

Not so Disconnected: Exchange Rates and the Capital Stock*

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Abstract

We investigate the link between stochastic properties of exchange rates and differences in capital-output ratios across industrialized countries. To this end, we endogenize capital accumulation within a standard model of exchange rate determination with non-traded goods. The model predicts that currencies of countries that are more systemic for the world economy (countries that face particularly volatile shocks or account for a large share of world GDP) appreciate when the marginal utility of traded goods is high. These currencies are better hedges against consumption risk faced by international investors because they appreciate in “bad” states of the world. As a consequence, more systemic countries face a lower cost of capital, accumulate more capital per worker, and pay higher wages than less systemic countries. We estimate our model using data from seven industrialized countries with freely floating exchange rate regimes 1984-2010 and show that cross-currency variation in the stochastic properties of exchange rates accounts for 72% of the cross-country variation in capital-output ratios. In this sense, the stochastic properties of exchange rates map to fundamentals in the way predicted by the model.

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1 Introduction

A large literature links variation in capital-output ratios across countries to differences in the cost of capital and technology. Such differences have been attributed to heterogeneity in the protection of property rights (Hall and Jones, 1997), taxation rates (Jorgenson, 1996), depreciation rates, the capital share of output (Neiman and Karabarbounis, 2014), and distortions in the allocation of resources (Hsieh and Klenow, 2009). In this paper, we argue that differences in the stochastic properties of countries' real exchange rates may be another important driver of variation in capital-output ratios across countries.

If firms are price-takers and use a Cobb-Douglas production technology, we can summarize the various drivers of heterogeneity across countries in capital-output ratios by equating the expected marginal product of capital to its marginal cost and writing

$$\mathbb{E}(Y_i/K_i) = \frac{(\mathbb{E}[\tilde{r}_i] + \delta_i) \tau_i}{\nu_i},$$

where $\mathbb{E}(Y_i/K_i)$ is the expected output per unit of capital installed in country i , δ_i is the depreciation rate, ν_i is the capital share of output, and τ_i summarizes distortions that may be caused by variation in tax rates, the protection of property rights, or dysfunctional markets. In contrast to these well-established factors, cross-country variation in the required net rate of return on capital, $\mathbb{E}[\tilde{r}_i]$, has received relatively little attention as a potential driver of differences in capital accumulation across countries.

Nevertheless, there are good reasons to believe that $\mathbb{E}[\tilde{r}_i]$ varies systematically across countries. A growing literature in international asset pricing argues that heterogeneity in the stochastic properties of countries' real exchange rates generate differences in one important component of $\mathbb{E}[\tilde{r}_i]$ – the risk-free rate of interest. In particular, a common theme in this literature is that countries whose currencies tend to appreciate in “bad” times pay lower expected returns (Lustig and Verdelhan, 2007) and have lower risk-free interest rates (Hassan, 2013; Martin, 2012).

In this paper, we endogenize capital accumulation within a canonical model of exchange rate determination and show that countries whose currencies appreciate when the shadow price of traded goods is high also have a lower $\mathbb{E}[\tilde{r}_i]$. As a result, these coun-

tries install higher capital-output ratios in equilibrium. When we calibrate our model to match data for seven OECD countries with freely-floating exchange rates, we find that heterogeneity in the stochastic properties of their exchange rates can account for 72% of the variation in their relative capital-output ratios.

Our model builds on the canonical model of exchange rate determination: households consume a bundle of a freely traded good and a country-specific nontraded good. The nontraded good is produced using capital and labor as inputs, subject to a country-specific productivity shock. This productivity shock induces variation in output of nontraded goods that allows the consumption price index to differ across countries and, as a result, induces variation in the real exchange rate. In addition, country-specific shocks to the inflation rate affect real exchange rates because a subset of households can only hold nominal bonds that make fixed payments in terms of the currency of the country in which they reside. The remaining households have access to complete international asset markets. In equilibrium, the real exchange rate thus responds to (real) shocks to the productivity in production of nontraded goods across countries and to (monetary) shocks to the inflation rates of national currencies.

Our most important addition to this canonical setup is heterogeneity across countries in volatility and size: some countries are affected by more volatile productivity or inflation shocks than others and some countries account for a larger share of world GDP than others (they comprise more or wealthier households).

Heterogeneity in volatility and size generates differences in the stochastic properties of countries' exchange rates. First, countries that experience more volatile productivity and inflation shocks tend to have more volatile bilateral exchange rates. Second, larger and more volatile countries tend to appreciate in "bad" times: households react to shocks by shipping traded goods across countries in an effort to share risk across borders. However, the shocks that affect larger or more volatile countries (for ease of reference, we refer to them as more "systemic" countries) are harder to diversify internationally. For example, when a country has a low per capita output of nontraded goods, its nontraded good becomes relatively more expensive, its real exchange rate appreciates, and it imports additional traded goods. However, a low output of nontraded goods in a larger country simultaneously triggers a larger rise in the world-wide shadow price of traded goods, while

a low output in the nontraded sector of a small country does not. As a consequence, the currencies of more systemic countries tend to appreciate when the shadow price of traded goods is high.

These differences in the stochastic properties of real exchange rates in turn are indicative of differential incentives for capital accumulation. Households around the world worry about states of the world in which more systemic countries suffer bad productivity shocks because these shocks are transmitted to the rest of the world in the form of higher shadow prices of traded goods. Although households cannot prevent these states from occurring, they can partially insure against them by accumulating more capital and thus raising the mean level of output of nontraded goods in systemic countries across all states. Accumulating more capital in more systemic countries thus softens the impact of adverse shocks in these countries and represents a good hedge against world-wide consumption risk. In other words, large countries and countries with volatile exchange rates have a low required rate of return on capital and attract investment because they have “safe” currencies that appreciate in “bad” times.

An important detail in this prediction of the model is that heterogeneity in the variance of shocks across countries affecting the traded sector has no effect on differential incentives for accumulating capital. The reason is that shocks affecting the traded sector are fully shared internationally and thus have no effect on the *relative* price of consumption in different countries and on real exchange rates. In this sense, differential incentives to accumulate capital are intimately linked to differences in the stochastic properties of real exchange rates rather than the stochastic properties of GDP or aggregate consumption.

The prediction that more systemic countries accumulate larger capital-output ratios is consistent with additional implications of the model for international asset prices that have been documented in earlier work: the currencies of more systemic countries have lower risk-free and nominal interest rates, and stocks in the nontraded sector of more systemic countries pay lower expected returns (Hassan, 2013). Our main theoretical contribution is thus to show that these predictions about cross-currency variation in asset returns also produce differential incentives for capital accumulation across countries.

In addition to deriving these qualitative results, we take several steps to make the model quantitatively viable and take it to the data. To this end, we first show formally

that the conclusions outlined above hold regardless of whether markets are complete or incomplete, and regardless of whether variation in exchange rates is driven by real or monetary shocks. This finding is particularly important because allowing for market incompleteness significantly improves the model's fit to the data. One long-standing challenge for models of exchange rate determination with complete markets is that they predict a perfect, negative correlation between appreciations of the real exchange rate and consumption growth (this is the seminal [Backus and Smith \(1993\)](#) puzzle). By contrast, inflation shocks induce a positive correlation between appreciations and consumption growth when markets are incomplete. Allowing for market incompleteness and both real and monetary shocks can thus generate a correlation between appreciations and differential consumption growth that is more consistent with the data. Moreover, allowing for market incompleteness increases the effect of a given productivity shock on the real exchange rate and raises its volatility in equilibrium, partially addressing another well-documented shortcoming of the complete-markets model.

We calibrate our model using annual data from seven industrialized countries that maintain freely floating exchange rates during the sample period, 1984-2010. Our analysis proceeds in three steps. First, we allow countries to differ only in size, measuring each country's size as the average share it contributes to the overall GDP of the seven countries under consideration during the sample period. We then estimate the variance of productivity and inflation shocks to match the average variance across countries of GDP growth and CPI inflation, constraining the variances of the two shocks to be equal across countries. Under this specification, the model accounts for 7% of the variation in cross-country differences in capital-output ratios.

In a second step, we repeat the same procedure but now allow heteroskedastic productivity and inflation shocks across countries: we estimate the relative size of each country's shocks to match the variance of its equal-weighted average real and nominal exchange rates, subject to the constraint that we again match the average variance across countries of GDP growth and CPI inflation. This heteroskedastic specification accounts for 72% of the variation in cross-country differences in capital-output ratios. In this sense, the stochastic properties of exchange rates, as proxied by differences in country size and the variances of exchange rates, can account for the majority of the differences in capital-

output ratios across the seven countries in our sample. At the same time, the model produces a small negative correlation between appreciations and consumption growth and thus is not obviously rejected by the data along this dimension.

Although the baseline model does remarkably well in predicting the relative differences in capital-output ratios across countries, it fails dramatically on magnitudes. For example, in the data, Canada has a 30% lower capital-output ratio than the United States whereas the model predicts a difference of around 0.03%. Similarly, although the model (by construction) matches the relative variances of the real exchange rate, it under-predicts the average variance of real exchange rates by an order of magnitude. We thus conjecture that the model's failure to match the quantitative magnitude of the differences in capital-output ratios may be related to the well-known failure of the canonical model to generate sufficient variability of the real exchange rate.

In a third step, we apply a series of modifications to the model that increase the variance of the real exchange rate, including the introduction of capital adjustment costs. The general pattern we find is that, consistent with our conjecture, modifications that increase the variance of the real exchange rate also tend to narrow the gap in magnitudes, although no single modification yields entirely satisfactory results.

The conclusion of our analysis is that the stochastic properties of countries' exchange rates contain information that may account for some of the large variation in capital-output ratios across countries. We make two main caveats to this interpretation. First, our results are based on a sample of relatively homogeneous developed countries that have similar institutions, tax systems, and well-functioning markets. This relative homogeneity may explain why we get a good fit to the data even without controlling for these other factors. In a sample comprised of a broader set of countries, especially developed and developing countries, we would expect the stochastic properties of exchange rates to play a relatively smaller role in explaining differences in capital-output ratios. For this reason, we make no claim about the importance of heterogeneity in stochastic properties of countries' real exchange rates relative to heterogeneity in these other factors. Second, our baseline model matches only relative differences, but falls far from the mark in matching the magnitude of absolute differences. For this reason, our empirical results should be interpreted with due caution.

Our work relates to a growing theoretical literature that centers on the idea that currencies that appreciate in “bad” times pay lower returns and have lower risk-free interest rates in equilibrium. This literature has explored various potential drivers of heterogeneity of the stochastic properties of countries’ real exchange rates, including differences in country size (Hassan, 2013; Martin, 2012; Govillot, Rey, and Gourinchas, 2010), the size and volatility of shocks affecting the nontraded sector (Tran, 2013), financial development (Maggiori, 2013), and patterns of production specialization (Ready, Roussanov, and Ward, 2013; Richmond, 2015). We contribute to this literature by showing that the same forces that generate differences in currency returns also generate differences in the incentive to accumulate capital across countries, linking this literature to the broader literature on differences in factor accumulation across countries.

Our paper also relates to a parallel empirical literature that shows evidence that is broadly consistent with the predictions of these models, including Lustig and Verdelhan (2007), Campbell, Serfaty-De Medeiros, and Viceira (2010), Menkhoff, Sarno, Schmeling, and Schrimpf (2012), and Lustig, Roussanov, and Verdelhan (2011).¹ Our contribution to this empirical literature is to show that information contained in the variance-covariance matrix of exchange rates can predict differences in capital accumulation between the seven industrialized countries in our sample.

Finally, our work also relates to the broader literature on violations of uncovered interest parity by arguing that persistent violations of uncovered interest parity have real effects on factor accumulation. For an overview of this literature see the surveys by Lewis (2011), and Engel (2014), as well as Hassan and Mano (2014).

The remainder of this paper is structured as follows: section 2 sets up and derives the equilibrium conditions of the model. Section 3 derives the main economic insights using a special case of the model with two periods, full depreciation, and Cobb-Douglas utility. Section 4 solves and estimates the fully dynamic model. Section 5 concludes.

¹Also see the evidence in some of the aforementioned papers, including Hassan (2013), Tran (2013), and Ready, Roussanov, and Ward (2013) and Richmond (2015).

2 A Model of Exchange Rates and Capital Accumulation

In this section, we develop an international asset pricing model in which the allocation of capital across countries and the stochastic properties of real exchange rates are jointly determined as a function of real and monetary shocks. The model generalizes the framework in [Hassan \(2013\)](#) by introducing a fully dynamic setting and endogenizing the accumulation of capital in the nontraded sector. It nests the canonical complete-markets of exchange rate determination ([Backus and Smith, 1993](#)) as well as a simplified version of the incomplete-markets model by [Alvarez, Atkeson, and Kehoe \(2002\)](#).²

To simplify the analysis, we assume that households receive transfer payments before trading in complete asset markets commences so that the initial distribution of wealth exactly decentralizes a Social Planner’s problem with unit Pareto weights. We therefore do not consider the effects that asymmetries in the stochastic properties of exchange rates may have on the distribution of initial wealth across countries.

2.1 Setup

There exists a unit measure of households $i \in [0, 1]$, partitioned into N subsets Θ^n of measure θ^n , $n = 1, \dots, N$. Each subset represents the constituent households of a country n . Households’ utility functions exhibit constant relative risk aversion according to

$$U(i) = \sum_{t=0}^T \frac{\beta^t}{1-\gamma} \mathbb{E} [C_t(i)^{1-\gamma}], \quad (1)$$

where β is the time discount factor, $C_t(i)$ is the quantity of a final good consumed by household i at time t , and T is a terminal period. Households are risk-averse with $\gamma > 0$. The final good can be used for consumption or investment in capital. Households produce

²Other closely related models of exchange rate determination include [Stockman and Dellas \(1989\)](#), [Tesar \(1993\)](#), [Stockman and Tesar \(1995\)](#), [Baxter, Jermann, and King \(1998\)](#), and [Collard, Dellas, Diba, and Stockman \(2007\)](#).

the final good using a traded and a nontraded (intermediate) good according to

$$Y_t(i) = [\tau I_{T,t}(i)^\alpha + (1 - \tau) I_{N,t}(i)^\alpha]^{\frac{1}{\alpha}}, \quad (2)$$

where $\tau \in (0, 1)$ is the weight of the traded good in the production function, $I_{T,t}(i)$ is the number of traded goods used as inputs by household i , $I_{N,t}(i)$ is the number of nontraded goods used, and $\alpha \in (0, 1)$ determines the elasticity of substitution between the two inputs, $\varepsilon_\alpha = 1/(1 - \alpha)$. Once produced, the final good is freely traded among all households (both active and inactive) within a given country.

At the start of each period, households receive a stochastic endowment of the homogeneous traded good. Shocks to endowments are common within each country such that all households within a country n receive the same amount $Y_{T,t}^n$ of the traded good with

$$\log [Y_{T,t}^n] \sim N(0, \sigma_{T,n}^2).$$

Throughout, we use the traded good as the numéraire, such that all prices and returns are accounted for in the same units.

In each country, a representative firm operates a technology for producing nontraded goods employing capital and labor as inputs. Each household inelastically supplies one unit of labor. The *per capita* output of nontraded goods is equal to

$$Y_{N,t}^n = \exp[\eta_t^n] (K_t^n)^\nu \quad (3)$$

where $0 < \nu < 1$ is the capital share in production, K_t^n is the *per capita* stock of capital in country n , and η_t^n is a productivity shock to the production of non-tradable goods in period t ,

$$\eta_t^n \sim N(0, \sigma_{N,n}^2).$$

The law of motion for per capita capital is

$$K_{t+1}^n = (1 - \delta)K_t^n + I_{K,t}, \quad (4)$$

where δ is the depreciation rate of capital and $I_{K,t}$ is the per capita level of investment in

period t . Goods cannot be stored and must be consumed or invested in the period they are produced or endowed.

A fixed proportion ϕ of households within each country own equal shares of the representative firm in their country and trade a complete set of state-contingent securities across all time periods in international markets. We label these households as “active.”

The remaining proportion $1 - \phi$ of households within each country is excluded from owning stocks or trading state-contingent securities. We label these households as financially “inactive.” Inactive households cede the claims to their endowments and wages in each period t to their country’s active households and receive a nominal bond in return. This nominal bond pays off a fixed number of units of the country’s nominal consumer price index in the following period ($t + 1$). In each period, each inactive household thus receives a payment of $P_t^n e^{-\mu_t^n} C_{ss}$ traded goods, where P_t^n is the number of traded goods required to purchase one real unit of country n ’s final good in period t , μ_t^n is an exogenous inflation shock to the nominal price of one unit of the traded good in terms of the currency of country n ,

$$\mu_t^n \sim N(0, \tilde{\sigma}_n),$$

and C_{ss} is a normalizing constant that equalizes consumption between active and inactive households at the deterministic steady state.

To simplify notation, let ω represent the state vector, recording the realization of endowments, inflation shocks, and capital stocks across countries within a given period. In what follows, we indicate the dependence on the state once for each variable and drop the argument thereafter whenever appropriate.

All households take prices as given. Active households maximize their lifetime utility (1) subject to their intertemporal budget constraint

$$\begin{aligned} & \sum_{t=0}^T \left[\int_{\omega} Q_t(\omega) \left(P_t^n(\omega) C_t(i, \omega) + \frac{1-\phi}{\phi} P_t^n(\omega) e^{-\mu_t^n} C_{ss} \right) d\omega \right] \\ & \leq \sum_{t=0}^{\infty} \left[\int_{\omega} Q_t(\omega) \frac{1}{\phi} (Y_{T,t}^n(\omega) + w_t^n(\omega) + \pi_t^n(\omega)) d\omega \right] + \kappa^n, \end{aligned} \quad (5)$$

where $Q_t(\omega)$ is the time-zero price of a state contingent security that pays one unit of

the tradable good in period t if state ω occurs, $P_t^n(\omega)$ denotes the price of country n 's final good in period t , $w_t^n(\omega)$ is the wage rate and $\pi_t^n(\omega)$ is the per capita share of profits accruing to the country's representative firm. Note that $\frac{1-\phi}{\phi}$ is the number of inactive households per active household in each country. To simplify the solution of the model, we also assume that active households receive a country-specific transfer κ^n before trading begins such that the decentralized problem coincides with the Social Planner's allocation with unit Pareto weights.

Inactive households also maximize (1), but subject to the constraints

$$P_t^n \hat{C}_t(i) \leq P_t^n e^{-\mu_t^n} C_{ss} \quad \forall n, t, \quad (6)$$

where $\hat{C}_t(i)$ is the consumption of final goods by inactive household i . Since inactive households cannot transfer goods across periods, they optimally choose to consume the entire payment received from their nominal bond, and thus

$$\hat{C}_t(i) = \hat{C}_t^n = e^{-\mu_t^n} C_{ss} \quad \forall i \in \Theta^n. \quad (7)$$

Each country's representative firm makes investment decisions to maximize its present discounted value of future profits, π_t^n , subject to the law of motion for capital (4). The firm's objective function is

$$\max_{\{K_t^n(\omega)\}_t} \sum_{t=0}^{\infty} \int_{\omega} Q_t(\omega) \theta^n (P_{N,t}^n(\omega) Y_{N,t}^n(\omega) - w_t^n(\omega) - P_t^n(\omega) I_{K,t}^n(\omega)) d\omega \quad (8)$$

where $P_{N,t}^n$ is the price of country n 's nontraded good.

The market clearing conditions for final goods, nontraded, and traded goods with a given period t are

$$\int_{i \in \Theta^n} C_t(i, \omega) + \theta^n I_{K,t}^n(\omega) = \int_{i \in \Theta^n} Y_t(i, \omega) \quad \forall n, \quad (9)$$

$$\int_{i \in \Theta^n} I_{N,t}(i, \omega) = \theta^n Y_{N,t}^n(\omega) \quad \forall n, \quad (10)$$

and

$$\int_{i \in \Theta^n} I_{T,t}(i, \omega) di = \sum_n \theta^n Y_{T,t}^n(\omega), \quad (11)$$

respectively. The economy is at an equilibrium when all households maximize utility subject to their budget constraints, firms maximize the present discounted value of profits subject to the law of motion of capital, and goods markets clear.

2.2 Solving the Model

Although financial markets are incomplete, the model's solution remains tractable because a subset of the population in each country (active households) have access to complete markets. As a result, we can solve for the equilibrium allocation of capital and goods by solving the Social Planner's problem, subject to the equilibrium behavior of inactive households. To characterize this behavior, it is useful to calculate the number of inputs of traded and nontraded goods used for the production of final goods consumed by each inactive household according to (7). Taking prices as given, inactive households use the optimal mix between traded and nontraded goods given by

$$\hat{I}_{T,t}^n = \frac{\exp(-\mu_t^n) P_t^n C_{ss}}{1 + \left[\frac{\tau}{1-\tau} (P_{N,t}^n)^\alpha \right]^{\frac{1}{\alpha-1}}}, \quad \hat{I}_{N,t}^n = \frac{\left[\frac{\tau}{1-\tau} (P_{N,t}^n) \right]^{\frac{1}{\alpha-1}} \exp(-\mu_t^n) P_t^n C_{ss}}{1 + \left[\frac{\tau}{1-\tau} (P_{N,t}^n)^\alpha \right]^{\frac{1}{\alpha-1}}}, \quad (12)$$

where \hat{I}_T^n and \hat{I}_N^n refer to the inputs of traded and nontraded goods used by inactive agents in country n , respectively.³

All active households within a given country are identical and receive the same endowments. In a symmetric equilibrium, they therefore consume the same number of final goods C_t^n , and use the same number of traded and nontraded goods inputs $(I_{T,t}^n, I_{N,t}^n)$. Using this simplified notation, we can write the Social Planner's problem as

$$V(K_t^n, Y_{T,t}^n, \eta_t^n, \mu_t^n) = \max_{\{I_{T,t}^n, I_{N,t}^n, I_{K,t}^n\}_{n,t}} \sum_n \phi \theta^n \frac{(C_t^n)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E} [V(K_{t+1}^n, Y_{T,t+1}^n, \eta_{t+1}^n, \mu_{t+1}^n)]$$

subject to (12) as well as the economy's resource constraints for final goods, nontraded

³See Appendix A for a formal proof.

goods, and traded goods.⁴

For further reference, we denote the Lagrange multipliers on these $2n + 1$ resource constraints as Λ_t^n , $\Lambda_{N,t}^n$, and $\Lambda_{T,t}$ and interpret them as the Social Planner's shadow prices of final, nontraded, and traded goods, respectively. Given this definition, we can re-write the prices in (12) as

$$P_{N,t}^n(\omega) = \frac{\Lambda_{N,t}^n(\omega)}{\Lambda_{T,t}(\omega)} \text{ and } P_t^n(\omega) = \frac{\Lambda_t^n(\omega)}{\Lambda_{T,t}(\omega)}, \quad (13)$$

completing the setup of the Social Planner's problem.

The first-order conditions with respect to $I_{T,t}^n$ equate the shadow price of the traded good across all active households in the world,

$$\tau (I_{T,t}^n)^{\alpha-1} (\tau (I_{T,t}^n)^\alpha + (1 - \tau) (I_{N,t}^n)^\alpha)^{\frac{1}{\alpha}-1} (C_t^n)^{-\gamma} = \Lambda_{T,t} \quad \forall n. \quad (14)$$

The first-order conditions with respect to $I_{N,t}^n$ define the shadow prices of nontraded goods within each country,

$$(1 - \tau) (I_{N,t}^n)^{\alpha-1} (\tau (I_{T,t}^n)^\alpha + (1 - \tau) (I_{N,t}^n)^\alpha)^{\frac{1}{\alpha}-1} (C_t^n)^{-\gamma} = \Lambda_{N,t}^n \quad \forall n. \quad (15)$$

In addition, it is useful to keep track of the (redundant) first-order condition with respect to C_t^n because it pins down the marginal utility of consumption of active households in each country,

$$(C_t^n)^{-\gamma} = \Lambda_t^n \quad \forall n. \quad (16)$$

By definition, the real exchange rate between two countries h and f equals the ratio of these shadow prices,

$$S_t^{f,h}(\omega) = \Lambda_t^f(\omega) / \Lambda_t^h(\omega).$$

Finally, combining the first-order conditions with respect to I_K with the corresponding envelope conditions yields an Euler equation for the accumulation of capital in each country,

$$(C_t^n)^{-\gamma} = \beta \mathbb{E} \left[(1 - \delta) (C_{t+1}^n)^{-\gamma} + \Lambda_{N,t+1}^n \eta_{t+1}^n \nu (K_{t+1}^n)^{\nu-1} \right] \quad \forall n. \quad (17)$$

⁴See Appendix B for the derivation of these constraints.

Combining these N Euler equations with the $3N$ resource constraints and the $3N$ first-order conditions (14) - (16) yields a recursive system of $7N$ equations that determines the sequence of endogenous variables $\{I_{T,t}^n, I_{N,t}^n, C_t^n, K_{t+1}^n, \Lambda_t^n, \Lambda_{N,t}^n, \Lambda_{T,t}^n\}_{n,t}$.

3 Exchange Rates and the Capital Stock

From (17), it is already apparent that heterogeneity in the effect of the country-specific productivity shock η_{t+1}^n on the shadow value of nontraded goods ($\Lambda_{N,t+1}^n$) induces variation in the incentive to accumulate capital across countries. To illustrate how such heterogeneity relates to heterogeneity in the stochastic properties of countries' real exchange rates, we first consider a simplified, two-period version of the model. In this simplified version, the production technology for final goods is Cobb-Douglas ($\varepsilon_\alpha = 1$), the economy is at its deterministic steady state at $t = 0$, and the world ends at $T = 1$ with $\delta = 1$. In this case, (17) simplifies to

$$K_1^n = \beta \frac{\nu}{(C_0^n)^{-\gamma}} \mathbb{E} [\Lambda_{N,1}^n Y_{N,1}^n].$$

Moreover, from (15), (16), and (30), it follows that $\Lambda_{N,1}^n Y_{N,1}^n = (1-\tau) (\Lambda_{t+1}^n)^{1-\frac{1}{\gamma}}$. Guessing that all endogenous variables are log-normally distributed in equilibrium, we can then write the difference in two countries' per capita stocks as

$$k_1^f - k_1^h = \frac{\gamma-1}{\gamma} \mathbb{E} \left(\lambda_1^f - \lambda_1^h - (\lambda_0^f - \lambda_0^h) \right) + \frac{1}{2} \left(\text{var} \left(\lambda_1^f \right) - \text{var} \left(\lambda_1^h \right) \right), \quad (18)$$

where lowercase variables stand for logs such that $x = \log(X)$. This equation shows that differences in capital accumulation between countries are a function of expected changes in the real exchange rate between the two countries (the first term on the right hand side) and of the difference in the variance of the marginal utility of consumption in the two countries (the second term). Plainly, both terms reflect stochastic properties of the real exchange rate between the two countries, $s_t^{f,h} = \lambda_t^f - \lambda_t^h$.⁵

Because the economy is at its deterministic steady-state in $t = 0$, cross-country differ-

⁵ Note that this condition is also closely related to very similar conditions that relate cross-country differences in expected currency returns to cross-country differences in the variances of stochastic discount factors, as in [Backus, Foresi, and Telmer \(2001\)](#). We discuss this analogy in detail in section 3.3

ences in the first term on the right-hand side of (18) can only arise as a result of differences in the second term. That is, the only reason for expected growth of marginal utilities to differ across countries is if households in these countries accumulate different levels of capital for use in period 1. The first term thus represents a feed-back effect that operates only if the second term is non-zero. The key driver of cross-country differences in capital accumulation is therefore the difference in the variance of the marginal utility of active households across countries.

To study this term in closed form, we log-linearize the model around the deterministic steady state, i.e., the point at which the variances of all shocks are zero ($\sigma_n, \tilde{\sigma}_n = 0 \forall n$) and all households have a capital stock that is fixed at the deterministic steady state level. That is, for the remainder of this section, we study the *incentives* to accumulate different levels of capital across countries while holding the capital stock fixed. Appendix C shows the log-linearized system of equations.

We proceed in three steps. The following subsection studies the effect of productivity shocks on the stochastic properties of exchange rates and capital accumulation by considering the special case in which financial markets are complete $\phi = 1$. We then consider the effect of inflation shocks while abstracting from productivity shocks before combining the insights from both special cases and discussing the joint effects of real and monetary shocks.

3.1 Real Shocks, Exchange Rates, and the Capital Stock

Equation (19) shows the equilibrium use of the traded good in an arbitrary country h for the special case in which financial markets are complete, $\phi = 1$ (recall that lowercase variables stand for logs):

$$i_T^h = \bar{y}_T + \frac{(\gamma - 1)(1 - \tau)}{(1 - \tau) + \tau\gamma} (\bar{y}_N - y_N^h), \quad (19)$$

where $\bar{y}_N = \sum_{n=1}^N \theta^n y_N^n$ is the average of the log output of non-tradables across countries and $\bar{y}_T = \sum_{n=1}^N \theta^n y_T^n$ is the average of the log world endowment of tradables. As one may expect, use of the traded good in the home country moves one for one with the world

supply. Since the traded good can be freely shipped around the globe, it is inconsequential which country has a better or worse endowment of tradables, as long as \bar{y}_T is constant. Households thus perfectly share risk when it comes to endowments of the traded good. However, the second term of (19) shows that they also use the traded good to insure against productivity shocks in the nontraded sector. Although non-tradables cannot be shipped, households purchase insurance in world markets in the form of compensating deliveries of traded goods if $\gamma > 1$.

This condition ensures that the cross-partial of the shadow price of the traded good with respect to the nontraded good is negative, i.e., that the relative price of a country's nontraded good falls when its supply increases. As most empirical applications of international asset pricing models find a relative risk aversion significantly larger than one and an elasticity of substitution around one, we follow the literature in assuming that this condition is met, but refer to it whenever it is relevant (see Coeurdacier (2009) for a detailed discussion). If relative risk aversion is sufficiently high, households in the home country thus receive additional tradables whenever they have a lower than average output of nontraded goods.

This risk-sharing behavior is reflected in the shadow price of traded goods,

$$\lambda_T = -((1 - \tau) + \tau\gamma) \sum_{n=1}^N \theta^n y_T^n - (1 - \tau)(\gamma - 1) \sum_{n=1}^N \theta^n y_N^n + \log(\tau). \quad (20)$$

The first term on the right-hand side shows that the shadow price unambiguously falls with the world supply of traded goods. The second term states that the same is true for the average output of nontraded goods, as long as $\gamma > 1$. Thus λ_T tends to be low in “good” states of the world when endowments of traded goods are high and countries experience positive productivity shocks in the nontraded sector.⁶ Note, however, that not every productivity shock has the same influence on λ_T . Shocks to the productivity of larger countries affect a larger measure of households and thus have a larger effect on λ_T . Similarly, countries with more volatile shocks have a larger impact on the variability of

⁶Because the capital stock is fixed at its deterministic steady state level we have that $y_N^n = \nu k^{ss} \eta^n$, such that differences in per capita output of non-traded goods are proportional to differences in the realization of productivity shocks.

λ_T . The following definition formalizes the notion of large and volatile countries:

Definition 3.1

Country f is **larger** than country h if $\theta^f > \theta^h$. It is **more volatile** than country h if

$$\sigma_{Nf} > \sigma_{Nh} \text{ and } \tilde{\sigma}_f > \tilde{\sigma}_h,$$

and it is **more systemic** than country h if

$$\theta^f \geq \theta^h \text{ and } \sigma_{Nf} \geq \sigma_{Nh} \text{ and } \tilde{\sigma}_f \geq \tilde{\sigma}_h$$

with two of the three weak inequalities holding strictly.

The real exchange rate between countries f and h is

$$s^{f,h} = \lambda^f - \lambda^h = \frac{\gamma(1-\tau)}{(1-\tau) + \gamma\tau} (y_N^h - y_N^f). \quad (21)$$

It depends only on the difference between the per-capita production of nontraded goods between the two countries. The country with the lower per-capita output of nontraded goods appreciates.

From inspecting (20) and (21), we can see that the model produces two kinds of heterogeneity in the stochastic properties of countries' exchange rates. First, more volatile countries – that is, countries that experience more volatile productivity shocks – tend to have more volatile exchange rates,

$$\text{var}(s^{f,h}) = \left(\frac{\gamma(1-\tau)}{(1-\tau) + \gamma\tau} \right)^2 (\sigma_{N,f}^2 + \sigma_{N,h}^2)$$

Second, more systemic countries – that is, countries that are either larger, more volatile, or both – tend to appreciate when the shadow price of traded goods is high. If country f is more systemic than country h , we have

$$\text{cov}(s^{f,h}, \lambda_T) = \frac{\gamma(\gamma-1)(1-\tau)^2}{(1-\tau) + \gamma\tau} (\theta^f \sigma_{N,f}^2 - \theta^h \sigma_{N,h}^2) > 0.$$

The intuition for this result is as follows: whenever any country suffers a low productivity

shock, the relative price of its nontraded goods rises and its real exchange rate appreciates. For a given percentage decline in productivity, this appreciation occurs independently of how large the country is. (Note that there is no θ in equation (21).) However, when a larger or more volatile country suffers this shock, more of it is transmitted to the rest of the world. For example, in states of the world in which the US (the largest economy in the world) draws a low productivity shock, it imports a lot of the world's traded goods, raising the shadow price of traded goods all over the world. As a result, the US dollar tends to appreciate when λ_T is high, producing a positive covariance between the US real exchange rate and λ_T .

This heterogeneity in the stochastic properties of countries' exchange rates in turn generates heterogeneity in the incentive to invest across countries. In particular, it generates a larger incentive to invest in the capital stock of more systemic countries. Interpreting $k_1^f - k_1^h$ in (18) as the differential incentive to accumulate capital across two countries, we can show the following proposition:

Proposition 1

If markets are complete, and $\gamma > 1$, more systemic countries have an incentive to accumulate more capital per worker:

$$k_1^f - k_1^h = \frac{\gamma^2(\tau - 1)^2}{4((\gamma - 1)\tau + 1)^2} (2(\gamma - 1)\tau (\theta^f \sigma_{N,f}^2 - \theta^h \sigma_{N,h}^2) + (\sigma_{N,f}^2 - \sigma_{N,h}^2)) > 0 \quad (22)$$

if country f is more systemic than country h .

Proof. In each of the two periods, the marginal utility of consumption of households in country n is

$$\lambda^n = -\frac{\gamma\tau}{\phi} \bar{y}_T - \frac{\gamma\tau(1-\tau)(\gamma-\phi)}{\phi(\gamma\tau + (1-\tau)\phi)} \bar{y}_N - \frac{\gamma(1-\tau)}{\gamma\tau + (1-\tau)\phi} y_N^n.$$

It follows immediately that the first term in (18) is equal to zero because $\lambda_0^n = \mathbb{E}\lambda_1^n = 0$. Differential incentives to accumulate capital thus only arise from the second term, that is, the difference in the volatility of marginal utilities of the two countries. These, in turn, result from differences in $cov(\bar{y}_N, y_N^n)$ (the first term in the brackets in (22)) and from differences in the volatility of y_N^n (the second term in the brackets). ■

It is efficient to accumulate more capital in more systemic countries because a larger capital stock in a more systemic country represents a good hedge against world-wide consumption risk. Households around the world worry about states of the world in which systemic countries suffer bad productivity shocks, because these shocks are transmitted to the rest of the world in the form of higher shadow prices of traded goods. Although households have no way of preventing these states from occurring, they can partially insure against them by accumulating more capital in more systemic countries. By doing so, they raise the unconditional mean level of output of nontraded goods in these countries across all states and thus soften the impact of adverse shocks.

Another way of thinking about this result is from the perspective of an international investor. Consider an investor who can buy units of capital all around the world. Other things equal, she will prefer to invest in the capital stock of a more systemic country because systemic countries' currencies tends to appreciate in states of the world when the shadow price of traded goods, and thus the marginal utility of consumption around the world, is high. In this sense, systemic countries attract investment because they have “safe” currencies that appreciate in “bad” states of the world.

This simple, complete-markets model of exchange rate determination thus predicts a link between the stochastic properties of countries' real exchange rates and differential incentives to accumulate capital across countries. Other things equal, larger countries and countries with more volatile exchange rates should have higher capital-output ratios in equilibrium.

3.2 Monetary Shocks, Exchange Rates, and the Capital Stock

While the complete-markets model is an important benchmark, it has a number of well-known empirical shortcomings. First, it predicts a (perfect) negative correlation between appreciations of the real exchange rate and aggregate consumption growth—a country appreciates when its aggregate consumption falls. This follows directly from (16) and (21) (Backus and Smith, 1993). Second, real exchange rates seem much too volatile to be rationalized exclusively by real (productivity) shocks (Chari, Kehoe, and McGrattan, 2002). As a result, many authors have argued for incomplete market models that allow

for an effect of monetary shocks on equilibrium real exchange rates.

In this subsection, we analyze the case in which markets are incomplete ($\phi < 1$). We will argue that allowing for market incompleteness can partially address both of the main empirical shortcomings of the complete-markets model outlined above, while preserving the main intuition that larger countries and countries with more volatile exchange rates should have higher capital-output ratios in equilibrium. To simplify the discussion, we first consider the case where productivity and endowments are deterministic $\sigma_{N,n}, \sigma_{T,n} = 0$ before discussing the joint effects of real and monetary shocks.

Inactive households hold nominal bonds denominated in their national currencies and are thus vulnerable to inflation shocks. A positive inflation shock acts as an “inflation tax” on these households: the higher inflation, the less their nominal bonds are worth and the less they are able to consume (see equation (12)). However, since monetary shocks have no bearing on the real endowments available for consumption, this reduction of consumption on the part of inactive households must go to the benefit of active households in equilibrium.

Active households’ equilibrium use of traded goods is

$$i_T^h = \frac{1 - \phi}{\phi} \mu^h + \frac{\gamma(\tau(1 - \phi) + \phi)(1 - \phi)}{(\gamma\tau + (1 - \tau)\phi)\phi} (\bar{\mu} - \mu^h), \quad (23)$$

where $\bar{\mu} = \sum_{n=1}^N \theta^n \mu^n$ is the weighted sum of inflation shocks in all countries. The first term on the right-hand side reflects the immediate rise in active households’ consumption, which is proportional to the number of inactive households per active household. However, risk-sharing among active households residing in different countries requires that some of the initial rise in consumption be shared internationally, which is reflected in the second term on the right-hand side: whenever domestic inflation exceeds weighted average inflation, $\bar{\mu}$, the home country ships traded goods to the rest of the world. This shipment of traded goods has two effects. First, it results in a fall in the relative price of nontraded goods in the domestic economy, such that the domestic currency *depreciates* in real terms whenever inflation is relatively high,

$$s^{f,h} = \lambda^f - \lambda^h = -\frac{\gamma(\tau - 1)(\phi - 1)(\mu^f - \mu^h)}{\gamma\tau + (1 - \tau)\phi}. \quad (24)$$

Second, it lowers the shadow price of traded goods in the world:

$$\lambda_T = -\frac{1-\phi}{\phi}\gamma \sum_{n=1}^N \theta^n \mu^n + \log(\tau). \quad (25)$$

This expression is similar to (20) in that inflation shocks unambiguously lower λ_T , but their impact is proportional to the size of the country in which they originate. Inflation shocks in larger countries thus have a larger impact on marginal utility than inflation shocks in smaller countries.

Inspecting (24) and (25) shows that, similar to the effect of productivity shocks, countries with more volatile inflation shocks tend to have more volatile exchange rates and that inflation shocks also result in a positive covariance between the exchange rates of more systemic countries and λ_T . The intuition for this result is also similar to the complete-markets case: whenever any country experiences a high inflation shock, active households receive more real resources, lowering their marginal utility of consumption, and thus leading to a depreciation of the real exchange rate. (The marginal utility of inactive agents is irrelevant for exchange rates because they do not access financial markets.) For a given inflation shock, this appreciation occurs independently of how large the country is. However, when a larger or more volatile country suffers this shock, more of it is transmitted to the rest of the world, as is apparent from (20). As a result, more systemic countries' exchange rates tend to depreciate when λ_T is low, again producing a positive covariance between their real exchange rate and λ_T .

Proposition 2

If financial markets are incomplete and the economy is affected only by inflation shocks, more systemic countries have an incentive to accumulate more capital per worker:

$$k_1^f - k_1^h = \frac{\gamma^2(1-\tau)(\phi-1)^2}{4\phi((\gamma-\phi)\tau+\phi)^2} (((1-\tau)\phi(\tilde{\sigma}_f^2 - \tilde{\sigma}_h^2) + 2\gamma\tau(\theta^f\tilde{\sigma}_f^2 - \theta^h\tilde{\sigma}_h^2))) > 0$$

if country f is more systemic than country h .

Proof. In each of the two periods, the marginal utility of consumption of active house-

holds in country n is

$$\lambda^n = -\frac{\gamma^2\tau(1-\phi)}{\phi(\gamma\tau+(1-\tau)\phi)}\bar{\mu} - \frac{\gamma((1-\phi)(1-\tau))}{\gamma\tau+(1-\tau)\phi}\mu^n, \quad (26)$$

where the remainder of the proof is identical to the one in Proposition 1. ■

The central insight from the complete-markets model thus holds even if markets are incomplete and variation in exchange rates is driven by monetary instead of real shocks. Appendix D shows that this is also true when the economy is simultaneously affected by both real and monetary shocks.

In addition to re-enforcing the main insight from the complete-markets case, allowing for market incompleteness also improves the fit of the model to the data in two critical dimensions. First, we demonstrate that the incomplete markets model with inflation shocks loosens or even reverses the negative correlation between appreciations of the real exchange rate and aggregate consumption growth. In a second step, we show that the incomplete markets model with inflation and productivity shocks may better fit the high volatility of real exchange rates observed in the data.

Corollary 3

If financial markets are incomplete and the economy is affected only by inflation shocks, real appreciations of the exchange rate are perfectly positively correlated with relative growth in aggregate consumption.

Proof. At the deterministic steady state we have log aggregate consumption is $\bar{c}^n = \phi c^n + (1-\phi)(-\mu^n)$. Using the fact that $c^n = -\lambda^n/\gamma$ in conjunction with (24) and (26), the covariance between the exchange rate and aggregate consumption is

$$\text{cov}(s^{f,h}, \bar{c}^f - \bar{c}^h) = \frac{\gamma^2(1-\tau)\tau(1-\phi)^2}{((1-\tau)\phi + \gamma\tau)^2}(\tilde{\sigma}_f^2 + \tilde{\sigma}_h^2) > 0$$

and their correlation is one. ■

To see the intuition for this result again consider a country that experiences a positive inflation shock. Because active households within the country now have more real resources available, they ship traded goods to active households residing in other countries. As a result of this outflow of resources, aggregate consumption drops while their coun-

try's currency depreciates. Inflation shocks thus induce a positive association between aggregate consumption growth and appreciations.

By contrast, the productivity shocks discussed in the previous sub-section induce a negative association between aggregate consumption growth and appreciations: a country that experiences a negative productivity shock in the nontraded sector experiences a fall in aggregate consumption and a simultaneous appreciation. Because of these opposite signs in the association between aggregate consumption and appreciations, the full model with incomplete markets, real and monetary shocks can produce any correlation between aggregate consumption and appreciations. By modulating ϕ , it may thus resolve the [Backus and Smith \(1993\)](#) puzzle.

Similarly, allowing for market incompleteness may go some of the way in addressing the second main empirical shortcoming of the complete-markets model by increasing the volatility of the real exchange rate. The following expression shows the equilibrium real exchange rate when markets are incomplete ($\phi < 1$) and the economy is affected by productivity and inflation shocks ($\sigma_{N,n}, \sigma_{T,n} > 0$).

$$s^{f,h} = \frac{\gamma(1-\tau)}{(1-\tau)\phi + \gamma\tau} (y_N^h - y_N^f) + \frac{\gamma(1-\tau)(1-\phi)}{(1-\tau)\phi + \gamma\tau} (\mu^h - \mu^f). \quad (27)$$

Comparing this expression with [\(21\)](#) shows two differences. First, and most obviously, the variance of the real exchange rate is now higher because inflation shocks represent an additional source of volatility. Second, market incompleteness increases the effect of a given productivity shock because fewer households engage in risk-sharing (the fraction in the first term is unambiguously larger than its counterpart in [\(21\)](#)).

We draw four main conclusions from the analysis of our simplified model. First, the stochastic properties of real exchange rates should predict differences in capital accumulation across countries: countries whose currencies appreciate when the shadow price of traded goods is high should accumulate more capital per capita. Second, absent a direct measure for the shadow price of traded goods, differences in country size and differences in the volatility of countries' real exchange rates serve as sufficient statistics for the relevant differences in the stochastic properties of real exchange rates. Third, these conclusions hold regardless of whether markets are complete or incomplete, and regardless of whether

variation in exchange rates is driven by real or monetary shocks. Fourth, allowing for market incompleteness may help to address some of the empirical shortcomings of the complete-markets model.

3.3 Link to the Asset Pricing Literature

Before moving to the analysis of our quantitative model, it is useful to relate our main theoretical results to similar results in the asset pricing literature. The previous subsections derived Propositions 1 and 2 directly from condition (17), which pins down the optimal allocation of capital across countries without considering asset prices. Nevertheless, the results in the propositions are closely linked to violations of uncovered interest parity and differences in the required rate of return on capital across countries. To see this link to the extant literature, consider pricing one unit of capital installed for the production of non-traded goods in country n that pays a stochastic return \tilde{R}^n in terms of country n 's final good. The consumption Euler equations of active households require that

$$1 = \beta \mathbb{E} \left(\frac{\Lambda_1^n}{\Lambda_0^n} \tilde{R}^n \right).$$

Using this condition for units of capital installed in two different countries h and f , one can show that

$$\mathbb{E}\tilde{r}_1^f + \mathbb{E}\Delta s^{f,h} - \mathbb{E}\tilde{r}_1^h = \frac{1}{2} \text{var}(\lambda_1^h + \tilde{r}_1^h) - \frac{1}{2} \text{var}(\lambda_1^f + \tilde{r}_1^f).$$

Since the price of capital is known at time 0, the only part of \tilde{r}_1^n that has a variance is its payoff, which we can write as $\frac{\Lambda_N^n}{\Lambda^n} \nu Y_N^n = \nu (1 - \tau) (\Lambda^n)^{-\frac{1}{\gamma}}$. Applying this condition in the equation above and performing a similar calculation for a risk-free asset that pays one unit of each country's final good with certainty, we can write

$$\begin{aligned} \mathbb{E}\tilde{r}_1^f + \mathbb{E}\Delta s^{f,h} - \mathbb{E}\tilde{r}_1^h &= \left(1 - \frac{1}{\gamma}\right)^2 (r^f + \mathbb{E}\Delta s^{f,h} - r^h) = \left(1 - \frac{1}{\gamma}\right)^2 \frac{1}{2} \left(\text{var}(\lambda_1^h) - \text{var}(\lambda_1^f)\right) \\ &= \left(1 - \frac{1}{\gamma}\right)^2 \left(k_1^h + \frac{\gamma - 1}{\gamma} \mathbb{E}\Delta s^{f,h} - k_1^f\right), \end{aligned}$$

where r^n is the risk-free rate of interest between periods 0 and 1 in country n and the third equality uses (18). The difference in incentives to accumulate capital across countries is thus one of many manifestations of the effect of heterogeneity in the stochastic properties of countries' real exchange, which also include violations of uncovered interest parity and differences across countries in the required return to capital in the non-traded sector.

The prediction that larger and more volatile countries accumulate higher capital-output ratios is thus intimately linked to similar theoretical results on violations of uncovered interest parity, as defined in (Backus, Foresi, and Telmer, 2001), and consistent with a number of empirical findings on cross-country variation in international asset returns. In particular, these results fit with the findings that high interest rate currencies depreciate when US consumption growth is low (Lustig and Verdelhan, 2007), that a portfolio that is long high interest rate currencies and short low interest rate currencies is a priced factor in international currency markets (Lustig et al., 2011), that large countries' currencies have lower interest rates and pay lower expected returns (Hassan, 2013), that stocks in the non-traded sector of larger countries pay lower expected returns (Hassan, 2013), and a number of similar results in the empirical asset pricing literature.

4 Dynamic Model and Estimation

4.1 Data

Our sample consists of annual data from all OECD member countries that had freely floating exchange rates from 1984 to 2010 according to the IMF. These are Australia, Canada, Japan, New Zealand, Switzerland, the United Kingdom, and the United States. We intentionally focus on this set of industrialized and relatively homogeneous countries with similar institutions, tax systems, and well-functioning markets to mitigate the concern that heterogeneity in these other factors may confound our analysis. We generate the change in log real and nominal exchange rates for each pair of these countries assuming free convertibility of currencies using nominal exchange rates against the US dollar and CPI deflators from the OECD database.

Following Hassan (2013), we interpret the size of country n 's economy as its average

share of GDP in the sample of seven countries,

$$\theta^n = \sum_t \frac{1}{T} \frac{GDP_t^n}{\sum_{j=1}^N GDP_t^j}.$$

We measure each country’s capital-output ratio by dividing its capital stock at constant 2005 national prices (converted to US Dollars at purchasing power parity) from the Penn World Tables version 8.1 by real GDP from the same source and averaging across our sample period. We also calculate the average difference between each country’s risk-free interest rate and the risk-free interest rate in the US using covered interest parity from data on forward and spot exchange rates following the procedure in [Hassan and Mano \(2014\)](#).

In addition, we use real GDP, CPI deflators, and population growth data from the OECD database to measure log per capita GDP growth and inflation growth for each country. Appendix [E.1](#) gives additional details on our data sources and construction.

4.2 Calibration and Estimation

We begin by calibrating the parameters of our model to a set of standard values used in the literature, summarized in [Table 1](#). We choose $\varepsilon_\alpha = 0.44$, $\beta = 0.99$, and $\delta = 0.1$, consistent with [Stockman and Tesar \(1995\)](#). In addition, we choose standard values for relative risk aversion ($\gamma = 10$) and the capital share ($\nu = 0.4$). We set the expenditure share of traded goods in the production of final goods to $\tau = 0.3$, reflecting that, in the data, a large proportion of the price of tradables accrues to nontradable retail services ([Burstein, Neves, and Rebelo \(2003\)](#) and [Goldberg and Campa \(2010\)](#)). The only parameter we have relatively little guidance on is the degree of market segmentation. For lack of a clear alternative, we set $\phi = 0.26$, equal to the share of households in the United States who directly own stocks according to [Giannetti and Koskinen \(2010\)](#).

Since the volatility of endowments of traded goods has no bearing on exchange rates or differential incentives to accumulate capital across countries, we choose $\sigma_T^n = 0.1\%$ for all countries. We consciously choose a small value for these shocks to maximize the ability of the model to match the volatility of equilibrium exchange rates, as will become

apparent below.

We estimate the volatility of the remaining fourteen productivity shocks and inflation shocks using a Simulated Methods of Moments procedure. Our estimator is the parameter vector $\hat{\Phi}$ that minimizes the distance between a vector of empirical moments, H_D , and the corresponding model moments, $H(\hat{\Phi})$. This estimator $\hat{\Phi}$ is given by:

$$\begin{aligned} \hat{\Phi} &= \arg \min_{\Phi} [H(\Phi) - H_D]' [H(\Phi) - H_D] \\ \text{s.t. } &\sigma_{N,n} > 0 \text{ and } \tilde{\sigma}_n > 0, \end{aligned}$$

For a given parameter vector, Φ , we create 1,000 synthetic time series, each of length equal to our sample size. On each sample, we calculate the sample moments of interest. The vector $H(\Phi)$ that enters the loss function is the average value of the sample moments across the synthetic time series. Consistent with our data, we assume agents make decisions at the annual frequency and derive the model's implications for data moments computed at a annual frequency.

In our baseline specification, the target vector H_D contains a total of 16 empirical moments. The first 7 moments are the variances of each country's equally-weighted average real exchange rate with all other countries in the sample normalized by the average of these variances across countries,

$$\frac{\text{var}_n \left(\sum_{j \neq n} \frac{1}{N-1} \Delta s_t^{n,j} \right)}{\sum_n \frac{1}{N} \text{var}_n \left(\sum_{j \neq n} \frac{1}{N-1} \Delta s_t^{n,j} \right)}.$$

The rationale for focusing on equally weighted average exchange rates rather than trade-weighted average exchange rates is to isolate the variance of real and monetary shocks specific to each country while averaging out the shocks affecting all other countries as in (Verdelhan, Verdelhan).⁷ The following 7 moments are the nominal counterparts of these moments: the variances of each country's equally-weighted average nominal exchange

⁷Using equal-weighted averages is not essential for our results. Nevertheless note from (27) we have that $\lim_{N \rightarrow \infty} \sum_{j \neq n} \frac{1}{N-1} s_t^{n,j} = -\frac{\gamma(1-\tau)}{(1-\tau)\phi + \gamma\tau} y_N^f - \frac{\gamma(1-\tau)(1-\phi)}{(1-\tau)\phi + \gamma\tau} \mu^f$, such that in a large sample of countries, the equally weighted average exchange rate isolates country-specific shocks, making this measure appealing on theoretical grounds.

rate with all other countries in the sample, again normalized by the average of these variances across countries. The purpose of normalizing with these averages is to avoid rewarding the estimator for choosing implausibly volatile real and monetary shocks in order to match the (high) average volatility of exchange rates in the data. (This is the well-known exchange rate volatility puzzle.) Instead, we anchor the average level of real and monetary shocks by including the average variance of GDP growth across countries and the average variance of CPI inflation across countries in the target vector.

Beyond these details, the crucial feature of the target vector is that it only contains information about the variability of exchange rates, output, and inflation, and no information about capital accumulation, interest rates, or any variable directly related to it. The number of parameters to be estimated depends on our assumptions about the covariance matrix for the shocks, and varies across different specifications.

4.3 Results

In our first empirical exercise, we constrain the variance of productivity and inflation shocks to be constant across countries ($\sigma_{N,n} = \sigma_N$ and $\tilde{\sigma}_n = \tilde{\sigma}$). Column 2 of Table 2 and column 1 of Figure 1 show the results of this “homoskedastic” calibration. The estimated standard deviation of productivity and inflation shocks are $\sigma_{N,n} = 1.96\%$ and $\tilde{\sigma} = 0.01\%$, respectively. Given these parameters, the model roughly matches the average standard deviation of output and inflation across countries (2.01% and 2.15% in the model versus 1.99% and 1.87% in the data, respectively) but produces a counterfactually large negative correlation between aggregate consumption growth and appreciations of the real exchange rate (-0.887 versus -0.012 in the data).

Because shocks are homoskedastic across countries, differences in country size are the only source of variation in the stochastic properties of countries’ exchange rates and thus the only potential driver of differences in capital-output ratios in this calibration. Panel A of Figure 1 shows a slightly positive association between the model-implied and empirical capital-output ratios. In this sense, heterogeneity in country size accounts for 7% of the variation in relative capital-output ratios across the seven countries in our sample. Similarly, Panel B shows an only slightly positive association between model-implied and

empirical mean risk-free interest rates across countries.

Panel C plots the model-implied and empirical variances of each country’s equally-weighted average real exchange rates. It shows a horizontal line, reflecting the fact that differences country size have no effect on the variance of exchange rates. The same is true for the variances of nominal exchange rates (not shown).

To motivate our next calibration, we regress differences in capital-output ratios on a constant term and our measure of country size. We then plot the residuals of this regression over the average variance of each country’s bilateral exchange rates in Figure 2. The plot shows a positive association between the two variables consistent with the predictions of Propositions 1 and 2: countries with more volatile exchange rates tend to have higher capital-output ratios, accounting for more than half of the residual variation that is not accounted for by differences in country size.

In our second, “heteroskedastic”, calibration we explore this possibility by examining how differences in the volatility of productivity and inflation can improve the model’s predictions. We repeat our estimation procedure, but now allow heteroskedastic productivity and inflation shocks across countries, such that the vector Φ now contains the pair $(\sigma_{N,n}, \tilde{\sigma}_n)$ for all n . The results of this estimation are in Column 3 of Table 2 and column 2 of Figure 1.

Column 3 of Table 2 shows that this heteroskedastic calibration again fits almost perfectly the cross-country averages of the standard deviation of GDP growth and inflation but now gives a better fit for the average correlation of consumption growth and appreciations in the data (-0.21 versus -0.01).

More importantly, Panel A of Figure 1 shows that allowing for heteroskedastic shocks, and thus heterogeneity in the volatility of exchange rates across countries, significantly increases the correlation between empirical and model-implied capital-output ratios. The regression line shown in the plot accounts for 72% of the variation in the data. In this sense, the stochastic properties of exchange rates, as proxied by differences in country size and the variances of exchange rates, can account for the majority of the differences in relative capital-output ratios across the countries in our sample. Panels B shows that the same is true for cross-country variation in mean risk-free rates ($R^2 = 0.57$), where countries with more volatile exchange rates also tend to have lower risk-free interest rates,

as predicted by the model. By design, the estimation now almost fully achieves its targets by matching the relative differences in the volatility of real and nominal exchange rates across countries. (Panel C shows an R^2 of 0.99 for the relative variances of real exchange rates. The corresponding R^2 for the nominal moments is also 0.99, not shown.)

Although the model does remarkably well in predicting the relative differences in capital-output ratios across countries, it fails significantly on magnitudes. The slope of the regression line in Panel A is orders of magnitude smaller than one (0.001), reflecting the fact that the absolute size of the model-predicted differences is small compared to those in the data. For example, in the data, Canada has a 30% lower capital-output ratio than the United States. The model predicts a difference of around 0.03%. Similarly, the slope coefficients in Panels B and C are 0.002 and 0.16, respectively, revealing that the model also under-predicts the absolute size of differences in risk-free interest rates and the average variance of real exchange rates by more than an order of magnitude. We thus conjecture that the model's failure to match the quantitative magnitude of the differences in capital-output ratios may be related to the well-known failure of the canonical model to generate (i) sufficiently large risk premia (and thus sufficiently large variation in risk-free interest rates across countries) and (ii) sufficient variability of the real exchange rate.

In Table 3, we attempt to address the latter issue by applying two modifications to the model that increase the variance of the real exchange rate. To this end we remove the average variances of GDP growth and inflation from the target vector and instead target the average variance of real and nominal exchange rates. In column 1 we add capital adjustment costs to our model of the form suggested by [Jermann \(1998\)](#), such that the equation of motion of the capital stock is now

$$K_{t+1} = (1 - \delta)K_t + I_t - G_t K_t$$

where

$$G_t = \frac{I_{K,t}}{K_t} - \left(\frac{v_1}{1 - \frac{1}{\xi}} \left(\frac{I_{K,t}}{K_t} \right)^{1 - \frac{1}{\xi}} + v_0 \right).$$

Under this specification it is costly to adjust the capital stock quickly, such that shocks are harder to absorb by increasing and decreasing the level of capital installed, increasing

the effect of a given shock on marginal utility. To maximize the effect of this modification to the model we set $\xi = 2$, which is the highest level of adjustment costs suggested by [Jermann \(1998\)](#).

The table shows that the model is now able to match the variance of real and nominal exchange rates almost perfectly (the slopes of regression lines relating model-implied and empirical variances of real and nominal exchange rates are now 0.94 and 0.96, respectively), albeit at the cost of significantly exceeding the variance of inflation (6.76% versus 1.98% in the data). With this modification, the absolute differences in capital-output ratios increase by a factor of five such that the slope corresponding to Panel A in [Figure 1](#) is now 0.005 rather than 0.001.

In column 2 we go one step further and increase market segmentation to $\phi = 0.04$, such that it is approximately equal to the share of the US population that directly holds foreign stocks in their financial portfolio ([Nechio, 2010](#)). The results of this calibration are similar but now produce a slope corresponding to Panel A in [Figure 1](#) of 0.01. Both modifications thus go part of the way in terms of generating more volatile real exchange rates (ii) but nevertheless fail to generate sufficiently large differences in capital-output ratios.

We believe that this latter failure persists because the model is unable to generate sufficiently large risk and equity premia (i) even when it generates volatile exchange rates. An interesting avenue for future research would be to augment the model with remedies developed in the literature on the equity premium puzzle, such as long-run risk, rare events, or habit formation. In closed economy models, these remedies effectively increase the variance of investors' marginal utility. It is thus plausible that they could also increase the absolute differences in variances of marginal utility across countries when applied to a setting similar to ours. However, implementing such remedies in a multi-lateral model of exchange rate determination with asymmetric countries poses considerable technical difficulties that are beyond the scope of this paper.

5 Conclusion

A growing literature in asset pricing centers on the idea that “safe haven” currencies appreciate in “bad” times, linking heterogeneity in the stochastic properties of exchange rates to violations of uncovered interest parity, differences in currency risk premia, and differences in equity returns across countries. In this paper, we have argued that, beyond affecting asset prices, this heterogeneity should have real effects: countries whose currencies appreciate when the shadow price of traded goods is high should have lower cost of capital for the production of nontraded goods and, as a consequence, accumulate higher capital-output ratios for the production of these goods. In a simple quantitative application of our model, we find strong support for this prediction – that differences in stochastic properties of real exchange rates can account for 72% of the variation in relative capital-output ratios across seven industrialized countries.

The tentative conclusion of our analysis is that variation in exchange rates may not be as disconnected from economic fundamentals as previously thought in the sense that the stochastic properties of countries’ exchange rates appear to line up with variation in capital-output ratios across countries in the way predicted by our model. Nevertheless, we interpret our results with due caution. First, our estimation is based on a sample of relatively homogeneous developed countries that have similar institutions, tax systems, and well-functioning markets. This relative homogeneity may explain why we get a good fit to the data even without controlling for these other factors. Second, our model matches only relative differences but falls far from the mark in matching the magnitude of absolute differences. We conjecture that this failure to match magnitudes is connected to the model’s failure to generate sufficient volatility in the real exchange rate and to match the magnitude of the equity premium, both of which are well-established issues in the literature. We believe a crucial next step may be to combine our framework with some of the remedies that have been developed for these issues, such as long-run risk, rare events, or habit formation, along the lines of recent work by Colacito, Croce, Ho, and Howard (2012) and Colacito, Croce, and Gavazzoni (2014).⁸

Beyond more quantitative applications, a link between the stochastic properties of

⁸See Colacito and Croce (2011), Verdelhan (2010), Heyerdahl-Larsen (2014), and Stathopoulos (2012) for other promising approaches that apply habit-formation or long-run risk in open-economy models.

exchange rates and factor allocation across countries also poses interesting normative questions because it may create incentives to manipulate these stochastic properties. In [Hassan, Mertens, and Zhang \(2015\)](#), we take a step in this direction by studying the effect of currency pegs on capital accumulation across countries.

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Table 1: Calibrated Parameters

Parameter	Value	Reference
$\epsilon_\alpha = 1/(1 - \alpha)$	0.44	Stockman and Tesar (1995)
β	0.99	Stockman and Tesar (1995)
δ	0.10	Stockman and Tesar (1995)
γ	10.00	
ν	0.40	
τ	0.30	Burstein, Neves, and Rebelo (2003)
ϕ	0.26	Giannetti and Koskinen (2010)
$\sigma_T(\%)$	0.10	

Notes: Parameters used in our homoskedastic and heteroskedastic calibrations. All calibrations are to annual data for seven industrialized economies. ϵ_α denotes the elasticity of substitution between traded and nontraded goods, β the time discount factor, δ the depreciation rate of capital, γ the coefficient of relative risk aversion, ν the capital share of output, τ the expenditure share of traded goods, ϕ the share of each country's population that has access to complete international asset markets, and σ_T the standard deviation of the log endowment of traded goods.

Table 2: Estimation Results

		(1)	(2)	(3)
		Data	Homoskedastic Calibration	Heteroskedastic Calibration
Panel A		<i>Estimated Parameters</i>		
Productivity Shocks	$\sigma_N^{AUS}(\%)$		1.96	1.99
	$\sigma_N^{CAN}(\%)$		1.96	1.52
	$\sigma_N^{CHE}(\%)$		1.96	2.22
	$\sigma_N^{GBR}(\%)$		1.96	1.69
	$\sigma_N^{JPN}(\%)$		1.96	2.91
	$\sigma_N^{NZD}(\%)$		1.96	1.21
	$\sigma_N^{USA}(\%)$		1.96	1.84
Inflation Shocks	$\tilde{\sigma}^{AUS}(\%)$		0.01	0.11
	$\tilde{\sigma}^{CAN}(\%)$		0.01	0.01
	$\tilde{\sigma}^{CHE}(\%)$		0.01	0.27
	$\tilde{\sigma}^{GBR}(\%)$		0.01	0.05
	$\tilde{\sigma}^{JPN}(\%)$		0.01	0.32
	$\tilde{\sigma}^{NZD}(\%)$		0.01	0.01
	$\tilde{\sigma}^{USA}(\%)$		0.01	0.16
Panel B		<i>Additional Targeted Moments</i>		
Avg. std(GDP growth)		1.99 (0.13)	2.01	1.99
Avg. std(inflation)		1.87 (0.09)	2.15	2.18
Panel C		<i>Non-targeted Moments</i>		
Avg. Corr(agg. cons. growth, rer)		-0.012	-0.94	-0.21

Notes: The table shows estimated parameters (Panel A) and moments generated (Panels B and C) by our homoskedastic and heteroskedastic calibrations. ‘Avg. std(x)’ stands for the average standard deviation of x across countries. We present the standard deviation of the average standard deviation of x in brackets. Standard errors on the empirical moments are bootstrapped (see Appendix E.2 for details). ‘Avg. Corr(agg. cons. growth, rer)’ stands for the average across countries of the correlation of aggregate consumption growth with real appreciations. Moments listed in columns 2 and 3 of Panels B and C are from simulated annual data for 4,000 years in 500 economies using Dynare++. We calculate the moments on each simulated economy and then average the simulated moments across economies. Column 1 shows the corresponding moments in the data.

Table 3: Model Extensions

	(1)	(2)
Modified Parameters	$\zeta = 2$	$\zeta = 2, \phi = 0.04$
Panel A: Difference in Log Capital-Output Ratio Relative to US		
<i>Slope</i>	0.006	0.011
R^2	0.75	0.63
Panel B: Variance of the Real Exchange Rate (Average for each Country)		
<i>Slope</i>	1.04	1.03
R^2	0.99	0.99
Panel C: Variance of the Nominal Exchange Rate (Average for each Country)		
<i>Slope</i>	0.95	0.94
R^2	1.00	1.00
Panel D: Additional Moments		
Avg. std(GDP growth) (%)	0.61	0.80
Avg. std(inflation) (%)	6.14	6.14
Avg. Corr(agg. cons. growth, real exchange rate)	-0.330	0.036

Notes: This table summarizes estimation results for each of the modifications to the model. In column 1 we add capital adjustment costs according to Jermann (1998). We change the equation of motion of the capital stock to

$$K_{t+1} = (1 - \delta)K_t + I_t - G_t K_t$$

where

$$G_t = \frac{I_{K,t}}{K_t} - \left(\frac{v_1}{1 - \frac{1}{\xi}} \left(\frac{I_{K,t}}{K_t} \right)^{1 - \frac{1}{\xi}} + v_0 \right)$$

We calibrate ξ to 2.0 and v_0 and v_1 are calibrated such that $G_t = 0$ and $\partial G_t / \partial (I_t / K_t) = 0$ at the deterministic steady state. In column 2 we implement the capital adjustment costs modification and additionally decrease the fraction of active households in the model to 0.04. For each modification, we report the slope and R^2 of the regression line from a regression of the simulated moment on the empirical moment. We calculate simulated moments by simulating annual data for 4,000 years in 500 economies using Dynare++. We calculate the moments on each simulated economy and the average the simulated moments across economies. Panel A reports regression results for capital-output differences. Panel B reports regression results for the variance of country n 's equal-weighted average real exchange rate. Panel C reports regression results for the variance of country n 's equal-weighted average nominal exchange rate. Panel D reports simulated moments corresponding to Panels B and C of Table (2). "Avg. std(x)" stands for the average standard deviation of x across countries. "Avg. Corr(agg. cons.growth, rer)" stands for the average correlation of aggregate consumption growth with real exchange rate appreciations.

Notes: Figure 1 plots the results of the empirical exercise. We calculate and plot simulated moments against their empirical counterparts. We calculate simulated moments by simulating annual data for 4,000 years in 500 economies using Dynare++. We calculate the moments on each simulated economy and the average the simulated moments across economies. Column (1) presents results for the homoskedastic calibration and column (2) presents results for the heteroskedastic calibration. The slope and R^2 of the regression lines are listed below each figure. Panel A plots differences in log capital-output ratios. In our model, the capital-output ratio in period t in country n is $\ln(K_t^n/Y_t^n) = (1-\nu) \ln(K_t^n/\theta^n)$. Panel B plots the interest rate differential relative to the United States dollar. Panel C plots the variance of each country's equal-weighted average real exchange rate.

Figure 1: Estimation Results

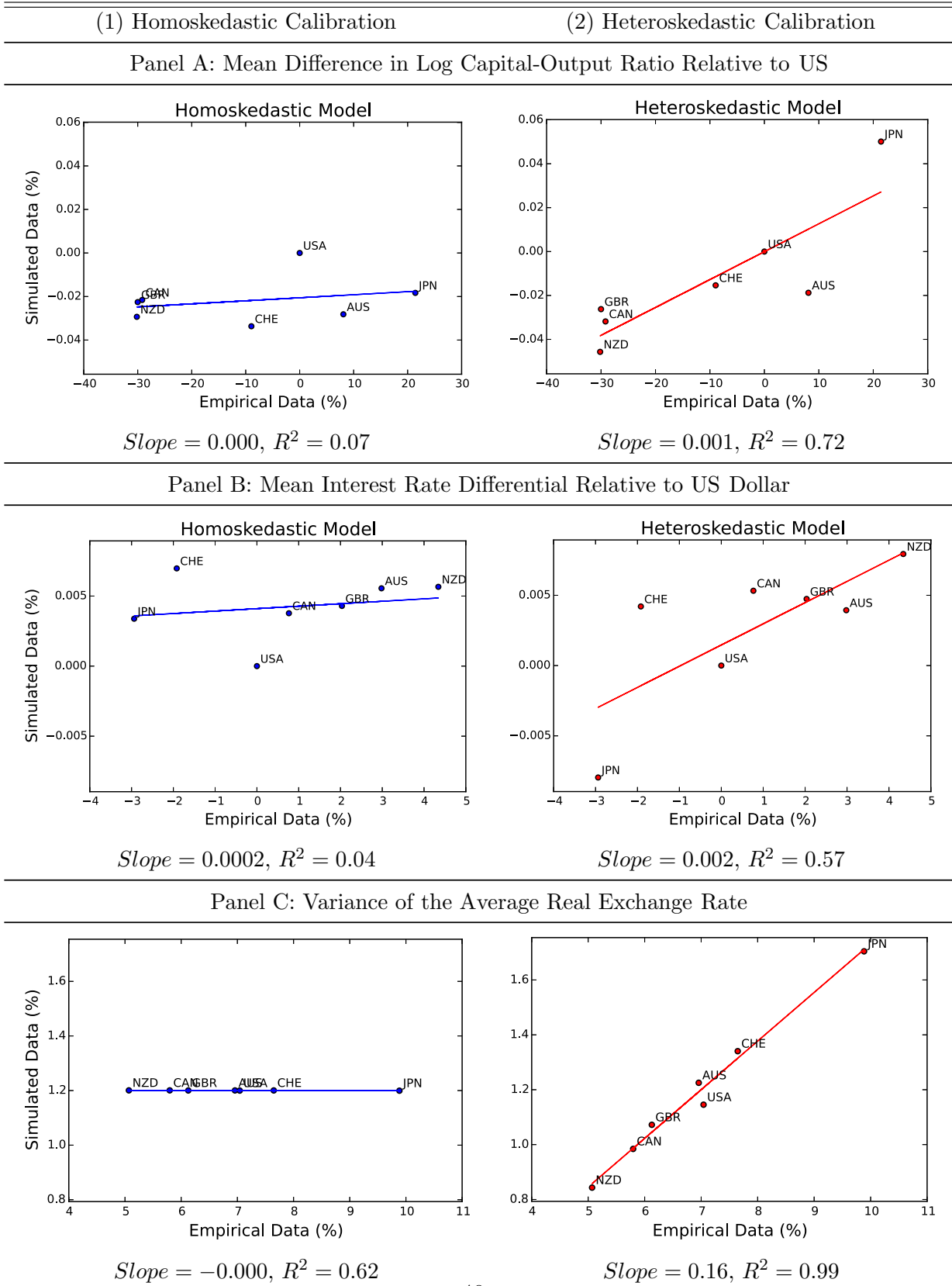
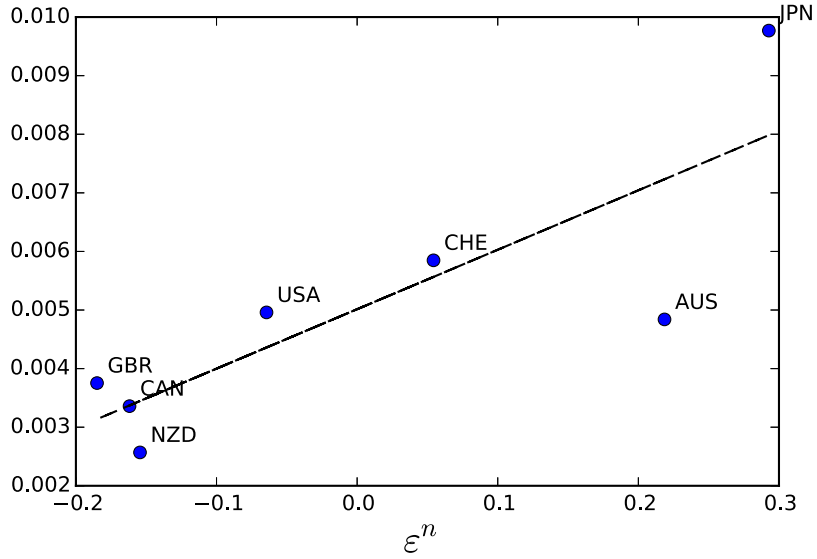


Figure 2: Reduced-form Results



Notes: The figure plots residuals of a regression of differences in capital-output ratios on a constant term and our measure of country size over the variance of each country's average exchange rate. Residuals are generated using the specification

$$\log(K^n/Y^n) - \log(K^{USA}/Y^{USA}) = \alpha_n + \beta_n \theta^n + \epsilon^n.$$

We calculate country n 's average real exchange rate in period t by calculating the equally weighted average of a country's log real exchange rate appreciations from $t - 1$ to t . We calculate the variance of this average real exchange rate. The regression line has a slope of 0.01 and an R^2 of 0.69.

A Equilibrium Consumption of Inactive Households

Inactive households maximize utility, defined in equation (1), in each state of the world by splitting their wealth $\exp(-\mu_t)P_t^n C_{ss}$ optimally between tradable and non-tradable goods. Their optimization problem can be written as maximizing (1) in each state subject to (2) and (6)

$$\begin{aligned} \max_{\hat{I}_{T,t}, \hat{I}_{N,t}} \frac{1}{1-\gamma} \left(\tau \hat{I}_{T,t}^\alpha + (1-\tau) \hat{I}_{N,t}^\alpha \right)^{\frac{1-\gamma}{\alpha}} \\ \text{s.t. } \hat{I}_{T,t} + P_{N,t} \hat{I}_{N,t} = \exp(-\mu_t) P_t^n C_{ss} \end{aligned} \quad (28)$$

We solve this problem by setting up a Lagrangian and taking first-order conditions with respect to $I_{T,t}$ and $I_{N,t}$ to get equation (12).

B Social Planner's Problem

The Social Planner maximizes the utility of active households, subject to the equilibrium behavior of inactive households, the resource constraint for tradable goods, the resource constraint for non-tradable goods and the resource constraint for final goods.

First, we derive the resource constraint for final goods. Final goods are produced by active and inactive households. Final goods are either consumed by inactive households, consumed by active households or used for investment. Hence, the resource constraint for final goods (9) can be written as

$$\phi \left[\tau (I_{T,t}^n)^\alpha + (1-\tau) (I_{N,t}^n)^\alpha \right]^{\frac{1}{\alpha}} + (1-\phi) \left[\tau (\hat{I}_{T,t}^n)^\alpha + (1-\tau) (\hat{I}_{N,t}^n)^\alpha \right]^{\frac{1}{\alpha}} = \phi C_t^n + (1-\phi) \hat{C}_t^n + I_{K,t}^n. \quad (29)$$

Inactive households optimize their utility by consuming their entire production

$$\left[\tau (\hat{I}_{T,t}^n)^\alpha + (1-\tau) (\hat{I}_{N,t}^n)^\alpha \right]^{\frac{1}{\alpha}} = \hat{C}_t^n.$$

Plugging inactive households' consumption into the resource constraint for the final good

(29) delivers

$$\theta^n (\phi C_t^n + (K_{t+1}^n - (1 - \delta)K_t^n)) = \phi \theta^n [\tau (I_{T,t}^n)^\alpha + (1 - \tau) (I_{N,t}^n)^\alpha]^\frac{1}{\alpha} \quad \forall n, \quad (30)$$

The resource constraint for the non-tradable good (10) can be written as

$$\theta^n (\phi I_{N,t}^n + (1 - \phi)\hat{I}_{N,t}^n) = \theta^n [\eta_t^n (K_t^n)^\nu] \quad \forall n, \quad (31)$$

and the resource constraint for the tradable good (11) can be written as

$$\theta^n (\phi I_{T,t}^n + (1 - \phi)\hat{I}_{T,t}^n) = \theta^n Y_{T,t}^n. \quad (32)$$

The Social Planner maximizes the utility of active households subject to the resource constraints (30), (31) and (32). We can thus write the Social Planner's value function as

$$\begin{aligned} V \left(\{K_t^n, Y_{Y,t}^n, \eta_t^n, \mu_t^n\}_{n=1}^N \right) &= \sum_n \frac{\phi \theta^n}{1 - \gamma} (C_t^n)^{1 - \gamma} + \beta \mathbb{E} \left[V \left(\{K_{t+1}^n, Y_{Y,t+1}^n, \eta_{t+1}^n, \mu_{t+1}^n\}_{n=1}^N \right) \right] \\ &\quad + \Lambda_{T,t} \sum_n \theta^n [Y_{T,t}^n - \phi I_{T,t}^n - (1 - \phi)\hat{I}_{T,t}^n] \\ &\quad + \sum_n \Lambda_{N,t}^n \theta^n [\eta_t^n (K_t^n)^\nu - \phi I_{N,t}^n - (1 - \phi)\hat{I}_{N,t}^n] \\ &\quad + \sum_n \Lambda_t^n \phi \theta^n \left([\tau (I_{T,t}^n)^\alpha + (1 - \tau) (I_{N,t}^n)^\alpha]^\frac{1}{\alpha} - C_t^n - \frac{1}{\phi} (K_{t+1}^n - (1 - \delta)K_t^n) \right). \end{aligned}$$

The first-order conditions with respect to $I_{T,t}^n$ and $I_{N,t}^n$ are given by (14) and (15), respectively. The first-order conditions with respect to next period's capital per capita (K_{t+1}^n) is

$$\theta^n (C_t^n)^{-\gamma} = \beta \mathbb{E} \left[\frac{\partial}{\partial K_{t+1}^n} V \left(\{K_{t+1}^n, Y_{Y,t+1}^n, \eta_{t+1}^n, \mu_{t+1}^n\}_{n=1}^N \right) \right] \quad (33)$$

The envelope condition is

$$\frac{\partial}{\partial K_t^n} V \left(\{K_t^n, Y_{Y,t}^n, \eta_t^n, \mu_t^n\}_{n=1}^N \right) = \theta^n (1 - \delta) (C_t^n)^{-\gamma} + \Lambda_{N,t}^n \theta^n \eta_t^n \nu (K_t^n)^{\nu - 1}. \quad (34)$$

C Log-linearized System of Equations

We log-linearize the system of equilibrium conditions around the point at which $\sigma_{N,t}^n, \tilde{\sigma}_n = 0 \forall n$. Therefore, we start with equation (14) which leads to

$$(1 - \gamma - \alpha)(\tau i_T^n + (1 - \tau)i_N^n) + \log \tau + (\alpha - 1)i_T^n = \lambda_T \forall n$$

and equation (15) yields

$$(1 - \gamma - \alpha)(\tau i_T^n + (1 - \tau)i_N^n) + \log(1 - \tau) + (\alpha - 1)i_N^n = \lambda_N^n \forall n.$$

Next we approximate the resource constraint for non-tradable goods (31) in which we substitute equilibrium consumption of inactive households (12) and replacing prices $P_{N,t}$ and P_t with the ratio of Lagrange multipliers from (13)

$$\phi i_N^n + (1 - \phi) \left(-\mu^n - \tau \frac{1}{1 - \alpha} \left((\lambda_N^n - \lambda_T) - \log\left(\frac{1 - \tau}{\tau}\right) \right) \right) = y_N^n \forall n$$

Lastly, we sum the resource constraint for tradable goods (31) over all countries and again substitute for the consumption of inactive households and prices as above to get

$$\phi \sum_{n=1}^N \theta^n i_T^n + (1 - \phi) \sum_{n=1}^N \left(-\mu^n + \frac{1}{1 - \alpha} (1 - \tau) \left((\lambda_N^n - \lambda_T) - \log\left(\frac{1 - \tau}{\tau}\right) \right) \right) = \sum_{n=1}^N \theta^n y_T^n.$$

D Differential Incentives for Capital Accumulation

When markets are incomplete and the economy is affected by both real and monetary shocks the differential incentive to accumulate capital can be written as

$$\begin{aligned} k_1^f - k_1^h &= \frac{\gamma(\gamma - 2((1 - \tau)\phi + \gamma\tau))}{2((1 - \tau)\phi + \gamma\tau)^2} (\sigma_{N,f}^2 - \sigma_{N,h}^2) \\ &+ \frac{\tau(1 - \tau)(\phi - \gamma)^2(\gamma - \gamma\tau - (1 - \tau)\phi)}{\phi((1 - \tau)\phi + \gamma\tau)^2} (\theta^f \sigma_{N,f}^2 - \theta^h \sigma_{N,h}^2) \\ &+ \frac{\gamma^2(1 - \tau)(1 - \phi)^2}{4\phi((\gamma - \phi)\tau + \phi)^2} ((1 - \tau)\phi(\tilde{\sigma}_f^2 - \tilde{\sigma}_h^2)a + 2\gamma\tau(\theta^f \tilde{\sigma}_f^2 - \theta^h \tilde{\sigma}_h^2)), \end{aligned}$$

which is unambiguously greater than zero if country f is more systemic than country h .

E Data Appendix

E.1 Data Sources

IMF Exchange Rate Class The IMF exchange rate regime classification was taken from the data section of Carmen Reinhart’s website. The web address is <http://www.carmenreinhardt.com/data-by-topic/topics/12/>

Nominal Exchange Rates Nominal exchange rates are taken from the National Accounts section of the OECD Statistics database. The exchanges rates are given as quarterly averages of national currency per US dollar.

Consumer Price Index We use a CPI deflator to translate nominal exchange rates to real exchange rates. We use the “Consumer prices - all items” variable from the Consumer Prices (MEI) subsection of the Prices and Purchasing Power Parities section of the OECD Statistics database. The base year for the CPI is 2010.

Real GDP (OECD) We use GDP data to generate a country’s Pareto weights. These data come from the OECD Statistics database. Real GDP is measured using the expenditure approach at the quarterly frequency in millions of national currency, in the OECD reference year prices and are seasonally adjusted.

Real GDP per Capita We also gather real GDP per capita data to calculate the standard deviation of the growth rate of real GDP per capita. The data are GDP per head of population measured in USD, constant prices at 2005 PPPs. They are taken from the OECD database under the subject heading T_GDPPOP.

Capital Stock Capital stock data is taken from the Penn World Tables version 8.1. We use “Capital stock at constant 2005 national prices (in mil. 2005US\$)”. The data can be accessed at:

<http://citaotest01.housing.rug.nl/febPwt/Dmn/AggregateXs.mvc/VariableCodeSelect>.

The capital stock data are given at the annual frequency.

Real GDP (PWT) We use the Penn World Table’s version of real GDP as the denominator for calculating the capital to output ratio across countries. This is “Real

GDP at constant 2005 national prices (in mil. 2005US\$". As with the capital stock data, these data are given at the annual frequency.

Population We use population data to generate a measure of GDP per capita. We use the variable "Population (hist5) All ages". Population data is given at the annual frequency.

Consumption Growth We obtain consumption growth data from the OECD Statistics database. We observe net GDP private final consumption expenditure growth from the previous period as a percentage. The subject code is NAEXKP02.

Forward Premia Data In order to calculate differences in real bond returns, we use forward premia data from HassanMano2015 FIXME. Under covered interest parity, the log forward premia between two countries is equal to the spread in the two countries' risk free log bond returns.

E.2 Moment Generation

We estimate our model using the following moments

1. The variance of each each country's average real exchange rate normalized by the average variance of the average real exchange rates
2. The variance of each each country's average nominal exchange rate normalized by the average variance of the average nominal exchange rates
3. The average variance of per capita GDP growth
4. The average variance of inflation

Real Exchange Rates

We calculate the real exchange rate between each pair of countries by adjusting their nominal exchange rate by the ratio of their CPIs. Specifically, this is

$$RER_t^{h,f} = NER_t^{h,f} \frac{CPI^f}{CPI^h}$$

where $RER_t^{h,f}$ is the real exchange rate given as the amount of currency h needed to buy a unit of currency f , $NER_t^{h,f}$ is the corresponding nominal exchange rate, and CPI^h, CPI^f are the price indices for each country.

We calculate each country's average exchange rate as the equally weighted sum of the change in that country's exchange rates against the other countries in the world. For country h , the average exchange rate is

$$\overline{rer}^h = \sum_{f=1}^N \frac{1}{N} \left[\log RER_{t+1}^{h,f} - \log RER_t^{h,f} \right]$$

Where $1 \dots N$ indexes the other countries in the model. Since we have seven total countries in the model, $N = 6$. We calculate the variance of the average real exchange rate by calculating the variance of the quantity derived above $Var(\overline{rer}^h)$.

We normalize the variance each exchange rate by the average variance average exchange rates in the sample. This allows us to match the relative volatility of exchange rates without having to match the level of exchange rates.

The theoretical counterpart to the variance of the real exchange rate between countries h and f is

$$Var(\overline{rer}^h) = Var\left(\sum_{f=1}^N \frac{1}{N} \left[p_{t+1}^h - p_{t+1}^f - (p_t^h - p_t^f) \right]\right)$$

where p_t^n represents the log price level of country n in period t

Nominal Exchange Rates

We calculate the variance of average nominal exchange rates in the same way as we calculated the variance of average real exchange rates. For country h , this is

$$Var(\overline{ner}^h) = Var\left(\sum_{f=1}^N \frac{1}{N} \left[\log NER_{t+1}^{h,f} - \log NER_t^{h,f} \right]\right)$$

The theoretical counterpart to the variance of the nominal exchange rate between

countries h and f is

$$Var(\overline{ner}^h) = Var\left(\sum_{i=1}^N \frac{1}{N} \left[-\mu_{t+1}^h + \mu_{t+1}^f + rer_{t+1}^{h,f} - (-\mu_t^h + \mu_t^f) - rer_t^{h,f}\right]\right)$$

where $rer_t^{h,f}$ represents the log real exchange rate and μ_t^n is the inflationary shock in country n at time t .

Average Variance of GDP Growth

We use annual GDP per capita data from the OECD. Call this $GDPPC_t$. We calculate the variance of GDP growth for country n by taking the variance of $\log GDPPC_{t+1}^n - \log GDPPC_t^n$. Finally, we average the variance of GDP growth across countries.

The theoretical counterpart to real GDP per capita is the value of all goods produced in that country in period t in terms of the final consumption bundle.

$$GDPPC_t = \frac{Y_{T,t} + P_{N,t}Y_{N,t}}{P_t}$$

Average Variance of Inflation

We measure inflation as the change in the CPI, $INF_{t+1} = CPI_{t+1} - CPI_t$. We calculate the variance of inflation for country n by taking the variance of $\log INF_{t+1}^n$. Finally, we average the variance of inflation across countries.

The theoretical counterpart to inflation is the change in the nominal price level.

$$INF_t^n = -\mu_{t+1}^n + P_{t+1}^n - P_t^n$$

Consumption Growth

We observe consumption growth at the annual frequency from the OECD Statistics library. We calculate log consumption growth and subtract out log population growth in order to measure per capita log consumption growth. This is

$$dc_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right) - \log\left(\frac{POP_{t+1}}{POP_t}\right)$$

where dc_{t+1} is per capita log consumption growth, C_{t+1}/C_t is observed from the OECD data and POP_{t+1}/POP_t is constructed from the population data.

The theoretical counterpart to log per capita consumption is the weighted sum of consumption by active and inactive households.

$$c_t^n = \log(\phi C_t^n + (1 - \phi)\hat{C}_t^n)$$

where C_t^n is the consumption of active households and \hat{C}_t^n is the consumption of inactive households.