Plants in Space

Ezra Oberfield (Princeton)
Esteban Rossi-Hansberg (Princeton)
Pierre-Daniel Sarte (Richmond Fed)
Nicholas Trachter (Richmond Fed)

NBER SI, July 2020

Introduction

- A fundamental part of a firm's production problem is to determine the number, size, and location of its plants
- More plants, closer to consumers, imply lower **transport costs** but larger **fixed and managerial span-of-control costs**
  - Plants **cannibalize** each other's markets, particularly if they are close
- There are **no general insights** on the solution to this problem **when locations are heterogeneous**
  - Large **combinatorial problem**, so most of the analysis is purely numerical
  - Solution is known when space is homogeneous
- Important to study this problem to understand firm characteristics as well as consumer access to a firm's output
- In equilibrium, plant location decisions are an essential determinant of local concentration and the distribution of economic activity in space

What We Do

- Propose a model of a firm's location decisions
  - Heterogeneous firms locate multiple plants in heterogeneous locations
  - Firms face transport, span-of-control, and fixed costs
  - Key tradeoff: **minimize transport cost** vs. **cannibalization**
- Problem of the firm is a large combinatorial discrete-choice problem
- Our contribution is to **propose a tractable limit case**
  - All key forces and trade-offs remain relevant
  - Solution method inspired by central place theory, discrete geometry
  - Amounts to a many-to-many matching problem that can be partially characterized analytically
- Limit case can be embedded in an equilibrium framework
  - Study effect of changes in technology and transport costs on sorting and local characteristics
- We verify many of the implications using the NETS data set

Related Literature

- Firm with multiple plants
  - Over time: Luttmer (2011); Cao, et al. (2019); Aghion, et al. (2019)
  - Across space: Rossi-Hansberg, et al. (2018), Hsieh and Rossi-Hansberg (2020)
- Multinationals and export platforms
  - Ramondo (2014), Ramondo and Rodriguez Clare (2013), Tintelnot (2017), Arkolakis, et al. (2018)
- Solutions to plant location problem
  - Homogeneous space: Christaller (1933), Fejes Toth (1953), Bollobas (1972)
  - Numerical: Jia (2008), Holmes (2011), Arkolakis and Eckert (2018), Hu and Shi (2019)
- Assignment in space
  - Firm location: Gaubert (2018), Ziv (2019)
  - Worker location: Behrens, et al. (2014), Eeckhout et al. (2014), Davis and Dingel (2019), Bilal and Rossi-Hansberg (2019)

The Environment

- Customers distributed across locations $s \in \mathcal{S} = [0,1]^2 \subset \mathbb{R}^2$
- Each location s characterized by
  - Exogenous local productivity, B_s
  - Residual demand, $D_s(p) = D_s p^{-\varepsilon}$, with $\varepsilon > 1$
    - Later, D_s a function of local price index and exogenous amenities
  - Wage rate, W_s
  - Commercial rent, R_s
- Firms take local equilibrium as given

Firms

- Each firm $j \in J$ produces a unique variety
- Chooses set of locations $O_j \in S$ where to produce
  - Let $N_j = |O_j|$, denote the number of locations where j produces
- Firm productivity in location $o \in O_j$ is $B_o Z(q_j, N_j)$
  - where q_j is an exogenous component of firm productivity
  - and $Z_N(q_j, N_j) < 0$ and $Z(q_j, 0) < \infty$ (**Span-of-control costs**)
- Each plant takes ξ units of commercial real estate, with rental cost R_s per unit of space
- Iceberg cost, $T(\delta)$, to deliver good to customer at distance δ

The Firm's Problem

- Minimal cost of delivering one unit to s is $\Lambda_{js}(O_j) \equiv \min_{o \in O_j} \frac{W_o T(\delta_{so})}{B_o Z(q_j, N_j)}$

- Optimal price is then $\max_{p_{js}} D_s(p_{js})(p_{js} - \Lambda_{js})$

- Total profit of firm j is then given by

$$\pi_j = \max_{O_j} \left\{ \int_s \max_{p_{js}} D_s p_{js}^{-\varepsilon} (p_{js} - \Lambda_{js}(O_j)) \, ds - \sum_{o \in O_j} R_o \xi \right\}$$

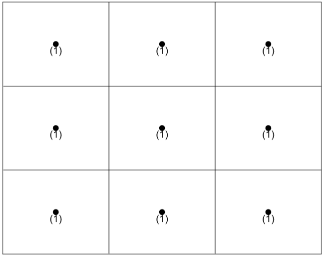
$$= \max_{O_j} \left\{ Z(q_j, N_j)^{\varepsilon-1} \frac{(\varepsilon - 1)^{\varepsilon-1}}{\varepsilon^{\varepsilon}} \int_s D_s \max_{o \in O_j} \left(\frac{B_o / W_o}{T(\delta_{so})} \right)^{\varepsilon-1} ds - \sum_{o \in O_j} R_o \xi \right\}$$

Catchment Areas

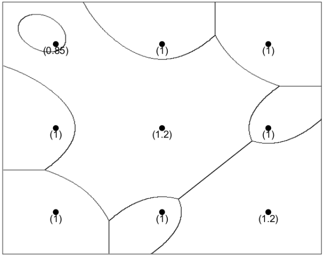
- **Given plant locations**, catchment areas only depend on $T(\delta_{so})$ and B_o/W_o

$$CA(o) = \left\{ s \in \mathcal{S} \text{ for which } o = \arg\max_{\tilde{o} \in O_j} \left\{ \frac{B_{\tilde{o}}/W_{\tilde{o}}}{T\left(\delta_{s\tilde{o}}\right)} \right\} \right\}$$

- Example: $T(\delta_{so}) = 1 + \delta_{so}$



$B_o/W_o = 1 \ \forall \ o$



B_o/W_o vary with o

A Limit Case

- In general, placement of plants in space is a hard problem
  - Catchment areas depend on local characteristics of plant locations
  - Plant locations depend on the whole distribution of demand across space
- Our approach is to study a limit case in which firms choose to have many plants, with small catchment areas
  - Consider an environment indexed by Δ, in which

$$\xi^{\Delta} = \Delta^2$$
$$T^{\Delta}(\delta) = t\left(\frac{\delta}{\Delta}\right)$$
$$Z^{\Delta}(q, N) = z(q, \Delta^2 N)$$

  - Study limit case as $\Delta \to 0$
- **Tradeoffs** between the fixed and span-of-control costs of setting up plants and the cost of reaching consumers **remain relevant**
  - Plants continue to cannibalize each other's customers
  - But forces will apply at local level

The Core Result

Proposition

Suppose that R_s, D_s, and B_s/W_s are continuous functions of s. Then, in the limit as $\Delta \to 0$,

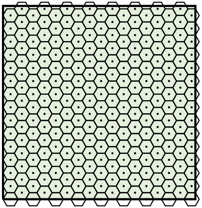
$$\pi_j = \sup_{n:\mathcal{S}\to\mathbb{R}^+} \int_s \left[x_s z\left(q_j, \int n_{\tilde{s}} d\tilde{s}\right)^{\varepsilon-1} n_s g\left(1/n_s\right) - R_s n_s \right] ds$$

where $x_s \equiv \frac{(\varepsilon-1)^{\varepsilon-1}}{\varepsilon^\varepsilon} D_s \left(B_s/W_s\right)^{\varepsilon-1}$ and where $g(u)$ is the integral of $t\left(\cdot\right)^{1-\varepsilon}$ over the distances of points to the center of a regular hexagon with area u.

- x_s combines local demand and effective labor costs
- $\kappa(n) \equiv ng\left(\frac{1}{n}\right)$ represents the **local efficiency of distribution**
- In the limit:
  - Maximum profits are attained by placing plants so that catchment areas are, locally, uniform infinitesimal hexagons
  - Firm's problem is one of calculus of variations which is much simpler

Elements of the Proof

- When economic characteristics are **uniform across space** solution is known
  - Fejes Toth (1953) shows that if number of plants grows large, catchment areas are uniform regular hexagons



- We show that logic can be generalized to **heterogeneous space**
  - Construct upper and lower bounds for π_j in original problem for all Δ
  - Use hexagons for the bounds, as $\Delta \downarrow 0$
  - Upper and lower bound approach the same limit value. Thus, so does π_j

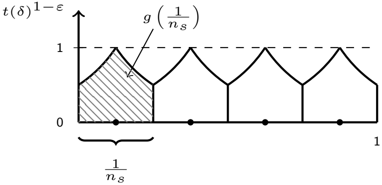
The Local Efficiency of Distribution

- We can write the firm's problem in the limit case as

$$\pi_j = \sup_{N_j, n_j:\mathcal{S}\to\mathbb{R}^+} \int_s \left[x_s z(q_j, N_j)^{\varepsilon-1} x_s \kappa(n_{js}) - R_s n_s\right] ds, \quad \text{s.t. } N_j = \int_s n_{js} ds$$

- The local efficiency of distribution, $\kappa(n) \equiv n g\left(\frac{1}{n}\right)$, is
  - $\kappa(0) = 0$
  - Strictly increasing and strictly concave
  - $\lim_{n\to\infty} \kappa(n) = 1$ (**Saturation**)
  - $1 - \kappa(n) \underset{n\to\infty}{\sim} n^{-1/2}$ (**Asymptotic power law**)

- If, additionally, $\lim_{\delta\to\infty} T(\delta)\delta^{-4/(\varepsilon-1)} = \infty$, then
  - $\kappa''(0) = 0$
  - $\kappa'(0) < \infty$ (**No INADA condition**)



FOCs and Span-of-Control Costs

- The problem in the limit case is

$$\pi_j = \sup_{N_j, n_j : \mathcal{S} \to \mathbb{R}^+} \int_s \left[x_s z(q_j, N_j)^{\varepsilon - 1} x_s \kappa(n_{js}) - R_s n_s \right] ds, \quad \text{s.t. } N_j = \int_s n_{js} ds$$

- Differentiating with respect to the number of plants in s, n_{js}, we obtain

$$x_s z(q_j, N_j)^{\varepsilon - 1} \kappa'(n_{js}) \leq R_s + \lambda_j, \qquad \text{with " = " if } n_{js} > 0$$

- Differentiating with respect to the total number of plants, N_j, we obtain

$$\lambda_j = -\frac{dz(q_j, N_j)^{\varepsilon - 1}}{dN_j} \int_s x_s \kappa(n_{js}) ds$$

  - where the Lagrange multiplier of the constraint, λ_j, can be interpreted as the **marginal span-of-control cost** of firm j

Sorting

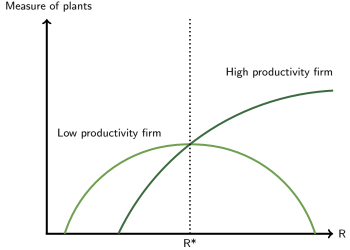
- The FOC implies that more productive firms have larger span-of control costs
  - That is, if $z_1 < z_2$ then $\lambda_1 < \lambda_2$, in fact $\frac{\lambda_1}{z_1^{\varepsilon-1}} < \frac{\lambda_2}{z_2^{\varepsilon-1}}$
- This implies that firms sort in space according to rents, namely,

Proposition

If $z_1 < z_2$, there is a unique cutoff $R^*(z_0, z_1)$ for which $\frac{R^*(z_1,z_2)+\lambda_2}{R^*(z_1,z_2)+\lambda_1} = \frac{z_2^{\varepsilon-1}}{z_1^{\varepsilon-1}}$.

- *If $R_s > R^*(z_1, z_2)$ then $n_{2s} \geq n_{1s}$, with strict inequality if $n_{2s} > 0$*
- *If $R_s < R^*(z_1, z_2)$ then $n_{1s} \geq n_{2s}$, with strict inequality if $n_{1s} > 0$.*
- *If $R_s = R^*(z_1, z_2)$ then $n_{1s} = n_{2s}$.*

- Firms also sort based on local profitability, x_s



Sorting and Span-of-Control

- In virtually all existing models, more productive firms enter more marginal markets
- Here, less productive firms have more plants in worse locations. **Why?**
  - Productive firms have more profits per plant, but also larger effective fixed costs

$$x_s z_j^{\varepsilon-1} \kappa'(n_{js}) = \underbrace{R_s + \lambda_j}_{\text{effective fixed cost}}$$

  - High productivity firms are less sensitive to rents since

$$\lambda_j \uparrow \Rightarrow \frac{d \ln(R_s + \lambda_j)}{d \ln R_s} \downarrow$$

    so they sort into high-rent locations

Equilibrium

We specify the rest of the economy as follows:

- Locations characterized by exogenous **amenities**, A_s, and **productivity**, B_s
- Mass $\mathcal{L}$ of workers
  - Freely mobile across s, live and work at same location, supply labor inelastically
  - Preferences from consumption and housing are given by $u(c, h, a) = Ac^{1-\eta}h^{\eta}$
    - ⋆ with c Dixit-Stiglitz with elasticity ε
  - Budget constraint is $P_s c + R_s^H h \leq W_s + \Upsilon$
    - ⋆ where Υ are the mutual fund proceeds from land and firms
  - Hence, $D_s = \mathcal{L}_s c_s P_s^{\varepsilon}$
- Unit measure of land in each location
  - Competitive developers rent land to firms and workers
  - $H_s + \mathcal{N}_s \leq 1$, where $H_s = h_s \mathcal{L}_s$ and $\mathcal{N}_s = \int_j n_{js} dj$

Aggregation

- In equilibrium we can define **local productivity** as

$$\mathcal{Z}_s \equiv \left(\int_j z_j^{\varepsilon-1} \kappa(n_{js}) dj \right)^{\frac{1}{\varepsilon-1}}$$

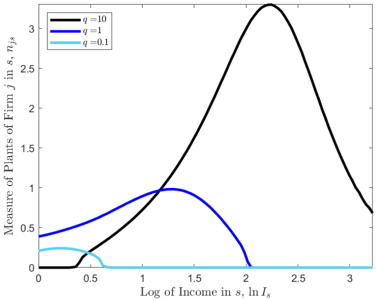
  - Then, the consumption bundle is given by $c_s = B_s \mathcal{Z}_s$
  - the price index by $P_s = \frac{\varepsilon}{\varepsilon-1} \frac{W_s}{B_s \mathcal{Z}_s}$
  - **local profitability** by $x_s = \frac{1}{\varepsilon-1} \frac{W_s \mathcal{L}_s}{\mathcal{Z}_s^{\varepsilon-1}}$
  - and the share of labor in location s is then

$$\frac{\mathcal{L}_s}{\mathcal{L}} = \frac{\left[A_s B_s^{1-\eta} \mathcal{Z}_s^{1-\eta} H_s \right]^{1/\eta}}{\int_s \left[A_{\tilde{s}} B_{\tilde{s}}^{1-\eta} \mathcal{Z}_{\tilde{s}}^{1-\eta} H_s \right]^{1/\eta} d\tilde{s}}$$

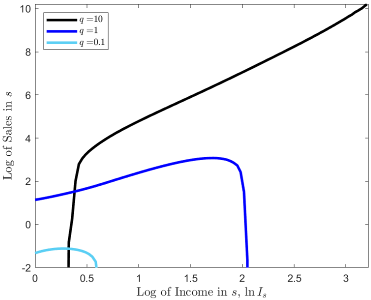
Numerical Illustration: Industry Equilibrium

- Continuum of industries and symmetric Cobb-Douglas preferences
- Total income, I_s, is truncated Pareto and $R(I_s) = e^{\log(I_s)^2}$
- Productivity, q_j, is also truncated Pareto with $z(q, N) = qe^{-N/\sigma}$
- Transportation costs are given by $t(\delta; \phi) = e^{\gamma/\sqrt{\phi}}$

Plants

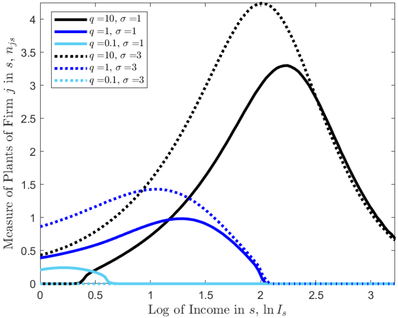


Sales

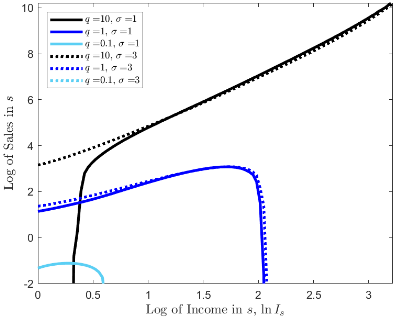


Improvements in Span-of-Control: $z(q, N) = qe^{-N/\sigma}$

Plants



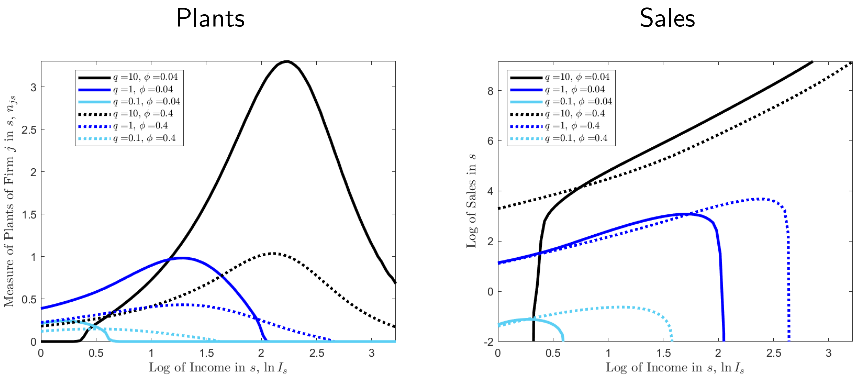
Sales



- **Top firms** expand to low income locations
  - Also, contract presence in top locations due to competition
- **Worse firms** exit

Effect on λ_j and x_s

Improvements in Transportation: $t(\delta;\phi) = e^{\gamma/\sqrt{\phi}}$



- **Increase in catchment areas** reduces effective fixed cost of new plants
  - **Top firms** suffer more from increased cannibalization and competition
  - **Worse firms** benefit disproportionately from lower fixed costs

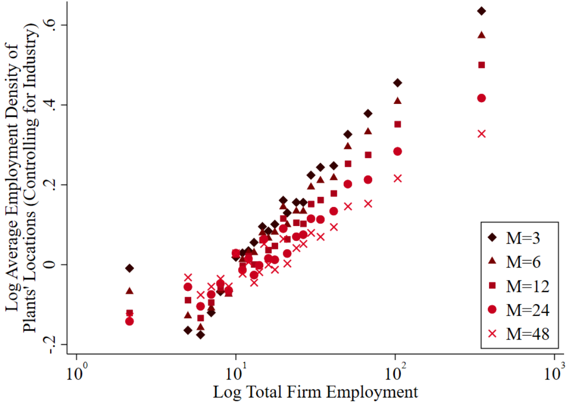
Empirical Evidence

- We use the National Establishment Time Series (NETS) dataset in 2014
  - Private sector source of business microdata (Dunn & Bradstreet)
  - We have establishment's precise location, employment, and link to parent company
    - ⋆ Drop plants with less than 5 employees
  - Definition of a location is a square of resolution M miles $\times$ M miles
- In the data we do not observe
  - Firm productivity: Monotone in total employment if $z(q, N) = qe^{-N/\sigma}$
  - Location 'quality' index, $A_s B_s^{1-\eta}$: Monotone in population density
- In Zillow data rents are increasing in population density

Rents and Density

Sorting in the Data: Average Density and Firm Size

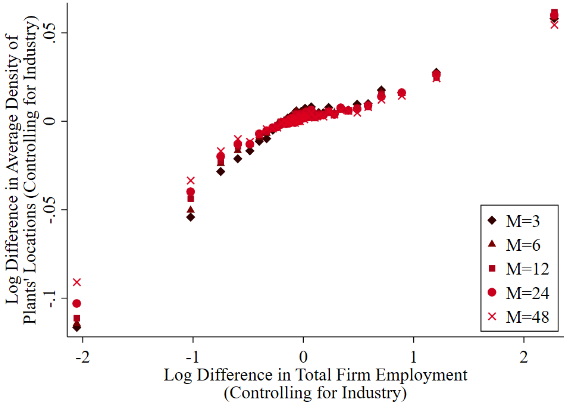
- Compute average weighted population density of plant locations of firms
  - In the baseline we use share of plants as weights



Alternative weights | Netting out firm's contribution | Sectors | Non-imputed data

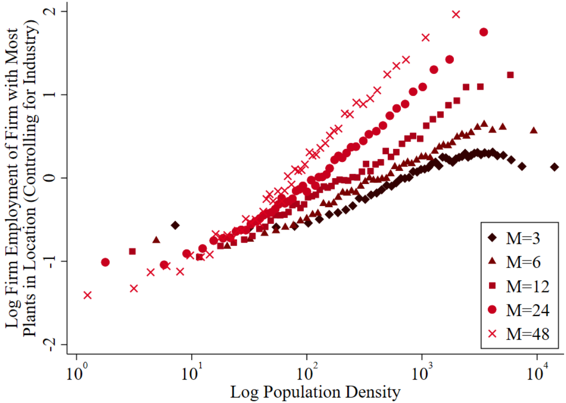
Sorting in the Data: Changes over Time

- Calculate change in firm's plant location density between 2000 and 2014
  - In both years, use 2000 local density levels
- Calculate change in total firm size between 2000 and 2014
- Subtract industry fixed effects from both variables



Sorting in the Data: National Size of the Top Firm in Town

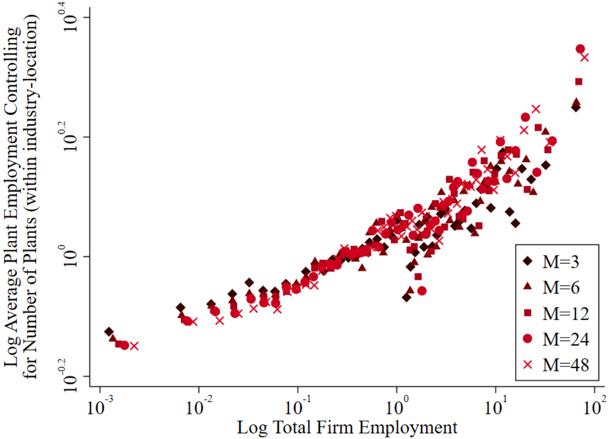
- For each location and industry, find firm with most plants



Alternative tie-breaking rules | Netting out location's contribution | Sectors | Non-imputed data

The Role of Span-of-Control in the Data

- If two firms with different z_j have the same n_{js}
  - Low z_j, low λ_j, firm's n_{js} is limited by low productivity
  - High z_j, high λ_j, firm's n_{js} is limited by high span-of-control cost
- Since firm size rises with z_j, **large firms have larger plants, given n_{js}**



Saturation in the Data

- The local size of a firm's plants is given by $l_{js} = (\varepsilon - 1) z_j^{\varepsilon-1} \frac{x_s}{W_s} \kappa(n_{js})/n_{js}$
  - Remember, $\kappa(n_{js})$ is increasing, convex, and converges to 1
- **The more saturation, the more cannibalization, and so the more plant size declines with extra plants**
  - Important to control for firm and location fixed effects

	(1) $\Delta \ln l_{js}$	(2) $\Delta \ln l_{js}$	(3) $\Delta \ln l_{js}$	(4) $\Delta \ln l_{js}$	(5) $\Delta \ln l_{js}$
$\ln n_{js,2000}$	-0.0792***	-0.0467***	-0.0402***	-0.0634***	-0.0661***
	(0.0260)	(0.0169)	(0.0136)	(0.0115)	(0.0101)
$\Delta \ln n_{js}$	0.0729***	0.0460**	0.0768***	0.0610***	0.0639***
	(0.0262)	(0.0190)	(0.0157)	(0.0135)	(0.0126)
$\ln n_{js,2000} \times \Delta \ln n_{js}$	**-0.0954*****	**-0.00369**	**-0.0194**	**-0.0192****	**-0.0225*****
	(0.0308)	(0.0203)	(0.0132)	(0.00970)	(0.00787)
Observations	20,230	30,583	41,246	49,888	56,170
R-squared	0.628	0.621	0.604	0.589	0.571
SIC8-location FE	✓	✓	✓	✓	✓
SIC8-firm FE	✓	✓	✓	✓	✓
M	3	6	12	24	48

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Local Characteristics and Plant Growth in the Data

- Suppose local profitability in s, x_s, increases, which makes rents, R_s, rise
- Remember the FOC, $x_s z(q_j, N_j)^{\varepsilon-1}\kappa'(n_{js}) \leq R_s + \lambda_j$
- Hence, conditional on n_{js}, and firm and local fixed effects
  - Nationally large firms expand the number of plants more when x_s rises
  - Rents are a smaller part of large firms' fixed costs, $R_s + \lambda_j$

	(1) Growth in n_{js}	(2) Growth in n_{js}	(3) Growth in n_{js}	(4) Growth in n_{js}	(5) Growth in n_{js}
$\ln n_{js,2000}$	0.00438	-0.00302	-0.00213	0.00150	0.00511*
	(0.00646)	(0.00457)	(0.00357)	(0.00303)	(0.00268)
$\ln L_{j,2000} \times \Delta \ln \mathcal{L}_s$	**0.0100***	**0.0126****	**0.0171*****	**0.0237*****	**0.0427*****
	(0.00540)	(0.00522)	(0.00549)	(0.00622)	(0.00717)
Observations	272,506	360,721	418,772	443,227	442,352
R-squared	0.793	0.788	0.782	0.780	0.775
SIC8-firm FE	✓	✓	✓	✓	✓
SIC8-location FE	✓	✓	✓	✓	✓
M	3	6	12	24	48

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Transport Efficiency and Plants in the Data

- Effect of **transport efficiency**, ϕ, on n_{js} is ambiguous
  - Higher transport efficiency enlarges CA but increases L_{js}
- **Saturation** ($\kappa'' < 0$) implies that **cross-effect of L_{js} and ϕ is negative**

VARIABLES	(1) $\ln n_{js}$	(2) $\ln n_{js}$	(3) $\ln n_{js}$	(4) $\ln n_{js}$	(5) $\ln n_{js}$	(6) $\ln n_{js}$	(7) $\ln n_{js}$	(8) $\ln n_{js}$
$\ln L_{js}$	0.276***	0.244***	0.276***	0.351***	0.129***	0.271***	0.271***	0.272***
	(0.00145)	(0.00131)	(0.00145)	(0.00219)	(0.00555)	(0.00152)	(0.00183)	(0.00150)
X Gini		-0.151***						
		(0.00171)						
X Ellison-Glaeser			-0.0413***					
			(0.00607)					
X Consumer Gravity				-0.0970***				
				(0.00150)				
X Freight Cost					0.0200***			
					(0.00584)			
X Trade Cost						0.0250***		
						(0.00118)		
X Speed Score							-0.00355***	
							(0.00131)	
X Travel Time								0.00881***
								(0.00117)
Observations	366,979	366,979	366,979	209,700	8,166	345,771	207,155	323,782
R-squared	0.747	0.769	0.748	0.798	0.692	0.750	0.769	0.753
SIC8-location FE	✓	✓	✓	✓	✓	✓	✓	✓
SIC8-firm FE	✓	✓	✓	✓	✓	✓	✓	✓
M	24	24	24	24	24	24	24	24

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

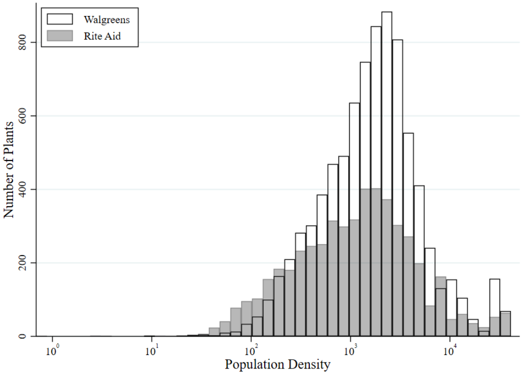
Robustness for M

Conclusions

- We propose methodology to analyze the location, number, and size of a firm's plants across heterogeneous locations
    - Original problem intractable but limit problem much simpler to analyze
    - Limit problem preserves all the relevant tradeoffs, but locally
    - Easy to incorporate in a quantitative spatial economic framework
- Problem yields multiple insights: Sorting, as well as the role of saturation, span-of-control, and transport technology
    - We corroborate these implications using U.S. NETS data
- We study numerically the effect of changes in span-of-control and transport technology in a 'small' industry
    - Interesting to study economy-wide or 'large' industry changes
    - In particular the effects of secular changes of technology on economic activity and local competition

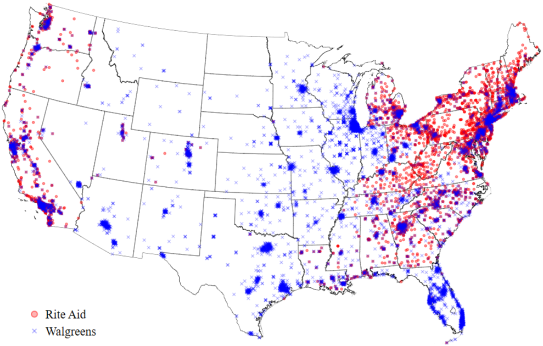
Thank You

Sorting Example: Drug Stores



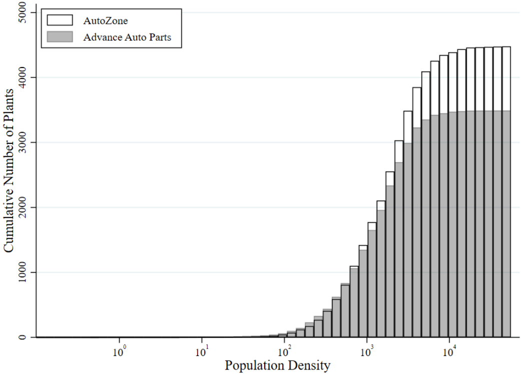
Back to Intro

Sorting Example: Drug Stores



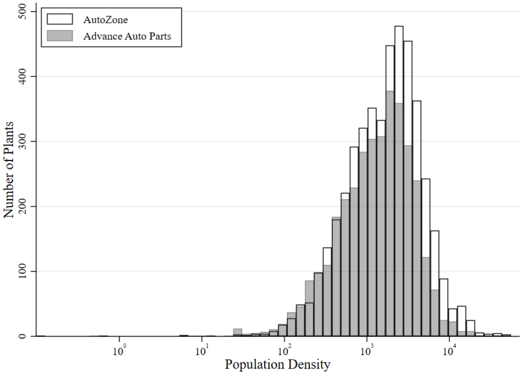
Back to Intro

Sorting



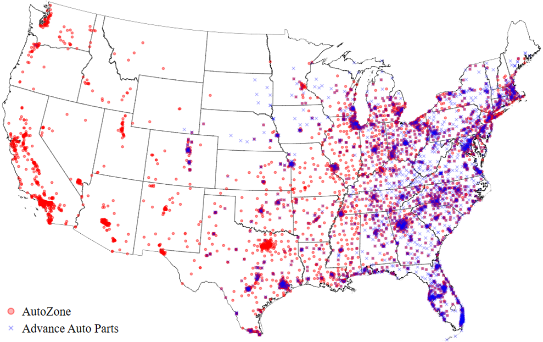
Density Map Back to Intro

Sorting Example: Auto Parts



Back to Auto Parts Back to Intro

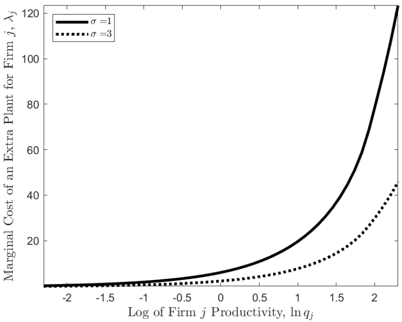
Sorting Example: Auto Parts



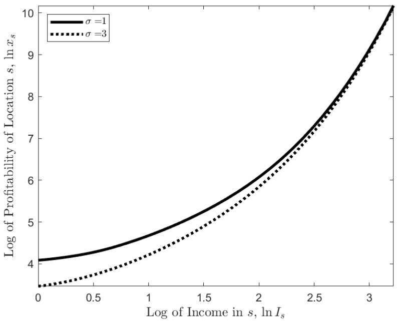
Back to Auto Parts
Back to Intro

Improvements in Span of Control: $z(q, N) = qe^{-N/\sigma}$

λ_j

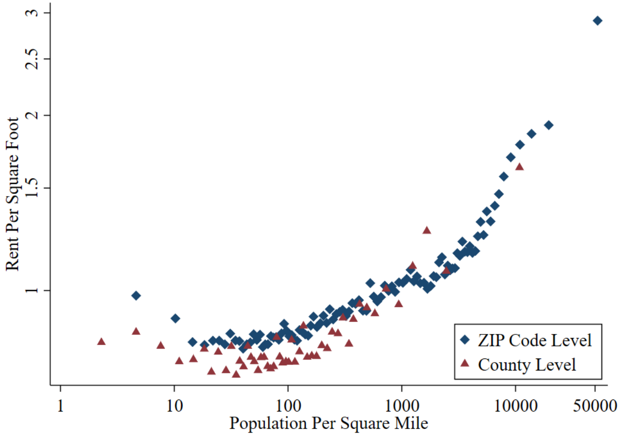


x_s



back

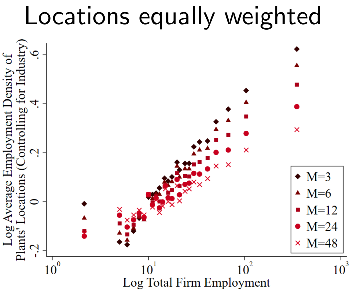
Denser Locations Have Higher Rents



Data source: Zillow

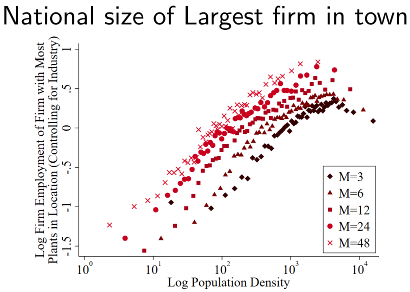
back

Sorting in the cross-section



back

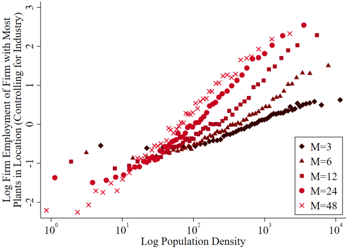
(Net) Sorting and National Size



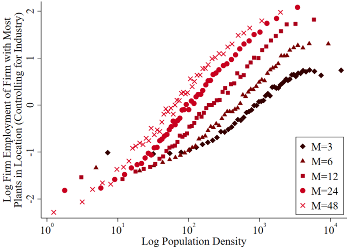
back to sorting
back to National Size

National Size, alternative tie-breakers

Discarding Locations with Ties

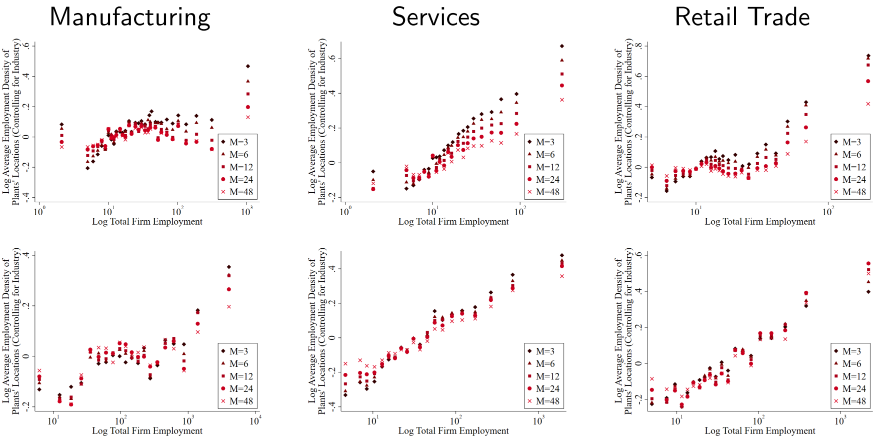


Using Largest Firm



back to National Size

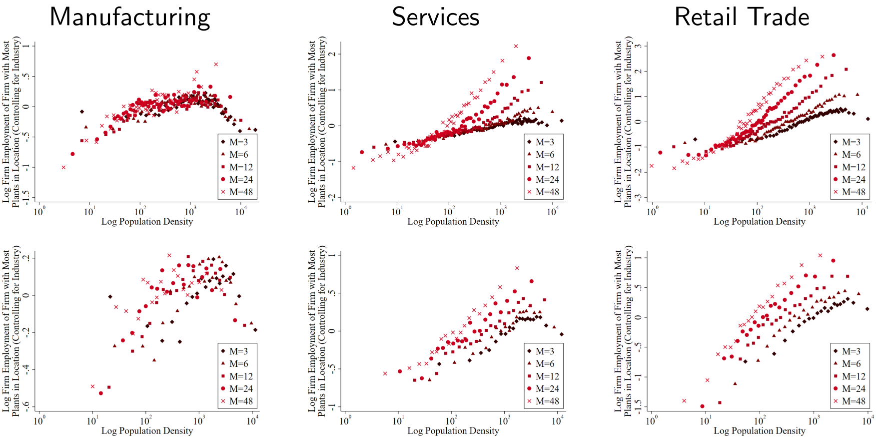
Sorting, by sector



- Second row excludes single-plant firms

back to Sorting

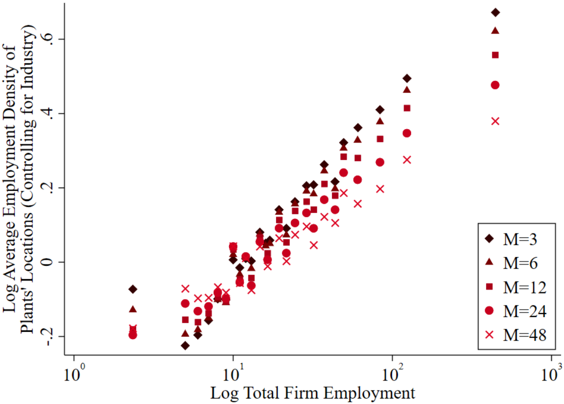
National size, by sector



- Second row excludes single-plant firms

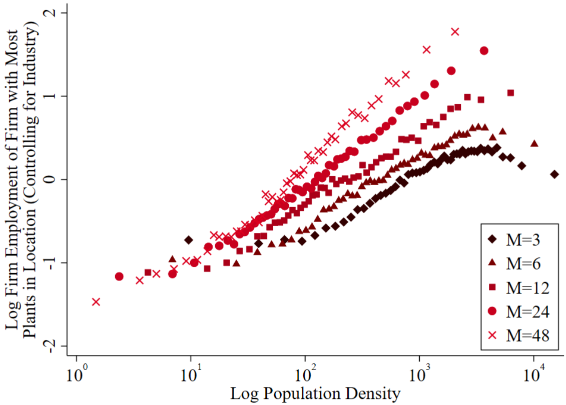
back to National Size

Sorting, non-imputed data



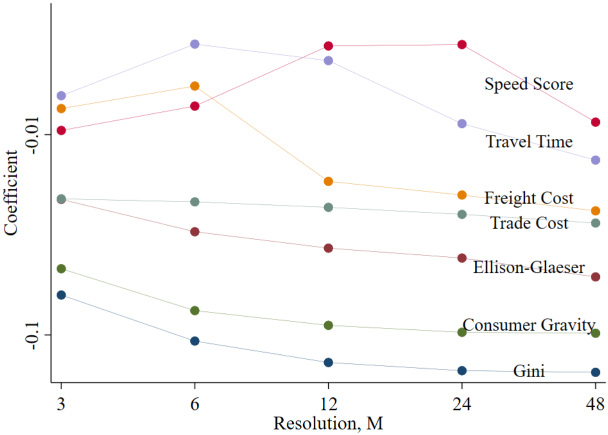
back to sorting

National Size of the Top Firm in Town, non-imputed data



back to National Size

Transportation efficiency, plants, and catchment areas



back