THE POLITICS OF SOVEREIGN DEFAULT UNDER FINANCIAL INTEGRATION*

Marina Azzimonti
Stony Brook University and NBER

Vincenzo Quadrini
University of Southern California and CEPR

VERY PRELIMINARY AND INCOMPLETE

Abstract

In this paper we study the role of portfolio diversification on optimal default of sovereign debt in a two-country model with large economies that are financially integrated. Financial integration increases the incentives to default not only because part of the defaulted debt is owned by foreigners (the standard redistribution channel), but also because the endogenous macroeconomic cost for the defaulting country is smaller when financial markets are integrated. We show that the sovereign default of one country may be triggered by higher debt (liquidity) issued by other countries. Because the macroeconomic costs of default spill to other countries, creditor countries may find it beneficial ex-post to bail-out debtor countries. Although bailouts create moral hazard problems, they can be welfare improving also ex-ante.

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1 Introduction

The last 30 years have been characterized by a dramatic increase in the international integration of financial markets. This allowed governments to ‘export’ their public debt, that is, to borrow from foreign countries. Figure 1 plots the share of public debt for the US and the largest EU countries during the 1997-2010 period and shows that this share has increased substantially during this period.

Figure 1: Share of public debt held abroad. Left axis for solid line countries. Right axis for dotted line countries. Source: Merler and Pisani-Ferry (2012).

During the same period, countries have also ‘imported’ foreign public debt as domestic residents increased their holdings of securities issued by foreign countries. Figure 2 plots the ownership of debt instruments issued by foreign countries, including foreign government debt, for several countries since 1980. As can be seen from the figure, the ownership of foreign debt has increased dramatically, especially since the mid 1990s.

The two figures illustrate an important trend in global financial markets: the cross-country diversification of financial portfolios. This is a general trend that is not limited to debt instruments but it extends to portfolio investments and FDI. In this paper, however, we focus on debt instruments
and, especially, sovereign debt, because of their role in providing liquidity. The goal is to understand how the international diversification of portfolios affects: (i) the incentive of governments to default on sovereign debt; (ii) the spillover of the macroeconomic costs of default to other countries; (iii) the benefits (ex-post and ex-ante) for the creditor countries to bail-out defaulting countries.

Let’s start with the effect of portfolio diversification on the government choice to default. Of course, if a larger share of sovereign debt is held by foreigners, the incentive to default for the debtor country increases since it redistributes wealth from foreign residents to domestic residents. This mechanism is well recognized in the literature although the study of Broner, Martin and Ventura (2010) challenges its relevance. In this paper, however, we explore a different mechanism through which financial diversification increases the incentive of a country to default. We show that the ‘macroeconomic cost’ of default declines when the country is (internationally) financially diversified.

Why is the macroeconomic cost of default smaller when the country is financially diversified? The central mechanism is the disruption of financial
markets induced by default. When a government defaults on its debt, the holders of government debt incur capital losses. To the extent that financial wealth held by some agents is important for economic decisions, this has a negative effect on aggregate economic activities. Notice that this effect is present independently of whether the country is integrated or operates in a regime of financial autarky. In the latter case, default redistributes wealth between domestic agents. Still, heterogeneity within a country implies that redistribution is not neutral for both economic activity and aggregate welfare (as in D’Erasmo and Mendoza (2016)). When financial markets are integrated (and portfolios diversified), however, domestic residents hold a smaller share of wealth in domestic assets and a larger share in foreign assets. This implies that, when the domestic government defaults, the wealth losses of domestic residents (and, therefore, domestic redistribution) are smaller, which causes a smaller macroeconomic contraction. Then, being the macroeconomic cost smaller, the government has higher incentive to default.

The mechanism described above points out that it is not only the quantity of domestic debt held by foreigners that matters for the choice of a country to default but also the debt issued by foreign countries held by domestic agents. Of course, the quantity of foreign debt held by domestic agents depends on the external supply of foreign debt. This introduces a channel through which the supply of foreign debt affects the incentive of a country to default. More specifically, an increase in the issuance of foreign debt implies that in equilibrium domestic agents hold more of this debt and they are more diversified. Higher diversification then implies that the macroeconomic cost of default is lower, which in turn increases the government incentive to default even if the quantity of domestic debt held by foreigners remains unchanged. This shows that an increase in the stock of debt issued by ‘foreign countries’ (higher international liquidity) could trigger the default of the ‘domestic country’.

The role played by external factors for the choice of a country to default is an important dimension in which our paper differs from a large body of literature. The majority of studies on sovereign default focus on the internal factors that lead a country to default. For example, a sequence of negative productivity or fiscal shocks leads a country to borrow more and, if the economic conditions continue to deteriorate, it becomes optimal or necessary for the country to default. In our paper, instead, we show that the factors that could cause a country to default may not originate domestically. In particular, the debt issued by other countries (higher international liquidity)
may also induce a country to default.

To illustrate the importance of external factors, we consider a model with two large countries: the home country $h$, which is riskier than the foreign country $f$. The issuance of debt and its repayment are chosen optimally by the governments of both countries. Using this model we consider an exogenous change in the debt issued by the ‘foreign’ country and study how this affects the incentive of the ‘home’ country to default. As the debt of country $f$ increases, residents in country $h$ acquire more foreign debt. Thus, the holding of safe, nondefaultable debt increases in country $h$. Consequently, if the home government defaults, domestic agents face a proportionally smaller loss in their financial wealth, which in turn implies that the macroeconomic consequences of default are smaller. This reduces the macroeconomic cost of default in country $h$ and increases the incentive of its government to default.

Greater financial diversification also means that foreigners hold more debt issued by the home country. Therefore, when the home country defaults, the foreign country experiences larger financial losses. In addition to the direct capital losses, the foreign country also experiences a macroeconomic contraction. Therefore, financial diversification creates the conditions for real macroeconomic spillover across countries, which bring us to the second issue studied in the paper, that is, how portfolio diversification affects the international transmission of default to the real sector of nondefaulting countries.

The international spillover has important policy implications: when a country defaults, the other country may have an incentive to bailout the debtor country in order to guarantee the repayment of the debt. In the model a bailout takes the form of a bargaining problem between creditor and debtor countries. The two countries negotiate a financial transfer from the creditor country to the debtor country against a higher repayment of the debt. In those states in which a country has an incentive to default, the ex-post bailout is Pareto improving. However, the anticipation of bailout also encourages the country to borrow more in the first period, which captures the typical moral hazard problem associated with bailout. Despite the moral hazard problem, bailouts may not be inefficient, that is, the ex-ante welfare without bailouts could be lower for both countries.

The possibility that bailouts could be efficient also ex-ante derives from the assumption that countries choose their own debt in period 1 without coordination. This implies that, when a country chooses its debt, it ignores the liquidity benefits that the debt brings to foreigners since part of the debt will be held by residents in the other country. The resulting equilibrium is
then characterized by sub-optimally low issuance of worldwide debt relative to the autarky regime. The anticipation of bailout encourages the more issuance of debt because in the event of default part of the debt will be repaid by the other country, effectively reducing the cost of borrowing. In this way, the anticipation of bailouts partially corrects for the (inefficient) low issuance of public debt induced by the lack of policy coordination.

2 Literature review

This paper builds on a large literature on public debt with incomplete markets. The main role of government debt in our paper is to partially complete the assets market when agents are subject to uninsurable idiosyncratic risk. The mechanism is similar to the one studied in Aiyagari and McGrattan (1998), Golosov and Sargent (2012), and Floden (2001), who study heterogeneous agents models without default. Closer to our paper is Azzimonti, de Francisco, and Quadrini (2014, AFQ henceforth), in which debt is held by agents for consumption smoothing (self-insurance). There are, however, three main departures from this paper. First, our economy is subject to both idiosyncratic and aggregate uncertainty, whereas AFQ considers only idiosyncratic uncertainty. Without aggregate uncertainty sovereign default would never arise in equilibrium. Second, the stock of public debt affects labor markets and hence the aggregate level of production while in AFQ aggregate production was fixed and, therefore, public debt did not have any macroeconomic implications. Finally, and contrary to AFQ, debt can be partially defaulted. Because of the possibility of default, our paper is also related to a growing literature on external sovereign default that builds on Eaton and Gersovitz (1981) (e.g. Aguiar and Amador (2013), Aguiar and Gopinath (2006), Arellano (2008), Cuadra, Sanchez, and Sapriza (2010), Pouzo and Presno (2014), Yue (2010), among others). Aguiar and Amador (2014) and Tomz and Wright (2012) provide recent reviews of this literature.

Our paper is also related to studies that make the cost of default endogenous by assuming that public debt provides liquidity and study the role of secondary markets (see Guembel and Sussman (2009), Broner, Martin, and Ventura (2010), Broner and Ventura (2011), Gennaioli, Martin, and Rossi (2014), Basu (2009), Brutti (2011), and Di Casola and Sichlimiris (2014)). Extending the work of Gennaioli, Martin, and Rossi (2014), some recent papers study the interaction between sovereign debt and domestic financial institutions (e.g. Sosa-Padilla (2012), Bocola (2014), and Perez (2015)). As in our paper, the cost of default is endogenous as it disrupts production and causes a recession. One important difference with our paper, however, is that these studies focus on small open economies which is the mainstream approach in the literature. Our paper, instead, emphasizes the importance of foreign factors by studying large open economies that operate in a globalized market. As emphasized in the introduction, this framework allows us to study how the world supply of financial assets affects the incentive to default and how macroeconomic consequences of default are transmitted to other countries (spillovers). International spillovers allow us to study the optimality of bailouts from the prospective of creditor countries.

Arellano and Bai (2008) also consider an environment in which the choices of debt and default affect other countries. The channel is based on the interest rate change. Our channel of transmission, instead, relies on the degree of portfolio diversification which is important for the international transmission of macroeconomic recessions. Our paper is also related to contributions that study debt restructuring through bargaining as Yan (2010) and Bai and Zhang (2009).

3 The model

We analyze a two-period economy composed of two large countries, ‘home’ and ‘foreign.’ We will use the superscripts $h$ and $f$ to denote, respectively, the home and foreign country. The governments of the two countries borrow in period 1 and repay the debt or default in period 2.

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1See also Borri and Verdelhan (2009), Park (2013), Lizarazo (2013) and Volkan (2013), Pouzo and Presno (2011).

2See Niepelt (2016) and Mihalache (2016) for an alternative renegotiation protocols.
The only possible ‘ex-ante’ difference between the two countries is in the degree of commitment to repay their public debt in period 2. In particular, we assume that with probability $\rho^i$, country $i \in \{h, f\}$ keeps its commitment to repay in period 2 and with probability $1-\rho^i$ it will opportunistically choose whether to default and repay a smaller amount. We denote the commitment state in period 2 by $\xi \in \{\text{Commit}, \text{Not Commit}\}$. One important factor that could affect the commitment of the government is political turnover. In period 2 countries could also differ in productivity as we will see below.

In each country there are two types of agents: a measure 1 of workers and a measure 1 of entrepreneurs. The assumption that the number of workers is the same as the number of entrepreneurs is without loss of generality. In the first period workers receive the endowment $e$ and entrepreneurs receive the endowment $a$. The endowments received by workers and entrepreneurs are the same in the two countries (and therefore, they are not indexed by the country superscript $i$). We can think of $e$ and $a$ as the wealth of workers and entrepreneurs accumulated up to period 1. Given their wealth, agents make consumption/saving decisions and move to the second period. In period 2 entrepreneurs produce with the input of labor hired from workers. Therefore, production takes place only in period 2. Workers also receive the endowment $e$ in period 2.

Workers value consumption and leisure with the utility

$$U(c^i_1) + \beta U\left(\varphi(c^i_2, h^i_2)\right),$$

where $c^i_1$ and $c^i_2$ denote consumption in period 1 and 2, respectively, and $h^i_2$ is the supply of labor in period 2. To simplify the analysis we assume that

$$U(.) = \log(.) \quad \text{and} \quad \varphi(c, h) = c - \frac{\alpha \nu}{1+\nu} h^{1+\nu}.$$

Workers receive lump-sum transfers from the government in period 1 and in period 2, and are excluded from financial markets (hand-to-mouth). Therefore, workers’ consumptions in the two periods are

$$c^i_1 = e + T^i_1,$$
$$c^i_2 = e + w^i_2 h^i_2 + T^i_2,$$

where $w^i_2$ denotes the wage rate earned in period 2, $T^i_1$ denotes the government transfers in period 1 and $T^i_2$ the government transfers in period 2. The stark
assumption that workers cannot borrow simplifies the exposition but it is not essential. Our results would hold in an environment in which workers have access to financial markets but they are subject to a borrowing limit.

The utility of entrepreneurs takes the form

\[ u(d_1^i) + \beta u(d_2^i). \]

where \( d_1^i \) and \( d_2^i \) denote their consumption levels in periods 1 and 2, respectively. Also for entrepreneurs we assume that their utility takes the logarithmic form, that is, \( u(.) = \log(.) \).

Entrepreneurs produce in the second period using a linear technology

\[ y_2^i = A(z_2^i, \varepsilon_2)l_2^i, \]

where \( l_2^i \) is the input of labor, \( z_2^i \) is an aggregate productivity shock which is country specific (and therefore, it is indexed by the superscript \( i \in \{h, f\} \)), and \( \varepsilon_2 \) is an idiosyncratic productivity shock.

We assume that the draw of aggregate productivity \( z_2^i \) takes place at the beginning of period 2 before entrepreneurs choose the input of labor. However, the idiosyncratic productivity \( \varepsilon_2 \) is observed only after hiring labor. The assumption that the idiosyncratic productivity is observed after the choice of the production scale makes the hiring decision risky for entrepreneurs. The importance of this assumption will become clear when we describe the labor market equilibrium.

There is no market for contingent claims and the only assets that entrepreneurs can trade are one-period government bonds. If financial markets are integrated, entrepreneurs can hold bonds issued by home and foreign governments. Without financial integration they can hold only bonds issued by their own government. But independently of financial markets regime, entrepreneurs cannot perfectly insure the idiosyncratic risk because government bonds are not contingent on the realization of these shocks.

Let \( B^h \) be the outstanding debt of the home country. Furthermore, denote by \( B^{hh} \) and \( B^{hf} \) the home debt purchased, respectively, by entrepreneurs in home and foreign countries. In equilibrium \( B^h = B^{hh} + B^{hf} \). Similarly, the outstanding debt of the foreign country is denoted by \( B^f \), in part purchased by entrepreneurs in the home country, \( B^{fh} \), and in part by entrepreneurs in the foreign country, \( B^{ff} \). Therefore, the first superscript indicates the nationality of the government that issued the debt and the second superscript indicates the nationality of the holders of debt (entrepreneurs).
4 Model in period 2

We characterize first the equilibrium in period 2 taking as given the debt issued by the two countries in period 1 and their holdings from entrepreneurs, as well as the governments’ commitments to repay $\xi^i$. This allows us to characterize some of the key results of the paper as if the model has only one period. Since in this section all variables refer to period 2, we abstract from time subscripts.

The aggregate states of the economy at the beginning of period 2 are $s = (B^{hh}, B^{hf}, B^{fh}, B^{ff}, z^h, z^f, \xi^h, \xi^f)$. They include the holdings of public debt, $B^{hh}, B^{hf}, B^{fh}, B^{ff}$, aggregate productivities, $z^h, z^f$, and the commitment state for repayment $\xi^h, \xi^f$.

The policy variables for the governments are the repayment rates $\pi = (\delta^h, \delta^f)$, where $\delta^i \leq 1$. We allow for partial default which is a common feature of the data (see Arellano, Mateos-Planas and Rios-Rull (2013)). The idiosyncratic shock, $\varepsilon$, is realized after the repayment decision of the government and after the hiring decisions of entrepreneurs. Following is the detailed description of timing.

1. The economy starts with debt holdings $B^{hh}, B^{hf}, B^{fh}, B^{ff}$, aggregate productivities $z^h, z^f$ and commitment states $\xi^h, \xi^f$.

2. Each government of country $i$ chooses the fraction of debt that will be repaid, $\delta^i$. With commitment ($\xi^i = \text{Commit}$) $\delta^i = 1$. Without commitment ($\xi^i = \text{NotCommit}$) $\delta^i$ is chosen optimally to maximize country $i$ welfare and, therefore, it could be smaller than 1.

3. Entrepreneurs choose the input of labor $l^i$ and workers choose the supply of labor $h^i$. The wage $w^i$ clears the labor market in each country $i \in \{h,f\}$.

4. The idiosyncratic productivity $\varepsilon$ is realized. Production and consumption take place.
4.1 Equilibrium for given policies

We start characterizing the competitive equilibrium for given repayment policies. The problem solved by workers is

\[
\max_{c^i, h^i} U\left(\varphi(c^i, h^i)\right) \tag{1}
\]
subject to
\[
c^i = e + w^i h^i - \delta^i B^i.
\]

The problem solved by entrepreneurs is

\[
\max_{l^i, d^i(\varepsilon)} \mathbb{E}_\varepsilon u\left(d^i(\varepsilon)\right) \tag{2}
\]
subject to
\[
d^i(\varepsilon) = \left[A(z^i, \varepsilon) - w^i\right]l^i + \delta^h b^{hi} + \delta^f b^{fi}.
\]

Notice that we have denoted by \(b^{hi}\) and \(b^{fi}\) the individual bonds holding of an entrepreneur in country \(i\) while the aggregate holdings are denoted in capital letters \(B^{hi}\) and \(B^{fi}\). Even though in equilibrium \(b^{hi} = B^{hi}\) and \(b^{fi} = B^{fi}\), we would like to derive the optimal decisions for any individual portfolio holding.

In solving the above optimization problems, workers and entrepreneurs take as given the governments’ policies, \(\pi = (\delta^h, \delta^f)\), and the wage rate \(w^i\). Following is the definition of a competitive equilibrium.

**Definition 1** A competitive equilibrium in period 2 for given aggregate states \(s\) and government policies \(\pi\) is defined by decision functions for workers, \(h^i = f^i_h(s; \pi)\) and \(c^i = f^i_c(s; \pi)\), decision functions for entrepreneurs, \(l^i = f^i_l(b^{hi}, b^{fi}, s; \pi)\) and \(d^i(\varepsilon) = f^i_d(b^{hi}, b^{fi}, s, \varepsilon; \pi)\), wage rate \(w^i = f^i_w(s; \pi)\), such that: (i) the decision functions of workers and entrepreneurs solve, respectively, problems (1) and (2); and (ii) labor markets clear, that is, for \(i \in \{h, f\}\), \(f^i_h(s; \pi) = f^i_l(B^{hi}, B^{fi}, s; \pi)\).

Because of the concavity of the utility function and the assumption that the hiring decision is made before observing the idiosyncratic productivity, entrepreneurs take into account the risk associated with production. The following lemma characterizes the optimal entrepreneurs’ policies.
Lemma 2 Let $\phi^i$ satisfy the condition $\mathbb{E}_z \frac{A(z, \varepsilon) - w^i}{1 + [A(z, \varepsilon) - w^i] \phi^i} = 0$. The entrepreneur’s policies in country $i$ are

$$h^i = \phi^i \left( \delta^h b^i + \delta^f b^f \right),$$

$$d^i(\varepsilon) = \begin{cases} 1 + \left( A(z^i, \varepsilon) - w^i \right) \phi^i \left( \delta^h b^i + \delta^f b^f \right) & \text{Big}. \end{cases}$$

Proof. See Appendix C. ■

Of special interest is the hiring policy of the firm (first equation in the lemma) which depends on the post-default wealth of the entrepreneur, $\delta^h b^i + \delta^f b^f$. This is because labor is risky and when individual wealth falls, the entrepreneur is less willing to take risks. To reduce the risk the entrepreneurs hires fewer workers. The aggregation of individual decisions then implies that equilibrium employment and wage depend positively on the debt repayment policies $\pi = (\delta^h, \delta^f)$. This is made precise by the following lemma.

Lemma 3 In the competitive equilibrium

1. The hiring factor $\phi^i$ is strictly decreasing in $\delta^h$ and $\delta^f$;

2. The wage rate $w^i$ and employment $h^i$ are increasing in $\delta^h$ and $\delta^f$.

Proof. See Appendix A. ■

Therefore, if the government of either country decides to default, that is, $\delta^h < 1$ and/or $\delta^f < 1$, both employment and wages decline. The central mechanism through which default generates a macroeconomic contraction is by destroying the financial wealth of entrepreneurs. This has two effects. The first effect is to redistribute wealth from entrepreneurs (who hold government debt) to workers (who pay taxes to repay the debt). Of course, lower is the holding of the debt issued by country $i$ by entrepreneurs in country $i$, and higher is the cross-country redistribution. The second effect, which is a consequence of the first, is to generate a macroeconomic contraction:

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$^3$The assumption that only workers pay taxes is not essential. The mechanism would still operate if taxes were equally paid by workers and entrepreneurs. What matters is that taxes are not proportional to the holding of public debt so that default implies that agents who hold the debt (entrepreneurs) experience a net loss while agents who do not hold the debt (workers) experience a net gain.
entrepreneurs end up with lower financial wealth and, since labor is risky, they hire fewer workers. For country \( i \), the macroeconomic effect decreases if a larger share of the debt issued by country \( i \) is held by entrepreneurs in the other country.

Although the redistributive effect of default is beneficial for workers, the recessionary effect has negative consequences for them: it reduces the demand for labor and, therefore, their wages. Thus, from the perspective of workers, government default implies a trade-off: the benefit is the reduction of taxes; the cost is the reduction of income. From the perspective of entrepreneurs, instead, government default implies only a cost: in addition to losing part of their financial wealth, they also earn lower incomes. The different welfare effects of default on workers and entrepreneurs will be key for understanding the optimal choice of government policies.

For comparison, we can also characterize the autarky environment in which entrepreneurs can only hold bonds issued by their own countries. This is the special case in which \( b_{ji} = B_{ji} = 0 \) if \( i \neq j \).

The key difference between autarky and financial integration is that in the latter entrepreneurs hold a portfolio of bonds issued by both home and foreign governments. This has three implications. First, since part of the public debt is held by foreigners, the home government may have a higher incentive to default. Second, the holding of foreign assets in the portfolio of home entrepreneurs (financial diversification) reduces the macroeconomic cost of default for the home country. An implication of this is that, when a foreign country issues more debt and this is associated with entrepreneurs in the home country holding a larger share of foreign debt (a property that we will see holds in our model), the macroeconomic cost of defaulting for the home country becomes smaller. This, in turn, rises the incentive of the home country to default. Third, the default of the home country affects employment and output in both countries. In other words, the macroeconomic consequences of sovereign default are exported to other countries (spillover). While the first implication is common to most of the sovereign default models proposed in the literature, the second and third implications are special features of our model.

### 4.2 Determination of government policies

Given the aggregate states \( s = (B^{hh}, B^{hf}, B^{fh}, B^{ff}, z^h, z^f, \xi^h, \xi^f) \), the government of country \( i \in \{h, f\} \) chooses the repayment rate \( \delta^i \). If the go-
ernment remains committed to repay, that is, $\xi^i = \text{Commit}$, then $\delta^i = 1$. However, if $\xi^i = \text{Not Commit}$, the government behaves opportunistically and will default if this improves social welfare. When $\xi^i = \text{Not Commit}$, the optimization problem for the government of country $i$ is

$$
\max_{\delta^i \leq 1} (1 - \Psi) U \left( \varphi(c^i, h^i) \right) + \Psi \mathbb{E}_\varepsilon u(d^i(\varepsilon)) \\
h^i = f^i_h(s; \pi) \\
c^i = f^i_c(s; \pi) \\
d^i(\varepsilon) = f^i_d(B^{hi}, B^{fi}, s; \pi)
$$

where $\Psi$ denotes the weight assigned to entrepreneurs and the variables $h^i, c^i, d^i(\varepsilon)$ are determined by the equilibrium functions as defined in the competitive equilibrium for given policies.

Appendix D shows that, using the conditions defining a competitive equilibrium, the optimization problem can be written as

$$
\max_{\delta^i \leq 1} (1 - \Psi) \ln \left( \bar{\nu}(w^i)^{1+\nu} - \delta^i B^i \right) + \\
\Psi \left[ \ln \left( \delta^h B^{hi} + \delta^f B^{fi} \right) + \mathbb{E}_\varepsilon \ln \left( 1 + \left[ A \left( z^i, \varepsilon \right) - w^i \right] \phi^i \right) \right],
$$

where $w^i$ and $\phi^i$ are determined by the equilibrium functions $f^i_w(s; \pi)$ and $f^i_\phi(s; \pi)$.

In solving this problem, the government of country $i$ takes as given the policy of the other country, $\delta^{-i}$. By choosing $\delta^i$ the government affects consumptions and labor through the equilibrium functions $f^i_w(s; \pi)$ and $f^i_\phi(s; \pi)$.

To characterize the solution (conditional on the repayment policy of the other country), consider first the relaxed optimization problem where the repayment of the government debt is not subject to the constraint $\delta^i \leq 1$. Assuming that the government objective is strictly concave, there will be a unique solution. The first order condition, derived in Appendix E, takes the form

$$
\Psi E_\varepsilon u' \left( d^i(\varepsilon) \right) = (1 - \Psi) U' \left( \varphi^i(c^i, h^i) \right) \Omega^i(s, \pi),
$$

where the prime denotes derivatives.

The term $\Omega^i(s, \pi)$ in general bigger than 1. However, in the special case in which $B^{hf} = B^{fh} = 0$—which corresponds to financial autarky—$\Omega^i(s, \pi) = 13$
1. Therefore, without financial integration, the government would equalize the marginal utilities of consumption for entrepreneurs and workers, re-scaled by their relative weights $\Psi$ and $1 - \Psi$. With financial integration, instead, since $\Omega^i(s, \pi) > 1$, it is as if the government assigns a higher weight to workers. This implies that, keeping everything else equal, the incentive to default is higher when countries are financially integrated.

There are two reinforcing channels that generate the higher incentive to default with financial integration. The first channel arises from the redistribution of wealth from foreigners to domestic agents. Because some of the domestic debt is held by foreigners, default redistributes wealth not only from domestic entrepreneurs to domestic workers but also from foreign entrepreneurs to domestic workers. The redistribution from foreigners to domestic agents is also a feature of many other sovereign default models studied in the literature. The focus of our paper, however, is not on this channel. Instead, our paper focuses on the second channel, which derives from the fact that default generates lower financial losses for domestic entrepreneurs thanks to their international diversification. Compared to the autarky regime, this implies that the macroeconomic impact of default for the domestic country is smaller. This mechanism, which is novel in the sovereign default literature, increases the incentive of the government to default.

Using the first order condition (5) we derive the following result.

**Proposition 4** Let $\epsilon = 0$ and $A(z^i, \epsilon) = z^i + \epsilon$. The unconstrained optimal repayment rate $\delta^i$, conditional on the repayment of the other country $\delta^{-i}$, is strictly increasing in aggregate productivity $z^i$.

**Proof.** See Appendix B

This result shows that the incentive to default—that is, the incentive to repay a lower fraction of the debt—is higher when the country is in a recession. By repaying less debt, the government makes the recession deeper: in addition to the direct impact of lower productivity on employment and output, the destruction of entrepreneurial wealth associated with the lower repayment further discourages the demand for labor. This implies that the optimal government policy amplifies the recession. Still, from the government point of view, the policy is welfare improving.

To understand why the government prefers to repay less when the productivity is low, we should reconsider the two effects of default described earlier.
The first effect is to redistribute wealth from entrepreneurs to workers. The second is to induce a macroeconomic recession: a lower repayment implies lower entrepreneurs’ wealth which reduces the demand for labor. This is also harmful for workers. However, since the consumption of workers is lower when productivity falls, their marginal utility is higher. From the perspective of the government, this increases the benefit of redistributing wealth toward workers and, therefore, the incentive to default (the first effect). Also, since labor is less productive, the loss of output from distorting the demand of labor is smaller (the second effect).

Denote by $\hat{\delta}_i(s, \delta^{-i})$ the unconstrained optimal repayment rate of country $i$, given the repayment rate of the other country, $\delta^{-i}$. Once we know the unconstrained optimal repayment, the solution to the constrained problem (4) is the minimum between $\hat{\delta}_i(s, \delta^{-i})$ and 1. Thus, the optimal policy is

$$g^i(s, \delta^{-i}) = \begin{cases} 1, & \text{if } \xi^i = \text{Commit} \\ \min \left\{ 1, \hat{\delta}_i(s, \delta^{-i}) \right\}, & \text{otherwise} \end{cases}$$

The function $g^i$ is the optimal response function of country $i$ to the policy of the other country. A Nash equilibrium is defined by the pair $(\delta^h^*, \delta^f^*)$ that satisfies the conditions

$$\delta^h^* = \tilde{g}^h(s, \delta^f^*),$$

$$\delta^f^* = \tilde{g}^f(s, \delta^h^*).$$

We denote the equilibrium policies by $\pi^{Nash}(s) = (\delta^h^*, \delta^f^*)$.

**Numerical example.** Since we are analyzing the equilibrium in period 2 for given states $s = (B^{hh}, B^{hf}, B^{fh}, B^{ff}, z^h, z^f, \xi^h, \xi^f)$, for the numerical example we only need to set the utility parameters, the workers’ endowment, the distribution of the idiosyncratic productivity shock and the welfare weights. The parameters for the utility of workers are set to $\nu = \alpha = 1$ and their endowment to $e = 1$. The idiosyncratic productivity shock takes the values $\{0.9, 1, 1.1\}$ with equal probabilities. The weight assigned by the government to entrepreneurs is $\Psi = 0.5$.

To simplify the presentation of the model’s properties, we start with the autarky equilibrium, that is, when $B^{hf} = B^{fh} = 0$. This allows us to characterize the equilibrium in period 2 as if there is only one country.
Figure 3 plots the government indirect utility in period 2 for two levels of aggregate productivity. The graph is constructed using a specific parametrization of the model. Since the graph is only meant to provide a numerical example, we will postpone the discussion of the parameter values.

Figure 3: Government weighted utility in period 2 as a function of outstanding debt $B$. There are four lines. The first line (continuous) is when aggregate productivity is low and the government does not commit to repay the debt. The second line (short dashed) is when aggregate productivity is low but the government commits to repay the debt in full. The third line (long dashed) is when aggregate productivity is high and the government does not commit to repay the debt. The fourth line (dotted and dashed) is when aggregate productivity is high and the government commits to repay the debt.

The solid line refer to the case in which productivity is low ($z = z_L$) and the government does not commit to repay the debt ($\xi^i = \text{Not Commit}$). The short-dashed line also refers to the case of low aggregate productivity ($z = z_L$) but the government cannot default ($\xi^i = \text{Commit}$). Focusing on these two lines we observe that they have both a maximum. However, when the government has the option to default, welfare becomes flat. This is because what matters for welfare is the actual repayment. Thus, if $B$ is bigger than the optimum, the government simply repays what is optimal. Effectively, this brings the debt to the optimal (lower) level and, therefore, beyond this point the welfare function is flat. However, if the option to default is not available, welfare starts to decline after the optimal level of $B$.

The other two lines are for the case in which aggregate productivity is high
In this case the debt level that maximizes welfare is higher than in the case of low aggregate productivity (that is, the pick in the dotted-shaded line is bigger than the pick in the short-dashed line). This is further shown in Figure 4 which plots the debt repayment as a function of the initial debt for different values of $z$ and $\xi$. The figure also shows the role of the relative weight $\Psi$ assigned by the government to entrepreneurs. A higher weight (left panel) is associated with higher repayments in absence of commitment.

![Figure 4: Debt repayment in period 2 conditional on aggregate shocks and for different welfare weight $\Psi$.](image)

We now move to the economy in which financial markets are integrated. We will make the following assumption about the compositions of portfolios. We assume that $B_{hf} = B_{hh}$ and $B_{fh} = B_{ff}$. This implies that the debt issued by the home country, $B^h$, half is held by domestic entrepreneurs and half is held by foreign entrepreneurs. The same for the debt issued by the...
foreign country, $B^f$. Although this assumption seems arbitrary at this stage, we will see later that, when entrepreneurs choose their portfolio optimally in period 1, this is an equilibrium outcome.

With this assumption, we now analyze how the repayment of the home country depends on the repayment of the foreign country.

Figure 5 plots the optimal repayment of the home country, $\delta^h B^h$, as a function of the repayment of the foreign country, $\delta^f B^f$. We first fixed the outstanding debt of the home country, $B^h$, and then we plot the optimal repayment $\delta^h B^h$ as we change the repayment of the foreign country, $\delta^f B^f$. In the left panel $B^h = 0.485$ and in the right panel $B^h = 0.51$. Each panel reports the best responses of the home government when home productivity is low ($z = Z_L$) and when home productivity is high ($z = Z_H$).

Figure 5: Optimal debt repayment in period 2 from home country as a function of foreign repayment.
The figure shows that, as the repayment of the foreign country increases, it is optimal for the home country to repay less. For low repayments from the foreign country, the repayment of the home country becomes flat because of the constraint $\delta^h \leq 1$. In this case the home country repays the full debt (which in the left panel is $B^h = 0.485$ and in the right panel is $B^h = 0.51$).

**Portfolio diversification and default.** Figure 5 illustrates the importance of portfolio diversification for the incentive to default.

As already discussed, international holdings of public debt affects the incentive to default through two channels. The first is the typical redistributive channel: higher is the ‘home’ debt held by foreign residents, and higher is the redistribution of wealth from foreign residents to home residents when the home government defaults. This is the standard channel which is also embedded in the majority of sovereign default models studied in the literature. The second channel, instead, operates through domestic portfolio diversification. Higher is the ‘foreign’ debt held by home residents and lower is macroeconomic disruption when the home government defaults. This is the novel mechanism that we emphasize in this paper.

To be more specific, the redistributive channel depends on $B^{hf}/B^h$, that is, the fraction of the debt issued by the home country and held by foreign countries. The second channel, instead, depends on $B^{fh}/(B^{fh} + B^{hh})$, that is, the fraction of wealth of home residents held in foreign countries. This is an indicator of portfolio diversification for home residents. In general, these two indices are highly correlated and, therefore, when $B^{fh}/(B^{fh} + B^{hh})$ increases, $B^{hf}/B^h$ also rises.

The numerical exercise illustrated in 5, however, isolates the portfolio channel from the redistributive channel. In fact, in the exercise, we keep the home debt $B^h$ fixed (0.485 in the first panel and 0.51 in the second panel). Since we assumed that $B^{hh} = B^{hf}$, the importance of the cross-country redistributive channel does not change. However, as we change the repayment from the foreign country, effectively we are increasing $B^{fh}$, which implies a higher diversification of portfolios for entrepreneurs in the home country. Thus, as the portfolio diversification in the home country increases, the home government finds optimal to repay less (higher incentive to default). To the extent that higher repayments from the foreign country derives from higher outstanding foreign debt, the exercise shows that the default of a country could be driven externally by the debt of other countries.
Although in this paper we have considered only financial assets issued by governments, the effect described above could also be driven by the expansion of private financial markets. For example, a financial boom in advanced economies, either private or public, could induce the default of some emerging countries even if financial and real conditions of these countries have not changed. It is in this sense that default could be externally driven and the mechanism through which this happens is through the domestic portfolio composition induced in equilibrium by financial expansions abroad.

4.3 Financial integration with bailouts

So far we have characterized the equilibrium under the assumption that the debt is not renegotiated when a country defaults. In many instances of sovereign default, debt is restructured and with restructuring there is some form of direct or indirect subsidies. For example, with rescue packages the defaulting country is able to borrow at rates that are lower than the market rates. In this section we allow for renegotiation and debt restructuring. This seems a natural assumption since both countries could gain from renegotiating the repayment of the debt.

Renegotiation takes place only if one of the two countries defaults, that is, it is optimal to choose $\delta^i < 1$ when $\xi^i = \text{Not Commit}$. With renegotiation the two countries bargain the debt repayment—which we denote by $\bar{\delta}^h$ and $\bar{\delta}^f$—and a transfer $\tau^i$ to country $i$ (paid by the other country). Thus $\tau^i = -\tau^{-i}$. To use a compact notation we denote the renegotiated policies as $\bar{\pi} = (\bar{\delta}^h, \bar{\delta}^f, \tau^h, \tau^f)$.

We assume that renegotiation takes the form of Nash bargaining. Let’s first define some key functions. Given the negotiated policies $\bar{\pi}$, the value for country $i \in \{h, f\}$ under renegotiation is

$$\mathcal{V}^i(s; \bar{\pi}) = (1 - \Psi) \ln \left( \nu^i w^i \right) - \delta^i B^i + \tau^i + \Psi \left[ \ln \left( \bar{\delta}^h B^{hi} + \bar{\delta}^f B^{fi} \right) + E^i \ln \left( 1 + \left[ A^i(\varepsilon^i, \varepsilon) - w^i \right] \phi^i \right) \right],$$ (6)

where $w^i$ and $\phi^i$ are determined by the equilibrium functions $f^i_w(s; \bar{\pi})$ and $f^i_\phi(s; \bar{\pi})$.

This is analogous to (4) but with repayments $\bar{\delta}^h$ and $\bar{\delta}^f$, and workers receive the transfer $\tau^i$ (with $\tau^i = -\tau^{-i}$).
To see how renegotiation affects the welfare of the two countries, consider the case in which the home country defaults. For both countries, higher repayment $\bar{\delta}^h$ implies positive effects coming from lower macroeconomic distortions (higher wages for workers and profits for entrepreneurs captured by the first and third terms of equations (6)) and higher repayment to entrepreneurs (captured by the second term). Higher repayments also imply a cost for the home country due to higher taxes that home workers have to pay (see first term in equation (6)). The transfer $\tau^h$, instead, is a benefit for the home country (if positive) since it reduces the tax burden of workers but it is a cost for the foreign country since foreign workers have to pay the transfer. Effectively, foreign workers help home workers to repay the debt of the home country. Of course the opposite arises if it is the foreign country that defaults.

The above analysis illustrates why the foreign country may gain from subsidizing the repayment of the home debt: since a higher $\bar{\delta}^h$ increases the welfare of the foreign country by facilitating higher demand of labor, the foreign government may be willing to pay $\tau^f = -\tau^h$ in order to induce a higher repayment from the home government. Since a higher repayment has also positive effects for the home country (in addition to the higher taxes for home workers), the foreign government can convince the home government to repay by ‘partially’ subsidizing the repayment. This will become clear in the numerical example we will show below. Before doing so, however, we need to define the bargaining problem formally.

It the two countries do not reach an agreement, they revert to the equilibrium without renegotiation as described in the previous section. Therefore, the threat values are those associated with the uncoordinated policies, that is, $\delta^h = \delta^h^*, \delta^f = \delta^f^*, \tau^h = \tau^f = 0$. Denoting by $\pi = (\delta^h^*, \delta^f^*, 0, 0)$ the uncoordinated policies, the threat value is $V^i(s; \pi)$.

The bargaining problem can then be written as

$$\max_{\tau^h, \delta^h, \delta^f} \left[ V^h(s; \bar{\pi}) - V^h(s; \pi) \right]^\eta \left[ V^f(s; \bar{\pi}) - V^f(s; \pi) \right]^{1-\eta}$$

subject to

$$\bar{\delta}^i = 1, \quad \text{if} \quad \xi^i = \text{Commit},$$

$$\bar{\delta}^i \leq 1, \quad \text{if} \quad \xi^i = \text{Not Commit},$$

$$\tau^f = -\tau^h,$$
where $\eta$ is the relative bargaining power for the home country.

As it is standard, the bargaining problem maximizes the weighted product of the net renegotiation surpluses of the negotiating parties. The feasible repayment depends on the commitment of the two countries. Obviously, when both countries commit, $\tau^h = \tau^f = 0$. We denote the solution to the bargaining problem by $\bar{\pi}^{\text{Ren}}(s)$.

**Numerical example.** To characterize the bargaining solution we resort to the numerical example of the previous section. Baseline parameters are identical to the ones described earlier. The only additional parameter is the bargaining weight which we set to $\eta = 0.5$.

Figure 6 plots the repayment of the debt (total and by the home country) when $\xi^h = \text{Not Commit}$ and $\xi^f = \text{Commit}$; that is, when only the home country does not commit to repay. Total repayment is the fraction $(\delta^h B^h + \delta^f B^f)/(B^h + B^f)$ without bailout and $(\tilde{\delta}^h B^h + \tilde{\delta}^f B^f)/(B^h + B^f)$ with bailout. The home repayment is the fraction $\delta^h$ without bailout and $(\tilde{\delta}^h B^h - \tau^h)/B^h$ with bailout. Figure 7 depicts the case in which both countries have no commitment to repay, $\xi^h = \xi^f = \text{Not Commit}$.

The two figures shows that with bailout the total debt $B^h + B^f$ is fully repaid for relatively low values of debt (that is not the case for large values of debt, not depicted in these figures). Furthermore, higher is the debt of the home country and lower is the fraction $(\tilde{\delta}^h B^h - \tau^h)/B^h$ repaid by the home country. Although not shown, the repayment of the home country increases with the debt of the foreign country. Essentially, starting with higher debt is similar to having more bargaining power. Therefore, when the foreign country starts with higher debt, the home country repays more. Lastly we observe that the home repayment is higher when the productivity of the home country is high and the productivity of the foreign country is low.

5 Full model

The analysis conducted so far focused on period 2, for given debts issued by the two governments and the portfolio composition of private agents. We now characterize the full two-period model. But doing so we will be able to characterize the optimal issuance of debt chosen by the two governments and the portfolios composition chosen by private agents.
Figure 6: Debt repayments with and w/o commitment when only the home country does not commit. The repayments are plotted as functions of the home debt, when the foreign debt is $B_f = 0.677$. ‘Total repay w/o bailout’ is the fraction $(\tilde{B}_h + \tilde{B}_f)/(B_h + B_f)$ and ‘Total repay with bailout’ is $(P_h + P_f)/(B_h + B_f)$. ‘Home repay w/o bailout’ is the fraction $\tilde{B}_h/B_h$ and ‘Home repay with bailout’ is the fraction $(P_h - \tau_h)/B_h$. Each line corresponds to different combinations of productivity in the two countries.

In solving the optimization problems in period 1, agents and governments anticipate the equilibrium that will prevail in period 2 as characterized in the previous section. In particular, they anticipate that the government policies and the portfolios compositions chosen in period 1 will affect the equilibrium in period 2.

In period 1 the government of country $i \in \{h, f\}$ issue bonds $B^i$ at price $1/R^i$ and distributes all the revenues to workers. Therefore the consumption of workers in period 1 is $c^i = e + B^i/R^i$, where $e$ is an exogenous endowment that workers receive in both countries. As we described in the previous section, workers will then be taxed in period 2 when governments repay the debt. Effectively, the government borrows on behalf of workers. Notice
Figure 7: Debt repayments with and w/o commitment when both countries do not commit. The repayments are plotted as functions of the home debt, when the foreign debt is $B_f = 0.677$. ‘Total repay w/o bailout’ is the fraction $(\tilde{B}_h + \tilde{B}_f)/(B_h + B_f)$ and ‘Total repay with bailout’ is $(P_h + P_f)/(B_h + B_f)$. ‘Home repay w/o bailout’ is the fraction $\tilde{B}_h/B_h$ and ‘Home repay with bailout’ is the fraction $(P_h - \tau_h)/B_h$. Each line corresponds to different combinations of productivity in the two countries.

that, since the debt issued in period 1, $B^i$, is not restricted to be positive, the government could choose to save. However, we will focus on parameter values for which $B^i$ is positive.

Entrepreneurs also receive an endowment in period 1, denoted by $a$, part of which is consumed and part is saved in the form of government bonds issued by both countries. Therefore, the budget constraint for entrepreneurs in country $i$ in period 1 is $d^i = a - b^{hi}/R_h - b^{fi}/R_f$. 

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5.1 Equilibrium for given policies

Denote by \( \pi(s) = (\delta^h(s), \delta^f(s)) \) the equilibrium governments’ policy function in period 2 as characterized in previous sections. The policy function depends on the aggregate states \( s \) and on the particular environment under consideration: (i) financial autarky; (ii) financial integration without renegotiation; and (iii) financial integration with renegotiation. The equilibrium policy function is taken as given by agents and governments when they solve their optimization problem in period 1. In addition, private agents take as given the government policies chosen in period 1, that is, \( B^h \) and \( B^f \).

In period 1 workers behave passively in the sense that they consume the endowment plus the transfers received from their own government. Therefore, the lifetime utility of workers is
\[
W(B^h, B^f) = u(c_1^i) + \beta \mathbb{E}_s U(\varphi(c_2^i, h^i)) 
\]
with
\[
c_1^i = e + \frac{B^i}{R^i}, \quad c_2^i = e + f^i_w(s; \pi(s)) f^i_h(s; \pi(s)) - \delta^i(s) B^i, \quad h^i = f^i_h(s; \pi(s)).
\]

The functions \( f^i_h(s; \pi(s)) \) and \( f^i_w(s; \pi(s)) \) are, respectively, the supply of labor and the equilibrium wage in period 2 as defined in the previous section.

Entrepreneurs, instead, optimize also in period 1. They solve the problem
\[
V^i(B^h, B^f) = \max_{b^hi, b^fi} \left\{ u(d_1^i) + \beta \mathbb{E}_s, \varepsilon \left( d_2^i \right) \right\} 
\]
subject to
\[
d_1^i = a - \frac{b^hi}{R^h} - \frac{b^fi}{R^f}, \quad d_2^i = \left[ A(z^i, \varepsilon) - f^i_w(s; \pi(s)) \right] f^i_l(b^hi, b^fi, s; \pi(s)) + \delta^h(s)b^hi + \delta^f(s)b^fi.
\]

The function \( f^i_l(b^{hi}, b^{fi}, s; \pi(s)) \) is the equilibrium demand of labor in period 2 and \( f^i_w(s; \pi(s)) \) is the equilibrium wage in period 2. The demand of labor also depends on individual asset holdings that entrepreneurs choose in period 1. In equilibrium, of course, \( b^{hi} = B^{ih} \) and \( b^{fi} = B^{if} \). However, to derive the optimal decisions of entrepreneurs we have to distinguish individual
variables from aggregate variables. The following lemma characterizes the optimal entrepreneurs’ policies.

**Lemma 5** The entrepreneur’s policies in country $i$ are

$$d^i_1 = a \left(1 - \theta^h - \theta^f\right),$$

$$b^{hi} = \theta^h R^h a,$$

$$b^{fi} = \theta^f R^f a,$$

where $\theta^h$ and $\theta^f$ solve

$$1 + \beta = \mathbb{E}_s \left( \frac{1}{\theta^h \delta^h(s) R^h + \theta^f} \right),$$

$$\frac{\beta}{1 + \beta} = \theta^h + \theta^f.$$

**Proof.** See Appendix C. 

Entrepreneurs allocate a fraction $1 - \theta^h - \theta^f$ of their initial wealth $a$ for consumption and the remaining fraction $\theta^h + \theta^f$ is saved. Savings are then invested in home bonds and foreign bonds according to $\theta^h$ and $\theta^f$.

An important feature of the entrepreneurs’ policies stated in Lemma (5) is that the terms $\theta^h$ and $\theta^f$ are not indexed by $i$. This means that home and foreign entrepreneurs choose the same saving rates and the same composition of portfolios, that is, the same ratio of bonds issued by home and foreign governments. The choice of the same portfolio derives from the assumption that the two countries are identical in preferences and technology (including the distribution of the idiosyncratic shock) and the fact that entrepreneurs do not pay taxes or receive transfers.\(^4\)

We are now ready to define a competitive equilibrium in period 1.

**Definition 6** A competitive equilibrium in period 1 for given policy $(B^h, B^f)$ is defined by price functions $R^i = \mathbf{f}_R^i(B^h, B^f)$ and decision functions for

\(^4\)If entrepreneurs paid taxes or received transfers, their disposable income would be affected by the realization of $\xi^i$. Because the probability $\rho^i$ is country-specific, this would induce asymmetry in their portfolio decisions. Another important assumption for the portfolio result is the absence income effects on the supply of labor from workers.
entrepreneurs, \( b^{hi} = \xi^h_i(B^h, B^f), \) \( b^{fi} = \xi^f_i(B^h, B^f), \) \( d^{h1}_i = \xi^{1h}_{d_i}(B^h, B^f) \) such that (i) entrepreneurs’ decisions solve problem (8) and take the forms specified in Lemma 5; (ii) asset markets clear, that is, \( B^i = b^{ih} + b^{if}. \)

5.2 Optimal government policies in period 1

The objective for the government of country \( i \) in period 1 is

\[
\max_{B^i} \left\{ (1 - \Psi)W^i(B^h, B^f) + \Psi V^i(B^h, B^f) \right\}
\]

where the functions \( W^i(B^h, B^f) \) and \( V^i(B^h, B^f) \) are the equilibrium lifetime utilities of workers and entrepreneurs defined, respectively, in (7) and (8).

Denote by \( g^i(B^{-i}) \) the best response function of country \( i \) to the debt issued by the other country, \( B^{-i} \). A Nash equilibrium in period 1 is defined as the pair \( (B^{h*}, B^{f*}) \) that satisfies the conditions

\[
B^{h*} = g^h(B^{f*}), \quad B^{f*} = g^f(B^{h*}).
\]

Numerical example. For the numerical characterization, we use the same parameters we used in the previous examples used to characterize the equilibrium in period 2. The additional parameters that we need to set are as follows. The discount factor is set to \( \beta = 0.9825 \), the period endowments of workers and entrepreneurs are \( e = a = 1 \). The aggregate productivity shocks \( z^h \) and \( z^f \) are independently and identically distributed across countries. They take two values, \{0.95, 1.05\}, with equal probability. The probability of commitment in period 2 is set to \( \rho^h = \rho^f = 0.5 \).

Figure 8 plots the best responses in period 1 for both countries as functions of the other country’s debt. We consider three regimes: autarky, financial integration without renegotiation and financial integration with renegotiation. The autarky regime is a special case of the equilibrium with mobility where the entrepreneurs’ problem (8) is also subject to the constraint \( b^{ii} = 0 \) if \( i \neq j \). In other words, entrepreneurs can only purchase domestic bonds.
Figure 8: Response functions in period 1 for autarky, mobility without bailout and with bailout. Equilibrium determined by the intersection of the two response functions.

In the autarky regime (top panel), the response functions are constant since the equilibrium of one country is not affected by the debt chosen by the other country. With mobility (bottom panels), instead, the optimal debt depends on the debt chosen by the other country. In particular we observe that the optimal debt of one country decreases as the debt of the other country increases. This is because, when the foreign country issues more debt, domestic entrepreneurs hold more financial wealth in equilibrium (as a result of diversification). Therefore, there is less need for liquidity in the domestic country.

The comparison of the top panel with the bottom panels of Figure 8 shows that the equilibrium worldwide supply of debt, $B^h + B^f$, is significantly bigger when the economies are closed than in the regimes with financial integration (with and without renegotiation). This happens because, with mobility, part
of the debt issued by one country is purchased by entrepreneurs residing in the other country and, therefore, it brings benefits to the other country. But each government does not internalize the liquidity benefits that its debt creates for the other country. This implies that each government issues less debt than under autarky. This negative externality results in inefficiently low levels of debt when countries are financially integrated.\(^5\)

We now discuss the differences between the mobility environments with and without renegotiation (with and without bailout). With bailout the equilibrium debt of the two countries (intersection of the response functions) is significantly larger than the debt without bailout. This is because, knowing that in period 2 there will be a probability of bailouts, each country has an incentive to borrow more in the first period. Although the debt level with bailout is bigger than in the environment without bailout, it is still smaller than in autarky regime. The next question is whether the anticipation of bailouts in period is efficient from a welfare point of view.

### 5.3 To bailout or not to bailout?

Although renegotiation is always efficient in period 2, we have seen that its anticipation in period 1 induces higher borrowing and, therefore, higher ex-post incentive to default. It is then natural to ask whether bailouts are also efficient from an ex-ante prospective.

Figure 9 plots the country welfare in period 1 as a function of debt assuming that the foreign debt is equal to home debt. Since the two countries are symmetric and, therefore, in equilibrium they would choose the same debt, these (symmetric) levels of debt represent possible equilibrium outcomes. The curves represent the weighted expected utility in period 1 for one of the two countries and for the three regimes: autarky, mobility without bailout and mobility with anticipated bailout. The graph also indicates with the vertical lines the equilibrium debt in each of the three regimes. These are determined by the intersection of the best response functions shown in Figure 8.

The first thing to observe is that the welfare with bailout dominates the

\(^5\)Because we have not modeled the potential beneficial effects of financial integration, a move from autarky to financially integrated markets results in a welfare loss for both countries in our model. The objective of this paper is not to analyze the trade-off between having an open or a closed economy, but rather to discuss the implications of external forces on the incentives of a country to default.
Figure 9: Social welfare as a function of public debt when both countries choose the same debt in period 1, for three regimes: autarky, mobility without bailout and mobility with bailout. Social welfare is the $\Psi$-weighted utility of workers and entrepreneurs. The vertical lines indicate the debt in the symmetric Nash equilibrium for each of the three regimes.

Why does bailout allow for higher welfare than no bailout? To see why it would be useful to compare the autarky equilibrium with the equilibrium in which financial markets are integrated but there is no bailout. More debt allows for greater efficiency in period 2. However, with financial integration, when a country issues more debt the benefits are shared with the other country. Therefore, the incentive to issue debt is lower. In autarky, instead, the country fully internalizes the benefits of issuing its own debt and, therefore, more debt is issued in equilibrium. The result is that the utility in the au-
tarky equilibrium is higher than in the equilibrium with integrated financial markets. Mobility creates the conditions for sub-optimal issuance of debt.

Bailouts alleviate this problem. Each country anticipates in period 1 that with some probability the other country will contribute to the repayment of the other country’s debt. Therefore, the incentive to borrow increases, bringing the allocation closer to autarky. Renegotiation acts as a compensating mechanism for the positive externality created by government debt. It makes the effective cost of debt for the issuing country lower since, in the case of default, the other country contributes to its repayment.

5.4 The role of commitment

Countries do not always default in period 2. Whether default occurs in equilibrium depends on the stock of debt with which each country enters period 2, the realization of productivities, \( z = (z^h, z^f) \), and commitment states, \( \xi = (\xi^h, \xi^f) \). In this section we examine how the probability of commitment \( \rho^i \) affects the equilibrium debt chosen in period 1.

Although \( \rho^i \) is exogenous in the model, it can be interpreted as the result of political turnover. In particular, in period 2 there is a probability of a new government taking power that assigns a much higher weight to entrepreneurs (who prefer higher repayment).\(^6\)

In the previous numerical example we have set the probability of commitment for each country to \( \rho^i = 50\% \). Now we consider two alternative scenarios with, respectively, a probability of commitment of 60\% (high commitment) and 40\% (low commitment).

Table 1 reports the equilibrium debt and social welfare for different levels of commitment and for three regimes: Autarky, Mobility w/o bailout and Mobility with bailout. Looking at the mobility regime with bailout we observe that lower commitment increases equilibrium debt and improves welfare. This is because, with lower commitment there is a higher probability that the debt issued by one country is partially repaid by the other country (through a bailout), which increases the incentive to borrow. This compensates in part for the under-issuance of debt, as the benefits of creating

\(^6\)Because the utility of entrepreneurs in country \( i \) is strictly increasing in the debt repaid in period 2, a government that assigns a weight \( \Psi^i = 1 \) to entrepreneurs would never find it optimal to default. That is, even if \( \xi^i = \text{Not Commit} \), the optimal repayment rate is \( \delta^i = 1 \) regardless of the realization of \( z^i \). Smaller values of \( \Psi^i \) would be associated with lower repayment values.
### Table 1: Equilibrium debt and welfare for different commitment

<table>
<thead>
<tr>
<th></th>
<th>High commitment ($\rho^h = \rho^f = 0.6$)</th>
<th>Baseline ($\rho^h = \rho^f = 0.5$)</th>
<th>Low commitment ($\rho^h = \rho^f = 0.4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Debt</td>
<td>Utility</td>
<td>Debt</td>
</tr>
<tr>
<td>Autarky</td>
<td>0.766</td>
<td>-0.411</td>
<td>0.768</td>
</tr>
<tr>
<td>Mobility w/o bailout</td>
<td>0.577</td>
<td>-0.454</td>
<td>0.567</td>
</tr>
<tr>
<td>Mobility with bailout</td>
<td>0.671</td>
<td>-0.418</td>
<td>0.693</td>
</tr>
</tbody>
</table>

financial assets of one country are shared with the other country. Thus, lower commitment brings the equilibrium closer to autarky where each country internalizes the full benefit of issuing debt.

### 6 Empirical analysis

A novel prediction of the theoretical analysis is that the portfolio composition of domestic agents affects a government’s decision to default. More specifically, we showed that when the level of external debt assets (e.g. foreign debt held by domestic agents) raises, the domestic government is more likely to default. In this section, we conduct an empirical investigation of this theoretical prediction using cross-country data for a panel of 31 countries in the period 1970-2011.

Before moving to the empirical specification, it is useful to summarize the determinants of default incentives in our model. There are three key portfolio-composition variables that affect the default decisions of government $j$: (i) foreign debt held by home entrepreneurs, $B^f_{ij}$; (ii) home debt held by home entrepreneurs, $B^{ji}$; and (iii) home debt held by the foreign country, $B^{jf}$. We are particularly interested in testing the hypothesis that $B^f_{ij}$ increases the incentives to default, controlling for $B^{ji}$ and $B^{jf}$. Recall that a higher level of $B^{ji}$, keeping everything else constant, increases the macroeconomic costs of default and hence reduces the incentives to renege on debt repayments. On the other hand, higher values of $B^{jf}$ make it cheaper for $j$’s government to default, as the cost is borne relatively more by foreign-
ers. Hence, our theory suggests that the empirical specification includes both variables as controls. In addition, we know from the model that default incentives are higher in recessions, suggesting that productivity must also be part of our set of controls.

**From model to data:** The data counterpart for our main variable of interest, $B^{Ij}$, is ‘External Debt Assets,’ obtained from the External Wealth of Nations Database by Lane and Milesi-Ferreti\(^7\). This annual series, constructed from BIS and IFS data, comprises ‘portfolio debt securities’ and ‘other investment’ (and excludes FDI intercompany debt). It is expressed in millions of current US dollars. It is important to note that this variable includes public as well as private debt. Ideally, we would like to have a measure including only external public debt assets, as this would correspond more closely to our model. Note, however, that an augmented version of our model that included private assets would have similar implications. The reason being that increases in any type of assets that are not defaultable by the domestic government would reduce the macroeconomic costs of a default in the home country and hence increase incentives to default.

Home debt held by foreigners, $B^{Ji}$, is proxied by ‘External Government Debt,’ obtained from Global Financial Data. The variable corresponds to ‘International Debt Securities, General Government’ Table C1 of the Debt Securities Statistics of the BIS. Home debt held by domestic entrepreneurs $B^{ji}$ is constructed as the difference between total government debt $B^j$ of country $j$ and $B^{Ji}$ described above. Total government debt is proxied by ‘Central Government Debt,’ obtained from Global Financial Data, which corresponds to ‘Total Debt Securities, General Government’ in Table C1 of the Debt Securities Statistics of the BIS. Both $B^{Ji}$ and $B^{ji}$ are measured in millions of current US dollars.

Due to the financial globalization process taking place during this time period, the series are non-stationary even when considered as percentages of GDP. In order to minimize co-integration problems, we use the following transformation for our variables of interest in the empirical specification: (i) external debt assets as a proportion of total public debt, $EDA_{jt}$ and (ii) external government debt as a percentage of total public debt $EGD_{jt}$. They

\(^7\)The database contains data on foreign assets and foreign liabilities for a large sample of countries for the period 1970-2011. See Lane and Milesi-Ferreti (2007) for details. The data can be downloaded from: http://www.philiplane.org/EWN.html
are constructed as follows

\[
EDA_{j,t} = \frac{B_{fj}}{B_{fj} + B_{jj}} \quad \text{and} \quad EGD_{j,t} = \frac{B_{ff}}{B_{fj} + B_{jj}}
\]

To check whether there are statistically significant links between the default risk of a given country and the portfolio composition of its residents, we regress interest rate spreads of public debt on \( EDA_{j,t} \) and \( EGD_{j,t} \), among other controls. The spread of country \( j \)'s in year \( t \), \( s_{j,t} \), is computed as the difference between the country's interest rate and the risk free rate

\[
s_{j,t} = r_{j,t} - r_t^*,
\]

where \( r_{j,t} \) corresponds to long-term ‘Government Bond Yields’ (10 years in most cases) of country \( j \) in period \( t \), obtained from Global Financial Data. The risk free rate is proxied by the US government bond yield of the same maturity.

**Empirical specification and results:** We estimate the following fixed effect regression equation:

\[
s_{j,t} = \alpha_j + I_t + \beta \cdot EDA_{j,t} \cdot \gamma \cdot EGD_{j,t} + \theta \cdot X_{j,t} + \epsilon_{j,t}. \tag{9}
\]

The variable \( \alpha_j \) denotes country fixed-effects whereas \( I_t \) captures year fixed-effects. Our set of additional controls \( X_{j,t} \) includes GDP growth, inflation rates, and an indicator for whether the country is in default. GDP growth is computed as the difference in the natural logarithm of GDP, obtained from Lane and Milesi-Ferreti’s dataset (measured in million of current US dollars). Inflation rates are obtained from the OECD dataset, and correspond to the series ‘CPI, % change in relation to same period of last year’. Finally, The default dummy was obtained from the Global Crises Dataset compiled by Reinhart, Rogoff, and Trebesch\(^8\). We use the variable ‘Domestic Debt in Default.’

The results are displayed in Table 2. The first specification includes only our variable of interest \( EDA_{j,t} \) as well as country and time fixed effects. Consistent with the model, a larger share of external debt assets is associated

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\(^8\)Their dataset can be obtained here: http://www.hbs.edu/faculty/initiatives/behavioral-finance-and-financial-stability/Pages/global.aspx
Table 2: Country fixed-effect regression. The dependent variable is spreads on long-term debt.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EDA&lt;sub&gt;j,t&lt;/sub&gt;</td>
<td>0.095***</td>
<td>0.093***</td>
<td>0.089**</td>
<td>0.083**</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.037)</td>
<td>(0.038)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>EGD&lt;sub&gt;j,t&lt;/sub&gt;</td>
<td>−1.79</td>
<td>−0.446</td>
<td>(1.54)</td>
<td>(0.967)</td>
</tr>
<tr>
<td>GDP growth</td>
<td>−0.10**</td>
<td>−0.050***</td>
<td>(0.04)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Inflation</td>
<td></td>
<td></td>
<td></td>
<td>0.134</td>
</tr>
<tr>
<td>Domestic Default</td>
<td></td>
<td></td>
<td></td>
<td>10.837***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(3.850)</td>
</tr>
<tr>
<td>Observations</td>
<td>492</td>
<td>469</td>
<td>469</td>
<td>469</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.13</td>
<td>0.21</td>
<td>0.32</td>
<td>0.51</td>
</tr>
<tr>
<td>Number of countries</td>
<td>31</td>
<td>27</td>
<td>27</td>
<td>27</td>
</tr>
</tbody>
</table>

The sample period is 1970-2011 and includes the following countries: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Colombia, Denmark, Finland, France, Germany, Greece, Italy, Japan, Mexico, Malaysia, the Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Russia, Singapore, Spain, South Africa, Sweden, Thailand, and the United Kingdom. Specifications (2)-(4) exclude: Argentina, Colombia, China, and Denmark because they have fewer than 8 observations. Robust standard errors are clustered per country and adjusted for autocorrelation and heteroskedasticity, represented are in parentheses.

* Significant at 10 percent. ** Significant at 5 percent. *** Significant at 1 percent.

with a higher spread, indicating additional default risk. In the second specification, we restrict the sample to include at least 8 years of observations. We can see that while the coefficient is basically unchanged the goodness of fit improves, as seen from the higher value of the $R^2$. The number of countries is only reduced from 31 to 27. In the third specification, we include GDP growth and $EGD_{j,t}$ as additional controls. The result is qualitatively similar. As expected, higher GDP growth is associated with lower spreads, also consistently with our model. The coefficient on $EGD_{j,t}$ is statistically insignificant. The $R^2$ increases from 0.2 in the previous specification to 0.31. Finally, the last specification includes inflation and the default indicator as additional controls. We find that the positive relationship between external debt assets and spreads is robust to the inclusion of these controls. The significant increase in the $R^2$ indicates that the last specification is the one that
best fits the data. We also included an indicator variable for sovereign default, but it did not change the results relative to specification (4). Including dummies for banking crises or stock market crashes (both available from the Reinhart, Rogoff, and Trebesch’s dataset) slightly improves the goodness of fit (the $R^2$ increases to 0.55), but does not change the size of the coefficient of $EDA_{j,t}$, so the results are omitted.

7 Conclusion

In this paper we have shown that sovereign debt default could be induced by excessive borrowing from other countries if financial markets are integrated. The integration of financial markets increases the incentive to default not only because part of the defaulted debt is owned by foreigners (as widely emphasized in the literature) but also because the ‘endogenous’ macroeconomic cost of default is smaller when the defaulting country is financially integrated.

In our model government debt is held by producers as an insurance instrument. When financial markets are integrated, producers also hold foreign government debt. Therefore, when the domestic government defaults, producers are only partially affected by default with smaller consequences for aggregate production in the domestic country. Furthermore, the higher the debt issued by the foreign country, the higher the incentive to default for the home since home producers hold more foreign debt and, therefore, are more insured. This implies that the sovereign default of a country could be externally induced by the excessive borrowing of foreign countries. From this perspective, the recent debt problems experienced by some European countries can be the result (at least in part) of the increased debt in ‘safe’ industrialized countries since the early 1980s.

We have also considered the possibility of debt renegotiation, which can be interpreted as a ‘bailout’ policy. We have shown that the anticipation of a bailout increases the indebtedness of both countries. However, the higher indebtedness is not necessarily welfare reducing. The reason being that the higher level of sustainable debt corrects for an externality that emerges when financial markets are integrated. The externality arises because each country ignores the benefits that public debt has for the other country in terms of liquidity to entrepreneurs. When bailouts are possible, countries anticipate that with some probability the other country will contribute to the repayment
of their own debt, effectively reducing the cost of borrowing, and creating the conditions for higher liquidity. This corrects the under-issuance of debt and makes the equilibrium more efficient. Therefore, bailouts could be Pareto efficient not only ex-post but also ex-ante.
Appendix

A Proof of Lemma 3

Replace eq. (??) into eq. (??) to obtain

$$E_\varepsilon \frac{A(z, \varepsilon) - \alpha \left( \phi \tilde{B} \right)^{1/\nu}}{1 + \left[ A(z, \varepsilon) - \alpha \left( \phi \tilde{B} \right)^{1/\nu} \right] \phi} = 0,$$

where dependence on the aggregate state $z$ has been omitted to ease readability. This can be written more compactly as the following implicit function

$$F(\phi, \tilde{B}) \equiv E_\varepsilon \left\{ \left[ A(z, \varepsilon) - \alpha \left( \phi \tilde{B} \right)^{1/\nu} \right]^{-1} + \phi \right\}^{-1} = 0.$$

Using the implicit function theorem,

$$\frac{\partial \phi}{\partial \tilde{B}} = -\frac{\partial F/\partial \tilde{B}}{\partial F/\partial \phi} = -\frac{E_\varepsilon G^{-2}(A(z, \varepsilon) - w)^{-2} \frac{w}{\nu \tilde{B}}}{E_\varepsilon G^{-2} \left[ (A(z, \varepsilon) - w)^{-2} \frac{w}{\nu \phi} + 1 \right]} < 0$$

since $G \equiv \left[ A(z, \varepsilon) - \alpha \left( \phi \tilde{B} \right)^{1/\nu} \right]^{-1} + \phi > 0$. This establishes the first result.

Differentiate eq.(??) to obtain $\frac{\partial w(\pi, z)}{\partial \tilde{B}}$. After some algebraic manipulations,

$$\frac{\partial w}{\partial \tilde{B}} = \frac{1}{\nu} \frac{w}{\tilde{B}} \left[ \tilde{B} \frac{\partial \phi}{\partial \tilde{B}} + 1 \right]$$

where $\frac{\partial \phi}{\partial \tilde{B}} \leq 0$ is the elasticity of the entrepreneurs’ labor share $\phi$ with respect to $\tilde{B}$. We will show that wages are increasing in $\tilde{B}$ by contradiction.
Suppose $\frac{\partial w}{\partial B} < 0$. Since $\tilde{B} \geq 0$ (by assumption), it must be the case that

$$\frac{\bar{B}}{\phi} \frac{\partial \phi}{\partial \tilde{B}} < -1.$$ 

Alternatively,

$$\frac{E \varepsilon G^{-2}(A(z, \varepsilon) - w)^{-2} w}{E \varepsilon G^{-2} [(A(z, \varepsilon) - w)^{-2} \frac{w}{\nu} + \phi]} > 1.$$

But this would imply that $E \varepsilon G^{-2} \phi < 0$, a contradiction.

Finally, using the fact that $H = l = (w/\alpha)^{\nu}$, we can show that

$$\frac{\partial H}{\partial \tilde{B}} = \nu H \frac{\partial w}{w} \frac{\partial \tilde{B}}{\partial \tilde{B}} \geq 0.$$

QED

**B Proof of Proposition 4**

Replacing $c_2$ and $d_2$ in eq. (??), defining $\psi = \frac{1}{\nu}$, and rearranging, we obtain

$$E^{-1} \sum_{1 + (A(z, \varepsilon) - w(\pi, z)) \phi(\pi, z)} = \psi \frac{\bar{B}}{\nu w(\pi, z)^{1+\nu} - \bar{B}}.$$ 

The left hand side is equal to 1 from the optimality condition of entrepreneurs. Hence,

$$\tilde{\nu} w(\pi, z)^{1+\nu} - \bar{B} = \psi \bar{B}. \Rightarrow w(\pi, z) = \left[ \frac{\bar{B}(1 + \psi)}{\tilde{\nu}} \right]^{\frac{1}{1+\nu}}.$$ 

Equating this to eq. (??) and simplifying delivers

$$\phi(\bar{B}) = \frac{\phi_0}{\bar{B}^{1+\nu}} \quad \text{where} \quad \phi_0 = \frac{1}{\alpha} \left[ \frac{1 + \psi}{\tilde{\nu}} \right]^{\frac{1}{1+\nu}}.$$ 

Wages become

$$w(\bar{B}) = \omega_0 \bar{B}^{\frac{1}{1+\nu}} \quad \text{where} \quad \omega_0 = \left( \alpha \phi_0 \right)^{\frac{1}{\nu}}.$$ 

(10)
Replacing eq. (10) and (11) into eq. (??), we obtain an implicit function \( \tilde{B}(z) \)

\[
F(\tilde{B}, z) = \mathbb{E}_\varepsilon \left\{ \frac{1}{1 + [A(z, \varepsilon) - w(\tilde{B})] \phi(\tilde{B})} \right\} - 1 = 0
\] (12)

We can obtain \( \frac{\partial \tilde{B}}{\partial z} \) using the implicit function theorem:

\[
\frac{\partial \tilde{B}}{\partial z} = -\frac{\partial F(\tilde{B}, z)}{\partial z} \frac{\partial \tilde{B}}{\partial F(\tilde{B}, z)},
\] (13)

where

\[
\frac{\partial F(\tilde{B}, z)}{\partial z} = -\mathbb{E}_\varepsilon [1 + [A(z, \varepsilon) - w] \phi]^{-1} \phi < 0
\] (14)

using that \( A(z, \varepsilon) = z + \varepsilon \) to replace \( \frac{\partial A}{\partial z} = 1 \), and

\[
\frac{F(\tilde{B}, z)}{\partial B} = -\mathbb{E}_\varepsilon [1 + [A(z, \varepsilon) - w] \phi]^{-1} \left[ -\phi \frac{\partial w}{\partial B} + [A(z, \varepsilon) - w] \frac{\partial \phi}{\partial B} \right].
\]

Using eqs. (10) and (11), we obtain

\[
\frac{\partial w}{\partial B} = \frac{1}{1 + \nu} \frac{w}{B} \quad \text{and} \quad \frac{\partial \phi}{\partial B} = -\frac{1}{1 + \nu} \frac{\phi}{B}.
\]

Replacing these equations in \( \frac{F(\tilde{B}, z)}{\partial B} \) above, we have

\[
\frac{F(\tilde{B}, z)}{\partial B} = \mathbb{E}_\varepsilon [1 + [A(z, \varepsilon) - w] \phi]^{-1} \frac{A(z, \varepsilon) \phi}{(1 + \nu) \tilde{B}} > 0.
\] (15)

Using eq. (14) and eq. (15) in eq. (13) establishes the result.

QED

\section{C \ Proof of Lemma 5}

The entrepreneurs’ maximization problem is

\[
\max_{x_i} \ln d_i^1(\pi) + \beta E_{s, \varepsilon} \ln d_2^s(\pi, s, \varepsilon)
\]

\[
d_i^1(\pi) = a - \frac{b^{hi}}{R^h(\pi)} - \frac{b^{fi}}{R^f(\pi)}
\]
\[ d_2^i(\pi, s, \varepsilon) = (A(z^i, \varepsilon) - w^i(\pi, s))l^i(\pi, s) + b^{hi}(s) + b^{fi}(s), \]

where \( x_i = \{d_1^i, d_2^i, l^i, b^{fi}, b^{hi}\} \) is their set of choices.

Their FOC are

\[
\frac{1}{d_1^i(\pi)} \frac{1}{R^h(\pi)} = \beta E_{s, \pi, \varepsilon} \frac{\delta^h(s)}{d_2^i(\pi, s, \varepsilon)} \tag{16}
\]

\[
\frac{1}{d_1^i(\pi)} \frac{1}{R^f(\pi)} = \beta E_{s, \pi, \varepsilon} \frac{\delta^f(s)}{d_2^i(\pi, s, \varepsilon)} \tag{17}
\]

\[
E_{s, \pi, \varepsilon} \frac{A(z^i, \varepsilon) - w^i(\pi, s)}{d_2^i(\pi, s, \varepsilon)} = 0 \tag{18}
\]

From eqs. (16) and (17) we obtain

\[
R^h(\pi) = \eta R^f(\pi) \quad \text{where} \quad \eta = \frac{E_{s, \pi, \varepsilon} \frac{\delta^f(s)}{d_2^i(\pi, s, \varepsilon)}}{E_{s, \pi, \varepsilon} \frac{\delta^h(s)}{d_2^i(\pi, s, \varepsilon)}}.
\]

Guess the following

\[
b^{hi}(\pi) = \theta^{hi}(\pi) R^h(\pi) a
\]

\[
b^{fi}(\pi) = \theta^{fi}(\pi) R^f(\pi) a
\]

\[
l_i(\pi) = \phi^i(\pi, s)[b^{hi}(\pi)\delta^h(s) + b^{fi}(\pi)\delta^f(s)]
\]

Under that guess (and abstracting from arguments to simplify notation)

\[
d_1^i(\pi) = a(1 - \theta^{hi}(\pi) - \theta^{fi}(\pi))
\]

\[
d_2^i(\pi) = ([A(z^i, \varepsilon) - w^i(\pi, s)]\phi^i(\pi, s) + 1) [b^{hi}(\pi)\delta^h(s) + b^{fi}(\pi)\delta^f(s)],
\]

Moreover,

\[
b^{hi}(\pi)\delta^h(s) + b^{fi}(\pi)\delta^f(s) = a[\theta^{hi}(\pi) R^h(\pi)\delta^h(s) + \theta^{fi}(\pi) R^f(\pi)\delta^f(s)]
\]
Replacing the equations above in eq. (18) and using the fact that $b^{hi}(\pi)\delta^h(s) + b^{fi}(\pi)\delta^f(s)$ is independent of $\varepsilon$ we obtain

$$E_\varepsilon \frac{A(z^i, \varepsilon) - w^i(\pi, s)}{[A(z^i, \varepsilon) - w^i(\pi, s)]} = 0$$

Multiplying by $\phi^i(\pi, s)$ and subtracting 1 from both sides, we get

$$E_\varepsilon \frac{1}{[A(z^i, \varepsilon) - w^i(\pi, s)]} + 1 = 1 \quad (19)$$

Replacing the guesses in eq. (17)

$$\frac{1}{1 - \theta^{fi}(\pi) - \theta^{hi}(\pi)} = \beta E_{s,\pi} \left\{ \frac{R^f(\pi)\delta^f(s)}{\theta^{hi}(\pi)R^h(\pi)\delta^h(s) + \theta^{fi}(\pi)R^f(\pi)\delta^f(s)} \right\}$$

From eq. 19, we know that for each $\{s, \pi\}$, the term involving $E_\varepsilon$ is equal to 1. Using the fact that $R^h(\pi) = \eta R^f(\pi)$,

$$\frac{1}{1 - \theta^{fi}(\pi) - \theta^{hi}(\pi)} = \beta E_{s,\pi} \left[ \frac{1}{\theta^{hi}(\pi)\eta\delta^h(s)/\delta^f(s) + \theta^{fi}(\pi)} \right] \quad (20)$$

Replace the guesses into eq. (16), and follow the same steps to obtain

$$\frac{1}{1 - \theta^{fi}(\pi) - \theta^{hi}(\pi)} = \beta E_{s,\pi} \left[ \frac{\eta\delta^h(s)/\delta^f(s)}{\theta^{hi}(\pi)\eta\delta^h(s)/\delta^f(s) + \theta^{fi}(\pi)} \right] \quad (21)$$

Multiply both sides of eq. (20) by $\theta^{fi}(\pi)$, and both sides of eq. (21) by $\theta^{hi}(\pi)$, and add the resulting expressions. This delivers,

$$\theta^{hi}(\pi) + \theta^{fi}(\pi) = \frac{\beta}{1 + \beta} \quad (22)$$

Substituting into eq. (20) and remembering that $\eta = R^h(\pi)/R^f(\pi)$ we obtain

$$\frac{1 + \beta}{\beta} = E_{s,\pi} \left[ \frac{1}{\theta^{hi}(\pi)\delta^h(s)R^h(\pi) + \theta^{fi}(\pi)R^f(\pi)} \right] \quad (23)$$

QED
D Derivation of government’s objective under FI

Let \( \tilde{B}_i(s) = B_i^s \delta_i(s) \). In the second period, country \( i \)'s government solves a problem like eq. (3), where allocations satisfy the conditions established by Lemma 5. Since all entrepreneurs are identical in \( t = 1 \), \( b^{ii} + b^{ij} = B^i \). This implies that

\[
\begin{align*}
  b^{ii}(\pi) = b^{ij}(\pi) = \theta^i(\pi)R^i(\pi) &\Rightarrow b^i(\pi) = \frac{B^i}{2},
\end{align*}
\]

for \( i, j \in \{h, f\} \). Using the results from Lemma 5 and noticing that \( \theta^i(\pi) \) is country-independent, we get

\[
R^h(\pi) = \frac{B^h}{2a\theta^h(\pi)} \quad \text{and} \quad R^f(\pi) = \frac{B^f}{2a\theta^f(\pi)}.
\]

We can use \( R^h(\pi) = \eta R^f(\pi) \) to obtain

\[
\eta = \frac{B^h \theta^f(\pi)}{B^f \theta^h(\pi)}.
\]

Replacing this into eq. (23) and simplifying, we get

\[
\theta^f(\pi) = \frac{\beta}{1 + \beta E_{s,\pi}} \left[ \frac{1}{1 + \frac{\tilde{B}^h}{\tilde{B}^f}} \right], \quad (24)
\]

Replacing this into eq. (22), we obtain \( \theta^h(\pi) \). Hence, the financial wealth of both home and foreign entrepreneurs is \( (\tilde{B}^h + \tilde{B}^f)/2 \).

Equating the aggregate supply of labor to the aggregate demand for labor we obtain

\[
h^i(\pi, s) = \left[ \frac{w^i(\pi, s)}{\alpha} \right]^{\nu} = \phi^i(\pi, s) \left[ \frac{\tilde{B}^h + \tilde{B}^f}{2} \right].
\]

Replacing \( b^h(\pi) \) and \( b^f(\pi) \) in the labor market equilibrium condition, delivers two equations determining \( w^i \) and \( \phi^i \),

\[
w^i(\pi, z^i) = \alpha \left( h^i(\pi, z^i) \right)^{1/\nu}
\]

\[
E_{\varepsilon} \frac{A(z^i, \varepsilon) - w^i(\pi, z^i)}{[A(z^i, \varepsilon) - w^i(\pi, z^i)] \phi^i(\pi, z^i) + 1} = 0,
\]

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with \( h^i(\pi, z^i) = \phi^i(\pi, z^i) \left[ \frac{\hat{B}^h + \hat{B}^f}{2} \right] \). The wage rate and, therefore, the factor that determines the demand of labor depend on country \( i \)'s productivity \( z^i \) and wealth of home entrepreneurs after government default \( (\hat{B}^h + \hat{B}^f)/2 \). Therefore, we will denote the wage as \( w^i(\pi, z^i) \) and the labor demand factor as \( \phi^i(\pi, z^i) \). Replacing these results in entrepreneurs' and workers' budget constraints delivers eq. (??). QED

### E Optimality condition under Financial Integration

When \( \xi^i = \text{Not Commit} \), country \( i \)'s first order condition of the relaxed problem (that is, ignoring the constraint \( \hat{B}^i \leq B^i \)) is

\[
\Psi \mathbb{E}_u'(d^i_2) \left[ -\frac{\partial w^i}{\partial B^i} h^i + \frac{1}{2} + (A^i - w^i) \frac{\partial h^i}{\partial B^i} \right] + (1 - \Psi)U'\left( \varphi(c^i_2, h^i_2) \right) \left\{ \frac{\partial \varphi}{\partial c^i_2} \left[ \frac{\partial w^i}{\partial B^i} h^i - 1 + w^i \frac{\partial h^i}{\partial B^i} \right] + \frac{\partial \varphi}{\partial h^i_2} \frac{\partial h^i}{\partial B^i} \right\} = 0.
\]

We can use the optimality condition of entrepreneurs and workers to further simplify this expression using steps similar to those under autarky (see Appendix ??). The equation is reduced to

\[
\Psi \mathbb{E}_u'(d^i_2) \left[ -\frac{\partial w^i}{\partial B^i} h^i + \frac{1}{2} \right] + (1 - \Psi)U'\left( \varphi(c^i_2, h^i_2) \right) \frac{\partial \varphi}{\partial c^i_2} \left[ \frac{\partial w^i}{\partial B^i} h^i - 1 \right] = 0.
\]

Letting \( F = \frac{\hat{B}^h + \hat{B}^f}{2} \) and defining \( \Omega^i(\pi, z^i) = \frac{1 - \frac{\partial w^i}{\partial h^i}}{1 - \frac{\partial w^i}{\partial h^i}} \geq 1 \) and re-arranging this equation delivers the optimality condition in the text.

QED
References


