Value without Employment

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Abstract

While young firms' contribution to aggregate employment has been underwhelming over the past decades, a similar trend is not apparent in their contribution to aggregate sales and aggregate stock market capitalization. We provide evidence that the weak job creation by young firms coincided with an increase in the ratio of firm-value-to-employment and sales-to-employment for these firms. The weak job creation suggests that recent firm cohorts have faced a comparatively lower marginal product of labor (for a fixed unit of labor) compared to their predecessors, while the higher firm value-to-employment ratios suggest a higher average product of labor. Motivated by these facts, we study the implications of the arrival of "high average / low marginal" product-of-labor firms in a stylized model of dynamic firm heterogeneity, and show that the model can account for a large number of facts related to the decline in "business dynamism". The decline in business dynamism – per se – is not an ominous sign for the behavior of long term consumption, except if it signifies an economy-wide increase in rents in the long run.

Keywords: Business dynamism, productivity, firm dynamics, entrepreneurship

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1 Introduction

An extensive body of research has documented a decline in the dynamism of the US economy in the last decades. The US economy has exhibited declining entry and exit rates, declining gross labor flows and declining contribution of young firms to aggregate employment. These disconcerting facts, which are based on employment data, are referred to as a "decline in business dynamism."

While labor is an input to the production function, rather than a direct measure of output, a concern in this literature is that the decline in such a critical input may eventually translate into a decline in aggregate output.

In this paper we document that, while measured in terms of employment the performance of recent firm-cohorts has been underwhelming, the sale-to-employment and the equity-value-to-employment ratio of these same firms appears to be higher than it was for their predecessors at similar ages. As a result, while the contribution of young firms to aggregate employment has been underwhelming, their contribution to aggregate wealth has remained relatively stable over time. Specifically, for publicly traded firms, Compustat data show a steady decline in the fraction of employment that is due to young firms, but no trend in the fraction of aggregate market value that is created by these firms over time. For the broader universe of young firms for which we can obtain a firm valuation (specifically firms that have been the target of an acquisition, or have gone through an IPO) data from Pitchbook suggest little difference in the aggregate market value of recent young firm cohorts (expressed as a fraction of the aggregate stock market capitalization at the time of their exit) compared to their predecessors at a similar age.

For the universe of firms where no market valuation exists, we use data from the National Establishment Time Series (NETS). We document a recent trend in the data (starting since mid-2000), whereby young firms purchasing an establishment with similar characteristics as an establishment purchased by an older firm operate the establishment with fewer workers than their predecessors, suggesting lower labor intensities for younger firm cohorts.

Using minimal assumptions and general production functions, we argue that all of the above findings point to changing relation between output creation and labor inputs for young firms. Specifically, the joint occurrence of weak performance in terms of employment, but rising valuation-to-employment (and sales-to-employment) ratios implies that the production functions of young firms have exhibited a low marginal, yet high average product of labor, when compared to their predecessors: The low marginal product of labor $\frac{\partial y}{\partial l}(l_t)$ (when evaluated at the same labor input

l_t as for an older firm) is necessary to explain why younger firms hire fewer workers than their predecessors. The higher average product of labor is necessary to account for the higher value-to-employment, or sales-to-employment ratios.

To study the economic implications of this low marginal, yet high average product of labor phenomenon, we embed this feature of the data in an intentionally stylized model of dynamic firm heterogeneity and perform a theoretical exercise to examine whether this feature alone can account for the decline in business dynamism.

We choose a stylized model to keep our exercise as transparent as possible. In the model firms arrive with heterogeneous labor productivities, experience idiosyncratic productivity shocks, and eventually perish either due to an exogeneous, geometrically distributed death shock, or because of endogenous bankruptcy. (Endogenous bankruptcy is driven by the presence of "operating leverage", i.e., the requirement that in order to remain operative a firm must pay a fixed labor cost per unit of time.) We characterize the steady state of this model analytically, and then perform a transition experiment, whereby from some time t_0 onward, the arriving firms exhibit a higher average, but lower marginal product of labor compared to the older firms. Along the transition path to the new steady state, both "old" and "young" firms co-exist. We show that along the transition path, this economy exhibits declining wages, a declining aggregate labor share (which is not driven by a decline in the average labor share), declining gross job creation and destruction for both older and younger firms, declining output creation by arriving firms, declining death rates by existing firms, greater cross-sectional dispersion in TFP, and an ageing population of firms.

Despite the decline in all these measures of dynamism, the behavior of long-term aggregate consumption is theoretically ambiguous. We provide a formula (in terms of in principle observable quantities) that provides a connection between the decline of output produced by young firms and the decline of consumption in steady state. We show that the elasticity between the decline of young firm output and steady state consumption is equal to the share of economic rents in production, which is likely a small number. The reason for this small elasticity is that the decline of output by young firms is likely to reflect transitory factor adjustments along the transition path (in addition to sectoral shifts in a more extended version of the model). The formula also contains an additive term whose magnitude could be large and depends on the extent to which the decline in the importance of labor also implies a rise in the importance of rents or not. In short the formula implies that it is not the decline in the output produced by young firms that is worrisome, but rather whether the high average / low marginal product of labor of young firms signifies not just a decline in the importance of labor for those firms, but also a rise in the importance of rents for these firms, which will eventually grow.

To explain the decline in labor reallocation, the model requires a diminished sensitivity of how firms' employment decisions react to their idiosyncratic shocks. Such a diminished sensitivity requires not just that the labor share declines, but also that the decline in the labor share is not completely offset by a rise in the capital share (that is –at least to some extent– the rent share must go up). Thus, to explain *jointly* the decline in labor reallocation, the decline in the labor share, the decline in young firm output, the rise in firm-value-to-employment and output-to-employment ratio for young firms, etc., the model requires a rising rent share. The quantitative decline of long term output is directly connected to the magnitude of the rise in this share.

In summary, the goal of this paper is to show how the high average / low average marginal product of labor for young firms can provide a simple, unified and parsimonious explanation for several facts that have been collectively referred to as a decline in business dynamism. The model also suggests that the decline in output by young firms may not be a disconcerting phenomenon (when viewed in isolation), except to the extent that the decline in the importance of labor for young firms may also signify a quantitatively significant rise in economic rents as these firms grow.

The paper is organized as follows. Section 2 presents the empirical facts concerning the rise in the value-to-employment and the sales-to-employment ratio. Section 3 presents the model and its analysis. All proofs, tables and figures are contained at the end of the paper.

1.1 Literature Review (Preliminary and Incomplete)

We contribute to the literature on declining dynamism. An extensive body of research had documented and analyzed the decline in dynamism of the US economy since the early 1980s. The main findings of this literature are declining gross labor flows (Decker et al. (2014)), declining firm entry and exit rates (Decker et al. (2016*a*)), declining contribution of young firms to aggregate employment (Decker et al. (2016*a*)), and since the early 2000s a decline in high growth firms (Decker et al. (2016*b*)). An overview of this literature is presented in Akcigit and Ates (2019*a*).

Recent research has proposed explanations for the declining dynamism. Akcigit and Ates (2019*b*) present a model of endogenous firm dynamics in which declining dynamism is driven by declining knowledge diffusion. In addition to matching the stylized facts of declining dynamism, Akcigit and Ates (2019*b*) match the increase in industry concentration, profits, and markups. Decker et al. (2018) present evidence that the decline in dynamism is driven by a decline in the responsiveness of firms to shocks, rather than a decline in the dispersion of shocks. The authors conclude that the decline in dynamism is consistent with an increase in frictions or distortions in the U.S. economy, potentially implying a drag on aggregate productivity. Consistent with an increase in frictions or distortions, Davis and Haltiwanger (2019) present empirical evidence that young-firm activity shares move strongly with local economic conditions and local house price growth. The authors provide evidence that the effects are driven by financing constraints that lead to a decline in the propensity to start new firms and expand young ones.

We make three contributions to the literature on declining dynamism. First, we present new evidence that shows that the decline in employment creation by new firms was not accompanied by a decline in aggregate firm value creation, which we believe is an interesting and possibly challenging fact to explain. Second, we propose a simple new explanation for the decline in dynamism that is based on changing characteristics of recent cohorts of firms. Specifically, we show that even in the most stylized model of firm dynamics, a number of facets of the decline in business dynamism can be explained in a simple, unified way by recent cohorts of firms having low marginal, yet high average product of labor. Third, we show that declining dynamism is not necessarily associated with or indicative of poor economic performance. Indeed, our model is able to produce declining dynamism even when output is held fixed.

We also contribute to the literature on the decline in the labor share. The period of declining dynamism in the U.S. coincides with a large decline in the labor share of income (Elsby, Hobijn and Şahin (2013); Karabarbounis and Neiman (2014)). There have been many proposed explanations for the decline in the labor share of income which can broadly be categorized into technological change (Karabarbounis and Neiman (2014); Brynjolfsson and McAfee (2014); and Acemoglu and Restrepo (2018)) and declining competition (Barkai (Forthcoming); De Loecker and Eeckhout (Forthcoming)). The two closest papers to ours are Autor et al. (Forthcoming) and Hartman-Glaser, Lustig and Xiaolan (2019). These two papers use firm level data to decompose the decline in the labor share into the within-firm and between-firm contributions. Our model is consistent with the view that the decline in the labor share was driven by an increase in the relative importance of firms that have low labor shares rather than by a decline in the labor share of the representative firm.

2 Motivating Facts

Studies referring to the decline in business dynamism typically study the amount of jobs in young firms as a fraction of aggregate employment. In this section, we study the contribution of young firms to aggregate wealth (rather than job) creation.

This section is structured as follows. In sections 2.1 and 2.2 we document that while the contribution of young firms to job creation has been underwhelming and below its historical levels in the last couple of decades, the aggregate firm valuations of recent cohorts (as well as their sales) have not shown a similar weakness. Section 2.1 documents this pattern in Compustat data and section 2.2 in Pitchbook data. In section 2.3 we show that the different behavior of market valuations and employment implies a declining marginal, yet increasing average product of labor for recent firm cohorts compared to their predecessors. Since only a fraction of firms undergo an IPO or acquisition, in section 2.4 we study all establishments in the National Establishment Time Series (NETS) data set, which covers a much larger set of firms and their establishments. We show that by studying the employment creation in similar establishments purchased by young firms (as compared to older firms), we can draw a similar conclusion about the behavior of marginal vs. average product of labor.

We study the implications of this empirical observation through the lens of a model of dynamic firm heterogeneity in section 3.

2.1 Employment, Sales, and Market Value Creation of IPO Cohorts: Compustat Data

In this section we study the employment, sales, and firm value creation of recent firm cohorts in the Compustat dataset. For the formation of cohorts, we use a 5-year bins of all the firms that went through an initial public offering (IPO) during that period. To avoid measuring the contribution of mature firms that happen to go public late, we use information on each firm's founding year and exclude firms that are founded more than 10 years prior to their IPO. We measure total employment, sales, and firm value of each 5-year IPO cohort and express it as a share, by dividing with the aggregate value of the respective quantity (employment, sales, firm value respectively) of all Computstat firms in the prior year.

To preview and summarize the results that follow, we show that there is a widening gap between the employment share of recent cohorts and their share of market value. More recent cohorts account for a progressively smaller share of employment, their share of of aggregate sales is slightly smaller, and there is no clear trend in the aggregate equity value share. To highlight some numbers, the employment share of the 2010–2014 IPO cohort bin is half the employment share of the 1985–1989 IPO cohort bin, its contribution to sales is only 10 percent less than the 1985–1989 IPO cohort, and its contribution to aggregate firm value is similar and even slightly larger than that of the 1985–1989 IPO cohort bin.

We next provide the details of our analysis.

2.1.1. Data and Sample Selection

We use firm level data on public U.S. firms from Compustat covering the period 1985–2014. We construct the sample of non-financial[1] U.S.[2] public firms that are traded on NYSE, AMEX, and NASDAQ. We further remove from the sample utilities, the United States Post Office, and firms that are classified as part of Public Administration.[3] Last, we require that firms have positive employment and market value.[4] In total, our sample consists of 7,559 firms and 83,189 firm×year observations.

We construct IPO years using the first year that a firm has non-missing information on the number and price of common shares in the Compustat database. The data contain precise IPO dates for about half of the firms in our sample. For the set of firms with precise IPO dates, our constructed measure of IPO year is within one year of the precise date in over 95% of cases. To avoid measuring the contribution of mature firms that go public, we use data on firm founding year and exclude firms that were founded more than 10 years prior to their IPO. As an example, Revlon was founded in the 1930s and went public in 1996 and we therefore exclude Revlon when measuring the contribution of the 1996 IPO cohort.

To ensure near complete coverage, data on firm founding year are taken from a large number of sources. These sources include Loughran and Ritter (2004), Jovanovic and Rousseau (2001), SDC Platinum, Crunchbase, Wikipedia, Bloomberg, Funding Universe, and Google. We adjust the firm founding years to account for spin-offs and mergers.[5] Our founding year data cover 90% of the

[1] We exclude firms with a two-digit SIC code equal to 60.

[2] We require that the firm is incorporated and is headquartered in the U.S.

[3] Utilities are firms with a two-digit SIC code equal to 49. Public Administration are firms with a two-digit SIC code equal to 90.

[4] Market value is the sum of debt and equity.

[5] We use the information from Wikipedia, Bloomberg, Funding Universe, and Google to identify spin-offs and mergers. If firm Y is spun-off from firm X, we assign firm Y the founding year of firm X. In the case of a merger, we attempt to determine the relative size of the two parties at the time of the merger using these same online sources and we then assign the founding year of the larger of the two parties of the merger.

firms in our sample. Importantly, our founding year data cover 96% of the firms that go public over the sample period. At the start of our sample in 1985, the set of firms for which we have founding year data are responsible for 94% of employment, 95% of market value, and 95% of total assets. For the year 2014 (last year that we use for cohort formation), the set of firms for which we have founding year data are responsible for 99.8% of employment, 99.5% of market value, and 99.4% of total assets.

We measure the employment, sales, and market value contributions of the year t IPO cohort as

$$\text{Employment Contribution}_t = \frac{\text{Employment of IPO Firms (Excluding Mature Firms)}_t}{\text{Total Employment}_{t-1}} \tag{1}$$

$$\text{Sales Contribution}_t = \frac{\text{Sales of IPO Firms (Excluding Mature Firms)}_t}{\text{Total Sales}_{t-1}} \tag{2}$$

$$\text{Market Value Contribution}_t = \frac{\text{Market Value of IPO Firms (Excluding Mature Firms)}_t}{\text{Total Market Value}_{t-1}} \tag{3}$$

where "Employment of IPO Firms (Excluding Mature Firms)$_t$" is employment of the IPO cohort in the year of the IPO, excluding firms that were founded more than 10 years prior to their IPO, "Total Employment$_{t-1}$" is total employment in the sample in the prior year. Sales contribution and market value (sum of debt and equity) contribution are defined similarly.

We present results by IPO cohort bin. We bin IPO cohorts into the following 5-year bins: 1985–1989, 1990–1994, 1995–1999, 2000–2004, 2005–2009, and 2010-2014. For each IPO cohort bin we construct the cumulative employment, sales, and market value contributions of the IPO cohort bin as follows:

$$\text{Employment Contribution}_{bin} = \sum_{i \in Bin} \text{Employment Contribution}_i \tag{4}$$

$$\text{Sales Contribution}_{bin} = \sum_{i \in Bin} \text{Sales Contribution}_i \tag{5}$$

$$\text{Market Value Contribution}_{bin} = \sum_{i \in Bin} \text{Market Value Contribution}_i. \tag{6}$$

2.1.2. Results

Figure 1 Panel A presents the logarithm of the employment, sales, and market value contributions of each IPO cohort bin since 1985. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the employment, sales, and market value contributions. Panel C presents the logarithm of the ratio of the sales (respectively market value) contributions to the employment contributions. Panel D presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales (respectively market value) contributions to the employment contributions.

To understand the units for these results, the log employment contribution of the 1985–1989 cohort bin to is -3.5. This means that the employment contribution of the 1985–1989 cohort bin is 3% and the average employment contribution of each IPO cohort in the 5-year bin is 0.6%.

The figure shows that there is a large decline in the employment contribution of IPO cohorts over time, but a more stable sales and market value contribution of IPO cohorts. This can be most easily seen in the normalized series presented in Panels B and D. Panel B shows that the sales and market value contribution of the most recent IPO cohorts in the sample (2010–2014) is in line with the contributions of the early cohorts in the sample. By contrast, the employment contribution of the most recent IPO cohorts in the sample is half that of the early cohorts in the sample.

As a consequence of the stable contribution to market values and declining contribution to employment, Panel D shows that the ratio of market value contribution to employment contribution has increased steadily over time. The ratio of market-value-contribution to employment-contribution has more than doubled over the sample period. While the 2000–2004 and 2005–2009 IPO cohort bins contributed less to employment, sales, and market value than previous cohorts, the relative drop in employment contribution is much larger than the drop in sales or market value contribution.

In terms of magnitudes, the employment contribution of the 2010–2014 IPO cohort bin is only 1.5%, half the employment contribution of the 1985–1989 IPO cohort bin. By contrast, the sales contribution of the 2010–2014 IPO cohort bin is 2.1%, only 10% below the contribution of the 1985–1989 IPO cohort bin, and the market value contribution of the 2010–2014 IPO cohort bin is 3.7%, slightly above the market value contribution of the 1985–1989 IPO cohort bin.

Appendix D shows that the same patterns that apply at the level of Compustat as a whole also apply when we perform the analysis at the level of individual sectors within Compustat. For the model we propose in section 3 it is not important whether the patterns occur within individual sectors, or reflect compositional changes across sectors (increasing importance of sectors with higer market value- to- employment, or sales-to-employment ratios). Therefore we relegate the details of the sectoral analysis to appendix D.

2.2 Exit Values by Cohort: Pitchbook Data

To further examine the finding that market valuations of recent firm cohorts have not behaved very differently than their predecessors, we next turn attention to the broader set of companies contained in Pitchbook. We use Pitchbook to compute the equity market valuations of young firms, which underwent an IPO, or were the target of a merger, or an acquisition. From the perspective of the original shareholders (and the various private equity funds that may have financed these firms at youth), an IPO or an acquisition is commonly referred to as an "exit" event. These exit events allow us to infer the market valuations of these firms. We find that recent cohorts of firms have aggregate exit valuations – expressed as a fraction of stock market capitalization at the time of exit – that are at least as large as earlier cohorts, controlling for age at exit.

2.2.1. Data and Sample

PitchBook is a financial data provider that covers private capital markets, including venture capital, private equity and M&A transactions. The data contain firm level information on both private and public companies including the line of business, key personnel, founding year, recent news, and detailed financing history. When a firm exits by IPO or M&A, the data provide a post-money valuation which we use to measure exit value.

From the PitchBook platform, we extract aggregate exit values in each year for each founding-year cohort. We limit our sample to U.S. firms and we separately extract exit values for firms exiting by IPO and for firms exiting by M&A. As an example, our data contain the aggregate exit value of all firms founded in 2002 that exited by IPO and M&A in 2016. We aggregate the data across exit types to construct total exit value by year and founding year cohort. We note that because we are interested in exits by both IPOs and M&A, in this section we form cohorts by founding year rather than by IPO year. This also serves as a robustness checks that our results don't depend on whether we form cohorts by IPO year (as with Compustat data) or founding year.

There are firms covered by PitchBook for which an exit occurs, but the exit value is missing. We ignore firms with missing values, that is we implicitly assume that the exit values of these firms are zero. The incidence of missing values appears to be more prevalent in recent years, and hence by assigning a value of zero, we are downplaying the market values of recent cohorts, which makes our estimates on the value creation of recent cohorts (compared to earlier cohorts) conservative.[6]

[6]In addition, firms with missing data are also more likely to be small, so they probably wouldn't affect aggregate valuations. The data provider continuously back-fills missing information and has incentives to provide exit values

To smooth out year-to-year fluctuations and facilitate the presentation of the results, we bin cohorts into 5-founding-year cohort-bins. Specifically, we present results using the following founding-year cohort-bins: 1990–1994, 1995–1999, 2000–2004, 2005–2009, 2010–2014, 2015–2019. We aggregate the data to construct a year-by-bin panel by summing the nominal exit values in year t of each of the firms belonging in the cohort-bin. For each cohort-bin, we define "age" as year t minus the birth year of the youngest cohort in the cohort-bin. For example, the age of the 2000–2004 cohort-bin in 2005 is 5.

2.2.2. Results

Figure 2 shows the cumulative sum of exit values of each cohort-bin (s) by age ($t - s$). Nominal exit values are measured in millions of U.S. Dollars. The figure shows that, with the exception of the 1995–1999 cohort, more recent cohorts of firms have nominal exit valuations that are larger than earlier cohorts. In nominal terms, by the age of four (the last age for which we have data for the 2015–2019 cohort-bin), the cumulative exit value of the 1990–1994 cohort-bin is \$3bil, the cumulative exit value of the 1995–1999 cohort-bin is \$175bil, the cumulative exit value of the 2000–2004 cohort-bin is \$55bil, the cumulative exit value of the 2005–2009 cohort-bin is \$126bil, the cumulative exit value of the 2010–2014 cohort-bin is \$168bil, and the cumulative exit value of the 2005–2009 cohort-bin is \$229bil. Each successive cohort-bin has created more market value than past cohort-bins.

To deflate the exit values in year t we use aggregate stock market capitalization at the end of year $t - 1$. Specifically, we use 2000 as a base year and construct :

$$\text{deflated exit value}_t = \text{nominal exit value}_t \times \frac{\text{market capitalization}_{2000}}{\text{market capitalization}_{\text{end of } t-1}}. \tag{7}$$

As with the definition of firm value contribution in the previous section, the presence of aggregate stock market capitalization in the denominator controls for aggregate fluctuations in stock market values.

Figure 3 shows the cumulative deflated exit values of each cohort-bin by age. The figure clearly shows that, with the exception of the 1995–1999 cohort, more recent cohorts of firms have (deflated) exit valuations similar or larger to those of earlier cohorts.

Figure 4 shows the cumulative deflated exit values of each cohort-bin by age and by exit type

for all firms with significant market valuations. We therefore consider it likely that firms with missing exit value are indeed small.

(separately for IPO and M&A exits.) To keep the figure easy to read, we only plot the four recent cohorts and the results are very similar when we plot all cohorts. The figure shows that exits through M&A are nearly twice as large in combined value as exit values through IPO. This points to the importance of using data that includes both IPO and M&A exits.

A notable outlier in the figure is the 1995-1999 cohort, which exhibits an unusual increase around the large stock market run-up of 1999. Similar to the Compustat data, the firms with birth years 2000-2004 have exhibited an underwhelming performance. One possible explanation is the collapse in venture capital in the aftermath of the stock market crash of 1999, along with the fact that the great recession was unfolding at the time when these companies would be ready for exit. However, firms with birth years since 2010 have not exhibited an alarmingly weak performance, when measured by deflated exit values. Overall, the deflated exit values in Pitchbook paint a very similar picture to Compustat: An exceptionally strong performance for firms that experienced their exits in the late 1990s, weakness in 2000s and a rebound post 2010. Other than these fluctuations that are likely to be cyclical, there is no noticeable time trend in these series.

It is also worth noting that while the number of IPOs has significantly declined in recent years, the total capitalization of IPOs by year has not experienced a similar decline. This shows that it was mostly the smaller market-capitalization IPOs that disappeared over the years.

2.3 Implications of the Empirical Findings

Before proceeding with the empirical analysis of the NETS data, we take stock of the implications of our findings for the behavior of the marginal and average product of labor of recent firm cohorts. Under some minimal assumptions, the observation that the market capitalization of younger firms (as a fraction of aggregate market capitalization) is not changing in any noticeable way, while their employment (as a fraction of aggregate employment) declines, implies that the ratio of the average product of labor to the prevailing wage must have gone up for these young firms.

To show this, let s be the time of birth of a firm, let t denote calendar time, and define the profits of a firm as

$$\pi_{t,s} \equiv y_{t,s}(l_{t,s}) - w_t l_{t,s},$$

where $y_{t,s}$ is the time -t output of a firm born at time s, $l_{t,s}$ is the labor input and w_t the prevailing wage at time t. Assume that the market value- to-profit ratio of a firm can be expressed as a product of some aggregate, time-varying process $A\left(t\right)$ and some positive age component $h\left(t-s\right)$, to capture firm-aging effects:

$$\frac{P_{t,s}}{\pi_{t,s}} = A(t)\, h(t-s)\,. \tag{8}$$

Under assumption (8) we have the following result:

Lemma 1 *Assume that (8) holds and let the ratio of the average product of labor* $\left(\frac{y_{t,s}}{l_{t\,s}}\right)$ *over the prevailing wage minus one be denoted by*

$$\delta_{t,s} \equiv \frac{\frac{y_{t,s}}{l_{t\,s}}}{w_t} - 1.$$

Letting Δ *denote the first-difference operator, we have*

$$\Delta \log\left(\frac{P_{t,t}}{\sum_{s\le t} P_{t,s}}\right) - \Delta \log\left(\frac{l_{t,t}}{\sum_{s\le t} l_{t,s}}\right) = \Delta \log\left(\frac{\delta_{t,t}}{\sum_{s\le t} \omega_{t,s}\delta_{t,s}}\right),$$

where $\omega_{t,s}$ *are positive weights that sum to one:*

$$\omega_{t,s} \equiv \frac{h(t-s)}{\sum_{s\le t} h(t-s)} \frac{l_{t,s}}{\sum_{s\le t} l_{t,s}}.$$

Remark 1 *When firms inside each cohort are heterogeneous and indexed by* i*, the conclusion of the above Lemma becomes*

$$\Delta \log\left(\frac{\sum_i P^i_{t,t}}{\sum_{s\le t}\sum_i P^i_{t,s}}\right) - \Delta \log\left(\frac{\sum_i l^i_{t,t}}{\sum_{s\le t}\sum_i l^i_{t,s}}\right) = \Delta \log\left(\frac{\sum_i \frac{l^i_{t,t}}{\sum_i l^i_{t,t}}\left(\delta^i_{t,t}-1\right)}{\sum_{s\le t}\sum_i \omega^i_{t,s}\left(\delta^i_{t,s}-1\right)}\right),$$

where

$$\omega^i_{t,s} = \left(\frac{h(t-s)}{h(0)}\right) \frac{l^i_{t,s}}{\sum_{s\le t}\sum_i l^i_{t,s}},$$

and

$$\delta^i_{t,s} \equiv \frac{\frac{y^i_{t,s}}{l^i_{t\,s}}}{w_t} - 1.$$

The term $\left(\frac{l_{t,t}}{\sum_{s\le t} l_{t,s}}\right)$ maps to our definition of the employment share of young firms (i.e., firms born in "period" t). Similarly, the term $\left(\frac{P_{t,t}}{\sum_{s\le t} P_{t,s}}\right)$ is the market value share of young firms. Lemma 1 shows that the discrepancy in the time-series evolution between a market capitalization-based measure of dynamism $\left(\frac{P_{t,t}}{\sum_{s\le t} P_{t,s}}\right)$ and an employment-based measure of dynamism $\left(\frac{l_{t,t}}{\sum_{s\le t} l_{t,s}}\right)$ has implications for the evolution of the average product of labor of young firms vs. older firms: When the logarithmic change in the market capitalization share of young firms exceeds the logarithmic change in the employment share of young firms, then the average product product of labor of young firms relative to the prevailing wage $\delta_{t,t} = \frac{\frac{y_{t,t}}{l_{t,t}}}{w_t} - 1$ must be growing faster for young firms relative to an appropriately weighted average of $\delta_{t,s}$ for older firms.

While the average product of labor must be growing, the fact that young firms employ less labor over time than older firms (i.e., the fact that $\frac{l_{t,t}}{\sum_{s\leq t} l_{t,s}}$ is decreasing as a function of time t) implies that over time the marginal product of labor of young firms declines (for a fixed amount of labor) compared to older firms. Specifically, equalization of the marginal product of labor implies

$$\frac{\partial y_{t,s}}{\partial l}\left(l_{t,s}\right) = \frac{\partial y_{t,t}}{\partial l}\left(l_{t,t}\right), \tag{9}$$

where $y_{t,s}$ is the time-t production function of a firm born at time s, and the notation $\frac{\partial y_{t,s}}{\partial l}\left(l_{t,s}\right)$ refers to its marginal product of labor evaluated at the level $l_{t,s}$.

Mutliplying and dividing the right-hand side of (9) by $\frac{\partial y_{t,t}}{\partial l}\left(l_{t,s}\right)$ the fact that $l_{t,t} < l_{t,s}$ in the data implies

$$\frac{\partial y_{t,s}}{\partial l}\left(l_{t,s}\right) = \left[\frac{\frac{\partial y_{t,t}}{\partial l}\left(l_{t,t}\right)}{\frac{\partial y_{t,t}}{\partial l}\left(l_{t,s}\right)}\right] \times \frac{\partial y_{t,t}}{\partial l}\left(l_{t,s}\right) > \frac{\partial y_{t,t}}{\partial l}\left(l_{t,s}\right),$$

where the last inequality follows the fact that $\frac{\partial y_{t,t}}{\partial l_{t,t}}$ is declining in its argument, so that the term inside square brackets is larger than one. A similar argument shows also that $\frac{\partial y_{t,s}}{\partial l_{t,s}}\left(l_{t,t}\right) > \frac{\partial y_{t,t}}{\partial l_{t,t}}\left(l_{t,t}\right)$. In words, the fact that an older firm uses more labor than a younger firm at time t, implies that the marginal product of its labor (evaluated at either $l_{t,t}$ or $l_{t,s}$) is higher than the marginal product of labor of the younger firm when evaluated at the same value of labor.

2.4 Micro-Level Evidence from Establishments: The National Establishment Time Series (NETS) Data

While the comparison between market valuations and employment across young and established firms is telling about the differential behavior of average and marginal product of labor, a natural concern is that these conclusions may be special to the subset of firms for which some form of equity valuation exists.

In this section we use an alternative set of assumptions to establish the joint occurrence of a higher average / lower marginal product of labor for young firms in instances where no firm valuation exists. This alternative approach relies on the comparison between the job creation in business establishments purchased by young and old firms.

To explain the key idea, we introduce a set of assumptions. Specifically, letting s denote the date of firm birth and t the current time period, suppose that a company confronted with the choice of purchasing a given establishment expects to reap profits equal to

$$\pi_{t,s} = \max_{l_{t,s}} \left[y_{t,s} - w_t l_{t,s} \right] = \max_{l_{t,s}} \left[w_t l_{t,s} \left(\delta_{t,s} - 1 \right) \right], \tag{10}$$

where $l_{t,s}$ is the labor that will be employed at that establishment and $y_{t,s}$ is the output that will be produced at the establishment. We let $\Pi_{t,s}$ denote the present value of the stream of profits $\pi_{t,s}$ in equation (10).

Next, consider two identical establishments "1" and "2", the first purchased by a firm born in period t and the second purchased by a firm born in period $s < t$. Being identical, the two establishments command the same price $P^{est.}_{1,t} = P^{est.}_{2,t}$. Assuming that buyers and sellers of establishments split the rents from a purchase in some fixed proportion, we have that

$$\phi \Pi_{t,t} = P^{est.}_{1,t} = P^{est.}_{2,t} = \phi \Pi_{t,s}, \tag{11}$$

where ϕ is a parameter that controls the fraction of rents going to sellers and $\Pi_{t,s}$ is the present value of profits from purchasing the establishment accruing to the firm born at time s. Assuming the same discount rate for the two firms from the profits produced by the establishment gives

$$\frac{\Pi_{t,t}}{\pi_{t,t}} = \frac{\Pi_{t,s}}{\pi_{t,s}}. \tag{12}$$

Combining (10) with (11) and (12) implies

$$\frac{w_t l_{t,t}}{w_t l_{t,s}} = \frac{(\delta_{t,s} - 1)}{(\delta_{t,t} - 1)}. \tag{13}$$

If we observe in the data that $l_{t,t} < l_{t,s}$, then (13) implies that $\delta_{t,t} > \delta_{t,s}$.

Using this idea, we examine situations in the data where an establishment with similar characteristics in the same year and geographical location is purchased by a young vs. an old firm. We show that employment creation in the subsequent year is smaller when the establishment is purchased by a young firm vs. an old firm post 2005, but not prior to that. In light of the equation (13), this suggests a higher average product of labor for young firms.

2.4.1. Data

We use for our analysis establishment-level data from the National Establishment Time Series (NETS) Database. The NETS data are constructed from annual snapshots of the Dun and Bradstreet (D&B) archival national establishment data and cover the period 1990–2015. Neumark, Zhang and Wall (2007) and Neumark, Wall and Zhang (2011) provide an extensive overview of the data and Haltiwanger, Jarmin and Miranda (2013), Barnatchez, Crane and Decker (2017), and Crane and Decker (2019) provide a comparison of NETS to the Longitudinal Business Database (LBD).

In the NETS data, each establishment is identified by a unique 9-digit DUNS Number. The data report annual establishment-level employment and industry. The data further provide the first address of each establishment and records significant moves.[7] In our analysis, we only use the first location of an establishment. D&B collect firm-level sales data, but these are not available in the NETS database. Instead, NETS provide estimates of establishment-level sales that are primarily determined by the estimated industry-level ratio of sales-to-employees. For this reason the data do not allow us to measure sales growth separate from employment growth. Therefore, we exclusively use employment data in our analysis.

All establishments have an identified headquarters in each year and firms are defined as the collection of all establishments with the same headquarters. We construct the founding year of a firm as the first year that the headquarters appear in the data and we adjust the founding year to account for firm reorganizations, spin-offs, and mergers.[8] We identify a change in the ownership of an establishment by a change in its reported headquarters.

We construct the sample of all private-sector[9] payroll[10] establishments that change its reported headquarters. We exclude reorganizations and spin-offs from our sample of changes in ownership.[11] We measure the log difference in employment from the year before the acquisition ($t-1$) to the year after the acquisition ($t+1$). This measure of employment growth implicitly requires that the establishment survives one year after the acquisition. We refer to the establishment that changes headquarters as the target establishment, and we refer to the new headquarters as the acquiring

[7] A significant move is defined as a change in an establishment's 5-digit ZIP Code and physical address. If the establishment changes address but at a later date returns to a previous address then the change is assumed to be temporary and therefore not a significant move.

[8] See Appendix E for a description of the classification of changes in ownership and of the adjustments to firm age.

[9] An establishment is private if it has a four digit SIC code less than 9000

[10] We require that an establishment has 2 employees (including the owner) in the year prior to the acquisition.

[11] The results are robust to including these in the sample.

firm.

Data limitations require the following additional filters. First, the headquarter variable in 1990 appears to be unreliable.[12] For this reason we construct our measure of firm founding year using the 1991–2015 data.

Second, our method of determining a firm's founding year is unable to determine the precise founding year of a firm that appears in the data at the start of the sample. Put simply, we cannot tell apart firms founded in 1950 from firms founded in 1990. For this reason, we start the analysis in 1998 and in this year we can definitively determine whether a firm is at least 8 years old.

Third, the data include many imputed values for employment.[13] We remove all acquisitions for which employment of the target establishment is imputed in the year before the acquisition or in the year after the acquisition.

Last, reported employment is sticky. In the NETS database, less than 20% of establishments report any change in employment over a two-year period. The stickiness of employment is discussed in Neumark, Wall and Zhang (2011). We restrict the sample to establishments that report a change in employment from the year before the acquisition to the year after the acquisition.

Table 1 presents a summary of our sample construction and reports the number of acquisitions in our sample after each data filter. Our analysis sample consists of 213,255 acquisitions over the period 1998–2014.

2.4.2. Results

Table 2 shows that establishments purchased by young firms grow slower than similar establishments purchased by older firms. The dependent variable is the log difference in employment from the year before the acquisition ($t-1$) to the year after the acquisition ($t+1$). The independent variables are an indicator Young Acquirer equal to one if the acquirer is less than 8 years old in the year of the acquisition and fixed effects that vary by column. The first column includes year fixed effects. The second column includes year$\times$industry fixed effects where industry is defined as a 4-digit SIC industry. The third column further accounts for geographic variation by including year$\times$industry$\times$state fixed effects. All standard errors are clustered by year$\times$industry$\times$state. The point estimates show that establishments purchased by young firms grow between 2.7 and 3.9

[12] The data show an abnormally high level of changes in ownership in 1991 that are not supported by external data. In addition, some of the headquarter identifiers that appear in 1990 do not have their own data entry as an establishment in 1990. Last, some of the headquarter identifiers that appear in 1990 are recycled later in the sample.

[13] See Crane and Decker (2019) for a discussion of imputed values in the NETS database.

percentage points slower than similar establishments purchased by older firms.

Table 3 reports our main results. In this analysis we split our sample into an early period (1998–2005) and late period (2006–2014). The table shows that, up until 2005, establishment purchase by young firms grew at a similar rate as establishments purchased by older firms. Only starting in 2005 do establishment purchased by young firms grow at a slower rate than similar establishments purchased by older firms. The point estimates show that since 2005 establishments purchased by young firms growth between 12 and 13.4 percentage points slower than similar establishments purchased by older firms.

One potential concern is that (since 2005) young firms systematically acquire establishments that differ in age and size from those acquired by older firms. To address this concern we repeat the analysis with controls for the age and size of the target establishment. To account for establishment age, we bin establishments into three age bins: [1,3], [4,7], and 8+. Table 4 reports the results. All columns include year×industry×state fixed effects. The first column repeats the analysis of Table 3. The second column includes establishment age bin dummies. The third column includes a control for log employment in the year before the acquisition ($t-1$). The fourth column includes both establishment age bin dummies and a control for log employment in the year before the acquisition ($t-1$). The results are very similar when we include establishment age bin dummies or a control for log employment at the target establishment in the year before the acquisition.

3 Model

3.1 Overview

In this section we study the implications of the arrival of firms with high average / low marginal product of labor through the lens of an intentionally stylized model. The model features firm birth and death. Productivity is subject to idiosyncratic shocks, and bankruptcy is an endogenous choice due to operating leverage. In section 3.2 we lay out the model in detail and in section 3.3 we derive its steady state. In section 3.4 we study the transition of this economy to a new steady state, when, starting from some time t_0, newly-arriving firms start employing technologies that feature a lower marginal and yet higher average product of labor. We show that this one feature alone (namely the arrival of firms with high average and low marginal product of labor) can account for a number of facts that have been collectively referred to as a decline in "business dynamism" (fewer jobs and output created by young firms, smaller gross job flows, larger fraction of the population employed by mature firms, larger dispersion in total factor productivity).

In sections 3.5 and 3.6 we provide a formula (in terms of observables) of the link between a decline in young firm output and the eventual decline (if any) of the new steady state output.

3.2 Setup

Time is continuous and indexed by t. Setting up the model in continuous time allows us to provide simple, closed-form solutions for several quantities. At each point in time a continuum of mass $\phi > 0$ of new firms arrives. Firms produce output utilizing labor. The production function exhibits decreasing returns to scale:

$$f\left(l_{it}, Z_{it}\right) = Z_{it} l_{it}^{\alpha}, \tag{14}$$

where l_{it} is the labor employed by firm i, $\alpha \in (0, 1)$ is the labor share and Z_{it} is a time-varying, firm-specific productivity shock that evolves according to

$$dZ_{it} = \mu Z_{it} dt + \sigma Z_{it} dW_{it}, \tag{15}$$

where μ and σ are parameters common to all firms. W_{it} denotes firm-specific Brownian motions, independent across firms. The assumption that Z_{it} follows a geometric Brownian motion is not essential for our results, but facilitates some derivations. More importantly, the assumption that productivity shocks are permanent, rather than transitory, helps make some of our (qualitative) results more surprising.

Remark 2 *While we model firms as employing a decreasing returns to scale technology in the production of a single good, an alternative formulation (provided in appendix B) would involve monopolistically competitive firms producing differentiated goods employing either a decreasing-returns to scale technology, or a technology that is linear in the labor input. In such an interpretation* $Z_{i,t}$ *would combine productivity and taste shocks. While relevant for the interpretation and source of economic rents, these distinctions are immaterial for the results of this section. Therefore we adopt the simpler approach of modeling firms as producting a homogenous good.*

Remark 3 *In this section we assume that the firm only employs labor. We generalize the model to allow for two factors of production (capital and labor) in section 3.5.*

The initial value of Z_{it} for each newly-born firm is drawn from some distribution $m(\cdot)$.

To operate, firms have to employ $\bar{l}$ units of labor per unit of time. This labor cost should be viewed as “overhead”. It does not impact the output of the firm, but it is a requirement for the firm to be able to exist. If a firm decides to shut down, then it terminates permanently.

A firm is also subject to an exogenous, idiosyncratic, exponentially-distributed death shock that arrives with intensity $\lambda > 0$. By the Law of Large Numbers, a fraction λ of firm perishes for exogenous reasons per unit of time dt.

Since there are no aggregate shocks in this model, all aggregate quantities are deterministic, and there is no risk premium. Therefore, each firm chooses its labor demand l_{it} and its optimal stopping time of termination, so as to maximize the expected present value of its profits

$$V_{i,t} = \max_{\tau, l_{iu}} E_t \int_t^{\tau} e^{-\int_t^u (r_s+\lambda)ds} \left[f\left(l_{iu}, Z_{iu}\right) - w_u \left(l_{iu} + \bar{l}\right)\right] du. \tag{16}$$

Maximizing over labor inputs leads to the optimal labor demand

$$l^* \left(Z_{it}; w_t\right) = \left(Z_{it}\right)^{\frac{1}{1-\alpha}} \left(\frac{w_t}{\alpha}\right)^{\frac{1}{\alpha-1}}. \tag{17}$$

Letting $g_t(Z)$ denote the measure of firms with productivity Z that are active at time t, labor market clearing requires that

$$\int_0^{\infty} g_t(Z) \left[l^* \left(Z; w_t\right) + \bar{l}\right] dZ = L, \tag{18}$$

where L is the labor supply in this economy. We assume a constant, inelastic labor supply equal to L, for simplicity.

The representative household has constant relative risk aversion preferences with an intertermporal elasticity of substitution (IES) equal to $\frac{1}{\gamma}$. The representative household collects all wages and profits, which equal to aggregate consumption. Since there are no aggregate shocks, the Euler equation implies that the interest rate is given by

$$r_t = \rho + \gamma \left(\frac{\dot{C}_t}{C_t}\right). \tag{19}$$

Finally, goods market clearing requires that aggregate consumption equal aggregate output

$$C_t = \int_0^{\infty} g_t(Z) f\left(l^* \left(Z; w_t\right), Z\right) dZ. \tag{20}$$

3.3 Analysis: Steady State

To illustrate the properties of the model, we start by solving for a steady state of this economy. Since in steady state $\dot{C}_t = 0$, we have that the interest rate is constant and equal to

$$r_t = \rho. \tag{21}$$

Furthermore, we define the steady state wage as $w_t = w$ and use the lowercase $z_{i,t} = \log(Z_{i,t})$ to refer to the logarithm of a firm's productivity. Ito's Lemma implies that

$$dz_{i,t} = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW_{i,t}. \tag{22}$$

The next proposition provides the cut-off level of log-productivity z^* below which a firm decides to terminate.

Proposition 1 *Let w denote the wage of the economy in steady state. Define ω_1 as the negative and ω_2 as the positive root of the quadratic equation*

$$\omega^2 \frac{\sigma^2}{2} + \omega \left(\mu - \frac{\sigma^2}{2}\right) - (r + \lambda) = 0, \tag{23}$$

which are given by

$$\omega_{1,2} = \frac{-\left(\mu - \frac{\sigma^2}{2}\right) \pm \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2 (\rho + \lambda)}}{\sigma^2}. \tag{24}$$

Assume that $\omega_2 > \frac{1}{1-\alpha}$. Then the value-maximizing termination decision for a firm is to stop operating the first time that z_{it} becomes smaller than

$$z^* \equiv (1-\alpha) \log\left(\frac{\omega_2 - \frac{1}{1-\alpha}}{\omega_2} \frac{\alpha}{1-\alpha} \bar{l}\right) + \log\left(\frac{w}{\alpha}\right). \tag{25}$$

Equation (25) provides the optimal termination threshold as a function of the equilibrium wage w. To determine the equilibrium wage w, we next determine the steady state distribution of firm productivity.

Proposition 2 *Define η_1 as the negative and η_2 as the positive root of the quadratic equation*

$$\frac{\sigma^2}{2}\eta^2 - \left(\mu - \frac{\sigma^2}{2}\right)\eta - \lambda = 0, \tag{26}$$

which are given by

$$\eta_{1,2} = \frac{\left(\mu - \frac{\sigma^2}{2}\right) \pm \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2\lambda}}{\sigma^2} \tag{27}$$

The steady-state distribution of productivity is given by

$$g\left(z; z^*\right) = \frac{2}{\sigma^2} \frac{\phi}{\eta_2 - \eta_1} \left[\begin{array}{c} \left(\int_{z^*}^{z} e^{\eta_1 (z-s)} m\left(s\right) ds + \int_{z}^{\infty} e^{\eta_2 (z-s)} m\left(s\right) ds\right) \\ - \left(\int_{z^*}^{\infty} e^{\eta_2 (x^*-s)} m\left(s\right) ds\right) e^{\eta_1 (z-z^*)} \end{array} \right], \tag{28}$$

where $m(\cdot)$ is the initial productivity distribution.

To complete the construction of equilibrium, we combine the labor demand curves (17) with the labor market clearing condition (18) to arrive at

$$\left(\frac{w}{\alpha}\right)^{\frac{1}{\alpha-1}} = \frac{L - \bar{l} \int_{z^*}^{\infty} g\left(z; z^*\right) dz}{\int_{z^*}^{\infty} \exp\left(\frac{1}{1-\alpha} z\right) g\left(z; z^*\right) dz}. \tag{29}$$

To ensure that the denominator of (29) is integrable, we assume that

$$\frac{1}{1-\alpha} < -\eta_1. \tag{30}$$

Remark 4 *Assumption (30) implies the condition $\frac{1}{1-\alpha} < \omega_2$ of Proposition 1, since $-\eta_1 < \omega_2$.*

By equation (25), the cutoff z^* is a function of w, so equation (29) is a non-linear equation in w. As the next Lemma asserts, it has a unique, positive solution w.

Lemma 2 *There is a unique $w > 0$, for which equation (29) holds.*

Letting w denote the unique solution to (29) it is now possible to solve for the equilibrium output Y_t by substituting w into (17), then into (14) and aggregating across all firms to obtain

$$Y_t = \left(L - \bar{l} \int_{z^*}^{\infty} g\left(z; z^*\right) dz\right)^{\alpha} \left(\int_{z^*}^{\infty} \exp\left(\frac{1}{1-\alpha} z\right) g\left(z; z^*\right) dz\right)^{1-\alpha}. \tag{31}$$

The expression for aggregate output resembles the production function of an individual firm. The first term on the right hand side of (31) is the quantity of labor employed in the production of output (rather than overhead activities) raised to the power α.

The second term is an aggregator of the productivities of individual firms in steady state. As α becomes small (the marginal product of labor is strongly diminishing) this aggregator resembles the cross-sectional average of the level of productivities. As α approaches one, and individual technologies start resembling linear functions of labor, this aggregator puts progressively higher weight on the highest productivities.

Combining (31) with (29) leads to the following expression for the aggregate labor share of output

$$\frac{wL}{Y_t} = \alpha \times \left(\frac{L}{L - \bar{l} \int_{z^*}^{\infty} g\left(z; z^*\right) dz} \right). \tag{32}$$

Equation (32) shows that in this economy the labor share is a product of α, which measures the curvature of the production function and $\frac{L}{L-\bar{l}\int_{z^*}^{\infty} g(z;z^*)dz}$, which is the inverse of the share of the labor force that is employed in the production of output (rather than "overhead" activities).

3.4 Transition to a New Steady State

In section 2 we documented that the ratio $\delta_{it} = \frac{\frac{y_{it}}{l_{it}}}{w_t} - 1$ appears to have increased for recent cohorts of entering firms. The functional form (14) implies a one-to-one mapping between δ_{it} and the curvature parameter α. Since in equilibrium the wage equals the marginal product of labor, we have $w_t = \alpha Z_{it} l_{it}^{\alpha-1} = \alpha \frac{y_{it}}{l_{it}}$. Therefore using the definition of δ_{it},

$$\delta_{it} = \frac{\frac{y_{it}}{l_{it}}}{w_t} - 1 = \frac{1}{\alpha} - 1.$$

Motivated by this observation, we consider the following stylized transition experiment: We assume that starting from the steady state that we analyzed previously, the new firms entering the economy start employing a production function with a lower curvature parameter $\alpha^* < \alpha$. Specifically, from time t_0 onward the firms that entered the economy prior to t_0 continue using the technology $Z_{it} l_{it}^{\alpha}$, while the firms that enter after t_0 use the technology $Z^c Z_{it} l_{it}^{\alpha^*}$, where Z^c is a common constant. The role of the constant Z^c is to ensure that we can differentiate between a decline in the marginal product of labor and an increase in the average product of labor.[14] Alternatively phrased, by considering a joint change in Z^c and α^*, the decline in output of young firms can be disentangled from the decline in their employment.

Our goal in this section is to illustrate that this simple, stylized way of modeling a decline (increase) in the marginal (average) product of labor is enough to reproduce a large number of the

[14] There is also a "dimensional" reason to consider a joint change in Z^c and α^*. Without a modification in Z^c, a change in α^* can either raise or lower the marginal product of labor, depending on whether l_{it} is above or below one. But since labor has no natural units (days, hours, seconds, etc.) a value of one is arbitrary and some change in Z^c is required – if for no other reason – to keep the units unchanged.

facts that are commonly referred to as a decline in business dynamism.

Deriving an analytical solution for the evolution of all equilibrium quantities on the way to the new steady state is infeasible. The key difficulty is that wages (and interest rates) become time-dependent as we transit between steady states. This makes the bankruptcy thresholds time-, and productivity-dependent, requiring a numerical algorithm to derive the bankruptcy threshold. We provide a brief description of the numerical algorithm in appendix C.

The parameter values for our numerical example are provided in table 5. We choose $\mu = 0$ and $\sigma = 0.15$ so that the model can match the properties reported in figures 1.3A and 1.3B of Haltiwanger et al. (2017). Specifically these figures in Haltiwanger et al. (2017) report a median employment (and output) growth around zero and an interdecile range that declines with age (staring from about 1.3 and declining to 0.5). By choosing $\sigma = 0.15$, the cross-sectional standard deviation of employment growth (excluding overhead) is approximately $1/(1-\alpha)\sigma = 0.32$, which results in an interdecile range slightly higher than 0.8, in line with the typical values for firms between 5-15 years. [15] For λ we choose a value of 0.07, which implies a 7% death rate for exogenous reasons. This death rate is mostly relevant for the mature firms, since in the early years several firms terminate due to endogenous bankruptcy in the model. For the discount rate we choose $\rho = 0.04$, to account for the low interest rates and the low weighted average cost of capital during the sample period. We choose the numbers for α and $\bar{l}/L$ to match a) a labor share of about 2/3 and b) a bankruptcy cutoff z^* approximately equal to the lowest possible value of the productivities of entering firms, which we normalize to zero. With this tight bankruptcy cutoff we can capture the very strong selection effects (several exits, high growth rates conditional on survival, etc.) that are typical of firms that enter the economy. We choose the initial distribution of productivities $m(z)$ so that the steady state distribution of the model can match the distribution of firm size (measured by employment) in 1995. Specifically, we assume that $m(z) \propto 2.5e^{-2.5z}$ where z takes values in [0,3]. With this choice the model can reproduce the fact that the firm at the top 0.003 percentile is approximately 500 times bigger than the firm with the lowest productivity, the firm at the top 10% is approximately 20 times bigger than the firms with the lowest productivity etc.[16]

For the firms that arrive after the onset of the transition, we choose α^* and Z^c to match three targets as close as we can: a) A drop in the labor share between the steady states of about 6%, a drop in employment by young firms of approximately 25%, and c) a drop in their output of

[15] Firm growth for very young firms (0-5 years) is substantially skewed both in the data and the model, so it makes more sense to match the cross-sectional standard deviation of firms past 5 years of age.

[16] Source: Small Business Administration.

approximately 10% at the onset of the transition.

Figure 5 plots the equilibrium wage, labor share and output for our example. The x-axis depicts years, with year $t_0 = 20$ being the year of transition from the old to the new steady state. We divide the wage and the output number by their values in the original steady state, so that these numbers can be interpreted as a fraction of their original steady state value. Not surprisingly, the labor share declines essentially monotonically (the small variations around this monotonic path are due to the fact that the model is simulated with a finite number rather than a continuum of firms). Wages decline by about 10% between the steady states, while aggregate output exhibits very small changes along the transition path. Since the behavior of long term output is the topic of the next two sections we postpone a discussion of the behavior of long term output until later.

While the decline of the labor share is not our primary focus, we point out that the model is consistent with the fact that while the aggregate labor share has declined in the last decades, the "average" labor share has increased. To see this, we perform a "time-share" decomposition. To start, we observe that the aggregate labor share can be expressed as

$$\frac{w_t L}{Y_t} = \frac{w_t \int_i l_{it} di}{Y_t} = \int_i \left(\frac{y_{it}}{Y_t}\right)\left(\frac{w_t l_{it}}{y_{it}}\right) di = \int_i \omega_{it} a_{it} di, \tag{33}$$

where $\omega_{it} \equiv \frac{y_{it}}{Y_t}$ is the output weight of firm i and $a_{it} \equiv \frac{w_t l_{it}}{y_{it}}$ is firm $i's$ labor share. In light of (33), we obtain the "time-share" decomposition

$$\begin{aligned}\frac{w_{t+1} L}{Y_{t+1}} - \frac{w_t L}{Y_t} &= \int_i \omega_{it+1} a_{it+1} di - \int_i \omega_{it} a_{it} di \\ &= \underbrace{\int_i \omega_{it} \left(a_{it+1} - a_{it}\right) di}_{\text{Average chage of the labor share}} + \\ &\quad \underbrace{\int_i \left(\omega_{it+1} - \omega_{it}\right) a_{it} di}_{\text{Share changes}} + \underbrace{\int_i \left(\omega_{it+1} - \omega_{it}\right)\left(a_{it+1} - a_{it}\right) di.}_{\text{Product of changes}}\end{aligned} \tag{34}$$

The above equation shows that the change in the labor share can be decomposed into three distinct terms. The first term is the output-share-weighted change in individual labor shares. The second term captures the effect of changing shares and the third term is a term that resembles a covariance term. Figure 6 shows that the first term is always positive both in the old steady state and in the transition phase. This is driven by the fact that in our model $\mu - \frac{\sigma^2}{2} < 0$ and hence for the "median" firm the log productivity declines slightly. Due to the presence of a fixed labor cost, the labor share for the typical firm increases. The decline in the labor share is driven mostly by the sum of the second and the third components of equation (34) (middle plot), which capture the effects of changing firm weights as the output weight of the more productive firms (which have the smaller labor shares) increases. In equation (34) we aggregate only over firms that are alive both at time t and $t+1$. The impact of births and deaths on labor share changes is depicted in the third plot of the figure.

Turning to measures of business dynamism, the left plot of figure 7 depicts $l_t^{\text{new}} = \int_{z^*}^{\infty} m(z)\, l_t(z)\, dz$, the employment by entering firms at time t. The right plot of the same figure depicts the output produced by young firms at time t, $Y_t^{\text{new}} = \int_{z^*}^{\infty} m(z)\, f(z_{t,} l_t(z))\, dz$. Both output and employment of young firms decline, with employment experiencing a larger percentage decline compared to output.

Figure 8 presents results for additional measures of business dynamism. The top left plot presents the sum of gross job creation and destruction defined in a similar manner to Davis and Haltiwanger (1991). These measures are computed as the employment-weighted average of $\frac{|l_{i,t+1}-l_{i,t}|}{\frac{1}{2}(l_{i,t+1}+l_{i,t})} 1_{\{l_{i,t+1}>l_{i,t}\}}$ (gross job creation) and $\frac{|l_{i,t+1}-l_{i,t}|}{\frac{1}{2}(l_{i,t+1}+l_{i,t})} 1_{\{l_{i,t+1}<l_{i,t}\}}$ (gross job destruction). Both measures show a decline along the transition path (and in the new steady state). This decline is driven by three distinct forces inside the model. First, there is less job creation by entering firms, as noted above. Second, there is less gross job reallocation among existing firms. The reason is that the increasing fraction of firms with a smaller value of the curvature parameter α implies smaller variation of the marginal product of labor (despite the volatility of idiosyncratic shocks remaining the same).[17] This decline in the the responsiveness of firms to shocks is consistent with the empirical evidence presented in Decker et al. (2018). Third, there is less job destruction due to exit, since the lower wages decrease the fixed cost $w_t \bar{l}$ that firms have to pay to remain alive. The bottom left plot of Figure 8 illustrates the lower death rates. These lower death rates are also responsible for the greater dispersion of TFP in the new steady state (bottom right plot). The combination of reduced exit and weaker job creation by entering firms naturally implies that an increasing share of the workforce is employed at older firms.

In summary, the arrival of firms that have a low marginal / high average product of labor (compared to their predecessors) allows the model to address a number of empirical patterns related to the decline in business dynamism as emanating from a common source. We would like to conclude

[17]To see this, note that the ratio of the volatility of (log) employment to the volatility of the (log) idiosyncratic shock is $\alpha - 1$. Hence a decline in α reduces the volatility of employment.

with several remarks:

First, the goal of this section was mostly to provide intuition rather than provide a large-scale calibration of the transitional dynamics in the data. To undertake the latter task one would have to take into account that the transition does not happen abruptly at some time t_0, but rather that there is gradual entry of firms employing different technologies. This would help overcome the discontinuous changes at time t_0.

Second, around the transition we assumed that the number of arriving firms does not change, so that we could isolate the decline in young-firm employment – even if the raw number of firms remains unchanged. It is straightforward for the model to incorporate a decline in the number of arriving firms.[18] We chose however to abstract from these forces, so that we could better isolate and study the implications of the arrival of firms with high average / low marginal product of labor.

Third, in this section we chose to focus on a single factor of production. With that choice a decline in α simultaneously reduces the labor share and increases the share of economic rents in production. The former (the decline in the labor share) is important for obtaining a decline in wages and the aggregate labor share. The latter (the rise in economic rents) is important for reducing the responsiveness of firms to their idiosyncratic shocks, and hence reducing labor re-allocation. We explain this point further in the next section, when we introduce a second factor of production (capital).

3.5 A Sufficient Statistic for the Behavior of Long Term Output

For simplicity, in the previous section we assumed that labor was the only factor of production. In this section we add a second factor of production, utilized by all firms, so that the production function takes the form

$$y_{i.t}\left(z_{i,t}, k_{i,t}, l_{i,t}; z^c\right) = \exp\{z_{i,t} + z^c\} k_{i,t}^{\beta} l_{i,t}^{\alpha}.$$

Our goal is to show that what matters for our results (and especially for the connection between the decline in dynamism and steady state output) is how the sum of $\beta + \alpha$ changes, rather than α in isolation.

Specifically, suppose that α and β are positive numbers satisfying $\alpha + \beta < 1$. The rental cost of capital is $r_t + \delta$, where δ is the depreciation rate of capital. Profits are

$$\pi_{i,t} = \max_{l_{i,t}, k_{i,t}} \left[y_{i,t} - w_t(l_{i,t} - \bar{l}) - (r_t + \delta) k_{i,t} \right] \tag{35}$$

[18] One approach taken in the literature is to reduce the number of entrepreneurs, due to demographic reasons.

There are no adjustment costs, aggregate capital K_t obeys the usual dynamics

$$\frac{\dot{K}_t}{K_t} = \frac{I_t}{K_t} - \delta$$

and market clearing is given by $Y_t = C_t + I_t$, where I_t is aggregate investment and Y_t is the total output of all firms. To ensure a well-defined equilibrium we need the additional assumption

$$\frac{1}{1-\alpha-\beta} < -\eta_1.$$

In this modified setup, we consider a joint change in α, β, z^c for firms arriving after t_0.

The next proposition provides a formula connecting the percentage change of output by entering cohorts at time t_0^+ with the eventual decline in steady state consumption.

Proposition 3 *At time t_0 consider a joint marginal change $d\alpha, d\beta$, and dz^c, so that $\alpha^* = a + d\alpha, \beta^* = \beta + d\beta$ for all firms entering from t_0 onward. Define the change dC^{SS} of the steady state consumption (as $t \to \infty$) as*

$$dC^{SS} = \frac{\partial C^{SS}}{\partial \alpha} d\alpha + \frac{\partial C^{SS}}{\partial \beta} d\beta + \frac{\partial C^{SS}}{\partial z^c} dz^c, \tag{36}$$

where all partial derivatives $\frac{\partial C^{SS}}{\partial \alpha}, \frac{\partial C^{SS}}{\partial \beta}, \frac{\partial C^{SS}}{\partial z^c}$ are evauated at the old values $\alpha = \alpha^*, \beta = \beta^*, z^c = 0$. Define the distribution of output among entering firms, $\widetilde{m}(z)$, and the stationary distribution of output among all firms, $\widetilde{g}(z)$, (evaluated at the old steady state), as

$$\widetilde{m}(z) \equiv \frac{m(z)\exp\left(\frac{z}{1-\alpha-\beta}\right)}{\int_{z^*}^{\infty} m(z)\exp\left(\frac{z}{1-\alpha-\beta}\right) dz}, \quad \text{and } \widetilde{g}(z) \equiv \frac{g(z,z^*)\exp\left(\frac{z}{1-\alpha-\beta}\right)}{\int_{z^*}^{\infty} g(z,z^*)\exp\left(\frac{z}{1-\alpha-\beta}\right) dz}. \tag{37}$$

Define the output produced by new firms at time t_0^+ as $Y^{new} = \exp(z^c) \times \int_{z^*}^{\infty} m(z)\exp(z)\, l^{\alpha^*}(z)\, l^{\beta^*}(z)\, dz$. Then as the discount rate $\rho \to 0$, we have that

$$\frac{dC^{SS}}{C^{SS}} = (1-\alpha-\beta)\frac{dY^{new}}{Y^{new}} + [G-1](d\alpha + d\beta), \tag{38}$$

where

$$G \equiv \int_{z^*}^{\infty} (\widetilde{g}(z) - \widetilde{m}(z)) \log l(z)\, dz. \tag{39}$$

The proposition shows a connection between the percentage change of the output of new firms at the onset of the transition (dY^{new}) and the percentage change in steady state consumption (dC^{SS}). There are several noteworthy observations about equation (38).

First, it is theoretically possible that dY^{new} and dY^{SS} have opposite signs. In particular, if $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ is negative, $\frac{dC^{SS}}{C^{SS}}$ can still be positive, as long as $[G-1](d\alpha + d\beta)$ is positive and sufficiently large. An appealing aspect of (38) is that G is measurable in the data, so that each term in equation (38) can be quantified. We postpone such a quantification exercise until the end of this section.

Second, equation (38) implies that the behavior of output of newly entering firms dY^{new} provides sufficient information for predicting the eventual reaction of output. Other quantities that may change during the transition (the decline in gross reallocation, the decline of employment in young firms, etc.) don't provide any additional information neither directly or indirectly.

Third, and somewhat surprisingly, the elasticity of the reaction of $\frac{dC^{SS}}{C^{SS}}$ to $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ is $1-\alpha-\beta$, which is smaller than one, and realistically closer to zero than to one. In other words the "pass through" of a decline of the output of young firms to steady state consumption is very limited.

To better understand this fact, it is useful to provide a heuristic derivation of equation (38). Ignoring heterogeneity and assuming that all firms entering are identical, the onset of the transition at time t_0 leads to the following change in the output of arriving firms

$$\begin{aligned} d\log Y^{\text{new}} \quad &= \quad dz^c + (\log l^{\text{new}})\, d\alpha + \alpha\, (d\log l^{\text{new}}) \\ &\quad + (\log k^{\text{new}})\, d\beta + \beta\, (d\log k^{\text{new}})\,, \end{aligned} \tag{40}$$

where l^{new} is the old steady state value of labor employed by entering firms in the production of output (rather than overhead activities) and k^{new} is the old steady state value of capital used by entering firms.

At the onset of the transition (t_0), the parameters dz^c, $d\alpha$, and $d\beta$ change for the arriving firms, but since the entering firms have measure zero the equilibrium wage does not change. Therefore using the familiar relation that the wage bill is a fraction α of Y^{new} (and similarly the capital bill is a fraction β), we obtain[19]

$$d\log l^{\text{new}} = \frac{d\alpha}{\alpha} + d\log Y^{\text{new}}\,,\ \text{and}\ d\log k^{\text{new}} = \frac{d\beta}{\beta} + d\log Y^{\text{new}} \tag{41}$$

Substituting (41) into (40) leads to

$$(1-\alpha-\beta)\, d\log Y^{\text{new}} = dz^c + (\log l^{\text{new}} + 1)\, d\alpha + (\log k^{\text{new}} + 1)\, d\beta. \tag{42}$$

[19] Throughout this section, we refer to l^{new} as employment in production, but not overhead activities.

Proceeding heuristically and ignoring heterogeneity, the change in steady state consumption is

$$d \log C^{SS} = dz^c + \left(\log l^{SS}\right) d\alpha + \log\left(k^{SS}\right) d\beta. \tag{43}$$

Notice the absence of terms involving $d \log l^{SS}$ and $d \log k^{SS}$ in equation (43). The reason for the absence of the term $d \log l^{SS}$ is that aggregate labor remains fixed across the steady states; the justification for $d \log k^{SS}$ is different: Because we are looking at steady state consumption (rather than output), the envelope theorem implies that any change in welfare arising from an adjustment in the (endogenous) quantity $d \log k^{SS}$ is of second order for steady state welfare. (The precise justification of all these steps is included in the formal proof of the proposition).

Combining (42) with (43) gives

$$\begin{aligned} d \log C^{SS} &= (1 - \alpha - \beta) \, d \log Y^{\text{new}} + \left(\log l^{SS} - \log l^{\text{new}} - 1\right) d\alpha \\ &\quad + \left(\log k^{SS} - \log k^{\text{new}} - 1\right) d\beta. \end{aligned} \tag{44}$$

Since we evaluate all quantities around the point $\alpha = \alpha^*$, $\beta = \beta^*$, $z^c = 0$, the capital-labor ratios across old and young firms are equalized and

$$\log k^{SS} - \log l^{SS} = \log k^{\text{new}} - \log l^{\text{new}},$$

implies that $\log k^{SS} - \log k^{\text{new}} = \log l^{SS} - \log l^{\text{new}}$ and therefore (44) becomes

$$d \log C^{SS} = (1 - \alpha - \beta) \, d \log Y^{\text{new}} + \left(\log l^{SS} - \log l^{\text{new}} - 1\right) (d\alpha + d\beta). \tag{45}$$

While derived heuristically, equation (45) is essentially the same equation as (38). The first term on the right hand side of (44) is identical to equation (38). In particular, the multiplicative term $(1 - \alpha - \beta)$ in front of $d \log Y^{\text{new}}$ reflects the fact that, on impact, factor quantities and prices don't change at the aggregate, since the entering firms are of measure zero. Hence the factor choices of entering firms react to the joint change in $d\alpha$, $d\beta$, and dz^c. By contrast, in the long run the wage and quantity of capital both adjust. Thus the reaction of the output of young firms in the short run largely reflects transitory factor adjustments.

This is the reason why only the fraction $(1 - \alpha - \beta)$ of the drop in new firm output $d \log Y^{\text{new}}$ passes through to eventual consumption. This fraction $1 - \alpha - \beta$ of output, which is not accounted by factors of production is the only fraction of output that is not due to an adjustment in factor employment and hence passes through to long-term welfare.

The second term on the right-had side of (45) is essentially the same term as $[G-1](d\alpha+d\beta)$ in (38). The main difference between equation (44) and (38) is that the term G accounts for the heterogeneity across firms (both in the entering and steady state distribution).

One attractive feature of equation (38) is that it involves observable quantities, and hence it can be used to perform some quantitative calculations. To compute G one needs information on the cross sectional distribution of employment among young and established firms. If the wage bill is a constant fraction of output, and firms face the same wage, then it doesn't matter whether one uses output or employment weights for the computation of G, with the latter being more readily available in the data. Defining young firms as firms 5 years or younger, G is a number around 2.7 in the data, and so $G-1$ is around 1.7.

A critical step in evaluating (38) is what one assumes about $d\alpha+d\beta$, not just $d\alpha$. If the labor coefficient (α) declines, but the capital coefficient (β) partially, or entirely, offsets the change, then $d\alpha+d\beta$ is likely to be small, and so will be the term $(G-1)(d\alpha+d\beta)$. The point is that it is not the decline in the labor coefficient that matters, but rather the increase in "rents" $1-(\alpha+\beta)$.

For instance, if one assumes that $1-(\alpha+\beta)$ increased from 0.05 to 0.15 (Barkai (Forthcoming)) then $(G-1)(d\alpha+d\beta)$ is equal to $1.7\times 0.1 = 17\%$ percentage drop in steady state consumption, a rather large number. By contrast, the first term of equation (38) $(1-\alpha-\beta)\frac{dY^{\text{new}}}{Y^{\text{new}}}$ is likely to be a small number by comparison. Assuming that $1-\alpha-\beta$ is around 0.05 in the old steady state, then even if one assumed that $\frac{dY^{\text{new}}}{Y^{\text{new}}}=0.4$, the end result would be a 2% drop in steady state output, which is a small number by comparison.

The above discussion can be summarized as follows: It is not the decline in young firm output –per se– that is an ominous sign for long term consumption. Since arriving firms make up a small part of the economy, and hence don't significantly affect factor prices or quantities, the reaction of their output reflects transitory factor adjustments. The disconcerting aspect of the decline in young-firm output is that it may reflect a rise in economic rents, which over time will affect the entire economy as more and more firms employing rent-intensive technologies arrive. This is what is captured by the second term of (38).

It is useful to remark that in order to explain the decline in output, employment and labor reallocation as joint phenomena inside our model, it is necessary to assume not just that α has declined, but also that $1-\alpha-\beta$ has increased. The easiest way to see this is to maximize (35) over capital and labor and express labor demand in terms of the firm-specific productivity $z_{i,t}$.

$$\log l_{i,t} = Const. + \frac{z_{i,t}}{1-\alpha-\beta}, \tag{46}$$

where the constant depends on the prevailing wages and interest rates. Note that the reaction of labor to a firm-specific productivity shock is given by $\frac{1}{1-\alpha-\beta}$, not just $\frac{1}{1-\alpha}$. In other words, the volatility of labor demand – which is the crucial quantity for the decline of labor reallocation– would remain unchanged if α decreased, but β offset this change to keep $1-\alpha-\beta$ constant.

3.6 Sectoral shifts

So far, we have assumed that all firms inside a cohort utilize the same technology. In this section we extend the analysis to allow for within-firm heterogeneity in the technologies used. This extension addresses the possibility that the patterns we documented in the empirical section may be driven (at least partially) by an increasing (output) weight of some sectors employing less labor-intensive technologies.

Specifically, in this section we extend equation (38) to allow for sectoral shifts. We assume that within each cohort there are J different "sectors", which are assumed to be just different technologies with different values of α_j, β_j, z_j^c, and also different parameters governing the productivity dynamics (μ, σ), initial distribution of productivities, etc.

Letting ω_j denote the sectoral output weights in the (old) steady state, equation (38) generalizes to

$$\begin{aligned} \frac{dC^{SS}}{C^{SS}} &= \sum_{j\in J} \omega_j \left((1-\alpha_j-\beta_j) \frac{dY_j^{\text{new}}}{Y_j^{\text{new}}} \right) \\ &\quad + \sum_{j\in J} \omega_j \left(\int_{z_j^*}^{\infty} \log l_j(z_j) \left(\widetilde{g}_j(z_j) - \widetilde{m}_j(z_j) \right) dz_j - 1 \right) (d\alpha_j + d\beta_j), \end{aligned} \tag{47}$$

where Y_j^{new} is the output of sector j, $l_j(z_j)$ is the labor employed by a firm in sector j having log productivity z_j. $\widetilde{m}_j(z_j)$ and $\widetilde{g}_j(z_j)$ are the sector-specific analogues of (37), namely

$$\widetilde{m}_j(z_j) \equiv \frac{m_j(z_j)\exp\left(\frac{z_j}{1-\alpha_j-\beta_j}\right)}{\int_{z_j^*}^{\infty} m_j(z_j)\exp\left(\frac{z}{1-\alpha_j-\beta_j}\right) dz_j}, \text{ and } \widetilde{g}_j(z_j) \equiv \frac{g_j\left(z_j, z_j^*\right)\exp\left(\frac{z_j}{1-\alpha_j-\beta_j}\right)}{\int_{z_j^*}^{\infty} g_j\left(z_j, z_j^*\right)\exp\left(\frac{z_j}{1-\alpha_j-\beta_j}\right) dz_j},$$

where $m_j(z_j)$ and $g_j(z_j)$ are the distributions of entering and stationary log productivity within the sector respectively.

An interesting special case of (47) is when $d\alpha_j = d\beta_j = 0$ for all j. Letting $\widetilde{\omega}_j$ denote the output weights of each sector in the entering firm cohort, we can then express the change in steady-state consumption as

$$\begin{aligned} \frac{dC^{SS}}{C^{SS}} &= \sum_{j \in J} \widetilde{\omega}_j \left(\frac{\omega_j (1 - \alpha_j - \beta_j)}{\widetilde{\omega}_j} \frac{dY_j^{\text{new}}}{Y_j^{\text{new}}} \right) \\ &= E^{(\widetilde{\omega}_j)} \left(\frac{\omega_j (1 - \alpha_j - \beta_j)}{\widetilde{\omega}_j} \frac{dY_j^{\text{new}}}{Y_j^{\text{new}}} \right) \end{aligned} \tag{48}$$

where $E^{(\widetilde{\omega}_j)}$ denotes an expectation operator that places the weight $\widetilde{\omega}_j$ on each of the sectors $i = 1..J$. Note that if $\frac{dY_j^{\text{new}}}{Y_j^{\text{new}}}$ is negative for all j, then $\frac{dC^{SS}}{C^{SS}}$ is negative.

Since the change $\frac{dC^{SS}}{C^{SS}}$ is an expectation of a product of random variables, we can write it as

$$\begin{aligned} \frac{dC^{SS}}{C^{SS}} &= E^{(\omega_j)} \left[(1 - \alpha_j - \beta_j) \right] \times \frac{dY^{\text{new}}}{Y^{\text{new}}} \\ &\quad + Cov^{(\widetilde{\omega}_j)} \left(\frac{\omega_j (1 - \alpha_j - \beta_j)}{\widetilde{\omega}_j}; \frac{dY_j^{\text{new}}}{Y_j^{\text{new}}} \right), \end{aligned} \tag{49}$$

where Y^{new} is the total output of the entering cohort, $E^{(\omega_j)}$ denotes an expectation operator that places the weight ω_j on each of the sectors $i = 1..J$, and the covariance $Cov^{(\widetilde{\omega}_j)}$ uses $\widetilde{\omega}_j$ as probability weights.

The right-hand side of equation (49) contains two terms. Assuming that $\frac{dY^{\text{new}}}{Y^{\text{new}}}$ is negative, the first term is negative since $(1 - \alpha_j - \beta_j) > 0$ for all j. The second term is the covariance between $\frac{\omega_j(1-\alpha_j-\beta_j)}{\widetilde{\omega}_j}$ and $\frac{dY_j^{\text{new}}}{Y_j^{\text{new}}}$. Roughly speaking, if the sectors that exhibit a relatively large share of rents are also sectors that experience above average values of $\frac{dY_j^{\text{new}}}{Y_j^{\text{new}}}$, then the covariance term is positive. In principle, it could be large enough to overwhelm the negative term $E^{(\omega_j)} \left[(1 - \alpha_j - \beta_j) \right] \times \frac{dY^{\text{new}}}{Y^{\text{new}}}$.

In addition, if this covariance is positive, the relative importance of firms with high values of $1 - \alpha_j - \beta_j$ rises, which implies a decline in the sensitivity by which the labor demand of the "representative" firm reacts to changes in its productivity, thus helping explaining the decline in reallocation.

In short, if one views the decline in business dynamism as arising from a sectoral shift towards firms with a high share of economic rents and a low labor share, this could be (at least qualitatively) consistent with a decline in re-allocation (at least over the medium term), a decline in the labor share and wages, a decline in employment creation of young firms etc. Moreover, the reaction of $\frac{dC^{SS}}{C^{SS}}$ would be quite small, since the first (negative) term on the right hand side of (49) is likely to be quantitatively small, and is reduced further in absolute value by the second term (the covariance term).

The main empirical obstacle of this benign view of the decline in dynamism is that the high average / low marginal product of labor patterns manifest themselves also within industry groups, at least using the conventional industry classifications. This implies that it is unlikely that the terms $d\alpha_j + d\beta_j$ are zero for all sectors, as we assumed in going from equation (47) to (48).

4 Conclusion

In the last decades, young firms have created fewer jobs than their predecessors at similar ages. Compustat and Pitchbook data suggest, however, that the contribution of these firms to aggregate stock market value and sales has not shown a similar weakness. NETS data suggest that young firms purchasing similar establishments to their predecessors create fewer jobs in the purchased establishments starting around 2005. Under weak assumptions – indeed almost by definition – these developments can be encapsulated by the statement that younger firms have faced a lower marginal / yet higher average product of labor.

Taking this observation as given, we consider its implications within an intentionally stylized model of dynamic firm heterogeneity. In particular, we consider an economy where entering firms start employing technologies that exhibit a lower marginal / higher average product of labor. We show that this fact alone can provide a unified explanation for a large number of facts that have been collectively referred to as a decline in business dynamism. This suggests that the decline in the labor share (and likely rise in the rent share) and the decline in business dynamism may be two closely related phenomena.

A surprising implication of the model is that there is no automatic link between the decline in business dynamism and aggregate output. Indeed, it is theoretically possible that aggregate consumption experiences no change in the long run, even though both employment and output of entering firms drop and all productivity shocks are permanent.

Our theoretical results provide a simple mathematical relation between the decline of young-firm output and the decline in steady state consumption. This mathematical relation shows that it is not the decline in young-firm output –per se– that is a disconcerting sign for long term output. The elasticity of the decline in long-term consumption to new-firm-output is close to zero, since most of the reaction of the output of young firms may just reflect factor adjustments on the way to the new steady state. The more disconcerting aspect is that if the decline in dynamism is indeed driven by a rise in economic rents (within every sector), then over the long term this rise in rents will permeate the entire economy, leading to a non-trivial drop in output.

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Figure 1: **Employment, Sales, and Market Value Contributions of IPO Cohorts**
Data on employment, sales, and market values of US public firms are taken from Compustat. Data on firm founding years are described in the text. We exclude from IPO cohorts all firms that were founded more than 10 years prior to their IPO. We measure the employment, sales, and market value contribution of an IPO cohort as a share of the total employment, sales, and market value of public firms in the prior year. We then measure the contribution of an IPO cohort bin as the sum of the contributions of the different IPO cohorts in the bin. Panel A presents the logarithm of the employment, sales, and market value contributions of each IPO cohort bin since 1985. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the employment, sales, and market value contributions. Panel C presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel D presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. See Section 2.1 for further details.

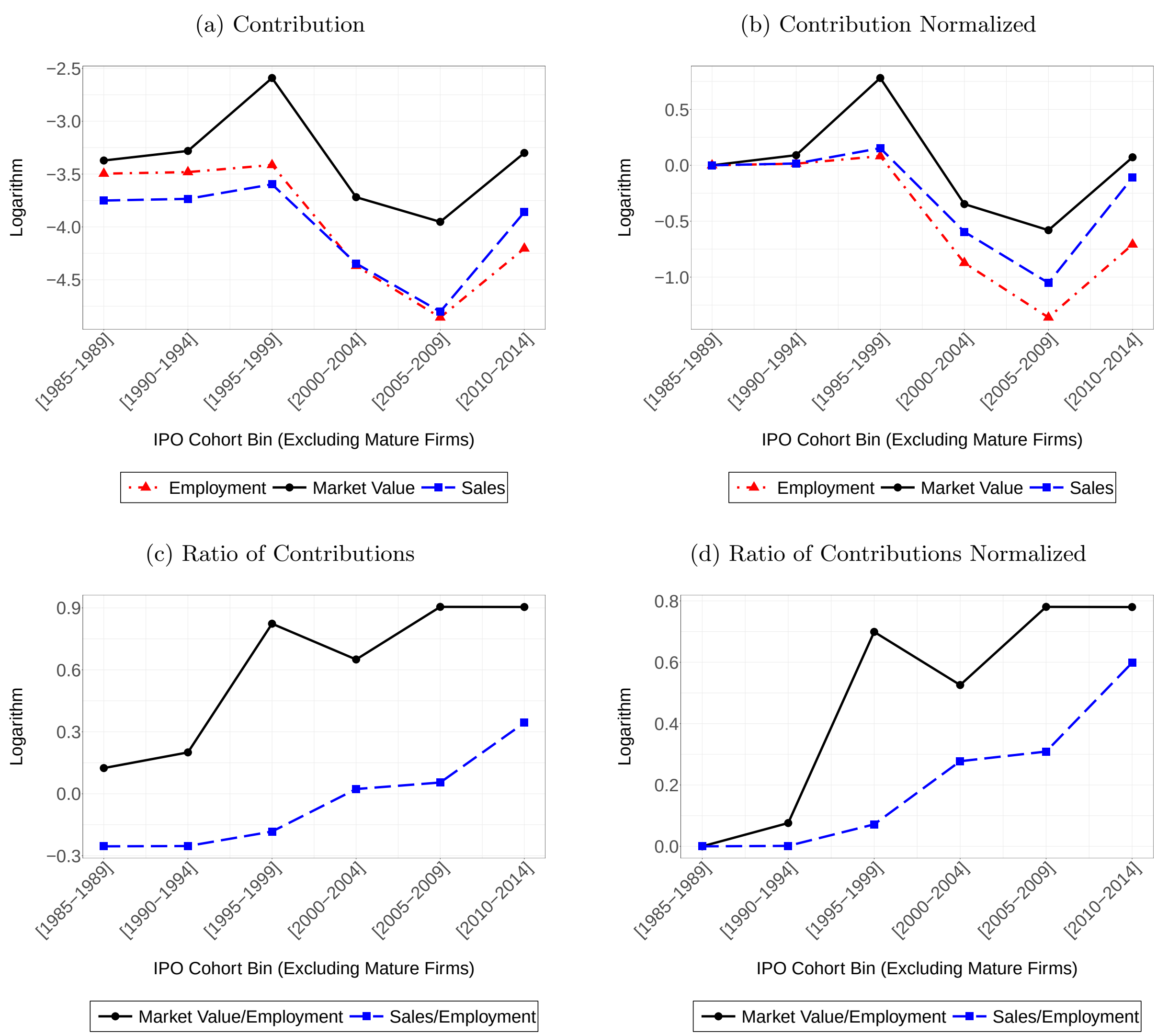
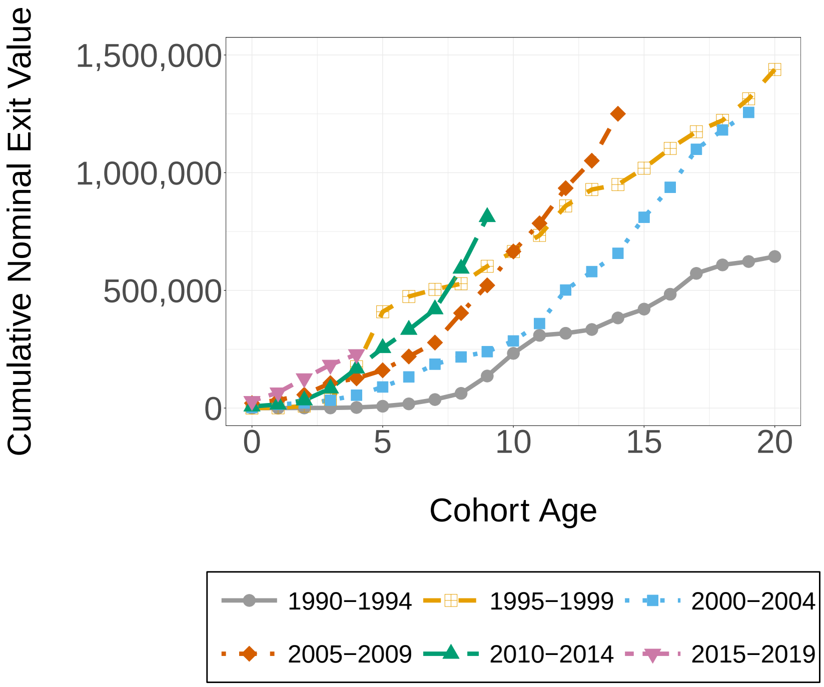


Figure 2: **Nominal Exit Values by Cohort**
The figure shows the cumulative nominal exit values of each cohort-bin by age. Exit values are measured in millions of U.S. Dollars. We bin cohorts into 5-year cohort-bins. Age is defined as year less youngest cohort in the cohort-bin. Exit value is post-money valuation. We include both IPO and M&A exits. Data on exit values are taken from PitchBook. See Section 2.2 for further details.

(a) All Cohorts



(b) Excluding 1995–1999 Cohort

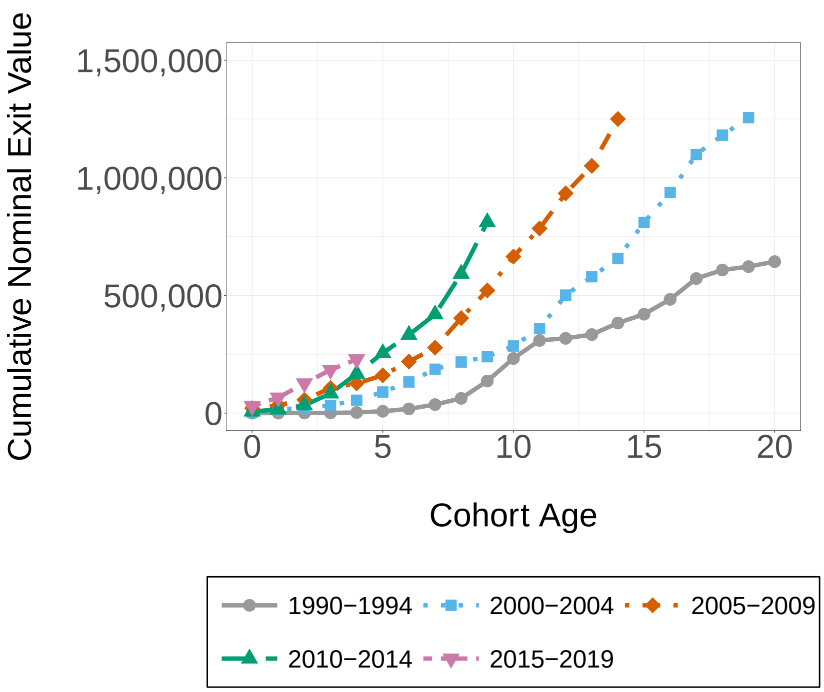
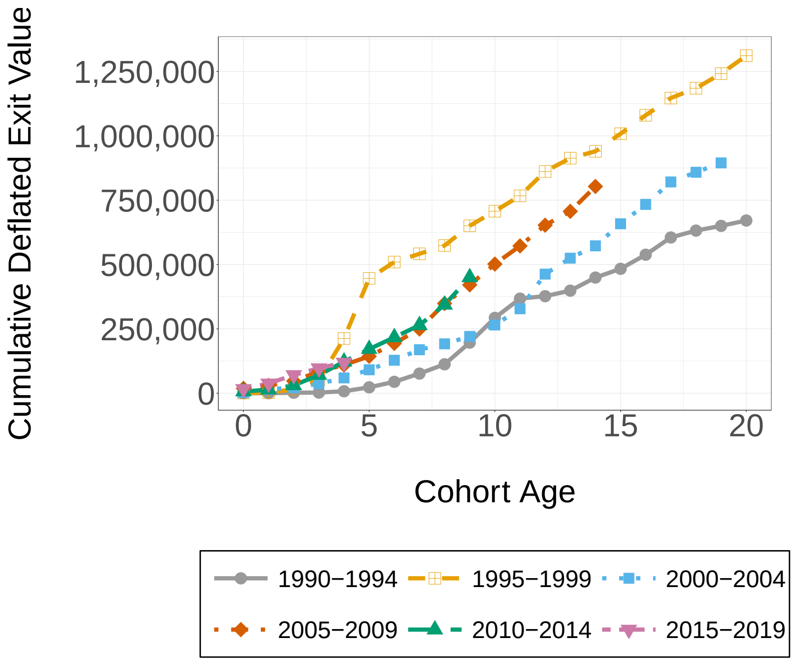


Figure 3: **Deflated Exit Values by Cohort**

The figure shows the cumulative deflated exit values of each cohort-bin by age. Nominal exit values are measured in millions of U.S. Dollars. Deflated exit values are nominal exit values deflated by the U.S. stock market capitalization, where the deflator is normalized to one at the start of 2000. We bin cohorts into 5-year cohort-bins. Age is defined as year less youngest cohort in the cohort-bin. Exit value is post-money valuation. We include both IPO and M&A exits. Data on exit values are taken from PitchBook. See Section 2.2 for further details.

(a) All Cohorts



(b) Excluding 1995–1999 Cohort

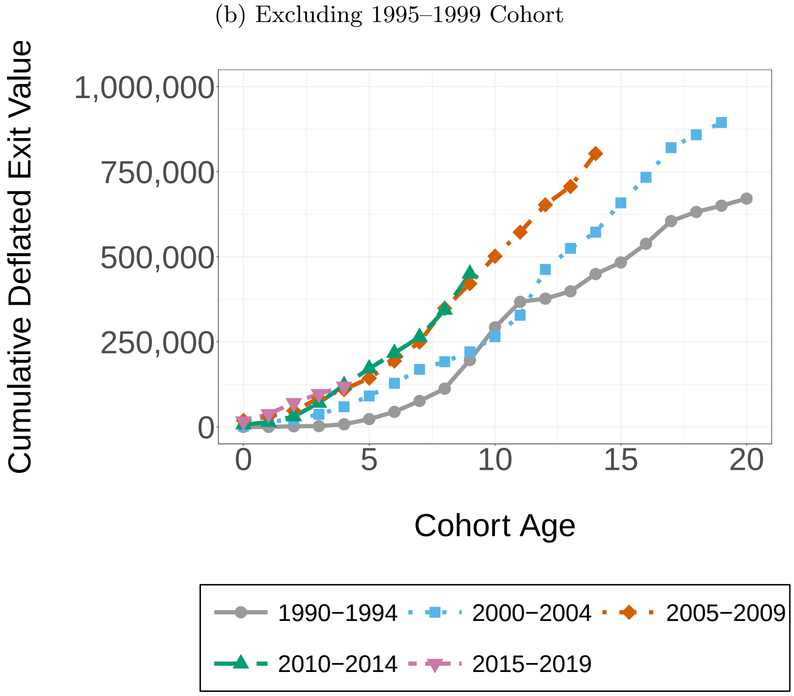


Figure 4: **Deflated Exit Values by Cohort and Exit Type**

The figure shows the cumulative deflated exit values of each cohort-bin by age. Nominal exit values are measured in millions of U.S. Dollars. Deflated exit values are nominal exit values deflated by the U.S. stock market capitalization, where the deflator is normalized to one at the start of 2000. We bin cohorts into 5-year cohort-bins. Age is defined as year less youngest cohort in the cohort-bin. Exit value is post-money valuation. The figure presents deflated exit values for IPO exits, M&A exits and Total exits. Data on exit values are taken from PitchBook. See Section 2.2 for further details.

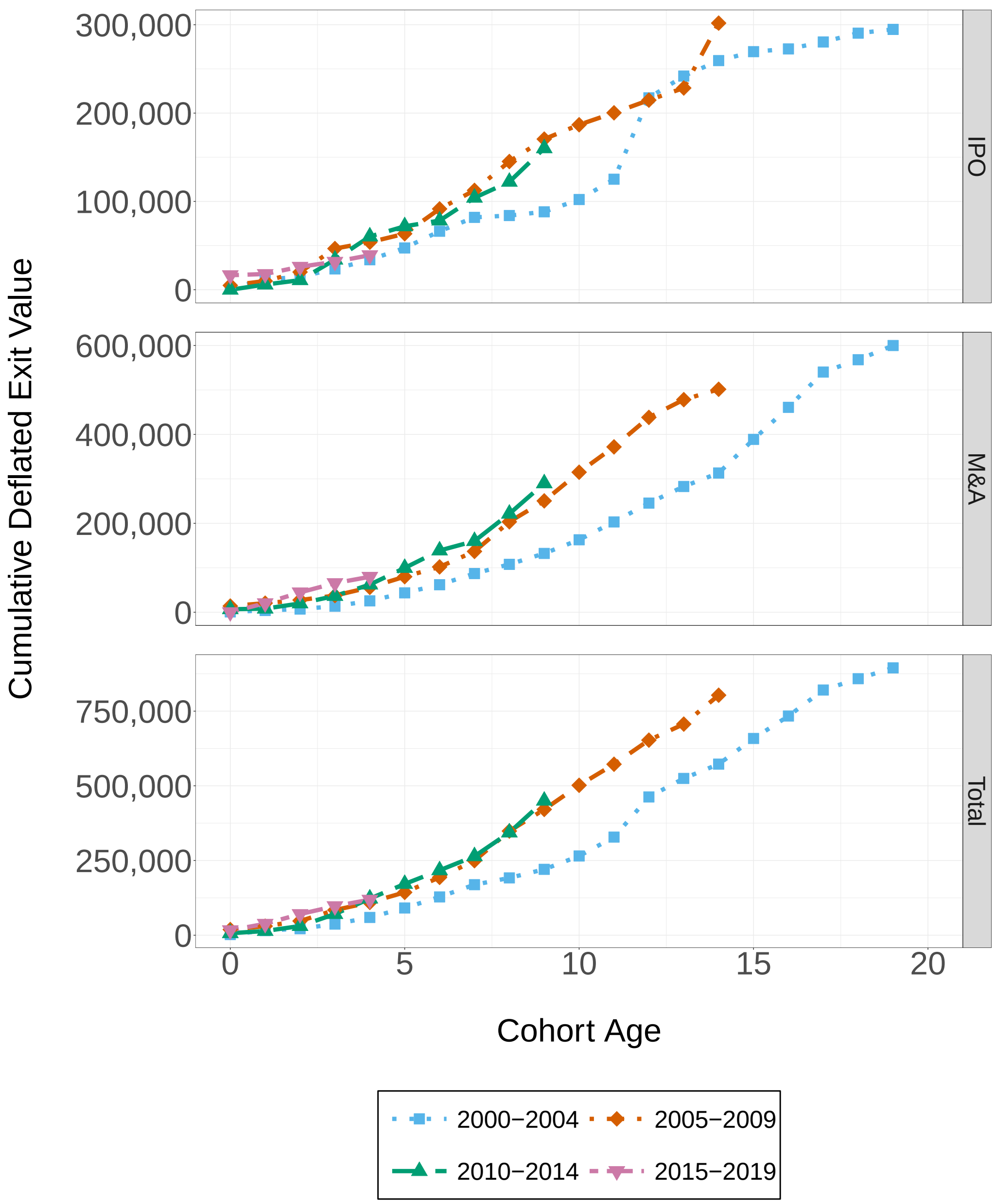


Figure 5: **Model-Implied Equilibrium Wage, Output, and Labor Share**. The dashed line denotes the onset of the transition. Output and the wage are normalized to one at the onset of the transition. The dashed line denotes the onset of the transition. Output is normalized to one at the onset of the transition. The “noisy” fluctuations in the figures is due to the use of Monte Carlo simulation to solve for the transition path.

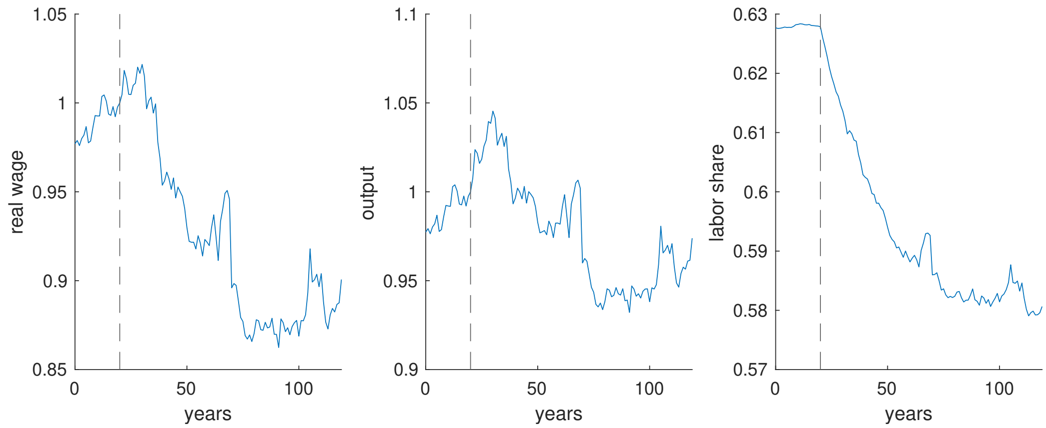


Figure 6: **Model-Implied Decomposition of the Labor share decline.** The left plot depicts the evolution of the term labeled “Average change of the labor share” in equation (34). The middle plot depicts the sum of “Share changes” and “Product of changes” (solid line). The dashed line depicts the term “Product of changes”. The last plot depicts the effect of deaths and births, i.e., the difference between the change in the labor share and the sum of the three components in (34). The vertical dashed line in all three plots depicts the onset of the transition. The “noisy” fluctuations in the figures is due to the use of Monte Carlo simulation to solve for the transition path.

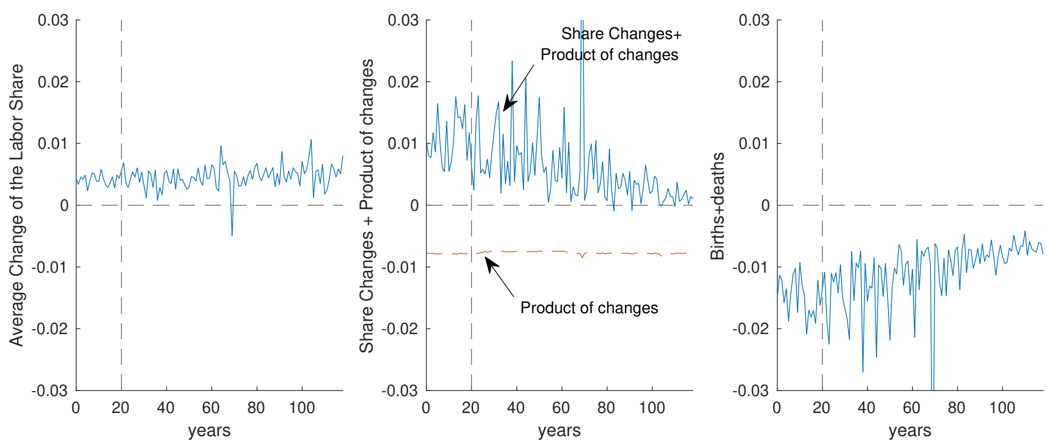


Figure 7: **Model-Implied Employment Share and Output of Entering Firms**. The left plot depicts the employment share of firms entering in year t. The right plot depicts output of these firms. The dashed line denotes the onset of the transition. Output is normalized to one at the onset of the transition. The "noisy" fluctuations in the figures is due to the use of Monte Carlo simulation to solve for the transition path.

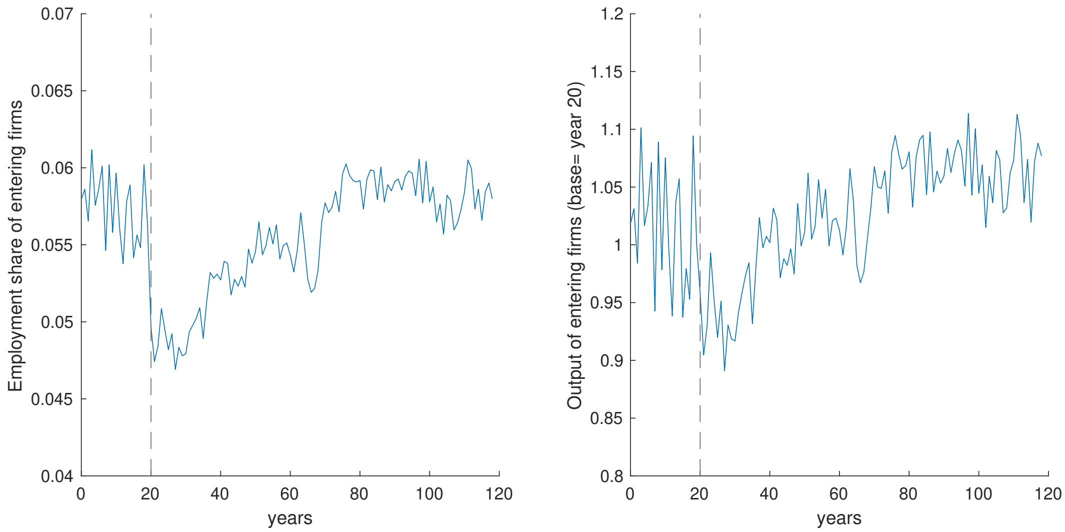


Figure 8: **Model-Implied Gross Job Creation and Destruction and TFP Dispersion.** The top left plot depicts gross job creation and destruction. The top right plot isolates job creation. These measures are computed as the employment-weighted average of $\frac{|l_{i,t+1}-l_{i,t}|}{\frac{1}{2}(l_{i,t+1}+l_{i,t})}1_{\{l_{i,t+1}>l_{i,t}\}}$ (gross job creation) and $\frac{|l_{i,t+1}-l_{i,t}|}{\frac{1}{2}(l_{i,t+1}+l_{i,t})}1_{\{l_{i,t+1}<l_{i,t}\}}$ (gross job destruction). The bottom left plot depicts the fraction of firms that exit every year. The bottom right plot depicts the cross-sectional standard deviation of the logarithm of total factor productivity. ($log(Z)$). The dashed line depicts the onset of the transition. The "noisy" fluctuations in the figures is due to the use of Monte Carlo simulation to solve for the transition path.

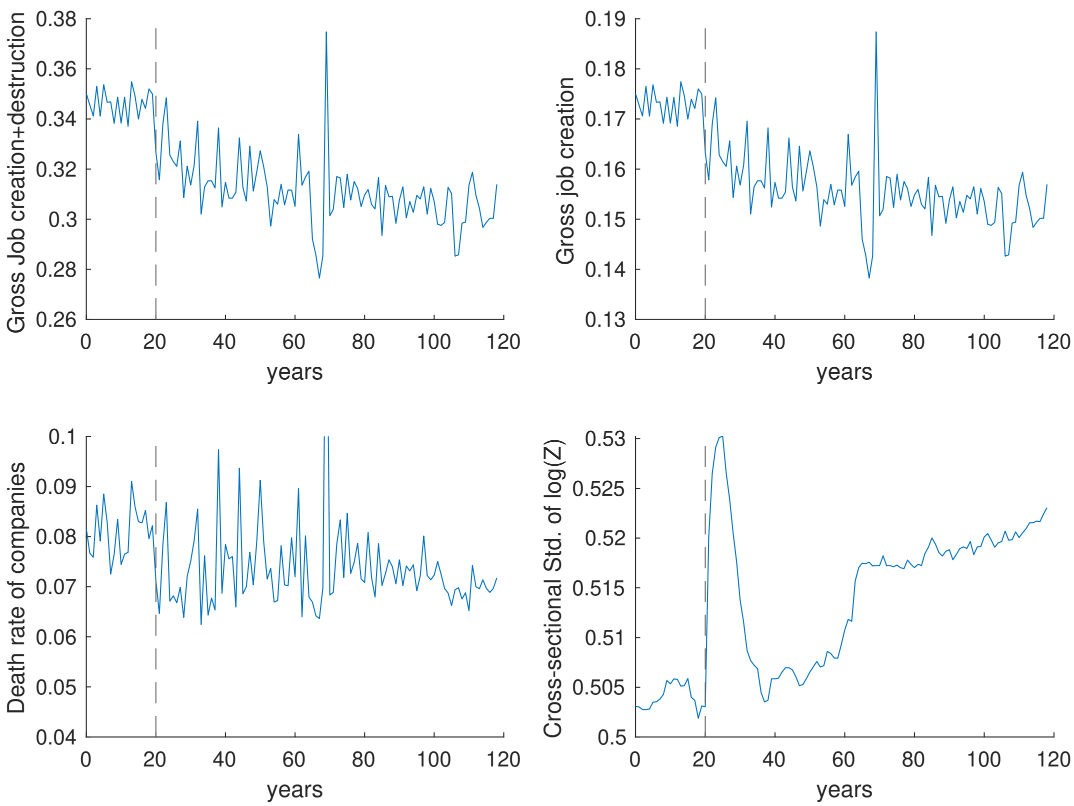


Table 1: **Sample of Switchers in NETS**
This table presents the number of NETS establishments the report a change in ownership over the years 1998–2014. Row 1 consists of all private payroll establishments. An establishment is private if it has a four digit SIC code less than 9000. Payroll establishments are those with at least 2 reported employees (including the owner) in the year prior to the acquisition. Row 2 removes establishments that exit the sample at the end of the year of the acquisition. Row 3 further removes transactions that we classify as a reorganization or spin-off. Row 4 further removes all observations where employment is imputed in the year before the acquisition or in the year after the acquisition. Row 5 (our analysis sample) further removes all establishments that report no change in employment from the year before the acquisition to the year after the acquisition. See Section 2.4 and Appendix E for further details.

Sample	N
Changes in ownership, all private payroll establishments	1,728,088
After removing exiting establishments	1,618,286
After further removing reorganizations and spin-offs	1,546,055
After further removing imputed employment	982,131
After further removing sticky employment	213,255

Table 2: **Employment Growth of Switchers in NETS** (First Specification)
This table reports results of regressions of changes in log-employment on an indicator equal to one if the acquiring firm is young. Data on establishment level employment and changes in ownership are taken from the National Establishment Time Series (NETS). The unit of observation is an establishment. The change in log-employment is measured from the year before the acquisition ($t-1$) to the year after the acquisition ($t+1$). The independent variables are an indicator Young Acquirer equal to one if the acquirer is less than 8 years old in the year of the acquisition and fixed effects that vary by column. The first column includes year fixed effects. The second column includes year×industry fixed effects where industry is defined as a 4-digit SIC industry. The third column further accounts for geographic variation by including year×industry×state fixed effects. All standard errors are clustered by year×industry×state. See Section 2.4 for further details.

	Dependent variable:		
	$\log L_{t+1} - \log L_{t-1}$		
	(1)	(2)	(3)
Young Acquirer	-0.039^{***}	-0.035^{***}	-0.027^{**}
	(0.010)	(0.010)	(0.012)
Fixed Effects	Year	Year×SIC4	Year×SIC4×State
S.E. Cluster	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State
Sample Period	1998–2014	1998–2014	1998–2014
Observations	213,255	213,255	213,255
R^2	0.015	0.119	0.504
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3: **Employment Growth of Switchers in NETS** (Main Specification)
This table reports results of regressions of changes in log-employment on an indicator equal to one if the acquiring firm is young and an interaction term equal to one if the acquiring firm is young and the acquisition occurs after 2005. Data on establishment level employment and changes in ownership are taken from the National Establishment Time Series (NETS). The unit of observation is an establishment. The change in log-employment is measured from the year before the acquisition ($t-1$) to the year after the acquisition ($t+1$). The independent variables are an indicator Young Acquirer equal to one if the acquirer is less than 8 years old in the year of the acquisition, an indicator Young Acquirer×Post-2005 equal to one if the acquirer is less than 8 years old in the year of the acquisition and the acquisition occurs after 2005, and fixed effects that vary by column. The first column includes year fixed effects. The second column includes year×industry fixed effects where industry is defined as a 4-digit SIC industry. The third column further accounts for geographic variation by including year×industry×state fixed effects. All standard errors are clustered by year×industry×state. See Section 2.4 for further details.

	Dependent variable:		
	$\log L_{t+1} - \log L_{t-1}$		
	(1)	(2)	(3)
Young Acquirer	-0.018^{*}	-0.018^{*}	-0.017
	(0.009)	(0.010)	(0.013)
Young Acquirer × Post-2005	-0.134^{***}	-0.127^{***}	-0.120^{**}
	(0.035)	(0.039)	(0.055)
Fixed Effects	Year	Year×SIC4	Year×SIC4×State
S.E. Cluster	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State
Sample Period	1998–2014	1998–2014	1998–2014
Observations	213,255	213,255	213,255
R^2	0.015	0.119	0.504
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 4: **Employment Growth of Switchers in NETS** (Target Age and Size Controls)

This table presents results of regressions of changes in log-employment on an indicator equal to one if the acquiring firm is young, an interaction term equal to one if the acquiring firm is young and the acquisition occurs after 2005, and additional controls for the age and size of the target establishment. Data on establishment level employment and changes in ownership are taken from the National Establishment Time Series (NETS). The unit of observation is an establishment. The first column repeats the specification presented in Table 3. The second column includes establishment age bin dummies, using the age bins [1,3], [4,7], and 8+. The third column includes a control for log employment of the target establishment in the year before the acquisition ($t-1$). The fourth column includes both establishment age bin dummies and a control for log employment of the target establishment in the year before the acquisition ($t-1$). All columns include year×industry×state fixed effects. All standard errors are clustered by year×industry×state. See Section 2.4 and Table 3 for further details.

	Dependent variable:			
	$\log L_{t+1} - \log L_{t-1}$			
	(1)	(2)	(3)	(4)
Young Acquirer	−0.017	−0.018	−0.029**	−0.023**
	(0.013)	(0.013)	(0.012)	(0.012)
Young Acquirer × Post-2005	−0.120**	−0.123**	−0.116**	−0.110**
	(0.055)	(0.055)	(0.050)	(0.050)
Fixed Effects	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State
Establishment Age Bins	No	Yes	No	Yes
Control for $\log L_{t-1}$	No	No	Yes	Yes
S.E. Cluster	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State	Year×SIC4×State
Sample Period	1998–2014	1998–2014	1998–2014	1998–2014
Observations	213,255	213,255	213,255	213,255
R^2	0.504	0.504	0.589	0.591

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5: **Parameter values for the model**

μ	0.00	α	0.54	$\bar{l}/L$	0.0169
σ	0.15	α^*	0.47	ρ	0.04
λ	0.07	$\log(Z^c)$	0.7	$m(\cdot)$	$\propto e^{-2.5z}1_{(z\in[0.3])}$

Appendix A Proofs

Proof of Lemma 1. Using (8) we obtain

$$\begin{aligned}
\frac{P_{t,t}}{\sum_{s\leq t} P_{t,s}} &= \frac{\pi_{t,t}}{\sum_{s\leq t}\left(\frac{h(t-s)}{h(0)}\right)\pi_{t,s}} = \frac{y_{t,t}-w_t l_{t,t}}{\sum_{s\leq t}\left(\frac{h(t-s)}{h(0)}\right)(y_{t,s}-w_t l_{t,s})} \\
&= \left(\frac{l_{t,t}}{\sum_{s\leq t} l_{t,s}}\right)\times\frac{\left(\frac{y_{t,t}}{l_{t,t}}-w_t\right)}{\sum_{s\leq t}\left(\frac{h(t-s)}{h(0)}\right)\left(\frac{l_{t,s}}{\sum_{s\leq t} l_{t,s}}\right)\left(\frac{y_{t,s}}{l_{t,s}}-w_t\right)}
\end{aligned}$$

Taking logarithms and then first differences on both sides

$$\begin{aligned}
\Delta\log\left(\frac{P_{t,t}}{\sum_{s\leq t} P_{t,s}}\right)-\Delta\log\left(\frac{l_{t,t}}{\sum_{s\leq t} l_{t,s}}\right) &= \Delta\log\frac{\frac{y_{t,t}}{l_{t,t}}-w_t}{\left(\sum_{s\leq t}\omega\left(t-s\right)\left(\frac{y_{t,s}}{l_{t,s}}\right)\right)-w_t} \\
&= \Delta\log\left(\frac{\delta_{t,t}-1}{\sum_{s\leq t}\omega_{t,s}\delta_{t,s}-1}\right)
\end{aligned}$$

■

Proof of Proposition 1. Substituting the optimal labor demand (17) into the profit function $\pi\left(Z_{i,t}\right)=f\left(Z_{i,t},l_{i,t}\right)-w\left(l_{i,t}+\bar{l}\right)$ of the firm gives the flow of profits:

$$\pi\left(z_{i,t}\right)=\left(1-\alpha\right)\left(\frac{w}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}e^{\frac{1}{1-\alpha}z_{i,t}}-\bar{l}w. \tag{50}$$

Before its termination, the value function of the firm solves the following differential equation:

$$V_{zz}\frac{\sigma^2}{2}+V_z\left(\mu-\frac{\sigma^2}{2}\right)-\left(r+\lambda\right)V+\pi\left(z_{i,t}\right)=0. \tag{51}$$

A particular solution V^P of this differential equation is

$$V^P\left(z\right)=\frac{2}{\sigma^2}\frac{1}{\omega_2-\omega_1}\left(\int_{z^*}^{z}e^{\omega_1(z-s)}\pi\left(s\right)ds+\int_{z}^{\infty}e^{\omega_2(z-s)}\pi\left(s\right)ds\right), \tag{52}$$

which can be verified by substituting (52) into (51). As a result, the general solution of (51) is

$$V\left(z\right)=C_1e^{\omega_1 z}+C_2e^{\omega_2 z}+V^P\left(z\right). \tag{53}$$

By standard arguments (value matching, smooth pasting, no bubble condition) we have that

$$V(z^*) = 0 \tag{54}$$

$$V_z(z^*) = 0 \tag{55}$$

$$\lim_{z\to\infty} V(z) = V^P(z) \tag{56}$$

Condition (56) implies that $C_2 = 0$ and condition (54) implies that

$$C_1 = -e^{-\omega_1 z^*} \frac{2}{\sigma^2} \frac{1}{\omega_2 - \omega_1} \left(\int_{z^*}^{\infty} e^{\omega_2(z^*-s)} \pi(s)\, ds \right). \tag{57}$$

Differentiating (53) with respect to z, evaluating the resulting expression at z^* and using (55) gives

$$\omega_1 C_1 e^{\omega_1 z^*} + \frac{2}{\sigma^2} \frac{\omega_2}{\omega_2 - \omega_1} \int_{z^*}^{\infty} e^{\omega_2(z-s)} \pi(s)\, ds = 0. \tag{58}$$

Using (57) inside (58) and re-arranging implies that

$$\int_{z^*}^{\infty} e^{-\omega_2 s} \pi(s)\, ds = 0. \tag{59}$$

Substituting (50) into (59) and integrating leads after some simplifications to (25). ■

Proof of Proposition 2. Letting $g(z)$ denote the mass of firms with log-productivity z in the steady state, the forward Kolmogorov equation implies that the density $g(z)$ obeys the differential equation

$$\frac{\sigma^2}{2} g_{zz} - \left(\mu - \frac{\sigma^2}{2}\right) g_z - \lambda g + \phi m(z) = 0 \tag{60}$$

subject to the boundary condition $g(z^*) = 0$ and $\lim_{z\to\infty} g(z) = 0$. Similar to the proof of Proposition 1, a particular solution of (60) is

$$g^P(z) = \frac{2}{\sigma^2} \frac{\phi}{\eta_2 - \eta_1} \left(\int_{z^*}^{z} e^{\eta_1(z-s)} m(s)\, ds + \int_{z}^{\infty} e^{\eta_2(z-s)} m(s)\, ds \right). \tag{61}$$

The general solution is therefore

$$g(z) = K_1 e^{\eta_1 z} + K_2 e^{\eta_2 z} + g^P(z) \tag{62}$$

The two boundary conditions $g(z^*) = 0$ and $\lim_{z\to\infty} g(z) = 0$ imply that $K_2 = 0$ and

$$K_1 = \frac{-g^P(z^*)}{e^{\eta_1 z^*}}. \tag{63}$$

Substituting (63) and $K_2 = 0$ into (62) leads to

$$g(z; z^*) = g^P(z) - g^P(z^*)\, e^{\eta_1(z-z^*)}, \tag{64}$$

which leads to (28). ■

Proof of Lemma 2. The left hand side of (29) is a decreasing function of w. Indeed, $\lim_{w\to 0}\left(\frac{w}{\alpha}\right)^{\frac{1}{\alpha-1}} = \infty$ and $\lim_{w\to\infty}\left(\frac{w}{\alpha}\right)^{\frac{1}{\alpha-1}} = 0$. The right hand size is increasing in w, which can be shown by differenating the right hand side of (29) with respect to z^* and then using the fact that z^* is equal to $\log(w)$ plus an additive constant, by (25). Specifically, using the fact that $g(z^*) = 0$, we have that

$$\frac{d}{dz^*}\int_{z^*}^{\infty} g(z; z^*)\, dz = \int_{z^*}^{\infty} \frac{\partial g(z; z^*)}{\partial z^*} dz =$$

$$= \left(\int_{z^*}^{\infty} e^{\eta_2(z^*-s)} m(s)\, ds\right)\left(\int_{z^*}^{\infty} e^{\eta_1(z-z^*)} dx\right)\frac{2\phi}{\sigma^2}\frac{\eta_1}{\eta_2-\eta_1} < 0 \tag{65}$$

and

$$\frac{d}{dz^*}\int_{z^*}^{\infty} \exp\left(\frac{1}{1-\alpha}z\right) g(z; z^*)\, dz = \int_{z^*}^{\infty} \exp\left(\frac{1}{1-\alpha}z\right)\frac{\partial g(z; z^*)}{\partial z^*} dz =$$

$$= \frac{2\phi}{\sigma^2}\frac{\eta_1}{\eta_2-\eta_1}\left(\int_{z^*}^{\infty} e^{\eta_2(z^*-s)} m(s)\, ds\right)\int_{z^*}^{\infty} e^{\eta_1(z-z^*)+\frac{1}{1-\alpha}z} dz < 0 \tag{66}$$

Using (65) and (66) implies that the right hand side of (29) is increasing in w and by inspection, the right hand side becomes strictly positive as $w\to\infty$. Combining all the above facts implies that the difference between the left and the right hand side of (29) tends to positive infinity as $w\to 0$ and to a negative limit as $w\to\infty$. By continuity, we conclude that there is a unique w, for which equation (29) holds. ■

Proof of Proposition 3. To prove proposition 3, we start by proving the following Lemma, which shows the correspondence of the decentralized equilibrium with a central planning problem.

Lemma 3 *Assume that $\rho = 0$ and consider the optimization problem of maximizing the utility of consumption*

$$\max_{l(z),k(z),x^*} U(Y^* - \delta K^*) \tag{67}$$

where

$$Y^* \equiv \int_{x^*}^{\infty} g(z, x^*)\exp(z)\, k^{\beta}(z)\, l^{\alpha}(z)\, dz, \tag{68}$$

and

$$K^* \equiv \int_{x^*}^{\infty} g(z, x^*)\, k(z)\, dz$$

subject to the constraint

$$\int_{x^*}^{\infty} g(z, x^*) \left(\bar{l} + l(z)\right) dz = L. \tag{69}$$

This "central planning" problem has the same solution as the market equilibrium, namely $x^* = z^*$, $k(z) = k^*(z)$ and $l(z) = l^*(z)$.

Proof of Lemma 3. Maximizing (67) over $k(z)$ gives

$$k(z) = \left(Z_{it} l_{it}^{\alpha}\right)^{\frac{1}{1-\beta}} \left(\frac{\delta}{\beta}\right)^{\frac{1}{\beta-1}}, \tag{70}$$

which is the same first order condition as for the market allocation when $r_t = 0$.

Substituting (70) into (67) reduces the optimization problem to

$$\max_{l(z), x^*} U\left(\int_{x^*}^{\infty} g(z, x^*)(1-\beta)\left(\frac{\delta}{\beta}\right)^{\frac{\beta}{\beta-1}} \exp\left(\frac{z}{1-\beta}\right)\left(l_{it}^{\frac{\alpha}{1-\beta}}\right) dz\right)$$

subject to the constraint (69). Define

$$\widehat{\alpha} \equiv \frac{\alpha}{1-\beta}, \text{ and } \widehat{z}(z) \equiv \log\left[(1-\beta)\left(\frac{\delta}{\beta}\right)^{\frac{\beta}{\beta-1}}\right] + \frac{1}{1-\beta} z,$$

and re-write the optimization problem more compactly as

$$\max_{l(z), x^*} U\left(\int_{x^*}^{\infty} g(z, x^*) \exp(\widehat{z}(z))\, l_{it}^{\widehat{\alpha}} dz\right)$$

Attaching a Lagrange multiplier φ to the constraint (69) and maximizing over $l(x)$ leads to

$$\exp(\widehat{z}(z))\, \widehat{\alpha} l^{\widehat{\alpha}-1}(z) = \varphi, \tag{71}$$

or after re-arranging

$$l(z) = \exp\left(\frac{\widehat{z}(z)}{1-\widehat{\alpha}}\right)\left(\frac{\varphi}{\widehat{\alpha}}\right)^{\frac{1}{\widehat{\alpha}-1}}. \tag{72}$$

Differentiating Y with respect to x^* and seting the derivative equal to zero leads to

$$\frac{\partial}{\partial x^*}\int_{x^*}^{\infty} g(z, x^*) \exp(\widehat{z}(z))\, l^{\widehat{\alpha}}(z)\, dz = \varphi \frac{\partial}{\partial x^*}\int_{x^*}^{\infty} g(z, x^*)\left(\bar{l} + l(z)\right) dz, \tag{73}$$

Using (28) and differentiating implies

$$\frac{\partial}{\partial x^*} g(z; x^*) = -\left(\int_{x^*}^{\infty} e^{\eta_2 (x^* - s)} n(s)\, ds\right) \frac{2\phi}{\sigma^2}\left[e^{\eta_1 (z - x^*)}\right]. \tag{74}$$

Using (74) inside (73) and noting that $g(x^*; x^*) = 0$, leads after some simplifications to

$$\int_{x^*}^{\infty} e^{\eta_1 z} \left[\exp\left(\widehat{z}(z)\right) l^{\widehat{\alpha}}(z) - \varphi\left(\bar{l} + l(z)\right)\right] dz = 0 \tag{75}$$

Substituting (72) inside (75) and re-arranging gives

$$\int_{x^*}^{\infty} e^{\eta_1 z} \left[(1-\widehat{\alpha})\left(\frac{\varphi}{\widehat{\alpha}}\right)^{\frac{1}{\widehat{\alpha}-1}} e^{\frac{\widehat{z}(z)}{1-\widehat{\alpha}}} - \varphi\bar{l}\right] dz = 0. \tag{76}$$

We guess (and verify shortly) that the Lagrange multiplier of the optimization problem φ equals the market clearing wage w. With this supposition, the expression inside square brackets in equation (76) is equal to the firm profits $\pi(z)$ in the presence of capital.[20] When $\rho = 0$, we have that $\eta_1 = -\omega_2$ and hence (76) coincides with $\int_{x^*}^{\infty} e^{\eta_1 z}\pi(z)\,dz = 0$, i.e., the optimality condition for z^* (equation (59)), leading to the conclusion that $x^* = z^*$. In addition, the supposition that $\varphi = w$ implies that $l(z) = l^*(z)$, upon comparing (72) with (17).

To summarize, setting the Lagrange multiplier $\varphi = w$, the optimal allocations chosen by the central planner coincide with those chosen for the market. Moreover, when $\varphi = w$, the labor market clears (since the market allocation is feasible), which implies that the optimal choices of the central planner are also feasible. Accordingly, $\varphi = w$ is the Lagrange multiplier associated with the central planning problem, completing the proof. ■

In light of Lemma 3, the competitive-market equilibrium is identical to the solution of a central planning problem, which maximizes steady state welfare. Letting $z^c = \log(Z^c)$, and noting that $U(\cdot)$ is monotone, the market equilibrium is the solution to the maximization problem

$$C^{SS} = \max_{l(z),k(z),x^*} \int_{x^*}^{\infty} g(z, x^*) \exp(z^c + z)\, k^{\beta}(z)\, l^{\alpha}(z)\, dz - \delta \int_{x^*}^{\infty} g(z, x^*)\, k(z)\, dz \tag{77}$$

subject to the constraint (69). Using the envelope theorem, noting that $x^* = z^*$, differentiating (77) with respect to α, z^c, and β and evaluating around $z^c = 0$ gives

$$\begin{aligned}
\frac{\partial C^{SS}}{\partial \alpha} &= \int_{z^*}^{\infty} g(z, z^*) \exp(z)\, k^{\beta}(z)\, l^{\alpha}(z) \log\left(l^*(z)\right) dz, \\
\frac{\partial C^{SS}}{\partial \beta} &= \int_{z^*}^{\infty} g(z, z^*) \exp(z)\, k^{\beta}(z)\, l^{\alpha}(z) \log\left(k^*(z)\right) dz, \\
\frac{\partial C^{SS}}{\partial z^c} &= \int_{z^*}^{\infty} g(z, z^*) \exp(z)\, k^{\beta}(z)\, l^{\alpha}(z)\, dz.
\end{aligned}$$

[20] To see this, note that when $\varphi = w$ and $r = 0$, the firm maximizes $\pi(z) = e^z k^{\beta} l^{\alpha} - \varphi\left(l + \bar{l}\right) - \delta k$. Solving this maximization problem for k and l gives the expression inside square brackets in equation (76).

Because the change in steady state output is dC^{SS}, we have

$$\frac{\partial C^{SS}}{\partial \alpha} d\alpha + \frac{\partial C^{SS}}{\partial \beta} d\beta + \frac{\partial C^{SS}}{\partial z^c} dz^c = dC^{SS}. \tag{78}$$

The implicit function theorem, along with equations (78), (17) and the definition of $\widetilde{g}(z, z^*)$ in the statement of the proposition imply that

$$\begin{aligned} dz^c &= \frac{dC^{SS}}{\frac{\partial C^{SS}}{\partial z^c}} - \frac{\frac{\partial C^{SS}}{\partial \alpha}}{\frac{\partial C^{SS}}{\partial z^c}} d\alpha - \frac{\frac{\partial C^{SS}}{\partial \beta}}{\frac{\partial C^{SS}}{\partial z^c}} d\beta & (79) \\ &= \frac{dC^{SS}}{C^{SS}} - \left(\int_{z^*}^{\infty} \widetilde{g}(z) \log l^*(z)\, dz\right) d\alpha - \left(\int_{z^*}^{\infty} \widetilde{g}(z) \log k^*(z)\, dz\right) d\beta, & (80) \end{aligned}$$

where we used (17) to obtain (80).

Equation (79) provides an expression for how z^c needs to change in response to $d\alpha$ and $d\beta$ in order to ensure that the change in steady state output equals dC^{SS}. For such a joint change, we are interested in how Y^{new} changes, which is given by

$$\begin{aligned} Y^{\text{new}} &= \int_{z^*}^{\infty} m(z) \exp(z^c + z) \left(k^*(z^c + z; \alpha, \beta)\right)^{\beta} \left(l^*(z^c + z; , \beta\alpha)\right)^{\alpha} dz \\ &= \int_{z^*}^{\infty} m(z) \exp(z^c + z) \exp\{\alpha \log l^*(z^c + z; \alpha, \beta)\} \exp\{\beta \log k^*(z^c + z; \alpha, \beta)\}\, dz \end{aligned}$$

Computing the total derivative of Y^{new}, evaluating around $z^c = 0$ and dividing by Y^{new} gives

$$\begin{aligned} \frac{dY^{\text{new}}}{Y^{\text{new}}}|_{z^c=0} &= dz^c + \left(\int_{z^*}^{\infty} (\log l^*(z))\, \widetilde{m}(z)\, dz\right) d\alpha + \alpha \left(\int_{z^*}^{\infty} \widetilde{m}(z)\, d\log(l^*(z))\, dz\right) & (81) \\ &+ \left(\int_{z^*}^{\infty} (\log k^*(z))\, \widetilde{m}(z)\, dz\right) d\beta + \beta \left(\int_{z^*}^{\infty} \widetilde{m}(z)\, d\log(k^*(z))\, dz\right). \end{aligned}$$

Next we note that $wl^*(z) = \alpha Y(z)$, $\delta k(z) = \beta Y(z)$, and therefore

$$\begin{aligned} d\log(l^*(z)) &= \frac{1}{\alpha} d\alpha + \left(\frac{dY(z)}{Y(z)}|_{z^c=0}\right) \\ d\log(k^*(z)) &= \frac{1}{\beta} d\beta + \left(\frac{dY(z)}{Y(z)}|_{z^c=0}\right) \end{aligned}$$

Accordingly (81) becomes

$$\begin{aligned} \frac{dY^{\text{new}}}{Y^{\text{new}}}|_{z^c=0} &= dz^c + \left(\int_{z^*}^{\infty} (\log l^*(z) + 1)\, \widetilde{m}(z)\, dz\right) d\alpha + \left(\int_{z^*}^{\infty} (\log k^*(z) + 1)\, \widetilde{m}(z)\, dz\right) d\beta & (82) \\ &+ (\alpha + \beta) \int_{z^*}^{\infty} \widetilde{m}(z) \left(\frac{dY(z)}{Y(z)}|_{z^c=0}\right) dz. \end{aligned}$$

Since $\widetilde{m}(z)$ are the output weights of new firms, we have that $\int_{z^*}^{\infty} \widetilde{m}(z) \left(\frac{dY(z)}{Y(z)}|_{z^c=0} \right) dz = \frac{dY^{\text{new}}}{Y^{\text{new}}}|_{z^c=0}$, which allows us to re-write (82) as

$$\begin{aligned} (1-\alpha-\beta) \frac{dY^{\text{new}}}{Y^{\text{new}}}|_{z^c=0} &= dz^c + \left(\int_{z^*}^{\infty} \left(\log l^* (z) + 1 \right) \widetilde{m}(z)\, dz \right) d\alpha \\ &\quad + \left(\int_{z^*}^{\infty} \left(\log k^* (z) + 1 \right) \widetilde{m}(z)\, dz \right) d\beta. \end{aligned} \tag{83}$$

Combining (83) with (80) gives

$$\frac{dC^{SS}}{C^{SS}} = (1-\alpha-\beta) \frac{dY^{\text{new}}}{Y^{\text{new}}}|_{z^c=0} + \left(\int_{z^*}^{\infty} \log l^* (z) \left(\widetilde{g}(z) - \widetilde{m}(z) \right) dz - 1 \right) (d\alpha + d\beta)$$

■

Appendix B Monopolistic competition

Throughout the text, we assumed the presence of one final good and various decreasing returns to scale technologies to produce it. Modifying the model to allow for differentiated products (produced by monopolistically competitive producers) is an alternative approach to obtaining the same results, even if the production technology features constant returns to scale. Similarly, in such a context the productivity process Z_t could be also interpreted as emanating from preference shocks. Since these points are well understood in the literature, we only provide a sketch of the arguments.

In the modified setup each firm employs a production technology of the form $y_{i,t} = Z^*_{i,t} l_{i,t}$, but that the representative consumer views the products as being differentiated. Her utility function takes the form

$$U\left(\int_{i\in I_t} \omega_{i,t} y_{i,t}^{\alpha^*} di\right) \tag{84}$$

where $\omega_{i,t}$ are preference weights, $y_{i,t}$ is quantity of each good, α^* controls the extent of substitution between different products, I is the set of firms alive at time t and U is the consumer's utility function.[21] Use labor as the numeraire and express the consumer's budget constraint as

$$\int_{i\in I_t} p_{i,t} y_{i,t} di = L + \int_{i\in I_t} \pi_{i,t} di \tag{85}$$

wher p_i is the price of each differentiated good, and

$$\pi_{i,t} = p_{i,t} y_{i,t} - \left(l_{i,t} + \bar{l}\right) \tag{86}$$

are the profits of each firm. We note here that upon substituting (86) into (85) and simplifying, we obtain Walra's law in the sense that (85) implies labor market clearing. Attaching a Lagrange multiplier λ to (84) and maximizing with respect to $y_{i,t}$ gives

$$U'(.)\, \alpha^* \omega_{i,t} y_{i,t}^{\alpha^*-1} = \lambda p_{i,t}. \tag{87}$$

Substituting (87) into (86) and using $y_{i,t} = Z^*_{i,t} l_{i,t}$ shows that the firm's maximization problem is equivalent to

$$\max_{l_{i,t}} \alpha^* \omega_{i,t} \left(Z^*_{i,t} l_{i,t}\right)^{\alpha^*} - \frac{\lambda}{U'(.)}\left(l - \bar{l}\right). \tag{88}$$

[21]In this context, one can think of exogenous firm death as a jump in $\omega_{i,t}$ to zero forever.

Maximizing (88) and comparing the first-order conditions for two firms i and j gives

$$\frac{\omega_{i,t}\left(Z_{i,t}^{*}\right)^{\alpha^{*}}\left(l_{i,t}\right)^{\alpha^{*}-1}}{\omega_{j,t}\left(Z_{j,t}^{*}\right)^{\alpha^{*}}\left(l_{j,t}\right)^{\alpha^{*}-1}}=1. \tag{89}$$

Equation (89) shows that the allocation of labor across firms has exactly the same form as in the text, except that now $\omega_{i,t}\left(Z_{i,t}^{*}\right)^{\alpha^{*}}$ plays the role of Z_t in the text and α^* plays the role of α. What we refer to as "productivity" in the text includes an influnence from preferences $\omega_{i,t}$ and the degree of substitutabity α^* across products acts in a manner similar similar to the production function curvature parameter α.

Appendix C Numerical algorithm for computing the transition dynamics

In this section we provide a brief description of our numerical algorithm to solve for the transition path. The key difficulty preventing a closed from solution along the transition path is that both the wage, the interest rate and the threshold level of productivity that leads to endogenous bankruptcy (for the two different kinds of firms) are now functions of time rather than constants. To solve for the transition dynamics, we start with an initial guess for the productivity thresholds that trigger bankruptcy. With that guess in hand, we simulate an economy whereby new firms arrive each year, with idiosyncratic shocks that follow the dynamics (15). The number of new firms is in principle irrelevant for the model, with a higher number reducing simulation error at the expense of computational power.[22]

Using the cross-section of simulated productivities at the end of each year, we start by forming a guess on the bankruptcy thresholds of the two types of firms. Taking these thresholds as given, we determine the market clearing wage and output. Fixing this time-series of wages and output, we then use a binomial tree with 200 years and time increment $dt = 0.1$ to solve for the optimal termination thresholds for the pre- and post-transition firms separately. Using these optimal termination policies, we repeat the wage and output calculation and iterate to convergence.

[22]For our simulation we choose that number to be 10,000.

Appendix D Additional Compustat Results

D.1 Compustat by Sector

Our measures of employment, sales, and market value contribution measures in Section 2.1 show that the ratio of the ratio of market value (or sales) contribution to employment contribution has increased over time. In this section we show that the same patterns hold when we perform the analysis at the level of individual sectors.

We construct two additional measures of employment, sales, and market value contribution that account for firms' sector. Our first alternative measure, presented in equation 90, separately measures the contribution of young firms from each sector relative to the universe of public firms. Our second alternative measure, presented in equation 91, separately measures the contribution of young firms from each sector relative to the mature public firms in the same sector.

In equation form, letting X denote either employment, sales, or market value, we define the contribution of the year t IPO cohort from sector s as:

$$\text{X Contribution in Total}_{s,t} = \frac{\text{X of IPO Firms (Excluding Mature Firms)}_{s,t}}{\text{Total X}_{t-1}} \tag{90}$$

$$\text{X Contribution in Sector}_{s,t} = \frac{\text{X of IPO Firms (Excluding Mature Firms)}_{s,t}}{\text{Sector X}_{s,t-1}} \tag{91}$$

Continuing with the format of our main results, for each of the two measures of sector specific contributions, we construct the cumulative employment, sales, and market value contributions of 5-year IPO cohort bin as follows:

$$\text{X Contribution in Total}_{s,bin} = \sum_{i \in Bin} \text{X Contribution in Total}_{s,i} \tag{92}$$

$$\text{X Contribution in Sector}_{s,bin} = \sum_{i \in Bin} \text{X Contribution in Sector}_{s,i} \tag{93}$$

Figure D.1 presents the ratio of the sales and market value contributions to the employment contributions for each sector. In Panels A and B, we measure the contribution of young firms from each sector of the economy relative to the universe of public firms. Panel A presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. In Panels C and D, we measure the contribution of young firms from each sector of the economy relative to mature public firms in the same sector. Panel C presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel D presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel E presents the number of firms going public in each sector and IPO cohort bin, after excluding firms that were founded more than 10 years prior to their IPO.

D.2 Operating Income

Figure D.2 presents a slightly modified version of the analysis of Section 2.1, in which we present results for employment, operating income, and market value. Operating income is Compustat variable OIBDP.

We measure the employment, operating income, and market value contribution of an IPO cohort as a share of the total market value and employment of public firms in the prior year. We then measure the contribution of an IPO cohort bin as the sum of the contributions of the different IPO cohorts in the bin.

Figure D.1: **Contribution of IPO Cohorts, By Sector**
Data on employment, sales, and market values of US public firms are taken from Compustat. Data on firm founding years are described in the text. We exclude from IPO cohorts all firms that were founded more than 10 years prior to their IPO. In Panels A and B, we measure the contribution of young firms from each sector of the economy relative to the universe of public firms. Panel A presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. In Panels C and D, we measure the contribution of young firms from each sector of the economy relative to mature public firms in the same sector. Panel C presents the logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel D presents the normalized (1985–1989 cohort = 0) logarithm of the ratio of the sales and market value contributions to the employment contributions. Panel E presents the number of firms going public in each sector and IPO cohort bin, after excluding firms that were founded more than 10 years prior to their IPO. See Section D.1 for further details. [Images are on the next five pages.]

Figure D.1: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(a) Ratio of Contributions, Relative to All of Public Firms

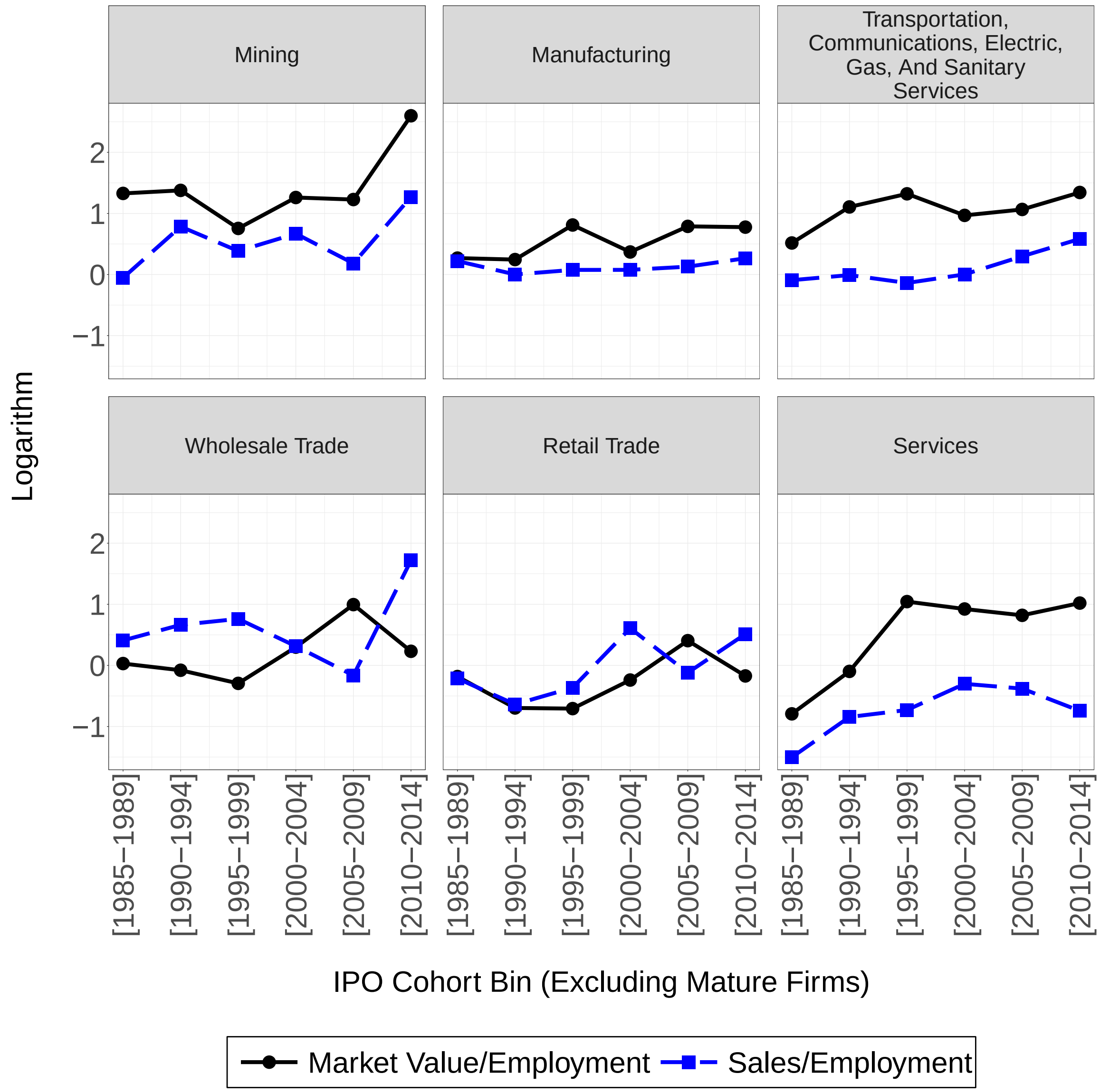


Figure D.1: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(b) Ratio of Contributions Normalized, Relative to All of Public Firms

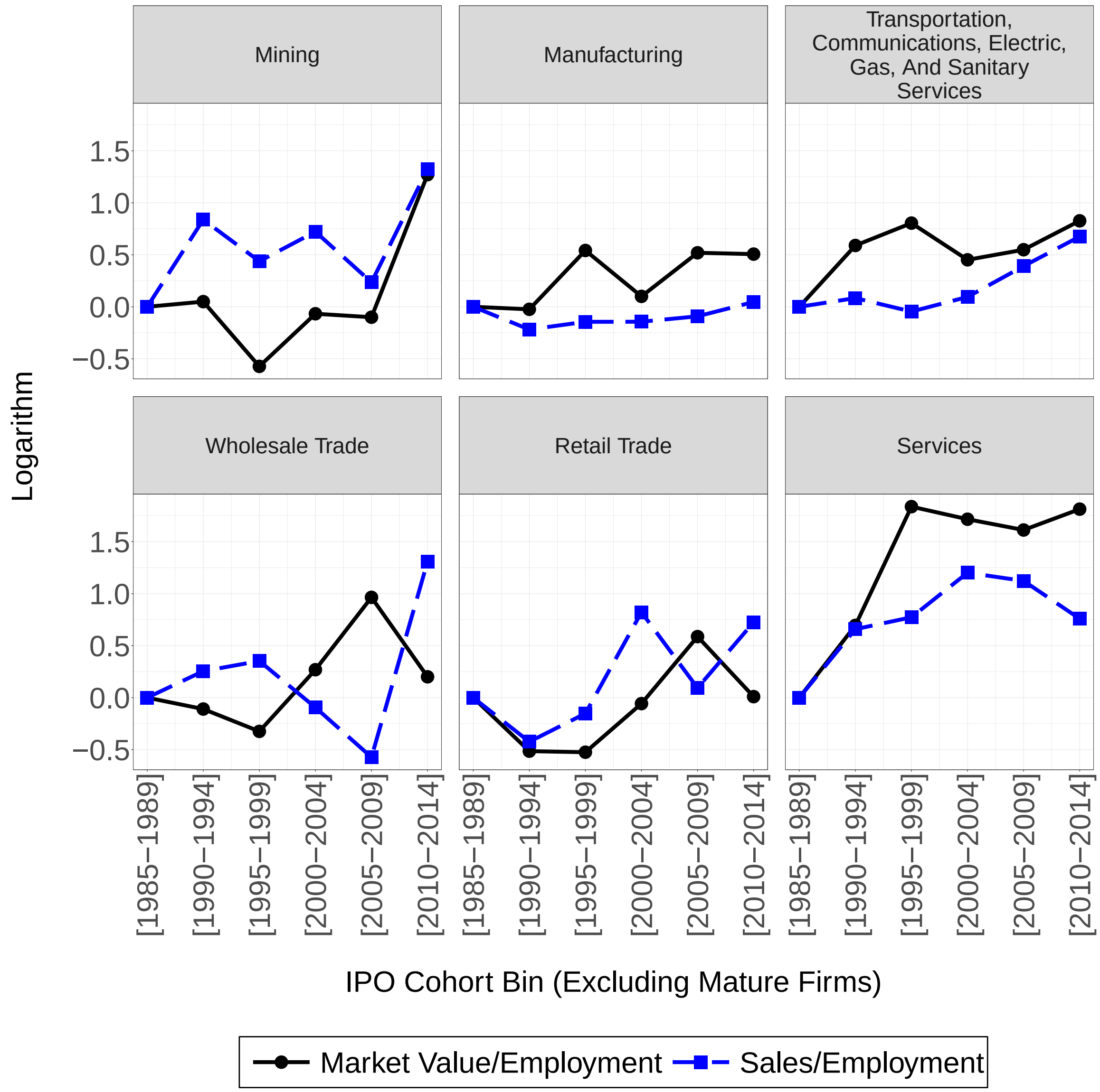


Figure D.1: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(c) Ratio of Contributions, Relative to Public Firms in Sector

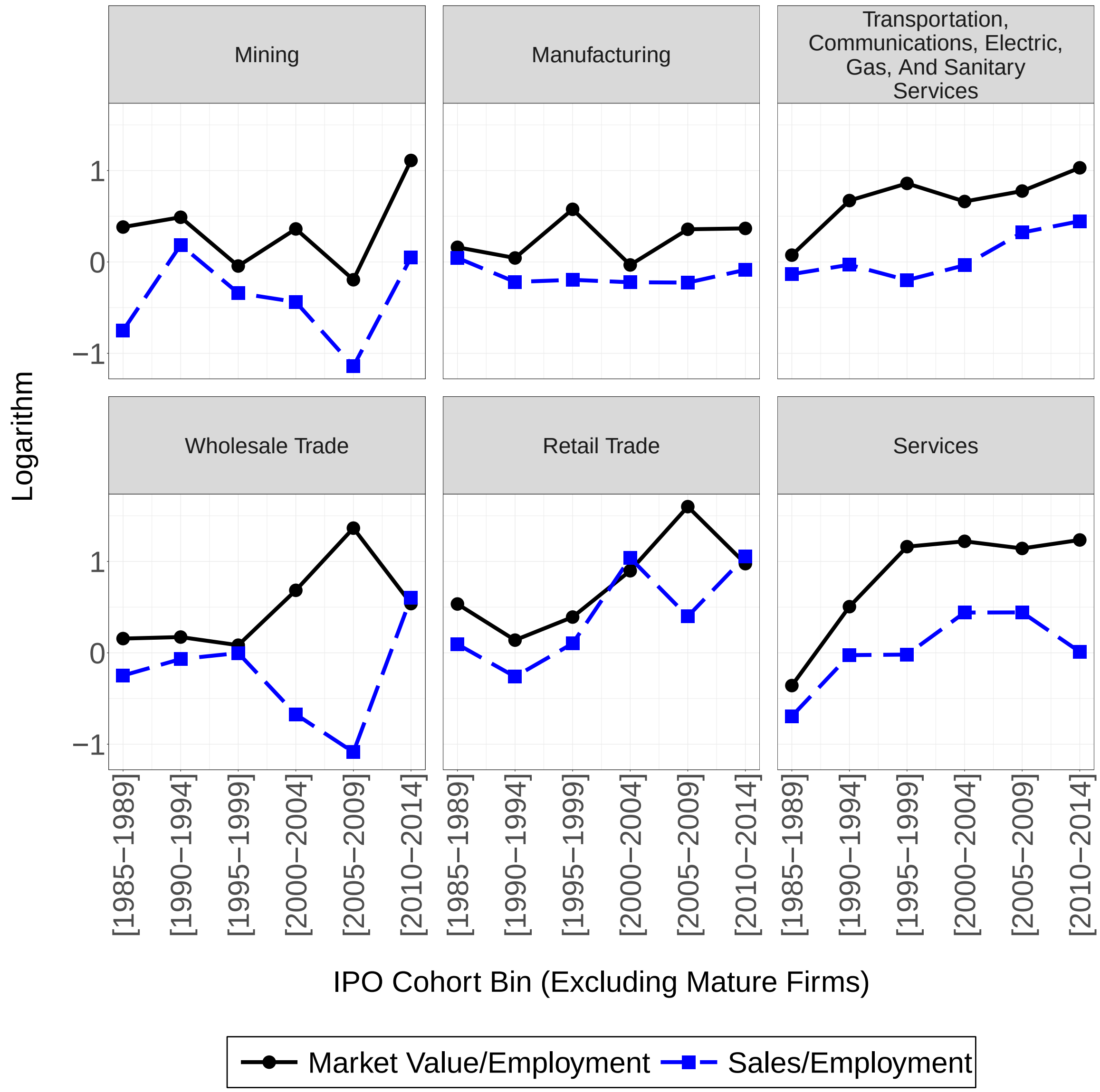


Figure D.1: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(d) Ratio of Contributions Normalized, Relative to Public Firms in Sector

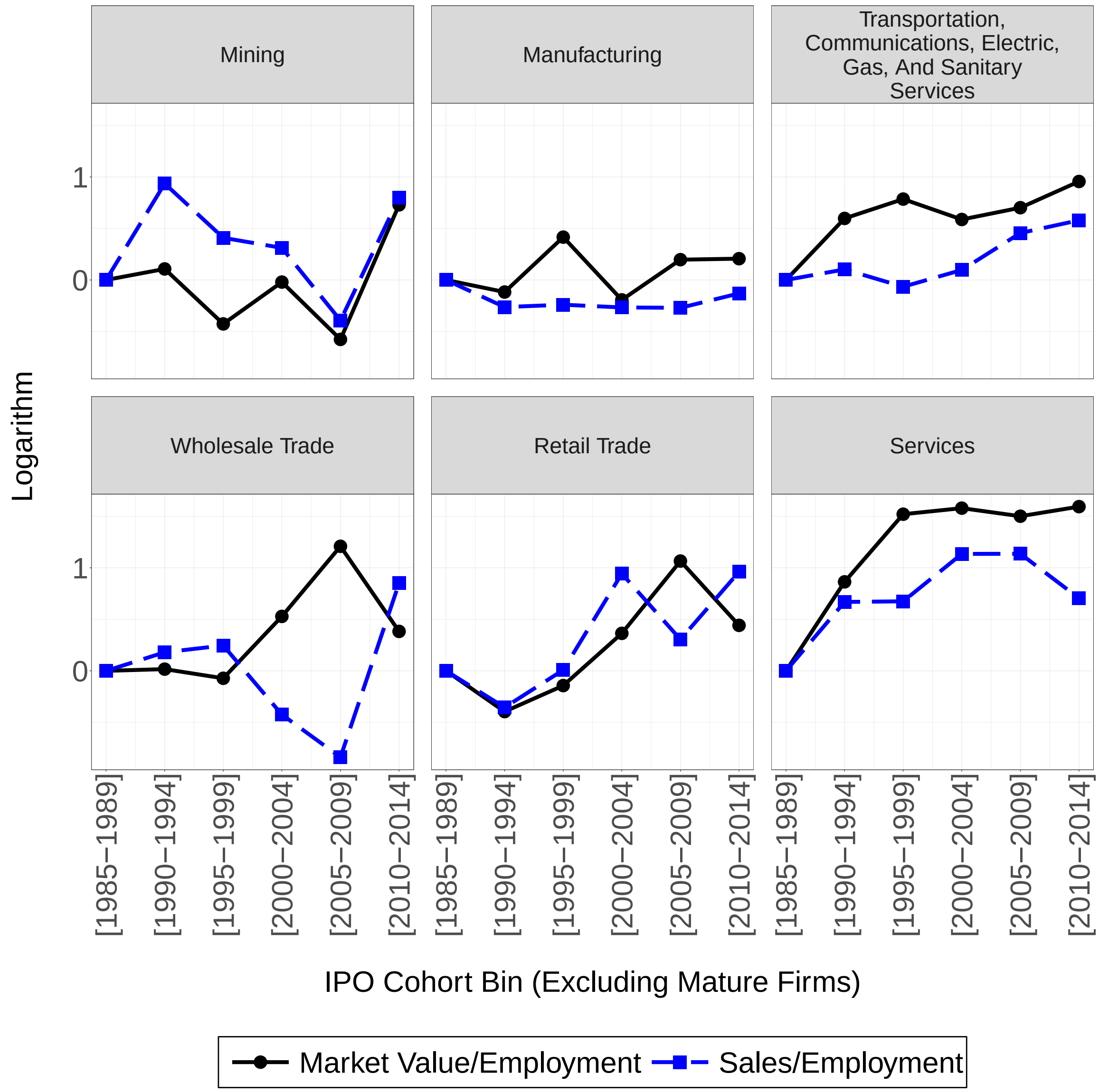


Figure D.1: **Contribution of IPO Cohorts, By Sector (Continued from Previous Page)**

(e) Number of Firms in each IPO Cohort, Excluding Mature Firms

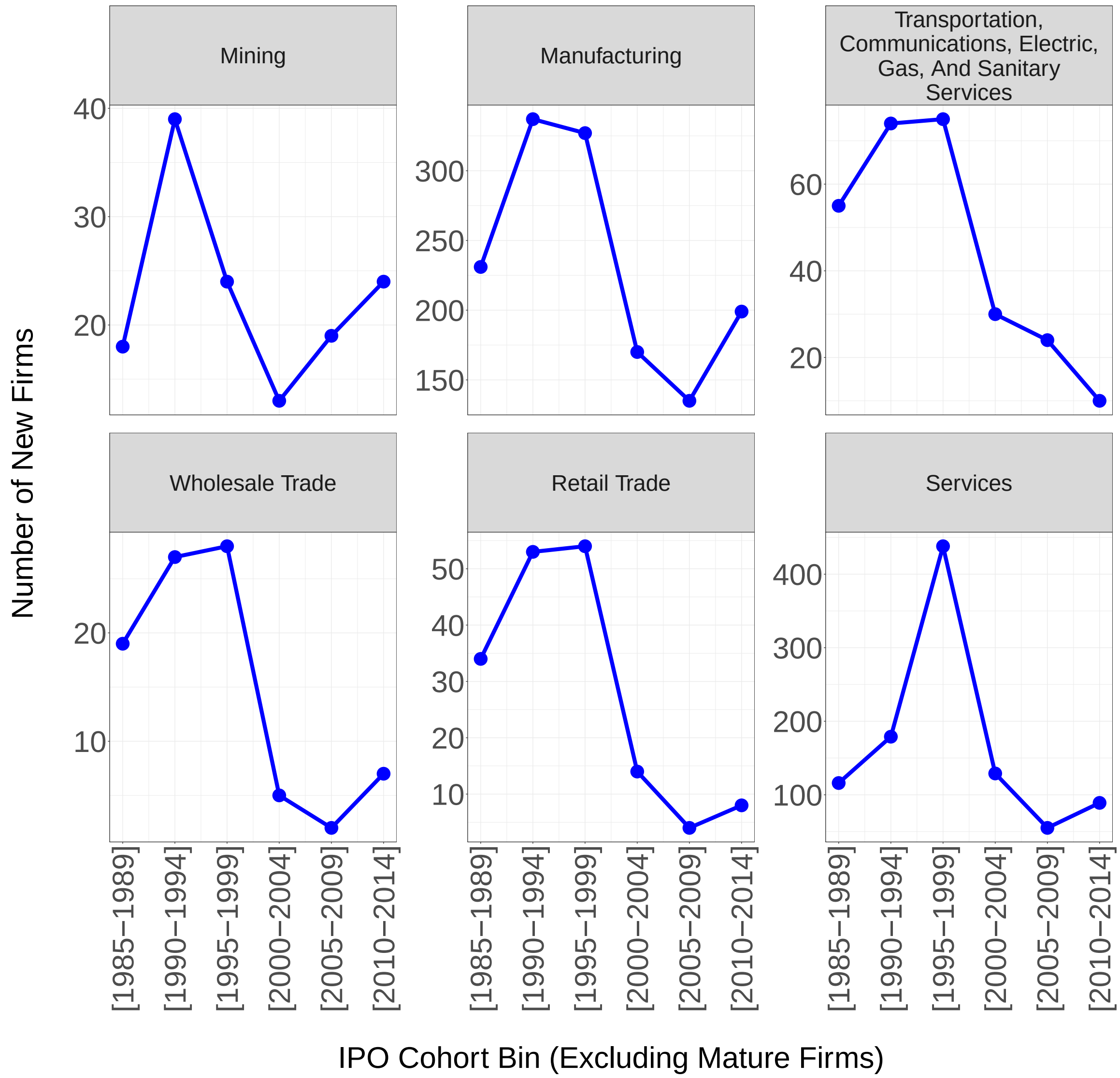
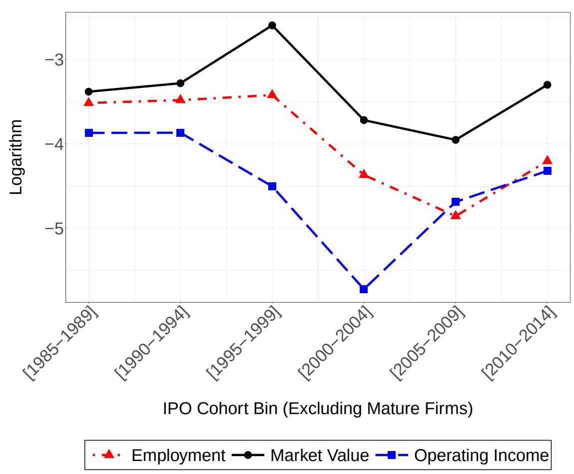


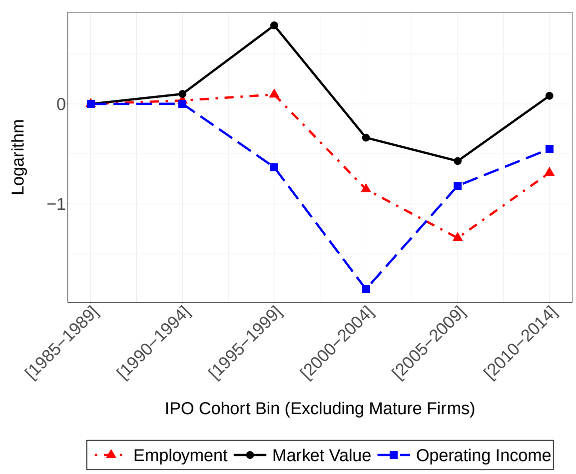
Figure D.2: **Employment, Operating Income, and Market Value Contributions of IPO Cohorts**

Data on employment, operating income, and market values of US public firms are taken from Compustat. Operating income is Compustat variable OIBDP. Data on firm founding years are described in the text. We exclude from IPO cohorts all firms that were founded more than 10 years prior to their IPO. We measure the employment, operating income, and market value contribution of an IPO cohort as a share of the total employment, operating income, and market value of public firms in the prior year. We then measure the contribution of an IPO cohort bin as the sum of the contributions of the different IPO cohorts in the bin. Panel A presents the logarithm of the employment, operating income, and market value contributions of each IPO cohort bin since 1985. Panel B presents the normalized (1985–1989 cohort = 0) logarithm of the employment, operating income, and market value contributions. Panel C presents the logarithm of the ratio of the operating income and market value contributions to the employment contributions. Panel D present the normalized (1985–1989 cohort = 0) logarithm of the ratio of the operating income and market value contributions to the employment contributions. See Section 2.1 for further details.

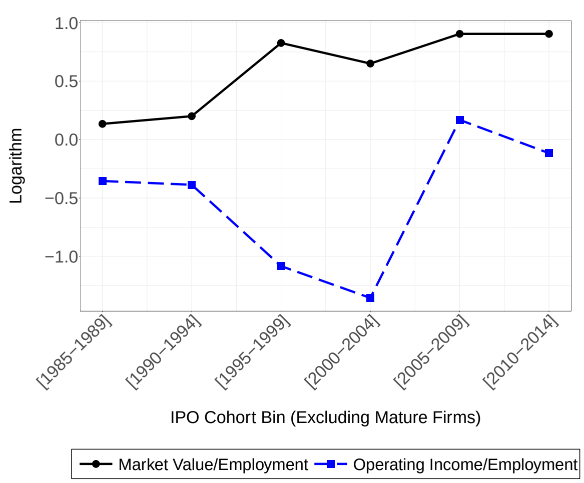
(a) Contribution



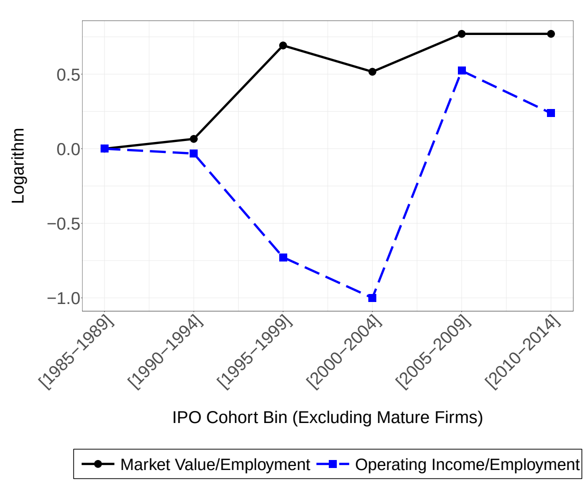
(b) Contribution Normalized



(c) Ratio of Contributions



(d) Ratio of Contributions Normalized



Appendix E Constructing Founding Year in NETS

This appendix describes our classification of changes in ownership and our adjustments to firm founding year that account for firm reorganizations, spin-offs, and mergers.

E.1 Classifying Changes in Ownership

We construct the set of all firms in year t that are destination of a switcher (destination) and all firms in year t-1 that are home of a switcher (home). Each home-destination pair is classified as one of the following mutually exclusive transactions.

1. **Reorganization** A home-destination pair is defined as a reorganization if all of the following are true:
   (a) The destination is a firm that had no establishments in year t-1.
   (b) The establishments of the destination firm are precisely the continuing establishments of the home firm.
2. **Spin-Off** A home-destination pair is defined as a spin-off if all of the following are true:
   (a) The destination is a firm that had no establishments in year t-1.
   (b) The establishments of the destination firm are a strict subset of the continuing establishments of the home firm.
3. **Merger** A home-destination pair is defined as a merger if all of the following are true:
   (a) The destination is a firm that had no establishments in year t-1.
   (b) The destination acquired establishments from more than one firm.
4. **Acquisition** A home-destination pair is defined as part of an acquisition if it is not a reorganization, spin-off, or merger. These are cases in which the destination is not a new firm.

E.2 Adjusting Firm Founding Year

We repeat the following process sequentially from the start to the end of the sample.

1. **Reorganization** In the case of a reorganization the destination firm is assigned the founding year of home firm.

2. **Spin-Off** In the case of a spin-off we distinguish between two possibilities. (1) If the spun-off destination is a new firm we assign the founding year of home firm. (2) If the spun-off destination had existed in the past we assign the minimum of the founding year of the home firm and the founding year of the previously existed firm. This second possibility arises in cases where a firm is purchased and then spun-off several years later.

3. **Merger** In the case of a merger the destination firm is assigned the founding year of largest of the home firms (measured by employment in year t-1).

E.3 Sample of Changes in Ownership

We exclude reorganizations from our sample of changes in ownership. The results are robust to including these in the sample.