Understanding the relationship between VIX

and the S&P 500 Index volatility

Irena Vodenska[1] and William J. Chambers[1]

[1] Administrative Sciences Department, Metropolitan College, Boston University, Boston, MA

Abstract

We study the VIX Index, often referred to as "the investor's fear gauge," relative to the observed volatility of the S&P 500 Index to investigate the relationship between these two measures of financial markets variability and to understand the directionality of their influence on one another. Calculated as a weighted average of put and call options on the S&P 500 Index, the VIX is considered as a forecasting indicator of the S&P 500 Index's volatility over a one-month period. We examine the daily VIX and S&P 500 Index volatility data for the 20-year period between 1990 and 2009 and find that VIX lags the S&P 500 one-month volatility for the period that we study. Furthermore, we analyze the VIX Index and the S&P 500 volatility for different time periods, when the financial markets exhibit: (i) higher level of stability with volatility below two standard deviations from the mean and (ii) lower stability regimes, with volatilities above two standard deviations from the mean. We find that in general, the VIX overestimates the S&P 500 Index volatility during the stable financial market regimes, and underestimates the S&P 500 Index volatility throughout high volatility periods.

1. Introduction

Volatility of the S&P 500 Index (SPX) is of importance to many financial market participants. Volatility can have negative impact on existing portfolio position, may require implementation of costly hedging strategies, and could adversely affect overall investment returns. Traders and portfolio managers incorporate volatility expectations in their investment strategy selections and actively adjust their positions to better manage risk associated with volatility fluctuations. Financial market participants often use the Chicago Board Options Exchange (CBOE) Implied Volatility Index (VIX) to forecast S&P 500 Index's future volatility over a short period of time and may employ it as a hedge against existing positions.

Given the importance of volatility forecasting, studies have reported several views and arguments about using VIX as a benchmark for SPX future short-term volatility. Numerous articles have examined the forecasting power of VIX since the Index was introduced by CBOE in 1993 and revised in 2003. Taking VIX as the respective set of implied volatilities, Chernov (2001) concludes that the un-biasedness of VIX cannot be rejected over the sample period from 1986 to 2000 and therefore contains information of future volatility (Chernov, 2001). In a related context, in 2003, Poon and Granger conclude that the construction of VIX is a good tool for model-based forecasting (Poon & Granger, 2003). In contrast, the study of Becker et al. (2006) rejects the notion that VIX contains any information for SPX volatility forecasting based on data from 2 January 1990 to 17 October 2003. Yet they conclude that VIX may be viewed as important combination of forecasting measures in model-based forecasting. (Becker, Clements, & White, 2006). Nevertheless, after a more detailed study, specifically examining the forecast performance of VIX, Becker and Clemens (2007) conclude that VIX cannot simply be viewed as a combination of various measures in model based forecasting either (Becker & Clements, 2007). Based on arguments on the forecasting performance of VIX and the financial markets turmoil in 2008, Whaley (2008) argues that VIX is forward-looking measurement of SPX volatility, representing expected future market volatility over the next 30 calendar days (Whaley, 2008). However, no statistical analyses have been undertaken directly between VIX and SPX volatility in his research study. Hung et al. (2009) find that combining VIX into a GARCH-type model can enhance the one-step-ahead volatility forecasts while evaluating the forecasting with different types of loss functions (Hung, Ni, & Cheng, 2009).

In this study, we investigate the VIX index relation with the SPX volatility to explore the association between these two measures of financial market volatility and to understand the directional influence between them. We examine the daily VIX and SPX returns for a 20-year period between 1990 and 2009 using data from CBOE and Yahoo-Finance databases. We perform a set of regression analyses to find out whether VIX has predictive power in regards to the SPX volatility for different time frames.

We regress VIX versus SPX future 22-trading day ("22-day") volatility[1] to examine the forecasting strength of VIX to SPX future one-month volatility. Furthermore, we perform regression analysis between VIX and SPX future 22-day volatility including VIX past 22-day volatility and 1-month T-bill rate, respectively, as additional independent variables to see if there is any incremental explanatory information from the 1-month T-bill rate in addition to the VIX historical volatility. We also analyze different volatility time frames for SPX volatility such as 11-day and 33-day volatility windows against VIX and find that VIX shows superior performance in predicting shorter time-period volatilities. Then based on 22-day volatility, we shift the starting point for calculating the SPX future volatility into the past and into the future by 11, 22, 33 and 44 days, in order to show the best relationship between VIX and SPX 22-day future volatility. Superior correlation coefficients and coefficients of determination (R-squared) are found when we shift the starting point for calculating SPX future volatility one month backwards (testing for VIX against SPX past 22-day volatility). Surpassing the commonly accepted conviction that VIX is a predictor of the SPX future 22-day volatility, we find that SPX historical 22-day volatility is better related to the VIX index with correlation coefficient and R-squared over 80% for the time period that we study. In other words, the VIX index is more reflective of recent past SPX volatility than predictive of future SPX volatility.

In addition, we analyze the VIX and SPX future 22-day volatility distributions for the period between 1990 and 2009 and examine the volatility values for periods within two standard deviations as the "normal" stock market regime versus the volatilities for periods exceeding two standard deviations representing "high volatility" regime. In addition, we also fitted the distribution graphs with the Exponential, Gaussian, and Weibull distributions to understand the characteristics of the VIX and SPX future 22-day volatility distributions for the 20-year time period that we study.

We then reexamine the results by using the model built on the actual data from 1990 to 2009 for the VIX Index and the SPX future 22-day volatility to estimate VIX in 2010 and statistically compare the forecast based on the empirical data with the actual VIX Index observed in 2010. We could reject the two hypotheses that (1) the means and (2) the variances of the two time series, estimated and actual VIX, for the 2010 test period are the same.

The rest of this paper is organized as follows. In section 2 we give an overview of the VIX Index and the SPX Volatility, in sections 3 and 4 we explain the data analysis and the results of our study, in section 5 we examine the distributions for VIX and SPX volatilities, and in section 6 we discuss the results and offer our concluding remarks.

[1] 22-trading day volatility is in fact essentially the same as a one-month or 30-calendar-day volatility, ignoring the weekends and holidays.

2. Background on VIX and SPX

2.1. CBOE Implied Volatility Index (VIX)

VIX is an index that is generally used to forecast the expected volatility of the S&P 500 Index over the future 30-day period and is often referred to as "investor's fear gauge." The CBOE Implied Volatility Index (VIX) was introduced by Robert E. Whaley in 1993 and was initially designed to measure the market's expectation of 30-day volatility implied by the at-the-money S&P 100 Index (OEX) option prices. CBOE, together with Goldman Sachs, revised the VIX calculation in 2003 to base the Index on call and put options on the S&P 500 Index (SPX), the core index for U.S. equities.

VIX is calculated on a real-time basis throughout each trading day by averaging the weighted prices of SPX put and call options over a wide range of strike prices. The only significant difference from the previous VIX Index that was based on the S&P 100 market index is that the revised VIX measures the volatility and not the price. Volatility is a crucially important variable when pricing derivative securities, and a good forecast of the volatility of asset prices over the investment holding period is essential for assessing investment risk (Poon & Granger, 2003) .VIX is an indicator that reflects the price of portfolio insurance (Whaley, 2008) and determines how much people are willing to pay for a stock, which essentially is measuring the future state of the economy (Abbate, 2008).

According to the CBOE VIX white paper (2009), VIX is a volatility index comprised of options rather than stocks, with the price of each option reflecting the market's expectation of future volatility. The VIX calculation could be divided in 3 steps;

The first step is selecting options to be used in the VIX equation by calculating the current "forward index level (F)" based on the options strike price at which the difference between the average of the bid and ask spreads for the put and the call options is the smallest. The second step is including the selected options in the VIX calculation:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

where, "i" is the strike price.

K_i represents the "i^{th}" strike price from the forward index level (F), and "K_i" is strike price of i^{th} out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$. Q(K_i) is the midpoint of the bid-ask spread for each option with strike K_i. K_0 is the first strike below the forward index level (F). T is the time until expiration as a proportion of the number of minutes in a year. R is risk-free interest rate based on Treasury yields with maturity matching the nearest option expiration time. Third, by taking the square root of the 30-day weighted average variance (σ^2) and multiplying it by 100 the VIX Index is obtained.

2.2. S&P 500 Index (SPX)

The Standard & Poor's, a division of The McGraw-Hill Companies[2], first offered its primary daily stock market index in 1923. It was the S&P 90 Index, a value weighted index based on 90 stocks. On March 4, 1957, the S&P 90 index was transformed into the S&P500 Index (SPX) representing and tracking the performance of a wide spectrum of the US stock market. S&P 500 is widely regarded as the best single gauge of the U.S. equities market; this world-renowned index is followed daily, hourly, and minutely by almost all participants in the US stock market (Abbate, 2008). In mid-2004, for index calculation purposes, Standard & Poor’s considered using only “freely floating shares” that exclude shares held by insiders, founding families, or governments which are not available for investors to purchase.

To obtain the S&P 500 Index, the S&P committee selects the largest publicly held US-based companies that are traded on either of the two largest American stock market exchanges; the New York Stock Exchange and the NASDAQ. The general guidelines considered for adding stocks to the index are market value, industry group classification, capitalization, trading activity, fundamental analysis, and emerging industries. On the other hand, the general guidelines for removing stocks to the index are mergers, acquisitions, LBOs, bankruptcies, restructuring, and lack of representation.

The S&P 500 index is calculated using market-value weighted or base-weighted aggregate methodology. The Index is generally referred to as a price return index. In general, the rate of return of the S&P 500 Index equals to the rate of return of a portfolio that holds all 500 stocks that are part of the Index in the proportion to their market values. The S&P 500 index is a price index and does not reflect any cash dividends

The formula to calculate the S&P 500 is;

$$Index\ Level = \frac{\sum_i Pi * Qi}{Divisor}$$

Where, the index level is the index value of that day. P_i is the ending prices for stocks on the day. Q_i is the number of outstanding shares or freely floating shares on the day.

[2] The portion of the McGraw-Hill Companies including Standard and Poor’s subsequently became part of McGraw-Hill Financial Inc.

In a value-weight index, the importance of individual stocks in the sample depends on the market value of the stock (Reilly & Brown, 2008). Moreover, the movements in the prices of an index component with higher market capitalizations (price of the stock times the number of shares available for public trading) have a greater effect on the index than companies with smaller market caps.

The S&P 500 index calculations take in consideration corporate actions such as stock splits, share issuance, stock dividends and restructuring events such as mergers or spinoffs. In addition, the divisor needs to be adjusted as a result of corporate financial actions or when a company is dropped and replaced by another company with a different market capitalization.

3. Data Analysis

3.1. Modeling VIX and SPX volatility directional relationship

There is a general belief that the VIX is a good predictor for the future 30-day volatility of the S&P 500 Index (Whaley, 2008). We start our analyses by performing regression analyses between VIX and S&P 500 volatility including 1-month and 3-month T-bill interest rates as additional independent variables, and find that the interest rates do not contribute to the explanatory power of the independent variable in cases where VIX is the independent variable and S&P 500 is the dependent variable and vice versa. We analyze different volatility time periods for the S&P 500 index such as 22-trading days, effectively equivalent to a 30 calendar day period, as well as 11-day, 33-day, 44-day, 55-day, and 66-day forward volatilities against the VIX index, and find that the VIX index is superior in explaining the forward 11-trading-day S&P 500 volatility compared to the other period lengths studied. Overall we find that the VIX is only a fair predictor of the subsequent SPX volatility. Conversely, we find that the historical 22-day S&P 500 volatility is a superior predictor of the VIX Index, with a coefficient of determination, R^2 of over 80%.

CBOE started publishing the VIX daily closing prices on January 2nd, 1990. Hence, we collect the daily prices of VIX, VIX_t from the CBOE web site from January 2nd, 1990 to December 31st, 2009. The adjusted daily close price of S&P 500 Index, SPX_t, is collected from Yahoo Finance from January 2nd, 1990 to December 31st, 2009. The daily rate of return of the SPX, SPR_t is calculated as follows:

$$\mathrm{SPR_t} = \ln\left(\frac{\mathrm{SPX_t}}{\mathrm{SPX_{t-1}}}\right) *100$$

We use SPX volatility over 22 working days as our primary time interval, to explore the VIX forecasting power of future 30 calendar days SPX volatility (Whaley, 2008). Using this time frame, we eliminate weekends when no daily close prices of VIX and

SPX can be obtained. We calculate SPX future 22-day volatility, $SPFV_t$, as follows:

$$SPFV_t = \sqrt{252} * \sqrt{\sum_{i=0}^{21} (SPR_{t+i} - (\sum_{i=0}^{21} \frac{SPR_{t+i}}{22}))^2}$$

To test the relationship between daily close price of VIX, VIX_t, and SPX future 22-day volatility, $SPFV_t$, we first postulate Model 1 describing the relationship between the VIX as independent variable and the SPX volatility as dependent variable:

$$\text{Model 1: } SPFV_t = \beta_0 + \beta_1 * VIX_t + \varepsilon_t$$

The estimated relation between daily close prices of the VIX and SPX future 22-day volatility is

$$SPFV_t = -3.03177 + 0.97188 * VIX_t$$

In Model 1, 5038 pairs of samples have been observed. The correlation coefficient between the VIX and the future 22-day S&P 500 Volatility is 0.8018 with p-value <.0001 and R-squared is 0.6428 at 99% confidence level. Figure 1 shows the regression plot of model 1.

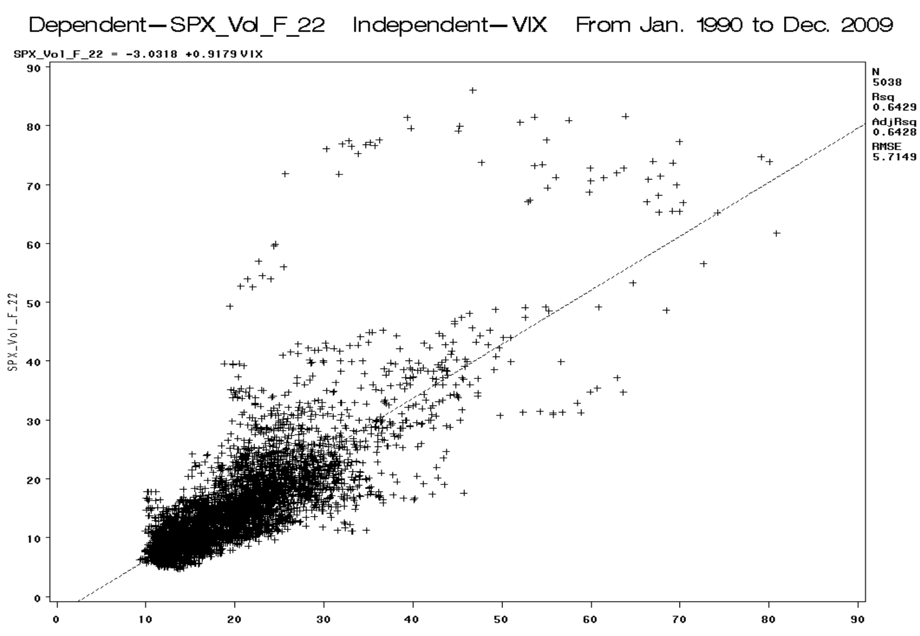

Figure 1: Regression plot between VIX and SPX future 22-day volatility

The regression result shows that VIX has a modest level of explanatory power towards the SPX future 22-working-day volatility. However, within the SPX high volatility periods, significant outliers are observed.

We test the performance of VIX in predicting the SPX volatility in Model 1, by using the data from January 1st to December 31st, 2010 as a test dataset. We calculate the estimated SPX future 22-day volatility by the formula in Model 1 and compare it with the actual SPX future 22-day volatility we observe in the real data.

Figure 2 shows the relation of actual SPX future 22-day volatility and estimated SPX future 22-day volatility in year 2010.

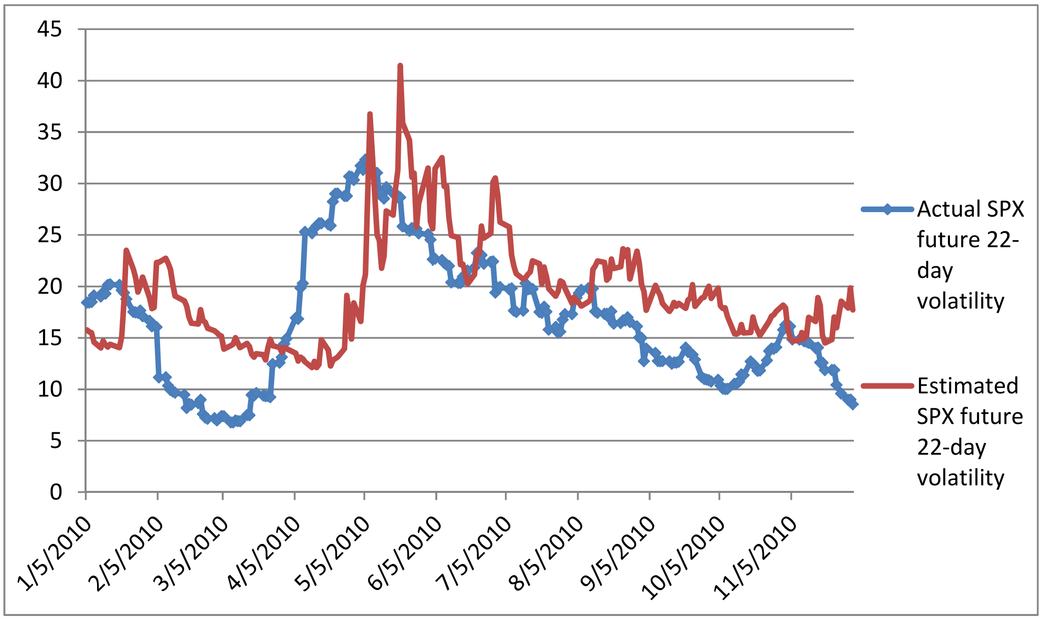

Figure 2: Actual SPX future 22-day volatility against estimated SPX future 22-day volatility in year 2010

The correlation coefficient between the actual SPX future 22-day volatility and estimated SPX future 22-day volatility is 0.4448. The real and the estimated means are 16.98 and 19.35 respectively. Using a t-test, we reject the hypothesis that the real and estimated means of SPX future 22-day volatility are similar at a p-value level <0.0001 and a t-critical value of 1.97. The variances of the real and estimated values of the SPX future 22-day volatility are respectively 42.19 and 26.18. Using an F-test, we also reject the hypothesis that the real and estimated variances of the SPX future 22-day volatility are similar with a p-value of 0.0002 and F-critical value of 1.243.

To test whether VIX is the best indicator of the SPX future volatility over the next 22 days, compared to other length of days, we perform regression analysis of the VIX against SPX future volatility over the subsequent 6, 11, 33, 44, 55, 66 days. Table 1 shows the regression results of VIX as an independent variable versus SPX future 6, 11, 22, 33, 44, 55, and 66-day volatilities as a dependent variable.

Table 1: Regression results of VIX as an independent variable versus SPX future 11, 22, 33, 44, 55, and 66-day volatility as a dependent variable.

Dependent Variable	Independent	Corr. coefficient	R-squared
SPX future 6-day volatility	VIX	0.7904	0.6247
SPX future 11-day volatility	VIX	0.8149	0.6641
SPX future 22-day volatility	VIX	0.8020	0.6428
SPX future 33-day volatility	VIX	0.7761	0.6024
SPX future 44-day volatility	VIX	0.7469	0.5579
SPX future 55-day volatility	VIX	0.7226	0.5222
SPX future 66-day volatility	VIX	0.7031	0.4944

Figure 3 shows the correlation coefficient and R-squared for different time periods represented in Table 1.

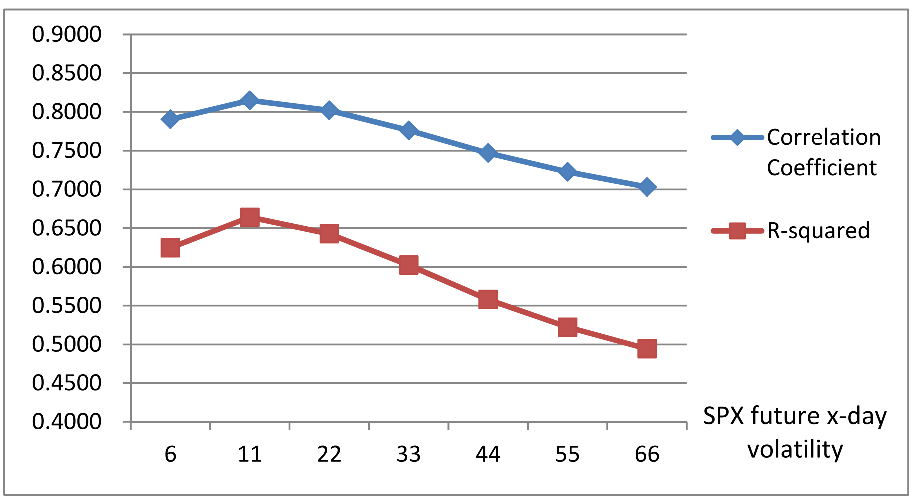

Figure 3: Correlation coefficients and r-squared values for SPX future volatility for 6, 11, 22, 33, 44, 55, and 66-day future calculation time periods

From table 1 and figure 3, we conclude that VIX shows superior predicting power of the SPX future 11-day volatility. If the time window becomes longer or shorter, the predictive performance of the VIX declines gradually.

The commonly accepted claim about VIX is that the Index is a good predictor of the future SPX 22-day volatility. Our regression results, with R-squared around 0.64, show that the VIX does have some explanatory power of the SPX future 22-day volatility. However, the concurrent relation between the VIX and SPX future 22-day volatility, starting on the same date, might not be the best one. To examine the best relationship between VIX and SPX future 22-day volatility, we perform regression analyses for the VIX and the SPX future 22-day volatility, $SPFV_{t+s}$, with the volatility calculations starting on different dates, e.g. shifted by different time periods such as 11, 22, 33, or 44 days backwards and forwards. The model is examined as follows:

$$\text{Model 1*:} \quad SPFV_{t+s} = \beta_0 + \beta_1 * VIX_t + \varepsilon_t$$

We set *s* in $SPFV_{t+s}$ equal to -44, -33, -22, -11, 0, 11, 22, 33 and 44 respectively. For example, if we have *s=-22*, we examine VIX of today, say May 1st, against SPX future 22-day volatility starting one month ago, that is, Apr. 1st, in which case we will be examining the past one-month SPX volatility against the current value of VIX. If we set *s=33* as another example, we examine VIX of today, say May 1st, against SPX future 22-day volatility starting one and a half month from now, that is, June 15th. If we set *s=0*, Model 1^* is exactly the same as Model 1, regressing VIX against SPX volatility starting at the same time point. In the following table, we show correlation coefficients and R-squared for different values of *s* based on Model 1^*:

Table 2: Regression between SPX future 22-day volatility at different starting points and VIX based on Model 1^*

Value of *s* in $SPFV_{t+s}$	Correlation coefficient	R-squared
s=-44	0.7884	0.6214
s=-33	0.8433	0.7111
s=-22	0.9001	0.8100
s=-11	0.8912	0.7941
s=0	0.8018	0.6428
s=11	0.6972	0.4859
s=22	0.6154	0.3785
s=33	0.5511	0.3035
s=44	0.5053	0.2550

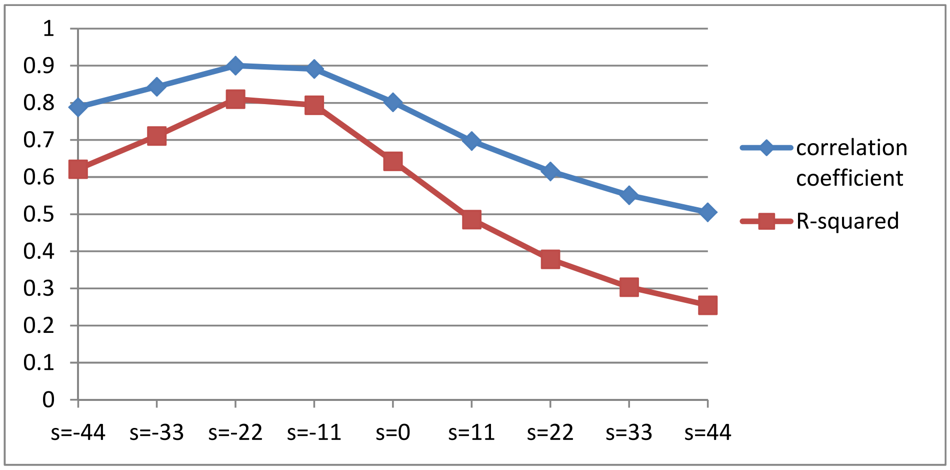

Figure 4: Correlation coefficients and R-squared values for VIX and SPX future 22-day volatility for shifted time frames from backward shift of 44 days up to forward shift of 44 days in 11-day increments.

We observe the best relationship between VIX and SPX future 22-day volatility when we shift the SPX volatility time series by 22-day backwards, which means that the past one month volatility of the SPX is well related to the VIX. This relationship implies that the SPX one month volatility is a good predictor of VIX as shown in table 2 and figure 4. From this superior performance, we can conclude that the VIX is more reflective of recent historical SPX volatility than it is predictive of future SPX volatility.

To test whether we can improve the predicting power of the SPX future 22-day volatility, we introduce a new variable, the actual historical volatility of VIX over the past 22 working days, into model 1. Here, we want to examine whether VIX past 22-day volatility gives any additional information as a secondary independent variable. First, we calculate the daily rate of return of $\mathrm{VIX_t}$, VXR_t as follows:

$$\mathrm{VXR_t} = \ln(\frac{\mathrm{VIX_t}}{\mathrm{VIX_{t-1}}})*100$$

The VIX past 22-day volatility, $VXPV_t$, is then calculated as shown in the equation below:

$$VXPV_t = \sqrt{252} * \sqrt{\sum_{i=1}^{22}(VIX_{t-i} - (\sum_{i=1}^{22}\frac{VIX_{t-i}}{22}))^2}$$

Therefore, the model for predicting SPX future 22-day volatility is enhanced to

$$\text{Model 2: } SPFV_t = \beta_0 + \beta_1 * VIX_t + \beta_2 * VXPV_t + \varepsilon_t$$

When VIX past 22-day volatility is included into the model, the R-squared of Model 2 is 0.6457, not showing significant improvement compared to the R-squared of Model 1, which was 0.6428. Meanwhile, the correlation coefficient between SPX future 22-day volatility and VIX past 22-day volatility is as low as 0.33967. Hence, we conclude that VIX past 22-day volatility does not contain incremental information in predicting SPX future 22-day volatility.

We also examine whether 1-month T-bill rate, R_t, as a secondary independent variable, provides any incremental information for predicting SPX future 22-day volatility. The data of 1-month T-bill rate is collected from the US Department of the Treasury website from Sept. 10th, 2001 to Dec. 31st, 2009, because Sept. 10th is the starting data point for the 1-month T-bill rate that the Government has published. Since the time period has been changed, we re-examine Model 1, add R_t into the Model and refer to the new model as Model 3, which is postulated as follows:

$$\text{Model 3: } SPFV_t = \beta_0 + \beta_1 * VIX_t + \beta_2 * R_t + \varepsilon_t$$

The R-squared of Model 3 is 0.6789, and by comparing it to the R-squared of Model 1, which was 0.6785, we conclude that the 1-month T-bill rate provides very little incremental information for the SPX future 22-day volatility.

3.2. Examining the best relationship between VIX and SPX 22-day volatility

To investigate the best relationship between VIX and SPX volatility, we first summarize the obtained results in Table 3. To better understand the relation between VIX and SPX historical 22-day volatility, we introduce a new variable, $SPPV_t$, to represent the SPX historical 22-day volatility starting today. We obtain the $SPPV_t$ by the following formula:

$$SPPV_t = \sqrt{252} * \sqrt{\sum_{i=1}^{22} (SPR_{t-i} - (\sum_{i=1}^{22} \frac{SPR_{t-i}}{22}))^2}$$

To examine the relationship between SPX past 22-day volatility and VIX, we propose

$$\text{Model 4: } VIX_t = \beta_0 + \beta_1 * SPPV_t + \varepsilon_t$$

In Model 4, the Correlation Coefficient is 0.89185 and R-squared is 0.7954, a notable improvement compared to 0.6428 R-squared in Model 1, in which VIX is examined against the SPX future 22-day volatility without a time shift. The results show that

VIX might be a better reflection of SPX past one-month volatility rather than being a predictor of SPX future one-month volatility. Hence, we suggest that the VIX could be a backward-looking rather than a forward-looking measurement of SPX volatility.

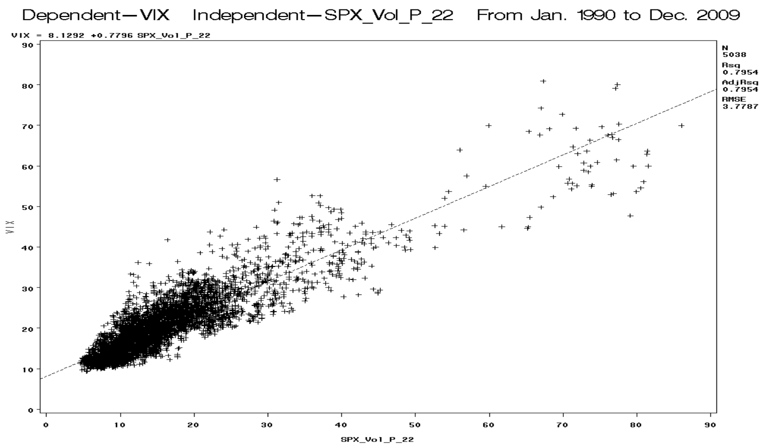

Figure 5: Regression plot between VIX and SPX past 22-day volatility as defined in Model 4.

Compared to Model 1, fewer outliers can be observed in Model 4 as shown in Figure 5. The outliers are mostly present in higher volatility periods. The estimated relation between VIX and SPX past 22-day volatility, based on Model 4 is

$$VIX_t = 8.1292 + 0.77959 * SPPV_t$$

Table 3: Summary of the results from Models 1, 1*, 2, 3, and 4

Model and Description	Dependent	Independent	Correlation coefficient	R-squared
Model 1: SPX future volatilities ($SPFV_t$) as a dependent versus VIX daily close price (VIX_t) as an independent	$SPFV_t$ + 11 days	VIX_t	0.8149	0.6641
	$SPFV_t$ + 22 days	VIX_t	0.8020	0.6428
	$SPFV_t$ + 33 days	VIX_t	0.7761	0.6024
	$SPFV_t$ + 44 days	VIX_t	0.7469	0.5579
	$SPFV_t$ + 55 days	VIX_t	0.7226	0.5222
	$SPFV_t$ + 66 days	VIX_t	0.7031	0.4944
Model 1*: SPX future 22-day volatilities at different time spot ($SPFV_{t+s}$) as	$SPFV_{t-44}$ + 22 days	VIX_t	0.7884	0.6214
	$SPFV_{t-33}$ + 22 days	VIX_t	0.8433	0.7111
	$SPFV_{t-22}$ + 22 days	VIX_t	0.9001	0.8100
	$SPFV_{t-11}$ + 22 days	VIX_t	0.8912	0.7941

a dependent versus VIX daily close price (VIX_t) as an independent	$SPFV_{t+11}$ + 22 days	VIX_t	0.6972	0.4859
	$SPFV_{t+22}$ + 22 days	VIX_t	0.6154	0.3785
	$SPFV_{t+33}$ + 22 days	VIX_t	0.5511	0.3035
	$SPFV_{t+44}$ + 22 days	VIX_t	0.5053	0.2550
Model 2: add VIX past 22-day volatility ($VXPV_t$) as a secondary independent	$SPFV_t$ + 22 days	VIX_t	0.8020	0.6457
		$VXPV_t$	0.3397	
Model 3: add 1-month Treasury bill rate (R_t) as a secondary independent	$SPFV_t$ + 22 days	VIX_t	0.8206	0.6789
		R_t	-0.2948	
Model 4: SPX past 22-day volatilities ($SPPV_t$) as an independent versus VIX daily close price (VIX_t) as a dependent	VIX_t	$SPPV_t$ (-22 days)	0.89185	0.7954

In Table 3, we show a summary of Models 1, 1*, 2, 3, and 4. In Model 1, we demonstrate that when we examine SPX future 22-day volatility ($SPFV_t$) as a dependent variable versus VIX as independent variable, we obtain R-squared of 0.6428 and correlation coefficient of 0.8020. We also observe that, compared to the 22-day future SPX volatility, VIX shows better predictability of the future 11-day SPX volatility ($SPFV_t$), with R-squared of 0.6641 and correlation coefficient of 0.8149. We then examine Model 1* where we shift the starting point for calculating the future 22-day SPX volatility from 44 days backwards to 44 days forward using 11-day increments. The results for shifted calculation starting points show that the best R-squared could be found for SPX future 22-day volatility when we shift the starting calculation point backwards by 22 days, which in essence represents the relationship between the VIX and the 22-day past SPX volatility. Thus, we could say that VIX is well related to the past 22-day SPX volatility and could rather be considered as a reflection of the SPX past one-month volatility rather than as a predictor of the future one-month SPX volatility.

In Model 4 we have SPX past 22-day volatility as independent variable and VIX as a dependent variable, and the results from model 4 confirm the result of Model 1*, where we observe R-squared value of 0.7954 and correlation coefficient of 0.89185 between the VIX and SPX past-22 day volatility.

In models 2 and 3, we use the future 22-day SPX volatility, $SPFV_t$, as dependent variable and VIX daily close price (VIX_t) as an independent variable with additional independent variables, VIX past 22-day volatility ($VXPV_t$) in model 2, and 1-month T-bill rate (R_t) in model 3. We observe a slight improvement in R-squared value by 0.0332 when we change a secondary independent variable from $VXPV_t$ in model 2 to R_t as in model 3. However, when we switch the dependent and independent variables in model 4, having SPX past 22-day volatilities ($SPPV_t$) as an independent variable and VIX daily close price (VIX_t) as dependent, we observe a significant improvement in R-squared value by 20% compared to model 2 and 3.

We use the 2010 real market data for VIX to test model 4. Figure 6 shows the relation between the actual VIX and estimated VIX from the model.

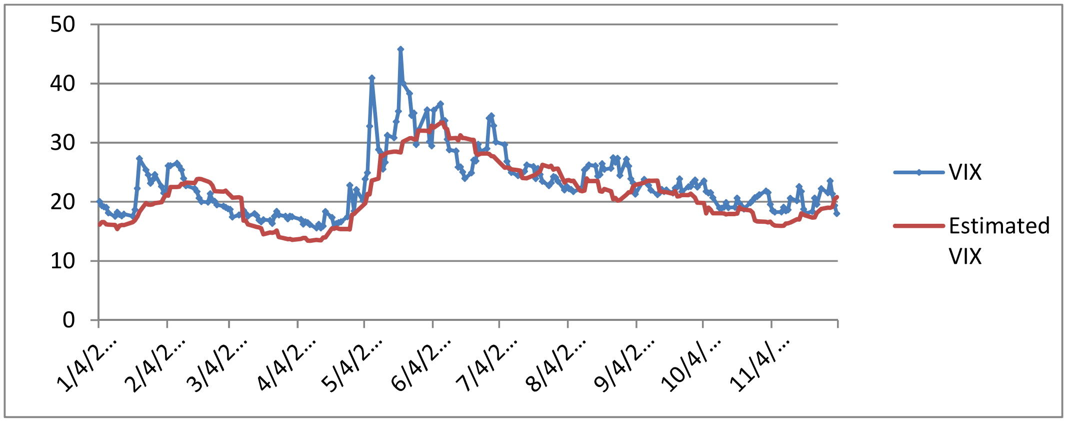

Figure 6: Comparison between 2010 actual VIX closing data and VIX data estimates based on model 4

The correlation coefficient between real VIX and estimated VIX is 0.8124. The means of real VIX and estimated VIX are respectively 22.98 and 21.30. The variance of real VIX and estimated VIX are respectively 27.56 and 25.66. Using a t-test we reject the null hypothesis that the means of the actual observed and estimated VIX are similar, with t-critical value of 1.97 and p-value <0.0001. On the other hand, using the F-test, we fail to reject the hypothesis that the variances of actual and estimated VIX are similar with F-critical of 1.074 and P-value of 0.293 under 99% significance level.

We compare the model test results of the relation between the actual VIX and estimated VIX of model 4, where we test how well the past 22-day SPX volatility predicts VIX, with the results of model 1, where we test VIX predictability of SPX future 22-day volatility for 2010, and observe that R-squared of model 4 is 0.8124 whereas the R-squared of model 1 is 0.4448. These results can be viewed as further confirmation that the VIX seems to be a reflection of SPX historical monthly volatility rather than a predictor of SPX future monthly volatility.

4. Regression analysis for SPX high volatility periods

To further understand the relations between VIX and SPX volatility, we divide our datasets to two different regimes, a high-volatility regime and a low-volatility or "normal" regime, based on SPX volatility levels. The mean and standard deviation for SPX future 22-day volatility for the entire dataset from January 2nd, 1990 to December 31st, 2009 are 15.5890 and 9.6514 respectively. We define any SPX future 22-day volatility > mean+2*standard deviation as a high volatility data point. We observe 202 high volatility data points in total and 201 of them are in the following 5 periods: (1) Oct. 16th, 1997 to Oct. 27th, 1997 (2) Aug. 3rd, 1998 to Sept. 1st, 1998 (3) June 24th, 2002 to July 29th, 2002 (4) Sept. 12th, 2002 to Oct. 1st, 2002 and (5) August 19th 2008 to Mar. 23rd 2009. There are total of 218 data points in these 5 periods, out of which, 92.2% are classified as high volatility points.

To examine the VIX predictive power of the SPX future 22-day volatility during the high volatility periods, we perform a regression analysis of the SPX future 22-day volatility, $SPFV_t$ as dependent variable versus the VIX closing price, VIX_t, as follows,

$$SPFV_t = \beta_0 + \beta_1 * VIX_t + \varepsilon_t$$

The correlation coefficient between VIX and SPX future 22-day volatility is 0.3791 and the R-squared is 0.1398.

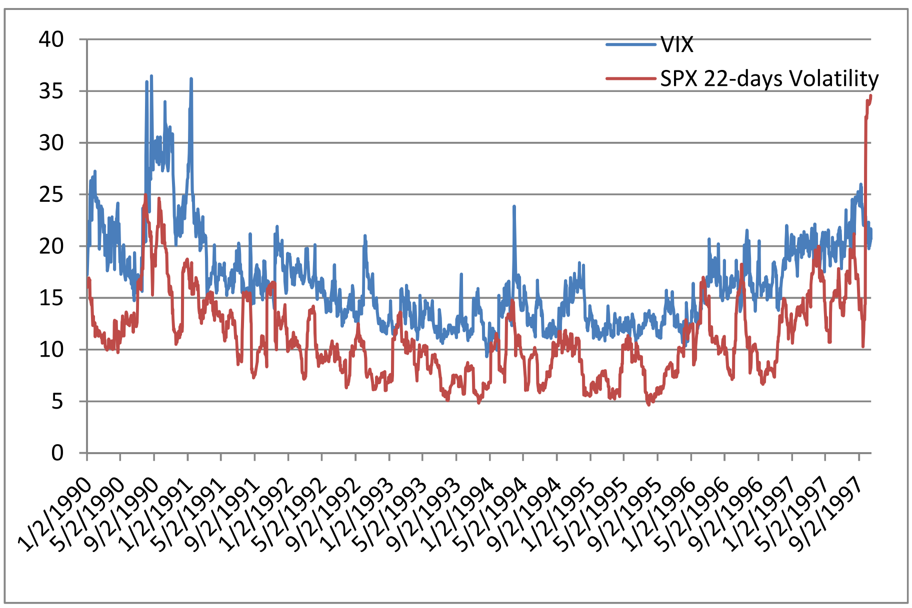

Figure 7: The SPX 22-day future volatility and VIX between Jan. 2nd, 1990 and Oct. 15th, 1997.

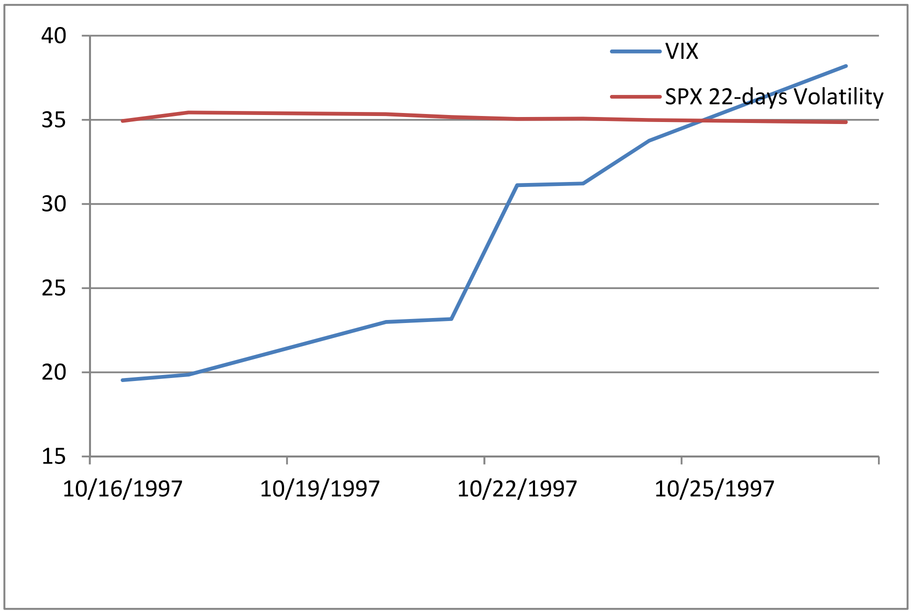

Figure 8: The SPX 22-day future volatility and VIX between Oct. 16th, 1997 and Oct. 27th, 1997.

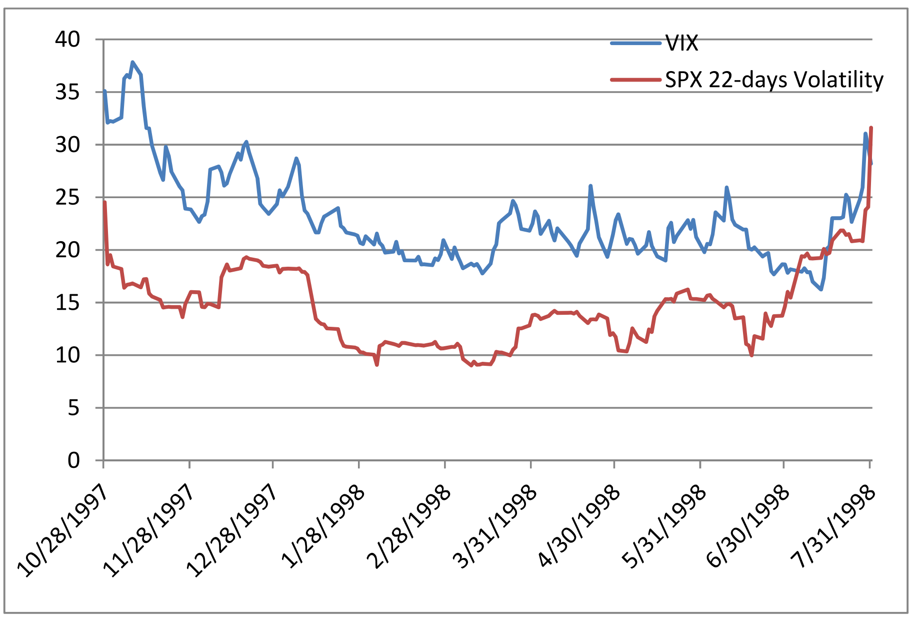

Figure 9: The SPX 22-day future volatility and VIX between Oct. 28th, 1997 and July. 31st, 1998.

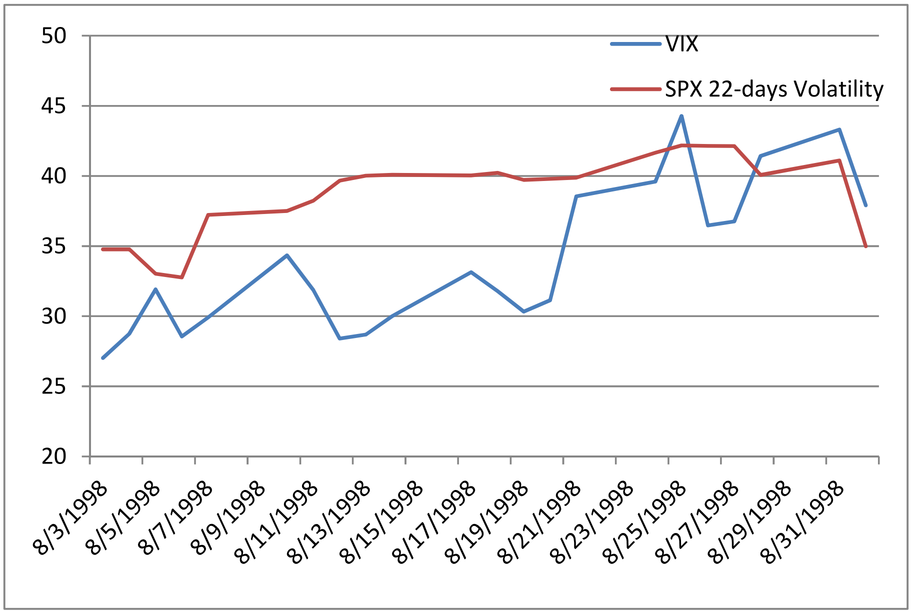

Figure 10: The SPX 22-day future volatility and VIX between Aug. 3rd, 1998 and Sept. 1st, 1998

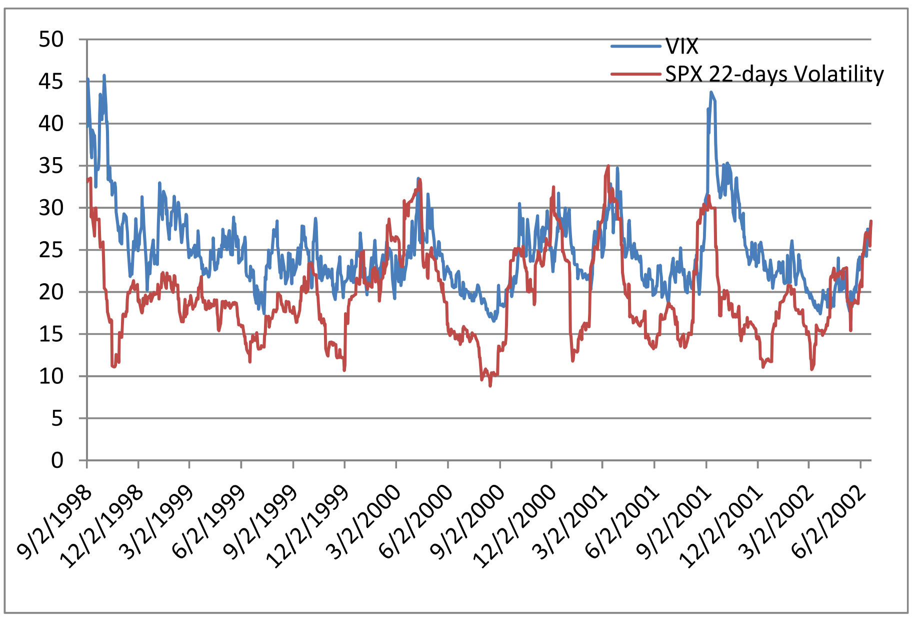

Figure 11: The SPX 22-day future volatility and VIX between Sept. 2nd, 1998 and June 21st, 2002.

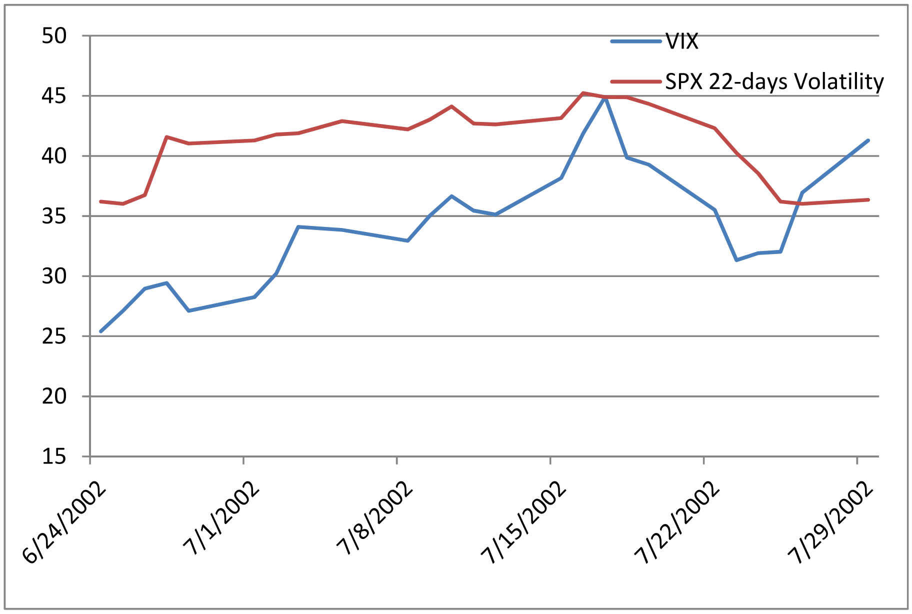

Figure 12: The SPX 22-day future volatility and VIX between June 24th, 2002 and July 29th, 2002.

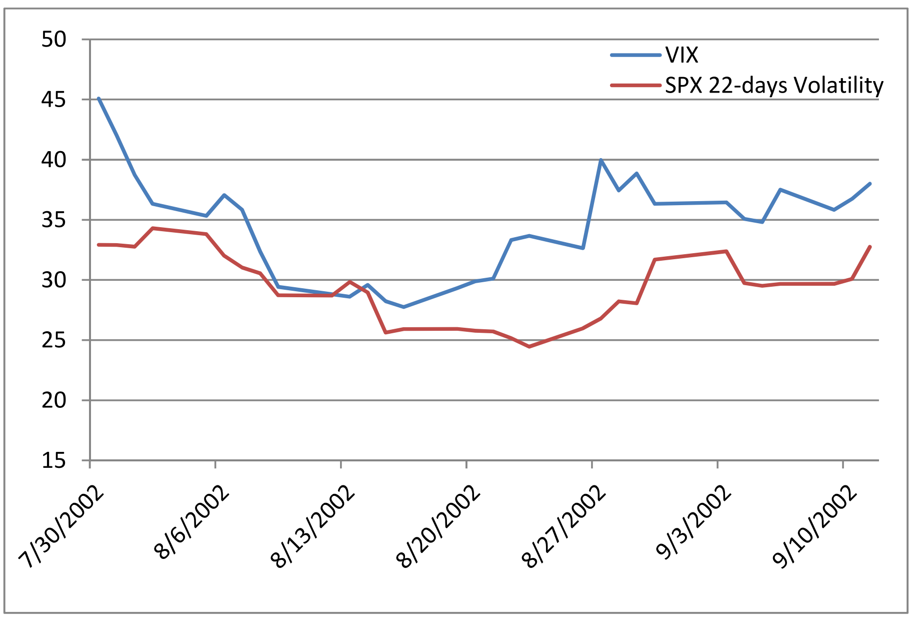

Figure 13: The SPX 22-day future volatility and VIX between July 30th, 2002 and Sept. 11th, 2002.

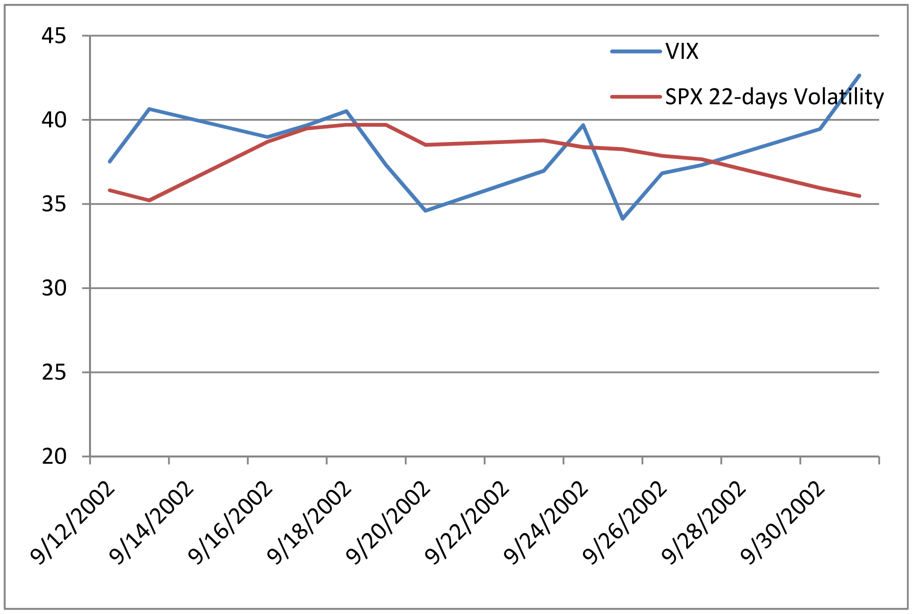

Figure 14: The SPX 22-day future volatility and VIX between Sept. 12th, 2002 and Oct. 1st, 2002.

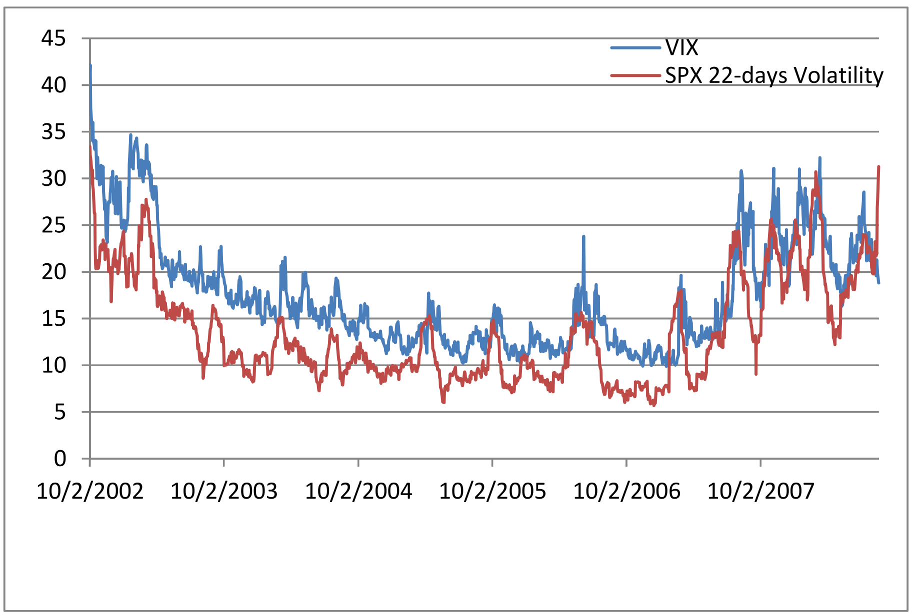

Figure 15: The SPX 22-dayfuture volatility and VIX between Oct. 2nd, 2002 and Aug. 18th, 2008.

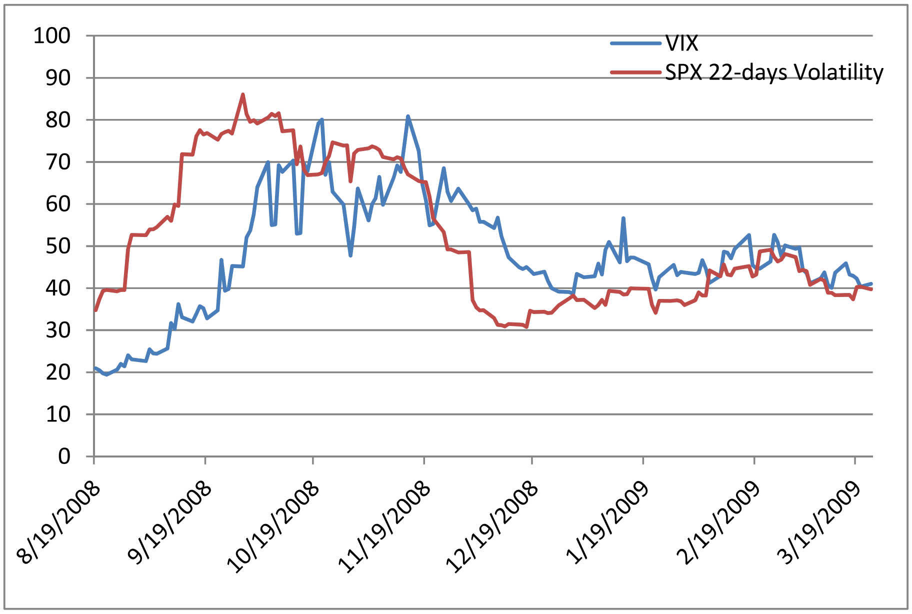

Figure 16: The SPX 22-day future volatility and VIX between Aug. 19th, 2008 and Mar. 23rd, 2009.

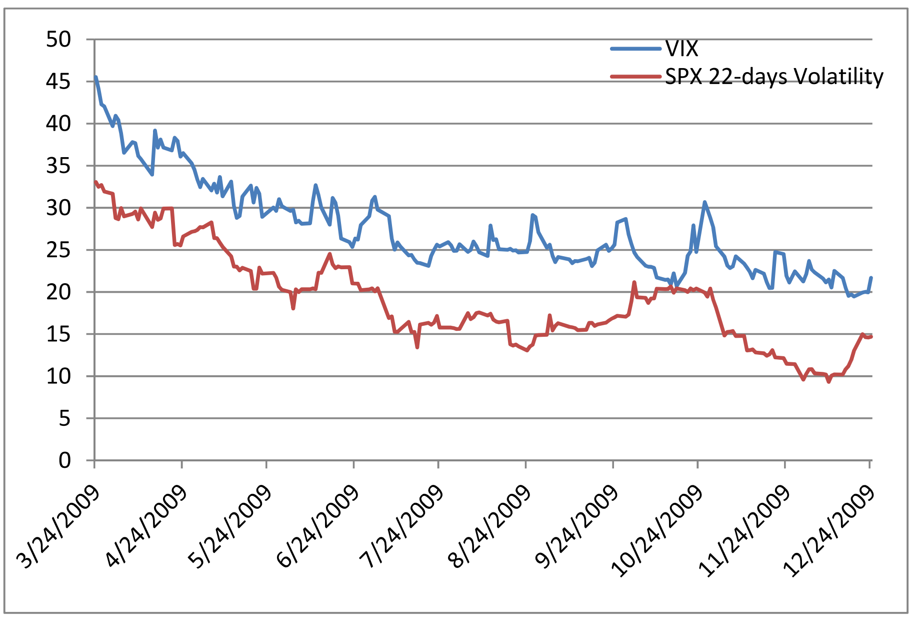

Figure 17: The SPX 22-day future volatility and VIX between Mar. 24th, 2009 and Dec. 30th, 2009.

Table 4: Summary of the VIX and SPX future 22-day volatility for 1990 through 2009

	Time period	Market Condition	VIX			SPX 22-day Volatility		
			Highest	Lowest	Average	Highest	Lowest	Average
1	01/02/1990-	Normal	36.47	9.31	16.53	34.58	4.62	11.19

	10/15/1997							
2	10/16/1997-10/27/1997	Highly Volatile	38.20	19.53	27.48	35.44	34.86	35.10
3	10/28/1997-07/31/1998	Normal	37.84	16.03	23.05	31.61	9.02	14.57
4	08/03/1998-09/01/1998	Highly Volatile	44.28	27.02	33.82	42.17	32.76	38.72
5	09/02/1998-06/21/2002	Normal	45.74	16.53	24.65	34.99	8.82	19.52
6	06/24/2002-07/29/2002	Highly Volatile	44.92	25.40	34.10	45.23	36.01	41.04
7	07/30/2002-09/11/2002	Normal	45.08	27.75	34.55	34.29	24.45	29.34
8	09/12/2002-10/01/2002	Highly Volatile	42.64	34.12	38.30	39.70	35.21	37.82
9	10/02/2002-08/18/2008	Normal	42.13	9.89	17.46	33.41	5.67	13.54
10	08/19/2008-03/23/2009	Highly Volatile	80.86	19.43	47.93	86.04	30.78	52.02
11	03/24/2009-12/30/2009	Normal	45.54	19.47	27.34	33.05	9.30	18.87

In figures 7 through 17, we show the relationship between the SPX future 22-day volatilities and VIX, separating the highly volatile and normal phases over the period of 1990-2009. By defining the highly volatile period as a period where we observe volatilities greater than the mean + 2*standard deviations based on the entire studied period, we obtain 5 highly volatile periods and 6 normal periods over the 20-year time frame.

In general, the figures show that during the normal periods of relatively low volatility, VIX appears generally to overestimate the SPX future 22-day volatility, while during the high volatility periods VIX appears to underestimate SPX future 22-days volatility. Moreover, we observe in Table 5 that the average and the lowest values of VIX during the normal periods are always higher than the average and the lowest values of the SPX future 22-day volatility. On the other hand, the average and the lowest values of VIX during the highly volatile periods are always lower than the average and the lowest values of SPX future 22-days volatility.

Interestingly, a sudden increase in SPX future 22-day volatility could be detected at the end of almost all normal volatility periods. In figure 7, for the period of normal volatility between Jan. 2nd, 1990 and Oct. 15th, 1997, SPX future 22-day volatility increases by 350% in a very short period of time while at the same time VIX decreases from 25 to 20. Figure 11 shows upward trends for both, the SPX 22-day future volatility and the VIX; we observe that VIX could not predict the very high 22-day future volatility for the SPX of 35 which was the highest volatility for this particular normal period. Thus, we might assume that VIX could not predict the sudden dramatic changes in SPX future 22-days volatility and could not predict when the next highly volatile period would occur.

5. Distribution Analysis

After looking into various models to determine the best relationship between the VIX and the SPX volatility, we also analyze the distributions of these indicators of financial markets variability to better understand their statistical characteristics. To generate the distribution graphs, we use the log binning method for reorganizing the SPX volatility and the VIX data for the 20-year period between 1990 and 2009. Log binning can reduce the time required to fit the data by orders of magnitude for large number of events (McManus, Blatz, & Magleby, 1987).

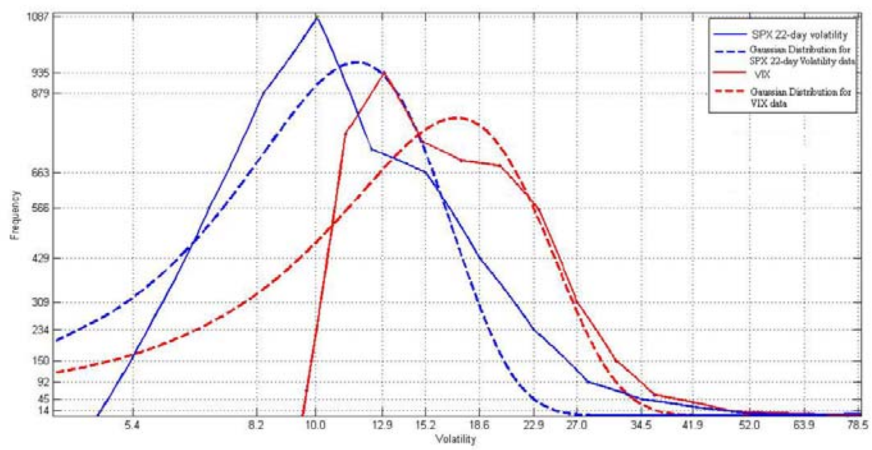

Figure 18: Distribution graph of SPX future 22-day volatility and the VIX using log binning method with 15 bins.

From the distribution graph, we observe that the highest SPX 22-day future volatility frequency is at 10.6069 with 1087 occurrences. On the other hand, the highest VIX frequency is at 12.9 with 935 occurrences.

We also plot the Gaussian distribution fit for VIX and the SPX future 22-day volatility and observe that, in general, the actual empirical distributions have fatter tails compared to the Gaussian plots, with the SPX future 22-day volatility distribution exhibiting larger kurtosis in regards to the Gaussian fit compared to the VIX fit to the Gaussian distribution.

Since we are especially interested in the relationship between the SPX volatility and the VIX during highly volatile periods, we separate the SPX volatility and VIX datasets into two subsets: (i) normal volatility periods on one hand, and (ii) highly volatile periods on the other, as determined in section 4. The mean and standard deviation for SPX 22-day volatility from January 2nd, 1990 to December 31st, 2009 are 15.5890 and 9.6514 respectively, and the mean and standard deviation for VIX from January 2nd, 1990 to December 31st, 2009 are 20.2862 and 8.35149 respectively.

Theoretically, VIX and SPX Volatility distribution graphs should be overlapping if VIX is a good predictor of SPX future volatility. Moreover, the tails[3] of both VIX and SPX Volatility should be similar if VIX is a reliable indicator of future SPX volatility during the high volatility periods.

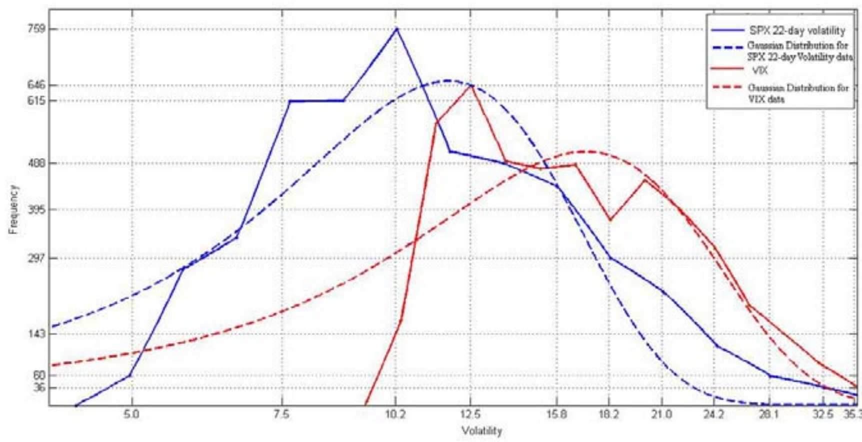

Figure 19: Distribution graph of SPX 22-day future volatility and VIX without tails using log binning method with 15 bins.

In Figure 19, we observe that in general during the normal volatility periods, the shapes of the VIX and SPX 22-day volatility distributions are similar, with the SPX distribution exhibiting wider distribution range compared to the VIX distribution.

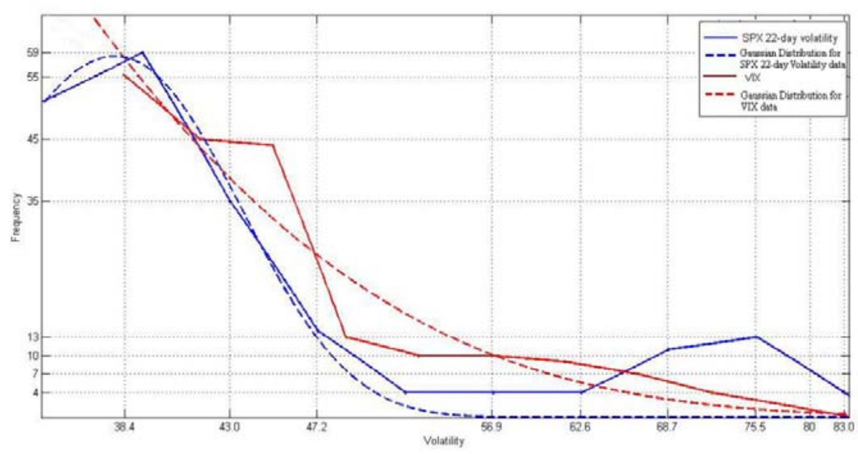

Figure 20: The "tails" distribution graph of SPX 22-day volatility and VIX data using log binning method with 10 bins.

[3] We define "Tails" as high volatility data points that exceed the mean plus two standard deviations

In Figure 20 on the other hand, when comparing the “tails” of both VIX and SPX 22-day future volatility data, while in the lower tail volatility ranges, up to 65, we observe similar distribution shapes for both datasets, in the range of volatilities above 65, the frequencies for VIX decrease steadily, while the SPX 22-day future volatility exhibits another peak around volatility of 75. All the volatility data between 65 and 83 occur in 2008. The very high SPX 22-day future volatility in this range is not matched by the VIX data, which could be interpreted as the VIX reduced predictive power of the SPX future 22-day volatility during extremely high volatility periods.

6. Discussion and Conclusion

In this paper we study the relationship between the VIX and the SPX volatility for the 20-year period between 1990 and 2009 to understand the directional relation between these two measures of financial market variability. In order to examine the best relationship between VIX and SPX volatility, we begin by testing the correlation coefficient and R-squared values between VIX as independent variable and SPX 22-day future volatility as dependent variable. We find that the regression R-squared is 0.6425 and therefore suggest that VIX does contain some explanatory power in regards to the SPX future 22-day volatility. However, within SPX high-volatile periods, we can easily observe a great number of outliers. When we introduce additional independent variables, besides VIX, such as VIX volatility and 1-month Treasury bill rate, we find that these additional variables do not add significant incremental information in explaining the SPX future volatility.

We then change the 22-day period for calculating future SPX volatility to 6, 11, 33, 44, 55, and 66-day volatility, and find that VIX is best at predicting the 11-day future SPX volatility and that its predictive ability decreases gradually when the time windows become shorter or longer.

We then conduct a further examination between VIX and SPX 22-day volatility based on the same linear regression model with different starting points for calculating the SPX 22-day volatility. We move the calculation starting point by 11, 22, 33, 44 working days backwards and forwards in order to get the best relationship between VIX and SPX 22-day volatility. We find that the best relationship between VIX and SPX 22-day volatility exists for VIX and SPX historical 22-day volatility, which can be interpreted to indicate that the VIX is a backward looking instead of a forward looking measure of SPX volatility.

To test the observation that VIX is a backward instead of a forward oriented index, we propose an additional linear regression in which we use SPX past 22-day volatility as a predictor of VIX. We find out that the R-squared of this regression is 0.7954. In addition, using 2010 as a test year, we find that the correlation between the real VIX and the estimated VIX using the past SPX 22-day volatility is 0.8154 compared with correlation coefficient of 0.4448 between the SPX future 22-day volatility and VIX.

In addition, we also study the low volatility or “normal” versus high volatility periods defining a high volatility period as a time range where the volatilities are two standard deviations above the mean. We observe that the VIX appears to underestimate SPX 22-days future volatility during periods of high volatility. Moreover, during the high volatility periods, VIX fails to predict the sudden change in SPX 22-day future volatility. During 2008, for example, SPX 22-day volatility dramatically increases in the beginning of August, while VIX remains at the lower level for several days before moving up following the SPX 22-day future volatility increase. In addition, when the SPX 22-day future volatility passed the peak and started to decline, VIX appeared to have missed the direction change and appears to overestimate the SPX 22-days future volatility at the end of 2008. In contrast, in periods of relatively low volatility, the VIX tends to overestimate actual future volatility.

Finally, we investigate the distributions for the VIX and SPX future 22-day volatility for the entire dataset, and separately for the two subsets: (i) including volatilities within two standard deviations from the mean (normal volatility periods) and (ii) periods where the volatility values are above two standard deviations from the mean (high volatility periods). We find that for the complete dataset, and for the subset including the volatilities within two standard deviations around the mean, both, the VIX and the SPX future 22-day volatilities exhibit similar behavior closely corresponding to the Gaussian distribution; however, for the tail volatilities, representing the volatilities in excess of two standard deviations from the mean, the VIX index still preserves the regular Gaussian shape of the distribution, while the SPX future 22-day volatility subset exhibits a bi-modal (double peak) distribution with one peak around 40% volatility similar to the VIX index, and another peak around 75% volatility which is not observed in the VIX index data.

Acknowledgments

We wish to thank the European Commission FET Open Project ‘FOC ’255987 and ‘FOC-INCO’ 297149 for financial support, and we would like to thank Borui Wang and Sompop Chaichanavitchakij for research assistance.

References

Abbate, Christine. (2008). Stock Market 101 - What is the S&P 500 and What Does it Represent?

Becker, R., & Clements, A. E. (2007, 6). Are combination forecasts of S&P 500 volatility statistically superior? National Center for Econometric Research Working Paper Series.

Becker, R., Clements, A. E., & White, S. I. (2006, 11). Does implied volatility provide any information beyond that captured in model-based volatility forecasts? Journal of Banking & Finance , pp. 2535-2549.

Chernov, M. (2001). Implied Volatilities as Forecasts of Future Volatility. unpublished manuscript, Columbia University.

HungJui-Cheng, NiRen-Xi, & ChengXin-Hua. (2009,10). Using VIX Index and Range-based Volatility to Enhance the Volatility Forecasting Performance of GARCH Models. Economics Bulletin, 2592-2604.

McManusB.O., BlatzL.A., & MaglebyL.K. (1987). Pflügers Archiv European Journal of Physiology. Sampling, log binning, fitting, and plotting durations of open and shut intervals from single channels and the effects of noise, 530-553.

Poon, S.-H., & Granger, C. W. (2003, June). Forecasting Volatility in Financial. Journal of Economic Literature , pp. 478-539.

ReillyK.Frank, & BrownC.Keith. (2008). Investment Analysis and Portfolio Management . South-Western College Publisher.

Whaley, R. E. (2008, 11 6). Understanding VIX. Social Science Research Network .