

Uncertainty and Economic Activity: Identification Through Cross-country Correlations[☆]

Ambrogio Cesa-Bianchi[†] M. Hashem Pesaran[‡] Alessandro Rebucci[§]

July 12, 2017

Abstract

This paper proposes a multi-country approach for the analysis of the interaction between uncertainty and economic activity both within and across economies. It considers two global factors, labeled “real” and “financial”, and identified assuming different patterns of cross-country correlations of country-specific innovations to output growth and stock market volatility. It is assumed that only the real factor is required to model cross-country correlations of growth innovations, but both factors (real and financial) are needed to model cross-country correlations of volatility innovations. The paper shows that these assumptions are in accordance with stylized facts of the underlying data and the estimated innovations. The identified factors are then used within a high-dimensional factor-augmented VAR to quantify the impact and the relative importance of common and country-specific shocks. It is found that most of the unconditional correlation between volatility and growth can be accounted for by the two common factors. While unconditionally volatility behaves countercyclically for all but one of the 32 countries in our sample, once we condition on the factors the growth-volatility correlation becomes essentially negligible in all countries with the exception of five emerging market economies. The paper also shows that the share of volatility variance explained by the real factor and by country-specific growth shocks is relatively small. Similarly, shocks to the financial factor explain only a small fraction of the country-specific growth variance. Finally, country-specific shocks are important for domestic volatility and growth, but have limited spillover effects to other countries.

Keywords: Business Cycle, Common Factors, Financial Cycle, Growth, Identification, Uncertainty, Volatility.

JEL Codes: E44, F44, G15.

[☆]Preliminary draft. The views expressed in this paper are solely those of the authors and should not be taken to represent those of the Bank of England.

[†]Bank of England and CfM. Email: ambrogio.cesa-bianchi@bankofengland.co.uk.

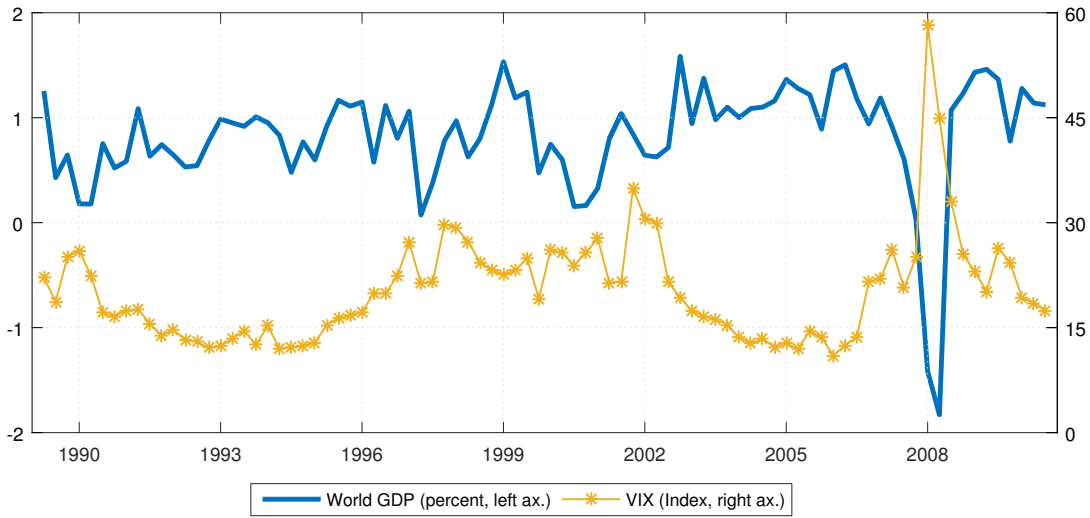
[‡]Department of Economics, USC Dornsife, INET, University of Southern California, and Trinity College, Cambridge. Email: pesaran@usc.edu

[§]Johns Hopkins University and NBER. Email: arebucci@jhu.edu.

1 Introduction

During the global financial crisis, the world economy experienced a sharp and synchronized contraction in economic activity and an exceptional increase in volatility. Indeed, after the VIX Index (a commonly used measure of market volatility) spiked during the second half of 2008, world growth collapsed (Figure 1). Since then, in part as a response to this dramatic event, there has been a renewed and strong interest in the relationship between uncertainty (defined and measured in various ways) and economic activity over the business cycle.¹

Figure 1 QUARTERLY WORLD GDP GROWTH AND VIX INDEX



NOTE. World GDP growth is a PPP-GDP weighted average of the quarter on quarter GDP growth (in percent) of 32 advanced and developing economies. See Appendix C for data sources.

It is now well established that empirical measures of uncertainty behave countercyclically in the United States and in most other countries around the world.² But interpreting this correlation in economic or structural terms is challenging because the direction of causation can run in both ways.

From a theoretical standpoint, uncertainty can cause economy activity to slow and contract through a variety of mechanisms, both on the household side via precautionary savings (Kimball (1990)) and on the firm side via physical frictions (see for instance Bernanke (1983), Dixit and Pindyck (1994) and, more recently, Bloom (2009)) or financial frictions (Christiano et al. (2014), Gilchrist et al. (2013),

¹The ensuing literature is now voluminous. Here we focus on the studies more directly related to our paper. See Bloom (2014) for a recent survey.

²See, for instance, Baker and Bloom (2013), Carriere-Swallow and Cespedes (2013), Nakamura et al. (2017).

Arellano et al. (2012)).³

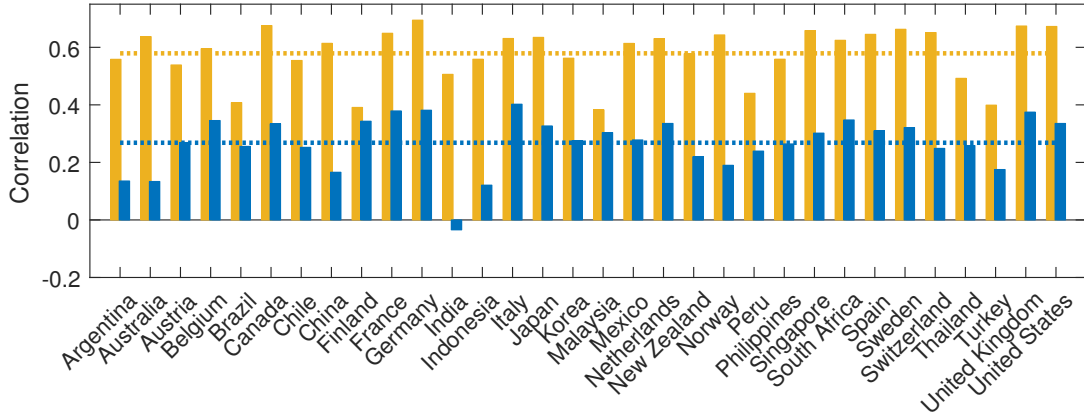
But it is also possible that uncertainty responds to fluctuations in economic activity. Indeed, the theoretical literature highlights numerous mechanisms through which spikes in uncertainty may be the reflection of adverse economic conditions rather than driving them. Examples based on information and financial frictions include Van Nieuwerburgh and Veldkamp (2006), Fostel and Geanakoplos (2012), Ilut et al. (2017), and Tian (2015). Yet, other models stress interaction effects between uncertainty and financial frictions *via* an increase in the risk premium (e.g., Christiano et al., 2014, Gilchrist et al., 2013, Arellano et al., 2012).

In this paper, we propose a multi-country approach to identification and modeling of the interaction between uncertainty and economic activity, without restricting the direction of economic causation. We measure uncertainty and activity with realized equity market volatility and GDP growth, respectively. We assume that uncertainty and activity are driven by two common factors, a “real” and “financial” factor, as well as country-specific volatility and growth shocks. We identify and estimate the two common factors by assuming that innovations to volatility and growth have different patterns of correlations across countries. Specifically we assume that only the real factor is required to model cross-country correlations of output growth innovations, but both factors (real and financial) are needed to model cross country correlations of volatility innovations. It is shown that these assumptions are in accordance with stylized facts of the underlying data and the estimated innovations. We then estimate the impact of shocks to these factors on country-specific volatility and growth as well as their importance relative to each other, and to country-specific shocks.

We start by assuming that all growth and volatility series share at least one common factor. This assumption is consistent with a standard, multi-country capital asset pricing model (CAPM) in which world technology drives the price and the variance of all country equity claims. To identify this factor, we further assume that volatility innovations are more correlated across countries than growth innovations. Technically, we assume *strong* cross-country dependence of volatility innovations, and *weak* dependence of growth innovations (in the sense of Chudik et al., 2011). This is equivalent to assuming that volatility innovations share at least one further strong common factor as compared

³Pricing frictions and the zero lower bound can amplify the impact of the shock and produce data-congruent comovements between investment and consumption (Fernandez-Villaverde et al. (2011), Basu and Bundick (2017), Born and Pfeifer (2014)).

Figure 2 CROSS-COUNTRY CORRELATION OF VOLATILITY AND GDP GROWTH



NOTE. Average cross-country pairwise correlation of volatility (light bars) and GDP growth (dark bars). The dotted lines correspond to the average pairwise correlations across countries, at 0.58 and 0.27 for volatility and GDP growth, respectively. See section 4 for data sources and measurement of volatility.

to growth innovations. As the paper shows, under this key assumption, the common factor can be approximated by world growth, up to an affine transformation. Thus, we label it a "real" factor. The key point here is that the identification assumptions apply to a large enough set of countries, as opposed to a single country considered in isolation. It allows a two-way causality between growth and volatility innovations at the individual country levels, and only requires the factors to enter the global economy in a triangular manner. The approach is simple and transparent, and can be applied in other contexts.

Our main identification assumption is in accordance with patterns of cross-country correlation that we document in the raw data, as well as formal testing on the cross-country dependence of the estimated country-specific volatility and growth innovations that we report in the paper. For instance, Figure 2 exhibits the average cross-country pairwise correlation of volatility and growth. The average pairwise correlation for the volatility series is more than twice the average for the growth series, at 0.58 and 0.27, respectively. As we shall see in the paper, we find comparable differences by using principal component analysis, and even more striking differences when we look at volatility and growth innovations.⁴

This evidence suggests that volatility series share additional common factors not shared by the

⁴Notice here that these patterns are consistent with those documented by [Tesar \(1995\)](#), [?](#), and [Lewis and Liu \(2015\)](#) for equity return and consumption growth correlations.

growth series. Under the additional and testable assumption that the volatility series share *only* one additional strong common factor, the loading matrix becomes triangular, with country-specific volatilities load contemporaneously on both factors growth rates, and growth rates loading only on the first factor.

The interpretation of this second factor is more problematic. It is well known that the international CAPM has a number of counterfactual predictions, and additional factors are needed to address these shortcomings. For instance, one could consider adding preference shocks, or altering the information assumptions, in the basic framework to improve its empirical performance. As we discuss in the paper, the second common factor in our model could be interpreted as the common jump component in an international disaster or long-run risk model.⁵ Alternatively, one could also think about the United States as “small and granular” in global production and consumption (because relatively close to trade in goods and services), but “large and dominant” in global finance (in the sense that all volatility series must load onto it as discussed by [Chudik and Pesaran \(2013\)](#) (because of its very deep and open capital markets). For this reason we shall label it “financial” factor. Nonetheless, the paper is ultimately agnostic on the deeper economic interpretation of this second factor that we identify from the data. The approach taken in the paper is to start from the properties of the data without taking a stance on the specific model that might be generating them.

For each country, we focus on real GDP growth and realized equity market volatility. To measure economic uncertainty, we build on the contributions of [Andersen et al. \(2001, 2003\)](#) and [Barndorff-Nielsen and Shephard \(2002, 2004\)](#), and we compute realized equity price volatility for a given quarter by using daily stock returns on 32 advanced and emerging economies representing more than 90 percent of the world economy. In the paper, we consider and discuss several other measures of volatility and argue that they are either not suitable for the purpose of our analysis or not readily available for a large collection of countries like the one we use in our analysis. We measure economic activity with quarterly GDP growth.

The multi-country nature of our framework allows us to investigate a number of different empirical issues. First, and most importantly, we find that most of the negative correlation between volatility

⁵[Lewis and Liu \(2017\)](#), show that a disaster risk model with both country-specific and common disaster risk can generate patterns of equity return and consumption growth correlations across countries consistent with the one we assume for growth and volatility.

and growth observed in the data can be accounted for by the real and financial factors. While unconditionally volatility behaves countercyclically for all but one of the 32 countries in our sample, when we condition on the two factors, the correlation between volatility and growth innovations becomes essentially negligible in all countries with the exception of five emerging market economies.

Second, the papers shows that, on average, country-specific volatility variance is explained largely by the financial factor (60%), and the own country-specific volatility shock (30%), and is only marginally affected by the real factor, or own country-specific growth shock. Thus one could argue that the endogenous component of volatility is small, or, equivalently, that volatility is largely exogenous.

In contrast, country-specific growth variance is mainly explained by own growth shocks (60%), and the real factor (25%). Even though it has contractionary effects on country-specific growths, shocks to the financial factor explain only a small fraction (12%) of the country-specific growth variance. This suggests that shocks to the financial factor can be potentially harmful for growth, although they tend to occur rather rarely. Finally, shocks to country-specific volatility explain a negligible share of growth variance.

The paper is related to two different strands of the literature. The first strands acknowledges that volatility may be endogenous and could be driven by business or financial cycles—see, for instance, [Ludvigson et al. \(2015\)](#), [Clark et al. \(2016\)](#), [Giglio et al. \(2016\)](#). A key difference relative to these contributions, is that the common real factor, which is the main source of endogeneity in our factor model, is common to both macroeconomic and financial variables (GDP growth and equity price volatility), as opposed being common only to the macroeconomic variables. Also, our identification strategy is simple and consistent with observable cross-country correlation properties of the data.

A second strand of the literature assumes that uncertainty is exogenous and considers the international dimension of the relation between uncertainty and activity, with findings consistent with those obtained for the United States. For instance, [Carriere-Swallow and Cespedes \(2013\)](#) estimate a battery of separate small open economy VARs for 20 advanced and 20 emerging market economies in which the VIX index is assumed to be determined exogenously. Using an unbalanced panel of 60 countries, [Baker and Bloom \(2013\)](#) provide evidence of the counter-cyclicality of different proxies for uncertainty, such as stock market volatility, sovereign bond yields volatility, exchange rate volatility and GDP forecast disagreement. Finally, [Hirata et al. \(2012\)](#) estimate a factor-augmented VAR (FAVAR), with factors

computed based on data for 18 advanced economies, and use a recursive identification scheme in which the volatility variable is ordered first in the VAR.

While these papers share with ours an international focus, the distinctive feature of our analysis is to consider a multi-country framework in which countries interact with each other, as opposed to a set of countries considered in isolation. The FAVAR model used by [Hirata et al. \(2012\)](#) does allow for interconnected countries through the factors, but it continues to rely on recursive structures at the country-specific level to identify volatility shocks. In contrast, our identification strategy allows us to quantify the extent to which country-specific volatility is exogenous as opposed to assuming it from the outset of the analysis.⁶

The rest of the paper is organized as follows. Section 2 discusses the model that we propose and the identification of the two factors. Section 3 extends the analysis to a fully dynamic and heterogeneous set up. Section 4 discusses the use of realized volatility as a proxy for uncertainty and explains how we compute it. Section 5 reports key stylized facts of the data, including evidence on the cross-country correlation structure. Section 6 reports the main empirical results. Section 7 concludes. Proof of our theoretical derivations and details on the data sources are in the Appendix. Selected country-specific results are reported in an online Supplement.

2 A Simple Static Factor Model

This section presents a static version of the dynamic model that we use in the empirical analysis and discuss its identification and interpretation. Omitting any dynamics and deterministic components to simplify the exposition, we posit the following common factor representation for the contemporaneous correlation between volatility and growth, for a large number of countries indexed by i :

$$v_{it} = \lambda_i f_t + u_{it}, \quad (1)$$

$$\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}, \quad (2)$$

⁶As we show in the paper, our multi-country set up can also be viewed as FAVAR model, with the difference that we allow for a large number of country-specific variables and estimate the factors as cross-country averages of output growth and volatility.

for $i = 1, 2, \dots, N$, and $t = 1, 2, \dots, T$. Here, v_{it} denotes a volatility (uncertainty) measure for country i , and Δy_{it} is an economic activity indicator for country i at time t (say, real GDP growth like in our application, simply called “output” or “growth” for brevity in the paper). The common factor, f_t , is assumed to have mean zero and a finite variance that is normalized to one for simplicity. The country-specific innovations, u_{it} and ε_{it} , are serially uncorrelated with zero means and finite variances, but can be correlated with each other both within and between countries. Conditional on the common factor, f_t , therefore, the correlation between u_{it} and ε_{it} captures any contemporaneous causal relation between volatility and growth, on which we do not impose any restriction.

This representation is quite general and can be motivated by both theory and empirical evidence. From a theoretical perspective, one can think of f_t as a fundamental factor, such as world technology, which affects all countries GDP growth rates and country equity price indexes volatilities at the same time. For example, in a general equilibrium version of the international CAPM with complete asset markets, f_t is world growth and it affects all country growth rates and return volatilities contemporaneously. While this frictionless general equilibrium benchmark cannot account for typical stylized facts of country equity returns, it provides a starting point to consider additional shocks and frictions for empirical modeling purposes.⁷ From an empirical perspective, as we will show below for our sample of countries, and as documented in the literature, GDP growth and volatility share a large and negative contemporaneous correlation at the country level, for all countries we consider.

2.1 Identifying the Fundamental Factor through Cross-Country Correlation

In order to identify the innovations u_{it} and ε_{it} , the common factor f_t , and the factor loadings λ_i and γ_i of (1)-(2) we need to impose some restrictions. The model (1)-(2) applies to all N countries. Consider the generic country i , like for instance the United States. If we consider the United States in isolation from the rest of the world, we cannot identify the parameters of the model above, even if f_t is assumed to be known, unless we exclude it from one of the two equations. To see this note that the covariance

⁷For a more detailed discussion, see, for instance Obstfeld and Rogoff (1996, Chapter 5).

matrix of v_{it} and Δy_{it} is given by:

$$\Theta_i = \begin{pmatrix} \lambda_i^2 + \sigma_{u,i}^2 & \lambda_i \gamma_i \\ \lambda_i \gamma_i & \gamma_i^2 + \sigma_{\varepsilon,i}^2 \end{pmatrix}. \quad (3)$$

This provides three independent restrictions, but the factor model contains four free parameters, namely, (λ_i, γ_i) , and $(\sigma_{u,i}^2, \sigma_{\varepsilon,i}^2)$. Their identification can be achieved only by imposing at least one *additional* exclusion restriction, e.g. either $\lambda_i = 0$ or $\gamma_i = 0$.⁸

The main idea of this paper is to achieve identification of all model parameters by placing restrictions on the degree of cross-country error correlations, namely the contemporaneous correlation of u_{it} and ε_{it} over $i = 1, 2, \dots, N$ at a given point in time t . The identification strategy that we propose is general and can be applied to any panel of time series with the same properties of cross-section correlation that we assume. The model could have more factors provided that we consider more observable variables across countries. However, what is crucial is to look at the whole set of N cross-sectional units, as opposed to one unit at a time, taken in isolation. As we will shall see, the identification strategy also requires that N is sufficiently large and no one country is dominant, in the sense defined by Chudik and Pesaran (2013), and to be made precise below.

To illustrate the strategy, let's define global volatility and GDP growth by $\bar{v}_{\omega,t}$ and $\Delta \bar{y}_{\omega,t}$, respectively, as follows:

$$\bar{v}_{\omega,t} = \sum_{i=1}^N \hat{w}_i v_{it}, \quad (4)$$

$$\Delta \bar{y}_{\omega,t} = \sum_{i=1}^N w_i \Delta y_{it}, \quad (5)$$

where $\hat{w}_i$ and w_i denote two sets of aggregation weights. These weights can be the same or differ for each variable. To achieve identification, we then make the following assumptions on the factor f_t , its loadings $(\lambda_i$ and $\gamma_i)$, the weights $(\hat{w}_i$ and $w_i)$, and the country-specific innovations $(u_{it}$ and $\varepsilon_{it})$:

Assumption 1 (*Loadings*) *The factor loadings, λ_i and γ_i , are distributed independently across i and*

⁸The model (1)-(2) is observationally equivalent to a standard structural vector autoregression model. Thus, it is subject to the same identification requirement.

the common factors f_t , for all i and t , with non-zero means λ and γ ($\lambda \neq 0$ and $\gamma \neq 0$), and satisfy the following conditions:

$$N^{-1} \sum_{i=1}^N \lambda_i^2 = O(1) \quad \text{and} \quad N^{-1} \sum_{i=1}^N \gamma_i^2 = O(1), \quad (6)$$

$$\lambda = \sum_{i=1}^N \dot{w}_i \lambda_i \neq 0 \quad \text{and} \quad \gamma = \sum_{i=1}^N w_i \gamma_i \neq 0, \quad (7)$$

for all N , and as $N \rightarrow \infty$.

Assumption 2 (*Weights*) Let $\mathbf{w} = (w_1, w_2, \dots, w_N)'$ and $\dot{\mathbf{w}} = (\dot{w}_1, \dot{w}_2, \dots, \dot{w}_N)'$ be $N \times 1$ vectors of non-stochastic weights with $\sum_{i=1}^N w_i = 1$ and $\sum_{i=1}^N \dot{w}_i = 1$. The growth weights, $\mathbf{w}$, are granular, in the sense that:

$$\|\mathbf{w}\| = O(N^{-1}), \quad \frac{w_i}{\|\mathbf{w}\|} = O(N^{-\frac{1}{2}}), \quad \forall i, \quad (8)$$

while volatility weights $\dot{\mathbf{w}}$ are left unrestricted, so they may or may not be granular.

Assumption 3 (*Cross-country correlations*) Let the variance-covariance matrices of the $N \times 1$ error vectors $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{Nt})'$ and $\mathbf{u}_t = (u_{1t}, u_{2t}, \dots, u_{Nt})'$ be $\boldsymbol{\Sigma}_{\varepsilon\varepsilon} = \text{var}(\boldsymbol{\varepsilon}_t)$ and $\boldsymbol{\Sigma}_{uu} = \text{var}(\mathbf{u}_t)$, respectively, and suppose that:

$$\varrho_{\max}(\boldsymbol{\Sigma}_u) = O(N), \quad (9)$$

$$\varrho_{\max}(\boldsymbol{\Sigma}_{\varepsilon}) = O(1). \quad (10)$$

where $\varrho_{\max}(\mathbf{A})$ denotes the largest eigenvalue of matrix $\mathbf{A}$.

Assumption 1 is standard in the factor literature (see, for instance, Assumption B in [Bai and Ng \(2002\)](#)). It ensures that f_t is a strong (or pervasive) factor for both volatility and growth so that it can be estimated consistently using principal components or the cross-section averages of country-specific observations (see [Chudik et al., 2011](#)). Assumption 2 requires that individual countries' contribution to world growth is of order $1/N$, while it leaves the volatility weights unrestricted.⁹ This implies that some country such as the United States could have a relatively large weight in global volatility that

⁹These weights could be predetermined. They need not be fixed.

does not necessarily tend to zero as $N \rightarrow \infty$. These assumptions about the weights are consistent with the notion that, since the 1990s, world growth has become a progressively more diversified process, while they leave open the possibility that financial markets have continued to be dominated by the largest and most developed countries like the United States.

Assumption 3 is crucial for our identification strategy. It states that the volatility innovations are strongly correlated across countries, while growth innovations are weakly correlated across countries. Weak cross-country correlation means that, asymptotically, as N becomes large, the average pairwise correlations across countries of output growth innovations tends to zero, since the largest eigenvalue of their variance-covariance matrix is bounded in N . On the other hand, strong cross-sectional correlation means that the largest eigenvalue of Σ_u grows with the size of the cross-section, N . As we shall see, this assumption is in accordance with the empirical properties of both the data we use and the residual that we obtain from the model estimation.¹⁰ The identification of f_t , up to an affine transformation, is set out in the following proposition.

Proposition 1 *Under Assumptions 1-3, for N sufficiently large, f_t can be identified (up to a scalar constant) by $\bar{y}_{\omega,t} = \sum_{i=1}^N w_i \Delta y_{it}$.*

Proof. Consider the model defined by (1)-(2) for $i = 1, 2, \dots, N$. Under Assumption 1-3, and using the definitions in (4)-(5), the following model can be obtained for the global variables:

$$\bar{v}_{\omega,t} = \lambda f_t + \bar{u}_{\omega,t}, \quad (11)$$

$$\Delta \bar{y}_{\omega,t} = \gamma f_t + \bar{\varepsilon}_{\omega,t}, \quad (12)$$

where $\bar{\varepsilon}_{\omega,t} = \bar{\mathbf{w}}' \boldsymbol{\varepsilon}_t$, and $\bar{u}_{\omega,t} = \mathbf{w}' \mathbf{u}_t$. Furthermore

$$\text{var}(\bar{\varepsilon}_{\omega,t}) = \mathbf{w}' \Sigma_{\varepsilon} \mathbf{w} \leq (\mathbf{w}' \mathbf{w}) \varrho_{\max}(\Sigma_{\varepsilon}). \quad (13)$$

Thus, under Assumption 3, we have:

$$\text{var}(\bar{\varepsilon}_{\omega,t}) = O(\mathbf{w}' \mathbf{w}) = O(N^{-1}), \quad (14)$$

¹⁰Different patterns of cross country correlations have been documented also for consumption growth and equity returns. See, for instance, [Tesar \(1995\)](#) and [Lewis and Liu \(2015\)](#).

and hence:

$$\bar{\varepsilon}_{\omega,t} = O_p\left(N^{-1/2}\right). \quad (15)$$

Using this in (12), and since $\gamma \neq 0$ under Assumption 1, we have:

$$f_t = \gamma^{-1} \Delta \bar{y}_{\omega,t} + O_p\left(N^{-1/2}\right), \quad (16)$$

which allows us to recover f_t from $\Delta \bar{y}_{\omega,t}$ up to the scalar $1/\gamma$. Note that to simplify the exposition we have assumed f_t to have mean zero. In general, where f_t might have a non-zero mean, it follows that f_t is an affine function of the global output variable, $\Delta \bar{y}_{\omega,t}$, up to probability terms of order $N^{-1/2}$. ■

A few remarks are in order here. First, if we assume that the volatility innovations are strongly correlated across countries, f_t cannot be recovered from $\bar{v}_{\omega,t}$. In fact, since $\varrho_{\max}(\Sigma_u) = O(N)$, it follows that $\text{var}(\bar{u}_{\omega,t}) = \mathbf{w}' \Sigma_u \mathbf{w}$ will generally not converge to zero. Under Assumptions 2 and 3, only the principal components or cross-section averages of output growth can be used to identify f_t (up to a scalar). Therefore, pooling observations on all volatility variables (or all volatility and growth variables) and then extracting their principal components would not be appropriate for the identification of f_t . Second, the difference in the way in which the volatility and GDP growth innovations are correlated across countries also ensures that we do not end up with a perfect relationship between v_t and Δy_t when N become sufficiently large. Third and finally, notice that equation (14) requires $\mathbf{w}' \mathbf{w} = O(N^{-1})$, i.e., that the weights in the global output measure are granular.

Finally, it is perhaps worth noting that whilst for mathematical precision we need $N \rightarrow \infty$, in practice N need not be too large for the theory to work reasonably well. The notion of $N \rightarrow \infty$, is best viewed as a counterfactual exercise which considers the extent to which the strength of connections across units declines with N . In our experience, working with many cross-country panels, N around 30 is often sufficiently large.

2.2 The Financial Factor

In view of condition (9) of Assumption 3, the cross-section of volatility innovations u_{it} must share at least one additional strong common factor that is not shared by growth innovations. To accommodate such a possibility we make the following additional assumption:

Assumption 4 *The volatility innovations, u_{it} , in equation 1 can also be decomposed into a strong factor (g_t) and a weak country-specific component (η_{it}), namely*

$$u_{it} = \theta_i g_t + \eta_{it}, \quad (17)$$

where η_{it} is a country-specific volatility innovation, which is cross-sectionally weakly correlated.

The above assumption is compatible with (9), and provides a specific factor structure for its characterization. Under this assumption, letting the variance-covariance matrix of the $N \times 1$ vector $\boldsymbol{\eta}_t = (\eta_{1t}, \eta_{2t}, \dots, \eta_{Nt})'$ be $\boldsymbol{\Sigma}_{\eta\eta} = \text{var}(\boldsymbol{\eta}_t)$, we have

$$\varrho_{\max}(\boldsymbol{\Sigma}_{\eta\eta}) = O(1), \quad (18)$$

and:

$$N^{-1} \sum_{i=1}^N \theta_i^2 = O(1), \quad (19)$$

which allows us to identify the second strong factor, shared only by the volatility series, from the data.

Proposition 2 *Supposed that u_{it} is given by (17) and Assumption 4 holds. Then:*

$$v_{it} = \lambda_i f_t + \theta_i g_t + \eta_{it}, \quad (20)$$

$$\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}, \quad (21)$$

and

$$g_t = \theta^{-1} \left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma} \Delta \bar{y}_{\omega,t} \right) + O_p \left(N^{-1/2} \right). \quad (22)$$

Proof. To see this, substitute (16) and (17) into (11) and apply to η_{it} the same reasoning used in Proposition 1. ■

It is now evident that the assumed, different pattern of correlation across countries of volatility and growth innovations implicitly provides a restriction on the factor loadings of the growth equations on the second strong factor g_t , and yields country models that are lower triangular in the factor loading

matrix.¹¹ In effect, the fact that a factor is shared by many countries is exploited to identify the unobserved factors and innovations to volatility and output growth. The key distinguishing feature of our identification scheme lies in the fact that no restrictions are made at the individual country levels, but on the cross-section correlation of all countries under consideration. Indeed, it is easy to see from (20) and (21) that for a given i it is not possible to identify the direction of casualty between v_{it} and Δy_{it} , despite its triangular factor specification. But if we consider a set of countries with N sufficiently large, it is possible to identify f_t and g_t up to affine transformations under the assumption that ε_{it} and η_{it} are cross-sectionally weakly dependent, even though for each i the innovations η_{it} and ε_{it} are correlated.

A second common factor in volatility and growth can be justified by asset pricing models with preference shocks.¹² Our identification assumptions on the cross country correlations of the growth and volatility innovations, however, imply that GDP growth does not load contemporaneously on the second factor g_t . Such a triangular loading structure is consistent, for instance, with the international disaster risk models of Colacito and Croce (2011) or Lewis and Liu (2017). Lewis and Liu (2017), in particular, show that a model with time-varying probability of disaster and both world and country-specific disaster risk generates patterns of cross-country consumption growth and equity return correlations consistent with the data and the assumptions we made on the correlation across countries of volatility and growth. In this framework, g_t can be interpreted as the common “jump” component of the model. More generally, we could interpret g_t as ‘exuberance’, ‘bubbles’, ‘panics’, consistent with models in which non-fundamental factors are included in the model to improve empirical asset pricing performance. An alternative way to interpret g_t is to think about the United States as “granular” in global production and consumption, but “dominant” in global financial markets (in the sense that all volatility series must load onto it as discussed by Pesaran and Chudik (2014)). In this case, g_t would simply be the volatility of the dominant market in the world. We remain agnostic on the interpretation of g_t as our main results do not depend on such interpretation. We will return to the issue of possible dominance of the US in global capital markets when we discuss US specific results.

¹¹It should be noted here that the triangular structure of factor loadings in our model does not lead to a triangular (i.e., recursive) structure for the observable variables Δy_{it} and v_{it} .

¹²For instance, a preference shock like the one studied by Albuquerque, Eichenbaum, Luo, and Rebelo (2015).

2.3 Model Estimation When the Factors are not Observed

used to obtain consistent estimates of the unobservable factors, f_t and g_t , for $t = 1, 2, \dots, T$. But for ease of interpretation it is standard to work with the orthogonalized version of the factors. In this regard, our task is simplified due to the triangular way the factors affect the global variables, $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, and the fact that, for sufficiently large N , the other interaction effects across countries vanish. Recall that this outcome does not apply if one country is considered at the time (i.e., when $N = 1$). Denote the estimated orthogonalized factors by $\tilde{f}_t$ and $\tilde{g}_t$, and write them in terms of f_t and g_t as:

$$\begin{aligned}\tilde{f}_t &= \alpha_f \Delta\bar{y}_{\omega,t} + O_p(N^{-\frac{1}{2}}) \\ \tilde{g}_t &= \alpha_{1g} \bar{v}_{\omega,t} - \alpha_{2g} \Delta\bar{y}_{\omega,t} + O_p(N^{-\frac{1}{2}})\end{aligned}$$

and then set $\alpha_g = (\alpha_{1g}, \alpha_{2g})'$, such that $T^{-1} \sum_{t=1}^T \tilde{f}_t \tilde{g}_t = 0$. This yields:

$$\frac{\hat{\alpha}_{2g}}{\hat{\alpha}_{1g}} = \frac{\sum_{t=1}^T \Delta\bar{y}_{\omega,t} \bar{v}_{\omega,t}}{\sum_{t=1}^T \Delta\bar{y}_{\omega,t}^2},$$

which is the OLS estimate of the coefficient of $\Delta\bar{y}_{\omega,t}$ in the regression of $\bar{v}_t$ on $\Delta\bar{y}_{\omega,t}$. The remaining parameters, α_f and α_{1g} are determined by requiring $\tilde{f}_t$ and $\tilde{g}_t$ to have unit in-sample standard deviations:

$$\hat{\alpha}_f^2 = \left(\frac{1}{T^{-1} \sum_{t=1}^T \Delta\bar{y}_{\omega,t}^2} \right)$$

and

$$1 = \hat{\alpha}_{1g}^2 \left(\frac{\sum_{t=1}^T \bar{v}_{\omega,t}^2}{T} \right) + \hat{\alpha}_{2g}^2 \left(\frac{\sum_{t=1}^T \Delta\bar{y}_{\omega,t}^2}{T} \right) - 2\hat{\alpha}_{1g}\hat{\alpha}_{2g} \left(\frac{\sum_{t=1}^T \bar{v}_{\omega,t} \Delta\bar{y}_{\omega,t}}{T} \right)$$

Hence

$$\hat{\alpha}_{1g}^2 = \frac{\left(\frac{\sum_{t=1}^T \Delta\bar{y}_{\omega,t}^2}{T} \right)}{\left(\frac{\sum_{t=1}^T \Delta\bar{y}_{\omega,t}^2}{T} \right) \left(\frac{\sum_{t=1}^T \bar{v}_{\omega,t}^2}{T} \right) - \left(\frac{\sum_{t=1}^T \bar{v}_{\omega,t} \Delta\bar{y}_{\omega,t}}{T} \right)^2}.$$

To ensure that $\tilde{f}_t$ and $\tilde{g}_t$ have zero means, we need to use $\Delta\bar{y}_{\omega,t} - \Delta\bar{y}_{\omega}$ and $\bar{v}_{\omega,t} - \bar{v}_{\omega}$, where $\Delta\bar{y}_{\omega} = T^{-1} \sum_{t=1}^T \Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega} = T^{-1} \sum_{t=1}^T \bar{v}_{\omega,t}$ in the above formula. These mean corrections will be applied automatically if intercepts are included in the country-specific models (20) and (21). In effect $\tilde{g}_t$ is the standardized residual from the OLS regression of $\bar{v}_{\omega,t}$ on $\Delta\bar{y}_{\omega,t}$, and as such measures

the orthogonalized effect of the second factor (financial) once the effect of the first factor (real or technological) is filtered out.

In summary, the key feature of our identification strategy is to exploit the differences in degree of cross-section correlation for the two series of innovations. In the case of Δy_{it} , this correlation is assumed to be weak and hence its effects on the global variable $\Delta \bar{y}_{\omega,t}$ vanish if N is reasonably large. In the case of v_{it} , the cross-country correlation is assumed to be strong and hence $\bar{v}_{\omega,t}$ does not vanish even if the cross section dimension is large. Under these assumptions the matrix of contemporaneous factor loadings is recursive, and observable and orthonormal factors can be estimated from the data simply by means of OLS. Importantly, as we shall see below, this identification scheme is in accordance with the stylized facts that characterize our data as well as existing evidence on the cross country correlations of consumption growth and equity return data—see, for instance, [Tesar \(1995\)](#), [Colacito and Croce \(2011\)](#), and [Lewis and Liu \(2015\)](#).

The identification strategy used here is general and can be applied to any panel of time series with similar cross-section correlation properties. The model could have more factors provided that we consider more cross sections of variables. What is crucial is to adopt a multi-country approach covering a large number of countries, as opposed to one unit at a time, taken in isolation.

3 A Multi-country Dynamic Heterogeneous Two-factor Model

Whilst the static model considered so far is helpful for pedagogical purposes, it is likely to be limited in empirical applications where there are important dynamics, and such dynamics differ across countries, due to institutional and policy differences across countries. It is particularly important to allow for dynamic interactions between volatility and growth to allow for lead or lagged effects that might exist. As we shall see, adding dynamics that differ across countries, while requiring additional regularity assumptions and derivations, does not alter our main results.

Consider the following first-order dynamics version of the static model given by (20) and (21):

$$v_{it} = a_{iv} + \phi_{i,11}v_{i,t-1} + \phi_{i,12}\Delta y_{i,t-1} + \lambda_i f_t + \theta_i g_t + \eta_{it}, \quad (23)$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21}v_{i,t-1} + \phi_{i,22}\Delta y_{i,t-1} + \gamma_i f_t + \varepsilon_{it}. \quad (24)$$

In matrix notation we have:

$$\mathbf{z}_{it} = \mathbf{a}_i + \mathbf{\Phi}_i \mathbf{z}_{i,t-1} + \mathbf{\Gamma}_i \mathbf{f}_t + \boldsymbol{\xi}_{it}, \quad \text{for } i = 1, 2, \dots, N; \quad t = 1, 2, \dots, T, \quad (25)$$

where $\mathbf{z}_{it} = (v_{it}, \Delta y_{it})'$ and

$$\mathbf{a}_i = \begin{pmatrix} a_{iv} \\ a_{iy} \end{pmatrix}, \quad \mathbf{\Phi}_i = \begin{pmatrix} \phi_{i,11} & \phi_{i,12} \\ \phi_{i,21} & \phi_{i,22} \end{pmatrix}, \quad \mathbf{\Gamma}_i = \begin{pmatrix} \lambda_i & \theta_i \\ \gamma_i & 0 \end{pmatrix}, \quad \mathbf{f}_t = \begin{pmatrix} f_t \\ g_t \end{pmatrix}, \quad \boldsymbol{\xi}_{it} = \begin{pmatrix} \eta_{it} \\ \varepsilon_{it} \end{pmatrix}. \quad (26)$$

The matrix $\mathbf{\Gamma}_i$ of contemporaneous factor loadings is triangular because, here, we have already imposed Assumptions 3 and 4. To accommodate the dynamic nature of the model, we now consider the following assumptions:

Assumption 5 (*Innovations*) The country-specific shocks, $\boldsymbol{\xi}_{it}$, are serially uncorrelated (over t), and cross-sectionally weakly correlated (over i), with zero means, positive definite covariance matrices, $\mathbf{\Omega}_i$, for $i = 1, 2, \dots, N$.

Assumption 6 (*Common factors*) The 2×1 vector of unobserved common factors, $\mathbf{f}_t = (f_t, g_t)'$, is covariance stationary with absolute summable autocovariances, and fourth order moments, distributed independently of the country-specific shocks, $\boldsymbol{\xi}_{it'}$, for all i, t and t' .

Assumption 7 (*Factor loadings*) The factor loadings λ_i , θ_i , and γ_i (i.e., the non-zero elements of $\mathbf{\Gamma}_i$) are independently distributed across i , and of the common factors, $\mathbf{f}_t$, for all i and t , with non-zero means λ , θ , and γ , and second-order moments. Further

$$\mathbf{\Gamma} = \mathbb{E}(\mathbf{\Gamma}_i) = \begin{pmatrix} \lambda & \theta \\ \gamma & 0 \end{pmatrix},$$

which is invertible (since $\gamma\theta \neq 0$).

Assumption 8 (*Coefficients*) The constants $\mathbf{a}_i$ are bounded, $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ are independently distributed for all i , the support of $\varrho(\mathbf{\Phi}_i)$ lies strictly inside the unit circle, for $i = 1, 2, \dots, N$, and the inverse of the polynomial $\mathbf{\Lambda}(L) = \sum_{\ell=0}^{\infty} \mathbf{\Lambda}_\ell L^\ell$, where $\mathbf{\Lambda}_\ell = \mathbb{E}(\mathbf{\Phi}_i^\ell)$ exists and has exponentially decaying coefficients, namely $\|\mathbf{\Lambda}_\ell\| \leq K\rho^\ell$, where K is a fixed constant and $0 < \rho < 1$.

Remark 1 *The above assumptions represent straightforward extension of the earlier assumptions made for the static case. The important additional assumption is to control the effects of aggregation of dynamics across the units by requiring that $\Lambda_\ell = E(\Phi_i^\ell)$ exists and has exponentially decaying coefficients. But it is easily seen that this latter assumption holds of if it is further assumed that $\sup_i E \|\Phi_i\| < \rho < 1$, where $\|\Phi_i\|$ is the spectral norm of Φ_i defined by $\varrho_{\max}(\Phi_i' \Phi_i)$.*

We now have the following result for the unobservable factors:

Proposition 3 *.Consider the following factor-augmented bivariate VAR model for country i in the observable 2×1 vector $\mathbf{z}_{it} = (v_{it}, \Delta y_{it})'$*

$$\mathbf{z}_{it} = \mathbf{a}_i + \Phi_i \mathbf{z}_{i,t-1} + \Gamma_i \mathbf{f}_t + \boldsymbol{\xi}_{it}, \quad (27)$$

for $i = 1, 2, \dots, N$, where

$$\mathbf{a}_i = \begin{pmatrix} a_{iv} \\ a_{iy} \end{pmatrix}, \quad \Phi_i = \begin{pmatrix} \phi_{i,11} & \phi_{i,12} \\ \phi_{i,21} & \phi_{i,22} \end{pmatrix}, \quad \Gamma_i = \begin{pmatrix} \lambda_i & \theta_i \\ \gamma_i & 0 \end{pmatrix}, \quad \mathbf{f}_t = \begin{pmatrix} f_t \\ g_t \end{pmatrix}, \quad \boldsymbol{\xi}_{it} = \begin{pmatrix} \eta_{it} \\ \varepsilon_{it} \end{pmatrix}.$$

Suppose that Assumptions 5-8 hold, and $\{\mathbf{z}_{it}, \text{ for } i = 1, 2, \dots, N\}$ have started some time before the start of the observations at $t = 1$. Then

$$f_t = b_f + \gamma^{-1} \Delta \bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty} \mathbf{c}'_{1,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + O_p(N^{-1/2}), \quad (28)$$

$$g_t = b_g + \theta^{-1} \left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma} \Delta \bar{y}_{\omega,t} \right) + \sum_{\ell=1}^{\infty} \mathbf{c}'_{2,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + O_p(N^{-1/2}), \quad (29)$$

where b_f and b_g are fixed constants,

$$\bar{v}_{\omega,t} = \sum_{i=1}^N w_i v_{it}, \quad \Delta \bar{y}_{\omega,t} = \sum_{i=1}^N w_i \Delta y_{it}, \quad \bar{\mathbf{z}}_{\omega,t} = (\bar{v}_{\omega,t}, \Delta \bar{y}_{\omega,t}),$$

$\{w_i, \text{ for } i = 1, 2, \dots, N\}$ are fixed weights that satisfy the granularity conditions under Assumption 2, and $\mathbf{c}'_{1,\ell}$ and $\mathbf{c}'_{2,\ell}$ are the first and the second rows of $\mathbf{C}_\ell = \Gamma^{-1} \mathbf{B}_\ell$, where $\Gamma = \mathbb{E}(\Gamma_i)$, $\mathbf{B}_\ell$ is defined by $\Lambda^{-1}(L) = \mathbf{B}_0 + \mathbf{B}_1 L + \mathbf{B}_2 L^2 + \dots$, $\Lambda(L) = \sum_{\ell=0}^{\infty} \Lambda_\ell L^\ell$, and $\Lambda_\ell = \mathbb{E}(\Phi_i^\ell)$, for all i .

Proof. See Appendix A.1. ■

We note here that $\mathbf{c}'_{1,\ell}$ and $\mathbf{c}'_{2,\ell}$ are the first and second rows of the 2×2 matrix $\mathbf{C}_\ell$ as defined in Appendix A.1. As shown in Pesaran and Chudik (2014) and Chudik and Pesaran (2015), if slope heterogeneity is not extreme (i.e., if these matrices of coefficients do not differ too much across countries) and $\mathbf{C}_\ell$ decays exponentially in ℓ , the infinite order distributed lag functions in $\bar{\mathbf{z}}_{\omega,t}$ in the above propositions can be truncated. In practice, these authors recommend a lag length ℓ equal to $T^{1/3}$, where T is the time dimension of the panel.

As we noted earlier, f_t and g_t are identified up to a 2×2 rotation matrix. To proceed, we impose again the normalization restriction that $\gamma = \theta = 1$. Normalization restrictions are innocuous and do not affect the final estimating equation that identifies the idiosyncratic shocks (which are also of interest). After truncating the infinite distributed lag functions with $p = 1/3$, we have

$$f_t = \Delta \bar{y}_{\omega,t} + \sum_{\ell=1}^p \mathbf{c}'_{1,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + O_p \left(N^{-1/2} \right), \quad (30)$$

$$g_t = \bar{v}_{\omega,t} - \lambda \Delta \bar{y}_{\omega,t} + \sum_{\ell=1}^p \mathbf{c}'_{2,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + O_p \left(N^{-\frac{1}{2}} \right). \quad (31)$$

Substituting these approximations in (25) we obtain the cross-section augmented country-specific VAR models:

$$v_{it} = a_{iv} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + \theta_i \bar{v}_{\omega,t} + \beta_i \Delta \bar{y}_{\omega,t} + \sum_{\ell=1}^p \psi'_{i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \eta_{it} + O_p \left(N^{-\frac{1}{2}} \right), \quad (32)$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + \gamma_i \Delta \bar{y}_{\omega,t} + \gamma_i \sum_{\ell=1}^p \mathbf{c}'_{1,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \varepsilon_{it} + O_p \left(N^{-\frac{1}{2}} \right) \quad (33)$$

where $\beta_i = \lambda_i - \theta_i \lambda$, and $\psi_{i\ell} = (\lambda_i \mathbf{c}_{1,\ell} + \theta_i \mathbf{c}_{2,\ell})$. Note that, as before, only $\Delta \bar{y}_{\omega,t}$ is included in the output growth equation.

The above model can now be estimated consistently by least squares so long as N and T are sufficiently large. This would yield estimated residuals and the factor loadings θ_i , β_i , and γ_i . But, as noted in the static case, coefficients might be difficult to interpret due to the possible non-zero correlation between $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$.¹³ A second complication is that the factors depend on lagged

¹³One could use the orthogonalized components of $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$ using the Cholesky factor. However, such a procedure could invalidate the triangular form of $\mathbf{\Gamma}_i$ applied to the true underlying factors, f_t and g_t . Also focusing on the

variables. It is therefore important to consider an orthonormal transformation that yields orthogonal factors once the effects of past values of $\bar{\mathbf{z}}_{\omega,t}$ are filtered out.

To this end we stack (30) and (31) by T and obtain

$$\mathbf{f} = \Delta \bar{\mathbf{y}}_{\omega} + \bar{\mathbf{Z}}_{\omega} \mathbf{C}_1 + O_p \left(N^{-1/2} \right), \quad (34)$$

$$\mathbf{g} = \bar{\mathbf{v}}_{\omega} - \lambda \Delta \bar{\mathbf{y}}_{\omega} + \bar{\mathbf{Z}}_{\omega} \mathbf{C}_2 + O_p \left(N^{-1/2} \right), \quad (35)$$

where $\Delta \bar{\mathbf{y}}_{\omega} = (\Delta \bar{y}_{\omega,1}, \Delta \bar{y}_{\omega,2}, \dots, \Delta \bar{y}_{\omega,T})'$, $\bar{\mathbf{v}}_{\omega} = (\bar{v}_{\omega,1}, \bar{v}_{\omega,2}, \dots, \bar{v}_{\omega,T})'$, and $\bar{\mathbf{Z}}_{\omega} = (\tau_T, \bar{\mathbf{z}}_{\omega,-1}, \bar{\mathbf{z}}_{\omega,-1}, \dots, \bar{\mathbf{z}}_{\omega,-p})$. Inclusion of τ_T in $\bar{\mathbf{Z}}_{\omega}$ ensures that the filtered factors have mean zeros. We can now establish the following proposition.

Proposition 4 *The orthogonalized filtered factors, $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$, can be recovered from the data as residuals of the the following OLS regressions:*

$$\Delta \bar{\mathbf{y}}_{\omega} = \bar{\mathbf{Z}}_{\omega} \hat{\mathbf{C}}_1 + \tilde{\mathbf{f}} + O_p \left(N^{-1/2} \right), \quad (36)$$

$$\bar{\mathbf{v}}_{\omega} = \lambda \tilde{\mathbf{f}} + \bar{\mathbf{Z}}_{\omega} \hat{\mathbf{C}}_2 + \tilde{\mathbf{g}} + O_p \left(N^{-1/2} \right). \quad (37)$$

Proof. See Appendix A.2. ■

Given the orthogonal factors, $\tilde{f}_t$ and $\tilde{g}_t$, and substituting them in (32)-(33), we can compute their impact and relative importance based on the following regressions:

$$v_{it} = a_{iv} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + \beta_{i,11} \tilde{f}_t + \beta_{i,12} \tilde{g}_t + \sum_{\ell=1}^p \psi'_{v,i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \eta_{it}, \quad (38)$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + \beta_{i,21} \tilde{f}_t + \sum_{\ell=1}^p \psi'_{\Delta y,i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \varepsilon_{it}. \quad (39)$$

Notice here that by construction the OLS estimates of η_{it} and ε_{it} in the above regressions will be numerically identical to those obtained using the non-orthogonalized factors $\Delta \bar{y}_{\omega}$ and $\bar{v}_{\omega}$. It is also important to note that, since $\tilde{f}_t$ and $\tilde{g}_t$ are based on residuals from regressions of f_t and g_t on τ_T and the lagged values $\bar{z}_{\omega,t-1}, \bar{z}_{\omega,t-2}, \dots, \bar{z}_{\omega,t-p}$, then it also follows that $\tilde{f}_t$ and $\tilde{g}_t$ will have zero means

orthogonalized components of $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, ignores the contributions of $\bar{\mathbf{z}}_{\omega,t-\ell}$ for $\ell = 1, 2, \dots, p$ to the estimation of f_t and g_t .

(in-sample) and for a sufficiently large value of p , they are also serially uncorrelated. Therefore, $\tilde{f}_t$ and $\tilde{g}_t$ can be viewed as global innovations (or shocks) to the underlying factors, f_t and g_t . In Appendix B we show how to compute impulse responses and variance decompositions to shocks, $\tilde{f}_t$ and $\tilde{g}_t$, as well as shocks to the country-specific innovations, η_{it} and ε_{it} .

We now turn to the empirical application of the above dynamic model to study the relation between volatility and growth, conditional on the estimated factors $\tilde{f}_t$ and $\tilde{g}_t$. But before doing that we need to discuss how we measure volatility in a multi-country setting.

4 Measurement of Volatility and Output Growth

As a proxy for uncertainty, in our application, we use realized measures of equity price volatility. Realized volatility has been used extensively in the theoretical and empirical finance literature and implicitly assumes that uncertainty and risk can be characterized in terms of probability distributions. It therefore abstracts from the Knightian notion of uncertainty, which refers to the idea that some types of risks cannot as such be characterized. Specifically, we use a measure of quarterly realized volatility based on the summation of daily squared stock price returns.¹⁴ This is a natural application of daily measures of volatility based on high frequency, within day price changes.

Denote the daily equity price of country i , measured at close of day τ in quarter t as $P_{it}(\tau)$. We compute the realized volatility for country i in quarter t as:

$$v_{it} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} (r_{it}(\tau) - \bar{r}_{it})^2} \quad (40)$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$, and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price changes in the quarter t , and D_t is the number of trading days in quarter t . The scaling factor $\sqrt{D_t}$ expresses the realized volatility at quarterly rates: in this way realized volatility is consistent with quarterly GDP growth. Note that, for most time periods, $D_t = 3 \times 22 = 66$, which is larger than the number of data points typically used in the construction of daily realized market volatility in the empirical finance literature. In the case of intra-day observations, for example, prices are usually sampled at 10-minutes intervals

¹⁴See, for example, Andersen et al. (2001, 2003), Barndorff-Nielsen and Shephard (2002, 2004))

which yield around 48 intra-daily returns in an 8-hour trading day.

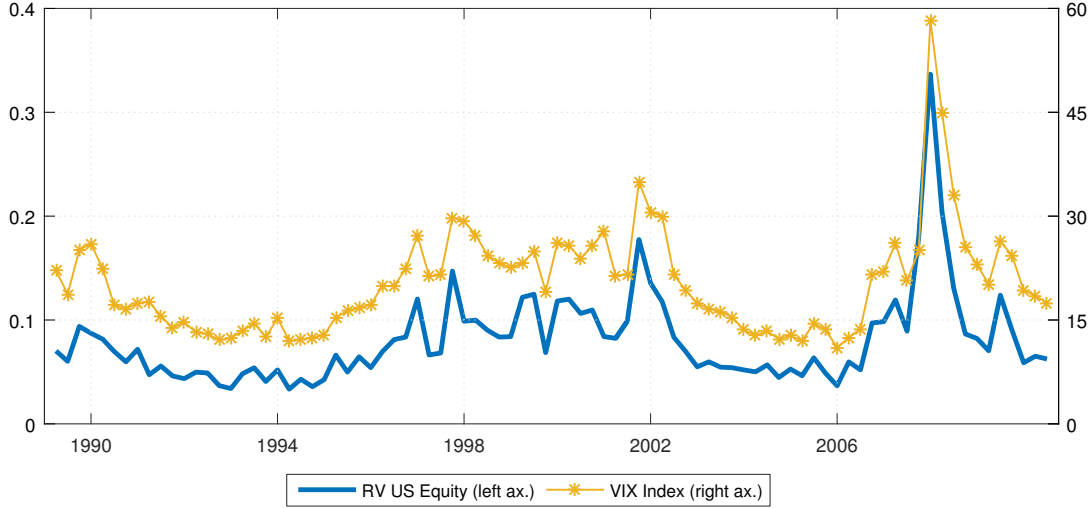
We computed realized volatility also in real terms, with $P_{it}(\tau)$ in the above expression replaced by $P_{it}(\tau)/P_{it}$, where P_{it} is the general price level in country i in quarter t (i.e., the *CPI* index), but it yields the same results, and in our application they are not reported. If we consider a panel of country equities, a different measure of volatility can be computed as the cross-sectional dispersion of equity prices. Cesa-Bianchi et al. (2014) show that realized volatility and cross-sectional dispersion are closely related in our data set. So, in our application, we will focus on realized volatilities.

Realized volatility and cross-sectional dispersion encompass most measures of uncertainty and risk proposed in the literature that would be suitable to implement our identification strategy. Schwert (1989b), Ramey and Ramey (1995), Bloom (2009), Fernandez-Villaverde et al. (2011) use aggregate time series volatility (i.e., summary measures of dispersion over time of output growth, stock market returns, or interest rates); Leahy and Whited (1996), Campbell et al. (2001), Bloom et al. (2007) and Gilchrist et al. (2013) use dispersion measures of firm-level stock market returns; Bloom et al. (2012) use cross-sectional dispersion of plant, firm, and industry profits, stocks, or total factor productivity.

In the finance literature, the focus of the volatility measurement has now shifted to implied volatility measures obtained from option prices, like the US VIX Index (see Figure 1). However, a key input for the implementation of our identification strategy is the availability of country-specific measures of uncertainty for a large number of countries for a long period of time, and implied volatility measures are not yet available for a meaningful number of countries. So, implied volatility is not suitable for our purposes. Figure 3 plots the U.S. realized volatility measure we constructed with the VIX index (as plotted in Figure 1) during the period over which they overlap. The chart shows that the two measures comove very closely with a correlation of about 0.9.

The literature has also used uncertainty measures based on expectation dispersion. While summarizing the range of disagreement among individual forecasters at a point in time, these measures do not give information about the uncertainty surrounding the individual's forecast. See, for instance, Zarnowitz and Lambros (1987), Popescu and Smets (2010), and Bachmann et al. (2013). Finally, model based measures, such as those Jurado et al. (2015) and Ludvigson et al. (2015) could be in principle computed for all countries in our sample, but the requirement of obtaining such measures for many countries over a long period of time, again, would pose tight constraints on their proper

Figure 3 QUARTERLY U.S. EQUITY REALIZED VOLATILITY AND THE VIX INDEX



NOTE. RV US Equity is the U.S. realized volatility measure we constructed in equation (40). The VIX Index is the quarterly average of the daily Chicago Board Options Exchange Market Volatility Index from Bloomberg. The sample period is 1990:Q1-2011:Q2.

implementation.

Defining and measuring a country's level of economic activity over the business cycle is, in principle, also open to debate. Consistent with the theoretical framework presented above, in our application, we shall use real GDP growth (calculated as the first difference of the log-level).

5 Data and Stylized Facts

This section briefly describes the data set we use in the empirical analysis and then reports some stylized facts on the unconditional properties of the data. Specifically, we consider the degree of over time persistence in the growth and volatility series, examine the patterns of cross-country correlations, which play an important role in the identification strategy, and investigate the correlations of country-specific volatility and growth.

The sources of the data and their sampling information are reported in Appendix C. To construct a balanced panel for the largest number of countries for which we have sufficiently long time series, we first collect daily stock prices for 32 advanced and emerging economies from 1979 to 2011. We then cut the beginning of the sample in 1993, as daily equity price data are not available earlier for two large

emerging economies (Brazil and China) and Peru up to that point. Better quality quarterly GDP data for China also became available from 1993. Our results, however, are robust to starting the sample period in 1988 and excluding these three countries. Moreover, some steps of the empirical analysis, like the computation of the factors ($\tilde{f}_t$ and $\tilde{g}_t$), can be implemented with the unbalanced panel from 1979 without any significance consequence for our main results.

5.1 Persistence

A battery of summary statistics on our volatility and real GDP series supports our model specification in terms of log-difference of real GDP and the level of realized volatility. It is well known that GDP levels are non stationary. This is also true in our sample. As Table D.1 in the Appendix shows, the persistence of the log-level of GDP is very high (on average around 1). Moreover, the null of a unit root is not rejected by a standard ADF test for any of the 32 countries in our sample. In contrast, the levels of realized volatility, even though highly persistent, tends to be mean reverting. Table D.2 in the Appendix shows that the first order auto-correlation coefficient for realized volatility is on average about 0.6. But standard ADF tests reject the null hypothesis that the volatility variables have a unit root. We also run a battery of tests for fractional integration, for comparison with the finance literature, and find that volatility series are stationary, and are best modeled in levels.¹⁵

5.2 Cross-country Correlations of Volatility and Growth

The differential pattern of cross-country comovement of the growth and volatility *innovations* is crucial for our identification strategy. Here we are concerned with the properties of the observed time series as in Figure 2. In order to gauge to what extent volatility and growth series comove across countries, we use two techniques: standard principal component analysis and pair-wise cross-country correlation analysis. The average cross-country or pairwise correlation in a panel of series over $i = 1, 2, \dots, N$ measures the degree of comovement of country i with all other $N - 1$ countries. The average for all N countries provides a summary measure of the overall degree of interdependence in the sample of countries under consideration.

¹⁵The stylized facts of volatility, as well as the other results of the paper, are unchanged if instead of the level of realized volatility we consider its logarithm.

Table 1 AVERAGE PAIRWISE CROSS-COUNTRY CORRELATIONS OF VOLATILITY AND GROWTH DATA

Panel A: Realized Volatility (Level)					
Pair. Corr.		Pair. Corr.		Pair. Corr.	
Argentina	0.56	India	0.51	Philippines	0.56
Australia	0.64	Indonesia	0.56	Singapore	0.66
Austria	0.54	Italy	0.63	South Africa	0.62
Belgium	0.59	Japan	0.63	Spain	0.65
Brazil	0.41	Korea	0.56	Sweden	0.66
Canada	0.68	Malaysia	0.38	Switzerland	0.65
Chile	0.55	Mexico	0.61	Thailand	0.49
China	0.61	Netherlands	0.63	Turkey	0.40
Finland	0.39	New Zealand	0.58	United Kingdom	0.67
France	0.65	Norway	0.64	United States	0.67
Germany	0.69	Peru	0.44		

Panel B: Growth (GDP Log-Difference)					
Pair. Corr.		Pair. Corr.		Pair. Corr.	
Argentina	0.14	India	-0.03	Philippines	0.26
Australia	0.13	Indonesia	0.12	Singapore	0.30
Austria	0.27	Italy	0.40	South Africa	0.35
Belgium	0.35	Japan	0.33	Spain	0.31
Brazil	0.26	Korea	0.28	Sweden	0.32
Canada	0.33	Malaysia	0.30	Switzerland	0.25
Chile	0.25	Mexico	0.28	Thailand	0.26
China	0.17	Netherlands	0.34	Turkey	0.18
Finland	0.34	New Zealand	0.22	United Kingdom	0.37
France	0.38	Norway	0.19	United States	0.34
Germany	0.38	Peru	0.24		

NOTE. Panel (A) refers to the realized volatility measures as computed in (40). Panel (B) refers to real GDP growth in log-differences. The overall average pairwise correlations across countries are 0.58 and 0.27 for volatility and growth, respectively, like exhibited in Figure 2.

The average pairwise correlation also relates to the strength and pervasiveness of the factors, as measured by the factor loadings. For example, consider the factor model

$$\Delta y_{it} = \gamma_i f_t + \varepsilon_{it},$$

where $\text{var}(f_t) = 1$ and $\text{var}(\varepsilon_{it}) = \sigma_i^2$. Then the population average pairwise correlation of Δy_{it} and Δy_{jt} , denoted by $\bar{\rho}_N$, is given by

$$\bar{\rho}_N = \frac{\left(\sum_{i=1} \sum_{j=1} \rho_{ij} - N \right)}{2N^2}, \quad (41)$$

where

$$\rho_{ij} = \begin{cases} \frac{\gamma_i \gamma_j}{\sqrt{\gamma_i^2 + \sigma_i^2} \sqrt{\gamma_j^2 + \sigma_j^2}} & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}.$$

Using the above expression for ρ_{ij} in (41), we have

$$\bar{\rho}_N = \frac{1}{2} \left(\frac{1}{N} \sum_{i=1}^N \frac{\tilde{\gamma}_i}{\sqrt{1 + \tilde{\gamma}_i^2}} \right)^2 - \frac{1}{2N},$$

where $\tilde{\gamma}_i = \gamma_i / \sigma_i$. Hence

$$\bar{\rho}_N = O \left(\frac{\sum_{i=1}^N \tilde{\gamma}_i}{N} \right), \quad (42)$$

where $N^{-1} \sum_{i=1}^N \tilde{\gamma}_i$ measures the degree of pervasiveness of the factor.

The attraction of pairwise correlation lies in the fact that it applies to multi-factor processes and, unlike the factor analysis, does not require the factors to be strong. $\bar{\rho}_N$ tends to be a strictly positive number if Δy_{it} contains at least one strong factor, otherwise it tends to zero as $N \rightarrow \infty$. Therefore, non-zero estimates of $\bar{\rho}_N$ are suggestive of strong cross-sectional dependence. Formal tests of cross-sectional dependence based on estimates of $\bar{\rho}_N$ is discussed in [Pesaran \(2015\)](#). For completeness, and to show that our analysis is robust, we also use standard principal component analysis.

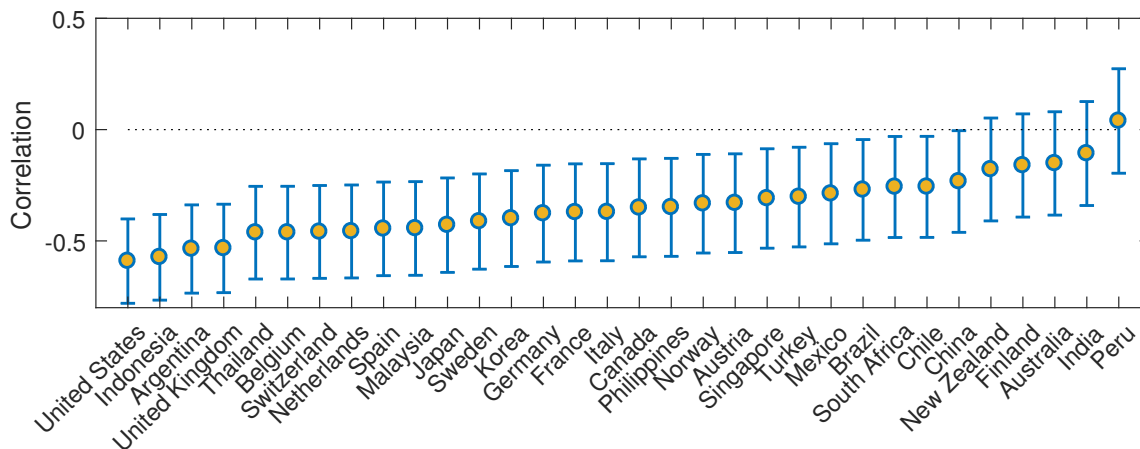
The average pairwise correlations are reported in Table 2 for all countries in our sample. The overall average is 0.58, like in Figure 2. In contrast, the overall average for the growth series is only 0.27. As we can see from Table 2, the pairwise correlations of volatility and growth have a similar values for different countries, but there is a clear difference between the two variables.

The principal component analysis yields similar results. The first principal component in our panel of realized volatility series explains 65 percent of the total variation in the level of v_{it} . Whilst the first principal component of the growth series accounts for only around 18 percent of the total variation. Like in the case of the pairwise correlation analysis, these results are in accordance with the identification assumptions made even though they are not direct evidence on their validity because the assumptions made are related to the innovations and not the realizations.

5.3 Country Correlations between Volatility and Growth

The countercyclical behavior of the U.S. stock market volatility is a well known stylized fact.¹⁶ Does this stylized fact hold in our sample of countries? Figure 4 plots the country-specific contemporaneous correlations between volatility and growth for all countries in our panel together with their 90-percent error band. It shows that, for most countries, there is a strong negative and statistically significant association between volatility and GDP growth. On average, this correlation is about -0.35 , ranging from a maximum slightly above -0.6 for the United States to a minimum of close to zero for Peru. As the confidence intervals illustrates most of these associations are also statistically significant except for New Zealand, Australia, Finland, India and Peru.

Figure 4 COUNTRY CORRELATION BETWEEN VOLATILITY AND GROWTH DATA



NOTE. The correlations are computed over the period 1992:Q1–2011:Q2. The dots represent the country-specific contemporaneous correlations, and the lines represent 95% confidence intervals.

6 Results

The above preliminary analysis supports the evidence of a negative association between output growth and volatilities in a significant majority of the countries, and suggests a stronger degree of cross-country correlation for volatility as compared to cross-country correlations of GDP growth rates. These findings

¹⁶See, for example, Schwert (1989a) and Schwert (1989b). On the volatility of firm-level stock returns see Campbell et al. (2001), Bloom et al. (2007) and Gilchrist et al. (2013); on the volatility of plant, firm, industry and aggregate output and productivity see Bloom et al. (2012) and Bachmann and Bayer (2013); on the behavior of expectations' disagreement see Popescu and Smets (2010) and Bachmann et al. (2013).

are compatible with the two-factor model advanced in the paper, which we now use to address the more difficult question of how to interpret the observed negative association between output growth and volatility.

We begin by first estimating the factors, $\tilde{f}_t$ and $\tilde{g}_t$, using (36) and (37). Armed with an estimate of $\tilde{f}_t$ and $\tilde{g}_t$, we then run OLS on the model (38)-(39) to recover, for each country, the country-specific volatility (η_{it}) and growth innovations (ε_{it}). Next, with these innovations, we compute conditional country and cross-country correlations. Finally we will compute impulse responses and variance decompositions to the factors ($\tilde{f}_t$ and $\tilde{g}_t$) and the country-specific shocks (η_{it} and ε_{it}).

6.1 Estimated Factors

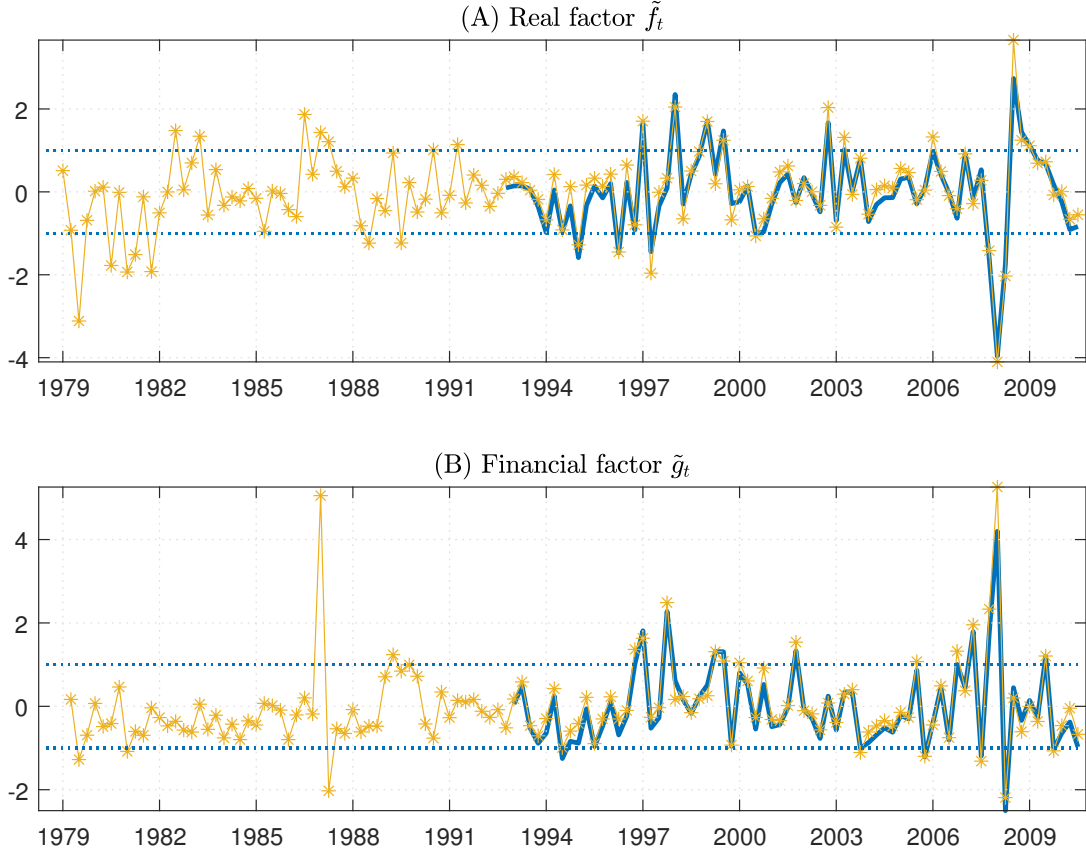
The factors $\tilde{f}_t$ and $\tilde{g}_t$ are recovered from the OLS estimation of (36) and (37). Figure 5 plots them when estimated using the unbalanced panel from 1979 (thin lines with asterisks), and when we use the balanced panel from 1993 (thick solid lines), so as to better illustrate their characteristics and time profile. A one-standard deviation band for the factors estimated on the 1993 sample are also reported.¹⁷ The factors are standardized and have zero mean and unit in sample variances. By construction, they also are serially uncorrelated and orthogonal to each other.

The figure shows that the largest swing in the real factor, $\tilde{f}_t$, was during time of the Lehman's collapse in 2008 and in correspondence to the second oil shock in 1979. Also note that after a sharp drop, $\tilde{f}_t$ rebounds in 2009 and displays a very unusual protracted period of decline associated with normal fluctuations in the financial factor. The largest movements in the financial factor, $\tilde{g}_t$, accords with the 1987 stock market crash and the 2008 Lehman's collapse. Our financial factor comoves positively, but not highly, with US measures of financial volatility, risk and investor sentiment. In general, it is more volatile and less persistent than these US measures. The correlation with the US Baa-Treasury credit spread and excess bond premium of Gilchrist and Zakrajsek (2012) is 0.2 and 0.35, respectively.¹⁸ The correlation with the index of financial volatility of Ludvigson et al. (2015) is about 0.4. The correlation with the US VIX index, however, is 0.5.

¹⁷As countries are progressively added over time, the two estimates get closer and closer.

¹⁸The Baa-Treasury credit spread is computed as the spread between yields on long-term Baa-rated industrial bonds and comparable maturity Treasury securities.

Figure 5 ESTIMATED REAL ($\tilde{f}$) AND FINANCIAL ($\tilde{g}$) FACTORS



NOTE. The factors ($\tilde{f}$ and $\tilde{g}$) are computed using (36) and 37) with one lag of $\mathbf{z}_{it}$, using the full unbalanced sample (thin lines with asterisks) and the shorter balanced sample (think solid lines). Both factors are orthonormalized. The dotted lines represent one-standard deviations bands around the mean.

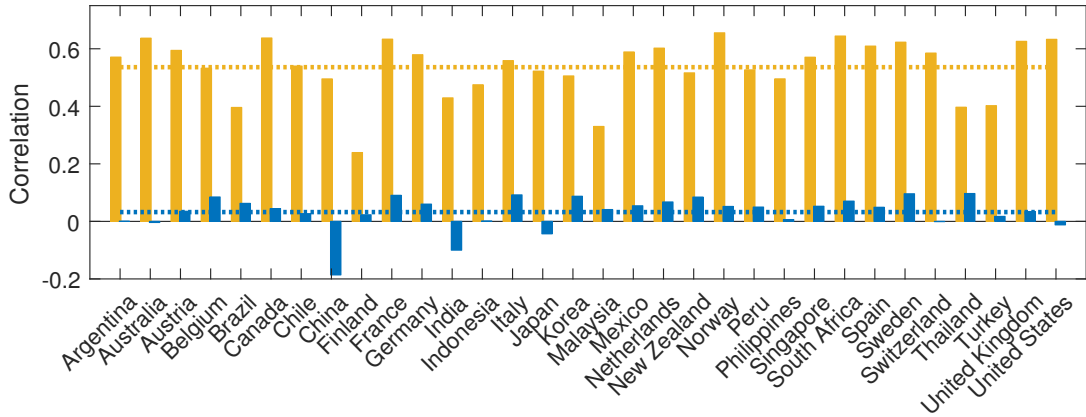
6.2 Cross-country Correlations of Country-specific Innovations

Although restrictions behind identification cannot be formally tested, our multi-country approach to the problem permits us to investigate the extent to which the implications of the identified model are in line with the identification scheme that we have adopted.¹⁹ To this end we explore the cross-country correlation of the residuals from the dynamic regressions (38) and (39), with and without explicit conditioning on the financial factor $\tilde{g}_t$. As we do not impose our identification restrictions directly on the cross-country correlation structure of these innovations, their empirical properties are informative as to whether our identification assumptions might be inconsistent with the results obtained.

¹⁹Note that without our identification restrictions $\tilde{f}_t$ and $\tilde{g}_t$ cannot be estimated. As a result, whilst we can directly estimate pairwise correlations of cross-country output growth and volatility series, we can not examine cross-country pairwise correlations of their innovations without the imposition of our identification conditions.

Figure 6 plots the average pairwise correlation of the volatility innovations ($\hat{u}_{it}$) and the growth innovations ($\hat{\varepsilon}_{it}$), when we condition only on $\tilde{f}_t$ in (38)-(39), rather than on both $\tilde{f}_t$ and $\tilde{g}_t$. The statistics reported are the same as in Figure 2. The blue (darker) bars are the pairwise correlations of the growth innovations. The yellow (lighter) bars are for the volatility innovations. The figure shows that, if we condition only on $\tilde{f}$ in (38)-(39), the volatility innovations display a pairwise correlation comparable to that of the raw data reported in Figure 2, of more than 0.5 on average. In contrast, the pairwise correlations of the growth innovations are negligible, with an overall average of 0.03.²⁰ Figure 7 reports the same statistics for the volatility innovations ($\hat{\eta}_{it}$) when we condition on both $\tilde{f}$ and $\tilde{g}$ in model (38)-(39).²¹ The Figure shows that the cross-country correlations of the volatility innovations now becomes negligible, as in the case of the growth innovations, with the same overall average of 0.03. For instance, in the case of the United States, the pairwise correlation of volatility innovations exceeds 0.6 conditional on f_t alone. But it drops to 0.03 if we condition on both factors. By comparison the US pairwise correlation of the growth innovations is -0.01 .

Figure 6 CROSS-COUNTRY CORRELATION OF VOLATILITY AND GROWTH INNOVATIONS CONDITIONAL ONLY ON $\tilde{f}$



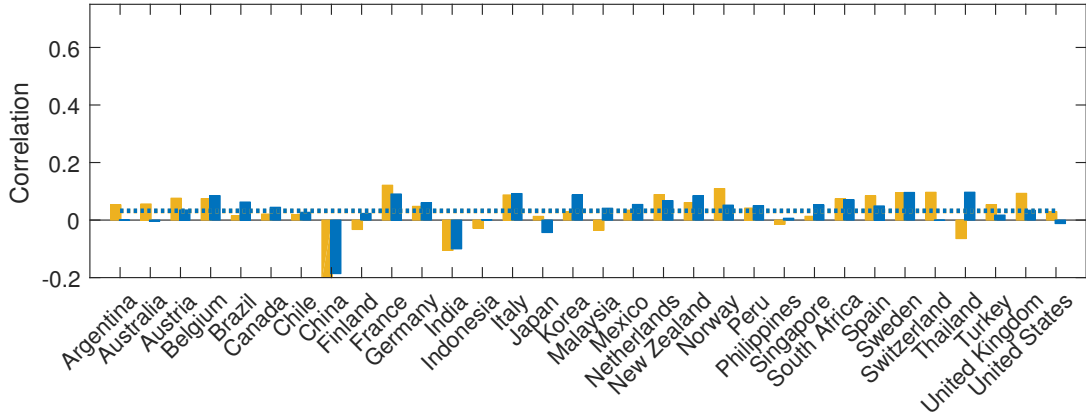
NOTE. Average cross-country pairwise correlation of volatility (yellow, lighter bars) and GDP growth (blue, darker bars) innovations conditional on $\tilde{f}$ only. The volatility measures are computed as in (40). The dotted lines are the overall averages, at 0.53 and 0.03 for volatility and growth, respectively. Country values are also reported in the online appendix in Table D.3.

Thus, Figures 6 and 7 illustrate that after conditioning on f_t alone—which is common to both growth and volatility series—not much commonality is left in the growth innovations, but the volatility innovations continue to share a strong common component. It is therefore interesting to test whether

²⁰Notable exceptions are those of China and India.

²¹Notice that the growth innovations are identical and are plotted again only for comparison.

Figure 7 CROSS-COUNTRY CORRELATION OF VOLATILITY AND GDP GROWTH INNOVATIONS CONDITIONAL ON $\tilde{f}$ AND $\tilde{g}$



NOTE. Average cross-country pairwise correlation of volatility (light bars) and GDP growth (dark bars) innovations conditional on $\tilde{f}$ and $\tilde{g}$. The volatility measures are computed as in (40). The dotted lines corresponds to the average pairwise correlations across countries, at 0.03 and 0.03 for volatility and GDP growth, respectively. Country values are also reported in the online appendix in Table D.4.

the two set of innovations satisfy a formal definition of weak and strong dependence as we have assumed for identification purposes.

To test statistically for weak and strong cross-section dependence we estimate the cross-sectional dependence statistic of Pesaran (2015) and the exponent of cross sectional dependence (α) as proposed by in Bailey et al. (2016). The CD statistic is normally distributed with zero mean and unit variance under the null of zero pairwise correlations. So, the critical value is around 2. When the null is rejected, Bailey et al. (2016) suggest estimating the strength of the cross-correlations dependence with an exponent denoted, α , with values in the range $(1/2, 1]$, with unity giving the maximum degree of cross sectional dependence, the lowest level being around $1/2$ and any number below 1, but significantly different from 1, suggesting weak dependence.²² So in what follows we present estimates of α for different series, together with their confidence intervals.

The results are summarized in Table 2 and are in accordance with the identification assumptions made. The table reports also test statistics for realized growth and volatility series the raw data for comparison. The CD statistic for the growth series (Δy_{it}) is 29.64 while the α exponent estimated at 1.00, with 99% confidence. The CD statistic for realized volatility data (v_{it}) is 53.95 with an

²²When estimating α one also need to take account of the sampling uncertainty which depends on the relative magnitude of N and T , and the null of weak cross dependence depends on the relative rates of N and T .

Table 2 TESTING FOR CROSS-SECTIONAL DEPENDENCE AND THE EXPONENT OF CROSS-SECTIONAL DEPENDENCE

	CD	$\hat{\alpha}_{0.05}$	$\hat{\alpha}$	$\hat{\alpha}_{0.95}$
v_{it}	53.95	0.94	1.00	1.06
Δy_{it}	29.64	0.82	1.00	1.17
u_{it}	49.76	0.92	0.99	1.06
ϵ_{it}	5.40	0.73	0.79	0.85
η_{it}	1.09	0.50	0.59	0.68
ϵ_{it}	5.40	0.73	0.79	0.85

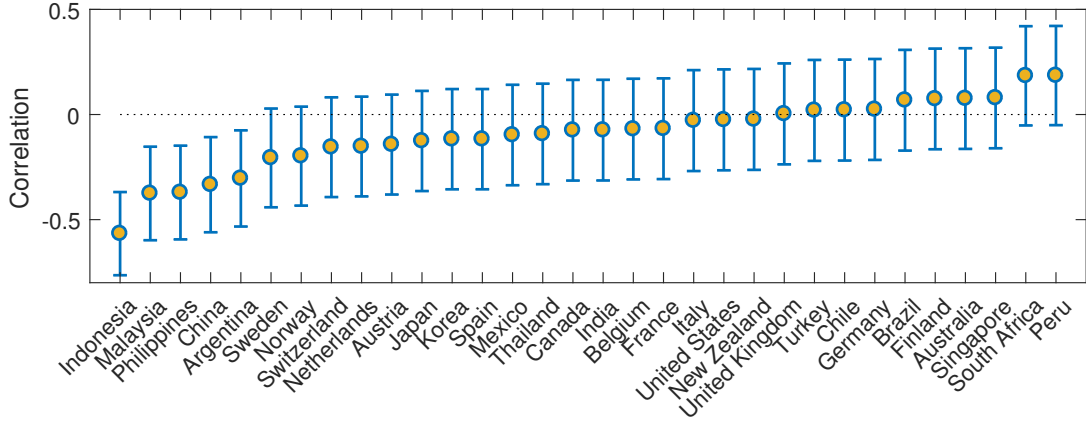
NOTE. CD is the cross-sectional dependence statistic of [Pesaran \(2015\)](#). α is the exponent of cross-sectional dependence as in [Bailey et al. \(2016\)](#), together with its 90-percent confidence interval $[\alpha_{0.05}, \alpha_{0.95}]$.

estimated α also at 1.00 with 99% confidence. Conditioning only on $\tilde{f}_t$, the CD statistic for the growth innovations (ϵ_{it}) drops to 5.40, close to its critical value under zero pairwise correlations, with an α value of 0.79, which is statistically different from 1. In sharp contrast, the CD statistic for the volatility innovations (u_{it}) remains close to that of the data at 49.76 with an α parameter clearly not distinguishable from one. However, when we condition on both $\tilde{f}_t$ and $\tilde{g}_t$, the CD statistic for the volatility innovations (η_{it}) falls to 1.09 with an estimated α of 0.59 and confidence interval of $[0.50, 0.68]$, while the CD statistic and α are the same for the growth innovations (ϵ_{it}).

6.3 Country Correlations Between Volatility and GDP Growth Innovations

We saw earlier in [Figure 4](#) that the volatility and growth series display a strong (and statistically significant) negative correlation at the country level. How much of that association is accounted for by the common factors $\tilde{f}_t$ and $\tilde{g}_t$? To answer this question, we compute the contemporaneous correlations between the volatility and growth innovations ($\hat{\eta}_{it}$ and $\hat{\epsilon}_{it}$) conditional on $\tilde{f}_t$ and $\tilde{g}_t$. The results are displayed in [Figure 8](#). The figure reports the estimated correlation together with a 95 percent confidence interval for all countries in our sample. The conditional correlation between volatility and growth innovations falls substantially when conditioned on both $\tilde{f}$ and $\tilde{g}$, and it is much weaker than the negative correlation in the underlying series. The confidence interval now includes zero for all but 5 countries that are very volatile emerging market economies. For the United States, for instance, this correlation essentially vanishes.

Figure 8 COUNTRY CORRELATION BETWEEN VOLATILITY AND GROWTH INNOVATIONS (CONDITIONAL ON $\tilde{f}_t$ AND $\tilde{g}_t$)



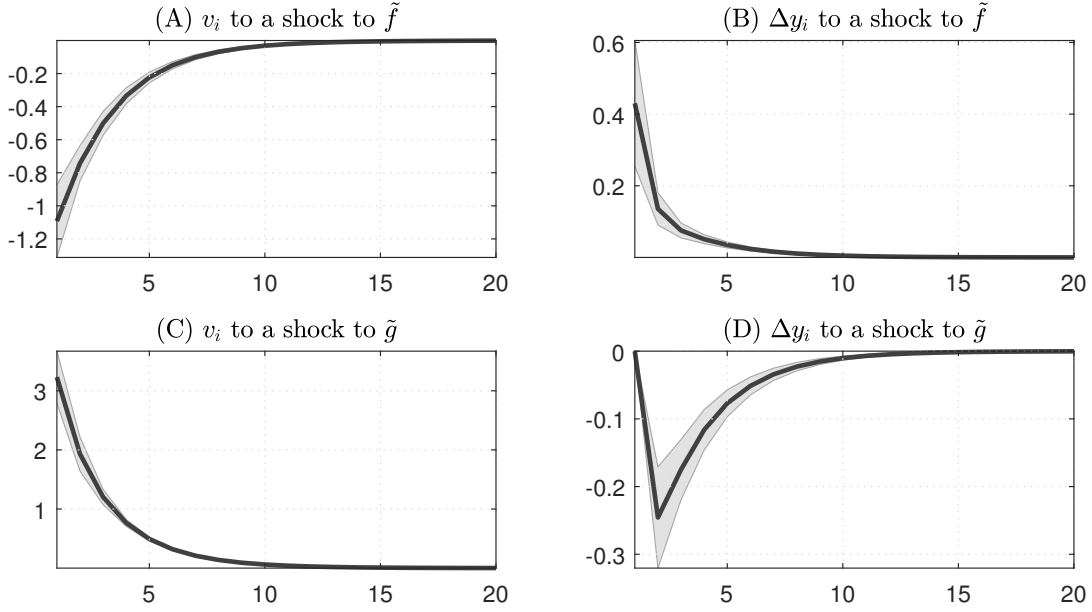
NOTE. Volatility innovations (η) are the residuals of equation (38) growth innovations (ε) are the residuals of equation (39). The dots represent the country-specific contemporaneous correlations, and the lines represent 95% confidence intervals.

6.4 Impulse Responses

A number of different impulse responses can be considered. Here we focus on the effects of positive unit shocks (one standard error) to the orthogonalized factors $\tilde{f}_t$ and $\tilde{g}_t$. The derivation of the impulse response functions associated with these shocks for a given country i is provided in Appendix B. Figure 9 reports a weighted average of the country-specific impulse responses (solid line) using PPP-GDP weights together with 2-standard deviations error bands (shaded areas). The error band gives the dispersion of the impulse responses across countries. The country-specific response are reported in Figure 10.

Panel (A) of Figure 9 plots the average volatility response of to a positive, one-standard-deviation shock to the common real factor $\tilde{f}_t$. The figure shows that a positive shock to the real factor increases growth and lowers volatility. This reflects an endogenous response of volatility to fundamental improvements in the world economy and it is consistent with volatility increasing when growth tanked during the global financial crisis in 2008. Note that the error bands around the typical response are very tight, reflecting the relatively homogeneous country responses to unexpected changes in $\tilde{f}_t$. We can see in fact from Figure 10 that for most countries responses have a very similar shape. The average response is sizable: a one standard error change in $\tilde{f}_t$ increases volatility by one percent per quarter on impact.

Figure 9 TYPICAL VOLATILITY AND GROWTH IMPULSE RESPONSES TO THE COMMON FACTORS



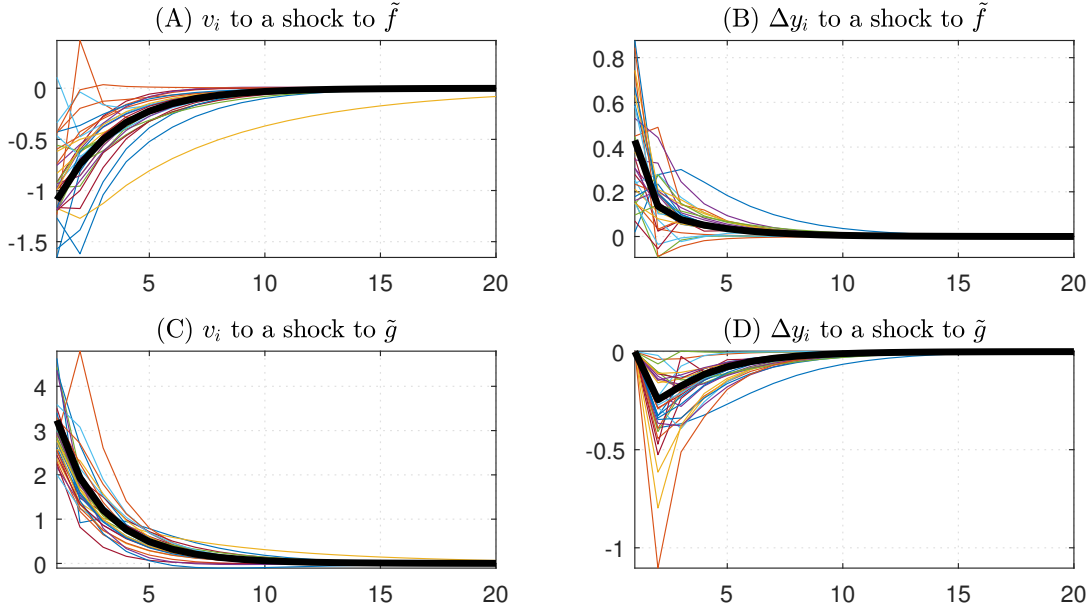
NOTE. One-Standard Deviation shock to $\tilde{f}_t$ and $\tilde{g}_t$. The solid line is the PPP-GDP weighted average of the country-specific responses, computed as in Appendix B in Eq. (B.12). The shaded areas are the 2 standard deviations confidence intervals computed as in Appendix B in Eq. (B.13). In percent.

Panel (B) plots the growth response to the same shock. It shows that, on average, growth loads positively on the fundamental factor $\tilde{f}_t$, with persistent effects up to 8-10 quarters. The size of the impact is smaller, with growth increasing only by about half a percentage point with a one standard error change in $\tilde{f}$. This is consistent with existing evidence on the international business cycle, which stresses an important role for the world factor, along with regional and country specific factors in driving the business cycle (e.g., [Kose et al. \(2003\)](#)).

Panels (C) and (D) of Figure 9 report the responses of volatility and growth to a positive, one standard deviation shock to the financial factor $\tilde{g}_t$. The responses suggest that a positive shock to $\tilde{g}_t$ is “bad news”, as volatility increases and growth slows. With a normal change in the financial factor, volatility increases three times as much as a change in the real factor. While growth declines only by about a quarter of percentage point compared to half a percentage point in response to a normal change in the real factor shock.

Note here that, the delayed response of growth to this shock follows from our identification assumptions through cross-country correlations, but it is not imposed directly on country-specific models

Figure 10 COUNTRY VOLATILITY AND GROWTH IMPULSE RESPONSES TO COMMON SHOCKS



NOTE. One-Standard Deviation shock to $\tilde{f}_t$ and $\tilde{g}_t$. Thin lines are individual countries responses. The solid line is the PPP-GDP Weighted average. Impulse responses computed as in Appendix B.

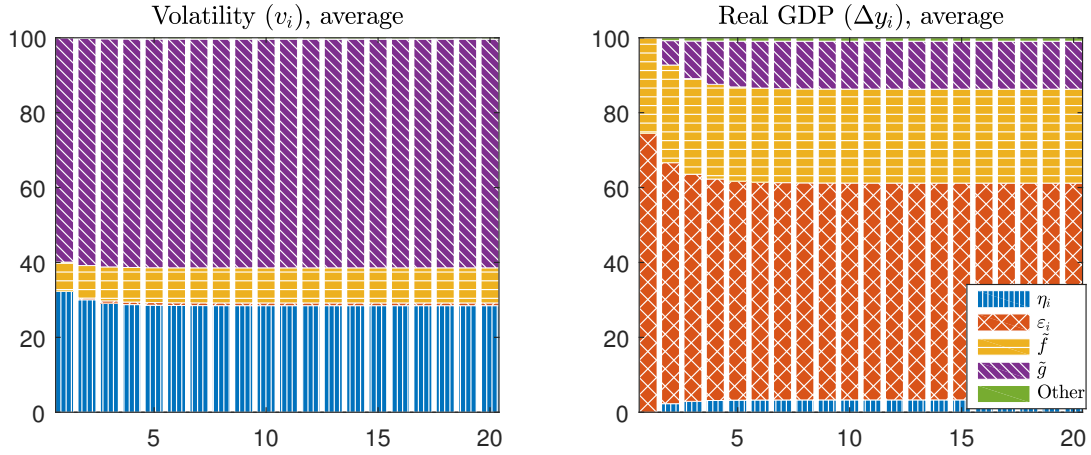
(unlike, for instance, in a Cholesky decomposition of the variance-covariance matrix with growth ordered first in a country VAR system). It is a consequence of the triangular structure of the factor loading matrix, which in turn derives from the cross-section correlation properties of the shocks. This is consistent with the idea that a more volatile environment takes some time to affect economic decisions, as it is often assumed in theoretical models of habit persistence in consumption and cost of adjusting investment. Notice also that the error bands of this impulse response are wider, reflecting larger cross-country differences in the incidence of the financial factor (see Figure 10).

6.5 Variance Decompositions

While impulse responses illustrate the transmission mechanism of a given shock identified in the model, variance decompositions speak to the importance of such a shock for the time-variation of the endogenous variables at different time horizons, relative to other shocks in the model. For all countries $i = 1, 2, \dots, N$, we decompose the total variance of volatility and growth and attribute it to shocks to the factors $\tilde{f}_t$ and $\tilde{g}_t$ as well as all country-specific shocks in the model $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$. Below we discuss an

”average” variance decomposition, weighting country-specific decompositions with PPP-GDP weights. Country specific results, are provided in the on-line Supplement.²³

Figure 11 AVERAGE VARIANCE DECOMPOSITION



NOTE. PPP-GDP weighted average of country variance decompositions. Country variance decompositions are reported in Figure D.2 and D.1. For both growth and volatility the figure plots the weighted average of the variance decompositions with respect to all the shocks in the model. ”Other” is an the sum of variance share of all other country-specific shocks other than the own country specific shock that is tracked separately. Variance decompositions computed as in Appendix B.

For the purpose of computing these decomposition, the country-specific innovations are assumed to be orthogonal not only to the factors, but also across countries $i = 1, 2, \dots, N$. The factors $\tilde{f}_t$ and $\tilde{g}_t$ are orthogonal to the country specific shocks and to each other by construction. The strong assumption that the country-specific shocks are also orthogonal to each other and across countries is justified by the empirical results reported in Figure 7 and 8. That evidence shows that most of the contemporaneous correlations between volatility and growth innovations, within or between countries, is absorbed by the common factors $\tilde{f}_t$ and $\tilde{g}_t$. For all but two emerging market countries, the cross-country correlations of volatility and growth were negligible (7), and even in the case of China and India they were below 0.2. Similarly, the conditional country correlation between volatility and growth was negligible in all but 5 emerging market countries (including China and 3 other Asian economies) (8).

Thus, in order to compute this variance decomposition, as a first approximation, we assume that the variance covariance matrix of the country-specific innovations is diagonal for the whole set of countries N . We then check the robustness of the decomposition by allowing for non-zero correlation

²³See Appendix B for the details on the derivations.

among all country specific innovations in the model. This is done by recomputing the variance error decompositions using the generalized method of [Pesaran and Shin \(1998\)](#) that allows for non-zero correlations and then comparing results. When we do this, we find that the results are essentially the same.

Consider first the variance decomposition of volatility averaged across countries reported on the left hand panel of Figure 11. The figure shows that volatility is driven largely by shocks to the financial factor (darkest, purple-shaded area) and country specific volatility shocks (lighter, blue-shaded area). Combined, these two shocks explain about 90 percent of the total variance of realized volatility over time, broken down in 60 percent for global shocks, and 30 percent for the country-specific volatility shock. Shocks to the real common factor (lightest-shaded, yellow area) explain only about 10 percent of the total variance of volatility. The “other” country-specific shocks, i.e., the country-specific growth shock as well as the country-specific volatility and growth shocks of all other countries in the sample, play essentially no role. Notice this does not follow mechanically from the assumptions that country specific shocks are orthogonal to each other and across countries. The results show that volatility is largely driven by shocks to the global financial factor and its own country specific shock. In short, according to our results, the endogenous component of volatility via common or country specific shocks is relatively small.

Consider now the average variance decomposition for growth reported in the left hand panel of Figure 11. The figure shows that, on average, growth variance is driven mostly by country-specific growth shocks and the common real factor, with a share combined share approaching 90 percent in the long run –lightest (or yellow) and darker (or orange) shaded areas. The country-specific growth shocks explains more than 60 percent of the total variance in the long-run, while the global real factor explains slightly less than 30 percent of the total growth variance for the average in our sample, in line with existing results in the international business cycle literature (see, for instance, [Kose et al. \(2003\)](#)). Shocks to the common financial factor explain only slightly more than 10 percent of country-specific growth variance on average. Their importance for growth variance increases quickly over time, but stabilizes within a year or so. Country specific volatility shocks explain only a very small share of the total variance, with a weight of about 1-2 percent, while all other country-specific shocks in the model combined explain a negligible share of variance.

7 Conclusions

The global financial crisis spurred renewed academic interest in quantifying the effects of uncertainty on macroeconomic dynamics. In this paper, we study the interrelation between realized equity price volatility and GDP growth without imposing restrictions on the direction of causation. To do so we assume volatility and growth are driven by two common factors. By taking a multi-country approach, as opposed to studying a single economy in isolation, we can identify and estimate these two factors, as well as country-specific volatility and growth shocks. Identification exploits different patterns of correlation across countries of volatility and GDP growth. The identification strategy that we propose can be applied in other contexts. Evidence based on the cross-country correlations of the raw data and estimated innovations is in strong accordance with the assumptions made to achieve identification.

Empirically, we find that shocks to the real and financial factors account for most of the unconditional correlation between volatility and growth in all but very few emerging market countries. But shocks to the real factor explain a small share of volatility variance, even though they do have a negative and statistically significant impact on growth when they realize, as shocks to the financial factor do for the variance of growth. Shocks to the financial factor are the most important drivers of the variance of country volatilities, while the country-specific growth shock accounts for the bulk of country growth variance. The results also show that, conditional on the real and financial factor, country specific shocks have negligible spillover effect because they explain no share of country volatility or growth variance.

These results imply that the endogenous component of country volatility is small and driven largely by global factors, that uncertainty is most important for the business cycle when the shocks are global in nature, and that financial and business cycle fluctuations share only a relatively small common component.

References

- ANDERSEN, T. G., T. BOLLERSLEV, F. X. DIEBOLD, AND P. LABYS (2001): “The Distribution of Realized Exchange Rate Volatility,” *Journal of the American Statistical Association*, 96, 42–55.
- (2003): “Modeling and Forecasting Realized Volatility,” *Econometrica*, 71, 579–625.
- ARELLANO, C., Y. BAI, AND P. KEHOE (2012): “Financial Markets and Fluctuations in Uncertainty,” Unpublished manuscript.
- BACHMANN, R. AND C. BAYER (2013): “‘Wait-and-See’ business cycles?” *Journal of Monetary Economics*, 60, 704–719.
- BACHMANN, R., S. ELSTNER, AND E. R. SIMS (2013): “Uncertainty and Economic Activity: Evidence from Business Survey Data,” *American Economic Journal: Macroeconomics*, 5, 217–249.
- BAI, J. AND S. NG (2002): “Determining the Number of Factors in Approximate Factor Models,” *Econometrica*, 70, 191–221.
- BAILEY, N., G. KAPETANIOS, AND M. H. PESARAN (2016): “Exponent of Cross-Sectional Dependence: Estimation and Inference,” *Journal of Applied Econometrics*, 31, 929–960.
- BAKER, S. R. AND N. BLOOM (2013): “Does Uncertainty Reduce Growth? Using Disasters as Natural Experiments,” NBER Working Papers 19475, National Bureau of Economic Research, Inc.
- BARNDORFF-NIELSEN, O. E. AND N. SHEPHARD (2002): “Estimating quadratic variation using realized variance,” *Journal of Applied Econometrics*, 17, 457–477.
- (2004): “Econometric Analysis of Realized Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics,” *Econometrica*, 72, 885–925.
- BASU, S. AND B. BUNDICK (2017): “Uncertainty Shocks in a Model of Effective Demand,” *Econometrica*, 85, 937–958.
- BERNANKE, B. S. (1983): “Irreversibility, Uncertainty, and Cyclical Investment,” *The Quarterly Journal of Economics*, 98, 85–106.
- BLOOM, N. (2009): “The Impact of Uncertainty Shocks,” *Econometrica*, 77, 623–685.
- (2014): “Fluctuations in Uncertainty,” *Journal of Economic Perspectives*, 28, 153–176.
- BLOOM, N., S. BOND, AND J. V. REENEN (2007): “Uncertainty and Investment Dynamics,” *Review of Economic Studies*, 74, 391–415.
- BLOOM, N., M. FLOETOTTO, N. JAIMOVICH, I. SAPORTA-EKSTEN, AND S. J. TERRY (2012): “Really Uncertain Business Cycles,” NBER Working Papers 18245, National Bureau of Economic Research, Inc.
- BORN, B. AND J. PFEIFER (2014): “Policy risk and the business cycle,” *Journal of Monetary Economics*, 68, 68–85.
- CAMPBELL, J. Y., M. LETTAU, B. G. MALKIEL, AND Y. XU (2001): “Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk,” *Journal of Finance*, 56, 1–43.

- CARRIERE-SWALLOW, Y. AND L. F. CESPEDES (2013): “The impact of uncertainty shocks in emerging economies,” *Forthcoming Journal of international Economics*.
- CESA-BIANCHI, A., M. H. PESARAN, AND A. REBUCCI (2014): “Uncertainty and Economic Activity: A Global Perspective,” *Tech. rep.*
- CHRISTIANO, L., R. MOTTO, AND M. ROSTAGNO (2014): “Risk Shocks,” *American Economic Review*, 104, 27–65.
- CHUDIK, A. AND M. H. PESARAN (2013): “Econometric Analysis of High Dimensional VARs Featuring a Dominant Unit,” *Econometric Reviews*, 32, 592–649.
- (2015): “Common correlated effects estimation of heterogeneous dynamic panel data models with weakly exogenous regressors,” *Journal of Econometrics*, 188, 393–420.
- CHUDIK, A., M. H. PESARAN, AND E. TOSETTI (2011): “Weak and strong cross-section dependence and estimation of large panels,” *Econometrics Journal*, 14, C45–C90.
- CLARK, T. E., A. CARRIERO, AND M. MASSIMILIANO (2016): “Measuring Uncertainty and Its Impact on the Economy,” *Working Paper 1622*, Federal Reserve Bank of Cleveland.
- COLACITO, R. AND M. M. CROCE (2011): “Risks for the Long Run and the Real Exchange Rate,” *Journal of Political Economy*, 119, 153–181.
- DIXIT, A. K. AND R. S. PINDYCK (1994): *Investment under Uncertainty*, Princeton University Press.
- FERNANDEZ-VILLAYERDE, J., P. GUERRON-QUINTANA, J. F. RUBIO-RAMIREZ, AND M. URIBE (2011): “Risk Matters: The Real Effects of Volatility Shocks,” *American Economic Review*, 101, 2530–61.
- FOSTEL, A. AND J. GEANAKOPOLOS (2012): “Why Does Bad News Increase Volatility and Decrease Leverage?” *Journal of Economic Theory*, 147, 501–525.
- GIGLIO, S., I. DEW-BECKER, AND D. BERGER (2016): “Contractionary Volatility or Volatile Contractions?” *Tech. rep.*
- GILCHRIST, S., J. SIM, AND E. ZAKRAJSEK (2013): “Uncertainty, Financial Frictions, and Irreversible Investment,” *Unpublished manuscript*.
- GILCHRIST, S. AND E. ZAKRAJSEK (2012): “Credit Spreads and Business Cycle Fluctuations,” *American Economic Review*, 102, 1692–1720.
- HIRATA, H., M. A. KOSE, C. OTROK, AND M. E. TERRONES (2012): “Global House Price Fluctuations: Synchronization and Determinants,” in *NBER International Seminar on Macroeconomics 2012*, National Bureau of Economic Research, Inc, NBER Chapters.
- ILUT, C., M. KEHRIG, AND M. SCHNEIDER (2017): “Slow to Hire, Quick to Fire: Employment Dynamics with Asymmetric Responses to News,” .
- JURADO, K., S. C. LUDVIGSON, AND S. NG (2015): “Measuring Uncertainty,” *American Economic Review*, 105, 1177–1216.
- KIMBALL, M. S. (1990): “Precautionary Saving in the Small and in the Large,” *Econometrica*, 58, 53–73.

- KOSE, M. A., C. OTROK, AND C. H. WHITEMAN (2003): “International Business Cycles: World, Region, and Country-Specific Factors,” *American Economic Review*, 93, 1216–1239.
- LEAHY, J. V. AND T. M. WHITED (1996): “The Effect of Uncertainty on Investment: Some Stylized Facts,” *Journal of Money, Credit and Banking*, 28, 64–83.
- LEWIS, K. K. AND E. X. LIU (2015): “Evaluating international consumption risk sharing gains: An asset return view,” *Journal of Monetary Economics*, 71, 84–98.
- (2017): “Disaster Risk and Asset Returns : An International Perspective,” Forthcoming in the *Journal of International Economics*.
- LUDVIGSON, S. C., S. MA, AND S. NG (2015): “Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?” NBER Working Papers 21803, National Bureau of Economic Research, Inc.
- NAKAMURA, E., D. SERGEYEV, AND J. STEINSSON (2017): “Growth-Rate and Uncertainty Shocks in Consumption: Cross-Country Evidence,” *American Economic Journal: Macroeconomics*, 9, 1–39.
- PESARAN, H. H. AND Y. SHIN (1998): “Generalized impulse response analysis in linear multivariate models,” *Economics Letters*, 58, 17–29.
- PESARAN, M. H. (2015): “Testing Weak Cross-Sectional Dependence in Large Panels,” *Econometric Reviews*, 34, 1089–1117.
- PESARAN, M. H. AND A. CHUDI (2014): “Aggregation in large dynamic panels,” *Journal of Econometrics*, 178, 273–285.
- POPESCU, A. AND F. R. SMETS (2010): “Uncertainty, Risk-taking, and the Business Cycle in Germany,” *CESifo Economic Studies*, 56, 596–626.
- RAMEY, G. AND V. A. RAMEY (1995): “Cross-Country Evidence on the Link between Volatility and Growth,” *American Economic Review*, 85, 1138–51.
- SCHWERT, G. W. (1989a): “Business cycles, financial crises, and stock volatility,” *Carnegie-Rochester Conference Series on Public Policy*, 31, 83–125.
- (1989b): “Why Does Stock Market Volatility Change over Time?” *Journal of Finance*, 44, 1115–53.
- TESAR, L. L. (1995): “Evaluating the gains from international risk sharing,” *Carnegie-Rochester Conference Series on Public Policy*, 42, 95–143.
- VAN NIEUWERBURGH, S. AND L. VELDKAMP (2006): “Learning asymmetries in real business cycles,” *Journal of Monetary Economics*, 53, 753–772.
- ZARNOWITZ, V. AND L. A. LAMBROS (1987): “Consensus and Uncertainty in Economic Prediction,” *Journal of Political Economy*, 95, 591–621.

A Appendix: Mathematical Derivations and Data Sources

A.1 Proof of Proposition 3 (Consistent estimation of unobservable factors in heterogeneous factor-augmented VARs)

Proof. Under Assumption 8, and noting that by assumption the processes $\{\mathbf{z}_{it}, i = 1, 2, \dots, N\}$ have been in operation for some time, we have:

$$\mathbf{z}_{it} = \boldsymbol{\mu}_i + \sum_{\ell=0}^{\infty} \boldsymbol{\Phi}_i^{\ell} \boldsymbol{\Gamma}_i \mathbf{f}_{t-\ell} + \boldsymbol{\zeta}_{it}, \quad (\text{A.1})$$

where:

$$\boldsymbol{\mu}_i = (\mathbf{I}_2 - \boldsymbol{\Phi}_i)^{-1} \mathbf{a}_i \quad \text{and} \quad \boldsymbol{\zeta}_{it} = \sum_{\ell=0}^{\infty} \boldsymbol{\Phi}_i^{\ell} \boldsymbol{\xi}_{i,t-\ell}. \quad (\text{A.2})$$

Pre-multiplying both sides of (A.1) by (w_i) and summing over i yields:

$$\bar{\mathbf{z}}_{\omega t} = \bar{\boldsymbol{\mu}}_{\omega} + \sum_{\ell=0}^{\infty} \mathbf{A}_{\ell,N} \mathbf{f}_{t-\ell} + \bar{\boldsymbol{\zeta}}_{\omega t} \quad (\text{A.3})$$

where:

$$\begin{aligned} \bar{\mathbf{z}}_{\omega t} &= \sum_{i=1}^N w_i \mathbf{z}_{it}, \quad \bar{\boldsymbol{\mu}}_{\omega} = \sum_{i=1}^N w_i \boldsymbol{\mu}_i, \\ \mathbf{A}_{\ell,N} &= \sum_{i=1}^N w_i \boldsymbol{\Phi}_i^{\ell} \boldsymbol{\Gamma}_i, \quad \text{and} \quad \bar{\boldsymbol{\zeta}}_{\omega t} = \sum_{i=1}^N w_i \boldsymbol{\zeta}_{it}. \end{aligned} \quad (\text{A.4})$$

Under Assumption 5, $\boldsymbol{\zeta}_{it}$ are cross-sectionally weakly correlated, given that the w_i are granular. Then using results in Pesaran and Chudik (2014), it readily follows that:

$$\bar{\boldsymbol{\zeta}}_{\omega t} = O_p \left(\sqrt{\sum_{i=1}^N w_i^2} \right) = O_p \left(N^{-\frac{1}{2}} \right), \quad \text{for each } t. \quad (\text{A.5})$$

Also, under Assumptions 7 and 8,

$$\mathbb{E} \left(\boldsymbol{\Phi}_i^{\ell} \boldsymbol{\Gamma}_i \right) = \mathbb{E} \left(\boldsymbol{\Phi}_i^{\ell} \right) \mathbb{E} \left(\boldsymbol{\Gamma}_i \right) = \boldsymbol{\Lambda}_{\ell} \boldsymbol{\Gamma},$$

and since $\boldsymbol{\Phi}_i$ and $\boldsymbol{\Gamma}_i$ are distributed independently across i , then using results in Pesaran and Chudik (2014) it also follows that

$$\mathbf{A}_{\ell,N} - \mathbb{E}(\mathbf{A}_{\ell,N}) = \sum_{i=1}^N w_i \left[\boldsymbol{\Phi}_i^{\ell} \boldsymbol{\Gamma}_i - \mathbb{E} \left(\boldsymbol{\Phi}_i^{\ell} \boldsymbol{\Gamma}_i \right) \right] = O_p \left(\sqrt{\sum_{i=1}^N w_i^2} \right) = O_p \left(N^{-\frac{1}{2}} \right). \quad (\text{A.6})$$

Using the above results, namely (A.5) and (A.6) in (A.3) we now have:

$$\begin{aligned}
\bar{\mathbf{z}}_{\omega t} &= \bar{\boldsymbol{\mu}}_{\omega} + \sum_{\ell=0}^{\infty} \boldsymbol{\Lambda}_{\ell} \boldsymbol{\Gamma} \mathbf{f}_{t-\ell} + O_p \left(N^{-\frac{1}{2}} \right) \\
&= \bar{\boldsymbol{\mu}}_{\omega} + \left(\sum_{\ell=0}^{\infty} \boldsymbol{\Lambda}_{\ell} L^{\ell} \right) \boldsymbol{\Gamma} \mathbf{f}_t + O_p \left(N^{-\frac{1}{2}} \right) \\
&= \bar{\boldsymbol{\mu}}_{\omega} + \boldsymbol{\Lambda}(L) \boldsymbol{\Gamma} \mathbf{f}_t + O_p \left(N^{-1/2} \right).
\end{aligned}$$

But under Assumptions 7 and 8, $\boldsymbol{\Gamma}$ and $\boldsymbol{\Lambda}(L)$ are both invertible and:

$$\mathbf{f}_t = \boldsymbol{\Gamma}^{-1} \boldsymbol{\Lambda}^{-1}(L) (\bar{\mathbf{z}}_{\omega t} - \bar{\boldsymbol{\mu}}_{\omega}) + O_p \left(N^{-\frac{1}{2}} \right),$$

where:

$$\begin{aligned}
\boldsymbol{\Gamma}^{-1} &= \begin{pmatrix} 0 & \gamma^{-1} \\ \theta^{-1} & -\frac{\lambda}{\theta\gamma} \end{pmatrix}, \\
\boldsymbol{\Lambda}^{-1}(L) &= \mathbf{B}_0 + \mathbf{B}_1 L + \mathbf{B}_2 L^2 + \dots
\end{aligned}$$

(note that $\mathbf{B}_0 = \boldsymbol{\Lambda}_0 = \mathbf{I}_2$). Hence,

$$\begin{aligned}
\mathbf{f}_t &= \boldsymbol{\Gamma}^{-1} (\bar{\mathbf{z}}_{\omega t} - \bar{\boldsymbol{\mu}}_{\omega}) + (\mathbf{C}_1 + \mathbf{C}_2 L + \mathbf{C}_3 L^2 + \dots) (\bar{\mathbf{z}}_{\omega, t-1} - \bar{\boldsymbol{\mu}}_{\omega}) + O_p \left(N^{-\frac{1}{2}} \right) \\
&= \mathbf{b} + \left(\sum_{\ell=0}^{\infty} \mathbf{C}_{\ell} L^{\ell} \right) \bar{\mathbf{z}}_{\omega, t} + O_p \left(N^{-\frac{1}{2}} \right),
\end{aligned}$$

where $\mathbf{C}_{\ell} = \boldsymbol{\Gamma}^{-1} \mathbf{B}_{\ell}$, for $\ell = 0, 1, 2, \dots$, and $\mathbf{b} = -\boldsymbol{\Gamma}^{-1} \boldsymbol{\Lambda}^{-1}(1) \bar{\boldsymbol{\mu}}_{\omega}$. Hence, given the lower triangular form of $\boldsymbol{\Gamma}^{-1}$, we have

$$f_t = \gamma^{-1} \Delta \bar{y}_{\omega, t} + \sum_{\ell=1}^{\infty} \mathbf{c}'_{1, \ell} \bar{\mathbf{z}}_{\omega, t-\ell} + O_p \left(N^{-\frac{1}{2}} \right), \quad (\text{A.7})$$

$$g_t = \theta^{-1} \bar{v}_{\omega, t} - \left(\frac{\lambda}{\theta\gamma} \right) \Delta \bar{y}_{\omega, t} + \sum_{\ell=1}^{\infty} \mathbf{c}'_{2, \ell} \bar{\mathbf{z}}_{\omega, t-\ell} + O_p \left(N^{-\frac{1}{2}} \right), \quad (\text{A.8})$$

where $\mathbf{c}'_{1, \ell}$ and $\mathbf{c}'_{2, \ell}$ are the first and the second rows of $\mathbf{C}_{\ell}$, respectively, and $\bar{v}_{\omega, t}$, $\Delta \bar{y}_{\omega, t}$, $\bar{\mathbf{z}}_{\omega, t}$ are defined as above.

Consider now $\mathbf{C}_{\ell}$ and note that $\|\mathbf{C}_{\ell}\| \leq \|\boldsymbol{\Gamma}^{-1}\| \|\mathbf{B}_{\ell}\|$, where $\|\boldsymbol{\Gamma}^{-1}\|$ is bounded for fixed non-zero

values of γ and θ . Further, $\mathbf{B}_\ell$ is given by the following recursions

$$\begin{aligned}\mathbf{B}_0 &= \mathbf{I}_2, \mathbf{B}_1 = -\mathbf{\Lambda}_1 \\ \mathbf{B}_2 &= -(\mathbf{\Lambda}_1 \mathbf{B}_1 + \mathbf{\Lambda}_2 \mathbf{B}_0), \\ &\vdots \\ \mathbf{B}_\ell &= -(\mathbf{\Lambda}_1 \mathbf{B}_{\ell-1} + \mathbf{\Lambda}_2 \mathbf{B}_{\ell-1} + \dots + \mathbf{\Lambda}_\ell \mathbf{B}_0).\end{aligned}$$

Hence, $\|\mathbf{B}_1\| \leq \|\mathbf{\Lambda}_1\| \|\mathbf{B}_0\|$, $\|\mathbf{B}_2\| \leq \|\mathbf{\Lambda}_1\| \|\mathbf{B}_1\| + \|\mathbf{\Lambda}_2\| \|\mathbf{B}_0\|$, and in general $\|\mathbf{B}_\ell\| \leq \|\mathbf{\Lambda}_1\| \|\mathbf{B}_{\ell-1}\| + \|\mathbf{\Lambda}_2\| \|\mathbf{B}_{\ell-1}\| + \dots + \|\mathbf{\Lambda}_\ell\| \|\mathbf{B}_0\|$, where $\|\mathbf{B}_0\| = 1$. However,²⁴

$$\|\mathbf{\Lambda}_\ell\| = \left\| \mathbb{E} \left(\mathbf{\Phi}_i^\ell \right) \right\| \leq \mathbb{E} \left\| \mathbf{\Phi}_i^\ell \right\| \leq (\mathbb{E} \|\mathbf{\Phi}_i\|)^\ell \leq \rho^\ell.$$

Hence, $\|\mathbf{B}_1\| \leq \rho$, $\|\mathbf{B}_2\| \leq \rho^2$, and so on, and as required $\|\mathbf{C}_\ell\| \leq \|\mathbf{\Gamma}^{-1}\| \rho^\ell$. ■

A.2 Derivation of orthogonalized factors

In this section we show how to extract orthogonal factors from the cross section averages, $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$. Consider the p^{th} order truncated approximation of the unobservable factors given by (A.7) and (A.8), and note that in matrix notations (abstracting from the probability order terms) we have:

$$\mathbf{f} = \Delta \bar{\mathbf{y}}_\omega + \bar{\mathbf{Z}}_\omega \mathbf{c}_1, \quad (\text{A.9})$$

$$\mathbf{g} = \bar{\mathbf{v}}_\omega - \lambda \Delta \bar{\mathbf{y}}_\omega + \bar{\mathbf{Z}}_\omega \mathbf{c}_2, \quad (\text{A.10})$$

where $\mathbf{f} = (f_1, f_2, \dots, f_T)'$, $\mathbf{g} = (g_1, g_2, \dots, g_T)'$, $\Delta \bar{\mathbf{y}}_{\omega,-l} = (\Delta \bar{y}_{\omega,1-l}, \Delta \bar{y}_{\omega,2-l}, \dots, \Delta \bar{y}_{\omega,T-l})'$, $\Delta \bar{\mathbf{y}}_\omega = \Delta \bar{\mathbf{y}}_{\omega,0}$, $\bar{\mathbf{v}}_{\omega,-l} = (\bar{v}_{\omega,1-l}, \bar{v}_{\omega,2-l}, \dots, \bar{v}_{\omega,T-l})'$, $\bar{\mathbf{v}}_\omega = \bar{\mathbf{v}}_{\omega,0}$, $\bar{\mathbf{Z}}_\omega = (\boldsymbol{\tau}_T, \bar{\mathbf{z}}_{\omega,-1}, \bar{\mathbf{z}}_{\omega,-2}, \dots, \bar{\mathbf{z}}_{\omega,-p})$, and $\bar{\mathbf{z}}_{\omega,-l} = (\Delta \bar{\mathbf{y}}_{\omega,-l} \bar{v}_{\omega,-l})$.²⁵ Then the orthogonalized filtered factors, $\tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$, can now be recovered as residuals from the following OLS regressions:

$$\Delta \bar{\mathbf{y}}_\omega = \bar{\mathbf{Z}}_\omega \hat{\mathbf{c}}_1 + \tilde{\mathbf{f}}, \quad (\text{A.11})$$

$$\bar{\mathbf{v}}_\omega = \hat{\lambda} \tilde{\mathbf{f}} + \bar{\mathbf{Z}}_\omega \hat{\mathbf{c}}_2 + \tilde{\mathbf{g}}, \quad (\text{A.12})$$

where $\hat{\mathbf{c}}_1$ is the OLS estimator of the regression coefficients in the regression of $\Delta \bar{\mathbf{y}}_\omega$ on $\bar{\mathbf{Z}}_\omega$, and $\hat{\lambda}$ and $\hat{\mathbf{c}}_2$ OLS estimators of the regression coefficients in the regression of $\bar{\mathbf{v}}_\omega$ on $\tilde{\mathbf{f}}$ and $\bar{\mathbf{Z}}_\omega$.

Proof. Let $\mathbf{M}_{\bar{\mathbf{Z}}_\omega} = \mathbf{I}_T - \bar{\mathbf{Z}}_\omega (\bar{\mathbf{Z}}_\omega' \bar{\mathbf{Z}}_\omega)^{-1} \bar{\mathbf{Z}}_\omega'$, and note that:

$$\begin{aligned}\mathbf{M}_{\bar{\mathbf{Z}}_\omega} \mathbf{f} &= \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \Delta \bar{\mathbf{y}}_\omega \\ \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \mathbf{g} &= \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \bar{\mathbf{v}}_\omega - \lambda \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \Delta \bar{\mathbf{y}}_\omega\end{aligned}$$

²⁴Note that for any matrix $\mathbf{A}$, $\|\mathbf{A}^p\| \leq \|\mathbf{A}\|^p$, and for any random variable X , $\|E(X)\| \leq E\|X\|$.

²⁵The inclusion of $\boldsymbol{\tau}_T$ in $\bar{\mathbf{Z}}_\omega$ ensures that the filtered factors have zero in-sample means.

since $\mathbf{M}_{\bar{\mathbf{Z}}_\omega} \bar{\mathbf{Z}}_\omega = \mathbf{0}$. We set the first normalized factor, that we label $\tilde{\mathbf{f}}$, to $\mathbf{M}_{\bar{\mathbf{Z}}_\omega} \mathbf{f} = \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \Delta \bar{\mathbf{y}}_\omega$; and set the second factor, that we label $\tilde{\mathbf{g}}$, as the linear combination of $\mathbf{M}_{\bar{\mathbf{Z}}_\omega} \mathbf{f}$ and $\mathbf{M}_{\bar{\mathbf{Z}}_\omega} \bar{\mathbf{v}}_\omega$ such that $\tilde{\mathbf{f}}' \tilde{\mathbf{g}} = 0$. This can be achieved selecting λ so that:

$$\tilde{\mathbf{f}}' \tilde{\mathbf{g}} = \Delta \bar{\mathbf{y}}_\omega' \mathbf{M}_{\bar{\mathbf{Z}}_\omega} (\mathbf{M}_{\bar{\mathbf{Z}}_\omega} \bar{\mathbf{v}}_\omega - \lambda \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \Delta \bar{\mathbf{y}}_\omega) = 0.$$

The value of λ that solves this equation is given by:

$$\hat{\lambda} = \frac{\Delta \bar{\mathbf{y}}_\omega' \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \bar{\mathbf{v}}_\omega}{\Delta \bar{\mathbf{y}}_\omega' \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \Delta \bar{\mathbf{y}}_\omega}.$$

Note that $\hat{\lambda}$ is the OLS estimator of the coefficient of the regression of $\mathbf{M}_{\bar{\mathbf{Z}}_\omega} \bar{\mathbf{v}}_\omega$ on $\mathbf{M}_{\bar{\mathbf{Z}}_\omega} \Delta \bar{\mathbf{y}}_\omega$. Hence, the orthogonalized factors are

$$\begin{aligned} \tilde{\mathbf{f}} &= \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \Delta \bar{\mathbf{y}}_\omega, \\ \tilde{\mathbf{g}} &= \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \bar{\mathbf{v}}_\omega - \hat{\lambda} \mathbf{M}_{\bar{\mathbf{Z}}_\omega} \Delta \bar{\mathbf{y}}_\omega. \end{aligned}$$

In practice, this implies that $\tilde{\mathbf{f}}$ can be recovered as residuals from the OLS regression of $\Delta \bar{\mathbf{y}}_\omega$ on an intercept and $\bar{\mathbf{z}}_{\omega,t-\ell}$, with $\ell = 1, 2, \dots, p$:

$$\Delta \bar{\mathbf{y}}_\omega = \bar{\mathbf{Z}}_\omega \hat{\mathbf{c}}_1 + \tilde{\mathbf{f}} \tag{A.13}$$

While $\tilde{\mathbf{g}}$ can be recovered as residuals from the OLS regression of $\bar{\mathbf{v}}_\omega$ on $\tilde{\mathbf{f}}$, an intercept, and $\bar{\mathbf{z}}_{\omega,t-\ell}$ with $\ell = 1, \dots, p$:

$$\bar{\mathbf{v}}_\omega = \hat{\lambda} \tilde{\mathbf{f}} + \bar{\mathbf{Z}}_\omega \hat{\mathbf{c}}_2 + \tilde{\mathbf{g}} \tag{A.14}$$

■

B Appendix: Computing impulse responses and variance decompositions

Consider the following factor-augmented country-specific VAR models also augmented with lagged cross section averages, $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, \dots, p$,

$$\mathbf{z}_{it} = \Phi_i \mathbf{z}_{i,t-1} + \sum_{\ell=1}^p \psi_{i,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \tilde{\Gamma}_i \tilde{\mathbf{f}}_t + \boldsymbol{\xi}_{it}, \text{ for } i = 1, 2, \dots, N, \tag{B.1}$$

where

$$= \begin{pmatrix} \tilde{\gamma}_{i,11} & \tilde{\gamma}_{i,12} \\ \tilde{\gamma}_{i,21} & 0 \end{pmatrix}, \tilde{\mathbf{f}}_t = \begin{pmatrix} \tilde{f}_t \\ \tilde{g}_t \end{pmatrix}.$$

Intercepts are omitted to simplify the exposition. Note also that $\bar{\mathbf{z}}_{\omega,t} = \sum_{i=1}^N w_i \Delta \mathbf{z}_{it} = \mathbf{W} \mathbf{z}_t$, where $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t}, \dots, \mathbf{z}'_{Nt})'$, and $\mathbf{W}$ is a $2 \times 2N$ matrix of weights. Stacking the VARs in (B.1) over i we obtain

$$\mathbf{z}_t = \Phi \mathbf{z}_{t-1} + \sum_{\ell=1}^p \psi_{\ell} \mathbf{W} \mathbf{z}_{t-\ell} + \mathbf{B} \tilde{\mathbf{f}}_t + \boldsymbol{\xi}_t, \quad (\text{B.2})$$

where $\boldsymbol{\xi}_t = (\boldsymbol{\xi}'_{1t}, \boldsymbol{\xi}'_{2t}, \dots, \boldsymbol{\xi}'_{Nt})'$ and

$$\Phi = \begin{pmatrix} \Phi_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \Phi_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \Phi_N \end{pmatrix}, \quad \psi_{\ell} = \begin{pmatrix} \psi_{1,\ell} \\ \psi_{2,\ell} \\ \vdots \\ \psi_{N,\ell} \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \tilde{\Gamma}_1 \\ \tilde{\Gamma}_2 \\ \vdots \\ \tilde{\Gamma}_N \end{pmatrix}.$$

The high-dimensional VAR in (B.2) can now be written as a standard FAVAR(p) model in $2N$ variables

$$\mathbf{z}_t = (\Phi + \psi_1 \mathbf{W}) \mathbf{z}_{t-1} + \sum_{\ell=2}^p \psi_{\ell} \mathbf{W} \mathbf{z}_{t-\ell} + \Gamma \tilde{\mathbf{f}}_t + \boldsymbol{\xi}_t. \quad (\text{B.3})$$

For example, when $p = 1$ we have the FAVAR(1)

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \Psi_1 \mathbf{L})^{-1} (\mathbf{B} \tilde{\mathbf{f}}_t + \boldsymbol{\xi}_t),$$

where $\Psi_1 = \Phi + \psi_1 \mathbf{W}$.

$$\mathbf{z}_t = (\mathbf{I} - \Psi_1 \mathbf{L})^{-1} \mathbf{B} \tilde{\mathbf{f}}_t + (\mathbf{I} - \Psi_1 \mathbf{L})^{-1} \boldsymbol{\xi}_t.$$

Note that by construction $\tilde{\mathbf{f}}_t$ and $\boldsymbol{\xi}_t$ are orthogonal, and for sufficiently large p , they are serially uncorrelated. Hence, the impulse response of shocks to elements of $\tilde{\mathbf{f}}_t$ and $\boldsymbol{\xi}_t$ can be computed using the following moving average representation

$$\mathbf{z}_t = \sum_{n=0}^{\infty} \mathbf{A}_n \tilde{\mathbf{f}}_{t-n} + \sum_{n=0}^{\infty} \mathbf{C}_n \boldsymbol{\xi}_{t-n}, \quad (\text{B.4})$$

where $\mathbf{A}_n = \Psi_1^n \mathbf{B}$, and $\mathbf{C}_n = \Psi_1^n$, for $n = 0, 1, 2, \dots$

B.1 Responses to common and country-specific shocks

Let $\mathbf{e}_i$ be a selection vector such that $\mathbf{e}'_i \mathbf{z}_t$ picks the i^{th} element of $\mathbf{z}_t$. Also let $\mathbf{s}_f = (1, 0)'$ and $\mathbf{s}_g = (0, 1)'$, the vectors that select $\tilde{f}_t$ and $\tilde{g}_t$ from $\tilde{\mathbf{f}}_t$; namely,

$$\mathbf{s}'_f \tilde{\mathbf{f}}_t \equiv \tilde{f}_t, \quad \mathbf{s}'_g \tilde{\mathbf{f}}_t \equiv \tilde{g}_t. \quad (\text{B.5})$$

Recall now that $\tilde{f}_t$ and $\tilde{g}_t$ have zero means, unit variances and are orthogonal to each other. Then the impulse responses to a positive unit shock to $\tilde{f}_t$ or $\tilde{g}_t$ are given by:

$$IR_{i,\tilde{f},n} = \mathbf{e}_i' \mathbf{A}_n \mathbf{s}_f \quad \text{and} \quad IR_{i,\tilde{g},n} = \mathbf{e}_i' \mathbf{A}_n \mathbf{s}_g \quad \text{for } n = 0, 1, 2, \dots, \quad (\text{B.6})$$

where $\mathbf{A}_n$ is given by the moving average representation, (B.4)

To derive impulse response functions for country-specific shocks, i.e., the j^{th} element of $\boldsymbol{\xi}_t$, we need to make assumptions about the correlation between volatility and growth innovations within each country and across countries. Since the elements of $\boldsymbol{\xi}_t$ are weakly correlated across countries, by construction, they have some, but limited correlation across countries (see Tables 2 and 4). We also documented that, conditional on the common factors $\tilde{f}_t$ and $\tilde{g}_t$, the country correlation of volatility and growth innovations are statistically insignificant for all except four countries.

As a first approximation, therefore, we will assume that the variance-covariance matrix of $\boldsymbol{\xi}_t$ in (B.3) is diagonal. Under this assumption, the impulse response function of a positive, unit shock to the j^{th} element of $\boldsymbol{\xi}_t$ on the i^{th} element of $\mathbf{z}_t$ is given by:

$$IR_{i,\xi_j,n} = \sqrt{\hat{\omega}_{jj}} \mathbf{e}_i' \mathbf{C}_n \mathbf{e}_j, \quad (\text{B.7})$$

where $\mathbf{C}_n$ is given by the moving average representation, (B.4), $\hat{\omega}_{jj}$ is the (estimate) of the variance of the j^{th} country-specific shock and $\mathbf{e}_j$ is a selection vector such that $\mathbf{e}_j' \mathbf{z}_t$ picks the j^{th} element of $\mathbf{z}_t$.

The above impulse responses can be compared to the generalized impulse responses of Pesaran and Shin (1998). The latter are given by:

$$GIR_{i,\xi_j,n} = \frac{\mathbf{e}_i' \mathbf{C}_n \hat{\Omega} \mathbf{e}_j}{\sqrt{\hat{\omega}_{jj}}}, \quad (\text{B.8})$$

where $\hat{\Omega} = (\hat{\omega}_{ij})$ is the estimate of the covariance of $\boldsymbol{\xi}_t$. The generalized impulse responses allow for non-zero correlations across the idiosyncratic errors. The two sets of impulse responses coincide if the variance-covariance matrix of $\boldsymbol{\xi}_t$ is diagonal.

B.2 Variance error decompositions

country specific shocks is diagonal, the relative importance of shocks to country volatility and growth for all countries (ξ_{jt} , for $j = 1, 2, \dots, 2N$) and shocks to the two common factors that $\tilde{f}_t$ and $\tilde{g}_t$, is easily characterized.

Let $VD_{i,\tilde{f},n}$ and $VD_{i,\tilde{g},n}$ be the share of the n -step ahead forecast error variance of the i^{th} variable in $\mathbf{z}_t$ that is accounted for by $\tilde{f}_t$ and $\tilde{g}_t$, respectively, and $VD_{i,j}$ the variance share of a generic country

specific shock, then:

$$VD_{i,\tilde{f},n} = \frac{\sum_{\ell=0}^n (\mathbf{e}'_i \mathbf{A}_\ell \mathbf{s}_f)^2}{\sum_{\ell=0}^n \mathbf{e}'_i \mathbf{A}_\ell \mathbf{A}'_\ell \mathbf{e}_i + \sum_{\ell=0}^n \mathbf{e}'_i \mathbf{C}_\ell \hat{\Omega} \mathbf{C}'_\ell \mathbf{e}_i}, \quad n = 1, 2, \dots, H, \quad (\text{B.9})$$

$$VD_{i,\tilde{g},n} = \frac{\sum_{\ell=0}^n (\mathbf{e}'_i \mathbf{A}_\ell \mathbf{s}_g)^2}{\sum_{\ell=0}^n \mathbf{e}'_i \mathbf{A}_\ell \mathbf{A}'_\ell \mathbf{e}_i + \sum_{\ell=0}^n \mathbf{e}'_i \mathbf{C}_\ell \hat{\Omega} \mathbf{C}'_\ell \mathbf{e}_i}, \quad n = 1, 2, \dots, H, \quad (\text{B.10})$$

$$VD_{i,j,n} = \frac{\hat{\omega}_{jj}^{-1} \sum_{\ell=0}^n (\mathbf{e}'_i \mathbf{C}_\ell \hat{\Omega} \mathbf{e}_j)^2}{\sum_{\ell=0}^n \mathbf{e}'_i \mathbf{A}_\ell \mathbf{A}'_\ell \mathbf{e}_i + \sum_{\ell=0}^n \mathbf{e}'_i \mathbf{C}_\ell \hat{\Omega} \mathbf{C}'_\ell \mathbf{e}_i}, \quad j = 1, 2, \dots, 2N, \quad n = 1, 2, \dots, H; \quad (\text{B.11})$$

where $VD_{i,\tilde{f},n} + VD_{i,\tilde{g},n} + \sum_{j=1}^{2N} VD_{i,j,n} = 1$ if $\hat{\Omega} = \text{diag}(\hat{\omega}_{11}, \hat{\omega}_{22}, \dots, \hat{\omega}_{2N,2N})$. As with impulse responses, we also compare these error variance decomposition results with the generalized variance error decompositions that employs the full matrix estimate, $\hat{\Omega} = (\hat{\omega}_{ij})$.

B.3 Average impulse responses

As a summary measure of the effects of shocks to the common (global) factors we computed the following average measures. Denote the impulse response of a particular shock on the j^{th} variable in country i at horizon h by IR_{ijh} . Let $w = (w_1, w_2, \dots, w_N)'$ be a vector of fixed weights such that that $\sum_{i=1}^N w_i = 1$. Then the average impulse response of the shock to variable j at horizon h is computed as

$$IR_{\omega,jh} = \sum_{i=1}^N w_i IR_{ijh}. \quad (\text{B.12})$$

and its dispersion is computed by

$$\sigma_{IR_{\omega,jh}} = \left[\sum_{i=1}^N w_i^2 (IR_{ijh} - IR_{\omega,jh})^2 \right]^{1/2}, \quad (\text{B.13})$$

assuming country-specific impulse responses are uncorrelated. The same formulae can be applied to compute average forecast error variance decompositions.

C Appendix: Data Sources

For equity prices we use the MSCI Index in local currency. We collected daily observations from January 1979 to June 2011, but the panel of countries is unbalanced with only 16 economies starting from the beginning of the sample. A balanced panel was also constructed with 32 countries from

1993:Q1. The data source for the daily equity price indexes is Bloomberg. The countries included in the sample are the following: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Norway, New Zealand, Peru, Philippines, Saudi Arabia, South Africa, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, and United States.

The list of Bloomberg tickers is as follows: MSELTAG, MSDLAS, MSDLAT, MSDLBE, MSELTBR, MSDLCA, MSELTCF, MSELTCH, MSDLFI, MSDLFR, MSDLGR, MSELTIA, MSELTINF, MS-DLIT, MSDLJN, MSELTKO, MXMY, MSELTMXF, MSDLNE, MSDLNO, MSDLNZ, MSELTPR, MSELTPHF, MSELTSA, MGCLSA, MSDLSG, MSDLSP, MSDLSW, MSDLSZ, MSELTTHF, MSELTTK, MSDLUK, MSDLUS.

Real GDP data come from standard sources. The data set is balanced and good quality quarterly data are also available for all countries from 1993:Q1. For more details see:

<https://sites.google.com/site/gvarmodelling/>.

D Online Supplement: Country-specific Results

In this online Supplement (not for publication), we report selected country-specific results, including summary statistics and pairwise correlation of growth and volatility. Table D.1 reports summary statistics on the log-level of real GDP for each country. Table D.2 reports the same set of statistics for the realized volatility series. Table D.3 reports the average pairwise correlation conditioning only on $\tilde{g}$). Table D.3 reports the average pairwise correlation conditioning both on $\tilde{f}$ and $\tilde{g}$.

For completeness, we also report all country impulse responses and forecast error variance decompositions for both volatility and growth. Figures 10 reports impulse responses. Figure D.1 and Figure D.2 reports the country variance decompositions of volatility and growth, respectively.

Table D.1 SUMMARY STATISTICS FOR REAL GDP LOG-LEVEL

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs	74	74	74	74	74	74	74	74	74	74	74
Mean	16.26	7.37	7.74	7.53	17.38	8.07	8.02	14.64	16.51	9.23	9.26
Max	46.42	24.92	31.02	24.35	46.94	33.90	23.80	41.53	77.18	28.01	33.59
Min	7.58	3.35	3.39	2.86	6.56	3.36	4.21	6.63	6.60	4.02	3.72
St. Dev.	6.95	3.31	5.16	4.27	8.76	4.69	3.45	6.70	10.35	4.22	4.29
Auto Corr.	0.41	0.58	0.77	0.70	0.57	0.68	0.41	0.58	0.47	0.60	0.51
ADF	-3.70	-2.24	-1.85	-2.54	-2.94	-2.65	-2.99	-2.72	-2.42	-2.46	-2.59
ADF(p)	0.01	0.00	0.00	0.00	0.05	0.10	0.05	0.10	0.00	0.00	0.00
	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs	74	74	74	74	74	74	74	74	74	74	74
Mean	12.06	13.90	9.90	10.03	13.26	9.04	11.50	8.77	8.20	10.08	13.44
Max	29.35	40.85	27.96	34.26	34.03	47.10	29.32	29.81	23.68	39.33	44.95
Min	5.41	4.11	3.61	4.26	5.93	2.64	5.11	3.60	3.76	4.74	6.14
St. Dev.	5.07	7.15	4.37	4.08	6.29	6.87	4.56	4.87	3.46	5.23	6.14
Auto Corr.	0.53	0.66	0.56	0.36	0.69	0.53	0.51	0.64	0.52	0.63	0.57
ADF	-2.90	-2.59	-2.36	-3.00	-1.90	-2.73	-2.61	-2.58	-2.55	-2.53	-2.22
ADF(p)	0.10	0.00	0.00	0.05	0.00	0.10	0.00	0.00	0.00	0.00	0.00
	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA	
Obs	74	74	74	74	74	74	74	74	74	74	
Mean	10.95	9.12	11.35	9.58	10.78	7.75	13.52	19.87	8.26	8.31	
Max	25.33	26.13	41.05	29.40	29.95	24.53	30.10	41.86	29.15	33.67	
Min	4.36	3.74	3.82	4.05	5.23	3.35	6.64	7.43	3.76	3.34	
St. Dev.	4.20	4.71	5.67	4.57	4.85	3.80	5.72	7.74	4.28	4.68	
Auto Corr.	0.42	0.62	0.47	0.56	0.65	0.55	0.59	0.57	0.61	0.68	
ADF	-2.64	-2.44	-2.97	-2.11	-2.37	-2.83	-2.15	-1.70	-2.51	-2.47	
ADF(p)	0.10	0.00	0.05	0.00	0.00	0.10	0.00	0.00	0.00	0.00	

NOTE. Summary statistics of the log-level of real GDP.

Table D.2 SUMMARY STATISTICS FOR THE COUNTRY REALIZED VOLATILITY LEVEL

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs	74	74	74	74	74	74	74	74	74	74	74
Mean	4.67	4.65	4.62	4.61	4.67	4.61	4.68	4.86	4.61	4.60	4.60
Max	5.12	4.93	4.79	4.76	5.00	4.81	5.07	5.89	4.84	4.74	4.73
Min	4.42	4.30	4.40	4.40	4.39	4.32	4.23	3.99	4.29	4.43	4.47
St. Dev.	0.19	0.19	0.12	0.11	0.17	0.16	0.23	0.57	0.16	0.10	0.07
Auto Corr.	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
ADF	0.64	-2.97	-1.26	-1.42	0.57	-1.96	-1.03	2.76	-1.63	-1.55	-0.90
ADF(p)	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs	74	74	74	74	74	74	74	74	74	74	74
Mean	4.74	4.74	4.60	4.63	4.67	4.66	4.60	4.59	4.92	4.54	4.72
Max	5.39	5.17	4.69	4.72	5.04	5.08	4.81	4.75	5.11	4.70	5.22
Min	4.16	4.37	4.46	4.55	4.19	4.15	4.31	4.36	4.63	4.23	4.24
St. Dev.	0.36	0.21	0.07	0.05	0.24	0.25	0.15	0.12	0.14	0.13	0.25
Auto Corr.	1.00	1.00	0.99	0.98	1.00	1.00	0.99	1.00	1.00	1.00	1.00
ADF	1.84	0.39	-2.12	-1.41	-1.14	-0.93	-0.52	-1.78	-1.76	-3.09	1.05
ADF(p)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00
	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA	
Obs	74	74	74	74	74	74	74	74	74	74	
Mean	4.70	4.65	4.68	4.62	4.62	4.62	4.73	4.66	4.62	4.61	
Max	5.11	5.18	4.98	4.83	4.84	4.79	5.05	5.07	4.80	4.77	
Min	4.33	4.06	4.39	4.37	4.36	4.48	4.37	4.27	4.37	4.33	
St. Dev.	0.23	0.30	0.18	0.15	0.14	0.10	0.19	0.22	0.13	0.14	
Auto Corr.	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00	
ADF	0.52	-0.16	-0.11	-1.63	-0.86	-0.03	-0.67	-0.72	-1.97	-1.80	
ADF(p)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	

NOTE. Summary statistics of the level of volatility.

Table D.3 AVERAGE PAIRWISE CORRELATIONS OF VOLATILITY AND GROWTH INNOVATIONS (CONDITIONAL ON $\tilde{f}$ ONLY)

Panel A: Volatility Innovations					
Pair. Corr.		Pair. Corr.		Pair. Corr.	
Argentina	0.57	India	0.43	Philippines	0.50
Australia	0.64	Indonesia	0.47	Singapore	0.57
Austria	0.59	Italy	0.56	South Africa	0.64
Belgium	0.53	Japan	0.52	Spain	0.61
Brazil	0.40	Korea	0.51	Sweden	0.62
Canada	0.64	Malaysia	0.33	Switzerland	0.59
Chile	0.54	Mexico	0.59	Thailand	0.40
China	0.50	Netherlands	0.60	Turkey	0.40
Finland	0.24	New Zealand	0.52	United Kingdom	0.63
France	0.63	Norway	0.66	United States	0.63
Germany	0.58	Peru	0.53		

Panel B: Growth Innovations					
Pair. Corr.		Pair. Corr.		Pair. Corr.	
Argentina	0.00	India	-0.10	Philippines	0.01
Australia	0.00	Indonesia	0.00	Singapore	0.05
Austria	0.04	Italy	0.09	South Africa	0.07
Belgium	0.08	Japan	-0.04	Spain	0.05
Brazil	0.06	Korea	0.09	Sweden	0.10
Canada	0.04	Malaysia	0.04	Switzerland	0.00
Chile	0.03	Mexico	0.05	Thailand	0.10
China	-0.19	Netherlands	0.07	Turkey	0.02
Finland	0.02	New Zealand	0.08	United Kingdom	0.03
France	0.09	Norway	0.05	United States	-0.01
Germany	0.06	Peru	0.05		

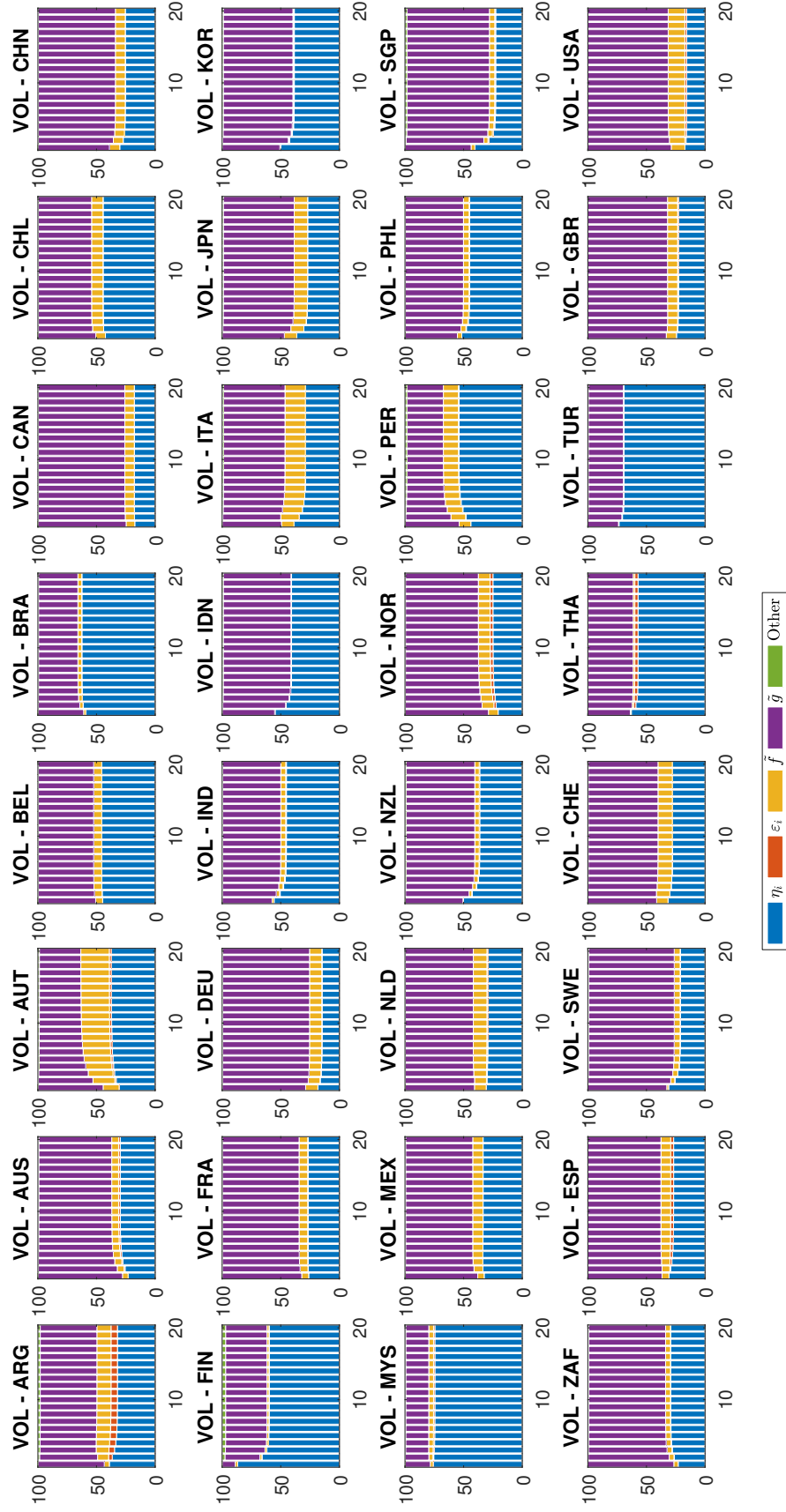
NOTE. Volatility innovations (u) are the OLS residuals of equation (38) without explicit conditioning on $\tilde{g}$. Growth innovations (ε) are the OLS residuals of equation (39) as specified above. The average pairwise correlation across countries is 0.54 and 0.03 for the volatility innovations and the growth innovations respectively. The statistics are the same as in Figure 6.

Table D.4 AVERAGE PAIRWISE CORRELATIONS OF VOLATILITY INNOVATIONS (CONDITIONAL BOTH ON $\tilde{f}$ AND $\tilde{g}$)

Pair. Corr.		Pair. Corr.		Pair. Corr.	
Argentina	0.05	India	-0.11	Philippines	-0.01
Australia	0.06	Indonesia	-0.03	Singapore	0.01
Austria	0.08	Italy	0.09	South Africa	0.07
Belgium	0.07	Japan	0.01	Spain	0.08
Brazil	0.02	Korea	0.03	Sweden	0.09
Canada	0.02	Malaysia	-0.04	Switzerland	0.10
Chile	0.02	Mexico	0.03	Thailand	-0.06
China	-0.21	Netherlands	0.09	Turkey	0.05
Finland	-0.03	New Zealand	0.06	United Kingdom	0.09
France	0.12	Norway	0.11	United States	0.03
Germany	0.05	Peru	0.04		

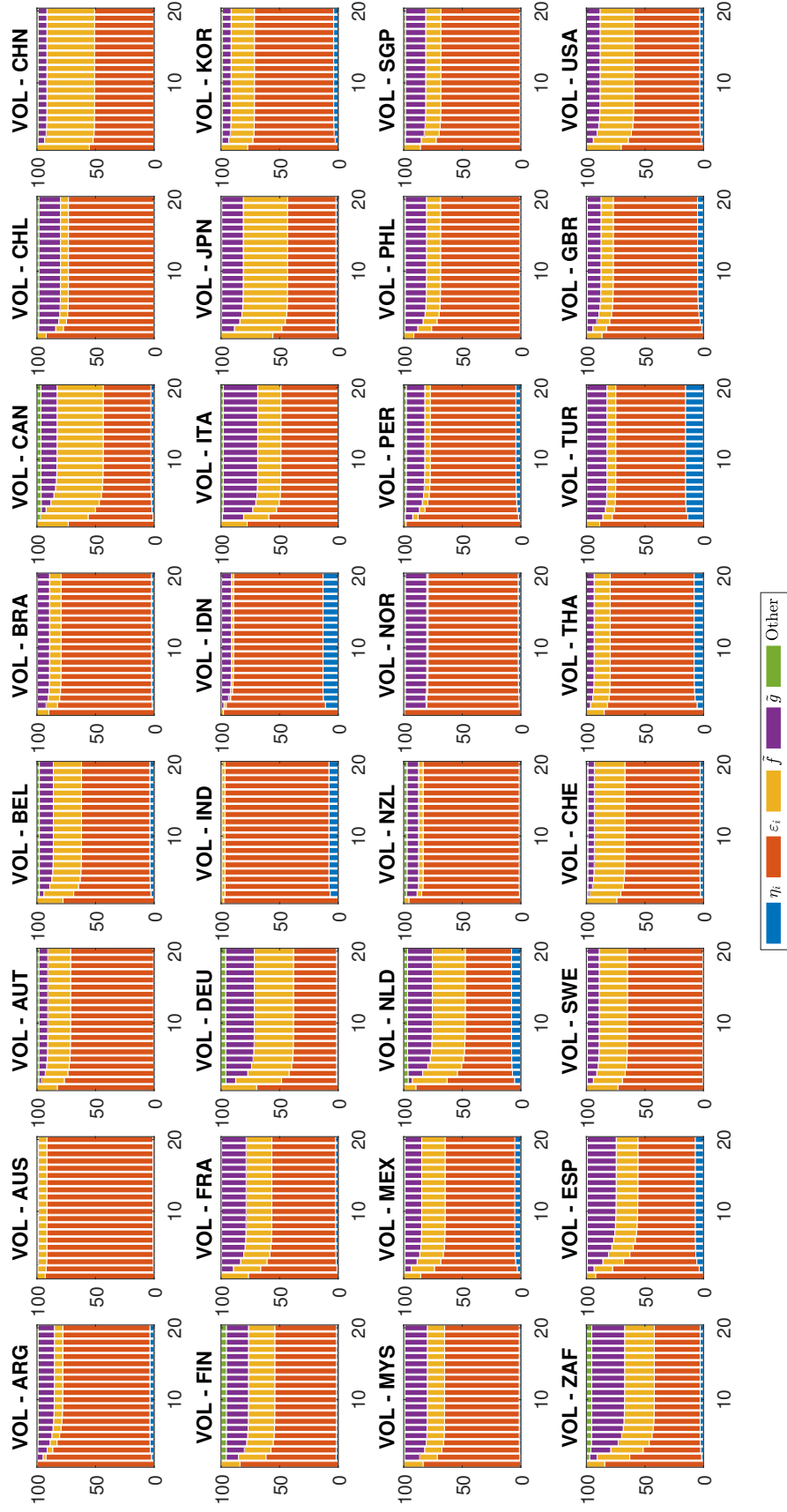
NOTE. Volatility innovations (η) are the residuals of equation (38). The average pairwise correlation across countries is 0.03. The statistics for the volatility innovations are the same as in Figure 7.

Figure D.1 VOLATILITY FORECAST ERROR VARIANCE DECOMPOSITIONS



NOTE. *Other* represents the variance share of all other country specific shocks in the model. Forecast error variance decompositions computed as in Appendix B.

Figure D.2 GDP GROWTH FORECAST ERROR VARIANCE DECOMPOSITIONS



NOTE. *Other* represents the variance share of all other country specific shocks in the model. Forecast error variance decompositions computed as in Appendix B.