College Pricing and Income Inequality

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The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
Introduction

• Price of US college tuition has risen fast in recent decades
• At the same time, income inequality has been rising
• Why is tuition rising so fast?
• Are smart low income students being priced out?
• To explore these questions, need a model of the college market
• Key Challenge: College is a club good:
  • Quality (desirability) of a given college depends on attributes (e.g. academic ability) of students who attend
  • Consumers are therefore an input in production
Tuition, Fees, Room & Board (College Board $2015)
Colleges as Clubs

- Club good feature complicates model analysis:
  - two colleges with different student bodies supply different products in different markets

- Lots of college variety $\Rightarrow$ lots of markets in general equilibrium

- $\Rightarrow$ existing literature assumes small number of different college types

- Potential concerns:
  - Counterfactual $\Rightarrow$ applied analysis difficult
  - Equilibrium existence problems (Scotchmer, 1997)
  - Price-taking assumption questionable – game theoretic oligopolistic price setting more natural
Model: Standard Elements

- Households differ by income and student ability
- Colleges differ by quality
- Quality depends on resources & avg. student ability
Model: Novel Element

Continuous distribution of college quality, with free entry (Ellickson, Grodal, Scotchmer, Zame, 1999)

- Entire distribution of college characteristics and prices can be compared to data
- College distribution can change smoothly and flexibly in response to changing drivers of college demand
- No existence problems
- Price taking natural
- No role for lotteries as in Cole and Prescott (1997) or Caucutt (1999)
Outline

• Model description

• A closed-form example

• Calibration and model-data comparison

• Applications: How do the following affect college pricing and college attendance
  1. Income inequality
  2. Subsidies to public universities
  3. Subsidies to all colleges
Model: Households

- Continuum of measure 2 of households, each containing a parent and a college-age child

- Heterogeneous wrt: (i) income $y$, (ii) student ability $a$

- Two ability levels, indexed $i \in \{l, h\}$, $a_l < a_h$, measure 1 of each level

- Continuous distribution for income, CDF $F^i(y)$

- Utility from non-durable consumption $c$ and quality $q$ of the college the child attends

$$u(c, q) = \log c + \varphi \log(\kappa + q)$$
Household Problem

• Make education choice $j \in \{0, 1, 2\}$:
  1. $j = 0$: No college
  2. $j = 1$: Public college, grant toward tuition $g_1$
  3. $j = 2$: Private college, grant toward tuition $g_2 < g_1$

• Take as given tuition functions $t^i_j(q; y)$

• Given idiosyncratic state $(y, i)$, solve

$$\max_{\{j, c, q \in Q^i\}} u(c, q)$$

s.t.
$$c + t^i_j(q; y) = y + g_j$$

• Solution: $s^i(y), c^i(y), q^i(y)$
Model: Colleges

- CRS technology for producing education of a given quality

- Quality (per student) reflects:
  
  (i) average ability of student body

  (ii) consumption good input (per student) $e$ (faculty etc)

  $$q = (\eta a_h + (1 - \eta) a_l)\theta e^{1-\theta}$$

  where $\eta$ is share of student body that is high ability

- Fixed consumption cost R&B $\phi$ per student admitted
Public versus Private Schools

- Assume all colleges profit maximize
  - minimize cost of supplying given value of education

- Observe income $y$ and child’s ability type $i$, take as given tuition schedules

- Colleges choose private or public status

- Public colleges must keep average tuition below a cap $T$

- No equilibrium tuition discrimination by income
  - If other colleges charge high income students more, a single profit-maximizing college would skim high income students
  - If other colleges are profit maximizing, a single college charging low income students less would incur negative profits
College Problem

1. Choose quality level
2. Choose public or private model to deliver $q$
3. Choose input mix and size

Input mix sub-problem for private college supplying mass 1 spots at $q > 0$

$$\max_{\eta,e} \left\{ t_2^h(q) \eta + t_2^l(q) (1 - \eta) - e - \phi \right\}$$

s.t.

$$q = (\eta a_h + (1 - \eta) a_l)^\theta e^{1-\theta}$$

- Public college problem similar s.t. additional constraint

$$t_1^h(q) \eta + t_1^l(q) (1 - \eta) \leq T$$
Profit Maximization Given $t^i_j(q)$

1. Fix quality $q$
2. Compute optimal input mix for unconstrained public college
   \[
   \frac{e_1(q)}{\eta_1(q)a_h + (1 - \eta_1(q)) a_l} = \frac{(1 - \theta) (t^l_1(q) - t^h_1(q))}{\theta \Delta a}
   \]
3. Check whether avg. tuition exceeds $T$.
   - If not, only public colleges at quality $q$
   - Else, compare profit from unconstrained private college to constrained public college, where $\eta_1(q)$ s.t.
     \[
     t^h_1(q)\eta_1(q) + t^l_1(q) (1 - \eta_1(q)) = T
     \]
4. Optimal size at each $q$:
   \[
   \begin{cases} 
   0 & \text{if } \pi_j(q) < 0 \\
   [0, \infty] & \text{if } \pi_j(q) = 0 \\
   \infty & \text{if } \pi_j(q) > 0 
   \end{cases}
   \]
Equilibrium

\( \chi_j(Q) \): measure of students in \( j \) type colleges with \( q \in Q \subset Q_j \)

Equilibrium is \( \chi_j(q), t^i_j(q), \eta_j(q), e_j(q), s^i(y), c^i(y), q^i(y) \) s.t.

1. Given \( t^i_j(q), s^i(y), q^i(y) \) & \( c^i(y) \) solve household’s problem
2. Given \( t^i_j(q), \eta_j(q) \) & \( e_j(q) \) solve college problem for \( j = 1, 2 \)
3. Zero profits: \( \forall Q, \pi_j(q) \leq 0 \ \forall q \in Q \) and
   \[
   \int_Q \pi_j(q) d\chi_j(q) = 0
   \]
4. Market clearing:
   \[
   \sum_{i=h,l} \int c^i(y) dF^i(y) + \sum_{j=1,2} \int (e_j(q) + \phi - g_j) d\chi_j(q) = \sum_{i=h,l} \int y dF^i(y)
   \]
   \[
   \int 1\{s^h(y)=j, q^h(y)\in Q\} dF^h(y) = \int_Q \eta_j(q) d\chi_j(q) \quad \forall Q, j = 1, 2
   \]
   \[
   \int 1\{s^l(y)=j, q^l(y)\in Q\} dF^l(y) = \int_Q (1 - \eta_j(q)) d\chi_j(q) \quad \forall Q, j = 1, 2
   \]
Properties of Tuition Functions

- At each quality level, \( t^h(q) < t^l(q) \)
  - Otherwise colleges would strictly prefer high ability students

- Tuition is increasing in quality: \( q_1 > q_2 \Rightarrow t^i(q_1) > t^i(q_2) \)
  - Otherwise no students would choose lower quality college

- Public schools dominate at low quality levels, private at high:
  - At low \( q \), if cap \( T \) non-binding, public schools can charge \( g_1 - g_2 \) more tuition
  - At high \( q \), cap binds tightly \( \Rightarrow \) private schools more profitable

- Sorting by income
  - Holding fixed ability, higher income households more willing to pay for higher quality colleges
Parametric Example

- Pure club good model: $\theta = 1 \Rightarrow q = \eta a_h + (1 - \eta)a_l$
  - Households sell and buy ability in college market

- Set $\varphi = 1 \Rightarrow u(c, q) = \log c + \log(\kappa + q)$

- No R&B: $\phi = 0$

- No grants, and no public schools

- Uniform income distribution:

$$y \sim U \left[ \mu_y - \frac{\Delta_y}{2}, \mu_y + \frac{\Delta_y}{2} \right]$$

$$F^h(y) = F^l(y)$$

- Let $\mu_a = \frac{a_h + a_l}{2}$, $\Delta_a = a_h - a_l$
Questions

1. What are $\chi(q)$, $t^h(q)$, $t^l(q)$?

2. How do these objects depend on $\Delta y$?

3. How does market for college differ from market for fish?
Digression: Modeling College Like Fish

- Households endowed with $a_l$ or $a_h$ units of ability
- Sell and buy ability at centralized market at per unit price $p$
- Household problem:

$$\max_{c,q} \{\log(c) + \log(\kappa + q)\}$$

s.t.

$$c + pq = y + pa_i$$

- Market clearing:

$$p = \frac{\mu_y}{\mu_a + \kappa}$$

- “Tuition” (net price) function:

$$t^i_F(q) = pq - pa_i = (q - a_i) \frac{\mu_y}{\mu_a + \kappa}$$

1. Net price functions are linear in $q$, and
2. Price function does not depend on income inequality $\Delta y$
The Club Good Model

- College distribution: \( \forall Q \subset (a_l, a_h) \)

\[
\chi(Q) = \frac{2}{\Delta a} \left( \frac{2}{4 + \pi} \right) \int_Q \left[ (1 - \eta(q))^2 + \eta(q)^2 \right]^{-2} dq
\]

\[
\chi(a_h) = \chi(a_l) = \frac{2}{4 + \pi} = 0.28
\]

- Tuition functions:

\[
t^i(q) = \mu_y \left( \frac{q - a_i}{\kappa + q} \right) \left[ 1 - \left( \frac{2}{4 + \pi} \right) \frac{\Delta_y}{\mu_y} \arctan (1 - 2\eta(q)) \right]
\]

- Competitive equilibrium is **Pareto efficient**

1. Distribution of quality independent of \((\mu_y, \Delta_y, \kappa)\)
2. Price functions **non-linear** in \(q\)
3. Price functions **depend on** \(\Delta_y\)
Sketch of Solution Method

1. Given any college distribution \( \chi(q) \), derive income of households attending college \( q \): \( y_i(q; \chi(.)) \)

2. Given \( y_i(q; \chi(.)) \), household’s FOC gives an ODE that pins down the college tuition function: \( t^i(q; \chi(.)) \)

\[
\frac{dt^i(q; \chi(.))}{dq} \frac{1}{y_i(q; \chi(.)) - t^i(q; \chi(.))} = \frac{1}{\kappa + q}
\]

3. Given \( t^i(q; \chi(.)) \), derive a college profit function:

\[
\pi(q; \chi(.)) = \eta(q)t^h(q; \chi(.)) + (1 - \eta(q))t^l(q; \chi(.))
\]

4. Solve for \( \chi(q) \) from the functional equation

\[
\pi(q; \chi(.)) = 0
\]

- This is a Volterra integral equation of the second kind with degenerate kernels, which has an analytical solution
Tuition
Tuition

**Low Ability, \( \kappa = 0.01 \)**

- \( \Delta_y = 0.01 \mu_y \)
- \( \Delta_y = 1.98 \mu_y \)
- fish

**High Ability, \( \kappa = 0.01 \)**

- \( \Delta_y = 0.01 \mu_y \)
- \( \Delta_y = 1.98 \mu_y \)
- fish

College Quality
More Properties of Club Good Equilibrium

1. At any quality level $q \in (0, 1)$ colleges have 2 types of customer:
   - high ability with relatively low income receiving subsidy
   - low ability with higher income paying positive tuition

2. Increasing $\Delta y$:
   - raises (lowers) $t^l(q)$ for $q \geq (\leq) \mu_a$
   - lowers (raises) $t^h(q)$ for $q \geq (\leq) \mu_a$
   - raises tuition differential for high $q$, lowers diff. for low $q$
Quantitative Example: Calibration

- Income distribution: Pareto Log-Normal:
  \[ \ln y \sim EMG(\mu_i, \sigma^2, \alpha) \]

- \( \sigma^2 = 0.4117 \) (SCF, 2007)
- \( \alpha = 1.8 \) (Piketty-Saez, 2014)
- \( \mu^i \) s.t. \( E[y] = 1 \) and

\[
\frac{E[y|_{i=h}]}{E[y|_{i=l}]} = \frac{67,000}{45,000}
\]

- (avg. family income conditional on child’s AFQT score being above / below median, 1997 NLSY).
Preferences and College Technology

Preferences \((\varphi, \kappa)\), Technology: \((\theta, \phi)\)

1. Enrollment: 37.0\% \Rightarrow \kappa = 0.034

2. Tuition + R&B $17,823 to Agg. Cons. \Rightarrow \varphi = 0.0235

3. Room and Board $10,881 \Rightarrow \phi = 0.019

4. Peers vs. goods equally important in quality \Rightarrow \theta = 0.5

(targets for 2015-17; all 4 yr colleges)
5. Federal and state grant aid: $3,204 for public colleges, $2,893 for private colleges \( \Rightarrow g_1 = 0.0057, g_2 = 0.0051 \)

6. Tuition cap \( T \) set to replicate public share of 4 year enrollment, 0.695 \( \Rightarrow T = 0.0250 \)

7. Ability gap \( a_h - a_l \) drives within-school tuition dispersion

- College Board reports avg. price paid net of all subsidies (federal, state and institutional grant aid)
- Assume (i) everyone gets “federal and state grant aid ” (ii) all institutional aid goes to high ability

\[
\frac{\text{ave. net low ability tuition}}{\text{ave. net tuition}} = \frac{\$24,676}{\$17,823}
\]

\( \Rightarrow a_l = 0.275 \quad (a_h = 1) \)
College Quality Distribution

$\chi(q)$
Tuition Schedules

\[ t_h(q) \]

\[ t_l(q) \]

\[ t_h(q)/q \]

\[ t_l(q)/q \]
Avg. Ability and Tuition by Quality

**Fraction of high ability**

- y-axis: Fraction of high ability
- x-axis: q
- Graph shows a steep increase in the fraction of high ability as q increases.

**Average tuition**

- y-axis: Average tuition
- x-axis: q
- Graph shows a continuous increase in average tuition as q increases.
Table 1: First Moments: Model and College Scorecard Data

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Public</td>
<td>Private</td>
</tr>
<tr>
<td>Enrollment</td>
<td>0.258</td>
<td>0.112</td>
</tr>
<tr>
<td>Sticker TFRB $</td>
<td>19,168</td>
<td>47,018</td>
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<tr>
<td>Net TFRB $</td>
<td>12,797</td>
<td>29,373</td>
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<tr>
<td>Avg. family income $</td>
<td>54,044</td>
<td>111,763</td>
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<tr>
<td>Avg. ability / SAT</td>
<td>0.76</td>
<td>0.88</td>
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<table>
<thead>
<tr>
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<tbody>
<tr>
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<td>Data</td>
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<tr>
<td>No Coll.</td>
<td>Public</td>
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<tr>
<td>High ability kids</td>
<td>0.467</td>
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<tr>
<td>Low ability kids</td>
<td>0.793</td>
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<tr>
<td></td>
<td>Model</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>var.(log avg. net TFRB)</td>
<td>0.164</td>
</tr>
<tr>
<td>var.(log sticker TFRB)</td>
<td>0.229</td>
</tr>
<tr>
<td>var.(log avg. fam income)</td>
<td>0.331</td>
</tr>
<tr>
<td>var.(log avg. SAT)</td>
<td>0.011</td>
</tr>
<tr>
<td>corr.(log net TFRB, log income)</td>
<td>0.987</td>
</tr>
<tr>
<td>corr.(log net TFRB, log SAT)</td>
<td>0.687</td>
</tr>
<tr>
<td>corr.(log income, log SAT)</td>
<td>0.790</td>
</tr>
</tbody>
</table>
Experiments

1. Move income distribution back in time from 2014 to 1984:
   - $\alpha^{1984} = 2.7$ instead of $\alpha^{2014} = 1.8$
   - Adjust $\mu$ to hold average income fixed

2. Eliminate additional $311 grant for public colleges

3. Eliminate all federal and state grants for colleges ($3,204 for public and $2,893 for private)
### Reducing Income Inequality

<table>
<thead>
<tr>
<th></th>
<th>Less Inequality</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Public</td>
</tr>
<tr>
<td>Enrollment (%)</td>
<td>44.2</td>
<td>32.1</td>
</tr>
<tr>
<td>Net TFRB $</td>
<td>13,474</td>
<td>11,388</td>
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<tr>
<td>High abil. part. (%)</td>
<td>63.9</td>
<td>42.9</td>
</tr>
<tr>
<td>Low abil. part. (%)</td>
<td>24.7</td>
<td>21.5</td>
</tr>
</tbody>
</table>
Reducing Income Inequality

\[ \chi(q) \]

Benchmark

Less Income Inequality
# Eliminating Public Subsidies

<table>
<thead>
<tr>
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<th>No Public</th>
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<th>Baseline</th>
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<tbody>
<tr>
<td></td>
<td>All</td>
<td>All</td>
<td>Public</td>
<td>Private</td>
</tr>
<tr>
<td>Enrollment (%)</td>
<td>35.9</td>
<td>37.0</td>
<td>25.8</td>
<td>11.2</td>
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<tr>
<td>Net TFRB $</td>
<td>18,435</td>
<td>17,815</td>
<td>12,797</td>
<td>29,373</td>
</tr>
<tr>
<td>High abil. part. (%)</td>
<td>51.8</td>
<td>53.3</td>
<td>34.7</td>
<td>18.6</td>
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<tr>
<td>Low abil. part. (%)</td>
<td>20.0</td>
<td>20.7</td>
<td>16.9</td>
<td>3.8</td>
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Eliminating Public Subsidies

\( \chi(q) \)

Benchmark
Remove Public Subsidy
## Eliminating All Subsidies

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<th>No Subsidies</th>
<th>Baseline</th>
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<tbody>
<tr>
<td></td>
<td>All</td>
<td>All</td>
</tr>
<tr>
<td>Enrollment (%)</td>
<td>27.5</td>
<td>37.0</td>
</tr>
<tr>
<td>Net TFRB $</td>
<td>23,340</td>
<td>17,815</td>
</tr>
<tr>
<td>High abil. part. (%)</td>
<td>40.4</td>
<td>53.3</td>
</tr>
<tr>
<td>Low abil. part. (%)</td>
<td>14.5</td>
<td>20.7</td>
</tr>
</tbody>
</table>
Eliminating All Subsidies

\[ \chi(q) \]

- Benchmark
- Remove All Subsidies
Conclusions

• Widening income inequality driving enrollment down, tuition up

1. rich demand higher quality colleges ⇒ average college quality goes up

2. marginal high ability become poorer and are priced out ⇒ high ability students become scarcer and more expensive ⇒ increased cost of producing quality

• Small subsidies to public colleges support large public sector, effective in supporting high ability enrollment
• Eliminating all subsidies would drastically shrink college enrollment, push up tuition