The Role of Industry, Geography and Firm Heterogeneity in Credit Risk Diversification*

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Abstract

In theory the potential for credit risk diversification for banks could be substantial. Portfolios are large enough that idiosyncratic risk is diversified away leaving exposure to systematic risk. The potential for portfolio diversification is driven broadly by two characteristics: the degree to which systematic risk factors are correlated with each other and the degree of dependence individual firms have to the different types of risk factors. We propose a model for exploring these dimensions of credit risk diversification: across industry sectors and across different countries or regions. We find that full parameter heterogeneity matters a great deal for capturing tail behavior in credit loss distributions, and that this tail behavior is often not captured using standard value-at-risk (VaR) measures. Instead, the coherent risk measure expected shortfall (ES) is needed. Symmetric shocks to observable risk factors result in asymmetric loss outcomes, and this asymmetry is especially pronounced when full parameter heterogeneity is allowed for. While neither industry nor regional (geography) fixed effects are sufficient to capture this firm-level heterogeneity, controlling for industry effects seems to generate results which are closer to the fully unrestricted heterogeneous model.

Key Words: Risk management, default dependence, economic interlinkages

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1 Introduction

In theory the potential for credit risk diversification for banks can be considerable. To the degree that different industries or sectors are more or less pro-cyclical, banks can alter their lending policy and capital allocation across those sectors. Similarly, internationally active banks are able to apply analogous changes across countries. In addition to such *passive* credit portfolio management, financial engineering, using instruments such as credit derivatives, enable banks (and other financial institutions) to engage in *active* credit portfolio management by buying and selling credit risk (or credit protection) across sectors and countries. Credit exposure to the U.S. chemical industry, say, can be traded for credit exposure to the Korean steel sector. One may therefore think of a global market for credit exposures where credit risk can be exported and imported.

Within such a global context, default probabilities are driven primarily by how firms are tied to business cycles, both domestic and foreign, and how business cycles are linked across countries. In order to implement such a global approach in the analysis of credit risk, we have developed in Pesaran, Schuermann and Weiner (2004), hereafter PSW, a global vector autoregressive macroeconometric model (GVAR) for a set of 25 countries accounting for about 80% of world output. Importantly, the foreign variables in the GVAR are tailored to match the international trade pattern of the country under consideration.

In Pesaran, Schuermann, Treutler and Weiner (2004), hereafter PSTW, we then link returns for a portfolio of 119 firms to this global macroeconometric model, which allows us to isolate macro effects from idiosyncratic shocks as they relate to default (and hence loss). The GVAR effectively serves as the economic engine reflective of the environment faced by an internationally active global bank. Domestic and foreign business cycle effects are allowed to impact each firm differently. In this way we are able to account for firm-specific heterogeneity in an explicitly interdependent global context. Since we are keenly interested in the impact of different yet equi-probable shock scenarios from observable risk factors, developing such a conditional modeling framework is key.

In this paper we extend the analysis of PSTW along three dimensions. First, we provide some analytical results on the limits of credit risk diversification. Second, we use this framework to understand the degree of diversfication with five models which differ in the their degree of parameter heterogeneity, from fully homogenenous, allowing for industry and regional heterogeneity but homogeneous factor sensitivities all the way to the fully heterogeneous model. Third, we have more than doubled the number of firms in the portfolio from 119 to 243 firms providing for more robust results and allowing us to explore the importance of exposure granularity. We go on to explore the impact of shocks to real equity prices, interest rates and real output on the resulting loss distribution as implied by the five different models.

Such conditional analysis using shock scenarios from observable risk factors is not possible in the most commonly used model in the credit risk literature, namely the Vasicek (1987, 1991, 2002) adaption of the Merton (1974) default model. In addition to being driven by a single and *un*observed risk factor, this model also assumes that risk factor sensitivities, analogous to CAPM-style betas, are the same across all firms in all regions and industries, yielding a fully homogeneous model. This single factor model also underlies the risk-based capital standards in the New Basel Accord, as shown in Gordy (2003).

We find that full parameter heterogeneity matters a great deal for capturing tail behavior in credit loss distributions, and that this tail behavior is often not captured using standard value-at-risk (VaR) measures. Instead, the coherent risk measure expected shortfall (ES) is needed. Symmetric shocks to observable risk factors result in asymmetric loss outcomes, and this asymmetry is especially pronounced when full parameter heterogeneity is allowed for. While neither industry nor regional (geography) fixed effects are sufficient to capture this firm-level heterogeneity, controlling for industry effects seems to generate results which are closer to the fully unrestricted heterogeneous model. Interestingly, adding firm fixed effects to the regional fixed effects model does not seem to improve matters. This points to the importance of firm heterogeneity in the risk factor sensitivities, the firm "betas," and neglecting such firm differences can either result in too much implied risk capital, under adverse shocks, or too little capital under benign shock scenarios.

The plan for the remainder of the paper is as follows: Section 2 provides a model of firm value and default, and Section 3 lays out how firm returns and hence default may be cross-sectionally correlated. Section 4 develops analytical results for the credit portfolio loss distribution. Section 5 presents the framework for conditional credit risk modeling including a brief overview of the global macroeconometric model. In Section 6 we introduce the credit portfolio and present the results from the multi-factor return regressions that link firm returns to the observable systematic risk factors from the macroeconomic engine. We present results for five models ranging from the homogeneous Vasicek model to one allowing for full heterogeneity, with industry and geography effects along the way. In Section 7 we analyze how those models impact the resulting loss distributions under a variety of macroeconomic scenarios, and we provide some concluding remarks in Section 8.

2 Firm Value and Default[1]

Any credit default model requires a model of the evolution of firm value as well as conditions under which default occurs. Specification of those conditions imply a model for a default threshold. In this section we provide a simple model of firm value, where the firm is leveraged (i.e. has debt), and its corresponding default threshold. Our approach adapts the option theoretic default model due to Merton (1974). Merton recognized that a lender is effectively writing a put option on the assets of the borrowing firm; owners and owner-managers (i.e. shareholders) hold the call option. If the value of the firm falls below a certain threshold, the owners will put the firm to the debt-

[1] This section follows closely the approach introduced in PSTW.

holders. Thus a firm is expected to default when the value of its assets falls below a threshold value determined by its liabilities.

The problem of modeling firm default inherits all the asymmetric information and agency problems between borrower and lender well known in the banking literature. The argument is roughly as follows. A firm, particularly if it is young and privately held, knows more about its health, quality and prospects than outsiders, e.g. lenders. Banks are particularly well suited to help overcome these informational asymmetries through relationship lending; learning by lending. Moreover, managers and owners of firms have an incentive to substitute higher risk for lower risk investments as they are able to receive upside gains (they hold a call option on the firm's assets) while lenders are not (they hold a put option); see the survey by James and Smith (2000) for a more extensive discussion, as well as Garbade (2001). If the firm is public, we have other sources of information such as quarterly and annual reports which, though accounting based, are then digested and interpreted by the market. Stock and bond prices serve as summary statistics of that information.

The scope for diversification of credit risk thus can enter through two channels: how firm value reacts to changes in the systematic risk factors and through differentiated default thresholds. Both channels need to be modeled. As we explore the impact of two dimensions of diversification, geography and industry or sector, we will denote a firm j in country or region i and sector s having asset values $V_{jis,t}$ at time t, and an outstanding stock of debt, $D_{jis,t}$. Under the Merton (1974) model default occurs at the maturity date of the debt, $t+H$, when the firm's assets, $V_{jis,t+H}$, are less than the face value of the debt at that time, $D_{jis,t+H}$. This is in contrast with the first-passage models where default would occur the first time that $V_{jis,t}$ falls below a default boundary (or threshold) over the period t to $t+H$.[2] Under both models the default probabilities are computed with respect to the probability distribution of asset values at the terminal date, $t+H$ in the case of the original Merton model, and over the period from t to $t+H$ in the case of the first-passage models. Although our approach can be adapted to the first-passage model, for simplicity we follow the Merton approach here.

The value of the firm at time t is the sum of debt and equity, namely

$$V_{jis,t} = D_{jis,t} + E_{jis,t}, \text{ with } D_{jis,t} > 0. \tag{1}$$

Conditional on time t information, default will take place at time $t+H$ if

$$V_{jis,t+H} \leq D_{jis,t+H}.$$

Because bankruptcies are costly and violations to the absolute priority rule in bankruptcy proceedings are so common, in practice the shareholders have an incentive to put the firm into receivership

[2] See Black and Cox (1976). More recent modeling approaches include direct strategic default considerations (e.g. Mella-Barral and Perraudin (1997)). For a review of these models, see, for example, Lando (2004, Chapter 3).

even before the equity value of the firm hits the zero value.[3] Similarly the bank might also have an incentive of forcing the firm to default once the firm's equity falls below a non-zero threshold.[4] Technical default definitions used by banks and bondholders are typically weaker than outright bankruptcy, a notion we follow here. Hence we assume that default takes place if

$$E_{jis,t+H} < C_{jis,t+H}, \tag{2}$$

where $C_{jis,t+H}$ is a positive default threshold, which could vary over time and with firm's particular characteristics (region and sector being two of them, of course). Natural candidates include quantitative factors such as leverage, profitability, firm age and perhaps size, most of which appear in models of firm bankruptcy,[5] as well as more qualitative factors such as management quality. Obviously some of these factors will be easier to observe and measure than others. The observable accounting-based factors are at best noisy and at worst reported with bias, highlighting the information asymmetry between managers (agents) and shareholders and debtholders (principals).[6]

To overcome these measurement difficulties and information asymmetries, we make use of a firm's credit rating $\mathcal{R}$.[7] This will help us specifically in nailing down the default threshold. Naturally rating agencies have access to, and presumably make use of, private information about the firm to arrive at their firm-specific credit rating, in addition to incorporating public information such as, for instance, financial statements and equity returns. Thus we make the assumption that rating agencies benchmark their ratings on past returns and volatilities of *all* firms that have been rated $\mathcal{R}$ in the past (say over the past 10 to 20 years, see below).[8]

Consider now a particular $\mathcal{R}-$rated firm at time t, and assume that the credit rating agency uses the following geometric random walk model of equity values:

$$\ln(E_{\mathcal{R},t+1}) = \ln(E_{\mathcal{R}t}) + \mu_{\mathcal{R}} + \sigma_{\mathcal{R}}\eta_{\mathcal{R},t+1}, \qquad \eta_{\mathcal{R},t+1} \sim IIDN(0,1), \tag{3}$$

with a non-zero drift, $\mu_{\mathcal{R}}$, and idiosyncratic Gaussian innovations with a zero mean and fixed volatility, $\sigma_{\mathcal{R}}$.[9] Hence, over the period $(t, t+H)$

$$\ln(E_{\mathcal{R},t+H}) = \ln(E_{\mathcal{R}t}) + H\ \mu_{\mathcal{R}} + \sigma_{\mathcal{R}}\sum_{s=1}^{H}\eta_{\mathcal{R},t+s},$$

[3] See, for instance, Leland and Toft (1996).

[4] For a treatment of this scenario, see Garbade (2001).

[5] See, for instance, Altman (1968), Lennox (1999) and Shumway (2001).

[6] Duffie and Lando (2001), with this in mind, allow for imperfect information about the firm's assets and default threshold in the context of a first-passage model.

[7] $\mathcal{R}$ may take on values such as 'Aaa', 'Aa', 'Baa',..., 'Caa' in Moody's terminology, or 'AAA', 'AA', 'BBB',..., 'CCC' in S&P's terminology.

[8] For an overview of the rating industry, see Cantor and Packer (1995); Jafry and Schuermann (2004) provide detailed default probability estimates by rating.

[9] Clearly non-Gaussian innovations can also be considered. But for quarterly data that we shall be working with Gausssian innovations seems a good first approximation.

and by (2) default occurs if

$$\ln(E_{\mathcal{R},t+H}) = \ln(E_{\mathcal{R}t}) + H\ \mu_{\mathcal{R}} + \sigma_{\mathcal{R}} \sum_{s=1}^{H} \eta_{\mathcal{R},t+s} < \ln\left(C_{\mathcal{R},t+H}\right), \tag{4}$$

Therefore, the default probability for the $\mathcal{R}-$rated firms at the terminal date $t+H$ is given by

$$\pi_{\mathcal{R}}(t,H) = \Phi\left(\frac{\ln\left(C_{\mathcal{R},t+H}/E_{\mathcal{R}t}\right) - H\ \mu_{\mathcal{R}}}{\sigma_{\mathcal{R}}\sqrt{H}}\right). \tag{5}$$

Denote the H-period forward log threshold-equity ratio to be $\lambda_{\mathcal{R}}(t,H) = \ln\left(C_{\mathcal{R},t+H}/E_{\mathcal{R}t}\right)$ so that

$$\lambda_{\mathcal{R}}(t,H) = H\mu_{\mathcal{R}} + Q_{\mathcal{R}}(t,H)\ \sigma_{\mathcal{R}}\sqrt{H},$$

where

$$Q_{\mathcal{R}}(t,H) = \Phi^{-1}\left[\pi_{\mathcal{R}}(t,H)\right],$$

is the quantile associated with the default probability $\pi_{\mathcal{R}}(t,H)$.

An estimate of $\lambda_{\mathcal{R}}(t,H)$ can now be obtained using past observations on returns, $r_{\mathcal{R},t+1} = \ln(E_{\mathcal{R},t+1}/E_{\mathcal{R}t})$, and the empirical default frequencies, $\hat{\pi}_{\mathcal{R}}(t,H)$, of $\mathcal{R}-$rated firms over a given period of say $t = 1,2,...,T$. Denoting the estimates of $\mu_{\mathcal{R}}$ and $\sigma_{\mathcal{R}}$ by $\hat{\mu}_{\mathcal{R}}$, and $\hat{\sigma}_{\mathcal{R}}$, respectively, we have

$$\hat{\lambda}_{\mathcal{R}}(t,H) = H\hat{\mu}_{\mathcal{R}} + \hat{Q}_{\mathcal{R}}(t,H)\ \hat{\sigma}_{\mathcal{R}}\sqrt{H}, \tag{6}$$

where

$$\hat{\mu}_{\mathcal{R}} = T^{-1}\sum_{t=1}^{T} r_{\mathcal{R}t},\ \hat{\sigma}^2_{\mathcal{R}} = T^{-1}\sum_{t=1}^{T}\left(r_{\mathcal{R}t} - \hat{\mu}_{\mathcal{R}}\right)^2,$$

and

$$\hat{Q}_{\mathcal{R}}(t,H) = \Phi^{-1}\left[\hat{\pi}_{\mathcal{R}}(t,H)\right]. \tag{7}$$

The estimates of $\hat{\mu}_{\mathcal{R}}$ and $\hat{\sigma}_{\mathcal{R}}$ can also be updated using a rolling window of size 7-8 years (the average length of the business cycle).

In practice, $\hat{\pi}_{\mathcal{R}}(t,H)$ might not provide a reliable estimate of $\pi_{\mathcal{R}}(t,H)$ as it is likely to be based on very few defaults over any particular period $(t, t+H)$. One possibility would be to use an average estimate of $\lambda_{\mathcal{R}}(t,H)$ obtained over a reasonably long period of 10 to 20 years (on a rolling basis).[10] For example, based on the sample observations $t = 1,2,...,T$ we would have

$$\hat{\lambda}_{\mathcal{R}}(H) = H\ \hat{\mu}_{\mathcal{R}} + \hat{Q}_{\mathcal{R}}(H)\ \hat{\sigma}_{\mathcal{R}}\ \sqrt{H}, \tag{8}$$

where the (average) quantile estimate $\hat{Q}_{\mathcal{R}}(H)$ is given by

$$\hat{Q}_{\mathcal{R}}(H) = T^{-1}\sum_{t=1}^{T}\left\{\Phi^{-1}\left[\hat{\pi}_{\mathcal{R}}(t,H)\right]\right\}. \tag{9}$$

[10] For a comparison of default estimation approaches, see Jafry and Schuermann (2004).

We assume that rating agencies use about a one-year horizon ($H = 4$ quarters) when assessing a firm.

We allow for the possibility of time-varying default thresholds, provided that $C_{jis,t+H}/E_{jis,t} = C_{\mathcal{R},t+H}/E_{\mathcal{R}t}$, meaning that the ratio of the future threshold to today's firm capital, is the same for all firms of a particular credit rating $\mathcal{R}$. Moreover, given sufficient data for a particular region or country i (the U.S. comes to mind) or sector s, one could in principle have π's varying over those dimensions as well. However, since a particular firm j's default is only observable once, multiple (serial) bankruptcies notwithstanding, it makes less sense to allow π to vary across j.[11] Empirically, then, we will abstract from possible variation in default rates across regions and sectors, so that probabilities of default vary only across credit ratings and over time.

An important source of heterogeneity is likely the large variation in bankruptcy laws and regulation across countries. However, by using rating agency default data, we use their homogeneous definition of default and are thus not subject to these heterogeneities.

2.1 Firm-Specific Defaults

We continue with the return of firm j in region i and sector s over the period t to $t+1$ denoted by $r_{jis,t+1} = \ln\left(E_{jis,t+1}/E_{jis,t}\right)$, and assume that conditional on the information available at time t, Ω_t, it can be decomposed as

$$r_{jis,t+1} = \mu_{jis,t} + \xi_{jis,t+1}, \tag{10}$$

where $\mu_{jis,t}$ is the (forecastable) conditional mean, and $\xi_{i,t+1}$ is the (non-forecastable) innovation component of the return process. It may contain firm-specific idiosyncratic as well as systematic risk factor innovations. Following the standard Merton model we shall assume that

$$\xi_{jis,t+1} \mid \Omega_t \sim N(0, \omega^2_{\xi,jis}). \tag{11}$$

We can now characterize the separation between a default and a non-default state with an indicator variable $I\left(r_{ji,t+1} < \lambda_{jis}(t,1)\right)$, where $\lambda_{jis}(t,1) = \ln\left(C_{jis,t+1}/E_{jis,t}\right)$ is the one period forward log default threshold-equity ratio, such that, using (4),

$$\begin{aligned} I\left(r_{jis,t+1} < \lambda_{jis}(t,1)\right) &= 1 \text{ if } r_{jis,t+1} < \lambda_{jis}(t,1) \Longrightarrow \text{ Default,} \\ I\left(r_{jis,t+1} < \lambda_{jis}(t,1)\right) &= 0 \text{ if } r_{jis,t+1} \geq \lambda_{jis}(t,1) \Longrightarrow \text{ No Default.} \end{aligned} \tag{12}$$

[11] To be sure, one is not strictly prevented from obtaining firm-specific default probabilities estimates at a given point in time. The bankrupcty models of Altman (1968), Lennox (1999) and Shumway (2001) are such examples, as is the industry model by KMV (Kealhofer and Kurbat (2002)). However, all of these studies focus on just one country at a time (the U.S. and U.K in this list) and do not address the formidable challenges of point in time bankruptcy forecasting with a multi-country portfolio.

Using the same approach as above, the one quarter ahead (with $T = 1$) default probability for firm j is given by

$$\pi_{jis,t} = \Phi\left(\frac{\lambda_{jis}(t,1) - \mu_{jis,t}}{\omega_{\xi jis}}\right). \tag{13}$$

$\mu_{jis,t}$ and $\omega_{\xi jis}$ can be estimated using the firm-specific multi-factor regressions. $\lambda_{jis}(t,1)$ will be estimated using the rating information of this firm at time t. If the firm is rated $\mathcal{R}$, then $\lambda_{jis}(t,1)$ will be estimated by $\hat{\lambda}_{\mathcal{R}}(t,1)$ as in (6), on the assumption that all $\mathcal{R}-$rated firms have the same default threshold-equity ratio. In this way variation in the firm-specific default likelihood $\pi_{jis,t}$ will be driven largely by that firm's return volatility $\omega_{\xi jis}$.

The default condition for firm j with credit rating $\mathcal{R}$ can now be written as

$$I\left(r_{jis,t+1} < \hat{\lambda}_{\mathcal{R}}(t,1)\right) = 1 \text{ if } r_{jis,t+1} < \hat{\lambda}_{\mathcal{R}}(t,1) \Longrightarrow \text{ Default}, \tag{14}$$

and is thus the same for all firms with rating $\mathcal{R}$. Once again due to the small number of defaults over a single period $(t, t+1)$, in practice it might be more appropriate to use a (rolling) average estimate such as $\hat{\lambda}_{\mathcal{R}}(H)$ defined by (8).

Mappings from credit ratings to default probabilities are typically obtained using corporate bond rating histories over many years, often 20 years or more, and thus represent some average across business cycles. The reason for such long samples is simple: default events for investment grade firms are quite rare; for example, the annual default probability of an '$\mathcal{A}$' rated firm is approximately one basis point for both Moody's and S&P rated firms. We will further make the indentifying interpretation of credit ratings as being "cycle-neutral" (Saunders and Allen (2002), Amato and Furfine (2004)), meaning that ratings are assigned only on the basis of firm-specific information and not on systematic or macroeconomic information.

Credit ratings from rating agencies, being neither buyers nor sellers of credit assets, have the potential of playing an important, albeit crude, informational role, especially since in assigning their rating, they typically have access to private (inside) information on the firm they are rating. In arriving at the firm-specific estimated default probability, $\hat{\pi}_{jis,t}$, we thus independently combine information from the perspective of the two main principals of the firm: the shareholder's view, namely stock returns which are public information, and the debtholder's view, given by rating agencies. This combination allows us to gain potentially rich insight into firms' behavior, particularly with respect to the default decision, by overcoming the inherent information asymmetries vis à vis the firm.

2.2 Multi-period Default Conditions

The default condition for one period ahead is trivially the same under the Merton and first-passage models. However, for multiple periods these two approaches diverge. To begin, consider the two-period problem. The Merton model considers default only at the terminal date. Firm j will default if

$$r_{jis,t+1} + r_{jis,t+2} < 2\hat{\mu}_{\mathcal{R}} + \hat{\sigma}_{\mathcal{R}}\sqrt{2}\hat{Q}_{\mathcal{R}}(2),$$

where the quantile estimate $\hat{Q}_{\mathcal{R}}(2)$ is given in (9). Matters are complicated a bit in the first-passage model. In this case firm j will default if either

$$r_{jis,t+1} < \hat{\mu}_{\mathcal{R}} + \hat{\sigma}_{\mathcal{R}}\hat{Q}_{\mathcal{R}}(1),$$

or

$$r_{jis,t+1} \geq \hat{\mu}_{\mathcal{R}} + \hat{\sigma}_{\mathcal{R}}\hat{Q}_{\mathcal{R}}(1) \quad \textit{and} \quad r_{jis,t+1} + r_{jis,t+2} < 2\hat{\mu}_{\mathcal{R}} + \hat{\sigma}_{\mathcal{R}}\sqrt{2}\hat{Q}_{\mathcal{R}}(2).$$

Extending this results to H periods is straight forward for the Merton model, namely

$$R_{jis,T+H} \equiv \sum_{\tau=1}^{H} r_{jis,t+\tau} < H\hat{\mu}_{\mathcal{R}} + \hat{\sigma}_{\mathcal{R}}\sqrt{H}\hat{Q}_{\mathcal{R}}(H). \tag{15}$$

3 Cross-Firm Return and Default Dependence

We now turn to the problem of considering cross-firm correlation through returns resulting in default correlation. This dependence arises through the systematic risk factors so that conditional on a realization of those factors, firm returns, and hence defaults, are independent. An extreme case is when all risk is idiosyncratic. Of course in this case there is no systematic risk, and in the limit the credit portfolio is fully diversified: unexpected loss vanishes as $N \to \infty$. As the number of exposures or obligors (firms) in the portfolio increases without bound, the portfolio default/loss will converge to the average firm-level default probability. If $LGD < 1$, then the portfolio loss will simply be proportionately less.

This extreme case is quite unrealistic as we would like to allow for some degree of cross-sectional correlation of returns. Indeed the credit risk literature has recognized for some time the importance of modeling correlated or dependent defaults (see for instance ch. 9 in Lando (2004)). The extent to which credit risk can be diversified crucially depends on the nature and degree of dependence in defaults across firms. The most widely used model of default dependence is the common factor model where the dependence is characterized in terms of a common set of risk factors, either directly via firm-specific default probabilities, or indirectly through firm returns. Vasicek (1987, 1991) was amongst the first to develop such a credit risk factor model, using a single factor model of firm returns.[12] This approach also forms the basis of New Basel Accord as outlined in detail by Gordy (2003).[13]

[12] Extensions to multiple factors were proposed by Wilson (1997a,b) and Gupton, Finger and Bhatia (1997) in the form of the industry credit portfolio model, CreditMetrics. For recent published accounts and further developments see Vasicek (2002) and Schönbucher (2000, 2002).

[13] There are also a number of other approaches to modeling default correlations which we do not consider in this paper. These include the correlated default intensities approach of Schönbucher (1998), Duffie and Singleton (1999)

Using (10) and suppressing the region (i) and sector (s) subscripts for convenience, the multifactor version of the return equation can be written as

$$r_{j,t+1} = \mu_{jt} + \boldsymbol{\gamma}_j' \mathbf{f}_{t+1} + \sigma_j \eta_{j,t+1}, \quad \eta_{j,t+1} \sim iid\mathcal{N}(0,1) \tag{16}$$

where $\mathbf{f}_{t+1}$ is an $m \times 1$ vector of common risk factors, $\boldsymbol{\gamma}_j$ is the associated vector of factor loadings, and $\eta_{j,t+1}$ is the firm-specific idiosyncratic shock, assumed to be distributed independently across j. In this section, along with much of the credit risk literature, we shall treat the common factors as unobserved and assume that returns are unpredictable, namely $\mu_{jt} = \mu_j$ and $\mathbf{f}_{t+1}$ are serially uncorrelated.[14] Specifically, we assume that $\mathbf{f}_{t+1} \sim iid\mathcal{N}(\mathbf{0}, \mathbf{I}_m)$, where $\mathbf{I}_m$ is an identity matrix of order m.[15].

Before exploring the implications of (16) for the credit risk modeling, some of the features of the multifactor model are worth emphasizing:

1. Under homogeneous factor loadings with $\boldsymbol{\gamma}_j = \boldsymbol{\gamma}$ for all j, the distinction between a single and multiple factors will become redundant. This is because under the homogeneity assumption $\boldsymbol{\gamma}_j'\mathbf{f}_{t+1} = \boldsymbol{\gamma}'\mathbf{f}_{t+1}$ can be treated as a single factor with a factor loading of $\sqrt{\boldsymbol{\gamma}'\boldsymbol{\gamma}}$.

2. The individual returns, $r_{j,t+1}$, are serially uncorrelated, but cross-sectionally correlated. Denoting the cross section correlation of returns of firm j and j', by $\rho_{jj'}$ we have

$$\rho_{jj'} = \frac{\boldsymbol{\delta}_{j'}'\boldsymbol{\delta}_j}{\left(1 + \boldsymbol{\delta}_{j'}'\boldsymbol{\delta}_{j'}\right)^{1/2}\left(1 + \boldsymbol{\delta}_j'\boldsymbol{\delta}_j\right)^{1/2}}, \tag{17}$$

   where $\boldsymbol{\delta}_j = \boldsymbol{\gamma}_j / \sigma_j$.

3. If $\boldsymbol{\delta}_j = \boldsymbol{\delta}$ for all j, then clearly the pair-wise correlations are the same across all firms, and $\rho_{jj'} = \rho$.

4. If $\boldsymbol{\delta}_j$ and $\boldsymbol{\delta}_{j'}$ are independently distributed, the average pair-wise correlation of asset returns is given by

$$E\left(\rho_{jj'}\right) = E\left(\frac{\boldsymbol{\delta}_j'}{\sqrt{1 + \boldsymbol{\delta}_j'\boldsymbol{\delta}_j}}\right) E\left(\frac{\boldsymbol{\delta}_{j'}}{\sqrt{1 + \boldsymbol{\delta}_{j'}'\boldsymbol{\delta}_{j'}}}\right),$$

and Duffie and Gârleanu (2001), the contagion model of Davis and Lo (2001), as well as Giesecke and Weber's (2003) indirect dependence approach, where default correlation is introduced through local interaction of firms with their business partners as well as via global dependence on economic risk factors. More general models of dependence, using copulas, have been discussed in Li (2000), Embrechts, McNeil and Straumann (2001), Frey and McNeil (2001) and Schönbucher (2002).

[14] The case of observable risk factors is discussed below, where $\mathbf{f}_{t+1}$ is linked to the variables in a global vector autoregressive model recently developed in Pesaran, Schuermann and Weiner (2004).

[15] The more general case where the factors may exhibit time varying volatility can be readily dealt with by allowing the factor loadings to be time varying, in line with market volatilities. But in this paper we shall not pursue this line of research, primarily because the focus of our empirical analysis is on quarterly and annual default risks: over such horizons asset return volatilities appear to be rather limited and of second order importance.

and will be exactly zero if $\boldsymbol{\delta}_j$ is symmetrically distributed around zero.

Consider now the correlation of defaults across firms. As before, let $z_{j,t+1}$ to be the default outcome for firm j, such that

$$z_{j,t+1} = I\left(r_{j,t+1} < \lambda_j\right), \tag{18}$$

and denote the pair-wise default correlation of firm j and j' by $\rho^*_{jj'}$. It is now easily seen that

$$\rho^*_{jj'} = \frac{E\left(z_{j,t+1}z_{j',t+1}\right) - \pi_j\pi_{j'}}{\sqrt{\pi_j(1-\pi_j)}\sqrt{\pi_{j'}(1-\pi_{j'})}} \tag{19}$$

where

$$\pi_j = E\left(z_{j,t+1}\right) = \Phi\left(\frac{\alpha_j}{\sqrt{1+\boldsymbol{\delta}_j'\boldsymbol{\delta}_j}}\right), \tag{20}$$

$$E\left(z_{i,t+1}z_{j,t+1}\right) = E\left[\Phi\left(\alpha_j - \boldsymbol{\delta}_j'\mathbf{f}_{t+1}\right)\Phi\left(\alpha_{j'} - \boldsymbol{\delta}_{j'}'\mathbf{f}_{t+1}\right)\right],$$

and

$$\alpha_j = \frac{\lambda_j - \mu_j}{\sigma_j}.$$

In the above expression, expectations are taken with respect to the distribution of the factors. Clearly, $\rho^*_{jj'} = 0$ if $\rho_{jj'} = 0$. For non-zero values of $\rho_{jj'}$ the relationship between $\rho^*_{jj'}$ and $\rho_{jj'}$ is non-linear, and depends on π_j, $\boldsymbol{\delta}_j$, and the probability densities assumed for $\varepsilon_{j,t+1}$ and $\mathbf{f}_{t+1}$, in a complicated manner. This relationship is simplified considerably under the homogeneous double-Gaussian case (where $\varepsilon_{j,t+1}$ and f_{t+1} are assumed to be jointly Gaussian) discussed by Vasicek. In this case using (17) and (20) we first note that

$$\boldsymbol{\delta}'\boldsymbol{\delta} = \frac{\rho}{1-\rho}, \text{ and } \alpha = \frac{1}{\sqrt{1-\rho}}\Phi^{-1}(\pi).$$

Hence the default correlation,

$$\rho^*_{ij} = \rho^*\left(\rho,\pi\right) = \frac{E_x\left[\Phi^2\left(\frac{1}{\sqrt{1-\rho}}\Phi^{-1}(\pi) - x\sqrt{\frac{\rho}{1-\rho}}\right)\right] - \pi^2}{\pi(1-\pi)}, \tag{21}$$

where expectations are taken with respect to $x \sim N(0,1)$, which can be carried out using stochastic simulations. Figure 1 provides simulated plots of $\rho^*\left(\rho,\pi\right)$ against ρ, for a few selected values of π. It is clear that the default correlation, ρ^*, is related non-linearly to ρ, and tends to be considerably lower than ρ. Also there is a clear tendency for the (ρ^*,ρ) relationship to shift downwards as π is reduced. For very small values of π, sizable default correlations are predicted by the double-Gaussian Vasicek model only for very high values of return correlations.

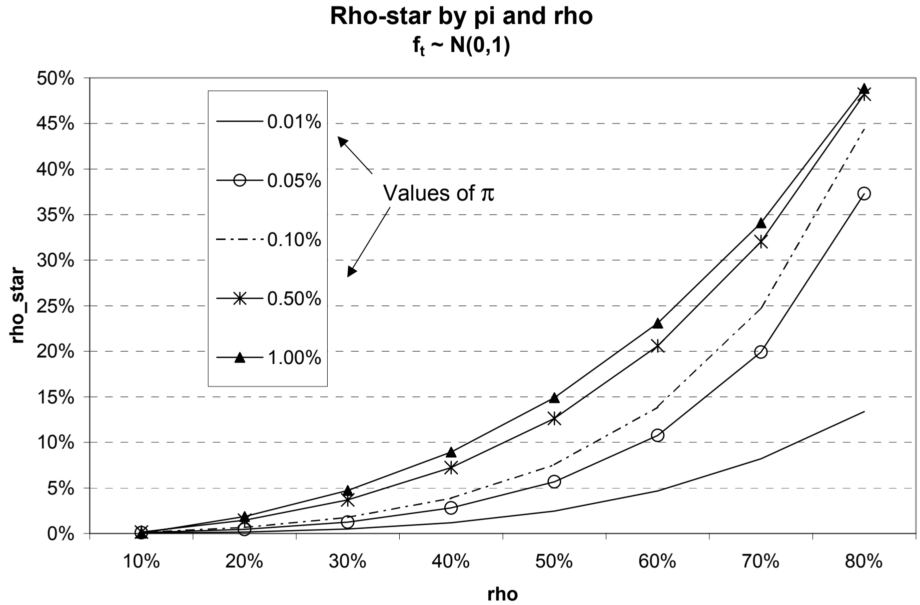

Figure 1: Default correlation (rho_star) as a function of return correlation in the Vasicek Model

4 Credit Loss Distribution

The complicated relationship between return correlations and defaults manifest itself at the portfolio level. Consider a credit portfolio composed of N different credit assets such as loans, each with exposures A_{jt}. Suppose further that loss-given-default (LGD) of obligor j is denoted by $\varphi_{j,t+1}$ assumed to lie in the range $[0, 1]$. The portfolio loss over the period t to $t+1$ is given by

$$L_{N,t+1} = \sum_{j=1}^{N} A_{jt}\varphi_{j,t+1}z_{j,t+1}, \tag{22}$$

where as before, $z_{j,t+1} = I(r_{j,t+1} < \lambda_{jt})$.

The loss defined as a fraction of total exposure, $A_t = \sum_{j=1}^{N} A_{jt}$, is given by

$$\ell_{N,t+1} = \sum_{j=1}^{N} a_{jt}\varphi_{j,t+1}z_{j,t+1},$$

where $\ell_{N,t+1} = L_{N,t+1}/A_t$, and $a_{jt} = A_{jt}/\sum_{j=1}^{N} A_{jt}$. To simplify the exposition we assume that the exposure shares, a_{jt}, and the LGD's, $\varphi_{j,t+1}$, are given and distributed independently of the default indicators, $z_{j,t+1}$. This is not a limiting restriction as far as the exposure shares are concerned, since they are set before the default outcomes are realized. The possible dependence of $\varphi_{j,t+1}$ on $z_{j,t+1}$, however, can not be ruled out and is a subject of ongoing research.[16] But in what

[16] One would expect loss severity to be higher in recessions than expansions (see Frye (2000) and Altman et al.

follows we abstract from this complication and assume that $\varphi_{j,t+1}$ is drawn exogenously from a Beta distribution whose parameters are calibrated to historical LGD data. Under these conditions we have

$$\ell_{N,t+1} = \left(\sum_{j=1}^{N} a_j \varphi_j\right)\left(\sum_{j=1}^{N} w_j z_{j,t+1}\right),$$

where $w_j = a_j\varphi_j / \sum_{j=1}^{N} a_j \varphi_j$. Therefore, the fraction of portfolio lost is equal to the product of the average LGD, $\sum_{j=1}^{N} a_j\varphi_j$, and the average fraction of defaults, $\sum_{j=1}^{N} w_j z_{j,t+1}$. To simplify the exposition we set the former to unity and consider the distribution of

$$\ell_{N,t+1} = \sum_{j=1}^{N} w_j z_{j,t+1},$$

where $w_j \geq 0$ and $\sum_{j=1}^{N} w_j = 1$.

In the extreme case, assuming firm returns are independently distributed, for the (unconditional) variance of $\ell_{N,t+1}$ we have[17]

$$Var\left(\ell_{N,t+1}\right) = \sum_{j=1}^{N} w_j^2 Var\left(z_{j,t+1}\right) < \frac{1}{4}\left(\sum_{j=1}^{N} w_j^2\right).$$

Hence, the effects of the remaining idiosyncractic shocks to the credit risk portfolio will be fully diversified if

$$\sum_{j=1}^{N} w_j^2 \to 0, \text{ as } N \to \infty. \tag{23}$$

A sufficient condition for this to hold is given by $w_j = O\left(N^{-1}\right)$, which is the standard "granularity" condition where no single exposure dominates the portfolio. This result is quite general and holds irrespective of whether the underlying processes of firm returns are homogeneous or not.

When the underlying returns are correlated there is a non-zero lower bound to $Var\left(\ell_{N,t+1}\right)$, and full diversification will not be possible. For example, under the Vasicek model

$$Var\left(\ell_{N,t+1}\right) = \pi(1-\pi)\left(\sum_{j=1}^{N} w_j^2\right) + \pi(1-\pi)\rho^*\left(\sum_{j\neq j'}^{N} w_j w_{j'}\right),$$

where $\pi = E\left(z_{j,t+1}\right)$ and ρ^* is defined by (21). Since, $\sum_{j=1}^{N} w_j = 1$, it is easily seen that

$$\sum_{j=1}^{N} w_j^2 + \sum_{j\neq j'}^{N} w_j w_{j'} = 1,$$

(2002)). Bankruptcies are pro-cyclical, flooding the market with distressed assets which drive down their price (or increasing severity).

[17] Note that $Var\left(z_{j,t+1}\right) = E\left(z_{j,t+1}^2\right) - \left[E\left(z_{j,t+1}\right)\right]^2 = \pi_{j,t+1}(1-\pi_{j,t+1}) \leq 1/4$.

and hence

$$Var\left(\ell_{N,t+1}\right) = \pi(1-\pi)\left\{\rho^* + (1-\rho^*)\sum_{j=1}^{N} w_j^2\right\}. \tag{24}$$

Under the granularity condition, (23), for N sufficiently large the second term in brackets become negligible, and $Var\left(\ell_{N,t+1}\right)$ converges to the first term which will be non-zero for $\rho^* \neq 0$. Hence, in the limit the unexpected loss is bounded by $\sqrt{\pi(1-\pi)\rho^*}$. For a finite value of N, the unexpected loss is minimized by adopting an equal weighted portfolio, with $w_j = 1/N$. For sufficiently large N, only the granularity condition (23) matters, and nothing can be gained by further optimization with respect of the weights, w_j.

The loss distribution associated with this perfectly homogeneous model is derived in Vasicek (1991, 2002). Denoting the fraction of the portfolio lost to defaults by x, he obtains the following limiting density (as $N \to \infty$)

$$f_\ell\left(x \mid \mathcal{I}_t\right) = \sqrt{\frac{1-\rho}{\rho}}\left\{\frac{\phi\left[\frac{\sqrt{1-\rho}\Phi^{-1}(x)-\Phi^{-1}(\pi)}{\sqrt{\rho}}\right]}{\phi\left[\Phi^{-1}(x)\right]}\right\}, \text{ for } 0 < x \leq 1, \ \rho \neq 0, \tag{25}$$

where $\phi\left(\cdot\right)$ is the density function of a standard normal. The associated cumulative loss distribution function is

$$F_\ell\left(x \mid \mathcal{I}_t\right) = \Phi\left[\frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(\pi)}{\sqrt{\rho}}\right].$$

Not surprisingly, Vasicek's limiting (as $N \to \infty$) distribution is also fully determined in terms of π and ρ. The former parameter sets the expected loss of the portfolio, whilst the latter controls the shape of the loss distribution. In effect one parameter, ρ, controls all aspects of the loss distribution: its volatility, skewness and kurtosis. It would not be possible to calibrate two Vasicek loss distributions with the same expected and unexpected losses, but with different degrees of fat-tailedness, for example.

Further, Vasicek's distribution does not depend on the portfolio weights so long as (23) is satisfied. Therefore, for sufficiently large portfolios that satisfy the granularity condition, (23), there is no further scope for credit risk diversification if attention is confined to the homogeneous return model that underlie Vasicek's loss distribution. Also, Vasicek's set up does not allow conditional risk modeling where the effects of macroeconomic shocks on credit loss distribution might be of interest. With these considerations in mind, we allow for macroeconomic factors and heterogeneity along several dimensions. These are: 1) multiple and observable factors, 2) firm fixed effects, 3) differentiated default thresholds, and 4) differentiated factor sensitivities (analogous to firm "betas") by region, sector or even firm-specific. If the Vasicek model lies at the fully homogeneous end of the spectrum, the model laid out in Section 2 above describes the fully heterogeneous end. How much does accounting for heterogeneity matter for credit risk? The outcomes we are interested in exploring are different measures of credit risk, be it means or volatilities of credit losses (expected and unexpected losses in the argot of risk management), as well as quantiles in the tails or value-at-risk (VaR). To get there we first introduce very briefly the macroeconomic or systematic risk model.

5 Conditional Credit Risk Modeling

5.1 The Macroeconomic Engine: GVAR

The conditional loss distribution of a given credit portfolio can now be derived by linking up the return processes of individual firms, initially presented in equation (10), explicitly to the macro and global variables in the GVAR model. The macroeconomic engine driving the credit risk model is described in detail in Pesaran, Schuermann and Weiner (2004), hereafter PSW. We only provide a very brief, non-technical overview here. The GVAR is a global quarterly model estimated over the period 1979Q1-1999Q1 comprising a total of 25 countries which are grouped into 11 regions, (shown in bold in Table 1 from PSTW, reproduced here for convenience). The advantage of the GVAR is that it allows for a true multi-country setting; however, it can become computationally demanding very quickly. For that reason we model the seven key economies of the U.S., Japan, China, Germany, U.K., France and Italy as regions of their own while grouping the other 18 countries into four regions.[18]

Table 1

Countries/Regions in the GVAR Model

U.S.A.	**Germany**	**Japan**	**China**
Western Europe	**South East Asia**	**Latin America**	**Middle East**
·Spain	·Korea	·Argentina	·Kuwait
·Belgium	·Thailand	·Brazil	·Saudi Arabia
·Netherlands	·Indonesia	·Chile	·Turkey
·Switzerland	·Malaysia	·Peru	
	·Philippines	·Mexico	
	·Singapore		
U.K.	**Italy**	**France**	

The output from these countries comprise around 80% of world GDP (in 1999).

In contrast to existing modeling approaches, in the GVAR the use of cointegration is not confined to a single country or region. By estimating a cointegrating model for each country/region separately, the model also allows for endowment and institutional heterogeneities that exist across the different countries. Accordingly, specific vector error-correcting models (VECM) are estimated for

[18] See PSW, Section 8, for details on cross-country aggregation into regions.

individual countries (or regions) by relating domestic macroeconomic variables such as GDP, inflation, equity prices, money supply, exchange rates and interest rates to corresponding, and therefore country-specific, foreign variables constructed exclusively to match the international trade pattern of the country/region under consideration. By making use of specific exogeneity assumptions regarding the 'rest of the world' with respect to a given domestic or regional economy, the GVAR makes efficient use of limited amounts of data and presents a consistently-estimated global model for use in portfolio applications and beyond.

The GVAR allows for interactions to take place between factors and economies through three distinct but interrelated channels:

- Contemporaneous dependence of domestic on foreign variables and their lagged values;
- Dependence of country specific variables on observed common global effects such oil prices;
- Weak cross-sectional dependence of the idiosyncratic shocks.

The individual models are estimated allowing for unit roots and cointegration assuming that region-specific foreign variables are weakly exogenous, with the exception of the model for the U.S. economy which is treated as a closed economy model. The U.S. model is linked to the outside world through exchange rates, which in turn are themselves determined by rest of the region-specific models. PSW show that the careful construction of the global variables as weighted averages of the other regional variables leads to a simultaneous system of regional equations that may be solved to form a global system. They also provide theoretical arguments as well as empirical evidence in support of the weak exogeniety assumption that allows the region-specific models to be estimated consistently.

The conditional loss distribution of a given credit portfolio can now be derived by linking up the return processes of individual firms, initially presented in equation (10), explicitly to the macro and global variables in the GVAR model. We provide a synopsis of the model developed in full detail in PSTW.

5.2 Firm Returns Based on Observed Common Factors Linked to GVAR

Here we extend the firm return model by incorporating the full dynamic structure of the systematic risk factors captured by the GVAR. We present a notationally simplified version of the model outlined in detail in PSTW; readers wishing details are asked to consult that paper. Accordingly, a firm's change in value (or return) is assumed to be a function of changes in the underlying macroeconomic factors (the systematic component), domestic and foreign, the exogenous global variables (in our application oil prices)[19] and the firm-specific idiosyncratic shocks $\eta_{jis,t+1}$:

$$r_{jis,t+1} = \alpha_{jis} + \boldsymbol{\gamma}'_{jis}\mathbf{f}_{t+1} + \eta_{jis,t+1},\ t = 1, 2, ..., T, \tag{26}$$

[19] In PSTW we allow for more than one global factor. For simplicity we restrict ourselves here to just one.

where $r_{jis,t+1}$ is the equity return from t to $t+1$ for firm j ($j = 1, ..., nc_i$) in region i and sector s, α_{jis} is a regression constant, $\boldsymbol{\gamma}_{jis}$ are the factor loadings (firm "betas"), and $\mathbf{f}_{t+1}$ collects all the *observed* macroeconomic variables plus oil prices in the global model (totaling 64 in PSW). To be sure, these return regressions are not prediction equations per se as they depend on contemporaneous variables. As in the unobserved factor model, f_{t+1} captures the common effects that induce correlated defaults.

The GVAR model provides forecasts of all the global variables that directly or indirectly affect the returns. If the model captures all systematic risk, the idiosyncratic risk components of any two companies in the model would be uncorrelated, namely the idiosyncratic risks ought to be cross-sectionally uncorrelated. In practice it will be hard to absorb all of the cross-section correlation with the systematic risk factors modeled by the GVAR, and so a model whose residuals are less correlated will in general be preferred to one where they are more correlated.

Note that we started by decomposing firm return into forecastable and non-forcastable components in (10), namely $r_{jis,t+1} = \mu_{jis,t} + \xi_{jis,t+1}$. The composite innovation $\xi_{jis,t+1}$ contains the idiosyncratic innovation $\eta_{jis,t+1}$, and innovations from the macroeconomic variables in the GVAR. The predictable component is likely to be weak and will depend on the size of the factor loadings, $\boldsymbol{\gamma}_{jis}$, and the extent to which the underlying global variables are cointegrating. In the absence of any cointegrating relations in the global model, none of the asset returns are predictable. As it happens the econometric evidence presented in PSW strongly supports the existence of 36 cointegrating relations in the global model and is, therefore, compatible with some degree of predictability in asset returns, at least at the quarterly horizon modeled here. The extent to which asset returns are predicted could reflect time-varying risk premia and does not necessarily imply market inefficiencies. Our modelling approach provides an operational procedure for relating excess returns of individual firms to all the observable macro factors in the global economy.

5.3 Expected Loss Due to Default

Given the value change process for firm j, defined by (26), and the log threshold-equity ratio, $\hat{\lambda}_{\mathcal{R}}(t, H)$, obtainable from an initial credit rating (see Section 2), we are now in a position to compute expected loss. Suppose we have data for firms and systematic factors in the GVAR for a sample period $t = 1, ..., T$. We need to define the expected loss to firm j at time T given information available to the lender (e.g. a bank) at time T, which we assume is given by Ω_T. Following (14), default occurs when the firm's value (return) falls below the default threshold-equity ratio $\hat{\lambda}_{\mathcal{R}}(T, 1)$. Expected loss at time T (but occurring at $T+1$), $E_T\left(L_{jis,T+1}\right) = E\left(L_{jis,T+1} \mid \Omega_T\right)$, is given by

$$E_T\left(L_{jis,T+1}\right) = \Pr\left(r_{jis,T+1} < \hat{\lambda}_{\mathcal{R}}(T, 1) \mid \Omega_T\right) \times A_{jis,T} \times E_T(\varphi_{jis,T+1}), \tag{27}$$

where $A_{jis,T}$ is the exposure assuming no recoveries (typically the face value of the loan) and is known at time T, and $\varphi_{jis,T+1}$ is the percentage of exposure which cannot be recovered in the event of default or *LGD*. Typically $\varphi_{jis,T+1}$ is not known at time of default and will be treated as a random variable over the unit interval. In the empirical application we make the typical assumption that $\varphi_{jis,T+1}$ are draws from a beta distribution with given mean and variance calibrated to (pooled) historical data on default severity.[20]

Substituting (26) into (27) we obtain:

$$E_T\left(L_{jis,T+1}\right) = \pi_{jis,T+1|T} \times A_{jis,T} \times E_T(\varphi_{jis,T+1}), \tag{28}$$

where

$$\pi_{jis,T+1|T} = \Pr\left(\alpha_{jis} + \boldsymbol{\gamma}'_{jis}\mathbf{f}_{T+1} + \eta_{jis,T+1} < \hat{\lambda}_{\mathcal{R}}(T,1) \mid \Omega_T\right),$$

is the conditional default probability over the period T to $T+1$, formed at time T. Our modeling framework allows us to derive an explicit expression for this probability, $\pi_{jis,T+1|T}$:

$$\pi_{jis,T+1|T} = \Pr\left(\xi_{jis,T+1} < \hat{\lambda}_{\mathcal{R}}(T,1) - \mu_{jis,T} \mid \Omega_T\right), \tag{29}$$

where $\xi_{jis,T+1}$ are the composite innovations (idiosyncratic and systematic) and $\mu_{jis,T}$ is the explained or expected component of firm returns containing the GVAR forecasts. Note that although the firm in question operates in country/region i, its probability of default could be affected by macroeconomic shocks worldwide.

Under the assumption that all these shocks are jointly normally distributed and the parameter estimates are given, we have the following expression for the probability of default over T to $T+1$ formed at T[21]

$$\pi_{jis,T+1|T} = \Phi\left[\frac{\hat{\lambda}_{\mathcal{R}}(T,1) - \mu_{jis,T+1|T}}{\sqrt{Var\left(\xi_{jis,T+1} \mid \Omega_T\right)}}\right]. \tag{30}$$

The expected loss due to default of a loan (credit) portfolio can now be computed by aggregating the expected losses across the different loans. Denoting the loss of a loan portfolio over the period T to $T+1$ by L_{T+1} we have

$$E_T\left(L_{T+1}\right) = \sum_{i=0}^{N}\sum_{j=1}^{nc_i} \pi_{jis,T+1|T} \times A_{jis,T} \times E_T(\varphi_{jis,T+1}), \tag{31}$$

where nc_i is the number of obligors (which could be zero) in the bank's loan portfolio resident in country/region i.

5.4 Simulation of the Loss Distribution

The expected loss as well as the entire loss distribution can be computed once the GVAR model parameters, the return process parameters in (26) and the thresholds in (8) have been estimated for

[20] The beta distribution is usually chosen since it is bounded, typically on the unit interval, with two shape parameters which can be expressed in terms of mean and standard deviation of losses.

[21] Joint normality is sufficient but not necessary for $\xi_{ji,t+1}$ to be approximately normally distributed. This is because $\xi_{ji,t+1}$ is a linear function of a large number of weakly correlated shocks (63 in our particular application).

a sample of observations $t = 1, 2, ..., T$. We do this by stochastic simulation using draws from the joint distribution of the shocks, $\epsilon_{T+1} = (\boldsymbol{\varepsilon}'_{T+1}, \boldsymbol{\eta}'_{T+1})'$, where $\boldsymbol{\varepsilon}_{T+1}$ is the vector of systematic shocks (associated with the macroeconomic variables in the GVAR model plus oil prices) and $\boldsymbol{\eta}_{T+1}$ is the vector of firm-specific shocks. These draws could either be carried out parametrically from normal or t-distributed random variables, or if sufficient data points are available, can be implemented non-parametrically using re-sampling techniques. Under the parametric specification the variance covariance matrix of ϵ_{t+1} is given by

$$Var\left(\epsilon_{T+1}\right) = \begin{pmatrix} \boldsymbol{\Sigma}_{\varepsilon} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{\eta} \end{pmatrix}, \tag{32}$$

where $\boldsymbol{\Sigma}_{\eta}$ is a diagonal matrix with elements $\omega^2_{\eta,jis}$, $j = 1, 2, ..., nc_i$, $i = 0, 1, ..., N$.

Denote the r^{th} draw of this vector by $\epsilon^{(r)}_{T+1}$, and compute the firm-specific return, $r^{(r)}_{iT,t+1}$, noting that

$$r^{(r)}_{ijs,T+1} = \mu_{jis,T+1|T} + \xi^{(r)}_{jis,T+1}, \tag{33}$$

where $\mu_{jis,T+1|T}$ is the return forecast, and

$$\xi^{(r)}_{jis,T+1} = \eta^{(r)}_{jis,T+1} + \boldsymbol{\theta}'_{jis}\boldsymbol{\varepsilon}^{(r)}_{T+1} \tag{34}$$

is the composite innovation, with $\boldsymbol{\theta}_{jis}$ being a vector of coefficients; details on the precise composition of $\boldsymbol{\theta}_{jis}$ can be found in Section 3 of PSTW. One may then simulate the loss in period $T + 1$ using (known) loan face values, $A_{jis,T}$, as exposures, and draws from a beta distribution for severities (as described above):

$$L^{(r)}_{T+1} = \sum_{i=0}^{N} \sum_{j=1}^{nc_i} I\left(r^{(r)}_{ijs,T+1} < \hat{\lambda}_{\mathcal{R}}(T, 1)\right) A_{jis,T}\ \varphi^{(r)}_{jis,T+1}. \tag{35}$$

The simulated expected loss due to default is given by (using R replications)

$$\bar{L}_{R,T+1} = \frac{1}{R} \sum_{r=1}^{R} L^{(r)}_{T+1}. \tag{36}$$

When $\epsilon^{(r)}_{T+1}$ are drawn from a multivariate normal distribution with a covariance matrix given by (32), then

$$\bar{L}_{R,T+1} \xrightarrow{p} E_T\left(L_{T+1}\right), \text{ as } R \to \infty.$$

The simulated loss distribution is given by ordered values of $L^{(r)}_{T+1}$, for $r = 1, 2, ..., R$. For a desired percentile, for example the 99%, and a given number of replications, say $R = 10,000$, credit value at risk is given as the 100^{th} highest loss.

6 Empirical Results

6.1 The Credit Portfolio

To analyze the effects of different model specifications, parameter homogeneity vs. heterogeneity, we constuct a fictitious large-corporate loan portfolio. This portfolio is an extended version used in PSTW and is summarized in Table 2. It contains a total of 243 companies, resident over 10 of the 11 regions. In order for a firm to enter our sample, several criteria had to be met. We restricted ourselves to major, publicly traded firms which had a credit rating from either Moody's or S&P. Thus, for example, Chinese companies are not included for lack of a credit rating. The firms should be represented within the major equity index for that country. We favored firms for which equity return data was available for the entire sample period, i.e. going back to 1979. Typically this would exclude large firms such as telephone operators which in many instances have been privatized only recently, even though they might now represent a significant share in their country's dominant equity index today. The data source is Datastream, and we took their Total Return Index variable which is a cum dividend return measure.

The column to the right in Table 2 indicates the inception of the equity series available for the multi-factor regressions. We allocated exposure by share of output of the region (in our "world" of 25 countries). Within a region, loan exposure is randomly assigned. The expected severity for loans to U.S. companies is the lowest at 20%, based upon studies by Citibank, Fitch Investor Service and Moody's Investor Service.[22] All other severities are based on assumptions, reflecting the idea that severities are higher in less developed countries. Table 2 gives the portfolio composition, regional weights, individual exposures and expected (μ_β) and unexpected (σ_β) severities.

[22] As cited in Saunders and Allen (2002).

Table 2a

The Composition of the Sample Portfolio for Regions

Region	# Obligors	Equity Series[1] Quarterly	Credit Rating[2] Range	Portfolio Per cent	Severity[3] Mean	S.D.
					(μ_β)	(σ_β)
U.S.	63	79Q1 - 99Q1	AAA to BBB-	20	20%	10%
U.K.	24	79Q1 - 99Q1	AA to BBB+	8	35%	15%
Germany	21	79Q1 - 99Q1	AAA to BBB-	10	35%	15%
France	14	79Q1 - 99Q1	AA to BBB	8	35%	15%
Italy	10	79Q1 - 99Q1	A to BBB-	8	35%	15%
W. Europe[23]	24	79Q1 - 99Q1	AAA to BBB+	11	35%	15%
Middle East[24]	4	90Q3 - 99Q1	B-	2	60%	20%
S.E. Asia[25]	34	89Q3 - 99Q1	A to B	14	50%	20%
Japan	35	79Q1 - 99Q1	AAA to B+	14	35%	15%
L. America[26]	14	89Q3 - 99Q1	A to B-	5	65%	20%
Total	243	-	-	100	-	-

1. Equity prices of companies in emerging markets are not available over the full sample period used for the estimation horizon of the GVAR. We have a complete series for all firms only for the U.S., U.K., Germany and Japan. For France, Italy and W. Europe, although some of the series go back through 1979Q1, data was available for all firms from 1987Q4 (France), 1987Q4 (Italy), 1989Q3 (W. Europe). For these regions the estimation of the the APT regressions were based on the available samples. For L. America we have a observations for all firms from 1990Q2.
2. The sample contains a mix of Moody's and S&P ratings, although S&P rating nomenclature is used for convenience.
3. Severity is drawn from a beta distribution with mean μ_β and standard deviation σ_β.

Table 2b provides summary information of sample size by industry.

[23] Western Europe is made up of Spain, the Netherlands, Belgium and Switzerland.
[24] The Middle East, while made up of Kuwait, Saudi Arabia and Turkey, contains only firms from Turkey.
[25] The countries in South East Asia are Indonesia, Korea, Malaysia, Philippines, Singapore and Thailand.
[26] Latin America is comprised of Argentina, Brazil, Chile, Mexico and Peru.

Table 2b: Breakdown by Industry	
	# of Firms
Agriculture, Mining & Construction	24
Communication, Electric & Gas	45
Durable Mfg.	30
FIRE†	71
Non-durable Mfg.	27
Service	6
Wholesale & Retail Trade	40
Total	243

† : FIRE: Finance, Insurance and Real Estate.

6.2 Risk, Return and Default by Credit Rating

In order to obtain estimates for the rating-specific default thresholds, we make use of the rating histories from Standard and Poor's spanning 1981-1999, roughly the same sample period as is covered by our GVAR model. The results are presented in Table 3 below for the range of ratings that are represented in our portfolio of firms, namely $\mathcal{AAA}$ to $\mathcal{B}$. The estimates of the four-quater ahead threshold-equity ratio, $C_{\mathcal{R},t+4}/E_{\mathcal{R}t}$, are computed using $exp(\hat{\lambda}_{\mathcal{R}}(H))$ with $H = 4$, where $\hat{\lambda}_{\mathcal{R}}(H)$ is defined by (8).[27] Empirical default probabilities, $\hat{\pi}_{\mathcal{R}}(t, H)$, are obtained using default intensity-based estimates detailed in Lando and Skødeberg (2002).

[27] Return means and standard deviations are computed daily for all U.S. firms of rating $\mathcal{R}$ alive in period t. Returns are computed using data from CRSP. Details on the estimation of $\hat{\mu}_{\mathcal{R}}$ and $\hat{\sigma}_{\mathcal{R}}$ are available from the authors upon request. Ratings and rating histories are from Standard and Poors CreditPro Database V. 6.2. We use the sample period 1981Q1-1999Q1.

Table 3

Rating-Specific Return and Equity-Threshold Estimation

S&P Rating	$\hat{\mu}_{\mathcal{R}}$	$\hat{\sigma}_{\mathcal{R}}$	$\hat{\mu}_{\mathcal{R}}/\hat{\sigma}_{\mathcal{R}}$	$\bar{\pi}_{\mathcal{R}}$ (in bp)	$\widehat{C_{\mathcal{R},4}/E_{\mathcal{R}}}$
$\mathcal{AAA}$	4.34%	13.87%	0.33	0.026	0.37
$\mathcal{AA}$	4.06%	15.16%	0.27	0.369	0.33
$\mathcal{A}$	4.13%	15.31%	0.27	0.714	0.35
$\mathcal{BBB}$	3.80%	17.37%	0.22	10.63	0.37
$\mathcal{BB}$	3.21%	24.12%	0.13	49.21	0.30
$\mathcal{B}$	2.04%	34.82%	0.06	351.66	0.30

$\widehat{C_{\mathcal{R},4}/E_{\mathcal{R}}}$ denotes the sample estimate of the four-quarter ahead default equity ratio, $\hat{\mu}_{\mathcal{R}}$ and $\hat{\sigma}_{\mathcal{R}}$ are the sample estimates of the mean and standard deviations of quarterly returns for R-rated firms, computed using daily data.

We note that average quarterly volatility, $\hat{\sigma}_{\mathcal{R}}$, increases monotonically as we descend the rating spectrum to the point where the volatility of a $\mathcal{B}$-rated firm is more than twice that of an $\mathcal{AA}$-rated firm. Average returns do not keep pace with the increasing volatility, resulting in similarly declining Sharpe ratios ($\hat{\mu}_{\mathcal{R}}/\hat{\sigma}_{\mathcal{R}}$). Annual default probabilities display the familiar pattern of increasing dramatically as we descend the credit spectrum, especially once the investment grade boundary crosses (i.e. $\mathcal{BB}$ and below).

Of particular interest is the behavior of the four-quarter forward threshold-equity ratio $\widehat{C_{\mathcal{R},4}/E_{\mathcal{R}}}$ which exhibits very little variation across ratings, and ranges from 0.30 for $\mathcal{BB}$ to 0.37 for $\mathcal{AAA}$ rated firms.

In order to examine the stability of these estimates over time, we take the maximum sample length available, 1981Q1 - 2002Q4, and split it evenly into two sub-periods of 11 years each. The results are summarized in Table 4.[28] Mean returns $\hat{\mu}_{\mathcal{R}}$ seem systematically lower in the second half than in the first with the exception of $\mathcal{B}$-rated firms whose mean return is slightly higher in the second sub-sample. Volatilities appear more stable, though higher for the two lowest rating categories, $\mathcal{BB}$ and $\mathcal{B}$. The Sharpe-ratio $\hat{\mu}_{\mathcal{R}}/\hat{\sigma}_{\mathcal{R}}$ declines monotonically across ratings and is quite consistent between the two sub-samples.

Not surpringly default probabilities exhibit considerable variations across the two sub-samples,

[28] Return means and standard deviations are computed quarterly for all U.S. firms of rating $\mathcal{R}$ alive in period t. Returns are computed data from CRSP. Ratings and rating histories are from Standard and Poors CreditPro Database V. 6.2.

especially for the higher credit grades ($\mathcal{AAA}$ to $\mathcal{A}$). [29] Nonetheless, despite this the four-quarter forward threshold-equity ratio is remarkably stable across the two sample periods. To be sure, the thresholds are systematically lower for the second sample period, indicating that it takes a higher level of capital to attain a similar default threshold than previously. This evidence is consistent with Blume, Lim and MacKinlay (1998) who report that credit rating agencies have raised their standards.

Table 4

Rating-Specific Return and Equity-Threshold Estimation

	Sample Range	$\mathcal{AAA}$	$\mathcal{AA}$	$\mathcal{A}$	$\mathcal{BBB}$	$\mathcal{BB}$	$\mathcal{B}$
$\hat{\mu}_{\mathcal{R}}$	1981Q1-1991Q4	4.72%	4.60%	4.55%	4.27%	3.42%	0.81%
	1992Q1-2002Q4	3.99%	3.34%	3.64%	2.74%	2.23%	1.26%
$\hat{\sigma}_{\mathcal{R}}$	1981Q1-1991Q4	12.49%	13.26%	14.80%	17.09%	23.84%	29.39%
	1992Q1-2002Q4	12.44%	12.13%	14.77%	17.94%	27.60%	42.09%
$\hat{\mu}_{\mathcal{R}}/\hat{\sigma}_{\mathcal{R}}$	1981Q1-1991Q4	0.38	0.35	0.31	0.25	0.14	0.03
	1992Q1-2002Q4	0.32	0.28	0.25	0.15	0.08	0.03
$\hat{\pi}_{\mathcal{R}}$ (in bp)	1981Q1-1991Q4	0.075	0.844	1.725	14.71	84.44	470.66
	1992Q1-2002Q4	0.007	0.031	0.541	10.16	61.61	534.77
$\widehat{C_{\mathcal{R},4}/E_{\mathcal{R}}}$	1981Q1-1991Q4	0.41	0.44	0.42	0.43	0.37	0.39
	1992Q1-2002Q4	0.36	0.38	0.37	0.37	0.28	0.27
# of obs.[30]	1981Q1-1991Q4	776	4,131	7,510	4,437	3,193	3,877
	1992Q1-2002Q4	569	2,785	7,593	8,300	6,610	5,780

6.3 Multi-factor Return Regressions: Specification and Selection

With the GVAR framework serving as the global economic engine, multi-factor return regressions are specified in terms of the observed macro factors in the GVAR model. A general form of such return regressions is given by (26). Given the diverse nature of the firms in our portfolio, one is tempted to include all the domestic, foreign and global factors (i.e. oil price changes) in the multi-factor regressions. Such a general specification may be particularly important in the case where a multinational is resident in one country, but the bulk of its operations takes place in the

[29] These probability values are very small (they are reported in basis points!) simply because there are so few defaults for the very high credit grades. In fact, there were no defaults by $\mathcal{AAA}$-rated firms at all. We use a duration approach to estimate transition matrices, the last column of which is the transition to default. In this way we may obtain a positive probability of default for highly rated obligors even though no default was observed during the sampling period. It suffices that an obligor migrated from, say, $\mathcal{AAA}$ to $\mathcal{AA}$ to $\mathcal{A}$, and that a default occurred from $\mathcal{A}$ to contribute probability mass to $\pi_{\mathcal{AAA}}$. See also Jafry and Schuermann (2004).

[30] These are firm-quarters.

global arena. However, because there is likely to be a high degree of correlation between some of the domestic and foreign variables (e.g. domestic and foreign real equity prices), it is by no means obvious that a general-to-specific model selection process would be appropriate, particularly considering the short time series data available relative to the number of different factors in the GVAR.

An alternative model selection strategy, which we adopt in this paper, is to view the 243 multi-factor regressions as forming a panel data model with heterogeneous coefficients. Such panels have been studied by Pesaran and Smith (1995) and Pesaran, Smith and Im (1996) where it is shown that instead of considering firm-specific estimates one could base the analysis on the means of the estimated coefficients, referred to as the mean group estimators (MGE). This approach assumes that the variations of factor loadings across firms in different regions are approximately randomly distributed around fixed means. This is the standard random coefficient model introduced into the panel literatue by Swamy (1970) and used extensively in the empirical literature.[31] The choice of the factors in the multi-factor regressions can now be based on the statistical significance of the (population) mean coefficients by using the MGE to select a slimmed-down regressor set.[32]

In addition to the above fully heterogeneous specification, to evaluate the quantitative importance of parameter heterogeneity, we also consider a number of specifications with differing degrees of slope and error variance homogeneity. It is worth emphasizing that the specifications considered here will be based on the same set of factors and only differ as far as the degree to which their parameters are allowed to vary across firms. The models, arranged from most to least homogeneous are:

M_1: The fully homogeneous model with the same "alpha" and "beta" across all the 243 firms in the portfolio, estimated by a pooled OLS regression. The default threshold λ is also identical for all firms and is calibrated to generate the same one-year expected loss as the fully heterogeneous model (M_5).

M_2: The pooled model with industry fixed effects; there are seven industry dummies.

M_3: The pooled model with regional effects; there are ten regional dummies.

M_4: The pooled model with regional and firm fixed effects (firm "alphas") model.

M_5: The fully heterogeneous model with firm "alphas" and "betas," firm specific error variances and rating-specific default thresholds.

Models 2 and 3 can be thought of allowing us to explore the impact of industry and regional effects on the resulting credit loss distribution, while Model 4 is a partial step to the fully heterogeneous specification. Only the last model allows for threshold heterogeneity by making use of

[31] A recent review of the random coefficient models is provided by Hsiao and Pesaran (2004).

[32] The appropriate test statistics for this purpose are given in PSTW, Section 6.

credit ratings along the lines described in Section 2.1. Although, as was noted earlier in Table 3, the default thresholds show little variations across firms with different credit ratings. This should be born in mind in comparing loss distribution outcomes across the different model specifications.

Another consideration in our comparative analysis is the extent to which the five alternative parametric specifications affect cross section correlations of the simulated returns. Since all the five models are based on the same set of observed factors, cross section correlations of the simulated returns will be affected significantly by parameter heterogeniety only if the differences of parameters across firms are systematic. In the case of pure random differences, it is easily seen that all specifications imply similar amount of cross return correlations.

It also follows that by pinning down the expected losses, one would also fix the unexpected losses if parameter differences across firms are non-systematic. This is because the loss distribution of heterogeneous factor models with purely random factor laodings and Gaussian shocks is primarily governed by the expected loss and the cross section correlation of asset returns.

6.4 Return Regression Results

In this section we present the estimation results for Models 1 through 5. To set the stage we begin with Model 5, presented in Table 5, which allows for full parameter heterogeneity. The pooled regression results for Models 1 through 3 are presented in Table 6. Based on the MG test results from the first stage the statistically most significant factors are, perhaps not surprisingly, changes in domestic and foreign real equity prices (Δq or Δq^*), domestic interest rate (Δr) and oil price changes (Δp^o). We ran two sets of multi-factor regressions (including the interest rate and oil price variables); one with Δq and another with Δq^*, and selected the regression with the higher $\bar{R}^2$. For three-quarters of the portfolio (183 firms) the domestic equity market return was chosen. To allow for easier comparison of models we kept this choice for Models 1 through 4 as well.

The summary of the final set of multi-factor regressions and the associated MG estimates are given in Table 5. In this specification changes in equity prices, interest rates and oil prices remain the key driving factors in the multi-factor regressions.

Table 5
Mean Group Estimates of Factor Loadings
Heterogeneous Model (Model 5)

Factors	MGE $\hat{\beta}$	S.E. of MGE $s.e.\left(\hat{\beta}\right)$	t-ratios
constant	0.022	0.002	10.175
$\Delta\tilde{q}_{t+1}$[33]	0.885	0.026	34.457
Δr	-2.997	0.528	-5.676
Δp^o	0.147	0.042	3.475
R^2		0.240	
# of firm quarters		17,114	

The portfolio equity "beta" is below one, indicating that our portfolio is somewhat less correlated than the global market. An increase in the rate of interest results in a decline in firm returns while the overall effect of the oil price changes is, positive. This seems a reasonable outcome for energy and petrochemical companies and for some of the banks, although one would not expect this result to be universal. In fact we do observe considerable variations in the individual estimates of the coefficients of oil prices changes across different firms in our portfolio. In the final regressions, of the 243 firm regressions, the coefficient on oil price changes was positive for 144 firms (about 59% of the total), and negative for the remaining firms. The MGE for each subset was also significant.

The lack of other observable systematic risk factors entering the return model confirms that most information relevant for firm returns is contained in the contemporaneous market return. Only interest rates and oil prices changes provided marginal explanatory power. To be sure, when forecasting the macroeconomic variables, and when conducting scenario analysis, the dynamics of *all* the variables modeled in the GVAR (all 63 of them, plus oil prices) can still affect returns through their possible impacts on equity returns and interest rates. A direct presence in the firm return equation is not necessary for real output, for example, to influence returns. Output shocks influence returns and credit losses to the extent that real output, interest rates and stock market returns are contemporaneously correlated.

Turning now to the results of the other models, we see in Table 6 (Models 1 to 3) that there is little variation in the firm equity "beta" across models, and that it is always quite close to the MGE beta of the heterogeneous model (Model 5). As far as the goodness of the fit of the alternative models is concerned, adding regional fixed effects (Model 3) seems to be somewhat more important than adding industry effects (Model 2) as the former has an R^2 of 0.151 while for the latter $R^2 = 0.145$, nearly the same as for the pooled, fully homogeneous model (0.144). The

[33] $\Delta\tilde{q}_{t+1}$ is equal to Δq_{t+1} or Δq^*_{t+1} depending on which yields a better in-sample fit.

only industry sector which has a strongly significant effect (at the 1% level) is Finance, Insurance and Real Estate (FIRE), while Wholesale & Retail Trade is marginally significant at the 10% level.

Regional effects are estimated relative to the U.S.. They are strongly significant (1% level) for Western Europe (positive), Japan (positive) and Latin America (positive), and significant at the 5% level for Italy (positive) and South East Asia (negative). If in addition to the regional effects we also add firm fixed effects, M_4, the overall R^2 increases to 0.160, and the factor sensitivities ("betas") are nearly identical to M_3.[34] Note that in all of the homogeneous models M_1 through M_4 the pooled coefficient on oil price changes is positive. Thus any heterogeneity to oil price changes by firms is lost. That this particular factor heterogeneity is important was borne out by the results from the fully heterogeneous model. This false restriction will carry forward into the loss distribution.

Finally, we computed the average pairwise return correlation across all firms in our portfolio. This turned out to be about 12%; recall this is quarterly data. The three factors used in the five model specifications are able to absorb a significant amount of the cross-firm dependence: the residual correlations are about 5%.

Before proceeding to the results, we recap each of the model specifications, starting with the most general specification, M_5.

6.4.1 Fully Heterogeneous Model (M_5)

The return for firm j in region i and sector s from t to $t+1$ is denoted $r_{jis,t+1}$. The fully heterogeneous model is

$$r_{jis,t+1} = \alpha_{jis} + \gamma_{1,jis}\Delta\tilde{q}_{t+1} + \gamma_{2,jis}\Delta r_{t+1} + \gamma_{3,jis}\Delta p^o_{t+1} + \xi_{jis,t+1}, \tag{37}$$

where $\Delta\tilde{q}_{t+1}$ is equal to Δq_{t+1} or Δq^*_{t+1} depending on which yields a better in-sample fit, and

$$\xi_{jis,t+1} \mid \Omega_t \sim N(0, \omega^2_{\xi,jis}),$$

are the compound innovations, allowing for firm-specific error variance heterogeneity. This is the only model where cross section error variance heterogeneity is allowed. Finally, default thresholds are differentiated by rating: $\lambda_{\mathcal{R}}$, although as discussed above the differences in $\lambda_{\mathcal{R}}$ across $\mathcal{R}$ is rather limited.

6.4.2 Fully Homogeneous Model (M_1)

Thus the fully homogeneous model, which we have called (perhaps inappropriately) the Vasicek model, will be specified as

$$r_{jis,t+1} = \alpha + \gamma_1\Delta\tilde{q}_{t+1} + \gamma_2\Delta r_{t+1} + \gamma_3\Delta p^o_{t+1} + \xi_{jis,t+1}, \tag{38}$$

[34] These results are not presented due to space constraint but are available from the authors upon request.

where the error variances are assumed to be the same for all firms: $\omega^2_{\xi,jis} = \omega^2_{\xi}$. Note that the same systematic risk factors appear in (38) as in (37). There is only one default threshold, $\bar{\lambda}$, which is calibrated to yield the same EL across models.

6.4.3 Industry Fixed Effects (M_2)

Next we allow for industry fixed effects:

$$r_{jis,t+1} = \alpha + \delta_1 I_s + \gamma_1 \Delta\tilde{q}_{t+1} + \gamma_2 \Delta r_{t+1} + \gamma_3 \Delta p^o_{t+1} + \xi_{jis,t+1}, \tag{39}$$

where I_s is a dummy variable which takes the value of 1 for industry/sector s and 0 otherwise, with the Communication, Electric & Gas as the omitted sector. This is the only difference to the fully homogeneous model in (38).

6.4.4 Region Fixed Effects (M_3)

Next we allow for region fixed effects:

$$r_{jis,t+1} = \alpha + \delta_2 R_i + \gamma_1 \Delta\tilde{q}_{t+1} + \gamma_2 \Delta r_{t+1} + \gamma_3 \Delta p^o_{t+1} + \xi_{jis,t+1}, \tag{40}$$

where R_i is a dummy variable which takes the value of 1 for region i and 0 otherwise, with the U.S. as the omitted region. This is the only difference to the fully homogeneous model in (38).

6.4.5 Firm and Region Fixed Effects (M_4)

Next we add firm fixed effects to the regional model. One needs to be careful to leave out one firm in each region in the fixed effects specification to avoid collinearity of the dummy variable and firm fixed effects. We may denote this as $\tilde{\alpha}_j$ in the fourth specification:

$$r_{jis,t+1} = \tilde{\alpha}_j + \delta_2 R_i + \gamma_1 \Delta\tilde{q}_{t+1} + \gamma_2 \Delta r_{t+1} + \gamma_3 \Delta p^o_{t+1} + \xi_{jis,t+1}. \tag{41}$$

Otherwise the model specification in (41) is the same as in (40).

7 Simulated Credit Loss Distributions

7.1 Model Heterogeneity and Baseline Losses

With the estimated GVAR model serving as the economic scenario generator and the fitted multi-factor regressions as the linkage between firms and the global economy, we simulated loss distributions one through four quarters ahead.[35] A one year horizon is typical for credit risk management

[35] The important issue of credit risk model evaluation, especially for a regulator under the New Basel Capital Accord, is beyond the scope of this paper; we plan to address it in subsequent work. See also Lopez and Saidenberg (2000). Lucas (2001) illustrates the difficulty of this validation process in the easier context of market risk.

and thus of particular interest. We carried out 100,000 replications for each scenario using Gaussian innovations. For the forecasts and shock scenarios, we computed expected loss results using the analytic formula (using (31)) as well as by stochastic simulations, (36). The two sets of estimates turn out to be very close indeed so we only report the simulated ones.

To make the simulated loss results from the different model specifications comparable, we calibrated the single default threshold used in models M_1 through M_4 to be such that the one-year expected loss, EL, was within one basis point of the one-year EL from the heterogeneous model, M_5; that turned out to be 42.7bp.

Table 7 gives summary statistics for the baseline (i.e. no macroeconomic shocks) loss distribution for the five models. EL is very close across models by construction. Unexpected losses (UL) vary little across the different specifications, and largely reflects the fact that all the specifications under consideration imply very similar simulated cross firm asset return correlations. Indeed the simulated loss volatilities are not only close to one another but also close to the asymptotic loss volatility implied by the Vasicek model as discussed in Section 4. This asymptotic expression, given in (24), is driven by the average default rate across the portfolio (equal to the EL if $LGD = 100\%$), π, and the default correlation, ρ^*, itself a function of π and the average return correlation of the firms in the portfolio, ρ, which is 12% for our portfolio; see (21) in Section (3). Thus for $\pi = 0.427\%$, this yields a default correlation of $\rho^* = 0.266\%$ and an asymptotic $UL = \sqrt{\pi(1-\pi)\rho^*} = 0.336\%$ or 33.6bp, somewhat below the simulated UL's, as it should be.

However, the behaviour of the loss distributions begin to diverge as we consider their tail properties. The 99.9% value-at-risk, or VaR, is lowest for the heterogeneous model at 184.6bp, though not the expected shortfall, ES, at the same percentile, which at 214.5bp is close to the figures obtained under M_1 and M_2, and somewhere between the figures for M_4 and M_3. Indeed this discrepancy between VaR and ES is best captured by their ratio which is around 1.1 for models M_1 to M_4, but much higher at 1.43 for the fully heterogeneous model.

It is by now well known that VaR is not a coherent risk measure (see Artzner, Delbaen, Eber, and Heath (1999)), plagued among other things by a lack of sensitivity to tail behavior. Simply put, VaR treats all beyond-VaR events the same and hence fails to capture tail heterogeneity. ES, being the expectation of the beyond-VaR tail, however is a coherent risk measure and thus of interest in this discussion. The ratio of ES to VaR in general exceeds one for fat-tailed distributions (it approaches one for the Gaussian distribution). It is clear from Table 7 that only the heterogenous model is able to capture the tail events properly, and that those are only measured with the ES risk measure, not with the VaR measure. Interestingly, more conventional measures of depatures from the normal distribution, such as skewness and kurtosis, also fail to reveal the tail differences that exist across the models. See the last two columns of Table 7. This could be due to the relatively small cross firm return correlations of the underlying portfolio, in turn reflecting the global nature of the portfolio being considered. It would be interesting to see that the same results prevail if we turn to exclusively domestic portfolios with higher cross from retun correlations.

7.2 Model Heterogeneity and Macroeconomic Shocks

One of the main advantages of our conditional modeling approach lies in the fact that allows us to consider the impact of different macroeconomic shock scenarios. The ability to conduct shock scenario analysis with observable risk factors is clearly important for policy analysis, be it business or public policy.

The risk factors in the firm return models are equity returns, interest rates and oil prices. In addition we shall explore the impact of business cycle heterogeneity across different countries, by considering shocks to real output, which (as noted earlier) can influence the loss distributions indirectly through their contemporaneous correlations with equity returns and interest rates. Accordingly, for each horizon we examined the following equi-probable scenarios, though others are possible, of course:[36]

- a -2.33σ shock to real U.S. equity, corresponding to a quarterly drop of 14.28%,
- a $+2.33\sigma$ shock to the German short term interest rate, corresponding to a quarterly rise of 0.33%,
- a -2.33σ shock to real U.S. output, corresponding to a quarterly drop of 1.85%.

In addition we experimented with a symmetric positive shocks to U.S. equity prices which are of particular interest here since their impacts on losses will not be (negatively) symmetric due to the nonlinearity of the credit risk model. In order to learn more about the tail properties of the various loss distributions, we also consider an extreme stress scenario for the U.S. equity market as reported in PSTW, namely an adverse shock of 8.02σ. This corresponds to a quarterly drop of 49% which is the largest quarterly drop in the S&P 500 index since 1928, that occurred over the three months to May of 1932. Finally, we include an intermediate negative equity shock of -5σ which corresponds to a quarterly decline of 30.64%. Details of how the macroeconomic shocks are generated and how they feed through firm returns to the loss distribution can be found in PSTW.

The results for the adverse shock scenarios can be found in Tables 8 and 9; we start the discussion with the -2.33σ shock to real U.S. equity summarized in Table 8a. Both EL and UL are similar across the resticted models M_1 through M_4, and they are systematically above the mean and volatility of losses implied by the heterogeneous model M_5. Industry fixed effects, M_2, seem to yield loss results that are closest to allowing for full parameter heterogeneity, yet even this model is unable to generate the tail behavior of M_5; the ratio ES/VaR is the same across models M_1

[36] 2.33σ corresponds, in the Gaussian case, to the 99% Value-at-Risk (VaR), a typical benchmark in risk management.

through M_4 at 1.08, much below the 1.48 figure obtained for the heterogeneous model. Despite this riskiness revealed only by the fully heterogeneous model, if capital were set using 99.9% VaR, the various restricted models would require between 23% and 29% more capital. Interestingly, this discrepancy shrinks to between 12% and 17% if capital were set using the coherent risk measure ES, though the implied capital remains lower with the fully heterogeneous model, presumably due to better portfolio diversification. Similar conclusions can be reached if we consider the other two adverse equi-probable shock scenarios: the increase in German interest rates and the drop in U.S. real output. The ratio of ES to VaR at the 99.9 percentile is around 1.4 for the heterogeneous model but only around 1.1 for the other models.

Next, we turn our attention to comparing the effects of benign and adverse shocks, and ask if symmetric shocks generate symmetric loss outcomes? The answer is clearly no, although the extent of asymmetry depends on model parameters. The asymmetry is especially pronounced for the fully heterogeneous model. In Table 9 we present the results for $\pm 2.33\sigma$ shocks to real U.S. equity returns. We repeat the results for the baseline and the adverse shock from Tables 7 and 8a for easy comparison. First, all models exhibit some degree of asymmetry: benign shocks do not mitigate losses as much as adverse shock increase them, relative to the baseline. This is the case wether measured by expected or unexpected losses, or even VaR or ES. Benign shocks generate loss distributions that are more skewed and have higher kurtosis relative to adverse shocks, a result that holds across all models. However, models M_1 through M_4 are all more "optimistic" when it comes to loss reduction from a benign shock when compared to the heterogeneous model M_5. Recall that baseline EL is calibrated to be nearly the same across models. The benign shock results in an EL of 33.5bp for the heterogeneous model, M_5, compared with 25.7 (M_4) to 27bp (M_3) for the restricted models. This discrepancy is especially noticable for VaR and ES: for example, while ES ranges from 171.2bp (M_2) to 178.5 (M_4), all are well below the 191.0bp for M_5. While the restricted models would assign too much capital for the adverse shocks, they would assign too little under the benign shock scenarios. Obviously firm parameter heterogeneity matters a great deal for credit risk, and fixed effects, either at the industry, regional or even firm level, do not seem to be sufficient to capture the heterogeneity relevant for credit risk.

Finally, we consider the effect of extreme shocks to the resulting distribution of credit losses under different model specfications. Tables 10a and 10b present results from two different U.S. real equity shock scenarios: -5.00σ and -8.02σ, the latter matching the largest quarterly drop in the S&P 500 index since 1928. To be sure, a shock as extreme as -8.02σ is, of course, outside the bounds of the estimated model. It would be unreasonable to believe that such a large shock would not result in changes to the underlying parameters. However, it is still instructive to examine the impact of an extreme shock, one way one might stress a credit risk model. Moreover, 5σ events are more common at higher frequencies than the quarterly data we have available to us, and in this way our results will likely underestimate the true loss outcomes.

Model parameter heterogeneity becomes especially important during such crisis scenarios. First, no matter how loss is measured, be it EL, UL, VaR or ES, model M_5 losses are always significantly below the other four restricted models. For instance, consider just EL for the -5σ shock in Table 10a: the fully heterogeneous model generates 85bp of EL while the other models range from 123.7 (M_2) to 141 (M_4). Once again industry fixed effects (M_2) seem to yield results that are closer to the fully heterogenenous model. Second, VaR is even a worse risk measure for these extreme shocks, though this is not apparent when using the restricted models. The ES to VaR ratio never exceeds 1.08 for models M_1 through M_4, but it is 1.56 for the -5σ and 1.85 (!) for the extreme -8.02σ shock under model M_5. Curiously for all models the resulting loss distribution becomes less skewed and fat-tailed, as measured by kurtosis, as the shocks sizes are increased.

7.3 Idiosyncratic Risk and Granularity

Portfolio-level results of credit risk models such as those discussed in Vasicek (1987, 2002) assume that the portfolio is sufficiently large that all idiosyncratic risk has been diversified away. More generally we consider a credit portfolio composed of N different credit assets such as loans, each with exposures or weights w_i, for $i = 1, 2, .., N$, such that the atomistic condition (23) holds. Recall that a sufficient condition for (23) to hold is given by $w_i = O\left(N^{-1}\right)$.[37] The lower the average firm return correlation, the greater the potential for diversification, but a larger N is required to attain that limit than if correlations are higher. A common rule of thumb for return diversification of a portfolio of equities is $N \approx 50$. But as seen in Section 3, default correlations are much lower than return correlations, meaning that more firms are needed to reach the diversification limits of credit risk.

Thus it seems reasonable to ask if a portfolio of $N = 243$ firms large enough to diversify away the idiosyncratic risk. To answer this question we took the simple Vasicek model with homogeneous factor loadings, a single default threshold and fixed LGD, and analyzed the impact of increasing N on simulated compared to analytic (asymptotic) unexpected loss (UL). The results are summarized in Table 11 below.

Table 11: **Impact of Granularity**

	# of loans in portfolio (N)			
	119	243	1000	10,000
Deviation from asymtotic lower bound	49%	27%	11%	5%

Fully homogeneous model with fixed LGD.

[37] Conditions (23) on the portfolio weights was in fact embodied in the initial proposal of the New Basel Accord in the form of the Granularity Adjustments which was designed to mitigate the effects of significant single-borrower concentrations on the credit loss distribution. See Basel Committee (2001, Ch.8).

The first value of $N = 119$ relates to the number of firms in the PSTW portfolio. By more than doubling N we cut idiosyncratic risk nearly in half. But to come within 5% of the asymptotic UL of the portfolio, more than 10,000 firms are needed! Thus credit portfolios or credit derivatives such as CDOs which contains rather fewer number of firms will likely still retain a significant degree of idiosyncratic risk, an observation also made by Amato and Remolona (2004). To be sure, these results may be different for different firm return specifications as the Vasicek model, for which closed form solutions for the asymptotic loss distribution are know, forces all firm-level variability into just two, in our case five, parameters (see (38)): "alpha," three risk factor sensitivities and the single default threshold.

8 Concluding Remarks

In this paper we made use of a conditional credit risk model with observable risk factors, developed in Pesaran, Schuermann, Treutler and Weiner (2004), to explore several dimensions of credit risk diversification: across industry sectors and across different countries or regions, either in a restrictive fixed effect or allowing for full firm-level heterogeneity. Specifically, we fix the number of risk factors – there are three: market equity returns and changes in domestic interest rates and oil prices – and only vary the amount of parameter heterogeneity across models. We find that allowing for full parameter heterogeneity can matter a great deal for capturing tail behavior in credit loss distributions. Moreover, this tail behavior is not captured using standard value-at-risk (VaR) measures; instead, the coherent risk measure expected shortfall (ES) is needed. We examine the effect of symmetric shocks to observable risk factors and show that they result in asymmetric loss outcomes, and this asymmetry is especially pronounced when full parameter heterogeneity is allowed for. Neglecting firm heterogeneity in the risk factor sensitivities, the firm "betas," can either result in too much implied risk capital, under adverse shocks, or too little capital under benign shock scenarios. While neither industry nor regional (geography) fixed effects are sufficient to capture this firm-level heterogeneity, controlling for industry effects seems to generate results which are closer to the fully unrestricted heterogeneous model.

The results raise a number of questions and issues that merit further exploration. Our portfolio, by virtue of being allocated across 24 countries in 10 regions, is already quite diversified as evidenced by an average pairwise return correlation of 12%. Concentrating all of the nominal exposure into just one region or one industry would undoubtedly have significant impact on the resulting loss distribution, in addition to yielding differences across models. A difficulty one would quickly encounter in exploring this problem are the rating or default probability differences across those dimensions. The average rating in the U.K., for instance, is much higher than for the Latin American obligors, especially if one follows the rule that an obligor rating cannot exceed the sovereign rating.[38]

It is also worth exploring the impact of fat-tailed innovations on the resulting loss distributions. The current application is limited to the double-Gaussian assumption (both idiosyncratic and systematic innovations are normal), but it seems reasonable to relax this assumption by considering, say, draws from Student-t distributions with low degrees of freedom.

[38]This rule seems quite reasonable when one considers debt denominated in, say, USD (or euros), but perhaps less so if the debt is exclusively in the local currency.

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Tables

Table 6: **Return Regression Results for Models 1-3**

	M_1: **Pooled**	M_2: **Pooled +**	M_3: **Pooled +**
		Industry Dummies	**Region Dummies**
constant	0.020 (0.001)***	-0.006 (0.005)	0.014 (0.005)***
$\Delta q/\Delta q^*$	0.869 (0.016)***	0.870 (0.016)***	0.865 (0.016)***
Δr	0.018 (0.050)	0.016 (0.050)	0.033 (0.050)
Δp^o	0.063 (0.021)***	0.063 (0.020)***	0.063 (0.020)***
Agric., Mining & Construction		-0.006 (0.005)	
Communication, Electric & Gas†		–	
Durable Manufacturing		-0.003 (0.004)	
FIRE[39]		0.020 (0.003)***	
Non-durable Manufacturing		0.006 (0.008)	
Service		0.003 (0.004)	
Wholesale & Retail Trade		0.009 (0.005)*	
U.S.†			–
U.K.			-0.007 (0.005)
Germany			0.008 (0.006)
France			0.007 (0.007)
Italy			0.010 (0.005)**
W. Europe[40]			0.113 (0.014)***
Middle East[41]			-0.005 (0.005)
S.E. Asia[42]			-0.008 (0.004)**
Japan			0.044 (0.008)***
L. America[43]			0.017 (0.002)***
R^2	0.144	0.145	0.151

Number of firm quarters: 17,114

* indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

†indicates omitted category.

Table 7

Baseline Scenario

Statistics of Simulated Losses for Models 1-5, Four Quarters Ahead

(in Basis Points of Exposure, where applicable)

Model	EL	UL	99.9% VaR	99.9% ES	ES/VaR	Skewness	Kurtosis
M1) Pooled	45.2	35.2	200.9	217.7	1.08	1.10	4.29
M2) Industry Fixed Effect	42.0	34.6	191.3	213.0	1.11	1.05	4.20
M3) Region Fixed Effect	43.5	35.7	205.8	224.5	1.09	1.10	4.30
M4) Region + Firm Fixed Effect	42.6	34.3	188.9	208.4	1.10	1.01	4.09
M5) Heterogeneous	43.7	34.6	184.6	214.5	1.43	1.05	4.25

Table 8a

-2.33σ Shock to Real U.S. Equity Returns: Quarterly Decline of 14.28%
Statistics of Simulated Losses for Models 1-5, Four Quarters Ahead
(in Basis Points of Exposure, where applicable)

Model	EL	UL	99.9% VaR	99.9% ES	ES/VaR	Skewness	Kurtosis
M1) Pooled	70.6	44.4	252.1	272.9	1.08	0.64	3.72
M2) Industry Fixed Effect	68.2	43.1	245.3	266.0	1.08	0.62	3.69
M3) Region Fixed Effect	72.8	44.8	257.1	277.5	1.08	0.62	3.69
M4) Region + Firm Fixed Effect	72.8	44.8	257.1	277.5	1.08	0.62	3.69
M5) Heterogeneous	56.3	39.0	199.2	237.5	1.48	0.75	3.81

Table 8b

+2.33σ Shock to German Interest Rates: Quarterly Increase of 0.33%

Statistics of Simulated Losses for Models 1-5, Four Quarters Ahead

(in Basis Points of Exposure, where applicable)

Model	EL	UL	99.9% VaR	99.9% ES	ES/VaR	Skewness	Kurtosis
M1) Pooled	46.9	36.3	200.8	219.0	1.09	0.89	3.93
M2) Industry Fixed Effect	46.2	36.0	202.7	221.0	1.09	0.93	4.06
M3) Region Fixed Effect	47.5	36.8	210.3	229.1	1.09	0.99	4.19
M4) Region + Firm Fixed Effect	46.9	35.8	203.2	223.3	1.10	0.94	4.09
M5) Heterogeneous	45.3	35.5	191.6	218.3	1.40	0.95	4.07

Table 8c

-2.33σ Shock to U.S. Real Output: Quarterly Decline of 1.85%

Statistics of Simulated Losses for Models 1-5, Four Quarters Ahead

(in Basis Points of Exposure, where applicable)

Model	EL	UL	99.9% VaR	99.9% ES	ES/VaR	Skewness	Kurtosis
M1) Pooled	41.1	34.6	199.2	215.6	1.08	1.18	4.40
M2) Industry Fixed Effect	40.6	33.9	189.4	212.2	1.12	1.10	4.30
M3) Region Fixed Effect	42.3	35.1	198.0	218.2	1.10	1.12	4.30
M4) Region + Firm Fixed Effect	41.7	34.1	191.8	211.6	1.10	1.08	4.23
M5) Heterogeneous	41.5	34.2	184.1	208.4	1.39	1.04	4.16

Table 9

±2.33σ Shock to U.S. Real Equity Returns

Statistics of Simulated Losses for Models 1-5, Four Quarters Ahead

(in Basis Points of Exposure, where applicable)

Model, shock type		EL	UL	99.9% VaR	99.9% ES	ES/VaR	Skewness	Kurtosis
M1) Pooled:	adverse	70.6	44.4	252.1	272.9	1.08	0.64	3.72
	Baseline	45.2	35.2	200.9	217.7	1.08	1.10	4.29
	benign	26.6	28.1	159.0	173.4	1.09	1.66	4.83
M2) Industry Fixed Effect:	adverse	68.2	43.1	245.3	266.0	1.08	0.62	3.69
	Baseline	42.0	34.6	191.3	213.0	1.11	1.05	4.20
	benign	26.7	28.0	155.3	171.2	1.10	1.62	4.77
M3) Region Fixed Effect:	adverse	72.8	44.8	257.1	277.5	1.08	0.62	3.69
	Baseline	43.5	35.7	205.8	224.5	1.09	1.10	4.30
	benign	27.0	28.6	159.4	178.5	1.12	1.71	4.91
M4) Region + Firm Fixed Effect:	adverse	72.8	44.8	257.1	277.5	1.08	0.62	3.69
	Baseline	42.6	34.3	188.9	208.4	1.10	1.01	4.09
	benign	25.7	27.2	152.3	172.1	1.13	1.74	4.99
M5) Heterogeneous:	adverse	56.3	39.0	199.2	237.5	1.48	0.75	3.81
	Baseline	43.7	34.6	184.6	214.5	1.43	1.05	4.25
	benign	33.5	30.9	170.1	191.0	1.37	1.28	4.45

Table 10a

Extreme Shocks to Real U.S. Equity Returns: -5σ, Quarterly Decline of 30.6%

Statistics of Simulated Losses for Models 1-5, Four Quarters Ahead: Baseline Scenario

(in Basis Points of Exposure, where applicable)

Model	EL	UL	99.9% VaR	99.9% ES	ES/VaR	Skewness	Kurtosis
M1) Pooled	133.6	58.6	358.6	385.0	1.07	0.33	3.40
M2) Industry Fixed Effect	123.7	56.4	341.1	368.9	1.08	0.37	3.44
M3) Region Fixed Effect	138.6	58.9	366.7	391.7	1.07	0.33	3.40
M4) Region + Firm Fixed Effect	141.0	59.2	361.5	386.3	1.07	0.31	3.32
M5) Heterogeneous	85.0	46.6	237.5	298.0	1.56	0.54	3.66

Table 10b

Extreme Shocks to Real U.S. Equity Returns: -8.02σ, Quarterly Decline of 49%

Statistics of Simulated Losses for Models 1-5, Four Quarters Ahead: Baseline Scenario

(in Basis Points of Exposure, where applicable)

Model	EL	UL	99.9% VaR	99.9% ES	ES/VaR	Skewness	Kurtosis
M1) Pooled	290.8	81.1	575.7	601.0	1.04	0.13	3.11
M2) Industry Fixed Effect	258.5	76.4	533.9	561.0	1.05	0.15	3.17
M3) Region Fixed Effect	301.7	80.8	585.7	617.3	1.05	0.14	3.13
M4) Region + Firm Fixed Effect	310.2	81.8	601.9	629.0	1.05	0.13	3.12
M5) Heterogeneous	169.7	60.3	291.1	427.1	1.85	0.29	3.38