Sudden Stops, Productivity, and The Exchange Rate*

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Abstract

Following a sudden stop, real exchange rates can adjust through a nominal exchange rate depreciation, lower domestic prices, or a combination of both. This paper makes three contributions to understand how the type of adjustment shapes the response of macroeconomic variables, in particular productivity, to such an episode. First, using Spanish micro data during two episodes, it documents that in a currency union unproductive firms exit more than in a floating regime. Second, it proposes a small open economy DSGE model featuring firm selection, variable markups and elastic labor supply to rationalize this finding. The model nests three mechanisms through which a sudden stop affects productivity: a pro-competitive, a cost, and a demand channel. While only the former operates when the nominal exchange rate adjusts, all three are active under a currency union. The model delivers general conditions under which the positive impact of the demand channel on productivity dominates. Third, it validates the model’s aggregate predictions against a wider set of economies. In particular, it shows that the decline in productivity after a sudden stop is increasing in the flexibility of the exchange rate.

JEL codes: D24, E52, F32, F41, O57.

Keywords: Sudden Stops, Monetary Policy, Productivity, Reallocation.

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1 Introduction

The benefits of flexible exchange rates during a balance of payment crisis have been widely discussed among generations of international macroeconomists. The arguments, however, mostly rely on an aggregate view of the economy. This contrasts with an increasing use of granular data and a stark emphasis on heterogeneity in theoretical frameworks across fields. Zooming into the micro-level response to exchange rate policy remains a pending assignment for this literature. This paper contributes towards closing the gap by pursuing an unexplored dimension of exchange rate policy: its effects on firm dynamics.

The recent European sovereign debt crisis makes for an excellent stage to rekindle this debate. As Greece admitted to have misreported the figures of its public debt in late 2009, the periphery of Europe experienced an unexpected reversal in capital flows. This phenomenon, often referred to as a sudden stop, had not yet been studied in the context of a currency union. In addition, sudden stops have been traditionally associated with declines in aggregate total factor productivity (TFP). However, with the exception of Greece, the periphery of Europe experienced a productivity improvement as shown in Figure I. It is well known that measuring TFP is particularly challenging in the aggregate and is often subject to compositional bias. Thus, in looking for explanations to this puzzling observation, firm-level heterogeneity emerges as a key element to consider when addressing the following questions: what is the relationship between sudden stops, productivity and the exchange rate regime? How does accounting for firm dynamics complement our understanding of fixed versus floating regimes?

This paper studies how the type of real exchange rate realignment shapes the response of macroeconomic variables to a sudden stop in the presence of firm heterogeneity. Using Spanish microdata during two balance of payment crises, it is the first paper to document differences in firm entry and exit across exchange rate regimes. An internal devaluation, as opposed to a nominal depreciation, is associated with greater exit of unproductive firms, contributing to TFP growth through a so-called cleansing effect. The paper rationalizes these patterns by incorporating firm dynamics to an otherwise standard small open economy model. The novel link between consumer labor income and firm profitability is crucial in explaining why firm exit is larger when wages fall. The model’s predictions apply to a wider set of countries as shown by the event study discussed at the end of the paper. This exercises looks at aggregate data by binning sudden stop episodes by the prevalent exchange rate regime.

Section 2 starts by inspecting micro evidence from the Spanish manufacturing sector. More specifically, I exploit survey firm-level data during the 2010-13 European sovereign debt crisis and contrast it to an earlier sudden stop that hit Spain in 1992-93: the Exchange Rate Mechanism crisis. Parallels in the onset but divergence in the observed cyclicity of productivity make for a relevant comparison.

The joint analysis of these episodes uncovers the following empirical patterns. First, changes
Figure I: Productivity Growth in Peripheral Europe 2002-2015

Notes: This graph plots the overall change in aggregate TFP for Greece, Ireland, Italy, Portugal and Spain for the 2002-08 and the 2009-15 periods. The latter coincides with the European sovereign debt crisis and is the period of interest, while the former is depicted for comparison. The data used is collected from the AMECO database.

Arguments based on disparities in the size of the construction bust, the uneven disruption of credit and opposing trends in the misallocation of (solely) capital empirically fail to fully explain these findings. There is, however, an obvious difference across episodes that cannot be ruled out: the response of exchange rate policy. While during the earlier sudden stop, the national currency, the peseta, depreciated on multiple occasions; during the latter, Spain was a member of a currency union and could only regain competitiveness by lowering wages. The rest of the paper is devoted to exploring this distinctness.

Based on the previous evidence, section 3 develops a small open economy model with a micro-
structure that builds on Melitz and Ottaviano (2008) to study the macroeconomic effects of a sudden stop. The use of quasi-linear quadratic preferences and firm heterogeneity in productivity generates firm selection into production and endogenous variable markups, as observed in the data. I extend this framework to include leisure in the utility function, thereby explicitly modeling the consumer’s labor supply decision. This means wages are allowed to respond to shocks, which is absent in the original framework but essential in studying internal devaluations. Moreover, this provides a new channel through which the wage level and individual firm profits interact.

To allow a role for policy, I introduce nominal rigidities in the wage-setting process. The central bank chooses the nominal exchange rate as its main policy tool. I focus on two extreme regimes: a currency union, characterized by a credible commitment to keep the nominal exchange rate constant; and a strict wage inflation targeting regime, where the flexible wage equilibrium is always implemented. A sudden stop is defined as a two-fold shock to the domestic economy. On the one hand, it involves an increase in the risk premium component of the interest rate that consumers pay when borrowing. By increasing the cost of borrowing abroad, the domestic economy is forced to deleverage internationally and increase net exports through a real exchange rate depreciation. On the other hand, it simultaneously features a decline in the common shifter of firm-level productivity, which leads to a contraction of domestic output despite the reversal in the current account.

Section 4 discusses the effects of a sudden stop shock on aggregate productivity as predicted by the model. The key insight is that aggregate productivity is proportional to a domestic productivity threshold. The threshold represents the minimum productivity level at which a firm can generate positive profits and, thus, select into the domestic market. It therefore suffices to understand how the threshold moves after a sudden stop to learn about its effect on aggregate productivity.

In equilibrium, the domestic threshold is determined by the number of active firms in the market and the wage level. Therefore, there are three endogenous mechanisms through which a shock can affect productivity. First, the threshold increases with the number of active firms, as greater competition lowers profit margins for all firms and, thus, requires a higher level of productivity to remain profitable. This is the pro-competitive channel. Second, higher wages increase the costs of production for all firms, lowering again their profit margin and calling for a higher productivity level. This is the cost channel. Third, higher wages also increase the demand for overall consumption by increasing households’ labor income. This, instead, increases the firm profit margin and relaxes the productivity requirement. This is the demand channel.

The effect of a sudden stop on the domestic productivity threshold will hinge on the relative strength of these conflicting forces. This, in turn, depends on how the real exchange rate adjusts.

A sudden stop is essentially a real exchange rate shock. To some extent, it is isomorphic to a specific trade policy mix: a simultaneous increase in export subsidies and import tariffs. I, thus, build on the New New Trade Theory, which has long studied the effects of trade liberalization on aggregate productivity through firm selection, to understand the impact of a sudden stop.
More precisely, on whether it takes place through the depreciation of nominal exchange rates or a lower wage level. For a simplified version of the model that can be solved analytically, I show that if the nominal exchange rate bears the full brunt of the adjustment, then only the pro-competitive channel is active, as fewer firms import and productivity falls unambiguously. In contrast, when the nominal exchange rate is fixed, the wage adjusts completely and all three channels operate, resulting in a quantitatively ambiguous overall effect. The simplified model delivers conditions under which the demand channel dominates, allowing a sudden stop to generate a productivity improvement in a currency union.

The rest of section 4 studies the properties of the full model through a numerical simulation exercise. I calibrate parameters using Spanish macroeconomic data as well as the firm-level evidence presented in section 2. Plotting the impulse response function of aggregate TFP confirms that the previous analytical results hold more generally: productivity falls under a floating arrangement and increases in a currency union following a sudden stop. This is not only robust to alternative parameterizations of the model, but also to a range of extensions presented in section 5.

The model is able to match the observed differences in the contribution of firm dynamics to overall productivity growth portrayed in section 2. The procyclicality of productivity at the firm-level (the intensive margin) impels the aggregate TFP decline in the first case, whereas a sizable cleansing effect (the extensive margin) is the main driver of the efficiency improvement in the second. In addition, the model generates the other stylized facts previously documented by the literature: a contraction in output, a reversal in the current account and a real exchange rate depreciation.

Section 6 explores the external validity of the paper by providing systematic evidence on the behavior of macroeconomic variables during a sudden stop for a wider set of economies during the 1990-2015 period. Using a standard criterion to identify sudden stops that captures both the episodes discussed previously in the literature as well as the recent Southern-European cases, I first confirm the established fact that TFP falls on average. Next, I show that when binning episodes by prevalent exchange rate regime, a new pattern emerges: the decline in productivity increases in the flexibility of the exchange rate as captured by the model. This is robust to alternative exchange rate classifications, detrending methods and controlling for crisis and country characteristics.

In comparing the response of other macroeconomic variables across regimes two more regularities provide additional empirical support for the working of the model. First, in a currency union there is a larger decline in employment in both absolute and relative to output terms. Second, there is also a greater decline in imports relative to the increase in exports, suggesting the increase in aggregate TFP comes at the expense of a greater domestic contraction.

**Relation to the literature** This paper contributes to several strands of the literature at the intersection of international finance, trade theory and firm dynamics.
First, it focuses on sudden stops, as defined by Calvo (1998), abrupt and unexpected reversals in foreign capital inflows. It follows the empirical research that documents regularities among historical sudden stop episodes including Calvo, Izquierdo and Mejía (2004), Guidotti et al. (2004), Calvo and Talvi (2005) and Kehoe and Ruhl (2009) and adds to the discussion by revisiting the established stylized facts when episodes are binned by the flexibility of the nominal exchange rate. I document the fall in productivity is increasing in the relative size of the nominal adjustment.

On the theoretical side, several articles propose amendments to the standard open economy neoclassical model in order to reconcile theoretical predictions with the observed behavior of macroeconomic variables. For example Meza and Quintin (2007) allow for endogenous factor utilization, Christiano, Gust and Roldos (2004), Neumeyer and Perri (2005) and Mendoza (2006) introduce advanced payments of inputs and Ates and Saffie (2016) incorporate endogenous technical change. My formalization of a sudden stop is somewhat close to Mendoza (2010), which features both a risk premium and productivity shock, although I abstract from financial frictions and generate amplification through selection into production. ²

The second strand of the literature to which this paper closely relates is trade models of heterogeneous firms à la Melitz (2003).³ My framework builds on Melitz and Ottaviano (2008) in featuring endogenous markups but departs along three dimensions. First, I explicitly model a labor supply choice, incorporating a new channel that affects firm entry decisions. Second, I allow for transition dynamics by embedding the steady-state version in a DSGE setting.⁴ Third, I introduce nominal rigidities and, thus, discuss the effects of monetary policy.⁵

Finally, this paper is connected to the literature that studies the contribution of reallocation to TFP growth. In particular, I provide empirical support for Caballero and Hammour (1994)'s cleansing hypothesis and discuss the conditions under which its magnitude is likely to be relevant in the context of a current account shock.⁶ Moreover, this work adds to the recent set of papers that link declining TFP and enhanced misallocation with capital inflows; see Reis (2013), Benigno and Fornaro (2014) and Gopinath et al. (2017) among others. While their focus is on an earlier period, I show that the negative relationship between productivity and flows holds when capital retrenches and propose a complementary explanation for changes in measured misallocation: variable markups.

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² Ates and Saffie (2016) also account for firm dynamics in their study of the productivity costs of sudden stops. Their focus is on the long run effects of entry distortions generated by an interest rate shock and how financial selection cushions the fall in endogenous productivity.

³For a review of the literature, refer to Melitz and Redding (2014).

⁴Ghironi and Melitz (2005) are the first to consider firm dynamics in an open economy setting. To gain tractability, however, they assume that all firms that enter the market generate positive profits and, thus, firm exit is exogenous.

⁵Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014) introduce price adjustment costs in a DSGE model with endogenous entry and product variety to study optimal monetary policy. They consider, however, a closed economy.

⁶The cleansing hypothesis is an interpretation of Schumpeter (1939)'s creative destruction argument that emphasizes the role of reallocation among new and incumbent firms at a business cycle frequency.
2 Spain: A Tale of Two Sudden Stops

In unraveling what might be behind the aggregate patterns summarized by Figure I, it is useful to look at more disaggregated data. In exploring the singularity of this episode, it is convenient to set it against a comparable sudden stop that features a TFP decline. I do both by exploiting firm-level data from two sudden stops in Spanish recent economic history: the 1992-93 Exchange Rate Mechanism (ERM) crisis and the 2010-2013 European sovereign debt crisis.

There are clear parallels between the two episodes regarding the onset. Both were preceded by periods of increasing capital inflows, declining international competitiveness and widening current account deficits. Economic growth was fueled by the construction sector, with steep increases in property prices and crawling private debt. Public finances, on the other hand, were in a similar good shape.

Foreign capital inflows abruptly reverted following a confidence crisis affecting the European integration project: the negative outcome of the Danish referendum on the Maastricht Treaty in the first case, and the Greek announcement of substantial upward revisions in the government budget deficit more recently. The flight of international investment led to an urgent correction of misaligned real exchange rates in order to expand net exports. As growth stalled and unemployment rose, austerity measures were put in place in order to curb the rising public deficits generated by automatic stabilizers. In addition, structural reforms aimed at increasing the flexibility of the labor market were passed during both episodes.\(^7\)

The response of exchange rate policy to these events, however, diverged significantly. While the peseta was devalued in three occasions during the 1992-93 crisis, Spain already shared a common currency with its largest trading partners since 2002 and underwent a process of internal devaluation.\(^9\) I take these episodes as representative of sudden stops under floating arrangements and currency unions, respectively, and use firm-level data to explore what is driving the observed aggregate TFP pattern.\(^10\)

\(^7\)There are two stark differences regarding these two sudden stops. First is the magnitude of the shock: Spain’s current account surplus as a share of GDP moved from -3.5% to -1.2% between 1991 and 1994 versus -4.3% to 1.0% between 2009 and 2014. However, the duration was longer in the second episode, such that, per year, the reduction was around 1.1% during both episodes. Second, the latter is an example of a twin crisis, defined as a simultaneous crisis in banking and currency, while the former is not. I partially address this concern by looking at the level of leverage of firms at the end of the section.

\(^8\)For a more detailed discussion on the comparability of these two sudden stops see Online Appendix A.1.

\(^9\)In 1992, the peseta was first devalued by 5% on September 17th, known as Black Wednesday, when the pound and the lira abandoned the ERM altogether. A further 6% was devalued on November 23rd, with a third devaluation taking place in May 1993.

\(^10\)It can be argued that Spain does not strictly classify as a floating exchange rate regime in 1992-93 as it remains a member of the ERM, a multilateral party grid of exchange rates established in 1979. However, the repeated realignments of its central rate against the deutsche mark and the substantial widening of the exchange rate fluctuation bands meant that the overall devaluation of its currency was even larger than that of floating currencies such as the pound. In order words, despite the formal membership of the ERM, the exchange rate effectively behaved as flexible.
2.1 Data

I use firm-level data from the Survey on Business Strategies (Encuesta sobre Estrategias Empresariales, ESEE, in Spanish) managed by the SEPI Foundation, a public entity linked to the Spanish Ministry of Finance and Public Administrations. The ESEE surveys all manufacturing firms operating in Spain with more than 200 workers and a sample of firms between 10 and 200 workers, providing a rich panel dataset with over 1,800 firms for the period 1990-2014. It covers around 35 percent of value added in Spanish manufacturing and provides information on each firm’s balance sheet together with its profit and loss statement.

The main advantage of ESEE, especially over the ORBIS dataset compiled by Bureau van Dijk Electronic Publishing (BvD), is that it closely captures the extensive margin of production.\(^\text{11}\) This is particularly true for the exit of firms as the dataset clearly differentiates between firms that decide not to collaborate in a given year, firms that exit the market and firms that are affected by a split-up, a merger or an acquisition process. In addition, firms that resume production or collaboration with the survey are re-included in the sample and properly recorded. As for entry, new firms are incorporated every year in order to minimize the deterioration of the initial sample. These include all entrants with more than 200 workers and a random selection representing 5% of those with 10 to 200 workers.

There are other advantages of the ESEE dataset that are also worth highlighting. It is the only dataset with reliable financial information going back as early as the beginning of the 1990s, allowing me to study the 1992-93 episode. It also provides firm-level records of the value of exports which is most often subject to stringent confidentiality rules in Spain.\(^\text{12}\) Finally, the ESEE dataset is intended for research purposes, with effort devoted to ensure consistency and accuracy during the data collection process.

Nonetheless, I am aware that the ESEE dataset has two key limitations that require discussion. First, large firms are over-represented in the sample. While this should bias results against a relevant role of exit, I partially address this concern by exploiting sampling weights as a robustness check. Second, the coverage of the sample is significantly worse than ORBIS for the later years. Despite the poor monitoring of firm exit, I confirm the results for the 2010-13 episode using data from ORBIS. All results are available on the Online Appendix A.6.

Details on the cleaning procedure and the deflating of nominal variables are relegated to On-

\(^{11}\)The other existing firm-level dataset, as used in García-Santana et al. (2020), is the Central Balance Sheet Data (Central de Balances Integrada, CBI, in Spanish) owned by the Bank of Spain and only accessible to in-house economists. This alternative dataset, however, is put together using the same source of data that constitutes the Spanish input for ORBIS, annual financial statements that firms are obliged to submit to the Commercial Registry, and, thus, is subjected to the same limitations. Almunia, López-Rodríguez and Moral-Benito (2018) provide extensive details.

\(^{12}\)To the best of my knowledge, the only available dataset is the foreign transactions registry collected by the Bank of Spain containing transaction-level data which can be aggregated to the firm-level using the firm’s fiscal identifier as done in Almunia et al. (2018). Given the administrative nature of the dataset, however, only large operations are recorded. Moreover, the minimum reporting threshold changed from 12,500 to 50,000 euros in 2008, hindering the possibility of correctly measuring the extensive margin of exports.
I estimate industry output elasticities for capital and labor using Ackerberg, Caves and Frazer (2015)'s algorithm and then compute firm-level productivity as a Solow residual.\textsuperscript{13}

2.2 Results

Aggregate TFP, defined as the employment-weighted average of firm-level TFP, decreased by 10.87\% during the 1992-1993 episode while increased by 10.02\% in the 2010-2013 period.\textsuperscript{14} The granularity of the data allows for a more detailed investigation regarding the drivers of productivity.

The Lower Tail

I first document changes in the distribution of firm-level productivity before and after each of the crises. A visual inspection of the kernel probability distribution estimate of log TFP before and after each of the two sudden stops confirms there is ample heterogeneity in TFP levels among firms in any given year as already highlighted by the literature. More surprisingly, the shape of the distribution is similar and remains unchanged throughout both crisis periods, with no major shifts. In fact, the lower tail concentrates most, if not all, of the action: it lengthens as TFP decreases in the former crisis while shortens as TFP increases in the latter case.

To see this graphically, Figure II presents the percentage change in average productivity for each percentile of the productivity distribution during the two sudden stops. On average, the difference in the change in productivity across episodes, the gap between the red and blue lines, is roughly constant across the entire distribution, with the notable exception of the 5\% percentile where TFP decreases by 44\% during 1991-1993 while increases by 8\% during 2009-2013. Although the error bands are admittedly wide in both cases, the difference relative to other percentiles is large enough to remain relevant - the gap is three times the average.

Estimated moments of the distribution support the predominant role of the lower tail with higher-order moments experiencing the largest swings.\textsuperscript{15} During the 1992-93 crisis firms display lower productivity on average and the dispersion of log TFP increases. The increase in dispersion, however, is asymmetric. The distribution of unproductive firms expands while that of productive firms changes little with the coefficient of skewness declining from -0.40 to -1.24. Moreover, increasing kurtosis, 7.04 versus 10.42, is associated with fatter tails as the probability mass moves

\textsuperscript{13}See Online Appendix A.3 for a more detailed review of production function estimation techniques.

\textsuperscript{14}I consider employment, as opposed to value added, weights when aggregating TFP for two reasons. First, I will be presenting a theoretical model with labor as the only factor of production where employment shares are the appropriate weight. Second, large firms in terms of employment are overstated in my sample, as explained above, and, thus, employment weights are consistent with the interpretation of my results as a lower bound. Results using value added weights, however, are reported in Online Appendix A.6.

\textsuperscript{15}Refer to Table A.I for further details.
Notes: This graph plots the growth in average TFP by percentile of the productivity distribution. It compares the average TFP of firms in a given percentile before and after each of the two sudden stops. As this is an unbalanced panel, firms are allowed to change percentiles and even exit the sample during the transition. The corresponding base and end years are 1991 and 1993 for the first episode; 2009 and 2013 for the second episode. To account for variability, the vertical lines represent error bands. The data used is collected from the ESEE dataset.

away from the shoulders of the distribution. Although the behavior of TFP exactly reverses during the 2010-2013 crisis - productivity increases while dispersion drops - it is still the tails, and especially, the lower tail, that changes the most. In this case, skewness increases from -2.37 to -0.89 while kurtosis shrinks from 27.92 to 7.13.

Decomposing Productivity Growth

While the above findings support a narrative of shifting productivity cutoffs, there is yet room for skepticism. It is often the case that firms at the lower end of the productivity scale are small in size and, thus, have negligible effects on the aggregate. A more formal test of growth patterns requires considering weighted measures. Moreover, it should aim at disentangling the role of incumbent, entering and exiting firms in shaping TFP changes.

I study this by performing a TFP growth decomposition exercise using the formulation proposed by Dias and Marques (2018), which I derive in Online Appendix A.4. Results for the two sudden stops are summarized in Table I. The decline in TFP in the 1992-1993 crisis is entirely driven by incumbents. In fact, net entry contributes to positive growth, although the magnitude is small. Among incumbents, there is some reallocation of market shares towards more productive
**Table I: Decomposition of Productivity Growth**

<table>
<thead>
<tr>
<th>Sudden Stops</th>
<th>1992-1993</th>
<th>2010-2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity Growth (%)</td>
<td>-10.87</td>
<td>10.02</td>
</tr>
</tbody>
</table>

Contribution to Productivity Growth

- **Incumbents’ Contribution**
  - Within-firm Contribution: -9.69, -2.41
  - Between-firm Contribution: 0.47, 3.75
  - Cross-term Contribution: -1.98, 1.71

- **Net Entry Contribution**
  - Entrants’ Contribution: -0.77, -0.72
  - Exiters’ Contribution: 1.10, 7.68

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Notes: Productivity growth refers to accumulated TFP growth for the stated period. Base and final years are 1991 and 1993 for the first episode; 2009 and 2013 for the second episode. Contribution of incumbents and net entrants add up to productivity growth. Contribution of within-firm, between-firm and cross-term components add up to incumbents’ contribution. Contribution of entrants and exiters add up to net entry contribution. The formal decomposition is given by:

\[
\Delta Z_i = \sum_{t \in \mathcal{T}} s_{i,t-1} \Delta Z_{i,t} + \sum_{t \in \mathcal{T}} Z_{i,t-1} \Delta s_{i,t} + \sum_{t \in \mathcal{T}} \Delta s_{i,t} \Delta Z_{i,t} + \hat{s}_t^N \left( Z_{i,t}^N - Z_{i,t}^C \right) - \hat{s}_t^X \left( Z_{i,t}^X - Z_{i,t-1}^C \right),
\]

where \( s_{i,t} \) is the employment share and \( Z_{i,t} \) is the productivity level of firm \( i \) in period \( t \) and \( C, N, X \) denote incumbents, entrants and exiters respectively. More details can be found in Online Appendix A.4. The data used is collected from the ESEE dataset.

...firms. However, it is far from enough to overcome the pronounced fall in within-firm productivity and the cross-term.

In contrast, the increase in TFP experienced during 2010-2013 is largely driven by net entry, in particular, by unproductive firms exiting the sample. The size of the effect is remarkable, especially given that small and medium firms are underrepresented in the sample. Delving deeper into the characteristics of exiting firms shows that during the 2010-2013 episode, firms that exit the market were, on average, bigger in terms of labor market share (7.01% versus 2.78) and three times as unproductive in relation to incumbents (27.16% versus 9.17%) than their 1992-1993 counterparts. Moreover, the annualized exit rate more than doubled from 4.47% to 9.19%.\(^{16}\) In other words, there is more and better exit.

Back to Table I, the contribution of incumbents, although half as important, is also noteworthy. It is still the case that, on average, the productivity of incumbents is procyclical, yet the positive effect of the between and cross terms dominate overall. The increase in market share reallocation and a stronger correlation between productivity and market share changes at the individual firm, together with the positive contribution of exiting firms, is consistent with a cleansing ef-

\(^{16}\)The corresponding averages for the entire sample are the following: the annualized exit rate is 7.71%, the employment share of exiting firms is 6.43% and the difference in TFP between exiting firms and incumbents is 14.09%.
fect of the 2010-13 sudden stop which is absent in the 1992-93 episode. The cleansing hypothesis, as discussed by Caballero and Hammour (1994), argues that crises are periods of accelerated productivity-enhancing reallocations, especially as resources are freed by the exit of unproductive firms. I turn to formally testing the firm-level implications of this interpretation in what follows.\footnote{A valid concern is that if firms are forward-looking, they might backload the decision to exit, and, thus, the duration of a crisis might be an important driver of results. I refer the reader to Online Appendix A.6, where I show that exit in the 2010-13 episode is not concentrated on the later years.}

**The Cleansing Hypothesis: A Formal Test**

According to the literature, there is a tight connection between firm exit, input growth and productivity: models of firm dynamics predict that exit is more likely among low productivity firms whereas high productivity firms are expected to grow by more every period. The cleansing hypothesis suggests that recessions accelerate these dynamics. One should therefore observe a stronger correlation between survival, employment growth and productivity levels during crises. To test whether this is the case for the two sudden stop episodes considered, I adjust the empirical specification proposed by Foster, Grim and Haltiwanger (2016) and run the following set of regressions:

\[
y_{it} = \lambda + \beta tfp_{it} + \delta ss_{1t} + \gamma ss_{1t} * tfp_{it} + \mu ss_{2t} + \theta ss_{2t} * tfp_{it} + \epsilon_{it},
\]

where \(y_{it}\) stands for a set of explanatory variables. It is a dummy variable with value one when a firm reports activity in period \(t\) and no activity in period \(t+1\) in the exit specification. It is a quantitative variable measuring employment growth in the regressions for input growth. The regressor \(ss_{1t}\) is a dummy variable for the 1992-93 sudden stop, \(ss_{2t}\) is a dummy variable for the 2010-13 sudden stop and \(tfp_{it}\) captures the log of firm-level productivity.

For the exit specification, the relationship between survival probability and productivity is expected to be positive and, thus, \(\beta < 0\). Under the cleansing hypothesis, this correlation should strengthen during a sudden stop episode and one would anticipate \(\gamma < 0\) and \(\theta < 0\). Note that the sign of parameters \(\delta\) and \(\mu\) provide additional insights regarding the interaction terms. They capture the change in exit rate during the sudden stops that is not correlated with productivity. For the input growth specification, the exact opposite applies.

Results of these regressions are summarized in Table II. The first column shows the relationship between productivity and the probability of exit. Consistent with earlier findings, firms that exit the market tend to feature lower productivity levels. Focusing on the interaction terms, there is evidence of a cleansing effect only during the second episode. Based on the estimates, 2010-2013 is a period of increasing exit rates, especially among the less productive firms.

The second and third columns support further the predictions of the cleansing hypothesis for the 2010-13 episode. First, note that there is a positive impact of productivity on labor growth as predicted by the literature. Of greater interest, this correlation is even higher during the second
2.3 Potential Explanations

While I have so far stressed marked divergence in the exchange rate policies implemented during the two sudden stops, there are a number of alternative stories that could potentially explain the contrast in firm dynamics documented in the previous section. In the following, I investigate differences in the construction bust, the simultaneity of a banking crisis, expenditure switching and balance sheet effects of a real depreciation and resource misallocation trends.

Table III documents characteristics of exiting firms and compares them with that of incumbents. These characteristics include age, size, type of employment or sector. The latter is particularly interesting in discerning the role of the construction bust. As the ESEE abstracts from the construction sector altogether, I instead measure the exposure of manufacturing firms to construction by creating a dummy that equals one if the sector in which the firm operates sells goods to the construction sector. It turns out that exiters are more likely to be exposed to the construction
TABLE III: CHARACTERISTICS OF EXITERS

<table>
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</thead>
<tbody>
<tr>
<td>Productivity</td>
<td>0.30</td>
<td>0.19</td>
<td>0.16</td>
<td>-0.05</td>
<td>2.43**</td>
<td>7.10***</td>
</tr>
<tr>
<td>Age</td>
<td>24.36</td>
<td>31.51</td>
<td>18.53</td>
<td>29.82</td>
<td>2.51**</td>
<td>1.57</td>
</tr>
<tr>
<td>Construction</td>
<td>0.20</td>
<td>0.30</td>
<td>0.20</td>
<td>0.04</td>
<td>0.14**</td>
<td>-3.45***</td>
</tr>
<tr>
<td>Employees</td>
<td>233</td>
<td>220</td>
<td>139</td>
<td>150</td>
<td>1.75*</td>
<td>2.10**</td>
</tr>
<tr>
<td>Part-time Share</td>
<td>0.02</td>
<td>0.04</td>
<td>0.02</td>
<td>0.07</td>
<td>-0.16</td>
<td>-0.18</td>
</tr>
<tr>
<td>Fixed-term Share</td>
<td>0.24</td>
<td>0.01</td>
<td>0.29</td>
<td>0.01</td>
<td>-1.80*</td>
<td>3.12***</td>
</tr>
<tr>
<td>Exports</td>
<td>0.50</td>
<td>0.56</td>
<td>0.45</td>
<td>0.56</td>
<td>0.26</td>
<td>5.57***</td>
</tr>
<tr>
<td>Imports</td>
<td>0.52</td>
<td>0.56</td>
<td>0.42</td>
<td>0.56</td>
<td>1.81*</td>
<td>4.73***</td>
</tr>
<tr>
<td>Imported Intermediates</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.49</td>
<td>3.60***</td>
</tr>
<tr>
<td>Debt</td>
<td>13.82</td>
<td>13.74</td>
<td>13.43</td>
<td>13.67</td>
<td>1.44</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Notes: Productivity measures firm-level productivity in logs. Age is in years. Construction=1 if the firm operates in “Timber”, “Plastic and rubber products”, “Non-metal mineral products”, “Basic metal products” and “Furniture”. Part-time and fixed-term shares refer to the percentage of total employees that have a part-time or fixed-term contract. Exports, imports and imp. intermediates are all dummy variables summarizing the firm’s foreign activity status. Debt measures log of real debt capital with financial institutions. An exiter is a firm that operates in 1991 (2009) but not in 1993 (2013). ** p < 0.01, * p < 0.05, and * p < 0.10 for two-sided test. The data used is collected from the ESEE dataset.

section than incumbents only in 2010-2013. As a robustness check, I repeat the analysis above restricting the sample to firms that do not operate in these sectors to find that reported differences remain.\textsuperscript{18}

Another important difference between 1992-1993 and 2010-2013 is that during the latter, Spain also experienced a banking crisis. In a credit crunch, one would expect highly leveraged firms to suffer the most and, thus, have a higher propensity to exit the market. While credit is often extended to more productive firms, to rule out this explanation I test explicitly whether exiting firms hold on average more debt than incumbents. The last row of Table III shows that there is no significant difference in the level of leverage between incumbents and exiting firms in both episodes. This implies that the banking crisis in the latter episode does not drive the difference in the exit pattern across sudden stops.

Other well-known effects of a real exchange rate depreciation include (i) an expenditure switching effect on imported intermediate inputs and (ii) balance sheet effects resulting from liability currency mismatches. While in the absence of a model it is ex-ante unclear whether these effects should be different across episodes, most economists tend to expect a greater impact whenever the currency depreciates. This would involve more exit in the first episode, which does not hold in the data. In addition, one should observe a higher share of imported intermediate inputs among exiting firms. Although data for imported intermediate inputs is only available for 2010-13, in-

\textsuperscript{18}Results available upon request.
### Table IV: Markups and Productivity

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm-level TFP</td>
<td>0.994***</td>
<td>0.992***</td>
<td>0.964***</td>
<td>0.960***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Aggregate TFP</td>
<td>0.022</td>
<td>-0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry TFP</td>
<td>-0.882***</td>
<td>-0.879***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.049)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>36,261</td>
<td>36,261</td>
<td>36,261</td>
<td>36,261</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.933</td>
<td>0.937</td>
<td>0.856</td>
<td>0.859</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Export status</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: This table reports the results of a cross-section regression of firm-level markups on different measures of productivity: at the firm level, at the industry level and at the economy level. All variables are measured in logs. Export status is a dummy equal to one whenever a firm reports a positive exporting revenue. Standard errors (in parentheses) are clustered by industry; ***$p < 0.01$, **$p < 0.05$, and *$p < 0.10$. The data used is collected from the ESEE dataset.

cumbents are more likely to import overall than exiting firms in 1991-1993 as reported by Table III. The balance sheet effect also fails to acknowledge that it is larger, more productive firms that are more likely to borrow in foreign currency. Unfortunately, the ESEE dataset does not provide information on debt denomination to confirm this.

Finally, I evaluate a popular complementary channel through which reallocation contributes to productivity growth - increased allocative efficiency. Following Hsieh and Klenow (2009), the degree of dispersion of firm-specific distortions is informative of the degree of misallocation in the economy. As distortions are unobservable in practice, I use marginal revenues products as proxies. Periods of higher TFP should be associated with periods of lower marginal revenue product dispersion and differences in the results for capital and labor can be interpreted as evidence of the different types of wedges that might prevail.

In this spirit, I estimate the within-sector standard deviations of marginal revenue products of capital (MRPK) and labor (MRPL) before and after each sudden stop for each two-digit industry. In most sectors, dispersion increases during the first episode and decreases by the end of the latter. Importantly, this pattern holds for both capital and labor, suggesting that while there are

---

19 This is not surprising as it is larger, more productive firms, which have a lower propensity to exit, that tend to engage in international trade.

20 See Online Appendix A.5 for a review of the argument and further details on how to construct these measures.

21 Take the biggest industry in the dataset, “Vehicles”, as an example. For the 1992-93 sudden stop the standard deviation for capital (labor) was 0.947 (0.342) the year before the crisis; it increased to 1.037 (0.439) by the end of the crisis. For the 2010-13 sudden stop the standard deviation for capital (labor) was 1.098 (0.704) the year before the crisis; it however decreased to 0.977 (0.368) by the end of the crisis. To derive an economy-wide measure of the standard
changes in the distortions over time, such distortions affect both factors of production simultaneously or, in Hsieh and Klenow (2009) lingo, it is changes in the output (and not the capital to labor ratio) wedge that are driving TFP movements.\footnote{\emph{While Gopinath et al. (2017) and García-Santana et al. (2020) have shown that increasing capital misallocation is responsible for the slowdown of productivity growth prior to the 2010-2013 crisis, these results rule out the possibility that it is the undoing of this phenomenon what drives the most recent improvement.}}

An alternative interpretation of this result, which implies moving away from the CES assumption, suggests the presence of firm-specific markups that are time-varying. I explore this possibility by computing markups at the firm level as the ratio of the output elasticity of labor to the labor share following De Loecker and Warzynski (2012)’s cost minimization approach. I find that the dispersion of firm-specific markups is substantial; the standard deviation is 0.47 and changes over time.\footnote{To calculate these numbers, I estimate standard deviation at the industry level and then compute an employment weighted average across industries.}

To study its relationship with productivity, I regress firm-specific markups on firm-level and aggregate TFP measures. Results are presented in Table IV. Columns (1) and (2) focus on a economy-wide measure of aggregate productivity while columns (3) and (4) restrict attention to productivity aggregated at the industry level. More productive firms set higher markups on average. This is consistent with most models of variable markups. In addition, lower markups are associated with higher levels of aggregate productivity at the industry level although the effect vanishes at a higher level.

In sum, the above findings call for a theory of sudden stops that features heterogeneously productive firms, selection into production and variable firm-specific markups. All of these elements, together with the exchange rate dimension, are featured in the theoretical framework that I develop next.

## 3 A Small Open Economy with Firm Dynamics

Consider an infinite-horizon small open economy. Time is discrete and indexed by $t$. The economy is populated by a representative household that consumes goods and leisure and engages in financial transactions with foreign investors. There is also a large number of differentiated firms that produce consumption goods using labor supplied by the household, and a monetary authority that sets the nominal exchange rate as the policy instrument.

### 3.1 A Representative Household

The representative household derives utility from leisure and the consumption of a set of differentiated goods, indexed by $\omega \in \Omega$, and supplies differentiated types of labor input, indexed by deviation, I then aggregate standard deviations at the industry level using time-invariant employment weights. Results, which are similar to those for “Vehicles”, are summarized in Table A.II.
The lifetime utility is given by:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U \left( q_t (\omega), L_t^i \right) \right],$$

where $E_t$ is the expectation operator conditional on the information set available at time $t$, $\beta$ is the discount factor, $q_t (\omega)$ is the consumption level of variety $\omega$ and $L_t^i$ is the labor supply of type $i$. The period utility function is based on Melitz and Ottaviano (2008):

$$U \left( q_t (\omega), L_t^i \right) = \alpha \int_{\omega \in \Omega} q_t (\omega) d\omega - \frac{1}{2} \gamma \int_{\omega \in \Omega} q_t (\omega)^2 d\omega - \frac{1}{2} \eta \left( \int_{\omega \in \Omega} q_t (\omega) d\omega \right)^2 - \int_0^1 L_t^i di,$$

where demand parameters $\alpha$, $\gamma$ and $\eta$ are strictly positive.

Melitz and Ottaviano (2008) preferences are appealing for three reasons. First, they capture love of variety through $\gamma$, which determines the level of product differentiation between consumption goods. As $\gamma$ increases, consumers place higher weight on the distribution of consumption across varieties. Second, the quadratic form gives rise to a linear demand function which ensures the existence of a choke price and an extensive margin of production even in the absence of fixed costs of production. Third, they generate endogenous variable markups, which capture the effect of market competition on firm sales (the so-called pro-competitive effect) as opposed to standard CES preferences.

Melitz and Ottaviano (2008) preferences also depict a second consumption good, which is homogeneous and assumed to be the numeraire, with a linear production technology that pins down the wage in the economy. As endogenous fluctuations in the wage level are relevant in this analysis, this feature of the original functional form is inconvenient. Moreover, in the context of an internal devaluation, it is also interesting to capture any changes in demand patterns that may arise from movements in wages. My approach is to explicitly model the labor supply decision by assuming preferences that are linear in leisure.\(^24\) The demand parameters $\alpha$ and $\eta$ therefore measure the substitutability between the consumption of differentiated goods and leisure.

The budget constraint of the representative agent in terms of domestic currency can be written as:

$$\int_{\omega \in \Omega} p_t (\omega) q_t (\omega) d\omega + e_t B_t = \int_0^1 W_t^i L_t^i di + \Pi_t + e_t R_{t-1}B_{t-1},$$

where $W_t^i L_t^i$ is the income derived from supplying differentiated labor input $i$, $\Pi_t$ is profit received from firms and $e_t$ denotes the nominal exchange rate, defined as units of domestic currency needed to buy one unit of foreign currency.

The representative household can only engage in financial transactions with foreign investors

\(^{24}\)Given the quasi-linear functional form, there is no income effect for differentiated varieties. However, changes in wages will affect demand through the substitution effect. In any case, the assumption of linearity in leisure can be easily relaxed.
by trading in risk-free foreign denominated nominal bonds $B_t$, which pay a debt elastic rate of return:

$$R_t = R^* + \phi \left( e^{B_t} - 1 \right) + \xi_t,$$

(3)

where $R^*_t$ is the world interest rate and $\bar{B}$ is the steady state level of debt.\footnote{Featuring a debt-elastic interest rate ensures a stationary solution to the model after detrending following Schmitt-Grohé and Uribe (2003). While I take this specification in reduced form, it can be micro-founded in models of limited international financial intermediation in the spirit of Gabaix and Maggiori (2015).} The only source of uncertainty so far is $\xi_t$, which is interpreted as a country risk premium shock, similar to that of Garcia-Cicco, Pancrazi and Uribe (2010), and assumed to follow an AR(1) process. A sudden stop in the model involves a positive realization of $\xi_t$: an unexpected increase in the cost of international borrowing that forces the domestic economy to deleverage internationally by expanding net exports.

Labor supply is differentiated. There is a unit continuum of labor types. Firms can aggregate labor types according to $L_t = \left( \int_0^1 L_i \frac{dL_i}{dL} i \right)^{1/\theta}$, where $\theta$ measures the elasticity of substitution. I assume that the representative household supplies all the differentiated labor inputs as in Woodford (2011).\footnote{Households are not allowed to trade in domestic bonds in the baseline model for the sake of simplicity. However, extending the model to include domestic bonds would be trivial as these would be in zero net supply.} Suppose, for example, that each member of the household specializes in one occupation. The representative household has monopoly power to set the wage for each labor type, $W^i_t$.

Each period the household chooses $q_t(\omega), B_t, L^i_t$ and $W^i_t$ to maximize the expected present discounted value of utility, equation (1), subject to the budget constraint, equation (2), and the demand for type $i$ labor input, which is given by:

$$L^i_t = \left( \frac{W_t}{W^i_t} \right)^{\theta} L_t.$$

**Optimality conditions** Given quadratic preferences, it may be the case that not all differentiated goods are demanded by the household. However, when a particular good $\omega$ is consumed, its inverse demand is determined by:

$$\lambda_t = \gamma q_t(\omega) - \eta Q_t = \lambda_t p_t(\omega),$$

(4)

where $Q_t$ is the consumption level over all varieties and $\lambda_t$ is the time $t$ Lagrangian multiplier. Consumption of a given variety decreases with price, the marginal utility of wealth and total consumption.

The optimal decision for the purchase of the foreign asset, $B_t$, delivers a standard Euler equa-
ation:
\[ \lambda_t = \beta R_t E_t \left[ \frac{\epsilon_{t+1} \lambda_{t+1}}{\epsilon_t} \right]. \]  
(5)

A higher interest rate and expectations of nominal exchange rate depreciation both increase the cost of borrowing internationally and, thus, encourage consumer savings.

Solving for the optimal wage for labor type \( i \) gives:
\[ W_i^t = \frac{\theta}{\theta - 1} \frac{1}{\lambda_t}. \]  
(6)

Intuitively, higher wages increase household’s wealth everything else equal. Given diminishing marginal utility, the Lagrangian multiplier falls. Equation (6) also implies that the optimal flexible wage is equalized across labor types i.e. \( W_i^{flex} = W_i \).

Finally, note that the representative household will be willing to satisfy firms’ labor demand as long as:
\[ \frac{W_i}{P_t} \geq \frac{1}{(\alpha - \eta Q_t) N_t - \gamma Q_t}. \]

3.2 Firms

There is a continuum of measure \( M \) of domestic firms, each deciding whether to produce a differentiated variety \( \omega \).\(^{28}\) Labor is the only factor of production and the unit cost is a concave function in the factor price i.e. \( C_t(z) = \frac{W_t^z}{Z_t} \), where \( 0 < \sigma \leq 1 \) is the labor income share.\(^ {29}\) Firm productivity has a common stochastic shifter, \( Z_t \), which follows an AR(1) in logs and a constant firm-specific component, \( z \), which is drawn from a Pareto distribution \( 1 - G(z) = (\frac{1}{z})^k \) with shape parameter \( k \) and minimum level equal to one.\(^ {30}\)

The main focus of the paper is the short-run and, as such, cross-country reallocation of firms is not allowed.\(^{31}\) This implies that the number of potentially active firms in the economy, \( M \), is fixed and there is no free entry condition. Firms only choose whether to produce or not in each period based on the profitability for the corresponding period.

Firms can sell their varieties in both the domestic and the export market. Markets are segmented and selling abroad requires incurring a per-unit trade cost \( \tau > 1 \). While domestic demand for variety \( z \), \( q_t^H(z) \), is given by equation (4), the foreign demand for a domestic variety \( z \), \( q_t^F(z) \),

\(^{28}\)The same is true for the foreign economy: there is a continuum of measure \( M^* \) potentially active foreign firms.

\(^{29}\)To rationalize this functional form, suppose there is a second factor of production, which is inelastically supplied by households and the production function is Cobb-Douglas. If the price of this second input, \( \kappa \), is assumed to be constant, the marginal cost is given by \( C_t = \left( \frac{W_t^z}{Z_t^2} \right)^{1-\sigma} \left( \frac{z}{1-\sigma} \right)^{1-\sigma} \). In section 5.1 I relax this assumption and show that explicitly considering two factors of production does not change the model’s predictions in any substantial way.

\(^{30}\)For completeness, the assumption is that \( Z_t^* = 1 \forall t \)

\(^{31}\)Note that this is only true for the baseline set-up. In section 5.3, I allow for firm entry and study long-run implications instead.
is given by:

$$ q^*_F(z) = A - B p^*_F(z), $$

where $A$ and $B$ are exogenous given a small-open economy setting. In the spirit of Demidova and Rodriguez-Clare (2009), I show in Online Appendix B.2 that this small open economy is a special case of the two economy framework where the share of potentially active firms in Home, $n = \frac{M}{M + M^*}$ approaches zero.\textsuperscript{32} 

**Optimality conditions** The profit maximization problem delivers the following set of first-order conditions:

$$ q^H_t(z) = \max \left\{ \frac{\lambda_t}{\gamma} \left[ p^H_t(z) - \frac{W_i^e}{z z_t} \right] , 0 \right\}, $$

$$ q^F_t(z) = \max \left\{ \frac{\lambda_t}{\gamma} \left[ p^F_t - \frac{\tau e_t (W_i^e)^2}{z} \right] , 0 \right\}, $$

$$ q^*_F(z) = \max \left\{ B \left[ p^*_F(z) - \frac{\tau W_i^e}{e_t z z_t} \right] , 0 \right\}, $$

where the expressions for domestically-produced, henceforth domestic goods, $q^H_t(z)$, and exported goods, $q^*_F(z)$, are given by the optimization of domestic firms while the expression for imported goods, $q^F_t(z)$, results from the optimization of foreign firms.

The labor demand for a domestic firm with productivity level $z$ is given by the firm’s cost minimization problem and reads:

$$ L_t(z) = \frac{\sigma}{W_i^{1-\sigma}} \frac{q_t(z)}{z z_t}, $$

where $q_t(z)$ will be either $q^H_t(z)$ or $q^*_F(z)$ depending on whether the labor input hired is used to serve the domestic or the export market.

### 3.3 Aggregation and Market Clearing
I aggregate firm-level variables and impose market clearing conditions as building blocks to define the competitive equilibrium.

**Productivity thresholds** Given that firm-level productivity follows a Pareto distribution, the aggregate productivity level for a given market is summarized by a productivity threshold.\textsuperscript{33} This is simply the productivity level of the marginal firm that is indifferent between producing or not for a specific market.

\textsuperscript{32}In the limit $z^*_F$ is unaffected by changes in Home, the term $A$ includes the price index, the number of consumed varieties and the marginal utility of wealth in Foreign while the term $B$ is proportional to the marginal utility of wealth in Foreign.

\textsuperscript{33}See section 4 for the formal proof.
On the supply side, the zero profit condition holds for the marginal firm: it optimally sets its price equal to its marginal cost. For example, for the domestic good \( p_t^H = \frac{W_t^r}{z_t^H} \). On the demand side, the linearity of consumer’s demand gives rise to a choke price. This is the maximum price that can be charged for a given variety; anything beyond which drives demand down to zero. Following the previous example, \( p_t^H = \lambda_t^{-1}(a - \gamma Q_t) = (\gamma + \eta N_t)^{-1}(a + \eta P_t) \). By combining these two sets of conditions, the equilibrium thresholds can be expressed as:

\[
z_t^H = \frac{\gamma + \eta N_t}{a\gamma + \eta P_t} \frac{W_t^r}{Z_t},
\]

(9)

\[
z_t^F = \frac{\gamma + \eta N_t}{a\gamma + \eta P_t} \tau \epsilon_t(W_t^*)^\sigma,
\]

(10)

\[
z_t^{EF} = \frac{B}{A} \tau W_t^*,
\]

(11)

where \( z_t^H \) is the productivity threshold for domestic firms serving the domestic market, \( z_t^F \) is the importer threshold and \( z_t^{EF} \) is the exporter threshold. Given the small open economy set-up, the productivity threshold for foreign firms serving the foreign market, \( z_t^{EF} \) is exogenously determined and irrelevant for the analysis.

All firm-level variables can then be written in terms of these thresholds. In particular:

\[
p_t^H(z) = \frac{1}{2} \frac{W_t^r}{Z_t} \left( \frac{1}{z_t^H} + \frac{1}{z} \right), \quad q_t^H(z) = \frac{1}{2} \frac{a^m}{\gamma} \frac{W_t^r}{Z_t} \left( \frac{1}{z_t^H} - \frac{1}{z} \right),
\]

\[
p_t^F(z) = \frac{1}{2} \tau \epsilon_t(W_t^*)^\sigma \left( \frac{1}{z_t^F} + \frac{1}{z} \right), \quad q_t^H(z) = \frac{1}{2} \tau \epsilon_t(W_t^*)^\sigma \left( \frac{1}{z_t^H} - \frac{1}{z} \right),
\]

\[
p_t^{EF}(z) = \frac{1}{2} \tau \epsilon_t \frac{W_t^r}{Z_t} \left( \frac{1}{z_t^{EF}} + \frac{1}{z} \right), \quad q_t^{EF}(z) = \frac{1}{2} \tau \epsilon_t \frac{W_t^r}{Z_t} \left( \frac{1}{z_t^H} - \frac{1}{z} \right),
\]

which are derived by combining the optimality conditions from the representative household and the firms and the corresponding definition of choke prices.

**Number of firms**  The number of active firms in the domestic market, \( N_t \), is the sum of domestic firms that serve the domestic market, \( N_t^H \), plus the number of foreign importers, \( N_t^F \). Given the number of existing firms in both markets, \( M \) and \( M^* \), and the Pareto distribution assumption, the number of active firms is given by:

\[
N_t = M \left( \frac{b}{z_t^H} \right)^k + M^* \left( \frac{b}{z_t^F} \right)^k,
\]

(12)

where \( \left( \frac{b}{z_t^H} \right)^k \) is the probability that an incumbent has a productivity level above the cutoff and, thus, generates positive profits. Note that because each firm specializes in a particular variety, \( N_t \)
is also the number of differentiated varieties available for consumption in the small open economy.

**Price index**  The aggregate price level is given by the sum of prices of all goods consumed domestically, that is, prices of domestically produced goods consumed domestically and import prices:

\[ P_t = N_t^H \int_{z_t^H}^{\infty} p_t^H(z) \frac{g(z)}{1 - G(z_t^H)} dz + N_t^F \int_{z_t^F}^{\infty} p_t^F(z) \frac{g(z)}{1 - G(z_t^F)} dz. \]

Combined with the optimal price expressions and the number of active firms in equilibrium, given by equation (12), the above expression is considerably simplified to read:

\[ P_t = \frac{2k + 1}{2k + 2} W_t^r N_t. \] (13)

The aggregate price level is determined by the number of firms and the average effective marginal cost. The former follows by definition, the latter from the individual firm’s optimization problem. Firms charge higher prices whenever their cost of production increase. This is the case when the wage level is high but also when the individual productivity level is low. As the average productivity level in the economy depends positively on the domestic threshold, the aggregate price level decreases in \( z_t^H \).

**Wage level**  I introduce nominal rigidities in the form of sticky wages à la Calvo (1983).\(^{34}\) Each labor type is able to reset its wage with a probability \( \mu \) each period. Thus, the labor type that adjusted its wage \( s \) periods ago would have chosen \( X_{t-s} \), such that:

\[ \log(X_{t-s}) = (1 - \beta(1 - \mu)) \sum_{j=0}^{\infty} (\beta(1 - \mu))^j \mathbb{E}_{t-s} \left( \log(W_t^{flex}) \right), \] (14)

where \( W_t^{flex} \) is the optimal flexible wage as defined by equation (6). This is a weighted average of the current and the expected future optimal wages as of time \( s \). Expectations farther in the future are given a lower weight not only because of discounting, but because there is a lower probability of the wage prevailing.

Given that the probability of updating is independent across labor types, the aggregate wage is simply:

\[ \log(W_t) = \mu \sum_{j=0}^{\infty} (1 - \mu)^j \log(X_{t-s-j}), \] (15)

\(^{34}\)The model could alternatively feature Rotemberg wage adjustment costs or sticky information à la Mankiw and Reis (2002) in the wage setting process. Note, however, that the model cannot feature downward wage rigidities as in Schmitt-Grohé and Uribe (2016). This would imply that wages are not set by households (or unions in their names), thus, preventing movements in wages to have an effect on demand, a key channel in this paper.
which combined with equations (6) and (14) yields a version of the wage Phillips curve,

\[
\Delta \log(W_t) = \beta E_t \Delta \log(W_{t+1}) + \frac{\mu (1 - \beta (1 - \mu))}{1 - \mu} \left( \log \left( \frac{\theta}{1 - \theta} \right) - \log(\lambda_t) - \log(W_t) \right)
\]

(16)

**Labor market clearing** To ensure that the labor market clears in equilibrium, aggregate labor demand must equal aggregate labor supply. To aggregate domestic individual labor demand given by equation (8), I sum across all active domestic firms using the Pareto distribution assumption. Labor market clearing then boils down to:

\[
L_t = k \left( \frac{k + 1}{k + 2} \right) \left( \frac{W_t}{z_t^H} \right)^2 M \left[ \frac{\lambda_t}{\gamma} \left( z_t^H \right)^{(k+2)} + B \left( \frac{\tau^2}{\epsilon_t} \right) \left( z_t^F \right)^{(k+2)} \right].
\]

(17)

**The balance of payments condition** Combining some of the equilibrium conditions above, together with the domestic firms’ aggregate profit equation and the consumer’s budget constraint gives the aggregate resource constraint of the economy, which, in an open-economy setting, is simply the balance of payments condition. In other words, it states that the current account must be equal to the capital account in equilibrium:

\[
EX_t - IM_t + \epsilon_t B_{t-1} (R_{t-1} - 1) = \epsilon_t (B_t - B_{t-1})
\]

(18)

where \( EM_t \) and \( IM_t \), the total export and import revenues in domestic currency terms, are given by:

\[
IM_t = N_t^F \int_{z_t^F}^\infty p_t^F(\omega) q_t^F(\omega) \frac{g(z)}{1 - G(z_t^F)} \, dz = \frac{b^k}{2(k + 2)} M^{*} \frac{\lambda_t}{\gamma} \frac{(\tau \epsilon_t W_t^*)^2}{2} \left( z_t^F \right)^{-(k+2)},
\]

(19)

\[
EX_t = \epsilon_t N_t^F \int_{z_t^F}^\infty p_t^F(\omega) q_t^F(\omega) \frac{g(z)}{1 - G(z_t^F)} \, dz = \frac{b^k}{2(k + 2)} M \frac{B}{\epsilon_t} \left( \frac{\tau W_t^*}{Z_t} \right)^2 \left( z_t^F \right)^{-(k+2)}.
\]

(20)

### 3.4 Exchange Rate Policy

To pin down the nominal variables of the model, I need to determine exchange rate policy. Suppose the central bank implements monetary policy by setting the nominal exchange rate. I consider two exchange rate regimes characterized by different targeting rules. First, consider a currency union. This is equivalent to assuming that the central bank can perfectly commit to a currency peg in which \( \epsilon_t = 1 \) at every period \( t \).

Second, assume a policy of strict zero wage inflation targeting. This rule simply offsets all the distortions originating from nominal rigidities in the economy by implementing the flexible wage equilibrium, which is given by equation (6). Any movements in the real exchange rate will
translate one-to-one into movements in the nominal exchange rate. This is the equivalent to a floating arrangement in this framework.\textsuperscript{35}

### 3.5 Equilibrium

I am now ready to define a rational expectations equilibrium as a set of stochastic processes \( \{z_H^t, z_F^t, z_{\beta}^F, IM_t, EX_t, L_t, N_t, B_t, P_t, \lambda_t, W_t\}_{t=0}^{\infty} \) satisfying equations (3), (5), (9)-(13) and (16)-(20) given the exogenous process \( \{\xi_t, Z_t\}_{t=0}^{\infty} \), initial conditions \( \{R_{-1}, B_{-1}, W_{-1}\} \) and the central bank’s policy \( \{\epsilon_t\}_{t=0}^{\infty} \). The foreign wage, \( W_F^t \), is normalized to one.

Online Appendix B.3 discusses the existence and uniqueness of the non-stochastic steady state.

### 4 Sudden Stops and Productivity

In order to study the effects of a sudden stop on aggregate productivity in this framework, I first define the variable of interest and discuss the channels through which a shock can potentially affect productivity. I then restrict attention to a version of the model, which delivers an analytical solution. Finally, I simulate the model numerically and show that the results hold more generally.

#### 4.1 Aggregate Productivity

The variable of interest is domestic aggregate productivity, which is given by:

\[
Z_H^t = N_H^t \int_{z_H^t}^{\infty} \Omega(z) z Z_t \frac{g(z)}{1 - G(z_H^t)} \, dz,
\]

where \( \Omega(z) \) is the weight used in the aggregation. It must satisfy:

\[
N_H^t \int_{z_H^t}^{\infty} \Omega(z) \frac{g(z)}{1 - G(z_H^t)} \, dz = 1.
\]

Aggregate productivity is often computed as: (i) the unweighted average, \( \Omega(z) = \frac{1}{N_H^t} \); (ii) the output-weighted average, \( \Omega(z) = \frac{q(z)}{Q_H^t} \), or (iii) the revenue-weighted average, \( \Omega(z) = \frac{r(z)}{R_H^t} \).\textsuperscript{36}

\textsuperscript{35}The exchange rate policy defined here can be easily generalized by assuming a rule such that:

\[
(\Pi_W^t)^{\phi_W} (\epsilon_t)^{1-\phi_W} = 1,
\]

where \( \Pi_W^t = \frac{W_t}{W_{-1}} \) is wage inflation and \( 0 \leq \phi_W \leq 1 \) is the weight that the monetary authority puts on wage stabilization. A currency union and a strict wage inflation target are the two extreme versions of this rule, with \( \phi_W \) set equal to zero and one, respectively.

\textsuperscript{36}\( Q_H^t \) is total domestic output given by:

\[
Q_H^t = N_H^t \int_{z_H^t}^{\infty} q(z) \frac{g(z)}{1 - G(z_H^t)} \, dz,
\]
following Lemma establishes that $z_H^t Z_t$ is the key statistic for measuring aggregate productivity independent of the weights used in the aggregation.

**Lemma 1.** Domestic aggregate productivity, $Z_t^H$, is an increasing function of the domestic productivity threshold, $z_t^H$ and the common shifter, $Z_t$.

**Proof.** See Appendix

In other words, changes in productivity in this model are partly governed by firms’ entry and exit dynamics. This is in contrast to alternatives in the literature that either model productivity exclusively as an exogenous shock to the economy, allow for variable capacity utilization or consider R&D decisions.

### 4.2 Pro-competitive, Cost and Demand Channels

For a given realization of the common shifter, the productivity threshold is determined by the number of firms in the market, the cost of production and the level of consumer demand; all three are potentially subject to change during a sudden stop episode. Let $\hat{X}_t$ define the log deviation of a some variable $X_t$ and $\overline{X}$ be its value at steady state.

**Proposition 1.** In equilibrium:

$$\hat{z}_t^H + \log Z_t = \frac{1}{2k + 2 a \gamma} \frac{\eta}{\hat{N} \lambda \hat{W}^\gamma} \hat{N}_t + \sigma \hat{W}_t + \hat{\lambda}_t.$$

**Proof.** See Appendix

The intuition follows next. In the first place, a larger number of active firms in the market, $\hat{N}_t > 0$, implies greater competition. Given the preferences considered, enhanced competition lowers individual firm demand. This forces less productive firms out of the market as profit margins are reduced at every level of productivity. This *pro-competitive effect* was first introduced by Melitz and Ottaviano (2008), which focuses on competition in the goods market.

Second, a higher wage, $\hat{W}_t > 0$, lowers the firm’s profit margin by increasing the cost of producing. Once again, a higher productivity level is then required for firms to remain profitable and select into production, therefore, aggregate productivity increases. This is what I denote the *cost* and $R_t^H$ is total domestic revenue given by:

$$R_t^H = N_t^H \int_{z_H^t}^{\infty} r(z) \frac{g(z)}{1 - G(z_H^t)} \, dz.$$
effect, which is the underlying mechanism in the canonical Melitz (2003) model, which focuses on competition in the labor market.

Finally, higher aggregate demand from consumers, $\hat{\lambda}_t < 0$, raises individual firm demand at all productivity levels and loosens the minimum productivity requirement. Less productive firms have a higher chance of entering or surviving in the market. This final channel, a novelty of this model, is referred to as the demand effect.\(^{37}\) It results from allowing competition in both the goods and the labor market.

### 4.3 An Analytical Result

Before proceeding to the full characterization of the model’s solution, it is useful to build intuition by providing some analytical results. In order to do this, I simplify the dynamics of the model in the following way. First, suppose the common productivity shifter remains constant, i.e., $Z_t = 1$ for all $t$. Second, consumers are no longer allowed to issue bonds but are instead required to pay a lump-sum tax to foreigners.\(^ {38}\) To ease the algebra, suppose the lump-sum tax is a fraction of total import revenues such that the balance of payment condition now reads:

$$\frac{EX_t}{IM_t} = (1 + \Delta_t), \quad (22)$$

where $\Delta_t$ is white noise. For now, let’s assume a sudden stop is simply a positive realization of $\Delta_t$. This will force an expansion of net exports and an improvement in international competitiveness. The following proposition considers its effect on productivity under the two alternative exchange rate regimes.

#### Proposition 2. Given a sudden stop,

1. In a floating arrangement, only the pro-competitive channel operates and productivity falls:

   $\hat{N}_t < 0$, $\hat{W}_t = 0$ and $\hat{\lambda}_t = 0$ so that $\hat{z}_t^H < 0$.

2. In a currency union, all three channel operate and the change in productivity is ambiguous:

   $\hat{N}_t < 0$, $\hat{W}_t < 0$ and $\hat{\lambda}_t > 0$ so that $\hat{z}_t^H \geq 0$.

#### Proof. See Appendix \(\Box\)

\(^{37}\)There is an implicit demand effect in the baseline Melitz (2003) model too. However, the assumption of fixed production costs introduces an additional fixed cost channel (on top of the variable cost channel here considered) that exactly offsets the demand effect.

\(^{38}\)The full model has three state variables, $B_{t-1}$, $R_{t-1}$ and $W_{t-1}$, which govern the dynamics. This simplification allows me to abstract from two of them.
First, suppose that the nominal exchange rate depreciates one-to-one with the real exchange rate, i.e. $e_t$ increases. Under this assumption, the cost and the demand effect are muted as the wage level remains unchanged. There is a fall, however, in the active number of firms in the domestic economy as the number of importers declines with the loss of competitiveness of foreign firms. There is an unambiguous fall in productivity as a result of this negative pro-competitive effect.

Suppose instead that the aggregate wage adjusts completely: $W_t$ falls while the nominal exchange rate remains unchanged. Under this alternative scenario, the negative pro-competitive effect prevails as there is still a decline in importing firms. The change in wages, in addition, leads to a negative cost effect, production of goods is cheaper, and a negative demand effect, households consume less.\(^{39}\) In other words, all three channels are operating.

In sum, the change in productivity after a sudden stop is ambiguous in the currency union and depends on parameter values. It is possible, nonetheless, to show under which parameterization, the demand effect dominates and productivity increases.

**Corollary 1.** Following a sudden stop in a currency union, a sufficient condition for $\hat{z}^H_t > 0$ is that

\[
\frac{1}{2} < \mu(1 - \beta(1 - \mu))\sigma(1 + k) < 1.
\]

**Proof.** See Appendix

There are three key parameters for this condition to hold: the share of labor income, $\sigma$, the degree of wage rigidities, $\mu$, and the shape parameter of the productivity distribution, $k$. The share of labor income governs the mapping between the wage level and the unit cost. As $\sigma$ increases, labor represents a greater share of the optimal input bundle and falling wages cheapen production costs by more. This reinforces the cost effect of a sudden stop. In the Melitz (2003) model, the cost channel is at its strongest featuring a production function which is linear in labor, $\sigma = 1$.

The degree of wage rigidities determines the size of the demand effect. A sudden stop here is simply an improvement in the domestic economy’s competitiveness through a decline in the wage level. As the level of wage stickiness increases and fewer labor-types are allowed to adjust, the decline in the optimal wage, $W^*_t$, that is required to achieve the desired overall wage adjustment is larger. This leads to a larger decrease in today’s consumer wealth and, thus, a stronger demand effect of a sudden stop.\(^{40}\)

The shape parameter measures the concentration of firms at the lower end of the productivity distribution. This represents the inverse of dispersion in firm-level productivity. As firms only

\(^{39}\)Recall that a negative demand effect is represented by a positive change in $\hat{\lambda}_t$.

\(^{40}\)The forward looking behavior of consumers further slows down the adjustment of the wage level: the fraction $\mu$ of labor types that adjust take into account that there is a possibility that they will keep the new wage in the future, where they expect no more shocks will materialize. While they discount the future at rate $\beta$, this prevents them from fully adjusting today, which explains the additional $(1 - \beta(1 - \mu))$ term.
A Qualitative Decomposition of Productivity

Table V reports the model predictions regarding the TFP growth decomposition exercise described in section 2.2. The previous results show that under the above parameter restriction, a positive shock to $\Delta_t$ leads to an increase (decrease) in productivity in a currency union (floating arrangement), which is summarized in the first row of Table V. The subsequent rows show that the overall pattern is driven by both net entrants and incumbents.

Regarding the extensive margin, the model matches the positive contribution of net entry in a currency union while it predicts a counterfactual negative contribution in a floating arrangement. Decomposing net entry further shows that this is driven by a particular feature of the model that prevents entry and exit occurring at the same time. While the model generates a negative
contribution of entrants in the floating arrangement in line with the data; it fails to fully offset it with a positive contribution of exiters. This caveat is not as important in the currency union regime because empirically it is exit, which the model is able to capture, that quantitatively dominates the overall contribution of net entry.

Regarding the intensive margin, the contribution of incumbent firms is exclusively driven by the reallocation of market shares. The exit of unproductive firms in a currency union frees up resources which are, at least partly, reallocated towards more productive survivors. The exact opposite holds in a floating arrangement. The model is silent about the within-firm and cross-term contribution because, so far, there is no firm-level productivity growth.

In sum, the current framework provides a fair representation of productivity patterns in a currency union but does not in a floating arrangement. This is not surprising as the empirical evidence concluded that the 2010-2013 increase in TFP was driven by a composition effect, which the model embraces, while the 1992-93 decline in TFP responded to a level adjustment, which is absent by construction. To improve performance, in what follows I will augment the definition of a sudden stop to allow for changes in firm-level productivity.

4.4 Numerical Simulation

As the full model cannot be solved analytically, I explore its properties by generating impulse response functions. To this end, I discuss how I model a sudden stop shock and calibrate parameters. Given the corollary result discussed above, I study the sensitivity of results to alternative calibrations. Finally, I quantify the contribution of the extensive vs. intensive margin to TFP growth as generated by the model.

Modelling a Sudden Stop Shock

The previous section depicts a sudden stop as an ad-hoc current account shock. In the full model, which allows for international borrowing, the natural extension is a positive realization of the risk premium shock $\xi_t$: an exogenous increase in the rate at which the economy borrows abroad, which forces international deleveraging and an expansion of net exports. However, I now augment the definition of a sudden stop to include a simultaneous decline in the common shifter of firm productivity $Z_t$.

For transparency, note that this improves the model fit in two dimensions. First, and as already anticipated above, it will better capture the contribution of the intensive margin to productivity growth, which is particularly important in a floating arrangement. Second, it circumvents the production boom that the model would otherwise generate. This technical limitation is common to many other papers in the sudden stop literature. Kehoe and Ruhl (2009) show that standard models that abstract from financial frictions are unable to reproduce observed decreases in output with an expansion of net exports. To fix this, the literature has considered featuring imported interme-
Table VI: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Calibration Target\Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) Discount factor</td>
<td>0.99</td>
<td>Annual real return on bonds is 4%</td>
</tr>
<tr>
<td>( \mu ) Index of wage rigidity</td>
<td>0.2</td>
<td>Galí and Monacelli (2016)</td>
</tr>
<tr>
<td>( \theta ) Elasticity of substitution (labor)</td>
<td>4.3</td>
<td>Galí and Monacelli (2016)</td>
</tr>
<tr>
<td>( \tau ) Iceberg trade cost</td>
<td>1.3</td>
<td>Ghironi and Melitz (2005)</td>
</tr>
<tr>
<td>( \gamma ) Preference parameter</td>
<td>10</td>
<td>Ottaviano (2012)</td>
</tr>
<tr>
<td>( \kappa ) Preference parameter</td>
<td>10</td>
<td>Ottaviano (2012)</td>
</tr>
<tr>
<td>( \eta ) Preference parameter</td>
<td>10</td>
<td>Ottaviano (2012)</td>
</tr>
<tr>
<td>( \phi ) Risk premium parameter</td>
<td>0.001</td>
<td>Garcia-Cicco, Pancrazi and Uribe (2010)</td>
</tr>
<tr>
<td>( \bar{B} ) Steady state level of debt</td>
<td>0</td>
<td>Steady state trade balance</td>
</tr>
<tr>
<td>( \sigma ) Labor share</td>
<td>0.64</td>
<td>National Accounts Spain</td>
</tr>
<tr>
<td>( n ) Relative size of SOE</td>
<td>0.12</td>
<td>Business Demographic Statistics</td>
</tr>
<tr>
<td>( k ) Shape productivity parameter</td>
<td>1.9</td>
<td>Estimated from ESEE data</td>
</tr>
<tr>
<td>( A ) Foreign demand parameter</td>
<td>0.01</td>
<td>Domestic productivity cutoff (1.55 )</td>
</tr>
<tr>
<td>( B ) Foreign demand parameter</td>
<td>0.33</td>
<td>Share of exporting firms (63.6%)</td>
</tr>
<tr>
<td>( M ) Number of total firms</td>
<td>173</td>
<td>Active domestic firms (75.86)</td>
</tr>
</tbody>
</table>

Notes: This table summarizes the baseline calibration for the model described in section 3. The first set of parameters are standard and set in line with the literature. The second set of parameters are set using Spanish aggregate or firm-level data directly. The third set of parameters are set to match the model’s predictions in steady state with moments of the Spanish firm-level data.

diate goods, labor frictions, variable capacity utilization, Greenwood, Hercowitz and Huffman (1988) preferences, and, as in this paper, exogenous TFP declines.

Calibration

Table VI provides a summary of the parameters of the model, their baseline values and the source or the empirical target. The first set of parameters are standard and, thus, values are set in line with the literature and, when possible, consistent with Spanish statistics taking the 2002-08 period as a reference. The time period of the model is a quarter. Accordingly, the discount factor \( \beta \) is chosen to be 0.99. The output elasticity parameter \( \sigma \) is set to 0.64, roughly the average labor share and within the range that is common in the literature. For the elasticity of substitution for labor types and the index of wage rigidities, values are taken from Galí and Monacelli (2016) which are based on empirical studies on European countries conducted by the OECD. In terms of trade costs, \( \tau \) is equal to 1.3 following Ghironi and Melitz (2005) and many others. The steady state level of debt, \( \bar{B} \), is assumed to be zero, such that trade is balanced in steady state. Regarding the preference parameters, \( \kappa \), \( \gamma \) and \( \eta \), I borrow the values used in Ottaviano (2012), all equal to 10.

The ESEE firm-level data presented in section 2 is then used to estimate the shape parameter of the Pareto distribution, following the approach proposed by Del Gatto, Mion and Ottaviano...
Given the observed cumulative distribution, \( G(z) \), I run the following regression for every year and industry:

\[
\ln(1 - G(z)) = \beta_0 + \beta_1 \ln(z) + \eta
\]

where, assuming a Pareto distribution, the slope coefficient, \( \beta_1 \), provides a consistent estimator for \( k \). For the 2002-08 period, \( k \) is estimated to be, on average, equal to 1.9, close to Del Gatto, Mion and Ottaviano (2006)’s result of 2 for a combination of European countries in the year 2000. In addition, the regression \( R^2 \), which is equal to 0.7, confirms that the Pareto distribution is a reasonable assumption in this setting.

The above estimation provides an additional coefficient, \( \beta_0 \), that maps one-to-one to the realized distribution’s cutoff, \( z^H \). I use the corresponding 2002-08 average as a first moment target in two different ways. On the one hand, I combine it with the 2002-08 average number of firms in the ESEE sample to back up the value of \( M \) given that the number of potentially active firms is unobservable. The corresponding expression is given by

\[
M = \left( \frac{h}{z^H} \right)^k \tilde{N}^H.
\]

On the other hand, I use \( z^H \) to determine the value of the foreign demand parameters, \( A \) and \( B \). To do so I proceed in three steps. First, I set the relative size of the domestic economy, \( n \), to match the 12% share of all Euro-area manufacturing firms that Spanish firms represent according to Eurostat’s Business Demography Statistics. Next, I take the average 2002-08 propensity to export as an additional first moment target which combined with \( z^H \) pins down \( z^F \) as

\[
\frac{\tilde{N}^F}{\tilde{N}^H} = \left( \frac{z^H}{z^F} \right)^k.
\]

Third, I back up the wage level that is consistent with the estimated cutoff using a combination of equilibrium conditions (9), (10), (13) and (12) in steady state. Parameter values for \( A \) and \( B \) then follow naturally using equation (11) and the trade balance condition.

The risk premium parameter, \( \phi \), is a theoretical shortcut to ensure stationarity in small open economy frameworks. It measures the sensitivity of the country interest-rate premium to debt. I follow Garcia-Cicco, Pancrazi and Uribe (2010) in choosing a very small value while I have also explored alternative parameterizations, which show that results are not sensitive to the choice.

**Impulse Responses Functions**

Figure III summarizes the model response of key macroeconomic variables to a sudden stop. All variables, but the current account, are expressed in log deviations from steady state. The current account is expressed in levels as trade balance is assumed to hold before the realization of the shock.

As expected, a sudden stop is characterized by a depreciation of the real exchange rate and a current account surplus. The model predicts a slight delay in the adjustment within a currency union. This is entirely driven by nominal rigidities as the model disregards additional policy instruments available under a currency union, such as public capital inflows, that might directly cushion the adjustment in the data.
The path of TFP clearly diverges across regimes. On the one hand, under the baseline calibration, the negative effect of a lower aggregate demand offsets the positive effect of lower production costs and fewer competing firms on the domestic productivity cutoff and, thus, TFP improves in the currency union. On the other hand, productivity falls unambiguously in the floating regime. I study the sensitivity of these results to alternative parameter values next.

Output and consumption are measured in real terms. The model predicts a fall in the two variables under both regimes although the decline is more pronounced in a currency union. Similarly, the decline in employment is only evident when productivity rises.

The current account surplus, denominated in domestic currency, is generated through an increase in export and a decline in import revenues. However, regimes differ in the relative magnitude of these simultaneous effects: in a floating regime the expansion of exports dominates while in a currency union the main driving force is the retrench of import revenues. This highlights the importance of the demand mechanism in the model as it is the larger domestic contraction generated by the adjustment of wages that additionally reduces imports in a currency union.41

Sensitivity of the TFP fact The analytical results of section 4 point to three structural parameters as the main determinants of the overall response of TFP: the degree of wage rigidities, $\mu$, the share

41Impulse response functions for all other endogenous variables in the model can be found in Figure A.IV.
Notes: All three figures plot the immediate response of TFP in log deviations to a one standard deviation shock to the country-specific risk premium and the common TFP shifter keeping all but one parameter values unchanged. The first figure focuses on different degrees of wage rigidities - higher \( \mu \) implies lower rigidities. The second figure allows for plausible calibrations of the labor share - higher \( \sigma \) implies a larger labor share. The last figure explores alternative values of the shape parameter of the Pareto distribution - higher \( k \) implies lower dispersion of productivity draws.

of labor income, \( \sigma \), and the shape parameter of the productivity distribution, \( k \). I next explore the role of each of these parameters in driving TFP behavior in the numerical model.

The first graph in Figure IV plots the immediate response of TFP, in log deviations from steady state, to a sudden stop shock under both the currency union and the floating arrangement regimes for different values of wage flexibility i.e. \( 0.1 \leq \mu \leq 0.9 \). By definition, under the floating arrangement wages are stabilized completely and, thus, there is no effect of wage frictions on macroeconomic variables whatsoever. For the currency union, nevertheless, higher wage flexibility (higher \( \mu \)) leads to a smaller increase in TFP as hinted by the analytical results.\(^{42}\)

Similarly the following two graphs in Figure IV repeat the exercise for the other two key parameters: the labor share and the shape parameter. In a currency union, the improvement of TFP is larger when there is a looser link between wages and the unit production cost (lower \( \sigma \)) and there is a lower concentration of firms at the lower end of the productivity scale (lower \( k \)) again in line with the intuition provided by the different channels. In a floating arrangement, the labor share is irrelevant as the adjustment of the exchange rate is not affected by the production structure of the economy whereas more heterogeneity (lower \( k \)) increases the decline in TFP.\(^{43}\)

All in all, the results depicted by Figure IV, show that the behavior of TFP is robust to different parameterizations of \( \mu, \sigma \) and \( k \) and the intuition developed along the analytical section is useful.

\(^{42}\)While Figure IV depicts the immediate effect of a sudden stop shock on TFP, conclusions remain true if the cumulative effect on TFP is considered.

\(^{43}\)The latter might sound counter-intuitive given Lemma 1. To rationalize this result, consider second round effects of a nominal depreciation - while a larger \( k \) initially leads to a larger decline in the number of importers, the resulting decline in the domestic threshold leads to a larger (cushioning) increase in the number of both domestic and importing firms. Under the current calibration the strength of such a cushioning effect is increasing in \( k \)
Table VII: Model Generated Quantitative Decomposition of Productivity Growth

<table>
<thead>
<tr>
<th>Exchange Rate Regime</th>
<th>Floating Arrangement</th>
<th>Currency Union</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity Growth (%)</td>
<td>-0.53</td>
<td>3.50</td>
</tr>
</tbody>
</table>

Contribution to Productivity Growth

**Incumbent’s Contribution**

- Within-firm Contribution: -0.53
- Between-firm Contribution: 3.00
- Cross-term Contribution: -1.00
- Net Entry Contribution: 4.04

**Entrants’ Contribution**

- Entrants’ Contribution: 0.01

**Exiters’ Contribution**

- Exiters’ Contribution: 0.51

Notes: This table reproduces the productivity growth decomposition exercise in section 2 but through the lens of the model described in section 3. It builds on the numerical results discussed in section 4.4, which are quantitative. Online Appendix B.4 provides more details on the required derivations.

A Quantitative Decomposition of Productivity

Table VII redoes the TFP growth decomposition exercise once again but attaching some numbers to the model’s predictions. As in the qualitative version, the full model generates a fall in productivity under a floating arrangement and an increase in the currency union. Here, however, the former is entirely driven by the contribution of incumbent firms and, more specifically, by the decline in the within-firm component. There is a positive, yet negligible, contribution of net entrants and a positive contribution of reallocation of market shares, both in line with the data.

In order to reconcile this result with the previous intuition, notice that it is the sum of the domestic threshold and the common productivity shifter, \( \log(Z_t) + \hat{z}_t^H \), that is now required to adjust given the deleveraging shock. Since the common shifter is assumed to fall simultaneously, it is possible that the exogenous decline in the first term suffices to ensure the adjustment of the sum i.e. \( \hat{z}_t^H \) might increase moderately. This is exactly the case depicted in Table VII.

The qualitative predictions for the currency union still hold in the fully-fledged model as shown in the last column of Table VII. Moreover, the exit of unproductive firms and the reallocation of idle resources is large enough to offset the exogenous decline in the firm-level productivity of incumbents.

While the model does a decent job in matching the overall behavior of productivity, in general, and the differences in the magnitude of the extensive margin across regimes, in particular, there
are two key limitations. First, it predicts a larger contribution of the between-firm component than what is observed in the data. This is particularly salient in the currency union case. Accounting for reallocation frictions in the model could partly address this concern. Second, under the current calibration, the results fully abstract from entry.

5 Extensions

This section briefly introduces a number of extensions to the baseline framework and discusses how (if anything) the previous results change.

5.1 A Second Factor of Production

The analysis has so far abstracted from explicitly modeling a second factor of production, in particular, capital. This is due to three reasons: first, for simplicity. Second, there is already a number of papers (Reis (2013), Gopinath et al. (2017) and García-Santana et al. (2020)) which have extensively studied the role of physical capital in the context of capital flows. Instead, this paper aims at incorporating an alternative yet complementary explanation to the discussion. Third, the firm-level evidence presented in section 2 is supportive of theories that focus on the composite variable input, and not only on capital. Nonetheless, this extension incorporates physical capital as a second factor of production to determine whether the concave cost assumption drives the baseline results.

The setting is standard: the production function is Cobb-Douglas in labor, $L_t$, and capital, $K_t$. Capital goods are owned by the representative consumer and rented to firms in exchange of a rental rate $k_t$. For the time being I assume the stock of capital is fixed - section 5.3 will incorporate investment decisions. Online Appendix C.1 formalizes this extension and provides details on the resulting equilibrium conditions.

This extension generates the same differences in TFP response across regimes, although the increase in the currency union is now smaller. This is explained by the decline in the rental price of capital as demand for capital collapses, which reduces the unit production cost by more than the wage level, i.e., reinforcing the cost channel. All other variables behave as in the baseline model. See Figure A.V for more details.

5.2 Imported Intermediate Inputs

Section 2.2 discusses imported intermediate inputs briefly. This extension augments the baseline framework to allow for an expenditure switching effect of exchange rate policy and study whether incorporating intermediate inputs in the production of differentiated varieties affects the model’s predictions.
Consider a Cobb-Douglas production function in labor, $L_t$, and a bundle of intermediate inputs, $x_t$. Intermediate inputs can be sourced domestically or can be imported. They are combined according to a CES aggregator: $x_t = \left[ (x_t^H)^{\frac{1}{\chi}} + (x_t^F)^{\frac{1}{\chi}} \right]^{\frac{\chi}{1}}$, where $x_t^H$ and $x_t^F$ measure domestic and imported intermediate inputs, respectively, and $\chi$ is the elasticity of substitution between them. Intermediate inputs are produced under perfect competition using only labor as a factor of production, i.e., prices are equal to the wage level in the source country. Online Appendix C.2 formalizes this extension and provides details on the resulting equilibrium conditions.

The macroeconomic effects of a sudden stop are qualitatively unchanged with the exception of output and employment. Under the current parameterization the decline in the common shifter of firm productivity is no longer enough to offset the production boom generated by the increase in exports.\(^44\) This explains the increase in labor demand. In any case, the TFP fact holds. In addition, the expenditure switching effect is captured by the shift in demand towards domestic intermediate inputs, as the relative price of imported intermediate inputs increases. This effect is present under the two regimes. However, differences in the size of the relative price change (the nominal exchange rate depreciation is larger) will lead to differences in how prominent this effect is. See Figures A.VI and A.VII for more details.

5.3 Long-run Analysis

This extension studies a long-run version of the baseline model that fully endogenizes the number of existing firms, $M_t$, in line with Ottaviano (2012). The previous framework is augmented by (i) allowing for investment in capital shares; (ii) introducing a new sector that produces capital; and (iii) imposing a fixed input requirement in terms of capital in the production of differentiated varieties.

In particular, the representative consumer is allowed to buy shares, $x_t$, of the economy’s capital stock, $K_t$, at price $V_t$. While capital is assumed to fully depreciate after one period; the investment entitles the representative consumer to a fraction of next period’s aggregate firm profit. The consumer budget constraint is correspondingly adjusted to read:

$$\int_{\omega \in \Omega} p_t(\omega) q_t(\omega) \, d\omega + e_t B_t + x_t V_t K_t = \int_0^1 W_t^i L_t^i \, di + x_{t-1} \Pi_t + e_t R_{t-1} B_{t-1} \,.$$

Capital is supplied under perfect competition by a second sector in the economy. A new unit of capital is produced by combining domestic and foreign units of labor using a Cobb-Douglas production technology: $K_t = \left( k_t^{H} \right)^{p} \left( k_t^{F} \right)^{1-p}$.\(^45\) Given the fixed capital requirement, the pro-

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\(^{44}\)One would need to impose that the shock to $Z_t$ is four times as big as the interest-rate shock to generate a decline in output.

\(^{45}\)I deviate from Ottaviano (2012) in two ways. First, I introduce foreign labor in the production of capital to ensure a direct role for the nominal exchange rate in firm entry. Second, I consider that while capital fully depreciates, all
duction of capital determines how many firms will be able to enter the market, \( M_t = \frac{K_f}{f} \). There is a one-period-time-to-build-lag such that firms that enter at time \( t \), will only be able to produce, provided that they satisfy the corresponding productivity threshold condition, in period \( t + 1 \).

Online Appendix C.3 describes this extension in greater detail and provides the full set of equilibrium conditions. It is relevant, however, to highlight one new optimality condition that emerges from this set-up:

\[
M_t = \left( \frac{\rho}{W_t} \right)^{\rho} \left( \frac{1 - \rho}{\epsilon_t} \right)^{1-\rho} \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \Pi_{t+1} \Pi_t \right]. \tag{23}
\]

Intuitively, a lower price of capital encourages investment and increases the number of existing firms. As capital is produced under perfect competition, price is equal to marginal cost and, thus, a function of the price of both types of labor. The price of foreign labor is equal to the foreign wage, which is normalized to one, in domestic currency units, i.e., the nominal exchange rate. In addition, the number of existing firms is also dependent on the discounted expected profits, as profits represent the return on capital investment. This inter-temporal dimension is missing in the previous analysis, however, solving for this long-run version of the model shows that the main conclusions derived above hold.

While the shape of responses is slightly changed because of the delay in adjustment caused by the new timing assumption, the predictions are qualitatively the same. The exception is output in the floating regime, which rises moderately as the sudden stop hits the economy. See Figure A.VIII for more details.

6 Other Sudden Stops

This section explores whether the model’s aggregate predictions apply beyond the Spanish experience. To systematically analyze a wider set of countries, I establish a criterion to identify sudden stops and use an event study approach to study the path of macroeconomic variables. Both these steps are standard in the literature. The novelty of the exercise relies on binning the episodes by exchange rate regime.

6.1 Data and Methodology

Following Cavallo and Frankel (2008), I define a sudden stop as an episode in which there is a substantial decline in the capital account surplus together with a recession.\(^46\) In particular, I clas-

\(^46\)The practice of conditioning on output contraction goes back as far as the canonical Calvo, Izquierdo and Mejía (2004) methodology. While I confine myself to what is standard in the literature, it is fair to acknowledge this is not strictly consistent with the model’s definition of a sudden stop.
sify as a sudden stop a period that contains at least one year during which (i) the financial account surplus has fallen at least one standard deviation below its rolling average and (ii) GDP per capita contracts.\textsuperscript{47} The start and end of each episode is marked by the first and last year within the period in which the financial account surplus is half a standard deviation below the rolling average.\textsuperscript{48} The latter requirement ensures that the capital flow reversals captured by the algorithm strictly qualify as sudden stops; first, by requiring that the financing disruption is accompanied by an appropriate macroeconomic adjustment, and second, by ruling out booming episodes that display similar characteristics, for example a positive trade shock. All data is collected from standard sources and, thus, its description is relegated to Online Appendix D.1

The total number of episodes is 78, representing 5.2\% of total available country/year observations in the sample. The full list of episodes per country, plus exchange rate classification, is given by Table A.X. The criterion successfully captures all traditional sudden stop episodes previously discussed by the literature - mostly occurring around the 1994/5 Tequila crisis, the 1997 Asian Financial Crisis, the 1998 Russian default - as well as the most recent balance of payment crisis in the peripheral economies of the European Union.\textsuperscript{49}

I build on Ilzetzki, Reinhart and Rogoff (2019) updated \textit{de facto} coding system in order to bin episodes by exchange rate flexibility. In my baseline results, I consider as prevalent the exchange rate regime that is in place during the last year of the sudden stop. There are four different cases: a currency union, a hard peg, a soft peg and a floating arrangement.\textsuperscript{50} Out of the 78 episodes identified, 11 occur within a currency union (8 in the Euro Area and 3 in the West African Economic and Monetary Union), 14 in a hard peg system, 26 in a soft peg regime and 25 in a floating arrangement.

Figures V and VI show the mean and median path of each of these aggregate variables during the episodes conditional on their exchange rate classification together with standard error bands.\textsuperscript{51} In order to capture the buildup and end phase of each episode, the plot depicts six-year windows that begin two years before the start of each reversal and marks the start and the average duration of a sudden stop with vertical lines. As is standard in this literature, I focus on the cyclical component of most of the variables by looking at its percentage deviation from an extrapolated pre-crisis

\textsuperscript{47}This contrasts with Cavallo and Frankel (2008), who also require an improvement in the current account deficit (or an equivalent decline in foreign reserves). As this is conceptually equivalent to the first condition, I drop it.

\textsuperscript{48}Refer to Online Appendix D.2 for further details.

\textsuperscript{49}The methodology does not account for changes in TARGET2 balances in the Eurozone and, thus, prevents me from measuring private capital flows accurately. However, this is not problematic for my purposes as the algorithm already identifies the GIIPS episodes.

\textsuperscript{50}In terms of the Ilzetzki, Reinhart and Rogoff (2019) fine classification, I deviate as follows: (1) I manually divide code 1 into currency union and no separate legal tender, (2) I group codes 2 to 4 under the hard peg category, (3) I group codes 5 to 11 under the soft peg category, (4) I group codes 12 to 14 under the floating arrangement and (5) I rename group 15 as 5, i.e., other categories.

\textsuperscript{51}Figure A.IX plots the same aggregate variables without conditioning on the exchange rate classification to confirm the stylized facts previously documented by the literature hold.
**Figure V: A Sudden Stop in a Floating Arrangement**

![Graphs showing the response of macroeconomic variables to a sudden stop under a floating arrangement.](image)

**Notes:** This figure plots the response of macroeconomic variables to a sudden stop under a floating arrangement. The black and red solid lines depict the mean and median path of the corresponding variables while the black dashed lines represent standard error bands. The two vertical lines show the start and end of an average episode. Output, consumption, employment, productivity, exports and imports are expressed in terms of percentage deviations from an extrapolated linear trend calculated from periods $t-5$ to $t-2$. Current account is expressed as a share of GDP and the real exchange rate (RER), calculated as an index, is expressed in levels. The data used is collected from IFS, WDI and the Total Economy Database.

### 6.2 Results

Figure V illustrates how domestic variables respond to an unexpected reversal of capital flows when the exchange rate is allowed to adjust freely. First, a sudden stop is associated with a contraction in output and consumption, with most of the decline occurring on impact or shortly after. There is also a small decline in employment levels, measured as the total number of hours worked, and a significant collapse in total factor productivity. The last four graphs capture the response of the external sector: capital outflows coincide with a depreciation of the real exchange rate, represented by a decline in the index. The current account deficit is reduced sharply, almost reaching trade balance as soon as one year after the start of the episode. Finally, the average duration is slightly less than two years.

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52The current account deficit, expressed as a share of GDP, and the real exchange rate index, with base $t-2$, are the exception.
The results for a currency union are summarized by Figure VI. The response of all variables but TFP is similar, in qualitative terms, to that depicted in the flexible exchange rate case. The unexpected reversal of flows is associated with a decline in output, consumption and employment. There is a gradual reduction in the current account deficit that yet persists four years after the onset of the crisis. In line with this result, the real depreciation is more gentle than in the previous case and the episodes last longer; on average, two and a half years.

The most notable difference across the plots is the behavior of TFP: whereas productivity clearly falls in the first case, it remains unchanged or, if anything, improves slightly within currency unions. The positive relationship between the size of the decline in TFP and the degree of exchange rate flexibility is in line with the model’s predictions.

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53 Given the reduced sample size, standard error bands are admittedly large to be able to conclude that TFP increases significantly.

54 For completeness, I present the results for the hard and soft pegs in Figures A.X and A.XI. It is still the case that the decline in productivity is increasing in the degree of flexibility: under a hard peg, there is an increase in productivity, although bands are much wider than within a currency union, and under a soft peg, there is some significant decline, especially on impact.
Moreover, there are additional, although arguably minor, differences in responses across regimes that are worth highlighting. Although a quantitative comparison is beyond the scope of this exercise, the decline in employment is more pronounced in Figure VI. This holds in both absolute and relative to output terms and is consistent with the predictions of the model. In addition, a closer look at the external sector shows that in floating arrangements the current account reversal is mostly driven by the increase in exports. In a currency union, however, the decline in imports almost matches in magnitude the increase in exports suggesting there is a larger contraction of domestic demand in line with the mechanisms at play in the model.

Finally, I conduct a battery of robustness checks to evaluate the consistency of the TFP finding including different approaches to exchange rate classification and removing the trend, alternative data sources and controlling for crisis and country characteristics. Results are available in Online Appendix D.3.

7 Conclusion

This paper revisits a classical question in international macroeconomics: how does exchange rate policy affect macroeconomic performance after a shock? While the literature provides many attempts at answering this issue, it has mostly overlooked the effect on firm dynamics. I study the question anew in the context of a sudden stop, emphasizing the divergence in TFP patterns that emerges across exchange rate regimes in the aggregate data and relating them to observed differences in firm exit at the micro level.

Taking the firm-level analysis of two sudden stops in Spain as a starting point, the paper argues that documented differences in the reallocation of resources from unproductive exiting firms to productive survivors might be related to the degree of currency appreciation vis-à-vis wage devaluation. A small open economy DSGE model featuring firm selection, variable markups and elastic labor supply formalizes the mechanism. Productivity is determined by the number of firms (pro-competitive channel), the marginal utility of wealth (demand channel) and the unit cost of production (cost mechanism). The relative magnitude of these forces depends on the exchange rate policy with a currency union generating quantitatively more cleansing because of a larger demand effect. Systematic analysis of the behavior of macroeconomic variables during sudden stops under different exchange rate regimes confirms that the model’s implications hold for a wide set of economies.

This paper provides a positive account of the effect of exchange rate policy on short-term productivity growth. However, it raises a new important question: how does productivity translate into welfare gains? Evaluating the trade-off between improving resource reallocation and undoing nominal rigidities seems key in understanding the normative implications of this type of model. In particular, what is the optimal weight policy should put on each of these remains an open question for future research.
References


Appendix

Proof of Lemma 1

Proof. Unweighted average productivity is given by
\[ \tilde{Z}_t = \int_{-\infty}^{\infty} z Z_t \frac{g(z)}{1 - G(z_t)} \, dz = \frac{k}{k - 1} z_t Z_t. \]

Average productivity weighted by output is given by
\[ \hat{Z}_t = \int_{-\infty}^{\infty} z Z_t \frac{q(z)}{q(\tilde{z}_t)} \frac{g(z)}{1 - G(z_t)} \, dz. \]

Noting that \[ \frac{q(z)}{q(\tilde{z}_t)} = \frac{z - z_t}{z - \tilde{z}_t}, \] the above expression simplifies to \( \hat{Z}_t = \tilde{Z}_t. \)

Average productivity weighted by revenue is given by
\[ \bar{Z}_t = \int_{-\infty}^{\infty} z Z_t \frac{r(z)}{r(\tilde{z}_t)} \frac{g(z)}{1 - G(z_t)} \, dz. \]
Noting that \( r(z) = \frac{z^2 - (z_H^t)^2}{(z_H^t)^2} \frac{(z_H^t)^2}{z^2} \), the above expression simplifies to
\[
Z_H^t = \frac{2k^3}{(2k-1)(k^2-1)} z_H^t Z_t .
\]

**Proof of Proposition 1**

*Proof.* By combining equations (9) and (13), the domestic productivity threshold can be rewritten as
\[
z_H^t Z_t = \frac{\lambda_t \omega}{\alpha \gamma} \left[ \gamma + \frac{\eta}{2k+1} N_t \right] .
\]

(24)

To derive the expression in Proposition 1 log-linearize equation (24) around its steady state.

**Proof of Proposition 2**

*Proof.* To see this formally, combine equations (9), (10) and (12) to rewrite the equilibrium number of active firms in the domestic market as
\[
N_t = \left( \frac{1}{z_H^t} \right)^k \left[ M + M^* \left( \frac{W_i^\omega}{\tau (W_i^\sigma)^\gamma} \eta_t \right)^k \right] ,
\]

(25)

and combine with the expression for \( z_H^t \) above, equation (24), to get
\[
z_H^t - \frac{\eta}{2k+2} \frac{\lambda_t \omega}{\alpha \gamma} \left( \frac{1}{z_H^t} \right)^k \left[ M + M^* \left( \frac{W_i^\omega}{\tau (W_i^\sigma)^\gamma} \eta_t \right)^k \right] = \frac{\lambda_t \omega}{\alpha} .
\]

(26)

Next, substitute equations (9), (10), (11), (19) and (20) into the new balance of payments condition, (22), which gives
\[
\gamma \frac{M^* A^{k+2}}{M^* B^{k+1}} \frac{e_i^{2k+1}}{\lambda_t W_t^{2\sigma(k+1)}} (z_H^t)^{k+2} = 1 + \Delta_t .
\]

(27)

We are now ready to summarize the model’s equilibrium in a single equation by combining (26) and (27) as
\[
\left[ M^* B^{k+1} \frac{\lambda_t \omega}{\alpha \gamma} \frac{1 + \Delta_t}{e_i^{2k+1}} \right]^{\frac{1}{k+2}} - \frac{\eta}{2k+2} \frac{\lambda_t \omega}{\alpha \gamma} b^k \left[ M^* B^{k+1} \frac{e_i^{2k+1}}{\lambda_t W_t^{2\sigma(k+1)}} \frac{\gamma}{\gamma + 1 + \Delta_t} \right]^{\frac{k}{k+2}} \left[ M + M^* \left( \frac{W_i^\omega}{\tau (W_i^\sigma)^\gamma} \eta_t \right)^k \right] = \frac{\lambda_t \omega}{\alpha} .
\]

(28)

From here it is straightforward to see that there is a positive relationship between \( \epsilon_t \) and \( \Delta_t \) as \( k > 1 \). It then follows that because there is a negative relationship between \( \epsilon_t \) and \( z_H^t \) given by
(26), an increase in $\Delta_t$, lowers $z_t^H$ unambiguously if wages remain unchanged.

The relationship between $W_t$ and $\Delta_t$ is less obvious. The right-hand side of equation (28) is decreasing in wages as $\lambda_t W_t^\tau \sim W_t^{\frac{1}{\rho+1}(1-s)}$ by Lemma 2. The left-hand side, however, depends on parameter values. Similarly the relationship between $W_t$ and $z_t^H$ given by (26) is also ambiguous.

\textbf{Lemma 2.} There is a negative relationship between the marginal utility of income and the wage level.

\textit{Proof.} Given the nature of the shock, $E_t \log(X_{t+1}) = \log(\bar{W})$, where $\bar{W}$ is the steady state wage level. It then follows from rewriting equation (14) for $s = 0$ that

$$\log(X_t) = (1 - \beta(1 - \mu)) \log(W_t^{\text{flex}}) + \beta(1 - \mu) \log(\bar{W}) .$$

Plugging the above into equation (15) and taking the exponential shows that $\lambda_t \sim W_t^{\frac{1}{\rho+1}(1-s)}$.

\textbf{Proof of Corollary 1}

\textit{Proof.} Suppose $\mu(1 - \beta(1 - \mu)) \sigma(1 + k) > \frac{1}{2}$, then the left-hand side of equation (28) is increasing in wages. Thus, there is an unambiguous negative relationship between $W_t$ and $\Delta_t$. If, in addition, $\mu(1 - \beta(1 - \mu)) \sigma(1 + k) < 1$, then (26) depicts a negative relationship between $W_t$ and $z_t^H$. Altogether, this ensure that an increase in $\Delta_t$, rises $z_t^H$ unambiguously if the exchange rate remains unchanged.