Universal patterns underlying ongoing wars and terrorism

Neil F. Johnson[1,6], Mike Spagat[2,6], Jorge Restrepo[3,6], Oscar Becerra[6], Juan Camilo Bohórquez[4], Nicolas Suárez[6], Elvira Maria Restrepo[5,6], and Roberto Zarama[4]

[1] *Department of Physics, University of Oxford, Oxford, U.K.*
[2] *Department of Economics, Royal Holloway College, University of London, Egham, U.K.*
[3] *Department of Economics, Universidad Javeriana, Bogotá, Colombia*
[4] *Department of Industrial Engineering, Universidad de los Andes, Bogotá, Colombia*
[5] *Faculty of Economics, Universidad de Los Andes, Bogotá, Colombia*
[6] *CERAC, Conflict Analysis Resource Center, Bogotá, Colombia*

ABSTRACT

We report a remarkable universality in the patterns of violence arising in two high-profile ongoing wars, and in global terrorism. Our results suggest that these quite different conflict arenas currently feature a common type of enemy, i.e. the various insurgent forces are beginning to operate in a similar way regardless of their underlying ideologies, motivations and the terrain they operate in. We provide a microscopic theory to explain our main observations. This theory treats the insurgent forces as a generic, self-organizing network, which is dynamically evolving through the continual coalescence and fragmentation of attack units.

The recent terrorist attacks in London, Madrid and New York (i.e. '9/11'), killed many people in a very short space of time. At the other extreme, the world currently hosts several longer-term 'local' conflicts within specific countries, in which there is a steady stream of new casualties every day. Examples include the war in Iraq, and the longer-term guerrilla war in Colombia which is taking place against a back-drop of drug-trafficking and Mafia activity. The origins, motivations, locations and durations of these various conflicts are quite different -- hence one would expect the details of their respective dynamical evolution to also be quite different.

Here we report the remarkable finding that identical patterns of violence are currently emerging within these different international arenas. Not only are the wars in Iraq and Colombia evolving to yield the *same* power-law behavior, but this behavior is of the *same* quantitative form as global terrorism in non-G7 countries. Our findings suggest that the dynamical evolution of these various examples of modern conflict has less to do with geography, ideology, ethnicity or religion and much more to do with the day-to-day mechanics of human insurgency; respective insurgent forces are effectively becoming identical in terms of how they operate. This in turn suggests that there is an underlying logic to insurgent warfare; conflicts that are extremely different from one another along numerous highly visible dimensions still display highly similar structural regularities. Viewing this result optimistically, one might hope that the successful conclusion of one of these conflicts might then simultaneously provide a solution for the other two. On the other hand, a pessimist could conclude that there is no hope of solving one problem without solving the others, since they are equally 'difficult' in terms of how the insurgent force is operating. At the end of the paper, we provide a microscopic mathematical model which describes how this 'common enemy' operates. In particular, our model features a dynamically evolving network of attack units and hence a continual ebb and flow of violent acts. The model's predicted power-law behavior is in excellent agreement with our main observations.

Power-law distributions are known to arise in a large number of physical, biological, economic and social systems[1]. In the present context, a power-law distribution means that the probability that an event will occur with x victims is given by the power-law distribution $p(x) = Cx^{-\alpha}$ over a reasonably wide range of x, with C and α positive coefficients. This in turn implies that a graph of $\log[P(X \geq x)]$ vs. $\log(x)$ will be a straight line over this range of x, with negative slope of magnitude $\alpha - 1$.[i] Previous studies have shown that the distribution obtained from 'old' wars between 1816-1980, exhibits a power-law with $\alpha = 1.80(9)$[1-4].[ii] However each data-point in these previous studies represents the aggregate casualty figure for a particular war. By contrast, our analysis for Iraq and Colombia looks at the pattern of casualties arising *within* a war. Casualty numbers in global terrorist events, 1968 to present, are also known to obey power laws where in this case each data point is a terrorist attack[5]: $\alpha = 2.5(1)$ for non-G7 countries while $\alpha = 1.71(3)$ for G7 countries[5]. We find that both Iraq and Colombia exhibit power-law behavior and that both power-law coefficients are approaching 2.5, which is exactly the value characterizing non-G7 terrorism.

Figure 1 shows log-log plots of the fraction of all recorded events with x or more victims, $P(X \geq x)$, versus x. The straight line fits over long ranges in Figure 1 suggest that both these wars follow a power-law. The Colombia data displays an extraordinary fit for a social science application while the Iraq data also fits well except for a bulge in the 150 to

[i] We will refer to $P(X \geq x)$ as the cumulative distribution obtained from $p(x)$.

[ii] Numbers in parentheses give the standard error on the trailing figure in each case.

600 range. Since we have many more Colombia events than Iraq ones, the superiority of the Colombia curve is not surprising. Nevertheless, the rest of the Iraq curve fits well enough so as to suggest that we should expect more events in the 150-600 range in the future..

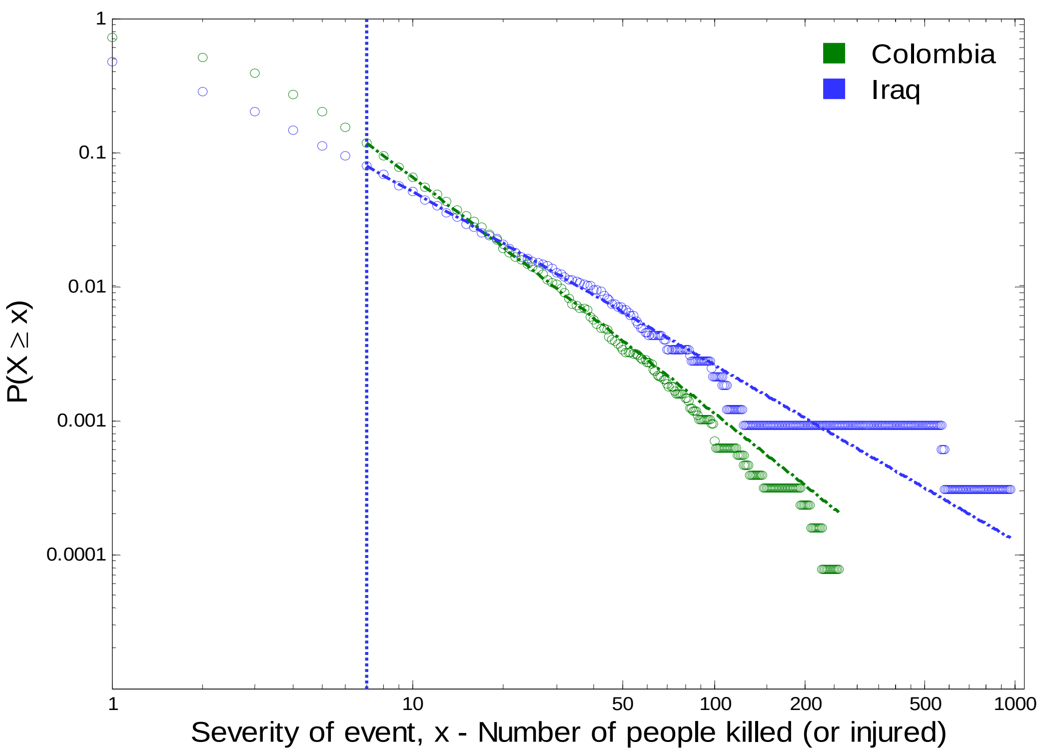

Figure 1 Log-log plots of cumulative distributions $P(X \geq x)$ describing the total number of events with severity greater than x, for the ongoing wars in Iraq (blue) and Colombia (green). For Iraq, the severity is taken to be the lower estimate of deaths from the CERAC Integrated Iraq Dataset. For Colombia, the severity is taken to be the total number of deaths plus injuries from the CERAC dataset[6]. Each line indicates the most likely power law that fits the data (see text).

As well as observing power-law behavior for aggregated data as in Figure 1, we find robust power-law behavior for data collected over smaller time-windows. This allows us to deduce the evolution of the power-law coefficient α over time by sliding this time-window through the data-series. Figure 2 shows these empirically-determined α values as a function of time for Iraq and Colombia. The α values in both cases are tending toward 2.5, which is the coefficient for global terrorism in non-G7 countries. The implication is that *both* these wars *and* global non-G7 terrorism are beginning to share a similar underlying structure.

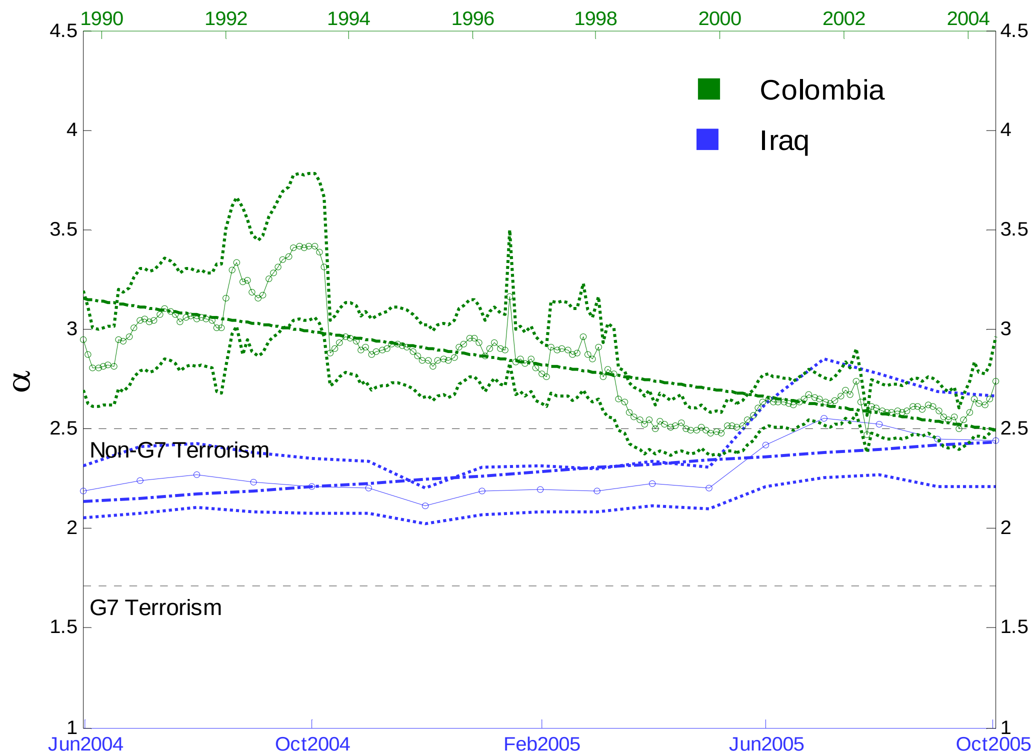

Figure 2 The variation through time of the power law coefficient α for Iraq (blue) and Colombia (green). The straight lines are fits through these points, and suggest a common value of approximately 2.5 for both wars. The values for G7 and non-G7 terrorism are also shown[5]. The dotted lines show asymptotic 95% confidence bands around the estimates.

But why should the same value of 2.5 emerge for Iraq, Colombia, and non-G7 global terrorism? Standard physical mechanisms for generating power laws make little sense in the context of Colombia or Iraq[1]. One might instead guess that casualties would arise in rough proportion to the population sizes of the places where insurgent groups attack: given that city populations may follow a power law[1], it is conceivable that this would also produce power laws for the severity of attacks. However, we have tested this hypothesis against our Colombia data and it is resoundingly rejected.

Figure 3 shows a model of modern 'generic' insurgent warfare which we have developed to understand this universal value of 2.5. Full details are given in the *Supporting Online Material* section. Our model proposes that the insurgent force operates as a dynamically evolving network of fairly self-contained units, which we call 'attack units'. Each attack unit has a particular 'attack strength' characterizing the average number of casualties arising in an event involving this attack unit. As time evolves, these attack units either join forces with other attack units (i.e. coalescence) or break up (i.e. fragmentation). Eventually this on-going process of coalescence and fragmentation reaches a dynamical steady-state which is solvable analytically as shown in the *Supporting Online Material*, yielding $\alpha = 2.5$. This value is in remarkable agreement with the α values to which both Colombia and Iraq appear to be tending (recall Figure 2). It also suggests that similar distributions of attack units are emerging in Colombia, Iraq and in non-G7 global terrorism, with each attack unit in an ongoing state of coalescence and fragmentation. Our model also offers the following interpretation for the dynamical evolution of α observed in Figure 2. The Iraq war began as a conventional confrontation between large armies, but continuous pressure applied to the Iraqis by coalition forces has fragmented the insurgency into a structure in which smaller attack units, characteristic of non-G7 global terrorism, now predominate. In Colombia, on the other hand, the guerrillas in the early 1990's had even less ability than global terrorists to coalesce into high-impact units but have gradually been acquiring comparable capabilities.

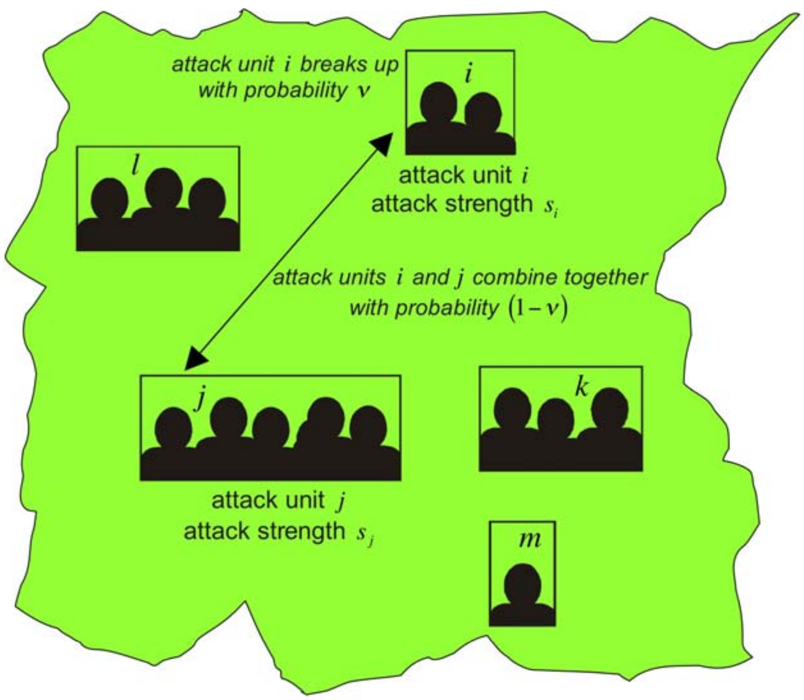

Figure 3

Figure 3 Our analytically-solvable model describing modern insurgent warfare. The insurgent force comprises *attack units*, each of which has a particular *attack strength*. The total attack strength of the insurgent force is being continually re-distributed through a process of coalescence and fragmentation.

Finally we note that the *Supporting Online Material* section gives full details of the databases and methods which we used to ensure accurate estimates of the power-law exponents, plus the extensive robustness checks which we have performed to double-check the accuracy of our findings.

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Acknowledgements The Department of Economics of Royal Holloway College provided funds to build the CERAC database. J. Restrepo acknowledges funding from Banco de la República, Colombia. J.C. Bohórquez acknowledges funding from the Department of Industrial Engineering of Universidad de los Andes.

Supporting Online Material
www.sciencemag.org
PART 1: Details of the model mentioned in the paper
PART 2: Table and figures confirming the robustness of our results
PART 3: Data and methods
Figs. S1, S2, S3, S4
Table. T1

Supporting Online Material

PART 1: Details of the model mentioned in the paper

Here we provide details of the model of modern insurgent warfare, which we introduced in the main paper. Our goal is to provide a plausible model to explain (i) why power-law behaviour is observed in the Colombia and Iraq wars, and in non-G7 global terrorism, and (ii) why the power-law coefficients for the Colombia and Iraq wars should both be heading toward this non-G7 terrorism value of 2.5.

Our model bears some similarity to a model of herding by Cont and Bouchaud[iii], and is a direct adaptation of the Eguiluz-Zimmerman model of herding in financial markets[iv]. The analytical derivation which we present, is an adaptation of earlier formalism laid out by D'Hulst and Rodgers[v], and also draws heavily on the material in the book *Financial Market Complexity* by Neil F. Johnson, Paul Jefferies and Pak Ming Hui (Oxford University Press, 2003). One of us (NFJ) is extremely grateful to Pak Ming Hui for detailed correspondence about the Eguiluz-Zimmerman model of financial markets, the associated formalism, and its extensions – and also for discussions involving the present model.

As suggested by Figure 3 in the paper, our model is based on the plausible notion that the total attack capability of a modern insurgent force is being continually re-distributed. Based on our intuition about such 'new' wars, we consider the insurgent force to be made up of ***attack units*** or ***cells*** which have certain ***attack strength*** (see below for a detailed

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[iii] R. Cont and J.P. Bouchaud, Macroeconomic Dynamics **4**, 170 (2000)

[iv] V.M. Eguiluz and M.G. Zimmerman, Phys. Rev. Lett. **85**, 5659 (2000)

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discussion). One might expect that the total attack strength for the entire insurgent force would change slowly over time. At any particular instant, this total attack strength is distributed (i.e. partitioned) among the various attack units -- moreover the composition of these attack units, and hence their relative attack strengths, will evolve in time as a result of an on-going process of coalescence (i.e. combination of attack units) and fragmentation (i.e. breaking up of attack units). Such a process of coalescence and fragmentation is realistic for an insurgent force in a guerilla-like war, and will be driven by a combination of planned decisions and opportunistic actions by both the insurgent force and the incumbent force. For example, separate attack units might coalesce prior to an attack, or an individual attack unit might fragment in response to a crackdown by the incumbent force. Here we will model this process of coalescence and fragmentation as a stochastic process.

Each attack unit carries a specific label $i, j, k, \ldots$ and has an attack strength denoted by $s_i, s_j, s_k, \ldots$ respectively. We start by discussing what we mean by these definitions:

attack unit or ***cell***: Here we have in mind a group of people, weapons, explosives, machines, or even information, which organizes itself to act as a single unit. In the case of people, this means that they are probably connected by location (e.g. they are physically together) or connected by some form of communications systems. In the case of a piece of equipment, this means that it is readily available for use by members of a particular group. The simplest scenario is to just consider people, and in particular a group of insurgents which are in such frequent contact that they are able to act as a single group. However we emphasize that an attack unit may also consist of a combination of people and objects – for example, explosives plus a few people, such as the case of suicide bombers. Such an attack unit, while only containing a few people, could have a high attack strength. In addition, information could also be a valuable part of an attack unit. For example, a lone suicide bomber who knows when a certain place will be densely populated (e.g. a military canteen at lunchtimes) and who knows how to get into such a place unnoticed, will also represent an attack unit with a high attack strength.

attack strength: We define the attack strength s_i of a given attack unit i, as the average number of people who are typically injured or killed as the result of an event involving attack unit i. In other words, a typical event (e.g. attack or clash) involving group i will lead to the injury or death of s_i people. This definition covers both the case of one-sided attacks by attack unit i (since in this case, all casualties are due to the presence of attack unit i) and it also covers two-sided clashes (since presumably there would have been no clash, and hence no casualties, if unit i had not been present).

We take the sum of the attack strengths over all the attack units (i.e. the total attack strength of the insurgent force) to be equal to N. From the definition of attack strength, it follows that N represents the maximum number of people which would be injured or killed in an event, on average, if the entire insurgent force were to act together as a single attack unit. Mathematically, $\sum_{i,j,k,\ldots} s_i = N$. For any significant insurgent force, one would expect $N >> 1$. The power-law results that we will derive do not depend on any particular choice of N. In particular, the power-law result which is derived in the Supporting Online Material section concerning the average number n_s of attack units having a given attack strength s, is invariant under a global magnification of scale (as are all power-laws).

The model therefore becomes, in mathematical terms, one in which this total attack strength N is dynamically distributed among attack groups as a result of an ongoing process of coalescence and fragmentation. As a further clarification of our terminology, we will now discuss the two limiting cases which we classify as the ‘coalescence’ and ‘fragmentation’ limits for convenience:

- ‘Coalescence’ limit: Suppose the conflict is such that all the attack units join together or *coalesce* into a single large attack unit. This is the limit of complete coalescence and would correspond to amassing all the available combatants and weaponry in a single place – very much like the armies of the past would amass their entire force on the field of battle. Hence there is one large attack unit, which we label as i and which has an attack strength N. All other attack units disappear. Hence $s_i \to N$. This ‘coalescence’ limit has the *minimum* possible number of attack units (i.e. one) but the *maximum* possible attack strength (i.e. N) in that attack unit.

- ‘Fragmentation’ limit: Suppose the conflict is such that all the attack units *fragment* into ever smaller attack units. Eventually we will have all attack units having attack strength equal to one. Hence $s_i \to 1$ for all $i = 1, 2, \ldots, N$. This would correspond to all combatants operating essentially individually. This ‘fragmentation’ limit has the *maximum* possible number of attack units (i.e. N) but the *minimum* possible attack strength per attack unit (i.e. one).

In practice, of course, one would expect the situation to lie between these two limits. Indeed, it seems reasonable to expect that these attack units and their respective attack strengths, will evolve in time within a given war. Indeed, one can envisage that these attack units will occasionally either break up into smaller groups (i.e. smaller attack units) or join together to form larger ones. The reasons are plentiful why this should occur: for example, the opposing forces (e.g. the Colombian Army in Colombia, or Coalition Forces in Iraq) may be applying pressure in terms of searching for hidden insurgent groups. Hence these insurgent groups (i.e. attack units) might either decide, or be forced, to break up in order to move more quickly, or in order to lose themselves in the towns or countryside.

Hence attack units with different attack strengths will continually mutate via coalescence and fragmentation yielding a ‘soup’ of attack units with a range of attack strengths. At any one moment in time, this ‘soup’ corresponds mathematically to partitioning the total N units of attack strength which the insurgent army possesses. The analysis which we now present suggests that the current states of the guerilla/insurgency wars in Colombia and Iraq both correspond to the steady-state limit of such an on-going coalescence-fragmentation process. It also suggests that such a process might also underpin the acts of terrorism in non-G7 countries, and that such terrorism is characteristic of some longer-term ‘global war’.

Against the backdrop of on-going fragmentation and coalescence of attack units, we suppose that each attack unit has a given probability p of being involved in an event in a given time-interval, regardless of its attack strength. For example, p could represent the probability that an arbitrarily chosen attack unit comes across an undefended target – or vice versa, the probability that an arbitrarily chosen attack unit finds itself under attack. In these instances, p should be relatively insensitive to the actual attack strength of the attack unit involved: hence the results which we shall derive for the distribution of attack strengths, should also be applicable to the distribution of events having a given severity. When obtaining our analytic and numerical results, we assume that the war has been underway for a long time and hence some kind of steady-state has been reached. This latter assumption is again plausible for the wars in Colombia and Iraq.

Given the above considerations, it follows that if there are, on average, n_s attack units of a given attack strength s, then the average number of events involving an attack unit of attack strength s will be proportional to n_s. We assume, quite realistically, that only one insurgent attack group participates in a given event. For example, an attack in which 10 people were killed is necessarily due to an attack by a unit of attack strength 10. In particular, it could not be due to two separate but simultaneous attacks by a unit of strength 6 and a unit of strength 4 (i.e. 6+4=10). Hence the number of events in which s people were killed and/or injured, is just proportional to n_s. In other words, the histogram, and hence power-law, that we will derive for the dependence of n_s on s, will also describe the number of events with s casualties versus s. Indeed, if we consider that an event will typically have a duration of T, and that there will only be a few such events in a given interval T, then these results should also appear similar to the distribution describing the number of intervals of duration T in which there were s casualties, versus s. This is indeed what we have found in our analysis of the empirical data.

Given these considerations, our task of analyzing and deducing the average number of events with s casualties versus s over a given period of time, becomes equivalent to the task of analyzing and deducing the average number n_s of attack units of a given attack strength s in that same period of time. This is what we will now calculate. We will start by considering a mechanism for coalescence and fragmentation of attack groups, before then finally deducing analytically the corresponding power-law behaviour and hence deducing a power-law coefficient equal to 2.5.

Consider an arbitrary attack unit i with attack strength s_i. At any one instant in time, labelled t, we assume that this attack unit may either:

a) fragment (i.e. break up) into s_i attack units of attack strength equal to 1. This feature aims to mimic an insurgent group which decides, either voluntarily or involuntarily, to split itself up (e.g. in order to reduce the chance of being captured and/or to mislead the enemy).

b) coalesce (i.e. combine) with another attack unit j of attack strength s_j, hence forming a single attack unit of attack strength $s_i + s_j$. This feature mimics two insurgent groups finding each other by chance (e.g. in the Colombian jungle) or deciding via radio communication to meet up and join forces.

To implement this fragmentation/coalescence process at a given timestep, we choose an attack unit i at random but with a probability which is proportional to its attack strength s_i. With a probability ν, this attack unit i with attack strength s_i *fragments* into s_i attack units with attack strength 1. A justification for choosing attack unit i with a probability which is proportional to its attack strength, is as follows: attack units with higher attack strength are likely to be bigger and hence will either run across the enemy more and/or be more actively sought by the enemy. By contrast, with a probability $(1-\nu)$, the chosen attack unit i instead *coalesces* with another attack unit j which is chosen at random, but again with a probability which is proportional to its attack strength s_j. The two attack units of attack strengths s_i and s_j then combine to form a bigger attack unit of attack strength $s_i + s_j$. The justification for choosing attack unit j for coalescence with a probability which is proportional to its attack strength, is as follows: it is presumably risky to combine attack units, since it must involve at least one message passing between the two units in order to coordinate their actions. Hence it becomes increasingly less worthwhile to combine attack units as the attack units get smaller.

This model is thus characterized by a single parameter ν. The set up of the model is shown schematically in the figure at the front of this *Supporting Online Material* section, and in Figure 3 of the paper. The connectivity among the attack units is driven by the dynamics of the model. For very small ν (i.e. much less than 1), the attack units steadily coalesce. This leads to the formation of large attack units. In the other limit of $\nu \to 1$, the system consists of many attack units with attack strength close to 1. A value of $\nu = 0.01$ corresponds to about one fragmentation in every 100 iterations. In what follows, we assume that ν is small since the process of fragmentation should not be very frequent for any insurgent force which is managing to sustain an ongoing war. Indeed if such fragmentation were very frequent, then this would imply that the insurgents were being so pressured by the incumbent force that they had to fragment at nearly every timestep. Hence that particular war would not last very long. It turns out that infrequent fragmentations are sufficient to yield a steady-state process, and will also yield the power-law behaviour which we observe for Colombia and Iraq.

A typical result obtained from numerical simulations, for the distribution of n_s versus attack strength s in the long-time limit (i.e. steady-state), is shown below in terms of n_s/n_1 :

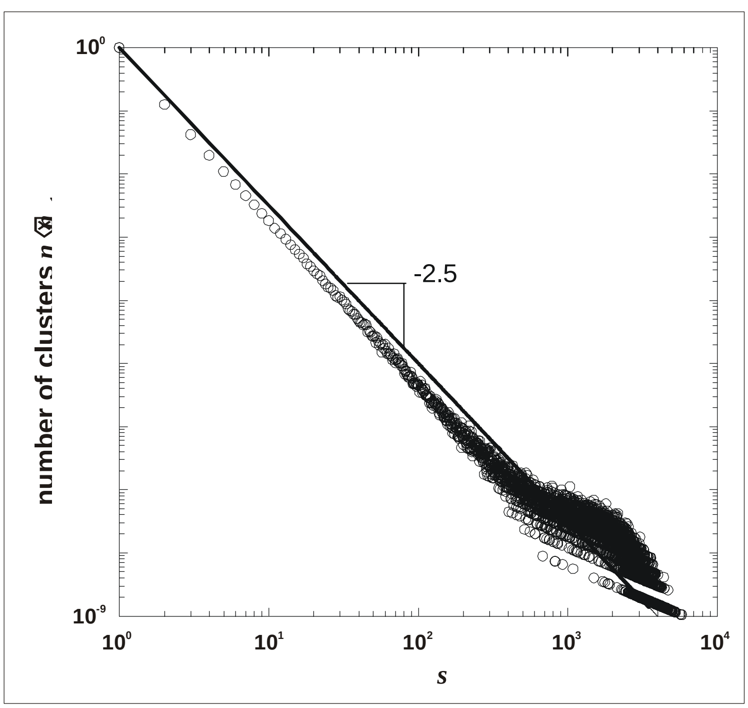

Figure S1: Log-log plot of the number of attack units with attack strength *s*, versus attack strength *s*. Here $N = 10{,}000$ and $\nu = 0.01$. The results are obtained from a numerical simulation of the model. The initial conditions of this numerical simulation are such that all attack units have size 1. As time evolves, these attack units undergo coalescence and fragmentation as described in the text. In the long-time limit, the system reaches a steady state with a power-law dependence as shown in the figure, and with an associated power-law coefficient of 2.5 (i.e. 5/2). The deviation from power-law behaviour at large *s* is simply due to the finite value of *N*: since there can be no attack unit with an attack strength greater than *N*, the finite size of *N* distorts the power-law as *s* approaches *N*.

We now provide an *analytic* derivation of the observed power-law behaviour, and specifically the power-law coefficient 2.5, in the steady-state (i.e. long-time) limit.

One could write a dynamical equation for the evolution of the model with different levels of approximation. For example, one could start with a microscopic description of the system by noting that at any moment in time, the entire insurgent army can be described by a partition $\{l_1, l_2, \ldots, l_N\}$ of the total attack strength N into N attack units. Here l_s is the number of attack units of attack strength s. For example $\{0, 0, \ldots, 1\}$ corresponds to the extreme coalescence case in which all the attack strength is concentrated in one big attack unit. By contrast, $\{N, 0, \ldots, 0\}$ corresponds to the case of extreme fragmentation in which all the attack units have attack strength of 1 (i.e. there are N attack units of attack strength 1). Clearly, the total amount of attack strength is conserved $\sum_{i=1}^{N} i l_i = N$. All that happens is that the way in which this total attack strength *N* is *partitioned* will change in time.

In principle, the dynamics could be described by the time-evolution of the probability function $p\left[l_1, l_2, \ldots, l_N\right]$: in particular, taking the continuous-time limit would yield an equation for $dp\left[l_1, l_2, \ldots, l_N\right]/dt$ in terms of transitions between partitions. For example, the fragmentation of an attack unit of attack strength s leads to a transition from the partition $\{l_1, \ldots, l_s, \ldots, l_N\}$ to the partition $\{l_1 + s, \ldots, l_s - 1, \ldots, l_N\}$. For our purposes, however, it is more convenient to work with the *average* number n_s of attack units of attack strength s, which can be written as $n_s = \sum_{\{l_1,\ldots,l_N\}} p[l_1, \ldots, l_s, \ldots, l_N] \cdot l_s$. The sum is over all possible partitions. Since $p\left[l_1, \ldots, l_N\right]$ evolves in time, so does $n_s[t]$. After the transients have died away, the system is expected to reach a steady-state in which $p\left[l_1, \ldots, l_N\right]$ and $n_s[t]$ become time-independent. The time-evolution of $n_s[t]$ can be written down either by intuition, or by invoking a mean-field approximation to the equation for $dp\left[l_1, l_2, \ldots, l_N\right]/dt$. Taking the intuitive route, one can immediately write down the following dynamical equations in the continuous-time limit:

$$\frac{\partial n_s}{\partial t} = -\frac{\nu s n_s}{N} + \frac{(1-\nu)}{N^2}\sum_{s'=1}^{s-1} s' n_{s'} (s - s') n_{s-s'} - \frac{2(1-\nu) s n_s}{N^2}\sum_{s'=1}^{\infty} s' n_{s'} \quad \text{for } s \geq 2 \qquad (0.1)$$

$$\frac{\partial n_1}{\partial t} = \frac{\nu}{N}\sum_{s'=2}^{\infty} (s')^2 n_{s'} - \frac{2(1-\nu) n_1}{N^2}\sum_{s'=1}^{\infty} s' n_{s'} \qquad (0.2)$$

The terms on the right-hand side of Equation (0.1) represent all the ways in which n_s can change. The first term represents a decrease in n_s due to the fragmentation of an attack unit of attack strength s: this happens only if an attack unit of attack strength s is chosen and if fragmentation then follows. The former occurs with probability $s n_s / N$ (see earlier discussion) and the latter with probability ν. The second term represents an increase in n_s as a result of the merging of an attack unit of attack strength s' with an attack unit of attack strength $(s - s')$. The third term describes the decrease in n_s due to the merging of an attack unit of attack strength s with any other attack unit. For the $s = 1$ case described by Equation

(0.2), the chosen attack unit remains isolated; thus Equation (0.2) does not have a contribution like the first term of Equation (0.1). The first term which appears in Equation (0.2) reflects the increase in the number of attack units of attack strength equal to 1, due to fragmentation of an attack unit. Similarly to Equation (0.1), the last term of Equation (0.2) describes the merging of an attack unit with attack strength 1, with an attack unit of any other attack strength. Equations (0.1) and (0.2) are so-called ‘master equations’ describing the dynamics within the model. Note that for simplicity, we are only considering fragmentation into attack units of attack strength 1. However this could be generalized – indeed, we will look at more general fragmentations in future publications.

In the long-time steady state limit, Equations (0.1) and (0.2) yield:

$$s n_s = \frac{(1-\nu)}{(2-\nu)N} \sum_{s'=1}^{s-1} s' n_{s'} (s-s') n_{s-s'} \quad \text{for } s \geq 2 \tag{0.3}$$

$$n_1 = \frac{\nu}{2(1-\nu)} \sum_{s'=2}^{\infty} (s')^2 n_{s'} \tag{0.4}$$

Equations of this type are most conveniently treated using the general technique of ‘generating functions’. As the name suggests, these are functions which can be used to generate a range of useful quantities. Consider

$$G[y] = \sum_{s'=0}^{\infty} s' n_{s'} y^{s'} \tag{0.5}$$

where $y = e^{-\omega}$ is a parameter. Note that $s n_s / N$ is the probability of finding an attack unit of attack strength s. If $G[y]$ is known, $s n_s$ is then formally given by

$$s n_s = \frac{1}{s!} G^{(s)}[0] \tag{0.6}$$

where $G^{(s)}[y]$ is the s-th derivative of $G[y]$ with respect to y. $G^{(s)}[y]$ can be decomposed as

$$G[y] = n_1 y + \sum_{s'=2}^{\infty} s' n_{s'} y^{s'} \equiv n_1 y + g[y] \tag{0.7}$$

where the function $g[y]$ governs the attack-units' attack-strength distribution n_s for $s \geq 2$. The next task is to obtain an equation for $g[y]$. This can be done in two ways. One could either write down the terms in $(g[y])^2$ explicitly and then make use of Equation (0.3), or one could construct $g[y]$ by multiplying Equation (0.3) by $e^{-\omega s}$ and then summing over *s*. The resulting equation is:

$$(g[y])^2 - \left(\frac{2-\nu}{1-\nu} N - 2n_1 y\right) g[y] + n_1^{\,2} y^2 = 0 \tag{0.8}$$

First we solve for n_1. From Equation (0.7), $g[1] = G[1] - n_1 = N - n_1$. Substituting $n_1 = N - g[1]$ into Equation (0.8) and setting $y = 1$, yields

$$g[1] = \frac{1-\nu}{2-\nu} N \tag{0.9}$$

Hence

$$n_1 = N - g[1] = \frac{1}{2-\nu} N \tag{0.10}$$

To obtain n_s with $s \geq 2$, we need to solve for $g[y]$. Substituting Equation (0.10) for n_1, Equation (0.8) becomes

$$(g[y])^2 - \left(\frac{2-\nu}{1-\nu} N - \frac{2N}{2-\nu} y\right) g[y] + \frac{N^2}{(2-\nu)^2} y^2 = 0 \tag{0.11}$$

Equation (0.11) is a quadratic equation for $g[y]$ which can be solved to obtain

$$\begin{aligned} g[y] &= \frac{(2-\nu)N}{4(1-\nu)} \left(1 - \sqrt{1 - \frac{4(1-\nu)}{(2-\nu)^2} y}\right)^2 \\ &= \frac{(2-\nu)N}{4(1-\nu)} \left(2 - \frac{4(1-\nu)}{(2-\nu)^2} y - 2\sqrt{1 - \frac{4(1-\nu)}{(2-\nu)^2} y}\right). \end{aligned} \tag{0.12}$$

Using the expansion[vi]

$$(1-x)^{1/2} = 1 - \frac{1}{2}x - \sum_{k=2}^{\infty} \frac{(2k-3)!!}{(2k)!!} x^k, \tag{0.13}$$

we have

$$g[y] = \frac{(2-\nu)N}{2(1-\nu)} \sum_{k=2}^{\infty} \frac{(2k-3)!!}{(2k)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2} y \right)^k . \tag{0.14}$$

Comparing the coefficients in Equation (0.14) with the definition of $g[y]$ in Equation (0.7), the probability of finding an attack unit of attack strength s is given by:

$$\frac{s n_s}{N} = \frac{(2-\nu)}{2(1-\nu)} \frac{(2s-3)!!}{(2s)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s . \tag{0.15}$$

It hence follows that the average number of attack units of attack strength s is

$$\begin{aligned} n_s &= \frac{(2-\nu)}{2(1-\nu)} \frac{(2s-3)!!}{s(2s)!!} \left(\frac{4(1-\nu)}{(2-\nu)^2} \right)^s N \\ &= \frac{(1-\nu)^{s-1}(2s-2)!}{(2-\nu)^{2s-1}(s!)^2} N \end{aligned} \tag{0.16}$$

The s-dependence of n_s is implicit in Equation (0.16), with the dominant dependence arising from the factorials. Recall Stirling's series for $\ln[s!]$:

$$\ln[s!] = \frac{1}{2}\ln[2\pi] + \left(s + \frac{1}{2} \right) \ln[s] - s + \frac{1}{12s} - \cdots . \tag{0.17}$$

Retaining the few terms shown in Equation (0.17) is in fact a very good approximation, giving an error of $< 0.05\%$ for $s \geq 2$. This motivates us to take the logarithm of both sides of Equation (0.16) and then apply Stirling's formula to each log-factorial term, as in

[vi] The 'double factorial' operator $!!$ denotes the product: $n!! = n(n-2)(n-4)\ldots$

Equation (0.17). We follow these mathematical steps (which were derived in the M.Phil. thesis of Larry Yip, Chinese University of Hong Kong, who was supervised by Prof. Pak Ming Hui). We hence obtain

$$\ln(n_s) \approx \ln\left(\frac{(1-\nu)^{s-1}}{(2-\nu)^{2s-1}} N\right) + \left(2s - \frac{3}{2}\right)\ln(2s-2) + \ln\left(e^2\right) - \frac{1}{2}\ln(2\pi) - (2s+1)\ln(s)$$

$$\approx \ln\left(\frac{e^2 4^s (1-\nu)^{s-1}}{2^{\frac{3}{2}}\sqrt{2\pi}(2-\nu)^{2s-1}} N\right) + \left(2s - \frac{3}{2}\right)\ln(s) - \left(3\text{s} - \frac{3}{2}\right)\frac{1}{s} - (2s+1)\ln(s)$$

Combining the terms on the right-hand side into a single logarithm, it follows that

$$n_s \approx \left(\frac{(2-\nu)e^2}{2^{3/2}\sqrt{2\pi}(1-\nu)}\right)\left(\frac{4(1-\nu)}{(2-\nu)^2}\right)^s \cdot \frac{(s-1)^{2s-3/2}}{s^{2s+1}} N. \qquad (0.18)$$

The s-dependence at large s can then be deduced from Equation (0.18):

$$n_s \sim N\left(\frac{4(1-\nu)}{(2-\nu)^2}\right)^s s^{-5/2} \quad . \qquad (0.19)$$

The above equation (0.19) can be re-written as follows:

$$n_s \sim N \exp\left(-s\ln\left[\frac{(1-\nu/2)^2}{(1-\nu)}\right]\right) s^{-5/2}$$

which shows that at large s, there will be an exponential cut-off. This makes sense since the coalescence process for attack units with very large attack strength will always be hampered by the fact that the total insurgent attack strength N is itself finite. However for sufficiently small values of ν, the dominant dependence on s over a wide range of intermediate s-values will be

$$n_s \sim s^{-5/2} \quad \text{hence} \quad n_s \sim s^{-2.5} \qquad (0.20)$$

We have therefore shown analytically that the distribution of attack strengths will follow a power-law with a coefficient 2.5 (i.e. 5/2). As discussed earlier, we assume that any particular attack unit could be involved in an event in a given time interval, with a probability p which is independent of its attack size. Hence these power-law results which we have derived for the distribution of attack strengths, will also apply to the distribution of attacks of severity x. (Recall that the attack strength s is a measure of the number of casualties in a typical event, and that the severity x of an event is measured as the number of casualties). In other words, the same power-law exponent 2.5 derived in Eq. (0.20), will *also* apply to the distribution of attacks having severity x.

Hence our model predicts that any guerrilla-like war which is characterized by an ongoing process of coalescence and fragmentation of attack units, and hence an ongoing re-distribution of the total attack strength, will have the following properties:

(i) **The distribution of events with severity *x* will follow a power-law. This finding is consistent with the behaviour observed for the aggregated data in the Iraq and Colombia wars (see Figure 1 of the paper).**

(ii) The power-law distribution will, in the steady-state (i.e. long-time) limit, have a coefficient of 2.5. This is precisely the value to which the results for Colombia and Iraq currently seem to be heading (see Figure 2 of the paper).

In the case of the Iraq war, we can go one step further by providing a simple generalization of the above model in order to offer an explanation for the evolution of the power-law coefficient throughout the war's entire history (recall Figure 2 of paper). The above model is characterized by the probability ν together with the mechanism for attack-unit coagulation and fragmentation. This value ν was chosen to be *independent* of the attack strength of the individual attack units involved. In this modification, we will keep the essential structure of the model, but we will add the modification that an attack unit will fragment with a probability which depends on its attack strength, and will coalesce with another attack unit with a probability depending on the attack strengths of the two attack units involved. With probability ν the randomly-chosen attack unit i (chosen with probability proportional to the attack strength) will fragment into attack units of attack strength 1, with a probability $f[s_i]$ which depends on s_i. With probability $(1-\nu)$ the attack unit of attack strength s_i coalesces with another randomly-chosen attack unit j having attack strength s_j, with probability $f[s_i]f[s_j]$. They remain separated otherwise. With the choice $f[s]=1$ the original model is recovered. Analytically, this particular formulation of the fragmentation and coagulation process can be readily treated by the generating function approach discussed earlier, as will be demonstrated below.

Before proceeding, we discuss why this probabilistic 'attack-unit-formation' process may indeed mimic certain aspects of guerrilla warfare. One such aspect is the effect of the arrival of opposing troops in the area. Imagine that at a given time step and with a given probability ν, the opposing army arrives in the vicinity of a given attack unit of attack strength s_i. If the overall conflict is such that the opposing army has the guerrillas/insurgents on the run, then this might suggest to the members of the insurgent attack unit that they should separate and move away from the area. However, if the state of the conflict is such that the guerrilla/insurgent force feels powerful, they are unlikely to just disband and run if they have a significant attack strength. Instead they will possibly stand their ground and fight. Hence their probability of fragmentation is likely to be a decreasing function of their attack strength. By contrast, with probability $(1-\nu)$, no opposing troops arrive in the vicinity of the attack unit. With probability $f[s_i]$ ($f[s_j]$) the attack unit i (j)

decide to join forces. Thus, the two attack units will coalesce with probability $f[s_i]f[s_j]$. Again, this need for coalescing is likely to be less if the two attack units involved already feel powerful. Hence we would expect the probability of coalescence of the two attack units to be a decreasing function of their attack strengths. It is therefore quite plausible that -- depending on the state of the war from the insurgent force's perspective -- the probabilities of fragmentation and coalescence should depend on $f[s_i]$ ($f[s_j]$), i.e. they depend on the attack strengths of the attack units involved.

Analytically, the master equations for the specific example case in which $f[s] \sim s^{-\delta}$ can readily be written down:

$$\frac{\partial n_s}{\partial t} = -\frac{\nu s^{1-\delta} n_s}{N} + \frac{(1-\nu)}{N^2}\sum_{s'=1}^{s-1}(s')^{1-\delta} n_{s'}(s-s')^{1-\delta} n_{s-s'} - \frac{2(1-\nu)s^{1-\delta} n_s}{N^2}\sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'} \quad \text{for } s \geq 2 \qquad (0.21)$$

$$\frac{\partial n_1}{\partial t} = \frac{\nu}{N}\sum_{s'=2}^{\infty}(s')^{2-\delta} n_{s'} - \frac{2(1-\nu)n_1}{N^2}\sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'} \qquad (0.22)$$

with the physical meaning of each term being similar to that for Equations (0.1) and (0.2). The steady state equations become

$$s^{1-\delta} n_s = A\sum_{s'=1}^{s-1}(s')^{1-\delta} n_{s'}(s-s')^{1-\delta} n_{s-s'} \qquad (0.23)$$

$$n_1 = B\sum_{s'=2}^{\infty}(s')^{2-\delta} n_{s'} \qquad (0.24)$$

The constant coefficients *A* and *B* are given by

$$A = \frac{1-\nu}{N\,\nu + 2(1-\nu)\sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'}} \quad \text{and} \quad B = \frac{N\,\nu}{2(1-\nu)\sum_{s'=1}^{\infty}(s')^{1-\delta} n_{s'}}$$

Setting $\delta = 0$ in Equations (0.23) and (0.24) recovers Equations (0.3) and (0.4) for the original model. A generating function

$$G[y] = \sum_{s'=0}^{\infty} (s')^{1-\delta} n_{s'} y^{s'} = n_1 y + g[y] \qquad (0.25)$$

can be introduced where $g[y] = \sum_{s'=2}^{\infty} (s')^{1-\delta} n_{s'} y^{s'}$ and $y = e^{-\omega}$. The function $g[y]$ satisfies a quadratic equation of the form

$$(g[y])^2 - \left(\frac{1}{A} - 2n_1 y\right) g[y] + n_1^2 y^2 = 0 \qquad (0.26)$$

which is a generalization of Equation (0.8). Using $n_1 + g[1] = \sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'}$ and Equation (0.26), n_1 can be obtained as

$$n_1 = \frac{(1-\nu)^2 - \nu^2 A^2 N^2}{4(1-\nu)^2 A} \qquad (0.27)$$

Solving Equation (0.26) for $g[y]$ gives

$$g[y] = \frac{1}{4A}\left(1 - \sqrt{1 - 4n_1 A y}\right)^2 \qquad (0.28)$$

Following the steps leading to Equation (0.19), we obtain n_s in the modified model:

$$n_s \approx N \left(\frac{4(1-\nu)\left((1-\nu) + \frac{N\nu}{\sum_{s'=1}^{\infty} (s')^{1-\delta} n_s}\right)}{\left(\frac{N\nu}{\sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'}} + 2(1-\nu)\right)^2} \right)^s s^{-(5/2-\delta)} \qquad (0.29)$$

For $\delta = 0$, $\sum_{s'=1}^{\infty} (s')^{1-\delta} n_{s'} = N$ and hence Equation (0.29) reduces to the result in Equation (0.19) for the original model. For $\delta \neq 0$, it is difficult to solve explicitly for n_s. However the summation simply gives a constant, and thus for small ν the dominant dependence on the attack strength s is $n_s \sim s^{-(5/2-\delta)}$ and hence equivalently $n_s \sim s^{-(2.5-\delta)}$.

Most importantly, we can see that by decreasing δ from $0.7 \to 0$ (i.e. by increasing the relative fragmentation/coalescence rates of larger attack units) we span

the entire spectrum of power-law exponents observed in the Iraq war from the initial value of 1.8, up to the current tendency towards 2.5. This effect of decreasing δ from $0.7 \rightarrow 0$ corresponds in our model to a relative increase in the tendency for larger attack units to either fragment or coalesce at each timestep. In other words, decreasing δ mimics the effect of decreasing the relative robustness or 'lifetime' of larger attack units.

Going further, we note that these theoretical results are consistent with, *and to some extent explain*, the various power-law exponents found for:

(1) Conventional wars. The corresponding power-law exponent 1.8 can now be interpreted through our generalized model with $\delta \approx 0.7$, as a tendency toward building larger, robust attack units with a fixed attack strength as in a conventional army -- as opposed to attack units with rapidly fluctuating attack strengths as a result of frequent fragmentation and coalescence processes. There is also a tendency to form a distribution of attack units with a wide spectrum of attack strengths – this is again consistent with the composition of 'conventional' armies from the past.

(2) Terrorism in G7 countries. The corresponding power-law exponent 1.7 can be interpreted through our generalized model with $\delta \approx 0.8$, as an even stronger tendency for robust units (e.g. terrorist cells) to form. There is also an increased tendency to form larger units – or rather, to operate as part of a large organization.

(3) Terrorism in non-G7 countries. The corresponding power-law exponent 2.5 can be interpreted through our model with $\delta = 0$, as a tendency toward more transient attack units (e.g. terrorist cells) whose attack strengths are continually evolving dynamically as a result

of an on-going fragmentation and coalescence process. Unlike a conventional army, there will be a tendency to form smaller attack units rather than larger ones.

Interestingly, we can now discuss the evolution of the wars in Colombia and Iraq in these terms:

War in Colombia. At the beginning of the 1990's, the power-law exponent was very high (3.5). Then over the following 15 years, it gradually lowered to the present value and hovers around 2.5 from 1999 onwards. Using our model, the interpretation is that the war at the beginning of the 1990's was such that the guerrillas favoured having small attack units. This is possibly because they lacked communications infrastructure, and/or did not feel any safety in larger numbers. The decrease toward the value 2.5, suggests that this has changed – probably because of increased infrastructure and communications, enabling attack units with a wide range of attack strengths to build up.

War in Iraq. At the beginning of the war, the power-law exponent was quite low (1.8) and was essentially the same value as conventional wars. This is consistent with the war being fought by a conventional Iraqi army against the Coalition forces. There is then a break in this value after a few months (i.e. the war ended) and following this, the power-law exponent gradually rises towards 2.5. This suggests that the insurgents have been increasingly favouring more temporary attack units, with an increasingly rapid fragmentation-coalescence process. This finding could be interpreted as being a result of increased success by the Coalition Forces in terms of forcing the insurgents to fragment. On the other hand, it also means that the Iraq War has now moved to a value, and hence character, which is consistent with generic non-G7 terrorism.

PART 2: Tables and figures confirming the robustness of our results

Estimates of power-law coefficients for the entire time-series

	Country	α	α Lower Confidence Band	α Upper Confidence Band	$x_{\min}$	Significance of KS test for Power Law dist.	Significance of KS test for Lognormal dist.
K	Colombia	2.9622	2.8847	3.0465	5	0.4860	<0.001
I	Colombia	2.7557	2.6345	2.8863	6	0.7780	<0.001
KI	Colombia	2.7896	2.7138	2.8722	7	0.5550	<0.001
K_{min}	Iraq	2.3135	2.1765	2.4766	7	0.9970	<0.001
K_{max}	Iraq	2.1612	2.1025	2.2313	3	0.4280	<0.001

Table T1 Shows two complementary estimates of the power-law coefficients for the variables K (reported deaths for Colombia), I (reported injuries for Colombia), KI (reported deaths plus injuries for Colombia), K_{min} (minimum reported deaths for Iraq) and K_{max} (maximum reported deaths for Iraq). Our α parameter estimate is the maximum likelihood estimator for a discrete power law. For further discussion, see PART 3 of this *Supporting Online Material*.

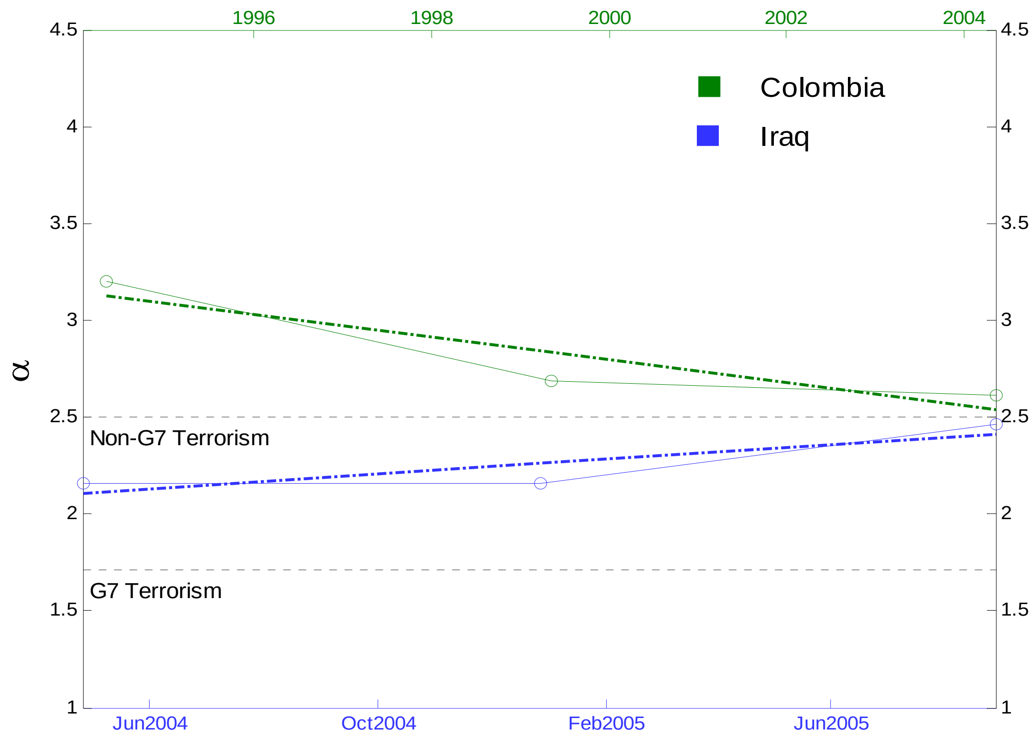

Figure S2 The variation through time of the power law coefficients for three 2,500 day intervals displaced by 60 months for the Colombian data and three 365 day intervals displaced every 8 months for the Iraq data. Despite this change in size of the windows and how they slide across time both curves seem to be tending toward 2.5, as in Figure 2 of the paper. For further discussion, see PART 3 of this *Supporting Online Material*.

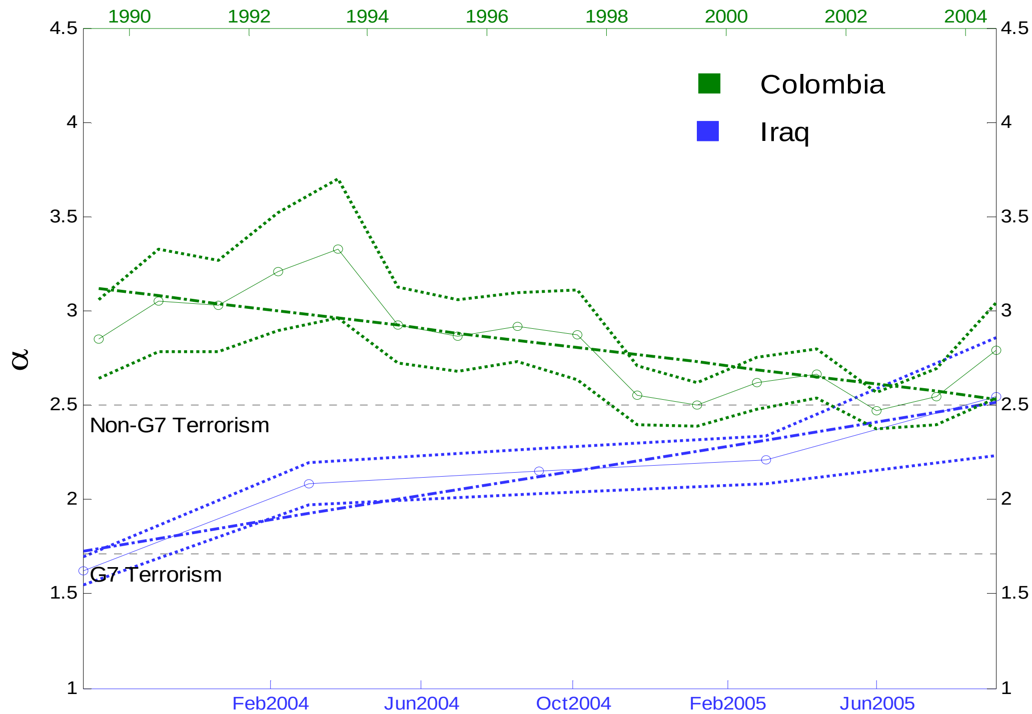

Figure S3 The variation through time of the power law coefficients for two year intervals displaced every year for Colombia and 250 day intervals displaced every 6 months for Iraq. Again, they both seem to be tending toward 2.5, as in Figure 2 of the paper. For further discussion, see PART 3 of this *Supporting Online Material*.

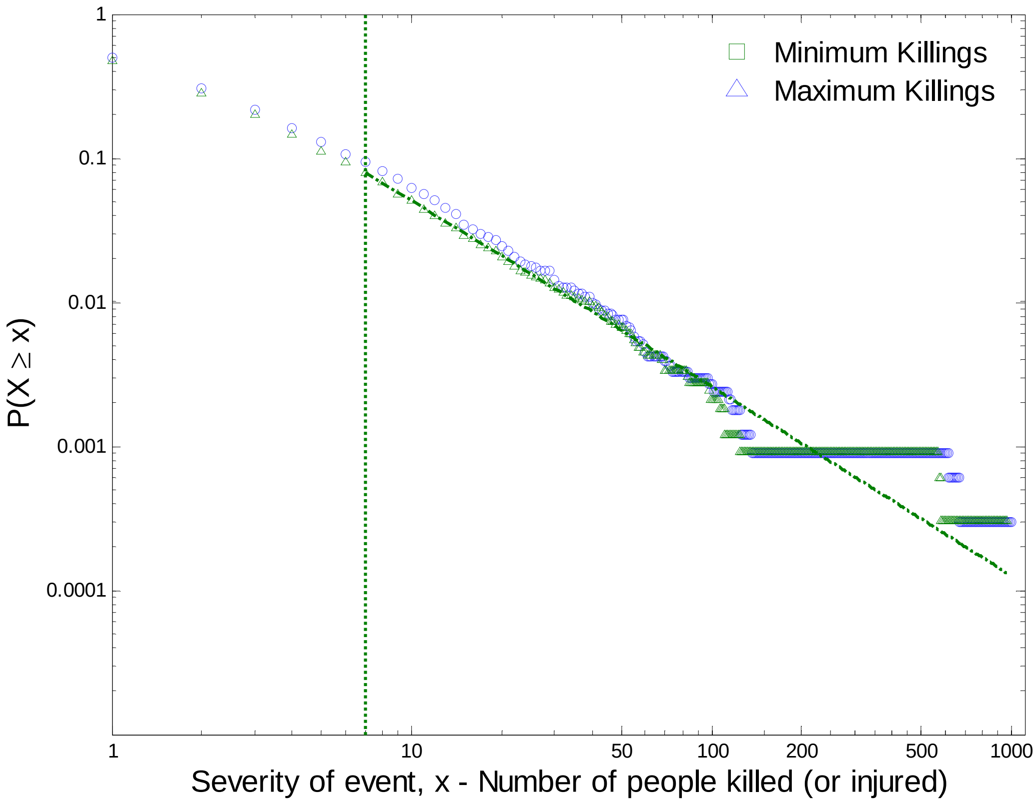

Figure S4 Log-Log plots of cumulative distributions $P(X \geq x)$ describing events greater than x, for the minimum possible value and maximum possible value of each event in the Iraq dataset. The results are very much the same across the two measures. For further discussion, see PART 3 of this *Supporting Online Material*.

PART 3: Data and methods

Data: For Colombia we are able to work with the very broad measure of all conflict-related killings plus injuries. For the Iraq data we work with killings as provided by the CERAC Integrated Iraq Dataset (CIID). The CERAC data builds on primary source compilations of violent events by Colombian human rights NGO's and from local and national press reports. We distil from this foundation all the clear conflict events, i.e., those that have a military effect and reflect the actions of a group participating in the armed conflict. For each event we record the participating groups, the type of event (massacre, bombing, clash, etc.), the location, the methods used and the number of killings and injuries of people in various categories (guerrillas, civilians, etc.). This data set covers the years 1988-2004 and includes 20,227 events. The CIID builds on the event description three datasets that monitor violence in Iraq: Iraq Body Count, ITERATE, and iCasualties. The Iraq Body Count Project monitors the reporting of more than 30 respected online news sources, recording only events reported by at least two of them. For each event they log the date, time, location, target, weapon, estimates of the minimum and maximum number of civilian deaths and the sources of the information. The concept of civilian is broad, including, for example, policemen. The list of events, posted online, covers the full range of war activity, including suicide bombings, roadside bombings, US air strikes, car bombs, artillery strikes and individual assassinations. The data set covers the period from May of 2003 to the present and includes 3,333 events. The ITERATE logs international terrorism events. iCasualties dataset logs the victimisation of members of coalition forces.

Methods: Let be X an $i.i.d.$ random variable that follows a discrete power law for values greater or equal to $x_{\min}$, hence, the probability function of X is:

$$p(x_k) = \frac{x_k^{-\alpha}}{\zeta(\alpha, x_{\min})} \qquad \text{for all } x_k \geq x_{\min}$$

where $\zeta(\alpha, x_{\min}) = \sum_{i=x_{\min}}^{\infty} x_i^{-\alpha}$ is the incomplete Riemann zeta function (Clauset and Young, 2005). The maximum likelihood estimator of the α parameter is obtained by maximizing the log-likelihood function:

$$\ell(\alpha \mid x_1, \ldots, x_n) = -\sum_{i=1}^{n} \left(\alpha \ln x_i + \ln[\zeta(\alpha, x_{\min})]\right)$$

We use a recursive estimation procedure in which we first estimate the α parameter for a wide range of X and then we calculate the Kolmogorov-Smirnov (KS) goodness-of-fit statistic for each of these values. The parameter $x_{\min}$ is selected as the value of X that minimizes the KS test. Additionally, we test the significance of the KS test using Montecarlo simulations for two distributions: the power law and the lognormal distribution.[vii] The 95% confidence bands for the α estimates were obtained by bootstrapping techniques.

To test the robustness of our findings in Figure 2 of the paper, we have repeated the calculation of α for several different sizes of the time windows. We also tried varying the way in which these windows slide forward in time. All these changes barely affected our results. As an example, in Part 2 of this document we have plotted the evolution of KI (deaths and injuries) for different time windows. For the Colombian data set we used a time window of 2,500 days moved every 60 months (see Figure S2); then a moving time window of two years displaced every year (see Figure S3), and finally a moving time window of 800 days, displaced every month (Figure 1). For the Iraq data set we used a 365

[vii] In the case of the lognormal null hypothesis, we evaluate the entire sample.

day time window displaced every 8 months (Figure S2), a 250 day interval displaced every 150 days (Figure S3), and a 400 day time window displaced every month (Figure 1). As can be seen, our results are essentially unchanged by these variations.

As a further test of the robustness of the results obtained in our paper, and in particular our main findings in Figures 1 and 2 of the paper, we ran the following variations of our calculations. For Colombia we used just killings and just injuries, rather than killings plus injuries as presented in the paper. For Iraq we used the maximum number of deaths rather than the minimum number of deaths as reported in the paper. These results are shown in Figure S4. As can be seen, these variations do not affect our findings. This is reassuring, and is actually not too surprising since the power-law coefficient α provides a statistical measure of the structure of the events' time-series, rather than the absolute number of killings and/or injuries. Hence the power-law coefficient α will be unaffected by any constant scale factors which are introduced as a result of a fixed ratio of injuries to killings.

An additional, but perhaps even more important, advantage of focusing on α, concerns possible over- or under-reporting of war casualties. In particular, α is insensitive to systematic over-reporting or under-reporting of casualties. This is because any systematic multiplication of the raw numbers by some constant factor has no effect on the α value which emerges from the log-log plot.