

# The Welfare Effects of Transportation Infrastructure Improvements\*

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PRELIMINARY AND INCOMPLETE

## Abstract

In this paper, we develop a framework to characterize the impact of infrastructure investment on welfare. We suppose infrastructure investment affects the cost of shipping goods between directly connected locations and the total bilateral cost between any two locations is determined by traders optimally traveling across the complete transportation network. Our approach comprises two distinct but complementary characterizations: First, we characterize how infrastructure investment between any two connected locations decreases the total trade costs between all pairs of locations. Second, we characterize how the cost reduction between any two locations changes affects welfare. We apply these results to shipment level data between U.S. cities to calculate the welfare effects of improving each portion of the U.S. Interstate Highway System (IHS). We find very heterogeneous welfare effects of improvements to different sections of the IHS – reducing the travel time by 30 minutes on I-95 South from New York to Philadelphia would increase aggregate U.S. welfare by 0.02%, whereas reducing the travel time by 30 minutes from Seattle to Salt Lake City along I-84 East would only increase aggregate U.S. welfare by one-hundredth of that.

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# 1 Introduction

More than a trillion dollars is spent on investment in transportation across the world each year (Lefevre, Leipziger, and Raifman, 2014). These investments on the transportation networks are designed to enhance economic activity by reducing the economic costs of moving goods and people across space. However, there is limited understanding both how investment in transportation networks affect economic costs and how changes in economic costs affect economic outcomes. Better understanding of both questions is necessary in order to determine how to optimally allocate transportation investment to optimize economic outcomes.

In this paper, we develop a new spatial framework incorporating infrastructure investment that admits an analytical solution for how infrastructure investment of any two connected locations decreases the cost of travel between all bilateral pairs. We then embed this framework in a standard spatial economic framework in order to characterize the general equilibrium impact on welfare of any infrastructure investment. Finally, apply the theory to detailed shipment level data on trade between U.S. cities in order to estimate the welfare effects of improving each portion of the U.S. Interstate Highway System.

To incorporate infrastructure investment, we suppose the world is composed of many locations arranged on a weighted graph. Trade between locations requires traveling across the network of the graph, incurring (iceberg) trade costs along the way. Infrastructure investment can be undertaken to reduce the cost of any connection on the graph, which reduces the bilateral trade costs both for the locations immediately connected and for all trade flows using that route. As a result, characterizing how infrastructure investment affects bilateral trade costs is quite difficult, as investment in infrastructure may potentially affect trade costs between all other locations and traders may endogenously alter the route they travel.

Previous applications of spatial models to understand the impact of transportation infrastructure have relied on computational algorithms such as Dijkstra (see e.g. Donaldson (2012); Donaldson and Hornbeck (2012); Redding (2012); Alder et al. (2014)) to solve for the optimal routes in graphs. Our approach, in contrast, is analytical. To make the problem tractable, we suppose that trade is undertaken by a continuum of heterogeneous traders, who choose the least cost route over the network but vary in their idiosyncratic cost of traveling over different paths. Using a combination of graph theory and properties of extreme value distributions (in the spirit of Eaton and Kortum (2002)), we show that this assumption admits a straightforward analytical relationship between the weighted graph of the network and the resulting endogenous expected bilateral iceberg trade costs. Moreover, the framework admits an intuitive expression for the elasticity of a bilateral trade cost between any

locations to an improvement in infrastructure between any two other locations.

We then show how this framework can be readily combined with standard general equilibrium spatial frameworks, including the economic geography framework of [Allen and Arkolakis \(2014\)](#) and the trade framework of [Anderson \(1979\)](#); [Anderson and Van Wincoop \(2003\)](#). For both these frameworks, the elasticity of equilibrium welfare to a change in bilateral trade costs between any two locations is equal to the fraction of world trade between those locations. Combining this elasticity with the elasticity of bilateral trade costs to infrastructure investments yields a straightforward expression of the welfare effect of any transportation infrastructure investment, which can be written as a “sufficient statistic” depending solely on observed trade flows and the bilateral trade cost where the infrastructure investment occurs.

We then apply our framework to examine the welfare impact of improving each segment of the U.S. Interstate Highway System (IHS). Using recently released shipment level micro data from the Commodity Flow Survey, we show that our framework closely matches observed trade flows, and both the average and variability of distance traveled across shipments between a given origin and destination. Finally, we estimate that the welfare impact of improvements to different segments of the IHS are very heterogeneous. Along the I-95 corridor on the eastern seaboard and along I-70 in Indiana and Ohio, the effects of infrastructure improvements are large; for example, reducing the travel time by 30 minutes on I-95 South from New York to Philadelphia (which we estimate is the most important highway in the U.S.) would increase aggregate U.S. welfare by 0.02%. However, infrastructure improvements elsewhere on the IHS have effects that are orders of magnitude smaller; for example, we estimate that reducing the travel time by 30 minutes from Seattle to Salt Lake City along I-84 East would only increase aggregate U.S. welfare by 0.0002%. Hence, the results highlight the need to appropriately target infrastructure improvements in order to maximize their benefit.

There is a recent expanding literature on evaluating the impact of infrastructure investment in general equilibrium, such as [Donaldson \(2012\)](#), [Allen and Arkolakis \(2014\)](#), [Donaldson and Hornbeck \(2012\)](#) in an inter-city context and the work of [Ahlfeldt, Redding, Sturm, and Wolf \(2012\)](#) in an intra-city context; see [Redding and Turner \(2014\)](#) for an excellent review of the literature. The application of these models to the study of spatial policy, however, has preceded more slowly, as the combination of many locations, general equilibrium forces, and flexible spatial linkages have made it difficult to characterize optimal policy. For example, [Ossa \(2014\)](#) and [Ossa \(2015\)](#) follow a computational approach to characterize optimal tariff and tax policies in spatial frameworks, [Alder et al. \(2014\)](#) uses a sequential algorithm to construct an approximation of the optimal transportation network, while [Allen, Arkolakis, and Takahashi \(2014\)](#) and [Allen, Arkolakis, and Li \(2015\)](#) derive first order necessary conditions for the characterizations of the optimal investment in infrastruc-

ture. However, despite the huge investment in transportation infrastructure, very little is known about the the impact of changes in the transportation network on welfare in a general equilibrium setup. Two notable exceptions are [Felbermayr and Tarasov \(2015\)](#), who consider infrastructure investment when the world is a line, and [Fajgelbaum and Schaal \(2016\)](#), who use tools from the transportation economics literature to derive the optimal transport network in the presence of congestion. Relative to [Felbermayr and Tarasov \(2015\)](#), our paper considers an arbitrary geography; whereas relative to [Fajgelbaum and Schaal \(2016\)](#), our setup falls within the class of general equilibrium spatial gravity models that have proven successful in matching observed empirical patterns and performing counterfactuals using real world data, see e.g. [Redding and Rossi-Hansberg \(2016\)](#).

The remainder of the paper proceeds as follows. The next section presents the framework for the endogenous transportation costs and derives a number of key properties. Section 3 embeds the transportation cost framework into a the spatial economic model and derives the elasticity of aggregate welfare

derives the elasticity of welfare to a change in any bilateral trade costs in both economic geography and trade frameworks. Section 4 then combines the two results to derive the elasticity of welfare to any infrastructure project. Section 5 concludes.

## 2 Endogenous transportation costs

In this section, we describe how to calculate the transportation costs between any two locations, accounting for the fact that agents endogenously choose the least cost route between locations.

### 2.1 Setup

Consider a world composed of a finite number of locations  $i \in \{1, \dots, N\}$ . Locations are organized on a weighted graph with an associated *infrastructure matrix*  $\mathbf{T} = [t_{ij} \geq 1]$ , where  $t_{ij}$  indicates the iceberg trade cost incurred from moving *directly* from  $i$  to  $j$  (if  $i$  and  $j$  are not directly connected in the graph, then  $t_{ij} = \infty$ ). The top left panel of Figure 1 provides an example of such a network, where  $N = 25$  and locations are arrayed in a two-dimensional grid, with locations that are directly connected (i.e.  $t_{ij}$  is finite) if they are adjacent and not connected (i.e.  $t_{ij}$  is infinite) otherwise.

Trade between  $i$  and  $j$  is undertaken by a continuum of heterogeneous traders  $\nu \in [0, 1]$  who travel along (endogenously chosen) paths to get from  $i$  to  $j$ . A *path*  $p$  between  $i$  and  $j$  is a sequence of locations beginning with location  $i$  and ending with location  $j$

$\{i = p_0, p_1, \dots, p_K = j\}$ , where  $K$  is the *length* of path  $p$ .<sup>1</sup> The aggregate trade cost from  $i$  to  $j$  on a path  $p$  of length  $K$ ,  $\tilde{\tau}_{ij}(p)$  is the product of the instantaneous trade costs along the path:

$$\tilde{\tau}_{ij}(p) = \prod_{k=1}^K t_{p_{k-1}, p_k} \quad (1)$$

Each trader also incurs a path-specific idiosyncratic trade cost shock  $\varepsilon_{ij}(p, \nu)$ , so the total cost to trader  $\nu$  of traveling along path  $p$  between  $i$  and  $j$  is  $\tilde{\tau}_{ij}(p) \varepsilon_{ij}(p, \nu)$ . Let  $\tau_{ij}(\nu)$  indicate the cost trader  $\nu$  incurs optimally choosing the path between  $i$  and  $j$  to minimize the iceberg trade costs incurred:

$$\tau_{ij}(\nu) = \min_{p \in P_{K,K} \geq 0} \tilde{\tau}_{ij}(p) \varepsilon_{ij}(p, \nu).$$

In what follows, we assume that  $\varepsilon_{ij}(p, \nu)$  is Frechet distributed with shape parameter  $\theta > 0$ , and we let  $P_{ij,K}$  denote the set of all paths of length  $K$  that go from  $i$  to  $j$ .

Notice in our formulation that we allow traders to choose any possible path to ship a good from  $i$  to  $j$  – including the most meandering of routes. The extent to which traders differ in which path they choose is determined by the shape parameter  $\theta$ . In a sense, the parameter  $\theta$  can be considered as capturing the possibility of mistakes and randomness in the choice of routes, with higher values indicating greater agreement across traders. In the limit case of no heterogeneity,  $\theta \rightarrow \infty$ , all traders choose the route with the minimum aggregate trade cost. The previous literature has focused exclusively on this limited case and relied upon computational methods to calculate this least cost route (i.e. using the Dijkstra algorithm as in [Donaldson \(2012\)](#) and [Donaldson and Hornbeck \(2012\)](#) or the Fast Marching Method as in [Allen and Arkolakis \(2014\)](#)). As will become evident below, the advantage of the framework proposed here is that the solution takes a convenient analytical form allowing for explicit characterizations for how changes in the infrastructure affect bilateral trade costs. For this, we view our formulation as relaxing the previous literature’s focus on the least cost route in order to to better understand the impact of changes in the investment matrix on realized trade costs. Finally, we should note that the concept of heterogeneous traders formulation has been utilized previously by [Allen and Atkin \(2015\)](#). However, their formulation does not consider the optimal route taken, as the the random trader shocks are realized on the expected cost matrix instead of the infrastructure matrix.

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<sup>1</sup>Following the literature on graph theory, we assume that  $t_{ii} = \infty$  to exclude paths that stay in the same location; however, we allow traders shipping goods from  $i$  to  $i$  to choose the “null” path where they travel nowhere and incur no trade costs (which is the only admissible path of length 0).

## 2.2 Optimal routes

Using the familiar derivations pioneered in [Eaton and Kortum \(2002\)](#) we can express the *expected trade cost*  $\tau_{ij}$  from  $i$  to  $j$  across all traders as:

$$\tau_{ij} \equiv E_{\nu} [\tau_{ij}(\nu)] = c \left( \sum_{K=0}^{\infty} \sum_{p \in P_K} \tilde{\tau}_{ij}(p)^{-\theta} \right)^{-\frac{1}{\theta}}, \quad (2)$$

where  $c \equiv \Gamma\left(\frac{\theta-1}{\theta}\right)$ . Substituting equation (1) into equation (2) yields:

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in P_K} \prod_{k=1}^K t_{p_{k-1}, p_k}^{-\theta}.$$

In what follows, it proves useful to characterize the *weighted adjacency matrix*  $\mathbf{A} = [a_{ij} \equiv t_{ij}^{-\theta}]$ . Note that  $a_{ij} \in [0, 1]$ , where 0 indicates there is no connection between  $i$  and  $j$ ,  $a_{ij} = 1$  indicates a cost-less connection, and  $a_{ij} \in (0, 1)$  indicates a costly connection. We can then write the expected trade cost as:

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} \sum_{p \in P_K} \prod_{k=1}^K a_{p_{k-1}, p_k}.$$

The summing over all paths of length  $K$  can be written in a more convenient form by explicitly summing across all locations that are traveled to first, second, etc. as follows:

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} \left( \sum_{k_1=1}^N \sum_{k_2=1}^N \dots \sum_{k_{K-1}=1}^N (a_{i, k_1} \times a_{k_1, k_2} \times \dots \times a_{k_{K-2}, k_{K-1}} \times a_{k_{K-1}, j}) \right),$$

where  $k_n$  is the sub-index for the  $n^{\text{th}}$  location arrived at on a particular path. This portion of the expression in the parentheses, however, is equivalent to the  $(i, j)$  element of the weighted adjacency matrix to the power  $K$ , i.e.:

$$\tau_{ij}^{-\theta} = c^{-\theta} \sum_{K=0}^{\infty} A_{ij}^K,$$

where  $\mathbf{A}^K = [A_{ij}^K]$ , i.e.  $A_{ij}^K$  is the  $(i, j)$  element of the matrix  $\mathbf{A}^K$ , i.e.  $\mathbf{A}$  to the matrix power  $K$ . Furthermore, as long as the spectral radius of  $\mathbf{A}$  is less than one,<sup>2</sup>the geometric

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<sup>2</sup>A sufficient condition for the spectral radius being less than one is if  $\sum_j t_{ij}^{-\theta} < 1$  for all  $i$ . This will necessarily be the case if either trade costs between connected locations are sufficiently large, the adjacency matrix is sufficiently sparse, or the heterogeneity across traders is sufficiently small (i.e.  $\theta$  is sufficiently

sum can be expressed as:

$$\sum_{K=0}^{\infty} \mathbf{A}^K = (\mathbf{I} - \mathbf{A})^{-1} \equiv \mathbf{B},$$

where we call  $\mathbf{B} = [b_{ij}]$  the *route cost matrix*. Finally, the expected trade cost from  $i$  to  $j$  can be written as a simple function of the route cost matrix:

$$\tau_{ij} = cb_{ij}^{-\frac{1}{\theta}}. \quad (3)$$

Equation (3) provides an analytical relationship between any given infrastructure matrix and the resulting expected trade cost, accounting for traders choosing the least cost route.

## 2.3 Properties of the endogenous trade costs

In this subsection, we derive four important properties of this endogenous trade cost framework. The first two properties will prove helpful in providing an analytical characterization of the welfare impact of transportation infrastructure improvements, whereas the latter two properties will prove helpful in estimation the model parameters from the data.

### 2.3.1 The probability of using a particular link along the optimal path

We first characterize the probability that trade going from location  $i$  to location  $j$  travels through the link  $t_{kl}$ , which we refer to as  $\pi_{ij}^{kl}$ . Given the extreme value distribution, the probability of taking any particular path  $p$  of length  $K$  can be written as:

$$\begin{aligned} \pi_{ij}(p) &= \frac{\tau_{ij}(p)^{-\theta}}{\sum_{K=0}^{\infty} \sum_{p' \in P_{ij,K}} \tau_{ij}(p')^{-\theta}} \iff \\ \pi_{ij}(p) &= \frac{1}{b_{ij}} \prod_{k=1}^K a_{p_{k-1}, p_k}, \end{aligned} \quad (4)$$

where the second line used the definition of  $\tau_{ij}(p)$  from equation (1) and the derivation for equation (3). As a result, we can calculate the probability of using link  $t_{kl}$  when traveling from  $i$  to  $j$  by summing across all paths from  $i$  to  $j$  that use the link  $t_{kl}$ :

$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in P_{ij,K}^{kl}} \prod_{k=1}^K a_{p_{k-1}, p_k}.$$

where  $P_{ij,K}^{kl}$  is the set of all paths from  $k$  to  $l$  of length  $K$  that use link  $t_{ij}$ .

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large).

Note that for any  $p \in P_{ij,K}^{kl}$ , there must exist some length  $B \in [1, 2, \dots, K-1]$  at which the path arrives at link  $t_{kl}$  so that this can be written as:

$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left( \sum_{p \in P_{i,k,B}} \prod_{k=1}^B a_{p_{k-1}, p_k} \right) \times a_{kl} \times \left( \sum_{q \in P_{k,j,K-B-1}} \prod_{k=1}^{K-B-1} a_{q_{k-1}, q_k} \right)$$

As above, we can then explicitly enumerate all possible paths from  $i$  to  $k$  of length  $B$  and all possible paths from  $l$  to  $j$  of length  $K-B-1$ :

$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left( \sum_{k_1=1}^N \cdots \sum_{k_{B-1}=1}^N a_{i,k_1} \times \cdots \times a_{k_{B-1},k} \right) \times a_{kl} \times \left( \sum_{k_1=1}^N \cdots \sum_{k_{K-B-1}=1}^N a_{l,k_1} \times \cdots \times a_{k_{K-B-1},j} \right),$$

which again can be expressed more simply as elements of matrix powers of  $\mathbf{A}$  :

$$\pi_{ij}^{kl} = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} A_{ik}^B \times a_{kl} \times A_{lj}^{K-B-1}$$

This is where it gets a little more difficult. Recall from matrix calculus that the derivative of the power of a matrix can be written as:

$$DS_K(\mathbf{A}) \mathbf{C} = \sum_{B=0}^{K-1} \mathbf{A}^B \mathbf{C} \mathbf{A}^{K-B-1}, \quad (5)$$

where  $S_K(\mathbf{A}) = \mathbf{A}^K$  and  $\mathbf{C}$  is an arbitrary matrix. Furthermore, recall from above that the geometric series of a power of matrices can be written as:

$$\sum_{K=0}^{\infty} \mathbf{A}^K = (\mathbf{I} - \mathbf{A})^{-1}.$$

Hence, we can right multiply both sides of equation (5) by  $\mathbf{C}$  and differentiate to yield:

$$\sum_{K=0}^{\infty} DS_K(\mathbf{A}) \mathbf{C} = DT(\mathbf{A}) \mathbf{C},$$

where  $T(\mathbf{A}) \equiv (\mathbf{I} - \mathbf{A})^{-1}$ . Recall from matrix calculus (see e.g. [Weber and Arfken \(2003\)](#)) that  $DT(\mathbf{A}) \mathbf{C} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{C} (\mathbf{I} - \mathbf{A})^{-1}$  so that we have:

$$\sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}^B \mathbf{C} \mathbf{A}^{K-B-1} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{C} (\mathbf{I} - \mathbf{A})^{-1}. \quad (6)$$



Define  $\mathbf{C}$  to be an  $N \times N$  matrix that takes the value of  $a_{kl}$  at row  $k$  and column  $l$  and zeros everywhere else. Equation (6) then implies that:

$$\begin{aligned} \pi_{ij}^{kl} &= \frac{b_{ik}a_{kl}b_{lj}}{b_{ij}} \iff \\ \pi_{ij}^{kl} &= \left( \frac{1}{c} \frac{\tau_{ij}}{\tau_{ik}t_{kl}\tau_{lj}} \right)^\theta. \end{aligned} \quad (7)$$

Equation (7) is intuitive: the numerator on the right hand side is the expected trade cost on the least cost route from  $i$  to  $j$ , whereas the denominator is the expected trade cost on the least cost route from  $i$  to  $j$  *through the transportation link*  $t_{kl}$ . Hence, the more costly it is to travel through the link  $t_{kl}$  relative to the unconstrained least cost route, the less likely a trader is to use the link  $t_{kl}$ .

As an example, consider the network of locations presented in Figure 1. If we assume that all connected locations have the same direct cost  $t_{ij}$ , then the top right panel depicts the probability that an agent going from  $i = 1$  (bottom left) to  $j = 25$  (top left) travels over each connection. As is evident, connections along the direct diagonal route are chosen much more often than other routes, and connections near the diagonal route are chosen more often than those further away. As an additional example, the bottom left panel depicts the probability for agents going from  $i = 1$  (bottom left) to  $j = 15$  (top center). In this case, there are four different paths that share the same least deterministic costs. As is evident, traders are much more likely to use links on these routes than other links. However, there is also heterogeneity in the probability traveled across these different links – for example, the connection from 1 to 7 is traveled more often than the connection from 2 to 8. This occurs because there are many more possible paths going from 1 to 15 that use the 1 to 7 link than that use the 2 to 8 link, so each trader is more likely to get an idiosyncratic draw that causes her to choose a path using the former link.

It turns out that this same intuition is helpful in interpreting how changes in an infrastructure link affect bilateral trade costs, which we turn to next.

### 2.3.2 How changes to infrastructure affect endogenous trade costs

Consider now how a change in the transportation link  $t_{kl}$  affects the bilateral trade costs from  $i$  to  $j$ , i.e.  $\frac{\partial \ln \tau_{ij}}{\partial \ln t_{kl}}$  for all  $i, j, k, l \in \{1, \dots, N\}$ . First note that this elasticity is equal to the elasticity of an element of the route cost matrix to a change in the element of the

weighted adjacency matrix  $\mathbf{A}$ :

$$\frac{\partial \ln \tau_{ij}}{\partial \ln t_{kl}} = \frac{\partial \ln \left( cb_{ij}^{-\frac{1}{\theta}} \right)}{\partial \ln \left( a_{kl}^{-\frac{1}{\theta}} \right)} = \frac{\partial \ln b_{ij}}{\partial \ln a_{kl}}.$$

To calculate this elasticity, we parameterize the weighted adjacency matrix as a function of a variable  $t$  follows:

$$\mathbf{A}_{kl}(t) = \begin{cases} a_{ij} & \text{if } k \neq i \text{ or } l \neq j \\ t & \text{if } k = i \text{ and } l = j \end{cases},$$

i.e. changing  $t$  just increases  $a_{kl}$ . By defining  $\mathbf{C}_{kl}(t) \equiv \mathbf{I} - \mathbf{A}_{kl}(t)$ , we have:

$$\frac{\partial \ln b_{ij}}{\partial \ln a_{kl}} = \left[ \frac{d\mathbf{C}_{kl}(t)^{-1}}{dt} \right]_{ij} \times \frac{a_{kl}}{b_{ij}} \quad (8)$$

Using the familiar expression for the derivative of an inverse of a parameterized matrix (see e.g. [Weber and Arfken \(2003\)](#)), we have:

$$\frac{d\mathbf{C}_{kl}(t)^{-1}}{dt} = -\mathbf{C}_{kl}(t)^{-1} \frac{d\mathbf{C}_{kl}(t)}{dt} \mathbf{C}_{kl}(t)^{-1}.$$

Note that  $\frac{d\mathbf{C}_{kl}(t)}{dt} = \begin{cases} 0 & \text{if } k \neq i \text{ or } l \neq j \\ -1 & \text{if } k = i \text{ and } l = j \end{cases}$  and  $\mathbf{C}_{kl}(t)^{-1} = \mathbf{B}$  so the derivative becomes:

$$\frac{d\mathbf{C}_{kl}(t)^{-1}}{dt} = \mathbf{B}\mathbf{E}_{kl}\mathbf{B},$$

where  $\mathbf{E}_{kl}$  is an  $N \times N$  matrix equal to one at  $(k, l)$  and zeros everywhere else. The  $(i, j)$  component of this inverse is hence:

$$\left[ \frac{d\mathbf{C}_{kl}(t)^{-1}}{dt} \right]_{ij} = [\mathbf{B}\mathbf{E}_{kl}\mathbf{B}]_{ij} = \left[ \sum_n \sum_m b_{im} E_{mn}^{kl} b_{nj} \right]_{ij} = b_{ik} b_{lj}, \quad (9)$$

so combining equations (8) and (9) and using the relationships  $a_{kl} = t_{kl}^{-\theta}$  and  $\tau_{ij} = cb_{ij}^{-\frac{1}{\theta}}$  yields a simple expression for how a change in the infrastructure matrix changes the endogenous trade costs:

$$\frac{\partial \ln \tau_{ij}}{\partial \ln t_{kl}} = \frac{b_{ik} a_{kl} b_{lj}}{b_{ij}} = \left( c \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lj}} \right)^\theta. \quad (10)$$

Note that the elasticity of trade costs from  $i$  to  $j$  to changes in the link  $t_{kl}$  from (10) is exactly equal to the probability that a route from  $i$  to  $j$  uses the  $t_{kl}$  infrastructure from (7). Hence, the more “out of the way” the transportation link  $t_{ij}$  is from the optimal path between  $k$  and  $l$ , the less frequently that path is used, and the smaller the effect an improvement in  $t_{ij}$  has on the reduction of trade costs between  $k$  and  $l$ .

### 2.3.3 The expected distance between two locations

We now characterize the expected distance traveled between two locations on the optimal route. While traders do not directly care about distance (distance only matters inasmuch as it affects trade costs), the following expressions will prove useful in estimating the model parameters from the data.

Let  $d_{ij}$  be the distance between locations  $i$  and  $j$ , where  $d_{ij}$  is finite if  $i$  and  $j$  are directly connected and infinite otherwise. Let  $\tilde{d}_{ij}(p)$  be the distance between  $i$  and  $j$  on path  $p$ :

$$\tilde{d}_{ij}(p) = \sum_{k=1}^K d_{p_{k-1}, p_k}$$

The expected distance from  $i$  to  $j$  is equal to the sum across all possible paths of the probability of traveling over that path times its distance:

$$E\left(\tilde{d}_{ij}(p)\right) = \sum_{K=0}^{\infty} \sum_{p \in P_K} \pi_{ij}(p) \sum_{k=1}^K d_{p_{k-1}, p_k}.$$

Applying expression (4) for the probability of a particular path yields:

$$E\left(\tilde{d}_{ij}(p)\right) = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \sum_{p \in P_K} \prod_{k=1}^K a_{p_{k-1}, p_k} \sum_{k=1}^K d_{p_{k-1}, p_k},$$

which as above we can write by explicitly accounting for all possible paths:

$$E\left(\tilde{d}_{ij}(p)\right) = \frac{1}{b_{ij}} \sum_{K=0}^{\infty} \left( \left( \sum_{k_1=1}^N \sum_{k_2=1}^N \dots \sum_{k_{K-1}=1}^N (a_{i, k_1} \times \dots \times a_{k_{K-1}, j}) (d_{i, k_1} + \dots + d_{k_{K-1}, j}) \right) \right).$$

As in Section 2.3.1, this expression can be rearranged to note that distance  $d_{kl}$  is incurred on any path that travels through link  $a_{kl}$ , allowing us to consider all paths that go through

link  $a_{kl}$  on the  $B^{th}$  stop along the way from  $i$  to  $j$ :

$$\begin{aligned}
E\left(\tilde{d}_{ij}(p)\right) &= \frac{c^{-\theta}}{\tau_{ij}^{-\theta}} \sum_{k=1}^N \sum_{l=1}^N d_{kl} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left( \sum_{k_1=1}^N \sum_{k_2=1}^N \cdots \sum_{k_{B-1}=1}^N (a_{i,k_1} \times \cdots \times a_{k_{B-1},k}) \times a_{kl} \times \right. \\
&\quad \left. \sum_{k_{B+1}=1}^N \sum_{k_{B+2}=1}^N \cdots \sum_{k_{K-1}=1}^N (a_{l,k_{B+1}} \times \cdots \times a_{k_{K-1},k}) \right) \iff \\
E\left(\tilde{d}_{ij}(p)\right) &= \frac{c^{-\theta}}{\tau_{ij}^{-\theta}} \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \left( \sum_{k=1}^N \sum_{l=1}^N A_{ik}^B \times d_{kl} a_{kl} \times A_{lj}^{K-B-1} \right) \quad (11)
\end{aligned}$$

Define the matrix  $(\mathbf{D} \circ \mathbf{A}) \equiv [d_{ij} a_{ij}]_{ij}$ . Note that  $[\mathbf{A}^B (\mathbf{D} \circ \mathbf{A}) \mathbf{A}^{K-B-1}]_{ij} = \sum_{k=1}^N \sum_{l=1}^N A_{ik}^B \times d_{kl} a_{kl} \times A_{lj}^{K-B-1}$ , so that we can rewrite expected distance from expression (11) in matrix notation as follows:

$$E\left(\tilde{d}_{ij}(p)\right) = \frac{1}{b_{ij}} \left[ \sum_{K=0}^{\infty} \sum_{B=0}^{K-1} \mathbf{A}^B (\mathbf{D} \circ \mathbf{A}) \mathbf{A}^{K-B-1} \right]_{ij}.$$

We can then apply equation (6) to write:

$$\begin{aligned}
E\left(\tilde{d}_{ij}(p)\right) &= \frac{1}{b_{ij}} [(\mathbf{I} - \mathbf{A})^{-1} (\mathbf{D} \circ \mathbf{A}) (\mathbf{I} - \mathbf{A})^{-1}]_{ij} \iff \\
E\left(\tilde{d}_{ij}(p)\right) &= \frac{[\mathbf{B} (\mathbf{A} \circ \mathbf{D}) \mathbf{B}]_{ij}}{[\mathbf{B}]_{ij}},
\end{aligned}$$

where the operation “ $\circ$ ” is the Hadamard (element-wise) product. This can be written equivalently in “trade notation” as:

$$E\left(\tilde{d}_{ij}(p)\right) = \sum_{k=1}^N \sum_{l=1}^N d_{kl} \left( \frac{1}{c} \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lk}} \right)^{\theta}. \quad (12)$$

Equation (12) is intuitive: from equation (7), the probability of a trader going from  $i$  to  $j$  traveling through link  $t_{kl}$  is  $\left( \frac{1}{c} \frac{\tau_{ij}}{\tau_{ik} t_{kl} \tau_{lk}} \right)^{\theta}$ ; as a result, the expected distance that a trader travels is simply the sum across all links multiplied by the probability each link is used.

### 2.3.4 The variation across traders in distance traveled between two locations

Finally, we derive a measure of the variation across traders in the distance traveled between two locations. This expression will prove helpful when comparing the model predictions to the data below. A natural measure of the variation would be the standard deviation of distance traveled across all traders going from  $i$  to  $j$ . We instead employ an alternative measure of the variation which admits a simple analytical formulation: the “LogMeanExp

minus mean”, which for a random variable  $x$  is defined as:

$$\text{”LogMeanExp minus mean of } x\text{”} \equiv \log E[\exp(x)] - E(x).$$

Because  $\log E[\exp(x)]$  is convex, the “LogMeanExp minus mean” of  $x$  is increasing in the variance of  $x$ .<sup>3</sup>

Using an analogous derivation as in Section 2.2 and applying equation (12) for the expected distance traveled, the LogMeanExp minus mean can be calculated as:

$$\log(E(\exp d_{ij}(p))) - E(\tilde{d}_{ij}(p)) = \log\left(\frac{[(\mathbf{I} - (\mathbf{A} \circ \exp(\mathbf{D})))^{-1}]_{ij}}{[\mathbf{B}]_{ij}}\right) - \frac{[\mathbf{B}(\mathbf{A} \circ \mathbf{D})\mathbf{B}]_{ij}}{[\mathbf{B}]_{ij}},$$

where  $\exp(\mathbf{D}) \equiv [\exp d_{ij}]$  is a matrix of the exponential of distances.

### 3 The welfare effects of changing bilateral trade costs

We now introduce a spatial economic setup which we incorporate with the endogenous transportation network setup presented in Section 2 in order to characterize the welfare effect of changing the transportation infrastructure. We consider two variants of the setup – one in which agents are perfectly immobile and one in which agents are perfectly mobile – and present new derivations of how a change in any bilateral trade cost affects the endogenous aggregate welfare.

#### 3.1 A Spatial Economic Framework

**Setup** Suppose that the  $N$  locations are inhabited by an exogenous measure  $\bar{L}$  of agents. On the production side, an agent residing in location  $i$  supplies her endowed unit of labor inelastically in order to produce  $A_i$  units of that locations differentiated variety, for which she is compensated with a wage  $w_i$ . Production occurs in a perfectly competitive framework with labor as the only factor of production. On the consumption side, agents use their wage to purchase a constant elasticity of substitution (CES) bundle of these differentiated varieties with elasticity of substitution  $\sigma$ . To purchase goods, workers are randomly matched with traders and hence face trade costs equal to the expected trade costs  $\tau_{ij}$  above.

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<sup>3</sup>Indeed, one can show that for any set of realizations of random variables,  $\log \frac{1}{N} \sum_{i=1}^n \exp(x_n) \in [\max x_n - \log n, \max x_n]$ , so that the “LogMeanExp minus mean” can be interpreted as measuring the difference between the maximum of a random value and its mean. We show below that the “LogMeanExp minus the mean” of distances traveled across traders is highly correlated (correlation of 0.89) with the standard deviation of distance traveled.

**Equilibrium** Given the perfect competition assumption, the price for a unit of a good from  $i$  for a consumer in location  $j$  is  $p_{ij} = \tau_{ij} \frac{w_i}{A_i}$ . Given the CES preferences of consumers, the value of goods shipped from location  $i$  to location  $j$  can be written as:

$$X_{ij} = \tau_{ij}^{1-\sigma} A_i^{\sigma-1} w_i^{1-\sigma} P_j^{\sigma-1} E_j, \quad (13)$$

where  $P_j$  is the ideal Dixit-Stiglitz price index and  $E_j$  is the total expenditure of agents in location  $j$ .

In equilibrium, the following three conditions will hold:

1. Total income  $Y_i$  is equal to total sales:

$$Y_i = \sum_{j=1}^N X_{ij} \quad (14)$$

2. Total expenditure  $E_i$  is equal to total purchases:

$$E_i = \sum_{j=1}^N X_{ji} \quad (15)$$

3. Total income is equal to total expenditure which is each to total payments to workers:

$$Y_i = E_i = w_i L_i, \quad (16)$$

where  $L_i$  is the measure of agents residing in location  $i$ .

The equilibrium welfare of a worker is given by  $W_i \equiv \frac{w_i}{P_i} u_i$ , where  $u_i$  is the amenity value of living in location  $i$ . Using this expression to substitute out for the price index and the third equilibrium to substitute for  $E_j$ , we can rewrite equation (13) for the value of bilateral trade as follows:

$$X_{ij} = \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^\sigma L_j. \quad (17)$$

Substituting the gravity equation (17) and the equilibrium condition (16) into equilibrium conditions (14) and (15) yields a system of equilibrium conditions:

$$w_i L_i = \sum_i \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^\sigma L_j \quad (18)$$

$$w_i L_i = \sum_{j=1}^N \left( \frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} w_j^{1-\sigma} W_i^{1-\sigma} w_i^\sigma L_i. \quad (19)$$

We do so under two different assumptions regarding labor mobility:

**Assumption 1.** Trade framework (Anderson, 1979; Anderson and Van Wincoop, 2003). *Workers are perfectly immobile, so the measure of workers in each location  $\{L_i\}$  is exogenous.*

**Assumption 2.** Economic geography framework (Allen and Arkolakis, 2014). *Workers are perfectly mobile, so the measures of workers in each location ensures that welfare is equalized across all locations, i.e.  $W_i = W$  for all  $i \in \{1, \dots, N\}$ .*

In the trade framework, equations (18) and (19) can be solved directly to determine the equilibrium wage and welfare in all locations. In the economic geography framework, the imposition of welfare equalization results in the following system of equilibrium equations:

$$w_i L_i = W^{1-\sigma} \sum_i \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} w_j^\sigma L_j \quad (20)$$

$$w_i L_i = W^{1-\sigma} \sum_{j=1}^N \left( \frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} w_j^{1-\sigma} w_i^\sigma L_i, \quad (21)$$

which can be solved for the equilibrium wage and population in all locations along with the aggregate welfare  $W$ .<sup>4</sup>

In what follows, we use the system of equilibrium conditions above to derive how changes in transportation infrastructure affects the equilibrium welfare of agents. As an intermediate step, we first derive the welfare effects of a changes in a particular (endogenous) trade cost.

### 3.2 Elasticity of welfare to changes in trade costs

We first examine how a change in a (endogenous) bilateral trade costs affects the aggregate welfare. To do so, we construct a planner problem which maximizes aggregate welfare subject to resource constraints and then show that the first order conditions of this problem are identical to the equilibrium conditions (18) and (19) in the trade framework or equilibrium conditions (20) and (21) in the economic geography framework. This alternative planner formulation of the equilibrium then allows us to directly apply the envelope theorem to calculate the elasticity of welfare to a change in trade costs.

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<sup>4</sup>See Allen, Arkolakis, and Takahashi (2014) for a detailed description of the properties of both types of equilibria.

### 3.2.1 Trade framework

Consider the following “planner” problem which chooses wages and welfare to maximize a weighted average of (log) welfare subject to the constraint that aggregate labor income is equal to aggregate trade flows:

$$\begin{aligned} \max_{\{w_i, W_i\}} \sum_{i=1}^N \omega_i \ln W_i \text{ subject to:} \\ \frac{1}{1-\sigma} \sum_{i=1}^N w_i L_i = \frac{1}{1-\sigma} \sum_{i=1}^N \sum_{j=1}^N \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^\sigma L_j \end{aligned} \quad (22)$$

where  $\omega_i \equiv w_i L_i$  is the Pareto weight placed on the welfare of location  $i$  and the constraint requires that total world labor income is equal to total world sales.

The associated Lagrangian is:

$$\mathcal{L} : \sum_{i=1}^N \omega_i \ln W_i - \frac{1}{1-\sigma} \lambda \left( \sum_i \sum_j \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^\sigma L_j - \sum_i w_i L_i \right)$$

The first order conditions of this maximization problem can be written as:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \ln W_i} = 0 : \omega_i = \lambda \sum_{j=1}^N \left( \frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} w_j^{1-\sigma} W_i^{1-\sigma} w_i^\sigma L_i \\ \frac{\partial \mathcal{L}}{\partial \ln w_i} = 0 : w_i L_i = (1-\sigma) \sum_{j=1}^N \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W_j^{1-\sigma} w_j^\sigma L_j + \frac{\sigma}{\lambda} \omega_i \end{aligned}$$

Substituting in the weight  $\omega_i = w_i L_i$  and noting in equilibrium the Lagrange multiplier  $\lambda = 1$ , we see immediately that the solution to the maximization problem (22) is the equilibrium system of equations (18) and (19). As a result, we can apply the envelope theorem to derive the elasticity of world welfare to a change in bilateral trade frictions  $\tau_{ij}$ :

$$\sum_{i=1}^N \left( \frac{Y_i}{Y^W} \right) \frac{\partial \ln W_i}{\partial \ln \tau_{ij}} = - \frac{X_{ij}}{Y^W}. \quad (23)$$

Equation (23) – which has been derived using alternative methods by [Atkeson and Burstein \(2010\)](#) and [Fan, Lai, and Qi \(2013\)](#) – shows that the effect of a change in bilateral trade friction between  $i$  and  $j$  on a weighted average of world welfare in a trade framework is simply proportional to the fraction of world income comprised by trade flows from  $i$  to  $j$ .



### 3.2.2 Economic geography framework

Now consider the slight variant of the “planner” problem (22) which chooses wages, population, and welfare to maximize the (equalized) log welfare across locations subject to the same constraint that aggregate labor income is equal to aggregate trade flows:

$$\begin{aligned} & \max_{\{\{w_i\}, \{L_i\}, W\}} \ln W \text{ subject to:} \\ & \frac{1}{1-\sigma} \sum_{i=1}^N w_i L_i = \frac{1}{1-\sigma} \sum_{i=1}^N \sum_{j=1}^N \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^\sigma L_j \end{aligned} \quad (24)$$

The associated Lagrangian is:

$$\mathcal{L} : \ln W - \frac{1}{1-\sigma} \lambda \left( \sum_i \sum_j \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^\sigma L_j - \sum_i w_i L_i \right)$$

The first order conditions of this maximization problem can be written as:

$$\frac{\partial \mathcal{L}}{\partial \ln W} = 0 : \frac{1}{W} = \lambda \sum_i \sum_j \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W^{-\sigma} w_j^\sigma L_j \iff$$

$$\lambda = \frac{1}{Y^W}$$

$$\frac{\partial \mathcal{L}}{\partial \ln w_i} = 0 : w_i L_i = (1-\sigma) \sum_{j=1}^N \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^\sigma L_j + \sigma \sum_j \left( \frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} w_j^{1-\sigma} W^{1-\sigma} w_i^\sigma L_i \quad (25)$$

$$\frac{\partial \mathcal{L}}{\partial \ln L_i} = 0 : w_i L_i = W^{1-\sigma} \sum_j \left( \frac{\tau_{ji}}{A_j u_i} \right)^{1-\sigma} w_j^{1-\sigma} w_i^\sigma L_i \quad (26)$$

Substituting equation (26) into equation (25) yields:

$$w_i L_i = W^{1-\sigma} \sum_j \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} w_j^\sigma L_j \quad (27)$$

Hence, the first order conditions (26) and (27) correspond to the equilibrium conditions (20) and (21). As a result, we can apply the envelope theorem to determine the elasticity of welfare to a change in the (endogenous) trade cost  $\tau_{ij}$ , which yields:

$$\frac{\partial \ln W}{\partial \ln \tau_{ij}} = \frac{\partial \mathcal{L}}{\partial \tau_{ij}} \times \tau_{ij} = \lambda \left( \frac{\tau_{ij}}{A_i u_j} \right)^{1-\sigma} w_i^{1-\sigma} W^{1-\sigma} w_j^\sigma L_j = \frac{X_{ij}}{Y^W}. \quad (28)$$

Hence, in both the case where labor is perfectly immobile and perfectly mobile, the elasticity of aggregate welfare to a change a bilateral trade cost,  $\frac{\partial \ln W}{\partial \ln \tau_{ij}}$ , is equal to the fraction of world income that is traded from that origin to that destination,  $\frac{X_{ij}}{Y^W}$ .

### 3.3 Elasticity of welfare to changes in infrastructure

A simple application of the product rule allows us to combine equation (10) – which provides the elasticity of bilateral trade costs between any pair of locations to a change in any infrastructure – with equations (23) and (28) – which give us the elasticity of welfare with respect to a change in bilateral trade costs in economic geography and trade frameworks, respectively – to determine the welfare effect of any change in infrastructure:

$$\frac{d \ln W}{d \ln t_{ij}} = \sum_{k=1}^N \sum_{l=1}^N \frac{d \ln W}{d \ln \tau_{kl}} \times \frac{d \ln \tau_{kl}}{d \ln t_{ij}}. \quad (29)$$

Note that because traders are optimally choosing their routes, any change in infrastructure will affect all bilateral trade flows. Substituting in equations (10) and either (28) or (23) into equation (29) yields:

$$- \frac{d \ln W}{d \ln t_{ij}} = \sum_{k=1}^N \sum_{l=1}^N \frac{X_{kl}}{Y^W} \frac{b_{ki} a_{ij} b_{jl}}{b_{kl}}, \quad (30)$$

which can be written in “trade notation” as:

$$- \frac{d \ln W}{d \ln t_{ij}} = c^\theta \sum_{k=1}^N \sum_{l=1}^N \frac{X_{kl}}{Y^W} \left( \frac{\tau_{kl}}{\tau_{ki} t_{ij} \tau_{jl}} \right)^\theta. \quad (31)$$

As an example, return to the two dimensional grid network considered in Figure 1 and suppose all locations have identical productivities and amenities and labor is mobile. The bottom right panel of Figure 1 depicts the elasticity of welfare to a change in each direct connection using equation (31). As is evident, reducing the cost of traveling over links in the center of the grid have larger impacts on welfare than those in the periphery. Intuitively, this is for two reasons: first, the connections in the center are more likely to be traveled than on the periphery (see the bottom left panel of Figure 1) and hence will have larger effects on bilateral trade costs; second, because they are more centrally located, locations in the center will have greater populations and economic output, so that the trade flows flowing through the central links will be larger on average. As a result, improvements to those links will have larger effects on welfare.

We can further express this equation solely in terms of data and  $(i, j)$  variables by noting

from gravity equation (17) that we can write:

$$\left(\frac{X_{kl}X_{ji}}{X_{ki}X_{jl}}\right) = \left(\frac{\tau_{kl}\tau_{ji}}{\tau_{ki}\tau_{jl}}\right)^{1-\sigma},$$

so that expression (30) becomes:

$$-\frac{d \ln W}{d \ln t_{ij}} = \left(\frac{t_{ij}\tau_{ji}}{c}\right)^{-\theta} X_{ji}^{\frac{\theta}{1-\sigma}} \sum_{k=1}^N \sum_{l=1}^N \left(\frac{X_{kl}}{Y^W}\right) \left(\frac{X_{ki}X_{jl}}{X_{kl}}\right)^{\frac{\theta}{\sigma-1}}. \quad (32)$$

Equation (32) provides a “sufficient statistic” of the welfare effect of any infrastructure improvement that depends only observed bilateral trade flows and knowledge of the expected trade cost between  $j$  and  $i$ ,  $\tau_{ji}$ , and the instantaneous trade cost,  $t_{ij}$ . Moreover, the result is intuitive: a larger value of  $\frac{X_{ki}X_{jl}}{X_{kl}}$  is indicative of low trade costs from  $k$  to  $i$  and low trade costs of  $j$  to  $l$  relative to trade costs between  $k$  and  $l$ , which implies that more trade from  $k$  to  $l$  travels through  $i$  and  $j$ . As a result, an improvement in the infrastructure between  $i$  and  $j$  will have a larger effect on trade flows between  $k$  and  $l$ . Finally, the larger the value of  $\theta$ , the more responsive traders are to changes in the relative cost of different paths; conversely, the larger the value of  $\sigma$ , the more responsive trade flows are to changes in trade costs (so a given ratio of trade flows is indicative of smaller differences in trade costs). As a result, the relevant measure of the effect of an infrastructure improvement between  $i$  and  $j$  on trade flows between  $k$  and  $l$  in terms of observables is  $\left(\frac{X_{ki}X_{jl}}{X_{kl}}\right)^{\frac{\theta}{\sigma-1}}$ . To calculate the effect of such an infrastructure improvement on total world welfare, we then simply construct a weighted average of this ratio, where the weights are the fraction of world trade between any two locations  $\frac{X_{kl}}{Y^W}$ .

## 4 The welfare effects of improving the U.S. Interstate Highway System

In this section, we apply the theoretical framework developed above to estimate the aggregate welfare effect of the U.S. of improving each segment of the Interstate Highway System (IHS). The IHS – depicted in Panel A of Figure 2 – is one of the largest infrastructure investments in history (Kaszynski, 2000). It took more than thirty five years to construct at an estimated cost \$560 billion (in 2007 dollars), and total annual maintenance costs of the highway system is approximately \$130 billion (CBO, 1982; FHA, 2008; NSTIFC, 2009). However, little is known about the relative importance of different segments of the IHS in terms of how each affects the welfare of the U.S. population. Such knowledge is crucial for appropriately

targeting future investments in the improvement of the IHS.

## 4.1 Data

To assess the relative importance of each segment of the IHS, we use shipment level data between 67 U.S. metropolitan statistical areas (which we refer to as “cities”) from the public use micro data of the 2012 Commodity Flow Survey (CFS, 2012). Shipments between these cities constitute 60% of the value of commodity flows within the United States. For each of the 696,021 shipments on for-hire trucks between these cities, we observe the origin city, the destination city, the value of the shipment, and the distance the truck traveled on its chosen route.<sup>5</sup> Consistent with the theoretical framework developed above, the CFS micro-data demonstrate that there exists substantial heterogeneity across shipments in the distance they travel between a particular origin city - destination city pair. Panel A of Figure 3 depicts the distribution of distance traveled for the twenty origin city - destination city pairs with the greatest observed trade flows. For example, the interquartile range of shipments from New York to Boston is 91 miles. While certainly part of this variation is due to differences in the particular origin and destination within the two cities, it seems likely that differences in the path chosen also are responsible for part of this variation.<sup>6</sup>

We proceed by constructing the graph representation of the IHS. We say that a pair of cities in the CFS is directly connected by the IHS if the shortest route between the two cities on the IHS does not pass through another city.<sup>7</sup> For each connected city pair, we then calculate both the distance traveled and the time it takes to traverse the direct route using Google Maps. The resulting graph of the IHS network is depicted in Panel B of Figure 2.

Finally, for all origin city - destination city pairs, we use the CFS micro data to construct an aggregate measure of the value of bilateral trade flows, the mean distance traveled between the pair, the standard deviation of distance traveled between the pair, and the difference between the log of the mean of the exponential and the mean distance traveled (the “LogMeanExp minus Mean”).<sup>8</sup> As discussed above, the “LogMeanExp minus Mean” is an alternative measure of the variability of a random variable; consistent with this, Panel B of

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<sup>5</sup>We constrain our analysis to for-hire trucks because the distance traveled for private trucks is top-coded at 500 miles.

<sup>6</sup>We examine the extent to which the model can capture the variation across cities in route distance variability despite treating each city as a single point below.

<sup>7</sup>We allow a small amount of flexibility in defining the shortest route. For example, without traffic, traveling from Denver, CO to Seattle, WA takes 18 hours and 16 minutes via Salt Lake City on I-80 W and I-84 W takes 18 hours and 31 minutes on I-25 N and I-90 W without passing through another CFS city. Because the direct route is nearly as fast, we classify Denver and Seattle as directly connected on the IHS.

<sup>8</sup>To control for outliers, we winsorize the distance traveled in the micro CFS data at the 10%/90% level before calculating the variability measures. Both variability measures are measured in hundreds of miles.

Figure 3 shows that the correlation between the standard deviation and the “LogMeanExp minus Mean” is very high (0.89).

## 4.2 Estimation

In order to assess the elasticity of aggregate welfare to improvements to different segments of the IHS, we need (1) a measure of the infrastructure matrix  $\mathbf{T} = [t_{ij} \geq 1]$  and (2) estimates of the shape parameter governing heterogeneity across routes  $\theta$  and the elasticity of substitution  $\sigma$ . To proceed, we assume that the trade cost incurred along the direct connection from  $i$  and  $j$  is a function of the time it takes to travel from  $i$  to  $j$  (in hours)  $time_{ij}$ :

$$t_{ij} = \exp(\kappa time_{ij}), \quad (33)$$

where  $\kappa > 0$ . The exponential functional form has been used extensively in the economic geography literature, and has a number of attractive properties including (1) infrastructure costs are always greater or equal to one, i.e.  $t_{ij} \geq 1$ ; (2) conditional on travel time, the number of locations through which a trader passes does not affect the trade costs that trader incurs; and (3) the effect of a reduction in travel time on the percentage change in welfare is proportional to the welfare elasticity to a change in the infrastructure cost, i.e.  $\frac{\partial \ln W}{\partial time_{ij}} = \kappa \frac{\partial \ln W}{\partial \ln t_{ij}}$ .

Given parametric assumption in equation (33), we just need to estimate the parameter triplet  $\{\kappa, \theta, \sigma\}$ . To proceed, we use a method of moments estimation strategy using three sets of moments: (1) the mean distance traveled between each origin city - destination city pair; and (2) the observed value of bilateral trade flows between each origin city - destination city pair; and (3) an average trade cost 20% ad valorem tariff equivalent, consistent with the evidence from the literature on domestic distribution costs in the U.S., see e.g. [Anderson and Van Wincoop \(2004\)](#). This provides us with  $2 \times N^2 + 1 = 8979$  moments to estimate the three parameters.

The three sets of moments match intuitively to each of the parameters. Given the parametric assumption in equation (33),  $a_{ij} = \exp(-(\theta\kappa) dist_{ij})$ , so that the matrices  $\mathbf{A}$  and  $\mathbf{B}$  depend only on the product  $\theta\kappa$ . From equation (12), this in turn implies that the expected distance between any origin city - destination city pair depends only on the product of  $\theta$  and  $\kappa$  as well, so the first set of moments (the mean in distance traveled) are sufficient to identify  $\theta\kappa$ . The second set of moments – the bilateral trade flows – can then be used to identify the ratio of the elasticity of substitution and the trader heterogeneity  $\frac{\sigma-1}{\theta}$ . To see this, note that substituting equation (3) into the gravity equation (17) and taking logs yields the following

expression for bilateral trade flows:

$$\ln X_{ij} = \left( \frac{\sigma - 1}{\theta} \right) \ln b_{ij} + \ln \gamma_i + \ln \delta_j,$$

where  $\ln \gamma_i$  and  $\ln \delta_j$  are origin and destination fixed effects, respectively. For any set matrix  $\mathbf{B} = [b_{ij}]$ , this implies that we can identify the ratio of parameters  $\frac{\sigma-1}{\theta}$  by regressing log trade flows on  $\ln b_{ij}$  with a set of origin and destination fixed effects. Finally, from equation (3), we see that the level of trade costs are differentially affected by  $\kappa$  and  $\theta$ , as  $b_{ij}$  depends on the product  $\kappa\theta$  but  $\tau_{ij} = c b_{ij}^{-\frac{1}{\theta}}$ , allowing us to find the level of the  $\theta$  that ensures the mean trade cost matches our assumed value.

### 4.3 Estimation Results

Table 1 presents the results of the estimation. We estimate an effect of travel time (in hours)  $\kappa = 0.0108$ , indicating a five hour trip incurs an ad valorem tariff equivalent trade cost of 5.5%. We estimate an elasticity of substitution  $\sigma = 7.92$ , implying a trade elasticity of about 7, consistent with a large literature (e.g. [Eaton and Kortum \(2002\)](#)). Finally, we estimate a trader heterogeneity parameter  $\theta = 136.13$ , indicating reasonably small heterogeneity across different paths, which suggests most trade occurs on paths close to the least cost route. The implied trade costs from these parameters are calibrated to have a mean of 20% ad valorem tariff equivalent, with a median of 16.9% and moderate variation (a 25th percentile of 10.4% and a 75 percentile of 27.1%).

Figure 4 shows that the model is able to closely match the observed average distance traveled in the data (with a correlation of 0.997) and the observed bilateral trade flows (with a correlation of 0.737). Moreover, we can also test the extent to which the model is able to capture the observed heterogeneity in distance traveled between a particular origin city - destination city pair. Reassuringly, the “LogMeanExp minus Mean” predicted by the model is positively and statistically significantly correlated with the “LogMeanExp minus Mean” observed in the data, albeit with a correlation of only 0.03. However, is able to closely capture the observed variation across origin and destination cities in variability of routes. To see this, we regress variability in traveled distance for a given origin-destination city pair across shipments on a set of origin and destination city fixed effects. Figure 5 shows the correlation between those fixed effects in the CFS data to those estimated in the model is positive and reasonably large (0.318 for origin fixed effects and 0.168 for destination fixed effects), although the model predicts substantially more variation in route distance than exists in

the data.<sup>9</sup> Note that this positive correlation exists despite the model not accounting for across-city variation in the size of the Metropolitan Statistical Area, which is likely driving part of variation in the data due to within-city differences in where the particular origin and destination was for a given shipment.

#### 4.4 The welfare effects of improving the U.S. Interstate Highway System

Given our estimated parameters, we can estimate the probability of using any particular connection for any origin city - destination city pair using equation (7). As an example, Figure 6 depicts the probability a trader going from Los Angeles to New York uses each part of the Interstate Highway System. As is evident, there is reasonable variation in the paths chosen, with traders employing both northern connections (e.g. Salt Lake City to Omaha) and southern connections (e.g. Phoenix to Oklahoma City) along the way. As traders continue eastward, however, the geographic dispersion of connections narrow, as routes converge through Indiana, Ohio, and Pennsylvania.

Finally, we estimate the welfare effects of improving the U.S. Interstate Highway System. To do so, we apply equation (30) to all connected origin city - destination city pairs to calculate  $\frac{\partial \ln W}{\partial \ln t_{ij}}$ . Figure 7 presents the results. The portions of the IHS for which improvements would have the largest impact on aggregate welfare are the I-95 corridor which connects the major eastern seaboard cities (Washington, Baltimore, Philadelphia, and New York) and the interstates in the states of Ohio and Indiana (“the crossroads of America”), through which much of the U.S. trade flows. Table 2 reports the top 20 IHS segments for which improvements would have the greatest welfare impacts. As is evident, the welfare impact of transportation infrastructure improvements is substantial: our parameter estimates imply that a 30 minute reduction in the travel time from New York to Philadelphia – which given our estimate of  $\kappa$  implies a fall in trade costs of about 0.5% – would increase aggregate U.S. welfare by 0.02%. However, there is substantial heterogeneity across various parts of the IHS – for example, a 30 minute reduction in travel time from Seattle to Salt Lake City along I-84 East would only increase aggregate U.S. welfare by 0.0002%. Hence, these results highlight the need to appropriately target funds for infrastructure improvements in order to maximize the benefit.

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<sup>9</sup>Indeed, if we also include the variability of distance traveled as a moment, the estimated trader heterogeneity  $\theta$  increases to 228, suggesting we may be potentially overstating the heterogeneity across traders.

## 5 Conclusion

TBD



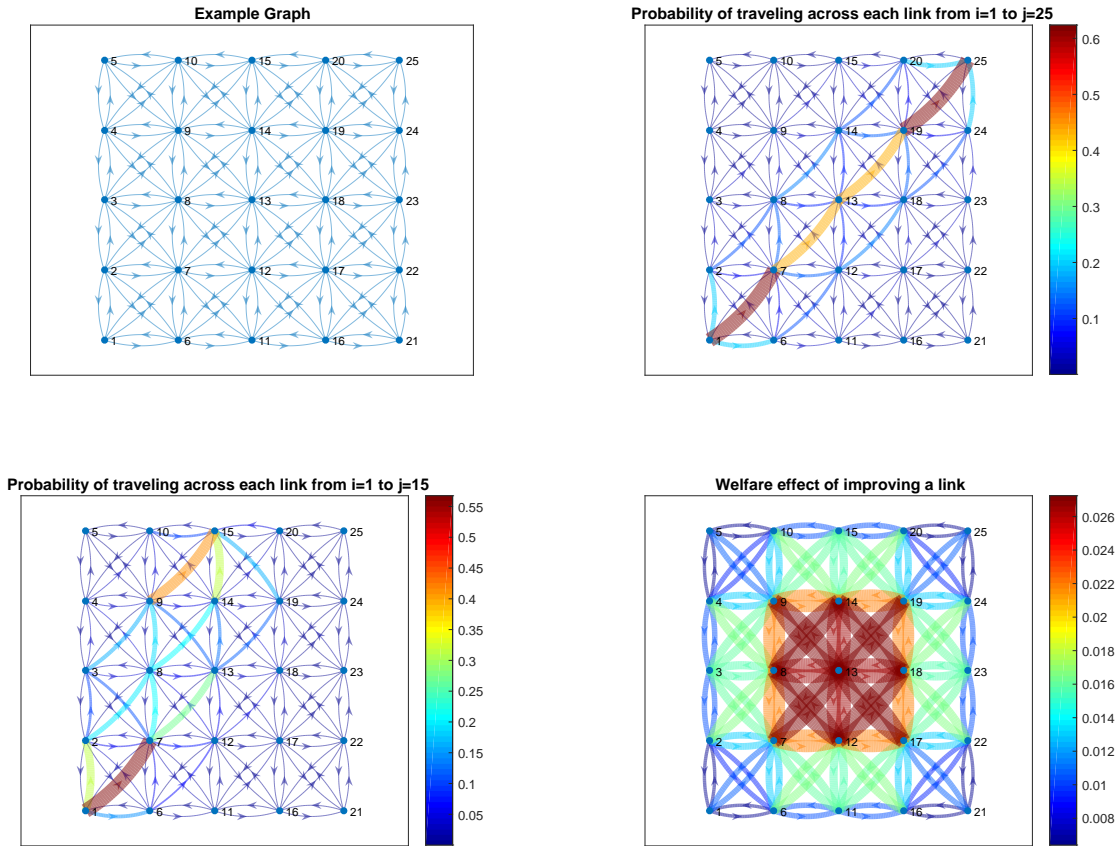
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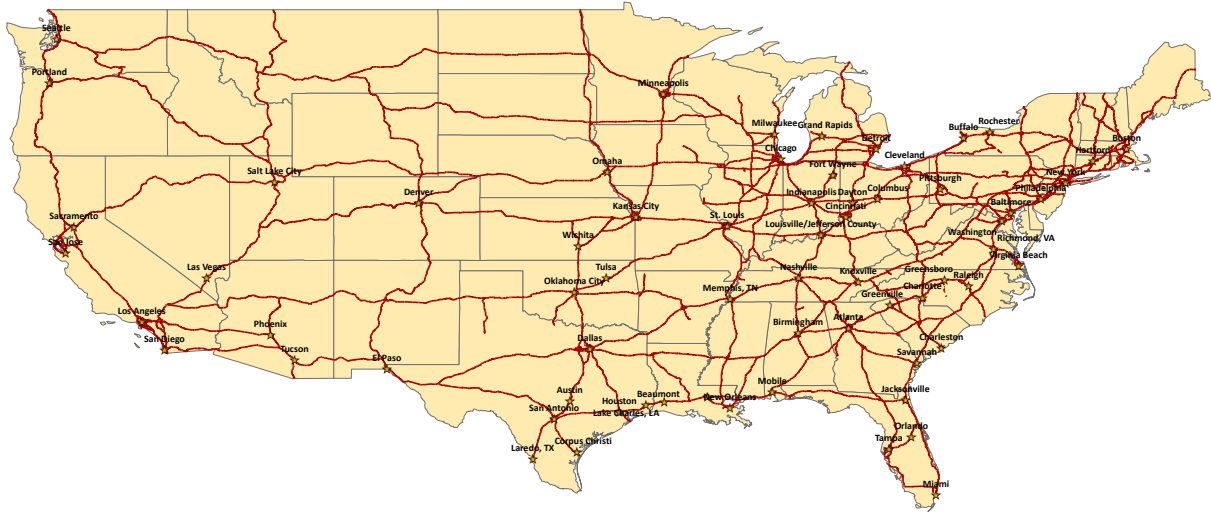
# Tables and Figures

Figure 1: AN EXAMPLE GEOGRAPHY

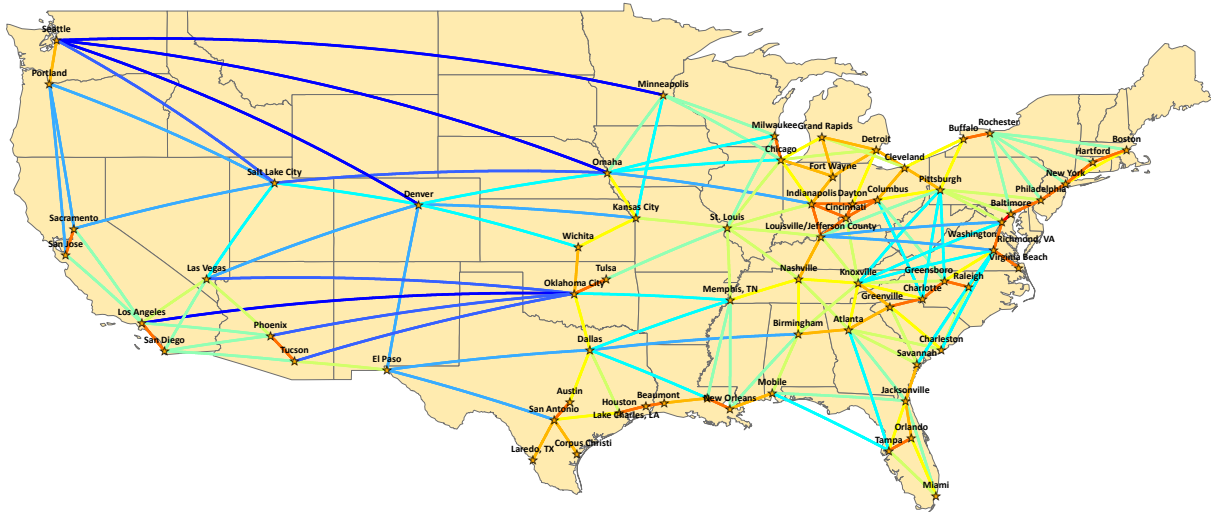


*Notes:* This figure provides an example geography. The top left frame depicts the graph of the connections between each of the 25 locations, where all connections are assumed to incur an equal cost. The top right depicts the probability of traveling along each connection when beginning at location 1 (bottom left) and traveling to location 25 (top right). The bottom left frame also depicts the probability of traveling along each connection when beginning at location 1 (bottom left) but now traveling to location 15 (top center). Finally, the bottom right figure depicts the elasticity of aggregate welfare to an improvement in each of the connections.

Figure 2: THE U.S. INTERSTATE HIGHWAY NETWORK  
 Panel A: *The Observed U.S. Interstate Highway Network*

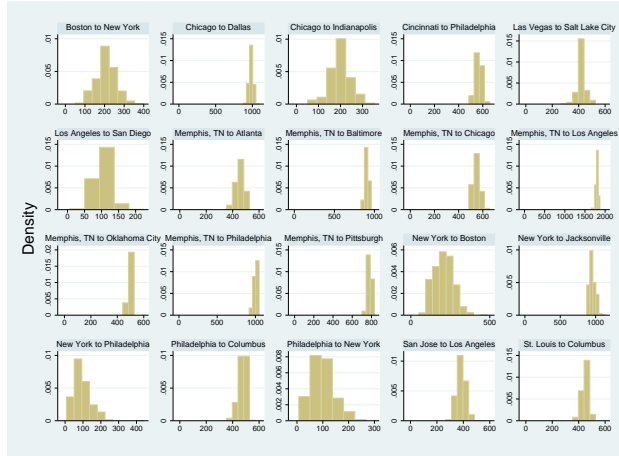


Panel B: *A Graphical Representation of the U.S. Interstate Highway Network*

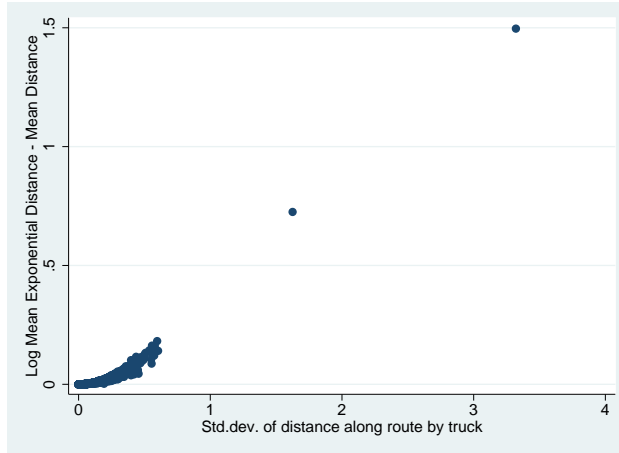


Notes: This figure depicts the U.S. Interstate highway network. Panel A shows the observed interstate highway network along with the cities included in the Commodity Flow Survey. Panel B depicts the graphical representation of the direct connections between cities along the interstate highway system. Two cities are considered connected if the fastest route between the locations along the interstate highway system does not pass through another city. The color of the links indicates the driving distance between the connected cities, with blue indicating a longer distance and red indicating shorter distances.

Figure 3: THE VARIABILITY OF ROUTE DISTANCES ACROSS SHIPMENTS  
 Panel A: *Variation in Distance Traveled between Major City Pairs*

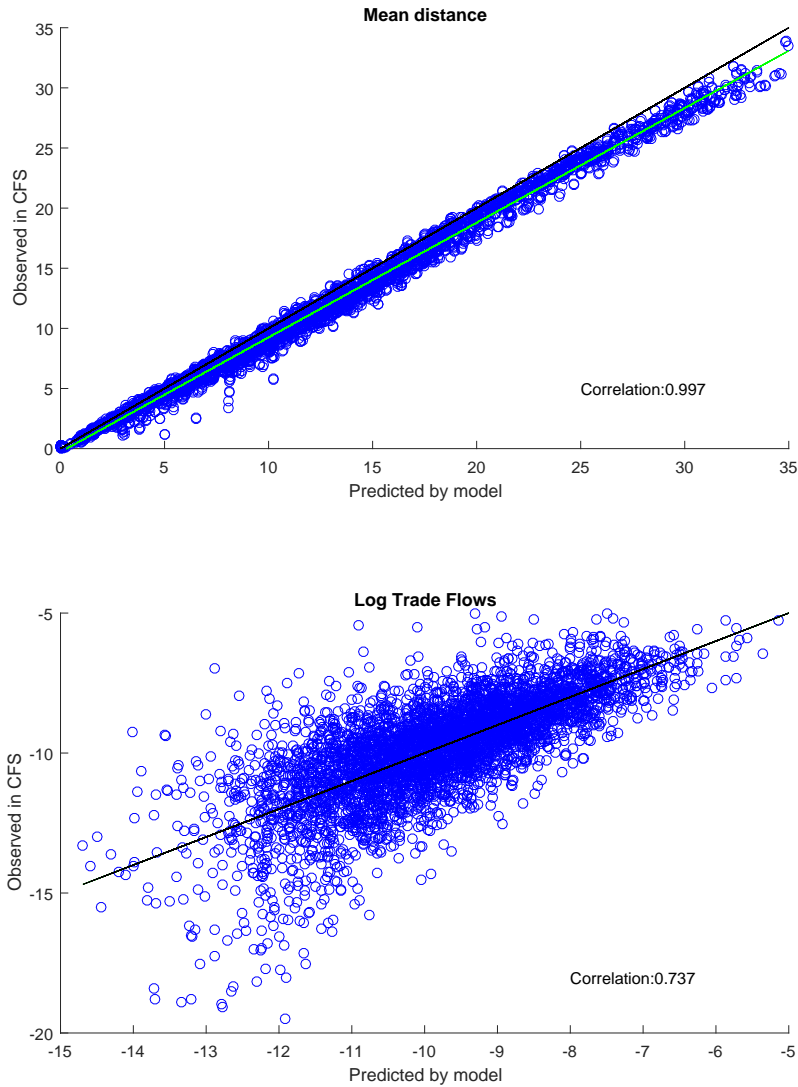


Panel B: *Correlation between Different Measures of Variability*



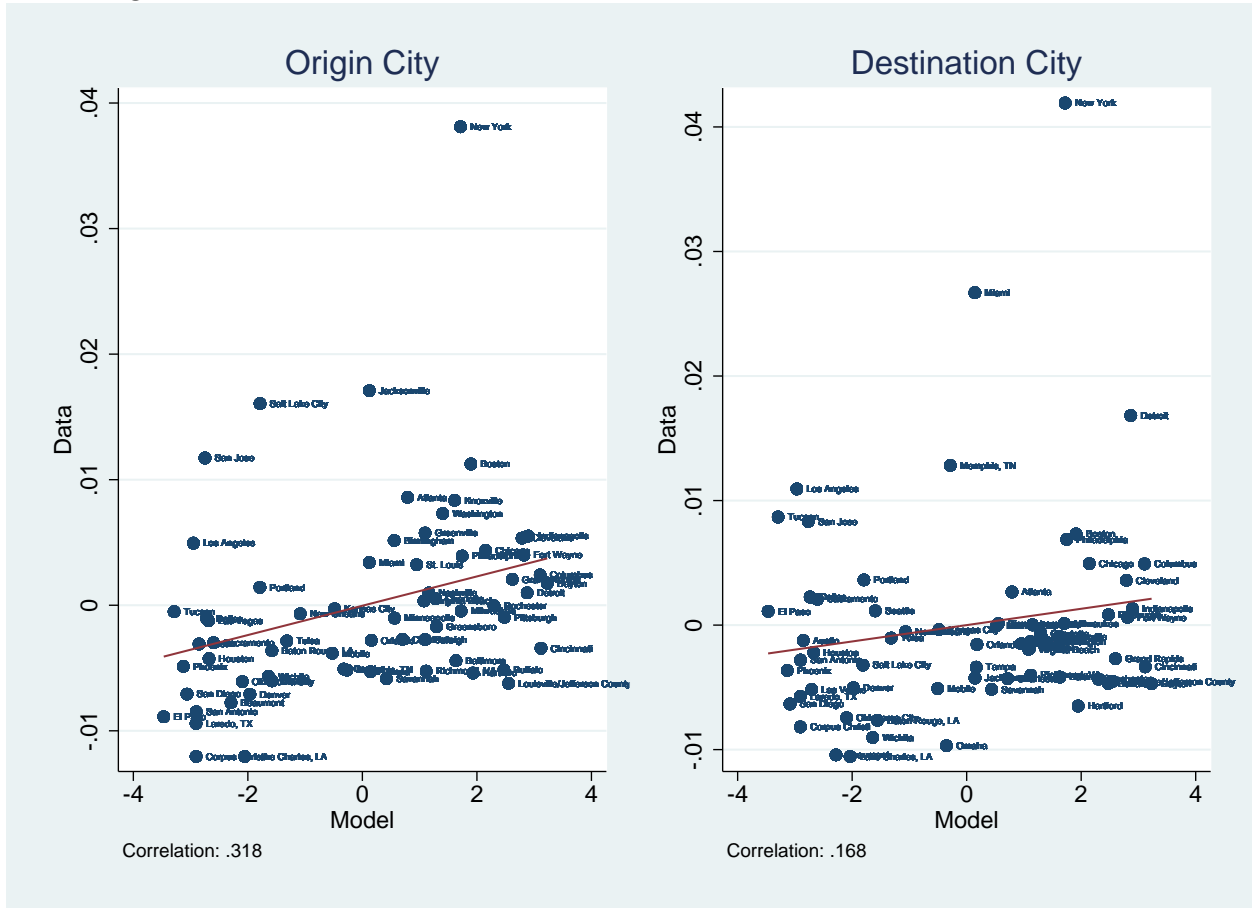
*Notes:* This figure shows the variability in distances traveled from a given origin to a given destination across shipments in the CFS micro data. Panel A presents the histogram of distances traveled for the twenty origin-destination city pairs with the largest observed value of shipments. Panel B presents the relationship between the standard deviation in distance traveled between a particular city pair to the log of the mean of the exponential distance traveled, where distances are measured in hundreds of miles. Both measures of variability are calculated across all for-hire truck shipments in the micro CFS data and winsorized at the 10%/90% level. The correlation between the two measures is 0.89. The two outlying origin-destination pairs are shipments from Jacksonville to Memphis and from Salt Lake City to El Paso.

Figure 4: MODEL FIT



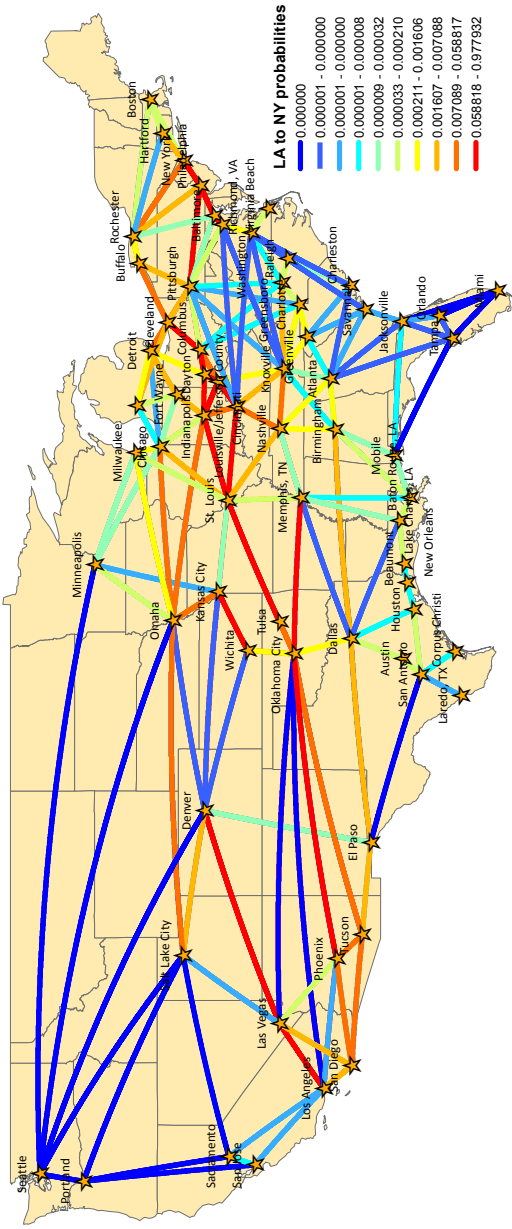
*Notes:* This figure depicts the ability of the model to match the moments in the data. The top panel reports the relationship between the model predicted and observed mean distance (in hundreds of miles) across all shipments within each city pairs. The middle panel reports the relationship between the model predicted and observed difference between the log of the mean of the exponential of the distance and the mean distance, which measures the heterogeneity in which routes are chosen between city pairs. The bottom panel reports the relationship between the observed trade flows and those predicted from a gravity regression with origin and destination fixed effects using the model predicted average bilateral trade costs.

Figure 5: PREDICTED AND OBSERVED VARIABILITY OF ROUTE DISTANCES



*Notes:* This figure compares the observed and model predicted variability of distance traveled by origin and destination cities. To do so, we regress variability in traveled distance for a given origin-destination city pair across shipments (measured as the difference between the log of the mean of the exponential distance traveled ) on a set of origin and destination city fixed effects. The figure depicts the relationship between those fixed effects in the CFS data to those estimated in the model.

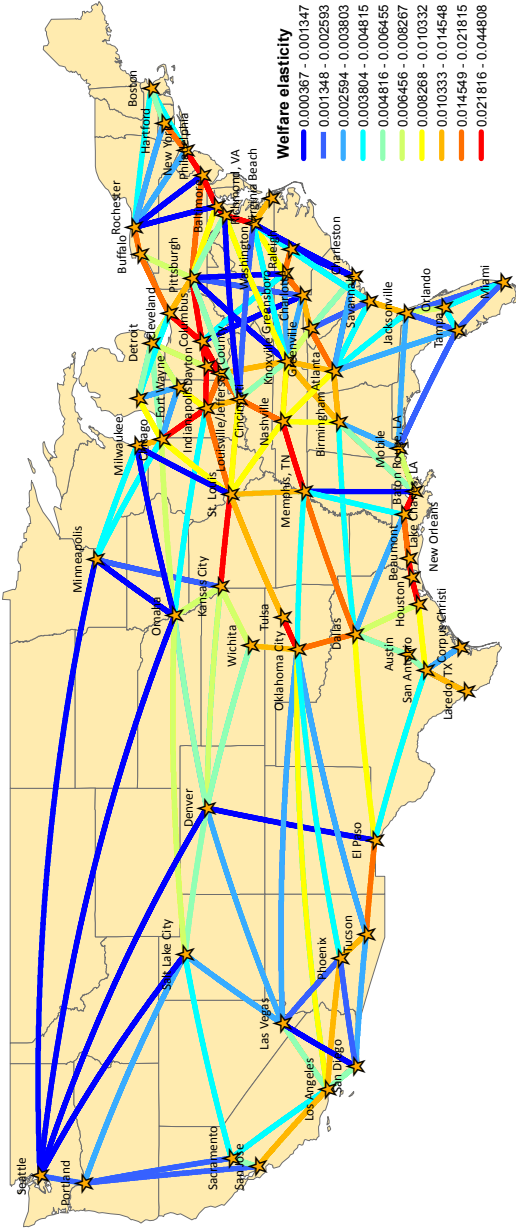
Figure 6: PROBABILITY OF USING DIFFERENT HIGHWAYS FROM LA TO NY



Notes: This figure depicts the probability of a trader going from Los Angeles to New York using each particular connection of the Interstate Highway System. This is equivalent to the elasticity of trade costs from LA to NY to a change in the cost of traveling over each connection.



Figure 7: THE WELFARE EFFECTS OF TRANSPORTATION INFRASTRUCTURE IMPROVEMENTS



*Notes:* This figure depicts the elasticity of total welfare in the U.S. to a change in the direct trade cost between any two locations, accounting for both how that change affects bilateral trade costs between all locations and the general equilibrium response to those changes in trade costs. The color of each link indicates its relative elasticity, with blue indicating a smaller welfare effect and red indicating a larger welfare effect.

Table 1: ESTIMATION

*Model parameters*

Parameter	Symbol	Moment	Estimated value	Correlation: predicted vs. observed
Effect of distance on direct trade cost	$\kappa$	Mean path distance	0.0108	0.997
Trader heterogeneity	$\theta$	Trade flows	136.13	0.737
Elasticity of substitution	$\sigma$	Trade costs	7.9237	1

*Implied trade costs*

Mean	25% perc.	Median	75% perc.
1.2 (calibrated)	1.104	1.169	1.271

*Notes:* This table reports the estimated parameter values and summary statistics for the implied bilateral trade costs. The three parameters were estimated to most closely match the three reported moments; note that the estimation procedure simultaneously estimated all parameters by minimizing all moments, so the assignment of parameter to moment above is heuristic.

Table 2: TOP 20 HIGHWAYS

	Origin City	Destination City	Interstate	Welfare Elasticity
1	New York	Philadelphia	95 (South)	0.04481
2	Baltimore	Washington	95 (South)	0.04477
3	Columbus	Dayton	70 (West)	0.0438
4	Dayton	Columbus	70 (East)	0.04094
5	Dayton	Cincinnati	75 (South)	0.04007
6	Philadelphia	Baltimore	95 (South)	0.03961
7	Cincinnati	Dayton	75 (North)	0.03691
8	Philadelphia	New York	95 (South)	0.03292
9	Washington	Baltimore	95 (North)	0.03069
10	Washington	Richmond, VA	95 (South)	0.03025
11	Beaumont	Houston	10 (West)	0.02874
12	Tulsa	Oklahoma City	44 (West)	0.02874
13	Pittsburgh	Columbus	70(West)	0.02868
14	Lake Charles, LA	Beaumont	10 (West)	0.02858
15	Baltimore	Philadelphia	95 (North)	0.02793
16	Columbus	Cincinnati	71 (South)	0.02742
17	Cincinnati	Louisville/Jefferson County	71 (South)	0.02662
18	Louisville/Jefferson County	Nashville	65 (South)	0.02659
19	Columbus	Cleveland	71 (North)	0.02629
20	Nashville	Memphis, TN	40 (West)	0.02617

*Notes:* This table reports the twenty (one-way) interstate links that have the greatest improvement on U.S. welfare for a given percentage reduction in trade costs on that connection.