Incentivizing Negative Emissions Through Carbon Shares*

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I analyze a novel climate policy instrument that attaches a transferable asset to each unit of carbon in the atmosphere. I show that this instrument improves on an emission tax by incentivizing both optimal emission reductions and optimal removal of past emissions. Emitters post a bond equal to the worst-case social cost of carbon, and the regulator deducts damages as they are realized over time. Quantitatively, a bond that is double the optimal emission tax is sufficient to provide optimal carbon removal incentives in 95% of cases.

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1 Introduction

Countries and corporations are increasingly adopting ambitious goals of eliminating or offsetting all greenhouse gas emissions by the middle of the century, if not sooner.[1] What happens once these targets are reached? Harm from carbon dioxide will not cease at this point, as earlier emissions will remain in the atmosphere. The role for policy should also not cease: it is unlikely that the optimal policy would stop precisely at zero emissions, forgoing additional use of the several technologies for removing carbon dioxide from the atmosphere.[2] In fact, many models suggest that achieving global temperature targets will require negative emissions over the latter part of the century (e.g., Clarke et al., 2014; Rogelj et al., 2015, 2018; Hilaire et al., 2019; Realmonte et al., 2019). But is unclear how to implement such pathways through market-based mechanisms.

The standard economic prescription for climate change requires taxing emissions (or, equivalently for present purposes, capping emissions) so that market actors account for the external costs that their emissions impose through global climate change.[3] However, an emission price contains a sharp discontinuity: it can incentivize emission reductions up to the point at which there are no further emissions from the present period, but it cannot incentivize further spending to remove emissions from past periods. Governments could directly subsidize carbon removal, but doing so could be prohibitively expensive, with costs potentially exceeding even the share of U.S. output spent on defense (Bednar et al., 2019).

Negative emissions need pose no problems if policy recognized that the social harm from carbon dioxide follows not from its emission but from the choice to leave it in the atmosphere. A carbon stock tax would charge firms period-by-period as long as their emissions remain

[1] As of April 23, 2021, 44 countries and the European Union (covering 70% of carbon dioxide emissions) had announced net-zero emission targets, with 10 of those countries having made the target a legal obligation (IEA, 2021).

[2] Carbon dioxide removal, or negative emission, strategies include chemically separating carbon dioxide from air ("direct air capture"), capturing emissions from power plants that burn biomass ("bioenergy with carbon capture and storage"), accelerating the weathering of rocks, enhancing uptake of carbon by forests or oceans, and more. See National Research Council (2015), Fuss et al. (2018), and National Academies of Sciences, Engineering, and Medicine (2018) for recent reviews. There is unlikely to be a discontinuity in costs at zero emissions because zero emission targets are unlikely to be achieved without substantial use of carbon removal.

[3] Much work has also discussed how the first-best policy is in fact a portfolio that also includes policies such as R&D subsidies that account for other market failures (e.g., Fischer and Newell, 2008; Nordhaus, 2008; Acemoglu et al., 2012; Lemoine, 2020). Most see Pigouvian emission pricing as critical to that portfolio. I here emphasize that corrective emission price.

in the atmosphere, with optimal charges equal to realized, contemporary marginal damage. This policy explodes the emission tax into its constituent strip of period-by-period marginal damages. Critically, it treats current and past emissions symmetrically, so it provides incentives not just to avoid emitting but also to remove old emissions from the atmosphere.

However, a carbon stock tax requires that emitters survive until removing their emissions becomes optimal, which could be decades after the time of emission. Further, it requires today's emitters to anticipate surviving indefinitely so that they internalize future charges when choosing current emissions. In reality, substantial market churn is likely over several decades, even for large energy firms. This problem is a version of judgment-proofness (Shavell, 1986): past emissions are a pure liability, potentially shed through firm death or bankruptcy. This possibility distorts the incentive to emit and can destroy the incentive to remove past emissions from the atmosphere.[4]

To avoid such problems, I propose a new type of policy. Each emitter posts a bond and receives an asset, called a carbon share, attached to the unit of emission. The emitter can choose whether to retain or sell its carbon share. Initially, the face value of the carbon share is the bond. In each subsequent period, the regulator pays a dividend to the holder of the share and deducts both that dividend and a damage charge from the face value of the share. The policy never requires revenue to be raised from taxpayers, instead generating public funds through the damage charges. If the owner of a share ever removes the unit of carbon attached to it, then the owner receives the remaining face value and the share is retired. In essence, the share is an option to recover the remaining face value, with the strike price being the cost of carbon removal. This policy converts past emissions into a valuable asset that investors want to own, whether or not the original emitter continues to exist.[5]

I show that the optimal carbon share policy combines the first-best emission and removal incentives of the idealized stock tax with the judgment-proof upfront payments of the emission tax. The optimal policy sets the initial bond equal to the worst-case social cost of carbon emissions and sets each period's damage charge equal to the marginal damage incurred in the same period, which was also the optimal stock tax.[6] The dividends return

[4]Plus there could be principal-agent problems preventing firms from fully internalizing charges that will arise decades down the line.

[5]In Section 4, I discuss how to implement carbon shares via caps on cumulative emissions (a quantity policy) rather than damage charges (a price policy).

[6]A policymaker could generate empirically grounded estimates of same-period marginal damages by combining the costs of realized weather events with attribution studies of how climate change altered the

the difference between an updated estimate of the worst-case social cost of carbon and the previous estimate. The shareholder thus receives substantial dividends if climate change turns out to impose small costs and few dividends if climate damages turn out to be large. Shareholders remove their carbon in order to recover the stream of future damage charges. They therefore weigh the cost of carbon removal against the expected remaining marginal harm from atmospheric carbon, as required for first-best carbon removal. Emitters pay the worst-case social cost of carbon but receive a valuable asset in return. I show that their net outlays are equal to the expected social cost of carbon, as required for first-best emissions.

I quantitatively assess the benefits of the proposed policy within a conventional economic model of climate change. I assume that the true costs of climate change are initially unknown but revealed in 2065. Their variance is calibrated to a recent expert survey (Pindyck, 2019). The first-best policy nearly always uses negative emissions at some future time and may even do so in 2065. If the policymaker required a bond double what the year 2015 emission tax would have been, then the policymaker could fund the ex post optimal series of charges in over 95% of damage realizations. By enabling negative emissions, the carbon share policy provides 10% more value than does an emission tax policy.

The recommendation to address climate change through Pigouvian emission pricing dates back to at least Nordhaus (1977). In fact, Nordhaus (1977) observes that there are two strategies for controlling carbon dioxide: reducing emissions and cleaning old emissions from the atmosphere. He restricts attention to the first in order "to avoid the odor of science fiction" (pg 343). More recently, Nordhaus (2019) evaluates carbon removal as unavailable at both scale and reasonable cost. Much other literature is more optimistic about the costs and scalability of carbon removal technologies, with several climate-economy models showing heavy use of such technologies around midcentury (e.g., Obersteiner et al., 2001; Azar et al., 2010; Clarke et al., 2014; Rogelj et al., 2015, 2018; Hilaire et al., 2019; Realmonte et al., 2019). Further, Microsoft and Stripe each recently committed to paying for carbon removal services. Despite the increasingly prominent discussion of carbon removal, I know of no work on market-based approaches to incentivizing optimal use of these technologies.[7] In the

weather events' probability. Whereas emission taxes require estimating expected marginal damage in all future periods, the carbon share policy's damage charges can be grounded in empirical analyses of realized losses.

[7]Conventional emission pricing policies could incentivize use of carbon dioxide removal technologies up to the point at which net emissions are zero. However, the European Union's flagship cap-and-trade program does not in practice provide the credits for carbon dioxide removal that could sustain even this limited

absence of alternative policy instruments, many assume that attaining aggregate negative emissions would require direct government subsidization of carbon removal, despite concerns about the fiscal burden such subsidies would impose (e.g., Bednar et al., 2019; Edenhofer et al., 2021). I investigate how to provide optimal incentives for carbon removal without requiring government expenditure.

Although climate change policy has been almost exclusively focused on controlling the injection of pollution into the atmosphere, the broader environmental policy literature grapples with the need to clean up pollution that has already been released.[8] This literature has discussed bonding and deposit-refund schemes as solutions to the problem of monitoring improper waste disposal (e.g., Bohm, 1981; Russell, 1987; Fullerton and Kinnaman, 1995; Torsello and Vercelli, 1998).[9] I construct a new type of dynamic refund instrument and obtain a sharp result: the required bond (or deposit) should be set to the worst-case social cost of carbon. Others have previously proposed that fees on materials or products be set to their most harmful possible environmental fate, with fees refunded in accord with the harmfulness of actual outcomes (e.g., Solow, 1971; Mills, 1972; Bohm and Russell, 1985; Costanza and Perrings, 1990; Boyd, 2002). These informal proposals employ arguments ranging from ambiguity aversion to difficulties in monitoring pollution to judgment-proofness. The long timespans over which carbon emissions affect the atmosphere make the judgment-proofness argument especially salient here. I formally show how the worst-case bond can be used to finance a transferable asset that reduces the bond's upfront cost to emitters, does not burden the regulator with cleaning up past emissions in the event that emitters forsake the bond, and provides first-best incentives for both emission and cleanup. This new policy should improve outcomes in other applications with stock externalities.[10]

The next section contains the theoretical analysis. Section 3 quantitatively explores

incentive (Scott and Geden, 2018; Rickels et al., 2020).

[8]Stock taxes have been proposed in the context of climate change (Lemoine, 2007), mine remediation (White et al., 2012; Yang and Davis, 2018), and space orbits (Rao et al., 2020).

[9]Deposit-refund schemes have also been understood as means to avoid the fiscal costs of subsidies and the distributional costs of taxes (Bohm, 1981). Here the motivation is to overcome an inefficiency in conventional tax policies without incurring additional fiscal costs from using the public purse to directly fund carbon removal.

[10]For instance, satellite owners could post a bond to fund an "orbital-use share" that would incentivize both optimal debris creation and optimal debris cleanup. Fees for launching satellites are the analogue of an emission tax. They fail to incentivize either active measures to avoid creating debris or cleanup of debris. Rao et al. (2020) propose orbital-use fees that, as a stock tax, are the analogue of the atmospheric rental policy discussed here. Orbital-use shares have the advantage of avoiding problems induced by market churn.

carbon shares. The final section discusses additional aspects. The appendix details the numerical model, contains additional theoretical results, and discusses political risk.

2 Theoretical Analysis

Consider a world with many small firms and infinitely many periods. Index firms by i, and normalize total firms to be of measure 1. Firm i's business-as-usual emissions in period t are $e_{it} > 0$. Firm i can choose to eliminate quantity $A_{it} \leq e_{it}$ of time t emissions before they reach the atmosphere. Abatement cost $C_{it}(A_{it})$ is strictly increasing and strictly convex, with $C'_{it}(0) = 0$ for convenience (where primes indicate derivatives). Each firm can also fund the removal of quantity $Z_{it} \geq 0$ of emissions from the atmosphere. It purchases this emission removal from a competitive industry with aggregate cost curve $G_t(Z_t)$, with $Z_t \triangleq \int_0^1 Z_{it}\,\mathrm{d}i$ and $G_t(\cdot)$ strictly increasing and strictly convex in Z_t.[11] Firms seek to minimize their expected present costs, subject to current and anticipated policies and discounted at per-period rate r.

A regulator begins to implement policy in period 0. Let cumulative emissions up to time t be $M_t = M_{t-1} + \int_0^1 \left[e_{i(t-1)} - A_{i(t-1)} - Z_{i(t-1)}\right] \mathrm{d}i$, with pre-policy cumulative emissions $M_0 \geq 0$ given. Time t warming is $T_t = \alpha \left[M_t + \int_0^1 (e_{it} - A_{it} - Z_{it})\,\mathrm{d}i\right]$. This representation recognizes that carbon dioxide is a globally mixed pollutant and follows recent scientific findings that global temperature is approximately a linear function of cumulative emissions (see Dietz and Venmans, 2019, among others).[12] Social damages from warming in period t are $D_t(T_t; \tilde{d}_t)$, with $D_t(\cdot; \tilde{d}_t)$ strictly increasing and weakly convex in T_t and $D'_t(0; \tilde{d}_t) = 0$. The value of the Markovian random variable $\tilde{d}_t$ is public knowledge in period t, with $D'_t(T_t; \tilde{d}_t)$ finite in expectation. The regulator chooses period t policy to minimize expected present social costs, with per-period discount rate r.

Define the time t social cost of carbon scc_t as

$$scc_t \triangleq \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} D'_{t+s}(T_{t+s}),$$

[11] This convexity reflects both the cost of removing carbon from the atmosphere and the potential scarcity of sites for storing carbon after removal.

[12] Allowing time t emissions to affect temperature only with a lag would not qualitatively change the results.

given trajectories for A_{it} and Z_{it}.[13] And define the worst-case social cost of carbon $\overline{scc}_t$ as

$$\overline{scc}_t \triangleq \sup_{\{\tilde{d}_{t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \alpha [D'_{t+s}(T_{t+s})].$$

2.1 First-Best Abatement and Emission Removal

Consider the first-best allocation of abatement and emission removal. In period t, a regulator who can directly prescribe firms' decisions solves

$$V_t^{fb}(M_t, \tilde{d}_t) = \min_{A_{it} \leq e_{it}, Z_{it} \geq 0} \left\{ \int_0^1 C_{it}(A_{it}) \, \mathrm{d}i + G_t(Z_t) + D_t(T_t; \tilde{d}_t) + \frac{1}{1+r} E_t \left[V_{t+1}^{fb}(M_{t+1}, \tilde{d}_{t+1}) \right] \right\},$$

where E_t indicates expectations at the time t information set. Repeatedly applying the envelope theorem,

$$\frac{\partial V_t^{fb}(M_t, \tilde{d}_t)}{\partial M_t} = E_t[scc_t^{fb}],$$

where scc_t^{fb} is evaluated along the first-best pathway for A_{it} and Z_t. At an interior solution, standard first-order conditions imply

$$C'_{it}(A_{it}) = E_t[scc_t^{fb}], \tag{1}$$
$$G'_t(Z_t) = E_t[scc_t^{fb}]. \tag{2}$$

These conditions equate the marginal private cost of abatement and emission removal to their marginal social benefits, as is familiar.[14]

[13] Some reserve "social cost of carbon" for a no-policy pathway. I here use it to refer to social cost along some particular pathway for abatement and emission removal.

[14] One might wonder why the marginal cost of abatement is equated to the sum of future marginal damages out to an infinite horizon if there is a chance of removing a unit of today's emissions at some future time. The reason is that such removal is not free: by equation (2), optimal use of emission removal equates its marginal cost to the sum of marginal damage over all remaining periods. Equating current marginal abatement cost to the sum of all future marginal damage thus incorporates both expected realized marginal damage and expected future spending on emission removal.

2.2 Conventional Policy Instruments

How can a regulator incentivize firms to undertake optimal emission reductions and optimal emission removal? Economists typically propose to price emissions by taxing them or by limiting their quantity in a cap-and-trade program. The optimal charge per unit of time t emissions is $E_t[scc_t^{tax}]$, the expected present value of marginal damage along the path induced by the optimal emission charge (see Appendix C.1). This is the familiar Pigouvian emission tax (e.g., Nordhaus, 1982, 1991; Farzin, 1996). It achieves first-best abatement and carbon removal defined by (1) and (2) as long as negative emissions could never be optimal. But if negative emissions turn out to be optimal ex post, the tax can incentivize firms to eliminate emissions only up to the point at which they have no more emissions to tax.[15]

The regulator could implement the first-best allocation if it could pair the emission tax with payments for emission removal, priced at $E_t[scc_t^{fb}]$. However, the magnitude of required payments is potentially enormous, exceeding emission tax revenues many times over (Bednar et al., 2019).[16] These payments would be especially burdensome if, as Edenhofer et al. (2021) fear, they are financed through distortionary taxation.

2.3 A New Instrument: Carbon Shares

I now propose a new type of policy instrument and show that it can incentivize optimal emissions and optimal emission removal while also raising revenue for the public purse. This policy requires an emitter to post a bond B_t per unit of time t emissions. The bond finances a transferable asset that the emitter receives from the regulator. This asset is attached to the unit of carbon emitted. I refer to the asset as a carbon share because it reflects a claim on a part of the carbon in the atmosphere. The face value of the carbon share in each period $t + s$ is $\theta_{t,t+s}$, with $\theta_{t,t} = B_t$. In each period subsequent to emission, shareholders decide whether to leave the unit of carbon in the atmosphere. If they remove it from the atmosphere in time $t + s$, they receive $\theta_{t,t+s}$ and return the share; otherwise they receive a

[15] Further, Appendix C.1 shows that $E_t[scc_t^{tax}] > E_t[scc_t^{fb}]$ if the negative emission constraint might eventually bind and damages are strictly convex. The optimal emission tax is distorted in earlier periods in order to offset the cost of the constraint.

[16] In particular, emission removal is likely to be needed at scale precisely when early estimates of expected marginal damage (which integrate over the possible $\tilde{d}_t$ and determine early periods' optimal emission tax) end up much smaller than later estimates of marginal damage (which determine optimal deployment of emission removal) because $\tilde{d}_t$ turned out large.

dividend $\delta_{t,t+s}$ and keep the share. The policymaker can also charge $\kappa_{t,t+s}$ to the face value of the share. I refer to the $\kappa_{t,t+s}$ as "damage charges" for reasons that will become clear. The face value evolves as $\theta_{t,t+s+1} = (1+r)(\theta_{t,t+s} - \delta_{t,t+s} - \kappa_{t,t+s})$. The policymaker cannot return or deduct any more than the current value of the share: $\delta_{t,t+s} + \kappa_{t,t+s} \leq \theta_{t,t+s}$. The policymaker must eventually allocate the entire original bond to either dividends or declared charges: $\lim_{s\to\infty} \theta_{t,t+s} = 0$.

The carbon share is an option to obtain the face value by spending on carbon removal. The option's holder receives the dividends $\delta_{t,t+s}$ whether exercising or holding the option, but the option's holder loses the charges $\kappa_{t,t+s}$ as long as the option is unexercised. The option's value is $\Omega_{t,t+s}$. Clearly, $\Omega_{t,t} \leq B_t$ and $\Omega_{t,t+s} \geq 0$. At the time of emission, the firm's net outlays per unit of non-abated emissions are $B_t - \Omega_{t,t} \geq 0$. If a firm that held a carbon share were to declare bankruptcy or otherwise liquidate, its creditors would want the carbon share so they could receive its dividends and have the option to eventually reclaim its face value.

The benefit from exercising the option to remove carbon in period $t+s$ is

$$\theta_{t,t+s} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_{t+s}[\delta_{t,t+s+j} + \kappa_{t,t+s+j}]. \tag{3}$$

The cost of exercising the option is the cost p_{t+s} of removing the unit of carbon plus the cost $E_{t+s}[\Omega_{t,t+s+1}]/(1+r)$ of losing the option in the future plus the cost of not receiving the dividend $\delta_{t,t+s}$. In a competitive equilibrium with abundant carbon shares, shareholders exercise their options up to the point at which the cost of removal absorbs the profits from exercise:

$$p_{t+s} = \theta_{t,t+s} - \frac{1}{1+r} E_{t+s}[\Omega_{t,t+s+1}] - \delta_{t,t+s}. \tag{4}$$

Agents may not compete away the entire face value of the carbon share (i.e., $p_{t+s} \leq \theta_{t,t+s} - \delta_{t,t+s}$) because they must be compensated for forgoing the right to exercise the option in future periods.

The following lemma establishes the equilibrium value of the carbon share:

Lemma 1. *In a competitive equilibrium,*

$$\Omega_{t,t+s} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_{t+s}[\delta_{t,t+s+j}]. \tag{5}$$

Proof. Conjecture that the value of the carbon share depends linearly on each $E_{t+s}[\delta_{t,t+s+j}]$ and $E_{t+s}[\kappa_{t,t+s+j}]$:

$$\Omega_{t,t+s} = \sum_{j=0}^{\infty} \Lambda_{t+s,t+s+j} E_{t+s}[\delta_{t,t+s+j}] + \sum_{j=0}^{\infty} \Phi_{t+s,t+s+j} E_{t+s}[\kappa_{t,t+s+j}],$$

for unknown sequences $\{\Lambda_{t+s,t+s+j}\}_{j=0}^{\infty}$ and $\{\Phi_{t+s,t+s+j}\}_{j=0}^{\infty}$, with the first subscript corresponding to the evaluation period and the second subscript corresponding to the period in which the dividend is received or the charge is incurred. The constant is zero because $\Omega_{t,t+s} \to 0$ as $\theta_{t,t+s} \to 0$. If the option is exercised in period $t+s$, its value is

$$\Omega_{t,t+s} = \theta_{t,t+s} - p_{t+s}.$$

Using equation (4) and substituting for $\Omega_{t,t+s+1}$, we find

$$\Omega_{t,t+s} = \delta_{t,t+s} + \frac{1}{1+r} \sum_{j=1}^{\infty} \Lambda_{t+s+1,t+s+j} E_{t+s}[\delta_{t,t+s+j}] + \frac{1}{1+r} \sum_{j=1}^{\infty} \Phi_{t+s+1,t+s+j} E_{t+s}[\kappa_{t,t+s+j}].$$

(Note that this condition is identical to the condition that holds if an option is optimally not exercised in period $t+s$.) Matching coefficients, $\Lambda_{t+s,t+s} = 1$, $\Phi_{t+s,t+s} = 0$, $\Lambda_{t+s,t+s+j} = \Lambda_{t+s+1,t+s+j}/(1+r)$, and $\Phi_{t+s,t+s+j} = \Phi_{t+s+1,t+s+j}/(1+r)$ for $j \geq 1$. Advancing the analysis by one timestep, we find $\Lambda_{t+s+1,t+s+1} = 1$ and $\Phi_{t+s+1,t+s+1} = 0$. Therefore $\Lambda_{t+s,t+s+1} = 1/(1+r)$ and $\Phi_{t+s,t+s+1} = 0$. The lemma follows from repeating these steps for later time periods. □

The equilibrium value of the carbon share is the expected present value of the dividends that it claims. Using equations (3) and (5) in (4), we find

$$p_{t+s} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_{t+s}[\kappa_{t,t+s+j}]. \tag{6}$$

Exercising the carbon share asserts a claim to the present value of expected remaining damage charges, but the equilibrium cost of exercising the share absorbs these benefits. The value of the carbon share in (5) does not directly reflect future damage charges because their value gets competed away in equilibrium. However, we will soon see that the value of the share does directly reflect good news about damages when dividends are set optimally.

We now come to the main result:

Proposition 1. *The carbon share policy achieves first-best abatement and emission removal if M_0 is sufficiently small, $B_t \geq \overline{scc}_t^{fb}$, and*

$$\kappa_{t,t+s} = \alpha[D'_{t+s}(T_{t+s})]. \tag{7}$$

Proof. Because $\delta_{t,t+s} + \kappa_{t,t+s} \leq \theta_{t,s+s}$, (7) requires

$$\theta_{t,t+s} \geq \sup_{\{\tilde{d}_{t+s+j}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \alpha[D'_{t+s+j}(T_{t+s+j})]$$

for all $s \geq 0$, which implies

$$B_t \geq \sup_{\{\tilde{d}_{t+j}\}_{j=0}^{\infty}} \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \alpha[D'_{t+j}(T_{t+j})]. \tag{8}$$

Substituting for equilibrium p_{t+s}, equation (6) implies that, as long as there are carbon shares outstanding,

$$G'_{t+s}(Z_{t+s}) = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_{t+s}[\kappa_{t,t+s+j}]. \tag{9}$$

Comparing to (2), time $t+s$ emission removal is first-best if $\kappa_{t,t+s+j}$ is as in (7) and later abatement and removal will be first-best. The cost of emitting in period t is $B_t - \Omega_{t,t}$, which from (5) is

$$B_t - \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t[\delta_{t,t+j}].$$

Combining this, $\lim_{s\to\infty}\theta_{t,t+s}=0$, and (8), we have:

$$B_t - \Omega_{t,t} = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t[\kappa_{t,t+j}]. \tag{10}$$

Comparing to (1), time t abatement is first-best if $\kappa_{t,t+j}$ is as in (7) and later abatement and removal will be first-best.

We have shown that time t abatement and removal are first-best if some shares are outstanding and later periods' abatement and removal will be first-best. By induction, time t abatement and removal are first-best if some shares will always be outstanding. If M_t is sufficiently small, then some emissions will not be abated because $D_t'(0;\tilde{d}_t)=0$ and $C_{it}'(0)>0$. So some shares are initially issued if M_0 is sufficiently small. Once shares are issued, some of them will never have their underlying emissions removed if M_0 is sufficiently small because $D_t'(0;\tilde{d}_t)=0$ and $G_t'(0)>0$. Thus if M_0 is sufficiently small, then there are always outstanding shares. In that case, the carbon share policy achieves first-best and inequality (8) becomes $B_t \geq \overline{scc}_t^{fb}$. □

The optimal carbon share policy achieves first-best as long as it is begun before too many emissions have accumulated (i.e., while M_0 is small). Waiting to replace an emission tax with carbon shares dilutes the gains from using carbon shares. Interpreting (7), the regulator should deduct the current period's marginal damage from the face value of any share attached to units of carbon that remain in the atmosphere.[17] This current-period marginal damage is in principle observable in data, in contrast to the stream of future marginal damage required for the optimal emission tax. The initial bond should be at least as large as the worst-case social cost of carbon. The dividends can be structured in any fashion so long as they do not reduce the face value of the share below the current estimate of the worst-case social cost of carbon. The dividend plan that returns the bond to shareholders in the most rapid fashion refunds the change in the worst-case social cost of carbon from period to period.[18] From (5), the value of the carbon share is then the difference between the worst-case social

[17]The optimal time $t+s$ damage charge is the same for all emission vintages t because damages here depend only on cumulative emissions.

[18]If the worst-case social cost of carbon is calculated correctly, then its value must weakly decline from period to period. If the worst-case social cost of carbon somehow increased over time, then the regulator could require shareholders to increment the posted bond. The regulator in essence marks the face value of the carbon share to the evolving worst-case damage estimate.

cost of carbon and the expected social cost of carbon. The value of holding a share derives from the possibility that some damages will never be realized, with the loss from emitting (in (10)) just the expected social cost of carbon.

A carbon share policy provides the same incentives as a tax on the stock of carbon,[19] but whereas a carbon stock tax can be avoided by declaring bankruptcy, carbon shares are valuable assets that investors want to hold. Each asset is financed at the time of emission by the initial bond. The potentially large bond does not distort firms' emission incentives because the value of the carbon shares that firms receive increases in the size of the bond. One might be concerned that the initial bond would challenge firms' liquidity (see Shogren et al., 1993). Note, however, that firms can immediately sell a new carbon share on. From (10), their net outlays per unit of emissions are the exact same outlays required by the traditional Pigouvian carbon tax. The carbon share policy therefore need not be any more financially challenging than a conventional carbon emission tax.[20]

3 Quantitative Evaluation

I now quantitatively assess the level of the optimal bond and the gains from using carbon shares instead of emission taxes. I extend the DICE-2016R climate-economy model of Nordhaus (2017) to allow for uncertainty about damages from warming.[21] Prior to 2065, the damage parameter is fixed and negative emissions are not allowed. In 2065, a random component of the damage parameter is realized and negative emissions become feasible. I calibrate the variance of damages to the expert survey of year 2066 losses from climate change in Pindyck (2019), following the implementation in Lemoine (2021) that adjusts for uncertainty about warming. In one case ("DICE Damages"), I fix the mean of the distribu-

[19] The optimal time $t+s$ stock tax on emissions from time t is $\kappa_{t,t+s}$ (see Appendix C.2).

[20] Gross outlays are also capped, for two reasons. First, the magnitude of the optimal bond is capped as long as optimal policy would avoid incurring very large marginal damage by removing sufficient carbon. Second, any firm could avoid posting the bond by reducing its emissions. The growing number of firms making zero emission pledges and recent cost projections for removal technologies both suggest that even the maximum gross outlays are limited to a reasonable scale.

[21] Because it allows abatement to exceed 100%, the abatement cost function in DICE-2016R implicitly accounts for carbon dioxide removal technologies. I maintain this cost function and focus on uncertainty about damages. Allowing for the possibility of cheaper carbon removal would increase the relative benefits of the carbon share policy. A full analysis would incorporate uncertainty about these costs and about other parameters, including those controlling economic growth and the sensitivity of the climate to emissions. This first analysis builds on evidence that uncertainty about damages is especially important (Lemoine, 2021).

tion to match damages in DICE-2016R, and in the other case ("Expert Damages"), I allow the mean to also be determined by the expert survey. The latter case implies more severe losses from warming. Appendix B provides the full equations and parameterization. It also plots optimal trajectories in a deterministic version of each calibration.

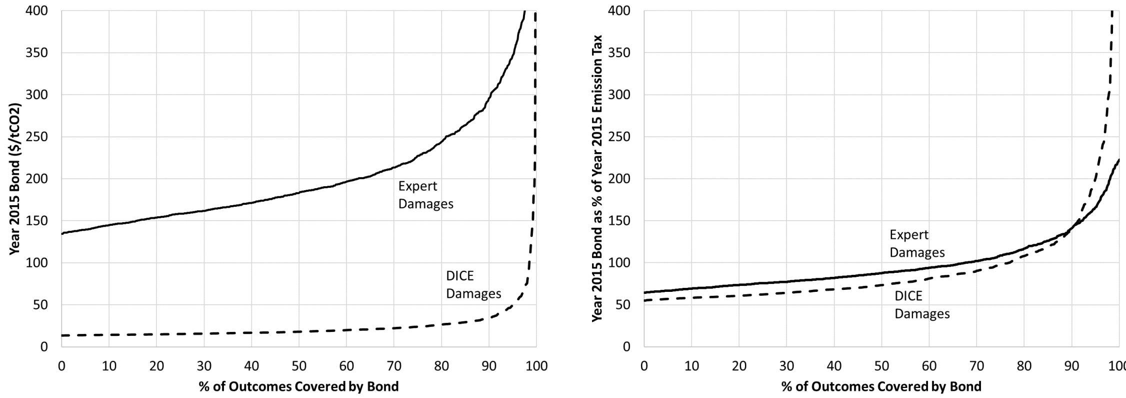
Negative emissions are relevant. In the case with DICE damages, emissions are negative in 2065 in 2.6% of cases, emissions are negative at some point after 2065 in 95% of cases, and those negative emissions are substantial enough to eventually remove some pre-2065 emissions in 71% of cases. In the case with expert damages, emissions are negative in 2065 in 48% of cases (including at the mean—see Appendix B), are always eventually negative at some point after 2065, and are nearly always substantial enough to eventually remove some pre-2065 emissions.

Figure 1 plots the percentage of cases in which bonds of varying sizes end up being large enough to fund the ex-post optimal sequence of damage charges $\kappa_{t,t+s}$. The left panel shows that the case with expert damages requires much larger bonds, reflecting its much larger emission charges. A bond of 300 $/t$CO_2$ covers 90% of outcomes under expert damages, whereas a bond of 50 $/t$CO_2$ covers 95% of outcomes in the case with DICE damages (all in year 2015 dollars). The right panel plots these same bonds as a percentage of the optimal year 2015 optimal emission tax, which is 210 $/t$CO_2$ in the case of expert damages and 25 $/t$CO_2$ in the case of DICE damages.[22] The two curves track each other remarkably closely until we get to the very highest damage realizations. Because the damage parameter is distributed lognormally, a bond equal to the initial emission tax covers the stream of damage charges more than 50% of the time. Requiring that firms post a bond equal to twice what the initial emission tax would have been has a better than 95% chance of covering the stream of optimal damage charges.[23]

Table 1 reports the balanced growth equivalent (BGE) increase in consumption from implementing policy (Mirrlees and Stern, 1972). Policy is far more valuable in the calibration to expert damages, providing expected benefits equivalent to a permanent 40% increase

[22]The optimal tax with DICE damages is slightly below the optimal tax of 34 $/t$CO_2$ from DICE-2016R (in year 2015 dollars), primarily because I update the carbon cycle and climate system in accord with recommendations in Dietz et al. (2020). With either damage model, the initial period's optimal tax under uncertainty is very close to the optimal tax without uncertainty.

[23]Experiments with DICE damages and a smaller utility discount rate of 0.1% per year (as in Stern, 2007) suggest that both this result and the expected loss from being constrained to weakly positive emissions are robust to that discount rate.



(a) Bond ($\$/tCO_2$) (b) Bond (as % of initial tax))

Figure 1: The percentage of cases in which a given bond is large enough to cover the ex-post optimal damage charges, with the bond measured in $\$/tCO_2$ (left) and as a percentage of the year 2015 emission tax that would be optimal in the absence of the bond (right).

in consumption as opposed to a permanent 1.5% increase in consumption. The second row constrains the policymaker from obtaining negative emissions in any period, as when implementing policy through an emission tax.[24] This constraint imposes expected losses of 7–9%. These losses likely understate the benefits from enabling negative emissions because they do not account for uncertainty about warming or for uncertainty about the cost of carbon removal. By enabling negative emissions, a carbon share policy can substantially increase the benefits of climate policy.

4 Discussion

I have analyzed a new type of climate change policy that replaces an emission tax with a bond used to fund a financial asset called a carbon share. I show that the bond should be set equal to the worst-case social cost of carbon, the share should pay out the difference between worst-case and realized damages as dividends, and the share's face value should be reduced for realized climate change damages and dividend payments. The share's remaining face value is refunded if a shareholder removes the underlying unit of carbon from the atmosphere.

[24] The regulator chooses policy in full knowledge of this constraint. To correct for the chance that the negative emission constraint will bind, the regulator increases the initial period's emission tax to 246 (26) $\$/tCO_2$ with expert (DICE) damages.

Table 1: Balanced growth equivalent gain from optimal carbon shares and from optimal emission taxes.

	Expert Damages		DICE Damages	
	BGE (%)	Loss (% of BGE)	BGE (%)	Loss (% of BGE)
Carbon Shares	40.3	-	1.51	-
Emission Taxes	37.6	6.8	1.37	9.1

Balanced growth equivalent gain (BGE) is relative to a case with abatement fixed at zero (but savings optimized). The BGE translates changes in welfare into the constant relative difference in consumption between two counterfactual consumption trajectories that grow at the same constant rate (Mirrlees and Stern, 1972).

This new policy improves on commonly proposed emission tax and cap-and-trade policies by optimally incentivizing both emission reductions and emission removal.

I have described the carbon share as a price instrument, but it could be implemented as a quantity instrument. Instead of announcing damage charges period by period, the regulator would announce a cap on cumulative emissions period by period. The number of outstanding shares must match that cap. When the cap is increasing, the regulator finds the damage charge at which the market clears with the correct number of new shares issued. Each new share is funded by an upfront bond, as before.[25] When the cap is decreasing, the regulator issues no new shares and each shareholder bids the damage charge above which they will retire their share by removing its underlying unit of carbon from the atmosphere. In either case, the regulator deducts the market-clearing charge from the face value of each outstanding share. Whether the regulator sets damage charges directly or discovers them via caps on cumulative emissions, the key is that carbon shareholders will not be forced to spend money after the time of emission and will trade off the cost of carbon removal against expected future charges. The regulator thereby divorces cleanup from emission decisions and optimally incentivizes each, enabling announced climate goals to be achieved through market-based policies.

A few objections may arise. First, one may wonder how the regulator is to develop an estimate of either the period-by-period charge or the worst-case social cost of carbon. In fact, the informational challenge is smaller under the carbon share policy than under conventional

[25] The regulator could discover the value of the bond by running a secondary market constrained by the most stringent possible cumulative emission outcome.

emission tax or cap-and-trade policies: specifying the worst-case social cost of carbon is less informationally demanding than specifying the expected social cost of carbon, the current period's charge is but one piece of the current period's expected social cost of carbon, and the regulator no longer should adjust current policy for the chance that a negative emission constraint will bind in the future. Further, whereas future damages from climate change are inherently out of sample and thus difficult to ground empirically, optimal damage charges can be plausibly estimated from recent data on weather and economic outcomes (Dell et al., 2014) and attribution studies that relate the probability of realized weather events to climate change (Otto, 2016; Stott et al., 2016).

Second, one may be concerned about political risk. On the one hand, the regulator may have an incentive to confiscate the bonds (see Shogren et al., 1993) or to set high per-period charges that raise revenue from inelastic, prior emission decisions. On the other hand, future changes in government may produce regulators unconcerned with climate change and shareholders may lobby for small per-period charges. Appendix A discusses political risk in more detail and outlines some solutions. Future work should further consider the design of regulatory institutions.

There are two additional benefits to the carbon share policy that I have not explored formally but which could be especially important. First, by establishing a larger market for carbon removal technologies, this policy should accelerate those technologies' development. If climate damages do end up warranting negative emissions, then innovators should receive a strong signal in the form of high per-period charges in advance of those technologies being needed. By directing innovation, the carbon share policy offers additional insurance against worst-case outcomes.

Finally, the public sector has to date borne both the burden of projecting future climate change damages for social cost of carbon calculations and the risk of paying for carbon removal should it become optimal. The proposed policy disperses this burden and risk throughout the private sector. Markets would perform price discovery, with futures and options markets emerging for future damage charges. Such markets would coordinate expectations throughout the economy and thereby facilitate lending for new removal technologies and installations. Private firms would have an incentive to become educated about future climate risks and to fund new monitoring and modeling systems. And these investments could in turn mitigate political risk, as improved knowledge of climate impacts may enable more durable and informed climate policy.

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Appendix to "Incentivizing Negative Emissions Through Carbon Shares"

Appendix A discusses ways to mitigate political risk. Appendix B details the numerical model. Appendix C formally derives familiar results regarding optimal emission and stock taxes and formally describes the challenge of inducing negative emissions in each case.

A Political Risk

Any climate policy seeks to redirect long-run investment in capital and research. To do so, it must be credible. Carbon emission taxes and carbon shares both suffer from credibility challenges, but with subtle differences. I here discuss some of these differences and how to design carbon shares to mitigate some of the concerns. For exposition, I consider political risk as the chance that a policy becomes overly lax for intervals of time (as in the span between elections).

Carbon emission taxes force firms to internalize estimated social costs at the time of emission. If the time t tax is set suboptimally, it directly affects emissions in only the same period. However, prior to time t, firms must form expectations of future taxes when making investment and research decisions that span many years. If firms anticipate that the emission tax could collapse for periods of time, then they underinvest in low-carbon infrastructure and technologies in advance of those collapses and overemit during the periods of collapse. In the short run, a carbon emission tax set at the socially optimal level does achieve socially optimal emissions despite political risk, but in the long run, political risk increases the emissions induced by any given emission tax.

Under the carbon share policy, emitters internalize only the market's expectation of damage charges. One benefit of a carbon share policy is enabling carbon removal incentives to respond to new information about the cost of carbon removal and about the damages from emissions. The latter benefit is achieved as the regulator updates the charge on time t emissions to match the true costs of those emissions as manifested at later times. This flexibility comes at the cost of increasing the impact of suboptimal regulatory decisions. If markets anticipate that future regulators may set suboptimally low damage charges, then the value of a carbon share will be relatively large and the incentive to reduce emissions will be relatively small (see equation (10)). This political risk distorts not just future emissions and infrastructure but also current emissions. Institutional design is therefore especially critical to a carbon share policy.

With carbon shares, the danger of suboptimal policy decisions can be mitigated by linking those decisions to some observable variable that correlates with information about climate damages. Whereas carbon emission taxes will always be limited by the inherent unobservability of future damages from climate change, the damage charges underpinning carbon shares depend only on realized damages. There are three promising possibilities for mitigating political risk. First, if realized damages can be estimated reliably through a transparent and credible empirical framework, then damage charges could be linked to these estimates much as many policies are linked to estimated inflation. Second, if damage charges correlate with established earth system metrics such as global temperature, sea level, or storm intensity, then regulators need only periodically establish multipliers for these metrics. Finally, because information about damages may not jump too much over intervals of a few years, a carbon share policy could constrain regulators' ability to deviate from the most recent damage charges or multipliers. Future work should consider these issues in more detail, both theoretically and quantitatively.

The required bond is also subject to political risk. A miscalculated bond causes problems if the bond ends up being too small to cover the realized sequence of damage charges. If carbon removal would be optimal at smaller sequences, then even this miscalculation does not cause problems. But if carbon removal would be optimal only at larger damage charges, then the suboptimally small bond reduces total carbon removal and, by reducing expected damage charges, increases emissions. Because the bond does not need to respond to information as flexibly as damage charges should, a carbon share policy may be able to constrain changes in the bond without strongly affecting the overall efficiency of the policy.

Finally, one benefit of carbon shares is that, as property rights, they would be subject to legal restrictions on takings. Such restrictions will make it hard to simply rescind issued carbon shares in order to confiscate the bonds underlying them. They may also imply judicial oversight over damage charges. Such oversight would be likely to establish criteria that the process of developing damage charges must meet in order to pass muster. Judicial oversight may thereby eventually mitigate the consequences of political risk, although potentially by restricting the adaptability of damage charges to new information.

B Numerical Model

This appendix gives the full equations for the model, which follows DICE-2016R (Nordhaus, 2017). The only modifications are to change the horizon, to allow uncertainty about a damage parameter, to allow negative emissions to begin as soon as that uncertainty is resolved, and to update the carbon cycle and climate system. Table A-1 reports the values of the model parameters. A Matlab implementation of DICE-2016R (with various extensions) can be found at `https://github.com/dlemoine1/DICE-2016R-Matlab`.

The DICE model is a Ramsey growth model coupled to a climate module. An infinitely lived representative agent aims to maximize the sum of the stream of discounted utility from consuming output. The timestep is Δ years and the horizon is here 400 years, or $\bar{t} = 400/\Delta$ periods.[26] I follow DICE-2016R in setting $\Delta = 5$. The initial year is 2015, denoted here as time 0. At time 0, the policymaker chooses the abatement rate μ_t and savings rate s_t to maximize a utilitarian expected welfare function of consumption C_t and population L_t:

$$\max_{\{\mu_t, s_t\}_{t=0}^{\bar{t}-1}} E_0 \left[\sum_{t=0}^{\bar{t}-1} \frac{1}{(1+\rho)^{\Delta\, t}} L_t \, u(C_t; L_t) \right], \tag{Welfare}$$

where expectations are taken at the time 0 information set. Per-period utility is:

$$u(C_t; L_t) = \frac{(C_t/L_t)^{1-\eta}}{1-\eta}, \tag{Utility}$$

with $\eta \geq 0, \neq 1$ is the inverse of the elasticity of intertemporal substitution and also the coefficient of relative risk aversion. Utility is discounted at annual rate ρ. As described below, the policymaker chooses abatement and savings rates as functions of information about damages (i.e., as closed-loop policies), not as functions of time.

To produce time t gross output Y_t^g, the agent combines capital K_t with labor L_t and technology A_t in a Cobb-Douglas production function:

$$Y_t^g = A_t \, (L_t/1000)^{1-\kappa} \, K_t^{\kappa}. \tag{Gross output}$$

Some of this output is lost to damages caused by surface warming T_t, so that output net of

[26] The horizon in DICE-2016R is 500 years. Shortening the horizon to 400 years does not sacrifice much but helps when optimizing under uncertainty because the number of controls becomes large.

damages is given by

$$Y_t^n = Y_t^g \left[1 - \min\{0.95, d_t \left[T_t\right]^2\}\right]. \tag{Net output}$$

The parameter d_t is constant and known prior to 2065, with value d. It is also constant from 2065 on, with value $\tilde{d}$. The policymaker does not know $\tilde{d}$ until 2065. In the DICE damage specification, $d = 0.00236$ and the distribution of $\tilde{d}$ is lognormal with mean 0.00236. The standard deviation of $\ln \tilde{d}$ is 1.286, from Appendix C.1 of Lemoine (2021). That calibration fits a distribution to the Pindyck (2019) expert survey of losses from climate change in fifty years after adjusting for uncertainty about warming. The expert damage specification increases both d and the mean of $\tilde{d}$ to 0.0228 in order to match the survey results and truncates the distribution from above at 0.1132 (see Lemoine, 2021). I cap the losses in any one period at 95%.

The policymaker allocates net output to consumption C_t, investment I_t, or spending Ψ_t on emission abatement. Industrial emissions (net of abatement) per timestep are:

$$E_t^I = \Delta\, \sigma_t (1 - \mu_t) Y_t^g, \tag{Industrial emissions}$$

where σ_t is the emission intensity of production at time t. Emissions E_t (net of abatement) per timestep are

$$E_t = E_t^I + \Delta\, E_t^{\sim I}, \tag{Emissions}$$

where $E_t^{\sim I}$ gives (exogenous) annual emissions from deforestation. Cumulative industrial emissions up to each time t are constrained by the stock of available carbon:

$$\sum_{t=0}^{\tau} \left[400 + \max\{0, E_t^I\}\right] \leq 6000 \quad \text{for all } \tau \in [0, T-1], \tag{Cumulative fossil constraint}$$

where E_t^I is measured in Gt C. The cost of abating fraction μ_t of industrial emissions is

$$\Psi_t = \psi_t Y_t^g [\mu_t]^{a_2}. \tag{Abatement cost}$$

The carbon tax is equal to marginal abatement cost. I constrain $\mu_t \leq 1$ prior to 2065.[27]

The economy's resource constraint is:

$$C_t + I_t + \Psi_t \leq Y_t^n. \qquad \text{(Resource constraint)}$$

Capital depreciates at annual rate δ_K:

$$K_{t+1} = K_t\,(1-\delta_k)^{\Delta} + \Delta\, I_t. \qquad \text{(Capital)}$$

Annual investment is determined by the savings rate s_t:

$$I_t = s_t[Y_t^n - \Psi_t]. \qquad \text{(Investment)}$$

The final fifty years' savings rate is fixed at 0.2583. I convert to year 2015 dollars using a deflator of 1.09, from the World Bank.

The model's exogenous economic processes are

$$\begin{aligned}
L_{t+1} =& L_t \left(\frac{L_\infty}{L_t}\right)^{g_L\,\Delta/5}, && \text{(Population)}\\
A_{t+1} =& A_t/(1-g_{A,t})^{\Delta/5}, && \text{(Production technology)}\\
g_{A,t+1} =& g_{A,0}\, e^{-\Delta\,(t+1)\,\delta_A}. && \text{(Production technology growth rate)}
\end{aligned}$$

The model's exogenous climate-related processes are

$$\begin{aligned}
\sigma_{t+1} &= \sigma_t e^{\Delta\, g_{\sigma,t}}, && \text{(Gross emissions per unit of output)}\\
g_{\sigma,t+1} &= g_{\sigma,t}(1+\delta_\sigma)^{\Delta}, && \text{(Growth rate of gross emissions per unit of output)}\\
\psi_{t+1} &= \frac{a_1(1-g_\psi)^{t\,\Delta/5}\,\sigma_{t+1}}{1000\,a_2}, && \text{(Abatement cost coefficient)}\\
E_{t+1}^{\sim I} &= E_0^{\sim I}\,(1-g_E)^{(t+1)\,\Delta/5}, && \text{(Emissions from deforestation)}\\
EF_{t+1} &= EF_0 + (EF_{100} - EF_0)\min\{\Delta\, t/(5*17), 1\}. && \text{(Non-CO}_2\text{ forcing)}
\end{aligned}$$

I now describe the carbon cycle and climate model, both of which deviate from DICE-

[27]In DICE-2016R, $\mu_t \leq 1$ for the first 145 years and $\mu_t \leq 1.2$ afterward.

2016R. The carbon cycle follows Joos et al. (2013, Table 5), as recommended and compiled by Dietz et al. (2020).[28] That carbon cycle has

$$\boldsymbol{M}_{t+1} = \boldsymbol{\Lambda}^{\Delta} M_t + \boldsymbol{b} E_t \tag{Carbon reservoirs}$$

where $\boldsymbol{M}$ is a 4×1 vector of atmospheric carbon reservoirs. The coefficient matrices are:

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9975 & 0 & 0 \\ 0 & 0 & 0.9730 & 0 \\ 0 & 0 & 0 & 0.7927 \end{bmatrix} \tag{Carbon transfer}$$

and

$$\boldsymbol{b} = \begin{bmatrix} 0.2173 \\ 0.2240 \\ 0.2824 \\ 0.2763 \end{bmatrix}. \tag{Emissions' fate}$$

The year 2015 values (in Gt C) are

$$\boldsymbol{M}_0 = \begin{bmatrix} 588 + 139.1 \\ 90.2 \\ 29.2 \\ 4.2 \end{bmatrix}, \tag{Carbon starting value}$$

where 588 Gt C is the stock of preindustrial carbon.

The parameters of the climate model come from Geoffroy et al. (2013), as compiled by Dietz et al. (2020). Additional atmospheric carbon dioxide (CO_2) increases radiative forcing $F_t(\boldsymbol{M}_t)$, which measures additional energy at the earth's surface due to CO_2 in the

[28] Dietz et al. (2020) additionally recommend using the FAIR model to capture carbon cycle feedbacks, but doing so would further increase the complexity of an already nontrivial optimization problem.

atmosphere. Forcing is

$$F_t(\boldsymbol{M}_t) = f_{2x} \frac{\ln\left(\sum_{i=1}^{4} M_t^i/588\right)}{\ln(2)} + EF_t, \tag{Forcing}$$

where M_t^i indicates element i of $\boldsymbol{M}_t$, EF_t is exogenous forcing from non-CO_2 greenhouse gases (defined above), and f_{2x} is forcing induced by doubling CO_2. Surface temperature evolves as

$$T_{t+1}^s = T_t^s + \frac{\Delta}{5}\phi_1 \left[F_{t+1}(\boldsymbol{M}_{t+1}) - \lambda T_t^s - \phi_3 \left(T_t^s - T_t^o\right)\right]. \tag{Surface temperature}$$

Ocean temperature evolves as

$$T_{t+1}^o = T_t^o + \frac{\Delta}{5}\phi_4 \left[T_t^s - T_t^o\right]. \tag{Ocean temperature}$$

Steady-state warming from doubled carbon dioxide ("climate sensitivity") is $f_{2x}/\lambda = 3.1$°C.

I solve the model by searching over contingent trajectories for μ_t, s_t, K_t, $\boldsymbol{M}_t$, T_t^s, and T_t^o, treating the transition equations as constraints. With this form, I can supply an analytic gradient for the objective and an analytic Jacobian for the constraints. I approximate the distribution over $\tilde{d}$ using quadrature with 5 nodes.[29] The trajectories are contingent because they vary by quadrature node. I solve the model in Matlab. When optimizing the full model, I search over 2,880 controls. When simulating the distribution of future outcomes, I use the year 2065 state reached along the optimal trajectory (defined by policy optimized under uncertainty) and take 1,000 draws from the damage distribution.

I calculate the bond required by each draw from the damage distribution, by combining the optimal year 2065 emission tax (as chosen upon learning the value of $\tilde{d}$ with the strip of pre-2065 charges. I calculate the pre-2065 charges by perturbing year 2015 emissions and calculating the change in each period's welfare.

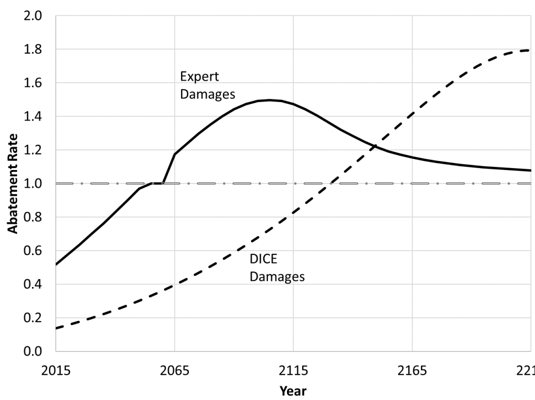
Figure A-1 reports the abatement, emission tax, and temperature trajectories in a deterministic model in which the damage parameter is fixed to its mean at all times (i.e., $\tilde{d} = d$). Negative emissions occur in midcentury in the case with expert damages and occur early

[29] I use the compecon toolbox to obtain Gaussian quadrature nodes for non-truncated distributions (Miranda and Fackler, 2002) and use the Fortran90 version of truncated_normal_rule (available at `http://people.math.sc.edu/Burkardt/c_src/truncated_normal_rule/truncated_normal_rule.html`) to obtain quadrature nodes for truncated distributions.

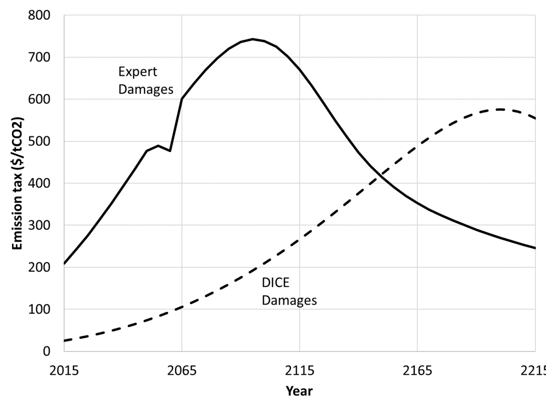
in the next century in the case with DICE damages.[30] Near-term abatement is greater in the case with expert damages, but long-term abatement is reduced because early abatement leaves a smaller stock of atmospheric carbon. The emission tax trajectory reveals corresponding effects on the implied emission price. Negative emissions eventually undo some warming in both calibrations, with the expert damage calibration never allowing warming to exceed 2°C.

The lower right panel explodes the initial emission tax into the strip of per-period marginal damages. These are the optimal carbon stock taxes per 5-year timestep and are also the optimal sequence of per-period damage charges that would be implemented under the carbon share policy. The sum of each set of points equals the optimal emission tax. The charges increase over the next decades as current emissions translate into warming and as higher temperatures interact with convex damages. The charges eventually decline due to the effect of discounting, the eventual decline in temperature, and the decay of initial emissions. The charges spread the emission tax's upfront payment over more than a century, with the peak charges comprising only a small fraction of the optimal emission tax.

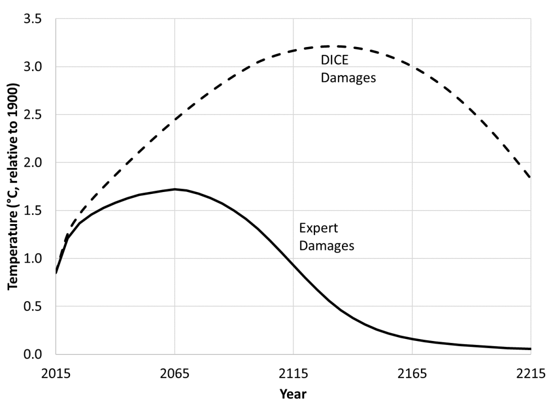
[30]The kink in the case with expert damages arises because the pre-2065 constraint that abatement be weakly less than 100% briefly binds. The tax declines over this interval because exogenously improving technology gradually reduces the tax needed to obtain 100% abatement.



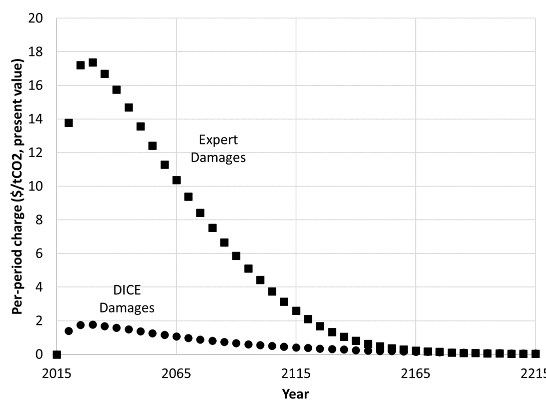
(a) Abatement Rate



(b) Emission Tax



(c) Temperature



(d) Damage Charge

Figure A-1: Optimal trajectories in deterministic versions of each damage calibration.

Table A-1: Parameters

Parameter	Value	Description
Δ	5	Timestep (years)
$\bar{t}$	80	Horizon (periods)
A_0	5.115	Initial production technology
$g_{A,0}$	0.076	Initial growth rate of production technology, per five years
δ_A	0.005	Annual decline in growth rate of production technology
L_0	7403	Year 2015 population (millions)
L_∞	11500	Asymptotic population (millions)
g_L	0.134	Rate of approach to asymptotic population level, per five years
σ_0	0.0955	Initial emission intensity of output (Gt C per trillion 2010$)
$g_{\sigma,0}$	-0.0152	Initial annual growth rate of emission intensity
δ_σ	-0.001	Annual change in growth rate of emission intensity
a_1	2016.7	Cost of backstop technology in 2015 (2010$ per ton of C)
a_2	2.6	Abatement cost function exponent
g_ψ	0.025	Decline rate of backstop cost, per five years
$E_0^{\sim I}$	0.71	Initial emissions from deforestation (Gt C per year)
g_E	0.115	Decline rate of deforestation emissions, per five years
EF_0	0.5	Year 2015 non-CO_2 forcing (W/m^2)
EF_{100}	1	Year 2100 non-CO_2 forcing (W/m^2)
κ	0.3	Capital share in production
δ_K	0.1	Annual capital depreciation rate
ρ	0.015	Annual utility discount rate
η	1.45	Inverse of elasticity of intertemporal substitution; also RRA
ϕ_1	0.386	Warming delay parameter
ϕ_3	0.73	Parameter governing transfer of heat from ocean to surface
ϕ_4	0.034	Parameter governing transfer of heat from surface to ocean
f_{2x}	3.503	Forcing from doubling CO_2 (W/m^2)
λ	1.13	Forcing per degree warming ($[W/m^2]/°C$)
$d, \tilde{d}$	see text	Damage parameters

Continued on next page

Table A-1 – continued from previous page

Parameter	Value	Description
K_0	223	Year 2015 capital (trillion 2010\$)
$\boldsymbol{M}_0$	see text	Year 2015 carbon reservoirs (Gt C)
T_0^s	0.85	Year 2015 surface temperature (°C, wrt 1900)
T_0^o	0.0068	Year 2015 lower ocean temperature (°C, wrt 1900)

C Additional Analysis

Let $D_t'(T_t)$ be increasing in $\tilde{d}_t$ and $\tilde{d}_t$ have support between d^L and d^H, where $d^L < d^H$. Equations (1) and (2) and the constraints implicitly define unique $A_{it}^{fb}(M_t, \tilde{d}_t)$ and $Z_t^{fb}(M_t, \tilde{d}_t)$.[31] Emission removal would be used while aggregate emissions are still positive if and only if there exists a firm i such that $C_{it}'(e_{it}) > G_t'(0)$.

C.1 Emission Tax Policy

Consider a regulator seeking to control emissions through emission taxes. Firms report their emissions net of any removal they fund and pay τ_t per unit in period t. Current emissions can be offset either by abatement or by removal, but abatement is the cheaper option for the first unit of emissions: $G_t'(0) > C_{it}'(0)$. Firm i solves:

$$\pi_{it}^{tax}(\tau_t) = \min_{A_{it} \le e_{it}, Z_{it} \ge 0} \left\{ C_{it}(A_{it}) + p_t Z_{it} + \max\{0, \tau_t[e_{it} - A_{it} - Z_{it}]\} + \frac{1}{1+r} E_t \left[\pi_{i(t+1)}^{tax}(\tau_{t+1}) \right] \right\},$$

where p_t is the cost of emission removal and where I suppress dependence of π_{it} on the random variables. Funding emission removal allows the firm to avoid paying a tax but does not entitle the firm to a subsidy if total removal exceeds $e_{it} - A_{it}$. Firms therefore never

[31] The first-best allocation does not specify which firms pay for emission removal because that allocation does not affect real outcomes. In contrast, the first-best allocation does specify that firms equalize marginal abatement costs.

choose $A_{it} + Z_{it} > e_{it}$. At an interior solution, the first-order conditions imply

$$\begin{aligned} C'_{it}(A_{it}) =& \tau_t, \\ p_t =& \tau_t. \end{aligned} \tag{A-1}$$

In equilibrium, $p_t = G'_t(Z_t)$, so the second condition implies:

$$G'_t(Z_t) = \tau_t. \tag{A-2}$$

Let A_{it}^{tax} and Z_{it}^{tax} indicate firms' choices. Both increase in τ_t. Because $A_{it}^{tax} + Z_{it}^{tax} \leq e_{it}$ for all τ_t, there is a tax $\bar{\tau}_{it}$ beyond which $A_{it}^{tax} + Z_{it}^{tax}$ is constant. Raising the tax above $\bar{\tau}_{it}$ does not affect firm i's net emissions because all emissions have either been eliminated or offset by emission removal. That maximum tax is the smallest τ_{it} such that

$$A_{it}^{tax} + Z_{it}^{tax} = e_{it}. \tag{A-3}$$

Use $\bar{\tau}_t$ to denote $\sup_i \bar{\tau}_{it}$. Assume, for convenience and in line with reality, that some firm would find using emission removal to be cheaper than abating all of its emissions: $Z_{it}^{tax} > 0$ for some i when $\tau_t = \bar{\tau}_t$.

The time t regulator solves:

$$V_t^{tax}(M_t, \tilde{d}_t) = \min_{\tau_t} \left\{ \int_0^1 C_{it}(A_{it}^{tax})\,\mathrm{d}i + G_t(Z_t^{tax}) + D_t(T_t; \tilde{d}_t) + \frac{1}{1+r} E_t[V_{t+1}^{tax}(M_{t+1}, \tilde{d}_{t+1})] \right\}.$$

The regulator's first-order condition is

$$\begin{aligned} 0 =& \int_0^1 \frac{\partial A_{it}^{tax}}{\partial \tau_t} \left[C'_{it}(A_{it}^{tax}) - \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})] \right] \mathrm{d}i \\ &+ \int_0^1 \mathbb{1}_{Z_{it}^{tax}>0} \frac{\partial Z_{it}^{tax}}{\partial \tau_t} \left[G'_t(Z_t^{tax}) - \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})] \right] \mathrm{d}i, \end{aligned}$$

where $\mathbb{1}$ is the indicator function and where I substitute from the envelope theorem. The partial derivatives are zero for all firms i such that $\tau_t \geq \bar{\tau}_{it}$. Substituting the other firms' first-order conditions yields

$$\tau_t = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})]. \tag{A-4}$$

I denote this tax $\breve{\tau}_t$. This is the instrument familiar from previous literature (e.g., Nordhaus, 1982, 1991; Farzin, 1996). It is the unique optimal tax as long as $\breve{\tau}_t \leq \bar{\tau}_t$. If $\breve{\tau}_t > \bar{\tau}_t$, then any $\tau_t \geq \bar{\tau}_t$ is an optimum.

Comparing the resulting firm first-order conditions to (1) and (2), we see that, given whatever emission decisions were made prior to time t, two conditions must hold for the regulator to implement the first-best allocation in time t. First, $\breve{\tau}_t$ must be weakly less than $\bar{\tau}_{it}$ for all firms i, so that no firms merely eliminate their emissions instead of undertaking negative emissions. This in turn happens if $\tilde{d}_t$ is sufficiently small. Second, $E_{t+s}[D'_{t+s}(T_{t+s})]$ must be as in first-best for all $s > 0$. This latter condition occurs if and only if either (i) $\bar{\tau}_{i(t+s)}$ cannot bind for any i at any $s \geq 0$ or (ii) $D_{t+s}(\cdot)$ is linear for all $s \geq j$, where j is the first time at which $\bar{\tau}_{ij}$ might bind for some i. If condition (i) does not hold, then there are states of the world in which some firms merely eliminate emissions, making T_{t+s} larger than first-best for all sufficiently large s. As a result, $E_t[D'_{t+s}(T_{t+s})]$ becomes larger than first-best for some $s > 0$ if condition (ii) also does not hold, making $\breve{\tau}_t$ larger than in first-best. Summing up, the unconstrained-optimal tax $\breve{\tau}_t$ obtains more abatement than in first-best when some time t firms merely eliminate their emissions instead of undertaking negative emissions and when future taxes might not obtain the future negative emissions potentially required by first-best.

The following proposition formalizes the foregoing discussion.

Proposition A-1.

1. Ex-Ante Optimality: *Looking forward from time 0, there exists $\bar{d} > 0$ such that $\{\breve{\tau}_t\}_{t=0}^{\infty}$ will achieve the first-best allocation in all states of the world if and only if $d^H \leq \bar{d}$.*

2. Ex-Post Optimality: *Looking backward from some time $s > 0$, $\{\breve{\tau}_t\}_{t=0}^{s}$ achieves the first-best allocation in periods 0 through s if and only if either (i) it achieves first-best ex-ante or (ii) each $D_t(\cdot)$ is linear for all t sufficiently large and each $\tilde{d}_j$ is sufficiently small that $\breve{\tau}_j \leq \bar{\tau}_{ij}$ for all firms i and all periods $j \in [0, s]$.*

Proof.

1. Follows from the foregoing analysis, defining $\bar{d}_t$ as the smallest $\tilde{d}_t$ such that $\breve{\tau}_t = \bar{\tau}_{it}$ for some firm i and defining $\bar{d}$ as the infimum of the $\bar{d}_t$.

2. Condition (i) follows by definition. For condition (ii), assume that $\{\breve{\tau}_t\}_{t=0}^{s}$ does not achieve first-best ex ante, which means that $\bar{\tau}_{it}$ binds in some state of the world and at some time t. Let k be the first time at which $\bar{\tau}_{it}$ binds for some i. $E_{t-j}[D_t'(T_t)]$ (for $j \in [0,t]$) is the same in equations (1), (2), and (A-4) if and only if each $D_t(\cdot)$ is linear for all $t \geq k$. And from equations (A-1) and (A-2), $\{\breve{\tau}_t\}_{t=0}^{s}$ implements A_{it}^{fb} and Z_t^{fb} from periods 0 through s if and only if, first, $E_{t-j}[D_t'(T_t)]$ (for $j \in [0,t]$) is the same in equations (1), (2), and (A-4) and, second, $\breve{\tau}_t \leq \bar{\tau}_{it}$ for all firms i and all $t \in [0,s]$. We know that $\breve{\tau}_t$ decreases in $\tilde{d}_t$, so $\breve{\tau}_t \leq \bar{\tau}_{it}$ if $\tilde{d}_t$ is sufficiently small. The proposition follows.

□

We have detected a new inefficiency when high damages imply $\breve{\tau}_t > \bar{\tau}_{it}$ for some firm i. The cost of the tax policy relative to first-best depends on the probability of wanting some firms to undertake negative emissions. It also depends on the convexity of abatement and emission removal costs in regions with negative emissions: if those costs are highly convex at the point where firm i chooses net zero emissions, then first-best may not obtain significant negative emissions from firm i and the loss from being unable to incentivize negative emissions may be small.

C.2 Stock Tax Policy (Atmospheric Rental Policy)

Now consider taxing the stock of carbon rather than the emission of carbon. I refer to the stock tax as an atmospheric rental policy to differentiate it from the standard use of "carbon taxes" to refer to emission taxes. The regulator now charges firms ψ_t for their ongoing use of atmospheric storage (i.e., for each unit of current or past emissions remaining in the atmosphere at the end of period t). Under familiar emission tax policies, firms pay a tax only in the period in which they emit; under the atmospheric rental policy, firms pay a tax in every period from the time of emission until the time of emission removal (should it occur).

Let M_{it} indicate firm i's cumulative emissions from time 0 up to time t:

$$M_{it} = \sum_{s=0}^{t-1} [e_{is} - A_{is} - Z_{is}].$$

At time t, firm i solves:

$$\pi_{it}^{rental}(\psi_t, M_{it}, M_t) = \min_{A_{it} \leq e_{it}, Z_{it} \geq 0} \Bigg\{ C_{it}(A_{it}) + p_t Z_{it} + \max\{0, \psi_t[e_{it} - A_{it} - Z_{it} + M_{it}]\} + \frac{1}{1+r} E_t[\pi_{i(t+1)}^{rental}(\psi_{t+1}, M_{i(t+1)}, M_{t+1})] \Bigg\},$$

where I again suppress dependence of π_{it} on the random variables. The maximization problem differs from that under the tax policy only in that payments here depend on the history of abatement and emission removal decisions. Firms never choose $A_{it} + Z_{it} > e_{it} + M_{it}$. Repeatedly applying the envelope theorem and substituting for equilibrium p_t, the first-order conditions satisfied by an interior solution become:[32]

$$C'_{it}(A_{it}) = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\psi_{t+s}],$$

$$G'_t(Z_t) = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\psi_{t+s}].$$

Let A_{it}^{rental} and Z_{it}^{rental} indicate firms' choices. Each increases in each expected charge $E_t[\psi_{t+s}]$, holding the other expected charges fixed. If the first-order conditions imply $A_{it}^{rental} + Z_{it}^{rental} > e_{it} + M_{it}$, then firm i chooses $A_{it}^{rental} + Z_{it}^{rental} = e_{it} + M_{it}$ and both A_{it}^{rental} and Z_{it}^{rental} are locally independent of all ψ_{t+s}.

The time t regulator solves:

$$V_t^{rental}(M_t, \tilde{d}_t) = \min_{\psi_t} \Bigg\{ \int_0^1 C_{it}(A_{it}^{rental}) \, \mathrm{d}i + G_t(Z_t^{rental}) + D_t(T_t; \tilde{d}_t) + \frac{1}{1+r} E_t[V_{t+1}^{rental}(M_{t+1}, \tilde{d}_{t+1})] \Bigg\}.$$

[32]Firms are small, so they do not account for their infinitesimal effect on M_{t+s} and thus on ψ_{t+s}.

Using the envelope theorem, the regulator's first-order condition is

$$0 = \int_0^1 \frac{\partial A_{it}^{rental}}{\partial \psi_t} \left[C_{it}'(A_{it}^{rental}) - \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D_{t+s}'(T_{t+s})] \right] \mathrm{d}i$$
$$+ \int_0^1 \mathbb{1}_{Z_{it}^{rental}>0} \frac{\partial Z_{it}^{rental}}{\partial \psi_t} \left[G_t'(Z_t^{rental}) - \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D_{t+s}'(T_{t+s})] \right] \mathrm{d}i.$$

The partial derivatives are zero for all firms i such that $A_{it}^{rental} + Z_{it}^{rental} \geq e_{it} + M_{it}$. Substituting the other firms' first-order conditions yields

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\psi_{t+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D_{t+s}'(T_{t+s})]. \tag{A-5}$$

Many sequences of ψ_{t+s} satisfy this condition for given t, but a time-consistent policy satisfies this condition for all t. The following proposition describes the optimal time-consistent policy:

Proposition A-2. *The unique time-consistent policy that satisfies* (A-5) *sets* $\psi_t = \alpha\, D_t'(T_t)$ *at every time* $t \geq 0$.

Proof. A time-consistent policy that satisfies (A-5) also satisfies:

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t+1}[\psi_{t+1+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t+1}[D_{t+1+s}'(T_{t+1+s})].$$

Taking expectations of both sides with respect to the time t information set and using the law of iterated expectations, we have:

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\psi_{t+1+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D_{t+1+s}'(T_{t+1+s})].$$

Using this, (A-5) becomes:

$$\psi_t = \alpha\, D_t'(T_t).$$

The proposition follows from observing that the choice of t was arbitrary and that condition (A-5) holds if $\psi_{t+s} = \alpha\, D_{t+s}'(T_{t+s})$ for all $s \geq 0$.

$\square$

I denote the charge derived in Proposition A-2 as $\breve{\psi}_t$.

In Section C.1, the regulator's desired emission tax $\breve{\tau}_t$ was the present value of the strip of marginal damages incurred by a unit of emissions. Using equation (A-4), we have:

$$\breve{\tau}_t = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\breve{\psi}_{t+s}]. \tag{A-6}$$

The optimal rental policy explodes this strip into its constituent pieces, charging firms only as damages are realized and only on the condition that their emissions remain in the atmosphere. Firms' interior solutions are the same whether they face $\breve{\tau}_t$ or the stream of $\breve{\psi}_{t+s}$.[33] Define $\bar{\bar{\tau}}_{it}$ as the smallest τ_t such that

$$A_{it}^{tax} + Z_{it}^{tax} = e_{it} + M_{it}. \tag{A-7}$$

We immediately have the analogue of the analysis in Section C.1: the rental policy achieves first-best abatement and emission removal in period t as long as

$$\sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[\breve{\psi}_{t+s}] \leq \bar{\bar{\tau}}_{it}$$

for all firms i and either the analogous condition holds for all later times at all feasible states or each $D_{t+s}(\cdot)$ is linear for all s sufficiently large.[34] Comparing equations (A-3) and (A-7), $\bar{\bar{\tau}}_{it} \geq \bar{\tau}_{it}$: the rental policy can obtain more abatement and emission removal than can the emission tax policy and can therefore achieve first-best in a weakly larger set of cases than can the emission tax policy.

Two points are of special policy relevance. First, note that the optimal allocation would never have $\int_0^1 A_{it}\,\mathrm{d}i + Z_t > M_t + \int_0^1 e_{it}\,\mathrm{d}i$ as $M_t \to 0$ (because $D_t'(0) = 0$). In that case, $\bar{\bar{\tau}}_t \triangleq \sup_i \bar{\bar{\tau}}_{it}$ never binds, so the rental charge policy can always attain the optimum. It is

[33]The result relies on forward-looking firms discounting the future at the same rate as the regulator. Appendix C.3 shows how policy can overcome firms using higher discount rates, there driven by bankruptcy risk. See Barrage (2018) for further analysis of differential social and private discounting.

[34]As in Section C.1, the regulator obtains more time t abatement and emission removal than in first-best if the analogous condition might not hold at some later time and $D_{t+s}(\cdot)$ is nonlinear at some sufficiently large s. Now, however, that additional abatement and emission removal arises not because the regulator implements a more stringent time t policy but because firms expect the regulator to implement a more stringent policy at the later time $t+s$.

important to begin implementing the rental policy early, when the preexisting emissions M_0 that escape later charges are still small. Second, $\bar{\tau}_t$ is independent of emission taxes chosen in periods $s < t$ but $\bar{\bar{\tau}}_t$ decreases in rental charges chosen in periods $s < t$. The gains from using a rental policy in time t vanish if those earlier charges were so large as to eliminate earlier emissions (implying $\bar{\bar{\tau}}_t = \bar{\tau}_t$), but the gains potentially become large if those earlier charges were so small that they left substantial emissions in the atmosphere (permitting $\bar{\bar{\tau}}_t \gg \bar{\tau}_t$). Thus, if policy is lax in some early periods (whether due to optimal choices or political constraints), then the gains from using a rental policy are potentially large. Putting these points together, it becomes especially important to immediately begin a rental policy precisely in the case in which policymakers insist on implementing an emission charge that is much smaller than the optimal charge. In such cases, high early emissions make negative emissions more likely to be desirable in later periods. Starting a rental policy earlier provides greater scope for obtaining these negative emissions through decentralized market incentives.

C.3 The Challenge of Market Churn

Instead of compiling the stream of expected marginal damages into a single emission charge, the rental policy requires firms to pay for marginal damages period by period. However, damages from climate change unfold over a very long time.[35] Negative emissions may not become optimal until midcentury or later. If firms declare bankruptcy before that time, then they will not be around to pay to remove their old emissions from the atmosphere. These bankruptcies erode the ability of a system of rental charges to incentivize negative emissions by eroding the base of emission liabilities that are subject to the charges.

Formally, if firm i has replaced an older firm that went bankrupt (or if firm i represents a firm that survived but shed its liabilities through bankruptcy), then M_{it} is reset and the right-hand side of equation (A-7) is smaller than it would have been in the absence of bankruptcy. At sufficiently large charges, firm i pays for less emission removal than if it were accountable for the full history of emissions by firms of type i.

Moreover, if firms anticipate that they may not be in business at some later time, then they may overemit in the near term because they do not fully internalize future rental charges. I now show that the rental charge policy can successfully force firms to internalize

[35] Appendix B shows that the rental charges that comprise the currently optimal emission tax remain significant for a century or more.

the social cost of their emissions if and only if bankruptcy risk is homogeneous across firms.

Let each firm have probability λ of declaring bankruptcy between any two periods, with λ homogeneous across firms to start. So as not to conflate issues, imagine that each firm is replaced by a similar firm, leaving aggregate business-as-usual emissions unaffected. The chance of bankruptcy reduces firm i's discount factor to $(1-\lambda)/(1+r)$. The chance of bankruptcy does not affect firms' decisions under the emission tax policy and thus does not affect the optimal emission tax. However, under the rental policy, bankruptcy risk leads firms to undertake less abatement and emission removal for a given sequence of anticipated charges. Moreover, the realization of bankruptcy also reduces M_{it} to 0, as the new firm i does not carry old emission liabilities. The maximum level of abatement plus emission removal that firm i will undertake therefore falls after bankruptcy.

For the regulator, equation (A-5) becomes:

$$\sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[\psi_{t+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})]. \tag{A-8}$$

The following proposition describes the optimal time-consistent policy:

Proposition A-3. *The unique time-consistent policy that satisfies* (A-8) *sets* $\psi_t = (1-\lambda)\breve{\psi}_t + \lambda\breve{\tau}_t$ *at every time* $t \geq 0$.

Proof. Rearrange (A-8):

$$\psi_t = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+s}(T_{t+s})] - \frac{1-\lambda}{1+r} \sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[\psi_{t+1+s}]. \tag{A-9}$$

A time-consistent policy that satisfies (A-8) also satisfies:

$$\sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_{t+1}[\psi_{t+1+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_{t+1}[D'_{t+1+s}(T_{t+1+s})].$$

Taking expectations of both sides with respect to the time t information set and using the law of iterated expectations, we have:

$$\sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[\psi_{t+1+s}] = \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D'_{t+1+s}(T_{t+1+s})].$$

Using this in (A-9), simplifying, and adding and subtracting $\lambda \, \alpha \, D_t'(T_t)$, we have:

$$\psi_t = (1-\lambda) \, \alpha \, D_t'(T_t) + \lambda \, \alpha \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} E_t[D_{t+s}'(T_{t+s})].$$

The proposition follows from observing that the choice of t was arbitrary and using the definitions of $\breve{\tau}_t$ and $\breve{\psi}_t$.

□

The optimal charge is a weighted average of the Pigouvian emission tax and the optimal rental charge in the absence of bankruptcy risk. As $\lambda \to 0$, we are back to the optimal rental charge analyzed in Section C.2. As $\lambda \to 1$, firms survive for only a single period and the optimal charge approaches the Pigouvian emission tax analyzed in Section C.1, forcing firms to pay for all future social costs at the time they emit. In between these two extremes, the optimal charge forces firms to immediately pay for the time t slice of marginal social costs associated with time t emissions and also forces them to pay for a share of future marginal social costs that reflects their chance of going bankrupt before paying future charges.

The following corollary establishes that firms' incentives to reduce emissions are as in first-best, as long as firms' solutions are interior:

Corollary A-4. *Under the policy from Proposition A-3, firms' interior solutions are defined by equations* (1) *and* (2)*.*

Proof. At an interior solution, firm i's first-order conditions imply

$$C_{it}'(A_{it}) = \sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[(1-\lambda)\breve{\psi}_{t+s} + \lambda \breve{\tau}_{t+s}],$$

$$G_t'(Z_t) = \sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t[(1-\lambda)\breve{\psi}_{t+s} + \lambda \breve{\tau}_{t+s}].$$

Using equation (A-6), the first-order conditions become:

$$C'_{it}(A_{it}) = \sum_{s=0}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t \left[(1-\lambda)\breve{\psi}_{t+s} + \lambda \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \breve{\psi}_{t+s+j} \right],$$
$$G'_t(Z_t) = \sum_{s=t}^{\infty} \frac{(1-\lambda)^s}{(1+r)^s} E_t \left[(1-\lambda)\breve{\psi}_{t+s} + \lambda \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \breve{\psi}_{t+s+j} \right].$$

Collecting terms, each right-hand side becomes:

$$\sum_{j=0}^{\infty} E_t[\breve{\psi}_{t+j}] \left[\frac{(1-\lambda)^j}{(1+r)^j}(1-\lambda) + \sum_{s=0}^{j} \frac{(1-\lambda)^s}{(1+r)^s} \frac{1}{(1+r)^{j-s}} \lambda \right],$$

which simplifies to

$$\sum_{j=0}^{\infty} E_t[\breve{\psi}_{t+j}] \left[\frac{(1-\lambda)^j}{(1+r)^j}(1-\lambda) + \sum_{s=0}^{j} \frac{(1-\lambda)^s}{(1+r)^j} \lambda \right].$$

Solving the geometric series in brackets, this becomes:

$$\sum_{j=0}^{\infty} E_t[\breve{\psi}_{t+j}] \left[\frac{(1-\lambda)^j}{(1+r)^j}(1-\lambda) + \frac{\lambda}{(1+r)^j} \frac{1-(1-\lambda)^{j+1)}}{1-(1-\lambda)} \right],$$

which simplifies to

$$\sum_{j=0}^{\infty} \frac{1}{(1+r)^j} E_t[\breve{\psi}_{t+j}].$$

The corollary follows from substituting for each $\breve{\psi}_{t+j}$ from Proposition A-2.

□

The optimal policy successfully forces firms to internalize the social cost of their emissions, despite the bankruptcy risk. However, as noted above, bankruptcy risk does generate distortions when negative emissions happen to be optimal.

In practice, the probability of bankruptcy will vary across firms. Denote this probability as λ_i. Adapting Corollary A-4, firm i's decisions are as in first-best if they are interior and the firm faces charges $\psi_t = (1 - \lambda_i)\breve{\psi}_t + \lambda_i \breve{\tau}_t$ at every time $t \geq 0$. However, the regulator cannot implement this policy unless it can differentiate the charges by firm: a new inefficiency arises when each firm knows its own probability of bankruptcy and the regulator is unable to tailor the charge based on this information. The optimal feasible charge will allow too-high emissions from some firms and too-low emissions from others. An emission tax could dominate this charge.

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