Money and Capital*

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Abstract

We continue the integration of modern monetary theory and mainstream macro. First, we pursue price taking as alternative to bargaining. Second, we add firms, capital and labor markets. Compared to previous attempts at this integration, our models have rich feedback from monetary policy to all markets. We calibrate the model and compute the effects of inflation, including the transition path, since policy affects capital accumulation. The cost of 10% inflation is between 1% and 4% of consumption, depending on the specification.

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1 Introduction

Much progress has been made over the last two decades on the microfoundations of money. There is by now a large and growing literature that goes beyond reduced-form approaches, like cash-in-advance or money-in-the-utility-function models, by providing explicit descriptions of preferences, technology, information, and so on, that imply certain objects arise endogenously as media of exchange, and that make money essential. Yet many economists continue to use reduced-form models. Why? Perhaps it is due to the fact that early search-based models abstract from many components of conventional macro models, and invoke some nonstandard assumptions, including strong restrictions on money holdings.[1]

More recent contributions, including Shi (1997) and Lagos and Wright (2005), have reduced the gap between monetary theory and mainstream macro by relaxing restrictions on money holdings. Shi does this by taking Lucas's (1980) idea of worker-shopper pairs to the extreme of infinite families; Lagos-Wright uses alternating centralized and decentralized markets, plus quasilinear utility. While these models allow one to analyze questions concerning, say, the effects of inflation, they still look different from the neoclassical growth model. One reason is that at least the benchmark versions of these models are still missing capital and labor markets, taxation, productivity shocks, etc. Another reason is that, rather than competitive pricing, they typically use bilateral bargaining.

The goal of this project is to continue the integration of modern monetary theory and mainstream macro. First, following Rocheteau and Wright (2005), we pursue the idea of using competitive pricing in these models, and comparing this to the outcome implied by bargaining. This helps disentangle which results come from explicitly incorporating frictions, like specialization, information, etc., into the physical environment, and which results come from imposing a particular pricing mechanism. Second, we add neoclassical firms that use labor and capital, traded in competitive markets.

Now labor and capital markets were already introduced into search-based models in Aruoba and Wright (2003).[2] But that model is very special, because it entails a strong *dichotomy*: one can solve independently for equilibrium allocations in the centralized and

[1] We have mind the models in Kiyotaki and Wright (1989,1993), Aiyagari and Wallace (1991), Masuyma, Kiyotaki and Matsui (1993), Shi (1995) or Trejos and Wright (1995), just to mention a few.

[2] As we do here, that paper worked with the Lagos-Wright framework; one can also pursue these issues in the Shi model, as in Shi (1999) or Faig (2001).

decentralized markets. This is problematic for several reasons. First, one could argue that this result means we have not *integrated* monetary theory and standard macro at all – at best, we have shown they can coexist without too much conflict (Howitt 2003). Second, this dichotomy has stark policy implications, because it means money has no impact on aggregate employment, consumption or investment in the centralized market. Here we show this result is not general: natural changes in the specification lead to rich feedback between the centralized and decentralized markets, and monetary policy affects both.

Finally, we continue recent efforts to quantify monetary theory. Especially once capital and labor markets are introduced, these models lend themselves to calibration much as the standard growth model. Of course, monetary models have some parameters that do not appear in nonmonetary theory, but this is no more true of monetary models with microfoundations than those without. Moroever, to calibrate the model, we make it more realistic by including taxes, and this allows us to see some interesting interactions between fiscal and monetary policy. An interesting part of the numerical work is that, in models that do not dichotomize, to compute the effects of policy changes we need to take into account transition paths, rather than simply comparing steady states. We find that taking into account the transition path is important for the welfare calculations.[3]

The rest of the paper is and some of the main results can be summarized as follows. Section 2 describes the basic model and defines equilibria under two different pricing assumptions: bargaining and price taking. Section 3 discusses some extensions. Section 4 presents the numerical approach. Section 5 present results on the effects of inflation; depending on the specification, we find that going from 10% inflation to constant prices is worth between 1% and 4% of consumption. Taking into account the transition is important for the results. We discuss how the results depend on various features of the model, including holdup problems in both money demand and investment. We also show there are very big welfare costs are associated with distorting taxes. Section 6 concludes.

[3]By contrast, in a reduced-form model Cooley and Hansen (1991, n. 13) find that "The transitional dynamics have a very minor effect on welfare."

2 The Basic Model

The environment is based on Lagos and Wright (2005), hereafter referred to as LW. There is a $[0, 1]$ continuum of infinite-lived agents. Time is discrete, and each period is divided into two subperiods. In one subperiod, there is a frictionless centralized market, called the CM in what follows; in the other, there is a decentralized market, called the DM, with various degrees of frictions depending on the version of the model. One friction in the DM in all versions is a *double-coincidence problem*. Another is that agents in the DM are *anonymous*, precluding private credit arrangements and making a medium of exchange essential (Kocherlakota 1998; Wallace 2001). Also, while the CM is always perfectly competitive, we consider two alternatives for the DM: price taking and bargaining.

In the CM, capital k and labor h are rented to competitive firms. As usual, profit maximization implies $r = F_K(K, H)$ and $w = F_H(K, H)$, where $F(K, H)$ is a constant returns CM aggregate production function, r is the rental rate and w the real wage.[4] In the DM this technology does not operate, but agents' labor ℓ may be used as an input to an individual technology $\phi(k, \ell)$. In the base model, capital cannot be traded in the DM (but see Section 3). The idea is that once put in place k cannot be physically moved to the location where the DM convenes. Moreover, claims to k cannot be traded for the same reason that personal IOUs cannot – anonymity, which means one could fake such a claim with no fear of retribution.

Although capital is not physically present in the DM, the technology $\phi(k, \ell)$ may still depend on it (as an example of capital that is productive even though not physically present, and hence not tradable at a location, think about logging on to your computer remotely). The only reason for making capital immobile is to prevent it from serving as commodity money, which is a very interesting issue but not one that concerns us here; see Waller (2004) and Lagos and Rocheteau (2004) for related models where capital can be used as money. A simple alternative would be to interpret k as human capital, which is nontradable, but this would change the empirical implications, and so we do not take that route.

We generate a double-coincidence problem in the DM as follows: With probability σ, each agent wants to consume but cannot produce, with the same probability each agent

[4] We follow the convention of letting lower (upper) case letters indicate individual (aggregate) variables. Also, it is easy to include a productivity shock, but here we focus here on the nonstochastic model.

can produce but does not want to consume, and with probability $1 - 2\sigma$ he can neither produce nor consume. This is equivalent for our purposes to the standard specification in the search literature, where there is a probability σ of meeting someone who can produce what you like. We frame things in terms of taste and technology shocks, rather than random matching, because it makes it slightly easier to compare the different pricing mechanisms.

Instantaneous utility in the CM is $U(x) - Ah$, where x is consumption and h hours. In the DM, with probability σ an agent is a consumer and his utility is $u(q)$, and with probability σ he is a producer and his utility is $-v(\ell)$. Assume $U(x)$, $u(q)$ and $v(\ell)$ have the usual properties. Linearity of utility in h is not important, in principle, but the trick of LW is that it generates a big gain in tractability.[5] It is convenient to rewrite the disutility of production as follows: given k, solve $q = \phi(k, \ell)$ for $\ell = \Phi(q, k)$ and let $c(q, k) = v[\Phi(q, k)]$. Notice $c_q > 0$, $c_k < 0$, $c_{qq} > 0$, and $c_{kk} > 0$ under the usual monotonicity and convexity assumptions on ϕ and v, and $c_{qk} < 0$ under the additional restriction $\phi_k \phi_{\ell\ell} < \phi_\ell \phi_{\ell k}$, which always holds if k is a normal input; see Appendix .

There is a government in the model that controls the money supply according to $M_{+1} = (1 + \tau)M$, where a subscript $+1$ denotes next period, and τ may or may not be constant over time. They also set taxes on labor and capital income in the CM, t_h and t_k, as well as sales taxes in both the CM and DM markets, t_x and t_q There is also a lump-sum tax T and government consumption G in the CM market. Their budget constraint is

$$G = T + t_h wH + t_k rK - \delta t_k K + t_x X + \sigma t_d \frac{M}{p} + \tau \frac{M}{p} \tag{1}$$

where δ is the depreciation rate on capital, which is tax deductable. Here p is the price level in the CM, so $\tau M/p$ is seniorage, and $\sigma t_d M/p$ is DM sales tax receipts, in units of the CM good (this follows because in equilibrium σ is the number of DM trades and in each trade M dollars changes hands).

Agents discount between the CM and DM at rate β, but not between the DM and CM.[6]

[5] An alternative is to assume general utility and indivisible labor, since as in Rogerson (1988) this gives rise to a quasi-linear reduced form; see Rocheteau et al. (2005) for details.

[6] Usually in this type of model, if agents discount between the CM and DM at rate β_1 and between the DM and CM at rate β_2, only the product $\beta_1\beta_2$ matters. This is not quite the case here due to the way capital enters; e.g. β_1 would appear without β_2 in front of the γ term in (18) below. For the experiments we consider, however, this was numerically irrelevant, so we set $\beta_2 = 1$ to reduce the notation.

If $W(m,k)$ and $V(m,k)$ are the value functions of agents in the CM and DM, we have

$$\begin{aligned} W(m,k) &= \max_{x,h,m_{+1},k_{+1}} \{U(x) - Ah + \beta V(m_{+1},k_{+1})\} \\ \text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + [1+(r-\delta)\,(1-t_k)]\,k - k_{+1} - T + \frac{m-m_{+1}}{p} \end{aligned}$$

Eliminating h using the budget equation, we have

$$\begin{aligned} W(m,k) = {} & \frac{A}{w\,(1-t_h)}\left\{\frac{m}{p} + [1+(r-\delta)\,(1-t_k)]\,k - T\right\} \\ & + \max_{x,m_{+1},k_{+1}}\left\{U(x) - \frac{A}{w\,(1-t_h)}\left[\frac{m_{+1}}{p} + (1+t_x)\,x + k_{+1}\right] + \beta V(m_{+1},k_{+1})\right\}. \end{aligned}$$

The first order conditions are[7]

$$\begin{aligned} x &: U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \\ m_{+1} &: \frac{A}{pw\,(1-t_h)} = \beta V_m(m_{+1},k_{+1}) \qquad (2) \\ k_{+1} &: \frac{A}{w\,(1-t_h)} = \beta V_k(m_{+1},k_{+1}). \end{aligned}$$

Notice the choice of (m_{+1},k_{+1}) is independent of (m,k). Hence, given $V\,(m,k)$ is strictly concave, for any distribution of (m,k) across agents entering the CM, the distribution entering the next DM is degenerate (assuming an interior solution for h; see LW for assumptions to guarantee this is valid). We also have the envelope conditions,

$$W_m(m,k) = \frac{A}{pw\,(1-t_h)} \qquad (3)$$

$$W_k(m,k) = \frac{A\,[1+(r-\delta)\,(1-t_k)]}{w\,(1-t_h)}, \qquad (4)$$

showing that W is linear in (m,k).

Moving to the DM market, we have

$$V(m,k) = \sigma V^b(m,k) + \sigma V^s(m,k) + (1-2\sigma)W(m,k), \qquad (5)$$

[7] The second order conditions are generally ambiguous since they involve second derivatives of V which can involve third derivatives of u and c in models with bargaining. We simply assume V is strictly concave here, but in the numerical exercises it is easy to check that this is true. Or, as in LW, one can prove it must be true if the buyers' bargaining power is close to 1, or under additional conditions on preferences. In the model with price taking, this is not an issue, as it is easy to see V is always strictly concave.

where

$$V^b(m,k) = u(q_b) + W(m - d_b, k) \tag{6}$$

$$V^s(m,k) = -c(q_s,k) + W[m + (1-t_q)\, d_s, k], \tag{7}$$

and q_b and d_b are output and money exchanged when buying, while q_s and d_s are output and money exchanged when selling net of the sales tax. Using (3), we have

$$V(m,k) = W(m,k) + \sigma\left[u(q_b) - \frac{Ad_b}{pw\,(1-t_h)}\right] + \sigma\left[\frac{Ad_s}{pw\,(1-t_h)} - c(q_s,k)\right].$$

Differentiation yields

$$\begin{aligned} V_m(m,k) &= \frac{A}{pw\,(1-t_h)} + \sigma\left[u'\frac{\partial q_b}{\partial m} - \frac{A}{pw\,(1-t_h)}\frac{\partial d_b}{\partial m}\right] \\ &\quad + \sigma\left[\frac{A}{pw\,(1-t_h)}\frac{\partial d_s}{\partial m} - c_q\frac{\partial q_s}{\partial m}\right] \end{aligned} \tag{8}$$

$$\begin{aligned} V_k(m,k) &= \frac{A\,[1+(r-\delta)\,(1-t_k)]}{w\,(1-t_h)} + \sigma\left\{u'\frac{\partial q_b}{\partial k} - \frac{A\,[1+(r-\delta)\,(1-t_k)]}{w\,(1-t_h)}\frac{\partial d_b}{\partial k}\right\} \\ &\quad + \sigma\left\{\frac{A\,[1+(r-\delta)\,(1-t_k)]}{w\,(1-t_h)}\frac{\partial d_s}{\partial k} - c_q\frac{\partial q_s}{\partial k} - c_k\right\}. \end{aligned} \tag{9}$$

It remains to specify how prices are determined in the DM, so that we can substitute for the derivatives in (8) and (9), and this will differ across the two versions of the model below. Before pursuing equilibrium, however, consider the planner's problem in an economy without anonymity, so that money is *not* essential:

$$\begin{aligned} J(K) &= \max_{X,H,q,K_{+1}} \{\sigma u(q) - \sigma c(q,K) + U(X) - AH + \beta J(K_{+1})\} \\ s.t.\ X &= F(K,H) + (1-\delta)K - K_{+1} - G \end{aligned} \tag{10}$$

The first order conditions are

$$\begin{aligned} H:&\quad A = U'(X)F_H(K,H) \\ K_{+1}:&\quad U'(X) = \beta J'(K_{+1}) \\ q:&\quad u'(q) = c_q(q,K) \end{aligned} \tag{11}$$

Also, using $J'(K) = U'(X)[F_K(K,H) + 1 - \delta] - \sigma c_k(q,K)$, we have

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1},H_{+1}) + 1 - \delta] - \beta\sigma c_k(q_{+1},K_{+1}). \tag{12}$$

Clearly, $q = q^*(K)$ where $q^*(K)$ satisfies $u'(q) = c_q(q,K)$. Then the other variables (K_{+1},H,X) satisfy standard consitions: the Euler equation (12), the first equation in (11), and the constraint in (10).

2.1 Bargaining

Suppose each agent with a desire to consume is matched with one who can produce. Since buyers are anonymous, trade must be quid pro quo, and here this means they must pay with cash. Given the buyer's state is (m_b, k_b) and the seller's (m_s, k_s), the terms of trade (q, d) are determined using the generalized Nash solution, with bargaining power for the buyer θ and threat points given by continuation values. The buyer's payoff from the trade is $u(q) + W(m_b - d, k_b)$ and his threat point is $W(m_b, k_b)$, so (3) implies his surplus is $u(q) - \frac{Ad}{pw(1-t_h)}$. The seller's payoff is $-c(q, k_s) + W[m_s + (1-t_q)d, k_s]$, his threat point is $W(m_s, k_s)$, and his surplus is $-c(q, k_s) + \frac{A(1-t_q)d}{pw(1-t_h)}$. Hence the generalized Nash solution is

$$\max_{q,d} \left[u(q) - \frac{Ad}{pw(1-t_h)} \right]^{\theta} \left[-c(q, k_s) + \frac{A(1-t_q)d}{pw(1-t_h)} \right]^{1-\theta} \text{ s.t. } d \le m_b.$$

As in LW, in any equilibrium the constraint $d \le m_b$ holds with equality. This implies $q \le q^*(k_s)$ where $q^*(k_s)$ is the solution to $u'(q) = c_q(q, k_s)$, with strict inequality unless $\theta = 1$ and we follow the Friedman rule. In any case, inserting $d = m_b$ and taking the first order condition for q, we get

$$\frac{m_b}{p} = \frac{g(q, k_s)w(1-t_h)}{A}, \tag{13}$$

where

$$g(q, k_s) \equiv \frac{\theta c(q, k_s)u'(q) + (1-\theta)u(q)c_q(q, k_s)}{(1-t_q)\theta u'(q) + (1-\theta)c_q(q, k_s)}. \tag{14}$$

Hence, the bargaining solution is $d = m_b$, and $q = q(m_b, k_s)$ as given by the solution to (13).

This means $\partial d/\partial m_b = 1$, $\partial q/\partial m_b = A/pw(1-t_h)g_q > 0$, and $\partial q/\partial k_s = -g_k/g_q > 0$, where

$$g_q = \frac{c_q u'[(1-t_q)\theta u' + (1-\theta)c_q] + \theta(1-\theta)[(1-t_q)u - c][u'c_{qq} - c_q u'']}{[(1-t_q)\theta u' + (1-\theta)c_q]^2} > 0 \tag{15}$$

$$g_k = \theta \frac{c_k u'[(1-t_q)\theta u' + (1-\theta)c_q] + c_{qk}(1-\theta)u'[(1-t_q)u - c]}{[(1-t_q)\theta u' + (1-\theta)c_q]^2} < 0, \tag{16}$$

while the other derivatives in (8) and (9) are all 0. Inserting these and imposing symmetry, $(m, k) = (M, K)$, (8) and (9) reduce to

$$V_m(m, k) = \frac{\sigma A u'(q)}{pw(1-t_h)g_q(q, K)} + \frac{(1-\sigma)A}{pw(1-t_h)}$$

$$V_k(m, k) = \frac{A[1 + (r-\delta)(1-t_k)]}{w(1-t_h)} - \sigma\gamma(q, K),$$

where

$$\gamma(q,K) = \frac{c_k(q,K)g_q(q,K) - c_q(q,K)\,g_k(q,K)}{g_q(q,K)} < 0.$$

Substituting these into the conditions for m_{+1} and k_{+1} in (2), as well as the equilibrium prices $p = A/w\,(1-t_h)\,g(q,K)$, $r = F_K(K,H)$ and $w = F_H(K,H)$, and using $M_{+1} = (1+\tau)\,M$, we arrive at equilibrium conditions:

$$\begin{aligned} g(q,K) &= \frac{\beta g(q_{+1},K_{+1})}{1+\tau}\left[1-\sigma+\sigma\frac{u'(q_{+1})}{g_q(q_{+1},K_{+1})}\right] &(17)\\ U'(X) &= \beta U'(X_{+1})\left\{1+\left[F_K(K_{+1},H_{+1})-\delta\right](1-t_k)\right\} &(18)\\ &\quad -\sigma\beta\,(1+t_x)\,\gamma(q_{+1},K_{+1}). \end{aligned}$$

The other equilibrium conditions are

$$\begin{aligned} U'(X) &= \frac{A\,(1+t_x)}{(1-t_h)\,F_H(K,H)} &(19)\\ X+G &= F(K,H)+(1-\delta)K-K_{+1}. &(20) \end{aligned}$$

A monetary equilibrium is now defined as (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (17)-(20), given initial K_0.[8] If τ is constant, it makes sense to look for a steady state, defined as a constant solution (q,K,H,X) to (17)-(20). This means prices grow at the same rate as M, so inflation equals τ. Defining the real interest rate ρ by $\beta = \frac{1}{1+\rho}$ and the nominal rate by $1+i = (1+\rho)(1+\tau)$, in steady state (17)-(18) reduce to

$$\begin{aligned} 1+\frac{i}{\sigma} &= \frac{u'(q)}{g_q(q,K)} &(21)\\ \rho &= \left[F_K(K,H)-\delta\right](1-t_k) - \sigma\,(1+t_x)\,\frac{\gamma(q,K)}{U'(x)} &(22) \end{aligned}$$

A special case is the specification in Aruoba and Wright (2003) where K is not used in the DM, $c(q,K) = c(q)$, which means $\gamma(q,K) = 0$. This version displays a strong dichotomy: (17) determines a path for q and then (18)-(20) determine paths for (K_{+1},H,X) independently. Notice that money affects the path for q but not (K_{+1},H,X). Assuming a unique steady state q exists, which is true under conditions given in LW, q is decreasing in i or τ, but this has no effect on (K_{+1},H,X) with this specification. Welfare is maximized at $i = 0$, but if $\theta < 1$ then $q < q^*$ even at $i = 0$, where q^* is the efficient outcome, $u'(q^*) = c'(q^*)$. Under

[8] A nonmonetary equilibrium also always exists, and satisfies $q = 0$ instead of (17), (18) with $\gamma(\cdot) = 0$, and (19)-(20). These are exactly the equilibrium conditions for the standard nonmonetary growth model in, e.g., Hansen (1985).

bargaining, there is a holdup problem in money demand: buyers bear the cost of acquiring cash in the CM, but when $\theta < 1$ they share the surplus this generates with sellers. This lowers the demand for money, and hence q, below the efficient level.[9]

The dichotomy in Aruoba and Wright is special, and does not hold when K enters the cost function, since then K and q both appear in (17) and (18) and we cannot solve independently for q and the other variables. In this case, the investment decision not only takes into account the fact that K affects productivity in the CM, but also in the DM. Since a change in i affects q, this affects the return to K. Intuitively, inflation reduces the gains to trading in the DM, which affects the value of capital in that market and hence investment. Since the same capital is used in both markets, this lowers productivity and output and employment in the CM.

Notice, however, that even when K enters the DM production funciton, if we set $\theta = 1$ then $\gamma(q, K) = 0$. Hence, when $\theta = 1$, the model is recursive if not dichotomous: (18)-(20) determine paths for (K_{+1}, H, X) independently of q, and the solution is the same as the standard nonmonetary model; then given the K path, (17) determines a q path. In this case, anything like taxes that influence K will affect q, but there is no feedback in the other direction. Again, monetary policy affects q but not investment, employment or consumption in the CM. Intuitively, when $\theta = 1$ sellers get none of the DM surplus, so they realize no cost savings from bringing extra capital to the this market, and therefore their investment decision is based solely on returns in the CM.

A holdup problem in the demand for capital actually arises for all $\theta > 0$, and generally means investment is inefficient. This is a distortion over and above the usual ones in monetary theory – the standard inefficiency due to $i > 0$, and the holdup problem in money demand due to $\theta < 1$. In some models holdup problems are resolved if one simply sets θ correctly (Hosios 1990). Here this is not possible: $\theta = 1$ resolves the holdup problem in the demand for money, but this is the worst possible case for investment; $\theta = 0$ resolves the holdup problem in investment, but then monetary equilibrium breaks down ($q = 0$). There is no θ that eliminates both problems.

[9] See Rocheteau and Waller (2005) for a recent discussion of alternative bargaining solutions and holdup problems in monetary theory.

2.2 Price Taking

With care, competitive price taking can be used in search-based monetary theory (Rocheteau and Wright 2005). In particular, we can adopt this pricing mechanism and still maintain anonymity to rule out credit in the DM and maintain a role for money. Competitive pricing eliminates both holdup problems, and is also slightly easier to use.[10] The value function for the DM market before the shocks are realized has the same form as in (5) but (6) and (7) change. The former becomes

$$V^b(m,k) = \max_{q_b} \ \{u(q_b) + W(m - P_D q_b, k)\} \text{ s.t. } P_D q_b \leq m$$

where P_D is the price level in the DM, while the latter becomes

$$V^s(m,k) = \max_{q_s} \{-c(q_s,k) + W\left[m + (1-t_q)\, P_D q, k\right]\}.$$

These are standard demand and supply problems. In equilibrium $q_b = q_s = q$ because we have assumed there are the same number σ of buyers and sellers (this is for convenience only, and not necessary). It is easy to show that the constraint $P_D q_b \leq m$ is binding in equilibrium, so $P_D = M/q$. Inserting this into the first order consition from the seller's problem, $c_q(q,k) = P_D\,(1-t_q)\, W_m(\cdot) = P_D\,(1-t_q)\, A/pw\,(1-t_h)$, we have

$$c_q(q,k) = \frac{A\,(1-t_q)\,M}{pqw\,(1-t_h)}. \tag{23}$$

Also,

$$\begin{aligned} V_m(m,k) &= \frac{\sigma u'(q)}{P_D} + \frac{(1-\sigma)A}{pw\,(1-t_h)} \\ V_k(m,k) &= \frac{A\left[1 + (r-\delta)\,(1-t_k)\right]}{w\,(1-t_h)} - \sigma c_k(q,k). \end{aligned}$$

Inserting these into (2) and rearranging yields the analogs to (17)-(18) for this model,

$$\begin{aligned} c_q(q,K)q &= \frac{\beta c_q(q_{+1},K_{+1})q_{+1}}{1+\tau}\left[1 - \sigma + \sigma\frac{(1-t_q)\,u'(q_{+1})}{c_q(q_{+1},K_{+1})}\right] && (24) \\ U'(X) &= \beta U'(X_{+1})\left\{1 + \left[F_K(K_{+1},H_{+1}) - \delta\right](1-t_k)\right\} && (25) \\ & \quad -\sigma\beta\,(1+t_x)\,c_k(q_{+1},K_{+1}) \end{aligned}$$

[10] Rocheteau and Wright (2005) also consider a third mechanism, which involves the combination of price posting and directed search. In the presence of "search externalities" this actually leads to better outcomes than Walrasian pricing, but for what we do they are equivalent.

The other equilibrium conditions are the same, and are repeated here for convenience:

$$U'(X) = \frac{A(1+t_x)}{F_H(K,H)(1-t_h)} \tag{26}$$

$$X + G = F(K,H) + (1-\delta)K - K_{+1}. \tag{27}$$

Monetary equilibrium with price taking is defined by (positive, bounded) paths (q, K_{+1}, H, X) satisfying (24)-(27), given K_0.

The difference between bargaining and price taking is seen by comparing (17)-(18) and (24)-(25); these differ in general because $g(q,K) \neq c_q(q,K)q$, $g_q(q,K) \neq c_q(q,K)$ and $\gamma(q,K) \neq c_k(q,K)$. Concentrating on steady states, with price taking the analogs of (21)-(22) are

$$1 + \frac{i}{\sigma} = \frac{(1-t_q)\,u'(q)}{c_q(q,K)} \tag{28}$$

$$\rho = [F_K(K,H) - \delta]\,(1-t_k) - \sigma\,(1+t_x)\,\frac{c_k(q,K)}{U'(X)}. \tag{29}$$

The first of these is the same as in the bargaining model iff $\theta = 1$, and the second is the same iff $\theta = 0$. Thus, competitive pricing eradicates the holdup problems in money demand and investment. The only remaining distortion, other than taxes, is the standard wedge associated with discounting that is eliminated if $i = 0$. Hence, if we use only the lump sum tax T and run the Friedman rule, in the price-taking model, we get the first best.

3 Extensions

3.1 An Example

We begin with an example to show the model is very tractable. Moreover, this example is is what we will calibrtate in the next section. To ease the presentation, we focus here on price taking. We use the following functional forms:[11]

$$U(X) = B\frac{X^{1-\varepsilon} - 1}{1-\varepsilon}$$

$$u(q) = \frac{(q+b)^{1-\eta} - b^{1-\eta}}{1-\eta}$$

$$F(K,H) = K^{\alpha}H^{1-\alpha}$$

$$c(q,K) = q^{\psi}K^{1-\psi}$$

[11] The cost function comes from $\eta(\ell) = \ell$ and $q = e^{\chi}k^{1-\chi}$ where $0 < \chi < 1$, so $\psi = 1/\chi > 1$. The other parameters satisfy B, ε, η, $b > 0$ and $0 < \alpha < 1$.

The only nonstandard parameter is b, which guarantees $u(0) = 0$ for any $\eta > 0$; here we typically set $b \approx 0$, so that $u(q)$ is approximately CRRA.

With these functional forms, (24)-(27) become

$$\frac{K^{1-\psi}}{q^{-\psi}} = \frac{\beta}{1+\tau}\left[(1-\sigma)\frac{K_{+1}^{1-\psi}}{q_{+1}^{-\psi}} + \sigma\left(1-t_q\right)\left(q_{+1}+b\right)^{-\eta}\psi q_{+1}\right] \tag{30}$$

$$\frac{X_{+1}^{\varepsilon}}{X^{\varepsilon}} = \beta\left[\alpha\left(\frac{K_{+1}}{H_{+1}}\right)^{\alpha-1} + 1 - \delta\right]\left(1-t_k\right) - \frac{\sigma\beta\left(1+t_x\right)\left(1-\psi\right)}{B}X_{+1}^{\varepsilon}\frac{K_{+1}^{-\psi}}{q_{+1}^{-\psi}} \tag{31}$$

$$X = \left[\frac{(1-\alpha)\left(1-t_h\right)}{A\left(1+t_x\right)}B\frac{K^{\alpha}}{H^{\alpha}}\right]^{1/\varepsilon} \tag{32}$$

$$X = K^{\alpha}H^{1-\alpha} + (1-\delta)K - K_{+1} - G \tag{33}$$

Letting $\Bbbk = K/H$, we can write (33) as

$$\frac{X}{H} = \left(\frac{K}{H}\right)^{\alpha} + (1-\delta)\frac{K}{H} - \frac{K_{+1}}{H_{+1}}\frac{H_{+1}}{H} - \frac{G}{H},$$

or, using (32),

$$\frac{\Bbbk}{K}\left[\frac{(1-\alpha)\left(1-t_h\right)}{A\left(1+t_x\right)}\Bbbk^{\alpha}\right]^{1/\varepsilon} = \Bbbk^{\alpha} + (1-\delta)\Bbbk - \frac{H_{+1}}{H}\Bbbk_{+1} - \frac{G}{K}\Bbbk.$$

Hence, in steady state

$$K = \frac{\Bbbk^{1-\alpha}\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}B\Bbbk^{\alpha}\right]^{1/\varepsilon}}{\left[1-\left(\delta+\frac{G}{K}\right)\Bbbk^{1-\alpha}\right]}. \tag{34}$$

Similarly, with $b \approx 0$, in steady state (30)-(32) reduce to

$$q = \left[\frac{\sigma\left(1-t_q\right)}{\psi\left(i+\sigma\right)}\right]^{\frac{1}{\psi+\eta-1}} K^{\frac{\psi-1}{\psi+\eta-1}} \tag{35}$$

$$X = \left[\frac{(1-\alpha)\left(1-t_h\right)}{A\left(1+t_x\right)}B\Bbbk^{\alpha}\right]^{1/\varepsilon} \tag{36}$$

$$\begin{aligned} 1 = {} & \beta\left[1+\left(\alpha\Bbbk^{\alpha-1}-\delta\right)\left(1-t_k\right)\right] \\ & -\frac{\sigma\beta(1-\psi)(1-\alpha)(1-t_h)}{A}\left[\frac{\sigma(1-t_q)}{\psi(i+\sigma)}\right]^{\frac{\psi}{\psi+\eta-1}}\Bbbk^{\frac{\alpha(\psi+\eta-1)-(1-\alpha)\psi\eta}{\psi+\eta-1}}\left\{\frac{1-\left(\delta+\frac{G}{K}\right)\Bbbk^{1-\alpha}}{\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}B\Bbbk^{\alpha}\right]^{1/\varepsilon}}\right\}^{\frac{-\psi\eta}{\psi+\eta-1}} \end{aligned} \tag{37}$$

Now (37) is one equation in $\Bbbk$, the RHS of which approaches infinity as $\Bbbk \to 0$ and approaches a value less than 1 as $\Bbbk \to \left[1-\left(\delta+\frac{G}{K}\right)\right]^{\frac{1}{1-\alpha}}$. So it has a solution. Moreover, this solution is unique if we assume $\alpha\left(\psi+\eta-1\right) < (1-\alpha)\psi\eta$, as this implies the RHS is strictly decreasing. Given the solution for $\Bbbk$, (34) yields K, (35) yields q, (36) yields X, and $H = \Bbbk/K$. This example not only shows the model is easy to study, it also shows exactly how we solve it in the computational exercises. The only complication is that under bargaining, insted of price taking, we cannot get the closed-form for q as a function of $\mathbb{k}$, so we procede numerically.

3.2 Two Capital Goods

So far, the same stock of physical capital k was an input to both CM and DM production. Suppose there are two types of capital: k is used in the CM and z is used in the DM (production of both capital goods still occurs in the CM here, but see below). This helps develop some more intuition for our results and illustrates how things can be generalized. Let the depreciate rates be δ_k and δ_z. Individuals do not earn income on z capital, and so it is not taxed directly.

The problem in the CM is now

$$\begin{aligned} W(m,k,z) &= \max_{x,h,m_{+1},k_{+1},z_{+1}} U(x) - Ah + \beta V(m_{+1},k_{+1},z_{+1}) \\ \text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + [1+(r-\delta_k)\,(1-t_k)]\,k - k_{+1} - T + \left(\frac{m-m_{+1}}{p}\right) \\ &\quad + (1-\delta_z)\,z - z_{+1}. \end{aligned}$$

Following the same methods as above, first order conditions are

$$\begin{aligned} x &: U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \\ m_{+1} &: \frac{A\,(1+t_x)}{pw\,(1-t_h)} = \beta V_m(m_{+1},k_{+1},z_{+1}) \\ k_{+1} &: \frac{A}{w\,(1-t_h)} = \beta V_k(m_{+1},k_{+1},z_{+1}) \\ z_{+1} &: \frac{A}{w\,(1-t_h)} = \beta V_z(m_{+1},k_{+1},z_{+1}), \end{aligned}$$

and the envelope conditions are for W_m, W_k and W_z are derived in the obvious way. The usual logic implies the distribution of (m,k,z) is degenerate for agents leaving the CM. In the DM, everything is as before except we replace $c(q,k)$ with $c(q,z)$.

Consider bargaining in the DM (price-taking is similar). The bargaining solution is the same except $g(q,z_s)$ replaces $g(q,k_s)$, and the DM value function and envelope conditions are derived in the usual way. This leads to the following equilibrium conditions:

$$
\begin{aligned}
g(q,Z) &= \frac{\beta g(q_{+1},Z_{+1})}{1+\tau}\left[1-\sigma+\sigma\frac{u'(q_{+1})}{g_q(q_{+1},Z_{+1})}\right] && (38)\\
U'(X) &= \beta U'(X_{+1})\left\{1+\left[F_K(K_{+1},H_{+1})-\delta_k\right](1-t_k)\right\} && (39)\\
U'(X) &= \beta U'(X_{+1})\left[1-\delta_z-\frac{(1+t_x)\,\sigma\gamma(q_{+1},Z_{+1})}{U'(X_{+1})}\right] && (40)\\
U'(X) &= \frac{A\,(1+t_x)}{F_H(K,H)\,(1-t_h)} && (41)\\
X+G &= F(K,H)+(1-\delta_k)\,K-K_{+1}+(1-\delta_z)\,Z-Z_{+1} && (42)
\end{aligned}
$$

Condition (38) is equivalent to (17) except Z replaces K. Condition (39) is the standard condition for K from the one-sector growth model, and in particular, in contrast to (18) it is not augmented by $\gamma(\cdot)$ since K is not used in the DM. The $\gamma(\cdot)$ term shows up in (40), the new condition for Z.

Even though K has no direct effect on the DM, Z does, and the latter is still produced in the CM. Hence this model does not dichotomize, and again an increase in i reduces Z, which affects (K_{+1},H,X).[12] For $\theta=1$, $\gamma(\cdot)=0$, and $i=0$ generates the efficient q conditional on Z since there is no holdup problem in money demand. But $\theta=1$ implies $Z=0$, since there is a big holdup problem in Z investment. This differs from the benchmark model, where K is used in both markets, and so there is reason to invest even if sellers get no surplus in the DM. With competitive pricing, however, again both holdup problems would vanish.

3.3 Capital Produced in DM

So far all investment occurs in the CM. We now consider the alternative, where k is produced in the DM, so that cash is needed to invest. The idea that at least some types of capital are traded in decentralized markets seems plausible, but it is also interesting in terms of the literature, since it has been known since Stockman (1981) that in cash-in-advance models it matters a lot whether money is needed to invest. Thus, here we assume agents do not consume the output of the DM, but use it as an intermediate input they transform one-for-one into k, which is then used for production in the CM.[13] Assume that each period a

[12] Consider, however, the special case where $\delta_z=0$, and focus on steady states. Then $Z_{+1}=Z$ drops out of (42), and (K,H,X) solves the steady-state versions of (39), (41) and (42), independently of (q,Z). Hence, in this special case, money affects (q,Z) but not (K,H,X).

[13] Shi (1999) discussed a related model.

fraction σ of agents can produce the intermediate input, the same fraction can transform it into capital, and none can do both. Also, we assume for now that k is not only not produced, it also not traded in the CM – but we argue below that this is not restrictive.

Then the CM problem is

$$\begin{aligned} W(m,k) &= \max_{x,h,m_{+1}} \{U(x) - Ah + \beta V(m_{+1},k)\} \\ \text{s.t. } (1+t_x)\, x &= w\,(1-t_h)\, h + [1+(r-\delta)\,(1-t_k)]\, k - T + \frac{m-m_{+1}}{p}. \end{aligned}$$

The first-order conditions are

$$x \quad : \quad U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \tag{43}$$

$$m_{+1} \quad : \quad \frac{A}{pw\,(1-t_h)} = \beta V_m(m_{+1},k), \tag{44}$$

and the envelope conditions are still given by (3) and (4). Notice agents do not adjust k here, they simply carry what they have to the DM.

Since k is obtained in the DM, now individual capital depends on one's history of and hence there is a distribution of k across agents, say $\mathcal{F}_k(k)$. Since the first-order condition for m_{+1} appears to depend on k it is not yet obvious if the distribution of m_{+1} is degenerate; we now show that it is. Consider bargaining in the DM (price-taking is similar). The buyer gives up d units of money for q units of the intermediate good which produces $k = q$. The usual methods imply $d = m_b$, and $q = q(m_b)$ is the solution to $\tilde{g}(q) = m_b/p$ where now[14]

$$\hat{g}(q) = \frac{\theta c(q) + (1-\theta) q c'(q)}{\theta \frac{A(1-t_q)}{w(1-t_h)} + (1-\theta)\frac{c'(q)}{1+(r-\delta)(1-t_k)}}.$$

Hence, $\partial q/\partial m_b = 1/p\hat{g}'(q)$, and q is independent of (m_s, k_b, k_s).

By the usual methods, the DM value function is

$$\begin{aligned} V(m,k) &= W(m,k) + \sigma \left\{ \frac{A\,[1+(r-\delta)\,(1-t_k)]\, q_b}{w\,(1-t_h)} - \frac{Ad_b}{pw\,(1-t_h)} \right\} \\ &\quad + \sigma \int \left\{ \frac{Ad_s(\tilde{m})}{pw\,(1-t_h)} - c\,[q_s\,(\tilde{m})] \right\} d\mathcal{F}_m(\tilde{m}) \end{aligned}$$

where in the last term we integrate with respect to the (marginal) distribution of money holdings across agents to whom one may sell, $\mathcal{F}_m(\tilde{m})$.[15] Differentiating and using $\partial q/\partial m_b =$

[14] Comparing this with the bargaining solution in the base model, given in (13) and (14), the difference is simply due to the fact that here agents are trading capital $k = q$ rather than consumption goods.

[15] This integral did not appear in the earlier models, because we knew from the CM problem that the distribution of (m,k) was degenerate. Here the distribution of k is nondegenerate, and we have yet to prove the distribution of m is degenerate.

$1/p\hat{g}'(q)$,

$$V_m(m,k) = \frac{A}{pw\,(1-t_h)}\left[1-\sigma+\sigma\frac{1+(r-\delta)\,(1-t_k)}{\hat{g}'(q)}\right].$$

Since $V_m(m,k)$ is independent of k, according to (44), everyone still chooses the same m_{+1}. Hence, in the DM, F_m is again degenerate even if F_k is not.

Inserting $V_m(m,k)$, $p = M/\tilde{g}(q)$, $w = F_H(K,H)$, $r = F_K(K,H)$ and $M_{+1} = (1+\tau)M$ into (44), we arive at

$$\frac{\hat{g}(q)}{F_H(K,H)} = \frac{\beta}{1+\tau}\frac{\hat{g}(q_{+1})}{F_H(K_{+1},H_{+1})}\left\{1-\sigma+\sigma\frac{1+\left[F_K(K_{+1},H_{+1})-\delta\right](1-t_k)}{\hat{g}'(q_{+1})}\right\}. \tag{45}$$

The other equilibrium conditions are

$$K_{+1} = (K+\sigma q)(1-\delta) \tag{46}$$

$$U'(x) = \frac{A\,(1+t_x)}{F_H(K,H)\,(1-t_h)} \tag{47}$$

$$X+G = F(K,H)+(K+\sigma q)(1-\delta). \tag{48}$$

This model does not dichotomize: we cannot solve for q without (K,H), since these determine r and w and hence the value of the capital being purchased with cash – they not only enter (45) directly, but also through $\hat{g}(q)$. Intuitively, increase in τ and hence i lowers the value of money, so sellers produce less intermediate goods in the DM, and K is lower. This is similar to Stockman (1981).

What if agents can trade k in the CM? This is a secondary market: no net investment occurs, only a reallocation of k. Let p^k denote the price of existing capital. In the CM problem, the first order condition for k_{+1} is

$$\frac{Ap^k}{w\,(1-t_h)} = \beta V_k(m_{+1},k_{+1}).$$

Using $V_k = W_k = Ap^k_{+1}\left[1+(r_{+1}-\delta)(1-t_k)\right]/w_{+1}(1-t_h)$, we have

$$\frac{p^k}{F_H(K,H)} = \frac{\beta p^k_{+1}\left\{1+\left[F_k(K_{+1},H_{+1})-\delta\right](1-t_k)\right\}}{F_H(K_{+1},H_{+1})}.$$

This is independent of individual k: it merely pins down the path for p^k in the secondary market. Agents are indifferent about trading k, given p^k, and the distribution F_k is not pinned down. Hence, it is not restrictive to not let k trade in the CM.[16]

[16] A detail here is that we need to assume we are at an interior solution, $0 < h < \bar{h}$, in the LW framework for many of the results. There are several ways around this problem, the simplest being to assume $U(x) = x$.

4 Quantitative Analysis

In this section we retrun to the basic model discussed Section 2 and consider numerical analysis. In particular, for each pricing mechanism, we calibrate the model, solve for the decision rules, and compute the welfare cost of inflation. The cost of inflation has been analyzed in related models elsewhere, of course.[17] But things are much more interesting in models with capital that do not dichotomize, since one needs to compute transition paths.

We begin with some simple accounting. The price levels in the DM and CM are $P_D = M/q$ and $P_C = p$, and so nominal outputs are $\sigma P_D q = \sigma M$ and $pF(K, H)$. As a convention, we adopt p as the unit of account in which we convert all nominal variables into real terms. Hence, real GDP is

$$Y = \sigma \frac{M}{p} + F(K, H). \tag{49}$$

The price level satisfies

$$p = \frac{AM}{(1 - t_h)\, g(q, K)\, F_h(K, H)} \tag{50}$$

in the bargaining version by virtue of (13), and satisfies

$$p = \frac{(1 - t_q)\, AM}{(1 - t_h)\, c_q(q, K)\, q F_h(K, H)} \tag{51}$$

in the price-taking version by virtue of (23).

4.1 Calibration

[To be completed]

4.2 Equilibrium and Welfare

From now on we focus on stationary equilibria, where M/p is constant, so that the inflation rate equals the rate of money creation, τ. As is standard, we let $\tilde{m} = m/M$ and $\tilde{p} = p/M$, so that the individual state variable becomes $(\tilde{m}, k, K)$. In equilibrium, $\tilde{m} = 1$ and $k = K$. A stationary (recursive) equilibrium is then described by a list of time-invariant functions $[q(K), K_{+1}(K), H(K), X(K)]$, solving (17)-(20) for the bargaining version and (24)-27) for the price-taking version, and a value function $V(K)$ solving (5). We solve these

[17] See Rocheteau and Craig (2005) for a recent survey of this work.

equations numerically using the Weighted Residual Method with Chebyshev Polynomials and Orthogonal Collocation.[18] Figure 3 plots the decision rules for a typical parametrization.

For expositional purposes, let $[\bar{q}(K,\tau), \bar{K}_{+1}(K,\tau), \bar{H}(K,\tau), \bar{X}(K,\tau)]$ and $\bar{V}(K;\tau)$ describe equilibrium and the value function given inflation rate τ. Given τ, steady state solves $K_\tau = \bar{K}_{+1}(K_\tau;\tau)$. Generally, there is a transition path after a change in τ, and our welfare computations take this in to account. Thus, our comparisons are between $V(K_{\tau_1};\tau_1)$ and $V(K_{\tau_1};\tau_2)$; $V(K_{\tau_2};\tau_2) - V(K_{\tau_1};\tau_2)$ is the welfare loss during the transition. For reporting welfare results we use the standard measure: we solve for the Δ such that agents are indifferent between τ_1 and τ_2 if under τ_2 we increase both DM and CM consumption by $\Delta - 1$ percent.

5 Results

5.1 Main Results

[To be completed]

5.2 Robustness

[To be completed]

6 Conclusions

[To be completed]

[18] See Judd (1992) for details, and Aruoba et al. (2003) for a recent comparison of different solution methods.

A Cost Function

Suppose $q = \phi(k, \ell)$ is strictly increasing and concave. Saying k is normal means that in the problem

$$\min \{w\ell + rk\} \text{ s.t. } \phi(k, \ell) \geq q,$$

the solution satisfies $\partial k/\partial q = \phi_\ell \phi_{\ell k} - \phi_k \phi_{\ell\ell} > 0$. We then have $\ell = \Phi(q, k)$, $\partial \ell/\partial q = \Phi_q = 1/\phi_\ell > 0$ and $\partial \ell/\partial k = \Phi_k = -\phi_k/\phi_\ell < 0$. Also, $\Phi_{qq} = -\phi_{\ell\ell}/\phi_\ell^3 > 0$, $\Phi_{kk} = -\left(\phi_\ell^2 \phi_{kk} - 2\phi_\ell \phi_k \phi_{k\ell} + \phi_k^2 \phi_{\ell\ell}\right)/\phi_\ell^3 > 0$, and $\Phi_{kq} = -\left[\phi_{\ell k}(k, \ell)\phi_\ell(k, \ell) - \phi_{\ell\ell}(k, \ell)\phi_k(k, \ell)\right]/\phi_\ell(k, \ell)^3$. Therefore $c_q = v'/\phi_\ell > 0$, $c_k = -v'\phi_k/\phi_\ell < 0$, $c_{qq} = \left[v'' (\eta')^2 \phi_\ell - v'\phi_{\ell\ell}\right]/\phi_\ell^3 > 0$, $c_{kk} = -\left[v'\left(\phi_\ell \phi_{kk} - 2\phi_\ell \phi_k \phi_{k\ell} + \phi_k^2 \phi_{\ell\ell}\right) - \phi_\ell \phi_k^2 \eta''\right]/\phi_\ell^3 > 0$ and $c_{qk} = -\left[v'' \phi_\ell \phi_k - v'\left(\phi_k \phi_{\ell\ell} - \phi_\ell \phi_{\ell k}\right)\right]/\phi_\ell^3$.

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