Growth and Volatility*

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Abstract

This paper presents an endogenous growth model that has predictions for the first and second moments of the growth rate of output and productivity at the aggregate level and for the first and second moments of the growth rate of sales and sales per worker at the firm level. These predictions are consistent with the data. The model does a better job than standard endogenous growth models in accounting for the observed trends in productivity growth and R&D at the aggregate level. It is also able to account for the opposite trends observed in the volatility of aggregate productivity growth and in the firm-level growth rate of sales per worker. In this sense, this model goes beyond both standard growth models and state of the art models of firm-level heterogeneity by proposing mechanisms that allow firms to interact in a way consistent with the evolution of volatility in the post-war period.

*Very Preliminary and Incomplete

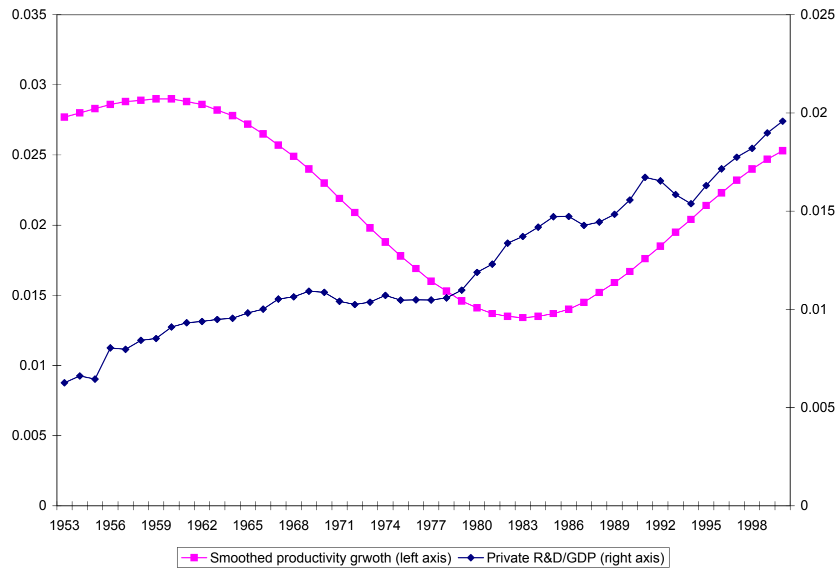

Figure 1: Evolution of (Smoothed) Productivity Growth and Private R&D share in GDP.

1 Motivation

Despite the enormous progress made by the endogenous growth literature in the last 15 years, there is still much to learn about the determinants of long run productivity growth. The state of the art models (Aghion and Howitt [1998 Ch. 12], Dinopoulos and Thompson [1998], Peretto [1998] and Young [1998]) predict a positive relationship between the growth rate of productivity and the share of R&D in GDP. Yet, this prediction does not seem to hold in the US post-war data. Figure 1 illustrates the evolution of the share of private R&D in GDP as measured by the NSF and the smoothed growth rate of productivity.[1] Figure 2 shows a scatter plot of the share of private R&D in GDP and the growth rate of productivity in the US. In both plots it is hard to find any relationship between R&D intensity and productivity growth. Examination of TFP growth or output growth results in similar conclusions.

In line with this observation, Comin [2003] shows that the expenditures that the NSF defines as R&D can account for a small fraction of the average post-war rate of productivity growth in the US. More specifically, there are other (probably) purposeful investments that lead to important improvements in productivity that are not embodied in new products and that, as a result, are not

[1] Specidically, we band-pass filter the annual growth rate of labor productivity keeping the frequencies associated to cycles with periods longer than 30 years.

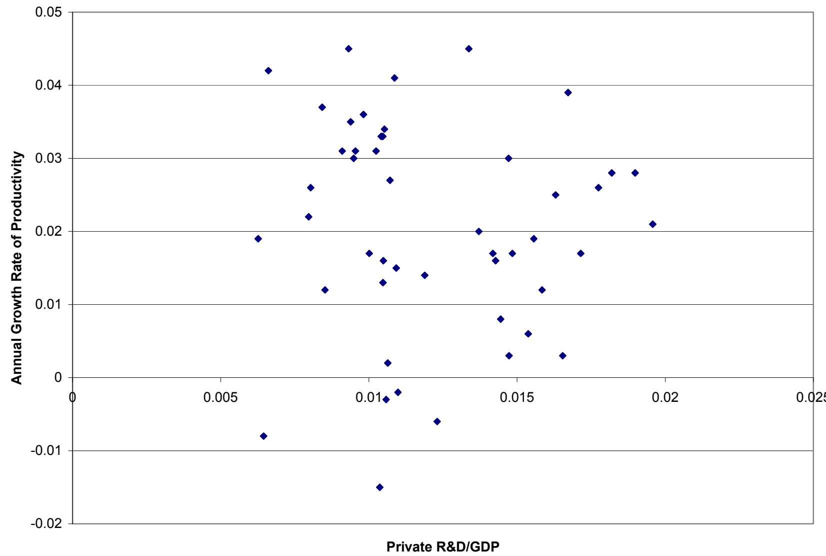

Figure 2: Private R&D share vs. Productivity Growth rate.

included in the NSF concept of R&D.

In addition to the problems encountered by existing growth theories to account for the first moments of the growth processes, the second moments have been left out of the scope of the literature as if their determinants where orthogonal to the determinants of the first moments. Yet this presumption is disputable in the light of the interesting dynamics of the volatility of productivity in the post-war period. Two empirical literatures have characterized the evolution of the volatility both at the aggregate and firm level. McConnell and Perez-Quiros [1999] and Stock and Watson [2003] have shown that the volatility of aggregate variables such as output, hours worked and labor productivity growth has declined during the post-war period. At the firm level, however, these same variables have become more volatile (Comin and Mulani [2003], Chenney et al. [2003] and this paper). Figure 3 illustrates these opposite trends for productivity. On the left axis, we plot the standard deviation of a 10 year-centered rolling window of annual productivity growth. On the right axis, we plot the evolution of the same variable averaged for firms in the COMPUSTAT data base.[2]

The goal of this paper is to build an endogenous growth model that can do a better job in accounting for the relationship between R&D and productivity growth and that has implications

[2]In Comin and Mulani [2003] we have shown that the upward trend in firm-level volatility is not the result of any compositional bias in the COMPUSTAT sample.

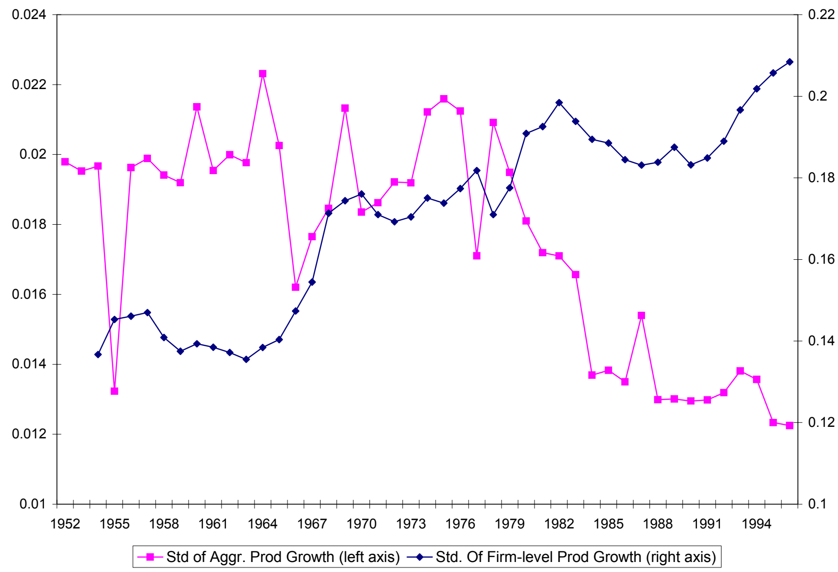

Figure 3: Evolution of the Aggregate and Firm-level Volatility of productivity

for firm and aggregate volatility that are consistent with the evidence.

The divergence in the volatility trends at the firm and aggregate level is quite puzzling for representative-agent models since they predict that the second moments of the aggregate and individual variables are identical. Models with firm heterogeneity such as Bertola and Caballero (1990) can accommodate different trends in aggregate and firm level volatility. However, Cheney et al. (2002) and Comin and Philippon (2004) provide evidence that these diverging trends are not just a coincidence. The goal then should be to build a model where in response to a shock, the firm and aggregate second moments respond in opposite ways. This is not the case in current models of firm heterogeneity because the interactions between firms embedded in these models are not adequate: Most of the models are partial equilibrium and treat firms as independent entities. The most recent of these models have incorporated general equilibrium interactions but even these seem insufficient to generate the diverging trends in volatility.

In this paper, we take a different route that is more promising. We build on the quality-ladder models of Aghion and Howitt [1992] and Grossman and Helpman [1991, Ch. 4]. In this context, standard R&D investments lead firms to develop new versions of existing products or new products that substitute the current market leaders. Such improvements in productivity lead to substantial firm-level volatility since incumbents incur losses while entrants enjoy capital gains. However, at the aggregate level, the effects of R&D investments on volatility are relatively minor since individual gains and losses negate each other.

To explain the movements in aggregate volatility, it seems necessary to consider a second type of innovations. For lack of a better name, we denote these as disembodied (or complementary) innovations. Disembodied innovations satisfy two properties. First, they affect symmetrically the firm that develops them and the rest of firms. Second, a firm that develops a disembodied innovation (by and large) cannot appropriate the benefits enjoyed by the other firms when adopting it. This is the case because disembodied innovations such as the mass production system, new personnel and accounting practices, the use of electricity as the source of energy in a plant,... are hard to patent and easy to reverse-engineer.

Most of the interesting implications of our model follow naturally from these two properties. The fact that a complementary innovation symmetrically affects all firms implies that it will have a large aggregate effect. Therefore, investments in the development of disembodied innovations may lead to substantial volatility in aggregate productivity growth. The fact that innovators cannot appropriate the social value of disembodied innovations implies that the private value of such innovations is proportional to the value of the firm. In equilibrium, disembodied innovations are conducted mostly by the more valuable firms i.e. current market leaders.

Now we are in a position to understand how the model can generate the facts described above. The value of market leaders is higher when the expected duration of their market leadership is longer. Interestingly, the market turnover is increasing in the R&D intensity. This means that a force that leads the economy to invest more in R&D may induce a decline in disembodied investments. Since productivity growth is an increasing function of the shares of both types of investment in GDP, the relationship between R&D intensity and productivity growth will be ambiguous.

In terms of the second moments, however there is no ambiguity. A decline in the intensity of investments in complementary innovations, leads to a decline in aggregate volatility. An increase in the R&D intensity leads to an increase in firm level volatility. This matches the picture of the US during the post-war period.

The structure of the paper is as follows. Section 2 formalizes these intuitions with a model. Section 3 calibrates the model and shows that the mechanisms highlighted are quantitatively significant. Section 4 provides evidence on the increase in the turnover rate of market leaders and on the positive relationship between firm-level volatility and R&D intensity at the 4-digit sector. Section 5 concludes.

2 Model

Next we lay out a model that delivers endogenous growth, and volatility at the aggregate and firm-level.

2.1 Set up

Preferences

The representative consumer enjoys a utility flow that is linear on the units of final output consumed (c_t). The present discounted value of utility can then be represented as

$$U = \int_0^\infty c_t e^{-rt} dt,$$

where r denotes the instantaneous discount rate. Consumers supply, inelastically, a mass of L units of labor.

Production

Final output (y) is produced by combining two goods denoted as leading (y_l) and standard (y_s) as specified in the following production function:

$$y = y_l^\beta y_s^{1-\beta}$$

This Cobb-Douglas aggregation of y_l and y_s simplifies the analysis later since it implies that the nominal sales of each good are proportional to aggregate output regardless of the good's quality. Formally, the demands for y_l and y_s are given by:

$$\begin{aligned} \beta y &= p_l y_l \\ (1-\beta) y &= p_s y_s, \end{aligned}$$

where the price of final output is normalized to 1.

Both leading and standard goods are produced competitively. The leading good is produced with labor (L_l) and leading intermediate good (x_l) according to the function:

$$y_l = q_l x_l^\alpha L_l^{1-\alpha},$$

where q_l denotes the productivity of the leading intermediate good.

The production of standard output entails the use of labor (L_s) and m different standard intermediate goods (x_{si}) as follows:

$$y_s = q_s \left(m^{\sigma-1} \sum_{i=1}^{m} x_{si}^{\sigma} \right)^{\frac{\alpha}{\sigma}} L_s^{1-\alpha},$$

where q_s is the efficiency of the standard intermediate goods and $1/\sigma$ is the elasticity of substitution between the different standard intermediate goods.

The production of a unit of intermediate good requires a_x units of labor. I assume that $a_x = q_l^{\psi_q}/h^{\psi_h}$, where h is a measure of the efficiency of the production process and $\psi_q, \psi_h > 0$.

Let p_l and p_s denote the prices of the leading and standard goods received by their producers. The producers of the leading and standard final goods have the following demands for intermediate goods and labor:

$$\begin{aligned} x_l &= L_l \left(\alpha \frac{p_l q_l}{p_l^x} \right)^{\frac{1}{1-\alpha}} \\ L_l &= (1-\alpha) p_l y_l / w \end{aligned}$$

$$\begin{aligned} x_{si} &= X_s \left(\frac{p_s^x}{p_{si}^x} \right)^{\frac{1}{1-\sigma}} \\ L_s &= (1-\alpha) p_s y_s / w, \end{aligned}$$

where w is the wage rate, p_l^x and p_{si}^x denote the prices of the leading and the i^{th} standard intermediate goods, $X_s = L_s \left(\frac{\alpha p_s q_s}{p_s^x} \right)^{\frac{1}{1-\alpha}}$, and $p_s^x = \left(\sum_{i=1}^{m} (p_{si}^x)^{\frac{-\sigma}{1-\sigma}} \right)^{\frac{-(1-\sigma)}{\sigma}}$.

Intermediate goods

In the economy, there is a fixed number of $m+1$ intermediate goods, each produced by one and only one producer at any moment in time. The producer of the leading intermediate good alone can produce a good with the highest quality (q_l). Intermediate good producers can try to develop an intermediate good with higher quality than the current leading one. In particular, after spending n_{si}^q units of final output, they face a probability $\lambda_i^q = \lambda_0^q n_{si}^q / y$ of developing a new leading good with quality $\delta_q q_l$ ($\delta_q > 1$). However, while they are behind the leader, they can only produce a (differentiated) standard intermediate good with a fixed quality q_s.[3]

[3]This formulation captures advantage of being the leader (i.e. being able to produce a more differentiated good)

In addition, intermediate goods producers also have the option of investing in improving the production process of their intermediate good (i.e. reducing the cost of production, a_x). Specifically, each intermediate goods firm can invest n^h units of final output and face a probability $\lambda^h = \max\left\{0, -f_0^h + \lambda_0^h \left(n^h/y\right)^{\rho_h}\right\}$, with $0 < \rho_h < 1$, of successfully increasing h by $(\delta_h - 1)h > 0$.

One important distinction between the two forms of innovation is that innovations in the production process are disembodied and therefore are immediately (and costlessly) adopted by all producers. The development of a new leading intermediate good, instead, has a more modest aggregate effect.

Given, the demand functions, the profits of the intermediate goods producers are:

$$\begin{aligned}
\pi_l &= (p_l^x - a_x w)x_l - \left(\left(\frac{\lambda_l^h + f_0^h}{\lambda_0^h}\right)^{\frac{1}{\rho_h}} + \frac{\lambda_l^q}{\lambda_0^q}\right) y \\
\pi_{si} &= (p_{si}^x - a_x w)x_{si} - \left(\left(\frac{\lambda_{si}^h + f_0^h}{\lambda_0^h}\right)^{\frac{1}{\rho_h}} + \frac{\lambda_{si}^q}{\lambda_0^q}\right) y.
\end{aligned}$$

Intermediate goods producers sell their goods monopolistically. Optimal pricing of the intermediate goods implies that $p_l^x = a_x/\alpha$, and that $p_{si}^x = a_x/\sigma$.

We denote the profits of the intermediate goods producers evaluated at the optimal pricing rule as $\bar{\pi}_l$ and $\bar{\pi}_s$. Let V^l and V^{si} denote the values of a intermediate goods producer of the leading good and of the i^{th} standard good. In a slight abuse of notation, let $\overrightarrow{\lambda_{-i}^z}$ denote the vector that contains the hazard rates for innovations on z (for $z = q, h$) for all the intermediate goods producers other than i (for $i = l, si$). Finally, we denote by G^z the law of motion for $\overrightarrow{\lambda_{-i}^z}$. After introducing this notation, we can define V^l and V^{si} with the following Bellman equations:

$$\begin{aligned}
V^l\left(q_l, h; \overrightarrow{\lambda_{-l}^q}, \overrightarrow{\lambda_{-l}^h}\right) &= \max_{\lambda_l^h, \lambda_l^q} \bar{\pi}_l + (1 + rdt)^{-1}[\lambda_l^q V^l(q_l\delta_q, h; \overrightarrow{\lambda_{-l}^{q\prime}}, \overrightarrow{\lambda_{-l}^{h\prime}}) + \sum_{i=1}^m \lambda_{si}^h V^{si}(q_l\delta_q, h; \overrightarrow{\lambda_{-l}^{q\prime}}, \overrightarrow{\lambda_{-l}^{h\prime}}) \\
&\quad (\lambda_l^h + \sum_{i=1}^m \lambda_{si}^h) V^l(q_l, h\delta_h; \overrightarrow{\lambda_{-l}^{q\prime}}, \overrightarrow{\lambda_{-l}^{h\prime}}) + \\
&\quad (1 - \lambda_l^q - \sum_{i=1}^m \lambda_{si}^q - \lambda_l^h - \sum_{i=1}^m \lambda_{si}^h) V^l(q_l, h; \overrightarrow{\lambda_{-l}^{q\prime}}, \overrightarrow{\lambda_{-l}^{h\prime}})] \\
&\quad s.t. \\
\overrightarrow{\lambda_{-l}^{q\prime}} &= G^q(q_l, h; \overrightarrow{\lambda_{-l}^q}, \overrightarrow{\lambda_{-l}^h});\ \overrightarrow{\lambda_{-l}^{h\prime}} = G^h(q_l, h; \overrightarrow{\lambda_{-l}^q}, \overrightarrow{\lambda_{-l}^h})
\end{aligned}$$

and avoids two potential complications. By not having to carry around the distribution of intermediate goods qualities, we can make substantial progress in solving the model analytically. In addition, the absence of entry and exit simplifies the computation of the second moments.

$$
\begin{aligned}
V^{si}\left(q_l, h; \overrightarrow{\lambda^q_{-si}}, \overrightarrow{\lambda^h_{-si}}\right) &= \max_{\lambda^h_{si}, \lambda^q_{si}} \bar{\pi}_s + (1+rdt)^{-1}[\lambda^q_l V^s(q_l\delta_q, h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}}) + (\sum_{i'\neq i} \lambda^q_{si'}) V^{si}(q_l\delta_q, h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}}) \\
&\quad + \lambda^q_{si} V^l(q_l\delta_q, h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}}) + (\lambda^h_l + \sum_{i=1}^m \lambda^h_{si}) V^{si}(q_l, h\delta_h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}}) + \\
&\quad (1 - (\lambda^q_l + \sum_{i=1}^m \lambda^q_{si}) - (\lambda^h_l + \sum_{i=1}^m \lambda^h_{si})) V^{si}(q_l, h; \overrightarrow{\lambda^{q\prime}_{-si}}, \overrightarrow{\lambda^{h\prime}_{-si}})] \\
&\quad s.t. \\
\overrightarrow{\lambda^{q\prime}_{-si}} &= G^q(q_l, h; \overrightarrow{\lambda^q_{-si}}, \overrightarrow{\lambda^h_{-si}});\ \overrightarrow{\lambda^{h\prime}_{-si}} = G^h(q_l, h; \overrightarrow{\lambda^q_{-si}}, \overrightarrow{\lambda^h_{-si}})
\end{aligned}
$$

These Bellman equations do not require much explanation. They simply capture the capital gains and losses suffered by each type of firm when an embodied or disembodied innovation arrives. The only noteworthy element is that firms take as exogenous the hazard rates generated by other firms' innovation activities.

2.2 Optimal investments and stationary symmetric equilibrium

Producers of standard intermediate goods have the option of challenging the current producer of the leading intermediate good by coming out with a higher quality good. Optimal investment in developing this superior intermediate good leads followers to equalize the marginal cost to the expected marginal benefit from the R&D investments:[4]

$$
\underbrace{y}_{\text{Marginal Cost}} = \overbrace{\lambda^q_0(V^l(q\delta_q, h) - V^{si}(q, h))}^{\text{Expected Mg. Benefit from Embodied Innovations}}
$$

Current leaders, in principle, can also increase the quality of their intermediate good. They face the same marginal cost as followers, but the expected marginal benefit is now equal to $\lambda^q_0(V^l(q\delta_q, h) - V^l(q, h))$. Note that this implies that, if in equilibrium $V^l > V^{si}$, only followers will invest in increasing the quality of the leading intermediate good as in standard quality ladder models.

[4] Second order condition:

$$
V^l_{\lambda^q}(q\delta_q, h) - V^s_{\lambda^q}(\delta_q, h) < 0
$$

Leaders may find incentives to come out with disembodied innovations that reduce the marginal cost of producing intermediate goods across the board. In an interior solution, the optimal investment in disembodied innovations by the leader results in the following equality:

$$\overbrace{\frac{y}{\rho_h}\left(\frac{\lambda^h + f_0^h}{\lambda_0^h}\right)^{\frac{1-\rho_h}{\rho_h}}}^{\text{Marginal Cost}} = \overbrace{\lambda_0^h(V^l\left(h\delta_h, q\right) - V^l\left(h, q\right))}^{\text{Expected Mg. Benefit from Disembodied Innovations}}$$

Followers, in principle, also can come out with disembodied improvements in productivity. In equilibrium, however, since the private value of these innovations is proportional to the value of the firm, their incentives to undertake these innovations are lower. In what follows, we assume for simplicity that followers do not find it profitable to indulge in investments that lead to disembodied innovations.[5]

Our analysis is restricted to the Stationary Symmetric Equilibrium (SSE) of this economy. The SSE is characterized by the optimality conditions derived thus far, by the equilibrium in the market for labor and by the fact that the optimal investments in the development of innovations lead to constant hazard rates over time and over intermediate goods producers for any given category (i.e. leader vs. standard, embodied vs. disembodied).

Labor market clearing implies that

$$L = L_l + L_s + L_l^x + \sum_{i=1}^{m} L_{si}^x \tag{1}$$

From the demands of the final output producers, the demands of the producers of leading and standard goods, and from the linear technology in the production of intermediate goods, we obtain the following expressions for the allocation of labor and for the profit flows:

$$\begin{aligned}
L^l &= \frac{(1-\alpha)\beta}{(1-\alpha)+\alpha(\alpha\beta+\sigma(1-\beta))}L \\
L^s &= \frac{(1-\alpha)(1-\beta)}{(1-\alpha)+\alpha(\alpha\beta+\sigma(1-\beta))}L \\
L_l^x &= \frac{\alpha^2\beta}{(1-\alpha)+\alpha(\alpha\beta+\sigma(1-\beta))}L \\
L_{s_i}^x &= \frac{\alpha\sigma(1-\beta)/m}{(1-\alpha)+\alpha(\alpha\beta+\sigma(1-\beta))}L
\end{aligned}$$

[5] This can be the case for several reasons. First, $\frac{y}{\rho_h}\left(\frac{f_0^h}{\lambda_0^h}\right)^{\frac{1-\rho_h}{\rho_h}}$ can be larger than $\lambda_0^h(V^{si}(q,\delta_h h) - V^{si}(q,h))$. Second, f_0^h can be significantly higher than for the leader, or λ_0^h can be significantly lower.

$$\bar{\pi}_l = y((1-\alpha)\alpha\beta - (\frac{\lambda^h + f_0^h}{\lambda_0^h})^{\frac{1}{\rho_h}})$$
$$\bar{\pi}_s = y(\frac{(1-\sigma)\alpha(1-\beta)}{m} - \frac{\lambda^q}{\lambda_0^q m})$$

Imposing the symmetry in the investments of standard intermediate goods producers, the result that only those invest in the development of embodied innovations and the assumption that only the leader undertakes disembodied innovations, we can solve for V^l and V^s in terms of the aggregate hazard rates for embodied and disembodied innovations, λ^q and λ^h.

$$V^s = \frac{1}{\Omega_s}\left[\pi^s + \frac{\lambda^q}{m}\frac{\triangle q \pi^l}{r + \lambda^q - \lambda^h(\triangle h - 1)}\right]$$
$$V^l = \frac{1}{\Omega_l}\left[\pi^l + \frac{\lambda^q \triangle q \pi^s}{r - \lambda^h(\triangle h - 1) - (\frac{m-1}{m}\triangle q - 1)\lambda^q}\right],$$

where

$$\triangle q = \delta_q^{\frac{\beta-\psi_q}{1-\alpha}}$$
$$\triangle h = \delta_h^{\frac{\psi_h}{1-\alpha}}$$
$$\Omega_s = r - \lambda^h(\triangle h - 1) - (\frac{m-1}{m}\triangle q - 1)\lambda^q - \frac{(\lambda^q \triangle q)^2}{m(r + \lambda^q - \lambda^h(\triangle h - 1))}$$
$$\Omega_l = r - \lambda^h(\triangle h - 1) + \lambda^q - \frac{(\lambda^q \triangle q)^2}{m(r - (\frac{m-1}{m}\triangle q - 1)\lambda^q - \lambda^h(\triangle h - 1))}$$

Statistics

Once we have characterized the SSE of the economy, we can compute the relevant statistics in terms of the hazard rates, λ^q and λ^h. The expected growth rate of both aggregate output ($E\gamma_y$) and labor productivity ($E\gamma_{y/L}$) is equal to

$$E\gamma_y = E\gamma_{y/L} = \frac{\beta - \psi_q}{1-\alpha}\ln(\delta_q)\lambda^q + \frac{\psi_h}{1-\alpha}\ln(\delta_h)\lambda^h.$$

This reflects the fact that both embodied and disembodied innovations may generate growth. Similarly, equation (2) provides the formula for the variance of the aggregate growth rate of the economy. Since embodied and disembodied productivity follow independent Poisson processes, the variance of the growth rate of aggregate output (and productivity) is linear in the hazard rates, where the coefficients of λ^q and λ^h are the square of the log-productivity gain in the event of an innovation.

$$var(\gamma_y) = var(\gamma_{y/l}) = \left(\frac{\beta - \psi_q}{1-\alpha}\ln(\delta_q)\right)^2 \lambda^q + \left(\frac{\psi_h}{1-\alpha}\ln(\delta_h)\right)^2 \lambda^h \tag{2}$$

At the firm level, disembodied innovations symmetrically affect the expected growth rate of sales for all firms. That is reflected in the effect of λ^h on $E\gamma_y$ in the following expression for the expected growth rate of sales for producers of the leading and standard intermediate goods:

$$E\gamma_{salesi} = \begin{cases} E\gamma_y - \lambda^q \ln(\beta m/((1-\beta))) \text{ for } i = l \\ E\gamma_y + \lambda^q/m \ln(\beta m/((1-\beta))) \text{ for } i = s \end{cases}$$

Embodied innovations also affect the expected growth rate of sales through the effect on $E\gamma_y$. However, the arrival of an innovation embodied in a new intermediate good generates market turnover that affects differently the expected growth rate of sales for different producers. Specifically, it generates a reduction in the sales of the leaders and a expected increase in the sales of the followers.

These same considerations help us understand the determinants of the expected growth rate of sales per worker. In addition to the effect of innovations on aggregate productivity growth, market turnover affects the firm sales per worker because market leaders charge higher markups than producers of standard intermediate goods. The possibility of a change of role in the market creates an expected gain (loss) in the sales per worker for standard (leading) intermediate goods producers as becomes clear in the next expression:

$$E\gamma_{salesi/L_i} = \begin{cases} \gamma_y - \lambda^q \ln(\sigma/\alpha) \text{ for } i = l \\ \gamma_y + \lambda^q/m \ln(\sigma/\alpha) \text{ for } i = s \end{cases}.$$

In terms of the second moments, the firm-level volatility of the growth rates of sales and sales per worker are affected by aggregate and firm-specific phenomena. On the one hand, the stochastic nature of the aggregate growth process will affect the volatility of firm sales. But, more importantly, the volatility of the growth rate of sales and sales per worker for a firm are affected by the risk of market turnover. It naturally follows then, that, for a given variance of aggregate growth, the firm-level volatility is increasing in the turnover rate (λ^q). This intuition is evident in expressions (3) and (**??**), where we take advantage of the Poisson nature of the stochastic elements to compute the weighted variances of the growth rate of sales and of sales per worker, where the weights are given by the share of the firm's sales in the total sales of intermediate goods in the economy.

$$wvar(\gamma_{sales_i}) = var(\gamma_y) + \lambda^q \left(\frac{1+\beta(m-1)}{m}\right)\left(\ln(\frac{\beta m}{(1-\beta)})\right)^2 \quad (3)$$

$$wvar(\gamma_{sales_i/L})) = var(\gamma_y) + \lambda^q \left(\frac{1+\beta(m-1)}{m}\right)(\ln(\sigma/\alpha))^2 \quad (4)$$

2.3 Comparative statics

Now that we have derived the equilibrium of the economy as well as the relevant statistics, we proceed to analyzing the comparative statics. Specifically, we are interested in understanding the effects of an increase in β. This can be interpreted as a move towards an environment where the market leaders obtain a larger share of the profits due to an homogenization of preferences or to globalization. A more detailed discussion of the interpretation of this force is left for section 4. Totally differentiating the two optimality conditions for the intensities of investment in the development of innovations, we obtain the following conditions:

$$0 = (\triangle h - 1)[v^l_\beta \hat{\beta} + v^l_{\lambda^h}\hat{\lambda}^h + v^l_{\lambda^q}\hat{\lambda}^q] - c''\hat{\lambda}^h \quad \text{(h)}$$

$$0 = \triangle q[v^l_\beta \hat{\beta} + v^l_{\lambda^h}\hat{\lambda}^h + v^l_{\lambda^q}\hat{\lambda}^q] - [v^s_\beta \hat{\beta} + v^s_{\lambda^h}\hat{\lambda}^h + v^s_{\lambda^q}\hat{\lambda}^q] \quad \text{(q)}$$

where $\hat{z}$ denotes the deviation of a generic variable z from steady state, $c'' \equiv \frac{1-\rho_h}{\left(\lambda^h_0\rho_h\right)^2}\left(\frac{\lambda^h + f^h_0}{\lambda^h_0}\right)^{\frac{1-2\rho_h}{\rho_h}}$, and v^b_z denotes the partial derivative of V^b (for $b = l, s$) with respect to variable z.

Isolating $\hat{\lambda}^h$ from expression (h) it follows that:

$$\hat{\lambda}^h = \frac{v^l_\beta \hat{\beta} + v^l_{\lambda^q}\hat{\lambda}^q}{\frac{c''}{\triangle h - 1} - v^l_{\lambda^h}}, \quad (5)$$

where $\frac{c''}{\triangle h-1} - v^l_{\lambda^h} > 0$ from the second order necessary condition.

This expression illustrates the basic forces at play when studying the effect of an increase in β on λ^h. On the one hand, there is a direct effect: *Ceteris paribus*, an increase in β raises the value of market leaders and the marginal value of disembodied innovations that is proportional to v^l. On the other hand, there is an indirect effect that operates through λ^q. *Ceteris paribus,* when β increases, so does the gap between the value of leaders and followers. As a result, followers invest more in R&D to take over the market leadership. This raises the turnover rate and reduces the value of market leaders and their marginal value of disembodied innovations. The net effect of β

on λ^h depends on which of these two forces dominates. We can gain further insight on this issue by studying condition (q). Isolating the term in squared brackets from condition (h) and plugging in condition (q) we obtain the following expression for $\hat{\lambda}^q$:

$$\hat{\lambda}^q = \frac{[-v^s_\beta + \Phi v^l_\beta]}{v^s_{\lambda^q} - \Phi v^l_{\lambda^q}} \hat{\beta}, \tag{6}$$

where $\Phi \equiv \frac{\frac{\triangle q c''}{\triangle h - 1} - v^s_{\lambda^h}}{\frac{c''}{\triangle h - 1} - v^l_{\lambda^h}}$. Almost from a second order condition in the optimal determination of λ^q, it follows that both Φ and $v^s_{\lambda^q} - \Phi v^l_{\lambda^q}$ are positive.[6] The intuition behind expression (6) for $\hat{\lambda}^q$ is that an increase in β increases the gap between the values of the leader and the followers and therefore the incentives of the latter to develop more sophisticated goods that allow them to displace current market leaders. Combining expressions (5) and (6) we obtain the following expression for $\hat{\lambda}^h$:

$$\hat{\lambda}^h = \frac{v^l_\beta v^s_{\lambda^q} - v^s_\beta v^l_{\lambda^q}}{\left[\frac{c''}{\triangle h - 1} - v^l_{\lambda^h}\right]\left[v^s_{\lambda^q} - \Phi v^l_{\lambda^q}\right]} \hat{\beta}. \tag{7}$$

Remember that from the second order conditions the bracketed terms in the denominators are positive. Hence the sign of $\hat{\lambda}^h$ depends on the sign of the numerator $(v^l_\beta v^s_{\lambda^q} - v^s_\beta v^l_{\lambda^q})$. Proposition 1 states some conditions that are sufficient for this term to be negative.

Proposition 1 *If $v^s_\beta < 0$ and $\frac{\triangle q}{\triangle h - 1} c'' - v^s_{\lambda^h} > 0$, λ^q increases with β. If, in addition, $\triangle q$ is close to 1, λ^h declines with β.*

To understand better the intuition of this proposition it is convenient to take a small detour. In standard models of R&D, where there is only one form of innovation, it is typically the case that when the market size increases, more resources are allocated in equilibrium to the innovation activity. This would be the case here too if, for example, only disembodied innovations where feasible. In that case $\hat{\lambda}^q$ would be zero and from expression (5), since v^l_β is positive, it follows that λ^h increases in response to an increase in β.

Competition through innovations embodied in new forms of leading products introduces an interesting effect. An expansion of the market for the leaders makes more attractive becoming the leader. Consider what would happen with the investments in disembodied innovations in a typical quality ladder environment à la Aghion and Howitt [1992]. There the value of being a follower (V^s)

[6] clarify

is always zero and therefore $v^s_{\lambda^q}$ and v^s_β are also equal to zero.[7] It follows then from expression (7) that $\hat{\lambda}^h$ is also zero while $\hat{\lambda}^q$ is positive. In that case, the increase in β induces followers to invest more in embodied innovations until the point where the value of a leader is unchanged. As a result the marginal value of a disembodied innovation for the leader remains constant and so does λ^h.

When σ is smaller than 1, the market value of a follower (V^s) is positive and varies with λ^q, λ^h and β. Now an increase in β not only increases V^l but also reduces V^s, if v^s_β is negative. In this case, an increase in β widens very much the gap between the values of the leader and a follower enhancing the response of λ^q. Further, in expression (6) we can see that the response of $\hat{\lambda}^q$ is higher the more negative is $v^s_{\lambda^q}$. $v^s_{\lambda^q}$ is given by the following expression:

$$v^s_{\lambda^q} = \frac{\overbrace{\frac{m-1}{m}V^s(\triangle q - 1)}^{\text{externality from emb. innov.}} - \overbrace{\frac{\lambda^q}{m}\triangle q \frac{(V^l - \triangle q V^s)}{r - \lambda^h(\triangle h - 1) + \lambda^q}}^{\text{loss from future overtaking}}}{\Omega_s}.$$

The first term in the numerator reflects the gain for a follower that some other follower succeeds in making an embodied innovation. The second term reflects the future loss for a current follower from having another follower taking over her when she becomes the market leader. The sign of $v^s_{\lambda^q}$ depends on the relative importance of these two terms. It is clear that as $\triangle q$ tends to 1, the first term converges to 0 and $v^s_{\lambda^q}$ becomes negative. Proposition 1 states that when this is the case, the response of λ^q to an increase in β is so large that it wipes out the direct positive effect of β on the marginal value of a disembodied innovation for the leader. As a result, an increase in the share of the leading good leads to decline in the investments in disembodied innovations.

Proposition 1 also helps us understand how the statistics of the economy vary with an increase in β. Productivity growth may come both from embodied and disembodied innovations. As can be appreciated in the following expression, the change in the expected growth rate of the economy associated with an increase in β depends on the relative size of the contributions of these innovations to growth.

$$\frac{\partial E\gamma_{y/L}}{\partial\beta} = \ln(\triangle q)\frac{\partial\lambda^q}{\partial\beta} + \ln(\triangle h)\frac{\partial\lambda^h}{\partial\beta}$$

In particular, an increase in β reduces expected productivity growth if and only if the relative productivity gain associated with an embodied innovation is relatively small: $\ln(\triangle q)/\ln(\triangle h) \leq -\frac{\partial\lambda^h}{\partial\beta}/\frac{\partial\lambda^q}{\partial\beta}$.

[7] That would naturally be the case in our model if σ was equal to 1 (i.e. perfect competition for standard goods) provided that we have a linear technology to develop embodied innovations.

Similarly, the effect of β on the variance of aggregate productivity growth is given by the following expression:

$$\frac{\partial var(\gamma_{y/L})}{\partial \beta} = (\ln(\triangle q))^2 \frac{\partial \lambda^q}{\partial \beta} + (\ln(\triangle h))^2 \frac{\partial \lambda^h}{\partial \beta}$$

This implies that aggregate volatility declines with β if and only if $(\ln(\triangle q)/\ln(\triangle h))^2 \leq -\frac{\partial \lambda^h}{\partial \beta}/\frac{\partial \lambda^q}{\partial \beta}$. However, if the productivity gain from an embodied innovation is smaller than the gain from a disembodied innovation (i.e. $\ln(\triangle q)/\ln(\triangle h) < 1$), the fact that the expected growth rate of the economy does not increase following an increase in β is sufficient to generate a decline in aggregate volatility.

At the firm level, the relevant variable to understand the comparative statics is the rate of turnover measured by λ^q. To see that, we reproduce below the expressions for the weighted variance of the growth rate of firm level sales and sales per worker.

$$wvar(\gamma_{sales_i}) = var(\gamma_y) + \lambda^q \left(\frac{1+\beta(m-1)}{m}\right)\left(\ln(\frac{\beta m}{(1-\beta)})\right)^2 \quad (8)$$

$$wvar(\gamma_{sales_i/L})) = var(\gamma_y) + \lambda^q \left(\frac{1+\beta(m-1)}{m}\right)(\ln(\sigma/\alpha))^2 \quad (9)$$

Aggregate demand affects directly sales and sales per worker, therefore the variance of aggregate output enters the first two expressions. However, the variance of aggregate output is approximately two orders of magnitude smaller than the variance of firm-level volatility. Hence, this term should be unimportant to understand the effect of β on firm-level measures of volatility. The important effects come from the second term. An increase in β increases investments in embodied productivity that trigger the turnover rate (λ^q). This is a first order effect that accounts for much of the increase in firm-level volatility. To justify this quantitative remark, note that the share of private R&D in GDP in the US has increased by about a factor of 3, while the variance of firm-level volatility measures has increased by a factor of 4. This means that the effect of β through the turnover rate accounts for about 75 percent of the observed increase in volatility. There is a second effect common to the three measures associated with the fact that as β increases, the volatility of the leader is given a higher weight when computing the weighted firm-level volatility. Since the leader is more volatile because it is more likely to experience a large capital loss, weighted firm-level volatility increases. The third relevant term is only present in the expression for the growth rate of sales. The increase in β implies that more is at stake in the competition for being the leader. As a result of this effect, the variance of firm-level volatility rises too. Note that this effect is absent in the other two measures. This implies that the upward trend in firm-level volatility should be steeper for the growth rate of sales than for the growth rate of sales per worker.

3 Calibration

Next we evaluate the power of the mechanisms highlighted by the model in explaining the effect of an increase in β on the evolution of the level and variance of productivity growth and the variance of the firm-level growth rate of sales per worker. To that end, we calibrate 10 parameters to match initial conditions in several statistics.[8] $\triangle h$ and $\triangle q$ are set to approximately match the initial levels and standard deviation of the growth rate of TFP. The initial level of β and the number of firms producing standard intermediate goods (m) is set to match the initial level of the standard deviations of the growth rate of sales and of sales per worker at the firm level. a and s are set to approximately match the profit rate. λ_0^q is set to 10 so that the initial ratio of investments in embodied innovations over GDP is about 2.5 percent. This implies that measured R&D investments by the NSF are about one third of actual efforts in taking over market leaders. f_0^h is set to one percent which is very close to minimum overhead cost necessary to prevent followers from investing in disembodied innovations in equilibrium.[9] ρ_h is set to 0.6 which is consistent with the lowest estimates obtained for the degree of diminishing returns for R&D. The discount rate is set to 10 percent.

We infer the change in β from the evolution of the R&D intensity. Specifically, we impose that the increment in β must be such that the R&D share in GDP increases by a factor of 2.5 as we have observed in the post-war period in the US. The results from the exercise are reported in Table 1.

year	1950	2000
$E\gamma_{TFP}$	0.0163	0.0122
$\sigma_{\gamma_{Y/L}}$	0.0196	0.0147
$\sigma_{\gamma_{sales/Li}}$	0.0942	0.1725

Table 1

In table 1 we can appreciate how the model is able to generate a decline in TFP growth in line with the data. The model is also successful in doubling the standard deviation of the firm-level

[8] We normalize λ_0^h to 1.

[9] This level is 0.0092.

growth rate of sales per worker. At the aggregate though, the model only accounts for half of the decline in the standard deviation of the growth rate of productivity.

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4 Discussion and Evidence

Globalization leads to the homogenization of preferences. In a world where preferences are more homogeneous the gap between leaders and followers increases. Similarly, a reduction in the transport costs and more generally in the costs of accessing markets, makes easier for leaders to appropriate the markets previously dominated by less sophisticated producers. For example, deregulation in the US banking industry since the early 80s allowed banks to compete in attracting new clients from other states. The internet has greatly increased competition in many service activities such as real estate, travel agencies, insurance, banking, legal services, consulting,... All of these forces tend, to a first order, to make easier for any potential customer to demand being serviced by the market leader.[10]

We have shown in proposition 1 that forces that increase the gap between the market value of leaders and followers trigger the incentives of the latter to become the new leaders. In figure 3 we have shown how the post-war period has been one where there has been a very drastic increase in firm-level volatility. This trend is due to a large extent to the increase in the turnover rate for market leaders. Figure 4 establishes this claim. For each 2 digit sector, we have ranked firms by the level of sales per worker. Once we have the vector of percentiles for every year in the postwar period, we measure the persistence in the rankings by computing the correlation between the vector of rankings in two years five years apart (i.e. 1950 and 1955). By doing that for all the years in the postwar period we have a time series for the turnover in market leadership. In table 4 we also report the evolution of the correlations of rankings for pairs of years 10 years apart. For both of these exercises we can observe how there has been a decline in the persistence of the firms' position in the market and equivalently an increase in market turnover. In the early 50's the correlation of rankings was 0.9 for the 5 year apart measure and 0.8 for the 10 years apart. These correlations have declined pretty much monotonically until reaching 0.71 and 0.66 respectively at the end of

[10] A second effect that complements the mechanisms highlighted in this draft is that these transformations also have reduced the cost faced by followers of developing a product that dominates the current leading product. That force unambiguously leads to an increase in λ^q and a reduction in λ^h.

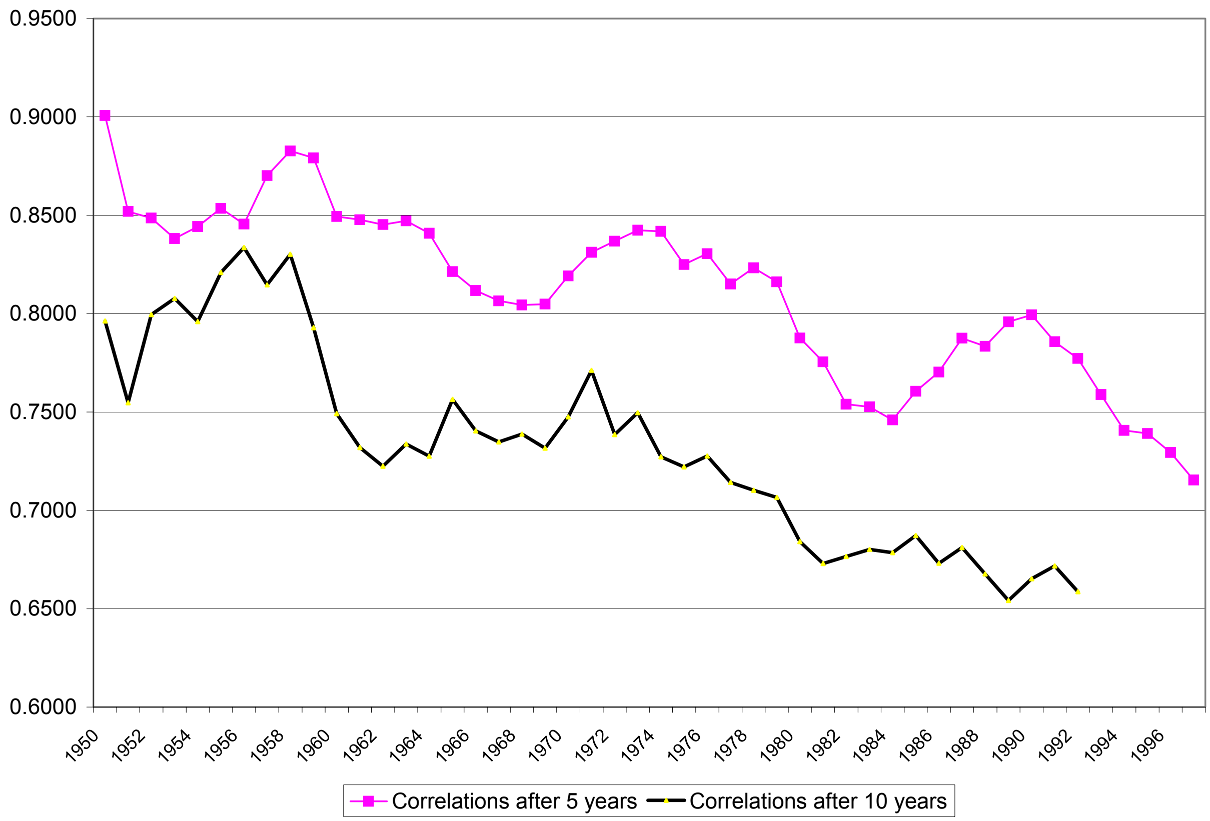

Figure 4: Correlations of firm percentiles by sales per worker.

the sample in 2002.

One important road to leadership is technological superiority. Baumol [2002] emphasizes the role of research and development of superior goods as a competitive mechanism far more important than competition in prices. Figure 1 has showed that one measure of these efforts in developing superior products (the NSF measures of non-federally financed R&D over GDP) has almost tripled since the early 50's.

R&D, however, is not the only way to bring a firm to the top of the market. Firms use advertising as a way to convince potential customers that their product is the leader. Interestingly, advertising expenses have also increased dramatically in the postwar period. The fact that product-specific knowledge has become more important, makes more attractive for workers that have acquired this product specific knowledge to quit and create a new firm that may drive out of business the original one because of its potentially superior knowledge. For example Mountain Hardwear was founded in 1993 by workers that left North Face and Sierra Designs. They justify their success as follows:

"we decided to take a fresh approach to making great gear. Figuring that if we made innovative, technologically advanced tents, outdoor clothing and sleeping bags, consumers would buy them. We were right. [...] But it wasn't just about making great gear. From all those years of working in the outdoor industry, we knew what we liked about the business, and we also knew what we wanted to change." In this case, it is not pure company R&D what leads to a new product that dominates the existing one but the acquisition of specific-knowledge within the market leader, but this new mechanism can be clearly understood in the context of our model.

Finally, another way to interpret our mechanism is as follows: The increasing complexity of products makes more necessary to consult experts before buying them. Experts are more informed and more aware of the different advantages of the products. In this sense, they increase the frequency of shifts in the demand for different products.

It is difficult to measure each of these margins and see if they lead to higher volatility at the firm level. Yet we can conduct a simple exercise focusing our attention again in R&D. We can use cross-sector variation in R&D intensity to test the implication of the model that a higher R&D intensity (in the sector) leads to more turnover and more firm-level volatility (in the sector). Table 2 reports the results from these regressions both with and without weighting each 4-digit sector by the share of their sales in GDP.

We observe there that as predicted by the model there is a positive association between the average R&D intensity in the 4-digit sector and the average firm-level volatility in the sector. This relationship holds even stronger when we weight the different sectors by the share of sales.

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5 Conclusion

To be written.

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Table 2: Relationship between Firm-Level Volatility & R&D intensity*

Independent Variable	Dependent Variable: Average firm-level Volatility in the 4-digit sector	
	Un-Weighted	Weighted
Constant	0.1652	0.0391
	(33.35)	(12.21)
R & D Expense / Sales	0.0532	1.3821
	(4.59)	(7.59)
R-Squared	0.0545	0.1363

* t-stats in parenthesis