Strategic Judgment Proofing*

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Abstract: A liquidity-constrained entrepreneur needs to raise capital to finance a business activity that may cause injuries to third parties — the tort victims. Taking the level of borrowing as fixed, the entrepreneur finances the activity with senior debt in order to shield assets from the tort victims in bankruptcy. Interestingly, senior debt serves the interests of society more broadly: it creates better incentives for the entrepreneur to take precautions than either junior debt or outside equity. Unfortunately, the entrepreneur will raise a socially excessive amount of senior debt, reducing his incentives for care and generating wasteful spending. Giving tort victims priority over senior debtholders in bankruptcy prevents over-leveraging but leads to suboptimal incentives. Lender liability exacerbates the incentive problem even further. A Limited Subordination Rule, where the firm may issue senior debt up to an exogenous limit after which any further borrowing is treated as junior to the tort claim, dominates these alternatives. Mandatory liability insurance and punitive damages are also discussed.

Keywords: the judgment proof problem, strategic judgment proofing, capital structure, subordination, lender liability, limited subordination

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1 Introduction

There was a striking 41% rise in the number of taxi and livery accidents in New York City in the 1990’s. As described in the New York Times, many of the victims — often bystanders on the sidewalk — found themselves unable to collect their awards after receiving favorable judgments at trial.\footnote{Drew and Newman (1998).} There were several reasons for this. First, most of New York City’s 12,000 taxi cabs were minimally insured. Second, the taxi industry is organized in such a way as to make taxi medallions — worth about $275,000 each — unreachable by the victims. The owners of the medallions often use them as collateral for loans, so that “even when the rare victim tries to seize a medallion in court, it is common to find that the owner has attached so much debt to it that there is little money left to recover.”\footnote{Drew and Newman (1998).} Furthermore, owners of large fleets often organize their operations into collections of much smaller taxi companies owning just two or three medallions, thereby protecting their assets from liability. In the words of Pam Liapakis, former president of the New York State Trial Lawyers Association, “When one owner can own 100 cars in different corporations, and then mortgage them to protect his assets from accident victims, that’s wrong ..... The purpose of the corporate law is being subverted.”\footnote{This practice was described by New York Supreme Court judge as “a legalized racket to avoid liability for payment of the negligent maiming and killing by taxicabs.” See Drew and Newman (1998).}

In an ideal world, companies engaging in risky activities would take appropriate precautions to avoid harming others. While legal liability for these harms surely provides some incentives for companies to behave prudently, these incentives may be suboptimal. When
firms are “judgment proof,” lacking the financial assets to fully compensate the victims, they tend to take too little care to avoid the harm.\textsuperscript{4} To a certain degree, the judgment-proof problem is unavoidable: the total resources held by private companies, public organizations, and individual people are limited while the potential for causing harm is (arguably) boundless. This paper is concerned, however, with the additional financial strategies that companies may adopt to further shield their assets from tort victims — “strategic judgment proofing.”

We consider the problem facing a liquidity constrained entrepreneur (the injurer) who needs capital to finance a risky activity that may harm others. The entrepreneur faces several important decisions: the level and method of financing the activity (outside equity, subordinated junior debt, secured senior debt), and (subsequently) the level of care to reduce the social harm from his operations. Taking the level of borrowing as fixed, we first show that the entrepreneur would choose to finance the project with secured senior debt. Secured senior debt enjoys the highest priority in bankruptcy, and can therefore be used to shield assets from tort victims. Interestingly, this form of strategic judgment-proofing enhances social welfare. Taking the level of outside financing as fixed, senior debt creates the best incentives for the entrepreneur to take precautions to reduce the harm to the victims. The reasoning is as follows. The secured senior debtholders face a lower risk of non-repayment than the holders of junior claims and, as a consequence, require a lower interest rate. This lower interest rate makes bankruptcy less likely, leading the entrepreneur to better internalize the social harm from the risky activity.\textsuperscript{5}

Unfortunately, the level of outside financing by entrepreneurs is not fixed. We show that the entrepreneur will secure an excessive amount of senior debt in order to further dilute the value of the tort claim. In the extreme, the entrepreneur could essentially reduce its liability to zero by issuing securities whose face value exceeds the upper bound on the future firm value. In contrast to the result that secured senior debt is desirable for fixed levels of financing, the over-leveraging motive leads social welfare to fall for two reasons. First, the firm will take too little care to avoid accidents and, second, may misallocate

\textsuperscript{4}See Shavell (1986) and Summers (1983).
\textsuperscript{5}This intuition is similar to Pitchford’s (1995) observation that lender liability increases the interest rate and consequently reduces the borrower’s precautions. The relationship with Pitchford is discussed below in more detail.
capital and engage in wasteful spending in order to shield the cash acquired through the leveraged transaction from the tort victims.

Within this general framework we can consider several different public policies that address the judgment-proof problem. First, suppose the victims were given priority over the secured senior debtholders in bankruptcy. This would, in effect, force the senior debt into a subordinated junior position. While this policy can prevent the over-borrowing problem identified above, the firm will still take too little care to avoid harm. (As described above, junior claimants require a higher interest rate to compensate them for the risk of non-repayment and so the entrepreneur’s incentives to take precautions are diluted.) Second, the senior debtholders can be held liable for the residual harms unpaid by the injurer. This policy also prevents excessive leveraging, but exacerbates the moral hazard problem even further. A rule that we call the Limited Subordination Rule dominates these other policies. Under this rule, the firm is constrained to offer senior debt up until a limit, after which any further borrowing is treated as junior to the tort claim. Limited subordination essentially gives the “best of both worlds.” The junior treatment of borrowing beyond the pre-set limit eliminates the incentives for over borrowing (since overborrowing does not help shield the firm from liability). At the same time, the senior status of the borrowing up to the pre-set limit implies that the firm can borrow at a low interest rate, giving better incentives for precaution taking.

Shavell (1986) gives one of the first formal treatments of the judgment-proof problem, arguing that injurers with limited assets will engage in risky activities too often and will take too little care while doing so. A number of possible solutions have been proposed. Shavell (1986, 2004) shows that requiring judgment-proof firms to purchase liability insurance can force them to internalize the costs of their risky activities, getting them to scale back on the levels of these activities. Mandatory insurance can also lead to better incentives for care when the insurer can observe the firm’s effort level. The literature on vicarious

6See also Summers (1983). Beard (1990) extended Shavell’s analysis to include a pecuniary effort choice, and shows that firms may in fact take too much rather than too little care in this setting. The reason is that corporate investments made out of cash reserves are subsequently not claimable by tort victims in bankruptcy. In other words, the tort victims effectively subsidize the firm’s pecuniary investments. See also Dari Mattiacci and De Geest (forthcoming).

7Shavell (2004) also argues that minimum asset requirements can serve a similar beneficial role, but may block efficient firms from entering.
liability makes a similar point: extending liability to a third party can improve social welfare when that third party is in a contractual relationship with the injurer and can effectively control the injurer’s actions. Absent the ability to monitor or otherwise control the injurer, however, extending liability can harm rather than help the incentive problem. In work that is the most closely related to ours, Pitchford (1995) considers the effects of imposing liability on lenders. Lenders, anticipating future liability, require a higher interest rate in compensation. This leaves less remaining wealth for the borrower to lose in the event of an accident, diluting the incentives for care. All of these papers take the degree of injurer’s judgment proofness as given, so they did not analyze strategic judgment proofing.

Our paper focuses on strategic judgment proofing by endogenizing the firm’s method and level of external financing. This gives new insights into the desirability of lender liability and allows us to consider important policy alternatives — namely debt subordination — that have been largely overlooked by the literature. Specifically, lender liability can be

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8 See, for example, Sykes (1984), Hiriart and Martimort (2003), and Boyer and Laffont (1997), Dari Mattiacci and Parisi (2003). Hay and Spier (forthcoming) consider extending liability to manufacturers of risky products when a consumer, while using the product, harms someone else. The possible benefits of extended liability include the reduction in the sales of risky products and the design of safer products.

9 Shavell (2004) also argues that if insurers cannot observe the firm’s care level, then mandatory insurance dulls incentives for care. First, insured agents don’t bear the downside and therefore underinvest; second, the increase in the insurance premium leaves less money for the firm to lose. Boyd and Ingberman (1997) argue that extended liability can distort capital investments and the pattern of business transactions. Boyd and Ingberman (1994, 1999) argue that punitive damages on judgment-proof firms can sometimes reduce their incentives for care.

10 Lewis and Sappington (2001) generalize Pitchford’s binary technology and give the lender more instruments with which to control the firm, including non-monotonic contracts. They contend that “... the deep pockets of a lender can often be employed to mitigate judgment proof problems ....” Balkenborg (2001) show that Pitchford’s results may no longer hold when the lender has bargaining power and extracts rents from the firm. Lewis and Sappington (1999) argue that social welfare would be even higher if penalties were decoupled from the victim’s actual damages.

11 A short article in the Harvard Law Review (Note, 2003) also considers the effect of debt subordination on the firm’s effort choice. The informal reasoning in that article leads to the opposite (and incorrect) conclusion, however. “Because secured credit subordination ... significantly diminishes payment of tort claims, the bankruptcy regime may undermine tort deterrence, which normally requires threatening a firm with liability for all tortious harm arising from accidents.” (Note, 2003, p. 2542.) Bebchuk and Fried (1996) more generally discuss the treatment of secured claims in bankruptcy and highlight potentially adverse effects on so-called “non-adjusting creditors.”
desirable in the presence of strategic judgment proofing, in contrast with the Pitchford’s conclusion, although elevating the bankruptcy status of tort claim weakly dominates lender liability. Furthermore, we identify a new rule that dominates both and attains a second-best welfare target.

Our paper also contributes to the literature the role of agency costs in the design of financial securities (Jensen and Meckling, 1976). Innes (1990), assuming a fixed capital requirement, showed that debt contracts dominate all monotonic alternatives in terms of the incentives they provide to the borrower to maximize the value of the venture. In Innes’ model, however, the firm’s effort choice affects the future cash flows of the organization, not harms to tort victims (who were absent from his analysis). The presence of tort victims in our model allows us to broaden the set of instruments to include both senior and junior claims, and allows us to consider the relevant public policies.

The topic of strategic judgment proofing has received considerable attention from legal scholars. In a 1996 article called “The Death of Liability,” Lynn LoPucki (then a law professor at Cornell) argued emphatically that the liability system in the United States was at risk of collapse. “The system by which money judgments are enforced is beginning to fail. The immediate cause is the deployment of legal structures that render potential defendants judgment proof ....” These legal structures fall into two broad categories. First, firms can use secured debt (collateralized with assets such as taxi medallions) to force tort victims into

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12 Modigliani and Miller’s (1958) famous result about the irrelevance of capital structure fails to hold in the presence of taxes, bankruptcy costs, and (as here) agency costs.

13 Innes assumed, as we do, that the lender’s payoff must be non-decreasing in firm profit. This can be motivated by the possibility that lenders could sabotage the firm’s results or that borrowers could misrepresent their cash flows.

14 Our paper also contributes to the small literature on the strategic use of debt. Perotti and Spier (1993) argue that debt can be used to credibly threaten future underinvestment and is an effective bargaining tool in extracting concessions from other creditors including labor unions. Spier and Sykes (1998) show how different forms of debt can alter the bargaining range when a firm is negotiating with a tort victim before a costly trial. None of these papers consider the issues addressed here.

15 Empirical work includes Ringleb and Wiggins (1990), who find a 20% increase in the number of small corporations between 1967 and 1980. They argue that this was due to large corporations shutting down their inhouse risky operations and outsourcing these activities to small firms instead. In contrast, Brooks (2002) argues that higher liability for oil spills in the oil industry led to a pattern of more vertical integration, not less. He argues that this might have been due to the need for higher levels of care under the new regime.

a subordinated position in bankruptcy. Second, firms can engage in asset securitization, or “the partial or complete segregation of a specific set of cash flows from a corporation’s other assets and the issuance of securities based on these cash flows.”

Corporations have securitized assets as diverse as equipment leases, franchise fees, and cash flows from oil and gas reserves (Harrel, Rice, and Shearer, 1997). These types of securities are sometimes referred to as “Bowie Bonds” after rock star David Bowie who issued securities that were backed by the future revenues from his previously-released music albums (Clark, 1997). Relatedly, the firm might effectively “sell” a machine or factory to a separate entity and then proceed to lease the machine back. As explained by Iacobucci and Winter (2005, pp. 164-5), asset securitization differs from traditional secured debt. The asset — the cash flow from oil and gas reserves, for example — are legally sold to a third party and therefore separated from the other assets of the firm. In a bankruptcy proceeding, the oil and gas reserves would not be considered part of the bankrupt firm’s estate.

The effectiveness of secured debt and asset securitization strategies for judgment-proofing purposes hinges on the ability of the entrepreneur to dispose of — or shield — the cash acquired through the transaction. There are several ways that the firm might do this. First, the proceeds from the sale may be used, for example to buy back equity or to pay

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17 This quote is from Iacobucci and Winter (2005, p. 161-2). Asset securitization has become quite pervasive, with the current securities exceeding $2.5 trillion (Lapica 2001, 292).

18 The initial growth in so-called structured finance occurred in the area of consumer debt. Home mortgages that are made by banks are usually sold on the secondary market. Purchasers of these mortgages include government-backed Federal National Mortgage Association (Fannie Mae), and the Federal Home Loan Mortgage Corporation (Freddie Mac), who then re-package them as securities in the bond market. Student loans, automobile loans, and credit card receivables are also commonly securitized. In 1990 the percentage of all consumer debt that was securitized was 10.5%. Just three years later, this percentage had risen to 15.5%. (LoPucki, p. 24. Quoting 80 Federal Reserve Bulletin, A-40 table 1.3 ll. 5, 12 Feb. 1994).

19 Although the firm no longer legally owns the assets, it may never the less continue to manage the assets and track the accounts receivable through a contracts with the new owners.

20 This third party — often called a special purpose vehicle (SPV) or a bankruptcy remote vehicle — may be a separate corporation or trust. Iacobucci and Winter (2005) argue that asset securitization can help to better control agency problems.

21 Schwarcz (1999), in a critique of Lopucki (1996), argued that asset securitization by itself would be ineffective, since it merely exchanges one asset (future cash flows) for another (a current cash infusion). If the cash is retained by the firm, then the cash would be acquired by the tort victims in bankruptcy.
dividends to shareholders. Under the principle of limited shareholder liability, the money is then protected from the future tort victims. Second, the firm might spend the money on higher salaries or corporate perks. Third, a firm might adopt a variety of exemption strategies to shield the money. Under a homestead exemption, for example, where a debtor can keep his home even while tort claims against the debtor are discharged in bankruptcy.\textsuperscript{22} So-called “floating exemptions” may be applied to other assets, including corporate jets. Perhaps surprisingly, some jurisdictions allow debtors to replace nonexempt property with exempt property just prior to bankruptcy.\textsuperscript{23} Finally, the managers of the firm may benefit from the cash infusion through higher salaries or corporate perks.

The paper is arranged as follows. Section 2 illustrates some of the key contributions of our paper in a simple example. Section 3 lays out the basic assumptions of the model and establishes a social welfare benchmark. Section 4 characterizes the financial decisions and effort choice of the firm. Section 5 considers public policy responses, including the elevation of tort victims in bankruptcy and lender liability. Section 6 discusses robustness of our findings and other remedies of judgment proofness.

\section{Example}

Consider an entrepreneur who needs to raise at least $300 to purchase capital — a “taxi medallion.” The capital market is competitive and the risk-free interest rate is normalized to zero. The taxi medallion, which does not depreciate in value, will generate an additional cash flow of $200 under the control of the entrepreneur. Although the cash flow is riskless, the business activity is risky in the sense that it may cause harm to other people. For the moment, let’s assume that there is an exogenous one-in-ten probability that the activity will cause $1,000 in damages to a tort victim. Notice that this business activity is inherently judgment proof: in the event of an accident, the total assets (the $300 medallion plus the $200 cash flow) are insufficient to compensate the tort victim for his loss.

For any fixed level of borrowing below the total value of the assets — say $300 — it is clear that the entrepreneur would choose to finance the business with senior debt.\textsuperscript{22} A taxi owner, for example, might borrow against his medallion and use the proceeds to purchase a house.\textsuperscript{23} See in re Reed, 12. B.R. 41 Bankr. N D Texas 1981. Cited in LoPucki (1996) p. 31.
With senior status, the lender is guaranteed repayment of his loan in the event of an accident and is therefore willing to issue the loan at the risk-free rate of 0%. In the event of an accident, the lender receives the $300 taxi medallion and the tort victims claim the $200 cash flow. Note that the entrepreneur’s equity has an expected value of $180 — the entrepreneur keeps the residual $200 cash flow 90% of the time and keeps nothing in the event of an accident. If the debt were junior to the tort claim, on the other hand, then the lender would not be repaid following an accident. A face value of (approximately) $333 would allow the lender to break even in expectation, corresponding to an interest rate of 11%. What is the entrepreneur’s expected payoff when issuing junior debt? If no accident occurs the entrepreneur’s payoff is $300 + $200 — $333 = $167; if an accident occurs the entrepreneur receives $0. His expected payoff is therefore 90% of $167, or $150. Therefore the entrepreneur’s expected payoff is $30 higher when he uses senior debt instead of junior debt.

Senior debt is an effective mechanism for transferring value from the tort victims to the entrepreneur: the entrepreneur is made better off by $30 and the tort victims are made worse off by $30. To see this, consider the expected payments to the tort victims. When the debt is senior, the taxi medallion is essentially taken “off the table” and the tort victims’ recovery is limited to $200. That is, the tort victims collect $20 in expectation. When the debt is junior, on the other hand, the tort victims can seize the taxi medallion worth $300 in addition to the $200 cash flow. So the tort victims’ recovery following an accident is $500, or $50 in expectation.

The method of financing does more than simply reallocate value among the different players, however. It can also effect the entrepreneur’s choice effort and hence the expected accident losses. To see this, suppose that there are two levels of precaution: low and high. The low level of effort is costless for the entrepreneur and leads to a 20% accident probability. The high level of effort requires the entrepreneur to make non-pecuniary investment of $18 and reduces the accident probability to 10%. Notice that the high level of effort is socially optimal here: the entrepreneur’s cost of effort, $18, is outweighed by the $100 reduction in the expected accident losses. It is easy to see that, with senior debt, the entrepreneur will take the high level of precaution. The 10% reduction in probability multiplied by the entrepreneur’s $200 out-of-pocket cost in the event of an accident outweighs

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2490% of $333 is approximately $300.
his $18 additional cost of effort. With junior debt, on the other hand, the entrepreneur will not take the high level of precautions. Suppose he did. Recall that the 11\% rate of interest required by the lender reduces the entrepreneur’s personal stake from $200 to $167. The additional cost of effort, $18, is higher than the benefit of this effort, (.1) ($167) = $16.7. This simple example illustrates that entrepreneur’s preferred method of financing — senior secured debt — is aligned with that of society more broadly. If the entrepreneur controlled the level of financing as well, he would issue securities that are backed by the $200 cash flow in addition to the $300 taxi medallion and can subsequently consume (or hide) the immediate cash infusion of $200. Since the lender expects to be repaid in full, the required rate of interest is 0\%. Now the company is totally judgment proof: there are no assets for the victims to claim in the event of an accident. The entrepreneur takes the low level of effort here and, in a richer framework, his precautions would be even lower than that.

What can society do to control this behavior? First, suppose that a law were passed that elevated the status of the tort victims in bankruptcy above that of the debtholders. This effectively forces debt into a junior position. On the positive side, this law would prevent the over-leveraging identified above. The entrepreneur will limit his borrowing to the $300 taxi medallion only. On the negative side, however, the 11\% interest rate demanded by the lender implies that the entrepreneur will take only the low level of effort. Suppose instead that the lender is held liable for 100\% of the accident victim’s losses. Assuming a high level of effort, the interest rate would would necessarily rise to 33\% — the first $300 of the $400 face value reflects the principal of the loan while the remaining $100 reflects the lender’s expected future liability (10\% of the $1,000 loss). From the entrepreneur’s perspective, the 10\% reduction in probability multiplied by his $100 loss following an accident is outweighed by the $18 cost of effort.\footnote{If there is no accident, the lender receives the $400 face value and the firm keeps $500 – $400 = $100.} Indeed, this example suggests that the entrepreneur’s incentives would be even worse with lender liability.

Our proposed Limited Subordination Rule, which allows the entrepreneur to issue senior debt up to a limit of $300 and forces further borrowing into a junior subordinated position, does better than either of these other remedies. The entrepreneur would borrow exactly $300 and no more, and would subsequently take the high level of precautions. The junior treatment of the additional cash flow eliminates the incentives for overborrowing since overborrowing cannot help to shield the entrepreneur from liability. At the same time, the
scheme allows for the senior status of debt up to the level required for productive use. This means that the firm can borrow on the terms that will leave it with best incentives to take precautions.

3 Model

- Primitives

Consider a privately owner-managed firm. The firm has a project that would generate a fixed cash flow of \( v > 0 \). The manager is capital constrained, so he requires an outside investment of \( k < v \).\(^{26}\) The project causes harm of \( x \) to the society. The size and likelihood of the harm depends on the effort (or precaution) made by the firm. Suppose that, given effort \( e \in \mathbb{R}_+ \), \( x \) is distributed over \( \mathcal{X} := [0, \pi] \), according to a cdf \( F(\cdot \mid e) \) which has positive density \( f(\cdot \mid e) \) in its support. We assume that \( e \) reduces \( x \), in the sense of \( f \) satisfying monotone likelihood ratio property in \((-x, e)\):

\[
(MLRP) \quad \frac{f(x' \mid e')}{f(x \mid e)} < \frac{f(x' \mid e)}{f(x \mid e)} \text{ for any } x' > x, e' > e, x', x \in \mathcal{X}.
\]

Assuming differentiability of \( F(\cdot \mid e) \) with respect to \( e \), \((MLRP)\) implies that \( F_e(\cdot \mid e) > 0 \). We further assume that \( F_{ee} \leq 0 \). An effort of \( e \) by the firm incurs the cost of \( c(e) \), where \( c(0) = 0, c'(e) \geq 0, c''(e) > 0, c'(0) = 0 \) and \( c'(\infty) = \infty \). The effort is unobservable to all parties other than the firm, so it cannot be directly contracted upon.

Initially, the firm can borrow any amount \( K \geq k \) from an outside investor. In particular, any borrowing in excess of its productive use \( k \) can be spent in a way not reachable by the tort victims or the investor. For instance, the amount can be immediately spent through executive compensation, salaries to the workers, dividends to the existing shareholders, or in a long term investment that does not generate cash flow in the short term. Further, the additional financing may be subject to extra scrutiny from the investor, and thus may require additional transactions costs for the firm. For these reasons, excess borrowing may earn less than the market rate (which is normalized to be zero). Hence, we assume that

\[^{26}\]The firm may have internal funds of \( w \) at its disposal for the investment, in which case the project requires total investments of \( k + w \), so that it requires outside investment of \( k \). In this sense, \( k \) is interpreted to be the minimal investment to be raised outside.
the firm realizes benefit of \( \phi(K - k; \theta) \), where \( \theta \in [0, 1] \) is a constant parameterizing the degree of efficiency in the use of the excess borrowing such that \( \phi_{12} > 0, \phi'(z; 0) = 0 \) and \( \phi'(z; 1) = 1 \). That is, a firm with \( \theta = 1 \) faces no loss in its use and a firm with \( \theta = 0 \) is unable to generate any return on the additional money. We assume that \( \phi(0; \theta) \equiv 0, \phi'(z; \theta) > 0, \phi''(z; \theta) \leq 0 \), and finally that \( \phi'(0; \theta) = 1 \) for all \( \theta \in (0, 1] \). This last assumption implies that any firm with \( \theta > 0 \) incurs almost no cost in using a small amount beyond the project requirement.\(^{27}\)

Once the cash flow is generated and the damages are realized, the victim of the harm can sue for damages. For the most part, we assume that the firm pays compensatory damages, which equals realized harm, \( x \), whenever the firm has sufficient cash flows after repaying the senior claims.

- Financial Instruments

The firm can finance its required capital, say \( K \geq k \), by issuing claims that may have different status at the time of bankruptcy. “Senior debt” specifies a fixed repayment rate \( r_S \leq v \), which has priority over the tort claims, so it is paid out first. Next, tort claims are paid out of cash flow left after paying \( r_S \). Finally, any “junior claims” are paid out last. As described below, these junior claims may be a function of the amount left.\(^{28}\)

As mentioned above, this specification is in keeping with the treatment of secured debt under U.S. bankruptcy law. A repayment contract, \( r_S < k \), could represent senior debt that is secured by the physical capital of the firm, for example. Our simple analytical framework is consistent with asset securitization strategies as well. A promise to repay the lender above and beyond the required capital, \( r_S > k \), could represent “Bowie Bonds” that are secured by the future cash flows of the company (rather than by physical assets). Similarly, we can interpret the senior debt in our model as actually being equity that is owned by a “parent,” while the firm (a “subsidiary”) rents the assets from the parent and controls the risky activity. As described in the introduction, these asset securitization

\(^{27}\)This assumption is made for analytical convenience; it “smooths out” the mathematical program and avoids corner solutions.

\(^{28}\)In practice, junior debt is often afforded the same status as tort claims in bankruptcy. The junior debtholders and the tort victims would then share, pro rata, in the value that remains after paying the senior claim. Our framework could be adapted to consider this intermediate case without changing the main conclusions.
strategies have the feature that the assets owned by the parent are not part of a bankruptcy proceeding when the subsidiary becomes insolvent. In short, our framework captures many different types of judgment-proofing strategies.

The junior claims mentioned above could be either junior debt or outside equity. In general, an arbitrary junior claim specifies any payout to the investor, $\rho_J(v - r_S - x)$, given a cash flow $v$, senior claims $r_S$, and tort claim of $x$, where $\rho_J(v - r_S - x) \in [0, v - r_S - x]$. Like Innes (1990), we may restrict the set of junior claims by requiring both the payment to the claimant, $\rho_J(z)$, and the payment to the firm $z - \rho(z)$ to be nondecreasing in the remaining cash flow $z$. We call the set, $\mathcal{R}$, of junior claims satisfying these properties standard junior claims. All well known junior claims belong to this set. For instance, a typical junior debt contract with repayment rate $r_J$ is described by $\rho_J(z) := \min\{r_J, z\}$. An outside equity claim is described by $\rho_J(z) = \mu z$, for some $\mu \in (0, 1]$, so again $\rho_J(z) \in \mathcal{R}$. It is easy to see that any mixture of junior debt and equity generates another $\rho_J \in \mathcal{R}$. The firm’s repayment terms can be summarized by a pair, $r := (r_S, \rho_J)$, such that $r_s < v$ and $\rho_J \in \mathcal{R}$. In case the firm carries only debt in its junior claim, then we will simply replace the third component by the repayment rate $r_J$. In sum, then a contact $r$ yields the ex post payoffs to the three parties as follows:

<table>
<thead>
<tr>
<th>payoffs</th>
<th>lender</th>
<th>tort victims</th>
<th>the firm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_S + \rho_J(v - r_S - x)$</td>
<td>$\min{v - r_S, x}$</td>
<td>$\max{v - r_S - \rho_J(v - r_S - x) - x, 0}$</td>
</tr>
</tbody>
</table>

The time line is as follows. At date $T = 0$, the firm chooses its financing contract $(K, r)$. At date $T = 1$, the firm chooses effort $e$. At date $T = 1.5$, the harm $x$ is realized. At date $T = 2$, the investor is repayed and the tort victim is compensated.

• Welfare Benchmark

Before proceeding, we will establish a useful social welfare benchmark. Suppose the social planner can simply choose the firm’s borrowing as well as its precaution level directly, and assume that the firm’s project is socially desirable given the optimal choice. Then, the
planner will choose \((K_{FB}, e_{FB})\) to maximize the social welfare,

\[
W(K, e) := v - K + \phi(K - k; \theta) - \int_X xf(x|e)dx - c(e),
\]

subject to the constraint that \(K_{FB} \geq k\). First, the optimal amount of borrowing should be \(K_{FB} = k\), since the firm has no productive use of funds beyond what is needed for the project. Next, to determine the first-best precaution level, we integrate the social welfare function by parts to obtain

\[
v - K + \phi(K - k; \theta) - \int_0^\infty \left[1 - F(x|e)\right]qdx - c(e),
\]

so the optimal precaution, \(e_{FB}\), is characterized by

\[
\int_0^\infty F_e(x|e)dx - c'(e) = 0. \tag{1}
\]

Throughout, we assume that \(W(k, 0) \geq 0\), so the project is socially valuable even with zero effort as long as the firm does not overleverage. This assumption will ensure the the project will be carried in the relevant cases studied below, thus simplifying our analysis.

4 The Firm’s Problem

We first study the firm’s behavior. While our focus is to analyze its behavior without any regulation, it is convenient for a later analysis to begin with a slightly general framework in which the lender may be subject to some liability. Specifically, suppose the firm picks \((K, r, e) \in [k, v] \times \mathbb{R}_+ \times \mathcal{R} \times \mathbb{R}_+ =: \mathcal{F}\) to initiate the project. We assume that, after the lender is repaid according to \(r\), he is liable to pay \(\ell(x)\) when the harm \(x\) is realized. Then, the lender’s ex post payoff is

\[
\pi(x, r, \ell) := r_S + \rho J(v - r_S - x) - \ell(x), \tag{2}
\]

when harm \(x\) is realized. If the lender expects the firm to choose \(e\), then his ex ante payoff becomes

\[
\Pi(r, e; \ell) := \int_X \pi(x; r, \ell) f(x|e)dx.
\]

Meanwhile, the firm receives ex post

\[
u(x; r) := \max\{v - r_S - \rho J(v - r_S - x) - x, 0\}, \tag{3}
\]
so its ex ante payoff given effort $e$ is
\[
U(K, r, e) := \phi(K - k; \theta) + \int_{X} u(x; r)f(x|e)dx - c(e).
\]

The firm then faces the problem:

\[
\begin{aligned}
\max_{(K, r, e) \in \mathcal{F}} & \quad U(K, r, e) \\
\text{subject to} & \\
(IR) & \quad \Pi(r, e; \ell) \geq K.
\end{aligned}
\]

and

\[
(IC) \quad e \in \arg \max_{e' \in \mathbb{R}_+} U(K, r, e').
\]

Condition (IR) ensures that the lender breaks even from the financial contract $(K, r)$, when the firm is expected to choose effort $e$. Condition (IC) means that the firm must have the incentive to choose $e$, facing the financial contract $(K, r)$. This is a constraint because the firm cannot commit to a level of precaution ex ante, even though it may wish to do so.\(^2^9\) We say that $(K, r, e) \in \mathcal{F}$ is feasible if it satisfies both (IR) and (IC) and \textit{optimal for the firm} if it solves the program $[\mathcal{P}(\ell)]$.

### 4.1 The unregulated behavior of the firm

We now analyze the unregulated behavior of the firm. Formally, we consider $[\mathcal{P}(0)]$: That is, no restriction is placed on the firm’s financial decision making (i.e., the amount of borrowing and its choice of financing instruments), and the lender bears no liability (i.e., $\ell(\cdot) = 0$). Therefore, the firm is free to choose the amount of borrowing, $K$, the financing instruments, $r$, and its precaution level $e$. Before proceeding, we characterize the optimal financial structure for the firm and its incentive for precaution, given that financial structure.

**Lemma 1 (Optimality of senior debt)** For any feasible $(K, r, e)$, with a non-debt structure there exists a feasible $(K, \hat{r}, \hat{e})$, with an all-debt structure $\hat{r}$ and $\hat{e} \geq e$, which the firm \(^2^9\)(IC) may bind since, starting at the solution of the relaxed program ignoring (IC), it may pay the firm to change $e$ in a way violating (IR).
prefers over \((K, r, e)\). For any feasible \((K, r, e)\) with an all-debt structure with \(r_J > 0\), there exists a feasible \((K, \hat{r}, \hat{e})\), with a senior-debt-only structure with \(\hat{r}_J = 0\) and \(\hat{e} \geq e\), which the firm prefers over \((K, r, e)\).

**Proof:** See the Appendix.

The proof is relegated to the appendix. Taken together, these two statements suggest that it is privately optimal for the firm to choose a senior-debt only structure. As mentioned, the optimality of a senior debt stems from its effectiveness as a judgment-proofing device. Suppose that the firm borrows \(K\) with (only) junior debt with the payment rate of \(r_J\), and assume \(v - r_J < x\) so that insolvency arises with positive probability, in which case the (junior) creditor does not always receive her payment \(r_J\). This scenario is depicted in Figure 1.1.

[PLACE FIGURE 1.1 ABOUT HERE.]

For a given level of harm, \(x\), the tort victim is paid \(\min\{v, x\}\), the junior creditor is paid \(\min\{v - x, 0\}\), and the firm receives \(\min\{v - x - r_J, 0\}\) (gross of effort cost and the cash received through the leveraged transaction). Notice that the repayment rate, \(r_J\), is inflated to reflect the risk of non-repayment: \(r_J > K\).

Suppose instead that the firm borrows \(K\) with senior debt, assuming for a moment the same payment rate \(r_S = r_J > K\). The firm would still receive \(\min\{v - x - r_J, 0\}\), but the rent is redistributed from the tort victim to the lender: the lender now receives \(r_J > K\) with certainty and the tort victim receives the remainder, \(\min\{v - x - r_J, x\}\). This redistributed rent can be easily shifted to the firm. Since the lender would receive a strictly higher payoff with senior rather than junior debt (holding the repayment rate fixed) she can be persuaded to charge a lower rate. In fact, the competitive capital market would drive the repayment rate down to a level that allows the lender to break even: \(r_S = K\). This is shown in Figure 1.2.

[PLACE FIGURE 1.2 ABOUT HERE.]

The firm extracts all of the redistributed rents, i.e., the entire gain from diluting the tort claims.
Interestingly, this judgement proofing strategy is socially desirable since the firm chooses a higher level of precautions with senior debt than with junior debt (or other junior claims). A senior claimant is assured repayment of the loan, unlike junior claimants, so the former charges a lower repayment rate the the latter. Hence, the firm is less likely to be insolvent with senior debt. Comparing Figure 1.1 to Figure 1.2 shows that, with senior debt, the firm is a residual claimant in more states of nature and thus has a greater incentive to reduce the harm to the tort victims.

Given Lemma 1, we can restrict attention to the senior-debt only financial structure for the firm. If the firm issues senior debt with any $K \in [k, v]$, the break-even repayment rate is simply $r_S = K$ since the debt-holder has the seniority over tort victims. From (3) above the firm’s ex post payoff is

$$u_0(x; K) := u(x; K, 0) = \max\{v - K - x, 0\}. \quad (4)$$

Hence, its ex ante payoff given effort $e$ is

$$U_0(K, e) := \phi(K - k; \theta) + \int_0^{v-K} (v-K-x)f(x|e)dx - c(e) = \phi(K - k; \theta) + \int_0^{v-K} F(x|e)dx - c(e),$$

where the equality follows from integration by parts.

The unregulated behavior of the firm, $(K_0, e_0)$, must then maximize $U_0(K, e)$. The behavior is characterized as follows. First, given any $K \geq k$, the firm’s effort $e_0(K)$ satisfies

$$\int_0^{v-K} F_e(x|e)dx = c'(e). \quad (5)$$

Given this effort choice, the firm borrows $K$ that satisfies

$$\phi'(K - k; \theta) - F(v - K|e_0(K)) - \lambda = 0, \quad (6)$$

where $\lambda \geq 0$ is a Lagrangian multiplier satisfying $\lambda(v - K) = 0$ and the envelope theorem is used.

These conditions reveal the nature of distortions on each choice. First, (5), when compared with (1), shows that the firm picks to little precaution: $e_0(K) < e^*$. The reason is that the firm internalizes the harm it causes only when it is solvent, i.e., when $x < v - K$. More interestingly, $e_0(K)$ is decreasing in $K$. As the firm raises its borrowing $K$, the likelihood of its insolvency rises, so its incentive for precautions worsens.
Next, (6) shows the firm borrows in excess of its productive use if $\theta > 0$ and $v - k < \bar{x}$ (the firm is inherently judgment proof). To see this, suppose to the contrary $K = k$. Then, $\lambda = 0$, $\phi'(0; \theta) = 1$ for $\theta > 0$ and $F(v - k) < 1$, which contradicts (6). This overborrowing result can be interpreted as “strategic judgment proofing” on the part of the firm. When the firm borrows in excess of $k$, it suffers from its “hiding cost” $1 - \phi'(K - k; \theta)$. But is can be more than compensated by the fact that, whenever the firm is insolvent, the additional repayment to the lender comes out of the fund that would have been used for the tort award, given the seniority of the debt. Hence essentially, each additional dollar borrowed is paid out of the tort victims’ pocket with probability $1 - F(v - K)$. Since the inefficiency cost is nil when borrowing slightly over $k$, the firm borrows more than $k$ in equilibrium.

The firm’s excess borrowing has two negative welfare effects. Not only does it entail direct welfare cost associated with inefficient use of funds, but it also exacerbates the firm’s judgment proofness, and thus worsens the firm’s underinvestment in precaution.

In fact, one can see that $U_0$ is supermodular in $(K, -e)$ and satisfies strict single crossing property with respect to $(K, -e; \theta)$. Hence, as the hiding efficiency $\theta$ rises, the firm borrows more and lowers its effort. In particular, there exists $\hat{\theta} \in (0, 1)$ such that if $\theta > \hat{\theta}$, then the firm completely judgment proofs itself by borrowing up to the cash flow limit, $v$, and thus chooses zero precaution.

**Proposition 1** Without any policy intervention, the firm issues senior debt, borrows more than its is productive use (i.e., $K_0 > k$), and takes too few precautions (i.e., $e_0 = e_0(K_0) < e_{FB}$). The firm’s excess borrowing increases and precaution decreases with $\theta$, such that if $\theta > \hat{\theta}$, for some $\hat{\theta} \in (0, 1)$, the firm borrows $v$ and chooses zero precaution.

Two remarks are worth noting. First, excessive borrowing is not necessarily limited to a situation where the firm is inherently judgment proof (i.e., $v - k < \bar{x}$). Even when $\bar{x} < v - k$, so the firm is not inherently judgment proof, it may create “artificial” judgment-proofness by choosing a sufficiently high $K$. In the extreme case with $\theta = 1$, for instance, the firm will completely judgment proof itself and pick zero precaution, even though there is no inherent judgment proofness. Second, it may be that $W(K_0, e_0) < 0$, so the firm’s project may not be socially justifiable. Yet, $U_0(K_0, e_0) > U_0(k, 0) \geq W(k, 0) \geq 0$, so the firm always operates. In other words, strategic judgment proofing may allow a socially unworthy project to be carried out.
5 Public Policy Responses

We consider various remedies to the firm’s judgment proofness/prooﬁng problem. The oft-discussed remedies involve extending liability beyond the injurer to the lenders or seniorizing the bankruptcy status of tort claims. These two remedies share a common purpose of expanding the recovery of damages for the victims from a judgment-proof injurer. Yet, it is not clear how these remedies affect the incentives for the precaution and borrowing. More importantly, it is unclear if either remedy or some other remedy is socially optimal. To this end, we ﬁrst establish more realistic welfare benchmark than the one established before, which will then serve as a welfare upper bound for alternative remedies.

5.1 Welfare Target with Moral Hazard

Suppose the social planner controls all aspects of the ﬁrm’s behavior, except for its precaution decision. Speciﬁcally, the planner chooses the amount of borrowing \( K \geq k \), the terms of ﬁnancial contract, \( r \), for the ﬁrm and imposes a liability of \( ℓ(\cdot) \) to the lender, subject only to the constraints that the lender must break even (i.e., (IR)) and the ﬁrm must have incentive to choose the precaution the planner wishes to implement. Formally, this planner’s problem is

\[
\text{[SW]} \quad \max_{(r,K,e,ℓ)} W(K,e)
\]

subject to

\((r,K,e,ℓ)\) satisﬁes \((IR)\) and \((IC)\).

Although the regulators probably do not have either the information or the power to control the amount of borrowing or the terms of ﬁnancial contracts of ﬁrms, the program [SW] yields a more realistic welfare target than the ﬁrst-best level.

The next proposition characterizes the optimal borrowing and precaution behavior, \((K^*,e^*)\), that the planner would wish to implement.

**Proposition 2** (Constrained efﬁciency) The solution of the problem [SW] involves \( K^* = k \) and \( e^* = e_0(k) \). No liability is imposed on the lenders, \( ℓ(x) = 0 \), and the ﬁnancial contract involves only senior debt, \( r = (k,0) \).
This result shows that the underinvestment in precaution chosen by the unregulated firm is attributed entirely to its excess borrowing. Had the firm borrowed \( K = k \), then the firm would have chosen the (constrained) efficient level of precaution \( e_0(k) \). The reason is the following. The unregulated firm dilutes the tort claims by choosing a senior claim over junior claim and also borrowing beyond the productive use of fund. As seen in Lemma 1, a senior debt in fact improves precaution. Excessive borrowing, however, worsens the precaution incentives. Proposition 2 thus suggests the crucial importance of curbing the overleveraging incentive. We show below that two oft-discussed remedies, subordination and lender liability, can discourage excessive borrowing, but introduce their own problems.

5.2 Subordination

Under this policy regime, the firm is allowed only to use financial claims that are junior in bankruptcy priority to the tort claims. Given Step 1 of Lemma 1\' (see the appendix), the firm would prefer to use only junior debt among all standard junior claims. Given the junior status of the debt, the tort victims have priority, meaning that they will receive up to the level allowed by the cash flow, or \( \min\{v, x\} \). This means that raising the level of borrowing cannot help the firm to avoid tort liability. Subordination controls the over-leveraging problem, with the firm choosing \( K_{sub} = k \).

The firm’s effort choice, \( e_{sub} \), is determined jointly as the repayment rate, \( r_{sub} \), as follows. Given the equilibrium choice \( e_{sub} \), \( r_{sub} \) must be chosen to satisfy the lender’s break-even condition \((IR)\):

\[
\int_0^v \min\{r_{sub}, v - x\} f(x|e_{sub}) dx = k.
\]  
(7)

Given the repayment rate, the effort choice must satisfy the firm \((IC)\), or the associated first-order condition:

\[
\int_0^{v-r_{sub}} F_{e}(x|e_{sub}) dx = e'(e_{sub}).
\]  
(8)

Subordination means that, unless the project is inherently not judgment proof \((\bar{x} \leq v - k)\), the lender must charge \( r_{sub} > k \) to break even. This latter fact means, however, that the firm is more likely to be insolvent relative to the senior debt case, thus leading to too little precaution, i.e., \( e_{sub} < e_0(k) \). Hence, the constrained efficient precaution level, \( e^* = e_0(k) \), is not attainable by subordination. In fact, comparison of (8) with (5) reveals
the following.

**Lemma 2** *Subordination improves effort (relative to unregulated firm behavior) if and only if $K_0 > r_{sub}$, i.e., the firm would engage in sufficient overleveraging absent regulation.*

This lemma leads us to the following conclusion.

**Proposition 3** *Suppose the firm is allowed to employ only (standard) junior claims. Then, the firm never borrows more than $K$, but chooses too little precaution $e_{sub} < e^*$. Subordination improves social welfare (relative to unregulated behavior) if and only if $\theta > \hat{\theta}_{sub}$ for some $\hat{\theta}_{sub} < \hat{\theta} < 1$.*

**Proof:** See the Appendix.

In sum, subordination trades off two sources of precaution incentives. On the one hand, it eliminates overleveraging, which improves the firm’s precaution incentives. But at the same time, the switch from a senior to a junior debt worsens the firm’s incentives. Subordination is thus desirable whenever the former effect outweighs the latter, i.e., when $\theta$ is sufficiently high.

### 5.3 Lender Liability

We next consider lender liability. In this system, the lender bears the entire residual liability for the damages suffered by the tort victims when the firm is unable to compensate them. Since the additional liability imposed on the lender causes him to raise its repayment rate to a point that will allow him to break even, the liability is in fact shifted to the firm. Under this system, the firm thus cannot avoid liability by raising its debt. Hence, the firm would never borrow more than its productive use, i.e., $K_U = K$, just like the case of subordination.

In fact, the residual liability on the part of the lender means that whatever the cash flow the firm generates must be first used to compensate the tort victims, so it effectively makes the lender’s (possibly senior) debt junior. Hence, the effect of lender liability is quite similar to “subordination.” In fact, if the harm never exceeds the cash flow (i.e., $\bar{x} \leq v$), then lender liability coincides with subordination, since, although the tort victims enjoy a
priority over financial claims in bankruptcy proceedings, its compensation is limited to the cash flow. If harm can exceed the cash flow (i.e., \( \overline{x} > v \)), however, the compensation and total liability will exceed the firm’s cash limit, so lender liability will not be the same.

To be more precise, let \((r_{ll}, e_{ll})\) be the equilibrium repayment rate and precaution choice under lender liability. Then, as before, we must have

\[
\int_0^{\overline{x}} \min\{r_{ll}, v-x\} f(x|e_{ll})dx = k, \tag{9}
\]

and

\[
\int_0^{v-r_{ll}} F_x(x|e_{ll})dx = c'(e_{ll}). \tag{10}
\]

If \( \overline{x} \leq v \), then comparison of (7) with (9) reveals \( r_{ll} = r_{sub} \), so \( e_{ll} = e_{sub} \). If \( \overline{x} > v \), however, the extra liability borne by the lender causes him to charge a higher rate, or \( r_{ll} > r_{sub} \), which means that the firm is more likely to be insolvent, and thus will have a lower incentive for precaution, i.e., \( e_{ll} < e_{sub} \).

**Proposition 4** If \( \overline{x} \leq v \), then unlimited lender liability yields the same outcome as subordination. If \( \overline{x} > v \), then unlimited lender liability induces lower precautions than subordination. In either case, the firm never borrows more than \( k \). Lender liability improves social welfare relative to unregulated behavior if and only if \( \theta > \hat{\theta}_{ll} \) for some \( \hat{\theta}_{ll} \in [\hat{\theta}_{sub}, \hat{\theta}) \).

**Proof:** Only the last statement requires a proof, which follows the same line of argument for Proposition 3 and is thus omitted.

Lender liability has a similar benefit as subordination in eliminating the incentive for over-leveraging. But lender liability leads to (weakly) lower incentives for the firm to take precautions. This suggests that subordination weakly dominates the lender liability. As stated in Proposition 3, subordination is not constrained efficient. Nevertheless, much like subordination, lender liability can strictly improve social welfare relative to the unregulated behavior. This possibility contrasts sharply with Pitchford (1995), who found lender liability to serve no value in any circumstances. This highlights the fact that the value of lender liability rests on its role of deterring over-leveraging.

We next propose a liability rule that does attain the constrained efficiency.
5.4 Optimal Liability Scheme: Limited Subordination Rule

Here, we introduce a liability rule that implements the constrained efficient outcome, \((K^*, e^*)\), as defined in Proposition 2. Under the optimal rule, the firm’s financial claims are “subordinated” to tort claims in the event of bankruptcy, but up to a certain limit, \(v - k\). More precisely, this rule would treat the firm’s financial debts as “junior” up to \(v - k\), but would distribute the remaining portion of cash flow, \(k\), according to the standard absolute priority rule. In other words, the tort victims will have the first “crack” at the firm’s asset up to \(v - k\), but the remaining asset would be disposed in the order of seniority of the claims: i.e., the senior debt will have the claim for the remaining asset \(k\), and if there is any thing left, the tort claim will get additional claim, and so on and so forth.

The effect of this rule can be analyzed as follows. First, note that the optimality of a senior debt (i.e., Lemma 1) extends to this rule, so there is no loss in restricting attention to a senior debt. Hence, suppose the firm obtains a senior debt of \(K \geq k\). Then, the equilibrium repayment rate \(\hat{r}(K)\) and precaution \(\hat{e}(K)\) is determined as follows. First, the repayment rate \(\hat{r}(K)\) must be no less than \(K\). Hence, under this rule, the lender will receive

\[
\begin{cases} 
  \min\{\hat{r}(K), v-x\} & \text{if } x \leq k, \\
  k & \text{otherwise.}
\end{cases}
\]

Given that the lender anticipates the firm to choose \(\hat{e}(K)\), the would break even if

\[
\hat{r}(K)F(v - \hat{r}(K)|\hat{e}(K)) + \int_{\hat{r}(K)}^{v-k} (v-x)f(x|\hat{e}(K))dx + k(1 - F(v - k|\hat{e}(K))) = K. \quad (11)
\]

Meanwhile, the firm’s incentive compatibility requires

\[
\int_{0}^{v-\hat{r}(K)} F_e(x|\hat{e}(K))dx - c'(\hat{e}(K)) = 0. \quad (12)
\]

Observe from (11) that \(\hat{r}(k) = k\). That is, if the firm borrows the productive requirement, \(k\), it does not bear any additional liability, so the repayment rate of \(k\) will break even. Substituting \(\hat{r}(k) = k\) into (12) shows that the firm’s precaution choice will be constrained efficient, i.e., \(\hat{e}(k) = e^*\).

It therefore remains to determine if the firm would have the incentive to borrow no more than \(k\). Suppose to the contrary that the firm borrows \(K > k\). Then, the firm’s ex
ante payoff will be (with integration by parts)
\[ \hat{U}(K) := \phi(K - k; \theta) + \int_{0}^{v - \hat{r}(K)} F(x|\hat{e}(K))dx - c(\hat{e}(K)). \]

Differentiate this with respect to \( K \), using the envelope theorem, to obtain:
\[ \hat{U}'(K) = \phi'(K - k; \theta) - F(v - \hat{r}(K)|\hat{e}(K))\hat{r}'(K). \] (13)

Next, differentiate totally (11) to obtain
\[ F(v - \hat{r}(K)|\hat{e}(K))\hat{r}'(K) + \left[ \int_{v - \hat{r}(K)}^{v - k} F_{\hat{e}}(x|\hat{e}(K))dx \right] \hat{e}'(K) = 1. \] (14)

Substituting (14) into (13) gives
\[ \hat{U}'(K) = \phi'(K - k; \theta) - 1 + \left[ \int_{v - \hat{r}(K)}^{v - k} F_{\hat{e}}(x|\hat{e}(K))dx \right] \hat{e}'(K) < 0, \]
where the last inequality holds since \( \phi'(\cdot; \theta) \leq 1 \), \( F_{\hat{e}} > 0 \) and \( e'(K) \leq 0 \). The following conclusion is then immediate.

**Proposition 5** The limited subordination rule with limit at \( v - k \) implements the constrained efficient outcome \((K, e) = (k, e^*)\).

### 6 Discussion

We conclude by discussing robustness of our findings as well as other remedies of judgment proofness.

#### 6.1 Robustness of Findings

- **Uncertain Cash Flows and Capital Requirements:**

  Thus far, we have assumed that the cash flow, \( v \), is deterministic and the productive requirement, \( k \), is known. These assumptions, made primarily for simplicity, may not hold in reality. Our results are largely robust to relaxing these assumptions, however. Suppose the cash flow \( v \) is a random variable, distributed according to some cdf \( G(\cdot) \)
over \([0, \infty)\). Then, the choice of senior claims is no longer limited to senior debt, for the repayment can vary with the realized level of cash flow. Innes (1990) showed that debt is preferable to all other “monotonic” financial claims (which as noted earlier include all plausible financial claims) in a model without tort victims.\(^{30}\) By analogy, there would be little loss in restricting attention to senior debt here as well.

More importantly, the firm’s preference for senior debt over junior claims and its tendency for over-leveraging remain unchanged in this case, since the “judgment proofing” benefits that these practices offer do not depend on the stochastic nature of the cash flow. Hence, the firm will choose senior secured debt and borrow in excess of its productive use. Some of the remedies to this problem – namely subordination and lender liability – will lead to the same tradeoffs as discussed before. The optimality of the limited subordination rule would extend to this new environment, except that the scope of the “junior treatment,” \(v - k\), would be random instead of deterministic.

Likewise, the capital requirement \(k\) need not be perfectly observable by the policy maker — a sufficiently accurate assessment of \(k\) could replace it without much loss. The important feature in deterring over-leveraging is not the accuracy of the capital requirement but the *exogeneity* of the scope of “senior treatment,” for the scope of senior treatment not to depend on the actual amount of borrowing. The actual implementation of the rule may thus mimic “prudency” test of capital reimbursement of utilities: Often the state public utilities commissions reimburse the capital expenditures by utilities based on the amount they *ought to have* spent rather than the amount they *actually* spent.\(^{31}\)

**Richer Contracting Possibilities and Lender Monitoring:**

We have considered a broad set of financing contracts that encompass most of the commonly observed financing arrangements. It is of (at least theoretical) interest to consider

\(^{30}\)In Innes (1990), higher level of effort increase the value of the firm’s assets directly, benefitting outside investors. Here, higher effort reduces the harm to tort victims. This can also benefit outside junior lenders or senior lenders who may be held liable.

\(^{31}\)Similarly, several scholars have suggested that the ideal remedy for breach of contract, “efficient expectation damages,” should not take into account the actual reliance of the breached-against party. Much like the prudency test, this remedy compensates the victim of breach only for the portion of the actual reliance by the victim that is justifiable from the social efficiency perspective. See Cooter and Eisenberg (1985), Craswell (1989), Spier and Whinston (1995) and Chung (1992).
even richer contracting possibilities. For example, we can imagine junior claims that do not satisfy the monotonicity properties assumed in $\mathcal{R}$, or senior claims whose payment requirements depend on realized harm $x$, or the investor may be able to observe some informative signal of the firm’s effort. While contracts outside $\mathcal{R}$ are not common in reality, they are at least theoretically interesting since often such contracts may dominate the ones in $\mathcal{R}$ in performance.\textsuperscript{32}

The model could also be extended to include the possibility of monitoring by the investor. Indeed, holding a principal liable for the misdeeds of an agent may be warranted when the principal has the ability to prevent the agent from engaging in a harmful act. Conversely, if the principal cannot observe or otherwise control the agent’s tort-causing behavior, then holding the principal liable will have no beneficial effect of preventing the harmful act.\textsuperscript{33} In fact, the negative view of Pitchford (1995) on lender liability can be seen as confirming this point, as recognized by Balkenborg (2001). Their views, however, were based on models where the injurer could not influence the degree of judgment proofness.

As shown in our paper, with the injurer is able to judgment proof herself, the investor’s monitoring ability is not necessary for lender liability to be socially valuable. Even when the principal lacks the ability to monitor the agent’s behavior, holding the principal liable for the harm caused by her agent can have a desirable effect on the contract signed with the agent. In our model, lender liability or the subordination of her claim causes the interest charges against the firm to rise, which not only deters the firm’s judgment proofing but also may improve its incentive for precaution.

In fact, investor monitoring and richer contracting possibilities will likely strengthen the main tenet of our paper, as they will worsen (rather than alleviate) the judgment proofing problem. Without any regulation, the lender’s improved ability to control the firm’s behavior via sophisticated contracting or monitoring will simply enable them to promote their joint interest more effectively. And judgment proofing does promote their

\textsuperscript{32}Innes (1990) shows that a financial claim which charges high repayment when the cash flow is low and a lower repayment when the cash flow is high does better than a debt. See Lewis and Sappington (2001) for a similar point.

\textsuperscript{33}See, however, Hay and Spier (forthcoming) where holding a manufacturer liable for the injuries caused by consumers while using the risky products reduces the level of the risky activity. For example, holding gun manufacturers liable for the deaths and injuries – including those caused by crimes – leads to higher gun prices and fewer guns sold.
joint interest at the expense of tort victims. Hence, the improved contracting ability will lead to more judgment proofing, not less.

The policy interventions discussed above — subordination, lender liability, and limited subordination — all have the property of deterring the firm's judgment proofing, thus forcing the firm to internalize the social harm it causes. This latter feature will prove more useful in the face of improved contracting between the lender and the firm. An improved contract between the two parties enables them to implement the level of precaution in their best joint interest, and the above policies align their joint interest with the social welfare. In the extreme case, if the lender can monitor the firm's precaution accurately, the agency problem between the two parties disappears. In this case, lender liability will yield the first-best outcome: The firm will borrow $K = k$ with a contract that punishes the firm whenever it does not pick the first-best effort. The parties will have the incentive to sign such a contract since lender liability will force the firm to internalize the entire social harm.

### 6.2 Other Remedies

In the text, we focused on three remedies of judgment proofness and strategic judgment proofing. These three remedies — debt subordination, lender liability, and limited subordination — all focused on the capital market. We will now discuss two additional remedies — mandatory liability insurance and punitive damages.

- **Mandatory Liability Insurance**

  Mandating that the injurers purchase liability insurance is a simple way of ensuring the tort victims’ recovery of their court awards. Given adequate coverage of liability insurance, victims would receive their court judgments despite the injurers’ insolvency and the horror stories with taxi accidents told in the introduction would be avoided. Will the insurance create the right incentives for precautions, though? As Shavell (2004) observed, the answer depends critically on the monitoring ability by the insurers. If a full-insurance provider can monitor and control the injurer's precaution level (say by conditioning its payout or insurance premium on this level), the provider will require the firm to choose a socially efficient level of precaution.

  In practice, however, insurance providers are unlikely to possess fully effective moni-
toring capabilities. For instance, in the context of the taxi accidents, precautions would take the form of a taxi company’s screening for careful drivers in hiring and its training of drivers, as well as the monitoring of their driving practices, all of which requires an intimate involvement with the management of the business. It is unlikely that insurance providers’ monitoring and influence would have such a reach.

Without monitoring, liability insurance would have a damaging, rather than beneficial, effect on injurers’ precautions. To see this most clearly, suppose \( \theta > \hat{\theta} \) so that the firm would borrow up to its cash flow, \( v \), absent any regulation. If the liability insurance has a partial coverage, and thus exposes part of the firm’s assets to tort liability, the firm would still borrow excessively to completely judgment proof itself.\(^3\) If the liability insurance has complete coverage, then there is no need for further judgment proofing. Either way, the firm has no incentive to invest in precautions, for it is completely shielded from ex post liability. In other words, the standard moral hazard problem associated with insurance compounds the judgment proofing problem, which aggravates the incentives. Even in the case where \( \theta < \hat{\theta} \), liability insurance is likely to generate even worse incentives than the three remedies discussed earlier, and will often worsen the firm’s unregulated behavior.

- **Damage Inflation: Punitive Damages**

The merits of punitive damages have been widely debated among legal scholars (see Polinsky and Shavell (1998) for a survey). Punitive damages are most compelling when compensatory damages, for various reasons, leaves victims undercompensated so that they do not provide adequate deterrence incentives to injurers.\(^3\) Since judgment proofness typically leaves victims undercompensated, punitive damages may be one possible way to hold the judgment proof defendant accountable. While inflating damages can do little to extract payment from a bankrupt injurer, it can raise payment when the injurer is not bankrupt. Hence, it will increase the injurer’s *ex ante* expected payments.

One may think that punitive damages or other ways of inflating damages may be a superior alternative to the policy remedies discussed above. This is not the case, however.

\(^3\)The idea here is that the firm can continue to borrow after signing the insurance contract, and that the insurance company cannot prevent this behavior.

\(^3\)Enforcement errors and exclusion of non-monetary harm can be reasons that victims may not recover full compensation from compensatory damage awards. See Polinsky and Shavell (1998).
First of all, even without judgment proofing strategies, damage inflation has a dubious effect on incentives in the presence of judgment proofness. Note that damage inflation does not affect the injurer’s payment when she is insolvent but is likely to increase her payment when she is solvent. Hence, damage inflation may well reduce the injurer’s incentives for precaution.

To illustrate, recall the example in Section 2, except that now the social harm is either $10 or $1,000, with probability .9 and .1, respectively, if the injurer makes an effort (that costs $18), and with probability .8 and .2, respectively, if the injurer does not make an effort. Assume further the injurer never borrows more than his productive requirement $300. Suppose there is no damage inflation. Given senior debt of $300, the injurer will enjoy the net payoff of $190 in the low damage state (cash flow of $500 less loan repayment of $300 and damage payment of $10) and zero in the high damage state. Hence, the expected gain from the effort is $19, the 10% increase in the probability that the firm receives $190. Since $19 exceeds the cost of effort, $18, the injurer will make the effort, given no damage inflation. Suppose now that damages are inflated to three times its compensatory damages (e.g., a compensatory damages plus 200% punitive damages). Then, the injurer’s payoff in the low damage state reduces to $170 whereas her payoff in the high damage state remains zero. Now, the expected gains from effort is $17, falling short of its cost, so the injurer would make no effort. Although the effect on incentives are less clear cut if there are more than two damage states, the negative effect on incentives remains important.36

Second, inflating damages creates more temptation for the firm to resort to judgment proofing. Inflated damages mean that the injurer has more to lose in the solvent state, thus motivating her to shield her asset by judgment proofing. (In this sense, the motive for judgment proofing is slightly different from the one highlighted earlier — i.e., the rent shifting in the insolvent state.) This point can be illustrated with a the slightly modified version of the above example: the cost of effort is now $16 (instead of $18) with everything else remaining the same. In this case, the injurer will make the effort even with the treble damages, if she borrows $300. Suppose, however, that the firm can borrow up to $500, with the excess debt generating value to the firm of only $\phi(500 - 300) = 140$, a $60$ dead-weight loss.

36See Boyd and Ingberman (1994) for a more general treatment. They do not consider the effect on strategic judgment proofing, though, which is discussed below.
Without damage inflation, if the injurer borrows $300 and makes the effort, then her expected payoff will be \((.9) \times 190 - 16 = 155 > 140\), so she will not overleverage and make the effort, which is socially desirable. Consider next the treble damages. If she borrows $300 and makes the effort (which dominates borrowing $300 and making no effort), then her expected payoff will be \((.9) \times 170 - 16 = 137 < 140\). Hence, the injurer will in this case choose judgment proofing and no effort. Inflating damages does not seem useful in the context of judgment proofness and judgment proofing.
7 Appendix

We prove a more general version of Lemma 1.

**Lemma 1’**: Fix any $\ell(\cdot) \geq 0$. For any feasible $(K, r, e)$, there exists a feasible $(\hat{K}, \hat{r}, \hat{e})$, with $\hat{r}$ containing only a senior debt and $\hat{e} \geq e$, such that the firm prefers the latter to the former.

**Proof of Lemma 1’**: The proof of the first statement consists of two steps:

**Step 1**: For any feasible $(K, r, e)$ with nondebt structure there exists a feasible $(K, \hat{r}, \hat{e})$ with all-debt structure $\hat{r}$ and $\hat{e} \geq e$, which the firm prefers over $(r, e)$.

Proof. Fix any feasible $(K, r, e)$ with nondebt structure (i.e., $\rho \neq 0$, $\rho \in \mathcal{R}$). We consider an all-debt structure $\hat{r} := (\hat{r}_S, \hat{r}_J)$, where $\hat{r}_J$ is chosen so that, so that

$$\Pi(\hat{r}, e, \ell) = \Pi(r, e, \ell),$$

$$\Leftrightarrow \int_0^{v-r_S-\hat{r}_J} \min\{\hat{r}_J, v - r_S - x\} dF(x | e) = \int_0^{v-r_S-x} \rho(v - r_S - x) dF(x | e).$$

Since $\rho \in \mathcal{R}$, $\hat{r}_J$ exists (recall the properties of $\mathcal{R}$). Further, there exists $\hat{x} \in [0, v - r_S]$ such that $\min\{\hat{r}_J, v - r_S - x\} \leq \rho(v - r_S - x)$ if $x \leq \hat{x}$ and $\min\{\hat{r}_J, v - r_S - x\} \geq \rho(v - r_S - x)$ if $x \geq \hat{x}$, (which again follows from the fact that $\rho \in \mathcal{R}$).

For any $e' < e$,

$$U(K, \hat{r}, e') - U(K, r, e')$$

$$= \int_0^{v-r_S} [\rho(v - r_S - x) - \min\{\hat{r}_J, v - r_S - x\}] f(x | e') dx$$

$$= \int_0^{\hat{x}} [\rho(v - r_S - x) - \min\{\hat{r}_J, v - r_S - x\}] f(x | e') dx$$

$$+ \int_{\hat{x}}^{v-r_S} [\rho(v - r_S - x) - \min\{\hat{r}_J, v - r_S - x\}] f(x | e') dx$$

$$= \int_0^{\hat{x}} [\rho(v - r_S - x) - \min\{\hat{r}_J, v - r_S - x\}] f(x | e) \left( \frac{f(x | e')}{f(x | e)} \right) dx$$

$$+ \int_{\hat{x}}^{v-r_S} [\rho(v - r_S - x) - \min\{\hat{r}_J, v - r_S - x\}] f(x | e) \left( \frac{f(x | e')}{f(x | e)} \right) dx$$

$$\leq \int_0^{\hat{x}} [\rho(v - r_S - x) - \min\{\hat{r}_J, v - r_S - x\}] f(x | e) \left( \frac{f(\hat{x} | e')}{f(\hat{x} | e)} \right) dx$$

$$+ \int_{\hat{x}}^{v-r_S} [\rho(v - r_S - x) - \min\{\hat{r}_J, v - r_S - x\}] f(x | e) \left( \frac{f(\hat{x} | e')}{f(\hat{x} | e)} \right) dx$$
The lone inequality follows from (MLRP), and the last equality follows from (15).

Since, by (15),

\[ U(K, \hat{r}, e) - U(K, r, e) = \int_0^{v-r_S} [\rho(v - r_S - x) - \min\{\hat{r}_J, v - r_S - x\}] f(x|e)dx = 0, \quad (17) \]

we have, for any \( e' < e \),

\[ U(K, \hat{r}, e) - U(K, \hat{r}, e') \geq U(K, r, e) - U(K, r, e') \geq 0, \quad (18) \]

where the first inequality follows from (16) and the second follows from the fact that \((K, r, e)\) satisfies \((IC)\). It follows from (18) that there exists \( \hat{e} \geq e \) such that \( \hat{e} \in \arg\max_{e \in \mathbb{R}} U(K, \hat{r}, \hat{e}) \). That is, \((K, \hat{r}, \hat{e})\) satisfies \((IC)\).

Observe next that

\[ \Pi(\hat{r}, \hat{e}; \ell) = \int_{\mathcal{X}} \pi(x, \hat{r}, \ell) f(x|\hat{e})dx \geq \int_{\mathcal{X}} \pi(x, \hat{r}, \ell) f(x|e)dx = \Pi(\hat{r}, e; \ell) = \Pi(r, e; \ell) \geq K, \]

where the first inequality holds since \( \pi \) is nonincreasing in \( x \) and \( f \) has (MLRP) in \((-x, e)\), the second equality follows from the construction of \( \hat{r} \), the third equality follows from (15), and the last inequality holds since \((K, r, e)\) satisfies \((IR)\). We thus conclude that \((K, \hat{r}, \hat{e})\) satisfies \((IR)\).

Thus far, we have shown that \((K, \hat{r}, \hat{e})\) is feasible. It therefore remains to show that the firm (weakly) prefers \((K, \hat{r}, \hat{e})\) to \((K, r, e)\), which holds since

\[ U(K, \hat{r}, \hat{e}) \geq U(K, \hat{r}, e) = U(K, r, e), \]

where the first inequality follows from the fact that \((K, \hat{r}, \hat{e})\) satisfies \((IC)\), and the equality follows from (17).

**Step 2:** For any feasible \((K, r, e)\) with all debt structure and \( r_J > 0 \), there exists a feasible \((\hat{r}, \hat{e})\), with senior-debt-only structure with \( \hat{r}_J = 0 \) and \( \hat{e} \geq e \), which the firm prefers over \((r, e)\).
Proof. Consider first a senior debt only structure $r' = (r'_S, 0)$ with $r'_S = r_S + r_J$. Observe for each $\hat{e} \in \mathbb{R}_+$,

$$U(K, \hat{e}, r') = \phi(K - k) + \int_0^{v-r'_S} (v - r'_S - x) f(x|e) dx - c(\hat{e})$$

$$= \phi(K - k) + \int_0^{v-r_S-r_J} (v - r_S - r_J - x) f(x|e) dx - c(\hat{e})$$

$$= U(K, \hat{e}, r),$$

so $(r', e)$ satisfies $(IC)$. Further,

$$\pi(x; r', \ell) = r'_S - \ell(x) = r_S + r_J - \ell(x) \geq r_S + \min\{r_J, z\} - \ell(x) = \pi(x; r, \ell).$$

Hence,

$$\Pi(r', e; \ell) = \int_{\mathcal{X}} \pi(x; r', \ell) f(x|e) dx \geq \int_{\mathcal{X}} \pi(x; r, \ell) f(x|e) dx = \Pi(r, e; \ell) \geq K, \quad (19)$$

proving that $(r', e)$ satisfies $(IR)$, and is thus feasible.

Since $\Pi(r, e; \ell)$ is continuous and strictly increasing in $r$, there exists a senior-debt only structure $\hat{r} = (\hat{r}_S, 0)$ with $\hat{r}_S \leq r_S + r_J$ such that

$$\Pi(\hat{r}, \hat{e}; \ell) = \Pi(r, e; \ell). \quad (20)$$

Consider any $e' \in \mathbb{R}_+$. Then,

$$U(K, \hat{r}, e') - U(K, r, e')$$

$$= \int_0^{v-r'_S} (v - \hat{r}_S - x) f(x|e') dx - \int_0^{v-r_S-r_J} (v - r_S - r_J - x) f(x|e') dx$$

$$= \int_0^{v-r'_S} F(x|e') dx - \int_0^{v-r_S-r_J} F(x|e') dx$$

$$= \int_{v-r_S-r_J}^{v-r'_S} F(x|e') dx \geq 0. \quad (21)$$

Furthermore, the last line is nondecreasing in $e'$, which implies that there exists $\hat{e} \geq e$ such that

$$\hat{e} \in \arg\max_{e' \in \mathbb{R}_+} U(K, e', \hat{r}). \quad (22)$$

Hence, $(\hat{r}, \hat{e})$ satisfies $(IC)$. It also satisfies $(IR)$, since

$$\Pi(\hat{r}, \hat{e}; \ell) \geq \Pi(\hat{r}, e; \ell) = \Pi(r, e; \ell) \geq K,$$

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where the first inequality follows since Π is nondecreasing in \( e \), the first equality follows from (25), and the second inequality follows from \((r, e)\) being feasible.

Since \((\hat{r}, \hat{e})\) is feasible, it suffices to show that the firm prefers \((\hat{r}, \hat{e})\) to \((r, e)\), which follows since

\[
U(K, \hat{r}, \hat{e}) \geq U(K, \hat{r}, e) \geq U(K, r, e),
\]

where the first inequality follows from (22), and the second follows from (21).

If the tort system is not fully compensating (i.e., \( v - r_S - r_J + \ell(x) < x \) for a positive measure of \( x \)), then at least one inequality of (24) is strict, which makes \( \hat{r}_S < r_S + r_J \). This in turn makes the second inequality of (23) strict. In other words, the firm strictly prefers \((\hat{r}, \hat{e})\) over \((r, e)\). By contrast, if the tort system is fully compensating (i.e., \( v - r_S - r_J + \ell(x) \geq x \) for all \( x \)), then the inequalities of (24) become equalities, in which case the firm is indifferent between the two contracts. □

**Proof of Proposition 2:** Lemma 1’ implies that the social planner would choose the senior-debt-only structure (i.e., with \( r_J = 0 \)). (The social planner would prefer to choose a structure that induces the highest precaution from the firm.)

We next show that the social planner would choose \( \ell(\cdot) = 0 \). To see this, fix any \((r, K, e, \ell)\) that satisfies \((IC)\) and \((IR)\), where \( K \in [k, v] \), \( r = (r_S, 0) \) and \( \ell(\cdot) \geq 0 \). We show that there exists \((\hat{r}, K, \hat{e}, \hat{\ell})\), with \( \hat{\ell}(\cdot) = 0 \) and \( \hat{e} \geq e \), satisfying \((IC)\) and \((IR)\).

To this end, consider first \((r, K, e, 0)\). Since this gives exactly the same payoff to the firm, it satisfies \((IC)\). Further,

\[
\pi(x; r, 0) = r_S \geq r_S - \ell(x) = \pi(x; r, \ell).
\]

Hence,

\[
\Pi(r, e; 0) \geq \Pi(r, e; \ell) \geq K,
\]

so \((r, K, e, 0)\) satisfies \((IR)\).

Hence as before, there exists \( \hat{r} = (\hat{r}_S, 0) \) with \( \hat{r}_S \leq r_S \) such that

\[
\Pi(\hat{r}, e; 0) = \Pi(r, e; \ell).
\]

Since \( \hat{r}_S \leq r_S \), the same argument as in Step 2 of Lemma 1’ proves that there exists \( \hat{e} \geq e \) such that \((\hat{r}, K, \hat{e}, 0)\) satisfies \((IR)\) and \((IC)\). Consequently, it is optimal for the social planner to choose \( \ell = 0 \).
Since the social planner chooses senior-debt only structure and imposes no liability to the lender, the social planner’s choice coincides with that of the unregulated firm, except $K$. In other words, $e_0(K)$ is precisely the precaution level the social planner induces with the choice of $K \geq k$. Hence, the social welfare level associated with $K \geq k$ is $W(K, e_0(K))$. It is straightforward to check that $W(K, e_0(K))$ is nonincreasing in $K$ for $K \geq k$. Hence, we conclude that $K^* = k$ and $e^* = e_0(k)$.

Proof of Proposition 3: That $e_{\text{sub}} < e^*$ is clear from the fact that $r_{\text{sub}} > k$. To prove the last statement, recall that $U_0$ is supermodular in $(K, -e)$ and satisfies strict single crossing property of with respect to $(K, -e; \theta)$. Hence, $K_0(\cdot)$ is nondecreasing. In particular, the first order condition (6) implies that $K_0(\theta)$ is strictly increasing in $\theta$ whenever $K_0(\theta)$. Hence, there exists a unique $\hat{\theta}_{\text{sub}}$ such that $K_0(\hat{\theta}) > r_{\text{sub}}$ if and only if $\theta > \hat{\theta}_{\text{sub}}$. It follows from the assumption $W(k, 0) \geq 0$ that $r_{\text{sub}} < v$. Since $K_0(\hat{\theta}) = v > r_{\text{sub}}$, $\hat{\theta}_{\text{sub}} < \hat{\theta}$. The statement then follows from Lemma 2.
References


Figure 1

Figure 1.1: Junior Debt

Figure 1.2: Senior Debt