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Testing Factor-Model Explanations of Market Anomalies

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- Abstract -

A set of recent papers attempts to explain the size and book-to-market anomalies with either: (1) conditional CAPM or Consumption-CAPM models with economically motivated conditioning variables, or (2) factor models based on economically motivated factors. The tests of these models use similar methodologies, similar test assets, and each test fails to reject the proposed model. This is surprising, as the correlation between the proposed factors is very small. We argue that many or all of these tests may fail to reject as a result of low statistical power. We propose an alternative test methodology which provides higher power against reasonable alternative hypotheses, and show that the new test methodology results in the rejection of several of the proposed factor models at high levels of statistical significance.

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1 Introduction

The Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model suggests that the expected returns of risky assets should be determined by the covariance of their returns with the returns on the market portfolio. However, studies by Fama and French (1992) and others uncover almost no relation between market betas and expected returns, and instead find a strong cross-sectional relation between characteristics like size and book-to-market and returns. The reason why this is the case has been debated for over 10 years, and there is still no consensus. One hypothesis is that a firm's size and book-to-market serve as proxies for the riskiness of the firm. Another possibility is that these characteristics proxy for mispricing, *i.e.*, high book to market stocks have higher expected returns because they are undervalued.

The results of Fama and French (1993) appear to provide support for the first hypothesis by showing that a set of factor-mimicking portfolios, MKT, SMB and HML, do a fairly good job of pricing the cross-section of stock returns. However, for several reasons this evidence is not particularly satisfying. First, the SMB and HML factors are not economically motivated; they are simply the returns from the financial assets that others conjecture are potentially mispriced. Indeed, Daniel and Titman (1997) argue that even if mispricing is responsible for the cross-sectional differences in returns, factor portfolios that are constructed in this way would still price size and book-to-market sorted portfolios.

Largely because of such concerns, a set of more recent papers have attempted to discover the underlying risks that might be responsible for the observed return patterns. If we abstract from liquidity and behavioral considerations, an asset's expected return is determined by the covariance between its realized returns and a representative agent's marginal utility, suggesting that the returns of the SMB and HML portfolios must in some ways capture the variation of more fundamental economic factors that are correlated with marginal utility. Researchers have investigated a number of such economically-motivated factor models.

We divide these models into two categories: *Conditional (C)CAPM Models* and *Alternative Factor Models*. The conditional versions of the CAPM and Consumption-CAPM (CCAPM) of Breeden (1979) are motivated by the strong rejections of tests of unconditional versions of these models. The conditional models retain the basic structure of the CAPM or CCAPM, but allow for time-variation in the covariation of asset returns with the market return (or consumption growth, in the case of the CCAPM), and time variation in the premium associated with this covariation.

Conditional CAPM models can be written as unconditional multifactor models where one factor is the market return, and the second factor is the market return interacted with a conditioning variable (Cochrane (1996)). Similarly, a Conditional-CCAPM model can be expressed as a multi-factor model with factors equal to consumption growth and consumption growth interacted with a conditioning variable. Thus, the models effectively augment the market return (or consumption growth) with an additional factor equal to the scaled market or to scaled consumption growth.

Alternative Factor Models propose a factor other than the standard market portfolio return or consumption growth as a pricing kernel. Some of the proposed models are unconditional (time-invariant), while others are conditional, meaning that the premium associated with the factor covariation, and the premium associated with this covariation are time varying. Based on the logic of Cochrane (1996), such conditional factor models can be tested as scaled multifactor models with additional factors equal to the factors scaled by instruments which capture the time variation.

Thus, both the Conditional (C)CAPM Models and Alternative Factor Models argue that it is some additional factor beyond the standard market return or consumption growth that is missing from the standard (C)CAPM, and that once this factor is accounted for, the model will capture the value effect.

The models that have been proposed and tested are based on a number of plausible economic stories for why value stocks might be riskier than growth; a subset of these tests are listed in Table 1. Moreover, based on the associated empirical tests, it appears that these models do each capture the value effect; these tests generally fail to reject these proposed factor models. Based on these results, it appears that there are a number of plausible economic factors can explain the value effect.

Table 1: **Proposed Factor Models**

This table lists a subset of the factor models examined in the finance literature, and the factors and conditioning variables considered in these tests.

Paper	Factor(s)	Cond. Vars.
Conditional (C)CAPM Models		
Ferson and Harvey (1999)	VW	S&P 500 Dividend Yield[1]
Lettau and Ludvigson (2001)	VW or Cons Growth	*cay*
Santos and Veronesi (2001)	VW + Labor Income Growth	Labor Income to Cons Ratio (s)
Petkova and Zhang (2003)	VW Index	$E[R_m]$ based on BC Vars
Alternative-Factor Models		
Fama and French (1993)	VW, HML, SMB	
Jagannathan and Wang (1996)	Labor Income Growth	DEF
Heaton and Lucas (2000)	Proprietary Income Growth	
Piazzesi, Schneider, and Tuzel (2003)	Cons Growth $+\Delta$ NH Expenditure Ratio ($\Delta log(\alpha)$)	Non-Housing Expenditure Ratio (α)
Lustig and Nieuwerburgh (2002)	Scaled Rental Price Change ($A\Delta log\rho$)	Housing Collateral Ratio
Aït-Sahalia, Parker, and Yogo (2003)	Luxury Good Consumption	
Li, Vassalou, and Xing (2002)	Sector Inv. Growth Rates	
Parker and Juillard (2003)	Innovations in Future Long Horizon Consumption Growth	
Campbell and Vuolteenaho (2004)	CF and DR news	

While at first glance these results appear promising, there are several reasons to question these findings. The first concern is that these results present a conundrum for anyone attempting to use the models. Which, if any, of these dozen or so models is the correct one to use in determining cost of capital? The results in these papers offer no answer to this question, as each of the proposed models appears to "work" reasonably well. If it were the case that each of the factor models would give about the same answer, this might not be a concern. However, the correlations between the proposed factors are actually very low, suggesting that the different models should yield very different expected returns/costs of capital.

A second related concern is that asset pricing theory dictates that there is a unique factor in the span of the asset return space that prices all assets.[2] This implies that the only way that two single factor models can each price the full cross-section of returns is if the projections of each of these factors onto the asset return space are equal. In other words, if the factors in two proposed single factor models both lie in the asset return space, then *at most one of the two models can be correct.*

A final concern is that several studies suggest problems with the proposed conditional (C)CAPM specifications. A recent paper by Lewellen and Nagel (2003) argues that the covariance of the conditional expected return on the market and of the conditional market betas of high and low book-to-market stocks is not high enough to explain the value effect. Also, Hodrick and Zhang (2001) find large specification errors for the Lettau and Ludvigson (2001) conditional CCAPM model. However, tests of these conditional CAPM models fail to reject the models, again suggesting the possibility that the failure to reject is a result of low test power.

In this paper we attempt to explain why so many different factors, with such a low average correlations between them, each appear to explain the cross-section of returns. In Section 4 we argue that the culprit is the test methodology: each of these tests have been done in a similar way, often using exactly the same test assets. Specifically, empirical tests of the proposed models are generally performed using the 25 size and book-to-market portfolios first examined in Fama and French (1993), and the test methodology generally used is

[2] Following Hansen and Richard (1987), and Hansen and Jagannathan (1991), while in incomplete markets, multiple pricing kernels ($\tilde{m}$'s) may exist which price all assets, but there exists a unique projection of each of these pricing kernels onto the space of asset returns $\tilde{m}^*$.

that proposed by Fama and MacBeth (1973).

The motivation for the use of book-to-market (BM) sorted portfolios as test assets is intuitively appealing: we know that there is a strong empirical relation between book-to-market and average returns. Thus, the returns of these portfolios should prove difficult to explain with any asset pricing model. Intuitively, it seems that such sorts should, *a priori*, produce higher test power.

However, BM is a "catch-all" variable, one that will proxy for sensitivity to a variety of macroeconomic innovations, sorting on BM will also produce a spread in loadings for a large set of factors. For example, because more of low-BM (growth) firm's value probably derives from its growth options, high and low BM firms are likely to have different sensitivities to business cycle innovations.

Thus, book-to-market sorted portfolios are likely to produce both variation in expected-returns and (correlated) variation in the loadings on any number of macroeconomic factors. Moreover, in grouping all of the assets with similar BM together, any variation in factor loading that is independent of BM is largely eliminated. The end result is that, even if the loadings on a proposed factor are only loosely correlated with the expected returns of the individual assets in the economy, the sorting procedure will result in a set of test portfolios where there is a close to perfect linear relation between loadings on the proposed factor and expected returns. The problem is that, in grouping all of the assets with similar BM together, any variation in factor loading that is independent of BM is washed out.

A slightly more technical way of seeing this is as follows: an asset pricing model will explain the average returns of a set of portfolios if and only if the pricing kernel implied by that model prices the test assets. Since the payoffs of any set of test assets will not span all sources of risk, there will not be a unique pricing kernel (a unique $\tilde{m}$). However, there is a unique pricing kernel that lies in the span of the payoffs/returns of the test assets (a unique $\tilde{m}^*$). Moreover, *any* model with an implied pricing kernel $\tilde{m}_i = \tilde{m}^* + \tilde{e}_i$, where $\tilde{e}_i$ is outside of the space spanned by the payoffs of the test assets, will properly price the set of test assets.

The problem with the use of the 25 size-BM sorted portfolios as test assets is that their payoffs lie in a low-dimensional subspace of the full payoff space. Specifically, Fama and French (1993) show that the average R^2 in time-series regressions of the returns of their 25 portfolios on their three stock market factors is 93%, and there is little variation in the loading on the market factor.[3] This means that, to a close approximation, the returns of the 25 FF portfolios lie in a 2-dimensional subspace spanned by HML and SMB.

Thus, many sources economic risk can be expected to lie outside the span of the returns of these test assets, even if these sources of risk could be hedged using other portfolios of stocks. Moreover, if the risk-premium associated with each factor is left as a free parameter, as is generally done in the Fama and MacBeth (1973) procedure, any factor which is loosely correlated with the m^* implied by HML and SMB will appear to properly price these test assets, even if this model would not properly price a fuller set of assets.

This means that a powerful test requires that the test assets span a higher dimensional space. Specifically, the test assets should be augmented by portfolios which are highly correlated with the proposed factor.

To construct such portfolios requires the use of an instrument which is correlated with variation in loading on the proposed factor, and which is imperfectly correlated with book-to-market. In our empirical tests below, we use two sets of instruments: first, we use estimates of lagged betas on the proposed factors; second, we use industry membership. Industry portfolios exhibit variation in factor loadings relative to a number of macroeconomic factors but this variation is, at least to some extent, unrelated to book-to-market ratios.

Using these instruments to form portfolios, we reexamine several of the models proposed in the literature. Based on this preliminary analysis, we argue that before any of these models can be acceped as an full explanation of the book-to-market effect more powerful

[3]See Table 6 of Fama and French (1993). Reported regression R^2s range from 83% to 97%. Reported loadings on $[RM(t) - RF(t)]$ range from 0.91 to 1.18.

tests, based on the framework we lay out here, need to be carried out.

The outline of the remainder of the paper is as follows. In Section 2 we discuss the conditional and unconditional tests and motivate the empirical design of the tests we discuss. In Section 4, we evaluate the power of the proposed tests both via some basic analytical results and a set of simulations. More formally, we do this by proposing an alternative hypothesis, and argue that this test methodology yields low power against this alternative. Second we propose a methodology that has higher statistical power. In Sections 5 and 6, we apply this new methodology to test several recent alternative factor models, and find that these model are rejected at high levels of significance with the new methodology. Section 7 concludes.

2 The Equivalence of Conditional (C)CAPM Tests and Multi-Factor Models

In this section, we show that conditional CAPM models can be written as unconditional multifactor models where one factor is the market return, and the second factor is the market return interacted with a conditioning variable (Cochrane (2000)). Similarly, a conditional-CCAPM model can be expressed as a multi-factor model with factors equal to consumption growth and consumption growth interacted with a conditioning variable.

The intuition behind this argument is best seen via an example: suppose that, the market is conditionally mean-variance efficient, which means that for all assets i:

$$(R_{i,t+1} - R_{f,t+1}) = \beta_{i,t}(R_{m,t+1} - R_{f,t+1}) + \epsilon_{i,t+1}$$

where $\epsilon_{i,t} \perp 1, (R_{m,t+1} - R_{f,t+1})$. Taking (conditional) expectations of each side gives:

$$E_t[R_{i,t+1} - R_{f,t+1}] = \beta_{i,t} E_t[R_{m,t+1} - R_{f,t+1}].$$

This is the usual statement of the conditional CAPM. However, suppose also that the expected return on the market varies over time. There is now substantial evidence consistent with this: market returns are high in business cycle troughs and low at business cycle peaks.

Notice that, if value stocks have a high beta (i.e., if $\beta >> 1$) when the market's expected return is high, and a low beta ($\beta << 1$) when the market return is low, then the average or unconditional beta of the value stock could be close to 1, but the unconditional expected return of the value stocks would be much higher than the market's. Intuitively, the value stocks are effectively "market timing," taking on more risk when the reward to risk ratio (*i.e.*, the expected return on the market) is higher. Thus, if the CAPM were tested unconditionally, it would be rejected.

The same argument suggests that, if the consumption-CAPM holds, but the premium to consumption beta varies over time and the consumption beta of value value stocks positively covaries with this premium, a test of the unconditional CCAPM will be rejected.

A remedy to this problem is to test a conditional version of the CAPM or CCAPM. This is typically done by assuming the asset's market beta is a linear function of a n-dimensional vector of instruments $\mathbf{Z}_t$ in the investors information set at t. The restriction that is then tested is that the conditional loadings on the factors explains the average returns of the test assets, *i.e.*, that:

$$(R_{i,t+1} - R_{f,t+1}) = (\boldsymbol{\beta}_i' \mathbf{Z}_t)(R_{m,t+1} - R_{f,t+1}) + \epsilon_{i,t+1}$$

where the restriction that is now tested is that $E[\epsilon_{i,t+1} \cdot (R_{m,t+1} - R_{f,t+1})\mathbf{Z}_t] = \mathbf{0}$.[4]

As has been pointed out in Cochrane (2000), this test is equivalent to a test of a multi-factor unconditional model in which the factors are the market, and a set of "scaled"

[4]Note that this is not the same as a *conditional* test of the conditional model. This would be a test of the restriction that $E[\epsilon_{i,t+1}\mathbf{Z}_t] = \mathbf{0}$. See also Appendix A.

market returns, that is:[5]

$$r_{i,t+1} = \beta_{1,i}\ r_{m,t+1} + \beta_{2,i}(r_{m,t+1}Z_{2,t}) + \cdots + \beta_{n,i}\ (r_{m,t+1}Z_{n,t}) + \epsilon_{i,t+1}$$

The important thing for us is that these tests are therefore equivalent to tests of the CAPM (or the CCAPM) plus one or more additional factors added on.

Thus, if the uncondtional CAPM fails and the conditional CAPM properly prices value and growth portfolios, it must be the case that it is covariation with the scaled market return that is responsible for the differences in expected return on the value and growth portfolios.

2.1 Conditional CAPM Model Tests

A number of (C)CAPM tests have now been proposed in the literature. One that has recently received a good deal of attention is that of Lettau and Ludvigson (2001). Lettau and Ludvigson argue that, based on the intuition behind the Campbell and Cochrane (1999) model, their *cay* variable should be a good proxy for the market risk premium.

In their 2001 paper, LL show that the betas of value/growth stocks are higher/lower when the expected return on the market is high (*i.e.*, when cay is low). Thus even if value stocks have a lower unconditional beta than do growth stocks, their betas are much higher when the expected return on the market is higher. Thus, were one to test the unconditionall CAPM on value/growth stocks, one could reject it. However, LL argue that once the conditional variation in beta and the expected return on the market is taken into account, the *conditional* CAPM and CCAPM do a good job of explaining these returns.

The methodology used by LL to test the conditional CAPM is similar to that used by many recent papers: they test whether the returns of the Fama and French (1993) 25

[5]Here, we are implicitly assuming that the first element of $\mathbf{Z}_t$ is one.

size/BM sorted portfolios can be explained by their conditional CAPM using Fama and MacBeth (1973) tests. They find that it does a very good job.

However, an alternative test of the LL conditional CAPM model is based on a Gibbons, Ross, and Shanken (1989)-style time-series regression (as employed by Fama and French (1993)). For example, one can examine whether return of the FF(93) HML portfolio can be explained using the single regression:

$$HML_t = \alpha + \beta_{vw} R^e_{vw,t} + \beta_{vwz} \ \widehat{cay}_{t-1} R^e_{vw,t} + e_t \tag{1}$$

For a conditional CAPM model such as this one, the interaction term $\widehat{cay}_{t-1} R^e_{vw,t}$ captures the extra return arising from the covariation of the HML beta with the expected return on the market, as discussed in the preceding subsection.

Using quarterly data over the period 1953:01-1998:04, the same period examined by LL, the estimated intercept ($\hat{\alpha}$) for this regression is 1.26%/quarter, ($t = 3.47$). This is both economically and statistically big. For comparison, without including the $\widehat{cay}_{t-1} R^e_{vw,t}$ interaction term, the α is 1.50% (t=4.16). The difference between the intercept terms in the two regressions is 0.24% /quarter, which is probably not statistically different from zero. This simple regression dramatically illustrates the point argued by Lewellen and Nagel (2003): while the betas of value stocks do increase in economic downturns, they don't increase anywhere near enough to explain the high returns of the HML portfolio.

However, given that this simple test so strongly rejects the LL model, why do the tests done by LL indicate such strong support for their model? The answer, we argue in the next section, is that the test methodology used by LL has extremely low power to reject their factor model.

Table 2: **Correlations of Candidate Factors**
This table presents the sample correlation matrix of a subset of the factors that have been proposed in the literature as explanations for the value premium. Each of the conditioning variables (DP, $\widehat{cay}$ and s) is demeaned. The sample correlations are each estimated using quarterly data over the period 1963Q4:1998Q3.

	HML	DP$\cdot r_m$	$\widehat{\text{cay}}\cdot r_m$	$s\cdot r_m$	$\widehat{\text{cay}}\cdot\Delta c$	Δy	Δ(prop)	$\Delta\log(\alpha)$	$-N_{CF}$
HML	1	-0.10	0.07	-0.05	0.06	0.01	0.07	0.11	0.27
DP$\cdot r_m$	-0.10	1	0.61	0.37	0.14	-0.01	0.04	0.00	-0.09
$\widehat{\text{cay}}\cdot r_m$	0.07	0.61	1	0.03	0.12	-0.03	-0.16	-0.00	-0.12
$s\cdot r_m$	-0.05	0.37	0.03	1	0.07	0.03	0.14	-0.07	0.07
$\widehat{\text{cay}}\cdot\Delta c$	0.06	0.14	0.12	0.07	1	0.13	0.10	-0.07	0.06
Δy	0.01	-0.01	-0.03	0.03	0.13	1	0.25	0.15	-0.10
Δ(prop)	0.07	0.04	-0.16	0.14	0.10	0.25	1	0.28	0.11
$\Delta\log(\alpha)$	0.11	0.00	-0.00	-0.07	-0.07	0.15	0.28	1	0.09
$-N_{CF}$	0.27	-0.09	-0.12	0.07	0.06	-0.10	0.11	0.09	1

3 Correlations of Candidate Factors

Table 2 shows the correlations of nine of the factors used in models that have been proposed as potential explanations for the value premium, specifically those listed in Table 1. The correlations are all calculated on a quarterly basis. Each of the conditioning variables (DP, $\widehat{cay}$ and s) is demeaned. The sample correlations are each estimated using quarterly data over the period 1963Q4:1998Q3.

Interestingly, the correlation matrix shows that the correlations of each of the factors with HML is low – the highest is a 27% correlation with $-N_{CF}$, the principal factor of Campbell and Vuolteenaho (2004). In addition, the correlations between the proposed factors is also for the most part quite small. Here, other than the correlations between DP, $\widehat{cay}$, and s interacted with the market, the maximum correlations between two factors is 28% (between the change in proprietary income and the log growth in the non-housing expenditure ratio.) Other than this the correlations are generally less than 20%.

4 The Power of Tests on Characteristic Sorted Portfolios

There are now more than a dozen factor models that explain the returns of portfolios sorted on size and book-to-market. It can't really be the case that all of these factor models are "correct," in the sense that they all explain the cross-section of stock returns. Thus, our first concern in this paper is explaining how it is that so many different factors, with such a low average correlation, seem to explain the cross-section of returns.

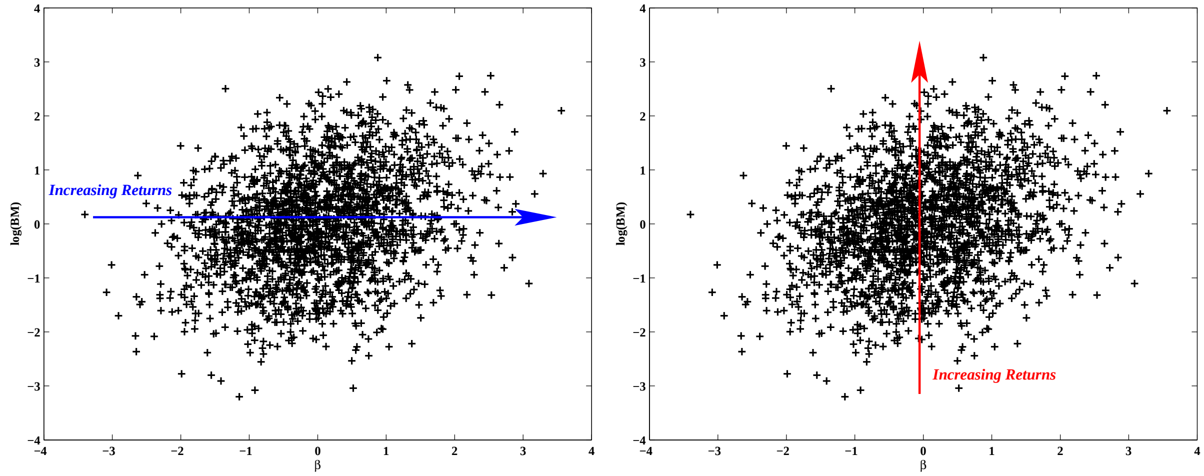
As we discussed in the introduction, our explanation for this is that these tests are designed in such a way that the test lack statistical power. Specifically we argue that, under very weak conditions, *any* factor will appear to explain the average returns of size and book-to-market sorted portfolios. That is, these tests will make it appear that returns are consistent with the factor model even when they are not.

To illustrate this point, this section presents a simulation that demonstrates how the FF-FM methodology can provide spurious support for a factor model. In addition, we use this simulation to motivate our approach for testing factor models against the characteristic alternative, an approach that we implement in our empirical tests in Section 5.

4.1 Simulation Results

The simulations presented here consider a factor that has been proposed to explain the observed cross-sectional relation between returns and characteristics. For example, innovations in housing price changes have been proposed as a factor that explains the book-to-market effect, which is known to be related to expected returns. To abstract from estimation problems we assume that we accurately measure both a firm's factor-beta and its expected return. In addition, we assume that factor betas are correlated with the characteristic.

Figure 1: Null and Alternative Hypotheses



In our simulations we randomly draw 2500 log book-to-market ratios and the single factor beta from a correlated normal distribution. Specifically, we draw from a multivariate normal distribution such that:

$$\begin{aligned} bm_i = \log(BM_i) &\sim \mathcal{N}(0,1) \\ \beta_i &\sim \mathcal{N}(0,1) \\ \rho(bm_i, \beta_i) &= \rho_{bm,\beta}. \end{aligned}$$

We assume a relatively weak correlation between the characteristic and the factor loading of $\rho = 0.3$, which is low enough to allow us to distinguish between the two hypotheses in an appropriately designed test.

Figure 1 illustrates the distribution of characteristics and factor loadings that are generated from the simulation. This figure has two plots. Consider the left side plot first. The vertical (y-axis) is the firm's log book-to-market ratio, and the horizontal (x-axis) is the factor beta. Each of the 2500 crosses in this figure represents a single firm or stock. The weak correlation can be seen in the distribution of the crosses: high β firms generally have high BM ratios, but there is considerable variation in βs that is unrelated to BM.

Figure 1 illustrates the null and alternative hypotheses we'll consider. The null hypothesis, that the factor model fully explains the cross-section of returns, is represented by the left plot. Under the null, a stock's expected return increases with β (as you move to the right in the plot), but is unrelated to book-to-market *after controlling for beta.* Of course, there is still an unconditional relation between book-to-market and expected returns.

This null hypothesis is a risk-based story for the book-to-market effect: a firm's book-to-market ratio forecasts its future return because it serves as is a proxy for systematic risk. Under this hypothesis, if the factor loading can be observed, it will better explain average returns than book-to-market ratios.

A test's power is defined as the probably of the test's rejecting the null hypothesis given that the alternative is true. Thus, test power can only be evaluated relative to an alternative hypothesis. The alternative hypothesis we propose is illustrated in the right hand side plot in Figure 1. Under the alternative, the expected return is linearly related to the log book-to-market ratio, but is not directly related to the factor beta. That is, the beta is related to returns only through its correlation with the characteristic.

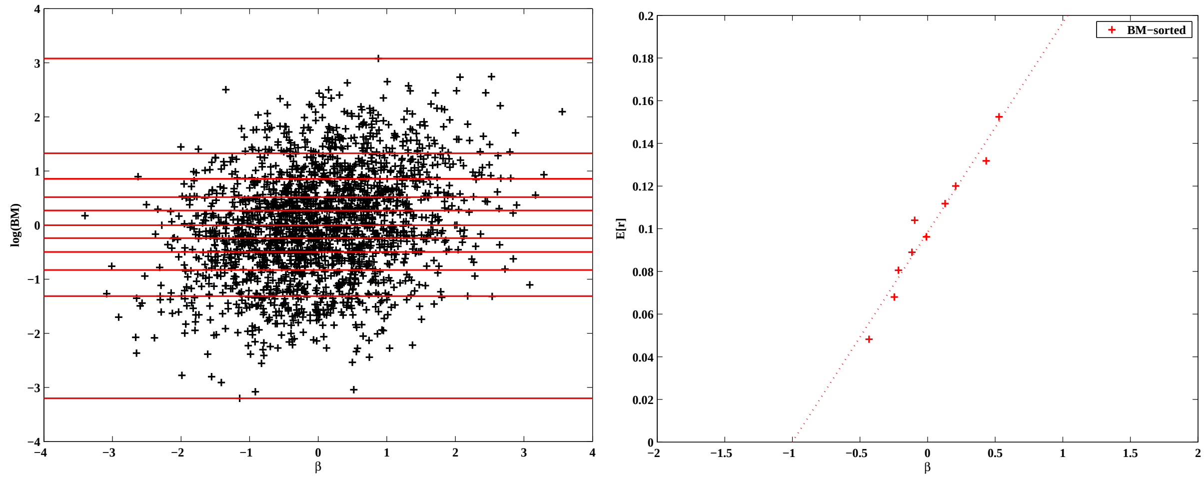
The intuition behind this alternative is two fold: first, book-to-market can be related to expected returns either because it serves as a mispricing-proxy, or as a proxy for misspecification in the asset pricing model being tested.[6] So one part of the alternative is that expected returns, for whatever reason, are directly linked to book-to-market.

The second part of the argument is that factor loadings on a variety of factors are likely to be correlated with book-to-market ratios. There are a number of good reasons why this should be the case. Theoretically, we know that a firm's book-to-market ratio is a good proxy for a firm's future growth (see, *e.g.*, Fama and French (1995), Cohen, Polk, and Vuolteenaho (2000)), and high- and low-growth firms are likely to have different sensitivities to a number of economic factors. Empirically, Table 1 shows that this is indeed the case.

The alternative captures the idea that these might not explain expected returns other than indirectly through their correlation with book-to-market.

[6]For the rational story, see Berk (1995). For the mispricing story, see Daniel, Hirshleifer, and Subrahmanyam (1998).

Figure 2: BM-Sorted Portfolio Formation



To evaluate power of the tests, we calculate expected returns under the different alternative hypothesis; that is:

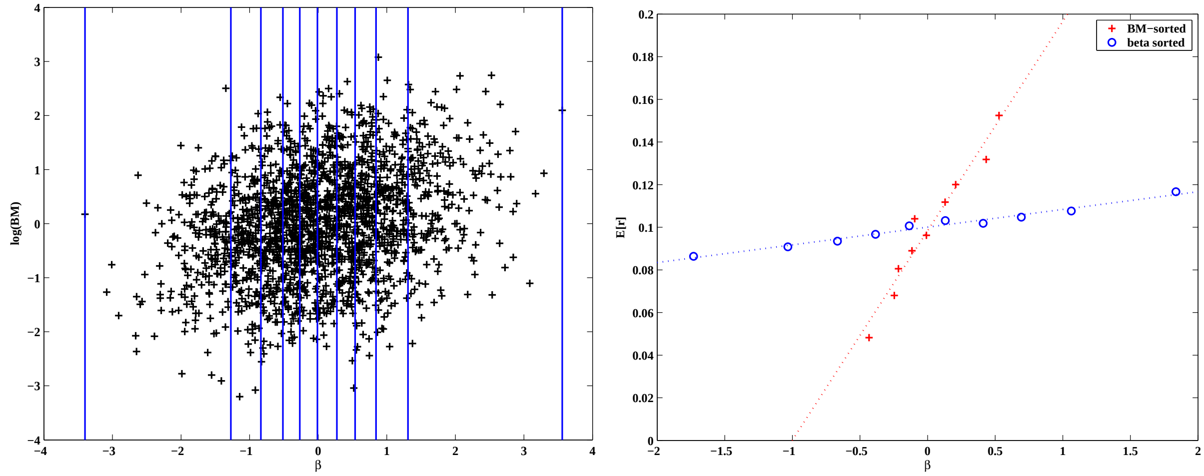
$$E[r] = \lambda_0 + \lambda_1 \log(B/M).$$

Following, the test procedures used in the literature, we then sort the 2500 (simulated) firms into 10 portfolios depending on their book-to-market ratios. This sort is illustrated in the left panel of Figure 2. Each horizontal line in the figure represents the cutoff between the BM deciles: the number of firms between any two lines is 250 (one-tenth of the sample).

This figure illustrates the problem that arises when characteristic sorted portfolios are used to test factor models. Notice that the top decile will have a high average BM ratio, and will therefore have a high expected return. In addition, it will have a high factor beta as a result of the correlation between beta and BM. As we move from the top to the bottom BM decile, the average return and the average (portfolio) beta declines.

Note also, that there is very little variation in betas in the different deciles, since differences in the betas that are not correlated with the characteristic is "diversified away" by the portfolio formation procedure. As a result, as we show in the right panel of Figure 2, which plots the expected returns and betas for these 10 portfolios, these variables are very highly correlated. The regression R^2 here is 94.4%. The reason is that for this set of characteristic-sorted portfolios, there is almost no independent variation in beta.

Figure 3: Beta-Sorted Portfolio and Cross-Sectional Test



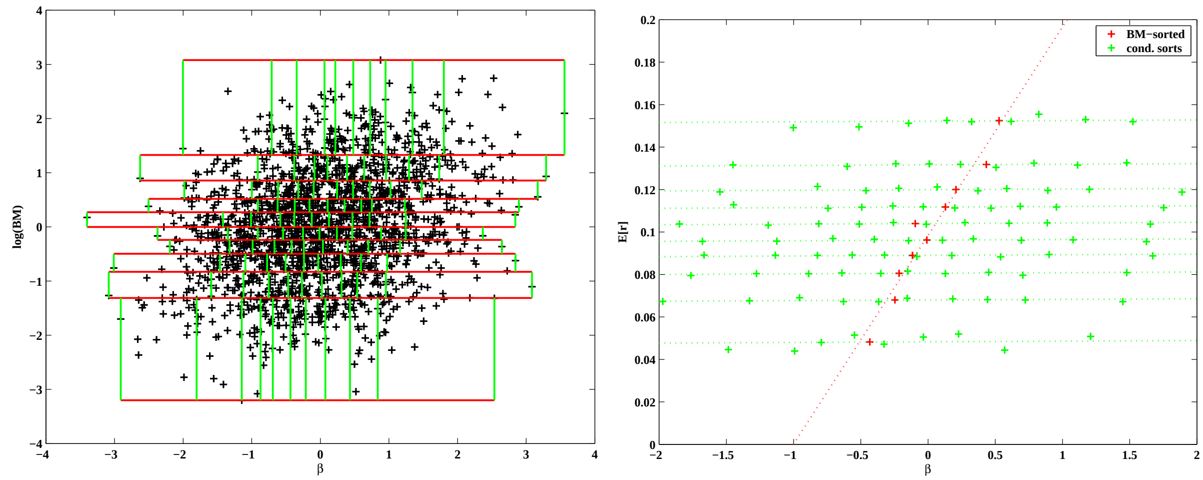
One can alternatively sort stocks into portfolios based on risk rather than characteristics. Figure 3 illustrates this formation method. In the left-hand plot the vertical lines show the cutoffs between beta-sorted deciles. The right side figure plots the expected returns and betas of these portfolios, and the regression line relating these two. The corresponding regression line for the BM sorted portfolios is also shown.

This plot shows that sorting portfolios in this way results in a lower estimated factor risk premium, but still yields a strong estimated relation between risk and return under the alternative, and again a good model fit, with a regression R^2 of 97.4%. Here, the problem is that high beta portfolios have, on average, high BM ratios and therefore high returns. Sorting in this way, there is almost no independent variation in BM across portfolios so that the betas and the characteristics are again almost perfectly correlated, making it impossible to discriminate between the two hypotheses.

In order to discriminate between to the two hypotheses one must construct test portfolios in a way that significant independent variation in betas and BM ratios. Figure 4 shows how this can be done with a multiple sort procedure.

The left panel of Figure 4 is similar to the corresponding panel in Figure 2: the horizontal lines again show the bounds of the 10 BM-sorted portfolios. Now, however we have

Figure 4: Characteristic/Beta Sorted Portfolio Test



superimposed on the plot a set of vertical lines which indicate the result of splitting each of the BM decile portfolios into 10 sub-portfolios based on beta; each of the 100 BM/beta sorted-portfolio contains 25 firms. With these portfolios there is substantial independent variation in beta, and therefore it is possible to discriminate between the null and alternative hypotheses.

The results of the regression tests on the three sets of portfolios are summarized in Table 3. Notice that the R^2s are high in each of the three tests. The R^2 is clearly not a good indicator of model fit. Notice also that, for the test on either the BM-sorted or β-sorted portfolios, the estimated risk premia are large and highly significant. Only when the test is done with the multiple-sort portfolios, and when dummy variables are included for the BM ranking, does the premium get close to the true value of zero. Interestingly, even in the third test, the estimated premium is still significant in the simulation. The reason is that, within each of the 100 portfolios, there is still some correlated variation in expected return and book-to-market.

Table 3: **Simulation Test Results Summary**

The first two rows of this table present the results of OLS regressions of $E[r_i] = \lambda_0+\lambda_1\beta_i+u_i$ for the simulated data, sorted into 10 portfolios according to BM (first row), and factor β (second row). The third row of the table shows the results of the OLS regression test for the 100 portfolios sorted first into deciles based on BM, and then into sub-portfolios based on on factor β. Here the regression is:

$$E[r_i] = \lambda_0 + \lambda_1\beta_i + \gamma_2 I(2) + \cdots + \gamma_{10} I(10) + u_i$$

where $I(2)$ is an indicator variables which is 1 if the firm is in BM decile 2, and zero otherwise.

	$\hat{\lambda}_0$	$\hat{\lambda}_1$	$R^2(\%)$
BM Sort	0.1006	0.0946	94.4%
	(11.6)	(39.8)	
β Sort	0.1010	0.0097	97.4%
	(187.0)	(17.4)	
Multiple Sort*	0.0463	0.0005	99.8%
	(92.5)	(3.1)	

*The coefficients and t-statistics associated with the 9 indicator variables are not shown for this regression.

5 Empirical Tests - Campbell and Vuolteenaho (2004)

Based on the results of the simulations in Section 4, we re-examine the Campbell and Vuolteenaho (2004, CV) tests of a multiple factor model using the multiply-sorted test portfolio procedure discussed in Section 4.

5.1 Description of the CV model

Campbell and Vuolteenaho (2004, CV) propose a version of the Merton (1973) Intertemporal CAPM as an alternative to the static CAPM as a way of explaining the size/book-to-market anomaly. They argue that the realized market return can be decomposed into the conditional expected return ($E_t[r_{t+1}]$) plus the component of the return attributable to news about the level of future cash flows ($N_{CF,t+1}$), plus the return component attributable to the news about the discount rates applied to these future cash flows by investors

$(-N_{DR,t+1})$:

$$r_{t+1} = E_t[r_{t+1}] + N_{CF,t+1} - N_{DR,t+1}.$$

Thus, CV show that one can calculate βs with respect the return components attributable to these two news components. Motivated by the representative agent model in Campbell (1993), who shows that:

$$E_t[r_{i,t+1}] - r_{r,t+1} + \frac{\sigma^2_{i,t}}{2} = \gamma\sigma^2_{p,t}\beta_{i,CF_p,t} + \sigma^2_{p,t}\beta_{i,DR_p,t}$$

where p is the portfolio the representative investor chooses to hold (*i.e.*, the market), γ is the coefficient of relative risk aversion of the representative agent, and $\beta_{i,CF_p,t}$ and $\beta_{i,DR_p,t}$ are the components of portfolio p's return attributable to cash-flow and discount-rate news, respectively.

CV find that over the 1929-1963 period, the standard CAPM prices the 25 FF portfolios, as value stocks have both a higher β_{DR} and a higher β_{CF} over this sample period.

However, over the 1963-2001 sample, the value stocks have a lower β_{DR}, but a higher β_{CF}. The measured CAPM β, which is the sum β_{DR} and β_{CF}, is lower for value stocks, but the average returns are higher because of the much higher premium attached to cash-flow risk. This, they argue, leads to a rejection of the standard CAPM. However the CV model, which separates out covariance with discount-rate and cash-flow shocks, is not rejected.

5.2 The Performance of the CAPM over the 1929-1963 Period

CV argue that the standard CAPM is not rejected over the 1929-1963 period. This is consistent with the results of Ang and Chen (2003), who also argue that the high βs of value stocks over this period explains their high returns, and further argue, like CV, that the CAPM cannot be rejected over this period.

We first re-examine the data from this period using portfolios which employ our sort methodology. Table 5 shows the average returns and post-formation betas for our portfolios. Our portfolio formation procedure closely follows Daniel and Titman (1997). We form 45 portfolios using the following procedure: First, we sort all firms into three portfolios on the basis of market capitalization (or size) as of December of year t, on the basis of NYSE breakpoints. Additionally, we sort of firms into three portfolios on the basis of the firm's book to market ratio. The book to market ratio is defined as the ratio of the firm's book value at the firm's fiscal year end in year t, divided by the firm's market capitalization as of December of year t.

Then, we sort each of the firms in these nine portfolios into five sub-portfolios based on the estimated pre-formation β_{mkt}s of these portfolios. We estimate the pre-formation betas by running regressions of individual firm excess monthly returns on the excess monthly returns of the CRSP value weighted index for 60 months leading up to December of year t. Sub-portfolio breakpoints are set so that, across each size-BM portfolio, there are an equal number of firms in each sub-portfolio.[7]

We then construct the realized returns for each of these 45 test portfolios. Even though the portfolios are formed using data up through the end of year t, we examine the returns from these portfolios starting in July of year $t+1$. The reason for this (following Fama and French (1993)) is that the book value data for the firm is unlikely to be publicly available as of January of year $t+1$, but it is almost certain to be available as of July. All of our portfolio returns are value-weighted. The portfolios are then rebalanced at the start of July of year $t+2$ using the new firm data from the end of year $t+1$.

The upper panel of Table 5 gives the average returns and t-statistics for each of our 45 portfolios. The lower panel gives the estimated post-formation betas for the realized returns, and the t-statistics associated with these betas. The final row of each of the two tables gives the average return/beta and the associated t-statistics for the "average portfolio," which is an equal weighted portfolio of each of the nine sub-portfolios in the same pre-formation beta group.

[7]However, note that the number of firms in the size-BM portfolios will vary because of the use of NYSE breakpoints in the size portfolio sort.

Finally, the last two columns of each table give the average return/beta and the associated t-statistics for the $5-1$ difference portfolio, that is a zero investment portfolio which buys one dollar of the high-estimated beta portfolio and shorts one dollar of the low-estimated beta portfolio.

A couple of important features of the data are evident in at the end in the two panels of this table. First, in the lower panel, note that our sort on preformation beta produces a large spread and in post–formation betas. This is important for the power of the test. Looking at the right two columns shows that sorting on pre-formation betas produces a large and highly statistically significant statistically spread in realized beta.

In contrast, the upper panel shows that the sort on preformation beta produces little spread in average returns. The average return and t-statistic in the lower right corner of the table shows that the mean return difference between the high beta and the low beta portfolios is only 0.02% per month. The nine entries directly above this show that the differences in beta don't produce a statistically significant difference in return for any of the nine size/BM portfolios.

It is important to note that the observation of Ang and Chen (2003) and Campbell and Vuolteenaho (2004) that that higher book to market is associated with higher beta in his early period is confirmed in our test: the lower panel of our table shows that, on average, higher book to market firms do indeed have higher betas. However, this positive correlation does not imply that the CAPM explains returns in this period. The reason is that differences in beta that are independent of differences in book to market are not associated with average return differences, at least at any statistically significant level.

We formally test the hypothesis that the CAPM explains the returns of these 45 portfolios by running time series regressions of the realized excess returns portfolios on the realized excess returns CRSP value-weighted portfolio returns over the 1929:07-1963:06 period. That is, the regressions are of the form:

$$(\tilde{r}_{i,t} - r_{f,t}) = \alpha_i + \beta_i(\tilde{r}_{m,t} - r_{f,t}) + \tilde{\epsilon}_{i,t}$$

The left part of Table 6 reports the estimated regression intercepts, and the right part presents the t-statistics associated with these intercepts. The last row of the table gives the intercepts and t-statistics for the average portfolio, and the last two columns give the estimated intercepts and t-stats for the $5-1$ difference portfolio. Consistent with the average returns and estimated betas reported in Table 5, the estimate alphas are negative, are economically large, and are highly statistically significant. Indeed, for the average-5–1 difference portfolio (the lower-right entry in the table), the t-statistic is -4.42.

However, recall that CV argue that it is *not* the CAPM beta that primarily determines expected returns, but the "bad" or "cash-flow" beta associated with the covariance between a portfolio's return and the component of the return on the market that is associated with news about future cash-flows. To test this hypothesis, we use discount-rate and cash-flow innovations as calculated by CV, calculate the βs with respect to these innovations for the 45 portfolios (as described in CV) and examine whether β_{CF} or β_{DR} explain the average returns of the 45 portfolios.

The results from these tests over this period are reported in Table 7. It is the numbers in the upper panel that are most relevant here. First, notice that there is indeed a large correlation between book-to-market and the cash-flow beta over this period: high BM firms do, on average, have higher cash-flow betas. However, sorting on pre-formation market beta produces a large spread in realized cash-flow betas (note the t-statistics in the last column of the table), and as we have already seen in Table 5, produces no statistically significant spread in returns. Thus, differences in β_{CF} that are unrelated to differences in BM are not priced.

Notice that the sort on pre-formation market beta also produces a large spread in realized discount-rate beta. Again, note the t-statistics in the last column of the lower panel of the table.

5.3 Late-Period Results

It is now well known (see, *e.g.*, Fama and French (1992)) that the CAPM fails to explain the returns of even size and book-to-market sorted portfolios since 1963. Over this period, value stocks have a low CAPM beta, and high average return, and growth stocks have a high CAPM beta, and low average returns.

As discussed earlier, CV argue that the reason for this rejection is that the two components of beta that they identify, β_{DR} and β_{CF}, have different premia. Specifically, CV argue that over the 1963-2001 sample, the value stocks have a lower β_{DR}, but a higher β_{CF}. The measured CAPM β, which is the sum β_{DR} and β_{CF}, is lower for value stocks, but the average returns are higher because of the much higher premium attached to cash-flow risk. This, they argue, leads to a rejection of the standard CAPM. However the CV model, which separates out covariance with discount-rate and cash-flow shocks, is not rejected for the 25 FF size and book-to-market sorted portfolios.

Here, we examine whether the CV model can explain the returns of the portfolios which are constructed in such a way as to introduce variation in cash-flow beta which is independent of BM. We do this by constructing 45 portfolios, as in Section 5.2.

Here though, we form the portfolios slightly differently than in Section 5.2. The reason is that sorting on pre-formation market beta produces a large spread in the discount rate beta in the later part of the sample, but no statistically significant differences in cash-flow betas.

Hence, instead of sorting on CAPM beta, we first form a time series of estimates of CF-innovations. We sort on a linear combination of the variables that CV use in their vector autoregression to estimate cash-flow news. Specifically, their VAR estimates of N_{CF} are equivalent to a linear combination of the RHS variables of their VAR:

$$N_{CF,t} = 0.004 + 0.60\ R_{m,t} + 0.40\ R_{m,t-1} + 0.01\ \Delta PE_t - 0.88\ \Delta TY_t\ - 0.28\ \Delta VS_t$$

We calculate our pre-formation estimates of β_{CF} by regressing individual firm excess returns on $N_{CF,t}$ in the 60 months leading up to December of year t (consistent with our method as described in Section 5.2). Other details of the portfolio formation procedure are as described in Section 5.2.

Table 8 presents the late period post-formation CF and DR betas and the associated t-statistics. Here, the spread is small that what is achieved in the early period sort on CAPM beta, but it is still statistically significant. Moreover, the CF beta spread in the $5-1$ portfolio is about as large as the spread in CF beta achieved via unconditional sorts on book-to-market: the lower right corner of the upper panel shows a spread of 0.068, with a t-statistic of 3.54.

However, while the sort produces a statistically significant difference in port-formation cash-flow betas, it produces no statistically significant difference in average returns. Table 9 reports the average returns of the 45 late-period portfolios. The mean return of the average difference portfolio is 0.09, with a t-statistic of 0.054.

6 Tests Using Industry Portfolios

The empirical tests in the previous section used pre-formation betas as an instrument for the risk-factor loadings as a way of introducing variation in the test assets' factor loadings which is uncorrelated with their size and book-to-market ratios. In this section we use an alternative approach, and examine industry portfolio returns. To the extent that sorting into portfolios on the basis of industry captures variation in risk-factor loadings that is unrelated to book-to-market ratio (and hence to expected returns, under the alternative hypothesis), such an approach should provide power against our characteristic-alternative.

In this section we re-examine the Conditional Consumption-CAPM (CCAPM) test of Lettau and Ludvigson (2001) in several ways. First, we reproduce their Fama MacBeth

Table 4: **Tests of the Lettau and Ludvigson (2001) Consumption CAPM Using Size/BM sorted and Industry Portfolios**
This table presents estimates of the premia (λs) associated with the LL factors. Fama-MacBeth t-statistics are given in parentheses.

Ports	Const	$\widehat{cay}_t$	Δc_{t+1}	$\widehat{cay}_t \cdot \Delta c_{t+1}$	R^2
25 FF SZ/BM	4.28	-0.12	0.02	0.0057	0.70
	(11.36)	(-0.66)	(0.23)	(3.10)	
48 FF Indust.	2.94	0.27	-0.10	0.0002	0.30
	(14.49)	(2.99)	(-2.26)	(0.24)	
38 FF Indust.	3.13	0.18	-0.07	0.0003	0.09
	(8.40)	(0.93)	(-0.92)	(0.17)	
11 FF Indust.	2.91	-0.02	0.03	-0.0033	0.51
	(7.08)	(-0.09)	(0.29)	(-1.84)	

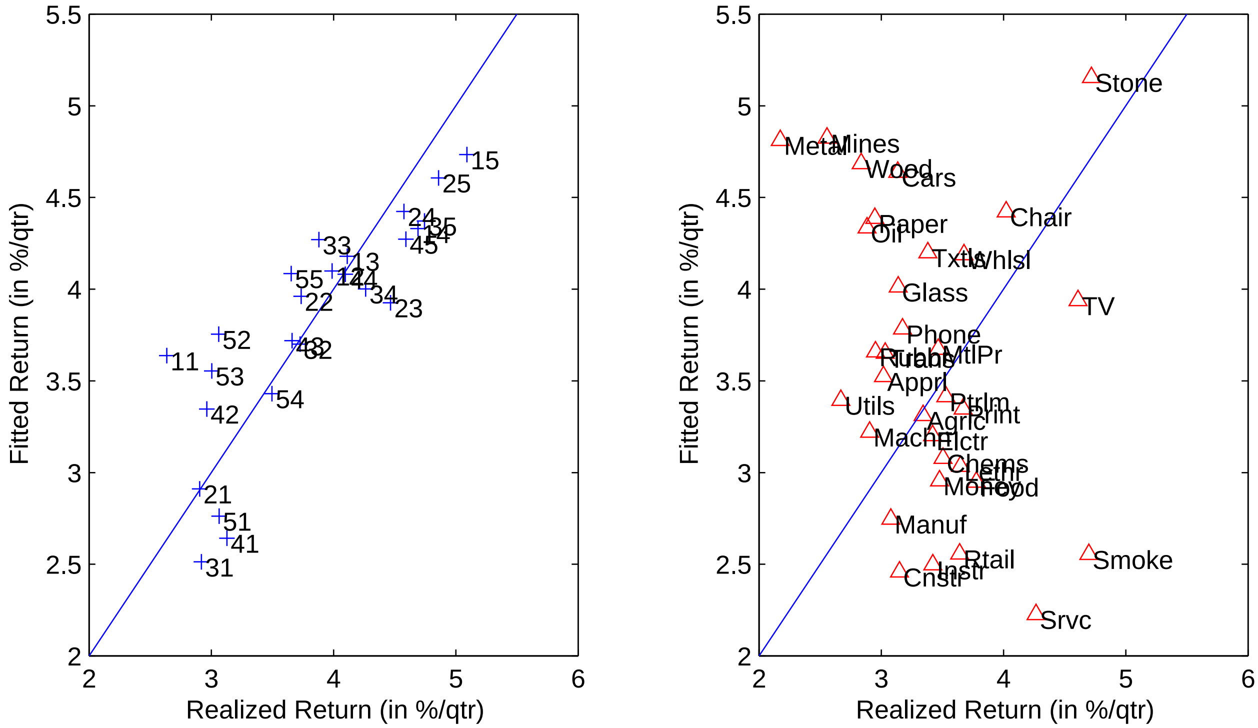
tests (in their Table 3), only using industry-sorted portfolio returns.[8] Table 4 shows the results of these sets of Fama-MacBeth regressions. The table shows that the estimated premia associated is different when these tests are done with industry portfolios than when they are done with the 25 FF portfolios as test assets.

In addition, we reproduce Lettau and Ludvigson's Figure 1, panel (d), here for the Fama and French 38 industry sorted portfolios. The left panel of Figure 5 is done per the LL methodology, and using the 25 FF test portfolios. Consistent with LL, we find that the conditional consumption CAPM does a good job pricing this set of test assets.

However, the right panel of the figure plots the fitted and realized returns for the 38 Fama and French industry portfolios. However, in constructing the fitted returns, we use the risk premia as estimated from the Fama-MacBeth regressions with the 25 sz/bm-sorted industry portfolios. Like the 25 sz/BM sorted portfolios, the industry sorted portfolios exhibit considerable variation in their loadings on the factors, and consequently very different fitted returns. However, the premia as estimated from the original test assets

[8] The returns to the sets of industry portfolios are taken from Ken French's web page. Details on the SIC codes associated with each of the industry breakdowns is available there.

Figure 5: Lettau and Ludvigson (2001) Conditional CCAPM Model – Realized and Fitted Returns - 25 Size/BM sorted Portfolios and 38 Industry Sorted Portfolios



are not consistent with the pricing of the industry portfolios.

This empirical analysis is very preliminary, but suggests that the covariation with the proposed factors outside of the original 25-FF portfolio return space is not priced is a manner consistent with the estimates for the original test assets.

7 Conclusions

We have shown that tests of factor models which are conducted on size and book-to-market sorted portfolios alone are unlikely to reject the null hypothesis that a factor model explains returns, even if the firms' factor loadings are only loosely correlated with book-to-market, and are not direct determinants of expected return. We argue that statistically powerful tests require that portfolios be formed on the basis additional sorts on some *ex-ante* variable which is a good forecaster of factor loadings.

We have presented two examples of such a test: we first redo the empirical examination of the models of Campbell and Vuolteenaho (2004) and of Ang and Chen (2003) using size, book-to-market and estimated future factor loadings, and come to decidedly different conclusions. In addition, we re-examine the Lettau and Ludvigson (2001) model using industry sorted portfolios, and again find evidence that appears inconsistent with their model.

We argue that to determine which if any of the numerous factor models that have been proposed as candidates to explain the book-to-market effect, it will be necessary to carry out the tests using this more powerful methodology.

Appendix A. Conditional Models and Conditional Tests

This appendix reviews results on conditional and unconditional models and tests of models.

A.1. Conditional and Unconditional Factor Models

In the absence of arbitrage, all assets are priced by a pricing kernel $\tilde{m}$ such that:

$$E_t[\tilde{m}_{t+1}\tilde{R}_{t+1}] = 1.$$

An *unconditional* k-factor model specifies that the pricing kernel is a linear function of a set of factors:

$$\tilde{m}_{t+1} = a + \mathbf{b}\,\tilde{\mathbf{f}}_{t+1} \tag{2}$$

where a and $\mathbf{b}$ are time-invariant. In contrast, a *conditional* k-factor model specifies that:

$$\tilde{m}_{t+1} = a_t + \mathbf{b}_t'\,\tilde{\mathbf{f}}_{t+1}$$

Here, in contrast to the specification in equation (2), a_t and $\mathbf{b}_t$ are not time invariant, but are adapted to the time t information set.

To test a conditional factor model, we generally specify that a_t and $\mathbf{b}_t$ are linear functions of a $(m\times 1)$ vector of instruments $\mathbf{Z}_t \in \mathcal{F}_t$:

$$\begin{aligned} a_t &= \mathbf{a}'\,\mathbf{Z}_t \\ \mathbf{b}_t &= \mathbf{b}\mathbf{Z}_t \end{aligned}$$

where $\mathbf{a}$ is $(m\times 1)$ and $\mathbf{b}$ is $(k\times m)$. This gives:

$$\tilde{m}_{t+1} = \mathbf{a}'\mathbf{Z}_t + (\mathbf{b}\mathbf{Z}_t)'\,\tilde{\mathbf{f}}_{t+1}$$

A.1..1 Interpreting Conditional Factor Models

As noted by Cochrane (2000), a conditional k-factor model with m conditioning variables is equivalent to a unconditional factor model with $(k\cdot m)$ factors.

For example, the unconditional CAPM specifies that:

$$\tilde{m}_{t+1} = a + b\tilde{r}_{m,t+1},$$

where a and b are time invariant. The Lettau and Ludvigson (2001) conditional CAPM specifies that

$$\tilde{m}_{t+1} = (\gamma_0 + \gamma_1 z_t) + (\eta_0 + \eta_1 z_t)\tilde{r}_{m,t+1} \tag{3}$$

where, in their quarterly tests, the instrument z_t is their *cay* variable measured at the start of the quarter. Notice that this model has the implication that, for a $(N \times 1)$ vector of asset returns from t to $t+1$, and given an observable risk-free rate:

$$(\tilde{\mathbf{r}}_{t+1} - \mathbf{1} r_{f,t+1}) = \boldsymbol{\beta}_m(\tilde{r}_{m,t+1} - r_{f,t+1}) + \boldsymbol{\beta}_{mz}(\tilde{r}_{m,t+1} - r_{f,t+1})z_t + \tilde{\boldsymbol{\epsilon}}_t \tag{4}$$

where $\tilde{\mathbf{r}}$, $\mathbf{1}$, $\boldsymbol{\beta}_m$, $\boldsymbol{\beta}_{mz}$, and $\tilde{\boldsymbol{\epsilon}}_t$ and $(N \times 1)$ vectors, and $r_{f,t+1}$ is the return on an efficient portfolio uncorrelated with the market portfolio return – it is the risk-free rate if it exists, or the (stochastic) return on a minimum-variance zero-beta portfolio.

Either equation (3) or equation (4) shows that this conditional CAPM is equivalent to a two factors model with factors equal to:

1. The excess market return, defined as the profit that results from investing \$1 in the market portfolio and shorting \$1 of the risk-free (or zero-beta) asset.
2. The *scaled excess-market return*, defined as the profit that results from investing \$$z_t$ in the market portfolio and shorting \$$z_t$ of the risk-free (or zero-beta) asset.

A.2. Conditional Tests of Factor Models

Any test of a factor model will be a test of the set of moment restrictions:

$$E_t[\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1}] = \mathbf{1}. \tag{5}$$

An *unconditional test* examines the moment restriction that results from taking an unconditional expectation of equation (2):

$$E[\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1}] = \mathbf{1}.$$

A *conditional test* examines additional restrictions implied by equation (5), specifically, that for any set of instruments $\mathbf{Z}_t$ in $\mathcal{F}_t$:

$$E\left[\left(\tilde{m}_{t+1}\tilde{\mathbf{R}}_{t+1} - \mathbf{1}\right) \otimes \mathbf{Z}_t\right] = \mathbf{0} \tag{6}$$

The set of papers that we consider here perform *unconditional* tests of *conditional* factor models. These papers generally do not test the additional moment restrictions implied by (6). In the language of Cochrane (2000), they don't augment the return space with scaled test assets – but they do augment the set of factors with scaled factors.

References

Aït-Sahalia, Yacine, Jonathan A. Parker, and Motohiro Yogo, 2003, Luxury goods and the equity premium, Princeton University Working Paper.

Ang, Andrew, and Joseph Chen, 2003, CAPM over the long-run: 1926-2001, Columbia University Working Paper.

Berk, Jonathan, 1995, A critique of size related anomalies, *Review of Financial Studies* 8, 275–286.

Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–96.

Campbell, John Y, 1993, Intertemporal asset pricing without consumption data, *American Economic Review* 83, 487–512.

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Campbell, John Y., and Tuomo Vuolteenaho, 2004, Bad beta, good beta, *American Economic Review* 94, 1249–1275.

Cochrane, John H., 1996, A cross-sectional test of an investment-based asset pricing model, *Journal of Political Economy* 104, 572–621.

———, 2000, *Asset Pricing* (Princeton University Press: NJ).

Cohen, Randolph B., Christopher K. Polk, and Tuomo Vuolteenaho, 2000, The value spread, University of Chicago working paper.

Daniel, Kent D., David Hirshleifer, and Avanidhar Subrahmanyam, 1998, Investor psychology and security market under- and over-reactions, *Journal of Finance* 53, 1839–1886.

Daniel, Kent D., and Sheridan Titman, 1997, Evidence on the characteristics of cross-sectional variation in common stock returns, *Journal of Finance* 52, 1–33.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

———, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

———, 1995, Size and book-to-market factors in earnings and returns, *Journal of Finance* 50, 131–156.

Fama, Eugene F., and James MacBeth, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Ferson, Wayne E., and Campbell R. Harvey, 1999, Conditioning variables and the cross section of stock returns, *Journal of Finance* 54, 1325 – 1360.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, A test of the efficiency of a given portfolio, *Econometrica* 57, 1121–1152.

Hansen, Lars, and Scott F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica* 55, 587–613.

Hansen, Lars P., and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99, 225–262.

Heaton, John C., and Deborah J. Lucas, 2000, Portfolio choice and asset prices: The importance of entrepeneurial risk, *Journal of Finance* 55, 1163–1198.

Hodrick, Robert J., and Xiaoyan Zhang, 2001, Evaluating the specification errors of asset pricing models, *Journal of Financial Economics* 62, 327–376.

Jagannathan, Ravi, and Zhenyu Wang, 1996, The CAPM is alive and well, *Journal of Finance* 51, 3–53.

Lettau, Martin, and Sydney Ludvigson, 2001, Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 109, 1238–1287.

Lewellen, Jonathan, and Stefan Nagel, 2003, The conditional CAPM does not explain asset-pricing anomalies, Working Paper, MIT.

Li, Qing, Maria Vassalou, and Yuhang Xing, 2002, An investment-growth asset pricing model, Columbia GSB working paper.

Lintner, John, 1965, The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13–37.

Lustig, Hanno, and Stijn Van Nieuwerburgh, 2002, Housing collateral, consumption insurance and risk premia, University of Chicago working paper.

Merton, Robert C., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

Petkova, Ralitsa, and Lu Zhang, 2003, Is value riskier than growth?, Rochester Working Paper.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel, 2003, Housing, consumption, and asset pricing, UCLA Working Paper.

Santos, Tano, and Pietro Veronesi, 2001, Labor income and predictable stock returns, University of Chicago working paper.

Sharpe, William F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Table 5: **BM/Size/Pre-β_{Mkt} Portfolios - $\bar{r}$s and Post-Formation βs**

For the 1933:07-1963:06 period, this table presents the average monthly returns (in %/month) and the post-formation market βs and the corresponding t-statistics for portfolios formed on the basis of independent sorts into 3 portfolios each based on size and book-to-market ratio, followed by dependent sorts into 5 sub-portfolios based on pre-formation betas. Size and book-to-market sorts are based on NYSE cutoffs. The final column of the table, labeled $5-1$, gives the average return and β for the zero-investment portfolio formed by buying \$1 of the high β portfolio, and selling \$1 of the low β sub-portfolio. The final row of the table, labeled "avg port.," gives the statistics for the equal-weighted portfolio of the 9 sub-portfolios listed directly above.

Chr Pt		$\bar{r}$ (%/mo)					$t(\bar{r})$					$\bar{r}$	$t(\bar{r})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	1.22	1.18	1.19	0.85	0.81	(3.48)	(2.54)	(2.57)	(1.47)	(1.39)	-0.40	(-0.96)
1	2	1.40	1.28	1.41	1.17	1.24	(4.40)	(3.42)	(3.38)	(2.32)	(2.41)	-0.16	(-0.51)
1	3	1.33	1.47	1.68	1.56	1.44	(3.91)	(3.36)	(3.26)	(3.19)	(2.36)	0.11	(0.30)
2	1	1.02	0.98	1.07	0.98	0.94	(4.40)	(3.45)	(3.03)	(2.56)	(2.18)	-0.08	(-0.28)
2	2	1.23	1.20	1.28	1.37	1.18	(5.09)	(4.03)	(3.71)	(3.56)	(2.57)	-0.05	(-0.18)
2	3	1.09	1.30	1.32	1.34	1.28	(3.53)	(3.21)	(3.11)	(2.75)	(2.39)	0.19	(0.54)
3	1	0.70	0.86	0.86	1.07	0.96	(3.46)	(3.75)	(3.16)	(3.50)	(2.77)	0.26	(1.19)
3	2	0.93	1.07	1.27	1.06	0.99	(4.37)	(4.02)	(3.99)	(3.14)	(2.47)	0.06	(0.20)
3	3	0.99	1.11	1.24	1.31	1.26	(3.14)	(2.94)	(2.87)	(2.92)	(2.48)	0.27	(0.88)
avg prt		1.10	1.16	1.26	1.19	1.12	(4.42)	(3.68)	(3.48)	(2.93)	(2.48)	**0.02**	**(0.09)**

Chr Prt		$\hat{\beta}_{Mkt}$					$t(\hat{\beta}_{Mkt})$					$\hat{\beta}_{Mkt}$	$t(\hat{\beta}_M)$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	1.03	1.40	1.30	1.71	1.76	(22.09)	(23.13)	(19.99)	(22.09)	(22.96)	0.73	(9.50)
1	2	1.06	1.24	1.40	1.66	1.70	(31.88)	(30.99)	(33.38)	(30.01)	(30.81)	0.64	(11.94)
1	3	1.07	1.40	1.66	1.61	1.88	(25.87)	(27.62)	(28.25)	(30.35)	(25.03)	0.82	(14.15)
2	1	0.78	0.99	1.25	1.36	1.53	(32.85)	(39.48)	(42.87)	(42.84)	(41.48)	0.75	(17.82)
2	2	0.82	1.05	1.24	1.40	1.63	(34.11)	(42.33)	(47.79)	(50.05)	(43.75)	0.81	(20.53)
2	3	0.99	1.39	1.47	1.63	1.80	(27.17)	(36.04)	(36.88)	(32.82)	(32.45)	0.81	(14.11)
3	1	0.70	0.82	0.99	1.13	1.29	(36.81)	(48.07)	(53.82)	(58.54)	(59.26)	0.58	(18.06)
3	2	0.70	0.89	1.11	1.21	1.40	(30.50)	(32.76)	(40.13)	(45.52)	(40.22)	0.70	(14.70)
3	3	1.04	1.27	1.44	1.56	1.71	(31.20)	(32.06)	(31.45)	(37.86)	(33.24)	0.66	(12.75)
avg prt		0.91	1.16	1.32	1.47	1.63	(53.20)	(56.75)	(53.61)	(50.75)	(47.68)	**0.72**	**(23.80)**

Table 6: **Early Period Times Series Regression Intercepts**

This table presents the results of the time-series regressions of the realized excess returns and t-statistics of the 45 portfolios on the realized excess returns of the CRSP value-weighted portfolio returns over the 1933:07-1963:06 period. The left part of the table reports the estimated regression intercepts, and the right part presents the t-statistics associated with these intercepts. The last row of the table gives the intercepts and t-statistics for the average portfolio, and the last two columns give the estimated intercepts and t-stats for the 5−1 difference portfolio, as described in the text.

Chr Prt		$\hat{\alpha}$					$t(\hat{\alpha})$					$\hat{\alpha}$	$t(\hat{\alpha})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.24	-0.15	*-0.05*	-0.78	-0.86	(1.01)	(-0.50)	*(-0.17)*	(-2.02)	(-2.26)	-1.10	(-2.88)
1	2	0.39	0.10	*0.07*	-0.41	-0.38	(2.34)	(0.50)	*(0.33)*	(-1.49)	(-1.39)	-0.77	(-2.87)
1	3	0.32	0.13	*0.10*	0.02	-0.36	(1.54)	(0.53)	*(0.33)*	(0.09)	(-0.96)	-0.67	(-2.34)
2	1	0.28	0.03	*-0.13*	-0.31	-0.51	(2.34)	(0.27)	*(-0.87)*	(-2.00)	(-2.80)	-0.79	(-3.79)
2	2	0.45	0.19	*0.09*	0.04	-0.38	(3.75)	(1.57)	*(0.73)*	(0.27)	(-2.04)	-0.83	(-4.20)
2	3	0.15	-0.03	*-0.08*	-0.22	-0.43	(0.85)	(-0.14)	*(-0.39)*	(-0.89)	(-1.57)	-0.59	(-2.05)
3	1	0.03	0.07	*-0.09*	-0.01	-0.26	(0.36)	(0.81)	*(-0.96)*	(-0.08)	(-2.43)	-0.30	(-1.84)
3	2	0.26	0.22	*0.20*	-0.09	-0.35	(2.27)	(1.61)	*(1.48)*	(-0.71)	(-2.03)	-0.61	(-2.57)
3	3	-0.01	-0.09	*-0.13*	-0.18	-0.37	(-0.05)	(-0.48)	*(-0.58)*	(-0.86)	(-1.46)	-0.36	(-1.40)
avg prt		0.23	0.05	*-0.00*	-0.22	-0.43	(2.75)	(0.52)	*(-0.01)*	(-1.49)	(-2.55)	**-0.67**	**(-4.42)**

Table 7: **Early Period Post-Formation CF and DR betas**

This table presents the estimated CF and DR betas from time-series regressions of the realized excess returns and t-statistics of the 45 portfolios on the component of the market return attributable to cash-flow and discount-rate news, as calculated by Campbell and Vuolteenaho (2004), over the 1933:07-1963:06 period. The left parts of the two panels report CF and DR betas, and the right parts of each panel present the t-statistics associated with these betas. The last row of the table gives the betas and t-statistics for the average portfolio, and the last two columns give the estimated betas and t-stats for the $5-1$ difference portfolio, as described in the text.

Chr Prt		$\hat{\beta}_{CF}$					$t(\hat{\beta}_{CF})$					$\hat{\beta}_{CF}$	$t(\hat{\beta}_{CF})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.20	0.26	0.26	0.38	0.45	(4.98)	(4.97)	(4.91)	(5.95)	(7.03)	0.26	(5.44)
1	2	0.22	0.28	0.29	0.36	0.42	(6.15)	(6.82)	(6.34)	(6.42)	(7.57)	0.21	(5.94)
1	3	0.31	0.35	0.42	0.41	0.47	(8.49)	(7.39)	(7.54)	(7.75)	(7.07)	0.16	(4.05)
2	1	0.13	0.18	0.24	0.26	0.27	(4.94)	(5.56)	(6.01)	(6.15)	(5.49)	0.14	(4.27)
2	2	0.18	0.24	0.25	0.30	0.34	(6.81)	(7.47)	(6.70)	(7.11)	(6.67)	0.16	(4.84)
2	3	0.28	0.37	0.39	0.43	0.45	(8.45)	(8.53)	(8.68)	(8.24)	(7.75)	0.17	(4.27)
3	1	0.09	0.13	0.16	0.16	0.24	(3.73)	(4.97)	(5.34)	(4.75)	(6.16)	0.15	(6.24)
3	2	0.13	0.21	0.21	0.25	0.29	(5.34)	(7.26)	(5.99)	(6.70)	(6.45)	0.16	(4.70)
3	3	0.22	0.31	0.32	0.34	0.42	(6.28)	(7.67)	(6.75)	(6.96)	(7.66)	0.20	(5.88)
avg prt		0.19	0.26	0.28	0.32	0.37	(7.13)	(7.57)	(7.19)	(7.29)	(7.56)	**0.18**	**(6.77)**

Chr Prt		$\hat{\beta}_{DR}$					$t(\hat{\beta}_{DR})$					$\hat{\beta}_{DR}$	$t(\hat{\beta}_{DR})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.84	0.99	1.02	1.13	1.27	(11.91)	(10.07)	(10.57)	(9.04)	(10.37)	0.43	(4.46)
1	2	0.77	0.91	1.02	1.11	1.19	(12.08)	(12.09)	(12.31)	(10.57)	(11.36)	0.42	(6.02)
1	3	0.82	1	1.08	1.15	1.24	(11.91)	(11.06)	(9.91)	(11.48)	(9.55)	0.42	(5.18)
2	1	0.63	0.72	0.91	0.96	1.02	(14.19)	(12.86)	(13.15)	(12.65)	(11.55)	0.39	(6.09)
2	2	0.62	0.75	0.89	0.97	1.13	(13.04)	(12.77)	(13.14)	(12.69)	(12.35)	0.51	(8.22)
2	3	0.68	0.86	0.93	1.04	1.17	(10.60)	(10.06)	(10.51)	(10.17)	(10.43)	0.48	(6.11)
3	1	0.54	0.61	0.69	0.82	0.90	(13.87)	(13.81)	(13.05)	(13.93)	(13.21)	0.36	(7.35)
3	2	0.49	0.52	0.75	0.79	0.91	(11.38)	(9.04)	(11.56)	(11.40)	(11.05)	0.42	(6.21)
3	3	0.65	0.86	0.91	1.01	1.08	(9.73)	(11.08)	(9.96)	(10.90)	(10.15)	0.43	(6.20)
avg prt		0.67	0.80	0.91	1.00	1.10	(14.12)	(12.87)	(12.78)	(12.29)	(12.11)	**0.43**	**(8.25)**

Table 8: **Late Period Post-Formation CF and DR betas**

This table presents the estimated CF and DR betas from time-series regressions of the realized excess returns and t-statistics of the 45 portfolios on the component of the market return attributable to cash-flow and discount-rate news, as calculated by Campbell and Vuolteenaho (2004), over the 1963:07-2001:12 period. The left parts of the two panels report CF and DR betas, and the right parts of each panel present the t-statistics associated with these betas. The last row of the table gives the betas and t-statistics for the average portfolio, and the last two columns give the estimated betas and t-stats for the $5-1$ difference portfolio, as described in the text.

Chr Prt		$\hat{\beta}_{CF}$					$t(\hat{\beta}_{CF})$					$\hat{\beta}_{CF}$	$t(\hat{\beta}_{CF})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.090	0.110	0.119	0.136	0.157	(2.62)	(3.04)	(3.04)	(3.08)	(3.25)	0.067	(2.54)
1	2	0.117	0.127	0.134	0.153	0.181	(4.55)	(4.46)	(4.16)	(4.20)	(4.22)	0.064	(2.63)
1	3	0.133	0.153	0.170	0.160	0.211	(4.89)	(5.25)	(5.24)	(4.47)	(5.12)	0.078	(3.63)
2	1	0.077	0.074	0.091	0.099	0.130	(2.82)	(2.39)	(2.72)	(2.74)	(2.86)	0.053	(1.93)
2	2	0.106	0.110	0.135	0.158	0.157	(4.71)	(4.14)	(4.94)	(5.19)	(4.23)	0.051	(2.10)
2	3	0.115	0.134	0.141	0.173	0.214	(4.83)	(4.99)	(4.71)	(5.41)	(5.49)	0.098	(3.54)
3	1	0.061	0.058	0.068	0.074	0.091	(2.49)	(2.27)	(2.42)	(2.41)	(2.50)	0.031	(1.27)
3	2	0.052	0.075	0.100	0.118	0.146	(2.28)	(3.04)	(3.74)	(4.15)	(4.39)	0.094	(3.70)
3	3	0.084	0.099	0.127	0.123	0.161	(3.50)	(4.00)	(4.48)	(3.95)	(4.67)	0.077	(2.90)
avg prt		0.093	0.104	0.121	0.133	0.161	(4.15)	(4.18)	(4.32)	(4.32)	(4.40)	0.068	(3.54)

Chr Prt		$\hat{\beta}_{DR}$					$t(\hat{\beta}_{DR})$					$\hat{\beta}_{DR}$	$t(\hat{\beta}_{DR})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	1.188	1.267	1.386	1.542	1.658	(17.05)	(17.46)	(17.77)	(17.32)	(16.82)	0.470	(7.28)
1	2	0.859	0.962	1.117	1.237	1.466	(15.90)	(16.07)	(16.92)	(16.37)	(16.55)	0.607	(10.79)
1	3	0.852	0.929	1.037	1.188	1.356	(14.22)	(14.56)	(14.61)	(15.71)	(15.32)	0.504	(9.93)
2	1	0.927	1.108	1.221	1.324	1.642	(16.71)	(18.46)	(18.78)	(18.85)	(18.47)	0.715	(11.33)
2	2	0.669	0.903	0.948	1.030	1.302	(13.35)	(16.44)	(16.63)	(15.91)	(17.34)	0.632	(11.49)
2	3	0.608	0.786	0.952	1.006	1.217	(10.80)	(12.89)	(14.65)	(14.28)	(14.11)	0.609	(9.11)
3	1	0.780	0.877	1.020	1.132	1.304	(15.21)	(17.21)	(18.72)	(19.45)	(18.25)	0.524	(9.16)
3	2	0.551	0.697	0.870	0.878	1.128	(10.41)	(12.69)	(15.27)	(14.07)	(16.35)	0.577	(9.52)
3	3	0.500	0.583	0.759	0.947	1.059	(8.58)	(9.97)	(11.56)	(13.81)	(13.93)	0.560	(8.71)
avg prt		0.770	0.901	1.034	1.143	1.348	(16.80)	(18.15)	(18.90)	(19.05)	(18.79)	0.577	(13.67)

Table 9: **Late Period Average Portfolio Returns**

For the 1963:07-2001:12 period, this table presents the average monthly returns (in %/month) and the corresponding t-statistics for portfolios formed on the basis of independent sorts into 3 portfolios each based on size and book-to-market ratio, followed by dependent sorts into 5 sub-portfolios based on pre-formation cash-flow betas. Size and book-to-market sorts are based on NYSE cutoffs. The final column of the table, labeled $5-1$, gives the average return for the zero-investment portfolio formed by buying $1 of the high β portfolio, and selling $1 of the low β sub-portfolio. The final row of the table, labeled "avg port.," gives the average return and t-statistic for the equal-weighted portfolio of the 9 sub-portfolios listed directly above.

Chr Prt		$\bar{r}$ (%/mo)					$t(\bar{r})$					$\bar{r}$	$t(\bar{r})$
SZ	BM	1	2	3	4	5	1	2	3	4	5	5−1	
1	1	0.53	0.57	0.77	0.53	0.24	(1.81)	(1.87)	(2.30)	(1.40)	(0.59)	-0.29	(-1.28)
1	2	0.71	0.85	0.88	0.76	0.95	(3.23)	(3.45)	(3.18)	(2.44)	(2.57)	0.23	(1.12)
1	3	1.04	0.95	1.07	1.02	1.11	(4.40)	(3.75)	(3.78)	(3.31)	(3.10)	0.07	(0.39)
2	1	0.66	0.64	0.55	0.42	0.49	(2.87)	(2.45)	(1.92)	(1.37)	(1.27)	-0.17	(-0.74)
2	2	0.63	0.62	0.80	0.66	0.93	(3.26)	(2.71)	(3.39)	(2.49)	(2.92)	0.29	(1.44)
2	3	0.67	0.90	1.02	0.95	1.02	(3.25)	(3.88)	(3.97)	(3.43)	(3.02)	0.35	(1.48)
3	1	0.57	0.55	0.54	0.48	0.52	(2.75)	(2.54)	(2.27)	(1.87)	(1.67)	-0.05	(-0.25)
3	2	0.51	0.61	0.48	0.57	0.77	(2.67)	(2.94)	(2.10)	(2.32)	(2.69)	0.25	(1.17)
3	3	0.67	0.57	0.73	0.75	0.79	(3.29)	(2.70)	(3.00)	(2.80)	(2.65)	0.11	(0.50)
avg prt		0.67	0.70	0.76	0.68	0.76	(6.26)	(5.73)	(5.38)	(4.60)	(4.10)	0.09	(0.54)