Sovereign Debt with Heterogeneous Creditors*

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Abstract

We develop a sovereign debt model with heterogeneous creditors (private and official) where the probability of default depends on both the level and the composition of debt. Higher exposure to official lenders improves incentives to repay due to more severe sanctions but carries extra costs in the form of a reduced value of the sovereign's default option. The model can account for the co-existence of private and official lending, the time variation in their shares in total debt as well as the low rates charged on both. It can also shed light on the joint default and debt composition choices observed during the recent sovereign debt crisis in Europe.

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1 Introduction

The recent sovereign debt crisis in the Eurozone has exhibited diverse patterns regarding default and debt–refinancing composition decisions: Heavily indebted Greece defaulted on its debt and completely switched financing from private to official (other Eurozone members and the IMF) funds that carried a low interest rate. Greece's default decision was encouraged by its official creditors. Similarly heavily indebted Italy did not default, did not receive any direct official loans and continued to rely on more expensive private funds. Other debt distressed countries, namely Ireland, Portugal and Spain, did not

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default, experienced a change in the composition of new funding towards cheaper official sources but nevertheless continued borrowing from private credit markets.

The canonical sovereign debt model ((Eaton and Gersovitz, 1981; Arellano, 2008)) contains only private creditors and is thus ill suited to address the choice of the composition of sovereign debt in terms of private and official credit as well as the interaction between this choice and the default decision. In this paper we add official creditors to the standard debt model and ask whether and how the modified model can help shed light on the patterns described above.

Creditor heterogeneity may take different forms. For instance, it may regard the type and extent of monitoring activities that accompany the loan, the characteristics of the debt contract (conditionality schemes and policy requirements), and so on. In our view, these differences derive primarily from and can be encapsulated by a single factor: Namely, the severity of the sanctions (costs) suffered by a sovereign when he defaults against a particular class of creditors. We show that the sovereign debt model can make sense of the empirical observations if it contains official creditors who enjoy superior "enforcement power" relative to their private counterparts: A sovereign borrower suffers higher costs when he defaults against official creditors than when he defaults against private creditors;[1] we elaborate on the justification for this assumption below. The more severe sanctions imply a lower probability of default on official funds and thus lower default risk premia and interest rates. This feature can thus account for the low interest rates charged on large official loans. But the low rates do not represent a "free lunch" for the borrower, otherwise borrowers would always prefer official to private credit. There is a countervailing force as official loans carry an extra cost in the form of loss of ex-post policy flexibility: More severe default costs imply that debt is repaid in some states of the world (say, during a protracted, severe recession) in which the sovereign would have opted for default were the debt owned by private creditors instead.[2] This trade off plays an important role in the "portfolio" choice of the sovereign.

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What does the availability of "cheap" official funds imply for the riskiness of private loans and the sovereign's demand for them? In order to address this question, it is instructive to break it into two parts. First, holding total debt fixed, how does substitution of official for private funds affect the probability of default on the latter? And second,

[1]That the identity of the creditor can make a difference for the cost of default and hence the riskiness of the loan seems rather undisputed. For instance, there is a widely shared view supported by anecdotal evidence that loans by Mafia carry lower risk and thus a low interest rate because the incentive to repay such loans is much stronger than the incentive to repay other creditors due to Mafia's more extensive set of enforcement tools. Note that the alternative to a Mafia loan is typically no loan at all, that is a loan with a prohibitively high rate. **?** discusses how Mafia's protection and guarantees of safe conduct substitute for lack of trust in society.

[2]See Zame (1993) for a discussion of the insurance benefits of *implicitly* state contingent debt. Under incomplete markets, a country may trade a higher interest rate on its debt for the option to declare default in states where debt repayment would have been very costly. The desire by sovereigns to maintain a wider default option may explain both Spain's resistance to accept official loans as well as Greece's recent attempts under the previous government to switch away from inexpensive official to much more expensive private sources of funds.

how is this probability of default affected by an increase in official—and consequently, total—funds when holding the quantity of private loans fixed.

Holding total debt constant, a switch from private to official funds may make *private* loans safer if the higher default costs associated with the official funds also apply to the private portion of total debt. Such an extension of "protection" could arise either directly from the existence of pari passu provisions in debt contracts;[3] or, indirectly from the form of default costs, for instance, from the existence of a fixed cost component. In both of these cases, private funds acquire the risk characteristics of official funds and they are priced accordingly.

Holding the quantity of private funds constant, an increase in the amount of official credit increases total sovereign debt. If higher total debt increases the probability of default against all creditors (such "dilution" of repayment is a standard property of the canonical sovereign debt model), then private loans become riskier. But the opposite effect is also possible if two conditions are satisfied: Official credit serves to enhance the debtor country's repayment capacity, for instance, by being linked to the adoption of structural reforms;[4] and this "collateral creation" by official credit is strong enough to also benefit the private creditors.[5]

Depending on the effect of official credit on the riskiness of total and private claims, official loans may crowd in (complementarity between the two sources of funds) or crowd out (substitutability between the two sources of funds) private loans.

Such extension of protection or collateral expansion effects seem to be present in the Eurozone debt crisis. The dispensation of official credit has been accompanied by a significant compression of the spreads on sovereign loans from the private sector (even in Greece in the period prior to the last elections). We interpret this as an indication that private claims were perceived to have been placed under official protection.[6] And also that the markets expect pressure by official creditors would force the debtor countries to

[3]Zettelmeyer, Trebesch and Gulati (2012, p. 25) report that the new bonds issued by Greece after the 2012 default include pari passu clauses and are subject to a "co-financing agreement that creates a symmetry in servicing debt to the new bondholders and to the EFSF (the EFSF notes and bills it received for the purposes of the debt exchange). In the event of a shortfall in payments by Greece, a common paying agent committed to distributing this shortfall pro rata between the EFSF and the bondholders. Hence, the co-financing agreement made it difficult for Greece to default on its bondholders without also defaulting on the EFSF."

[4]The establishment of a credit relationship with official creditors has been invariably associated with measures that create–expand collateral (monitoring, conditionality, etc.).

[5]It should be noted that such extension of "protection" obtains independent of whether default costs take the form of pure social costs suffered by the sovereign in the case of default; or, the form of resources—collateral—seized by the creditors. See the appendix.

[6]Anxious to avoid a crowding out of private funding, official lenders conceded that safeguards should be put in place to impede ex-post discrimination against their private counterparts. Consistent with this intention, the Greek debt exchange in Spring 2012 put private and official lenders (the EFSF) on an equal footing, see Zettelmeyer et al. (2012, p. 25). The Wall Street Journal (June 29, 2012, Investors Cheer Europe Deal) reports that Angela Merkel's agreement "to make ESM loans to Spain equal to Spanish bonds in creditors' pecking order was largely a recognition by Germany that this was necessary to protect Spain's ability to sell bonds ...." In another but related context, the recent New York court decision in the dispute between Argentina and Elliott Management regarding Argentina's default in 2002 has undermined the ex post preferred creditor status of certain lenders and provided a boost for pari passu.

undertake measures (such as a reduction in the public sector, the liberalization of markets etc.) that enhance their repayment ability.

When official creditors first got involved in the Eurozone debt crisis, the countries affected had different levels of debt overhang. Such differences have interesting implications for the interaction between a country's decision to default and its choice of debt composition. First, long-term debt overhang may induce a sovereign to collude with prospective official creditors in order to wipe out *outstanding* privately held long term debt, rendering freshly issued official loans safer and cheaper and thus benefitting both official creditors and the sovereign. While this implication is well known in the literature as it applies to any situation with distinct groups of creditors, it is accentuated in our model by the superior enforcement power of the official creditors. The borrower has a stronger incentive to default because in addition to eliminating the debt overhang, he also gets the chance to borrow at more favorable terms than if all classes of creditors had equal enforcement power. That is, inexpensive official funding in the presence of debt overhang simply aggravates default incentives. This seems consistent with the Greek default experience. The second and more novel implication is that, under pari passu, a sovereign with large future obligations to private creditors who chooses *not* to renounce them in the present will also try to stay clear of official loans in order to maintain the—large—option value of renouncing these claims (defaulting) in the future. This implication suggests that the differences in the choice of the composition of sovereign debt observed across the distressed Eurozone countries (as described in the opening paragraph above) may well have reflected differences in the size of outstanding debt as well as a country's assessment of the benefits from current versus future default.

Endowed with these two features, namely, more severe sanctions for default against official creditors and differences in the stock of outstanding privately held long term debt, the standard sovereign debt model can account for the fact that substantial amounts of inexpensive loans have been made to debt distressed Eurozone countries by both official and private creditors. It can also shed light on the nexus of default–debt composition patterns observed. In particular, the case of Greece (with a high debt overhang) conforms with the prediction that in periods of debt distress, default is more likely when the debt overhang is large and that when it occurs, it is accompanied by a switch from private towards official sources of funding. The cases of Italy, Ireland, Portugal and Spain (in decreasing order of debt overhang) seem consistent with the implication that absent a default, the share of official funds in fresh borrowing depends negatively on the stock of outstanding long-term debt.[7] More generally, the model implies that countries that have large borrowing needs will favor borrowing from official creditors, in particular when they also face acute credibility problems.

Related Literature The literature on the composition of sovereign debt by type of creditor is scant. Boz (2011) reviews the literature on IMF lending, summarizes empirical evidence and presents a quantitative model of a sovereign that may borrow from private

[7] IMF (2012, p. 17) contains debt data. Note that Spain ended up drawing 41 billion Euro out of the 100 billion allocated to her by the rescue fund.

lenders and the IMF. In her model, private lending is subject to default risk while IMF lending is default risk free. The cost of IMF funds exceeds the risk free rate by an exogenous surcharge. It predicts modest, countercyclical and intermittent IMF lending with *complete* crowding out of private funding. We think that the last feature makes the model unsuitable for analyzing the recent Euro debt crisis as private and official new lending co-exist and this is a key fact that needs to be explained.[8]

Bolton and Jeanne (2011) analyze the interaction between multiple sovereigns of different credit quality and the banking system in a financially integrated area. They argue that a country issuing 'safe haven' government debt may derive rents from exploiting its position as monopolistic supplier of this safe asset. The model proposed here, can also allow for non-competitive rents, but in contrast to Bolton and Jeanne (2011), it could have (official) lenders rather than the borrower extract rents.

Broner, Erce, Martin and Ventura (2013) develop a model without official creditors but with multiple classes of private creditors (domestic vs. foreign). Focusing on the effect of potential discrimination against foreigners on the crowding out of physical investment Broner et al. (2013) argue that the probability of default is lower for domestically held sovereign debt.[9] In the Greek experience, however, no discrimination against foreign debt holders occurred, in spite of the fact that this would have been possible, for instance by allowing domestic residents to use capital losses in lieu of tax payments (as some members of the Greek government considered).[10]

Niepelt (2011) analyzes the composition of sovereign debt across maturities rather than lenders, as considered here, and Diamond and He (2012) analyze the implications of the maturity structure of debt overhang on investment decisions. Tirole (2012) distinguishes between ex-post bailouts that aim at avoiding collateral damage and ex-ante risk-sharing (for example joint-and-several liability) among sovereigns.

The rest of the paper is organized as follows. Section 2 provides empirical support for the main premises of the model. The model is set up in section 3. The equilibrium is characterized in section 4. Section 5 analyzes the consequences of debt overhang for default and debt composition decision. Section 6 concludes.

2 Empirical Foundations of the Model

The key new element in our model relative to the standard sovereign debt setup concerns the existence of differential enforcement powers across groups of creditors. Such differen-

[8]Boz's model also has the unsatisfactory property that accessing the IMF lending facilities changes preferences by assumption by triggering an increase in the sovereign's discount factor.

[9]In the language of the sovereign debt literature this corresponds to an assumption that the default costs are higher when default occurs against foreign than against domestic holders of debt.

[10]Broner et al. (2013) support their assumption of potential discrimination by arguing that the share of domestic to foreign held debt increased as the crisis became worse. According to their figures, this largely reflects holdings of sovereign debt by domestic banks. In Greece, the purchases of short term debt by Greek banks reflected the fact that the Greek banking system was —and is still being—-forced to absorb newly issued debt by the government which owns-controls most of the Greek banking system; and also that Greek banks were insolvent and gambled for resurrection by holding risky, high return Greek debt. There exist no perceptions that Greek banks will be treated more favourably in case of default.

tial powers may arise from various factors. One is that the credit relationship may be part of a broader set of relations between the borrower and the lender, as it is the case with participation in the same club. Consider, for instance, the relationship between Greece and the other members of the Eurozone. A Greek default on official loans from those countries could trigger retaliation and lower Greece's benefits from club membership in the European Monetary Union (EMU) or even the European Union (EU): Structural fund payments and other transfers might be cut; Greece might be forced to leave the Eurozone; official lenders might be tempted to adopt policies that are less favorable to Greek interests; support for Greek foreign policy positions might wither; and so on.

As the ongoing crisis constitutes the first instance in which certain members of the Eurozone have borrowed large amounts from other members, and since no default against official funds has occurred we cannot yet know whether official lenders would be in a position to inflict sanctions of the type described above and if they were, whether they would actually choose to do so.[11] But what matters for the behavior of agents in our model—and hence for the properties of equilibrium—is the *perception* of the existence and likely use of such sanctioning powers, rather than the use itself.[12] In our view, the public debate in Europe and statements by policy makers provide ample evidence supporting a widely shared belief that superior sanctioning powers do exist and official lenders would be willing to use them.

For example, in Germany which provides most of the official financing, the statements of politicians, the debates in parliament and the public reaction all conjure the impression that the loans were perceived to face a low probability of default. In fact, such a perception was a sina qua non for large German loan provision at low rates to be politically feasible in the first place, given voters' expressed antipathy to solidarity (transfers) towards Greece. This perception was also founded in the knowledge that a default by Greece on debt held by official creditors amounts to violating EU treaties and breaking national laws, leaving Greece in uncharted and treacherous political territory regarding its future within the EU.[13] Naturally, time consistency is an issue as it would also be costly for Germany to impose sanctions ex post. But repeat business within the club (lending to Portugal, Ireland and Spain is but one example) makes reputational considerations important, and not imposing sanctions following a Greek default could undermine Germany's credibility.[14] Note also that in order to ensure broad political support for enforcement ex post, Germany has required club-wide participation in the official lending operations.

[11]Superior power certainly existed during the times when mighty countries would use military force to enforce repayment (for instance, when the British navy bombarded Athens).

[12]Naturally, in a model with asynchronous borrowing and default decisions of multiple borrowers, default by one country could reveal the existence of such powers and affect perceptions in those countries that have not made a default decision yet.

[13]The German government spokesman Steffen Seibert argued that the countries of the Eurozone could not accept a reduction in the value of their loans to Greece because this would contradict EU treaties as well as national legislation in Germany and other countries that prohibits member countries to assume the debts of other countries (Kathimerini, November 27, 2012).

[14]Steffen Seibert has argued that debt forgiveness would lead to a huge loss of credibility for Germany and could encourage other countries with debt problems to ask for similar treatment (Kathimerini, November 27, 2012).

Similar perceptions about the additional, severe cost of Greek default as a consequence of Eurozone loans are also held in Greece. All major parties have voiced their support for Greece's honoring its debt obligations to its official creditors and their concern about the disastrous ramifications of failure to do so. In our view, the evidence points to a widely shared belief in both the existence of superior sanctioning powers on the part of official lenders and their willingness to use them.

3 The Model

The economy lasts for two periods, $t = 1, 2$. It is inhabited by a representative agent, a government and foreign investors. In period t the representative agent receives an exogenous endowment, y_t, which may be stochastic. The agent has time- and state-additive preferences over consumption with strictly increasing and concave felicity function $u(\cdot)$ and discount factor $\delta \in (0, 1)$ but neither saves nor borrows.[15]. The economy starts with a zero initial asset position vis-a-vis the rest of the world (in the next section, we allow for debt overhang of different maturities). Foreign investors are risk neutral, require a risk free gross interest rate $\beta^{-1} > 1$ and hold all government debt (since taxpayers do not save).[16] They consist of private and official lenders. Private lenders are competitive. Official lenders may or may not be competitive. Either as a consequence of this, or due to differences in the cost of funds across classes of lenders, the interest rate charged by official lenders could differ from that charged by private lenders. In the benchmark model we have opted to assume that both types of lenders charge the same interest rate as this does not matter for the qualitative properties of the results. It is straightforward to solve the model under the assumption that the two interest rates differ either in an exogenous (a fixed markup or markdown) or an endogenous manner (for instance, due to bargaining between creditors and the borrower).

The government is benevolent and maximizes the welfare of the representative agent. Without loss of generality, public spending other than debt repayment is normalized to zero. In period $t = 1$, the government issues zero-coupon, one-period debt, b, of which b^e is purchased by official and $b - b^e$ by private creditors at the price q; and (residually) levies taxes. Crucially, the government cannot commit its successors (or future selves). Short-sales are ruled out. In period $t = 2$, the government chooses either a common repayment rate on both debt tranches; or, a creditor specific rate. We think that the former better fits the Eurozone debt experience, see discussion in the introduction, so we solve the model under the assumption of a common repayment rate. This could arise from either an official pari passu provision or the structure of sanctions.

Let the repayment rate on debt maturing in period $t = 2$ be denoted by r_2 with $r_2 = 1$ representing full and $r_2 < 1$ partial repayment (default). Suppose that the cost of default is given by a loss of output whose size depends on the realization of a random

[15] Mankiw (2000) or Matsen, Sveen and Torvik (2005) analyze fiscal policy in economies with "savers" and "spenders."

[16] The assumption that the sets of taxpayers and investors do not "overlap" simplifies the analysis and does not matter for the main results.

variable $\tilde{L}$ that is known before the default decision is made. The cumulative distribution function of $\tilde{L}$ is a function of the state variables in period $t = 2$, $\tilde{F}(\tilde{L}; y_2, b, b^e)$. In addition to the level of debt, its composition as well as the level of output may matter for this distribution and thus for the risk of default. This implies that changes in b and/or b^e have two conceptually distinct effects on default risk and debt prices. First, the usual dilution effect: For *given* values of the other state variables, an increase in b raises the probability of default. And second, there is a novel "credibility" effect: If a change in the quantity of debt or its composition alters the *distribution* function $\tilde{F}$, then it also alters the probability of default for a given amount of maturing debt. For example, if an increase in b^e, holding b constant shifts probability mass from low to high realizations of $\tilde{L}$ then it also increases the likelihood of repayment.

We employ a simple specification of $\tilde{F}$ such that an increase in b^e shifts the distribution function $\tilde{F}$ "to the right" and also assume that the other state variables do not matter for the shape of $\tilde{F}$. Formally, we let $\tilde{L} \equiv L + \phi(b^e), \phi' > 0$ and assume that L is distributed according to a standard cdf F which is *not* a function of the state variables. The cost of default thus contains two components: A random one; and a deterministic one that increases with the sovereign's exposure to official credit. Since the default cost is a fixed cost with respect to the repayment rate, default will be either zero or full (r_2 is either unity or zero) in equilibrium and symmetric across debt tranches, even in the absence of a formal pari passu provision. This implies that when private and official credit co-exist, private loans are placed under the superior protective power of the official creditors. These features are not critical for the general distinction between dilution and credibility effects contained in the paper, but they simplify the analysis. In appendix A, we present the more general case. There, we also show that the distinction between dilution and credibility effects is independent of whether default is associated with pure social losses—as we assume here—or with transfers to lenders.

The objective function of the government in period $t = 1$ takes the form

$$G_1(b, b^e) = u(y_1 + qb) + \delta \mathbb{E}_1 G_2(b, b^e) \tag{1}$$

where

$$G_2(b, b^e) = \max_{r_2} u(y_2 - \mathbf{1}_{[r_2=1]} b - \mathbf{1}_{[r_2<1]}(L + \phi(b^e)) \tag{2}$$

and $\mathbf{1}_{[x]}$ is the indicator function that takes the value of one when choice x has been made and zero otherwise.

Since that private creditors are competitive and risk neutral they price their loans at $q = \beta \mathbb{E}_1 r_2$.

4 Equilibrium

4.1 Choice of Repayment Rate

We characterize equilibrium by backward induction starting with the choice of the repayment rate, r_2. Due to the specification of the default costs, the marginal cost of lowering

r_2 is zero when $r_2 < 1$, so the optimal repayment rate equals either zero or unity. In particular,

$$r_2 = \begin{cases} 1 & \text{if } L \geq b - \phi(b^e) \\ 0 & \text{if } L < b - \phi(b^e) \end{cases} . \tag{3}$$

Condition (3) states that the government chooses to default when the resulting income losses, $L + \phi(b^e)$, are smaller than the amount of debt due. The equilibrium price of funds is then

$$q = \beta \mathbb{E}_1 r_2 = \beta(1 - F(b - \phi(b^e))) \tag{4}$$

where F is the cdf of L. The price q is decreasing in the quantity of debt issued, b.

4.2 Choice of Debt: Private Lenders

Issuing an extra unit of debt to private lenders has two effects on the funds obtained. First, it raises funds from this marginal unit of debt in proportion to its price, that is, one unit of b raises q units of funds in the present. And second, by lowering the price of debt (equation (4)), it reduces the amount of funds raised from inframarginal units of debt. This latter effect is the direct consequence of the government's lack of commitment and reflects the endogeneity of subsequent repayment decisions. More formally, the total effect is given by the slope of the "debt-Laffer" curve as

$$\frac{\partial (qb)}{\partial b} = q + \frac{\partial q}{\partial b} b = \beta[(1 - F) - fb] \tag{5}$$

where $f = F'$ is the pdf of L.

Funding is maximized at the top of the debt-Laffer curve, that is at the point where $\frac{\partial qb}{\partial b} = 0$. A perfectly myopic government ($\delta = 0$) would opt for the level of debt that corresponds to the maximum of the debt-Laffer curve. A non-myopic government ($\delta > 0$), in contrast, selects a lower level of debt because it cares also about future consumption.

Let λ and μ denote the multipliers associated with the short-sale constraints $b^e \geq 0$ and $b \geq b^e$, respectively. The effect of a marginal increase in debt issued to private lenders on the government's objective is given by

$$\frac{\partial G_1}{\partial b} = u'(c_1) \frac{\partial (qb)}{\partial b} + \delta \frac{\partial \mathbb{E}_1 G_2}{\partial b} + \mu$$

which can be expressed as[17]

$$(1 - F)(\beta u'(c_1) - \delta \mathbb{E}_1 u'(y_2 - b)) - u'(c_1) b \beta f + \mu. \tag{6}$$

[17] We use the fact that

$$\begin{aligned} & \frac{\partial \mathbb{E}_1 G_2}{\partial b} = \\ & \quad \frac{\partial}{\partial b} \int_0^{b - \phi(b^e)} \mathbb{E}_1[u(y_2 - L - \phi(b^e)) | L] dF(L) + \frac{\partial}{\partial b} \int_{b - \phi(b^e)}^{\infty} \mathbb{E}_1[u(y_2 - b)] dF(L) \\ & = \quad \mathbb{E}_1[u(y_2 - b)] f - \mathbb{E}_1[u(y_2 - b)] f - (1 - F) \mathbb{E}_1[u'(y_2 - b)]. \end{aligned}$$

The first part of this marginal effect represents the consumption smoothing benefit from the marginal unit of debt. It differs from the corresponding expression in the case without default risk because the price of debt equals $\beta(1-F)$ rather than β and because debt repayment occurs with probability $1-F$ rather than always.[18] The second part of the marginal effect arises because the repayment probability depends on the quantity issued: Each extra unit of debt lowers the price of all inframarginal units or, equivalently, raises the interest rate on them. This increase in the interest rate—which would be absent in a model with commitment—makes first period consumption more expensive. As a consequence, the equilibrium amount of debt issued (conditional on b^e) tends to be smaller than that under commitment. The final part of the marginal effect, the multiplier μ, is strictly positive if the short-sale constraint $b \geq b^e$ is binding, and equals zero otherwise.

It may seem surprising that the negative welfare effect associated with the reduction of funds raised from inframarginal units of debt (the second part discussed above) is not balanced by a positive welfare effect from the reduced repayment probability of these inframarginal units in the future. In fact, this effect is present. However, it does not appear in (6) because it is equal in absolute value to another welfare effect of opposite sign, reflecting the increased risk of future social losses in the wake of default.[19] It is these social losses that are at the source of the reduced incentive (relative to the commitment case) for the government to issue debt. Niepelt (2011) contains a detailed discussion in the context of a model with multiple maturities.

4.3 Choice of Debt: Official Lenders

Issuing debt to official lenders while holding total debt constant (that is, substituting official for private debt) raises the output losses of the borrowing country in case of default, thus reducing default risk. The resulting increase in the price of debt q has a positive effect on the amount of funds procured. Formally,

$$\frac{\partial(qb)}{\partial b^e} = b\frac{\partial q}{\partial b^e} = b\beta f \phi'$$

where $\phi' = d\phi/db^e$. The effect of substituting official for private funds on the government's objective is given by

$$\frac{\partial G_1}{\partial b^e} = u'(c_1)\frac{\partial(qb)}{\partial b^e} + \delta\frac{\partial \mathbb{E}_1 G_2}{\partial b^e} + \lambda - \mu$$

where the multipliers reflect the two short-sale constraints. This can be re-written as[20]

$$\phi'\left(u'(c_1)\beta f b - \delta E_1 \int_0^{b-\phi(b^e)} u'(y_2 - L - \phi(b^e))dF(L)\right) + \lambda - \mu. \tag{7}$$

[18]With risk free debt, the marginal effect would reduce to $\beta u'(c_1) - \delta\mathbb{E}_1[u'(y_2 - b)]$.

[19]Higher debt issuance increases subsequent default risk and thus, the risk of future output losses when default occurs. The corresponding first-order welfare effects that operate through the continuation value are zero. This is a consequence of an envelope condition—the successor government is indifferent at the margin between bearing the costs of debt repayment or suffering the income losses from default (see footnote 17).

[20]Note that $\partial\mathbb{E}_1 G_2/\partial b^e = -\phi'\mathbb{E}_1\left[\int_0^{b-\phi(be^e)} u'(y_2 - L - \phi(b^e))dF(L)\right]$ (see footnote 17).

The first part of this marginal effect reflects the benefit from higher credibility (more severe sanctions). A larger share of official debt generates stronger repayment incentives and hence lowers the default risk. This raises q and the amount of funds that can be obtained in the present (for given b), allowing the country to consume more in that period. The second part of the marginal effect reflects the cost of reduced flexibility. A larger share of official debt translates into additional income losses if default occurs. The income losses are of greater consequence when marginal utility in the default states is high. Conditional on the distribution F, the cost of reduced flexibility therefore is a more important concern when income y_2 is negatively correlated with L, that is, if default states are states with low consumption.

4.4 Discussion

The marginal conditions derived above make clear that the resulting optimal choice of the quantity and composition of sovereign debt as well as the default decision depend in general on factors such as the intensity of the borrowing needs, as manifested in the ratio β/δ, the steepness of the output profile, the distribution function of output losses, F, preferences and the enforcement technology, ϕ. The properties of the equilibrium can be characterized analytically only under a limited set of specifications, for instance, under linear utility, linear default costs ϕ, and uniform F. By its very nature, however, the linear-uniform specification produces corner solutions. Consequently, while it is adequate for certain purposes (for instance, for capturing the essence of the interaction between debt overhang and the default/debt composition decisions, see the next section), it is not useful for studying the possible co-existence of the two types of debt instruments in a sovereign's portfolio. As this represents an important empirical fact, we explore the properties of the equilibrium in the model under the assumption of a concave (logarithmic) utility function. In order to be able to see clearly the main forces at work we also use a discrete pdf for L and a linear enforcement function, ϕ. Naturally, the key qualitative properties of the equilibrium are not affected by this choice.[21]

In particular, suppose that L takes the value of zero with probability $1-\pi$ and the value of $\bar{L}$ with probability π. Let also $\phi(b^e) = \phi b^e, \phi > 0$. Note that in the case of only private credit, the maximum loan that can be extended at a non-zero price is $\bar{b} = \bar{L}$ and its price $q = \beta\pi$. For official credit only, the maximum loan is $\bar{b}^e = \frac{\bar{L}}{1-\phi}$ and its price is $q = \beta\pi$. When both private and official funds co-exist, then the debt portfolio must satisfy the repayment condition (3), $b \leq \bar{L} + \phi b^e$, both tranches are priced at $q = \beta\pi$, and the total quantity of debt lies between $\bar{b}$ and $\bar{b}^e$. That is, in all cases, the country defaults if the realization of L is low and pays back when it is high.

The Langrangean associated with the government's optimization problem then is

$$\mathcal{L} = \ln(y_1 + \beta\pi b) + \delta\pi \ln(y_2 - b) + \delta(1-\pi)\ln(y_2 - \phi b^e) + \nu(\bar{L} + \phi b^e - b)$$

(subject to the short-sales constraints) where ν denotes the multiplier associated with the repayment constraint. In order to guarantee that the commitment problem is operative,

[21] See appendix B for the continuous pdf case.

we assume that the optimal amount of (state contingent) debt under commitment, $\tilde{b} = \frac{\beta y_2 - \delta y_1}{\beta(1+\delta\pi)}$ exceeds $\bar{b}$. If this is the case, $\nu > 0$ and the optimal values for b, b^e solve the equations

$$\frac{\beta\pi}{y_1 + \beta\pi b} - \frac{\delta\pi}{y_2 - b} - \frac{\delta(1-\pi)}{y_2 + \bar{L} - b} = 0 \tag{8}$$

$$b^e - \frac{b - \bar{L}}{\phi} = 0. \tag{9}$$

An interior solution with strictly positive b and b^e exists if the left-hand-side of equation (8) is less than zero at $b = \bar{L}/(1-\phi)$ and greater than zero at $b = \bar{L}$. For a numerical example, use $\beta = 0.9, \delta = 0.5, \pi = 0.6, y_1 = 1, y_2 = 1.5, \bar{L} = 0.4, \phi = 0.3$. This produces $b = 0.47$ and $b^e = 0.23$. Noting that $\bar{b} = 0.4$ and $\bar{b}^e = 0.57$, this example reveals the key determinants of the debt portfolio issued by a sovereign, and in particular the trade off between obtaining more funds now at the expense of decreasing future flexibility. In our example, the sovereign turns to official creditors in order to improve his repayment credibility and thus alleviate his borrowing constraint. But because official loans are costlier in the case of default, the sovereign shows restraint when tapping official funds. The trade off between alleviating the thirst for current funds and sacrificing future flexibility (i.e. constraining the default option) is resolved at a level of total debt (0.47) that exceeds the level that would have maximized future flexibility ($b \leq 0.40, b^e = 0$) but falls short of the one that maximizes current funds (0.57).

How does this trade off vary with the parameters of the model? Let us consider how the share of official funds in total funds as well as the total amount of debt varies with the intensity of the borrowing needs (the value of δ) of the sovereign. Using equation (8) it can be shown that as δ decreases, the optimal quantity of debt, b, increases; and using equation (9), that the share of official loans to total loans ($s = b^e/b$) is decreasing in δ also. The former property is standard. The latter derives from the former: A higher relative valuation of present consumption (a lower δ) induces the sovereign to sacrifice more of future consumption in order to get more consumption now. Official credit accomplishes this. This is a general property of the model and obtains whether the price of debt is a continuous (as it would be with a continuous pdf) or discontinuous (as it is in this example) function of debt. We discuss this property in greater detail in appendix B. Sovereigns may have to shift their portfolio in favor of official loans when they seek a large amount of funds (when δ is low) in order to "flatten" the debt-Laffer curve and lower borrowing costs. This is costly because it requires accepting a reduction in the option value of future default (due to the higher default costs associated with official debt) but a lower δ means a smaller concern for such costs. This key implication of the model, namely, that sovereigns will tend to favor official over private credit when their borrowing ability is impaired by low debt prices on private credit markets (as it would be in the case of a debt crisis) accords well with extant empirical observations.

Similar comparative static exercises can be conducted with regard to the enforcement power of the official creditors (the value of ϕ), the expected rate of growth (y_2/y_1) and the probability of high realizations of sanction costs (π). Consider the sign of $ds/d\phi$. Equation (8) implies that total debt is independent of ϕ. Using this fact in equation (9) gives that

$ds/d\phi < 0$. That is, the higher ϕ the less b^e is needed in order to support a given total level of debt and thus the lower its share. Similarly, the effect of higher expected growth on s is positive. This is due to the fact that higher future relative to current output increases desired borrowing and this higher borrowing can only materialize by going to the official creditors. Finally, a lower π makes official debt less desirable because it makes it more likely that a sovereign will default and thus suffer the higher costs associated with official funds. That is, $ds/d\pi > 0$.

5 Debt Overhang

We now explore the implications of debt overhang. Let us assume that the economy starts in period $t = 1$ with inherited quantities of privately held sovereign debt b_{01} and b_{02} that are due in periods $t = 1$ and $t = 2$ respectively. Besides choosing a portfolio of debt instruments in period $t = 1$, there is an additional decision, namely, whether to honor outstanding debt or not. We assume that default on b_{01} is accompanied by default on b_{02}. Such "acceleration" represents standard practice in actual defaults involving multiperiod debt. As we demonstrate below, acceleration has interesting consequences for the interaction of default and debt composition decisions.

We will assume that default on privately held debt does *not* lead to exclusion from credit markets and only carries the cost discussed above. The default decision in period $t = 1$ requires the comparison of expected utility streams rather than simply the comparison of the current default cost to the amount of debt due in that period. In general, there is a critical value $\hat{L}_1$, such that the sovereign will choose to default on both b_{01} and b_{02} in period $t = 1$ whenever $L < \hat{L}_1$; and to honor debt repayment otherwise.

In order to best illuminate the role of debt overhang for the interaction of default and debt composition decisions we use a specification of the model that favors corner solutions in the optimal choice of the debt instrument. This specification produces switches from one source of funding to another and brings out the interaction between the two decisions most clearly. We assume linear utility with $u'(c) = 1$, linear sanctions with $\phi(b^e) = \phi b^e, 0 \leq \phi < 1$; and a uniform cdf for L so that the probability of default in the second period, F, is given by $F = f \cdot (b - \phi b^e)$ if there has been default in the first period (that also wiped out b_{02}) and $F = f \cdot (b + b_{02} - \phi b^e)$ otherwise, where $f = F'$.

In period $t = 1$, the government's objective function is

$$G_1(b, b^e, r_1) = y_1 - b_{01}\mathbf{1}_{[r_1=1]} - \mathbf{1}_{[r_1<1]}L_1 + \beta(1 - f \cdot (b + \mathbf{1}_{[r_1=1]}b_{02} - \phi b^e))b + \delta\mathbb{E}_1 G_2(b, b^e)$$
$$G_2(b, b^e) = \int_0^{b+\mathbf{1}_{[r_1=1]}b_{02}-\phi b^e} -(L + \phi b^e)f dL - (1 - f \cdot (b + \mathbf{1}_{[r_1=1]}b_{02} - \phi b^e))(b + \mathbf{1}_{[r_1=1]}b_{02}). \quad (10)$$

The optimal choice of b and b^e is determined by

$$\frac{\partial G_1}{\partial b} = (1 - F)(\beta - \delta) - \beta f b, \quad (11)$$
$$\frac{\partial G_1}{\partial b^e} = \phi(\beta f b - \delta F). \quad (12)$$

Holding b^e constant, G_1 is concave in b. The determinant of the Hessian is negative, so the Hessian is indefinite. This implies that any interior critical point of (10) constitutes a saddle point and consequently the equilibrium is in a corner. We consider the two corner equilibria—one with private debt and the other with official debt—in turn. The third equilibrium with zero debt is of no interest and is ruled out by assuming that δ/β is sufficiently small.

If all new sovereign debt is exclusively funded from private sources then the equilibrium level of debt is given by

$$b^{\mathrm{PR}} = \frac{1}{f}\frac{\beta-\delta}{2\beta-\delta}(1 - f b_{02}\mathbf{1}_{[r_1=1]}).$$

Less debt—by a factor of $(1-fb_{02}\mathbf{1}_{[r_1=1]})$—is issued relative to the case without long-term debt overhang. This is due to the fact that outstanding long-term debt already places the country higher up on the debt-Laffer curve, making default more likely, and thus new debt issuance more costly and less attractive.

When all new debt is financed by official sources then the equilibrium debt level is given by

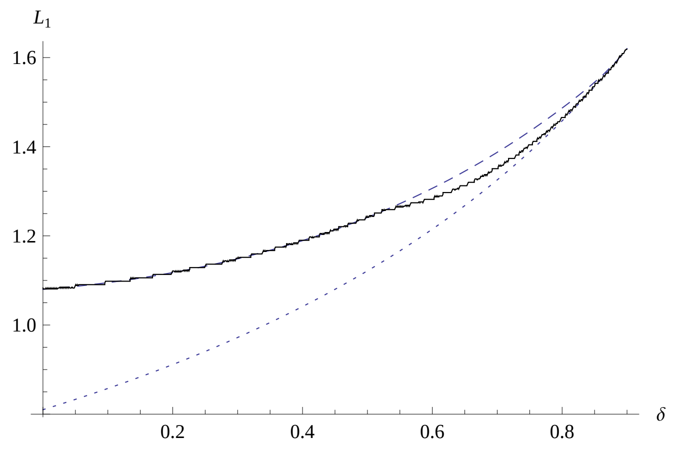
$$b^{\mathrm{OF}} = \frac{1}{f}\frac{\beta-\delta}{2\beta-\delta(1-\phi)}\frac{1}{1-\phi}(1 - f b_{02}\mathbf{1}_{[r_1=1]}) - \frac{1}{f}\frac{\delta\phi f b_{02}\mathbf{1}_{[r_1=1]}}{2\beta-\delta(1-\phi)}\frac{1}{1-\phi}.$$

As in the case with private financing only, outstanding long-term liabilities reduce the incentive to issue new debt because they place the borrowing country higher up on the debt-Laffer curve. This effect is reflected in the wedge $(1 - fb_{02}\mathbf{1}_{[r_1=1]})$. But long-term debt overhang makes official financing particularly unattractive. This is because debt overhang already makes default more likely, so any debt instrument that carries high costs of default becomes less attractive. Consequently, optimal new debt issuance is even lower, a fact captured by the right-most term in the expression for b^{OF}.

Let us now turn to the default decision in the first period and its interaction with the debt composition decision. It is instructive to start with a situation where these two are independent of each other because there is only one source of funds available to the sovereign. Figure 1 illustrates the default decision in this case. Ignore for the time being the solid line. The default threshold line $\hat{L}_1^{\mathrm{PR}}$ applies to the case when only private funds are available; and the threshold $\hat{L}_1^{\mathrm{OF}}$ applies when only official funds are available. Default occurs for realizations of default costs in period $t = 1$ below the relevant loci. For $b_{02} = 0$, the default thresholds are independent of δ and the two loci coincide and are flat at the level b_{01}. For $b_{02} > 0$, as in the example illustrated in the figure (where $b_{01} = 0, b_{02} = 2$), the loci have a non-zero slope because default reduces b_{02} to zero, and the effect of this change on the value of the government's program depends on δ.[22] More to the point, the figure shows that default in period $t = 1$ is always more likely when only official rather than private funds are available for refinancing (the default threshold locus in the latter case lies below that in the former case).

[22] A reduction of b_{02} affects the price of new debt, the equilibrium quantity of debt issued as well as the amount of long- and short-term debt to be serviced in the future. The price, the quantity, and the weight attached to the future all depend on δ.

Figure 1: $\hat{L}_1^{\mathrm{PR}}$ (dotted), $\hat{L}_1^{\mathrm{OF}}$ (dashed), $\hat{L}_1$ (solid) as functions of δ

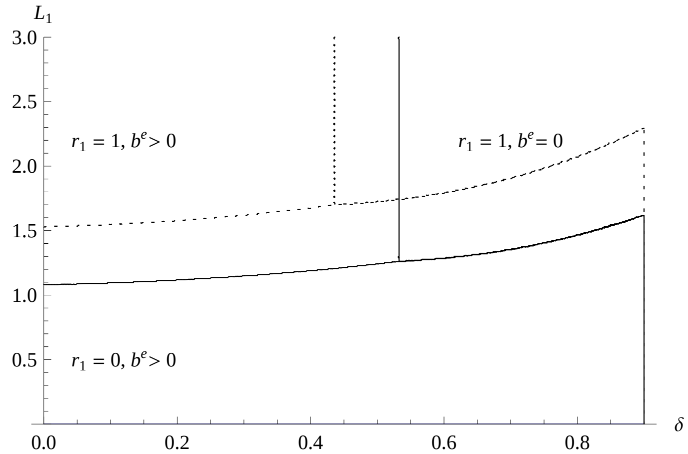


The solid line in figure 1 represents the equilibrium default threshold $\hat{L}_1$ when *both* sources of funds are available for refinancing, and the sovereign can select optimally which source to tap (recall that the equilibrium is always in a corner). It coincides with the default threshold $\hat{L}_1^{\mathrm{OF}}$ (the upper, dashed line), whenever the government *chooses* to borrow from official sources independently of the realization of L_1; and with the default threshold $\hat{L}_1^{\mathrm{PR}}$ (the lower, dotted line) whenever the government *chooses* to borrow from private sources independently of the realization of L_1. For instance, consider the situation with a low δ. In this case, the sovereign is hungry for funds in the present and cares little about maintaining flexibility in the future. Official funding can support a larger loan and represents the best choice in this case independent of the default decision in the current period (whether L_1 is high or low). When the solid line coincides with either of the two threshold loci, the financing and default decisions are independent of each other.; otherwise the two decisions are connected. In particular, when the solid line lies in between $\hat{L}_1^{\mathrm{OF}}$ and $\hat{L}_1^{\mathrm{PR}}$, the type of debt instrument available matters for the default decision. In that region, default in period $t = 1$ occurs more often than if only private debt were available but less often than if only official debt were available.

Figure 1 shows that the type of debt available impacts on default decisions. Figure 2 clarifies the interaction by depicting how the default decision correlates with the debt choice. Let us focus on the solid lines for the time being. This line has both horizontal and vertical segments. The horizontal segment corresponds to the solid line in figure 1 so it marks the default decision: For realizations of L_1 above it there is no default and for realizations of L_1 below it there is default. The vertical lines represent the threshold for the choice of type of debt. To the right, the government chooses private debt and to the left it opts for official. The main thing to notice here is that the two vertical segments do not lie on top of each other. That is, the debt choice depends on the default decision, with default in the first period favoring official and no default favoring private debt (the bottom vertical segment lies to the right of the top one). The distance between the two vertical segments gives the range of δ (borrowing needs) over which default makes a difference for the type of debt instrument selected. For instance, for $\delta = 0.65$, the sovereign selects official if he defaults in the current period but private if he does not default.

The dotted lines correspond to a higher level of long term debt overhang. Comparison of the dotted to the solid lines helps highlight the role of overhang for the default-debt choice nexus. The horizontal segment of the dotted lines lies above its solid lines counterpart, which simply signifies that the higher the amount of debt overhang the more likely a default. The more interesting part, though, concerns the distance between the two vertical segments. As debt overhang increases, so does this distance. But this distance represents the range of loans over which the default and debt choice decisions interact. The more to the left a vertical upper segment lies (the larger the debt overhang) the more likely that a sovereign will opt for private debt if he does not default, even when borrowing needs are large (δ is low).

Figure 2: Default and official lending regions for $b_{02} = 2$ (solid) and $b_{02} = 3$ (dashed)



In sum, figure 2 illustrates how default and refinancing decisions interact. In our view, it provides a compact characterization of the diverse experiences during the recent Eurozone sovereign debt crisis. The model predicts the following: Countries that choose to default favor refinancing from official sources. The experience of Greece supports this prediction. But if a country chooses not to default then its choice between private and official credit will be affected by its outstanding quantity of long term debt overhang. A high quantity works in favor of private refinancing while a low quantity in favor of official refinancing. The experiences of Ireland, Italy and Portugal seem consistent with this prediction.[23]

[23] At the time when official funds were provided, the debt levels of Ireland and Portugal (which received refinancing from official sources) were lower than the level of Italy which did not receive such refinancing. The case of Spain is somewhat more complicated. Spain had an even lower debt. While it did not actually receive official funds, the Eurogroup committed to provide up to 100 billion Euro. IMF (2012, p. 17) contains deficit and debt data.

6 Concluding Remarks

During the recent debt crisis in the Eurozone, sovereign default and debt composition decisions have varied significantly across the distressed countries. While Greece defaulted and sought refinancing from official sources, Ireland, Portugal and Spain did not default but still drew official funds (or accepted a Eurogroup commitment for conditional support), and Italy neither defaulted nor relied on official loans. Moreover, the rates charged on both official and private funds were low relative to what they had been before the entry of official creditors (with a brief exception for Greece).

Our model helps shed light on these patterns by extending the standard sovereign debt framework to include multiple creditors with differential enforcement powers. In the presence also of long term, private debt overhang and under explicit—or, implicit through the structure of sanctions—pari passu the model produces default and debt composition decisions as well as debt prices that are in line with the empirical evidence. It generates default accompanied by inexpensive official lending; substitution of official for private debt in times of debt distress, but without necessarily a complete crowding out and with a tendency towards larger debt quantities when funds are drawn from official sources; and reluctance to draw official funds at favorable rates by troubled but non-defaulting countries with large long-term debt obligations. And so on. The model also predicts that official would-be creditors may encourage a sovereign to default on outstanding private debt before they provide funds.

The model has two general implications regarding a country's debt portfolio choice. First, the larger a country's financing needs, the more likely the use of official credit. And second, the higher the value of the default option (say, because of expected poor economic outlook) the less likely that the sovereign will seek official funds. The role played by the sovereign's level of credibility for this choice is more nuanced. While a high level of credibility favors private, low credibility may favor either private or official funds depending on the country's borrowing needs. This is due to the fact that there is a trade off between the present benefits of official funds (more credit) and its future costs (more severe sanctions). This trade off varies with the country's borrowing needs.

An important advantage of our model is its simplicity. In particular, it generates rich cross-country variation of government choices as well as within country correlations without any need for political economy considerations.[24] An additional advantage is its generality: It can be easily applied to the study of other credit relationships that do not involve sovereign debt as long as the relationship contains classes of creditors that differ in terms of the punishment they can inflict on delinquent debtors.

Two extensions of the analysis seem worth pursuing. First, to examine the properties of the model (for instance, the optimal ratio of private to total debt) in the absence of pari passu. And second, to address normative questions, for example, how the pari passu provision affects the welfare of the borrower.

[24] Ardagna and Caselli (2012) offer an informal, political economy perspective on the Greek default.

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A Dilution Versus Credibility

For a general treatment of dilution and credibility effects, consider an economy with a set $\mathcal{I} = \{1, \ldots, I\}$ of international investors (creditors) who are competitive and require an expected gross return β^{-1}. The state at the beginning of period t includes (x_t, b_t) as well as time if the horizon is finite. Vector $b_t = \{b_{1t}, b_{2t}, ..b_{It}\}$ represents the quantities of zero coupon bonds held by the I creditors. Vector x_t includes the exogenous income, y_t, as well as the realization of the scalar random variable, L_t, that determines the consequences of a default. Income y_t follows an exogenous law of motion and L_t is distributed according to the cdf $F(\cdot; y_t, b_t,)$ which may depend on the other states, in particular on b_t *and* its composition. This could be due to the fact that a default on specific creditor groups is more or less costly for the domestic economy (or, interpreted more generally, the ruling government). Or it may reflect the fact that specific creditor groups induce the government to implement policies that increase the stock of collateral or the bargaining power of creditors in a renegotiation.[25]

The sovereign chooses creditor specific repayment rates and fresh debt issuance, $(r_t, b_{t+1}) \in \mathcal{R} \times \mathcal{B}$ after observing the state where $\mathcal{R} = [0, 1]^I$ and $\mathcal{B} \subseteq \mathbb{R}^I_+$. Scalar $g(x_t, b_t, r_t) \geq 0$ with $g(x_t, b_t, 1) = 0$ represents the adverse consequences of default for the domestic economy which take the form of temporary income losses.[26] The default costs $g(x_t, b_t, r_t)$ may amount to social losses (as is typically assumed in the sovereign debt literature) or to transfers. Social losses are present if the default costs reduce the consumption possibilities in the domestic economy without corresponding gain for creditors. If the costs are associated with gains for creditors, for example because default triggers a renegotiation

[25] Structural reforms under an IMF program may be interpreted in this light.

[26] The assumption of temporary income losses in the wake of a default is consistent with empirical evidence (see for example **?**). The setup can be extended to capture other negative consequences of default. For example, default induced exclusion from financial markets (potentially sustaining trigger strategies) can be modeled by augmenting x_t with a state variable that summarizes the history of default choices and letting the choice set for debt issuance be a function of the state and the repayment rate, $\mathcal{B}(x_t, r_t)$.

that eventually gives rise to a compensation payment or because default implies a loss of collateral, then the costs have a transfer component. Let $\rho_t^i(x_t, b_t, r_t)b_t^i$ denote the compensation payment or collateral receipt by creditor i after a default. The polar case with complete social losses corresponds to vector $\rho_t(x_t, b_t, r_t) = 0$ for all (x_t, b_t, r_t); the opposite polar case with no social losses corresponds to $g(x_t, b_t, r_t) = \rho_t(x_t, b_t, r_t) \cdot b_t$ for all (x_t, b_t, r_t). In equilibrium, creditors earn the required rate of return such that

$$q_t(x_t, b_t, r_t, b_{t+1}) = \beta \mathbb{E}[r_{t+1}(x_{t+1}, b_{t+1}) + \rho_{t+1}(x_{t+1}, b_{t+1}, r_{t+1}(x_{t+1}, b_{t+1}))|\iota_t] \tag{13}$$

for all (x_t, b_t) where ι_t denotes the information set at the end of period t, including (x_t, b_t, r_t, b_{t+1}). Since the sovereign chooses b_{t+1} independently of r_t (and investors are forward looking), $r_t(\cdot)$ satisfies

$$r_t(x_t, b_t) = \arg\max_{r_t \in \mathcal{R}} -g(x_t, b_t, r_t) - b_t \cdot r_t \tag{14}$$

for all (x_t, b_t).

To simplify the notation we assume that the income losses in the wake of a default, $g(x_t, b_t, r_t)$, have a fixed cost character; that is, reducing the repayment rate on a debt tranche from a starting value strictly smaller than unity has no effect on $g(x_t, b_t, r_t)$ if the repayment rates on all other debt tranches are held constant. In the case with complete social losses, $\rho_t(x_t, b_t, r_t) = 0$, this implies that the equilibrium repayment rates either equal zero or unity.

A.1 Dilution Versus Credibility Effects

A change in the quantity of debt or its composition has two types of effects on default choices (and thus, debt prices) and the government's value. Both types only arise when the government lacks commitment. First, "dilution effects." They arise because changes in the debt stock or composition alter the ex-post optimal default or rollover decisions of subsequent governments. Dilution effects are discussed at length in the sovereign and corporate debt literature. Second, "credibility (enhancing/reducing) effects." They arise if the distribution function of the random variable L_t varies with the state, in particular with b_t.

Formally, let $\hat{L}^i(y_t, b_t)$ denote the threshold value of the random variable L_t at or above which the repayment rate on tranche b_t^i equals unity (we assume that this threshold value is unique, for example because $g(x_t, b_t, r_t)$ is the sum of tranche specific default costs). The government then repays tranche b_t^i in full with probability $F(\hat{L}^i(y_t, b_t); y_t, b_t)$, and it defaults with the complementary probability. A change of b_t, from $b_t = \varphi$ to $b_t = \psi$ say, triggers dilution effects by altering the default threshold value from $\hat{L}^i(y_t, \varphi)$ to $\hat{L}^i(y_t, \psi)$ and the compensation payment/collateral receipt in default states from $\rho_t^i(x_t, \varphi, r_t(x_t, \varphi))$ to $\rho_t^i(x_t, \psi, r_t(x_t, \psi))$; and it triggers credibility effects by altering the probability distribution of L_t from $F(\cdot; y_t, \varphi)$ to $F(\cdot; y_t, \psi)$.

A.2 Dilution and Credibility Effects on Debt Returns

Assume that $F(\cdot; y_t, b_t)$ is differentiable with respect to L_t, with pdf $f(\cdot; y_t, b_t)$; and that this density function (as well as the other equilibrium objects) is differentiable with respect to b_t. The conditionally expected return on debt tranche b_t^i then equals

$$\int_0^{\hat{L}^i(y_t,b_t)} \rho_t^i(x_t, b_t, r_t(x_t, b_t))\mathrm{d}F(L_t; y_t, b_t) + \int_{\hat{L}^i(y_t,b_t)}^{\infty} 1\mathrm{d}F(L_t; y_t, b_t).$$

The term on the left-hand side represents the probability of default times the conditional expectation of the "settlement" in this case; the term on the right-hand side represents the probability of full repayment. The effect of a marginal change of debt structure on the expected return can be expressed as

$$\begin{aligned}
&\int_0^{\hat{L}^i(y_t,b_t)} \frac{\mathrm{d}\rho_t^i(x_t, b_t, r_t(x_t, b_t))}{\mathrm{d}b_t^j}\mathrm{d}F(L_t; y_t, b_t) \\
&\quad + \frac{\partial \hat{L}^i(y_t, b_t)}{\partial b_t^j} f(\hat{L}^i; y_t, b_t)(\rho_t^i(x_t, b_t, r_t(x_t, b_t))_{|\hat{L}^i} - 1) \\
&\quad + \int_0^{\hat{L}^i(y_t,b_t)} \left(\rho_t^i(x_t, b_t, r_t(x_t, b_t)) - 1\right) \frac{\partial f(L_t; y_t, b_t)}{\partial b_t^j}\mathrm{d}L_t.
\end{aligned} \tag{15}$$

The first two terms in (15) represent the conventional dilution effects and the third term the credibility effect due to the altered distribution function of L_t.

The first dilution effect, corresponding to the first term in (15), reflects modified debt returns in default states. It is only present if $\rho_t^i(x_t, b_t, r_t(x_t, b_t))$ varies with b_t and thus in particular, if $\rho_t^i(x_t, b_t, r_t(x_t, b_t)) > 0$.[27] The second type of dilution effect, corresponding to the second term in (15), arises due to reduced probability of repayment in full. When L_t equals $\hat{L}^i(y_t, b_t)$ a marginal increase of b_t^j implies a lower repayment rate r_t^i. When default triggers complete social losses the repayment rate drops to zero. Otherwise it falls by a smaller amount. Absent any social losses from default the second type of dilution effect is not present.

[27] Suppose that lenders are on an equal footing such that returns in default states are identical across debt tranches,

$$\rho_t^i(x_t, b_t, r_t(x_t, b_t)) = \rho_t^j(x_t, b_t, r_t(x_t, b_t)) = \frac{\theta_t(x_t)}{\sum_{k\in\mathcal{I}} b_t^k},\ i, j \in \mathcal{I},$$

for some function $\theta_t(\cdot)$. An increase in the quantity of one debt tranche then reduces the return on all tranches in all default states. Suppose instead that the first lender ($i = 1$) is senior, receiving a return $\rho_t^1(x_t, b_t, r_t(x_t, b_t)) = \min[\theta_t(x_t)/b_t^1, 1]$, while junior creditors receive

$$\rho_t^i(x_t, b_t, r_t(x_t, b_t)) = \rho_t^j(x_t, b_t, r_t(x_t, b_t)) = \frac{\max[\theta_t(x_t) - b_t^1, 0]}{\sum_{k\in\mathcal{I}\setminus 1} b_t^k},\ i, j \in \mathcal{I} \setminus 1,$$

in default states. A larger volume of senior debt now reduces the return on junior loans (not on all loans as in the case with equal footing) unless default on the latter can be ruled out; and a larger volume of debt also reduces the return on the tranche itself (as in the case with equal footing) in default states.

The third term in (15) represents the credibility effect. Suppose that a marginal increase of a debt tranche increases the probability of a specific realization of $L_t < \hat{L}^i(y_t, b_t)$ for which the sovereign does not repay b_t^i in full, and it decreases the probability of a realization with full repayment. Depending on the return in the default state, $\rho_t^i(x_t, b_t, r_t(x_t, b_t))$, this change of probability mass lowers the expected repayment rate on the debt tranche by more or less. The third term in (15) comprises the sum of all effects due to changes in probability mass.

With complete social losses, $\rho_t(x_t, b_t, r_t(x_t, b_t)) \equiv 0$, the credibility effect simplifies considerably under the assumption that a change in b_t "shifts" the density functions F and f in the sense that there exists a differentiable function $\alpha(\cdot)$ such that $F(L_t; y_t, b_t) \equiv F(L_t - \alpha(b_t); y_t, 0)$ for all $b_t \geq 0$. Exchanging the order of integration and differentiation we then have

$$\int_0^{\hat{L}^i(y_t,b_t)} (0-1)\frac{\partial f(L_t; y_t, b_t)}{\partial b_t^j} \mathrm{d}L_t = -\int_0^{\hat{L}^i(y_t,b_t)} \frac{\partial f(L_t - \alpha(b_t); y_t, 0)}{\partial b_t^j} \mathrm{d}L_t =$$

$$= -\frac{\partial F(\hat{L}^i(y_t, b_t) - \alpha(b_t); y_t, 0)}{\partial b_t^j}|_{\hat{L}^i} = \frac{\partial \alpha(b_t)}{\partial b_t^j} f(\hat{L}^i(y_t, b_t) - \alpha(b_t); y_t, 0). \tag{16}$$

Intuitively, a shift of the probability density function in combination with a fixed default threshold is equivalent to an unchanged probability density function in combination with a reduced default threshold. A parallel result holds if L_t has a discrete distribution. We use this result in the body of the paper.

A.3 Dilution and Credibility Effects on the Continuation Value

Consider the implications of debt structure b_t for expected utility in the repayment period. To simplify the notation, we assume that the cost function $g(x_t, b_t, r_t)$ is such that the default decision is perfectly correlated across debt tranches, with the default threshold denoted by $\hat{L}(y_t, b_t)$; we also disregard the utility flow from subsequent periods. Expected utility conditional on y_t then equals

$$\begin{aligned} & F(\hat{L}(y_t, b_t); y_t, b_t)\times \\ & \qquad \mathbb{E}[u(y_t - g(x_t, b_t, 0) + b_{t+1}(x_t, b_t) \cdot q_t(x_t, b_t, 0, b_{t+1}(x_t, b_t)))|\iota_{t-1}, y_t, L_t < \hat{L}(y_t, b_t)] \\ & + \quad (1 - F(\hat{L}(y_t, b_t); y_t, b_t))u(y_t - b_t \cdot 1 + b_{t+1}(x_t, b_t) \cdot q_t(x_t, b_t, 1, b_{t+1}(x_t, b_t))). \end{aligned}$$

Dropping some arguments of the utility function as well as conditioning variables for legibility, the dilution effect due to a change of debt structure from $b_t = \varphi$ to $b_t = \psi$ equals

$$\begin{aligned} & F(\hat{L}(y_t, \psi); y_t, \varphi)\mathbb{E}_\varphi[u(-g(x_t, \psi, 0))|L_t < \hat{L}(y_t, \psi)] + (1 - F(\hat{L}(y_t, \psi); y_t, \varphi))u(-\psi \cdot 1) \\ & -F(\hat{L}(y_t, \varphi); y_t, \varphi)\mathbb{E}_\varphi[u(-g(x_t, \varphi, 0))|L_t < \hat{L}(y_t, \varphi)] + (1 - F(\hat{L}(y_t, \varphi); y_t, \varphi))u(-\varphi \cdot 1) \end{aligned}$$

and the credibility effect is given by

$$\begin{aligned} & F(\hat{L}(y_t, \psi); y_t, \psi)\mathbb{E}_\psi[u(-g(x_t, \psi, 0))|L_t < \hat{L}(y_t, \psi)] + (1 - F(\hat{L}(y_t, \psi); y_t, \psi))u(-\psi \cdot 1) \\ & -F(\hat{L}(y_t, \psi); y_t, \varphi)\mathbb{E}_\varphi[u(-g(x_t, \psi, 0))|L_t < \hat{L}(y_t, \psi)] + (1 - F(\hat{L}(y_t, \psi); y_t, \varphi))u(-\psi \cdot 1). \end{aligned}$$

The dilution effect can again be decomposed into two components, one related to the change of default costs, $g(x_t, \psi, 0)$ versus $g(x_t, \varphi, 0)$, and the other related to the change of default threshold, $\hat{L}(y_t, \psi)$ versus $\hat{L}(y_t, \varphi)$. The credibility effect operates through a reallocation of probability mass between default and no-default states. It is absent if the probability measures before and after the change of debt structure coincide, $F(\cdot; y_t, \psi) = F(\cdot; y_t, \varphi)$.

Suppose that default costs constitute complete social losses and consider a change of debt structure from $b_t = \varphi$ to $b_t = \psi$ that reallocates probability mass among realizations of L_t that all fall short of the default threshold (before and after the change), $\hat{L}(y_t, b_t)$. Such a change does not affect the expected return on debt and thus, debt prices or domestic consumption in the period when the debt is issued; but it does affect the expected utility of domestic consumption in the repayment period if $g(x_t, \psi, 0) \neq g(x_t, \varphi, 0)$.

B Optimal Debt Portfolio with a Continuous PDF

One expects that the main determinant of the probability of default, namely the shape of the cdf of L would play a critical role for the properties of equilibrium. In this section, we study this relationship for the case of a continuous cdf. We consider a quadratic distribution function because the shape of this distribution corresponds well to that of the discrete case considered in section!4.4, something that expedites and facilitates the interpretation of the results.

Figure 3 shows three parametric examples from this distribution, A, B, and C. Table 1 reports the corresponding optimal debt portfolio and equilibrium default risk for each of these examples, computed numerically by solving the marginal conditions with regard to b and b^e, equations (8)–(9). Except for the distribution function, the values of the parameters of the model are the same as those in section 4.4.[28]

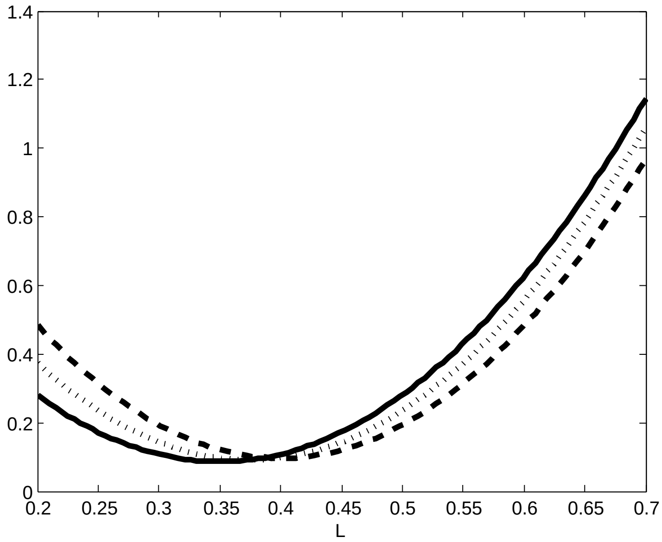
Table 1: Optimal debt portfolio: Effect of the distribution function

Case	A	B	C
b	0.583	0.577	0.546
b^e	0.421	0.236	0.034
$F(b - \phi b^e)$	0.184	0.228	0.277

The shifting of mass from high to low realizations of L (moving from case A to case C) increases the probability of default. This has two opposing effects on the desirability of official funds. On the one hand, official funds become more desirable because they help partly offset the increase in default risk and thus support more borrowing in the present. On the other hand, they become less desirable because they increase the cost suffered by the borrower in case of default, and the probability of default has increased due to the shift in mass. Which effect will prove stronger is in general ambiguous, depending on the curvature of the utility function, the ratios δ/β and y_2/y_1, the properties of the cdf

[28]That is: $\beta = 0.9, \delta = 0.5, y_1 = 1, y_2 = 1.5, \phi = 0.3$.

Figure 3: Quadratic pdf: Case A (solid), case B (dotted), case C (dashed)



and so on. For the family of distribution functions depicted in figure 3, the latter effect dominates, so the share of official funds in total funds decreases when moving from case A to case C. We saw a similar pattern in the analysis of debt overhang. There it was the debt overhang that made the credit relationship with official creditors perilous, here it is the exogenous change in the cdf of L.

Table 2 illustrates the role played by the size of borrowing needs (the value of δ) for the optimal composition of debt. For a given cdf of L, the sovereign chooses the private corner when borrowing needs are sufficiently small ($\delta = 0.55$). As they increase, the sovereign starts drawing funds also from official creditors ($\delta = 0.50$) and eventually switches completely to official funds when these needs become sufficiently high ($\delta = 0.4$).

Table 2: Optimal debt portfolio (case B): Effect of borrowing needs

δ	0.55	0.50	0.40
b	0.500	0.577	0.690
b^e	0.000	0.236	0.690