a job once searching is $W - U$. Figure 5 plots the two elements as functions of the Frisch elasticity. The units in both cases are months of production by the worker (because the wage is always about 97 percent of production, they are essentially in units of months of wages).

The incentive to become a searcher is measured on the left axis. At my calibration, with a Frisch elasticity of labor supply of 0.7, a non-worker gains about 56 months or almost 5 years of earnings by starting to look for work. At Hagedorn and Manovskii’s calibration, with a Frisch elasticity of 1.5, the gain is 11 months of earnings. The calibration based on less
elast labor supply implies about five times the desire to find work. The right axis measures the incentive for a searcher to take an available job rather than continuing to search and to take the next available one. This incentive is small in both cases—it is about two weeks of earnings in the less elastic case and about three days of earnings in the more elastic case. A searcher making this decision finds the next job relatively rapidly and enjoys the non-work flow value of 73 or 95 percent of earnings in the interim, so the loss is quite small.

### 5.1 The job-taking rate in the MP model

In the MP model and many of its progeny, job-seekers do not make a resource decision about how hard to look for work. Nonetheless, their desire to find work has an important influence on the job-taking rate. Hungrier job-seekers have lower reservation wages and thus, under a Nash bargain, give a larger benefit to the employer upon taking a job.

Figure 6 shows the payoff to the search strategy, the job-taking rate, and the unemployment rate in the earlier calibration of the MP model, with the Frisch elasticity of labor supply on the horizontal axis. In Figures 1 and 5, I calibrated to 5.5 percent unemployment by finding the corresponding value of the vacancy flow cost, $c$. In Figure 6, I keep the flow cost of a vacancy at the value calibrated earlier for a Frisch elasticity of 0.7. The Frisch elasticity controls the desire to find work, as measured by the search payoff, $U - Z$. The payoff falls dramatically with the Frisch elasticity. The job-taking rate is much higher when the
payoff is high and job-seekers are hungry—it is 80 percent per month at the lowest elasticity and 13 percent at the highest. Unemployment is 3.6 percent in the high-payoff case and 19.2 percent in the low-payoff case.

6 Factors that Amplify the Response of Unemployment to Productivity in the MP Model

6.1 Non-labor income

The individuals considered so far have no non-labor income apart from cash benefits received while unemployed. Significant permanent non-labor income erodes the desire to find work, as shown in Figure 7. The figure shows the results of calibrating the MP model with the preferences derived in Section 2 (with Frisch elasticity of labor supply of 0.7), but with non-labor income included in the budget constraint of equation (13). The horizontal axis shows amount of non-labor income, in units of months of production.

Above 0.24 units of non-labor income, the individual will not look for or accept a job paying the 0.97 units that is the wage in the MP model. The payoff to adopting a search strategy, $U - Z$, shown on the left vertical axis, goes to zero at this level of income. The flow
The strong income effect on labor supply shown in Figure 7 mirrors the calculation of the derivative of static labor supply with respect to non-labor income, which is $-0.72$ at the calibration point. I calculate the static labor supply derivative as discussed at the beginning of Section 2. This figure is at the upper end of the findings of the labor-supply literature surveyed in Blundell and MaCurdy (1999). The Frisch elasticity of labor supply is the primary determinant of the income effect. At an elasticity of 0.3, the income derivative of static labor supply is $-0.36$ and at an elasticity of 0.1, it is $-0.17$, both within the range of estimates in the labor-supply literature.

### 6.2 Selective job formation

In the simple MP model, jobs form from every match between a job seeker and an employer. This property flows from the homogeneity of both workers and jobs. In more realistic versions of the MP model, workers or jobs are heterogeneous. Here I will consider the case of job heterogeneity. When a job-seeker and employer encounter one another, they draw a random variable $x$ and the flow value of the potential job is $px$. Both parties learn the value of $x$. If $x$ meets a threshold or reservation value $R$, they make a wage bargain and the job begins.
With $G(\cdot)$ denoting the counter-cdf of $x$, the probability of a satisfactory value of $x$ is $G(R)$. 

With selective job formation, the model distinguishes between the contact rate, $f$, determined as before by the matching technology, and the job-taking rate, $fG(R)$.

The MP model has two crucial pre-match expected values. One is the expected part of the surplus accruing to the worker, $W - U$. Once $x$ becomes known and is found to meet the threshold, the worker gains by $W(x) - U$. $W$ is the expectation of $W(x)$ conditional on $x \geq R$. The second pre-match expected value is the employer’s expected part of the surplus. I make the convenient assumption of a symmetric Nash bargain, so both the realization and expectation of the employer’s part is the same as the worker’s part.

I take the distribution of potential match productivity to be exponential, starting at $\bar{x}$:

$$G(x) = \min(1, e^{-(x-\bar{x})/d}).$$  
(23)

The truncated expectation is

$$E(x|x \geq R) = R + d.$$  
(24)

The following equations describe the stationary equilibrium of the model, in Shimer’s notation:

*Value of unemployment state*, with discount $\delta$, expected value of employment state $W$, vacancy/unemployment ratio $\theta$, and contact rate $f(\theta)$:

$$U = z + \delta [f(\theta)G(R)W + (1 - f(\theta)G(R))U].$$  
(25)

*Expected value of employment state*, with expected flow wage $w$ and exogenous separation rate $s$:

$$W = w + \delta [(1 - s)W + sU].$$  
(26)

*Expected value of filled job*:

$$J = p(R + d) - w + \delta(1 - s)J.$$  
(27)

*Equal splitting of surplus*:

$$J = W - U.$$  
(28)

*Zero-profit condition for vacancy creation*:

$$0 = -c + \delta \frac{f(\theta)}{\theta} G(R)J.$$  
(29)
Figure 8. Effect of Job Heterogeneity on the Response of Unemployment to Productivity

Threshold for non-negative surplus (job creates flow value equal to job-seeker’s opportunity cost, the interest rate times the unemployment value):

\[ pR = (1 - \delta)U. \]  

(30)

I calibrate so that the fraction of contacts that become jobs is \( G(R) = 0.5 \) and the selection premium is \( d = 0.1 \). The rest of the calibration matches the earlier one. I double the matching efficiency parameter, \( \phi \), so that the contact rate doubles and the job-taking rate \( fG(R) \) is the same as before. I choose \( p \) so that expected productivity, \( p(R + d) \), is one. In the base case with Frisch elasticity of labor supply of 0.7, the calibrated value of the reservation productivity, \( R \), is 3.17, so \( p = 0.306 \).

Figure 8 compares the response of unemployment to a drop in productivity in the new version of the MP model with heterogeneous jobs with the response in the original model shown in Figure 1. At higher elasticities of labor supply, the response is substantially stronger. The reason is that, with elastic supply, the fraction of jobs that meet the minimum requirement of searchers rises sensitively with total productivity. The job-taking rate rises not just because of the increase in the contact rate, \( f \), but also because of the rise in the fraction meeting the threshold, \( G(R) \).
6.3 Variable search effort

In the standard version of the MP model, job-seekers have a passive role. Employers expend resources to attract workers, but the unemployed do not make a resource decision. Here I investigate how a model with variable search effort alters the properties of the model, especially in the response of unemployment to changes in productivity. The desire to find work has a major role in the new model. An increase in productivity raises the desire to find work—especially in a population with more elastic labor supply—and the resulting increase in search and job-taking amplifies the response of unemployment to productivity.

Nagypál (2005) provides a simple framework relating job-seeker search effort to the payoff from taking a job. I will return to the case of homogeneous workers and jobs to investigate this subject. The job-seeker can exert effort $\epsilon$ that raises the contact rate in proportion. Let $\bar{\epsilon}$ be the average search effort of other job-seekers. The contact (and job-taking) rate is

$$ f = \epsilon \phi \left( \frac{\theta}{\bar{\epsilon}} \right)^{0.5}. $$

The marginal cost of search effort is $k$. Recall that the gain accruing to a searcher upon taking a job is $W - U$. The first-order condition for optimal search effort is

$$ (W - U) \phi \left( \frac{\theta}{\bar{\epsilon}} \right)^{0.5} = k. $$

The equilibrium job-taking rate, with $\bar{\epsilon} = \epsilon$, is

$$ f = \frac{\phi^2(W - U)}{k} \theta. $$

The recruiting rate is

$$ \frac{f}{\theta} = \frac{\phi^2(W - U)}{k}. $$

The rest of the MP model can be solved directly. The zero-profit condition for job creation is $qJ = c$ or, with $J = W - U$ according to the symmetrical Nash bargain,

$$ \frac{\phi^2 J^2}{k} = c, $$

which determines $J$ and $W - U$ in terms of the given parameters $\phi$, $k$, and $c$. Then the present-value condition,

$$ J = p - w + \delta(1 - s)J, $$

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determines the wage. Finally, the difference between the Bellman equations for $W$ and $U$,

$$W - U = w - z + \delta(1 - s - f)(W - U),$$

(37)

can be solved for $\theta$ and thus unemployment and other variables of interest.

I calibrate the new model at the point where search effort $\epsilon$ is one; this is a normalization. I calibrate so that the Bellman values are the same is in the standard MP model discussed early in the paper. I obtain the value of the marginal cost of search, $k$, from equation (32).

It is 0.24 in the base case with Frisch elasticity of labor supply of 0.7.

Figure 9 compares the responses of unemployment to productivity in the model with variable search effort to the response in the standard model with fixed search effort.

7 Concluding Remarks

The model developed in this paper gives a precise answer to the question of how much a non-working individual gains from starting work. If individuals are fully insured against their idiosyncratic risks, the answer is particularly simple, because the gain is the same for all workers. In the more realistic case of partial insurance, the answer depends in principle upon the worker’s history as summarized by current wealth, but over most of the distribution
of wealth, the gain is close to constant, so the model with full insurance is a reasonable guide to labor-market decisions of individuals.

Calibration of a flexible class of preferences to the findings of a wide range of studies of individual work and consumption data concludes that the flow value of non-work is around 70 percent of the flow value of work, for a representative individual. The desire to find work is powerful. In the standard MP model with this value of non-work, the response of unemployment to productivity shifts is far too low to account for an interesting part of unemployment volatility.

Doubling the Frisch elasticity of labor supply underlying my calibration raises the response to productivity substantially, though not to the level needed to explain all unemployment volatility. This elasticity is at the outside limit of the findings of research on individual labor supply, however.

Three other factors also raise the response to productivity. One is non-labor income. If individuals have moderate amounts of non-labor income, the response of unemployment to productivity is stronger. The second is heterogeneity in jobs. Job-seekers accept a higher fraction of available jobs when aggregate productivity is higher, so the job-taking rate rises and unemployment falls more that it does in a model with homogeneous jobs. The third is variations in job-seeking efforts of the unemployed. Higher productivity stimulates more active search, which raises the job-taking rate and further sensitizes unemployment to productivity.

The right combination of somewhat higher elasticity of labor supply, some non-labor income, some heterogeneity of jobs, and some variation in search effort is probably enough to rationalize the observed volatility of employment and unemployment.

Other forces may operate as well. On-the-job search may inhibit the response of unemployment to productivity. If wages are governed by principles other than the symmetric Nash bargain considered throughout this paper, wage stickiness may amplify the response, either because the Nash bargaining power of the job-seeker is weak or because the bargain is not Nash.
References


