Evidence on the Non-Linear Impact of Management

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Abstract
Management practices have been shown to have a significant and economically large impact on firm output after controlling for a range of standard factors such as other inputs, industry etc. We investigate non-linearities in the impact of management practices on firm performance using Gaussian process and a continuous piece-wise linear approach with probabilistically smoothed endogenous breaks. In all cases we find significant evidence of a U-shaped relationship, with the biggest returns to management occurring when management practices are at their highest levels.

1 Introduction

In their seminal paper [2] establish that management practices have a large and statistically significant impact on output on manufacturing of firms in the USA, UK, France and Germany. The result is established primarily through the OLS estimation of an augmented Cobb-Douglas production function. The Cobb-Douglas framework is standard in the literature because it has proved to be well behaved in practice and approximates more flexible functional forms (see [8]).1

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1The [2] results are robust to considering return on assets and Tobin’s Q as dependent variables and to alternative estimation approaches like [7].
The Bloom and Van Reenen approach has had significant policy impact and has been extended to manufacturing firms in over twenty countries across Europe, Asia and the Americas (see [1]) and to healthcare, retail and schools. Details can be found at the World Management Survey website\(^2\).

The augmented Cobb-Douglas framework yields an estimated 4% increase in revenue for a one unit increase in the management index, or about a 16% increase in revenue from the 5th to 95th percentile on the management index (holding other inputs constant). Much theorizing in both Economics and Management predicts that this impact should not be constant across the distribution of firms but rather that the impacts of small improvements should be modest for most firms and that the big payoff comes from getting the whole bundle of practices right (see [3] for survey and overview of this extensive literature).

We apply this essentially non-linear (and possibly convex) perspective of management practices to the original Bloom and Van Reenen analysis by allowing increased flexibility in the impact of the management term while maintaining the rest of the original specification and sample. Specifically, we first consider a Gaussian processes approach in which each firm has its own management function and secondly a piecewise-linear approach with probabilistically smoothed endogenous breaks. Both methods reaffirm the original finding on the importance of management practices but show the impact is significantly non-linear and is strongest when firms have most of their practices right.

\section{The Models}

\subsection{The Data}

The survey was collected by telephone interview. Trained MBA students interviewed a member of a firm’s senior management about management practices in their firm. The responses were then scored by two scorers using 18 dimensions and a 5 point scale. Publicly reported financial data was then merged with the management data\(^3\). Details of the dimensions and the scor-

\footnote{\url{http://worldmanagementssurvey.org}}

\footnote{The issues with estimating production functions from financial/accounting data are well known but have not proved unacceptable to industrial economics research or national statistical agencies. Augmenting the accounting data with management practices seems}
ing method can be found in Bloom and Van Reenen (2007). The choice of the dimensions to focus on and the details of the scoring is necessarily some what arbitrary but was constructed to reflect expert opinion from an interna-
tional management consulting firm and has proved to work well statistically
in understanding the impact of management practices.

2.2 Features of Interest

One of the unique features of this dataset is that it allows the analysis of output (as measured by sales) as a function of management practices in addition to the normal inputs: labor, capital and intermediate goods. To gain an impression of the relationship between management and performance, consider the histograms and plot in Figure (1) which shows the key distributions and relationships.

Sales for this set of firms, as is typical, are highly skewed with a long right tail. Some of this skew is presumably part of the well documented persistent performance difference among seemingly similar firms ([4]), but some is quite simply due to differences in scale and input use. Thus sales are a poor subject to analyse directly to see the impact of management on firm performance.

To remove the influence on sales of these normal inputs, we regress log sales on a constant and these factors. Total factor productivity (TFP), calculated from the constant and residuals of a log-log regression of sales on inputs represents the differences in firm performance in efficiency terms. The log-log regression estimates a Cobb-Douglas production function

\[ S_i = A_i L_i^{\beta_1} K_i^{\beta_2} G_i^{\beta_3} \]

as

\[ \ln S_i = \ln A + \beta_1 \ln L + \beta_2 \ln K + \beta_3 \ln G + \varepsilon_i \]  

\hspace{1cm} (1)

where \( A_i = A e^{\varepsilon_i} \) is total factor productivity, \( L \) is labor, \( K \) is capital and \( G \) is intermediate inputs. Thus the constant plus the residual from this regression, which we will denote by \( a_i = \ln A + \varepsilon_i \), represents log TFP as the (log of) sales unexplained by these factors but which may be explained by other factors such as management practices. The histogram of \( a_i \) estimated from (1) is plotted in the top left hand panel in Figure (1). There clearly remains a strong right skew in the distribution of \( a_i \), although the effect is much less than would appear in the histogram for Sales.

to address some of the deficiencies in earlier work.
Recognizing the potential importance of management practices, [2] pioneer the analysis in a very natural way by augmenting the standard log-log specification with the management index, \( m_i \), specifically

\[
\ln S_i = \ln A_i + \beta_1 \ln L + \beta_2 \ln K_i + \beta_3 \ln G_i + \beta_4 m_i + \varepsilon_i.
\]

The management score is taken by simply averaging a firm’s score across the 18 categories and is then normalized as a z-score by subtracting the sample mean and dividing by the sample standard deviation. That is, the z-statistic computed as an affine function of the management index;

\[
z_i = \frac{m_i - \bar{m}}{s_m}
\]

where \( \bar{m} \) is the sample mean of \( m_i \) and \( s_m \) is the estimated sample standard deviation. The distribution for the management z-scores is shown in the bottom right panel of Figure (1). In contrast with the distribution for \( a_i \), the distribution of \( z_i \) looks relatively symmetric although there is a slight left skew. It is difficult to reconcile the symmetry of the distribution of \( z_i \) with the skew in the distribution of \( a_i \) without looking for other causes of firm performance or considering a nonlinear relationship between these variables. As the range of variables in the Bloom and Van Reenen (2007) is already extensive, we take up investigation of the latter possibility.

Looking at these histograms for total factor productivity \( a_i \) and the management score as \( z_i \) gives only an incomplete picture of their relationship as each plot is only one-dimensional. We need to consider two-dimensional representations of this relationship. The obvious choice of plot - a scatterplot of these two variables - is, however, even less informative than the histograms. A more useful impression of this relationship is given in the top right panel of Figure (1) which plots the median bands plots (essentially a non-parametric summary of the scatter plot) of \( a_i \) against \( z_i \). To produce this plot, the values of \( z_i \) are grouped into nine bands and the median coordinates (median \( a_i \) and median \( z_i \)) from each band are connected by a line. We see in this image the first evidence of form of the function describing the relationship between \( a_i \) and \( z_i \) which this paper investigates. The most important feature of the function is the U-shape. The results appear quite nonlinear with a fall at bottom, little variation across the middle and, in particular, a sharper increase in performance as the index of management practices approaches the upper values.
Figure 1: Productivity and Management
The median band plot is a non-parametric summary but it is largely arbitrary due to the need to choose the number of bands, and the appearance of the curve does change with the number of bands. It takes TFP as given and then compares it to management rather than including management in the estimation of the production function and of course in being agnostic about the structure of an underlying relationship it is limited in terms of inference we can obtain. In the following sections we estimate two alternative approaches which are something of a middle road, allowing more flexibility than the linear model while providing more structure and inferential opportunities than the graphical analysis.

3 Empirical Results

We investigate formal estimates of the functional relationship between firm performance and management practices. We approach the empirical investigation by beginning with a reasonably general model specification to get an impression of any likely nonlinearity. The estimate of this model provides clear evidence that the relationship is not linear and suggests points at which we might expect changes in the response of sales to changes in the management score. This evidence informs us what structure to impose for the second model which, due to the increased structure, permits formal inference and more precise estimates of the form of the response of firm performance to the management score.

Gaussian Process: We begin with an exploratory investigation of the likely form of the relationship between the response variable of interest and the management index in which we impose no structure beyond smoothness. The response variable of interest, such as log sales, we denote by $y_i$. As we are interested in modelling the response of $y_i$ to the management index score, $z_i$, without making any strong assumptions on the form of this response, we begin by modelling the response in a very flexible but smooth form using a Gaussian process. To permit comparison with the results in Bloom and Van Reenen (2007), we take $z$ on the $x$–axis. Using a Gaussian process is sometimes called a nonparametric estimate as no functional form is assumed for the mean of $y_i$. The model estimated by a Gaussian process is

\[
y_i = \mu_i + x_i \beta + \varepsilon_i
\]

\[
\varepsilon_i \sim N(0, \sigma^2).
\]
The coefficient \( \mu_i = \mu (z_i) \) varies smoothly with \( z_i \) but is not otherwise given a specific functional form. The smoothness is achieved via the correlation structure among the \( \mu_i \). To describe the process more explicitly, collect the \( N \) different \( \mu_i \) into an \( N \times 1 \) vector \( \mu = (\mu_1, \mu_2, \ldots, \mu_N)' \) and give \( \mu \) the prior distribution \( \mu \sim N (0, R) \) where \( R \) is an \( N \times N \) matrix of correlations. The strength of the correlation between any two points \( i \) and \( j \) is determined by the distance, \( d_{ij} = \|z_i - z_j\| \), between the two points \( z_i \) and \( z_j \). There are many ways to specify the correlation structure in \( R \); but for our purposes we take a squared exponential form such that the correlations are given by

\[
\rho_{ij} = \exp \left\{ -\frac{\alpha}{2} h(d_{ij}) \right\}
\]

where \( h_i = h (\|z_i - z_j\|) > 0 \) is a monotonically increasing function of the distance \( d_{ij} \). By construction the resulting function will be continuous.

Figure (2) presents the posterior average of the prediction

\[
E (y_i | z_i, x_i = \bar{x}, \mu, \beta, \sigma^2) = \mu_i + \bar{x} \beta
\]

obtained using the Gaussian process. In Bayesian analysis, the parameters are treated as random and unknown, so functions of the parameters, such as \( E (y_i | z_i, x_i = \bar{x}, \mu, \beta, \sigma^2) \), are also random and have their own distributions. We denote this function as \( E (y_i | GP) \). and the lower and upper bounds of the 95% credible interval around \( E (y_i | GP) \) are denoted as \( E (y_i | GP) \) 5% and \( E (y_i | GP) \) 95% respectively. On the \( x - axis \) is the \( z - statistic \) computed from the management index. We also plot the estimated function

\[
E (y_i | z_i, x_i = \bar{x}) = \hat{\mu} z_i + \bar{x} \hat{\beta}
\]

where \( \hat{\mu} \) and \( \hat{\beta} \) have been estimate using OLS as in Bloom and Van Reenen (2007), shown as \( E (y_i | BvR) \).

The bounds around the nonparametric function \( E (y_i | GP) \) largely support the linear estimate in \( E (y_i | BvR) \) through the middle region from, say, \( z_i = -1.37 \) through to \( z_i = 1.07 \). Outside this range the estimates disagree significantly with \( E (y_i | GP) > E (y_i | BvR) \). It is in the end regions that we begin to see evidence of deviation from the strictly linear form, although the deviation could be well described by new linear functions over subregions. Based upon these observations, and for further reasons discussed below, we choose to treat the form of the nonlinearity as a sequence of continuous,
Figure 2: Plot of the Gaussian Processes (GP) and Linear OLS as in Bloom and Van Reenen (2007), BvR (2007). For the GP, the plot shows $E(y_i|GP)$ and the lower and upper bounds of the 95% credible interval denoted as $E(y_i|GP) - lower$ and $E(y_i|GP) - upper$ respectively. On the $x$ - axis is the $z$ - statistic computed from the management index.

piecewise linear functions with a small number of breaks in the linear function.

**Continuous Piecewise Linear Model (CPLM):** Although visual inspection of the estimation using the Gaussian Process specification suggests strong non-linearity, this model does not provide much formal evidence of non-linearity nor an economically useful indication of key turning points in the relationship. To obtain estimates of the level at which increases in $z_i$ lead to more pronounced increases in the response variable, we need to impose a simpler structure on the model which will also permit formal inference on the response. We therefore propose a piece-wise linear form. However, we wish to maintain the continuous structure of the model and so we use the form with $B$ breaks in the continuous piecewise linear mean as a function of $z_i$. Define the support of $z_i$ as $M = [\underline{z}, \overline{z}]$. This support is partitioned into
$B + 1$ segments with boundaries at the break points $z^{(j)}$ where $z^{(0)} = -\infty$ and $z^{(B)} < \max \{z_i\}$. We will denote each segment by $S^{(j)}$ where $z_i \in S^{(j)}$ if $z^{(j-1)} < z_i < z^{(j)}$. The specification of the CPLM is given by

$$y_i = z_i \mu + \sum_{j=1}^{B} \left( z_i - z^{(j)} \right)^+ \delta_j + x_i \beta + \varepsilon_i \quad \varepsilon_i \sim N \left( 0, \sigma_j^2 \right)$$

where $\left( z_i - z^{(j)} \right)^+ = \max \left( z_i - z^{(j)}, 0 \right)$.

The model with $B$ breaks will have two main blocks of parameters; the $k_B \times 1$ vector $\gamma_B = (\mu, \delta_1, \ldots, \delta_B, \beta')'$ and the vector of $B$ variances, $\sigma^2 = (\sigma_1^2, \ldots, \sigma_B^2)$. The first element in $x_i$ is 1.

In this model, the response of $E \left( y_i | z_i, x_i, \gamma_B, \sigma^2 \right)$ to $z_i$ is non-linear in that it depends upon the value of $z_i$. Specifically,

$$\frac{\partial E \left( y_i | z_i, x_i, \gamma_B, \sigma^2 \right)}{\partial z_i} = \mu + \sum_{j=1}^{B} \delta_j \mathbf{1} \left( z_i > z^{(j)} \right) = \theta_j \left( z_i \right). \quad (2)$$

However, conditional upon being within a segment, the response is linear in that segment. In other words, for $z_i \in S^{(j)}$

$$\frac{\partial E \left( y_i | z_i, x_i, \gamma_B, \sigma^2 \right)}{\partial z_i} = \mu + \sum_{j=1}^{J} \delta_j = \theta_j \quad (3)$$

where $\theta_j = \sum_{j=1}^{J} \delta_j$.

Were the number, $B$, and location of the break points, $z^{(j)}$, known, this specification would constitute a simple regression exercise. However, there is no theoretical guidance as to where the breaks would be or how many there should be, so the parameters $B$ and $z^{(j)}$ must be estimated. Collect all of the break points into a vector $z = \left\{ z^{(1)}, z^{(2)}, \ldots, z^{(B)} \right\}$. Different values of $(B, z)$ define different models and we have only informal guidance on what their values should be. We again take a Bayesian approach as this allows a natural way to incorporate this model uncertainty.

We compare the support for the models by looking at the posterior probability that each model is true. We adopt an inverse gamma prior for each of the $\sigma_j^2$ in $\sigma^2$, and a standard prior for the $k_B \times 1$ vector $\gamma_B \sim N \left( 0, I_{k_B} \sigma_0^2 \right)$. 

9
The $z^{(j)}$ are assumed unknown and we give them a conditional uniform distribution over the area from the previous break point, $z^{(j-1)}$, to the end of the sample. Constraints are imposed to ensure all breaks have non-degenerate support.

We have chosen a piecewise linear form as this will provide us with clear estimates of where the relationship between management score and performance change, and how this relationship changes. However, as any linear combination of piecewise linear forms will approximate to a given degree of accuracy any smooth form, then the mean of all CPLM will be less informative on the important 'change-points' and will approximate just a smooth function. We therefore focus upon the modal model, that is the model $(B, \mathbf{z})$ with the highest posterior probability of being true and estimate of the relationship from this model. To provide a measure of uncertainty about the value in (2), we also report the full posterior of $\theta_j(z_i)$ averaged over all models. We explain this concept in more detail below.

The parameter estimates for the modal model are given in Table 1. The modal model has three breaks, however the first break occurs at the lowest value of $z_i$, $z_i = -1.6461$, which is observed for the first 1.1% of the observations. It is reasonable, therefore, to consider that the function only has two breaks or three pieces to the function. In Table 1, $j$ denotes the segment or piece in the function. So $j = 1$ denotes the region before the first break, $j = 2$ denotes the region after the first break but before the second, and so on. In the second column, $z^{(j)}$ denotes the point at which the function breaks between each piece. The column $E(\theta_j)$ gives the estimate of the term in (3) and the standard deviation of this estimate is given as $\sqrt{Var(\theta_j)}$. The final column gives what looks like a t-statistic and indeed it measures the distance $E(\theta_j)$ is, in standard deviations, from zero.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$z^{(j)}$</th>
<th>$E(\theta_j)$</th>
<th>$\sqrt{Var(\theta_j)}$</th>
<th>$\frac{\theta_j}{\sqrt{Var(\theta_j)}}$</th>
</tr>
</thead>
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<tr>
<td>0</td>
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<td>0.1132</td>
<td>0.0062</td>
<td>9.8973</td>
</tr>
<tr>
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<td>-1.6461</td>
<td>-0.3973</td>
<td>0.1006</td>
<td>-3.9477</td>
</tr>
<tr>
<td>2</td>
<td>-1.4046</td>
<td>0.0346</td>
<td>0.0438</td>
<td>5.6214</td>
</tr>
<tr>
<td>3</td>
<td>0.92845</td>
<td>0.1715</td>
<td>0.0438</td>
<td>3.9193</td>
</tr>
</tbody>
</table>

Table 1: Estimates of break location, $z^{(j)}$, and slope in each range.

We ignore the value of $E(\theta_j)$ in the first row as it plays no role in the model and, for reasons we discuss later, we also do not discuss the results for $z_i < -1.4046$. The largest segment or linear piece in this model extends from
$z_i = -1.4046$ to $z_i = 0.92845$. In this range, covering roughly three quarters of the data, the estimate of the effect of management score on log sales at 0.0346 is very close to the estimates of Bloom and Van Reenen (2007). Above this range the slope of the function is much steeper, 0.1715, suggesting increasing returns to management practices at higher values. In the following discussion we demonstrate that we have a high level of confidence in the general form of this estimated relation over the region $z_i > -1.4046$.

In Figure (3) we again plot the mean and upper and lower bands for the Gaussian process, but this time we also plot the modal estimate of the continuous piecewise linear, $E(y|CPL – mode)$, model. The bounds around $E(y|GP)$ encompass $E(y|CPL – mode)$ for all of the middle and upper region. The left hand part of the $E(y|CPL – mode)$ function begins well above, outside the bounds. We discuss this feature further below and give details now on the estimates from the modal model.

Figure 3: Comparison of the Gaussian Process and modal model from the set of Continuous Piece-Wise Linear Models, $CPL – mode$.

The above results relate to the *a posteriori* most probable, or modal, model. As mentioned earlier, there is uncertainty about which is the true
model such that the estimates from one model will under-represent the true uncertainty about the estimate. Generally, there is information to be gained from considering all estimated models and so to incorporate this evidence we use Bayesian model averaging (BMA). We can use BMA to obtain an estimate of, for example, the expected level of log sales for different values of the management score, as presented in Figures (2) and (3). In contrast to these earlier results, the estimate from BMA will not depend upon a single model. BMA has been used extensively in a range of applications in, for example, finance and macroeconomics (see, e.g., Koop, Potter and Strachan (2008), Koop, Léon-Gonzalez and Strachan (2012) and Strachan and Van Dijk (2013)).

We provide a brief overview of BMA, further details can be obtained from a range of textbooks such as Koop (2003). First let there be $M$ models and denote model $s$ by $M_s$. We compute the posterior probability that model $M_s$ is correct, $P(M_s|y)$.$^4$ Next compute the value of $E_s(y_i) = E(y_i|x, x_i = \bar{x}, \gamma_B, \sigma^2, M_s)$ for this model. Denote the posterior density of $E_s(y_i)$ from model $M_s$ by $p(E_s(y_i)|M_s, y)$. This value $E_s(y_i)$ has an interpretation that is constrained because it is only valid for model $M_s$. If $M_s$ is not a good model then it may be that $p(E_s(y_i)|M_s, y)$ is a very good representation of the expected of log sales for a given level of management practices. To remove this dependence on $M_s$, we use BMA to average the $E_s(y_i)$ from all models according to the weights or probabilities associated with each model. The BMA estimate of the posterior density of $E_s(y_i)$ is then

$$p(E(y_i)|y) = \sum_{i=1}^{M} p(E_s(y_i)|M_s, y) P(M_s|y).$$

Note that this output still depends upon the data $y$ which is desirable, but does not depend upon any one model, also an attractive feature. Rather than simply reporting the mean, we can also report the full distribution of $E(y_i)$ averaged over all models. Figure (4) plots the estimate $E_s(y_i)$ from the modal model (again denoted as $E(y_i|CPL - mode)$) against the 5th, 50th, and the 95th percentiles of the distribution of $E(y_i)$ averaged over all models, $p(E(y_i)|y)$. We see that the modal value and the percentiles are similar

$^4$A range of sophisticated approaches exist to estimate $P(M_s|y)$. In our model we have a closed form expression and so can compute this probability exactly.

$^5$Recall that as this function depends upon the parameters $\gamma_B$ and $\sigma^2$ which are random, then the function itself is random and has a distribution.
around the middle and upper regions of $z_i$, above about $z_i = -1.25$. This suggests that the densities of many of our models are located around the same area and so we can have a high degree of confidence in how log sales will respond to increases in the management score over this range.

There is considerable divergence, however, at lower values of $z_i$. Given that there are far fewer observations of $z_i$ at these lower values (see the bottom left panel in Figure (1)), it is not surprising that there is much more uncertainty about the response of log sales to increases of the management score at these low levels than in the middle. However, although there are more than at lower values, there are also few observations at high values of $z_i$ where the function is accurately estimated. One possible explanation for this difference is that the model that is not performing well at the lower values. Explaining the relationship in this region we therefore leave to further research.

Figure 4: In this figure are the modal estimate of the CPL model, and the BMA median and 5th and 95th percentile values of the distribution of $E(y_i)$ averaged over all models.

We conclude with a visual diagnostic of how well the continuous piecewise
linear model (CPLM) has performed. To do this, we compare the location of the posterior density \( p(E(y_i)|y) \) from the CPLM to the location of the density for the same parameters from the Gaussian Process which assumed no functional form beyond there being a smooth relationship. Figure (5) plots the 5\(^{th}\) and 95\(^{th}\) percentiles of the distributions of \( E(y_i|GP) \) from the Gaussian process (dashed lines) and the BMA estimate \( E(y_i|CPLM) \) of all of the CPL models (the solid lines). These agree very closely except at the very low values of \( z_i \). This agreement suggests that the simpler model, the CPL, captures the general form of the relationship well compared to a much more relaxed or general form as implied by the Gaussian process.

![Figure 5](image)

**Figure 5:** This figure plots the 5\(^{th}\) and 95\(^{th}\) percentiles of the response of log sales to the management index for the Gaussian process model and the CPL model averaged over all models by BMA.
4 Conclusion

The World Management Survey research agenda has been successful in demonstrating that good management is, to some degree, measurable and that the measured management has a significant impact on economic output. Motivated by theory, we investigated the possibility of a non-linear relationship between management and output. We found significant evidence of a U-shaped relationship from the graphical median bands plot between TFP and management; from a very flexible Gaussian processes model which allowed each firm to have its own output related function of management (with these functions varying smoothly between close firms) and finally from a piecewise-linear approach with probabilistically smoothed endogenous breaks.

All these approaches allowed sufficient flexibility to avoid our conclusions being driven by assumed functional forms. Taken together the evidence suggests that the original finding of [2], that management has a positive impact on output is broadly correct. Rather importantly, the returns to improving management for most of the sample are relatively flat compared to the very steep, positive returns for firms with the high existing levels of management. These results are consistent with theories of managerial complementarities, but are also consistent with management as a (approximately) weakest link or superstar technology. A weakest link technology would mean that the highest returns would require all the practices to improve together. The literature on superstars emphasizes the returns to performance can be highly non-linear with the classic example being the (almost) winner-takes-all nature of sporting competitions. To the degree that sales reflects innovation and quality, better management might be reflected in increased benefit per unit, not reduced cost per unit.

The U-shaped relationship also helps reconcile the findings on the impact of management with the large literature on persistent performance differences (see [4] for a survey). If firms are only able to make incremental improvements in management due perhaps to change costs or institutional constraints then firms with low and medium management scores will have much less incentive to invest in change than high management score firms and as a result the status quo will be maintained, i.e. top performers will tend to stay ahead. This area of empirical research is in its infancy and there is ample opportunity for new studies and surveys to throw further light on the considerable complexities of the internal organization of firms.
References


