The Investment Network, Sectoral Comovement, and the Changing U.S. Business Cycle

CHRISTIAN VOM LEHN
Brigham Young University and IZA

THOMAS WINBERRY
Wharton and NBER

NBER Winter EF&G Meeting
February 26, 2021
Overview of Our Project

- **Question**: how are sector-specific shocks propagated to macroeconomic aggregates?

- Focus on role of the *investment network*: distribution of investment of production and purchases across sectors.
Overview of Our Project

• **Question**: how are sector-specific shocks propagated to macroeconomic aggregates?

• Focus on role of the *investment network*: distribution of investment of production and purchases across sectors

• **Answer**: investment network important propagation mechanism
  1. Empirically, investment network is *extremely concentrated*
  2. In multisector RBC model: shocks to hubs and their key suppliers have *large effect on aggregate employment*
  3. Shocks to hubs + suppliers account for larger share of fluctuations since 1980s  \( \Rightarrow \) *acyclicality of labor productivity*
The Investment Network
Data Sources for 37 Sector-Level Coverage

Extend various BEA data sources 1947-2018 to include finer disaggregation of manufacturing

<table>
<thead>
<tr>
<th>Industry Type</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mining</td>
<td></td>
</tr>
<tr>
<td>Construction</td>
<td></td>
</tr>
<tr>
<td>Wood products</td>
<td></td>
</tr>
<tr>
<td>Primary metals</td>
<td></td>
</tr>
<tr>
<td>Machinery</td>
<td></td>
</tr>
<tr>
<td>Electrical equipment manufacturing</td>
<td></td>
</tr>
<tr>
<td>Other transportation equipment</td>
<td></td>
</tr>
<tr>
<td>Misc. Manufacturing</td>
<td></td>
</tr>
<tr>
<td>Textile manufacturing</td>
<td></td>
</tr>
<tr>
<td>Paper manufacturing</td>
<td></td>
</tr>
<tr>
<td>Petroleum and coal manufacturing</td>
<td></td>
</tr>
<tr>
<td>Plastics manufacturing</td>
<td></td>
</tr>
<tr>
<td>Retail trade</td>
<td></td>
</tr>
<tr>
<td>Information</td>
<td></td>
</tr>
<tr>
<td>Professional and technical services</td>
<td></td>
</tr>
<tr>
<td>Administrative and waste management services</td>
<td></td>
</tr>
<tr>
<td>Health care and social assistance</td>
<td></td>
</tr>
<tr>
<td>Accommodation</td>
<td></td>
</tr>
<tr>
<td>Other services</td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td></td>
</tr>
<tr>
<td>Real estate and rental services</td>
<td></td>
</tr>
<tr>
<td>Non-metallic minerals</td>
<td></td>
</tr>
<tr>
<td>Fabricated metals</td>
<td></td>
</tr>
<tr>
<td>Computer and electronic manufacturing</td>
<td></td>
</tr>
<tr>
<td>Motor vehicles manufacturing</td>
<td></td>
</tr>
<tr>
<td>Furniture and related manufacturing</td>
<td></td>
</tr>
<tr>
<td>Food and beverage manufacturing</td>
<td></td>
</tr>
<tr>
<td>Apparel manufacturing</td>
<td></td>
</tr>
<tr>
<td>Printing products manufacturing</td>
<td></td>
</tr>
<tr>
<td>Chemical manufacturing</td>
<td></td>
</tr>
<tr>
<td>Wholesale trade</td>
<td></td>
</tr>
<tr>
<td>Transportation and warehousing</td>
<td></td>
</tr>
<tr>
<td>Finance and insurance</td>
<td></td>
</tr>
<tr>
<td>Management of companies and enterprises</td>
<td></td>
</tr>
<tr>
<td>Educational services</td>
<td></td>
</tr>
<tr>
<td>Arts, entertainment, and recreation services</td>
<td></td>
</tr>
<tr>
<td>Food services</td>
<td></td>
</tr>
</tbody>
</table>
Empirical Investment Network

- **Investment network** = share of investment expenditure of sector $j$ purchased from sectors $i$ in year $t$

- BEA provides some pairwise capital flows data, but:
  1. Only small set of years (most recently in 1997)
  2. Does not include most of intellectual property
  3. Not consistent classification of sectors over time

- Construct estimates of pairwise flows using asset-level data
  - Compute how much of each asset purchased by sector $j$
  - Estimate how much of asset produced by sector $i$
  - Follows BEA practice + benchmarked to production data

- Result: annual time series of investment network consistent w/ current national accounting regarding intellectual property
• Entry \((i,j)\) = share of investment in sector \(j\) supplied by sector \(i\)
• Compute year-by-year, then average over sample

Changes over time
Empirical Investment Network

- Four investment hubs: construction (structures), machinery and motor vehicles (equipment), and professional/technical services (intellectual property)
• Hubs produce nearly 70% of aggregate investment, but only 10%-15% of aggregate value added, employment, or intermediates.
• Three times as concentrated as the intermediates network.
Empirical Investment Network

- Investment hubs more volatile at cyclical frequencies
- And more correlated with aggregate business cycle
Model and Calibration
Production

- Fixed number of sectors $j \in \{1, \ldots, N = 37\}$ (same as in data)

- Gross output $Q_{jt}$ produced according to

$$Q_{jt} = A_{jt} \left( K_{jt}^{\alpha_j} L_{jt}^{1-\alpha_j} \right)^{\theta_j} M_{jt}^{1-\theta_j}$$

- Intermediates input-output network

$$M_{jt} = \prod_{i=1}^{N} M_{ijt}^{\gamma_{ij}}, \quad \text{where} \quad \sum_{i=1}^{N} \gamma_{ij} = 1$$

- TFP shocks follow $\log A_{jt+1} = \rho_j \log A_{jt} + \varepsilon_{jt+1}$
  - Linear model solution $\implies$ certainty equivalence
  - Will feed in realizations of $\log \varepsilon_{jt}$ from data
Investment

- Capital accumulation technology
  \[ K_{jt+1} = (1 - \delta_j)K_{jt} + I_{jt} \]

- Investment network
  \[ I_{jt} = \prod_{i=1}^{N} \lambda_{ij}, \quad \text{where} \quad \sum_{i=1}^{N} \lambda_{ij} = 1 \]
  - Investment hubs \( i \) have high \( \lambda_{ij} \) for many \( j \)
  - In calibration, \( \lambda_{ij} = \) entry in empirical investment network
Household and Equilibrium

• Representative household with preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t - L_t), \quad \text{where } C_t = \prod_{j=1}^{N} C_{jt}^{\xi_j} \text{ and } \sum_{j=1}^{N} \xi_j = 1 \]

• Study the competitive equilibrium, which is efficient

\[ Q_{jt} = C_{jt} + \sum_{k=1}^{N} M_{jkt} + \sum_{k=1}^{N} l_{jkt} \]

\[ L_t = \sum_{j=1}^{N} L_{jt} \]
Calibration Overview

- Will later measure shocks from data and feed into model

- Now calibrate all parameters other than shocks so that model’s steady state \( \approx \) data’s long-run averages
  - Isolates role of changes in shock process
  - But results robust to allowing for structural change

- Cobb-Douglas \( \Rightarrow \) most parameters pinned down by average expenditure shares
  1. Production parameters from input-output database
  2. Preference parameters from final use tables
  3. Investment network from earlier measurement
Role of Investment Network in Propagating Sectoral Shocks
What Determines the Response of Employment?

Proposition

Employment is proportional to the household’s value of output:

\[
L_{jt} \propto \sum_{k=1}^{N} \ell_{jk} \frac{p_{kt}C_{kt}}{C_{t}} + \sum_{k=1}^{N} \ell_{jk} \sum_{m=1}^{N} \lambda_{km} \frac{p_{mt}l_{mt}}{C_{t}}
\]

where \( p_{mt} = \prod_{k=1}^{N} \left( \frac{p_{kt}}{\lambda_{km}} \right)^{\lambda_{km}} \) is the investment price index and \([L]_{jk} = \ell_{jk}\) is the Leontief inverse.

[Derivation] [Real GDP]
What Determines the Response of Employment?

**Proposition**

*Employment is proportional to the household’s value of output:*

\[ L_{jt} \propto \sum_{k=1}^{N} \ell_{jk} \frac{p_{kt}C_{kt}}{C_t} + \sum_{k=1}^{N} \ell_{jk} \sum_{m=1}^{N} \lambda_{km} \frac{p_{mt}I_{mt}}{C_t} \]

where \( p_{mt} = \prod_{k=1}^{N} \left( \frac{p_{kt}}{\lambda_{km}} \right)^{\lambda_{km}} \) is the investment price index and \([\mathcal{L}]_{jk} = \ell_{jk}\) is the Leontief inverse.

- Household’s valuation of consumption constant due to Cobb-Douglas preferences \( \sum_{k=1}^{N} \ell_{jk} \frac{p_{kt}C_{kt}}{C_t} = \sum_{k=1}^{N} \ell_{jk} \xi_k \)
- “Consumption supply shocks” are neutral w.r.t. employment!
- Shock increases \( MRPL_{kt} \) and \( C_t \) by same proportion
  \( \implies \) generate offsetting income and substitution effects
What Determines the Response of Employment?

**Proposition**

*Employment is proportional to the household's value of output:*

\[
L_{jt} \propto \sum_{k=1}^{N} \ell_{jk} \frac{p_{kt}C_{kt}}{C_t} + \sum_{k=1}^{N} \ell_{jk} \sum_{m=1}^{N} \lambda_{km} \frac{p^I_{mt}l_{mt}}{C_t}
\]

where \( p^I_{mt} = \prod_{k=1}^{N} \left( \frac{p_{kt}}{\lambda_{km}} \right) \lambda_{km} \) is the investment price index and \( [\mathcal{L}]_{jk} = \ell_{jk} \) is the Leontief inverse.

- **Derivation**
- **Real GDP**
- **C-D accumulation**

- Household’s valuation of investment fluctuates because capital accumulation technology not Cobb-Douglas:

\[
K_{jt+1} = (1 - \delta_j)K_{jt} + I_{jt}
\]

- “Investment supply shocks” only sources of employment fluctuations! (weaken income effect on labor supply)
- **Leontief-adjusted investment network**
Which Shocks Matter for Investment/Employment?

- For each sector $i$, simulate model in response to $\sigma(\log A_{it}) = 1\%$ (reduced-form elasticity of employment w.r.t. sectoral shock)
Which Shocks Matter for Investment/Employment?

• **Investment hub** shocks have the largest effect on aggregate employment
Which Shocks Matter for Investment/Employment?

- **Other important sectors** are important rows in Leontief-adjusted investment network (key suppliers to investment hubs)

  - Why?
  - Responses of Household’s Valuation of Investment
  - Supporting Evidence
Implications for Changing Business Cycles

\[
d \log Y_t = \sum_{j=1}^{N} \left( \frac{p_j Q_j}{P Y} \right)^* d \log A_{jt} + (1 - \bar{\alpha}^*) \sum_{j=1}^{N} \left( \frac{L_j}{L} \right)^* d \log L_{jt}
\]

- Pre-1980s sample dominated by aggregate shocks
- Post-1980s sample dominated by sector-specific shocks
  - Shocks to hubs/suppliers have biggest aggregate effects
  \[\implies\] primary source of fluctuations post-1980s
Changes in Business Cycles
Since 1984
Quantitative Exercise

• Procedure: feed in realized shocks from data, but hold other parameters fixed over time

• Measure realizations of sector-level TFP $A_{jt}$ as Solow residual, log-polynomially detrended

\[
\text{Var}(\Delta \log A_t) = \sum_{j=1}^{N} (\omega_{jt})^2 \text{Var}(\Delta \log A_{jt}) + \sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt} \omega_{ot} \text{Cov}(\Delta \log A_{jt}, \Delta \log A_{ot})
\]

<table>
<thead>
<tr>
<th>Measured TFP</th>
<th>Value Added</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-84</td>
<td>Post-84</td>
</tr>
<tr>
<td>1000\text{Var}(\Delta \log A_t)</td>
<td>0.41</td>
</tr>
<tr>
<td>Variances</td>
<td>0.08</td>
</tr>
<tr>
<td>Covariances</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Quantitative Exercise (cont’d)

• Helpful special case for interpretation: $\log A_{jt} = \log \hat{A}_t + \log \hat{A}_{jt}$
• Declining covariances $\implies$ aggregate shock less volatile
• Consistent with principal components analysis (also see Foerster et al. 2011) Detail
Quantitative Exercise (cont’d)

• Helpful special case for interpretation: $\log A_{jt} = \log \hat{A}_t + \log \hat{A}_{jt}$
  
• Declining covariances $\Rightarrow$ aggregate shock less volatile

• Consistent with principal components analysis (also see Foerster et al. 2011)

• To reduce capital reallocation, allow for imperfect substitution (Huffman-Wynne 1999):

$$Q_{jt} = C_{jt} + \sum_{i=1}^{N} M_{ijt} + \left( \sum_{i=1}^{N} I_{ijt}^{-\rho} \right)^{-\frac{1}{\rho}}$$

• $\rho \leq -1$ controls degree of imperfect substitution

• Set $\rho = -1.04$ to match degree of reallocation in data
Model Matches Aggregate Business Cycle Patterns

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-1984</td>
<td>Post-1984</td>
</tr>
<tr>
<td>( \sigma(\Delta y_t) )</td>
<td>3.18%</td>
<td>1.98%</td>
</tr>
<tr>
<td></td>
<td>Pre-1984</td>
<td>Post-1984</td>
</tr>
<tr>
<td>( \Delta y_t )</td>
<td>3.95%</td>
<td>2.42%</td>
</tr>
</tbody>
</table>

- Model generates decline in aggregate GDP volatility due to declining correlation of shocks
Model Matches Aggregate Business Cycle Patterns

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.18%</td>
<td>1.98%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.56</td>
<td>0.28</td>
</tr>
<tr>
<td>$\rho(y_t^{hp} - l_t^{hp}, y_t^{hp})$</td>
<td>0.52</td>
<td>0.14</td>
</tr>
</tbody>
</table>

- Model generates decline in aggregate GDP volatility due to declining correlation of shocks
- Model generates decline in cyclicality of labor productivity
Model Matches Aggregate Business Cycle Patterns

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th></th>
<th>Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.18%</td>
<td>1.98%</td>
<td>3.95%</td>
<td>2.42%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.56</td>
<td>0.28</td>
<td>0.52</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\rho(y_{t_{hp}} - l_{t_{hp}}, y_t)$</td>
<td>0.52</td>
<td>0.14</td>
<td>0.53</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td><strong>0.83</strong></td>
<td><strong>1.01</strong></td>
<td><strong>0.90</strong></td>
<td><strong>1.03</strong></td>
</tr>
</tbody>
</table>

- Model generates decline in aggregate GDP volatility due to declining correlation of shocks
- Model generates decline in cyclicality of labor productivity due to rise in relative employment volatility
## Model Matches Aggregate Business Cycle Patterns

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.18%</td>
<td>1.98%</td>
<td>3.95%</td>
<td>2.42%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.56</td>
<td>0.28</td>
<td>0.52</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\rho(y_{t_{hp}} - l_{t_{hp}}, y_{t_{hp}})$</td>
<td>0.52</td>
<td>0.14</td>
<td>0.53</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.83</td>
<td>1.01</td>
<td>0.90</td>
<td>1.03</td>
</tr>
<tr>
<td>$\sigma(\Delta i_t)/\sigma(\Delta y_t)$</td>
<td><strong>2.25</strong></td>
<td><strong>3.10</strong></td>
<td><strong>3.78</strong></td>
<td><strong>4.11</strong></td>
</tr>
</tbody>
</table>

- Model generates decline in aggregate GDP volatility due to declining correlation of shocks
- Model generates decline in cyclicality of labor productivity due to rise in relative employment volatility
- Model generates rise in relative investment volatility due to rising importance of hub + supplier shocks
Model Matches Aggregate Business Cycle Patterns

- Model matches timing of change in labor productivity cyclicality (14-year forward-looking rolling windows)
Role of Investment Network

<table>
<thead>
<tr>
<th>( \sigma(\Delta y_t) )</th>
<th>( \sigma(\Delta l_t) / \sigma(\Delta y_t) )</th>
<th>( \rho(\Delta y_t - \Delta l_t, \Delta y_t) )</th>
<th>( \sigma(\Delta y_t) )</th>
<th>( \sigma(\Delta l_t) / \sigma(\Delta y_t) )</th>
<th>( \rho(\Delta y_t - \Delta l_t, \Delta y_t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.95%</td>
<td>0.90</td>
<td>0.52</td>
<td>3.16%</td>
<td>0.88</td>
<td>0.59</td>
</tr>
<tr>
<td>2.42%</td>
<td>1.03</td>
<td>-0.01</td>
<td>1.72%</td>
<td>0.90</td>
<td>0.48</td>
</tr>
</tbody>
</table>

**Identity Network** sets investment network to \( \Lambda = I \)

\( \Rightarrow \) small decline in cyclicality of labor productivity

- Structural Change
- CES production/preferences
- Labor Reallocation Frictions
- Capital Frictions
- Maintenance Investment
- Detrending
- Shock size
- Shock Decomposition
Supporting Evidence
Changing Cycles Driven by Changing Comovement

<table>
<thead>
<tr>
<th></th>
<th>Aggregated</th>
<th></th>
<th>Within-Sector</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.56</td>
<td>0.28</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.83</td>
<td>1.01</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

**Model**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.52</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.90</td>
<td>1.03</td>
</tr>
</tbody>
</table>
Changing Cycles Driven by Changing Comovement

<table>
<thead>
<tr>
<th>Data</th>
<th>Aggregated</th>
<th></th>
<th>Within-Sector</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.56</td>
<td>0.28</td>
<td>0.69</td>
<td>0.67</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.83</td>
<td>1.01</td>
<td>0.76</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.52</td>
<td>-0.01</td>
<td>0.79</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.90</td>
<td>1.03</td>
<td>0.52</td>
<td>0.43</td>
</tr>
</tbody>
</table>

- Sector-level cycles stable within sector
  $\implies$ aggregate changes driven by comovement across sectors

Existing Explanations

Alternative Weights
Changing Cycles Driven by Changing Comovement

\[
\frac{\text{Var}(\Delta l_t)}{\text{Var}(\Delta y_t)} \approx \omega_t \frac{\sum_{j=1}^{N} (\omega^l_{jt})^2 \text{Var}(\Delta l_{jt})}{\sum_{j=1}^{N} (\omega^y_{jt})^2 \text{Var}(\Delta y_{jt})} + (1-\omega_t) \frac{\sum_{j=1}^{N} \sum_{o \neq i} \omega^l_{jt} \omega^l_{ot} \text{Cov}(\Delta l_{jt}, \Delta l_{ot})}{\sum_{j=1}^{N} \sum_{o \neq i} \omega^y_{jt} \omega^y_{ot} \text{Cov}(\Delta y_{jt}, \Delta y_{ot})}
\]

- Variance weight
- Variances
- Covariances
## Changing Cycles Driven by Changing Comovement

\[
\frac{\text{Var}(\Delta l_t)}{\text{Var}(\Delta y_t)} \approx \omega_t \left( \frac{\sum_{j=1}^{N}(\omega_{jt}^l)^2 \text{Var}(\Delta l_{jt})}{\sum_{j=1}^{N}(\omega_{jt}^y)^2 \text{Var}(\Delta y_{jt})} \right) + (1 - \omega_t) \left( \frac{\sum_{j=1}^{N} \sum_{o \neq i} \omega_{jt}^l \omega_{ot}^l \text{Cov}(\Delta l_{jt}, \Delta l_{ot})}{\sum_{j=1}^{N} \sum_{o \neq i} \omega_{jt}^y \omega_{ot}^y \text{Cov}(\Delta y_{jt}, \Delta y_{ot})} \right)
\]

### Variances

<table>
<thead>
<tr>
<th></th>
<th>Pre-84</th>
<th>Post-84</th>
<th>Cont.</th>
<th>Pre-84</th>
<th>Post-84</th>
<th>Cont.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\text{Var}(l_t)}{\text{Var}(y_t)})</td>
<td>0.68</td>
<td>1.04</td>
<td><strong>100%</strong></td>
<td>0.81</td>
<td>1.05</td>
<td><strong>100%</strong></td>
</tr>
<tr>
<td>Variances</td>
<td>0.41</td>
<td>0.48</td>
<td>15%</td>
<td>0.75</td>
<td>0.57</td>
<td>10%</td>
</tr>
<tr>
<td>Covariances</td>
<td>0.72</td>
<td>1.19</td>
<td>85%</td>
<td>0.82</td>
<td>1.15</td>
<td>90%</td>
</tr>
<tr>
<td>Variance Weight</td>
<td>0.12</td>
<td>0.21</td>
<td></td>
<td>0.10</td>
<td>0.17</td>
<td></td>
</tr>
</tbody>
</table>

\(\omega_t = \sum_{j=1}^{N}(\omega_{jt}^y)^2 \text{Var}(y_{jt}) / \text{Var}(y_t)\)

- Comovement of value added falls \(\implies\) aggregate volatility falls
- Comovement of employment stable \(\implies\) agg. volatility stable

### Derivation

- Decomposition accuracy
- Comovement
- NBER-CES
- Model Fit
Conclusion
Our contributions

**Investment network important propagation mechanism**
for business cycle analysis
Our contributions

**Investment network important propagation mechanism**
for business cycle analysis

1. Empirical investment network *dominated by investment hubs* which are highly cyclical, especially post-1984

2. Standard multisector business cycle model implies that *hub + supplier shocks* generate large changes in employment

3. Quantitatively, these shocks account for rising share of fluctuations and *explain declining cyclicality of labor productivity*
Our contributions

Investment network important propagation mechanism for business cycle analysis

1. Empirical investment network dominated by investment hubs which are highly cyclical, especially post-1984

2. Standard multisector business cycle model implies that hub + supplier shocks generate large changes in employment

3. Quantitatively, these shocks account for rising share of fluctuations and explain declining cyclicality of labor productivity
Appendix
Construction of the Data Set

1. **Value added, gross output, and intermediates** from BEA industry database, 1947 - 2018 (37 NAICS sector level)

2. **Investment** and capital stocks from BEA fixed asset tables aggregated to sector level using shares of capital types, 1947 - 2018 (37 NAICS sector level)

3. **Employment** from two sources, harmonized using Fort-Klimek (2016) crosswalk
   - 1997-2018: all sectors in BEA industry database
   - 1948-1997, non-manufacturing: BEA industry database
   - 1948-1997, manufacturing: historical supplements + Fort-Klimek crosswalk
Measurement of Investment Network

- All **non-residential structures** produced by construction, except for mining (following BEA practice)

- **Intellectual property** also follows BEA practice (McGrattan 2020)
  - Pre-packed software and part of artistic originals from info
  - Other software and R&D investment from prof/technical
  - Adjust for margin payments (wholesale trade, retail trade, transportation and warehousing)

- All **residential investment** purchased by real estate
  - 1997 - 2018: observe production
  - Before 1997: impute production as share of aggregate

- **Equipment production** combines three BEA cases
  - 1997 - 2017: BEA provides bridge file
  - 1987 + 1992: BEA provides SIC bridge, use Fort-Klimek
  - Remaining years: extrapolate based on observed bridge files and total production
Changes in Investment Network Over Time

Weighted Outdegree of Investment Hubs

- Construction
- Machinery
- Vehicles
- Prof/Tech

Year: 1940 to 2020
Changes in Investment Network Over Time
Concentration of Investment Network

Intermediates Network

Empirical Investment Network

<table>
<thead>
<tr>
<th></th>
<th>Eigenvalue Centrality</th>
<th>Weighted Outdegree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment net.</td>
<td>3.22</td>
<td>2.57</td>
</tr>
<tr>
<td>Intermediates net.</td>
<td>1.42</td>
<td>0.68</td>
</tr>
</tbody>
</table>
Investment Hubs are Highly Cyclical

<table>
<thead>
<tr>
<th></th>
<th>Investment Hubs</th>
<th>Non-Hubs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_{st})$</td>
<td>9.13%</td>
<td>9.18%</td>
</tr>
<tr>
<td>$\sigma(\Delta l_{st})$</td>
<td>6.14%</td>
<td>4.83%</td>
</tr>
<tr>
<td>$\sigma(y_{st}^{hp})$</td>
<td>5.64%</td>
<td>6.29%</td>
</tr>
<tr>
<td>$\sigma(l_{st}^{hp})$</td>
<td>4.08%</td>
<td>3.21%</td>
</tr>
</tbody>
</table>

- $y_{st} = \log$ of real value added
- $l_{st} = \log$ of employment
- $\Delta = $ first differences
- $^{hp} = $ HP-filtered w/ smoothing $\lambda = 6.25$
- Statistics are averaged across sectors unweighted
Investment Hubs are Highly Cyclical

- $y_{st} = \log$ of real value added
- $l_t = \log$ of aggregate employment
- $\Delta = \text{first differences}$
Investment Hubs are More Cyclical than Other Manufacturing Sectors

<table>
<thead>
<tr>
<th></th>
<th>Investment Hubs</th>
<th></th>
<th>Non-Hubs</th>
<th></th>
<th>Non-Hub Manuf.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-84</td>
<td>Post-84</td>
<td>Pre-84</td>
<td>Post-84</td>
<td>Pre-84</td>
</tr>
<tr>
<td>$\sigma(\Delta y_{st})$</td>
<td>9.13%</td>
<td>9.18%</td>
<td>6.63%</td>
<td>5.51%</td>
<td>9.14%</td>
</tr>
<tr>
<td>$\sigma(\Delta l_{st})$</td>
<td>6.14%</td>
<td>4.83%</td>
<td>3.81%</td>
<td>3.14%</td>
<td>5.12%</td>
</tr>
</tbody>
</table>

- $y_{st} =$ log of real value added
- $l_{st} =$ log of employment
- $\Delta =$ first differences
- Statistics are averaged across sectors unweighted
Investment Hubs are More Cyclical than Other Manufacturing Sectors

\[ y_{st} = \log \text{ of real value added} \]
\[ l_t = \log \text{ of aggregate employment} \]
\[ \Delta = \text{first differences} \]
• $y_{st} = \log$ of real value added
• $y_t = \log$ of real GDP
• $\Delta = \text{first differences}$
Calibration of Production Parameters

\[ Q_{jt} = A_{jt} \left( K_{jt}^{\alpha_j} L_{jt}^{1-\alpha_j} \right)^{\theta_j} M_{jt}^{1-\theta_j} \]

where \( X_{jt} = \prod_{i=1}^{N} M_{ijt}^{\gamma_{ij}} \)

1. **Primary inputs shares** \( \theta \): average value added as share of gross output (BEA I-O database 1947 - 2018)
Calibration of Production Parameters

\[ Q_{jt} = A_{jt} (K_{jt}^{\alpha_j} L_{jt}^{1-\alpha_j})^{\theta_j} M_{jt}^{1-\theta_j} \]

where \( X_{jt} = \prod_{i=1}^{N} M_{ijt}^{\gamma_{ij}} \)

1. **Primary inputs shares** \( \theta \)
2. **Labor shares** \( \alpha \): average labor compensation as share of total costs adjusted for self-employment

(BEA I-O database extended back to 1947 - 2018)
Calibration of Production Parameters

\[ Q_{jt} = A_{jt}(K_{jt}^{\alpha_j} L_{jt}^{1-\alpha_j})^{\theta_j} X_{jt}^{1-\theta_j} \quad \text{where} \quad X_{jt} = \prod_{i=1}^{N} M_{ijt} \]

1. **Primary inputs shares** $\theta$
2. **Labor shares** $\alpha$
3. **Intermediates input-output network** $\Gamma$: average intermediates cost from $i$ as share of total intermediates costs for $j$ (BEA I-O database 1947-2018)
Calibration of Investment Parameters

\[ K_{jt+1} = (1 - \delta_j)K_{jt} + l_{jt} \quad \text{where} \quad l_{jt} = \prod_{i=1}^{N} \lambda_{ij} \]

1. **Depreciation rate** \( \delta_j \): average annual depreciation (BEA fixed assets 1947 - 2017)
Calibration of Investment Parameters

\[ K_{jt+1} = (1 - \delta_j)K_{jt} + l_{jt} \quad \text{where} \quad l_{jt} = \prod_{i=1}^{N} \lambda_{ij}^{ijt} \]

1. **Depreciation rate** \( \delta_j \)
2. **Investment input-output network** \( \Lambda \): already constructed
Calibration of Preference Parameters

\[ E_0 \sum_{t=0}^{\infty} \beta^t (\log C_t - \chi L_t) , \quad \text{where} \quad C_t = \Pi_{j=1}^{N} C_{jt} \quad \text{and} \quad \sum_{j=1}^{N} \xi_j = 1 \]

1. **Discount factor** \( \beta = 0.96 \) (annual)
Calibration of Preference Parameters

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\log C_t - \lambda L_t), \quad \text{where } C_t = \prod_{j=1}^{N} C_{jt} \text{ and } \sum_{j=1}^{N} \xi_j = 1
\]

1. **Discount factor** \( \beta = 0.96 \) (annual)

2. **Consumption shares** \( \xi_j \): average consumption expenditure on \( j \) as share of total consumption expenditure
   (BEA I-O database 1947 - 2018)
Detrending Sector-Level Data

- Sector-level data is not well-described by linear trend
- Choose log-polynomial trend with order = 4 in order to balance:
  1. Flexibility of the trend ( → higher order)
  2. Overfitting of the data ( → lower order)
Measured Intermediates Shares

Value added shares $\theta_j$
Measured Labor Shares

Labor shares 1 – \( \alpha_j \)
Measured Depreciation Rates

Depreciation rates $\delta_j$

- Mining
- Util Const
- Wood
- Minerals
- Primary Metals
- Fabricated Metals
- Machinery
- Computers
- Electrical
- Vehicles
- Transport
- Furniture
- Misc Mfg
- Food
- Textile
- Apparel
- Paper
- Printing
- Petroleum
- Chemical
- Plastics
- WT
- RT
- TW
- Info
- Real estate
- Prof/Tech
- Mgmt
- Admin
- Educ
- Health
- Arts
- Accom
- Services
- Other

Values:
- 0
- 0.02
- 0.04
- 0.06
- 0.08
- 0.1
- 0.12
- 0.14
- 0.16
Persistence of TFP $\rho_j$
Helpful special case to interpret change in shock process:

\[ \log A_{jt} = \log \hat{A}_t + \log \hat{A}_{jt} \]

aggregate shock

sector-specific shock

Characterize using principal components analysis:

<table>
<thead>
<tr>
<th>Sample period</th>
<th>1000(\text{Var}(\Delta \log A_t))</th>
<th>Due to 1st component</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949-1983</td>
<td>0.41</td>
<td>0.31 (75%)</td>
<td>0.10 (25%)</td>
</tr>
<tr>
<td>1984-2018</td>
<td>0.09</td>
<td>0.03 (35%)</td>
<td>0.06 (65%)</td>
</tr>
</tbody>
</table>

Volatility of aggregate factor falls in half, but volatility of idiosyncratic factor stable
Proposition

Up to first order, the impact effect of a sector-specific shock $A_{it}$ on real GDP $Y_t$ is given by

$$d \log Y_t = \sum_{j=1}^{N} \left( \frac{p_j Q_j}{P^Y Y} \right)^* d \log A_{jt} + (1 - \bar{\alpha}^*) \sum_{j=1}^{N} \left( \frac{L_j}{L} \right)^* d \log L_{jt}$$

where $1 - \bar{\alpha}^* = \left( \frac{WL}{P^Y Y} \right)^*$ is the steady state labor share.
Proposition

Up to first order, the impact effect of a sector-specific shock $A_{it}$ on real GDP $Y_t$ is given by

$$d \log Y_t = \sum_{j=1}^{N} \left( \frac{p_j Q_j}{P^Y Y} \right)^* d \log A_{jt} + (1 - \bar{\alpha}^*) \sum_{j=1}^{N} \left( \frac{L_j}{L} \right)^* d \log L_{jt}$$

where $1 - \bar{\alpha}^* = \left( \frac{WL}{P^Y Y} \right)^*$ is the steady state labor share.

- Effect on $d \log TFP_t$ given by steady state Domar weight $\left( \frac{p_j Q_j}{P^Y Y} \right)^*$

- **Hulten’s theorem**: due to Cobb-Douglas technology and competitive + frictionless markets

- Effect on $d \log L_t$ is endogenous and depends on $\{d \log L_{jt}\}_{j=1}^{N}$

Derivation
Aggregating Sector-Level Value Added

- Straightforward to compute aggregate employment $L_t = \sum_{j=1}^{N} L_{jt}$, but real GDP difficult due to changes in relative prices

- Compute real GDP using a **Divisia index**
  1. Begin with definition of nominal GDP is $P_t^Y Y_t = \sum_{j=1}^{N} p_{jt}^Y Y_{jt}$
  2. Then compute growth rate, holding prices fixed

$$d \log Y_t = \sum_{j=1}^{N} \left( \frac{p_{jt}^Y Y_{jt}}{P_t^Y Y_t} \right) d \log Y_{jt}$$

- Sector-level value added depends only on TFP and primary inputs:

$$d \log Y_{jt} = \frac{1}{\theta_j} d \log A_{jt} + \alpha_j d \log K_{jt} + (1 - \alpha_j) d \log L_{jt}$$
Derivation of Employment Allocation

- Equilibrium is efficient, so allocation of employment satisfies
  \[ \alpha_j \theta_j \frac{p_{jt} Q_{jt}}{L_{jt}} = C_t \quad \implies \quad L_{jt} = \alpha_j \theta_j \frac{p_{jt} Q_{jt}}{C_t} \]

- Focus on characterizing the household’s value of output \( \frac{p_{jt} Q_{jt}}{C_t} \)
Derivation of Employment Allocation

- Equilibrium is efficient, so allocation of employment satisfies
  \[ \alpha_j \theta_j \frac{p_{jt} Q_{jt}}{L_{jt}} = C_t \quad \implies \quad L_{jt} = \alpha_j \theta_j \frac{p_{jt} Q_{jt}}{C_t} \]

- Focus on characterizing the household’s value of output \( \frac{p_{jt} Q_{jt}}{C_t} \)

- Define the **input-output matrix**
  \[ \Gamma = \begin{bmatrix}
  \gamma_{11}(1 - \theta_1) & \cdots & \gamma_{1N}(1 - \theta_N) \\
  \vdots & \ddots & \vdots \\
  \gamma_{N1}(1 - \theta_1) & \cdots & \gamma_{NN}(1 - \theta_N)
  \end{bmatrix} \]

  - Entry \((i,j) = j’s expenditure share on intermediates from i\)
  - Cobb-Douglas technology \( \implies \) expenditure shares constant

- **Leontief inverse** \( \mathcal{L} = (I - \Gamma)^{-1} = I + \Gamma + (\Gamma)^2 + \ldots, [\mathcal{L}]_{ij} = \ell_{ij} \)
  - Captures importance of \( i \) as supplier to \( j \) directly + indirectly
• Market clearing condition for gross output multiplied by price:

\[ P_{jt}Q_{jt} = P_{jt}C_{jt} + \sum_{i=1}^{N} P_{jt}M_{jit} + \sum_{i=1}^{N} P_{jt}l_{jit} \]

• Use Cobb-Douglas demand functions for consumption + inputs:

\[ P_{jt}Q_{jt} = \xi_j C_t + \sum_{i=1}^{N} (1 - \theta_i) \gamma_{ji} P_{it}Q_{it} + \sum_{i=1}^{N} \lambda_{ji} P_{it}^l l_{it} \]

• Divide by total consumption expenditure and solve system of equations:

\[ \frac{P_{jt}Q_{jt}}{C_t} = \sum_{k=1}^{N} \ell_{jk} \xi_k + \sum_{k=1}^{N} \ell_{jk} \left( \sum_{m=1}^{N} \frac{P_{mt}^l l_{mt}}{C_t} \right) \]
Proposition
(Rossi-Hansberg and Wright 2007) Consider the version of our model in which $K_{jt+1} = K_{jt}^{1-\delta_j} l_{jt}^{\delta_j}$. Then the household’s valuation of investment $\frac{p_{mt}^l m_{mt}}{C_t}$ is constant, so employment $L_{jt}$ is constant over time.

- C-D technology: investment expenditure $\propto$ total income
  C-D preferences: consumption expenditure $\propto$ total income
  $\implies$ investment expenditure proportional to total consumption

- Standard technology increases elasticity of capital w.r.t investment, breaking proportionality

- NB: full depreciation $\delta_j = 1$ special case of C-D technology
Proposition

Fluctuations in sector-level employment $L_{jt}$ are given by, up to first order,

$$d \log L_{jt} = \sum_{m=1}^{N} \omega_{jm} \left( \frac{P_m^{l} l_m}{P_j Q_j} \right) \ast d \log \left( \frac{p_m^{l} l_m}{C_t} \right), \quad \text{where } \omega_{jm} = \sum_{k=1}^{N} \ell_{jk} \lambda_{km}. $$
Proposition

Fluctuations in sector-level employment $L_{jt}$ are given by, up to first order,

$$d \log L_{jt} = \sum_{m=1}^{N} \omega_{jm} \left( \frac{P^l_m I_m}{P^l_j Q_j} \right) \ast d \log \left( \frac{p^l_{mt} I_{mt}}{C_t} \right),$$

where $\omega_{jm} = \sum_{k=1}^{N} \ell_{jk} \lambda_{km}$.

- Role of sector $j$ in supplying investment goods determined by 

**Leontief-adjusted investment network** $\Omega = \mathcal{L} \Lambda$
The Leontief-Adjusted Investment Network

Proposition

Fluctuations in sector-level employment $L_{jt}$ are given by, up to first order,

$$d\log L_{jt} = \sum_{m=1}^{N} \omega_{jm} \left( \frac{P_{m}^{l}}{P_{j}Q_{j}} \right)^{\ast} d\log \left( \frac{p_{mt}^{l}}{C_{t}} \right),$$

where $\omega_{jm} = \sum_{k=1}^{N} \ell_{jk} \lambda_{km}$.

- Role of sector $j$ in supplying investment goods determined by Leontief-adjusted investment network $\Omega = \mathcal{L} \Lambda$.

- Given fluctuations in $d\log \left( \frac{p_{mt}^{l}}{C_{t}} \right)$, Leontief-adjusted network $\Omega$ determines response of $L_{jt}$. 
Proposition

Fluctuations in sector-level employment $L_{jt}$ are given by, up to first order,

$$d \log L_{jt} = \sum_{m=1}^{N} \omega_{jm} \left( \frac{P_{m}^{l} l_{m}}{P_{j} Q_{j}} \right)^{*} d \log \left( \frac{p_{mt}^{l} l_{mt}}{C_{t}} \right),$$

where

$$\omega_{jm} = \sum_{k=1}^{N} \ell_{jk} \lambda_{km}.$$ 

• Role of sector $j$ in supplying investment goods determined by Leontief-adjusted investment network $\Omega = \mathcal{L}\Lambda$.

• Given fluctuations in $d \log \left( \frac{p_{mt}^{l} l_{mt}}{C_{t}} \right)$, Leontief-adjusted network $\Omega$ determines response of $L_{jt}$.

• Will now show that Leontief-adjusted network $\Omega$ also determines which shocks generate fluctuations in $d \log \left( \frac{p_{mt}^{l} l_{mt}}{C_{t}} \right)$.
Relationship to Networks Literature

- Primarily studies static models without investment
- Our model without investment $\implies$ employment is constant, so:
  1. Hulten’s holds for real GDP: \[ d \log Y_t = \sum_{j=1}^{N} \left( \frac{p_j Q_j}{P Y Y} \right) \ast d \log A_{jt} \]
  2. Hulten’s globally true (Domar weights constant)
- Networks literature breaks Hulten’s theorem with deviations from
  - Cobb-Douglas production (e.g. Baqae-Farhi 2019)
  - Competitive + frictionless markets (e.g. Baqae-Farhi 2020)
- Our paper: investment breaks strong Hulten’s theorem as well
  1. Employment and GDP also depend on Leontief-adjusted investment network $\Omega$
  2. Domar weights fluctuate reflecting changes in household’s valuation of investment (relevant for higher-order)
Which Shocks Matter for Investment/Employment?

\[
\frac{p_{mt}^l}{C_t} = \beta \mathbb{E}_t \left[ \alpha_j \theta_j \frac{p_{m,t+1}}{C_{t+1}} \frac{Q_{m,t+1}}{K_{m,t+1}} + (1 - \delta) \frac{p_{m,t+1}^l}{C_{t+1}} \right]
\]

MC

MB = MRPK + value of undepreciated capital
Which Shocks Matter for Investment/Employment?

\[
\frac{p_{mt}}{C_t} = \mathbb{E}_t \left[ \alpha_j \theta_j \frac{p_{m,t+1}}{C_{t+1}} \frac{Q_{m,t+1}}{K_{m,t+1}} + (1 - \delta) \frac{p_{m,t+1}'}{C_{t+1}} \right]
\]

\[\text{MC} = \text{MRPK} + \text{value of undepreciated capital}\]

- Shock to hub/supplier ≈ aggregate investment supply shock

- Direct effect on price, holding primary input prices fixed:
  \[d \log p_{mt} = - \sum_{j=1}^{N} \omega_{jm} d \log A_{jt}\]
  - Hubs/suppliers have high \(\omega_{jm}\) for many \(m\)
Which Shocks Matter for Investment/Employment?

\[
\frac{p^l_{mt}}{C_t} = \beta E_t \left[ \alpha_j \theta_j \frac{p_{m,t+1}}{C_{t+1}} \frac{Q_{m,t+1}}{K_{m,t+1}} + (1 - \delta) \frac{p^l_{m,t+1}}{C_{t+1}} \right] \]

\[MB = \text{MRPK + value of undepreciated capital}\]

- **Shock to hub/supplier** \(\approx\) **aggregate investment supply shock**
  - Direct effect on price, holding primary input prices fixed:
    \[
d \log p^l_{mt} = - \sum_{j=1}^{N} \omega_{jm} d \log A_{jt}\]
    - Hubs/suppliers have high \(\omega_{jm}\) for many \(m\)
  - Shock to other sectors \(\approx\) **idiosyncratic investment demand shock**
    - Primarily increases MRPK in own sector, but small effect on other sectors
Responses of $\frac{P_t^l l_t}{C_t}$ similar to responses of employment $N_t$

Heterogeneity driven by Leontief-adjusted investment network

- Heterogeneity in depreciation $\delta_j$ and capital shares $\alpha_j$ also matter for construction
## Supporting Evidence for Role of Key Suppliers

### Data

<table>
<thead>
<tr>
<th>Data</th>
<th>Investment Hubs</th>
<th>Suppliers</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-84</td>
<td>Post-84</td>
<td>Pre-84</td>
</tr>
<tr>
<td>$\sigma(\Delta y_{st})$</td>
<td>9.13%</td>
<td>9.18%</td>
<td>8.03%</td>
</tr>
<tr>
<td>$\sigma(\Delta l_{st})$</td>
<td>6.14%</td>
<td>4.83%</td>
<td>6.04%</td>
</tr>
</tbody>
</table>

### Model

<table>
<thead>
<tr>
<th>Data</th>
<th>Investment Hubs</th>
<th>Suppliers</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-84</td>
<td>Post-84</td>
<td>Pre-84</td>
</tr>
<tr>
<td>$\sigma(\Delta y_{st})$</td>
<td>12.92%</td>
<td>9.63%</td>
<td>9.02%</td>
</tr>
<tr>
<td>$\sigma(\Delta l_{st})$</td>
<td>9.37%</td>
<td>6.65%</td>
<td>5.93%</td>
</tr>
</tbody>
</table>

- $y_{st} = \log$ of real value added
- $l_{st} = \log$ of employment
- $\Delta = \text{first differences}$
- Statistics are averaged across sectors unweighted
• $y_{st} = \log$ of real value added
• $l_t = \log$ of aggregate employment
• $\Delta = \text{first differences}$
• Existing literature on effects of **investment-specific technology shocks** (e.g. Greenwood et al. 2000, Justiniano et al. 2010)
  • Like our model without intermediates network \( \Rightarrow \quad \Omega \approx \Lambda \)
  \[
  d \log L_{jt} = \sum_{m=1}^{N} \lambda_{jm} \left( \frac{P_{m} l_{m}}{P_{j} Q_{j}} \right)^{*} \left( d \log P'_{mt} l_{mt} - d \log C_{t} \right)
  \]

• **Comovement problem** between consumption and investment
  • Inconsistent with empirical degree of comovement \( \Rightarrow \) smaller agg effects of investment-specific shocks
  • Typical solution: strong nominal or real rigidities
  • Our model generates comovement through intermediates network (Hornstein and Praschnik 1997)

• Debate about **measurement of investment-specific shocks**
  • Our model: TFP shocks to hubs + key suppliers
Labor Productivity in Response to Sectoral Shocks

- The impact effect of shock on aggregate labor productivity is
  \[ d \log LP_t = d \log TFP_t - \alpha^* d \log L_t \]
- Effect on \( d \log TFP_t \) determined by Domar weight \( \frac{p_j Q_j}{P Y Y} \)
- Effect on \( d \log L_t \) depends on role in Leontief-adjusted investment network \( \Omega \)
Labor Productivity in Response to Sectoral Shocks

- The impact effect of shock on aggregate labor productivity is
  \[ d \log LP_t = d \log TFP_t - \alpha^* d \log L_t \]

- Effect on \( d \log TFP_t \) determined by Domar weight \( \frac{p_jQ_j}{PY_j} \)
- Effect on \( d \log L_t \) depends on role in Leontief-adjusted investment network \( \Omega \)
Capital Reallocation Frictions

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1000 \times \mathbb{E}[\Delta</td>
<td>\frac{p^l_{jt}}{\sum_{k=1}^{N} p^l_{kt}}</td>
<td>]$</td>
<td>2.0%</td>
</tr>
<tr>
<td>$1000 \times \sigma \left(\frac{p^l_{jt}}{\sum_{k=1}^{N} p^l_{kt}}\right)$</td>
<td>2.7%</td>
<td>14.1%</td>
<td>2.7%</td>
</tr>
</tbody>
</table>

• Baseline model generates excessive volatility in the composition of investment purchases across sectors

• Extend model to allow for imperfect substitution (Huffman-Wynne 1999):

$$Q_{jt} = C_{jt} + \sum_{i=1}^{N} M_{ijt} + \left(\sum_{i=1}^{N} l_{ijt}^{-\rho}\right)^{-\frac{1}{\rho}}$$

• $\rho \leq -1$ controls degree of imperfect substitution
• Set $\rho = -1.04$ to match degree of reallocation in data
Postwar Time Series: First Differences
Postwar Time Series: HP Filtered

GDP

Employment

Investment

Consumption

% Deviation

Years

Years

Years

Years

Years
Structural Change: Transition Path Approach

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th></th>
<th>Changing Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.95%</td>
<td>2.42%</td>
<td>4.57%</td>
<td>2.10%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.52</td>
<td>-0.01</td>
<td>0.50</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.90</td>
<td>1.03</td>
<td>0.89</td>
<td>1.02</td>
</tr>
</tbody>
</table>

- **Changing parameters** also allows for changes in: production function $\alpha_{jt}$, $\theta_{jt}$, intermediates network $\gamma_{ijt}$, investment network $\lambda_{ijt}$, depreciation rates $\delta_{jt}$, and preferences $\xi_{jt}$
  - Compute fourth-order trend of these parameters
  - Assume perfect foresight over path using Maliar et al. (2015)

- Parameter changes also generate fluctuations, so increase investment production frictions to $\rho = -1.3$
### Structural Change: Simulation Approach

<table>
<thead>
<tr>
<th>Simulated Moments</th>
<th>Changing Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-1984</strong></td>
<td><strong>Post-1984</strong></td>
</tr>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.63%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.76</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.83</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)$</td>
<td>4.07%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.80</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.83</td>
</tr>
</tbody>
</table>

- **Simulated moments** from model with estimated covariance matrix of shocks pre-1984 vs. post-1984
  - Must collapse non-durable manufacturing to one sector (30 total)

- **Changing parameters** allows for different parameter values in pre-1984 vs. post-1984 subsample: production function $\alpha_j$, $\theta_j$, intermediates network $\gamma_{ij}$, investment network $\lambda_{ij}$, depreciation rates $\delta_j$, preferences $\xi_j$, and persistence $\rho_j$
CES Production and Preferences

- Extend model to incorporate CES production function

\[ Y_{jt} = \left( \theta_{jt}^{\frac{1}{\sigma_v}} V_{jt}^{\frac{1}{\sigma_v}} + (1 - \theta_j) \right) \left( \frac{1}{\sigma_v} M_{jt}^{\frac{1}{\sigma_v}} \right)^{\frac{\sigma_v}{\sigma_v - 1}}, \sigma_v = 0.8 \text{ (Oberfield-Raval 2020)} \]

\[ V_{jt} = A_{jt} \left( \alpha_j^{\frac{1}{\sigma_k}} K_{jt}^{\frac{1}{\sigma_k}} + (1 - \alpha_j) \right) \left( \frac{1}{\sigma_k} L_{jt}^{\frac{1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1}}, \sigma_k = 0.6 \text{ (Oberfield-Raval 2020)} \]

\[ M_{jt} = \left( \sum_{i=1}^{N} \gamma_{ij}^{\sigma_m} M_{jt}^{\sigma_m} \right)^{\frac{\sigma_m}{\sigma_m - 1}}, \sigma_m = 0.1 \text{ (Atalay 2017)} \]

- Also allow for CES preferences over the bundle

\[ C_t = \left( \sum_{j=1}^{N} \xi_j^{\sigma_c} C_{jt}^{\sigma_c} \right)^{\frac{\sigma_c}{\sigma_c - 1}}, \sigma_c = 0.75 \text{ (Oberfield-Raval 2020)} \]
<table>
<thead>
<tr>
<th></th>
<th>Cobb-Douglas</th>
<th></th>
<th>CES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.89%</td>
<td>2.79%</td>
<td>4.31%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.52</td>
<td>-0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.90</td>
<td>1.05</td>
<td>0.96</td>
</tr>
<tr>
<td>$\sigma(\Delta i_t)/\sigma(\Delta y_t)$</td>
<td>3.78</td>
<td>4.09</td>
<td>3.73</td>
</tr>
</tbody>
</table>

- CES structure raises overall level of employment volatility, but generates similar changes over time.
## CES Production and Preferences

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_c$ only</th>
<th></th>
<th>$\sigma_k$ only</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.87%</td>
<td>2.79%</td>
<td>4.43%</td>
<td>2.98%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.54</td>
<td>-0.14</td>
<td>0.24</td>
<td>-0.35</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.89</td>
<td>1.04</td>
<td>0.97</td>
<td>1.08</td>
</tr>
<tr>
<td>$\sigma(\Delta i_t)/\sigma(\Delta y_t)$</td>
<td>3.79</td>
<td>4.11</td>
<td>3.71</td>
<td>4.01</td>
</tr>
</tbody>
</table>

|                  | $\sigma_v$ only |         | $\sigma_m$ only |         |
|                  |                 |         |                 |         |
| $\sigma(\Delta y_t)$ | 3.85%    | 2.76%    | 3.86%    | 2.79%    |
| $\rho(\Delta y_t - \Delta l_t, \Delta y_t)$ | 0.50     | -0.20    | 0.55     | -0.12    |
| $\sigma(\Delta l_t)/\sigma(\Delta y_t)$ | 0.90     | 1.06     | 0.89     | 1.04     |
| $\sigma(\Delta i_t)/\sigma(\Delta y_t)$ | 3.81     | 4.13     | 3.77     | 4.14     |

- Higher volatility under CES primarily due to capital-labor complementarity
### CES Production and Preferences

<table>
<thead>
<tr>
<th></th>
<th>First Order CD</th>
<th>Second Order CD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.89%</td>
<td>2.79%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.52</td>
<td>-0.15</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.90</td>
<td>1.05</td>
</tr>
<tr>
<td>$\sigma(\Delta i_t)/\sigma(\Delta y_t)$</td>
<td>3.78</td>
<td>4.09</td>
</tr>
</tbody>
</table>

|                  | First Order CES | Second Order CES |
| $\sigma(\Delta y_t)$ | 4.31%          | 2.94%           | 4.38%          | 3.07%           |
| $\rho(\Delta y_t - \Delta l_t, \Delta y_t)$ | 0.30           | -0.38           | 0.29           | -0.43           |
| $\sigma(\Delta l_t)/\sigma(\Delta y_t)$ | 0.96           | 1.08            | 0.96           | 1.10            |
| $\sigma(\Delta i_t)/\sigma(\Delta y_t)$ | 3.73           | 4.17            | 3.98           | 4.52            |

- Nonlinearities fairly unimportant for these unconditional statistics (but may generate different conditional behavior)
Allowing for Labor Reallocation Frictions

### Baseline

<table>
<thead>
<tr>
<th></th>
<th>Pre-1984</th>
<th>Post-1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.95%</td>
<td>2.42%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.52</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.90</td>
<td>1.03</td>
</tr>
<tr>
<td>$\sigma(\Delta i_t)/\sigma(\Delta y_t)$</td>
<td>3.78</td>
<td>4.11</td>
</tr>
</tbody>
</table>

### Labor Reallocation Frictions

<table>
<thead>
<tr>
<th></th>
<th>Pre-1984</th>
<th>Post-1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.63%</td>
<td>2.21%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.71</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.83</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma(\Delta i_t)/\sigma(\Delta y_t)$</td>
<td>3.49</td>
<td>3.81</td>
</tr>
</tbody>
</table>

- Modify preferences to become

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \left( \sum_{j=1}^{N} L_{jt}^{\tau+1} \frac{\tau}{\tau+1} \right) \right) \right]$$

- $\tau$ controls substitutability of labor across sectors
- Set $\tau$ to match $\sigma(\Delta l_t)/\sigma(\Delta y_t)$ in pre-84 period
### Role of Investment Production Frictions

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>No Frictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(\Delta y_t) )</td>
<td>3.95%</td>
<td>2.42%</td>
</tr>
<tr>
<td>( \rho(\Delta y_t - \Delta l_t, \Delta y_t) )</td>
<td>0.52</td>
<td>-0.01</td>
</tr>
<tr>
<td>( \sigma(\Delta l_t)/\sigma(\Delta y_t) )</td>
<td>0.90</td>
<td>1.03</td>
</tr>
<tr>
<td>( \sigma(\Delta i_t)/\sigma(\Delta y_t) )</td>
<td>3.78</td>
<td>4.11</td>
</tr>
</tbody>
</table>

### Large Frictions

<table>
<thead>
<tr>
<th></th>
<th>( \sigma(\Delta y_t) )</th>
<th>( \rho(\Delta y_t - \Delta l_t, \Delta y_t) )</th>
<th>( \sigma(\Delta l_t)/\sigma(\Delta y_t) )</th>
<th>( \sigma(\Delta i_t)/\sigma(\Delta y_t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td>3.86%</td>
<td>0.57</td>
<td>0.88</td>
<td>3.74</td>
</tr>
<tr>
<td>Post-1984</td>
<td>2.38%</td>
<td>0.10</td>
<td>1.00</td>
<td>4.16</td>
</tr>
</tbody>
</table>

- **No frictions** assumes \( \rho = -1 \) as in theory section
- **Large frictions** assumes \( \rho = -1.5 \) (baseline \( \rho = -1.04 \))
### Allowing for Maintenance Investment

<table>
<thead>
<tr>
<th></th>
<th><strong>Baseline</strong></th>
<th></th>
<th><strong>Maintenance Investment</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><em>Pre-1984</em></td>
<td><em>Post-1984</em></td>
<td><em>Pre-1984</em></td>
<td><em>Post-1984</em></td>
</tr>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.95%</td>
<td>2.42%</td>
<td>3.49%</td>
<td>2.10%</td>
</tr>
<tr>
<td>$\rho(\Delta y_t − \Delta l_t, \Delta y_t)$</td>
<td>0.52</td>
<td>-0.01</td>
<td>0.70</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.90</td>
<td>1.03</td>
<td>0.82</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma(\Delta i_t)/\sigma(\Delta y_t)$</td>
<td>3.78</td>
<td>4.11</td>
<td>4.02</td>
<td>4.38</td>
</tr>
</tbody>
</table>

- McGrattan and Schmitz (1999) estimate maintenance $\approx 30\%$ as big as new investment, but no systematic estimates available.
- Open question: from which sectors is maintenance purchased?
  - One extreme is same mix as new investment.
  - Another is all from own-sector output.

- **Maintenance investment** assumes half of maintenance is done out of own-sector investment.
### Alternative Trends for Sector-Level TFP

#### Baseline (4th order)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(\Delta y_t) )</td>
<td>3.95%</td>
<td>2.42%</td>
<td>3.75%</td>
<td>2.30%</td>
</tr>
<tr>
<td>( \rho(\Delta y_t - \Delta l_t, \Delta y_t) )</td>
<td>0.52</td>
<td>-0.01</td>
<td>0.66</td>
<td>0.23</td>
</tr>
<tr>
<td>( \sigma(\Delta l_t)/\sigma(\Delta y_t) )</td>
<td>0.90</td>
<td>1.03</td>
<td>0.85</td>
<td>0.98</td>
</tr>
<tr>
<td>( \sigma(\Delta i_t)/\sigma(\Delta y_t) )</td>
<td>3.78</td>
<td>4.11</td>
<td>3.65</td>
<td>3.95</td>
</tr>
</tbody>
</table>

#### 5th order

<table>
<thead>
<tr>
<th></th>
<th>Pre-1984</th>
<th>Post-1984</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(\Delta y_t) )</td>
<td>3.88%</td>
<td>2.66%</td>
</tr>
<tr>
<td>( \rho(\Delta y_t - \Delta l_t, \Delta y_t) )</td>
<td>0.47</td>
<td>0.07</td>
</tr>
<tr>
<td>( \sigma(\Delta l_t)/\sigma(\Delta y_t) )</td>
<td>0.92</td>
<td>1.01</td>
</tr>
<tr>
<td>( \sigma(\Delta i_t)/\sigma(\Delta y_t) )</td>
<td>3.87</td>
<td>4.07</td>
</tr>
</tbody>
</table>

- Results robust to allowing for different detrending (third order similar)
Role of Changing Shock Size

<table>
<thead>
<tr>
<th>Full Model</th>
<th>Pre</th>
<th>Post</th>
<th>Uniform Var.</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>3.95%</td>
<td>2.42%</td>
<td>$\sigma(\Delta y_t)$</td>
<td>1.76%</td>
<td>1.29%</td>
</tr>
<tr>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.90</td>
<td>1.03</td>
<td>$\sigma(\Delta l_t)/\sigma(\Delta y_t)$</td>
<td>0.88</td>
<td>1.03</td>
</tr>
<tr>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.52</td>
<td>-0.01</td>
<td>$\rho(\Delta y_t - \Delta l_t, \Delta y_t)$</td>
<td>0.55</td>
<td>0.03</td>
</tr>
</tbody>
</table>

- **Uniform variances** standardizes shocks to have SD = 1% pre-1984 and post-1984 (only shock comovement changes, not size of shocks)
### Rising Importance of Sector-Specific Shocks

<table>
<thead>
<tr>
<th></th>
<th><strong>Agg. Shocks Only</strong></th>
<th><strong>Sectoral Shocks Only</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(\Delta y_t) )</td>
<td>3.46%</td>
<td>1.67%</td>
</tr>
<tr>
<td>( \sigma(\Delta l_t) )</td>
<td>2.74%</td>
<td>1.42%</td>
</tr>
<tr>
<td>( \rho(\Delta y_t - \Delta l_t, \Delta y_t) )</td>
<td>0.85</td>
<td>0.79</td>
</tr>
<tr>
<td>( \sigma(\Delta i_t)/\sigma(\Delta y_t) )</td>
<td>3.31</td>
<td>3.48</td>
</tr>
</tbody>
</table>

### All Shocks

<table>
<thead>
<tr>
<th></th>
<th><strong>Pre-1984</strong></th>
<th><strong>Post-1984</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(\Delta y_t) )</td>
<td>3.95%</td>
<td>2.42%</td>
</tr>
<tr>
<td>( \sigma(\Delta l_t) )</td>
<td>3.55%</td>
<td>2.48%</td>
</tr>
<tr>
<td>( \rho(\Delta y_t - \Delta l_t, \Delta y_t) )</td>
<td>0.52</td>
<td>-0.01</td>
</tr>
<tr>
<td>( \sigma(\Delta i_t)/\sigma(\Delta y_t) )</td>
<td>3.78</td>
<td>4.11</td>
</tr>
</tbody>
</table>

- Aggregate shock = first principal component (as in empirics)
Existing Explanations for Changing Business Cycles

1. **Changing shock process:**
   - Reallocation shocks become more important: Garin, Pries, and Sims (2018)

2. **More flexible labor markets:** Barnichon (2010), Gali-van Rens (2013)

3. **Selective hiring/firing:**
   - Streamline in recessions: Koenders-Rogerson (2005); Berger (2018)
   - Labor hoarding: Gali-Gambetti (2009); Bachmann (2012)

4. **Mismeasurement of inputs or outputs:**
   - Non-measured intangible investment is procyclical: McGrattan-Prescott (2007, 2012); McGrattan (2020)
Existing Explanations for Changing Business Cycles

1. **Changing shock process:**
   - Reallocation shocks become more important: Garin, Pries, and Sims (2018)

2. **More flexible labor markets:** Barnichon (2010), Gali-van Rens (2013)

3. **Selective hiring/firing:**
   - Streamline in recessions: Koenders-Rogerson (2005); Berger (2018)
   - Labor hoarding: Gali-Gambetti (2009); Bachmann (2012)

4. **Mismeasurement of inputs or outputs:**

Existing mechanisms abstract from sectoral heterogeneity, so cannot speak to empirical results.
Existing Explanations for Changing Business Cycles

1. **Changing shock process:**
   - Reallocation shocks become more important: Garin, Pries, and Sims (2018)

2. **More flexible labor markets:** Barnichon (2010), Gali-van Rens (2013)

3. **Selective hiring/firing:**
   - Streamline in recessions: Koenders-Rogerson (2005); Berger (2018)
   - Labor hoarding: Gali-Gambetti (2009); Bachmann (2012)

4. **Mismeasurement of inputs or outputs:**

Existing mechanisms abstract from sectoral heterogeneity, ➞ need new explanation for falling cyclicality of labor productivity
Within vs. Between Sector Cycles

<table>
<thead>
<tr>
<th></th>
<th>Fixed Weights</th>
<th></th>
<th>Unweighted</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(y_t - l_t, y_t)$</td>
<td>0.68</td>
<td>0.69</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>$\sigma(l_t)/\sigma(y_t)$</td>
<td>0.78</td>
<td>0.78</td>
<td>0.66</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Decomposition on Role of Comovement

\[ \text{Var}(x_t) = \sum_{j=1}^{N} (\omega_{jt}^X)^2 \text{Var}(x_{jt}) + \sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^X \omega_{ot}^X \text{Cov}(x_{jt}, x_{ot}) \]

- within-sector
- between-sector
Decomposition on Role of Comovement

\[
\text{Var}(x_t) = \sum_{j=1}^{N} (\omega_{jt}^x)^2 \text{Var}(x_{jt}) + \sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^x \omega_{ot}^x \text{Cov}(x_{jt}, x_{ot})
\]

\[
\text{Var}(y_t) = \sum_{j=1}^{N} (\omega_{jt}^y)^2 \text{Var}(y_{jt}) + \sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^y \omega_{ot}^y \text{Cov}(y_{jt}, y_{ot})
\]
Decomposition on Role of Comovement

\[
\frac{\text{Var}(x_t)}{\text{Var}(y_t)} = \frac{\sum_{j=1}^{N} (\omega_{jt}^x)^2 \text{Var}(x_{jt})}{\sum_{j=1}^{N} (\omega_{jt}^y)^2 \text{Var}(y_{jt}) + \sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^y \omega_{ot}^y \text{Cov}(y_{jt}, y_{ot})} 
+ \frac{\sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^x \omega_{ot}^x \text{Cov}(x_{jt}, x_{ot})}{\sum_{j=1}^{N} (\omega_{jt}^y)^2 \text{Var}(y_{jt}) + \sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^y \omega_{ot}^y \text{Cov}(y_{jt}, y_{ot})}
\]
Decomposition on Role of Comovement

\[
\frac{\text{Var}(x_t)}{\text{Var}(y_t)} = \frac{\sum_{j=1}^{N} (\omega_{jt}^y)^2 \text{Var}(y_{jt})}{\text{Var}(y_t)} \cdot \frac{\sum_{j=1}^{N} (\omega_{jt}^x)^2 \text{Var}(x_{jt})}{\sum_{j=1}^{N} (\omega_{jt}^y)^2 \text{Var}(y_{jt})}
\]

\[+ \frac{\sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^x \omega_{ot}^x \text{Cov}(x_{jt}, x_{ot})}{\sum_{j=1}^{N} (\omega_{jt}^y)^2 \text{Var}(y_{jt}) + \sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^y \omega_{ot}^y \text{Cov}(y_{jt}, y_{ot})}\]
\[
\frac{\text{Var}(x_t)}{\text{Var}(y_t)} = \frac{\sum_{j=1}^{N} (\omega_{jt}^y)^2 \text{Var}(y_{jt})}{\text{Var}(y_t)} \frac{\sum_{j=1}^{N} (\omega_{jt}^x)^2 \text{Var}(x_{jt})}{\text{Var}(y_jt)}
\]
\[
+ \frac{\sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^y \omega_{ot}^y \text{Cov}(y_{jt}, y_{ot})}{\text{Var}(y_t)} \frac{\sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^x \omega_{ot}^x \text{Cov}(x_{jt}, x_{ot})}{\sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^y \omega_{ot}^y \text{Cov}(y_{jt}, y_{ot})}
\]
Accuracy of Decomposition

\[
\frac{\text{Var}(l_t)}{\text{Var}(y_t)} \approx \underbrace{\omega_t \frac{\sum_{j=1}^{N} (\omega_j^l)^2 \text{Var}(l_{jt})}{\sum_{j=1}^{N} (\omega_j^y)^2 \text{Var}(y_{jt})}}_{\text{within weight \ (within-sector)}} + (1 - \omega_t) \underbrace{\left( \frac{\sum_{j=1}^{N} \sum_{o \neq j} \omega_j^l \omega_o^l \text{Cov}(l_{jt}, l_{ot})}{\sum_{j=1}^{N} \sum_{o \neq j} \omega_j^y \omega_o^y \text{Cov}(y_{jt}, y_{ot})} \right)}_{\text{between-sector}}
\]

<table>
<thead>
<tr>
<th></th>
<th>Pre-84</th>
<th>Post-84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual, variance</td>
<td>0.68</td>
<td>1.02</td>
</tr>
<tr>
<td>Approximation, variance</td>
<td>0.68</td>
<td>1.04</td>
</tr>
<tr>
<td>Actual, standard deviation</td>
<td>0.83</td>
<td>1.01</td>
</tr>
<tr>
<td>Approximation, standard deviation</td>
<td>0.83</td>
<td>1.02</td>
</tr>
</tbody>
</table>
Sectoral Comovement

\[
\rho^x_\tau \equiv \frac{\sum_{i=1}^N \sum_{j=i+1}^N \omega_i^x \omega_j^x \text{Corr}(x_{it}, x_{jt} | t \in \tau)}{\sum_{i=1}^N \sum_{j=i+1}^N \omega_i^x \omega_j^x}
\]

- \( x_{jt} \) is HP-filtered + logged variable of interest
- \( \omega_{i\tau}^x = \mathbb{E}[\frac{x_{jt}}{x_s}] \) are sectoral weights
- \( \tau \in \{\text{pre 1984, post 1984}\} \) is time period

<table>
<thead>
<tr>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Employment</strong></td>
<td><strong>Value added</strong></td>
</tr>
<tr>
<td>1951-1983</td>
<td>0.50</td>
</tr>
<tr>
<td>1984-2012</td>
<td>0.49</td>
</tr>
<tr>
<td>Difference</td>
<td>-0.01</td>
</tr>
</tbody>
</table>
Decomposition at 450 Sector Level
(NBER-CES Manufacturing Data)

\[
\frac{\text{Var}(l_t)}{\text{Var}(y_t)} \approx \frac{\sum_{j=1}^{N} (\omega_{jt})^2 \text{Var}(l_{jt})}{\sum_{j=1}^{N} (\omega_{jt})^2 \text{Var}(y_{jt})} + (1 - \omega_t) \frac{\sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^l \omega_{ot}^l \text{Cov}(l_{jt}, l_{ot})}{\sum_{j=1}^{N} \sum_{o \neq j} \omega_{jt}^y \omega_{ot}^y \text{Cov}(y_{jt}, y_{ot})}
\]

<table>
<thead>
<tr>
<th></th>
<th>Pre-84</th>
<th>Post-84</th>
<th>Contribution of entire term</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\text{Var}(l_t)}{\text{Var}(y_t)} )</td>
<td>0.40</td>
<td>0.57</td>
<td>100%</td>
</tr>
<tr>
<td>Within Sector</td>
<td>0.34</td>
<td>0.20</td>
<td>1.4%</td>
</tr>
<tr>
<td>Between Sector</td>
<td>0.37</td>
<td>0.60</td>
<td>98.6%</td>
</tr>
<tr>
<td>Within Weight</td>
<td>0.03</td>
<td>0.06</td>
<td></td>
</tr>
</tbody>
</table>

\( \omega_t = \sum_{j=1}^{N} (\omega_{jt})^2 \text{Var}(y_{jt})/\text{Var}(y_t) \)
Model Fit to Sector-Pair Level Changes

- Plot sector-pair level “diff-in-diff” $\Delta \text{Cov}(n_{jt}, n_{ot}) - \Delta \text{Cov}(y_{jt}, y_{ot})$
- Model’s $R^2 = 53\%$!
Implications for Investment Stimulus Policy

- **Stimulus policy** = shock $sub_t$ to cost of capital

$$\left(1 - sub_t\right) \times \nu_{jt}$$

- Reduced-form subsidy captures class of fiscal stimulus, e.g. investment tax credits or accelerated depreciation

- Assume financed from outside the economy

- Study effect of one-time $sub_t = 1\%$ subsidy
• Increases aggregate investment by 6.5% and employment by 2%
• Employment increase concentrated in hubs + key suppliers
• More uniform effect in model without investment network  
  \[\Rightarrow\]  network creates uneven effect on production/employment  
  • Despite investment *purchases* being equally subsidized
Easier to see uniformity looking at percentage changes