Partial Equilibrium Thinking in General Equilibrium*

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Abstract

We develop a theory of "Partial Equilibrium Thinking" (PET), whereby agents fail to understand the general equilibrium consequences of their actions when inferring information from endogenous outcomes. PET generates a two-way feedback effect between outcomes and beliefs, which can lead to arbitrarily large deviations from fundamentals. In financial markets, PET equilibrium outcomes exhibit over-reaction, excess volatility, high trading volume, and return predictability. We extend our model to allow for rationality of higher-order beliefs, general forms of model misspecification, and heterogenous agents. We show that more sophisticated agents may contribute to greater departures from rationality. We also draw a distinction between models of misinference and models with biases in Bayesian updating, and study how these two departures from rationality interact. Misinference from mistakenly assuming the world is rational amplifies biases in Bayesian updating.

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The idea that financial markets aggregate dispersed information is central to the Efficient Market Hypothesis, and can be traced back to Hayek (1945). In a world where each agent receives a piece of private news about the future payoff of a risky asset and trades on it, the equilibrium price of the risky asset should aggregate all relevant information. However, correctly inferring this information from prices is far from straightforward, as this requires agents to perfectly understand what equilibrium forces generate the price they observe. The Rational Expectations Equilibrium (REE) achieves this level of understanding by assuming common knowledge of rationality.[1]

This paper studies the general equilibrium effects of relaxing this assumption when agents infer information from prices. We allow agents to have incorrect beliefs about other agents' actions, and thus to misunderstand the mapping from prices to other agents' information. Therefore, agents use a misspecified model of the world to extract information from prices, leading to misinference. Since agents trade on this incorrect information, the equilibrium price deviates from the REE benchmark.

For example, consider a trader who sees the price of Apple Inc.'s stock rise, but does not know what caused this price change. They may think that some informed agents received positive news about Apple and decided to buy, pushing up the price. Given this thought process, our uninformed trader infers positive news about Apple, and also decides to buy. However, agents who think in this way fail to realize that every other uninformed agent may be thinking and behaving just like them. Therefore, while part of the price rise they observe is due to the buying pressure of informed agents trading on new information, part of it may be due to the buying pressure of other uninformed agents. If instead uninformed agents attribute the whole price rise to positive information alone, they infer news which is better than in reality.

In this setting, different equilibrium concepts make different assumptions as to what uninformed traders in our example would do when they see the price of Apple rise. At one extreme, the Competitive (or Cursed) Equilibrium (CE) assumes all agents trade on their private information alone, and do not infer information from the price rise they observe, even though this price change contains valuable information (Eyster et al. (2019)). In the aggre-

[1] Grossman (1976), Hellwig (1980), Grossman and Stiglitz (1980), Admati (1985), among others.

gate, this leads to under-reaction to news. At the opposite extreme, the REE assumes that agents have common knowledge of rationality: not only do all agents infer valuable information from prices, but they also perfectly understand the process that generates the price change they observe. Therefore, since agents use the correct mapping to infer information from prices, they are able to recover the right information, which is then correctly aggregated into REE prices (Grossman (1976)). However, a vast literature, surveyed by Crawford, Costa-Gomes and Iriberri (2013), documents that the assumption of common knowledge of rationality is unrealistic in games with strategic thinking, and that instead agents exhibit low levels of rationality of higher order beliefs.

In this paper, we relax the assumption of common knowledge of rationality and propose a new equilibrium concept, Partial Equilibrium Thinking (PET), which leads to a psychologically plausible bias that lies in between the CE and the REE: while PET agents do learn information from prices, they fail to realize that all other uninformed agents do so too. In this sense, our agents are "partial equilibrium thinkers" in that they fail to realize the general equilibrium consequences of their actions. This leads PET agents to misunderstand the process that generates the price change they observe, and to attribute any price change to new information alone, just as we saw in the Apple example. Since PET agents then trade on their incorrect beliefs, PET gives rise to over-reaction relative to the REE benchmark.

Section 1 starts by formalizing the intuition behind the Apple example. We model a continuum of agents with constant absolute risk aversion (CARA) utility over terminal wealth who solve a portfolio choice problem between a risky and a riskless asset. A fraction of agents receive news about the future fundamental value of the asset and are thus informed. The remaining agents are uninformed but can infer information from prices. In this setup, PET is equivalent to assuming that all uninformed agents think that they are the only ones extracting information from prices, when in reality all uninformed agents are extracting information from prices using the same misspecified model of the world.

Importantly, prices play a dual role in our model. First, prices have an informational role, and this introduces strategic complementarities: higher prices reflect better fundamentals, leading uninformed agents to increase their asset demand, which in turn increases prices further, thus fuelling a feedback effect between outcomes and beliefs. Second, standard general equilibrium forces mean that prices also have a role as a measure of scarcity, and this introduces strategic substitutabilities: higher prices make the asset more expensive, leading all agents to demand less of it, which in turn puts downward pressure on prices, and dampens the feedback effect between outcomes and beliefs. The interaction of these two forces determines the overall strength of the feedback effect, and this in turn shapes the properties of equilibrium outcomes: the stronger the informational role relative to the scarcity role, the stronger the feedback effect, and the larger the deviations from REE outcomes. If the feedback effect is strong enough, PET can lead to arbitrarily large deviations from the REE, and in the extreme case when the feedback effect is explosive, the equilibrium is unstable.

Our notion of Partial Equilibrium Thinking is similar in nature to the concept of naïve herding developed in Eyster and Rabin (2008), Eyster and Rabin (2010) and Gagnon-Bartsch and Rabin (2017), which belongs to a broader literature on social learning, correlation neglect, and misinference.[2] Unlike our framework, these papers consider environments where the outcomes that agents learn from only have a purely informational role, and there is no general equilibrium market feedback effect.[3] Outside of the social learning literature, two papers are closest in spirit to ours. Glaeser and Nathanson (2017) apply naïve herding to explain patterns of momentum, reversal and excess volatility in house prices. They consider a model where prices are determined by the willingness to pay of risk-neutral buyers, and once again only have an informational role. Greenwood and Hanson (2015) develop a model specifically geared to the dry bulk shipping industry and focus on a particular type of misspecification where firms neglect the responses of their competitors. We contribute to this literature by proposing a tractable and general way to study model misspecification when prices affect inference via a general equilibrium role, and we characterize the properties of the feedback effect between outcomes and beliefs that misinference generates, including their possibly explosive nature.

[2] See Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992), on rational herding, DeMarzo, Vayanos and Zwiebel (2003), Eyster and Weizsacker (2010), Levy and Razin (2015), Gagnon-Bartsch and Rabin (2017), Enke and Zimmermann (2019) on correlation neglect, and Bohren (2016), Bohren and Hauser (2020), Frick, Iijima and Ishii (2020) on misinference, among others.

[3] Eyster et al. (2014) presents a model of social learning with *congestion costs*, which is a form of market feedback. However, this paper only considers purely rational learning.

Section 2 considers the implications of PET, and we show that it provides a micro-foundation for over-reaction to news. Interestingly, despite the fact that the assumptions which define PET are in between those of the CE and the REE, the equilibrium price in PET does not lie in the convex set of the CE and REE outcomes. While CE leads to under-reaction to information relative to the REE benchmark, PET leads to over-reaction, just as we saw in the Apple example. In the context of financial markets, PET also speaks to the leading asset pricing puzzles of excess volatility, excess trading volume, and return predictability, and predicts that all three patterns are more pronounced in periods when there are relatively fewer informed agents, and news has lower information content relative to agents' prior beliefs.[4]

Section 3 extends our equilibrium concept to allow for rationality of higher order beliefs, as in Stahl and Wilson (1994) and Nagel (1995). In particular, we can think of the CE as $1-$level thinking, of PET as $2-$level thinking, and of the REE as $\infty-$level thinking. Since PET equilibrium outcomes do not lie in the convex set of the CE and REE outcomes, in this section we consider whether $K-$level thinking converges to the REE as we allow for higher order beliefs. We find that this is not always the case, and that more sophisticated agents may exacerbate departures from the REE.[5] We also show how, in the presence of misinference, the way under-/over-reaction arises in equilibrium is determined by the nature of the mapping used by agents to extract information from prices: when misspecified agents think that on average all other agents are underreacting(overreacting) more than they really are, they attribute any price change to more(less) extreme news than in reality, which translates into over-(under-)reaction in equilibrium. This makes the path in going from the CE to the REE, passing through all possible levels of higher order beliefs, non-monotonic.[6]

Section 4 generalizes our model further by allowing uninformed agents to use a general

[4]See Shiller (1981) and LeRoy and Porter (1981) on the excess volatility puzzle, Campbell and Shiller (1988), Cochrane (2011), and Greenwood and Shleifer (2014) on return predictability, and French (2008) on excess trading volume.

[5]Bohren and Hauser (2020) also finds that a higher level of reasoning can perform strictly worse than a lower level of reasoning in a sequential learning environment.

[6]This is in contrast to Angeletos and Lian (2017), García-Schmidt and Woodford (2019), Farhi and Werning (2019) and Iovino and Sergeyev (2020), who study K-level thinking in *complete information* setups where agents have limited cognitive capacity to think through the effects of monetary policy shocks on future outcomes. These models do not feature the inference problem that is central in our paper.

linear (misspecified) mapping to extract information from prices, and we show that this specification is flexible and tractable enough to account for heterogenous agents with a wide range of behavioral and cognitive biases. Even in this more general framework, we are able to recover many of the insights uncovered in PET. For example, we show how the possibility of an explosive feedback effect between outcomes and beliefs, and the existence of an unstable equilibrium are pervasive features of situations where agents infer information using misspecified models, and not just a curiosity of our baseline setup. Moreover, we exploit this general framework to make clear the distinction between models of misinference and models of non-rational Bayesian updating, and to study how the two mistakes may interact: crucially, correct inference in models of non-rational Bayesian updating requires the very strong assumption that agents must recognize non-rational behavior in others, while failing to acknowledge irrationality in their own behavior.[7] Relaxing this assumption can lead to extreme and unstable outcomes, something which is not explored in models of non-rational Bayesian updating. For example, if agents fail to recognize their own mistakes in others, misinference due to the incorrect belief that others are rational always amplifies the original bias. This suggests that, in the presence of misinference, smaller departures from rational Bayesian updating may be required to match empirical evidence which is at odds with rational benchmarks.

Section 5 discusses further applications, and concludes. While prices are a natural general equilibrium variable agents may learn from, the feedback effect which PET generates can be applied much more broadly in a variety of macro and finance setups. In Bastianello and Fontanier (2020*a*) we build a dynamic model of financial markets featuring PET agents, and provide a feedback-loop theory of bubbles, and in Bastianello and Fontanier (2020*b*) we explore the implications of misinference in explaining several features of credit cycles.

[7]In models of misinference agents do correct Bayesian updating, but do so with the wrong information they learn. Instead, models of non-rational Bayesian updating assume that agents possess the correct information, but use it in the wrong way, as in models of *inattention* (Gabaix (2020)), of *overconfidence* (Odean (1998), Daniel et al. (1998) or Daniel et al. (2001)), or of *underinference*, as in Eyster et al. (2019), where agents do not make an error in the information they extract, but fail to fully incorporate this information into their expected utility.

1 Model

We consider a setup similar to Grossman and Stiglitz (1980), after information acquisition costs have been incurred. Agents solve a portfolio choice problem between a risky and a risk-free asset. The risky asset is in fixed supply Z, and has a terminal (next-period) payoff of v per share. The unconditional distribution of v is normal with mean μ_0 and variance σ_0^2. We let P be the price of the risky asset. The risk-free asset is in zero net supply and we normalize its price and its gross rate of return to 1.

There is a continuum of measure one of agents with constant absolute risk aversion (CARA) utility over terminal wealth. A fraction ϕ of agents are Informed (I) and receive a common noisy signal $s = v + \epsilon$, where ϵ is normally distributed with mean zero and variance σ_s^2, and is independent of v. The remaining fraction $1 - \phi$ of agents are Uninformed (U), and do not directly observe the realization of s.[8] The setup and the moments which define the conditional distribution of $v|s$ are common knowledge.

CARA utility implies that agent i's portfolio choice problem reduces to:[9]

$$\max_{X_i} \left\{ X_i \left(\mathbb{E}[v|\mathcal{I}_i] - P \right) - \frac{1}{2} A X_i^2 \text{Var}[v|\mathcal{I}_i] \right\} \tag{1}$$

with first-order condition:

$$X_i = \frac{\mathbb{E}[v|\mathcal{I}_i] - P}{A\text{Var}[v|\mathcal{I}_i]} \tag{2}$$

where A is the coefficient of absolute risk aversion, X_i is the dollar amount that agent i invests in the risky asset, and $\mathcal{I}_i$ is agent i's information set at the start of the period. While the optimal asset demand function in (2) is extremely standard, it also makes clear that in order to compute agents' demand functions, we need to determine what information set agents condition on in forming their beliefs. In particular, agents can either condition on their private information alone, or they may also condition on the equilibrium price, which aggregates agents' private information.[10]

[8] We use this information structure to illustrate our notion of PET in the simplest possible framework. In Appendix F we show that the intuitions of our model go through even when we allow for a symmetric information structure where all agents receive a private signal, and also when we allow for prices to be only partially revealing.

[9] See Campbell (2018) p. 25-26 for a derivation of this expression.

[10] Agents benefit from conditioning on prices as these aggregate informed agents' private information, and

However, specifying whether agents condition or not on prices is not enough to determine their information set: as illustrated in the Apple example in the introduction, the information uninformed agents extract from prices depends on their beliefs about the process that generates the price they observe, as this determines the mapping they use to extract information. In this respect, the Competitive/Cursed equilibrium (CE) assumes that agents do not extract information from prices, and instead trade on their private information alone. At the opposite extreme, the Rational Expectations Equilibrium (REE) assumes common knowledge of rationality: not only do agents condition on prices, but they also use the correct mapping to extract information from them, thus leading to $\mathcal{I}_I^{REE} = \{s, P\} = \{s\}$ and $\mathcal{I}_U^{REE} = \{P\} = \{s\}$.[11,12] Partial Equilibrium Thinking (PET) relaxes the assumption of common knowledge of rationality to rationality of second order beliefs: while PET agents do condition on prices, they fail to realize the general equilibrium consequences of their actions. Therefore, PET agents use a misspecified mapping to extract information, and $\mathcal{I}_I^{PET} = \{s, P\} = \{s\}$ and $\mathcal{I}_U^{PET} = \{P\} = \{\tilde{s}\}$, where $\tilde{s}$ is the (wrong) signal they infer from prices. More generally, we use $\tilde{\cdot}$ to refer to uninformed agents' beliefs about a variable.

Finally, we assume that agents do standard rational Bayesian updating: given a signal s_i (with $s_I = s$, and $s_U = \tilde{s}$), agent i's posterior beliefs are normally distributed with mean $\mathbb{E}[v|s_i] = \frac{\tau_s}{\tau_s+\tau_0} s_i + \frac{\tau_0}{\tau_s+\tau_0}\mu_0$ and variance $\mathrm{Var}[v|s_i] = (\tau_s + \tau_0)^{-1}$, where $\tau_0 = 1/\sigma_0^2$ and $\tau_s = 1/\sigma_s^2$ are the prior and signal precisions, respectively.

Once we have specified the mapping uninformed agents use to extract information from prices, we define the equilibrium as follows.

Definition 1. *An equilibrium in our economy is a combination of beliefs and prices* $(\tilde{s}, P)$ *such that:*

(i) *Agents optimize, according to* (2).

therefore allow uninformed agents to obtain a more precise estimate of the fundamental value of the asset, thus reducing the uncertainty they face.

[11] Notice that I agents know that the common signal they receive, s, is the only new information about v. Therefore, given that they observe this signal directly, they know there is no additional information left for them to learn from prices.

[12] Uninformed agents are able to perfectly recover s from prices because in our economy prices are fully revealing: since the risky asset is in fixed supply, the only random shock in the model comes from the signal that informed agents receive, and there exists a one-to-one mapping from prices to signals. All the intuitions of our model carry through to a model with partially revealing prices, as detailed in Appendix E.

(*ii*) *The market for the risky asset clears:* $\phi X_I + (1-\phi)X_U = Z$.

(*iii*) *Agents' beliefs are consistent with the equilibrium price they observe, given their (mis-specified) model of the world.*

In what follows, we solve for the PET equilibrium with three steps, and we solve for the CE and REE benchmarks in the same way in Appendix A.1.

Step 1: Mapping from Prices to Extracted Signals. We specify the mapping PET agents use to extract information from prices: for any given price P, this mapping returns the signal that uninformed agents extract, $\tilde{s}$. This is illustrated by the dashed line in Figure 1.

Step 2: True Price Function. Given any pair of information sets, $\mathcal{I}_I = \{s\}$ and $\mathcal{I}_U = \{\tilde{s}\}$, we compute agents' asset demand functions and impose market clearing. This gives us the market clearing price P as a function of s and of $\tilde{s}$, as shown by the solid line in Figure 1.

Step 3: Equilibrium Price and Extracted Signal. The equilibrium is given by the (P^{PET}, $\tilde{s}^{PET}$)−pair such that uninformed agents' beliefs are consistent with the equilibrium price they observe. This corresponds to the intersection of the dashed and solid lines in Figure 1.

1.1 Partial Equilibrium Thinking (PET)

With Partial Equilibrium Thinking, all uninformed agents extract information from prices, but they mistakenly think they are the only ones doing so.

1.1.1 Mapping from Prices to Extracted Signals

To construct the mapping that PET agents use to extract information from prices, we need to determine the market clearing condition which PET agents think is generating the price that they observe: $\phi \tilde{X}_I + (1-\phi)\tilde{X}_U = Z$. This, in turn, requires us to specify PET agents' beliefs about other agents' asset demand functions, $\tilde{X}_I$ and $\tilde{X}_U$.

In particular, uninformed PET agents think that I agents trade on $\tilde{s}$, and have the following demand function:[13]

$$\tilde{X}_I = \frac{\mathbb{E}[v|\tilde{s}] - P}{A\text{Var}[v|\tilde{s}]} = \tau_0 \frac{\mu_0 - P}{A} + \tau_s \frac{\tilde{s} - P}{A}, \tag{3}$$

Moreover, uninformed PET agents think that all other U agents do not receive the signal, and do not extract information from prices. Instead, they think that all other U agents trade on their prior information alone, with the following demand function:

$$\tilde{X}_U = \frac{\mathbb{E}[v] - P}{A\text{Var}[v]} = \tau_0 \frac{\mu_0 - P}{A} \tag{4}$$

Importantly, all agents are atomistic and do not consider the effect of their own asset demand on prices. Therefore, PET agents think that the equilibrium price is the result of the following market clearing condition:

$$\phi \underbrace{\left(\tau_0 \frac{\mu_0 - P}{A} + \tau_s \frac{\tilde{s} - P}{A}\right)}_{\tilde{X}_I} + (1-\phi) \underbrace{\left(\tau_0 \frac{\mu_0 - P}{A}\right)}_{\tilde{X}_U} = Z. \tag{5}$$

which gives U agents' misspecified mapping, linking prices to new information:

$$P^{Mis}(\tilde{s}) = \frac{\phi\tau_s}{\phi\tau_s + \tau_0}\tilde{s} + \frac{\tau_0}{\phi\tau_s + \tau_0}(\mu_0 - AZ\tau_0^{-1}), \tag{6}$$

Intuitively, (6) shows U agents' beliefs that a fraction ϕ of agents (I) trade on $\tilde{s}$ with precision τ_s, and a fraction 1 of agents (I and U) trade on the prior with precision τ_0. Therefore, for a given observed price P, PET agents extract $\tilde{s}$ by inverting (6):

$$\tilde{s} = \frac{\tau_0 + \phi\tau_s}{\phi\tau_s}P - \frac{\tau_0}{\phi\tau_s}(\mu_0 - AZ\tau_0^{-1}). \tag{7}$$

This mapping is shown by the dashed line in Figure 1: given any price P on the vertical axis, this mapping returns the signal PET agents extract, $\tilde{s}$, on the horizontal axis. The slope of this mapping is informative of the nature of misinference induced by PET. In particular,

[13] Recall that PET agents think that the signal they are extracting is the correct one. Therefore $\tilde{s}$ denotes PET agents' beliefs about the signal I agents receive and trade on.

(6) can be rewritten as:

$$P^{Mis}(\tilde{s}) = \gamma\tilde{s} + (1-\gamma)(\mu_0 - AZ\tau_0^{-1}) \tag{8}$$

where $\gamma \equiv \frac{\partial P^{Mis}(\tilde{s})}{\partial \tilde{s}} = \frac{\phi\tau_s}{\phi\tau_s+\tau_0}$ is uninformed PET agents' perceived sensitivity of prices to new information. Appendix A.1 shows that when agents use the correct mapping to infer information from prices, this slope is $\gamma_{REE} = \frac{\tau_s}{\tau_s+\tau_0}$, and since $\phi \in [0,1]$:

$$\gamma < \gamma_{REE} \tag{9}$$

Moreover, this wedge is decreasing in the fraction of informed agents, ϕ. Specifically, PET agents think that the price is less responsive to new information than it truly is, and the wedge between γ and γ_{REE} reflects the extent of misinference. By failing to take into account the general equilibrium consequences of their actions, PET agents attribute any price change they observe to new information alone, and extract a more extreme signal than in reality, as in the Apple example.

Finally, notice that (6) is exactly the price function which arises as the solution to the CE (derived in Appendix A.1), as PET agents (incorrectly) think that everyone is trading on their private information alone, as if they were fully cursed.

1.1.2 True Price Function

To determine the true price function, we must specify the true market clearing condition. In reality, all U agents extract information from prices using the mapping described above and trade on $\tilde{s}$, unlike PET agents' beliefs that they are trading on the prior alone. Moreover, in reality, I agents trade on the true signal s.[14]

[14]By trading on s, I agents are best responding, irrespective of other agents' beliefs and actions: given the optimal asset demand function in (2), all agents are only concerned with forecasting the fundamental value of the asset, $v|I_I$. Other agents' beliefs and actions affect agents' *realized* asset demand via the equilibrium price, but do not affect the demand *function* used by agents. In other words, other agents' actions affect the realization, but not the strategy used by I agents.

In equilibrium, the true market clearing condition is given by:

$$\phi \underbrace{\left(\tau_0 \frac{\mu_0 - P}{A} + \tau_s \frac{s - P}{A}\right)}_{X_I} + (1-\phi) \underbrace{\left(\tau_0 \frac{\mu_0 - P}{A} + \tau_s \frac{\tilde{s} - P}{A}\right)}_{X_U} = Z, \tag{10}$$

and the resulting market clearing price function is:

$$P^{True}(s, \tilde{s}) = \frac{\phi \tau_s}{\tau_0 + \tau_s} s + \frac{(1-\phi)\tau_s}{\tau_0 + \tau_s} \tilde{s} + \frac{\tau_0}{\tau_0 + \tau_s}(\mu_0 - AZ\tau_0^{-1}), \tag{11}$$

which shows that a fraction ϕ of agents (I) trade on s with precision τ_s, a fraction $(1-\phi)$ of agents (U) trade on $\tilde{s}$ also with precision τ_s, and a fraction 1 of agents (I and U) trade on the prior with precision τ_0. This mapping is shown by the solid line in Figure 1: given a fixed true signal s, this function provides the market clearing price that arises when I agents trade on s, and all U agents trade on $\tilde{s}$. We can rewrite (11) as:

$$P^{True}(s, \tilde{s}) = \alpha s + \beta \tilde{s} + (1 - \alpha - \beta)(\mu_0 - AZ\tau_0^{-1}) \tag{12}$$

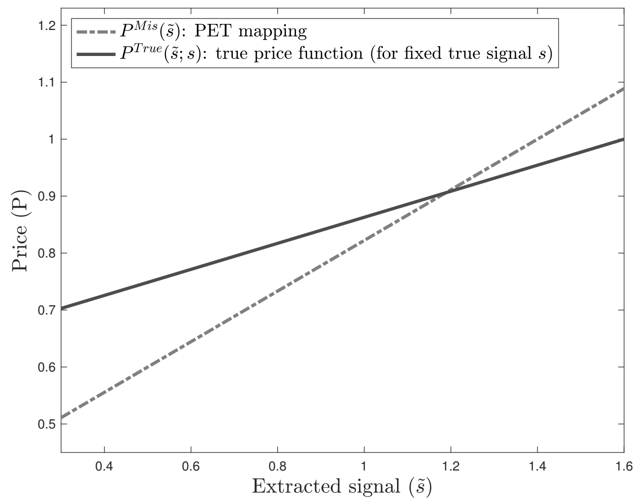
where $\alpha \equiv \frac{\partial P^{True}(s,\tilde{s})}{\partial s} = \frac{\phi \tau_s}{\tau_s + \tau_0}$ and $\beta \equiv \frac{\partial P^{True}(s,\tilde{s})}{\partial \tilde{s}} = \frac{(1-\phi)\tau_s}{\tau_s + \tau_0}$ reflect the influence on prices of I and U agents' beliefs, respectively. Given these definitions, notice that $\gamma_{REE} = \frac{\tau_s}{\tau_s + \tau_0} = \alpha + \beta$, as further discussed in Appendix A.1.

1.1.3 Equilibrium Price and Extracted Signal

In equilibrium, the signal U agents extract must be consistent with the equilibrium price they observe. We therefore solve for the equilibrium price and extracted signal $(P^{PET}, \tilde{s}^{PET})-$pair such that $P^{PET} \equiv P^{Mis}(\tilde{s}^{PET}) = P^{True}(s, \tilde{s}^{PET})$ in (7) and (11), as shown by the intersection of the two mappings in Figure 1. The resulting equilibrium price and extracted signal are given by:

$$P^{PET} = \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0} s + \left(1 - \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{13}$$

Figure 1: Equilibrium Price and Extracted Signal. This Figure plots the mapping used by uninformed PET agents to extract information from prices (dashed line), and the price function that arises when all I agents trade on a fixed signal s, and U agents extract a signal $\tilde{s}$ (solid line). The intersection of these two mappings gives the PET equilibrium price and extracted signal for a given s. Notice that, in $(P, \tilde{s})-$space, changing s shifts the true price function (solid line) and leaves the mapping from $\tilde{s}$ to P unchanged, as can be seen from the expressions in (7) and (11).



$$\tilde{s}^{PET} = s + \frac{\left(\frac{1-\phi}{\phi^2}\right)\tau_0}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0} \left(s - \left(\mu_0 - AZ\tau_0^{-1}\right)\right) \tag{14}$$

This shows that PET leads to misinference whenever $\phi \neq 1$ and $s \neq \mu_0 - AZ\tau_0^{-1}$. Intuitively, when $\phi = 1$, all agents are informed, and when $s = \mu_0 - AZ\tau_0^{-1}$ the equilibrium price is $P = \mu_0 - AZ\tau_0^{-1}$, leaving no scope for misinference in (6).

Before proceeding to comparing the PET equilibrium outcomes with those from the CE and REE benchmarks, we introduce two key properties of PET equilibria: the implicit feedback effect between outcomes and beliefs, and a notion of stability.

1.2 PET Feedback Effect and Stability

In the Apple example in the introduction, we alluded to a two-way feedback effect between outcomes and beliefs: following a positive shock, higher prices lead to more optimistic beliefs, which feed into even higher prices, more optimistic beliefs, and so on. Not only does the strength of this feedback effect determine the properties of the PET equilibrium, but it also sheds light on the core mechanism at the heart of PET.

To understand how this feedback effect operates in practice, and for ease of exposition, consider the case with $\mu_0 - AZ\tau_0^{-1} = 0$, so that we can write the system of the misspecified and true mappings in (7) and (11) as:

$$\tilde{s} = \frac{1}{\gamma} P \tag{15}$$

$$P = \alpha s + \beta \tilde{s} \tag{16}$$

where $\gamma \equiv \frac{\partial P^{Mis}(\tilde{s})}{\partial \tilde{s}} = \frac{\phi \tau_s}{\phi \tau_s + \tau_0}$, $\alpha \equiv \frac{\partial P^{True}(s,\tilde{s})}{\partial s} = \frac{\phi \tau_s}{\tau_s + \tau_0}$ and $\beta \equiv \frac{\partial P^{True}(s,\tilde{s})}{\partial \tilde{s}} = \frac{(1-\phi)\tau_s}{\tau_s + \tau_0}$.

Starting from a steady state with $P_0 = \mu_0 - AZ\tau_0^{-1} = \tilde{s}_0 = 0$, consider shocking the economy by increasing the signal I agents receive to $s > 0$, as illustrated by the upward shift in the true price function in Figure 2. By definition, the resulting PET equilibrium is given by the intersection of this new shifted true price function with the mapping used by U agents to extract information from prices:

$$P^{PET} = \frac{\alpha}{1 - \frac{\beta}{\gamma}} s \tag{17}$$

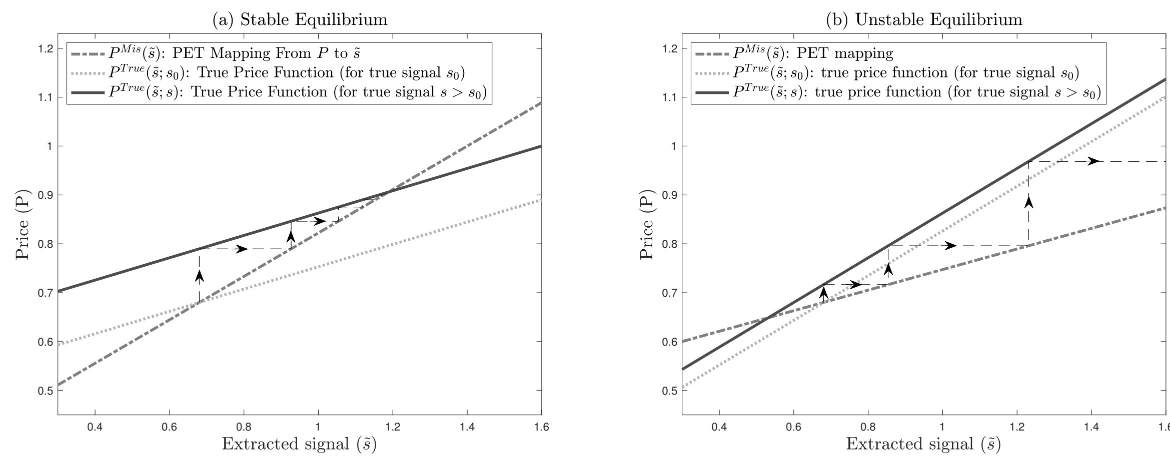
$$\tilde{s}^{PET} = \frac{\alpha}{\gamma - \beta} s \tag{18}$$

To see how this equilibrium embeds a two-way feedback effect between prices and beliefs which amplifies shocks, let us analyze the tâtonnement process in going from P_0 to P^{PET}. These are depicted by the arrows in Figure 2. When I agents receive a signal s, (16) implies that on impact $P_1 = \alpha s$. Given this price, PET agents extract a signal $\tilde{s}_1 = \frac{1}{\gamma} P_1 = \frac{1}{\gamma} \alpha s$ using (15). These new (more optimistic) beliefs then generate a higher market clearing price via (16): $P_2 = \alpha s + \frac{\beta}{\gamma} P_1 = \left(1 + \frac{\beta}{\gamma}\right) \alpha s$. However, since $P_2 \neq P_1$, U agents extract a different signal $\tilde{s}_2 = \frac{1}{\gamma} P_2 = \frac{1}{\gamma}\left(1 + \frac{\beta}{\gamma}\right) \alpha s$, and given these beliefs, the market clearing price becomes $P_3 = \alpha s + \frac{\beta}{\gamma}\left(1 + \frac{\beta}{\gamma}\right) \alpha s = \left(1 + \frac{\beta}{\gamma} + \frac{\beta^2}{\gamma^2}\right) \alpha s$. Iterating this further, we find that:

$$P_n = \sum_{i=0}^{n-1} \left(\frac{\beta}{\gamma}\right)^i \alpha s \tag{19}$$

This expression illustrates two key points. First, $\frac{\beta}{\gamma}$ corresponds to the size of the feedback effect between outcomes and beliefs. Second, when $\frac{\beta}{\gamma} < 1$, the feedback effect is weak enough that its influence on equilibrium outcomes dies out as it gets compounded. In other words, the geometric sum in (19) is well defined, and the tâtonnement process just described converge to the PET equilibrium in (17), where U agents' beliefs are consistent with the price they observe. This is illustrated in the left panel of Figure 2. If instead $\frac{\beta}{\gamma} > 1$, the influence of the feedback effect gets stronger as it gets compounded, and prices and beliefs do not converge to the PET equilibrium, unless $P_0 = P^{PET}$. Instead, they become extreme and decoupled from fundamentals, with $\lim_{n\to\infty} P_n = \pm\infty$, as in the right panel of Figure 2.

Figure 2: PET Feedback Effect and Stability. This figure shows how equilibrium outcomes evolve following a positive shock to informed agents' signal, $s > s_0$. Specifically, the positive shock is captured by an upward shift in the true price function from the dotted line ($P^{True}(s_0, \tilde{s})$) to the bold line ($P^{True}(s, \tilde{s})$). The misspecified mapping used by uninformed PET agents to extract information from prices is given by the dashed line ($P^{Mis}(\tilde{s})$), and this remains unchanged. The old and new steady states are given by the intersection of $P^{Mis}(\tilde{s})$ with $P^{True}(s_0, \tilde{s})$ and $P^{True}(s, \tilde{s})$, respectively. Finally, the arrows trace out the *tâtonnement* process which, starting from the original steady state, leads to (19) following the shock. Panel (a) provides an example of a stable equilibrium ($^{\beta}/_{\gamma} < 1$), and shows that in this case the tâtonnement process converges to the new steady state. Conversely, Panel (b) depicts a case where the equilibrium is unstable ($^{\beta}/_{\gamma} > 1$), and shows that in this case the tâtonnement process leads prices and beliefs to diverge away from the steady state.



In light of this discussion, we define the PET equilibrium to be stable if, regardless of the starting point, the process just described converges to the PET equilibrium. Otherwise, we define the PET equilibrium to be unstable.

Proposition 1 (Stability). *The PET equilibrium is stable if and only if:*

$$\frac{\beta}{\gamma} < 1 \iff \frac{\partial P^{Mis}(\tilde{s})}{\partial \tilde{s}} > \frac{\partial P^{True}(s, \tilde{s})}{\partial \tilde{s}} \tag{20}$$

In our setup, this is equivalent to:

$$\phi > \left(1 + \sqrt{1 + \frac{\tau_s}{\tau_0}}\right)^{-1} \tag{21}$$

Therefore, the stability region is increasing both in the fraction of informed agents, ϕ, and in the informativeness of the signal relative to the prior, τ_s/τ_0.

Proof. All proofs are in Appendix A.2, unless stated otherwise. $\square$

As shown in Figure 2, this condition requires the true price function to be steeper than the misspecified mapping in $(P, \tilde{s})-$space. Intuitively, for a given correct sensitivity γ^{REE}, the discrepancy between the true and the REE mapping is decreasing in γ, as anticipated in (9). Therefore, a higher γ reduces the extent of misinference. Moreover, a lower β diminishes the influence that PET agents have on equilibrium prices. Both of these effects contribute to stability.

To understand why the PET equilibrium can be unstable, notice that in this framework prices play a dual role: an informational role, as well as their traditional role as a measure of scarcity. Their informational role introduces strategic complementarities: U agents associate higher prices with more valuable fundamentals. These more optimistic beliefs lead them to increase their asset demand, and this translates into even higher prices, thus fuelling the feedback effect between prices and beliefs which PET generates. On the other hand, the traditional role as a measure of scarcity introduces strategic substitutabilities: higher prices make the asset more expensive, thus inducing all agents to reduce their asset demand, which translates into lower prices, and this weakens the strength of the feedback effect. The way these two effects interact in shaping equilibrium outcomes is particularly clear when writing down the PET aggregate excess demand function for the asset:

$$X_{TOT} - Z = \phi \underbrace{\left(\frac{\frac{\tau_s}{\tau_s+\tau_0}s + \frac{\tau_0}{\tau_s+\tau_0}\mu_0 - P}{A(\tau_s+\tau_0)^{-1}}\right)}_{X_I} + (1-\phi)\underbrace{\left(\frac{\frac{\tau_s}{\tau_s+\tau_0}\tilde{s} + \frac{\tau_0}{\tau_s+\tau_0}\mu_0 - P}{A(\tau_s+\tau_0)^{-1}}\right)}_{X_U} - Z \tag{22}$$

This shows P's role as a measure of scarcity, as it enters negatively into both agents' demand functions. However, we know from (7) that $\tilde{s}$ is also an increasing function of P, as U agents infer information from prices. Making use of this, and rearranging:

$$X_{TOT} - Z = \frac{1}{A(\tau_s + \tau_0)^{-1}} \left(\underbrace{\frac{(1-\phi)\tau_s}{\tau_s + \tau_0} \left(\frac{\phi\tau_s}{\phi\tau_s + \tau_0} \right)^{-1}}_{\text{informational role}} \underbrace{- \quad 1}_{\text{scarcity role}} \right) P + constant \tag{23}$$

Whenever $\frac{(1-\phi)\tau_s}{\tau_s+\tau_0}\left(\frac{\phi\tau_s}{\phi\tau_s+\tau_0}\right)^{-1} > 1$, which is equivalent to $\frac{\beta}{\gamma} > 1$, the informational role of prices dominates over their role as a measure of scarcity, and the feedback effect between prices and beliefs is so strong that the resulting aggregate excess demand function is upward sloping, and the equilibrium unstable. If we were to give this excess demand function to a Walrasian auctioneer, they would never reach the PET equilibrium, unless they were to start from it.[15] Instead, when the condition is reversed and $\frac{\beta}{\gamma} < 1$, the effect of strategic substitutabilities is greater than that of strategic complementarities, the excess demand function is downward sloping, the equilibrium stable, and a Walrasian auctioneer would converge to the PET equilibrium.[16]

The strength of the feedback effect between outcomes and beliefs shapes the properties of equilibrium outcomes, and Section 4.1 shows that the possible unstable nature of equilibrium outcomes is widespread in misinference models, and not just a curiosity of PET. In the rest of the paper, we focus on the properties of the stable equilibrium, which is consistent with excess demand functions being downward sloping in equilibrium. However, it is worth noticing that the possible unstable nature of the PET equilibrium is interesting in itself as it is the result of an extreme manifestation of the two-way feedback effect which PET generates. This feature, which is absent in the CE and REE benchmarks, lends itself to explaining periods where

[15]In addition, when the equilibrium is unstable, the resulting equilibrium price is *decreasing* in the level of the true signal: $\partial P^{PET}/\partial s < 0$. This is simply a manifestation of Samuelson's Correspondence Principle. See the proof in Appendix A.2.7 for more details.

[16]Notice that our Walrasian auctioneer is endowed with the excess demand function $X_{TOT} - Z$. Implicitly, this requires that no matter what price the Walrasian auctioneer calls out, U agents treat this price as if it were arising from the (misspecified) market clearing condition in (5), so that, for any price P, their demand function is given by:

$$X_U = \frac{1}{A(\tau_s + \tau_0)^{-1}} \left(\frac{\tau_s}{\tau_s + \tau_0} \left(\frac{\phi\tau_s}{\phi\tau_s + \tau_0} \right)^{-1} - 1 \right) P + constant_U \tag{24}$$

which we obtain from substituting the mapping in (14) into agents' FOC in (2) and rearranging.

outcomes and beliefs are extreme and decoupled from fundamentals.[17]

2 Properties of PET Equilibrium Outcomes

In this section we compare the properties of the PET equilibrium with the CE and REE benchmarks, and we discuss how PET outcomes relate to empirical patterns in asset pricing. Specifically, we compare beliefs, prices, trading volume, and returns across equilibrium concepts, and we show that PET outcomes do not lie in the convex set of their CE and REE counterparts, despite the fact that the assumptions which define PET lie in between those which characterize the CE and the REE.[18] We also show that PET is able to speak to some of the leading asset pricing puzzles: excessive trading volume, predictability of returns, and excess volatility. These properties are the result of the interaction of two crucial aspects of the PET equilibrium. First, PET's misinference leads agents to hold *incorrect* beliefs, which translate into over-reaction and predictability of realized returns. Second, PET generates endogenously *heterogenous* beliefs without relying on heterogeneous priors or "agree-to-disagree" assumptions. This leads to positive trading volume and variability in expected returns. While PET is not the only theory which addresses the asset pricing puzzles described above, it offers an alternative explanation for them, and we view its ability to do so as further corroborating the plausability of this behavioral bias.

Over-reaction and Excess Volatility. As anticipated in the Apple example, and as shown in (14), PET naturally leads to beliefs that are more extreme than under fully rational inference. Therefore, PET provides a micro-foundation to over-reaction to news, as is clear

[17] We explore this further in Bastianello and Fontanier (2020*a*) where we develop a dynamic version of PET, and we fully exploit the properties of the unstable region.

[18] Recall that the CE assumes rationality of first order beliefs, PET assumes rationality of first and second order beliefs, and the REE assumes common knowledge of rationality. Specifically, in our setup, rationality of first order beliefs is equivalent to agents having optimal demand functions, as in (2), given their private information. When all agents are rational in this sense, prices aggregate all agents' private information, and therefore contain valuable information. Rationality of second order beliefs additionally requires that agents understand that all other agents are rational, and therefore that prices contain valuable information which other agents also wish to extract.

from comparing equilibrium outcomes from Section 1.1.3 and Appendix A.1:

$$P^{CE} = \frac{\phi\tau_s}{\phi\tau_s + \tau_0}s + \left(1 - \frac{\phi\tau_s}{\phi\tau_s + \tau_0}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{25}$$

$$P^{REE} = \frac{\tau_s}{\tau_s + \tau_0}s + \left(1 - \frac{\tau_s}{\tau_s + \tau_0}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{26}$$

$$P^{PET} = \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}s + \left(1 - \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{27}$$

Figure 3 plots these expressions, and shows that PET gives rise to over-(under-)pricing following positive(negative) risk-adjusted signals.

Proposition 2 (Over-reaction)**.** *When the equilibrium is stable, PET delivers over-reaction to news, while the CE delivers under-reaction:*

$$\frac{\partial P^{CE}}{\partial s} < \frac{\partial P^{REE}}{\partial s} < \frac{\partial P^{PET}}{\partial s} \tag{28}$$

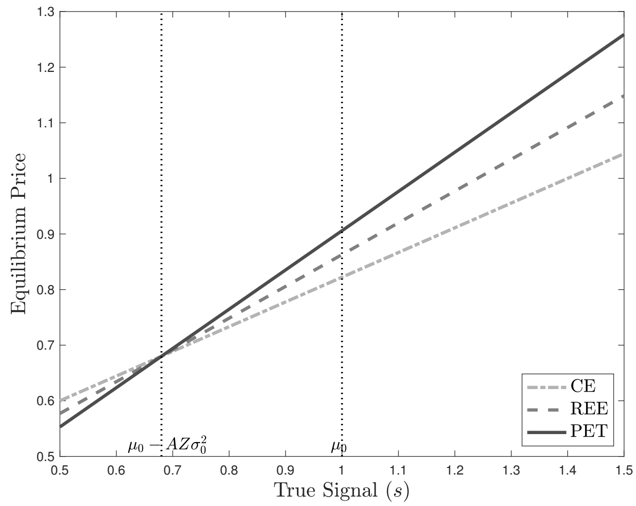
Even though the assumptions which define PET lie in between those that characterize the REE and CE equilibrium benchmarks, the resulting PET equilibrium outcomes do not lie in the convex combination of the REE and CE outcomes.

PET also speaks to the long-standing "excess volatility" puzzle (Shiller (1981) and LeRoy and Porter (1981)): PET naturally delivers excess volatility since beliefs are excessively volatile relative to fundamentals.[19]

The Identity of Contrarians. The implications of PET differ sharply from those of the REE and CE when we look at the identity of agents who trade in the direction of the signal, and those who trade as contrarians. In the REE, the asset is correctly priced, making all agents indifferent on how much of the asset to hold at the equilibrium price. On the other hand, the CE predicts that informed agents trade in the direction of the signal because they understand the asset is under-priced, and uninformed agents trade as contrarians. PET

[19]Bordalo, Gennaioli, LaPorta and Shleifer (2020) use survey data to show that beliefs about future fundamentals are sufficiently volatile to account for Shiller's excess volatility puzzle.

Figure 3: CE, PET, and REE equilibrium prices. This Figure plots the CE, PET, and REE equilibrium price functions in (25) (26) and (27) over the true signal, s. Parameters ϕ and τ_s/τ_0 satisfy the restrictions in (21), ensuring that the PET equilibrium is stable. The thin dotted lines mark the risk-adjusted neutral signal $s = \mu_0 - AZ\tau_0^{-1}$, and the average signal $s = \mu_0$.



makes the opposite prediction: following good news, informed agents know the asset is over-priced, while uninformed agents believe they live in a cursed world, and therefore think the asset is under-priced.

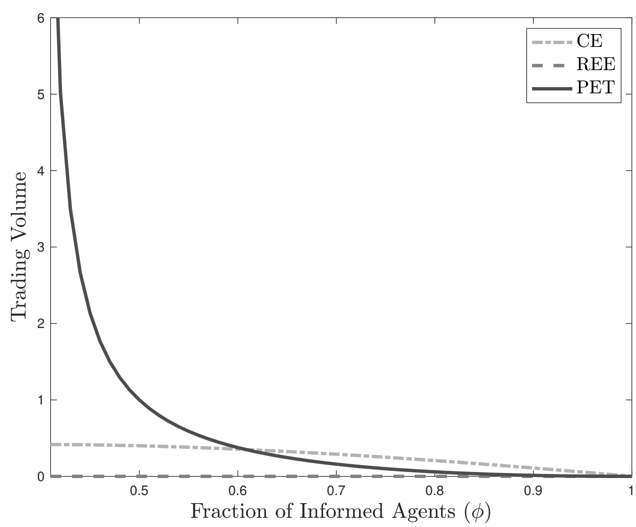
Proposition 3 (Contrarians)**.** *Unlike the CE and REE benchmarks, informed PET agents are contrarians and uninformed PET agents trade in the direction of the signal.*

Trading Volume. A well-documented fact in financial markets is that trading volume is far greater than what rational models can justify (Tirole (1982), Milgrom and Stokey (1982), French (2008), Barberis (2018)). Starting from common priors, PET generates endogenously heterogenous beliefs, and therefore positive trading volume in equilibrium. Moreover, the amount of trading volume may be arbitrarily large if the size of the feedback effect between outcomes and beliefs is strong enough.

Proposition 4 (Volume)**.** *For any fixed $s \neq \mu_0 - AZ\tau_0^{-1}$, PET trading volume is increasing in the size of the feedback effect, and $\lim_{\frac{\beta}{\gamma}\to 1^+} V^{PET} = \infty$.*

Proposition 4 highlights that PET does not rely on extreme signals to achieve excessive amounts of trading volume: indeed, Proposition 1 shows that the size of the feedback effect between outcomes and beliefs only depends on the fraction of informed agents, ϕ, and on the informativeness of the signal relative to the prior, τ_s/τ_0. To illustrate this point, Figure 4 compares CE, REE, and PET trading volume when informed agents receive the average signal. Even for $s = \mu_0$, PET trading volume can be arbitrarily large when the fraction of informed agents decreases towards the unstable region: misinference becomes more accentuated, beliefs become more dispersed, disagreement more pronounced, and trading volume more excessive. Another implication of Proposition 4 is that PET can generate even larger trading volume than its CE counterpart, even though PET agents actively extract information from prices and have beliefs which are consistent with the equilibrium price they observe.

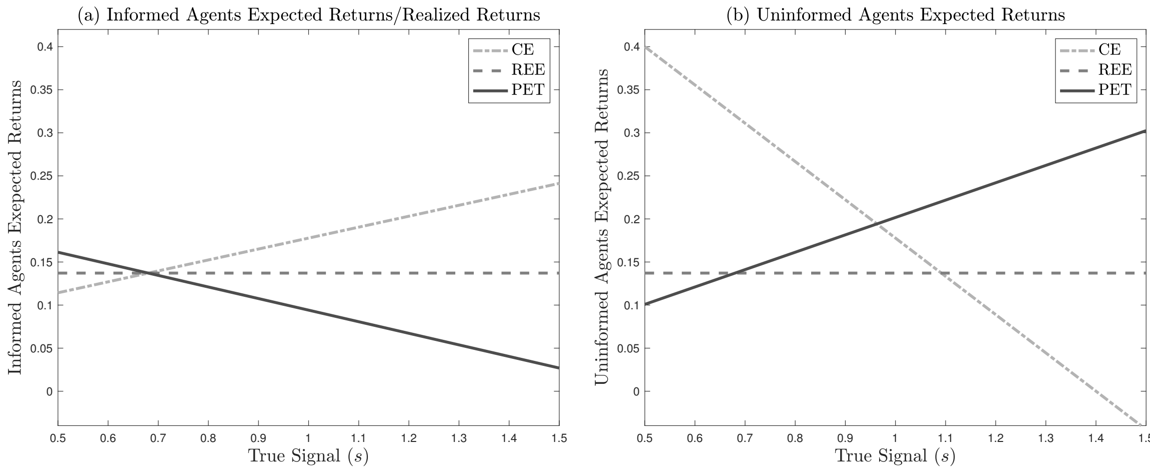
Figure 4: CE, PET and REE Trading Volume over ϕ, for the fixed true signal $s = \mu_0$. This Figure plots equilibrium CE, PET and REE trading volume for a fixed true signal $s = \mu_0$ over the range of fractions of informed agents, ϕ, such that (21) holds and the PET equilibrium is stable.



In addition, not only is PET able to generate large trading volume, but it also predicts that agents trade because they mistakenly think they have an information advantage. This is consistent with empirical evidence in Liu et al. (2020), who combine survey and transaction data of retail investors in China to elicit trading motives, and find that the two most dominant drivers of excessive trading are a misperceived information advantage and gambling preferences.

Finally, in Appendix C we show that if agents were to learn about ϕ and s from observing both prices and volume, they would learn the correct model of the world. However, notice that our model is attentionally stable, in the sense of Gagnon-Bartsch et al. (2020), and

Figure 5: Comparing CE, PET, and REE Expected and Realized Returns. This Figure plots the equilibrium CE, PET, and REE expected returns by I agents (left panel), and expected returns by U agents (right panel) for different values of the true signal, s. Notice that average realized returns are equal to expected returns by I agents since they hold rational beliefs.



agents need not realize their model of the world is misspecified.[20]

Realized and Expected Returns. We now turn to expected and realized returns:

$$\mathbb{E}_i[R]^K = \mathbb{E}[v|\mathcal{I}_i^K] - P^K = \mathbb{E}[v|\mathcal{I}_i^K] - \left(\bar{\mathbb{E}}[v]^K - \bar{\text{Var}}[v]^K\right), \tag{29}$$

where $i \in \{I, U\}$ and $K \in \{CE, REE, PET\}$, and $\bar{\mathbb{E}}[v]^K$ and $\bar{\text{Var}}[v]^K$ aggregate agents' posterior beliefs.[21] Conditional on s, realized returns are equal to I agents' expected returns, on average. Figure 5 plots these quantities over the true signal.

Figure 5 shows that REE expected returns are constant for both I and U agents, as prices perfectly adjust to account for changes in beliefs.[22] Turning to the CE, informed agents understand that equilibrium outcomes under-react, and therefore *correctly* expect

[20]Since prices are fully revealing, if agents were to extract information from equilibrium outcomes using the true model of the world, the equilibrium price would be a sufficient statistic and trading volume would provide no additional information. Conversely, prices are not a sufficient statistic when agents use the wrong model of the world. However, since agents think they are using the correct model of the world, they (incorrectly) believe that the equilibrium price is a sufficient statistic, and have no reason to pay attention to trading volume.

[21]Let $\tau_i^K = Var[v|\mathcal{I}_i^K]$ for $i \in \{I, U\}$ and $K \in \{CE, REE, PET\}$. Then $\bar{\mathbb{E}}[v]^K = \frac{\phi\tau_I^K}{\phi\tau_I^K + (1-\phi)\tau_U^K}\mathbb{E}[v|\mathcal{I}_I^K] + \frac{(1-\phi)\tau_U^K}{\phi\tau_I^K + (1-\phi)\tau_U^K}\mathbb{E}[v|\mathcal{I}_U^K]$, and $\bar{Var}[v]^K = (\phi\tau_I^K + (1-\phi)\tau_U^K)^{-1}$. By inspection of (25), (26) and (27), it follows that $P^K = \bar{\mathbb{E}}[v] - \bar{Var}[v]AZ$.

[22]This follows immediately from (29) as all agents update their beliefs in the same (correct) way so that $\mathbb{E}[v|\mathcal{I}_i^{REE}] = \bar{\mathbb{E}}[v]^{REE}$, and the average conditional variance is independent of s.

higher returns in good times, when the asset is under-priced. On the other hand, U agents' beliefs are independent of s, and they expect higher returns in bad times, when the price is lower.[23] Finally, in PET, U agents' misinference gives rise to over-reaction. Following good news, U agents become over-optimistic ($\mathbb{E}[v|\mathcal{I}_I^{PET}] < \bar{\mathbb{E}}[v]^{PET} < \mathbb{E}[v|\mathcal{I}_U^{PET}]$), and (29) then shows that they therefore expect higher payoffs and higher returns than their rational counterparts. The argument is symmetric following bad news. On the other hand, I agents understand the extent of over-reaction, and expect higher returns in bad times, when the asset is under-priced. Similarly, on average, realized returns are high in bad times, and low in good times. Therefore, by failing to realize the general equilibrium consequences of their actions, U PET agents have higher expected returns in good times, and they are systematically *disappointed*. Unlike the CE and REE, PET can rationalize the empirical evidence in Greenwood and Shleifer (2014).[24]

Proposition 5 (Returns)**.** *The stable PET can reconcile the empirical evidence in Greenwood and Shleifer (2014). On average:*

1. $\frac{\partial \mathbb{E}_U[R]^{PET}}{\partial s} > 0$*: U agents' expected returns are higher in good times;*

2. $\frac{\partial R^{PET}}{\partial s} < 0$*: realized returns are lower in good times.*

Similarly, using survey data Giglio et al. (2020) show that investors report low expected returns when they expect a higher probability of a disaster. Our model has a natural explanation: when investors expect a high probability of a disaster in the future, they fail to realize that other investors hold the same beliefs as them, and that the equilibrium price is lower to reflect this. This leads them to expect lower returns following bad news, just as in Figure 5.

[23] In the CE, I agents correctly update their beliefs conditional on s, while U agents' beliefs are independent of the signal. Therefore, when there is good news, $\mathbb{E}[v|\mathcal{I}_I^{CE}] > \bar{\mathbb{E}}[v]^{CE} > \mathbb{E}[v|\mathcal{I}_U^{CE}]$, and conversely when there is bad news. The rest follows from (29).

[24] Notice that the combination of two key ingredients allows us to speak to the empirical patterns in Greenwood and Shleifer (2014): heterogeneous agents, and over-reaction. One without the other would not be enough in our setup. For example, models of over-reaction with a representative agent are not able to generate any variation in expected returns, unless they allow for the perceived variance of the fundamental to vary: with $\bar{Var}[v]$ constant, homogenous beliefs imply that $\mathbb{E}[v|\mathcal{I}_i] - \bar{\mathbb{E}}[v] = 0$, so that expected returns in (29) must also be constant. Conversely, rational and heterogeneous agents in this setup can generate time-varying expected returns, but rational agents are right on average, unlike the empirical evidence in Greenwood and Shleifer (2014), which documents a decoupling of expected and realized returns.

Moreover, empirical studies that have looked for a relation between risk-premia (equivalently expected returns in our model) and future returns have shown that conventional wisdom on risk-based explanations for time-varying risk premia fail.[25] Proposition 5 shows how PET breaks the classic risk/return tradeoff. Since $\mathbb{E}[v|\mathcal{I}_i^{PET}] - \bar{\mathbb{E}}[v]^{PET}$ for $i \in \{I, U\}$ changes with s, our model delivers time-varying expected returns that do *not* ultimately compensate investors for bearing risk.

Comparative Statics. We conclude this section with a general characterization of how equilibrium quantities vary with the parameters of the model. In Proposition 4, we showed that the wedge between REE and PET trading volume is increasing in the strength of the feedback effect $\frac{\beta}{\gamma}$, which is itself decreasing in ϕ and in $\frac{\tau_s}{\tau_0}$. Moreover, we showed that as the strength of the feedback effect approaches the unstable region, the discrepancy between PET and REE volume can becomes arbitrarily large. This empirical prediction generalizes to all PET outcomes considered so far.[26]

Proposition 6 (Comparative Statics)**.** *When the PET equilibrium is stable, the wedge between PET and REE equilibrium outcomes ($\tilde{s}$, P, $^{\partial P}/_{\partial s}$, V, $\mathbb{E}_i[R]$, etc) is increasing in the size of the feedback effect between outcomes and beliefs, $\frac{\beta}{\gamma}$. Therefore, the discrepancy is greater when the fraction of I agents (ϕ) is low, and when news are uncertain relative to the prior (i.e. when $\frac{\tau_s}{\tau_0}$ is low). This wedge becomes arbitrarily large in the limit as $\frac{\beta}{\gamma} \to 1^+$.*

This is somewhat striking given that at the individual level the nature of the psychological bias itself is fixed: regardless of the environment, PET truncates common knowledge of rationality to rationality of second order beliefs. However, despite the nature of the bias being fixed, varying the environment (ϕ and $^{\tau_s}/_{\tau_0}$) leads to very different sized wedges between PET and REE outcomes, and Proposition 6 suggests that these wedges can be arbitrarily large even for reasonable (i.e. bounded) parameter values. This is a direct implication of the properties of the feedback effect between outcomes and beliefs which PET generates.

[25] Moreira and Muir (2017) show that a strategy that manages volatility so as to take *less* risk in recessions and crisis still earns high average returns, contrary to what leading models would predict.

[26] Further details are in Appendix A.2.6.

3 K-Level Thinking

In Section 2, we refer to the CE as $1-$level thinking, to PET as $2-$ level thinking, and to the REE as $\infty-$level thinking. The natural next step is to consider what happens as we allow for rationality of higher order beliefs, whereby a $K-$level thinker believes that all other agents are $(K-1)-$level thinkers.[27] For example, a $3-$level thinker believes that all other agents think they are the only ones extracting information from prices. In light of our finding in Proposition 2 that PET equilibrium outcomes do not lie in the convex set of the CE and REE, we analyze whether $K-$level thinking converges to the REE in the limit of common knowledge of rationality.

3.1 Solving for K-Level Thinking

We solve for $K-$level thinking recursively by following the same steps outlined in Section 1.1. First, in extracting information from prices, all U agents *believe* that all other U agents are $(K-1)-$level thinkers, and that the price they observe is generated by the $(K-1)-$level thinking equilibrium price function:

$$P^{Mis}(\tilde{s}) = \gamma_{K-1}\tilde{s} + (1-\gamma_{K-1})(\mu_0 - AZ\tau_0^{-1}) \tag{30}$$

where γ_{K-1} is the sensitivity of the price to the true signal in the $(K-1)-$level thinking equilibrium, and $\tilde{s}$ here denotes a $K-$level thinker's belief about s.

Second, in *reality*, all I agents trade on the true signal s, and all U agents are $K-$level thinkers and trade on the signal $\tilde{s}$, which they extract from prices using the mapping in (30). Given these beliefs, the true market clearing condition leads to:

$$P^{True}(s,\tilde{s}) = \alpha s + \beta\tilde{s} + (1-\alpha-\beta)(\mu_0 - AZ\tau_0^{-1}) \tag{31}$$

where $\alpha \equiv \frac{\phi\tau_s}{\tau_s+\tau_0}$, $\beta \equiv \frac{(1-\phi)\tau_s}{\tau_s+\tau_0}$, and are constant and independent of K.

Finally, in *equilibrium*, agents' (misspecified) beliefs must be consistent with the price

[27] Many papers have tested $K-$level thinking and estimated low levels of K, between 0 and 3 (Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2006), Crawford et al. (2013))

they observe. Defining 1−level thinking to be the CE, so that $\gamma_1 \equiv \frac{\phi\tau_s}{\phi\tau_s+\tau_0}$, and solving for the $(P^K, \tilde{s}^K)$−pair which satisfy (30) and (31) jointly, provides us with a recursive solution for $K > 1$:

$$P^K = \gamma_K s + (1-\gamma_K)(\mu_0 - AZ\tau_s^{-1}) \tag{32}$$

$$\tilde{s}^K = \frac{\gamma_K}{\gamma_{K-1}} s + \left(1 - \frac{\gamma_K}{\gamma_{K-1}}\right)\left(\mu_0 - AZ\tau_0^{-1}\right) \tag{33}$$

where:

$$\gamma_K = \frac{\alpha}{1 - \frac{\beta}{\gamma_{K-1}}} \tag{34}$$

Solving (34) forwards yields the following equilibrium sensitivity for $K > 1$:

$$\gamma_K = \frac{\alpha}{1 + \sum_{j=1}^{K-2}\left(-\frac{\beta}{\alpha}\right)^j - \left(-\frac{\beta}{\alpha}\right)^{K-2}\frac{\beta}{\gamma_1}} = \frac{\tau_s}{\tau_s + \tau_0 - \left(-\frac{1-\phi}{\phi}\right)^K \tau_0} \tag{35}$$

where the second equality uses the expressions for α and β.

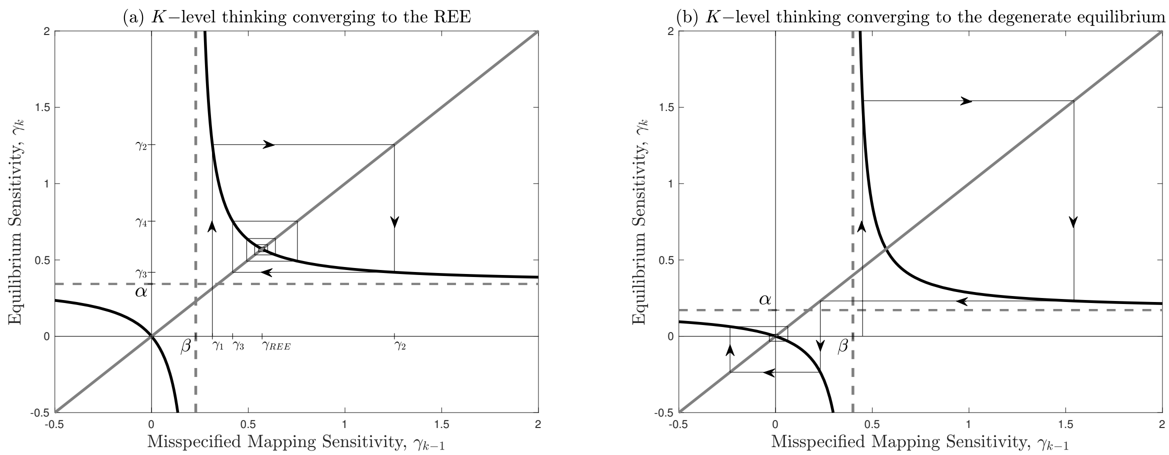
3.2 Properties of K-level Thinking

Let us start by studying the properties of the recursive relationship in (34), as this fully characterizes K−level thinking equilibrium outcomes in (32) and (33). Figure 6 plots this relationship in bold, together with the 45° line. The intersections of these two lines correspond to two fixed points such that $\gamma_{K-1} = \gamma_K$: $\gamma_K = \alpha + \beta > 0$ is the REE, and $\gamma_K = 0$ is a degenerate fixed point where the price is unresponsive to the signal. The arrows then show the recursive evolution of γ_K as we allow for rationality of higher order beliefs. Figure 6 makes clear three properties of the K−level thinking equilibrium, two of which generalize the intuitions on stability and over-reaction we uncovered in Section 1.2 and in Section 2, and the third relates to convergence to the REE as we allow for rationality of higher order beliefs. We discuss them in turn.

Proposition 7 (Stability and K−Level Thinking)**.** *The K−level thinking equilibrium is stable if and only if $\beta/\gamma_{K-1} < 1$. Otherwise, the equilibrium is unstable.*

This result generalizes our findings in Proposition 1. Intuitively, as argued in Section 1.2, when $\gamma_{K-1} > 0$ and $\beta/\gamma_{K-1} > 0$, the informational role of prices introduces strategic

Figure 6: Convergence of $K-$level thinking. This Figure plots the recursive relationship in (34) in bold: the $x-$axis tracks the sensitivity of the misspecified mapping of a level K thinker, and the $y-$axis tracks the corresponding sensitivity of the $K-$level thinking equilibrium price. The diagonal line is the 45° line. The intersection of these two lines correspond to two fixed points such that $\gamma_{K-1} = \gamma_K$: $\gamma_K = \alpha + \beta > 0$ is the REE, and $\gamma_K = 0$ is a degenerate fixed point. The arrows show the recursive evolution of γ_K as we allow for rationality of higher order beliefs, starting from γ_1. The left panel depicts a case where $\alpha > \beta$, and the left panel shows a case for $\alpha < \beta$.



complementarities which work against the strategic subsitutabilities from the scarcity role of prices, and push towards an upward sloping aggregate excess demand function. When this is the case, $\beta/\gamma_{K-1} < 1$ simply ensures that the scarcity role dominates over the informational role, so that the demand function is downward sloping, and the equilibrium is stable. On the other hand, notice that with $K-$level thinking we may now also encounter cases when $\gamma_{K-1} < 0$, according to which the informational role of prices introduces strategic *substitutabilities*. When this is the case, there is no tension between the two roles of prices: instead, the informational role reinforces the downward sloping nature of the demand function. Not only does this ensure stability, but it also *dampens* the general equilibrium adjustment, thus leading to under-reaction relative to the REE, as shown in Figure 6.[28,29]

[28]The results from Proposition 7 can be seen in Figure 6 by noticing that $\gamma_K > 0$ if and only if $\gamma_{K-1} > \beta$ or $\gamma_{K-1} < 0$. Moreover, when $\gamma_{K-1} < 0$, $\gamma_K < \gamma_{REE}$ and the equilibrium exhibits under-reaction. Appendix A.2.7 shows formally the equivalence between a positive sensitivity of the equilibrium price to the true signal, and equilibrium stability.

[29]The fact that $K-$level thinking can either dampen or amplify GE effects is consistent with Angeletos and Lian (2017), who study a complete information setup where agents have limited capacity to think through the response of others to a known shock. A different approach to studying wedges between partial and general equilibrium effects is in Angeletos and Lian (2018), Angeletos and Sastry (2020) and Angeletos and Huo (2020). They maintain common knowledge of rationality but add higher-order uncertainty: agents engage in a beauty contest style of thinking to forecast other agents' beliefs and responses to shocks, but there is no inference from endogenous outcomes.

Next, we turn to the properties of $K-$level thinking when the equilibrium is stable.

Proposition 8 ($K-$level Thinking Over-/Under-reaction)**.** *When the equilibrium is stable, $K-$level thinking outcomes exhibit over-reaction relative to the REE if and only if $\gamma_{K-1} < \gamma_{REE}$. Otherwise, they exhibit under-reaction.*

This result is clear in Figure 6: only when $\beta < \gamma_{K-1} < \gamma_{REE}$ is the $K-$level thinking equilibrium stable and $\gamma_K > \gamma_{REE} = \alpha + \beta$. Intuitively, focusing on stable outcomes, when $\gamma_{K-1} < \gamma_{REE}$, U agents think that all other agents are, on average, underreacting, and that the equilibrium price is less responsive to signals than it really is. Therefore, they attribute any price change they observe to more extreme news than in reality. Since all U agents behave in the same way, this translates into aggregate level over-reaction, just as in PET. Conversely, when $\gamma_{K-1} > \gamma_{REE}$, U agents think that other agents are, on average, overreacting, and their combined response leads to under-reaction in equilibrium. Therefore, in the specific case we consider (with level-1 being the CE), $K-$level thinking alternates between under-reaction when K is odd (and U agents think others are overreacting) and over-reaction when K is even (and U agents think others are underreacting). Section 4.1 makes this result about individual level inference more general and shows that it is independent of K.

We end this section by considering the conditions for $K-$level thinking to converge to the REE, as we allow for rationality of higher order beliefs.

Proposition 9 (Convergence to the REE)**.** *$K-$level thinking converges to the REE as we allow for rationality of higher orders beliefs if and only if $\alpha > \beta$. Otherwise, it converges to the degenerate fixed point with $\gamma_K = 0$.*

This result is clear from equation (35), where $\lim_{K\to\infty} \gamma_K = \alpha + \beta = \gamma_{REE}$ if $\alpha > \beta$ and $\lim_{K\to\infty} \gamma_K = 0$ if $\alpha < \beta$. Moreover, notice how convergence is independent of the initial size of the bias, γ_1. Instead, for $K-$level thinking outcomes to converge to the REE, we simply need the equilibrium influence on prices of I agents (α) to be greater than the influence on prices of U agents (β).[30] Figure 6 depicts examples of convergence to the REE

[30] When this is the case, the size of the bias shrinks as it gets compounded with K. Conversely, when $\alpha < \beta$, the extent of over-reaction in PET (2$-$level) is greater in absolute value than the extent of under-reaction in CE (1$-$level). The mapping used by 3$-$level agents then deviates more from the REE than the one used by 2$-$level thinkers, thus translating into even greater equilibrium deviations from the REE outcomes, and to even more misspecified mappings for higher level thinkers.

and degenerate fixed points, in the left and right panels, respectively. The right panel of Figure 6 also show that when $\alpha < \beta$ there always exists a K large enough such that the $K-$level thinking equilibrium becomes unstable.

In our framework the condition for convergence reduces to $\phi > 1/2$. This suggests that a market populated by more U agents than I agents may exhibit smaller deviations from the REE when agents have *lower* levels of K. If one were to interpret agents with higher levels of K as being more sophisticated than their low K counterparts, the above result implies that greater levels of sophistication may contribute to greater (rather than lower) mispricing and to instability. Only when $\alpha > \beta$ and the market is populated by more I than U agents do greater levels of sophistication bring us closer to the REE. We summarize these results in Corollary 1.

Corollary 1 (Deviations from REE and Rationality of Higher Order Beliefs)**.** *When $\alpha > \beta$, the deviations of $K-$level thinking equilibrium outcomes from the REE are decreasing in K. Conversely, when $\alpha < \beta$, the deviations of $K-$level thinking equilibrium outcomes from the REE are increasing in K for stable equilibria, and there exists a K large enough such that the equilibrium becomes unstable.*

Empirically, this suggests that we are likely to observe greater mispricing when the market is populated by a large fraction of U agents with high levels of rational higher order beliefs. In these scenarios, agents are more likely to chase mispricing rather than information. More generally, our results illustrate the fragility of the REE assumption in settings where agents are learning from endogenous outcomes.

4 Generalization and Robustness

Our theory of Partial Equilibrium Thinking departs from the common knowledge of rationality assumption in a deliberately specific way. We now allow for more general forms of model misspecification. This serves three different purposes. First, it clarifies how our insights on over-reaction and instability generalize. Second, it makes it possible to nest other types of biases, such as errors in Bayesian updating, and to consider the interaction of these biases with mistakes in inference. Third, it allows us to illustrate the tractability of our model and to explore a setup with heterogenous levels of sophistication. This section concludes by highlighting how the results uncovered so far are robust to setups where the supply of the risky asset is stochastic and prices are only partially revealing, as well as to setups with a symmetric information structure where all agents receive noisy private signals.

4.1 Misinference and Incorrect Bayesian Updating

Consider a setup similar to the one in Section 3 , except that we now allow for the true and misspecified models to be arbitrary linear functions of s and $\tilde{s}$:[31]

$$P^{Mis}(\tilde{s}) = \tilde{\gamma}\tilde{s} + (1-\tilde{\gamma})(\mu_0 - AZ\tau_0^{-1}) \tag{36}$$

$$P^{True}(s,\tilde{s}) = \hat{\alpha}s + \hat{\beta}\tilde{s} + (1-\hat{\alpha}-\hat{\beta})(\mu_0 - AZ\tau_0^{-1}), \tag{37}$$

where $\tilde{\gamma}$, $\hat{\alpha}$, and $\hat{\beta}$ are positive constants.[32] Specifically, denote by α and β the sensitivities of the true model of the world when agents perform rational Bayesian updating, and let $\hat{\alpha} \neq \alpha$ and $\hat{\beta} \neq \beta$ reflect departures from rational Bayesian updating. The intersection of (36) and (37) gives us the following equilibrium outcomes:

$$P = \frac{\hat{\alpha}}{1-\frac{\hat{\beta}}{\tilde{\gamma}}}s + \left(1 - \frac{\hat{\alpha}}{1-\frac{\hat{\beta}}{\tilde{\gamma}}}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{38}$$

$$\tilde{s} = s + \left(\frac{\hat{\alpha}+\hat{\beta}-\tilde{\gamma}}{\tilde{\gamma}-\hat{\beta}}\right)\left(s - (\mu_0 - AZ\tau_0^{-1})\right) \tag{39}$$

The expression in (39) makes clear that for agents to perform correct inference we must have $\tilde{\gamma} = \hat{\alpha} + \hat{\beta}$: only then do U agents recover the correct signal $\tilde{s} = s$, for any true signal s. Instead, when $\tilde{\gamma} \neq \hat{\alpha} + \hat{\beta}$, agents' signal extraction results in misinference. Specifically,

[31]While not completely general, the linear models we consider here are able to nest many other departures from rationality arising from errors in Bayesian updating, as discussed below.

[32]Notice that in both equations (36) and (37), the constant coefficients of the price involving the terms in $(\mu_0 - AZ\tau_0^{-1})$ could be replaced by arbitrary constants. Specifying the model in this more general form would nest a variety of cases, and would allow agents to have different perceptions of risk aversion, of liquidity, heterogeneous priors, and would even allow some agents to have trading motives which are orthogonal to the information we are interested in. However, we abstract from this generalization as it requires the introduction of additional notation that is not necessary for our purpose of discussing over-reaction and stability in this more general framework.

when $\tilde{\gamma} < \hat{\alpha} + \hat{\beta}$, U agents think that the price is less sensitive to new information than in reality, and therefore they extract a signal which is more extreme, as in PET, and conversely for $\tilde{\gamma} > \hat{\alpha} + \hat{\beta}$.

Moreover, fixing $\tilde{\gamma} = \hat{\alpha} + \hat{\beta}$, so as to shut down misinference, we see that whenever $\hat{\alpha}+\hat{\beta} > \alpha+\beta$ equilibrium outcomes are too sensitive to new information, as in models of over-confidence (Daniel et al. (1998) and Odean (1998)), or diagnostic expectations (Bordalo et al. (2018)). Conversely, when $\hat{\alpha}+\hat{\beta} < \alpha+\beta$ equilibrium outcomes underreact to new information, as in models of dismissiveness (Banerjee et al. (2009), Banerjee and Kremer (2010), and Banerjee (2011)), cursedness (Eyster et al. (2019)), or inattention (Gabaix (2020)).

Lemma 1 (Departures from Bayesian Updating and Misinference)**.** *Let α and β be the sensitivities of the price to, respectively, the true and extracted signals in the true model of the world when agents perform rational Bayesian updating. Then:*

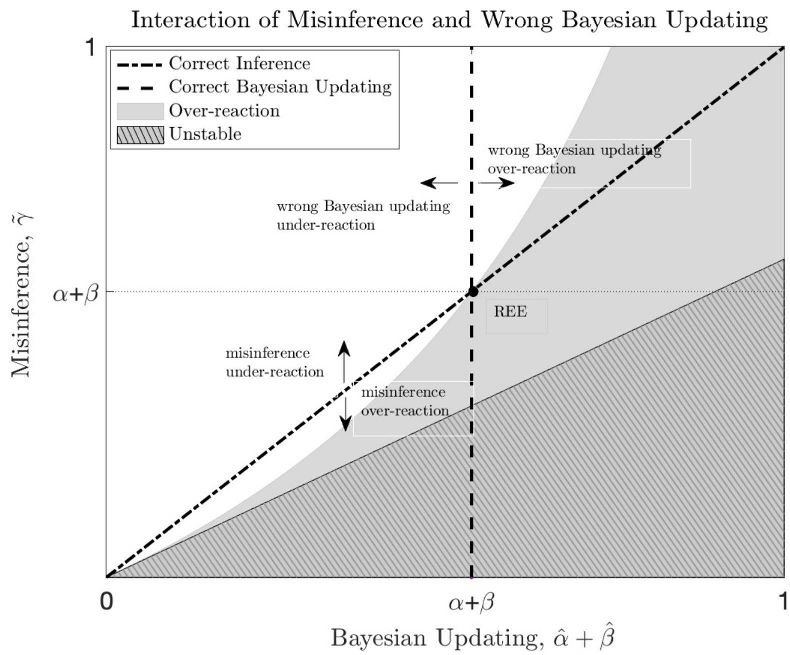
- *$\hat{\alpha} \neq \alpha$ and $\hat{\beta} \neq \beta$ represent departures from rational Bayesian updating.*
- *$\tilde{\gamma} \neq \hat{\alpha} + \hat{\beta}$ represents departures from correct inference.*

This is shown in Figure 7, where the two thick dashed lines correspond to regions where agents perform correct Bayesian updating and correct inference, respectively.[33] The REE then corresponds to the intersection of these two lines. Away from this intersection, the two lines split the plane into four quadrants, where the two departures from rationality interact in shaping the properties of equilibrium outcomes.

A key point to notice is just how plausible it is to have both mistakes in play in any setup where agents perform non-rational Bayesian updating *and* also extract information from endogenous outcomes. Models of non-rational Bayesian updating generally assume that agents do correct inference, implicitly assuming that agents are aware of the mistakes of others, and yet fail to recognize those exact same mistakes in themselves. For example, a common assumption in the over-confidence literature is that agents perform correct signal extraction, and must therefore be aware that others are overconfident, even though they are themselves overconfident regarding their own information (Odean (1998), Daniel et al.

[33] Details on how we vary $\hat{\alpha}$ and $\hat{\beta}$ are illustrated in Appendix D.

Figure 7: General Model Misspecification: misinference, non-rational Bayesian updating, and their interaction. This Figure illustrates how non-rational Bayesian updating and misinference interact in generating under- and over-reaction. The dashed vertical line at $\hat{\alpha}+\hat{\beta}=\alpha+\beta$ corresponds to the region where all agents perform rational Bayesian updating, and the dash-dotted diagonal line with $\tilde{\gamma}=\hat{\alpha}+\hat{\beta}$ corresponds to the region where agents perform correct inference. The REE is at the intersection of these two lines. The horizontal axis captures departures from correct Bayesian updating, while the vertical axis captures misinference. The horizontal dotted line at $\tilde{\gamma}=\alpha+\beta$ corresponds to the case where agents infer information from prices under the (incorrect) assumption that all agents are rational. The white and the gray areas illustrate combinations of parameters which lead to under-reaction and over-reaction, respectively. The dark gray area shows regions where the equilibrium is unstable.



(1998)). If instead agents believe that they live in a rational world, thus failing to realize that all other agents are also overconfident, they underestimate the sensitivity of the equilibrium price to new information, and they extract a more extreme signal than the true one, leading to even stronger departures from the REE. Similarly, in Bordalo, Gennaioli, Kwon and Shleifer (2020), diagnostic agents infer the correct signal from prices, which requires agents to recognize that other agents are diagnostic, even though they fail to realize that they themselves are not rational. In what follows, and in an example in Appendix D, our analysis shows that these assumptions can result in extreme outcomes and unstable regions, features which are normally absent in this literature.

Turning to the properties of equilibrium outcomes, Proposition 10 generalizes the conditions for stability uncovered in PET and in $K-$level thinking.

Proposition 10 (Stability with General Model Misspecfication)**.** *Given a general linear mapping, as in* (36) *and a true model of the world as in* (37)*, the equilibrium is stable if and only if* $\frac{\hat{\alpha}}{1-\frac{\hat{\beta}}{\tilde{\gamma}}} > 0$*. When* $\hat{\alpha} > 0$*, this condition is equivalent to* $\frac{\hat{\beta}}{\tilde{\gamma}} < 1$*.*

The relevant parameters in determining the strength of the feedback effect are $\tilde{\gamma}$ and $\hat{\beta}$. Instead, stability is not directly related to the rational parameters α and β.

Turning to the equilibrium properties relative to the REE, $\hat{\alpha} + \hat{\beta} > \alpha + \beta$ and $\tilde{\gamma} < \hat{\alpha} + \hat{\beta}$ both push towards over-reaction, while reversing these inequalities pushes towards under-reaction. Intuitively, fixing $\hat{\alpha}$ and $\hat{\beta}$, a lower $\tilde{\gamma} < \hat{\alpha} + \hat{\beta}$ gives rise to greater over-reaction in the form of misinference, as U agents attribute any price change to a signal which is more extreme than in reality. Moreover, for a given level of misinference, greater $\hat{\alpha} - \alpha > 0$ and $\hat{\beta} - \beta > 0$, give rise to greater over-reaction in the form of non-rational Bayesian updating. As is clear from Figure 7, these forces may either point in the same direction, in which case they compound and amplify each other (quadrants 1 and 3), or they push in opposite directions (quadrants 2 and 4). Proposition 11 shows that the resulting properties of equilibrium outcomes depends on the relative strength of the two mistakes.

Proposition 11 (Over- and Under-reaction with General Model Misspecfication)**.** *Given a general linear mapping as in* (36)*, a true model of the world as in* (37)*, and rational coefficients* α *and* β*, a stable equilibrium displays over-reaction if:*

$$\frac{\hat{\alpha}}{1 - \frac{\hat{\beta}}{\tilde{\gamma}}} > \alpha + \beta \iff \tilde{\gamma} < (\alpha + \beta)\left(\frac{\hat{\beta}}{\beta - (\hat{\alpha} - \alpha)}\right) \tag{40}$$

and under-reaction if the inequality is reversed.

When agents perform correct Bayesian updating, this condition is equivalent to $\tilde{\gamma} < \alpha + \beta = \gamma_{REE}$. In light of Proposition 11, Figure 7 can be used to illustrate how discrepancies from the REE arising from wrong Bayesian updating are always amplified when agents misinfer information from prices because they assume the world is rational when it is not, a result we anticipated earlier in this section. The thin dotted line with $\tilde{\gamma} = \alpha + \beta$ in Figure 7 corresponds to the region where agents infer information from prices under the (possibly incorrect) assumption that the world is rational. Importantly, this line always lies either in the top-left, or in the bottom-right quadrants described above, and wrong Bayesian updating and misinference amplify each other in both these regions. Figure 7 also shows that this type of misinference (with $\tilde{\gamma} = \alpha + \beta \neq \hat{\alpha} + \hat{\beta}$) can even lead to unstable outcomes, as is clear from the fact that, for large enough $\hat{\alpha} + \hat{\beta}$ the thin dotted line enters the unstable region.

Proposition 12 (Misinference from (mistakenly) assuming correct Bayesian updating)**.** *Consider a scenario where agents perform non-rational Bayesian updating, $\hat{\alpha} + \hat{\beta} \neq \alpha + \beta$. Misinference due to the incorrect assumption that the world is rational, $\tilde{\gamma} = \alpha + \beta$, amplifies the bias due to wrong Bayesian updating:*

- *If $\hat{\alpha} + \hat{\beta} < \alpha + \beta$, $\tilde{\gamma} = \alpha + \beta$ amplifies under-reaction relative to $\tilde{\gamma} = \hat{\alpha} + \hat{\beta}$;*
- *If $\hat{\alpha} + \hat{\beta} > \alpha + \beta$, $\tilde{\gamma} = \alpha + \beta$ either amplifies over-reaction relative to $\tilde{\gamma} = \hat{\alpha} + \hat{\beta}$, or it makes the equilibrium unstable.*

This discussion bears important quantitative implications for models which aim to assess the importance of these behavioral biases in empirical settings: smaller departures from Bayesian updating may be needed to obtain empirically meaningful departures from rationality. Similarly, we should expect the same bias in Bayesian updating to have a greater influence on aggregate outcomes in settings where agents infer information from endogenous outcomes. Moreover, unpacking the true driver of these departures from rationality is important as the two types of biases introduce different frictions, and have different policy implications: while misinference can be attenuated with greater transparency and better communication, as in Angeletos and Sastry (2020), errors in Bayesian updating would be unaffected by these measures and generally require more direct changes in agents' incentives (with, for instance, taxes and subsidies as in Farhi and Gabaix (2020)).

4.2 Heterogenous Agents

So far, we have considered the case where all U agents extract information from prices using the same misspecified model of the world, be it PET, $K-$level thinking, or any general form of misspecified model. Giglio et al. (2020) document a large amount of heterogeneity among agents' actions and beliefs. The tractability of our model allows us to consider the interaction of heterogeneous agents, who differ in their level of sophistication in extracting information from prices.[34]

The key point in solving these models is to notice that we have to specify the mapping used by each type of agent who extracts information from prices. Appendix B develops a general setup and allows for N types of agents, each with their own different misspecified model.[35] Appendix B.2 then considers an example where we study the interaction of three types of agents with different levels of sophistication, and we study the comparative statics which emerge from it.

When we extend our setup in this way, we find that the main insights in the paper generalize to a setup with heterogeneous agents. At the *individual* level, agents who believe the equilibrium price is less(more) responsive than it *truly* is attribute a given price change to a more(less) extreme signal, and overreact(underreact).[36] This also corroborates the results in Section 4.1 that agents who believe the world is rational need not necessarily be better off. At the *aggregate* level, outcomes display over-(under-)reaction if, on average, agents believe the equilibrium price is less(more) sensitive than in the REE.[37] Therefore, aggregate level rationality need not imply individual level rationality if agents' mistakes cancel out on average. Finally, we show that with heterogenous levels of thinking, having a higher level of rationality of higher order beliefs does not guarantee a higher level of realized welfare.

[34]Most papers studying model misspecification assume that all individuals share the same bias. Notable exceptions include Gagnon-Bartsch (2016); Frick et al. (2019); and Bohren and Hauser (2020), who characterizes long-run beliefs in general settings with model heterogeneity.

[35]For the sake of clarity we consider the case where each type is performing correct bayesian updating, but we can easily extend this exercise to include errors in bayesian updating along the lines of Section 4.1.

[36]Therefore, when there are heterogeneous types of uninformed agents, to understand responses at the individual level, one needs to compare agents' misspecified mappings with the resulting equilibrium mapping, and not with the REE one. For example, if the equilibrium exhibits over-reaction, an agent who believes the world is rational underreacts. Similarly, even agents who think that there is aggregate level over-reaction may overreact if they underestimate the extent of aggregate level over-reaction. This is reminiscent of the results in Section 4.1.

[37]In Appendix B, we maintain rational Bayesian updating. We could easily generalize these results further by allowing for non-rational Bayesian updating, as in Section 4.1. With non-rational Bayesian updating, correct inference is not enough to ensure the recovery of the REE at the aggregate level.

4.3 Robustness

Our model can be enriched along several other dimensions, all of which keep the basic intuitions we developed intact. In Appendix E we relax the assumption that prices are fully revealing, and allow for the supply of the risky asset to be stochastic, as in Diamond and Verrecchia (1981). In Appendix F we maintain the assumption of partially revealing prices and consider the symmetric case where all agents receive a noisy private signal of the risky asset's payoff. All the main intuitions of our results go through, thus showing that our results do not rely on the particular information structure that we adopt in the baseline model.

5 Conclusions

We develop a theory of "Partial Equilibrium Thinking (PET)" where agents fail to understand the general equilibrium consequences of their actions when inferring information from endogenous outcomes. Specifically, we model a financial market where all uninformed agents extract information from prices under the incorrect belief that they are the only ones doing so. Since PET agents fail to understand the general equilibrium forces which generate the prices they observe and instead attribute any price change to new information alone, they use a misspecified mapping to infer information from prices, and this results in misinference. We show that PET provides a micro-foundation to over-reaction to news relative to the Rational Expectations Equilibrium (REE), and that this simple deviation from rational expectations is also able to speak to some of the leading asset pricing puzzles, such as excess volatility, excessive trading volume, and return predictability.

Moreover, PET delivers a very natural feedback effect between outcomes and beliefs: higher prices translate into more optimistic beliefs, which in turn lead to even higher prices and optimistic beliefs. The strength of this feedback effect is decreasing in the fraction of informed agents and in the informativeness of new information relative to the prior, and it determines the properties of equilibrium outcomes: the stronger the feedback effect, the greater the discrepancies from rational outcomes, and these may become arbitrarily large. This feedback effect can even be explosive, in which case the equilibrium becomes unstable.

When we allow for higher order beliefs, we find that $K-$level thinking does not always converge to the REE in the limit of common knowledge of rationality. Moreover, agents with higher rational order beliefs may exacerbate instead of attenuate discrepancies from REE outcomes. Both these results highlight the fragility of the REE equilibrium concept.

Our model is tractable enough to allow for more general forms of model misspecification, as well as heterogenous agents with different levels of sophistication and cognitive limitations. Throughout the different frameworks we consider, we find that agents who believe the equilibrium price is less(more) responsive than it *truly* is overreact(underreact). We also show that the possibility of an explosive feedback effect between outcomes and beliefs is pervasive in models of misinference, and not just a curiosity of our baseline framework. Moreover, we exploit our general framework to study the interaction of misinference and non-rational Bayesian updating. We show that misinference from failing to recognize one's own irrationality in others amplifies the bias in non-rational Bayesian updating. This bears important implications as it suggests that when agents infer information from endogenous outcomes, empirical patterns which are at odds with rational expectations may be reconciled with smaller biases in Bayesian updating, and distinguishing between these two channels may have important policy implications.

More broadly, our mechanism can be generalized to any setup where agents use a misspecified model of the world to extract information from a general equilibrium outcome, thus lending itself to many other macro and finance applications. In Bastianello and Fontanier (2020*a*), we extend the framework dynamically, and we show that PET can give rise to self-sustained price increases, followed by an endogenous burst of the eventual bubble. This is achieved by showing that, in a dynamic framework, the strength of the feedback effect between prices and beliefs can be endogenously time-varying. Therefore, the feedback effect can be very strong at first, and weaken thereafter, thus allowing prices to converge back to fundamentals following a period where they are decoupled from them. In Bastianello and Fontanier (2020*b*), we apply our general framework of model misspecification to a setup where banks finance projects of unobservable quality. We show that when lenders misunderstand the relationship between credit volume and the underlying quality of borrowers, credit cycles naturally emerge. Credit booms are accompanied by a deterioration of borrower quality, weakening lending standards, but also decreasing credit spreads.

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Appendices

A Proofs and Derivations

A.1 Solving for the CE and REE benchmarks

A.1.1 Cursed/Competitive Equilibrium (CE)

In the Cursed (or Competitive) Equilibrium, agents trade on their own private information alone. Therefore, I agents trade on the signal s they receive, and U agents do not extract information from prices. This translates into the following market clearing condition:

$$\phi \underbrace{\left(\tau_0 \frac{\mu_0 - P}{A} + \tau_s \frac{s - P}{A}\right)}_{X_I^{CE}} + (1-\phi) \underbrace{\left(\tau_0 \frac{\mu_0 - P}{A}\right)}_{X_U^{CE}} = Z, \tag{A.1}$$

and the resulting equilibrium price P^{CE} is given by:

$$P^{CE} = \frac{\phi\tau_s}{\tau_0 + \phi\tau_s} s + \left(1 - \frac{\phi\tau_s}{\tau_0 + \phi\tau_s}\right)(\mu_0 - AZ\tau_0^{-1}). \tag{A.2}$$

where the sensitivity of the equilibrium price with respect to the signal is increasing in the fraction of I agents, ϕ.

Notice that (A.2) shows that, not only do prices aggregate all private information, but they are also fully revealing: P^{CE} is a linear function of the true signal s, which is the only shock in this economy, while all other parameters are common knowledge to all. One can interpret the fact that U agents do not extract information from prices as them failing to realize that prices contain valuable information. In this respect, we can think of the CE as $1-$thinking: agents are rational, but fail to realize that others are too, and that, as a consequence, prices contain valuable information which they could condition on to obtain more precise beliefs about the terminal payoff, v.

Moreover, notice that the expression in (A.2) is the same price function that PET agents use to extract information from prices in (6): this is precisely the price function agents would invert to extract s from prices if they believed that no other agent were extracting information from prices. As shown in Section 1.1, if all agents extract information from prices in this way as in PET, then the resulting equilibrium price will be different.

A.1.2 Rational Expectations Equilibrium (REE)

In the REE, U agents have common knowledge of rationality, and extract information from prices. As we did in PET, we can solve for the REE equilibrium price and extracted signals by finding the intersection between the mapping used by U agents to extract information from prices, and the true price function.

Mapping from Prices to Extracted Signals. Since agents have common knowledge of rationality, U agents believe that everyone is trading on the same signal $\tilde{s}$: they believe I agents receive the signal $\tilde{s}$, and U agents extract this information from prices. Therefore, they believe that the price they observe is generated by the following market clearing condition:

$$\phi \underbrace{\left(\tau_0 \frac{\mu_0 - P}{A} + \tau_s \frac{\tilde{s} - P}{A}\right)}_{\tilde{X}_I^{REE}} + (1-\phi) \underbrace{\left(\tau_0 \frac{\mu_0 - P}{A} + \tau_s \frac{\tilde{s} - P}{A}\right)}_{\tilde{X}_U^{REE}} = Z \tag{A.3}$$

which results in the following mapping:

$$P^{Mis}(\tilde{s}) = \frac{\tau_s}{\tau_0 + \tau_s}\tilde{s} + \left(1 - \frac{\tau_s}{\tau_0 + \tau_s}\right)(\mu_0 - AZ\tau_0^{-1}). \tag{A.4}$$

REE agents then invert this price funciton to extract the signal $\tilde{s}$:

$$\tilde{s} = \frac{\tau_0 + \tau_s}{\tau_s} P - \frac{\tau_s}{\tau_0}(\mu_0 - AZ\tau_0^{-1}). \tag{A.5}$$

Therefore, notice that $\gamma_{REE} \equiv \frac{\partial P^{Mis}(\tilde{s})}{\partial \tilde{s}} = \frac{\tau_s}{\tau_s + \tau_0} = \alpha + \beta$, where $\alpha \equiv \frac{\partial P^{True}(s,\tilde{s})}{\partial s} = \frac{\phi \tau_s}{\tau_s + \tau_0}$ and $\beta \equiv \frac{\partial P^{True}(s,\tilde{s})}{\partial \tilde{s}} = \frac{(1-\phi)\tau_s}{\tau_s + \tau_0}$, as can be seen from (A.7) below.

True Price Function. The true price function which occurs whenever I agents trade on s and U agents trade on $\tilde{s}$ is the same one we used in (11):

$$P^{True}(s,\tilde{s}) = \frac{\phi\tau_s}{\tau_0+\phi\tau_s+(1-\phi)\tau_s}s + \frac{(1-\phi)\tau_s}{\tau_0+\phi\tau_s+(1-\phi)\tau_s}\tilde{s} + \frac{\tau_0}{\tau_0+\phi\tau_s+(1-\phi)\tau_s}(\mu_0 - AZ\tau_0^{-1}). \quad \text{(A.6)}$$

or simply:

$$P^{True}(s,\tilde{s}) = \frac{\phi\tau_s}{\tau_s+\tau_0}s + \frac{(1-\phi)\tau_s}{\tau_s+\tau_0}\tilde{s} + \frac{\tau_0}{\tau_s+\tau_0}(\mu_0 - AZ\tau_0^{-1}). \quad \text{(A.7)}$$

Equilibrium Price and Extracted Signal. The equilibrium $(P^{REE}, \tilde{s}^{REE})$−pair is at the intersection of the two mappings in (A.5) and (A.7), such that $P^{REE} = P^{Mis}(\tilde{s}^{REE}) = P^{True}(s, \tilde{s}^{REE})$

$$\tilde{s}^{REE} = s, \quad \text{(A.8)}$$

and

$$P^{REE} = \frac{\tau_s}{\tau_s+\tau_0}s + \left(1 - \frac{\tau_s}{\tau_s+\tau_0}\right)(\mu_0 - AZ\tau_0^{-1}), \quad \text{(A.9)}$$

which reflects the fact that all agents trade on s with precision τ_s and on the prior with precision τ_0: I agents trade on s because they receive the signal, and U agents trade on s because they perfectly extract this information from prices. Therefore the sensitivity of the price with respect to the signal is independent of the fraction of informed agents, ϕ.

Equations (A.8) and (A.9) show that in a world with common knowledge of rationality, REE agents extract the right information from prices. The reason why this is the case is that in a world with common knowledge of rationality, agents who believe that everyone has common knowledge of rationality are indeed correct. This means that they invert the right price function to extract information from prices, as can be seen from the fact that (A.4) is the same function as (A.9).

This is a more general result: for uninformed agents to extract the right signal from prices, it must be the case that the mapping they use to extract information coincides with the price function that realizes in equilibrium. As we show in Section 3, this can be formalized into a fixed point argument.

A.1.3 Equilibrium Price and Extracted Signal across Benchmarks

For reference, we summarize below the equilibrium price and extracted signal of the three equilibrium benchmarks, which we derived in Section 1.1 and in Appendix A.1. Prices are given by:

$$P^{CE} = \frac{\phi\tau_s}{\phi\tau_s + \tau_0}s + \left(1 - \frac{\phi\tau_s}{\phi\tau_s + \tau_0}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{A.10}$$

$$P^{REE} = \frac{\tau_s}{\tau_s + \tau_0}s + \left(1 - \frac{\tau_s}{\tau_s + \tau_0}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{A.11}$$

$$P^{PET} = \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}s + \left(1 - \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{A.12}$$

and extracted signals can be expressed as:

$$\tilde{s}^{REE} = s \tag{A.13}$$

$$\tilde{s}^{PET} = s + \frac{\left(\frac{1-\phi}{\phi^2}\right)\tau_0}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}\left(s - (\mu_0 - AZ\tau_0^{-1})\right) \tag{A.14}$$

A.2 Proofs

A.2.1 Proposition 1: Stability

Starting from the condition for stability, which is proved in the main text:

$$\frac{\beta}{\gamma} < 1 \iff \frac{\partial P^{True}(s, \tilde{s})}{\partial \tilde{s}} < \frac{\partial P^{Mis}(\tilde{s})}{\partial \tilde{s}} \iff \frac{(1-\phi)\tau_s}{\tau_s + \tau_0} < \frac{\phi\tau_s}{\phi\tau_s + \tau_0} \tag{A.15}$$

Simplifying this inequality and rearranging the above inequality holds iff:

$$\phi^2\tau_s + 2\phi\tau_0 - \tau_0 > 0 \iff \tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0 > 0 \iff \phi > \left(1 + \sqrt{1 + \frac{\tau_s}{\tau_0}}\right)^{-1} \tag{A.16}$$

which is precisely the expression in (21). Comparative statics follow by inspection. □

A.2.2 Proposition 2: Over-reaction

From (A.10), (A.11) and (A.12), the equilibrium sensitivities of the equilibrium price to the true signal, across the three equilibrium benchmarks, can be rewritten as:

$$\frac{\partial P^{CE}}{\partial s} = \frac{\tau_s}{\tau_s + \tau_0 + \left(\frac{1-\phi}{\phi}\right)\tau_0} \quad , \quad \frac{\partial P^{REE}}{\partial s} = \frac{\tau_s}{\tau_s + \tau_0} \quad , \quad \frac{\partial P^{PET}}{\partial s} = \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}. \tag{A.17}$$

To prove the inequality in Proposition 2, we start by proving the following Lemma.

Lemma 2 (Stability and PET sensitivity)**.** *The sensitivity of the PET equilibrium price to new information is positive if and only if the PET equilibrium is stable.*

Proof.

$$\frac{\partial P^{PET}}{\partial s} > 0 \iff \tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0 > 0 \tag{A.18}$$

From (A.16), we see that this expression is positive if and only if the equilibrium is stable. □

Therefore, when the PET equilibrium is stable, all three quantities in (A.17) are positive (the CE and REE sensitivities are always positive). For $\phi \in (0,1)$, it immediately follows that:

$$\frac{\tau_s}{\tau_s + \frac{1}{\phi}\tau_0} < \frac{\tau_s}{\tau_s + \tau_0} < \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}. \tag{A.19}$$

This proves the first part of our statement.

Finally, the assumptions which define PET lie in between those which define the CE and REE benchmarks as we can reframe the CE as being 1−level thinking, PET as being 2−level thinking, and the REE as ∞ level thinking, as discussed in the introduction. □

A.2.3 Proposition 3: Contrarians

According to the first order condition in (2), agents' asset demand function conditional on their beliefs is given by:

$$X_i^K = \frac{\mathbb{E}[v|\mathcal{I}_i^K] - P^K}{A\text{Var}[v|\mathcal{I}_i^K]} \tag{A.20}$$

for $i \in \{I, U\}$, and $K \in \{CE, REE, PET\}$. We are now interested in understanding when, in equilibrium, this quantity is increasing or decreasing in the true signal s.

REE. $\mathbb{E}[v|\mathcal{I}_i^{REE}] = \frac{\tau_s}{\tau_s+\tau_0}s + \frac{\tau_0}{\tau_s+\tau_0}\mu_0$, and $\mathrm{Var}[v|\mathcal{I}_I^{REE}] = (\tau_s + \tau_0)^{-1}$, for both agents $i \in \{I, U\}$. Substituting these expressions and (A.11) into (A.20), we find that $X_I^{REE} = X_U^{REE} = Z$, so that:

$$\frac{\partial X_I^{REE}}{\partial s} = \frac{\partial X_U^{REE}}{\partial s} = 0 \tag{A.21}$$

In other words, agents' demands for the risky asset are in equilibrium independent of the signal, s, as the asset is (perceived) as being correctly priced at all times by both agents.

CE. Informed agents' beliefs are such that: $\mathbb{E}[v|\mathcal{I}_I^{CE}] = \frac{\tau_s}{\tau_s+\tau_0}s + \frac{\tau_0}{\tau_s+\tau_0}\mu_0$, and $\mathrm{Var}[v|\mathcal{I}_I^{CE}] = (\tau_s + \tau_0)^{-1}$. Substituting these expressions together with (A.10) into (A.20), and differentiating with respect to s, we find that:

$$\frac{\partial X_I^{CE}}{\partial s} = \frac{(1-\phi)\tau_s\tau_0}{A(\phi\tau_s + \tau_0)} > 0 \tag{A.22}$$

Similarly, uninformed agents' beliefs are such that: $\mathbb{E}[v|\mathcal{I}_U^{CE}] = \mu_0$, and $\mathrm{Var}[v|\mathcal{I}_I^{CE}] = \tau_0^{-1}$. Substituting these expressions together with (A.10) into (A.20), and differentiating with respect to s, we find that:

$$\frac{\partial X_U^{CE}}{\partial s} = -\frac{\phi\tau_s\tau_0}{A(\phi\tau_s + \tau_0)} < 0 \tag{A.23}$$

Therefore, in the CE, informed agents' equilibrium asset demand is increasing in the true signal s, while uninformed agents' equilibrium asset demand is decreasing in the true signal s. The same argument and comparative statics immediately go through when we refer to agents' net positions, $X_i^{CE} - Z$.

PET. Informed agents' beliefs are such that: $\mathbb{E}[v|\mathcal{I}_I^{PET}] = \frac{\tau_s}{\tau_s+\tau_0}s + \frac{\tau_0}{\tau_s+\tau_0}\mu_0$, and $\mathrm{Var}[v|\mathcal{I}_I^{PET}] = (\tau_s + \tau_0)^{-1}$. Substituting these expressions together with (A.12) into (A.20), and differentiating with respect to s, we find that in a stable equilibrium:

$$\frac{\partial X_I^{PET}}{\partial s} = -\frac{\left(\frac{1-\phi}{\phi}\right)^2 \tau_s\tau_0}{A\left(\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2\right)\tau_0} < 0 \tag{A.24}$$

Similarly, uninformed agents' beliefs are such that: $\mathbb{E}[v|\mathcal{I}_U^{CE}] = \frac{\tau_s}{\tau_s+\tau_0}\tilde{s} + \frac{\tau_0}{\tau_s+\tau_0}\mu_0$, and $\text{Var}[v|\mathcal{I}_I^{CE}] = (\tau_s + \tau_0)^{-1}$. Substituting these expressions together with (A.10) and (A.14) into (A.20), and differentiating with respect to s, we find that:

$$\frac{\partial X_U^{PET}}{\partial s} = \frac{\left(\frac{1-\phi}{\phi}\right)\tau_s\tau_0}{A\left(\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0\right)} > 0 \tag{A.25}$$

Therefore, in PET, informed agents' asset demand in equilibrium is decreasing in the true signal s, while uninformed agents' asset demand in equilibrium is increasing in the true signal s. The same argument and comparative statics immediately go through when we refer to agents' net positions, $X_i^{CE} - Z$.

Comparing the results in (A.21), (A.22), (A.23), (A.24) and (A.25) completes the proof.

□

A.2.4 Proposition 4: Trading Volume

Trading volume for $K = \{CE, REE, PET\}$ is formally defined as:

$$V^K = \phi|Z - X_I^K| + (1-\phi)|Z - X_U^K| = 2\phi(1-\phi)|X_U - X_I|, \tag{A.26}$$

where the last equality uses the market clearing condition, $\phi X_I + (1-\phi)X_U = Z$.

To find trading volume in PET, let's first make use of the fact that both informed and uninformed agents' demand functions use the same precision, so that (A.26) simplifies to:

$$V^{PET} = \frac{2\phi(1-\phi)\tau_s}{A}|\tilde{s} - s|. \tag{A.27}$$

This makes it clear that PET volume is proportional to, $|\tilde{s} - s|$, which is the amount of belief disagreement between I and U agents. Moreover, we see that risk-aversion scales volume, which is simply due to the fact that it scales all demand functions.

Substituting the PET equilibrium extracted signal in (A.14) into equation (A.27) yields an expression of volume as a function of fundamentals:

$$V^{PET} = \frac{2(1-\phi)^2\tau_s\tau_0}{A\phi\left(\tau_s+\tau_0-\left(\frac{1-\phi}{\phi}\right)^2\tau_0\right)}|s-\mu_0+AZ\sigma_0^2| \tag{A.28}$$

$$= \frac{2\phi(1-\phi)\tau_s}{A}\left(\frac{\frac{\alpha}{\gamma}-\left(1-\frac{\beta}{\gamma}\right)}{1-\frac{\beta}{\gamma}}\right)|s-\mu_0+AZ\sigma_0^2| \tag{A.29}$$

where $\alpha \equiv \frac{\partial P^{True}(s,\tilde{s})}{\partial s} = \frac{\phi\tau_s}{\tau_s+\tau_0}$, $\beta \equiv \frac{\partial P^{True}(s,\tilde{s})}{\partial \tilde{s}} = \frac{(1-\phi)\tau_s}{\tau_s+\tau_0}$, and $\gamma = \frac{\partial P^{Mis}(\tilde{s})}{\partial \tilde{s}} = \frac{\phi\tau_s}{\phi\tau_s+\tau_0}$, as in the main text. First, notice that when $s = \mu_0 - AZ\sigma_0^2$, trading volume is zero, as discussed in Section 2. Moreover, from (A.29), it is clear that V^{PET} is increasing in the size of the feedback effect $\frac{\beta}{\gamma}$, and that $\lim_{\frac{\beta}{\gamma}\to 1} = \infty$. This completes our proof. □

Moreover, notice how this result contrasts to the CE and REE cases. In the REE, (A.26) immediately shows that trading volume is zero, as all agents have the same asset demand, so that $X_U = X_I$, and $V^{REE} = 0$. We can also compute CE trading volume by substituting CE agents' asset demands into (A.26). This yields:

$$V^{CE} = \frac{2\phi(1-\phi)\tau_s\tau_0}{A(\phi\tau_s+\tau_0)}|s-\mu_0+AZ\tau_0^{-1}|, \tag{A.30}$$

which, for finite τ_s and τ_0, is always bounded. By contrast, PET allows for arbitrarily large trading volume using reasonable parameter values.

A.2.5 Proposition 5: Returns

Expected returns are defined additively, for $i \in \{I, U\}$, and $K \in \{CE, PET, REE\}$:

$$\mathbb{E}_i[R]^K = \mathbb{E}[v|\mathcal{I}_i^K] - P^K = \mathbb{E}[v|\mathcal{I}_i^K] - \left(\bar{\mathbb{E}}[v] - \bar{Var}[v]AZ\right) \tag{A.31}$$

where $\bar{\mathbb{E}}[v]$ and $\bar{Var}[v]$ are weighted averages of agents' beliefs and precisions, respectively.[38]

Since I agents all have the same beliefs across CE, REE and PET, the differences in I agents' expected returns across equilibrium concepts is purely driven by differences in

[38] Let $\tau_i = Var[v|\mathcal{I}_i]$ for $i \in \{I, U\}$. Then $\bar{\mathbb{E}}[v] = \frac{\phi\tau_I}{\phi\tau_I+(1-\phi)g}\mathbb{E}[v|\mathcal{I}_I] + \frac{(1-\phi)g}{\phi\tau_I+(1-\phi)g}\mathbb{E}[v|\mathcal{I}_U]$, and $Va\bar{+}r = (\phi\tau_I+(1-\phi)g)^{-1}$. The fact that $P^K = \bar{\mathbb{E}}[v] - \bar{Var}[v]AZ$ then follows by inspection of Equations (25), (26) and (27).

equilibrium prices. After some algebra, we find that, conditional on a signal s, I agents' expected returns are given by:

$$\mathbb{E}_I[R]^{REE} = AZ(\tau_s + \tau_0)^{-1} \tag{A.32}$$

$$\mathbb{E}_I[R]^{CE} = \left(\frac{\tau_s}{\tau_s + \tau_0} - \frac{\phi\tau_s}{\phi\tau_s + \tau_0}\right)\left(s - \left(\mu_0 - AZ\tau_0^{-1}\right)\right) + AZ(\tau_0 + \tau_s)^{-1} \tag{A.33}$$

$$\mathbb{E}_I[R]^{PET} = -\frac{(1-\phi)\tau_s}{\tau_s + \tau_0}\left(\frac{\left(\frac{1-\phi}{\phi^2}\right)\tau_0}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2\tau_0}\right)\left(s - \left(\mu_0 - AZ\tau_0^{-1}\right)\right) + AZ(\tau_0 + \tau_s)^{-1} \tag{A.34}$$

Moreover, since I agents' beliefs are rational, their expected returns are equal to average realized returns, conditional on s. The corresponding expected returns of U agents are given by:

$$\mathbb{E}_U[R]^{REE} = AZ(\tau_s + \tau_0)^{-1} \tag{A.35}$$

$$\mathbb{E}_U[R]^{CE} = -\frac{\phi\tau_s}{\phi\tau_s + \tau_0}\left(s - \left(\mu_0 - AZ\tau_0^{-1}\right)\right) + AZ\tau_0^{-1} \tag{A.36}$$

$$\mathbb{E}_U[R]^{PET} = \frac{\phi\tau_s}{\tau_s + \tau_0}\left(\frac{\left(\frac{1-\phi}{\phi^2}\right)\tau_0}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2\tau_0}\right)\left(s - \left(\mu_0 - AZ\tau_0^{-1}\right)\right) + AZ(\tau_0 + \tau_s)^{-1} \tag{A.37}$$

Proposition 5 then simply follows by inspection, as the denominator in (A.37) and (A.34) is positive when the equilibrium is stable, as shown in Lemma 2. Specifically:

$$\frac{\partial \mathbb{E}_U[R]^{PET}}{\partial s} = \frac{\phi\tau_s}{\tau_s + \tau_0}\left(\frac{\left(\frac{1-\phi}{\phi^2}\right)\tau_0}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2\tau_0}\right) > 0 \tag{A.38}$$

$$\frac{\partial R^{PET}}{\partial s} = -\frac{(1-\phi)\tau_s}{\tau_s + \tau_0}\left(\frac{\left(\frac{1-\phi}{\phi^2}\right)\tau_0}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2\tau_0}\right) < 0 \tag{A.39}$$

$\square$

A.2.6 Proposition 6: Comparative Statics

The equilibrium signal PET agents extract from prices, $\tilde{s}^{PET}$, can be written as:[39]

$$\tilde{s}^{PET} = s + \frac{\frac{\alpha}{\gamma} - \left(1 - \frac{\beta}{\gamma}\right)}{1 - \frac{\beta}{\gamma}} \left(s - \left(\mu_0 - AZ\tau_0^{-1}\right)\right) \tag{A.40}$$

This expression makes clear that the discrepancy of PET with the REE, $\tilde{s} - s$, is $i)$ increasing in β/γ, and $ii)$ $\lim_{\beta/\gamma \to 1} \tilde{s}^{PET} - s = \infty$.

Turning to the wedge between PET and REE sensitivities of the equilibrium price with respect to the true signal s, we can write this as:[40]

$$\frac{\partial P^{PET}}{\partial s} - \frac{\partial P^{REE}}{\partial s} = \left(\frac{\alpha}{1 - \frac{\beta}{\gamma}}\right) - (\alpha + \beta) = \frac{(\alpha + \beta - \gamma)\frac{\beta}{\gamma}}{1 - \frac{\beta}{\gamma}} \tag{A.42}$$

Moreover, in our setup, $\alpha + \beta = \frac{\tau_s}{\tau_s + \tau_0} > \gamma = \frac{\phi\tau_s}{\phi\tau_s + \tau_0}$. Therefore, by inspection, the wedge between the PET and REE sensitivity is also $i)$ increasing in β/γ, and $ii)$ $\lim_{\beta/\gamma \to 1} \frac{\partial P^{PET}}{\partial s} - \frac{\partial P^{REE}}{\partial s} = \infty$.

In what follows we show that the wedge between PET and REE outcomes is proportional to the difference between the extracted signal $\tilde{s}^{PET}$ and the true signal s. Starting with equilibrium prices, we can express the wedge between PET and REE prices using equation (11) as:

$$P^{PET} - P^{REE} = \beta(\tilde{s}^{PET} - s) \tag{A.43}$$

It follows immediately from the discussion above that this wedge is also $i)$ increasing in β/γ, and $ii)$ $\lim_{\beta/\gamma \to 1^+} P^{PET} - P = \infty$.

Finally, the wedge in expected returns,

$$\mathbb{E}_U[R]^{PET} - \mathbb{E}_U[R]^{REE} = \frac{\tau_s}{\tau_s + \tau_0}(\tilde{s} - s) - (P^{PET} - P^{REE}) \tag{A.44}$$

[39] To see this, one can either rely on the derivation in Section 4.1, or simply compare the expression in (A.40) with (A.14), after having substituted in for the values of $\alpha = \frac{\phi\tau_s}{\tau_s + \tau_0}$, $\beta = \frac{(1-\phi)\tau_s}{\tau_s + \tau_0}$, and $\gamma = \frac{\phi\tau_s}{\phi\tau_s + \tau_0}$.

[40] Notice that this wedge is also related to the wedge between $\tilde{s} - s$:

$$\tilde{s}^{PET} - s = \frac{1}{\beta}\left(\frac{\partial P^{PET}}{\partial s} - \frac{\partial P^{REE}}{\partial s}\right)\left(s - \left(\mu_0 - AZ\tau_0^{-1}\right)\right). \tag{A.41}$$

$$= (\alpha + \beta)(\tilde{s} - s) - \beta(\tilde{s} - s) \tag{A.45}$$

$$= \alpha(\tilde{s} - s) \tag{A.46}$$

where the second equality uses the fact that $\frac{\tau_s}{\tau_s+\tau_0} = \alpha+\beta$ and the result in (A.43). Similarly,

$$\mathbb{E}_I[R]^{PET} - \mathbb{E}_I[R]^{REE} = -(P^{PET} - P^{REE}) \tag{A.47}$$

$$= -\beta(\tilde{s} - s) \tag{A.48}$$

where the second equality uses the result in (A.43). Therefore, (A.46) and (A.48) confirm that the following properties hold for the discrepancy between PET and REE expected and realized returns as well. They are $i)$ increasing in β/γ, and $ii)$ $\lim_{\beta/\gamma \to 1} \mathbb{E}_i[R]^{PET} - \mathbb{E}_i[R]^{REE} = \infty$ for $i \in \{I, U\}$. This also holds for realized returns as, on average, realized returns are equal to informed agents' expected returns. □

These results are depicted graphically in Figure 8, which shows equilibrium quantities when $s = \mu_0$ as we increase the fraction of informed agents, ϕ. The lower ϕ, the greater is the feedback effect, and the greater the discrepancy.[41]

A.2.7 Proposition 7: Stability and $K-$level Thinking

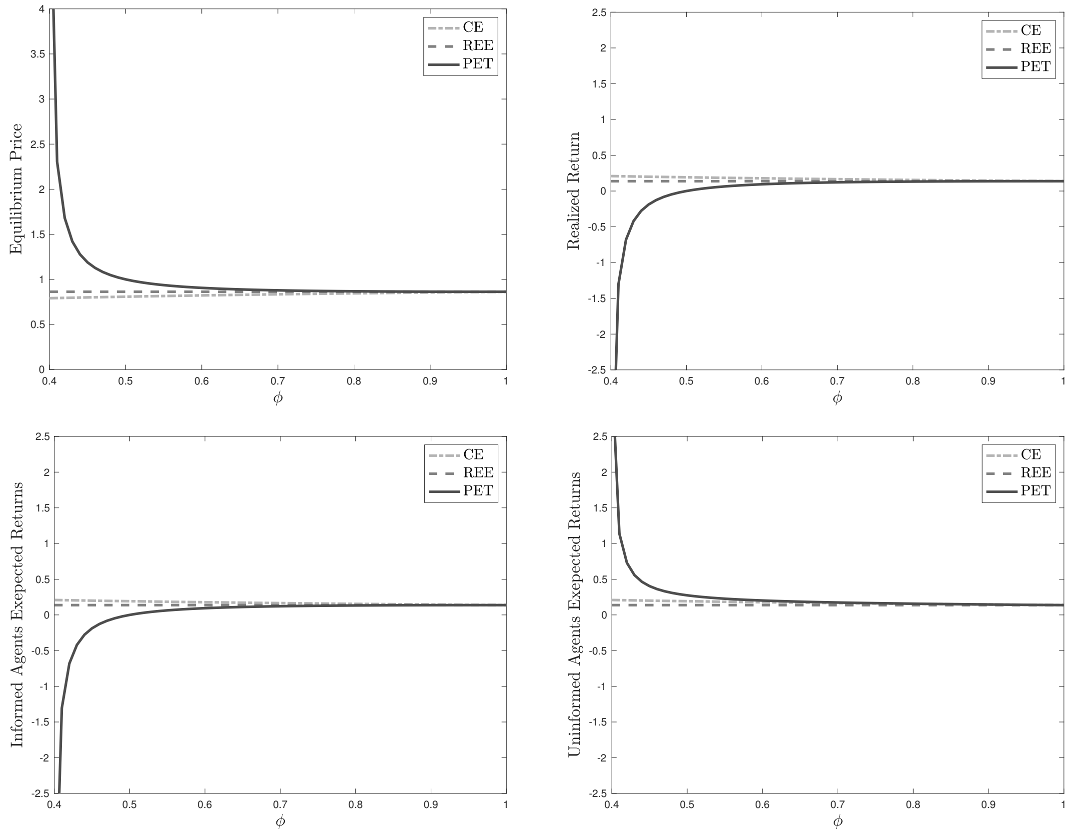
In Section 1.2, we showed that for an equilibrium to be stable, the aggregate excess demand function for the risky asset must be downward sloping. For $K-$level thinking, the aggregate excess demand function takes the following form:

$$X^K_{TOT} - Z = \phi\left(\frac{\frac{\tau_s}{\tau_s+\tau_0}s + \frac{\tau_0}{\tau_s+\tau_0}\mu_0 - P^K}{A(\tau_s + \tau_0)^{-1}}\right) + (1 - \phi)\left(\frac{\frac{\tau_s}{\tau_s+\tau_0}\tilde{s}^K + \frac{\tau_0}{\tau_s+\tau_0}\mu_0 - P^K}{A(\tau_s + \tau_0)^{-1}}\right) \tag{A.49}$$

Moreover, from (30), we know that $K-$level uninformed agents use the following mapping to extract information from prices: $\tilde{s}^K = \frac{1}{\gamma_{K-1}}P^K - \frac{1-\gamma_{K-1}}{\gamma_K}(\mu_0 - AZ\tau_0^{-1})$. Substituting this

[41] Decreasing ϕ has two effects on equilibrium outcomes: first, it increases the number of uninformed PET agents making a mistake; second, it increases the size of the mistake PET agents make: since PET agents are wrong about the actions of U agents, the more U agents there are, the more agents each PET agents is wrong about. Both these effects go in the same direction, so that our results are amplified as we decrease ϕ.

Figure 8: Comparative Statics over ϕ. This Figure plots CE, PET, and REE equilibrium prices (top left), realized returns (top right), expected returns by Informed agents (bottom left) and expected returns of Uninformed agents (bottom right), for different values of ϕ such that the equilibrium is stable and (21) is satisfied, and for a fixed true signal, at $s = \mu_0$. We let $\phi \in (0.5, 1]$. All parameter values are consistent with those in all other figures with $\sigma_0^2 = 0.16$, $\sigma_s^2 = 0.12$, $\mu_0 = 1$, $Z = 1$, and $A = 2$.



into (A.49), and rearranging, we get:

$$X_{TOT}^{K} - Z = \frac{1}{A(\tau_s + \tau_0)^{-1}} \left(\frac{(1-\phi)\tau_s}{\tau_s + \tau_0} \frac{1}{\gamma_{K-1}} - 1 \right) P^K + constants \tag{A.50}$$

and since $\frac{(1-\phi)\tau_s}{\tau_s+\tau_0} = \beta$, we can rewrite this as:

$$X_{TOT}^{K} - Z = \underbrace{\frac{1}{A(\tau_s + \tau_0)^{-1}} \left(\frac{\beta}{\gamma_{K-1}} - 1 \right)}_{\text{slope of excess demand function}} P^K + constants \tag{A.51}$$

Therefore, the $K-$level thinking equilibrium is stable if and only if the slope of this excess demand function is negative, which is equivalent to:

$$\left(\frac{\beta}{\gamma_{K-1}} - 1\right) < 0 \iff \frac{\beta}{\gamma_{K-1}} < 1 \tag{A.52}$$

Since $\beta > 0$, this condition is satisfied either if $\gamma_{K-1} < 0$, or if $0 < \beta < \gamma_{K-1}$. Otherwise, the excess demand function is upward sloping, and the equilibrium unstable. $\square$

Finally, notice that, as in Lemma 2 for PET, the $K-$level thinking sensitivity of the equilibrium price to the true signal is positive if and only if the equilibrium is stable. We summarize this in the following Lemma.

Lemma 3 (Stability and $K-$level thinking sensitivity)**.** *The sensitivity of the $K-$level thinking equilibrium price to the true signal is positive if and only if the $K-$level thinking equilibrium is stable.*

Proof. Since $\alpha > 0$, by inspection of (32) and (34):

$$\frac{\partial P^K}{\partial s} > 0 \iff 1 - \frac{\beta}{\gamma_{K-1}} > 0 \tag{A.53}$$

which is precisely the condition for stability uncovered in (A.52). $\square$

A.2.8 Proposition 8: $K-$level Thinking Over-/Under-reaction

The equilibrium sensitivity of the $K-$level thinking equilibrium price to the true signal is given in equation (34):

$$\gamma_K = \frac{\alpha}{1 - \frac{\beta}{\gamma_{K-1}}} \tag{A.54}$$

From Lemma 3, we also know that the $K-$level thinking equilibrium is stable if and only if $\gamma_K > 0$. Since $\alpha, \beta > 0$, this is equivalent to $\gamma_{K-1} > \beta$. The REE sensitivity is given by:

$$\gamma_{REE} = \alpha + \beta. \tag{A.55}$$

By definition, the $K-$level thinking equilibrium exhibits over-reaction if $\gamma_K > \gamma_{REE}$:

$$\frac{\alpha}{1-\frac{\beta}{\gamma_{K-1}}} > \alpha+\beta \iff \gamma_{K-1} < \alpha+\beta = \gamma_{REE} \tag{A.56}$$

where the last inequality uses the fact that the equilibrium is stable. Therefore, for a stable equilibrium to exhibit over-reaction, we need: $\beta < \gamma_{K-1} < \gamma_K$. $\square$

A.2.9 Proposition 9: Convergence to the REE

We start with the recursive formulation of the equilibrium sensitivity (34):

$$\gamma_K = \frac{\alpha}{1-\frac{\beta}{\gamma_{K-1}}} \tag{A.57}$$

Solving for the fixed point of this equation, such that $\gamma_K = \gamma_{K-1}$, yields two solutions, $\gamma_K = \alpha+\beta = \gamma_{REE}$ and $\gamma_K = 0$ (these are also shown in Figure 6). Define the convergence function as $f(\gamma_{K-1}) = \frac{\alpha}{1-\frac{\beta}{\gamma_{K-1}}}$. The REE fixed point is attractive when the following condition is met:

$$|f'(\gamma_{REE})| < 1 \tag{A.58}$$

Straigthforward algebra yields that this is verified when:

$$\frac{\alpha\beta}{(\gamma_{REE}-\beta)^2} < 1 \iff \frac{\alpha\beta}{(\alpha+\beta-\beta)^2} < 1 \iff \alpha > \beta \tag{A.59}$$

Similarly, the attractive fixed point is the degenerate one if:

$$|f'(0)| < 1 \tag{A.60}$$

which requires:

$$\frac{\alpha\beta}{(0-\beta)^2} < 1 \iff \alpha < \beta \tag{A.61}$$

$\square$

Using the values of α and β, we also notice that, in our setup, (A.59) is satisfied if and only if $\phi > 1/2$.

A.2.10 Proposition 12: Misinference from (mistakenly) assuming correct Bayesian updating

Consider the case with $\hat{\alpha}+\hat{\beta} \neq \alpha+\beta$, so that some agents make a mistake in their Bayesian updating. We need to compare two scenarios: *i*) agents understand that others do wrong Bayesian updating, and use the actual price sensitivity to extract information from prices, $\tilde{\gamma} = \hat{\alpha}+\hat{\beta}$; *ii*) agents fail to realize other perform incorrect Bayesian updating, and mistakingly assume that all other agents are rational on this dimension, $\tilde{\gamma} = \alpha+\beta$.

Starting with case *i*), and setting $\tilde{\gamma} = \hat{\alpha}+\hat{\beta}$ in (38) and (39), we have:

$$P^{\text{Case } i)} = (\hat{\alpha}+\hat{\beta})s + (1-\hat{\alpha}-\hat{\beta})(\mu_0 - AZ\tau_0^{-1}) \tag{A.62}$$

$$\tilde{s}^{\text{Case } i)} = s \tag{A.63}$$

Turning to case *ii*), and setting $\tilde{\gamma} = \alpha+\beta$ in (38) and (39), we have:

$$P^{\text{Case } ii)} = \frac{\hat{\alpha}}{1-\frac{\hat{\beta}}{\alpha+\beta}} s + \left(1 - \frac{\hat{\alpha}}{1-\frac{\hat{\beta}}{\alpha+\beta}}\right)(\mu_0 - AZ\tau_0^{-1}) \tag{A.64}$$

$$\tilde{s}^{\text{Case } ii)} = s + \left(\frac{\hat{\alpha}+\hat{\beta}-\alpha-\beta}{\alpha+\beta-\hat{\beta}}\right)\left(s - (\mu_0 - AZ\tau_0^{-1})\right) \tag{A.65}$$

Wrong Bayesian updating and under-reaction: $\hat{\alpha}+\hat{\beta} < \alpha+\beta$. Consider the case with $\hat{\alpha}+\hat{\beta} < \alpha+\beta$, so that wrong Bayesian updating pushes towards under-reaction. From (A.62) and (A.63) we see that Case *i*) leads to correct inference, but under-reaction relative to the REE since $\frac{\partial P^{\text{Case } i)}}{\partial s} = \hat{\alpha}+\hat{\beta} < \alpha+\beta = \frac{\partial P^{REE}}{\partial s}$.

Moreover, from (A.64) and (A.65), we notice that $\hat{\alpha}+\hat{\beta} < \alpha+\beta$ implies:

$$\frac{\partial P^{\text{Case } ii)}}{\partial s} < \frac{\partial P^{\text{Case } i)}}{\partial s} < \frac{\partial P^{REE}}{\partial s} \tag{A.66}$$

Therefore, when $\hat{\alpha}+\hat{\beta} < \alpha+\beta$ Case *ii*) leads to more under-reaction than Case *i*). In other words, misinference from the incorrect belief that the world is rational amplifies the bias which arises from wrong Bayesian updating alone.

Intuitively, since $\tilde{\gamma} = \alpha + \beta > \hat{\alpha} + \hat{\beta}$, agents mistakingly believe that the equilibrium price is more responsive to the true signal than it really is. Therefore, they attribute any price change they observe to news which is less extreme than in reality. The under-reaction in signal extraction then compounds with the under-reaction in Bayesian updating.

Wrong Bayesian updating and over-reaction: $\hat{\alpha} + \hat{\beta} > \alpha + \beta$. Now consider the case with $\hat{\alpha} + \hat{\beta} > \alpha + \beta$, so that wrong Bayesian updating pushes towards over-reaction. From (A.62) and (A.63) we see that Case *i*) leads to correct inference, but over-reaction relative to the REE since $\frac{\partial P^{\text{Case } i)}}{\partial s} = \hat{\alpha} + \hat{\beta} > \alpha + \beta = \frac{\partial P^{REE}}{\partial s}$.

Moreover, from (A.64) and (A.65), we notice that if $\alpha + \beta > \hat{\beta}$, the equilibrium is stable and we can make the following comparisons:

$$\frac{\partial P^{\text{Case } ii)}}{\partial s} > \frac{\partial P^{\text{Case } i)}}{\partial s} > \frac{\partial P^{REE}}{\partial s} \tag{A.67}$$

Therefore, when the equilibrium is stable, Case *ii*) leads to more over-reaction than Case *i*). In other words, misinference from the incorrect belief that the world is rational amplifies the bias which arises from wrong Bayesian updating alone.

Intuitively, since $\tilde{\gamma} = \alpha + \beta < \hat{\alpha} + \hat{\beta}$, agents mistakingly believe that the equilibrium price is more responsive to the true signal than it really is. Therefore, they attribute any price change they observe to news which is more extreme than in reality. The over-reaction in signal extraction then compounds with the over-reaction in Bayesian updating.

Moreover, from (A.64) and (A.65), we also see that when $\alpha + \beta < \hat{\beta}$, misinference due to the incorrect belief that agents perform correct Bayesian updating can be so strong as to make the equilibrium unstable, something which could not arise with wrong Bayesian updating alone. □

B Heterogeneous Agents

Giglio et al. (2020) document a large amount of heterogeneity among agents' actions and beliefs. In this section we illustrates how the tractability of our model allows us to consider heterogeneous agents who differ in their level of sophistication in extracting information from prices. The key point in solving these models is to notice that we have to specify the mapping used by each type of agent who extracts information from prices. We start by solving the models with a general framework and then offer some examples.

B.1 Solving the Model with Heterogeneous Agents

Let there be a fraction of informed agents, and a fraction of uninformed agents. Moreover, let there be N different types of uninformed agents, each using a different misspecified model of the world to extract information from prices.

Misspecified model of the world for each agent $n = 1, 2, \ldots, N$. To solve the model, we need to specify the mapping used by each type of agent to extract information from prices. In this section we allow agents to have general *linear* mappings.[42,43] We denote these models as:[44]

$$P^{Mis_1} = \gamma_1 \tilde{s}_1 + (1 - \gamma_1)\zeta_1 \tag{B.1}$$

$$\vdots$$

$$P^{Mis_N} = \gamma_N \tilde{s}_N + (1 - \gamma_N)\zeta_N \tag{B.2}$$

So that for $n = \{1, 3, ..., N\}$, inverting prices gives the following extracted signal:

$$\tilde{s}_n = \frac{1}{\gamma_n} P^{Mis_n} - \left(\frac{1 - \gamma_n}{\gamma_n}\right) \zeta_n \tag{B.3}$$

[42] As shown in the main text, the linearity assumption is unrestrictive for CE, PET, REE and $K-$level thinking, and in section 4.1 we also discussed how this assumption nests many types of behavioral biases studied in the literature.

[43] In a setup where there are heterogeneous agents extracting information from prices, this linearity assumption simply requires that each agent n thinks that the (misspecified) mappings used by other agents are linear. When this is the case, agent n believes that other agents' (wrong) extracted signals are linear functions of the true signal. Therefore, when each agent n aggregates his/her perception of all other agents' linear asset demands, the resulting market clearing price function which determines his/her (misspecified) mapping is also linear. In this case, the details of what each agent n thinks about other agent's extracted signals determines γ_n, but not the functional form of their (misspecified) mapping.

[44] Notice that ζ_i could simply be equal to $(\mu_0 - AZ\tau_0^{-1})$ as in the analysis in the main text. In this section we leave it general as, for example, agents may have different priors.

True Model of the World. Given these N types of agents, the true price function takes the following form:

$$P^{True} = \alpha s + \sum_{n=1}^{N} \beta_n \tilde{s}_n + \delta, \tag{B.4}$$

where α, β_n for $n \in \{1, \ldots, N\}$ and δ are constant coefficients that depend on the composition of agents and fundamental parameters of the economy. We assume that each type of agent performs correct Bayesian updating, so that misinference is the only mistake agents eventually make.[45,46]

Equilibrium Price and Extracted Signals. We solve for the equilibrium price and extracted signals by setting $P = P^{Mis_1} = P^{Mis_2} = ... = P^{Mis_N}$ and solving these N equations in N unknowns (P, $\tilde{s}_1$, $\tilde{s}_2$, ..., $\tilde{s}_N$). Substituting (B.3) into (B.4), we get:

$$P = \alpha s + \sum_{n=1}^{N} \beta_n \left(\frac{1}{\gamma_n} P - \frac{1-\gamma_n}{\gamma_n} \zeta_n \right) + \delta \tag{B.5}$$

Rearranging gives us the equilibrium price P:

$$P = \frac{\alpha}{1 - \sum_{n=1}^{N} \frac{\beta_n}{\gamma_n}} s + \frac{\delta - \sum_{n=1}^{N} \frac{\beta_n (1-\gamma_n)}{\gamma_n} \zeta_n}{1 - \sum_{n=1}^{N} \frac{\beta_n}{\gamma_n}} \tag{B.6}$$

We can also find the equilibrium extracted signal by substituting this expression into (B.3).

$$\tilde{s}_n = \frac{\frac{\alpha}{\gamma_n}}{1 - \sum_{m=1}^{N} \frac{\beta_m}{\gamma_m}} s + \frac{1}{\gamma_n} \left(\frac{\delta - \sum_{m=1}^{N} \frac{\beta_m (1-\gamma_m)}{\gamma_m} \zeta_m}{1 - \sum_{m=1}^{N} \frac{\beta_m}{\gamma_m}} - (1-\gamma_n)\zeta_n \right) \tag{B.7}$$

Notice that the resulting equilibrium is strikingly similar to the basic framework we explored in the main text.[47] In what follows, we show how stability and over-/under-reaction generalize to this setup with heterogeneous agent.

Proposition 13 (Stability with Heterogeneous Agents)**.** *In an economy with N different*

[45] This implies that the REE is achieved when $\gamma_n = \alpha + \sum_{m=1}^{N} \beta_m$ for all n.

[46] We can easily generalize this to account for non-rational bayesian updating.

[47] To add generality we did not take a stance on the coefficients ζ_n and δ of the price functions. It is easy to verify that if we assume that $\zeta_i = \mu_0 - AZ\tau_0^{-1}$ for all i, and $\delta = \left(1 - \alpha - \sum_{n=1}^{N} \beta_n\right)(\mu_0 - AZ\tau_0^{-1})$, we are back to the case we described in details in Section 4.1.

types of agents who extract information from prices using the linear mappings described in (B.4)*, the equilibrium is stable if and only if:*

$$\sum_{m=1}^{N}\left(\frac{\beta_m}{\gamma_m}\right) < 1 \tag{B.8}$$

Proof. This follows immediately from applying Lemma 2 to (B.6) with $\alpha > 0$. □

Notice that while (B.8) may seem more restrictive than the equivalent condition in Proposition 1, this is not necessarily the case. To see why, recall from (B.4) that β_n captures the relative influence on equilibrium prices of agent n's beliefs: the greater N is, the smaller β_n is, on average. Similarly, from (B.4), γ_n is agent n's perceived sensitivity of the equilibrium price to new information: the greater the number of agents extracting information from prices, the more likely it is that uninformed agents think the equilibrium price is more sensitive to new information.

Next, we turn to characterize over-and under-reaction at the individual and aggregate levels in the following two propositions.

Proposition 14 (Over- and Under-Reaction at the Individual Level)**.** *When the equilibrium is stable, an agent of type n overreacts to changes in the unobserved signal s if and only if:*

$$\gamma_n < \frac{\alpha}{1 - \sum_{m=1}^{N} \frac{\beta_m}{\gamma_m}} = \frac{\partial P}{\partial s} \tag{B.9}$$

and underreacts when the inequality is reversed.

Proof. This immediately follows from solving $\partial \tilde{s}_n / \partial s > 1$ and using our derivation in equation (B.7) together with the condition for stability in (B.8). □

This proposition shows that to determine an individual's level of over-/under-reaction, it is enough to compare the sensitivity of their mapping to a single aggregate statistic. Intuitively, individual agents extract the correct signal from prices if they use the true sensitivity of the equilibrium price to new information in their mapping. Agents who mistakingly think that the sensitivity is lower than in reality attribute a given price change to a more extreme signal, and overreact. Conversely, if agents think the sensitivity if greater than in reality they underreact.

When there is a single type of uninformed agent, this condition boils down to $\gamma_n < \alpha+\beta = \gamma_{REE}$, which is consistent with our earlier result in Proposition 11. However, whenever there are more than one type of agents, Proposition 14 shows that the correct benchmark to assess individual level over-reaction is the true equilibrium mapping, and not the REE one.

We now turn to the behavior of aggregate prices.

Proposition 15 (Over- and Under-Reaction at the Aggregate Level)**.** *The aggregate price function overreacts to changes in the signal s relative to the rational counterfactual if and only if:*

$$\frac{\alpha}{1-\sum_{m=1}^{N}\frac{\beta_m}{\gamma_m}} > \alpha + \sum_{m=1}^{N}\beta_m \iff \sum_{n=1}^{N}\left(\frac{\beta_n}{\sum_{m=1}^{N}\beta_m}\frac{1}{\gamma_n}\right) > \frac{1}{\gamma_{REE}} \tag{B.10}$$

and it underreacts when the inequality is reversed.

Proof. This proposition follows from inspecting $\partial P/\partial s$ from equation (B.6), and using the fact that the rational counterfactual is such that $\gamma_n = \alpha + \sum_{m=1}^{N}\beta_m$ for all n. □

The LHS of (B.10) is the weighted average of $1/\gamma_n$ across agents, where the weights are given by the relative influence that each agent has on equilibrium prices, $\beta_n/\sum_{m=1}^{N}\beta_m$. If we call this weighted average $1/\bar{\gamma}$, then the above expression implies that equilibrium prices exhibit over-reaction relative to the REE if and only if $\bar{\gamma} < \gamma_{REE}$. This is reminiscent of the result we found in Section 3, which now generalizes to a setup with heterogeneous agents: if, on average, agents' sensitivity is lower than γ_{REE}, then, on average, they extract signals which are more extreme than in reality, leading to over-reaction in equilibrium outcomes.

Moreover, the first inequality in (B.10) also makes clear that when the equilibrium is stable the extent of over-reaction is increasing in the strength of the aggregate feedback effect, $\sum_{n=1}^{N}\frac{\beta_n}{\gamma_n}$.

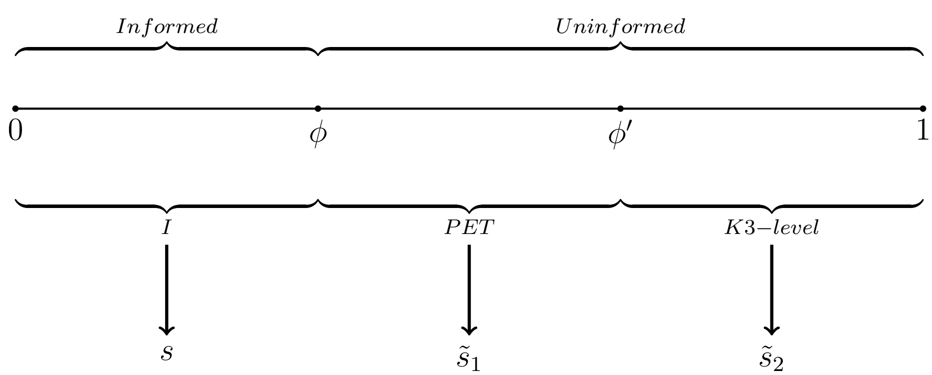
We flesh out a specific example to apply these results in Appendix B.2.

B.2 Combining Heterogeneous levels of Thinking

To illustrate the results of the framework with heterogenous agents, consider a world populated by 2−level thinkers (PET agents) and 3−level thinkers. More specifically, let us assume that the distribution of agents is as shown in Figure 9:

- *Informed agents*: a fraction ϕ of agents are informed and trade on s;
- *PET agents*: a fraction $(\phi'-\phi)$ of agents are uninformed and believe that all uninformed agents are cursed. The resulting signal is $\tilde{s}_1$;
- $K3-$*level agents*: a fraction $(1-\phi')$ of agents are 3−level thinkers, and believe that all other uninformed agents are PET. The resulting extracted signal is $\tilde{s}_2$.

Figure 9: Distribution of Heterogeneous Agents. This diagram shows the distribution of heterogeneous agents and the signal that they trade on, respectively. A fraction ϕ of agents are Informed and trade on the true signal s. Of the fraction $1-\phi$ of Uninformed agents, $\phi'-\phi$ are PET agents and extract and trade on signal $\tilde{s}_1$, and $1-\phi'$ are 3−level thinkers and trade on signal $\tilde{s}_2$.



Since both PET and $K3$ agents extract information from prices, we need to specify the two misspecified models of the world they use to do so. Consistent with notation in Section B.1, we let type $n=1$ refer to PET agents, and type $n=2$ refer to $K3$ agents.

PET's Misspecified Model of the World. PET agents believe that all other uninformed agents are cursed and only trade on their prior. Given this misspecified model of the world, PET agents believe the mapping from prices to signals is given by inverting:

$$P^{Mis1} = \frac{\phi\tau_s}{\phi\tau_s+\tau_0}\tilde{s}_1 + \left(1-\frac{\phi\tau_s}{\phi\tau_s+\tau_0}\right)(\mu_0 - AZ\tau_0^{-1}), \tag{B.11}$$

which yields the extracted signal by PET agents:

$$\tilde{s}_1 = \frac{\phi\tau_s + \tau_0}{\phi\tau_s} P^{Mis1} - \frac{\tau_0}{\phi\tau_s}(\mu_0 - AZ\tau_0^{-1}) \tag{B.12}$$

Therefore, to keep with notation from section B.1, let $\gamma_1 = \frac{\phi\tau_s}{\phi\tau_s+\tau_0}$.

3−**Level Thinkers' Misspecified Model of the World.** 3−level thinkers believe that all other uninformed agents are PET. Given this misspecified model of the world, they use the following mapping to extract information from prices:

$$P^{Mis2} = \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0} \tilde{s}_2 + \left(1 - \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}\right)(\mu_0 - AZ\tau_0^{-1}), \tag{B.13}$$

which yields the extracted signal by 3−level agents as:

$$\tilde{s}_2 = \frac{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}{\tau_s} P^{Mis2} - \frac{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}{\tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0}(\mu_0 - AZ\tau_0^{-1}) \tag{B.14}$$

Therefore, to keep with notation from section B.1, let $\gamma_2 = \frac{\tau_s}{\tau_s+\tau_0-\left(\frac{1-\phi}{\phi}\right)^2\tau_0}$.[48]

True Model of the World. Given that a fraction ϕ of agents are I and trade on s, a fraction $(\phi' - \phi)$ of agents are PET and trade on $\tilde{s}_1$, and a fraction $(1 - \phi')$ of agents are 3−level tinkers and trade on $\tilde{s}_2$, we get the following true price function:

$$P^{True} = \frac{\phi\tau_s}{\tau_s + \tau_0} s + \frac{(\phi' - \phi)\tau_s}{\tau_s + \tau_0} \tilde{s}_1 + \frac{(1 - \phi')\tau_s}{\tau_s + \tau_0} \tilde{s}_2 + \frac{\tau_0}{\tau_s + \tau_0}(\mu_0 - AZ\tau_0^{-1}) \tag{B.15}$$

Therefore, to keep with notation from section B.1, let $\alpha := \frac{\phi\tau_s}{\tau_s+\tau_0}$, $\beta_1 := \frac{(\phi'-\phi)\tau_s}{\tau_s+\tau_0}$, $\beta_2 := \frac{(1-\phi')\tau_s}{\tau_s+\tau_0}$ and $\delta := \frac{\tau_0}{\tau_s+\tau_0}(\mu_0 - AZ\tau_0^{-1}) = (1 - \alpha - \beta_1 - \beta_2)(\mu_0 - AZ\tau_0^{-1})$.[49]

[48] We assume that the parameters are such that the PET equilibrium would be stable, such that $\gamma_3 > 0$.

[49] Notice that PET, 3−level thinking are nested in this price function:

– PET when $\phi' = 1$: all uninformed agents are PET;

– 3−level when $\phi' = \phi$: as all uninformed agents are 3−level thinkers.

Equilibrium Price and Extracted Signals. Given the misspecified and true models of the world in equations (B.12), (B.14) and (B.15), we obtain the equilibrium price and extracted signals by setting $P = P^{True} = P^{Mis2} = P^{Mis3}$, and solving these three equations in three unknowns. This is a direct application of our general theory, and thus the equilibrium price and extracted signals are given by equations (B.6) and (B.7), with $N = 2$, $n = 1$ being PET agents, $n = 2$ being $K3$ agents, and with α, β_1, β_2, γ_1 and γ_2 as described above.

$$P = \gamma^* s + (1-\gamma^*)\left((\mu_0 - (AZ\tau_0^{-1}))\right) \tag{B.16}$$

$$\tilde{s}_1 = s + \frac{\gamma^* - \gamma_1}{\gamma_1}\left((s - (\mu_0 - AZ\tau_0^{-1}))\right) \tag{B.17}$$

$$\tilde{s}_2 = s + \frac{\gamma^* - \gamma_2}{\gamma_2}\left((s - (\mu_0 - AZ\tau_0^{-1}))\right) \tag{B.18}$$

where:

$$\gamma^* = \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi^2}\right)\left((\phi' - \phi) - (1-\phi')\left(\frac{1-\phi}{\phi}\right)\right)\tau_0} \tag{B.19}$$

Properties of Equilibrium Outcomes. We can refer to the propositions in Section B.1 to understand the properties of equilibrium outcomes.

First, at the *individual level*, we see that $3-$level agents always underreact:

$$\frac{\partial \tilde{s}_2}{\partial s} < 1 \tag{B.20}$$

Proof. From (B.18). we see that $3-$level agents underreact if and only if:

$$\gamma_2 > \gamma^* \tag{B.21}$$

$$\begin{aligned}\iff \quad & \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0} \\ & > \frac{\tau_s}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi}\right)^2 \tau_0 - \left(\frac{1-\phi}{\phi^2}\right)\left(-(1-\phi) + (\phi' - \phi) - (1-\phi')\left(\frac{1-\phi}{\phi}\right)\right)\tau_0}\end{aligned} \tag{B.22}$$

By comparing the denominators, it is clear that this hold iff:

$$\iff \frac{(1-\phi')}{\phi} > 0 \tag{B.23}$$

which is indeed always the case.

□

Intuitively, $K3$ agents are right about PET agents, and instead are wrong about other $K3$ agents. Specifically, they think that other $K3$ agents are PET and have a higher sensitivity than their true one. Therefore, $K3$ agents over-estimate the aggregate sensitivity, and underreact.

Whether PET agents over- or under-react depends on whether the fraction of $K3$ agents is large enough relative to the fraction of I agents:

$$\frac{\partial \tilde{s}_1}{\partial s} > 1 \iff \phi > 1 - \phi'. \tag{B.24}$$

Proof. Using the result in Proposition 14, PET agents over-react if and only if:

$$\gamma_1 < \gamma^* \tag{B.25}$$

$$\frac{\tau_s}{\tau_s + \tau_0 + \left(\frac{1-\phi}{\phi}\right)\tau_0} < \frac{\tau_s}{\tau_s + \tau_0 + \left(\frac{1-\phi}{\phi}\right)\tau_0 - \left(\frac{1-\phi}{\phi^2}\right)\left(\phi + (\phi' - \phi) - \frac{(1-\phi)(1-\phi)}{\phi}\right)\tau_0} \tag{B.26}$$

$$\iff \phi > 1 - \phi' \tag{B.27}$$

□

Intuitively, recall that we argued that $K3$ agents always underreact. If there are enough of these agents, and $\phi < (1 - \phi')$, then the equilibrium sensitivity may be even lower than the cursed sensitivity.[50] When this is the case, PET agents under-estimate the extent of under-reaction in the economy. This leads them to believe that the equilibrium price is more responsive to news than it really is, which results in them extracting more extreme signals

[50]In Proposition 9 we showed that in a world were all uninformed agents are the same type of $K-$ level thinker, the extent of under-reaction with $3-$level thinking can be greater than the extent of under-reaction with CE agents when $\phi < (1-\phi) \iff \phi < 1/2$. This condition becomes harder to satisfy when there are

than in reality and overreacting.

At the *aggregate level*, whether there is over- or under-reaction depends on the composition of agents: the equilibrium price exhibits over-reaction only if the fraction of PET agents is high enough relative to the fraction of $3-$level agents.

Proof. Using the result in Proposition 15, the equilibrium price exhibits over-reaction if and only if:

$$\left(\frac{\beta_1}{\beta_1+\beta_2}\right)\frac{1}{\gamma_1}+\left(\frac{\beta_2}{\beta_1+\beta_2}\right)\frac{1}{\gamma_2}>\alpha+\beta_1+\beta_2 \tag{B.29}$$

which simplifies to:

$$(\phi'-\phi)>(1-\phi')\left(\frac{1-\phi}{\phi}\right)\iff(\phi'-\phi)(\phi-(1-\phi'))>(1-\phi')^2 \tag{B.30}$$

where the last inequality uses the fact that $(1-\phi)=(1-\phi')-(\phi'-\phi)$. □

Notice that if $\phi<(1-\phi')$, then we know from (B.24) that PET agents underreact. Therefore, in this case both $K3$ and PET agents underreact, and there is under-reaction in equilibrium. When instead $\phi>(1-\phi')$, PET agents overreact, and $K3$ agents underreact. In this case, equilibrium outcomes exhibit over-reaction if there are enough PET agents $(\phi'-\phi)$ overreacting relative to $K3$ agents $(1-\phi')$ underreacting.

Welfare. Another question we can address with this is example is whether agents always benefit from having higher levels of rational higher order beliefs. Specifically, our example is populated with $2-$level and $3-$level thinkers. In what follows we show that $3-$level thinkers are not always better off, despite having more rational higher order beliefs.

PET agents as well as $3-$level thinkers populating the economy as:

$$\underbrace{\frac{\tau_s}{\tau_s+\tau_0+\left(\frac{1-\phi}{\phi}\right)^3\tau_0}}_{\gamma_{\text{All }3K}}<\underbrace{\frac{\tau_s}{\tau_s+\tau_0+\left(\frac{1-\phi}{\phi}\right)^3\tau_0-\left(\frac{1-\phi}{\phi^3}\right)(\phi'-\phi)}}_{\gamma^*} \tag{B.28}$$

Clearly, the more PET agents there are, $(\phi'-\phi)>0$, the less likely that $\gamma_{CE}=\frac{\tau_s}{\tau_s+\tau_0+\left(\frac{1-\phi}{\phi}\right)\tau_0}>\gamma^*$, so the less likely that PET agents underreact. Specifically $\phi<1/2$ isn't enough to ensure that $\gamma^*<\gamma_{CE}$, and this is reflected in the fact that for PET agents to underreact in (B.24), we need even fewer informed agents $\phi<\underline{\phi}<1/2$.

To address the question about welfare, we ought to refer back to agents' objective function. Agents' realized utility is maximized when they are able to best forecast the fundamental value, on average. Therefore, since the signal I agents receive is unbiased, uninformed agents' utility is maximized, on average, when they are able to correctly infer this signal from prices. In other words, the smaller the discrepancy $|\tilde{s}_i - s|$, the greater agent i's welfare.

Computing these quantities for $i \in \{1, 2\}$, we have that:

$$|\tilde{s}_1 - s| = \left| \frac{\left(\frac{1-\phi}{\phi^2}\right)\left(-\frac{1-\phi'}{\phi} + 1\right)\tau_0}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi^2}\right)\left((\phi' - \phi) - (1-\phi')\left(\frac{1-\phi}{\phi}\right)\right)\tau_0}(s - \mu_0 + AZ\sigma_0^2) \right| \tag{B.31}$$

$$|\tilde{s}_2 - s| = \left| \frac{\left(\frac{1-\phi}{\phi^2}\right)\left(-\frac{1-\phi'}{\phi}\right)\tau_0}{\tau_s + \tau_0 - \left(\frac{1-\phi}{\phi^2}\right)\left((\phi' - \phi) - (1-\phi')\left(\frac{1-\phi}{\phi}\right)\right)\tau_0}(s - \mu_0 + AZ\sigma_0^2) \right| \tag{B.32}$$

By comparing the numerators of these expressions, we see that PET agents can be better off than $K3$ agents if:

$$\begin{aligned} |\tilde{s}_1 - s| < |\tilde{s}_2 - s| \iff & \left| -\frac{1-\phi'}{\phi} + 1 \right| < \left| -\frac{1-\phi'}{\phi} \right| && \text{(B.33)} \\ \iff & |\phi - (1-\phi')| < |1-\phi'| && \text{(B.34)} \\ \iff & \phi < 2(1-\phi') && \text{(B.35)} \end{aligned}$$

which shows that PET ($K2$) agents are better off than $K3$ agents if there are enough $K3$ agents relative to I agents. Intuitively, increasing the fraction of $K3$ agents, increases individual level under-reaction by $K3$ agents, and decreases individual level over-reaction (which may even turn into under-reaction) by PET agents.

More generally, an agent has higher welfare when their model of the world is closest to the true model of the world, and having higher levels of rational higher order beliefs does not guarantee this, and may in fact be detrimental.

C Expected and Realized Volume

In this section we consider whether volume realized in the PET equilibrium is consistent with the amount of volume PET agents expect given their misspecified model of the world. In other words, would PET agents realize that they are making a mistake by observing trading volume?

To answer this question, we first need to compute the volume PET agents expect in equilibrium given their misspecified model of the world. Since they believe *i*) that they live in a CE world and *ii*) that the signal is $\tilde{s}$, we can obtain PET expected volume by simply evaluating $V^{CE}(\tilde{s})$. Starting from the definition of trading volume in (A.26), we can write PET realized and expected volume as follows:[51]

$$V^{PET} = \frac{2\phi(1-\phi)}{A(\tau_s+\tau_0)^{-1}}\left|\frac{\tau_s}{\tau_s+\tau_0}\left(\tilde{s}-s\right)\right| \tag{C.1}$$

$$\mathbb{E}[V^{PET}] = \frac{2\phi(1-\phi)}{A\left(\phi(\tau_s+\tau_0)^{-1}+(1-\phi)(\tau_0)^{-1}\right)}\left|\frac{\tau_s}{\tau_s+\tau_0}\left(\tilde{s}-(\mu_0-AZ\tau_0^{-1})\right)\right| \tag{C.2}$$

These expressions make clear that realized and expected volume may differ via two competing channels. First, by focusing on the denominator of the first term in both expressions, we see that PET agents think that the aggregate risk-baring capacity is lower than in reality.[52] This leads them to expect lower trading volume in equilibrium, as they think agents are trading less aggressively than they really are. Second, comparing the terms inside the absolute value operators, we see that PET agents expect greater belief dispersion. This channel leads them to expect higher trading volume in equilibrium.

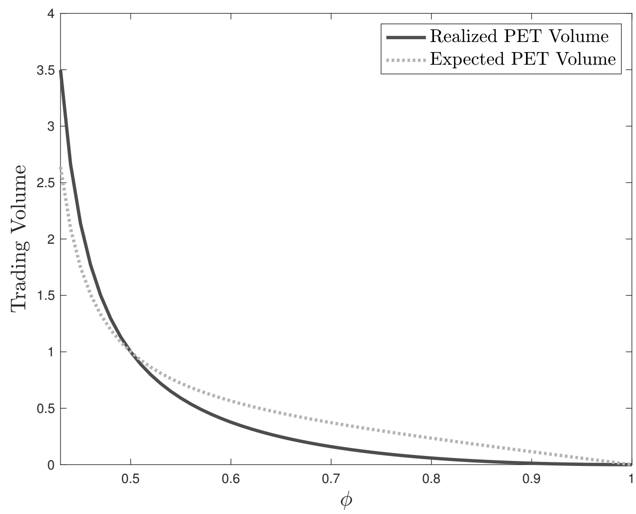
Figure 10 shows how realized and expected PET volume vary with the fraction of informed agents, ϕ. The two channels described above balance out for $\phi = 1/2$. When this is the case, even by looking at trading volume, uninformed agents would not realize their mistake. Instead, when $\phi > 1/2$, the second channel dominates and PET agents expect higher volume than what is realized. Conversely, when $\phi < 1/2$, PET agents under-estimate the amount of

[51]The expression for realized volume is less simplified than that in (A.28), so as to make the comparison with expected volume more explicit.

[52]PET agents think that a fraction $(1-\phi)$ of agents are trading on their prior alone, and therefore with lower precision.

trading volume.

Figure 10: Comparing PET Expected Volume by Uninformed Agents and PET Realized Volume. This Figure plots Expected Volume by PET agents (dashed line) and realized PET volume (full line) for different values of ϕ when the equilibrium is stable and (21) is satisfied, and for a fixed true signal, at $s = \mu_0$. Realized PET volume is the PET equilibrium trading volume, while the expected volume by PET agents is obtained by evaluating the CE equilibrium trading volume at $s = \tilde{s}$.



Therefore, when $\phi \neq \frac{1}{2}$, if agents were to learn about both ϕ and s from prices and volume, they would learn the correct model of the world. However, notice that our model is attentionally stable, as in Gagnon-Bartsch et al. (2020). Since prices are fully revealing, agents have no reason to pay attention to volume: according to their (misspecified) model of the world, volume should not convey any additional information. Therefore, agents need not realize that their model of the world is misspecified.

D Misinference from Assuming the World is Rational

In this section we illustrate a specific example where we consider the implications of misinference in models of wrong Bayesian updating. As explained in Section 4.1, inferring the correct information from prices when wrong Bayesian updating is pervasive implies that agents must fully understand other agents' mistakes, while failing to recognize those same mistakes in themselves. For example, consider agents who have diagnostic expectations, as in Bordalo et al. (2018). For these agents to correctly infer information from endogenous outcomes, they must fully understand that all other agents are diagnostic. If instead agents fail to recognize their same (to them unknown) mistake in others, then agents infer information as if they lived in a rational world, and this leads to misinference. The same is true of setups with overconfidence, as in Odean (1998), or if some agents are inattentive.

To study the implications of this type of misinference, consider a setup similar to our benchmark framework, but assume instead that the misspecified and true price functions are given by:

$$P^{Mis}(\tilde{s}) = \frac{\tau_s}{\tau_s + \tau_0}\tilde{s} + \frac{\tau_0}{\tau_s + \tau_0}\left(\mu_0 - AZ\tau_0^{-1}\right) \tag{D.1}$$

$$P^{True}(s, \tilde{s}) = \frac{\phi\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}s + \frac{(1-\phi)\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}\tilde{s} + \frac{\tau_0}{\tilde{\tau}_s + \tau_0}\left(\mu_0 - AZ\tau_0^{-1}\right) \tag{D.2}$$

where $\tilde{\tau}_s \neq \tau_s$. For example, when $\tilde{\tau}_s > \tau_s$, this true price function corresponds to the case where both I and U agents trade using a precision that is too high relative to the true precision of the signal, as would arise if agents were diagnostic or if they were over-confident about the precision of new information.

The dotted line in Figure 7 depicts this scenario. In this case $\hat{\alpha} + \hat{\beta} = \frac{\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}$, and changes in $\hat{\alpha} + \hat{\beta}$ along the $x-$axis come from varying $\tilde{\tau}_s$. Here we provide an example of how combining wrong Bayesian updating with misinference can be so detrimental so as to give rise to instability, something which could not arise if we had wrong Bayesian updating alone. Specifically, an immediate application of Proposition 10 tells us that the equilibrium is unstable if and only if:

$$\frac{\tau_s}{\tau_s + \tau_0} < \frac{(1-\phi)\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}. \tag{D.3}$$

When $(1-\phi)\tau_0 - \phi\tau_s > 0$, this condition reduces to:

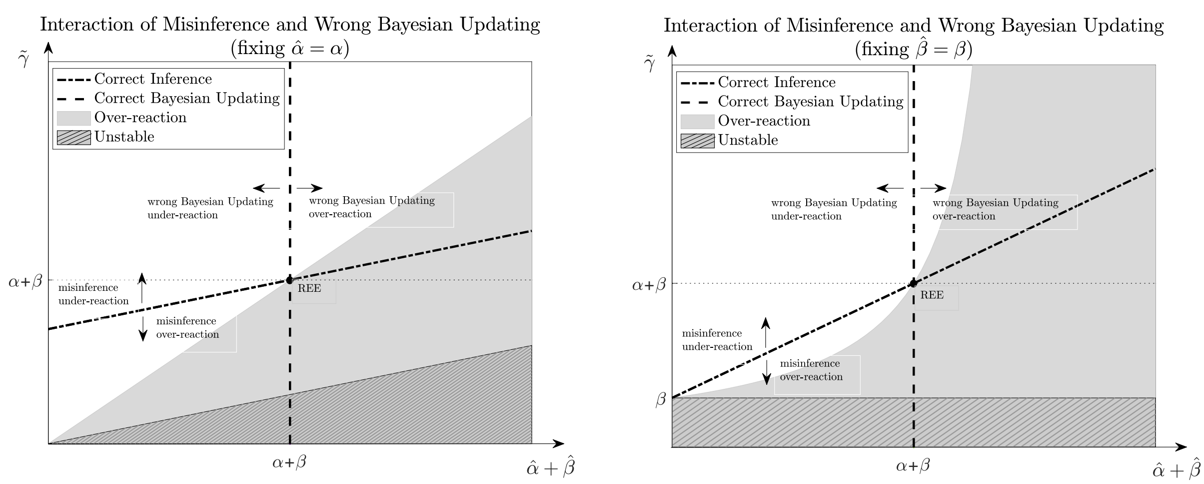
$$\frac{\tilde{\tau}_s}{\tau_s} > \frac{\tau_0}{(1-\phi)\tau_0 - \phi\tau_s} \tag{D.4}$$

Therefore, for a high enough $\tilde{\tau}_s$, misinference due to the fact that agents believe in a rational world when it is not, can indeed lead to instability. This is shown on the part of the light dotted line in Figure 7 which is in the shaded unstable region.

The intuition behind this example is as follows. Since uninformed agents fail to realize that everyone is over-reacting in the form of $\tilde{\tau}_s > \tau_s$, they extract information from prices using a misspecified model of the world that is less responsive to new information than the true model of the world. This leads them to extract a signal which is more extreme than in reality. Having extracted this extreme signal, uninformed agents then over-react themselves, leading to even further mispricing. Finally, notice that while informed agents are using the correct signal so in a way their over-reaction contributes to stabilizing the equilibrium, their over-reaction also worsens uninformed agents' misinference.

Finally, notice that incorrect bayesian updating can occur along two dimensions: $\hat{\alpha} \neq \alpha$ and $\hat{\beta} \neq \beta$, so that there really are two degrees of freedom in determining $\hat{\alpha} + \hat{\beta}$. In Figure 7 we specified a particular way of varying $\hat{\alpha}+\hat{\beta}$: in the example considered above, all agents are over-confident and varying $\tilde{\tau}_s$ changes both $\hat{\alpha}$ and $\hat{\beta}$, so that $\hat{\alpha} + \hat{\beta} = \frac{\phi\tilde{\tau}_s}{\tilde{\tau}_s+\tau_0} + \frac{(1-\phi)\tilde{\tau}_s}{\tilde{\tau}_s+\tau_0} = \frac{\tilde{\tau}_s}{\tilde{\tau}_s+\tau_0}$. However, $\hat{\alpha}$ and $\hat{\beta}$ need not be linked in this specific way. For example, in Figure 11 we perform a similar exercise, with a different type of variation in $\hat{\alpha} + \hat{\beta}$: the left panel fixes $\hat{\alpha}$, so that all variations in $\hat{\alpha} + \hat{\beta}$ come from changes in $\hat{\beta}$, and the right panel fixes $\hat{\beta}$, so that all variations in $\hat{\alpha} + \hat{\beta}$ come from changes in $\hat{\alpha}$.[53]

Figure 11: Instability, Under- and Over-reaction regions with General Model Misspecification. In the left panel we fix the parameter $\hat{\alpha}$ to its rational value α, and let $\hat{\beta}$ vary. In the right panel $\hat{\beta}$ is kept fixed at β and $\hat{\alpha}$ varies.



[53] In general, $\hat{\alpha}$ and $\hat{\beta}$ are likely to be linked. For example, if informed agents were rational and only uninformed agents were over-confident, this would still lead to $\hat{\alpha} \neq \alpha$, as both $\hat{\alpha}$ and $\hat{\beta}$ are scaled by the average precision across agents. Therefore, fixing $\hat{\alpha}$ and letting $\hat{\beta}$ vary does *not* necessarily amount to fixing the bias of I agents and letting the bias of U agents vary. We show these cases for completeness and for intuition.

E Partially Revealing Prices

This section generalizes PET to environments where prices are only partially revealing. This serves two purposes. First, it verifies that the insights we uncovered are robust to small variations in the information structure. Second, it allows us to expand our setup to a symmetric case where all agents receive a private signal, as shown in Appendix F.[54]

To do so, we model the supply of the risky asset as being stochastic, for example because of the presence of noise traders. Therefore, the supply shock z is now drawn from a normal distribution, $z \sim N(\bar{z}, \tau_z^{-1})$. All other assumptions are the same as in our baseline framework.

To construct their demand functions, informed traders need to compute the distribution of $v|s$ (using simple Bayesian updating as we did throughout the main text), while uninformed traders need to compute the distribution of $(v, s, z)|P$. Moreover, uninformed traders still think that they lived in a cursed world. Using the same notation as in Section 4.1, we can write uninformed agents' misspecified model as:

$$P^{Mis} = \gamma\tilde{s} + (1-\gamma)(\mu_0 - Az\tau_0^{-1}), \tag{E.1}$$

Uninformed traders cannot observe z, but they can use standard Bayesian updating to extract the maximum-likelihood signal. Following Diamond and Verrecchia (1981), we first write down the distribution of the information structure of the uninformed agents, $(v, s, z, P^{Mis}) \sim N(m, M)$, which is multivariate normal with mean m and variance-covariance matrix M, where:[55]

$$m = (\mu_0, \mu_0, \bar{z}, \mu_0 - (1-\gamma)A\bar{z}\tau_0^{-1})^T + \tag{E.2}$$

[54] It also allows us to circumvent the Grossman-Stiglitz paradox. In a setup with fully revealing prices, no agent would have an incentive to pay to acquire information, since all information can be inferred freely from prices. This is not the case anymore with partially revealing prices, which can rationalize why only a fraction ϕ of agents acquire information and are thus informed.

[55] Note that:

$$Var(P^{Mis}) = \gamma^2(\tau_0^{-1} + \tau_s^{-1}) + (1-\gamma)^2\tau_z^{-1}.$$

While the covariances are given by:

$$Cov(P^{Mis}, v) = \gamma\tau_0^{-1},$$
$$Cov(P^{Mis}, s) = \gamma(\tau_0^{-1} + \tau_s^{-1}),$$
$$Cov(P^{Mis}, z) = (1-\gamma)\tau_z^{-1}.$$

$$M = \begin{pmatrix} \tau_0^{-1} & \tau_0^{-1} & 0 & \gamma\tau_0^{-1} \\ \tau_0^{-1} & \tau_0^{-1}+\tau_s^{-1} & 0 & \gamma(\tau_0^{-1}+\tau_s^{-1}) \\ 0 & 0 & \tau_z^{-1} & (1-\gamma)\tau_z^{-1} \\ \gamma\tau_0^{-1} & \gamma(\tau_0^{-1}+\tau_s^{-1}) & (1-\gamma)\tau_z^{-1} & \gamma^2(\tau_0^{-1}+\tau_s^{-1})+(1-\gamma)^2\tau_z^{-1} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \tag{E.3}$$

where M_{11} is 3×3 (making the distinction between what is in the information set of the agent and what is not). Similarly, let $m = (m_1, m_2)$, with m_1 being 3×1.

Then $(v, s, z)|P$ is jointly normal, with mean m^*, and covariance matrix M^*, where:

$$m^* = m_1 + M_{12}M_{22}^{-1}(P - m_2) \tag{E.4}$$

$$M^* = M_{11} - M_{12}M_{22}^{-1}M_{21}. \tag{E.5}$$

The first element in m^* and in M^* characterize uninformed agents' belief about the fundamental value of the asset, given their mapping: We can then write uninformed agents beliefs as follows:

$$\mathbb{E}[v|\mathcal{I}_U] = \mu_0 + \frac{\tau_s/\gamma}{\tau_0 + \tau_s + \left(\frac{1-\gamma}{\gamma}\right)^2 \frac{\tau_0\tau_s}{\tau_z}}(P - \mu_0 + (1-\gamma)A\bar{z}\tau_0^{-1}) \tag{E.6}$$

$$= \mu_0 + \frac{g}{\gamma}(P - \mu_0 + (1-\gamma)A\bar{z}\tau_0^{-1}) \tag{E.7}$$

$$\mathrm{Var}[v|\mathcal{I}_U] = \tau_0^{-1} - \frac{\tau_s\tau_0^{-1}}{\tau_0 + \tau_s + \left(\frac{1-\gamma}{\gamma}\right)^2 \frac{\tau_0\tau_s}{\tau_z}} = \tau_0^{-1}(1-g) \tag{E.8}$$

where g is the Kalman gain:[56]

$$g = \frac{\tau_s}{\tau_0 + \tau_s + \left(\frac{1-\gamma}{\gamma}\right)^2 \frac{\tau_0\tau_s}{\tau_z}} \tag{E.9}$$

[56] When $\tau_z = \infty$, we recover the Kalman gain with fixed supply: $\tau_s/(\tau_0 + \tau_s)$.

Putting the above components together, U agents' demand function is given by:[57]

$$X_U = \frac{\mu_0 + \frac{g}{\gamma}\left(P - \mu_0 + (1-\gamma)A\bar{z}\tau_0^{-1}\right) - P}{A\tau_0^{-1}(1-g)}. \tag{E.10}$$

Moreover, I agents' demand function is simply given by:

$$X_I = \frac{\mu_0 + \frac{\tau_s}{\tau_s+\tau_0}(s-\mu_0) - P}{A(\tau_s+\tau_0)^{-1}} \tag{E.11}$$

As in our baseline case, the equilibrium price must solve the following market clearing condition, $z = \phi X_I + (1-\phi)X_U$:

$$z = \phi\left(\frac{\mu_0 + \frac{\tau_s}{\tau_s+\tau_0}(s-\mu_0) - P}{A(\tau_s+\tau_0)^{-1}}\right) + (1-\phi)\left(\frac{\mu_0 + \frac{g}{\gamma}\left(P - \mu_0 + (1-\gamma)A\bar{z}\tau_0^{-1}\right) - P}{A\tau_0^{-1}(1-g)}\right) \tag{E.12}$$

Before solving for the equilibrium price, notice that, just as in Section 1.1, prices still play a dual role: their informational role gives rise to strategic complementarities, while their scarcity role gives rise to strategic substitutabilities. The interaction of these two forces determines whether the equilibrium is stable, as is clear from the excess demand function:

$$X_{TOT} - z = \frac{1}{A}\left(\underbrace{\frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\gamma}}_{\text{information role}} \underbrace{- \frac{\phi}{(\tau_0+\tau_s)^{-1}} - \frac{(1-\phi)}{\tau_0^{-1}(1-g)}}_{\text{scarcity role}}\right)P + constants \tag{E.13}$$

As in Section 1.2, the complementarities introduced by the informational role of prices give rise to a two-way feedback effect between outcomes and beliefs. The strength of this feedback effect is now given by:

$$\text{strength of feedback effect } = \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\gamma}, \tag{E.14}$$

and this is increasing in the influence that uninformed agents have on prices $\left(\frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\right)$,

[57] The algebra is less straightforward than in our baseline case, because PET agents are now using a lower precision than informed agents: their inference is weakened by the noisy supply.

and decreasing in the sensitivity of the misspecified mapping γ.[58] When this feedback effect is strong enough, the aggregate excess demand function becomes upward sloping and the equilibrium unstable, just as we saw in Proposition 10. In particular, (E.13) shows that the equilibrium is stable if and only if:

$$\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} > \frac{(1-\phi)g}{\tau_0^{-1}(1-g)\gamma} \tag{E.15}$$

We can now solve for the equilibrium price:

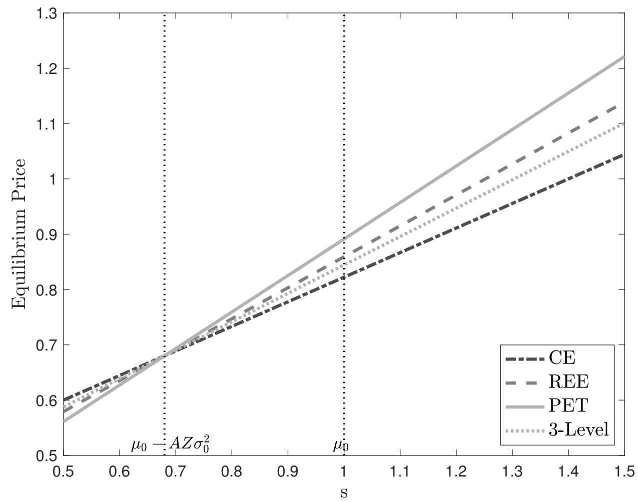
$$\begin{aligned} P = {} & \frac{\phi\tau_0 + \frac{(1-\phi)\left(1-\frac{g}{\gamma}\right)}{\tau_0^{-1}(1-g)}}{\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} - \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\gamma}}\mu_0 + \frac{\phi\tau_s}{\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} - \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\gamma}}s \\ & + \frac{\frac{(1-\phi)g(1-\gamma)}{(1-g)\gamma}}{\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} - \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\gamma}}A\bar{z} - \frac{1}{\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} - \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\gamma}}Az \end{aligned} \tag{E.16}$$

In Figure 12, we plot this price function against the signal informed agents receive, s, together with the equilibrium price function of the CE and REE in this setup. Our benchmark results are preserved: PET features over-reaction, and lies outside the convex set of the CE and REE prices. As in Section 3, we can extend this argument to allow for $K-$level thinking. Figure 12 plots the equilibrium price for $3-$level thinking and we see that this once again delivers under-reaction. Overall results are qualitatively and quantitatively similar to our fully revealing benchmark. For the sake of clarity, and for ease of exposition, we focus on the fully revealing case in the main text.

Finally, notice how in this particular setup the PET equilibrium concept is much easier to solve than the REE. Indeed, to solve for the REE one needs to postulate a price function, use Bayesian updating to find uninformed agents' demands conditional on the price function, and then work through market clearing to obtain the equilibrium price function. After this,

[58] Notice that increasing g has two effects: first, it reduces the conditional variance of uninformed agents, which makes them less risk averse, and this effect strengthens both the scarcity role and the informational role of prices in equal proportions. Second, it increases the weight that uninformed agents put on prices when performing Bayesian updating, and this increases the strength of the informational role. Therefore, the net effect of an increase in g is to strengthen the informational role relative to the scarcity role, and this contributed to instability. Moreover, as in Section 1.2, the strength of the feedback effect is decreasing in γ, so that the insights we uncovered in the fully revealing case are preserved.

Figure 12: Comparing Equilibrium Prices as we allow for rationality of Higher Order Beliefs in the Partially Revealing Case. This Figure plots the equilibrium prices which arise for different levels of $K-$thinking as we vary the value of the true signal, s. The CE is $1-$level thinking, PET is $2-$level thinking, and REE is $\infty-$level thinking. Here, ϕ is fixed at 0.6, $\tau_0^{-1} = 0.16$, $\tau_0^{-1} = 0.12$, $\mu_0 = 1$, $\bar{Z} = 1$, $\tau_z^{-1} = 0.1$, and $A = 2$.



agents still need to solve a fixed-point problem (that is far from trivial) where they equate the postulated price functions to the equilibrium price function. In this respect, PET agents are solving a much simpler problem, which requires far fewer cognitive skills than the REE.[59].

F Private signals

In this section we consider an alternative setup where agents are ex-ante symmetric, and all receive a different stochastic private signal. The price aggregates all private information, and agents infer the information received by others by conditioning on the equilibrium price. Specifically, there is a continuum of agents of measure 1, denoted by $i \in [0, 1]$. Supply is stochastic as in Appendix E. Each agent i receives a signal:

$$s_i = v + \epsilon_i \tag{F.1}$$

with $\epsilon_i \sim^{i.i.d.} \mathcal{N}(0, \tau_s^{-1})$. By the law of large numbers, the average signal received by the population corresponds to the period-1 payoff of the asset, $\int_0^1 s_i di = v$.

The misspecified model of the world corresponds to the case where all agents trade on

[59]Computationally, the PET equilibrium is the first-step in solving the fixed-point process of the REE.

their private information alone (the CE benchmark), which yields the following misspecified price function:

$$P^{Mis} = \gamma v + (1-\gamma)(\mu_0 - Az\tau_0^{-1}), \tag{F.2}$$

with $\gamma = \tau_s/(\tau_0 + \tau_s)$. All agents condition on s_i and P to update their prior. As previously, we start by constructing the variance-covariance structure:

$$Cov(v, s_i) = \tau_0^{-1}, \tag{F.3}$$

$$Cov(P^{Mis}, s_i) = \gamma\tau_0^{-1}. \tag{F.4}$$

Which gives the following covariance matrix for the triplet (v, s_i, P):

$$M = \begin{pmatrix} \tau_0^{-1} & \tau_0^{-1} & \gamma\tau_0^{-1} \\ \tau_0^{-1} & \tau_0^{-1} + \tau_s^{-1} & \gamma\tau_0^{-1} \\ \gamma\tau_0^{-1} & \gamma\tau_0^{-1} & \gamma^2\tau_0^{-1} + (1-\gamma)^2\tau_z^{-1} \end{pmatrix}. \tag{F.5}$$

We now need to define two Kalman gain coefficients, as all agents condition both on the price and on the private signal they receive:

$$g_s = \frac{(1-\gamma)^2\tau_s}{\gamma^2\tau_z + (1-\gamma)^2(\tau_0 + \tau_s)}, \tag{F.6}$$

$$g_v = \frac{\gamma^2\tau_z}{\gamma^2\tau_z + (1-\gamma)^2(\tau_0 + \tau_s)}. \tag{F.7}$$

Therefore, agent's posterior beliefs of the fundamental value of the asset are normal, and characterized by the following posterior mean and variance:

$$E[v|s_i, P] = \mu_0 + g_s(s_i - \mu_0) + g_v\left(\frac{P - \mu_0 - (1-\gamma)A\bar{z}\tau_0^{-1}}{\gamma}\right) \tag{F.8}$$

$$Var[v|s_i, P] = \tau_0^{-1}(1 - g_s - g_v). \tag{F.9}$$

Individual i's demand function is then given by:

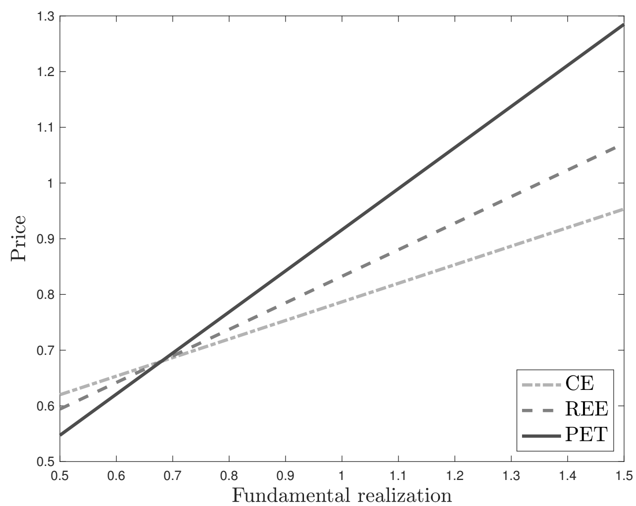
$$X_i = \frac{\mu_0 + g_s(s_i - \mu_0) + g_v\left(\frac{P-\mu_0+(1-\gamma)A\bar{z}\tau_0^{-1}}{\gamma}\right) - P}{A\tau_0^{-1}(1-g_s-g_v)}. \tag{F.10}$$

We can now impose market clearing and use the law of large numbers to aggregate private signals. This leads to the following equilibrium price expression:

$$P = \frac{1-g_s-g_v/\gamma}{1-\frac{g_v}{\gamma}}\mu_0 + \frac{g_s}{1-\frac{g_v}{\gamma}}v - \frac{(1-g_s-g_v)}{1-\frac{g_v}{\gamma}}Az\tau_0^{-1} - \frac{g_v\frac{1-\gamma}{\gamma}}{1-\frac{g_v}{\gamma}}A\bar{z}\tau_0^{-1} \tag{F.11}$$

Figure 13 displays how the equilibrium price varies with the value of the fundamental. The results are qualitatively and quantitatively similar to our baseline framework.

Figure 13: Comparing Equilibrium Prices with Symmetric Private Signals. This Figure plots the equilibrium prices as we vary the value of the fundamental realization, v. Here, $\tau_0^{-1} = 0.16$, $\tau_s^{-1} = 0.32$, and $\tau_z^{-1} = 0.95$ so that the condition which guarantees that the equilibrium PET price is stable is satisfied.



Notice how the price sensitivity to the fundamental realization in (F.11) has exactly the same form as the sensitivity of the price to new information in the general case with model misspecification in (38). With private signals, the feedback effect between outcomes and beliefs is now represented by the ratio g_v/γ, since g_v pertains to how agents incorporate into prices the information they get by learning from prices – the exact counterpart of $\hat{\beta}$ in our main framework. As such, all the insights we uncovered in the main text remain valid here. We conclude with a remark on how the volatility of supply relates to the stability of the equilibrium.

Corollary 2. *In this setup with symmetric agents and stochastic supply, the equilibrium is stable, and the price is increasing in the fundamental, if and only if:*

$$\gamma > g_v \iff \tau_z < \frac{\tau_0^2}{\tau_s} \tag{F.12}$$

In other words, once we allow for a symmetric information structure, the stochastic supply needs to be volatile enough for the equilibrium to be stable. Intuitively, when prices are close to fully revealing, the signal agents can recover from prices is infinitely more informative than their own private signal. Therefore, in this extreme case, agents do not use their own signal in their demand function. However, if all agents act in this way, no private signal is incorporated into prices, and the resulting price becomes completely uninformative. Since PET agents fail to realize that all other agents are also learning from prices (and therefore that they are not incorporating their own private signal in them), they treat the price as informative even when it is not, and this contributes to unstable outcomes.