Efficient Fiscal Policy and Amplification*

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Abstract

We provide a rationale for the observed pro-cyclicality of tax policies in emerging markets and present a novel mechanism through which tax policy amplifies the business cycle. Our explanation relies on two features of emerging markets: limited access to financial markets and limited commitment to tax policy. We present a small open economy model with capital where a government maximizes the utility of a working population that has no access to financial markets and is subject to endowment shocks. The government’s insurance motive generates pro-cyclical taxes on capital income. If the government could commit, this policy is not distortionary. However, we show that if the government lacks the ability to commit, the best fiscal policy available exacerbates the economic cycle by distorting investment during recessions. We characterize the mechanism through which limited commitment generates cycles in investment in an environment where under commitment investment would be constant. We extend our results to standard productivity shocks and to the case where the government has access to intra-period insurance markets. Lastly, we conjecture that our results would hold as well if the government could issue debt subject to borrowing constraints.

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Introduction

Fiscal policy appears to play an important role in exacerbating economic volatility in developing countries. This view is supported by empirical evidence on the procyclicality of fiscal policy in developing countries. Kaminsky, Reinhart and Vegh (2004), Gavin and Perotti (1997), among others, have documented that governments in emerging market economies tend to increase spending and reduce taxes during expansions; and the reverse during contractions. However, the question of why a government would follow a fiscal policy that exacerbates economic volatility remains an open question.

We provide a rationale for pro-cyclical tax policy and amplification in this paper. We study a small open economy model with capital where a government seeks to maximize the utility of a population that has no access to capital markets. We show that if the government lacks the ability to commit to future fiscal policies, the best fiscal policy available exacerbates the economic cycle. In our model, the government’s credibility regarding taxes on capital in the future varies with the current state of the business cycle. If the economy is in a recession, the government has an increased incentive to tax capital in the future. This reduces capital investment during a recession, amplifying and prolonging the downturn.

The main contribution of this paper is to show that a parsimonious model of capital taxation with limited commitment can explain an important element of the fiscal behavior of emerging markets economies. This is done without recourse to political economy considerations. Our model is simple enough to allow for a clear characterization of a plausible mechanism behind the observed procyclicality and amplification.

Our baseline model has an economy with two types of agents: workers and capitalists. The workers are risk averse, supply labor inelastically and do not have access to financial markets. The capitalists are risk neutral. They invest in capital, own the domestic firms and can access financial markets. Our economy is small and open and capitalists can transact at an exogenously determined world interest rate. In addition, there is a government that cares only about the workers and uses linear taxes/subsidies to redistribute income. The government is assumed to run a balanced budget. These assumptions generate a sparse structure that isolates our mechanism. We show that the mechanism remains relevant in richer settings.

The economy is subject to shocks. In particular, we assume that the workers in addition to their net of tax wage income, receive a stochastic endowment every period. Before this endowment shock is realized, capital is invested. The endowment shock generates a risk that the workers cannot insure. The government plays the role of providing insurance to
the workers by using a combination of taxes on capital income and labor income. A useful expositional feature of the setup is that the endowment shock does not affect the marginal product of capital. That is, the first-best capital stock is acyclical. This feature of the model allows us to starkly study the role of fiscal policy in generating investment fluctuations that amplify and prolong the business cycle.

If the government could commit, the optimal fiscal policy (the Ramsey solution) does not distort the capital margin in this economy (similar to Judd (1985), Chamley (1986), and Atkeson et al. (1999)) but does provide insurance to the workers. In the Ramsey solution, the government completely insures the workers against the intra-period uncertainty by taxing capital when the endowment is low and subsidizing it when the endowment is high. However, given that the government lacks a financial asset to smooth consumption intertemporally, consumption in the Ramsey solution will vary over time. Specifically, if the shocks are persistent, consumption will be higher following a high endowment realization.

The Ramsey insurance scheme exploits the fact that capital is perfectly elastic ex ante but inelastic ex post. The first best program therefore imposes taxes on capital that vary across states of nature but have an expected payment of zero (as in Zhu (1992) and Chari et al. (1994)). In the Ramsey solution, investment is constant. Note that to provide insurance, the government drives down the ex post return to capital if the ex post endowment shock is low. We observe this implemented in practice in many ways—higher taxes, failure to pay out nominal promises on contracts, confiscation, etc. Nevertheless, investors ex ante are willing to bear this risk as long as the returns to capital are high if times are good.

What if the government cannot commit to the Ramsey plan? Given that the capital stock is fixed for one period, the government is tempted to tax capital at the highest possible rate and to redistribute the proceeds to the workers. We follow Chari and Kehoe (1990) and use sustainable equilibria as our solution concept.

The best sustainable equilibrium for the government is supported by the threat that any deviation will be punished by reversion to the worst sustainable equilibrium. Specifically, we show that the sustainable equilibrium that delivers the lowest payoff to the government is one where taxes on capital are at their highest possible level for all histories. To be an incentive compatible allocation, the government cannot benefit by deviating from the prescribed allocation taxes when facing as a punishment indefinite reversion to the worst equilibrium. The best sustainable equilibrium can then be characterized by the incentive compatible allocation that maximizes the utility of the government.

The government’s ability to commit to the Ramsey allocation depends on the gains from
deviating from the promised consumption. When shocks are i.i.d. promised consumption is independent of the previous state. This implies that in an i.i.d. world future capital tax promises are independent of the current state. Investment may be suboptimal, but will be constant.

A main result of the paper relates to the case when the endowment shocks instead have persistence. We show that, given a shock today, the government has lower incentives to deviate from the Ramsey policy the higher the endowment shock was yesterday. Consequently, distortions on the capital margin start appearing first after low endowment shocks.

The intuition behind this result is as follows. In a world with persistent shocks, the current state affects the future promises of consumption. In particular, consumption in the Ramsey plan will be higher following a boom as compared to a recession. However, consumption following a deviation from the Ramsey plan is independent of the previous state and so are the continuation values. Consequently, the gains to deviating from the Ramsey plan and taxing capital are greater in any state following a recession. The best sustainable equilibrium allocation would yield to this by prescribing higher expected average taxes following a recession. This reduces the return to investment and therefore capital stocks following a recession will be depressed. The asymmetry in distortions between booms and recessions will result in investment levels that are positively correlated with the business cycle. Note that these cyclical changes in investment are purely the result of the lack of commitment: in the Ramsey solution, investment was constant.

We show that the best fiscal policy under commitment also amplifies the cycle even when the government has access to static insurance (i.e. it cannot borrow or save, but can insure across states period-by-period), as long as financial contracts face the same commitment problems as the tax policy. This highlights the importance of limited commitment in generating the result.

We also consider the more standard case when the shock affecting the economy is a multiplicative productivity shock. The complication that results is that now the first best level of capital will vary with the state as long as the state is persistent, and this affects the incentives to deviate directly (more capital implies more temptation to tax it). The conditions under which distortions first appear in a recession now depend on the shape of the utility function, production function and the extent of persistence. We show that these conditions are likely to hold for empirically relevant specifications. Lastly, we discuss the case when the government has access to a bond. Tax policy continues to be pro-cyclical and we present a conjecture that as long as we have the realistic case when financial access is less than perfect and consumption is higher following a high shock relative to a low shock,
we should obtain distortions and amplification similar to the case with budget balance.

In an important paper in the international business cycle literature, Kehoe and Perri (2001) consider a model of risk sharing across two countries with limited commitment. Differently from Kehoe and Perri (2001), we emphasize the role of the government in generating amplification in emerging markets and derive an analytical characterization of the mechanism behind it. In the literature on fiscal policy, Talvi and Vegh (2000) study a model where accumulation of surpluses by the government is assumed to be costly, and derive an optimal fiscal policy that is procyclical. Alesina and Tabellini (2005) present a political economy model where voters are partially informed about the state of the economy. Politicians who face these partially informed voters behave in a myopic way and procyclical fiscal policy obtains.

The paper is organized as follows. In Section 1 we present the model with endowment shocks and describe the full commitment solution. Section 2 presents the limited commitment results. Section 3 extends the model to static insurance markets and productivity shocks. Section 4 presents a conjecture for the case where the government has access to a bond and Section 5 concludes.

1 Model

Time is discrete and runs to infinity. The economy is composed of a government and two types of agents: workers and capitalists. Workers are risk averse, supply inelastically $l$ units of labor every period, and collect a wage. In addition they receive an endowment shock, $z$, every period that follows a markov process. We let $z \in Z$, and let $z^t = \{z_0, z_1, \ldots z_t\}$ be a history of endowment shocks up to time $t$. Denote by $q(z^t)$ the probability that $z^t$ occurs.

The expected lifetime utility of workers is given by

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t q(z^t) u(c(z^t))$$

where $c(z^t)$ is their consumption in history $z^t$.

**Assumption (Segmented Capital Markets).** Workers have no access to financial markets. Their consumption is given by

$$c(z^t) = z_t + w(z^t) l + T(z^t)$$
where \( T(z^t) \) are transfers received at history \( z^t \).

There is a mass of risk-neutral capitalists that supply capital, but no labor. The capitalists own competitive domestic firms that produce by hiring labor in the domestic labor market and using capital. The production function \( F \) is of the standard neoclassical form:

\[
    y = F(k, l)
\]

\( F \) is constant returns to scale with \( F_{kl} \geq 0 \).

The capitalists have access to financial markets. We assume a small open economy where the capitalists face the exogenous world interest rate of \( r^* \). We assume that capital is installed before the endowment shock and tax rate are realized and cannot be moved until the start of the next period. We assume the depreciation rate is 0.

The government in this economy plays a redistributive role.

**Assumption (Redistributive Government).** The government’s objective function is to maximize the lifetime utility of the workers.

The government taxes capitalists profits at a linear rate \( \tau(z^t) \) and transfers the proceeds to the workers \( T(z^t) \). The government runs a balanced budget at every state

\[
    \tau(z^t) \pi(z^t) = T(z^t)
\]

where \( \pi(z^t) \) are the aggregate profits generated by the firms. Specifically, profits of the firms are

\[
    \pi(z^t) = F(k(z^{t-1}), l) - w(z^t) l
\]

where \( w(z^t) \) is the competitive wage at history \( z^t \).

Given a tax rate plan \( \tau(z^t) \), firms maximize profits taking as given the the tax rate,

\[
    E_0 \sum_i \left( \frac{1}{1 + r^*} \right)^i (1 - \tau(z^t)) \pi(z^t)
\]

Profit maximization by capitalists imply the following two first order conditions:

\[
    F_l(k(z^{t-1}), l) = w(z^t) \quad (1)
\]
\[ r^* = E \left[ \left( 1 - \tau (z^t) \right) | z^{t-1} \right] F_k \left( k \left( z^{t-1} \right), l \right), \]  

(2)

where \( E[.|z^{t-1}] \) indicates expectation conditional on history \( z^{t-1} \) and \( F_i \) denotes the partial derivative of \( F \) with respect to \( i = k, l \).

According to equation (2), the expected return to capitalists from investing in the domestic economy should equal the world interest rate, \( r^* \). Given the additive nature of the endowment shock, optimal capital is a constant in a world without taxes. We now proceed to characterize the optimal fiscal policy under commitment.

1.1 Optimal Taxation under Commitment

Under commitment, the government can commit at time 0 to a tax policy \( \tau(z^t) \) for every possible history of shocks \( z^t \). This plan is announced before the initial capital stock is invested. In the Ramsey problem, the government chooses \( c(z^t), k(z^t) \) and \( \tau(z^t) \) to maximize

\[
\sum_{t=0}^{\infty} \sum_{z^t} \beta^t q(z^t) u \left( c \left( z^t \right) \right)
\]

subject to

\[
c \left( z^t \right) = z_t + w \left( z^t \right) l + T \left( z^t \right)
\]

(3)

\[
\tau \left( z^t \right) \left( F \left( k \left( z^{t-1} \right), l \right) - w \left( z^t \right) l \right) = T \left( z^t \right)
\]

(4)

\[
F_i \left( k \left( z^{t-1} \right), l \right) = w \left( z^t \right)
\]

(5)

\[
r^* = E \left[ \left( 1 - \tau (z^t) \right) | z^{t-1} \right] F_k \left( k \left( z^{t-1} \right), l \right)
\]

(6)

By combining the workers and governments budget constraint and the labor choice condition for firms (equation 5), we obtain

\[
c \left( z^t \right) = z_t + F \left( k \left( z^{t-1} \right), l \right) - \left( 1 - \tau \left( z^t \right) \right) F_k \left( k \left( z^{t-1} \right), l \right) k \left( z^{t-1} \right)
\]

(7)

We have used the constant returns to scale assumption and Euler's theorem, \( F(k, l) = F_k k + F_l l \). Taking expectations of the previous equation and substituting in equation (2) we obtain a single aggregate constraint in expectation,

\[
E \left[ z_t | z^{t-1} \right] + F \left( k \left( z^{t-1} \right), l \right) - E \left[ c \left( z^t \right) | z^{t-1} \right] - r^* k (z^{t-1}) = 0
\]

(8)

The sum of expected endowment and produced output should equal the sum of expected
consumption and payment to capitalists.

The following lemma helps in simplifying the constraint set.

**Lemma 1** For any $c(z^t)$ and $k(z^{t-1})$ that satisfy (8), there exists a function $\tau(z^t)$ such that (7) and (6) are satisfied.

**Proof.** Just define $\tau(z^t)$ as the solution to (7) for given $c(z^t)$ and $k(z^{t-1})$. The fact that (6) holds follows. ■

The problem of the government under commitment is then to maximize

$$
\sum_{t=0}^{\infty} \sum_{z^t} \beta^t q(z^t) u(c(z^t))
$$

subject to (8).

**Proposition 1** Under commitment, the optimal fiscal policy provides full intra-period insurance to the workers:

$$
c(\{z_t, z^{t-1}\}) = c(\{z'_t, z^{t-1}\}) \text{ for all } (z_t, z'_t) \in Z_t \times Z_t \text{ and } z^{t-1} \in Z^{t-1}
$$

and at the beginning of every period, the expected capital tax payments are zero:

$$
E[\tau(z^t) | z^{t-1}] = 0
$$

**Proof.** The Lagrangian of the problem is

$$
\sum_{t=0}^{\infty} \beta^t \left[ \sum_{z^t} q(z^t) u(c(z^t)) + \sum_{z^{t-1}} q(z^{t-1}) \lambda(z^{t-1}) \begin{cases} 
E[z_t | z^{t-1}] + F(k(z^{t-1}), l) \\
-E[c(z^t) | z^{t-1}] - (r^*) k(z^{t-1})
\end{cases} \right]
$$

where $z^{t-1}$ evaluated at $t = 0$ refers to the initial information set. Notice that if $\lambda(z^{t-1})$ is non-negative the Lagrangian is concave on $c, k$. The first order conditions for the maximization of the Lagrangian are

$$
u'(c(z^t)) = \lambda(z^{t-1})$$

$$F_k(k(z^{t-1}), l) = r^*$$
where the first condition implies that \( c(\{z_t, z_{t-1}\}) = c(\{z'_t, z'_{t-1}\}) \) for all \((z_t, z'_t) \in Z_t \times Z_t\) and the second condition implies that \( E[\tau(z^t) | z^{t-1}] = 0 \). 

Proposition 1 shows that the government can insure all the intra-period risk the workers are facing without distorting the investment margin,

\[
F_k (k (z^{t-1}), l) = r^* 
\]

In this purely redistributive model it is efficient to set expected tax payments on capital equal to zero, a result well known in the Ramsey taxation literature (Judd (1985), Chamley (1986) and the stochastic version in Zhu (1992)). Chari, Christiano and Kehoe (1994) obtain a similar result in a business cycle model.

A quick corollary follows,

**Corollary 1** Under commitment, (i) realized capital taxes are countercyclical:

\[
\tau (z_t, z^{t-1}) > \tau (z'_t, z'_{t-1}) \quad \text{for } z_t < z'_t 
\]

(ii) If \( E[z_t | z^{t-1}] \) is increasing in \( z^{t-1} \), then \( \tau (z_t, z^{t-1}) \) is increasing in \( z^{t-1} \).

**Proof.** From (7) it is possible to solve for the tax rate

\[
\tau (z_t, z^{t-1}) = \frac{E[z_t | z^{t-1}] - z_t}{r^* k}
\]

Since \( k \) is independent of \( z_t \) and \( z_{t-1} \), the results follow.

The government will tax capitalists and transfer to workers in recessions and transfer from workers to capitalists in booms, in such a way that the expected tax burden on capital is zero and the workers are fully insured intra-temporally. The government exploits the fact that capital is ex post inelastic to allocate capital income across states so that worker’s consumption is equalized. The ex ante elasticity of capital provides the necessary incentive to keep average tax payments at zero. The results in this section tell us that a government with commitment would not amplify the cycle through its tax policy, even though capital taxes are countercyclical. Investment will be a constant at the optimal level.

We now turn to the important question of what is the best policy a government can implement in the absence of commitment.
2 Optimal Taxation with Limited Commitment

Once the investment decision by the capitalists has been made at the beginning of a period, for any possible realization of the endowment shock, the government would like to tax capital as much as possible and redistribute the proceeds to the workers. Thus, the optimal tax policy under commitment might not be dynamically consistent. As is standard in the literature, we model the economy as a game between the capitalists and the government and use sustainability (Chari and Kehoe 1990) as our solution concept. We characterize the efficient sustainable equilibria of the game and show that investment and hence produced output will display cycles.

We assume the following

**Assumption 1 (A Maximum Tax Rate)** *At any state*, \( z \), *the tax rate on capital cannot be higher than* \( \bar{\tau} \)

Let \( h_{t-1} \) be the history of tax policies and endowment shocks up to the beginning of period \( t \): \( h_{t-1} = \{(s, z_s) | s = 0, \ldots, t - 1\} \) (we do not need to incorportate the capitalists previous investment decisions, see Chari and Kehoe 1990). A government’s policy rule at time \( t \) is a function \( \tau_t(h_{t-1}, z_t) \) that maps previous history into a corresponding tax rate smaller than \( \bar{\tau} \). A capitalist’s investment rule at time \( t \) is a function \( k(h_{t-1}) \) that maps previous history into a corresponding capital level.

A government policy plan is a sequence of policy rules \( \sigma = \{\tau_1, \tau_2, \ldots\} \). A capitalist’s investment plan \( \kappa = \{k_1, k_2, \ldots\} \) is a sequence of investments rules. For any \( (\sigma, \kappa) \) we can compute the associated consumption level of the workers after any history, called the consumption allocation by \( c(\sigma, \kappa) \).

**Definition 1** A *sustainable equilibrium* is a pair \( (\sigma, \kappa) \) such that:

(i) Given a policy plan \( \sigma \) and any history \( h_{t-1} \), the associated investment rule under \( \kappa \), \( k_t(h_{t-1}) \), is the value of \( k \) that solves

\[
q^* = E \left[ (1 - \tau(h_{t-1}, z_t)) F_k(k, l) | z^{t-1} \right]
\]

(ii) Given \( \kappa \), for any history \( (h_{t-1}, z_t) \), the continuation of the policy plan \( \sigma \) maximizes the expected lifetime utility of the workers from \( t \) onwards.

We will focus attention now on a particular sustainable equilibrium.
2.1 The Worst (Markov) Equilibrium

Suppose that the government after any history sets tax rates equal to $\bar{\tau}(z_t)$. Let $\sigma_M$ be the respective plan under such a policy. Suppose that capitalists always believe that they will be taxed at the maximum rate and invest $k_m(z_{t-1})$ where $k_m(z_{t-1})$ solves $r = E[(1 - \bar{\tau}(z_t))F_k(k, l)|z_{t-1}]$. Let $\kappa_M$ be the respective investment plan. The following then holds

**Proposition 2 (Worst Equilibrium)** The pair $(\sigma_M, \kappa_M)$ is a sustainable equilibrium. In particular, of all sustainable equilibria, after any history $h_{t-1}$, $(\sigma_M, \kappa_M)$ generates the lowest utility to the government.

**Proof.** To show that $(\sigma_M, \kappa_M)$ is an equilibrium, note that if the capitalists believe that the government will tax at the maximum rate in the next period, then investing $k_m(z_{t-1})$ is a best response. Note that if after any history $z_t$ and any investment $k(z_{t-1})$, if the government believes that the capitalists will follow the investment plan $\kappa_M$ in the future; then it is optimal for the government to tax at the maximum rate today.

To show that this equilibrium is a lower bound for the government’s utility, note first that in any equilibrium at any possible history we have that $k(z^t) \geq k_m(z_t)$. Given that $F(k, l) - (1 - \bar{\tau})F_k(k, l)$ is increasing in $k$, by taxing at $\bar{\tau}$ the government can guarantee a payoff at least as high as the one generated by $(\sigma_M, \kappa_M)$.

In this Markov equilibrium, clearly, the government will always set the capital tax rate at the maximum possible level. This will generate distortions in capital investment in all states of the world.

Let $V_M(z^{t-1})$ be the payoff to the government at the beginning of period $t$ after a history of shocks $z^{t-1}$ under the equilibrium $(\sigma_M, \kappa_M)$. We can use this function $V_M$ to generate efficient equilibria in a recursive fashion by following Abreu, Pearce and Stachetti (1990). We turn to the characterization of the equilibria in the next subsection.

2.2 The Best Sustainable Equilibria

We can characterize the best equilibrium recursively as follows:

\[ W(z_{t-1}) = \max_{k,c} E \left[ u(c(z_t)) + \beta W(z_t) \mid z_{t-1} \right] \] (10)

11
subject to

\[ E[z_{t}|z_{t-1}] + F(k(z_{t-1}), l) - E[c(z_{t})|z_{t-1}] - r^*k(z_{t-1}) = 0 \] (11)

\[ u(c(z_t)) + \beta W(z_t) \geq u(\bar{c}(z_t, k)) + \beta V^M(z_t) \] (12)

for

\[ \bar{c}(z_t, k) = z_t + F(k, l) - (1 - \bar{\tau})F(k, k) \] (13)

and where \( V^M \) is the value function of the government in the worst equilibrium as previously described.

Equation (11) is the aggregate resource constraint of the government and the inequality (12) is the participation constraint. One problem when trying to characterize the best equilibrium is that the constraint set in the maximization above is not convex. The presence of choice variables on both sides of constraint (12), implies that first order conditions will not be sufficient conditions for the optimum.

However, since the Bellman operator in (10) is monotone, for a numerical implementation we can iterate down to the best equilibrium with the initial guess for the value function being the full commitment value. Subsection 2.5 describes the results of a numerical example. However, before entering into the simulation, it is still possible to provide more information about the optimal equilibrium analytically. We start by proving a Folk theorem.

**Proposition 3** There exists a \( \beta^* \in (0, 1) \) such that for all \( \beta \geq \beta^* \) the Ramsey solution is sustainable and it is not sustainable for \( \beta \in [0, \beta^*) \)

**Proof.** First we show that if for \( \beta_0 \) the Ramsey allocation is sustainable, then it is sustainable for all \( \beta \in [\beta_0, 1] \). Note that the Ramsey allocation is independent of the value of \( \beta \). Note also that the Markov allocation is independent of the value of \( \beta \) as well. Let \( \Omega(z_{t-1}) \) be the Ramsey value minus the Markov value and define by \( c^R \) and \( c^M \) the consumption allocations under the Ramsey and the Markov plan respectively. Then we can represent \( \Omega(z_{t-1}) \) as

\[ \Omega(z_{t-1}, \beta) = W(z_{t-1}) - V(z_{t-1}) \]

\[ = \{u(c^R(z_{t-1})) - E[u(c^M(z_t)|z_{t-1}] \} + \beta E[\Omega(z_t, \beta)|z_{t-1}] \]

Taking derivatives with respect to \( \beta \) we get

\[ \Omega_\beta(z_{t-1}, \beta) = \beta E[\Omega_\beta(z_{t-1}, \beta)|z_{t-1}] + \beta E[\Omega(z_t, \beta)|z_{t-1}] \]
which solves for
\[ \Omega_\beta(z_{t-1}, \beta) = \sum_{z \in Z} a(z|z_{t-1}) E[\Omega(z_t, \beta)|z_{t-1}] \]
for some \( a(z|z_{t-1}) \geq 0 \). Given that \( u(c^R(z_{t-1})) \geq E[u(c^M(z_t)|z_{t-1})]\) this implies \( \Omega(z_t, \beta) \geq 0 \); and this that \( \Omega(z_t, \beta) \) is increasing in \( \beta \). So the participation constraint at the Ramsey allocation
\[ u(c^R(z_{t-1})) - u(c(z_t, k^R(z_{t-1}))) \geq -\beta \Omega(z_t, \beta) \]
is monotonically relaxed as \( \beta \) increases. When \( \beta = 0 \), it is clearly not satisfied for some \( z \). When \( \beta = 1 \), it is clearly satisfied with slackness (the right hand side is minus infinity). So there exists a \( \beta \in (0, 1) \) for which above that \( \beta \) the Ramsey solution is sustainable and below it isn’t. ■

When the government is patient enough, the Ramsey solution is sustainable. As before, this will imply a fiscal policy that does not affect the business cycle. The interesting question is however, what happens when the government is not patient enough to sustain the Ramsey solution, nor impatient enough that the punishment equilibrium is the unique sustainable one.

**Definition 2** Let \( k(z) \) and \( c(z'|z) \) be the respective policy rules that solve the Bellman problem at state \( z \).

The following lemmas help towards an answer.

**Lemma 2** For any given state \( z_{t-1} \) if the participation constraints (12) are not binding for a subset \( Z_o \subset Z \) then \( c(z|z_{t-1}) = c(z'|z_{t-1}) \) for all \( (z, z') \in Z_o \times Z_o \).

**Proof.** For given \( k \) the problem is convex on \( c \). Optimality over \( c \) will yield the result. ■

So, if the participation constraints do not bind tomorrow for two states, the planner will equalize consumption in those states. If consumption is not equalized across two states tomorrow, it is because a participation constraint is binding. We have the following result.

**Lemma 3 (Distorting Capital Down)** The following holds,

(i) for any given state \( z_{t-1} \),
\[ F_k(k(z_{t-1}), l) \geq r^* \]

\[ ^1 \text{Note, that this is after } k \text{ has adjusted to the Markov level.} \]
(ii) if for some \( z, z' \in Z \times Z \) we have that \( c(z|z_{t-1}) \neq c(z'|z_{t-1}) \) then

\[ F_k(k(z_{t-1}), l) > r^* \]

**Proof.** A necessary condition for an optimum is that there exists a \( \lambda(z) \geq 0 \) and \( \gamma \) such that

\[ \gamma \{ F_k(k, l) - r^* \} - \sum_{z_t} \lambda(z_t) u'(\tilde{c}(z_t, k)) \tilde{c}_k(z_t, k) = 0 \] (14)

Another necessary condition for an optimum is that

\[ (q(z_t|z_{t-1}) + \lambda(z_t)) u'(c(z_t)) - \gamma q(z_t|z_{t-1}) = 0 \] (15)

\[ \Leftrightarrow (1 + \lambda(z_t)/q(z_t))u'(c(z_t)) = \gamma \] (16)

This implies that \( \gamma \geq 0 \). Using the definition of \( \tilde{c}_k \) (equation 13), we have that \( \tilde{c}_k > 0 \). Equation (14) then implies (i).

For part (ii), note that if \( c(z_t) \) is not constant for all \( z_t \in Z \) at an optimum (by the hypothesis of the second part of the lemma) then \( \lambda(z) > 0 \) for some \( z \). Given then that \( \lambda(z) \geq 0 \) with strict inequality for at least one \( z \in Z \) we have the proof of (ii). \( \blacksquare \)

Benhabid and Rusticini (1997) have shown that in a deterministic closed economy model of capital taxation without commitment, there are situations where capital is subsidized in the long run, and the steady state level of capital is higher than the first best level. In our case, with an open economy, such a situation never arises. The previous lemma tells us that capital is always distorted downwards (taxed) and capital at any point in time cannot be greater than the first best level. It also says that if consumption is not equalized across states then, in an efficient allocation, capital will be distorted. Now the question is, in which states will capital become distorted? Let us first analyze a simple case, where the endowment shocks are i.i.d.

### 2.3 The Case of i.i.d. Shocks

It is easy to see that if the endowment shocks follow an i.i.d. process, then the value functions \( V^M \) and \( W \) are constants. Then the following result follows

**Proposition 4 (IC binds in high states)** Let the endowment shock follow an i.i.d. process. In an optimal allocation, if an incentive constraint binds for any \( z \in Z \), then it also binds
for any $z' \in Z$ such that $z' > z$

**Proof.** Suppose that an IC constraint is slack for some $z_2$ but it is binding for some $z_1$, where $z_2 > z_1$.

$$u(c(z_1)) = u(\bar{c}(z_1, k)) + \beta(V^M - W)$$

$$u(c(z_2)) > u(\bar{c}(z_2, k)) + \beta(V^M - W)$$

Given that $\bar{c}(z, k)$ increases in $z$, this implies that $c(z_2) > c(z_1)$. Now, create a new allocation by increasing $c(z_1)$ and reducing $c(z_2)$ such that the expected consumption does not change. For small enough change, this is incentive compatible. However, the new allocation attains strictly higher utility than the previous one, which is a contradiction. ■

This is a fairly standard result. In an i.i.d. world, incentive constraints bind in the high states. These are the states where the government is called to subsidize capital and what it really desires to do is to increase the transfer to workers. However, as will be explained below, this intuition is incomplete when analyzing the model in an economy with persistent shocks. In an i.i.d. world the future capital tax promises are independent of the current states and hence the current state should not affect next period taxes nor current period investment. However, in a world with persistent shocks, the current state does affect the future promises of taxation, and will affect the level of investment. This is where our attention turns to next.

### 2.4 Persistent Shocks and Amplification

With i.i.d. shocks, the current state of the economy did not affect next periods promises of taxation, nor next periods expected endowment shocks.

In a world where the current endowment shocks are signals about the distribution of endowment shocks tomorrow, the promises of taxation will be functions of the current state. Whether the economy is in a boom or recession, this will affect the expected future state of the economy and affect the tax promises the government will have to make to achieve full static insurance and maintain an efficient level of investment. How do these promises change over the cycles? Is it harder for a government to make promises of not taxing capital in good times or in bad times? How would this affect the business cycle?

We now make the following assumption that holds for the remainder of the paper.
Assumption 2 (Persistent Shocks) The endowment shocks are such that $E(z_{t-1})$ is strictly increasing in $z_{t-1}$.

Our main result will state that for any $z_t$, the incentive constraint is more likely to bind at time $t$, if the state of the economy was low at $(t-1)$. A low state today thus signals tighter incentive constraints tomorrow and will imply distortions on the investment margin during bad times.

Consider the commitment solution. Consumption under full commitment can be written as:

$$c^*(z_t|z_{t-1}) = E(z_t|z_{t-1}) + F(k^*, l) - rk^*$$

where $k^*$ is such that $F_k(k^*(z), l) = r$.

As stated before, consumption at time $(t)$ under commitment is independent of the realization of the endowment shock, $z_t$ (perfect intra-period insurance).

Autarkic consumption similarly can be written as

$$\bar{c}(z_t, k^*) = z_t + F(k^*, l) - (1 - \bar{r})F_k(k^*, l)k^*$$

Define

$$\Delta(z_{t-1}, z_t) = u(c^*(z_t|z_{t-1})) - u(\bar{c}(z_t, k^*))$$

Under the assumption that the first best is implementable, the incentive constraints can be written as

$$\Delta(z_{t-1}, z_t) \geq \beta(V(z_t) - W(z_t))$$

If $\Delta(z_{t-1}, z_t)$ is increasing in $z_{t-1}$ then as $\beta$ decreases, incentive constraints bind first in states where the previous endowment shock was low. This is formalized below.

Proposition 5 (Distortion in Bad States) Suppose that $\Delta(z_{t-1}, z_t)$ is increasing in $z_{t-1}$ for all $z_t$. Then in an optimal allocation if $k(z) = k^*$ for some $z \in Z$ then $k(z') = k^*$ for all $z' > z$.

Proof. The fact that $k(z) = k^*$ implies that the first best capital level is attained immediately after a $z$ shock. We know from lemma (2) that consumption the period after a $z$ shock will be constant and equal to $c^*$. So, it is the case then that

$$\Delta(z, \hat{z}) \geq \beta(V(\hat{z}) - W(\hat{z}))$$
for all $\tilde{z} \in Z$. Given the monotonicity condition this implies that

$$\Delta (z', \tilde{z}) \geq \beta (V(\tilde{z}) - W(\tilde{z}))$$

for all $z' > z$. So, first best capital is attained also after a $z'$ shock and $k(z') = k^*(z')$. ■

When is $\Delta (z_{t-1}, z_t)$ increasing in $z_{t-1}$? The following result follows directly from the definition of $\Delta$ and $c^*$.

**Proposition 6 (Persistence)** $\Delta (z_{t-1}, z_t)$ will increase in $z_{t-1}$ if and only if $E(z_t|z_{t-1})$ is increasing in $z_{t-1}$.

The intuition behind the result in the propositions is as follows. If shocks have positive persistence, consumption in the Ramsey plan will be higher following a higher endowment shock. Thus, the gains to deviating and taxing capital at the maximum possible rate $\hat{r}$ will be greater in any state following a recession. Consequently, the government is less able to commit to first best taxes and capitalists expect average taxes to be positive. This reduces the return from investing in capital and therefore capital stocks in a recession will be distorted down. Since distortions first appear in recessions, capital in a boom can be undistorted and at the first best level, while the capital stock in a recession will be strictly less than the first best capital stock. This mechanism generates cycles in investment and therefore cycles in produced output, as now the capital stock $k(z)$ will vary with the underlying shock $z$.

We now turn our attention to a numerical analysis that illustrates these results.

### 2.5 Numerical Example

In this subsection we present a simple numerical example of the best sustainable equilibrium that illustrate the amplification generated by the efficient fiscal policy. We consider two discrete values for $z$: $z_H$ and $z_L$. To solve the problem numerically, we iterate on (10), where the initial guess $W^0(z)$, is the value function for the case with full commitment. Since the value in the case with full commitment will necessarily be at least as great as the value with limited commitment, and since the bellman operator is monotone, starting with $W^0(z)$ we should converge monotonically down to the maximized value with limited commitment. Given the continuation value $W^0(z)$, the government chooses tax rates as functions of today’s shock and the previous one: $t(z_t|z_{t-1})$, such that it maximizes (10) subject to (11) and (12).
This generates a new value $W^1(z)$. We repeat this procedure until $|W^{i+1} - W^i| < \varepsilon$, where $\varepsilon$ is a small number.

Table 1 in Appendix B lists the values for the numerical example. Table 2 compares the taxes under full and limited commitment. In this example, the first best is attained following a high state, while the expected tax rates are strictly positive following a low state. Capital is distorted in the low states and un-distorted in the high states. That is, now capital is cyclical.

Figure 1 relates the ratio of the capital stock following a high state to that following a low state to the discount factor, $\beta$. As $\beta$ converges to 1, the first best level of capital stock is attained regardless of the state. The ratio accordingly is 1. At the other extreme, when $\beta$ converges to 0, the only sustainable equilibrium is the Markov (punishment equilibrium) and the capital stock in each state equals the same constant. For intermediate values of $\beta$, capital in the two states diverge, with capital in the high state always being greater than that in the low state.

Remark If the capitalists are defined to be foreign nationals, we can define the trade balance for this economy as $(z + F - c - I)$, where $c$ is the consumption of the workers. The trade balance is therefore the difference between the income capitalists receive $(z + F - c)$ and the amount they invest. In the case with full commitment, investment is constant. While
consumption is procyclical, it is still the case that the capitalists receive more on average in high states and the trade balance to GDP ratio, \(1 - \frac{c_t + I}{z_t + F}\), is pro-cyclical. In our numerical example this correlation is 0.56. However, in the case with limited commitment, investment is now procyclical. In our numerical example, the investment effect dominates and we obtain a negative correlation between the ratio of the trade-balance to GDP and GDP of -0.19, which is consistent with the data for emerging markets.

3 Extensions

3.1 Static Insurance Markets

An extension we consider is to determine if the government could have improved on the limited commitment outcome if it had access to static insurance markets. That is, suppose the government can buy and sell insurance claims \(a(z^t)\) with \(E(a(z^t)|z^{t-1}) = 0\). This insurance can be used to smooth the consumption of workers across states within a period, but not across periods. We show, that as long as the government has the same limited commitment issues related to the insurance contracts as it does with the tax contracts, the availability of static insurance markets will not improve on the equilibrium outcome previously described. That is, any welfare level that can be attained through the use of static insurance contracts can be replicated through the tax and transfer policy.

Consumption is now given by

\[
c(z^t) = z_t + F(k(z^{t-1}, l)) - (1 - \tau(z^t))F_k(k(z^{t-1}, l))k(z^{t-1}) + a(z^t)
\]  
(17)

If the government deviates, it also loses its insurance claims

\[
\bar{c}(z_t, k) = z_t + F(k, l) - (1 - \bar{\tau})F_k(k, l)k
\]  
(18)

We can now state the following proposition.

**Proposition 7**  For any equilibrium with \(\{\tau(z^t), a(z^t)\}\) there exists an equilibrium \(\bar{\tau}(z^t)\) that uses no insurance \((\bar{a}(z^t) = 0)\) and delivers the same utility at any history.

**Proof.** Define \(\bar{\tau}\) as

\[
c = z + F(k, l) - (1 - \tau)F_k(k, l)k + a \equiv z + F - (1 - \bar{\tau})F_k(k, l)k = \bar{c}
\]
This implies
\[ \tilde{\tau} = \tau - \frac{a}{F_kk} \]
By construction, \( c = \tilde{c} \). Since \( E(\tilde{\tau}) = E(\tau) \) capital stock is the same under both allocations. The deviation consumption \( \tilde{c}(z, k) \) is unchanged. So the new \( \tilde{\tau} \) is an equilibrium delivering the same allocation. ■

So, having access to static insurance markets does not change the incentive compatible allocations available to the government, and the results in the previous section still hold.

3.2 Productivity Shocks

This far we have modeled the shocks \( z \) as an endowment shock. We now consider the case where \( z \) is a productivity shock. The production function is
\[ y = zF(k, l) \]  \hspace{0.5cm} (19)
The consumers budget constraint is simply
\[ c(z^t) = w(z^t)l + T(z^t) \]
Profit maximization by firms and capitalists investment decision imply the following two conditions:
\[ z_tF_l(k(z^{t-1}),l) = w(z^t) \] \hspace{0.5cm} (20)
\[ r^* = E[(1 - \tau(z^t))z_t|z^{t-1}]F_k(k(z^{t-1}),l) \] \hspace{0.5cm} (21)
The main deviation from the previous set up is that now the optimal level of capital will vary with the state \( z^{t-1} \), as long as there is some persistence in the state. However, all the previous Lemmas and Propositions, with the exception of Proposition (6), follow through with small alterations to the proof. For instance, as in Proposition (1), when the government has full commitment, workers are completely insured intra-period. Further ex ante taxes, \( E(z_t\tau(z^t)) \), on capital equal 0 and capital is at its first best level. \( k^*(z) \) is increasing in \( z \).

Proposition (6) changes because \( \Delta_{z_{t-1}}(z_{t-1}, z_t) \) needs to take into account the fact that \( k^*(z_{t-1}) \) is increasing in \( z_{t-1} \).
Consumption under full commitment can be written as:

\[ c^*(z_t|z_{t-1}) = E(z_t|z_{t-1}) F(k^*(z_{t-1}), l) - r^* k^*(z_{t-1}) \]

where \( k^*(z) \) satisfies equation (21). As stated before, consumption at time \( t \) under commitment is independent of the realization of the productivity shock at time \( t, z_t \).

Autarkic consumption similarly can be written as

\[ \tilde{c}(z_t, k^*(z_{t-1})) = z_t F(k^*(z_{t-1}), l) - \frac{z_t(1 - \bar{\sigma})}{E(z_t|z_{t-1})} r^* k^*(z_{t-1}) \]

Since there is more capital following a boom, in the first best case, there can be greater temptation to deviate following a boom, since there is more to tax. \( \tilde{c}(z_t, k^*(z_{t-1})) \) is increasing in \( k^* \) \( (\tilde{c}_k(z_t, k^*(z_{t-1})) = \bar{\tau} z_t k^*) \).

Proposition (6) can now be restated as follows.

**Proposition 8** Suppose \( E(z_t|z_{t-1}) \) is increasing in \( z_{t-1} \). Then \( \Delta(z_{t-1}, z_t) \) is increasing in \( z_{t-1} \) if any of the following holds:

(i) The utility function is of the form \( u(c) = \frac{c^\theta}{\theta} \) with \( \theta \leq 0 \) and the expected capital share \( \frac{r^* k^*(z_t)}{E(z_t|z_{t-1}) F(k^*(z_{t-1}), l)} \) is weakly decreasing in \( z_{t-1} \).

(ii) The production function is Cobb Douglas, \( F(k, l) = k^\alpha l^{1-\alpha} \) and the shocks are such that \( \frac{z_t}{E(z_t|z_{t-1})} \leq \frac{(1-\alpha)}{\alpha} \).

**Proof.** See Appendix A. ■

Proposition (8) provides sufficient conditions for our main result that distortions begin to appear first in recessions. Since the expected capital share is simply the constant \( \alpha \) when the production function is Cobb Douglas, condition (i) states that as long as the CRRA utility function has a risk aversion parameter greater than or equal to 1, if the production function is Cobb Douglas it is necessarily the case that capital is first distorted in a recession. In the case when we do not require the utility function to have the CRRA form, then we must place restrictions on the persistence of the productivity process relative to the curvature of the Cobb Douglas production function.
4 The Role of Debt

An important ingredient for the amplification effect is that in the Ramsey plan, higher shocks today generate higher levels of consumption next period. This cyclical behavior in the level of consumption next period makes the Ramsey plan harder to sustain after a lower shock, and distortions in the capital margin appear first after low endowment states. As easily observed, the balanced budget restriction imposed on the government is fundamental in delivering the cyclical behavior of next period consumption under the Ramsey plan. If the government had access to inter-temporal financial instruments, it would smooth out that variation. However, by restricting ourselves to the balanced budget case, we were able to maintain sufficient tractability so as to highlight the mechanism behind amplification.

Consider now the case where the government has access to a risk free bond which it can use to smooth consumption across time. With this instrument, in addition to the taxes, the government can completely smooth worker’s consumption across time and across states if it had commitment. Taxes will be counter-cyclical, as before. However, now, with promised consumption no longer a function of the previous state, the government’s incentives to deviate from the Ramsey prescription will be independent of that state. Consequently, distortions will appear everywhere simultaneously.

Note that this result does not directly over-turn our previous results. It is expected then that a situation where the government’s access to financial markets is not perfect, will also be characterized by a fiscal policy that distorts capital first in the low states and amplifies the business cycle. Since the level of financial access of emerging economies is arguably far from perfect, one can conjecture that the realistic case is somewhere in the middle between budget balance and full access to financial markets, where promised consumption will still be cyclical and distortions will still appear first after lower realizations of the endowment.

To clarify the arguments, we present in this section the case where the government has access to a risk free bond and full commitment to both taxes and debt.

The value function of the government under full commitment now solves the following Bellman equation:

\[
W(z_{t-1}, b(z_{t-1})) = \max_{k, c(z), b(z)} E \left[ u(c(z_t, b(z_t)) + \beta W(z_t, b(z_t)) \right] z_{t-1}
\]

subject to

\[
E \left[ c(z_t) \right] = E \left[ z_t | z_{t-1} \right] + F \left( k(z_{t-1}, l) - r^* k(z_{t-1}) + (1 + r^*) b(z_{t-1}) - E \left[ b(z_t) \right] \right) \quad (22)
\]
where \( b(z^{t-1}) \) is the level of assets accumulated at the end of period \( t-1 \). Define \( \lambda(z^{t-1}) \) to be the lagrange multiplier on (22). The first order conditions are

\[
c(z^t): \\ u'(c(z_t, b(z^t))) = \lambda(z^{t-1}) \\
\]

(23)

\[
b(z^{t-1}): \\ F_k (k(z^{t-1}), l) = r^* \\
\]

(24)

\[
k(z^{t-1}): \\ \lambda(z^{t-1}) = \beta W_b(z_t, b(z_t)) \\
\]

(25)

The envelope condition is,

\[
W_b(z_{t-1}, b(z^{t-1})) = (1 + r^*) \sum_{z_t} q(z_t | z^{t-1}) \lambda(z^{t-1}) \\
\]

(26)

Combining (25), (23) and (26) we obtain

\[
u'(c(z_t, b(z^t))) = \beta (1 + r^*) E_t [u'(c(z_{t+1}, b(z^{t+1})))] \\
\]

(27)

Also, from (23), we have that consumption is equalized across states.

\[
c \left( \{ z_t, b(z_t, z^{t-1}), z^{t-1} \} \right) = c \left( \{ z'_t, b(z'_t, z^{t-1}), z^{t-1} \} \right) \\
\]

(28)

If we assume \( \beta(1 + r^*) = 1 \), from (27) and (28) we have that consumption is equalized across time and across states. From (24) we have that capital is at the first best level and constant.

From the constraint (22), recursive substitutions of \( b(z^t) \) and the law of iterated expectations, we can solve for the constant level of consumption in the Ramsey plan:

\[
c^* = r^* b(z^{t-1}) + F(k^*, l) - r^* k^* + \left( \frac{r^*}{1 + r^*} \right) E \left[ \sum_{n=0}^{\infty} \left( \frac{1}{1 + r^*} \right)^n z_{t+n} | z_{t-1} \right] \]

(29)

Since \( F(k^*, l) - r^* k^* \) is a constant and \( E \left[ \sum_{n=0}^{\infty} \left( \frac{1}{1 + r^*} \right)^n z_{t+n} | z_{t-1} \right] \) is increasing in \( z_{t-1} \), this implies that \( b(z^{t-1}) \) is decreasing in \( z^{t-1} \).
We can solve for the level of savings/borrowing using equation (29),

\[ b(z_t, z^{t-1}) - b(z_{t-1}, z^{t-2}) = \left( \frac{1}{1 + r^*} \right) \times \]

\[ \times \left( E \left[ \sum_{n=0}^{\infty} \left( \frac{1}{1 + r^*} \right)^n z_{t+n} \left| z_{t-1} \right. \right] - E \left[ \sum_{n=0}^{\infty} \left( \frac{1}{1 + r^*} \right)^n z_{t+n+1} \left| z_t \right. \right] \right) \]

which implies that \( b(z_t, z^{t-1}) - b(z_{t-1}, z^{t-2}) = 0 \) if \( z_t = z_{t-1} \).

Note that in the case with persistence, the government in the Ramsey plan will dissave in a high state following a low state and will save in a low state following a high state.

Using the budget constraint,

\[ c(z^t) = z_t + F(k(z^{t-1}), l) - (1 - \tau(z^t))F_k(k(z^{t-1}), l)k(z^{t-1}) + (1 + r^*)b(z^{t-1}) - b(z^t) \]

and the constant consumption equation (29) we can solve for the tax rate,

\[ \tau(z^t) = \frac{1}{r^*k} \left[ \frac{r^*}{1 + r^*} E \left[ \sum_{n=0}^{\infty} \left( \frac{1}{1 + r^*} \right)^n z_{t+n} \left| z_{t-1} \right. \right] - z_t + b(z_t, z^{t-1}) - b(z_{t-1}, z^{t-2}) \right] \]

It follows that in the Ramsey plan, the government taxes capitalists in low states and subsidizes them in high states as before. Accordingly, taxes are counter-cyclical.

In the case when a high state follows a low one, the government borrows and subsidizes capitalists. When a low state follows a high state, the government taxes capitalists and also saves for the future. The feature that the government borrows in a high state and saves in a low state is consistent with the evidence of counter-cyclical budget balances that is observed in the data for developing country governments. With this policy, the government achieves a perfectly smoothed consumption profile for the workers and does not distort the capital margin.

What about the incentives to deviate from the Ramsey plan? Suppose that following a deviation, the government losses its assets and does not repay its debts. The value after a deviation is then as before (given by \( V(z_t) \)). So, the incentive constraint is now

\[ u(c(z^t)) + \beta W(z^t) \geq u(\bar{c}(k, z_t)) + \beta V(z^t) \]
Note that under the Ramsey plan,

\[ u(c(z^t)) + \beta W(z^t) = \frac{u(c^*)}{1 - \beta} \]

which is constant independent of \( z^t \) and \( t \). The gains to deviating from the Ramsey plan at any state \( z^t \) is independent of the previous shocks \( z^{t-1} \), and the arguments in previous sections do not directly apply.

**Remark** Note that this analysis of the incentive constraints only applies when \( \beta(1 + r^*) = 1 \). In this case, there are parameter values where the Ramsey plan is sustainable, and we can ask the question (as in previous sections) of in which states the incentive constraints would bind “first”. However, when \( \beta(1 + r^*) < 1 \), the Ramsey plan would require a falling consumption profile, and it would never be incentive compatible. So when \( \beta(1+r^*) < 1 \), some incentive constraints will always bind in any continuation game for any parameter values. We conjecture that in this case, as the government would eventually always hit borrowing limits, fiscal policy would amplify the cycle.

To summarize, we find that at one extreme, when the government cannot borrow or save, in the case with limited commitment, distortions first appear following a low state. At the other extreme, when the government can perfectly insure consumption across time and states, distortions would appear everywhere simultaneously. Consequently, we conjecture that in the intermediate and more realistic case when financial access is less than perfect and consumption is higher following a high shock relative to a low shock, we should obtain distortions and amplification similar to the budget balance case. We do not however prove this in this paper and leave it for future research.

### 5 Conclusion

In this paper we have explored the question of why is fiscal policy procyclical in developing countries and under what circumstances does this amplify the cycle. Our explanation is based on two features that characterize developing markets: imperfect access to financial markets and high impatience rates that limits the governments commitment to its tax policy.

To provide a clear exposition of our mechanism underlying procyclicality and amplification, we considered a parsimonious specification in our benchmark model. The workers in this model are subject to endowment shocks that they cannot insure. The government, who
cares only about the workers provides insurance through the use of linear taxes on labor and capital. The government taxes capital and transfers to the workers in recessions. To prevent capital distortions, the government then taxes labor and subsidizes capital in booms. The insurance motive then generates counter-cyclical taxes or pro-cyclical fiscal policy. When the government has full commitment to its tax policy, it is able to provide intra-period insurance to the workers without distorting capital, by setting the expected tax rate on capital to zero. In this environment, the capital stock is a constant.

It is when the government lacks commitment that we find that fiscal policy can be distortionary and investment varies with the realization of the endowment shock. We show that the incentive to deviate in any state today depends not only on the realized state but also on the path the economy experienced before arriving at this state. This result arises because when the government is restricted to running a balanced budget, consumption in any period is greater following a boom than a recession, as long as there is some persistence in the endowment shock. Consequently, the gains to deviating and expropriating capital at the maximum possible rate is greater following a recession. In this environment, the government is less able to commit to not expropriating capital following recessions and distortions in capital first appear here.

Since an important part of the amplification effect arises because a higher shock today leads to higher consumption tomorrow, we discuss how the results would change when the government is not restricted to running a balanced budget. We conjecture that, as long as financial access is less that perfect, and the government cannot perfectly smooth consumption over time, our amplification mechanism should hold. We present a brief analysis of this in Section 4 and leave the proof of the conjecture to future research.
Appendix A: Proof for Proposition 14.

Suppose $E(z_t|z_{t-1})$ is increasing in $z_{t-1}$, $\Delta(z_{t-1}, z_t)$ is increasing in $z_{t-1}$ if any of the following statements hold: (i) The utility function is of the form $u(c) = \frac{c^\theta}{\theta}$ with $\theta \leq 0$ and the expected capital share $\frac{r^*k^*(z_{t-1})}{E(z_t|z_{t-1})F(k^*(z_{t-1}), l)}$ is weakly decreasing in $z_{t-1}$.

Proof for part (i): We need to show that the difference between first best consumption and deviation consumption at time $t$ is increasing in $z_{t-1}$. First best consumption in any state $z_t$ is

$$c^*(z_t) = E(z_t|z_{t-1})F(k^*(z_{t-1}), l) - r^*k^*(z_{t-1})$$

Deviation consumption is,

$$\bar{c}(z_t|z_{t-1}) = z_tF(k^*(z_{t-1}), l) - (1 - \bar{\tau}) z_l F_k(k^*(z_{t-1}), l) k^*(z_{t-1})$$

Alternatively, we need

$$\Delta = u [z_t (F - F_k k + \bar{\tau} F_k k)] - u [E (z_t|z_{t-1}) (F - F_k k)]$$

to be decreasing in $z_{t-1}$.

$$\Delta = (F - F_k k)^\theta \left[ u(z_t) \left[ 1 + \bar{\tau} \frac{1}{F_k k} - 1 \right] - u (E(z_t|z_{t-1})) \right]$$

$$\log \Delta = \theta \log (F - F_k k) + \log \left[ u(z_t) \left[ 1 + \bar{\tau} \frac{1}{F_k k/F} - 1 \right] - u (E(z_t|z_{t-1})) \right]$$

In the preceding equations we have used the utility function form, $u(c) = c^\theta / \theta$.

Now, $F - F_k k = F_il$, and as long as $\theta$ is negative, $(F - F_k k)^\theta$ would be decreasing in $k$. Given that $k$ is increasing in $z_{t-1}$, then $(F - F_k k)^\theta$ would be decreasing in $z_{t-1}$ as well.

Now, $u (E(z_t|z_{t-1}))$ is increasing in $z_{t-1}$. So, we need that

$$u(z_t) \left[ 1 + \bar{\tau} \frac{1}{F_k k/F} - 1 \right]$$
be decreasing in $z_{t-1}$. Given $\theta < 0$, and $u(z)$ is negative, this implies

$$1 + \bar{\tau} \frac{1}{\frac{1}{z_{t-1}}} - 1$$

should be decreasing in $z_{t-1}$. Note that $1 + \bar{\tau} \frac{1}{\frac{1}{z_{t-1}}}$ is decreasing in $\frac{F_k k}{F}$, the capital share.

So, if the capital share is weakly decreasing in $k$ then, $u(z_t) \left[1 + \bar{\tau} \frac{1}{\frac{1}{z_{t-1}}}\right]^{\theta}$ is also weakly decreasing in $k$ as long as $\theta$ is negative and the proposition goes through. A special case is when the production function is Cobb-Douglas and the share of capital is a constant.

Proof for part (ii) Suppose the production function is of the Cobb Douglas form.

$$y = z k^\alpha l^{1-\alpha}$$

Since,

$$E(z_t | z_{t-1}) F_k (k^*(z_{t-1})) = E(z_t | z_{t-1}) \alpha (k^*)^{\alpha-1} = r^*$$

we can rewrite the first best consumption as

$$c^*(z_t) = \left(\frac{1}{\alpha}\right) r^* k^*$$

$$\Delta (z_{t-1}, z_t) = u \left(\left(\frac{1}{\alpha}\right) r^* k^*\right) - u (z_t k^*(z_{t-1})^{\alpha} [(1 - \alpha) + \bar{\tau} \alpha]) \equiv H(k^*)$$

The preceding equation is a function of $k^*$. Since $k^*$ is increasing in $z_{t-1}$, we only need to check for the conditions under which $H'(k^*) \geq 0$.

$$H'(k^*) = u' (c^*) \left(\frac{1}{\alpha}\right) r^* - u' (\bar{c}) [(1 - \alpha) + \bar{\tau} \alpha] z_t \alpha (k^*)^{\alpha-1} \geq 0$$

$$= k^* u' (c^*) c^* - u' (\bar{c}) \bar{c} \alpha \geq 0$$

This requires

$$\frac{u' (c^*) c^*}{u' (\bar{c}) \bar{c}} \geq \alpha$$

(30)

Since $\bar{c} > c^*$, $\frac{u'(c^*)}{u'(\bar{c})} \geq 1$, a sufficient condition is $\frac{c^*}{\bar{c}} \geq \alpha$.

$$\frac{c^*}{\bar{c}} = \frac{(1 - \alpha) E(z_t | z_{t-1})}{z_t (1 - \alpha) + \bar{\tau} \alpha} \geq \frac{(1 - \alpha) E(z_t | z_{t-1})}{z_t}$$
A further sufficient condition is \( \frac{(1-\alpha)E(z_t|z_{t-1})}{z_t} \geq \alpha \), which implies

\[
\frac{z_t}{E(z_t|z_{t-1})} \leq \frac{1 - \alpha}{\alpha}
\]
Appendix B: Numerical Example

Table 1: Numerical Example: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>World interest rate</td>
<td>0.1</td>
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<tr>
<td>Depreciation rate</td>
<td>0</td>
</tr>
<tr>
<td>Time preference rate</td>
<td>0.7</td>
</tr>
<tr>
<td>Risk Aversion</td>
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</tr>
<tr>
<td>Punishment Tax</td>
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</tr>
<tr>
<td>Capital Share</td>
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</tr>
<tr>
<td>$z_H$</td>
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</tr>
<tr>
<td>$z_L$</td>
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</tr>
<tr>
<td>$q_{HH}$</td>
<td>0.98</td>
</tr>
<tr>
<td>$q_{LL}$</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 2: Numerical Example: Capital Tax Rates

| $(Z_t|Z_{t-1})$ | Full Commitment | Limited Commitment |
|----------------|-----------------|--------------------|
| $\tau(H|H)$    | -0.007          | -0.007             |
| $\tau(L|H)$    | 0.330           | 0.330              |
| $\tau(H|L)$    | -0.236          | -0.029             |
| $\tau(H|L)$    | 0.101           | 0.063              |
| $E(\tau|H)$    | 0               | 0                  |
| $E(\tau|L)$    | 0               | 0.04               |
References


