Aggregate Implications of Firm Heterogeneity:
A Nonparametric Analysis of Monopolistic Competition

Rodrigo Adao
Booth School of Business

Costas Arkolakis
Yale University

Sharat Ganapati
Georgetown University

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What are the aggregate implications of firm heterogeneity?

- The study of firm heterogeneity has transformed the trade field
  - Cornerstone observation: Correlation between firm attributes and trade performance
  - Emergence of workhorse monopolistic competition model of firm heterogeneity

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  - Parametric assumptions restrict aggregate predictions of the model
- This paper: Firm heterogeneity **without parametric restrictions**
  - Theoretically and empirically characterize role of firm heterogeneity for aggregate outcomes
  - Nonparametric counterfactuals and inversion of fundamentals, as well as semiparametric estimation

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5. **Quantification of Gains from trade/EU integration 2004-2014**
   - Larger gains for countries with a higher share of exporter firms

Related literature

- **Estimation of the workhorse model of firm heterogeneity**
  - Chaney '08, Arkolakis '10, Eaton Kortum Kramarz '11, Bas Mayer Thoenig '17, Fernandez et al '19
- **Welfare gains from trade debate**
  - Arkolakis, Costinot, Rodriguez-Clare '12, Costinot Rodriguez-Clare '14, Head Mayer Thoenig '14, Ossa '15, Melitz Redding '15
- **Nonparametric counterfactuals in trade**
  - Adao Costinot Donaldson '17, Barteleme, Lan, Levchenko '19
- **Nonlinear Elasticities**
  - Novy '13, Fajgelbaum Khandelwal '16, Lind Romondo '18, Kehoe Ruhl '13

Outline

- **Workhorse model of firm heterogeneity**
- Semiparametric gravity equations for firm exports
- Nonparametric counterfactuals and identification of fundamentals
- Semiparametric gravity estimation
- Empirical results
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Workhorse model of firm heterogeneity: Setup

- N locations (denote i the origin j the destination)
- Monopolistic competitive firms
  - Firms are unique world monopolists, each producing one variety ω
  - Linear production function and iceberg shipping. Fixed cost of selling to each market
- Consumers
  - CES Preferences

Firm Revenue and Cost

- Firm ω's demand is

$$R_{ij}(\omega) = \underbrace{\bar{b}_{ij} b_{ij}(\omega)}_{\text{Firm taste shifter}} \underbrace{(p_{ij}(\omega))^{1-\sigma}}_{\text{Firm price}} \left[E_j P_j^{\sigma-1}\right]$$

where E_j is spending and P_j is CES price index over available varieties, Ω_{ij}

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- The cost of firm ω from i to sell q units in j

$$C_{ij}(q,\omega) = \underbrace{\frac{\tau_{ij}(\omega)}{a_i(\omega)} \frac{\bar{\tau}_{ij}}{\bar{a}_i} w_i}_{\text{Firm variable cost in } j} q + \underbrace{f_{ij}(\omega) \bar{f}_{ij} w_i}_{\text{Firm fixed cost in } j}$$

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- **Previous literature** has used these wedges to match distribution of productivity, sales, and entry across firms and destinations

Firm-specific revenue and entry potentials

- In monopolistic competition with CES, constant markup. Revenue:

$$R_{ij}(\omega) = \underbrace{\left[b_{ij}(\omega) \left(\frac{\tau_{ij}(\omega)}{a_i(\omega)} \right)^{1-\sigma} \right]}_{\text{Revenue potential, } r_{ij}(\omega)} \underbrace{\left[\left(\frac{\sigma}{\sigma - 1} \frac{\bar{\tau}_{ij}}{\bar{a}_i} \right)^{1-\sigma} \bar{b}_{ij} \right]}_{\text{Bilateral shifter,} \bar{r}_{ij}} \left[\left(\frac{w_i}{P_j} \right)^{1-\sigma} E_j \right]$$

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- Firm ω of i enters j (i.e., $\omega \in \Omega_{ij}$) if, and only if, $\pi_{ij}(\omega) \geq 0$. So,

$$\underbrace{\frac{r_{ij}(\omega)}{f_{ij}(\omega)}}_{\text{Entry potential, } e_{ij}(\omega)} \geq \underbrace{\left[\frac{\sigma \bar{f}_{ij}}{\bar{r}_{ij}}\right]}_{\text{Bilateral entry shifter,} \bar{e}_{ij}} \left[\frac{w_i^{\sigma}}{P_j^{\sigma-1} E_j}\right]$$

General Equilibrium

- Firms hire $\bar{F}_i$ workers to independently draw $v_i(\omega) \equiv \{b_{ij}(\omega), \tau_{ij}(\omega), f_{ij}(\omega), a_i(\omega)\}_j$:

$$v_i(\omega) \sim G_i(v)$$

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- **Equilibrium:** $\{w_i, N_i, P_i, \{\Omega_{ij}\}_j\}_i$ satisfying **(i) CES demand, (ii) export decision**,
  - **iii) Free Entry:** N_i firms enter with an expected profit of zero,

$$w_i \bar{F}_i = \sum_j E\left[\max\{\pi_{ij}(\omega);\ 0\}\right]$$

  - **iv) Market Clearing:** from trade balance,

$$E_i = w_i \bar{L}_i = \sum_j \int R_{ij}(\omega) d\omega$$

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Distributions of revenue and entry potentials

- Without loss of generality, we can think of firms as

$$r_{ij}(\omega) \sim H^r_{ij}(r|e) \quad \text{and} \quad e_{ij}(\omega) \sim H^e_{ij}(e)$$

- **Assumption 1:** $H^e_{ij}(e)$ is continuous and strictly increasing in $\mathbb{R}_+$ with $\lim_{e\to\infty} H^e_{ij}(e) = 1$

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- It is key to notice that we do not make specific assumption about the correlation of the draws across markets
    - Such restrictions are not needed to either estimate or do counterfactuals with the model

Gravity Equations: extensive and intensive margin of firm exports

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- **Extensive margin of firm-level exports:**

$$\ln \bar{\epsilon}_{ij}(n_{ij}) = \ln(\sigma \bar{f}_{ij}/\bar{r}_{ij}) + \ln w_i^{\sigma} - \ln E_j P_j^{\sigma-1}$$

  - $\bar{\epsilon}_{ij}(n) \equiv (H_{ij}^{e})^{-1}(1-n)$ is *cost-to-sales ratio* supporting entry in j of n% of i firms
  - Slope of $\bar{\epsilon}_{ij}(n)$ controls dispersion in entry potential: $\varepsilon_{ij}(n_{ij}) = \frac{\partial \ln \bar{\epsilon}_{ij}(n_{ij})}{\partial \ln n} < 0$

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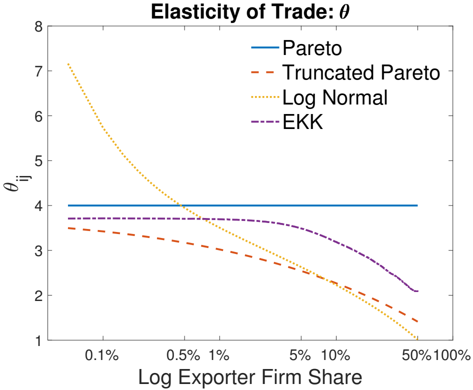
- **Intensive margin of firm level exports:**

$$\ln \bar{x}_{ij} - \ln \bar{\rho}_{ij}(n_{ij}) = \ln \bar{r}_{ij} + \ln w_i^{1-\sigma} + \ln E_j P_j^{\sigma-1}$$

  - $\bar{x}_{ij}$ is average sales of firms from i in j , $\bar{\rho}_{ij}(n) \equiv \frac{1}{n}\int_0^n E[r|e = \bar{\epsilon}_{ij}(n)]\, dn$ is the *avg. revenue potential* if n% of i firms enter j
  - Slope of $\bar{\rho}_{ij}(n)$ controls difference between marginal and incumbent firms: $\varrho_{ij}(n_{ij}) = \frac{\partial \ln \bar{\rho}_{ij}(n_{ij})}{\partial \ln n}$

Firm heterogeneity distribution $\implies$ Trade elasticity varies with n_{ij}

$$\theta_{ij}(n_{ij}) \equiv -\frac{\partial \ln X_{ij}}{\partial \ln \bar{\tau}_{ij}} = (\sigma - 1)\left(1 - \frac{1 + \varrho_{ij}(n_{ij})}{\varepsilon_{ij}(n_{ij})}\right)$$



- **Decreasing** trade elasticity: bilateral trade responds **less** to shocks when n_{ij} **is high**

Elasticity functions Elasticity Functions

Sufficient Statistics of Firm Heterogeneity

- **Lemma 1.** We can re-state $(w_i, N_i, P_i, \{X_{ij}, n_{ij}\}_j)$ in general equilibrium as a
  - function of the shifters $\left(\bar{T}_i, \bar{F}_i, \bar{L}_i, \bar{f}_{ij}, \bar{\tau}_{ij}\right)$
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- **Takeaway 1:** All dimensions of heterogeneity can be folded into our two elasticity functions $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$

- Looking ahead: we will exploit Takeaway 1 to
  - i) characterize model counterfactuals using $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$
  - ii) estimate firm heterogeneity with the semiparametric gravity equations of firm exports

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Nonparametric Counterfactuals and Identification of Fundamentals

- We now aim to use the characterization above to conduct counterfactuals and identification of economic fundamentals
  - Without parametric assumptions on the distribution of economic fundamentals
- Let us fix some terminology
  - $\left(\bar{T}_i, \bar{F}_i, \bar{L}_i, \bar{f}_{ij}, \bar{\tau}_{ij}\right)$ are "economic fundamentals" (or shifters)
  - $(\sigma, \bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$ are "elasticities"
  - $(w_i, P_i, N_i, X_{ij}, n_{ij})$ are "economic outcomes" (wage, price index, entry, bilateral trade/ export share)
  - Denote with a hat a change in a variable from its initial value e.g. $\hat{w}_i \equiv w_i / w_i^0$

Counterfactual Outcome Responses to Changes in Fundamentals

- **Proposition 1.** Given
  1. Counterfactual economic fundamentals: $(\hat{\bar{T}}_i, \hat{\bar{F}}_i, \hat{\bar{L}}_i, \hat{\bar{f}}_{ij}, \hat{\bar{\tau}}_{ij})$,
  2. Data in initial equilibrium: $\boldsymbol{X}^0 \equiv \{X_{ij}^0\}$ and $\boldsymbol{n}^0 \equiv \{n_{ij}^0\}$,
  3. Elasticities: substitution σ, and functions $(\bar{\epsilon}(\bar{\boldsymbol{n}}), \bar{\rho}(\bar{\boldsymbol{n}}))$,

  $\Rightarrow$ compute changes in **outcome** $\left\{\hat{w}_i, \hat{P}_i, \hat{N}_i, \{\hat{n}_{ij}, \hat{X}_{ij}\}_j\right\}_i$. GE system

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- Multiple dimensions of heterogeneity matter only through extensive and intensive margin
  - **Key Insight:** It is all about these elasticity functions!

Aggregate Implications: Is all About Shape of the Elasticity Functions!

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- **Proposition 2.** Let $Y_i \equiv \{w_i, P_i, N_i, \{X_{ij}\}_j\}$
  - The elasticity of elements of Y_i to changes in trade costs is a function of $(\sigma, \boldsymbol{\theta}(\boldsymbol{n}^0), \boldsymbol{X}^0)$,

$$\frac{d \ln Y_i}{d \ln \bar{\tau}_{od}} = \Psi_{i,od} \left(\sigma, \boldsymbol{\theta}(\boldsymbol{n}^0), \boldsymbol{X}^0\right)$$

  - For **small changes**: firm heterogeneity **only matters through $\boldsymbol{\theta}(\boldsymbol{n}^0)$** (a la ACR)

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  - The elasticity of n_{ij} is a function of $\left(\sigma, \boldsymbol{\theta}(\boldsymbol{n}^0), \boldsymbol{X}^0\right)$ and $\varepsilon_{ij}(n_{ij}^0)$:

$$\frac{d \ln n_{ij}}{d \ln \bar{\tau}_{od}} = \Gamma_{ij,od}\left(\sigma, \boldsymbol{\theta}(\boldsymbol{n}^0), \boldsymbol{X}^0, \varepsilon_{ij}(n_{ij}^0)\right)$$

  - For **large changes**: Need to compute change in $\theta_{ij}(n_{ij}^0)$ due to change in n_{ij}, so also need to know $\varepsilon_{ij}(n_{ij}^0)$

Firm Heterogeneity Matters=Variable Elasticities

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  - Heterogeneity plays a role (Melitz Redding '15, Head Mayer Thoenig '14)
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  - Firm heterogeneity only matters for counterfactuals through σ and $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$.
  - For small shocks, $(\bar{\rho}_{ij}(n), \bar{\epsilon}_{ij}(n))$ matter only through their combined effect in $\bar{\theta}_{ij}(n)$.
  - When elasticities are constant, $\bar{\rho}_{ij}(n) = n^{\varrho_{ij}}$ and $\bar{\epsilon}_{ij}(n) = n^{\varepsilon_{ij}}$, *aggregate trade elasticities θ_{ij} are sufficient to compute counterfactual responses to shocks*

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  - Thus, heterogeneity only matters when elasticities vary *and* shocks are large

Changes in Fundamentals to Changes in Outcomes

- We can show that we uniquely invert fundamentals given data without parametric restrictions on firm heterogeneity ▸ Return

- **Proposition 3:** Given
  1. Data in initial equilibrium: $\boldsymbol{X}^0 \equiv \{X_{ij}^0\}$ and $\boldsymbol{n}^0 \equiv \{n_{ij}^0\}$,
  2. Observed changes: $\{\hat{\boldsymbol{n}}, \hat{\bar{\boldsymbol{x}}}, \hat{\boldsymbol{X}}, \hat{\boldsymbol{w}}\}$,
  3. Elasticities: substitution σ, and functions $(\bar{\boldsymbol{\epsilon}}(\bar{\boldsymbol{n}}), \bar{\boldsymbol{\rho}}(\bar{\boldsymbol{n}}))$,

$\Rightarrow$ We uniquely identify shocks in fundamentals $\{\hat{\bar{T}}, \hat{\bar{L}}, \hat{\bar{F}}, \hat{\bar{\boldsymbol{f}}}, \hat{\bar{\tilde{\boldsymbol{r}}}}\}$ with $\hat{\tilde{r}}_{ij} = \hat{\bar{r}}_{ij}/\hat{\bar{r}}_{jj}$.

$\Rightarrow$ Observing the change in the price index $\hat{P}_j$ uniquely identifies the domestic revenue shock $\hat{\bar{r}}_{jj}$ in country j.

Do we Still Have Sufficient Statistics for Welfare Changes?

- Gains of reallocating resources from low to high entry potential firms (i.e., $\downarrow n_{ii}$)

$$\ln\left(\frac{\hat{w}_i}{\hat{P}_i}\right) = \frac{1}{\sigma - 1}\ln\left(\frac{\bar{\epsilon}_{ii}(n_{ii}\hat{n}_{ii})}{\bar{\epsilon}_{ii}(n_{ii})}\right)$$

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- Gains from consuming foreign varieties ($\downarrow$ domestic spending share x_{ii}):

$$d\ln\frac{w_i}{P_i} = -\frac{1}{\theta_{ii}(n_{ii})}d\ln\left(x_{ii}/N_i\right)$$

  - Similar formula in ACR '12 and Melitz-Redding '15, but here the trade elasticity is a function of n_{ij}.
  - We need to know **correlation between** $\theta_{ii}(n_{ii})$ **and** $d\ln\left(x_{ii}/N_i\right)$**.**

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- Conclusion: Takeaways 2–3 constitute a synthesis of the gains from trade debate

Extensions

- **Multiple-Sectors/Factors/Input-Output:** as in Costinot and Rodriguez-Clare '14
  - Sector-specific semiparametric gravity equations of firm exports
- **Zeros in bilateral flows**: as in Helpman-Melitz-Rubinstein '08:
  - Extensive margin gravity equation has a censoring structure
- **Import tariffs**: Need to keep track of tariff revenue
- **Multi-product firms**: Bernard-Redding-Schott '11, Arkolakis-Ganapati-Muendler '20
  - Another semiparametric gravity equation for average number of products
- **Non-CES preferences**: generalizing Arkolakis et al. '19, Matsuyama-Uschev '17
  - Generalized gravity equations implied by similar inversion argument

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- Nonparametric counterfactuals and identification of fundamentals
- **Semiparametric gravity estimation**
- Empirical results
- Quantifying the Gains from Trade

How Can We Measure Variable Elasticities?

- Recall definitions and notice that we can write the two elasticity functions as:
  - **Extensive margin gravity elasticity** $\bar{\epsilon}_{ij}(n)$

$$\ln \bar{\epsilon}_{ij}(n_{ij}) = (\sigma - 1) \ln \bar{\tau}_{ij} + \ln \bar{f}_{ij} + \delta_i^{\epsilon} + \zeta_j^{\epsilon} \tag{1}$$

  - **Intensive margin gravity elasticity** $\bar{\rho}_{ij}(n)$

$$\ln \bar{x}_{ij} - \ln \bar{\rho}_{ij}(n_{ij}) = -(\sigma - 1) \ln \bar{\tau}_{ij} + \delta_i^{\rho} + \zeta_j^{\rho} \tag{2}$$

    - where origin and Destination fixed-effects contain endogenous outcomes (w_i, P_i, N_i)

- **Takeaway 4:** Use semiparametric equations (1), (2) to estimate the elasticity functions

Estimation with three moments

- Calibrate the elasticity of substitution σ

- Assumptions

Estimation with three moments

- Calibrate the elasticity of substitution σ
- OLS estimator of pass-though from observable shifter z_{ij} to observable trade cost τ_{ij}:

$$\ln \tau_{ij} = \kappa^{\tau} z_{ij} + \delta_i^{\tau} + \zeta_j^{\tau} + \eta_{ij}^{\tau},$$

  - In our empirical application, τ_{ij} is freight cost and z_{ij} is log-distance

- Assumptions

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- Given $\tilde{\kappa}^{\tau} \equiv (\sigma - 1)\kappa^{\tau}$, use z_{ij} to estimate $(\kappa^{\epsilon}\gamma_{g,k}^{\epsilon}, \gamma_{g,k}^{\rho})$ from

$$\begin{bmatrix} z_{ij} \\ \ln \bar{x}_{ij} + \tilde{\kappa}^{\tau} z_{ij} \end{bmatrix} = \sum_{k=1}^{K} \begin{bmatrix} \kappa^{\epsilon}\gamma_{g,k}^{\epsilon} f_k(\ln n) \\ \gamma_{g,k}^{\rho} f_k(\ln n) \end{bmatrix} + \begin{bmatrix} \delta_i^{\epsilon} + \zeta_j^{\epsilon} \\ \delta_i^{\rho} + \zeta_j^{\rho} \end{bmatrix} + \begin{bmatrix} \eta_{ij}^{\epsilon} \\ \eta_{ij}^{\rho} \end{bmatrix}$$

  - g: group of origin-destination pairs

- Assumptions

Estimation with three moments

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  - g: group of origin-destination pairs

- Estimate of pass-though from z_{ij} to $\bar{f}_{ij}$ using $\zeta_j^{\epsilon} = \zeta_j^{\rho}\kappa^{\epsilon}$ (entry cost paid in origin)
- Assumptions

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Data Return

- Pool estimation: Countries in WIOD to obtain complete trade matrix $\{X_{ij}\}$.

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  - n_{ii}: 1-year survival rates for manufacturing firms (OECD SDBS)
  - N_{ii}: Active manufacturing firms (OECD SDBS, OECD SSIS, World Bank ES)
  - Compute

$$N_i = N_{ii}/n_{ii}$$

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  - N_{ij} and $\bar{x}_{ij}$: number of exporters and total exports for subset of manufacturing firms (OECD TEC, World Bank EDD) Empirical Distribution

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- Use distance as trade cost shifter z_{ij} (CEPII)
- Use freight cost as observed trade costs τ_{ij} (OECD freight cost database).

Semiparametric gravity estimates

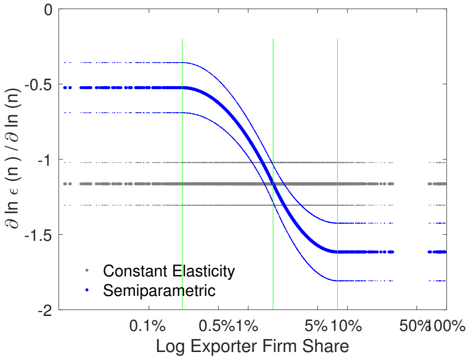


Figure: Elasticity of $\bar{\epsilon}(n)$

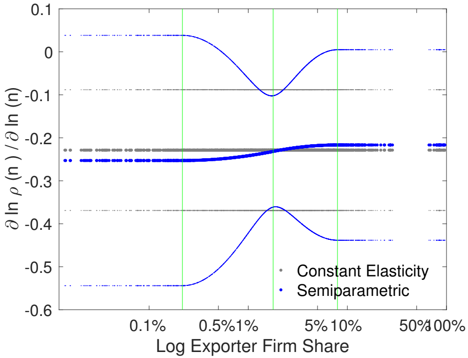
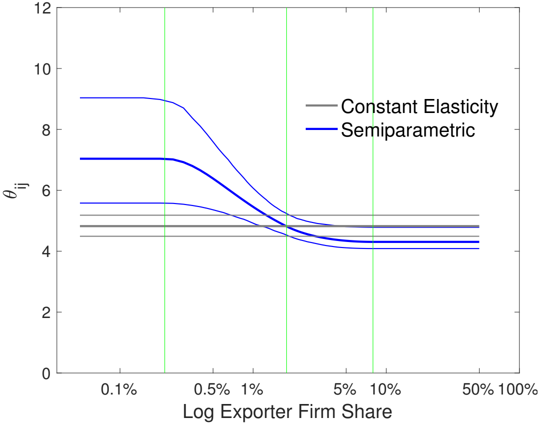


Figure: Elasticity of $\bar{\rho}(n)$

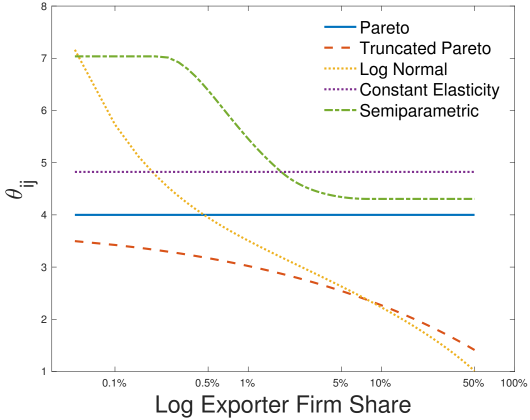
- Decreasing elasticity of $\bar{\epsilon}_{ij}(.)$: Entry is more sensitive to shocks if n_{ij} is low
- Flat elasticity of $\bar{\rho}_{ij}(.)$: Marginal entrants are similar in revenue potential to incumbents
- Data FirstStage GeneralizedPareto Reduced Form

Semiparametric gravity: Implied gravity trade elasticity $\theta(n)$



- Decreasing trade elasticity in $n_{ij} \implies$ Higher gains from trade because n_{ii} is high
- Estimation by country income levels Heterogeneity

Our semiparametric trade elasticity function differs from elasticity in existing literature matching cross-section variation in firm outcomes



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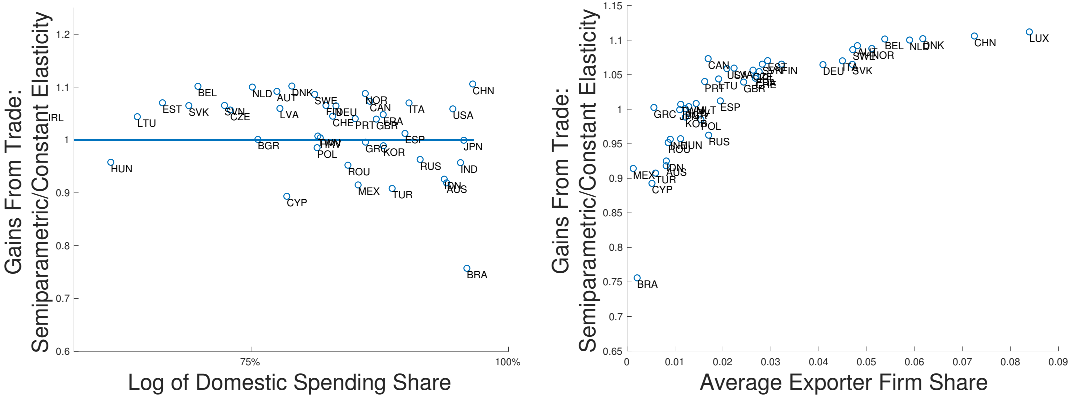
Understanding the impact of firm heterogeneity on the Gains from Trade



- Left: Domestic trade share does not explain deviations
- Right: Higher avg. exporter firm share ⇒ Larger Gains from Trade Scatter Plot

EU Expansion: Role of Firm Heterogeneity

- Sizable Differences between Semiparameteric and Constant Elasticity Gains Details Results

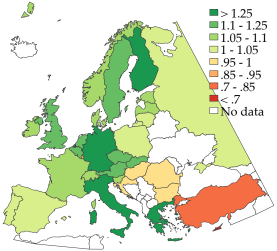


Figure: $\hat{\hat{f}}_{ij}$ and $\hat{\hat{r}}_{ij}$ for $i \neq j$

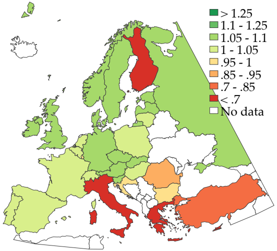


Figure: $\hat{\hat{r}}_{ij}$ for $i \neq j$

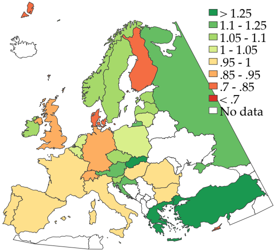


Figure: $(\hat{\hat{\tau}}_{ij})^{1-\sigma} = \hat{\hat{r}}_{ij}/\hat{\hat{r}}_{ii}$

Concluding Remarks

- Distribution of firm fundamentals determines elasticity of extensive and intensive margins of firm exports as functions of exporter firm share
- **Nonparametric counterfactuals:** Two elasticity functions are sufficient to compute impact of trade shocks on aggregate outcomes
- **Semiparametric estimation:** Flexibly estimate these functions using semiparametric gravity equations of firm exports
- The non-constant elasticities imply an average change in grains from trade of 10%. Gains are larger for countries with higher firm export shares.

Extensive/Intensive margin of trade elasticity

- **Extensive margin elasticity:** if endogenous macro outcomes are constant,

$$\frac{\partial \ln n_{ij}}{\partial \ln \bar{\tau}_{ij}} = \left(\left.\frac{\partial \ln \bar{\epsilon}_{ij}}{\partial \ln n}\right|_{n=n_{ij}} \right)^{-1} (\sigma - 1)$$

  - In Melitz-Pareto, entry elasticity is a negative constant for all (i,j). It is still negative, but may vary with n_{ij} across (i,j).

- **Intensive margin elasticity:** if endogenous macro outcomes are constant,

$$\frac{\partial \ln \bar{x}_{ij}}{\partial \ln \bar{\tau}_{ij}} = \underbrace{(1-\sigma)}_{\text{Inframarginal firms}} + \underbrace{\left(\left.\frac{\partial \ln \bar{\rho}_{ij}}{\partial \ln n}\right|_{n=n_{ij}} \right) \left(\frac{\partial \ln n_{ij}}{\partial \ln \bar{\tau}_{ij}} \right)}_{\text{Selection of firms into } (i,j)}$$

  - In Melitz-Pareto, this elasticity is zero for all (i,j). We allow the sales elasticity in (i,j) to take any sign and vary with n_{ij}. Return

Entry & revenue potential functions $\Longrightarrow$ General Equilibrium, $\{w_i, P_i, N_i\}$

- **Bilateral trade outcomes:**

$$\bar{\epsilon}_{ij}(n_{ij}) = \frac{\sigma \bar{f}_{ij}}{\bar{r}_{ij}} \left(\frac{w_i}{P_j}\right)^{\sigma} \frac{P_j}{w_j L_j} \quad \text{and} \quad \frac{\bar{x}_{ij}}{\bar{\rho}_{ij}(n_{ij})} = \bar{r}_{ij} \left(\frac{w_i}{P_j}\right)^{1-\sigma} (w_j \bar{L}_j)$$

- **CES price index:**

$$P_j^{1-\sigma} = \sum_i (N_i n_{ij}) \left(\bar{r}_{ij} w_i^{1-\sigma} \bar{\rho}_{ij}(n_{ij})\right)$$

- **Free Entry:**

$$N_i = \left[\sigma \frac{\bar{F}_i}{\bar{L}_i} + \sum_j \frac{n_{ij}\bar{x}_{ij}}{w_i \bar{L}_i} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}(n)}{\bar{\epsilon}_{ij}(n)}\, dn}{\int_0^{n_{ij}} \frac{\rho_{ij}(n)}{\bar{\epsilon}_{ij}(n_{ij})}\, dn}\right]^{-1}$$

- **Market Clearing:**

$$w_i \bar{L}_i = \sum_j N_i n_{ij} \bar{x}_{ij}$$

Return

Entry & revenue potential functions $\implies$ General Equilibrium, $\{w_i, P_i, N_i\}$

- **Bilateral trade outcomes:**

$$\frac{\bar{\epsilon}_{ij}(n_{ij}\hat{n}_{ij})}{\bar{\epsilon}_{ij}(n_{ij})} = \frac{1}{\hat{\bar{r}}_{ij}} \left(\frac{\hat{w}_i}{\hat{P}_j}\right)^{\sigma} \frac{\hat{P}_j}{\hat{w}_j} \quad \text{and} \quad \hat{\bar{x}}_{ij} = \hat{\bar{r}}_{ij} \frac{\bar{\rho}_{ij}(n_{ij}\hat{n}_{ij})}{\bar{\rho}_{ij}(n_{ij})} \left(\frac{\hat{w}_i}{\hat{P}_j}\right)^{1-\sigma} (\hat{w}_j)$$

- **CES price index:**

$$\hat{P}_j^{1-\sigma} = \sum_i x_{ij} \hat{\bar{r}}_{ij} (\hat{w}_i)^{1-\sigma} \left(\hat{n}_{ij}\hat{N}_i\right) \frac{\bar{\rho}_{ij}(n_{ij}\hat{n}_{ij})}{\bar{\rho}_{ij}(n_{ij})}$$

- **Free Entry:**

$$\hat{N}_i = \left[1 + \sum_j y_{ij} \frac{\bar{\epsilon}_{ij}(n_{ij})}{\int_0^{n_{ij}} \rho_{ij}(n)} \int_{n_{ij}}^{n_{ij}\hat{n}_{ij}} \frac{\rho_{ij}(n)}{\bar{\epsilon}_{ij}(n)} \, dn\right]^{-1}$$

- **Market Clearing:**

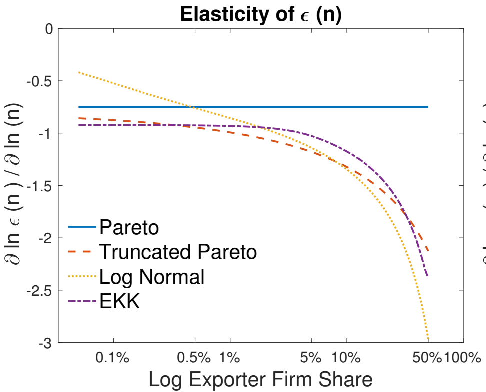
$$\hat{w}_i = \sum_j y_{ij} \left(\hat{N}_i \hat{n}_{ij} \hat{\bar{x}}_{ij}\right)$$

Return

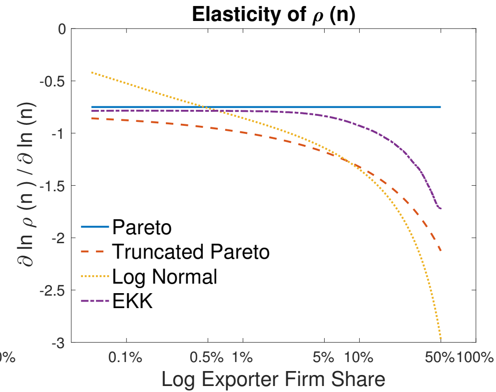
Margins of the Trade Elasticity Function

Return

$$\frac{\partial \ln n_{ij}}{\partial \ln \bar{\tau}_{ij}} \propto \left(\frac{\partial \ln \bar{\epsilon}_{ij}}{\partial \ln n} \right)^{-1}$$



$$\frac{\partial \ln \bar{x}_{ij}}{\partial \ln \bar{\tau}_{ij}} \propto \frac{\partial \ln \bar{\rho}_{ij}}{\partial \ln n}$$



- **Decreasing** elasticity of $\bar{\epsilon}_{ij}(n)$: Entry is **less sensitive** to shocks when n_{ij} **is high**
- **Decreasing** elasticity of $\bar{\rho}_{ij}(n)$: New entrants and incumbents are **more** different when n_{ij} **is high**

Gain from trade

- Gains from trade:

$$\frac{\hat{x}_{ii}^{A}}{\hat{N}_{i}^{A}} = \hat{n}_{ii}^{A} \frac{\epsilon_{ii}\left(n_{ii}\right)}{\epsilon_{ii}\left(n_{ii}\hat{n}_{ii}^{A}\right)} \frac{\bar{\rho}_{ii}\left(n_{ii}\hat{n}_{ii}^{A}\right)}{\bar{\rho}_{ii}\left(n_{ii}\right)}$$

$$\frac{1}{\hat{N}_{i}^{A}} - 1 = \sum_{j} y_{ij} \frac{\epsilon_{ij}(n_{ij})}{\int_{0}^{n_{ij}} \rho_{ij}(n)} \int_{n_{ij}}^{n_{ij}\hat{n}_{ij}^{A}} \frac{\rho_{ij}(n)}{\epsilon_{ij}(n)}\, dn$$

Return

Estimation: Full Estimating Equation

- Extensive Margin:

$$\ln \epsilon_{ij}(n_{ij}) = \underbrace{\left[\ln\left(\frac{\bar{f}_{ij}\bar{\tau}_{ij}^{\sigma-1}}{\bar{b}_{ij}}\right)\right]}_{\text{Bilateral shifter}} + \underbrace{\left[\ln \sigma w_i \left(\frac{\sigma}{\sigma-1}\frac{w_i}{\bar{a}_i}\right)^{\sigma-1}\right]}_{\text{Exporter shifter}} + \underbrace{\left[\ln\left(\frac{P_j^{1-\sigma}}{E_j}\right)\right]}_{\text{Importer shifter}}. \quad (3)$$

- Intensive Margin:

$$\ln \bar{x}_{ij} - \ln \bar{\rho}_{ij}(n_{ij}) = \underbrace{\left[\ln\left(\bar{\tau}_{ij}^{1-\sigma}\bar{b}_{ij}\right)\right]}_{\text{Bilateral shifter}} + \underbrace{\left[\ln\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\bar{a}_i}\right)^{1-\sigma}\right]}_{\text{Exporter shifter}} + \underbrace{\left[\ln\left(P_j^{\sigma-1}E_j\right)\right]}_{\text{Importer shifter}} \quad (4)$$

- Return

Inverting the Economic Fundamentals

- We established how to conduct counterfactuals for rich set of economic fundamentals
  - Challenge that lies ahead: how to measure changes in economic fundamentals
  - We show how to do so from observed data without parametric restrictions on firm heterogeneity

- Key relationships

$$\hat{f}_{ij}^{t}=\frac{\hat{x}_{ij}^{t}}{\hat{w}_{i}^{t}}\frac{\bar{\epsilon}_{ij}(n_{ij}^{0}\hat{n}_{ij}^{t})/\bar{\epsilon}_{ij}(n_{ij}^{0})}{\bar{\rho}_{ij}(n_{ij}^{0}\hat{n}_{ij}^{t})/\bar{\rho}_{ij}(n_{ij}^{0})},\quad \frac{\hat{r}_{ij}^{t}}{\hat{r}_{jj}^{t}}=\frac{\hat{x}_{ij}^{t}/\hat{x}_{jj}^{t}}{\left(\hat{w}_{i}^{t}/\hat{w}_{j}^{t}\right)^{\sigma-1}}\frac{\bar{\rho}_{jj}(n_{jj}^{0}\hat{n}_{jj}^{t})/\bar{\rho}_{jj}(n_{jj}^{0})}{\bar{\rho}_{ij}(n_{ij}^{0}\hat{n}_{ij}^{t})/\bar{\rho}_{ij}(n_{ij}^{0})}$$

Return

(Standard) Assumptions for gravity estimation

- **Assumption 2**
  1. We observe a component of variable trade cost, τ_{ij} (i.e., freight costs or tariffs)
  2. We observe a shifter of trade costs, z_{ij} (i.e., distance):

$$\begin{aligned} \ln \tau_{ij} &= \kappa^{\tau} z_{ij} + \delta_i^{\tau} + \zeta_j^{\tau} + \eta_{ij}^{\tau} \\ \ln \bar{f}_{ij} &= \kappa^{f} z_{ij} + \delta_i^{f} + \zeta_j^{f} + \eta_{ij}^{f} \end{aligned}$$

  where **identification** requires $\kappa^{\tau} \neq 0$ (first-stage coefficient)

- Return to Estimation

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  where **identification** requires $\kappa^{\tau} \neq 0$ (first-stage coefficient)

- **Assumption 3**

$$E[\eta_{ij}^{\tau} | z_{ij}, D_{ij}] = E[\eta_{ij}^{f} | z_{ij}, D_{ij}] = 0$$

  where D_{ij} is a vector of origin and destination fixed-effects

  - Orthogonality assumption is the basis of gravity approach (see Head Mayer '13)
- Return to Estimation

Flexible specification of main functions

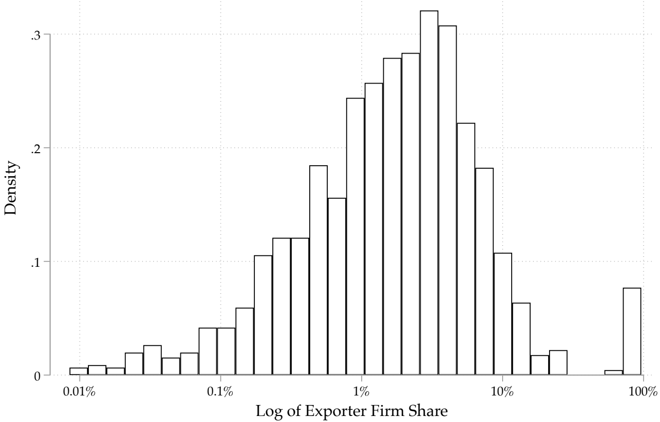
- **Assumption 4.** Origin-destination pairs divided into groups g such that, for $(i,j) \in g$,

$$\begin{bmatrix} \ln \bar{\rho}_{ij}(n) \\ \ln \bar{\epsilon}_{ij}(n) \end{bmatrix} \equiv \sum_k \begin{bmatrix} \gamma^{\rho}_{g,k} f_k(\ln n) \\ \gamma^{\epsilon}_{g,k} f_k(\ln n) \end{bmatrix}$$

  where $f_k(\ln n)$ denotes restricted cubic splines over intervals $\mathcal{U}_k \equiv [u_k, u_{k+1}]$.

  - Explore variation across origin-destination pairs by restricting shape of $\bar{\rho}_{ij}$ and $\bar{\epsilon}_{ij}$ to be **identical** within country groups.
  - Use flexible functional forms to approximate the shape of $\bar{\rho}_g$ and $\bar{\epsilon}_g$.
  - Return to Estimation

Empirical distribution of ln n_{ij}, 2012



- OECD sample with all sectors: fully populated trade matrix without zero flows
- Right tail mass: domestic entry

Return

Estimation: Pass-through of distance to freight costs Return

$$\log \tau_{ij,t} = \kappa^{\tau} \log z_{ij} + \delta^{\tau}_{i,t} + \zeta^{\tau}_{j,t} + \epsilon_{ij,t},$$

	Dep. Var.: Log of Freight Cost		
	(1)	(2)	(3)
Log of Distance	0.351***	0.349***	0.359***
	(0.062)	(0.085)	(0.103)
R^2	0.471	0.725	0.821
Fixed-Effects:			
Year	Yes	Yes	No
Origin, Destination	No	Yes	No
Origin-Year, Destination-Year	No	No	Yes

Note. Standard errors clustered by origin-destination pair. *** $p < 0.01$

Constant-elasticity benchmark: $\bar{\epsilon}_{ij}(n) = n^{\varepsilon}$ and $\bar{\rho}_{ij}(n) = n^{\varrho}$

ε	ϱ	θ
-1.13	-0.21	4.94
(0.03)	(0.03)	

Note. Sample of 1,479 origin-destination pairs in 2012.
$\sigma = 3.9$ from Hottman et al. (2016).
Robust standard errors in parenthesis.

- $\varepsilon = -1.1$: 1% higher trade costs $\implies$ $(1-\sigma)/\varepsilon = 2.6\%$ lower firm entry
- $\varrho = -0.2$: 1% more firm entry $\implies$ 0.2% lower revenue potential of marginal entrants
- $\epsilon \neq \varrho \Rightarrow$ rejects Melitz-Pareto due to intensive margin response

Reduced Form Return

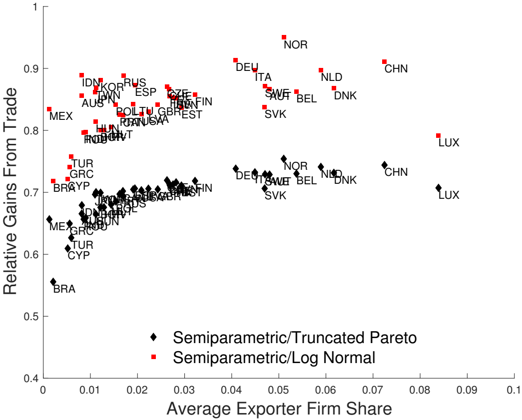
Estimation: Log-linear gravity

Dep. Var.:	$\ln n_{ij}$	$\ln \bar{x}_{ij}$	$\ln X_{ij}$
	(1)	(2)	(3)
Panel A: Log-linear gravity estimation			
Log of Distance	-1.192***	-0.374**	-1.566***
	(0.052)	(0.135)	(0.131)
R^2	0.905	0.846	0.853

Note. Sample of 8,603 origin-destination-year triples. Use $\sigma = 3.9$ from Hottman et al. (2016) and . Standard errors clustered by origin-destination. ** p $<$ 0.05 *** p $<$ 0.01

- Return

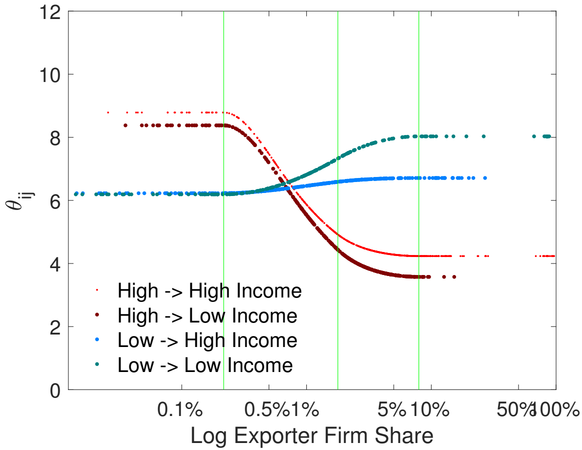
Understanding the importance of using semiparametric gravity estimates



- The “average” trade elasticity partially explains mean average deviation in each case

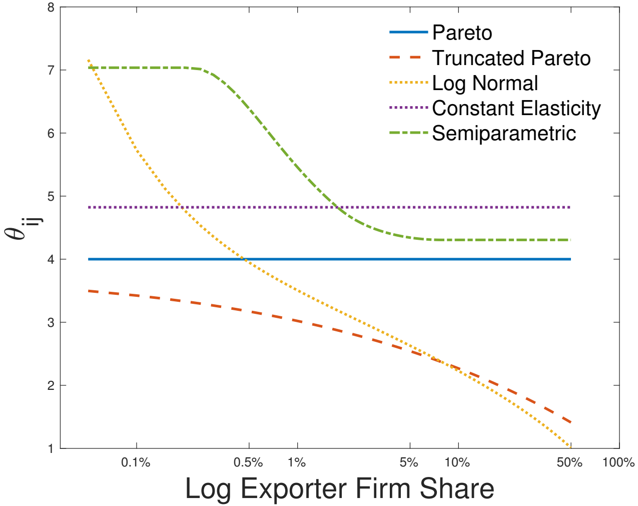
Trade Elasticities

Rich vs Poor Countries: Implied $\theta(n)$



- Return

Semiparametric gravity estimates: Theta Comparison



- Return

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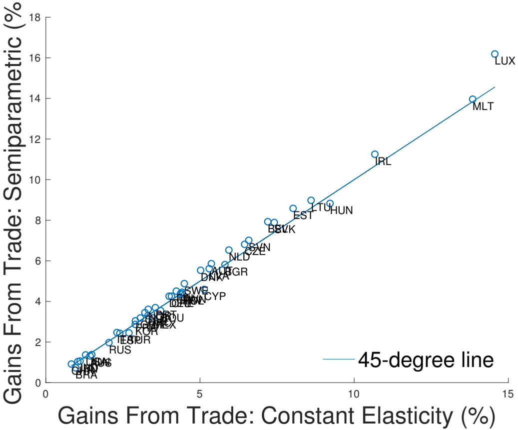
Importance of firm heterogeneity: Gains from Trade



- Highly correlated: Domestic trade share important in both scenarios
- But no longer sufficient statistic: mean change in gains from trade is 10%.
  - For some countries, gains from trade increase or decrease by more than 20% Return

Simulating the EU Expansion

- Unique Nonparametric Inversion $\rightarrow$ Recover $\hat{r}_{ij}$ and $\hat{f}_{ij}$ for $i \neq j$ from 2004-2014
  - Whereby i, j include all EU member states as of 2014. Look at averages over j Return

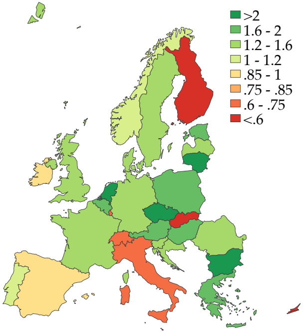


Figure: Average $\hat{\hat{f}}_{ij}$ for $i \neq j$

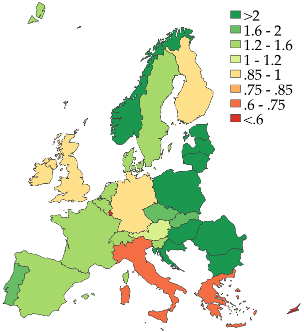


Figure: Average $\hat{\hat{r}}_{ij}$ for $i \neq j$

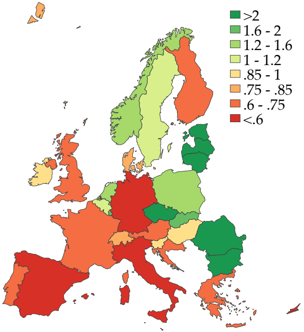


Figure: Average $\hat{\hat{r}}_{ij}/\hat{\hat{r}}_{ii}$ for $i \neq j$

Looking At Welfare (% Changes): EU Expansion

- Feed changes in $\hat{r}_{ij}$ and $\hat{\hat{f}}_{ij}$ in the EU on 2004 data and simulate forward
  - In aggregate, are generally positive
  - But if you normalize exporter productivity by domestic productivity -> EU gains disappear in Western Europe Return

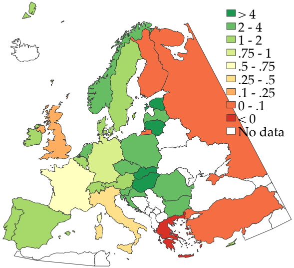


Figure: $\hat{\hat{f}}_{ij}$ and $\hat{\hat{r}}_{ij}$ for $i \neq j$

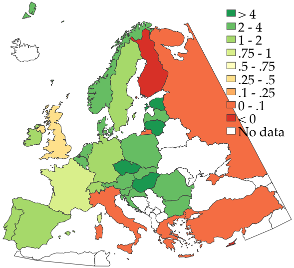


Figure: $\hat{\hat{r}}_{ij}$ for $i \neq j$

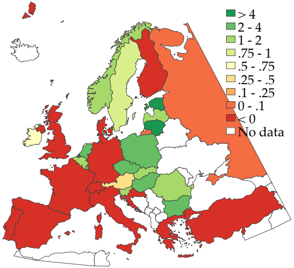


Figure: $(\hat{\hat{\tau}}_{ij})^{1-\sigma} = \hat{\hat{r}}_{ij}/\hat{\hat{r}}_{ii}$