Quality Pricing and Endogenous Entry A Model of Exchange Rate Pass-Through*

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Abstract

This paper develops a model of international trade under perfect competition and flexible prices that accounts for the slow and incomplete pass through of exchange rate fluctuations into consumer prices. We build an extension of the Mussa and Rosen (1978) model of quality pricing. Exporters sell goods of different quality to consumers with heterogeneous preferences for quality. In equilibrium, higher quality goods are more expensive. We derive three testable predictions. First, exchange rate fluctuations are only partially passed through to consumers. Second, there is more pass through in the long run than in the short run, and more pass through for aggregate prices than for individual prices. Third, there is more pass through for low quality goods than for high quality goods. When the exchange rate of an exporting country appreciates, existing exporters scale down their production, driving prices up. In the long run, low quality exporters pull out, driving prices up even further. Since those goods are inexpensive, aggregate prices go up more than individual prices. This exit of low quality exporters has a larger impact on the price of low quality goods than on the price of high quality goods. Low quality goods prices adjust more than high quality goods prices.

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1 Introduction

Why are movements of relative costs brought about by exchange rate fluctuations passed through to consumers partially and only gradually? We develop a model of international trade under perfect competition and perfectly flexible prices that accounts for the slow and incomplete pass through of exchange rate fluctuations into consumer prices.

Our model generates three testable predictions. First, exchange rate fluctuations are only partially passed through to consumers. Second, prices adjust more in the long run than in the short run. Third, there is more pass through for low quality goods than for high quality goods. These predictions arise in our model even though we consider a perfectly competitive setting, where firms do not set their prices strategically. Unlike existing models of incomplete pass through based on incomplete competition, these predictions are robust to having many firms competing in any given market and to free entry of firms.

We develop an extension of the Mussa and Rosen (1978) model of quality pricing. We depart from their model in two important dimensions. First, we consider a perfectly competitive setting, as opposed to the original monopoly setting. Second, we allow for the marginal cost of producing a given good to depend on the total supply of that good. This allows for a feedback of exchange rate shocks into both prices and quantities, which is at the heart of our model. Firms offer goods of different qualities. They are matched with consumers with heterogeneous preferences for quality. In equilibrium, higher quality goods are matched with higher valuation consumers. The key result is that the price schedule for goods with different qualities depends on the consumer valuation that goods are matched with. Observed prices are higher when the valuations of consumers in the market are higher. From this model of matching quality and consumer valuation, we derive three testable predictions for the pass through of exchange rate shocks.

First, exchange rate shocks are only partially passed through to consumers. When an exporting country is hit by a negative exchange rate shock, say an appreciation of its exchange rate vis à vis its trading partners, exporting firms scale down their exports. The relative scarcity of goods forces the lowest valuation consumers out of the market. As a consequence, exporters are matched with higher valuation consumers, which drives the export prices up. Part of the exchange rate shock is passed through to consumers. How much pass through there is depends on how heterogeneous consumers are in terms of valuation. With no heterogeneity in preferences, firms are always matched with consumers with the same valuation, prices are an affine function of quality, and there is minimal pass through. The more heterogeneous the consumers are, the larger the impact on prices of the shift towards higher valuation consumers, and the more pass through there is. To summarize, in our model, incomplete pass through of exchange rate shocks relies on two assumptions. First, firms face decreasing returns to scale, so that they scale down their exports in response to a negative exchange rate shock. Second, prices of goods in a given market depend on the heterogeneity of preferences, so that when fewer goods are supplied, only high valuation consumers remain.

Second, we predict that there is more pass through of exchange rate shocks into prices in the long run than in the short run, and that in the long run, aggregate prices respond more than individual prices. These two predictions are due to the endogenous selection of exporters, and to the composition effect of goods with different prices. In the presence of fixed entry cost into foreign markets, only the highest quality firms are able to export. When hit by a negative exchange rate shock, in the long run, the lowest quality firms pull out of the export market. These firms happen to be those firms that are charging the lowest price. This exit of low quality low price exporters has two effect, on individual prices, and on aggregate prices. First, the exit of low quality exporters shrinks the total supply of goods, driving out low valuation consumers. The remaining firms are matched with higher valuation consumers, so that each individual price increases further. Second, since only the low quality low price exporters pull out, the average price among the remaining exporters increases, so that aggregate prices increase even more than individual prices.

Third, we predict that there is more pass through for low quality goods than for high quality goods. This prediction relies on a subtle argument. After a negative exchange rate shock, there are two forces driving all prices up. First, the exit of low quality firms shrinks the total supply of goods, forces the lowest valuation consumers out of the market, so that the average valuation of the remaining consumers increases, and all prices increase. Second, all firms scale down their production, which also shrinks the total supply of goods and pushes all prices up. The relative strength of this second effect is larger for higher quality goods. For the lowest quality goods actually exported, the first effect is the main source of price increase. The lowest quality exporting firm exactly breaks even, so that its price is exactly equal to its cost. Following an appreciation of the exchange rate, which amounts to a negative productivity shock for exporters, low quality firms exit until the last firm exactly breaks even. So for the lowest quality goods, prices move almost one for one with the exchange rate[1]. The price of higher quality goods on the other hand depends on the overall tightness of the market, which determines which consumer they are matched with. In the limit, infinitely high quality goods prices are, in relative terms, not affected at all by the exit of low quality firms. Their price increases only because all firms scale down their production. The pass through of exchange rate shocks is higher for low quality goods than for high quality goods.

The predictions of our model match the existing stylized facts in the empirical literature on exchange rate pass through. First, exchange rate shocks are only incompletely passed through to consumers. Second, there is a relative consensus that there is more pass through of exchange rate shocks in the long run than in the short run. Third, there is some evidence that there is more pass through of exchange rate shocks for low quality goods than for high quality goods.

Campa and Goldberg (2006) gives an up to date review of the evidence on incomplete pass through, and on the larger pass through in the long run than in the short run. Even though there is almost full pass through of exchange rate shocks for prices at the dock, there is much more limited pass through for consumer prices. The order or magnitude is 40% in the short run to 60% in the long run. To explain this discrepancy between the pass through at the dock and the pass through into consumer prices, Burstein, Neves and Rebello (2003), Burstein, Eichenbaum and Rebello (2005), and Campa and Goldberg (2006) argue that non tradable inputs play a key role. Burstein et al. (2003) note that for a typical consumption good in the US, distribution margins account for more than 40% of the final price. Campa and Goldberg (2006) also note that distribution margins do not remain stable during real exchange rate fluctuations. A 1% real exchange rate depreciation leads to a .47% reduction in distribution margins. To capture these facts, we introduce a two tiered production function similar to Bacchetta and van Wincoop (2003). Exporters must not only ship goods, but also assemble them locally. The assembly process is subject to decreasing returns to scale. This gives rise to incomplete pass through of exchange rate shocks, despite full pass through at the dock, and to fluctuations in the distribution margin in response to exchange rate movements.

We point out the potential importance of composition effects in estimating exchange rate

[1] Note that the price of the lowest quality good actually exported moves exactly one for one with the exchange rate. However, because some firms exit, the lowest quality good is no longer the same after an exchange rate appreciation. The good that becomes the lowest quality good exported after the exchange rate appreciation was above the lowest quality before the exchange rate shock. Therefore, its price increases less than one for one with the exchange rate. To simplify, this is the basic intuition for the *incomplete* pass through of exchange rate.

pass through. In our model, since low quality firms gradually pull out of the export market following an appreciation of the exchange rate, and since low quality goods command a low price, aggregate prices gradually go up in part due to this composition effect. Burstein, Eichenbaum and Rebello (2005) suggest one specific composition effect, flight from quality. They point out that following a large devaluation, consumers stop buying high quality goods. Our predictions on this flight from quality are ambiguous. Indeed, following a devaluation, we predict that overall, since fewer quality goods are imported, many consumers switch from quality goods to generic goods. However, among those consumers that still buy quality goods, they will typically buy higher quality goods, at a higher price. This second part entirely relies on the assumption of partial equilibrium that we make. We do not consider the impact of exchange rate fluctuations on the disposable income, so that consumers in our model are never prevented from buying quality goods because of their budget constraint. This is an important limitation of our model, but it makes the whole analysis much simpler. We believe that this model describes normal exchange rate movements well enough, but may miss what happens during very large fluctuations such as large devaluations.

There is only scarce evidence of the relative pass through of exchange rate shocks for goods of different quality. Gagnon and Knetter (1995) study the exchange rate pass through for car exports from three main automobiles exporters. This remains however an understudied area. Despite a growing literature on measuring the quality of exports, there is at this time little evidence on the degree of exchange rate pass through for exports of different quality.

The existing theoretical literature on exchange rate pass through and pricing to market has so far relied entirely on two alternative assumptions: either price stickiness, or imperfect competition. We take a radically different approach. In our model, prices are fully flexible, and markets are perfectly competitive. Incomplete pass through arises in a competitive equilibrium because of decreasing returns to scale, and because of the endogenous selection of exporters, and not because prices are sticky or because oligopolists strategically adjust their prices.

Betts and Devreux (1996), Taylor (2001) or Bacchetta and van Wincoop (2003) assume that prices are sticky. However, there is some evidence that exporters have the ability to change their prices rather easily, as shown by Burstein, Eichenbaum and Rebello (2005), and Bills and Klenow (2001).

Most existing models of exchange rate pass through rely on incomplete competition. The seminal papers of Krugman (1987) and Dornbusch (1987) have been followed by more elaborate models, such as Yang (1997), Melitz and Ottaviano (2005) or Atkeson and Burstein (2006). All these models rely on the fact that when firms adjust their prices, they move along the demand curve, and face a different demand elasticity. Monopolists, taking the shape of the demand curve into account, will endogenously adjust their prices so as to maximize their profits. Under some conditions on the shape of the demand curve, exporters will adjust their markups so as to dampen price fluctuations, leading to pricing to market and incomplete pass through of exchange rate shocks. We chose to depart from these assumptions for two reasons. First, those models rely on extreme assumptions of imperfect competition. As the number of firms competing in a sector increases, the pricing to market predictions quickly become negligible[2]. Moving directly to a competitive setting provides more robust predictions. Second, those models rely on specific assumptions on the shape of the third derivative of the utility function, which are not generic. By not relying on such subtle properties of the demand function, we are able to describe how competitive forces lead to pricing to market and incomplete exchange rate pass through.

Finally, our model delivers predictions for the composition effect of prices that are in stark contrast with the existing trade literature with heterogeneity in productivity. In the Melitz (2003) model of trade with heterogeneous firms, the most productive firms charge the lowest price. When hit by a negative productivity shock, such as an appreciation of the exchange rate, the firms that exit the export market are the least productive firms, that is the firms that charge the highest price. This exit of firms leads to a *reduction* of aggregate prices. We get the exact opposite prediction for the direction of the composition effect of the endogenous entry and exit of exporters. In our model, the highest quality goods are sold at the highest price. When hit by a negative productivity shock, the lowest quality exporters exit the market. Since their goods are the cheapest ones, this leads to an *increase* of aggregate prices[3]. We find this property of our model appealing, since it predicts that in the long run, as the set of firms active on the export market adjusts, there is more pass through of exchange rate shocks than in the short run.

The remaining of the paper is organized as follows. In section 2, we present the general set up of our model of quality pricing. In section 3, we analyze a specific example and provide closed form

[2] Note that the model developped in Melitz and Ottaviano (2005) stands out from this literature. Its predictions are robust to having more than a few firms competing. The action in their model comes from the specific utility function they use, even though they also rely on a monopolistic setting.

[3] It should be noted that once endogenous entry of new firms into the domestic market is allowed, as in Ghironi and Melitz (2005), a positive productivity shock may lead to an *appreciation* of the terms of trade. Since we do not consider the endogenous entry into the domestic market, we cannot directly compare our predictions to those.

solutions. In section 4, we derive the predictions of our model for exchange rate pass through. Section 5 concludes.

2 Model

In this section, we develop a model of quality pricing and international trade.

There are two countries, home and foreign. The two countries are respectively populated by a mass L_H and L_F of consumers that share the same preferences. There are two sectors, $\mathcal{A}$ and Q. The $\mathcal{A}$ sector produces a homogeneous good, which may be freely traded. We will only consider equilibria where all consumers in each country consume some of this numeraire good. We can therefore normalize the price of this good to unity in each country. The Q sector produces a continuum of goods that differ in terms of quality. For simplicity, we assume that Q goods are differentiated by country of origin.

We consider a perfectly competitive setting. There is a continuum of atomistic firms producing each type of good. Those firms are price taker. Firms in the Q sector are heterogeneous in terms of the quality of the good they produce. They face an increasing returns to scale technology. In addition, in order to enter foreign market, they must pay a fixed entry cost. There is a continuum of (heterogeneous) consumers buying those goods. The consumers are price taker too.

The timing is the following. First, firms receive their quality draw. Second, they decide whether or not to enter each market, home and foreign. Third, given the prices that they expect, they decide how much output to produce. Finally, prices are determined so as to clear all markets. The strategies of firms and consumers are the following. Firms maximize expected profits, given their expectation for prices. Consumers maximize their utility, given the set of goods available to them, and given the prices they observe.

Preferences

Consumers can consume a continuum of $\mathcal{A}$ goods. For the consumption of Q goods, we consider a discrete choice model. Consumers can consume either zero or one unit of domestic Q good, and either zero or one unit of foreign Q good. Different Q goods have different quality, and different consumers have different valuation for quality. A consumer with valuation v for quality, who consumes one unit of home good with quality q_H and one unit of foreign good with quality q_F, and A units of the homogenous good, derives a utility,

$$U_v(q_H, q_F, A) = v(q_H + q_F) + A/a \quad (1)$$

where the marginal utility of the homogenous good, $1/a$, is a positive constant. For simplicity, if a consumer does not consume one of the Q goods, we set its quality to zero.

Valuations for quality, v, are distributed over all consumers according to,

$$v \sim F_v(v) \quad (2)$$

where F_v is the cumulative distribution of the v's. The density of consumers at any level v of valuation is $f_v(v)$. Valuations are distributed over the interval $[\bar{v}, v^{\max}]$[4]. We assume that there is a strictly positive density over the entire domain: $f_v(v) > 0$ for $v \in [\bar{v}, v_{\max}]$. We also assume that the distribution of income is such that consumers can always afford to buy one unit of Q good[5].

The main property of these preferences is that valuation and quality are complementary: the higher a consumer's valuation, the more she values quality, and the more she will be willing to pay for quality. This property allows us to derive two important results. First, there is assortative matching between consumers and goods, that is higher valuation consumers will buy higher quality goods. Second, the pace at which prices increase with quality is exactly determined by the valuation of consumers. We state and prove formally these two results in the following two propositions.

Proposition 1 (assortative matching) *If an equilibrium exists, consumers' valuations and goods' quality are matched assortatively:*

$$v_1 > v_2 \Rightarrow q_1 \geq q_2$$

where consumer $i = 1, 2$ with valuation v_i is matched with a good of quality q_i.

Proof. *See appendix A, page 8.* ■

Given the complementarity between quality and valuation built into the preferences, assortative matching is a very intuitive result. High valuation consumer gain benefit more from quality.

[4] In principle, we allow for $v^{\max} = +\infty$. In our closed form example in section 3, we consider unbounded from above supports for the distribution of valuation draws.

[5] Implicitly, we assume that high valuation consumers also have a high income, so that they can afford the high price for the Q good they will buy in equilibrium.

It would not be optimal to allocate high quality goods to low valuation consumers, and hence any market equilibrium must allocate higher quality goods to higher valuation consumers.

A direct corollary of this assortative matching is that, locally, relative prices are pinned down by a no arbitrage condition on the consumer side. Higher quality goods are more expensive. Moreover, prices increase with quality exactly according to the valuation of the consumers. The following proposition states this result formally.

Proposition 2 *If an equilibrium exists, the mapping from goods quality to prices is continuously differentiable. The prices are determined locally by the valuation of consumers in the following way,*

$$p'(q) = av(q)$$

where $v(q)$ is the valuation of the consumer matched with a good of quality q, $p(q)$ is the price of this good, and $p'(q)$ is the derivative of this price schedule.

Proof. *See appendix A, page 20.* ■

It is straightforward to see from the previous two propositions that prices are increasing and convex in quality. This property of prices is reminiscent of the Mussa and Rosen (1978) model of quality pricing. Whether goods are supplied by a monopolist, as in Mussa and Rosen (1978), by oligopolists as in Champsaur and Rochet (1989), or by atomistic price taking firms as in this model, prices must increase at an accelerating pace in order to prevent high valuation consumers from buying low quality goods.

In the next section, we describe the production technology, and the behavior of firms.

Production

Production in the $\mathcal{A}$ sector is made under constant returns to scale. The labor productivity at home (abroad) is Z_H (Z_F). We will only consider equilibria in which both countries produce the $\mathcal{A}$ good. Labor can freely move between sectors. So the wage w_H (w_F) of domestic (foreign) workers, in units of the $\mathcal{A}$ numeraire good, is simply equal to Z_H (Z_F).

Goods' quality: In the Q sector, there is a continuum of mass M_H (M_F) of firms in the home (foreign) country. Each of these firms produces a good of a specific quality. Firms randomly draw a quality shock from a stochastic distribution given by,

$$q \sim F_q(q) \tag{3}$$

where F_q is the cumulative distribution of the q's. The density is f_q. Qualities are distributed over the interval $[\bar{q}, q_{\max}]$[6].

Technology: Despite their differences in quality, all firms face the same technology for producing Q goods. They are subject to decreasing returns to scale. The cost for supplying S units of Q goods is given by $w_H C(S)$, with $C(\cdot)$ increasing and convex. We denote the marginal cost of supplying the S^{th} unit of good by $w_H c(S) = w_H C'(S)$, $c'(S) > 0$. For simplicity, we assume that firms produce goods for the domestic market independently from goods for the export market. The cost function applies to each type of production separately. This allows us to study sequentially the domestic production decision and the foreign production decision.

Trade barriers: In order to export abroad, domestic firms must overcome both a variable cost for shipping each unit of good abroad, and a fixed cost of entering the foreign market. Those costs are symmetric. The variable cost takes the traditional form of iceberg transportation costs. A fraction $(\tau - 1)$ of all shipments disappears on the way, with $\tau > 1$. So in order to sell 1 unit abroad, a firm must produce τ units. The fixed cost of entry is equal to $\tau w_H f^E$, which is paid in units of the $\mathcal{A}$ numeraire good.

We now consider the decision of a domestic firm who decides to export abroad. Leaving aside for the moment the question of whether or not it is profitable to pay the fixed entry cost, we characterize the quantity an exporter would supply abroad. Firms are price taker, so they decide to increase their supply of goods until their marginal cost equals the price of their good. In equilibrium, a firm that expects a price p for its good supplies $S(p)$ units abroad, with $S(p)$ defined by, $\tau w_H c(S(p)) = p$. We can rewrite this optimality condition as,

$$S(p) = c^{-1}\left(\frac{p}{\tau w_H}\right) \tag{4}$$

where c^{-1} is the inverse of the cost function. Note that the marginal cost of selling the S^{th} unit of good abroad is the marginal cost of production multiplied by τ. To sell one unit abroad, a firm must export τ units, each at a cost $w_H c(S)$. The marginal cost is strictly increasing in the quantity supplied, so that the quantity supplied S is strictly increasing in the price p. All firms follow the same strategy and supply a quantity which depends of the price they expect to receive for their quality.

Entry decision: Firms must decide whether or not to pay the fixed entry cost into the foreign market. They compare the profits they would earn from exporting to the fixed entry cost.

[6] In principle, we allow for $q^{\max} = +\infty$. In our closed form example in section 3, we consider unbounded from above supports for the distribution of quality shocks..

Only those firms whose gross profits are above the entry cost export. There is minimum price $p_{\min}$ below which it is not profitable to export. The minimum price is given by the following zero profit cutoff condition,

$$p_{\min} S\left(p_{\min}\right) - \int_0^{S(p_{\min})} \tau w_H c\left(s\right) ds - \tau w_H f^E = 0 \tag{5}$$

It states that the net profit from exporting if the price abroad is $p_{\min}$ is exactly zero. Since $c\left(\cdot\right)$ and $S\left(\cdot\right)$ are strictly increasing, $p_{\min}$ is uniquely determined by Eq. (5).

Note that for the moment, we know the price of the lowest quality exported, but we still have not determined the actual level of the lowest quality exported. It is determined in equilibrium, which we define in the next section.

Equilibrium

An equilibrium consists of a price schedule such that the goods market clears if consumers optimally chose which good to consume, if any, and if firms optimally chose how much to produce and whether or not to enter the foreign market. We will construct the equilibrium in the following way. First, we match goods to consumers. Given this matching, we define the price schedule matching quality to price, up to a constant. We then identify the quality of the good matched with the lowest valuation consumer.

First, note that there are potentially three possible types of equilibrium: a sellers' market where there are more consumers than goods, a buyers' market where there are more goods than consumers, or a third case where neither all exporting firms sell their good, nor all consumers buy a Q good. We prove in appendix B that in equilibrium, we are always in the first case, a sellers market. So in equilibrium, all exporting firms sell their goods, but not consumer buy a Q good.

We can rewrite the matching implied by proposition 1 and define formally the matching between quality and consumers. A good of quality q will be matched to a consumer with quality $v\left(q\right)$, according to,

$$N_H \int_q^{q_{\max}} S\left(p\left(\chi\right)\right) f_q\left(\chi\right) d\chi = L_F \int_{v(q)}^{q_{\max}} f_v\left(v\right) dv \tag{6}$$

for any $q \in [q_{\min},\ q_{\max}]$, where $q_{\min}$ is the lowest quality exported, and $S\left(p\left(\chi\right)\right)$ is the quantity of good supplied by a firm with quality χ. The left hand side is the number of goods with quality q and above, whereas the right hand side is the number of consumers with valuation $v\left(q\right)$ and above. For any level of quality q, these two must be equal.

Given the matching between goods and consumers, we can derive prices from proposition 5. Integrating prices over quality, we get the price the price $p(q)$ of a good of quality q,

$$p(q) = a \int_{q_{\min}}^{q} v(\chi)\, d\chi + p_{\min} \tag{7}$$

for any $q \in [q_{\min},\ q_{\max}]$, where $v(\chi)$ is the valuation of the consumer matched with quality χ given in Eq. (6), and $p_{\min}$ is the price of the lowest quality exported, given by the zero cutoff profit condition (5).

We now have to determine the quality of the good matched with the lowest valuation consumer. Since we are in a sellers' market, some consumers will not buy any Q goods. The last consumer buying must exactly break even. She must be indifferent between buying and not buying good $q_{\min}$, or in other words, she must be indifferent between buying $q_{\min}$ or buying $\mathcal{A}$ goods instead. The lowest quality exported $q_{\min}$ is defined by,

$$av(q_{\min})\, q_{\min} = p_{\min} \tag{8}$$

where $v(q_{\min})$ is the valuation of the consumer matched with quality $q_{\min}$ given in Eq. (6), and $p_{\min}$ is the price of the lowest quality exported, given by the zero cutoff profit condition (5).

An equilibrium price schedule will be solution to the zero cutoff profit condition (5), the matching equation (6), the pricing equation (7), and to equation (8) defining the lowest quality exported. The following proposition states the existence of such an equilibrium.

Proposition 3 *There exists a* $(p(\cdot),\ v(\cdot),\ p_{\min},\ q_{\min})$ *solution to Eqs. (5), (6), (7) and (8), not necessarily unique.*

Proof. *See appendix A, page 21.* ■

In order to derive closed form solutions for the path of exchange rate pass through, we introduce a specific functional form for the distribution of valuation and quality draws. We present this example in the next section.

3 A closed form example

In order to analyze the properties of exchange rate pass through in our model, we consider a specific example. We are able to derive closed form solutions for the equilibrium and for all variable of interest in this case.

First, we assume that both valuation shocks and quality shocks are Pareto distributed. The distribution of both shocks are as follows,

$$\begin{cases} F_v(v) = 1 - \left(\frac{v}{\bar{v}}\right)^{-\lambda_v} \\ F_q(q) = 1 - \left(\frac{q}{\bar{q}}\right)^{-\lambda_q} \end{cases} \tag{9}$$

Next, we assume that the marginal cost function takes the following form,

$$c(S) = \tau w_H + \tau w_H S^{1/\eta} \tag{10}$$

Implicitly, we assume a two tiered production function. In order to sell one unit of Q good, a firm must first ship its good to the destination market, and then assemble those goods locally. We assume that firms have a fixed installed capacity (that they acquired when they paid the fixed cost of entry), and that assembly is subject to decreasing returns to scale. The first term, τw_H, in the cost function in Eq. (10) corresponds to the cost of shipping one additional unit of good abroad. The second term, $\tau w_H S^{\eta}$, corresponds to the cost of assembly. Because of decreasing returns, the cost of assembling one additional unit of good increases with the total quantity supplied, S. This simple functional form for the marginal cost ensures that in equilibrium, the supply elasticity will be constant and equal to η for all firms.

Firms equalize their marginal cost to the price they face, so that we have the following expression for the supply of Q goods as a function of price,

$$S(p) = \left(\frac{p}{\tau w_H} - 1\right)^{\eta} \tag{11}$$

We are now able to solve for the equilibrium price schedule, as the following proposition shows.

Proposition 4 *If the entry cost is such that* $f^E = \left(\frac{\lambda_q - \eta}{(1+\eta)(\lambda_q + \lambda_v)}\right)^{1+\eta}$*, then there exists a unique equilibrium price schedule, lowest quality exported, and lowest price, defined as,*

$$\begin{cases} p(q) = \gamma (\tau w_H)^{\eta/(\lambda_v+\eta)} q^{(\lambda_v+\lambda_q)/(\lambda_v+\eta)} + \tau w_H \\ q_{\min} = \gamma' (\tau w_H)^{\lambda_v/(\lambda_v+\lambda_q)} \\ p_{\min} = \left(\frac{\lambda_q+\lambda_v}{\lambda_q-\eta}\right) \tau w_H \end{cases}$$

with γ *and* γ' *some constants*[7].

Proof. *See appendix A, page 23.* ■

[7] $\gamma = \left(a^{\lambda_v} \frac{\lambda_v \bar{v}^{\lambda_v}}{\lambda_q \bar{q}^{\lambda_q}} \left(\frac{\lambda_v+\eta}{\lambda_v+\lambda_q}\right)^{\lambda_v} \frac{\lambda_q-\eta}{\lambda_q+\lambda_v} \frac{L_F}{N_H}\right)^{1/(\lambda_v+\eta)}$ and $\gamma' = \left(\frac{\lambda_v+\eta}{\lambda_q-\eta}\right)^{(\lambda_v+\eta)/(\lambda_v+\lambda_q)}$.

As expected, the price schedule that maps quality to prices is increasing and convex in quality. But we can now describe the impact of each parameter of the model on the actual shape of the price schedule.

Asymptotically, the elasticity of the price with respect to quality converges to $\frac{\lambda_v+\lambda_q}{\lambda_v+\eta} > 1$. The more elastic the supply of goods by each individual exporter, that is the larger η, the less responsive are prices to changes in quality. If the technology of production is such that large changes in the quantity supplied are needed to generate some change in the marginal cost of production (η high), then firms with a higher quality will supply much larger quantities than firms with a lower quality. Instead of the price adjusting to make demand for and supply of quality meet, most of the adjustment will come through quantities. The price of higher quality goods will not be very high.

The other two key parameters that determine how prices are responsive to changes in quality are the measure of the fatness of the tails of the distributions of quality and valuation for quality, λ_q and λ_v. If the quality of firms is more homogenous (λ_q high), or if the valuation of consumers is more heterogeneous (λ_v small), prices will be more responsive to changes in quality. This is entirely driven by the sensitivity of either supply or demand to changes in prices. If firms are very homogenous, that is if most of the mass of firms is concentrated around the bottom of the distribution, higher qualities are very scarce. The price of those higher qualities will therefore be high. By the same token, if the distribution of consumers' valuations is very dispersed, there are relatively many consumers that a high valuation for quality, and who are therefore willing to pay a high price for higher qualities. The price of higher quality goods will therefore be high.

The equilibrium price schedule is presented on Figure 1, which plots the log of quality versus the log of price. $\ln \tau w_H$ is the cost of shipping one unit of good abroad, absent of any assembly cost. Because of the existence of a fixed entry cost, firms must sell more than one unit of good in order to generate enough profit to recover this entry cost. There is a minimum quality, $q_{\min}$, that commands a minimum price $p_{\min}$, and this minimum price is strictly above $\ln \tau w_H$. Below that price, no firm is willing to export. So any firm with a quality below $q_{\min}$ will not export its good abroad. The equilibrium price schedule starts at $p_{\min}$ and is then increasing and convex, and it converges asymptotically to a log linear relationship.

Now that we have characterized the equilibrium price schedule, we can describe the impact of exchange rate shocks on prices.

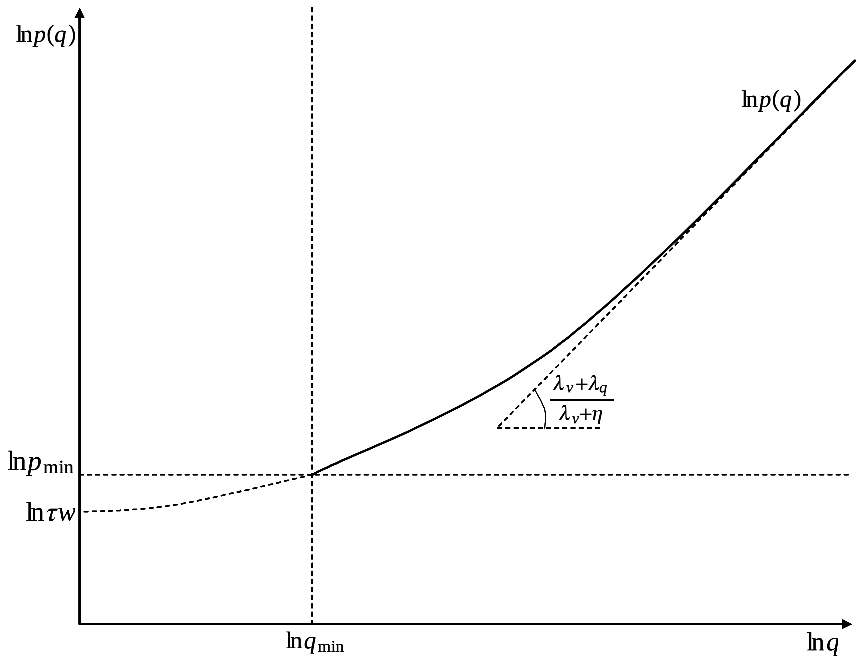

Figure 1: Equilibrium price-quality schedule.

4 Exchange rate pass through

In this section, we describe the impact of exchange rate shocks on prices. We first define exchange rate shocks as shocks to real wages arising from productivity shocks. We then characterize the response to those shocks of individual prices, of the composition of exporters, and of aggregate prices.

We define a shock to the exchange rate of the home country as a shock to the domestic wage in terms of the international numeraire $\mathcal{A}$ good. When the domestic productivity in the $\mathcal{A}$ sector Z_H increases, as labor is freely mobile between sectors, the domestic wages will have to increase proportionally with the productivity. For firms in the Q sector, this amounts to a negative productivity shock: firms must pay their workers a higher wage, in units of the numeraire. In this section, we will therefore define an appreciation of the domestic exchange rate as an increase in the real wage w_H.

What is the response of export prices to such an exchange rate shock? There are two margins that will adjust to such an exchange rate shock. First, firms, facing a higher marginal cost, scale down their production and export smaller quantities abroad. This is the intensive margin of adjustment. Second, facing this higher cost, some low quality firms stop exporting altogether.

This is the extensive margin of adjustment. Those two margins lead to an overall reduction of the total quantity of Q goods exported, so a relative scarcity of home Q goods abroad. Low valuation consumers are pushed out of the market and stop buying Q goods altogether. Overall, goods are matched with higher valuation consumers, so that prices increase. This is the source of exchange rate pass through into prices in our model. As fewer goods are exported, prices increase. Because some low quality firms exit the export market, and because supply responds to changes in marginal cost with some finite elasticity, the pass through is incomplete.

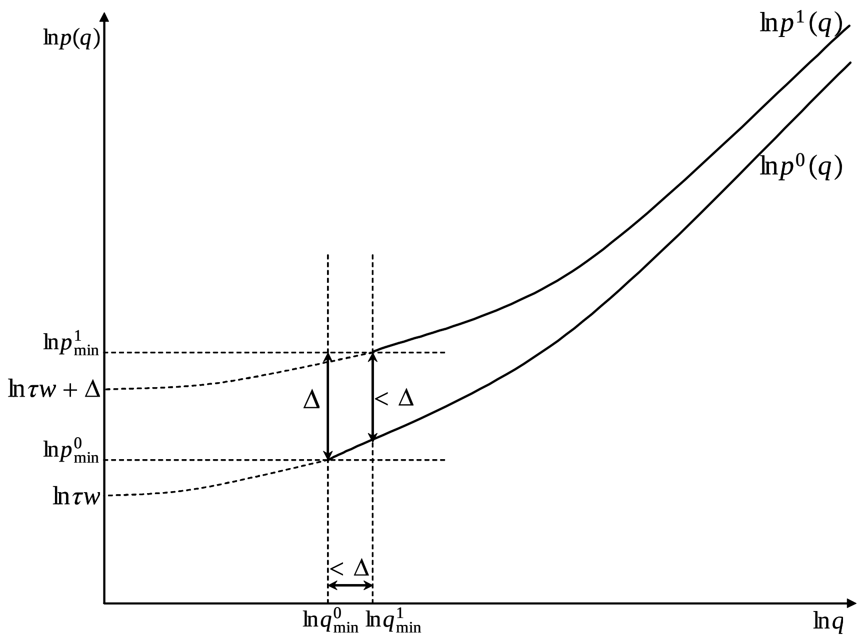

Figure 2: Exchange rate pass through.

The response of prices to an exchange rate shock is depicted on Figure 2, which plots the log of quality versus the log of price for two levels of the exchange rate. The exchange rate appreciates from τw at date $t = 0$, to $\tau w \times \Delta$ at date $t = 1$, with the constant $\Delta > 1$. Following an appreciation of the exchange rate, the price of the lowest quality exported, $p_{\min}$, increases proportionally with the exchange rate. However, the lowest quality firms pull out of the export market, so that the lowest quality exported, $q_{\min}$, increases. This exit of firms, as well as the reduction in the quantities exported by all firms, leads to an increase of the prices charged for exports. For every level of quality, the price increase is less than proportional to the exchange rate. Moreover, the price increase is lower for higher quality goods.

In the remaining of this section, we describe formally the response of individual prices to exchange rate shocks, the composition effect of exchange rate shocks, and the response of aggregate prices.

Proposition 5 (exchange rate pass through) *There is incomplete pass through of exchange rate shocks into the price of individual goods. The lower the quality of a good, the higher the pass through.*

Proof. *See appendix A, page 23.* ■

When the exchange rate appreciates, firms scale down their production, and some low quality exporter exit the export market altogether. There are two forces that drive all prices up. First, the lowest quality exported is now higher. The valuation of the consumer buying the lowest quality good increases. Since this consumer is willing to pay a higher price for the Q good she buys, the price of the low quality goods increase. Second, the overall supply of Q goods abroad shrinks, so that goods are now matched with higher valuation consumers. The slope of the price schedule gets steeper, and all prices increase. Prices of goods at different level of quality are affected by these two forces in different ways. For very high quality goods, the exit of low quality firms and the effect this has on prices is negligible. Only the second force, the overall tightening of the market, matters. For low quality goods on the other hand, both the change in the lowest quality exported, and the overall tightening of the supply matter. In relative terms, low quality goods prices increase more than high quality goods prices. There is more pass through for low quality goods.

In order to understand the composition effect due the endogenous selection of firms into the export market, we have to characterize precisely the response of the extensive margin of trade to exchange rate fluctuations. The following proposition describes how both the lowest price and the lowest quality exported respond to exchange rate shocks.

Proposition 6 *The price of the lowest quality exported moves one for one with the exchange rate. The lowest quality exported increases less than proportionally with the exchange rate.*

Proof. *See appendix A, page 26.* ■

As the exchange rate appreciates, both the price of the lowest quality exported, $p_{\min}$, and the actual lowest quality exported, $q_{\min}$, increase. Mechanically, since the fixed entry cost is paid in foreign labor, $p_{\min}$ goes up one for one with the exchange rate. Therefore, the lowest minimum price at which any firm is willing to export increases one for one with the exchange rate. However, because of the increase in the marginal cost of production, some firms exit the export market altogether, so that the lowest quality exported increases. Therefore, even for the lowest quality exporter, the price charged abroad increases less than one for one with the exchange rate. After an appreciation of the exchange rate, the new lowest quality exporter has a quality higher than that of the lowest quality exporter prior to the exchange rate shock. Since the price strictly increases with the quality, the new lowest quality good exported experiences an increase in its price that is less than proportional to the exchange rate shock.

Despite the fact that all prices increase less than proportionally with the exchange rate, we prove that aggregate prices increase exactly proportionally with the exchange rate. This is due to the composition effect of low quality exporters pulling out of the export market. Since those exporters charge the lowest price, their exit drives down aggregate prices. The following proposition states this result formally.

Proposition 7 (aggregate pass through) *Aggregate prices are proportional to the exchange rate, where aggregate prices are defined as a weighted average of individual prices, with consumer expenditure shares used as weights.*

Proof. *See appendix A, page 27.* ■

In this section, we have proved three main results. First, following a shock to the real exchange rate, there is only incomplete pass through into prices for all goods. Second, the pass through of exchange rate shocks is higher for lower quality goods. Third, the composition effect due to the exit of low quality exporters, implies that there is more pass through at the aggregate level than at individual levels. In the specific example that we have developed, the composition effect is such that there is complete pass through of exchange rate shocks for aggregate prices.

5 Conclusion

The contribution of this paper is to explain how – in the presence of complete markets and perfect competition – cost changes brought about by movements of the real exchange rate are transmitted internationally and what factors explain the magnitude of pass-through. Our model delivers three main predictions. First, there is incomplete pass through of exchange rate shocks into consumer prices of imported goods. Second, there is more pass through for low quality goods than for high quality goods. Third, there is more pass through in the long run than in the short run, and more pass through at the aggregate level than at the individual good level.

We first develop a perfectly competitive economy featuring heterogeneity of both good qualities and of consumer valuations. In equilibrium, high valuation customers and high quality firms are matched, and the relative scarcity of goods of different qualities leads to pricing-to-market, with prices determined by the local tightness of competition. We analyze how of changes in the relative cost of production affect prices in the short and in the long run. In the short run, the set of firms active in the export sector is fixed, but each firm accommodates changes in the relative cost brought about by a change in the exchange rate by adjusting the quantity of its exports. Since the quantities supplied decrease when the home currency appreciates, export markets get relatively less crowded and thus prices measured in foreign currency increase, leading to partial exchange rate pass-through in the short run. In the long run, the range of firms that are actively exporting changes. In the presence of fixed costs of market access, some low quality firms no longer export. While the short run change in the intensive margin (volume of exports per firm) affects all firms equally, this change in the extensive margin affects low quality firms relatively more, with two associated consequences. First, fewer firms are active in the export sector. Second, low quality goods prices respond more to exchange rate shocks than high quality goods prices.

A further consequence of the long run change in the set of exporters is that the average composition of firms changes, leading to an even larger pass through when evaluating aggregate data. An appreciation of the home currency drives out low quality firms that receive a low price for their goods. The observed aggregate price is hence averaged over a set of higher priced firms, leading to an overestimation of long term pass through when using aggregate data. Incorporating this finding, we show that a researcher estimating pass-through in the long run might actually arrive at the conclusion that long run pass through is equal to 100%. These results differ drastically from the existing literature. We model firm heterogeneity as heterogeneity in good quality and not in productivity. In our model, firms producing higher priced goods are more profitable, because high quality goods command high prices. In the long run, the composition effect hence tends to magnify the initial exchange rate movement, which is the opposite of what models with heterogeneity in productivity would predict.

Appendix A: proofs

Proof of proposition 1 (assortative matching)

Proposition 1 (reminded) *If an equilibrium exists, consumers' valuations and goods' quality are matched assortatively:*

$$v_1 > v_2 \Rightarrow q_1 \geq q_2$$

where consumer $i = 1, 2$ *with valuation* v_i *is matched with a good of quality* q_i*.*

Proof. By way of contradiction, assume there is an equilibrium such that,

$$v_1 > v_2 \text{ and } q_1 < q_2$$

In such a case, consumer 1 with valuation for quality v_1 is willing upgrade quality by exchanging his good of quality q_1 against consumer 2's good of quality q_2 and in addition pay her as much as $av_1 (q_2 - q_1)$ units of the $\mathcal{A}$ good. Consumer 2 on the other hand is willing to downgrade his quality by exchanging his good q_2 against good q_1 in exchange for at least $av_2 (q_2 - q_1)$ units of the $\mathcal{A}$ good. Note that

$$v_1 > v_2 \text{ and } q_2 > q_1 \Rightarrow av_1 (q_2 - q_1) > av_2 (q_2 - q_1)$$

so that both consumers will agree to exchange their goods and at least one of them will be strictly better off. This cannot be an equilibrium. Hence, in any equilibrium, it must be that

$$v_1 > v_2 \Rightarrow q_1 \geq q_2$$

■

Proof of proposition 2

Proposition 2 (reminded) *If an equilibrium exists, the mapping from goods quality to prices is continuously differentiable. The prices are determined locally by the valuation of consumers in the following way,*

$$p'(q) = av(q)$$

where $v(q)$ *is the valuation of the consumer matched with a good of quality* q*,* $p(q)$ *is the price of a good of quality* q*, and* $p'(q)$ *is the derivative of this price schedule.*

Proof. Suppose that an equilibrium exists. Take any two consumers with valuation $v_1 > v_2$, who are matched respectively with goods of quality q_1 and q_2, with prices p_1 and p_2. Given those prices, consumer 1 would strictly prefer to buy q_2 instead of q_1 if $v_1(q_1 - q_2) > (p_1 - p_2)/a$. In the same way, consumer 2 would strictly prefer to buy q_1 instead of q_2 if $v_2(q_1 - q_2) < (p_1 - p_2)/a$. If we are in equilibrium, given prices, consumers must not be willing to change their consumption bundles. So it must be that,

$$av_2 \leq \frac{p_1 - p_2}{q_1 - q_2} \leq av_1$$

These inequalities must hold for any q_1 and q_2, which implies that for any $q_0 \in \left[q^{\min}, q^{\max}\right]$,

$$\lim_{q \to q_0^+} \frac{p(q) - p(q_0)}{q - q_0} = \lim_{q \to q_0^-} \frac{p(q) - p(q_0)}{q - q_0} = p'(q) = av(q)$$

where $q^{\min}$ is the lowest quality actually consumed in equilibrium, with the left derivative only for $q_0 = q^{\min}$, and the right derivative only for $q_0 = q^{\max}$.

Therefore, prices increase with quality. The price schedule mapping qualities to prices is continuous and continuously differentiable. And the derivative of the price schedule is exactly equal to the valuation for quality, denominated in units of marginal utility of the $\mathcal{A}$ good. ■

Proof of proposition 3

Proposition 3 (reminded) *There exists a* $(p(\cdot),\ v(\cdot),\ p_{\min},\ q_{\min})$ *solution to Eqs. (5), (6), (7) and (8), not necessarily unique.*

Let us assume for simplicity that the cost function is quadratic, so that $c^{-1}(p) = p$, and that $\tau w_H = 1$. As pointed out by Rochet and Stole (2002, p. 282, footnote 10), this is not a restrictive assumption. As they argue, the cost function could be any strictly convex function: "since the measurement of units of consumers' [valuations] and product qualities are not intrinsic, they can be redefined in such a way that costs are quadratic [...]".

Before turning to the proof of proposition 3, it will be useful to first prove the following lemma.

Lemma 1 *There exists a unique* α *solution to,*

$$\begin{cases} \alpha = \frac{p_{\min}}{aF_v^{-1}\left[\frac{N}{L} p_{\min} F_q(\beta(\delta(\alpha)))\right]} \\ \beta(\delta) = \frac{p_{\min}}{aF_v^{-1}\left[\frac{N}{L}\delta\right]} \\ \delta(\alpha) = a\int_{\alpha}^{q_{\max}} F_v^{-1}\left[\frac{N}{L}\int_{\chi}^{q_{\max}} p_{\min} f_q(q)\, dq\right] d\chi + p_{\min} \end{cases}$$

Proof. In the third equation, the function δ is continuously decreasing in α. Since F_v^{-1} is a decreasing function, in the second equation, the denominator is decreasing in α, so that the function β is increasing in $\delta(\alpha)$. In the first equation, the counter-cumulative function F_q is decreasing, F_v^{-1} is decreasing, so that the denominator is increasing in β. $\beta(\alpha)$ is increasing in α, so that in the first equation, the denominator is increasing in α. Therefore the right hand side of the first equation continuously decreases in α, crossing the 45° line only once. ■

We can now turn to the proof of the existence of an equilibrium.

Proof. It is straightforward to prove that the zero cutoff profit condition (5) determines a unique price $p_{\min}$. Eq. (6) mechanically defines the matching function $v(\cdot)$. We now prove that there exists a solution $(p(\cdot),\ q_{\min})$ to Eqs. (7) and (8).

Let E be the set of continuous functions from any interval $I \subset [\alpha, \beta]$ to $[\gamma, \delta]$ normed by $\|(p, q_{\min})\| = \sqrt{\sup_q |p(q)|^2 + |q_{\min}|^2}$. α, β, γ and δ are positive real numbers (defined below). Let Γ be a mapping from $S = E \times [\alpha, \beta]$ into itself (proven below), such that $\Gamma(p_1, q_1) = (p_2, q_2)$ is defined as follows,

$$\begin{cases} p_2(q) = a \int_{q_1}^{q} \bar{F}_v^{-1} \left[\frac{N}{L} \int_{\xi}^{q_{\max}} p_1(\chi) f_q(\chi) d\chi \right] d\xi + p_{\min}, \ \forall Q \in [q_1, q_{\max}] \\ q_2 = \frac{p_{\min}}{a F_v^{-1}\left[\frac{N}{L} \int_{q_1}^{q_{\max}} p_1(q) f_q(q) dq\right]} \end{cases}$$

α is defined in Lemma 1. δ is defined by $\delta(\alpha)$ as in Lemma 1. $\gamma = p_{\min}$ and β is defined by $\beta(\delta)$ as in Lemma 1.

- S is a Banach space: the set of continuous functions over a closed interval of the real line, normed by the sup norm, is a Banach space; the Cartesian product of this space and a closed interval with the Euclidean norm is a Banach space too. Since Cauchy sequences converge in both E with the sup norm, and in $[\alpha, \beta]$ with the absolute value norm, then Cauchy sequences converge in S with the conjugated norm.

- Γ maps S into itself, or, if $(p_1, q_1) \in S$, then $\Gamma(p_1, q_1) = (p_2, q_2) \in S$:

  - if $p_1 \in E$, then by construction, $\bar{F}_v^{-1}$ and f_q being continuous, p_2 is continuous.
  - F_v^{-1} takes only positive values, so for $q \in [q_1, q_{\max}]$, $p_2(q) \geq p_{\min} = \gamma$.
  - F_v^{-1} takes only positive values, so for $q \in [q_1, q_{\max}]$, $p_1(q) \geq p_{\min}$. F_v^{-1} is decreasing and takes only non negative values, so that $p_2(q) \leq a \int_{\alpha}^{q_{\max}} \bar{F}_v^{-1} \left[\frac{N}{L} \int_{\xi}^{q_{\max}} p_{\min} f_q(\chi) d\chi \right] d\xi + p_{\min} = \delta$.
  - for any $q \in [q_1, q_{\max}]$, $p_1(q) \geq p_{\min}$. Therefore, for $q_1 \in [\alpha, \beta]$, $\int_{q_1}^{q_{\max}} p_1(\chi) f_q(\chi) d\chi \geq p_{\min} F_q(\beta)$. F_v^{-1} is decreasing, so that $q_2 \geq \alpha$.

– for any $q \in [q_1, q_{\max}]$, $p_1(q) \leq \delta$. Moreover, f_q is a well defined density function, so that $\int_{q_1}^{q_{\max}} p_1(\chi) f_q(\chi) d\chi \leq \frac{N}{L}\delta$. F_v^{-1} is decreasing, so that $q_2 \leq \beta$.

– We have therefore proven that if $(p_1, q_1) \in S$, then $\Gamma(p_1, q_1) = (p_2, q_2) \in S$: $p_2 \in E$ (it is a continuous function that from an interval included in $[\alpha, \beta]$ into $[\gamma, \delta]$), and $q_2 \in [\gamma, \delta]$.

- Γ is continuous, or $\forall \varepsilon > 0$, $\exists \delta > 0$ s.t. if $\|(p_1, q_1) - (p_1', q_1')\| \leq \delta$, then $\|\Gamma(p_1, q_1) - \Gamma(p_1', q_1')\| \leq \varepsilon$, for any (p_1, q_1) and (p_1', q_1') in S. TO BE DONE.

Applying Schauder fixed point theorem, there exists a fixed point (not necessarily unique) $(p, q_{\min})$ such that $(p, q_{\min}) = \Gamma(p, q_{\min})$ ■

Proof of proposition 4

Proposition 4 (reminded) *If the entry cost is such that* $f^E = \left(\frac{\lambda_q - \eta}{(1+\eta)(\lambda_q + \lambda_v)}\right)^{1+\eta}$, *then there exists a unique equilibrium price schedule, lowest quality exported, and lowest price, defined as,*

$$\begin{cases} p(q) = \gamma(\tau w_H)^{\eta/(\lambda_v+\eta)} q^{(\lambda_v+\lambda_q)/(\lambda_v+\eta)} + \tau w_H \\ q_{\min} = \gamma'(\tau w_H)^{\lambda_v/(\lambda_v+\lambda_q)} \\ p_{\min} = \left(\frac{\lambda_q+\lambda_v}{\lambda_q-\eta}\right)\tau w_H \end{cases}$$

with γ *and* γ' *some constants*[8].

Proof. An equilibrium is defined by the following 4 equations,

$$\begin{cases} v(q) = \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\int_q^{\infty} c^{-1}\left(\frac{p(x)}{\tau w_H}\right) f_q(x)\, dx\right) \\ p(q) = a\int_{q_{\min}}^{q} v(\chi)\, d\chi + p_{\min} \\ p_{\min} = a v(q_{\min}) q_{\min} \\ \tau w_H f^E = p_{\min} S(p_{\min}) - \int_0^{S(p_{\min})} \tau w_H c(s)\, ds \end{cases}$$

where $\bar{F}_v$ is the "counter cumulative distribution" of valuations v. In our closed form example, we have the following functional forms,

$$\begin{cases} \bar{F}_v^{-1}(m) = \bar{v} m^{-1/\lambda_v} \\ f_q(q) = \lambda_q\left(\frac{q}{\bar{q}}\right)^{-\lambda_q} q^{-1} \\ c^{-1}\left(\frac{p}{\tau w_H}\right) = \left(\frac{p}{\tau w_H} - 1\right)^{\eta} \end{cases}$$

[8] $\gamma = \left(a^{\lambda_v}\frac{\lambda_v \bar{v}^{\lambda_v}}{\lambda_q \bar{q}^{\lambda_q}}\left(\frac{\lambda_v+\eta}{\lambda_v+\lambda_q}\right)^{\lambda_v}\frac{\lambda_q-\eta}{\lambda_q+\lambda_v}\frac{L_F}{N_H}\right)^{1/(\lambda_v+\eta)}$ and $\gamma' = \left(\frac{\lambda_v+\eta}{\lambda_q-\eta}\right)^{(\lambda_v+\eta)/(\lambda_v+\lambda_q)}$.

We guess that the equilibrium price schedule is of the following form,

$$p(q) = \alpha \tau w_H q^{\beta} + \tau w_H$$

with α and β some positive constant to be determined. We have now 5 equations and 6 unknowns $(v(\cdot), p(\cdot), p_{\min}, q_{\min}, \alpha, \beta)$. This system is underidentified. Generically, there will not be a solution that satisfies our guess. We will therefore need to impose one additional condition on the size of the fixed entry cost, f^E.

Plugging the equilibrium conditions into our guess for the price schedule, the following simple algebra gives us,

$$\begin{aligned}
p(q) &= a \int_{q_{\min}}^{q} v(\chi)\, d\chi + p_{\min} \\
&= a \int_{q_{\min}}^{q} \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\int_{\chi}^{\infty} c^{-1}\left(\frac{p(x)}{w_H\tau}\right) f_q(x)\, dx\right) d\chi + p_{\min} \\
&= a \int_{q_{\min}}^{q} \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\int_{\chi}^{\infty} \left(\frac{p(x)}{\tau w_H} - 1\right)^{\eta} f_q(x)\, dx\right) d\chi + p_{\min} \\
&= a \int_{q_{\min}}^{q} \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\int_{\chi}^{\infty} \alpha^{\eta} x^{\beta\eta} \lambda_q \left(\frac{x}{\bar{q}}\right)^{-\lambda_q} x^{-1} dx\right) d\chi + p_{\min} \\
&= a \int_{q_{\min}}^{q} \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\frac{\alpha^{\eta}\lambda_q}{\lambda_q - \eta\beta}\chi^{\eta\beta - \lambda_q}\right) d\chi + p_{\min} \\
&= a \frac{\bar{v}}{\bar{q}^{\lambda_q/\lambda_v}}\left(\frac{N_H}{L_F}\frac{\alpha^{\eta}\lambda_q}{\lambda_q - \eta\beta}\right)^{-1/\lambda_v} \int_{q_{\min}}^{q} \chi^{(\lambda_q - \eta\beta)/\lambda_v} d\chi + p_{\min} \\
&= a \frac{\bar{v}}{\bar{q}^{\lambda_q/\lambda_v}}\left(\frac{N_H}{L_F}\frac{\alpha^{\eta}\lambda_q}{\lambda_q - \eta\beta}\right)^{-1/\lambda_v} \frac{\lambda_v}{\lambda_v + \lambda_q - \eta\beta}\left(q^{(\lambda_q+\lambda_v-\eta\beta)/\lambda_v} - q_{\min}^{(\lambda_q+\lambda_v-\eta\beta)/\lambda_v}\right) + p_{\min}
\end{aligned}$$

For our guess to be correct for any quality, it must be that,

$$\begin{aligned}
\beta &= \frac{\lambda_v + \lambda_q}{\lambda_v + \eta} \\
\tau w_H \alpha &= \left(a^{\lambda_v}\frac{\lambda_v \bar{v}^{\lambda_v}}{\lambda_q \bar{q}^{\lambda_q}}\left(\frac{\lambda_v + \eta}{\lambda_v + \lambda_q}\right)^{\lambda_v}\frac{\lambda_q - \eta}{\lambda_q + \lambda_v}\frac{L}{N}\right)^{1/(\lambda_v+\eta)} \times (\tau w_H)^{\eta/(\lambda_v+\eta)}
\end{aligned}$$

This gives us the following expression for the equilibrium price schedule,

$$\begin{aligned}
p(q) &= \gamma (\tau w_H)^{\eta/(\lambda_v+\eta)} q^{(\lambda_v+\lambda_q)/(\lambda_v+)} + \tau w_H \\
\text{with } \gamma &= \left(a^{\lambda_v}\frac{\lambda_v \bar{v}^{\lambda_v}}{\lambda_q \bar{q}^{\lambda_q}}\left(\frac{\lambda_v + \eta}{\lambda_v + \lambda_q}\right)^{\lambda_v}\frac{\lambda_q - \eta}{\lambda_q + \lambda_v}\frac{L}{N}\right)^{1/(\lambda_v+\eta)}
\end{aligned}$$

Note that $\frac{\lambda_v+\lambda_q}{\lambda_v+\eta} > 1$ iif $\lambda_q > \eta$. We need the assumption that $\lambda_q > \eta$, otherwise, there are too many large firms (λ_q small), or large firms are too big (η large), and our integrals would not converge.

If this equilibrium price schedule holds for every quality, it holds for the lowest quality $q_{\min}$, so that

$$p_{\min} = \gamma \left(\tau w_H\right)^{\eta/(\lambda_v+\eta)} q_{\min}^{(\lambda_v+\lambda_q)/(\lambda_v+)} + \tau w_H$$

This together with the equation defining the lowest valuation $q_{\min}$, we get a solution for the lowest price and for the lowest valuation,

$$\begin{cases} p_{\min} = \left(\frac{\lambda_q+\lambda_v}{\lambda_q-\eta}\right) \tau w_H \\ q_{\min} = \gamma' \left(\tau w_H\right)^{\lambda_v/(\lambda_v+\lambda_q)} \\ \text{with } \gamma' = \left(\frac{\lambda_v+\eta}{\lambda_q-\eta}\right)^{(\lambda_v+\eta)/(\lambda_v+\lambda_q)} \end{cases}$$

However, $p_{\min}$ is independently defined by the zero profit cutoff condition,

$$\begin{aligned} \tau w_H f^E &= p_{\min} S\left(p_{\min}\right) - \int_0^{S(p_{\min})} \tau w_H c\left(s\right) ds \\ p_{\min} &= \left(1 + \left(\left(1+\eta\right) f^E\right)^{1/(1+\eta)}\right) \tau w_H \end{aligned}$$

For our guess to be correct, we need that,

$$f^E = \left(\frac{\lambda_q - \eta}{\left(1+\eta\right)\left(\lambda_q + \lambda_v\right)}\right)^{1+\eta}$$

■

Proof of proposition 5 (exchange rate pass through)

Proposition 5 (reminded) *There is incomplete pass through of exchange rate shocks into the price of individual goods. The lower the quality of a good, the higher the pass through.*

Proof. Formally, define $\sigma_{p(q)}$ as the elasticity of the price $p\left(q\right)$ of a quality q good with respect to the exchange rate,

$$\sigma_{p(q)} \equiv \frac{\partial \ln p\left(q\right)}{\partial \ln \tau w_H}$$

From the definition of the equilibrium price schedule in proposition 4,

$$p\left(q\right) = \gamma \left(\tau w_H\right)^{\eta/(\lambda_v+\eta)} q^{(\lambda_v+\lambda_q)/(\lambda_v+\eta)} + \tau w_H$$

and differentiating with respect to τw_H, we immediately get that,

$$\begin{aligned} \sigma_{p(q)} &= \frac{\partial \ln p\left(q\right)}{\partial \ln \tau w_H} \\ &= 1 - \frac{\lambda_v}{\lambda_v - \eta} \times \frac{1}{1 + \gamma^{-1}\left(\tau w_H\right)^{\lambda_v/(\lambda_v+\eta)} q^{-(\lambda_v+\lambda_q)/(\lambda_v+\eta)}} \end{aligned}$$

From this expression, it is straightforward to prove that,

$$\begin{aligned}\frac{\partial \sigma_{p(q)}}{\partial q} &< 0 \\ \lim_{q\to+\infty} \sigma_{p(q)} &= \frac{\eta}{\lambda_v+\eta} \\ \lim_{q\to 0} \sigma_{p(q)} &= 1\end{aligned}$$

Since the lowest quality is strictly above 0, we know that for any $q \geq q_{\min}$, we have,

$$\frac{\eta}{\lambda_v+\eta} < \sigma_{p(q)} < 1$$

There is incomplete pass through of exchange rate shocks into the prices of individual goods (the elasticity $\sigma_{p(q)}$ is smaller than 1 for all goods), and the lower the quality of a good, the higher the pass through (the elasticity $\sigma_{p(q)}$ is increasing with the quality q). ■

Proof of proposition 6 (aggregate pass through)

Proposition 6 (reminded) *The price of the lowest quality exported moves one for one with the exchange rate. The lowest quality exported increases less than proportionally with the exchange rate.*

Proof. Formally, define $\sigma_{q_{\min}}$ as the elasticity of the lowest quality exported ($q_{\min}$) with respect to the exchange rate, and $\sigma_{p_{\min}}$ as the elasticity of the lowest price ($p_{\min}$) with respect to the exchange rate,

$$\begin{aligned}\sigma_{p_{\min}} &\equiv \frac{\partial \ln p_{\min}}{\partial \ln \tau w_H} \\ \sigma_{q_{\min}} &\equiv \frac{\partial \ln q_{\min}}{\partial \ln \tau w_H}\end{aligned}$$

From the definition of the equilibrium price schedule in proposition 4,

$$\begin{aligned}q_{\min} &= \gamma' \left(\tau w_H\right)^{\lambda_v/(\lambda_v+\lambda_q)} \\ p_{\min} &= \left(\frac{\lambda_q+\lambda_v}{\lambda_q-\eta}\right)\tau w_H\end{aligned}$$

and differentiating with respect to τw_H, we immediately get that,

$$\begin{aligned}\sigma_{q_{\min}} &= \frac{\lambda_v}{\lambda_v+\lambda_q} < 1 \\ \sigma_{p_{\min}} &= 1\end{aligned}$$

■

Proof of proposition 7

Proposition 7 (reminded) *Aggregate prices are proportional to the exchange rate, where aggregate prices are defined as a weighted average of individual prices, with consumer expenditure shares used as weights.*

Proof. Formally, define the Consumer Price Index as the weighted average of individual prices, where the weights are the aggregate consumer expenditure shares,

$$CPI = \frac{\int_{q_{\min}}^{\infty} f_q(q)\, S(q) \times p(q)\, dq}{\int_{q_{\min}}^{\infty} f_q(q)\, S(q)\, dq}$$

Some simple algebra, and using the of the expression for the price schedule and for the lowest quality exported from proposition 4, we get

$$\begin{aligned} CPI &= \gamma'' \tau w_H \\ \text{with } \gamma'' &= \frac{(\lambda_q - \eta)(\lambda_v - 1)}{(\lambda_q - \eta)(\lambda_v - 1) - (\lambda_v - \eta)} \gamma \end{aligned}$$

So aggregate prices are exactly proportional to the exchange rate. ■

Appendix B: buyers' market

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