Trade, Jobs, and Worker Welfare*

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Abstract

We study welfare effects of trade on workers across different regions and sectors by introducing a new dynamic discrete choice model of labor mobility with the endogenous number of choices. In our general equilibrium model, trade shocks impact worker welfare not only through wages, but also via the number of job opportunities available to workers in different labor markets. First, we exploit differential exposure of sectors and regions to destination-specific demand shocks to estimate the impacts of export shocks on wages, employment and labor mobility, using detailed employer-employee panel data for Brazil. Second, we employ the same empirical strategy to estimate structural parameters and the different components of the change in model-implied worker welfare. Third, we use our model and the estimated structural parameters to perform counterfactual policy simulations. Our counterfactual simulations show that the endogenous number of job opportunities channel maginfies the welfare gains from a trade shock by roughly 30%.

Keywords: Trade shocks, jobs, wages, worker mobility, adjustment costs, worker welfare.

JEL Classification: F16, F66, J6

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1 Introduction

How do trade shocks impact workers? Answering this question requires an understanding of how trade shocks affect their wages and job options they can choose from. In this paper, we develop and estimate a new dynamic discrete choice model with endogenous number of choices to quantify the impacts of trade shocks on labor mobility along various dimensions and on worker's lifetime utility. Dynamic models of trade-induced labor mobility have explored wage differentials and idiosyncratic utility as drivers of mobility across sectors, regions and occupations. We emphasize an additional important motive of mobility: *the number of job opportunities* provided by different sectors and regions.

This new channel matters for workers for two main reasons. First, if a worker can choose her job out of more opportunities, it is more likely that the best one delivers higher welfare. Second, even when she is hit by a negative labor demand shock in the future, it is more likely that she will be able to find another job without having to move to a different region or sector. Thus, a region-sector pair (henceforth referred to as labor market) receiving a positive trade shock will attract more workers not just because it provides a higher wage, but also because of the larger number of job opportunities that are created there. In addition, a labor market with a positive trade shock will see a larger internal churning, *i.e.*, more job switching within the labor market, which has been largely overlooked in the literature. Our model clearly shows how trade shocks affect worker's welfare differently through labor mobility between and within labor markets. The model delivers a structural equation of trade-induced relative change in worker's lifetime welfare between labor markets, which can be conveniently estimated through an instrumental-variable strategy exploiting differential exposure of sectors and regions to destination-specific import demand shocks. We then quantify the magnitude of the welfare effect of a trade shock through the full simulation of the model.

Our framework is motivated by reduced-form evidence on the effects of export shocks on labor markets. The empirical analysis draws on rich employer-employee panel data combined with customs records on export transactions from Brazil in the period 2003-2015. To account for the endogeneity of exports, we construct an instrument at the labor market level, exploiting variation over time in sectoral import demand directed to the region—defined as trade-weighted sectoral imports of the initial set of destinations, sourced from all countries other than Brazil. The empirical analysis draws on data on more than 500 regions and 3 broad sectors. The IV estimates reveal a positive causal effect of exports on residual wages, employment, worker inflows, and job turnover rates within the corresponding labor market. This evidence supports the relevance of the various drivers of between- and within-market labor mobility induced by trade.

Building on this reduced-form evidence, we develop a new dynamic general equilibrium model of labor mobility with an endogenous number of choices. Different labor markets offer different wages and different numbers of job opportunities to workers. A worker chooses the job which gives her the highest utility, where the number of jobs in each labor market is endogenously determined. This is a distinctive feature of our framework compared to previous dynamic models which assume that workers choose a labor market and that the number of different jobs is exogenously fixed, and the same across labor markets. In our model, a worker's job choice determines which labor market she belongs to. Both the wage and the set of job opportunities provided by each labor market are factored into her optimal job choice.

In a labor market with relatively more job opportunities, workers can choose optimally out of more potential jobs, to each of which workers attach idiosyncratic preference. This is the first channel through which a labor market with more job opportunities provides workers of a greater utility, because the maximum utility will be higher with more options. The second channel stems from frictions to worker mobility. We assume that a job switch requiring a change of labor market implies incurring a higher switching cost compared to a job switch within a labor market. Therefore, a growing labor market with more job opportunities reduces the risk of having to pay a higher switching cost in the future. The prospect of job switch generates an option value in worker's welfare. Our model further decomposes this option value into the option value associated alternative job opportunities within the current labor market and the option value from having alternative jobs in all other labor markets.

Our model delivers a structural equation of changes in relative worker welfare which is a function of only the estimated probability of moving between labor markets and the labor supply elasticity. The welfare result does not depend on the moving cost structure, observed changes in future wages, or moving probabilities across jobs within a labor market. The effects of a trade shock are fully embedded in the gross flows between labor markets. This is a powerful result which greatly simplifies the analysis of the welfare impacts of trade shocks.

For the welfare analysis, we structurally estimate the model using the worker-firm data from Brazil. In the first stage of the estimation, we pin down the common value attached to each labor market and the moving cost between labor markets for each worker group using a gravity-like equation. The implied probability of moving between labor markets is then calculated with the estimated value of each labor market and the estimated moving cost. We find the the regional moving cost coefficient in the gravity equation is approximately equal to -1, which is consistent with the migration literature. We find that the moving cost between sectors is equivalent to one time loss of approximately 65% of annual wage, which is also consistent with the estimates of Dix-Carneiro (2014). In the second stage of the estimation, we pin down the labor supply elasticity of our model. We first derive an estimable equation describing the relationship between a change in the transformed value of the labor market and a change in wages, with the labor supply elasticity governing the responsiveness of the former with respect to the latter. We exploit variation in residual wages induced by the instrument we used earlier: the trade-weighted change in import demand directed to the labor market. Armed with the estimate of the labor supply elasticity, we estimate the effect of trade shocks in Brazil on workers' welfare, employment and wages using the same instrument. We find that, during the sample period, a 10% increase in exports increases the lifetime welfare of a median formal sector worker by 3.39% of the annual wage.

Finally, we turn to the full general equilibrium simulation of our model to study the effect of a trade shock on workers' welfare, labor allocation, real wages, and the number of job opportunities. Our benchmark trade shock is a 30% permanent decline of trade costs in the manufacturing sector from Brazil to each of its trading partners. This positive export shock to the Brazilian manufacturing sector reallocates labor from the agriculture/resource sector toward the manufacturing and non-tradable service sectors. The lifetime utility of workers increases due to this shock on average. The amount of total lifetime welfare increase is equivalent to a 120.43% one-time and temporary increase of the annual wage of workers. Workers in the manufacturing sector experiences 23.4% larger welfare increases than workers in the agriculture/resource sector.

We highlight the importance of our job opportunity channel by comparing our benchmark model to an alternative specification without this channel. We show that the benchmark model with the job opportunity channel generates a larger labor allocation between individual labor markets defined as a pair of region and sector by increasing the likelihood that workers find a better job in response to a positive shock. In addition, our channel shows that a trade shock generates within-market mobility as well. As a result, it generates approximately 30% larger welfare effect from the same trade shock.

In the last set of counterfactual analyses, we explore the role of labor mobility frictions by quantifying the effect of potential labor market policies mitigating the degree of mobility frictions faced by workers. We show that a 20% lower mobility frictions across regions and sectors increases welfare gains from the same benchmark shock by 16.5%. Our model can be also used to quantify the effect of a policy targeting either sector-level or region-level mobility frictions separately. The welfare-enhancing effect of a policy of the same magnitude is greater when the policy targets the regional frictions than the sectoral frictions.

Related literature. This paper bridges dynamic models of labor mobility and reduced-form differential exposure methods of quantifying the impacts of trade shocks on worker welfare. Artuç, Chaudhuri, and McLaren (2010, ACM, henceforth), Dix-Carneiro (2014) and Traiberman (2019) study the dynamic transmission of international trade shocks on labor markets via labor mobility by modeling worker's idiosyncratic preference for a labor market with an extreme value distribution.[1] Using these models, they structurally estimate labor market frictions on the basis of differential labor market outcomes across sectors or occupations, such as wages or labor flows. We follow the convention of this literature when modeling worker preferences, but we introduce a new channel which affects worker welfare: the number of job opportunities within each labor market. Workers choose their job, and which labor market they belong to is a consequence of their choice. In our model, the number of job opportunities in each labor market is endogeneous. Hence, trade shocks can affect labor mobility and thus welfare not just through wages but also through the number of job opportunities.

By endogenizing the number of jobs and bringing it to the welfare analysis of trade shocks, we combine the main strength of dynamic models of labor mobility with that of the reduced-form literature on local labor market effects of trade, including influential contributions by Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013), Autor, Dorn, and Hanson (2015), McLaren and Hakobyan (2016), Dix-Carneiro and Kovak (2015), Dix-Carneiro and Kovak (2017) and Dix-Carneiro and Kovak (2019). This literature builds on the existence of frictions to spatial labor mobility to establish a strong reduced-form relationship between trade shocks and employment changes in local labor markets. However, the reduced-form approach is unable to estimate the

[1] McLaren (2017) offers a review of this literature. A related strand of work develops trade models featuring labor market frictions generated by search, including Davidson, Martin, and Matusz (1999), Coşar, Guner, and Tybout (2016), Helpman, Itskhoki, Muendler, and Redding (2017) and Ritter (2015). In standard search models explored in the trade literature, the number of jobs matters as it affects employment probabilities. In our model, however, workers are matched to multiple jobs in a sense that they are allowed to compare those jobs before they finally choose the best one. Therefore, our framework can account for the welfare effects generated by changes in both employment probabilities and number of job options. The model remains tractable and allows for the estimation of deep parameters using simple and transparent reduced-form econometric methods. Fajgelbaum (2020) studies job-to-job transitions by allowing for on-the-job search. Our paper models the value of jobs from worker's perspective instead of firm's and studies mobility frictions of various layers.

implications of these effects for worker lifetime welfare.[2] Our model answers this welfare question by bringing this employment channel into a structural dynamic model of labor mobility and providing a welfare equation. We estimate this welfare equation using an instrumental-variable strategy analogous to that used in the reduced-form literature, followed by the quantification of the magnitude of the welfare effect based on a full simulation of the model.

In a recent important contribution, Caliendo, Dvorkin, and Parro (2019, CDP, henceforth) embed the dynamic discrete choice worker problem in ACM into a multi-country, multi-region, and multi-sector general equilibrium model with trade and migration costs. In important departures from CDP, we endogenize the number of job opportunities available to workers in each labor market, and structurally estimate the welfare effects and other important primitives of the model using an instrumental variables strategy. These structural estimates allow us to perform interesting policy simulations. Since we explicitly estimate the moving cost for each time period, we can examine how specific policy interventions influencing different frictions to labor mobility impact workers' welfare gains from trade. By separately changing the distance-related component and the sector-related component of the moving cost, our model can shed light on which dimension policies should target with a prioritize in order to mitigate losses in workers' imposed by trade shocks.[3]

Interestingly, the welfare equation we derive relates closely to several others in the trade literature. Arkolakis, Costinot, and Rodriguez-Clare (2012) provide a sufficient statistic of welfare gains from trade, which is consistent with various classes of trade models. Since workers are assumed to be homogeneous in the trade models covered by that sufficient statistic result, each worker has the same welfare gains from trade. In our model, worker welfare depends on their actual mobility, and thus is allowed to differ across workers. However, we still maintain the same spirit of this paper by deriving a new formulation of the change in the welfare experienced by

[2] A related body of literature uses reduced-form methods to examine the effects of trade shocks on labor market outcomes at the industry, firm or worker level, including contributions by Revenga (1992), Verhoogen (2008), Brambilla, Lederman, and Porto (2012), Amiti and Davis (2012), Bertrand (2004), Hummels, Jøorgensen, and Xiang (2014), Autor, Dorn, Hanson, and Song (2014), Acemoglu, Autor, Dorn, Hanson, and Price (2016), Pierce and Schott (2016) and Frías, Kaplan, Verhoogen, and Alfaro-Serrano (2018). Harrison, McLaren, and McMillan (2011) provide an overview of the literature on trade and inequality.

[3] Galle, Rodriguez-Clare, and Yi (2017) develop a static multi-sector gravity model with heterogeneous workers to quantify the aggregate and group-level welfare effects of trade, and estimate a key structural parameter using the China shock as in Autor, Dorn, and Hanson (2013). Also in static setting, Adao, Arkolakis, and Esposito (2019) exploit the same source of variation in a model implied optimal IV to estimate the labor allocation elasticity. In a key departure from their work, we develop and structurally estimate a dynamic trade model of labor mobility with an endogenous number of choices. This framework allows us to emphasize the importance of the changing number of job opportunities for the dynamics of labor reallocation across regions and sectors following a trade shock, and quantify the implications of this channel for worker lifetime welfare.

workers following a trade shock. The structural welfare equation we derive makes it possible to compare the welfare implications of the previously discussed existing models, isolating the relative importance of each transmission channel of trade shocks.

Roadmap. The paper is organized as follows. In Section 2 we document various reduced-form evidence about the effect of export shocks on labor market outcomes using the RAIS database from Brazil. Motivated by the reduced-form evidence, in Section 3, we introduce a new framework of dynamic labor mobility with international trade and the endogenous number of job opportunities. In Section 4, we discuss how we estimate the key structural parameters of the model and evaluate trade-induced welfare changes in a reduced-form way by combining the local labor market approach with our structural model. We then discuss how we simulate our model and counterfactual exercises in Section 5. In Section 6, we validate our mechanism with macro-level data, before summarizing our main conclusions in Section 7.

2 Data and Reduced-form Evidence

2.1 Data Sources

The empirical analysis in this paper combines and examines several sources of panel data from Brazil spanning the period 2003-2015. In this section, we provide a brief description of each data source, while giving further details in Appendix A.1.

The main source of data is *Relacao Anual de Informacoes Sociais* (RAIS), a labor census gathering longitudinal data on the universe of formal workers and firms in Brazil, covering the period 2003-2015. RAIS is a high-quality administrative census of formal employees and employers collected every year by the Brazilian Ministry of Labor. These records are used by the government to administer several government benefits programs. Workers are required to be in RAIS in order to receive payments of these programs and firms face fines for failure to report.

RAIS covers virtually all formal workers and provides yearly information on demographics (age, gender, and schooling), job characteristics (detailed 6-digit occupation, wage, hours worked), as well as hiring and termination dates. For each job, the RAIS annual record reports average yearly earnings, as well as the monthly wage in December. We use the information on the December wage, so as to ensure that all labor market outcomes are measured at the same time and avoid potential mismeasurement for workers that did not work full year. RAIS also includes information on a number of establishment-level characteristics, notably number of employees, geographical location (municipality) and industry code (according to the 5-digit level of the Brazilian National Classification of Economic Activities). Unique identifiers (tax identification numbers) for workers and establishments make it possible to follow them over time. The establishment identifier contains 12 digits and the first 8 digits make it possible to uniquely identify the firm. Therefore, it is possible to identify and track multi-establishment firms.

While the RAIS data cover also segments of the public sector, we restrict the analysis to the private sector. We will use the detailed classification of occupations as a measure of the number of different jobs available in a labor market. The Brazilian Classification of Occupations changed in 2002 (CBO-2) and has been reported consistently since 2003. Although the RAIS data are available for earlier years, we restrict the analysis to the post-2003 period in order to ensure that this important variable is defined in a consistent way throughout the period of analysis. There are 2637 occupation codes at the 6-digit level during this period. We use information on the establishment's location (municipality) and industry, and worker-level data on gender, age, education and December wage. We focus on workers aged 16 to 64 years old. As in Dix-Carneiro and Kovak (2017), we use the "microregion" concept of the Brazilian Statistical Agency (IBGE) to define regional boundaries. This definition groups together economically integrated contiguous municipalities with similar geographic and productive attributes. We consider a set of 558 consistently defined microregions, grouping the 5571 municipalities in the data. To ensure a consistent definition of microregions over time, when necessary we merge microregions whose boundaries changed over the period of analysis.

We merge the RAIS with customs records on export transactions by microregion, industry and destination in each year. These customs records are administrative data collected by *Secretaria do Comercio Externo* (SECEX) of the Ministry of Development, Industry and Foreign Trade. These data are originally defined at the level of the municipality, detailed product category and destination market, and are available since 1997. For consistency with the RAIS data, we restrict the analysis to the post-2003 period and aggregate the customs records up to microregion-sector level. To construct an instrument for exports, we further use yearly data on the industry-level imports of each Brazilian destination (sourced from all countries except Brazil). There are 189 destinations in total reported in the customs data, to which we link information on sectoral import demand from the UN COMTRADE.

2.2 Econometric Model

We now describe the econometric strategy for examining the effects of export shocks on labor markets. We adopt the following baseline specification:

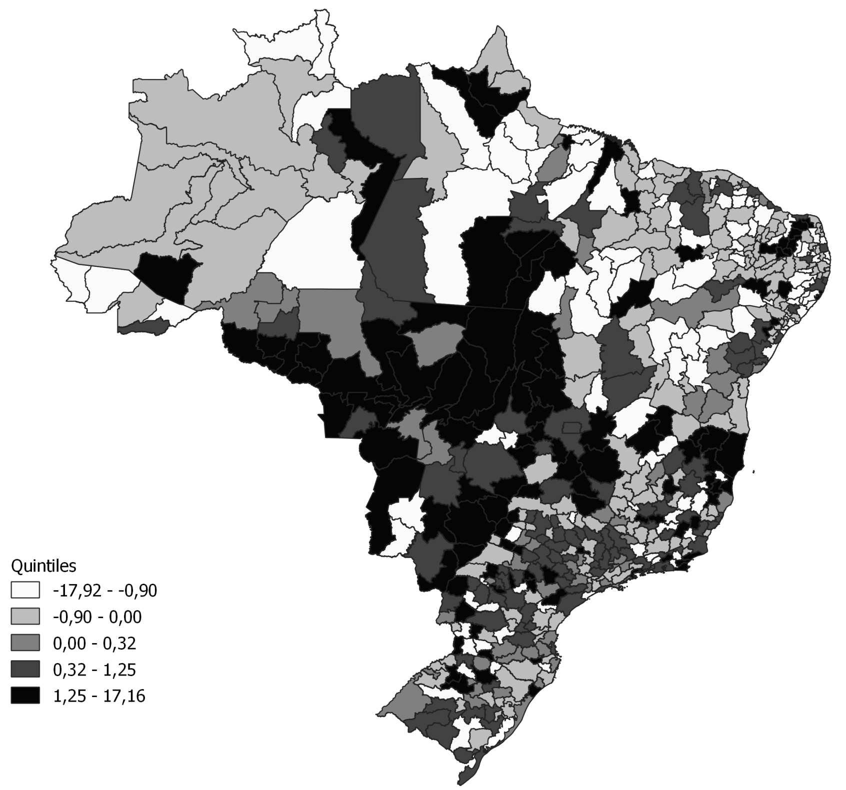
$$\Delta y_t^k = \tilde{\beta} \Delta Z_t^k + \lambda_t + \epsilon_t^k, \qquad (1)$$

where y_t^k denotes the log of the outcome variable of interest in the sector-region pair k in year t, Z_t^k denotes the log of export revenue originated in the same labor market, λ_t denotes a year fixed effect, and ϵ_t^k is the error term. The Δ operator denotes the change in a variable between year t and year $t-1$.

Figure 1 depicts the change in export revenue observed in different microregions over the period 2004-2014. It reveals that there exists substantial heterogeneity in the direction and the magnitude of the change in exports across space, which is convenient for identification. Notice also that because some labor markets were initially more export-oriented than others, they differ in the extent to which they are exposed to a given percentage change in export revenue. This heterogeneity is illustrated in Figure A1 in the Appendix, which depicts the distribution of export revenue per worker across microregions in 2003. Initial exports per worker also vary across sectors within each microregion. The same percentage change in exports would therefore be expected to have a stronger impact on labor market outcomes in labor markets where exports per worker were higher to begin with. To account for this, each observation of ΔZ_t^k is weighted by the export revenue per worker observed in the corresponding labor market in 2003.

An important concern is that changes in exports are potentially endogenous to changes in labor market outcomes. For example, lower wages or growing job turnover might cause an increase in export activity. Changes in both variables could also reflect the role of omitted variables, such as underlying changes in infrastructure or technology. To address potential endogeneity in the relationship between exports and labor market outcomes, we adopt an instrumental variables approach. An important challenge in constructing the instrument is to identify a source of variation at the microregion-sector level. Our strategy relies on variation over time in sectoral import demand directed to the region. This strategy builds on the fact that changes in external demand in a particular destination do not matter equally for all labor markets; it matters more for labor markets that initially shipped a larger share of their exports to that destination. Our instrument is therefore defined as the trade-weighted sectoral imports of the initial set of destinations of the

Figure 1: Change in export revenue, 2004-2014



Notes: Figure depicts the change in log of (1+exports) in Brazilian microregions during 2004-2014.

microregion-sector (sourced from all countries other than Brazil), where the weights are the export shares of each destination within each microregion-sector cell in 2003. Formally, the change of import demand directed to labor market k in year t can be written as:

$$\Delta \bar{Z}_t^k = \Delta \sum_d \gamma_{d,k,2003} IM_{dI_k t}, \tag{2}$$

where $IM_{dI_k t}$ denotes each destination d's total imports (excluding Brazil) in sector I_k in year t and $\gamma_{d,k,2003}$ the share of exports of labor market k to destination d in 2003. The Δ operator denotes the change in a variable between year t and year $t-1$. We take logs on the summation before taking first differences. Since different labor markets tend to serve different destinations, they vary in the degree to which they are exposed to changes in sectoral import demand from different countries. This heterogeneity is illustrated in Figure A2 in the Appendix, which depicts the main export destination of each microregion in 2003.[4] For the reasons discussed above, we weight each observation of the instrument by the corresponding export revenue per worker observed in 2003. The intuition behind this instrumental variables approach follows closely that adopted in the local labor markets literature, including Topalova (2010), Kovak (2013), Autor, Dorn, and Hanson (2013), Dix-Carneiro and Kovak (2017) and Dix-Carneiro and Kovak (2019). It is also closely related to earlier works using trade-weighted relative prices or import demand as the source of variation in imports or exports at the industry level, such as Revenga (1992) and Bertrand (2004); as well as with more recent work exploiting similar sources of variation at the firm-level, including Brambilla, Lederman, and Porto (2012), Bastos, Silva, and Verhoogen (2018) and di Giovanni, Levchenko, and Mejean (2018).

2.3 Summary Statistics and Results

Table 1 presents summary statistics on the variables used in the empirical analysis. Each variable is in log change and summarized by sectors across microregions and over time. Employment growth was relatively higher in services, followed by manufacturing, and agriculture and mining. These differential job dynamics across sectors is also reflected in the number of entrants and exiters, as well as in the number of job switchers within the labor market—defined as the number of workers who switched either occupation or establishment within the labor market. Changes in residual wages tended to be relatively more similar across sectors, on average. As detailed in the appendix, this variable is first computed at the individual-level, purging wages from the effects of age, gender and education, and then taking the average at the labor market level. Exports originated from firms whose main activity is within tradable sectors shows significant growth, in line with the growth in external demand.

Table 2 reports the first stage estimates relating changes in log export revenue originated from each labor market to changes in external demand directed to the labor market, as defined in equation (2). The econometric results reveal that our instrument provides a suitable source of variation for examining the impact of plausibly export shocks on labor market outcomes. The coefficient of interest is 0.793 indicating that a 10% increase in external demand directed to the labor market leads to a 7.7% increase in exports. This relationship is precisely estimated, with a Kleibergen-Paap rk Wald F-stat of 29.87, which is indicative of a strong instrument.

[4] Notice that the change in exports, initial exports per worker and the relative importance of each destination also vary across sectors within each microregion. This heterogeneity is exploited in the analysis, but is not reported in the figures to avoid clutter.

Table 1: Summary statistics, 2004-2011

Variable	Agriculture and mining (1)	Manufacturing (2)	Services (3)	All (4)
Δ Employment	0.033	0.053	0.073	0.053
	(0.278)	(0.236)	(0.138)	(0.226)
Δ Residual wage	0.032	0.025	0.025	0.028
	(0.106)	(0.097)	(0.065)	(0.091)
Δ # exiters	0.023	0.053	0.059	0.045
	(0.712)	(0.667)	(0.475)	(0.625)
Δ # entrants	0.038	0.042	0.071	0.051
	(0.719)	(0.745)	(0.493)	(0.660)
Δ # job switchers	0.013	0.025	0.091	0.046
	(0.659)	(0.749)	(0.358)	(0.599)
Δ Export revenues	0.043	0.04	-0.008	0.039
	(10.506)	(0.944)	(20.404)	(10.284)
$\Delta\bar{Z}$	0.096	0.055	0.052	0.072
	(0.240)	(0.150)	(0.293)	(0.202)

Notes: Table reports summary statistics on the unrestricted estimation sample. Means are reported in plain text, and standard deviations are in parentheses.

We proceed by examining the causal effects of export shocks on a range of labor market outcomes. Table 3 presents the instrumental-variables estimates of (1), using the strategy discussed above. Panel A measures effects on total employment and residual wages in the labor market. The estimates reveal that a 10% increase in exports leads to a 2.3% increase in employment and 3.1% increase in average residual wages. The estimates in Panel B reveal that export shocks have important implications for gross worker flows across and within labor markets. A positive export shock leads to a reduction in the number of workers leaving the labor market, an increase in the number of workers entering the labor market, and a significant increase in the number of workers switching occupation and/or establishment within the labor market. The last result implies that a positive export shock increases internal churning within the labor market.

3 Model

In this section, we first introduce a dynamic labor mobility model with the endogenous number of job opportunities. Each labor market is defined by a pair of region and sector. Each labor market offers a different number of job opportunities as well as a different wage. Conditional on wage, the number of job opportunities, and their idiosyncratic preference, workers optimally choose a

Table 2: First stage estimates

Dependent variable:	Δ Exports
$\Delta\bar{Z}$	0.806
	(0.100)
Kleibergen-Paap rk Wald F-stat	27.92
Sectors	3
Microregions	558
Labor markets	857
Observations	4008
Year effects	Y

Notes: Table reports first stage estimates. For each dependent and independent variable, we take the log before computing the first differences. Changes in log exports and sectoral export demand are weighted by exports per worker in the labor market in 2003. Standard errors clustered by microregion and year are presented in parentheses.

job which belongs to a certain labor market at every period to maximize their expected utility. Next, we introduce international trade to the framework to show how wage and the number of job opportunities in each labor market are endogenously determined at each period. Changes in labor market condition due to trade shocks endogenously impact the wage level and the number of job opportunities of each labor market. As a result, labor mobility between labor markets is endogenously determined. The labor mobility decision generates the dynamics of the model, while the international trade part of the model determines static economic environments period by period.

3.1 Labor Mobility Model with the Endogenous Number of Jobs

Consider an economy with a continuum of workers with mass L. Each worker is in a discrete state $k \in \{1, 2, .., K\}$ which is a region-sector labor market index.[5] The number of workers in labor market k at time t is denoted as L_t^k with $\sum_k L_t^k = L_t$. We denote R_k as the region of labor market k, S_k as the sector of labor market k. The total number of regions in this economy is R, and the total number of sectors is S, both of which we assume to be fixed over time. In most

[5]We denote a labor market with a single index k instead of a pair of region index and sector index. This notation is particularly convenient when we estimate the model with the data, because not all region-sector pairs are populated in the data. The maximum number of labor market we can have is a product of the number of regions and the number of sectors in the data, but the actual K is not necessarily equal to this maximum number.

Table 3: IV estimates on the impact of export shocks on labor markets

A. Dependent variable:	Δ Employment (1)	Δ Residual wage (2)
Δ Exports	0.230	0.318
	(0.039)	(0.034)
Sectors	3	3
Microregions	558	558
Labor markets	857	857
Observations	4008	4008
Year effects	Y	Y

B. Dependent variable:	Δ # leaving (f) (1)	Δ # leaving (r) (2)	Δ # entering (3)	Δ # switching jobs (4)
Δ Exports	-0.960	-0.148	0.403	0.460
	(0.134)	(0.056)	(0.080)	(0.101)
Sectors	3	3	3	3
Microregions	558	558	558	558
Labor markets	857	857	857	857
Observations	4008	4008	4008	4008
Year effects	Y	Y	Y	Y

Notes: Table reports IV results of equation (1) in text, using the baseline estimation sample. For all dependent and independent variables, we take the log before computing the first differences. Changes in log exports are instrumented by changes in sectoral import demand. All explanatory variables are weighted by exports per worker in the labor market in 2003. Standard errors clustered by microregion and year are in parentheses.

papers in the literature on dynamic discrete choice model of labor mobility, workers are assumed to choose a labor market directly. On the other hand, we model worker's problem as a choice of a *job*.

There are many job opportunities workers can compare in each labor market, and workers choose one job in every period. Which labor market a worker belongs to is an outcome of her optimal choice of a job. We denote the labor market which job j belongs to by $k = K_j$. In addition to that this assumption is much more realistic, it introduces an important dimension through which labor markets are affected by aggregate shocks. The status of an economy impacts not only relative wages across labor markets but also the number of job opportunities from which workers can choose. Depending on the nature of the shock and the differential exposure of each labor market to the shock, the impact of trade shocks on the number of jobs will be different between labor markets.

Instead of choosing a job from job opportunities available in each labor market, workers have an option to move to the residual labor market. Empirically, unemployment, home employment, and working in the informal labor market all belong to this choice.[6] Each worker compares the option to move to the residual labor market with all the other job opportunities available in each labor market. We denote the choice to be in the residual labor market by $j = I$. The residual labor market is assumed to offer only one job opportunity, and thus we do not need to distinguish a job from a labor market for the residual labor market. Therefore, we can express $K_I = I$ without loss of generality.

All job opportunities within labor market k are identical apart from the iid utility shock associated with them. We denote the real wage in labor market K_j at time t by $w_t^{K_j}$. If the current job j is I, then the real wage is w_t^I. Specifically, an agent who is indexed with h and who is attached to job j within labor market K_j receives instantaneous utility u_t^h at time t defined as

$$u_t^h = w_t^{K_j} + \varepsilon_t^{h,j}, \tag{3}$$

where $\varepsilon_t^{h,j}$ is distributed Gumbel with mean κ and the scale parameter $\nu > 0$.[7]

Workers start period t attached to a job j, and receive $w_t^{K_j}$ in the beginning of the period. After workers receive their wages, they sample job opportunities from each labor market and learn about the iid shocks associated with each of them, the current job, and the residual labor market. Conditional on this idiosyncratic draw, each worker chooses the best available job and receives the iid shock at the end of the current period. Then, the period t ends. In each period t, a worker can sample N_t^k job opportunities from labor market k. The number of job opportunities that can be sampled increases with the number of total job opportunities offered in the labor market. This relationship is determined at the general equilibrium which we will discuss after characterizing international trade. We assume that workers choose a job within a region-sector jointly with full information about the idiosyncratic component of their utility from any given job opportunity across all labor markets. In addition, they have full information about the idiosyncratic preference

[6] As explained in the previous section, the RAIS database covers only formal labor markets of Brazil. We introduce the residual market to account for the informal labor market that is not covered in the data we are going to use for quantification of the model.

[7] We can assume a log wage in the utility function as well, and this alternative specification does not change our main results.

of the current period for the residual labor market. They do not know the exact future values of the idiosyncratic component, but they form rational expectations. Therefore, each worker compares the expected utility from the current job and $\sum_k N_t^k + 1$ potential alternative jobs which include the option to move to the residual labor market with the expected utility from the current job at every period.[8]

Switching a job and switching a labor market are both subject to frictions. When a worker moves from a job j to a different job j' at time t, she pays the moving cost δ_t. This cost incurs even when the job switching is within the same labor market. If the job switching involves a switch to a different labor market, i.e., $K_j \neq K_{j'}$, then she pays an additional moving cost $C_t(K_j, K_{j'}) \geq 0$.[9] Since the option to be in the residual labor market is compared with job opportunities in the formal labor market, we assume that switching between the residual labor market and any other job in the formal labor market is subject to both types of switching costs. In other words, if a worker moves from a job j in a labor market K_j to the residual labor market, the total moving cost is $\delta_t + C_t(K_j, I)$, and it is $\delta_t + C_t(I, K_j)$ for the opposite direction.

Based on the moving cost structure, we derive the labor-market-specific present discounted utility of the agent h with job j in the beginning of t as

$$U_t^{j,h} = w_t^{K_j} + \max_{j'} \left\{ \beta E_t V_{t+1}^{K_{j'}} - \left(C_t(K_j, K_{j'}) + \delta_t \right) \mathbf{1}(j \neq j') + \varepsilon_t^{j',h} \right\}. \tag{4}$$

By taking an expectation over the idiosyncratic component, we define the expected present discounted value for the workers in labor market K_j in the beginning of time t as

$$\begin{aligned} V_t^{K_j} &= E_\varepsilon U_t^{j,h}, \\ &= w_t^{K_j} + E_\varepsilon \max_{j'} \left\{ \beta E_t V_{t+1}^{K_{j'}} - \left(C(K_j, K_{j'}) + \delta_t \right) \mathbf{1}(j \neq j') + \varepsilon_t^{j',h} \right\}. \end{aligned}$$

[8] We effectively assume that $N_t^I = 1$ in every t.

[9] Effectively, we assume $C_t(k, k) = 0$.

3.2 Equilibrium Labor Mobility and Option Values

Using the assumption of Gumbel distribution for the idiosyncratic shock, the probability that a worker moves from a labor market k to a labor market l is derived as

$$\begin{aligned} m_t^{kl} &= \frac{\mathbf{1}_{l=k}\exp\left(\beta E_t V_{t+1}^l/\nu\right) + N_t^l \exp\left(\frac{E_t\beta V_{t+1}^l - \delta_t - C_t(k,l)}{\nu}\right)}{\exp\left(\frac{\beta}{\nu}E_t V_{t+1}^k\right) + N_t^k\exp\left(\frac{E_t\beta V_{t+1}^k-\delta_t}{\nu}\right) + \sum_{l'\neq k} N_t^{l'}\exp\left(\frac{\beta E_t V_{t+1}^{l'}-\delta_t - C_t(k,l')}{\nu}\right) + \exp\left(\frac{\beta E_t V_{t+1}^I - \delta_t - C_t(k,I)}{\nu}\right)} \\ &= \frac{\mathbf{1}_{l=k}\lambda_{0,t}^k + \mathbf{1}_{l\neq k\wedge l\neq I}\lambda_{1,t}^l \exp\left(-\frac{C_t(k,l)}{\nu}\right) + \mathbf{1}_{l=I}\lambda_{I,t}^k}{\lambda_{0,t}^k + \lambda_{1,t}^k + \lambda_{2,t}^k + \lambda_{I,t}^k}, \end{aligned} \tag{5}$$

where we define

$$\lambda_{0,t}^k \equiv \exp\left(\frac{\beta}{\nu}E_t V_{t+1}^k\right) \tag{6}$$

$$\lambda_{1,t}^k \equiv N_t^k \exp\left(\frac{E_t\beta V_{t+1}^k - \delta_t}{\nu}\right) \tag{7}$$

$$\lambda_{2,t}^k \equiv \sum_{l'\neq k} N_t^{l'} \exp\left(\frac{\beta E_t V_{t+1}^{l'} - \delta_t - C_t(k,l')}{\nu}\right) \tag{8}$$

$$\lambda_{I,t}^k \equiv \exp\left(\frac{\beta E_t V_{t+1}^I - \delta_t - C_t(k,I)}{\nu}\right) \tag{9}$$

for notational simplicity. From this probability of moving from one labor market to another, the role of the number of job opportunities across labor markets becomes clear. Workers are more likely to move to a labor market in which they can sample more jobs, i.e., a larger N_t^l, conditional on the expected net value of a labor market. Since workers will choose the job which gives them the highest present discounted value, the expectation of the maximum of the idiosyncratic component should increase in the number of job opportunities that workers can sample, which makes it more likely for workers to move into a labor market with more job opportunities. Similarly, if formal labor markets offer more job opportunities, then it becomes relatively less likely for workers to move to the residual labor market, conditional on the relative expected net value.

For notational convenience, we define additional probabilities of switching. First, we denote the probability of moving from labor market k to labor market l conditional on changing jobs but staying in a formal labor market by $\widetilde{m}_t^{K_j K_{j'}}$ for $j \neq j'$.[10] In addition, we denote the probability of

[10] More formally, this conditional probability for $l \neq I$ is

$$\widetilde{m}_t^{kl} = \frac{\mathbf{1}_{l\neq k\wedge l\neq I}\lambda_{1,t}^l \exp\left(-\frac{C_t(k,l)}{\nu}\right)}{\lambda_{1,t}^k + \lambda_{2,t}^k},$$

staying in the same job, thus in the same labor market k, by $\mu^k_{0,t}$, the probability of changing jobs but staying in the same labor market k by $\mu^k_{1,t}$, and the probability of changing labor markets from k to any other labor market $l \neq k$ (thus also changing jobs) by $\mu^k_{2,t}$. Finally, we denote the probability of moving from labor market k to the residual labor market by $\mu^k_{I,t}$, which is effectively equal to m^{kI}_t.[11] The job opportunity channel that we introduce into the model will allow for internal–i.e., within-market– churning between jobs, which is measured by $\mu^k_{1,t}$. Since we assume that the residual labor market I is degenerate with a single job opportunity, $\mu^I_{0,t} = \mu^I_{I,t}$. With the new notations, we can re-write the labor-market-specific value as

$$V^k_t = w^k_t + \beta E_t V^k_{t+1} - \nu \log(\mu^k_{0,t}), \tag{10}$$

where $-\nu \log(\mu^k_{0,t})$ is an option value of moving. This option value can be decomposed into internal and external option values. The external option value is defined as the option value from alternative jobs in a different labor market. In order to net out the effect from switching to a different job in the same labor market, we need to divide $\mu^k_{0,t}$ by $\mu^k_{0,t} + \mu^k_{2,t} + \mu^k_{I,t}$ in the option value term, which gives the external option value term as

$$-\nu \log(\mu^k_{0,t}) + \nu \log(\mu^k_{0,t} + \mu^k_{2,t} + \mu^k_{I,t}). \tag{11}$$

The internal option value is the value from alternative job opportunities within the labor market, which is given as the difference between the total option value and the external option value, i.e.,

$$-\nu \log(\mu^k_{0,t} + \mu^k_{2,t} + \mu^k_{I,t}). \tag{12}$$

3.3 Relative Welfare and the Number of Jobs

The model delivers simple formulae for changes in the relative welfare between workers in two different labor markets and for changes in the number of jobs in each labor market in response to the change in an underlying policy variable denoted by x. First, we assume that a change in a

where $\lambda^l_{1,t}$, $\lambda^k_{1,t}$, and $\lambda^k_{2,t}$ are as defined above.

[11] By construction, $\mu^k_{0,t}+\mu^k_{1,t}+\mu^k_{2,t}+\mu^k_{I,t} = 1$ should hold for any t and k. Using the definition of $\lambda^k_{0,t}$, $\lambda^k_{1,t}$, $\lambda^k_{2,t}$, and $\lambda^k_{I,t}$ in equations (6)-(9), each probability can be expressed as $\mu^k_{0,t} = \frac{\lambda^k_{0,t}}{\lambda^k_{0,t}+\lambda^k_{1,t}+\lambda^k_{2,t}+\lambda^k_{I,t}}$, $\mu^k_{1,t} = \frac{\lambda^k_{1,t}}{\lambda^k_{0,t}+\lambda^k_{1,t}+\lambda^k_{2,t}+\lambda^k_{I,t}}$, $\mu^k_{2,t} = \frac{\lambda^k_{2,t}}{\lambda^k_{0,t}+\lambda^k_{1,t}+\lambda^k_{2,t}+\lambda^k_{I,t}}$, and $\mu^k_{I,t} = \frac{\lambda^k_{I,t}}{\lambda^k_{0,t}+\lambda^k_{1,t}+\lambda^k_{2,t}+\lambda^k_{I,t}}$.

policy variable x does not change the moving cost.

If we further assume that workers receive their wage in the beginning of time t before any change in policy parameter x is realized, then the change in the relative welfare of workers in labor market k compared to workers in labor market l is equal to

$$\Delta_x \left(V_t^k - V_t^l\right) = \Delta_x \left[\nu \left(\log \widetilde{m}_t^{lk} - \log \widetilde{m}_t^{kk}\right) - \nu \left(\log \left(1 - \mu_{0,t}^k - \mu_{I,t}^k\right) - \log \left(1 - \mu_{0,t}^l - \mu_{I,t}^l\right)\right)\right], \tag{13}$$

where Δ_x denotes the change induced by a change of x.

In the next section, we estimate changes in $\widetilde{m}_t^{lk}$ due to trade shocks using variations explored in the local labor market approach. This result also suggests that the change in the relative welfare due to a policy change depends on the relative wage and the relative number of job opportunities between labor markets, but their impact on the welfare change is entirely captured by the change in switching probabilities.

Next, our model shows that the change in the number of job opportunities in each labor market driven by a change in x can be written as

$$\Delta_x \log N_t^k = \Delta_x \left(\log \mu_{1,t}^k - \log \mu_{0,t}^k\right). \tag{14}$$

Equations (13) and (14) suggest that the labor supply elasticity and changes in moving probabilities due to a policy change ν are the sufficient statistics for relative changes in welfare and changes in the number of job opportunities.[12]

3.4 International Trade with Love for Variety of Jobs

The dynamic structural labor mobility model that we introduce can be used to quantify the effect of various labor demand shocks on worker welfare. In this paper we focus particularly on the effect of international trade. Different wages and different numbers of job opportunities across labor markets are two key driving forces which generate labor mobility in our model. In order to characterize how labor mobility is affected by trade shocks, we introduce international trade to our model, where trade shocks endogenously affect both wage and the number of job opportunities

[12] The absolute magnitude of changes in the welfare or the number of job opportunities can be pinned down only after the full general equilibrium model is solved. We will discuss the quantitative strategy for the level effect in Section 5.

at the labor market level at every period. We characterize production and international trade as a static problem of which solution affects workers' dynamic problem on mobility.

The international trade part of our model is based on the Eaton and Kortum (2002) model. This assumption will make international trade driven only from the standard Ricardian force. We assume that there are N countries ($n = 1, \dots, N$), but only country 1 is populated with more than one regions. For other countries $n \neq 1$, there is only one region which is the country itself. In other words, $R > 1$ for country 1 and $R = 1$ for all other countries. In order to characterize the country-level trade while we still have multiple regions in the country that we are interested in, we assume that there is a national aggregator for each sector in that particular country of interest. The aggregator of each sector can source any variety within that sector from the lowest cost region in the country at no trade cost. This way the national price level of each product is determined regardless of the consumption location. Then this aggregator trades each product with partner countries. If no region in country 1 is the lowest cost supplier of a certain product for consumers and producers of country 1 after taking into account cross-country differences in production cost as well as bilateral trade costs, then country 1 imports that product, and no region in this country produces that product at the equilibrium. In order to simplify our analysis, we assume no production in the residual labor market of country 1 and further assume that there is only formal sector in other countries $n' \neq 1$.

Labor, fixed factor, and composite intermediate inputs are used for production of each product variety ω in country 1. We assume that there is a continuum of product varieties in $[0, 1]$, and that each product variety is traded between countries. Products can be used either as intermediate inputs or for final consumption by consumers. We assume a Cobb-Douglas production function, where the production technology for a variety ω in labor market k of country 1 at time t is:

$$Q_{1,t}^{k}(\omega) = z_{1}^{k}(\omega)(\widetilde{l}_{1,t}^{k})^{\gamma_l}(M_{1,t}^{k})^{\gamma_m}(B_{1}^{k})^{\gamma_b},$$

where $M_{1,t}^{k}$ is a composite intermediate input and B_{1}^{k} is a fixed factor used in the labor market k of country 1. We assume that this fixed factor is specific to a labor market and does not change over time. To simplify the analysis, we do not impose an explicit input-output structure between sectors. Instead, each sector and each region considers the aggregate price index $P_{1,t}$ as the price of composite intermediate inputs in country 1 at time t. As a result, the share of a particular sector in the composite intermediate inputs is the same across all sectors that demand intermediates. This assumption is to simplify the production and trade structure and focus on the dynamics generated from the worker side.

In the production function, $\tilde{l}^k_{1,t}$ is in terms of the number of efficiency units provided by all workers in labor market k of country 1 at time t. This variable is different from $L^k_{1,t}$ which is the actual number of workers in labor market k of country 1 at time t. Here we introduce a notion of *task* which produces labor efficiency units. Producers within each labor market k will decide how to assign the total labor force into different tasks. Empirically, this allocation can be across differentiated occupations, different establishments, or both. We assume that workers are equally productive regardless of the task they are assigned, as long as they are in the same labor market. Therefore, producers will simply choose the total mass of a continuum of assignments to operate as well as the total efficiency units of labor they use.

For each labor market k, we assume that $\tilde{l}^k_{1,t}$ is a CES aggregate of all efficiency units provided by each task in labor market k. We define a continuum of tasks operated in labor market k as Ω^k_t and denote the mass of Ω^k_t as O^k_t. Then, the total labor aggregate is

$$\tilde{l}^k_{1,t} = \left[\int_{\tau \in \Omega^k_t} (l^k_{1,t}(\tau))^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}} d\tau \right]^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}},$$

where $l^k_{1,t}(\tau)$ is the total efficiency units provided by task τ in labor market k of country 1; and $\tilde{\sigma} > 1$ is the elasticity of substitution between tasks. In addition, we assume that the marginal cost of operating task is $\tilde{c} > 0$. Workers are equally productive at each task thus are paid the same wage regardless of assignment into a task or choice of a job. This assumption is the key difference from the standard search model. Under this assumption, $\tilde{l}^k_{1,t}$ can be rewritten as $\tilde{l}^k_{1,t} = L^k_{1,t}(O^k_t)^{\frac{1}{\tilde{\sigma}-1}}$, where $L^k_{1,t}$ is the total number of workers in labor market k at time t.[13]

If $\tilde{\sigma} > 1$ as we assumed, then there are more tasks in a cell if the optimal demand for total labor force $\tilde{l}^k_{1,t}$ is larger, given the labor supply $L^k_{1,t}$. This is the love for variety of tasks channel, which is analogous to the familiar love for variety of products in the Armington trade model. Therefore, if there is an exogenous shock which expands the labor market k, then the total labor demand in k will increase and thus the number of tasks operated in that labor market also will increase. Empirically, producers may post new type of tasks to operate a more differentiated task

[13] The assumption of the same productivity across tasks makes it optimal for producers to evenly allocate labor forces into different tasks, i.e., $l^k_{1,t}(\tau) = L^k_{1,t}/O^k_t$ for all $\tau \in \Omega^k_t$. In addition, we assume constant returns to scale in observable factors after substituting $\tilde{l}^k_{1,t}$ with $L^k_{1,t}$ and O^k_t. Therefore, $\tilde{\sigma} > \gamma_l + 1$ is a necessary condition for the constant returns to scale assumption.

structure or open new establishments to meet the increased demand.

In order to link this number of tasks to the number of job opportunities that workers perceive when they solve their problem of job choice, we assume that

$$N_t^l \;=\; \rho\left(O_t^l\right),$$

where N_t^l is a positive integer.[14] We assume that $\rho(.)$ is a monotonically increasing mapping in O_t^l. In other words, the number of job opportunities perceived by workers in a certain labor market is assumed to be increasing in the number of tasks operated in the labor market. For simplification, we assume that the function $\rho(.)$ is identical across labor markets.

We assume that the wage in the residual labor market is a fraction of the average wage of all formal labor markets, i.e., $w_{1,t}^I = \frac{\eta}{K}\sum_{k=1}^K w_{1,t}^k$, where $0 < \eta < 1$. In addition to the wage-induced labor mobility, the effect of a labor demand shock on labor mobility through the number of job opportunities is two-fold. First, a positive labor demand shock to a certain labor market increases the number of tasks operated by producers through the love for variety of tasks channel and thus increases the number of job opportunities that workers can compare. As a result, a labor market with a positive labor demand shock attracts more workers by providing more opportunities. Second, a positive labor demand shock in a non-residual labor market will decrease the probability of moving to the residual labor market. This second channel is similar to the standard search model, where the effect of a macro shock on welfare operates only through unemployment margin. In our model, a labor demand shock affects labor mobility and thus workers' welfare not only through the transition between formal and residual labor markets but also through higher utility from being able to compare more opportunities to choose the best one.

Other countries $n \neq 1$ have a simpler production function using only aggregate labor and composite intermediate inputs for each labor market k:

$$Q_{n,t}^k(\omega) = z_n^k(\omega)(\bar{L}_{n,t}^k)^{\bar{\gamma}_{n,l}}(M_{n,t}^k)^{1-\bar{\gamma}_{n,l}}.$$

For other countries $n \neq 1$, they all have the same number of sectors $\bar{S}$ such that $\bar{S}$ is the number of unique S_k over all k in country 1, so the sector-level trade flows between country 1 and $n \neq 1$ are well-defined. In addition, for all k in country $n \neq 1$, $R_k = 1$, as there is only one region in each country. Therefore, the superscript k effectively denotes a sector for country $n \neq 1$. At the general

[14] We will define the sampling function $\rho(.)$ formally in the appendix. We can think of it as a step function mapping positive real numbers to positive integers.

equilibrium, $\sum_{k}^{\bar{S}} \bar{L}_{n,t}^k = \bar{L}_n$ should hold for every (n,t), where $\bar{L}_n$ is the total labor endowment of country n which is assumed to be time-invariant. $M_{n,t}^k$ is the composite intermediate inputs demanded by sector k of country n at time t. $\bar{\gamma}_{n,l}$ is the value-added share of country n.

Factor-neutral productivity for each product variety is randomly drawn from a Fréchet distribution. For each country n, $z_n^k(\omega)$ is randomly drawn from

$$F_n^k(z) = \exp(-T_n^k z^{-\theta}).$$

As in the EK model, the productivity draws are independent across countries and labor markets. Since the labor market is defined in a different way between country 1 and country $n \neq 1$, k's are effectively sectors in country $n \neq 1$ but a pair of region and sector in country 1. The factor-neutral productivity is assumed to be time-invariant.[15]

There are iceberg trade costs between countries, $d_{nn',t}^s$, for products of sector s shipped from country n to n' at time t. If $n = 1$ or $n' = 1$, then this is trade cost between the national aggregator in country 1 and its partner country. We assume that there is no trade cost between regions within country 1. In our counterfactual exercises, we will introduce an exogenous trade shock by changing this iceberg trade cost parameter.

3.5 Equilibrium Trade Flows and Price Indices

In our model, producers in country 1 decide not only how much of each factor to employ but also how to allocate total labor force into different tasks. For the latter, producers have to decide the optimal number of tasks. There is a clear trade-off of having more tasks, i.e., more specialized labor technology. Having more tasks is beneficial because it increases the total efficiency units of labor conditional on $L_{1,t}^k$. However, to have more diversified production technology, producers have to pay a higher cost for such as training or building a new establishment.

The Cobb-Douglas production function gives us the following unit cost function for all firms in labor market k of country 1 at time t,

$$c_{1,t}^k = \Upsilon_1 (\widetilde{w}_{1,t}^k \tilde{c}^{\frac{1}{\tilde{\sigma}-1}})^{\gamma_l} (P_{1,t})^{\gamma_m} (b_{1,t}^k)^{\gamma_b}, \tag{15}$$

where $\widetilde{w}_{1,t}^k$ is the nominal wage of workers in labor market k of country 1 at time t, $P_{1,t}$ is the

[15] We can easily relax this assumption to study the effect of an exogenous change in T_1^k on endogenous variables such as labor market outcomes and workers' welfare.

aggregate price index in country 1 at time t, and $b^k_{1,t}$ is the price of the fixed factor for labor market k of country 1 at time t.[16] Υ_1 is the Cobb-Douglas constant. Similarly, the unit cost function for all producers in country $n' \neq 1$ at time t is

$$c_{n',t} = \Upsilon_{n'} (\widetilde{w}_{n',t})^{\bar{\gamma}_l} (P_{n',t})^{1-\bar{\gamma}_l}, \tag{16}$$

with the assumption of perfect labor mobility between sectors for all countries $n' \neq 1$.

At the equilibrium under perfect competition as in EK, bilateral trade share in sector s between regions r and r' of country 1 at time t is determined by

$$\lambda^s_{(1,r),(1,r'),t} = \frac{T^{(r,s)}_{1,t} (c^{(r,s)}_{1,t})^{-\theta}}{\sum_{r''} T^{(r'',s)}_{1,t} (c^{(r'',s)}_{1,t})^{-\theta} + \sum_{n' \neq 1} T^s_{n',t} (c_{n',t} d^s_{n'1,t})^{-\theta}} = \frac{X^s_{(1,r),(1,r'),t}}{X^s_{(1,r'),t}}. \tag{17}$$

[17] From the assumption of no trade cost between regions in country 1, $\lambda^s_{(1,r),(1,r'),t}$ is equalized across all r' of country 1. Similarly, the equilibrium trade flow of sector s from country $n \neq 1$ to region r of country 1 at time t is determined by

$$\lambda^s_{n,(1,r),t} = \frac{T^s_{n,t} (c_{n,t} d^s_{n1,t})^{-\theta}}{\sum_{r''} T^{(r'',s)}_{1,t} (c^{(r'',s)}_{1,t})^{-\theta} + \sum_{n' \neq 1} T^s_{n',t} (c_{n',t} d^s_{n'1,t})^{-\theta}} = \frac{X^s_{n,(1,r),t}}{X^s_{(1,r),t}}. \tag{18}$$

Since all regions take the sector-level price index as given for the use of intermediate inputs and consumers have the identical preference regardless where they live or in which sector they work, incoming trade shares should be the same across all regions. The actual demand level between regions depends only on their real income which is allowed to be different across regions in our setting. As a result, we have $\lambda^s_{n,1,t} = \bar{R} \lambda^s_{n,(1,r),t}$ for sector-level trade shares from country $n \neq 1$ to country 1 in aggregate.

The reverse trade flow from region r of country 1 to a country $n \neq 1$ is

$$\lambda^s_{(1,r),n,t} = \frac{T^{(r,s)}_{1,t} (c^{(r,s)}_{1,t} d^s_{1n,t})^{-\theta}}{\sum_{r'} T^{(r'',s)}_{1,t} (c^{(r'',s)}_{1,t} d^s_{1n,t})^{-\theta} + \sum_{n' \neq 1} T^s_{n',t} (c_{n',t} d^s_{n'n,t})^{-\theta}} = \frac{X^s_{(1,r),n,t}}{X^s_{n,t}}. \tag{19}$$

[16] Note that the wage is per worker, not per efficiency unit.

[17] In order to make a clear distinction between region and sector in trade flow equations, each labor market k is denoted as (r, s).

Finally, trade flows between countries $n \neq 1$ and $n'' \neq 1$ are derived as

$$\lambda^s_{n,n'',t} = \frac{T^s_{n,t}(c_{n,t}d^s_{nn'',t})^{-\theta}}{\sum_{r'} T^{(r'',s)}_{1,t}(c^{(r'',s)}_{1,t}d^s_{1n'',t})^{-\theta} + \sum_{n'\neq 1} T^s_{n',t}(c_{n',t}d^s_{n'n'',t})^{-\theta}} = \frac{X^s_{n,n'',t}}{X^s_{n'',t}}. \tag{20}$$

The exact price index for sector s in country 1 at time t is

$$P^s_{1,t} = \bar{\Gamma}\left[\sum_{r''} T^{(r'',s)}_{1,t}(c^{(r'',s)}_{1,t})^{-\theta} + \sum_{n'\neq 1} T^s_{n',t}(c_{n',t}d^s_{n'1,t})^{-\theta}\right]^{-\frac{1}{\theta}}, \tag{21}$$

and the exact price index for sector s in country $n \neq 1$ at time t is

$$P^i_{n,t} = \bar{\Gamma}\left[\sum_{r'} T^{(r'',s)}_{1,t}(c^{(r'',s)}_{1,t}d^s_{1n,t})^{-\theta} + \sum_{n'\neq 1} T^s_{n',t}(c_{n',t}d^s_{n'n,t})^{-\theta}\right]^{-\frac{1}{\theta}}, \tag{22}$$

where $\bar{\Gamma}$ is the same constant for both cases.[18] We assume that all consumers have an identical nested CES preference with a common elasticity of substitution σ at the variety level and a Cobb-Douglas aggregation across sectors with expenditure share ϕ^s. Then, the aggregate price index is given by $P_{n,t} = \prod_s (\frac{P^s_{n,t}}{\phi^s})^{\phi^s}$ for each country n.

3.6 Market Clearing

To simplify the general equilibrium, we assume that country-level trade deficit $D_{n,t}$ is exogenously fixed as the share of world GDP. We further assume that the country 1's trade deficit is distributed across consumers in different labor markets of country 1 proportionally to the income share. The trade deficit distributed to labor market k is denoted by $D^k_{1,t}$. The total expenditure on sectors s products by all agents in region r of country 1 is

$$X^s_{(1,r),t} = \phi^s \gamma_m \sum_{s'}\left(\sum_{r'} \lambda^{s'}_{(1,r),(1,r'),t} X^{s'}_{(1,r'),t} + \sum_{n'\neq 1} \lambda^{s'}_{(1,r),n',t} X^{s'}_{n',t}\right) + \phi^s \left(\sum_{k\in\{k|R_k=r\}} \left(\widetilde{w}^k_{1,t} L^k_{1,t} + D^k_{1,t}\right)\right), \tag{23}$$

where $\{k \mid R_k = r\}$ is the set of labor markets within region r. Note that we can denote $X^s_{1,t} \equiv \sum_r X^s_{(1,r),t}$. Similarly, the total expenditure on sector s products by all agents in country

[18] More precisely, $\bar{\Gamma} \equiv \left[\Gamma(\frac{\theta+1-\sigma}{\theta})\right]^{1/(1-\sigma)}$. We assume $\sigma < \theta + 1$ so that the price index is well-defined.

$n \neq 1$ at time t is

$$X_{n,t}^{s} = \phi^{s}(1-\bar{\gamma}_l)\sum_{s'}\left(\sum_{r'}\lambda_{n,(1,r'),t}^{s'}X_{(1,r'),t}^{s'}+\sum_{n'\neq 1}\lambda_{n,n',t}^{s'}X_{n',t}^{s'}\right)+\phi^{s}\left(\widetilde{w}_{n,t}\bar{L}_{n,t}+D_{n,t}\right). \quad (24)$$

Finally, the set of market clearing conditions is given by

$$\widetilde{w}_{1,t}^{k}L_{1,t}^{k} = \gamma_l\left(\sum_{r'}\lambda_{(1,R_k),(1,r'),t}^{S_k}X_{(1,r'),t}^{S_k}+\sum_{n'\neq 1}\lambda_{(1,R_k),n',t}^{S_k}X_{n',t}^{S_k}\right), \quad (25)$$

$$b_{1,t}^{k}B_{1}^{k} = (1-\gamma_l-\gamma_m)\left(\sum_{r'}\lambda_{(1,R_k),(1,r'),t}^{S_k}X_{(1,r'),t}^{S_k}+\sum_{n'\neq 1}\lambda_{(1,R_k),n',t}^{S_k}X_{n',t}^{S_k}\right) \quad (26)$$

for each labor market k of country 1, and

$$\widetilde{w}_{n,t}\bar{L}_{n,t} = \bar{\gamma}_l\sum_{s'}\left(\sum_{r'}\lambda_{n,(1,r'),t}^{s'}X_{(1,r'),t}^{s'}+\sum_{n'\neq 1}\lambda_{n,n',t}^{s'}X_{n',t}^{s'}\right), \quad (27)$$

for country $n' \neq 1$. The equilibrium labor supply for each labor market of country 1, $L_{1,t}^{k}$, is pinned down by the labor model.

To complete the model, we now characterize the residual labor market further. Workers in the residual labor market receive a fraction η of the average formal wagefixed wage. A worker who was in the residual labor market but will be in a formal labor market in the next period can draw iid shocks for available job opportunities and choose a job like the incumbent formal workers.

A temporary equilibrium at each period t is a vector of wages $\widetilde{\mathbf{w}}_t = (\widetilde{w}_{1,t}^{1},\ldots,\widetilde{w}_{1,t}^{K},\widetilde{w}_{2,t},\ldots,\widetilde{w}_{N,t})$ and a vector of prices of the fixed factor in country 1 $\widetilde{\mathbf{b}}_t = (\widetilde{b}_{1,t}^{1},\ldots,\widetilde{b}_{1,t}^{K})$ which satisfy (15)-(27), conditional on the labor supply $\mathbf{L}_t = (L_{1,t}^{1},\ldots,L_{1,t}^{K},\bar{L}_2,\ldots,\bar{L}_N)$ and other fundamental parameters. A sequential competitive equilibrium is the period-by-period sequence of $\mathbf{L}_t$, $\widetilde{\mathbf{w}}_t$, $\mathbf{b}_t$, and $\mathbf{m}_t = \left\{m_{1,t}^{kl}\right\}_{k=1,l=1}^{\infty,\infty}$ which solve the labor mobility model at each time period t, conditional on the initial labor allocation $\mathbf{L}_0$ and fundamental parameters of the model.

4 Estimation

We estimate our structural model with the same sample we used for the reduced-form analysis in Section 2. In addition to the sample selection rules discussed previously, we restrict the sample to the labor markets where at least 100 workers move in and out respectively in every time period.

Since the identification of moving probabilities are based on the worker mobility, values of labor markets with little labor mobility cannot be identified. Unlike ACM or CDP, we allow corridors with zero mobility. For example, if there are less than 100 workers move out of (or move into) labor market A, we drop labor market A. However, if there are zero workers moving from labor market B to labor market C, but there are more than 100 workers total moving into other cells from labor market B, then we keep labor market B and also keep the B-C corridor in the estimation. The idea is similar to dropping small countries in the gravity estimation of trade flows, where it is common to keep corridors with zero flows. After this restriction, we end up with 857 labor markets.[19]

The main objects to be estimated are the moving probabilities and the structural parameters such as moving costs and the labor supply elasticity ν. We combine the local labor market approach to estimate the labor supply elasticity parameter. With these estimates in hand, we then turn to our variables of interest: changes in welfare, option values, and the number of job opportunities due to trade shocks. In our reduced-form analysis, we did not study the effect of trade shocks on these welfare-related variables, because there are no data counterparts to these variables. Once we estimate our model, we can use our model structure which shows that these welfare-related variables are functions of observables. In other words, we can study the effect of trade shocks on these welfare-related variables in a reduced-form way based on our model structure, before doing full general equilibrium counterfactuals in the next section.

4.1 Estimation of the Flows

As we showed in Section 3, changes in welfare or the number of jobs due to a policy change depend on the following four factors: the probability of switching a labor market conditional on changing a job and staying in a formal labor market ($\widetilde{m}_t^{kl}$), the probability of staying in the current job ($\mu_{0,t}^k$), the probability of moving to the residual labor market ($\mu_{I,t}^k$), and the labor supply elasticity (ν). In practice, it is not possible to get $\log \widetilde{m}_t^{kl}$ directly from data without any estimation. The bin-estimator, i.e. imputing the probability of moving by dividing the number of switchers by the total number of workers, only works with a large sample size and a small number of choices. We have 857 labor markets, and each labor market offers more than one job opportunities, which makes the bin-estimator infeasible. This problem is more serious for the probability of switching, especially for the bilateral mobility, than for the probability of staying. Instead of using the simple

[19] All main results that we present in this paper are robust to the mobility cutoff.

bin-estimator, we can estimate $\log \widetilde{m}_t^{kl}$ by imposing a structure on the moving costs.

The probability of moving from a labor market k to a labor market l conditional on changing jobs is equal to

$$\log \widetilde{m}_t^{kl} = \widetilde{V}_t^l - \widetilde{C}_t(k,l) + \widetilde{\Gamma}_t^k - \log \widetilde{L}_t^k, \tag{28}$$

where

$$\begin{aligned} \widetilde{V}_t^l &\equiv E_t \frac{\beta}{\nu} V_{t+1}^l - \log \mu_{0,t}^l + \log \mu_{1,t}^l \\ \widetilde{C}_t(k,l) &\equiv \frac{C_t(k,l)}{\nu} \\ \widetilde{\Gamma}_t^k &\equiv -\log \sum_{l'} \exp\left(\widetilde{V}_t^{l'} - \widetilde{C}_t(k,l')\right) + \log \widetilde{L}_t^k, \end{aligned}$$

and $\widetilde{L}_t^k$ is the number of workers who change jobs within labor market k or move out of the labor market k but stay in a formal labor market. After estimating $\widetilde{C}_t(k,l)$ and $\widetilde{V}_t^l$, it is straightforward to calculate the implied probabilities using (28). If we define the number of workers observed in the sample moving from a labor market k to l conditional on changing jobs but staying in the formal labor market by $\widetilde{y}_t^{kl}$, then we have $\widetilde{L}_t^k = \sum_l \widetilde{y}_t^{kl}$. This result means that the likelihood function is equal to

$$\mathcal{L} = \prod_k \prod_l \left(\widetilde{m}_t^{kl}\right)^{\widetilde{y}_t^{kl}}, \tag{29}$$

or, alternatively,

$$\log \mathcal{L} = \sum_k \sum_l \widetilde{y}_t^{kl} \left[\widetilde{\Gamma}_t^k + \widetilde{V}_t^l - \widetilde{C}_t(k,l) - \log(\widetilde{L}_t^k)\right], \tag{30}$$

using (28). As discussed above, if $L_t^k \to \infty$, then $\widetilde{L}_t^k \to L_t^k(\mu_{1,t}^k + \mu_{2,t}^k)$, and $\widetilde{y}^{kl}/\widetilde{L}_t^k \to \widetilde{m}_t^{kl}$. Therefore, as the sample size goes to infinity, the maximum likelihood (ML) estimator becomes equivalent to the bin-estimator. We use the Poisson pseudo-maximum-likelihood (PPML) method to estimate $\widetilde{\Gamma}_t^k$, $\widetilde{V}_t^l$, and $\widetilde{C}_t(k,l)$ for each period, because we can write $\widetilde{y}_t^{kl}$ as

$$\widetilde{y}_t^{kl} = \exp\left(\widetilde{\Gamma}_t^k + \widetilde{V}_t^l - \widetilde{C}_t(k,l)\right) + \epsilon_t^{kl}, \tag{31}$$

where ϵ is a sampling error; $\widetilde{\Gamma}_t^k$ is the origin fixed effect; $\widetilde{V}_t^l$ is the destination fixed effect; and

$\widetilde{C}_t(k,l)$ is the moving cost between labor markets. The expected number of workers who move from labor market k to l is equal to $E_t\widetilde{y}_t^{kl} = \widetilde{m}_t^{kl}\left(\mu_{1,t}^k + \mu_{2,t}^k\right)L_t^{s,k}$.

Guimaraes, Figueirdo, and Woodward (2003) prove that the maximum likelihood estimation of gravity equation based on Fréchet distribution is identical to PPML. The same intuition applies to our model for Gumbel distribution as well. In the appendix, we prove that the PPML and MLE are equivalent herein.

In the estimation, we consider a simple moving cost structure as follows:

$$\widetilde{C}_t(j,k) = \widetilde{c}_{1,t}D^{jk} + \widetilde{c}_{2,t}\mathbf{1}_{S_j \neq S_k} + \widetilde{c}_{3,t}\mathbf{1}_{S_j \neq S_k \wedge R_j \neq R_k}, \tag{32}$$

where D^{jk} is the log of distance between labor markets j and k, and $\mathbf{1}_{S_j \neq S_k}$ is an indicator function that is equal to one if labor markets j and k are associated with different sectors, and $\mathbf{1}_{S_j \neq S_k \wedge R_j \neq R_k}$ is an indicator function that is equal to one if k and j are associated with different sectors and regions. We impose $D^{jj} = 0$ for every j and $D^{jk} = 0$ if $R_j = R_k$. All coefficients in equation (32) are divided by ν to be consistent with the definition of $\widetilde{C}_t(j,k)$.[20]

Table 4 reports the estimated moving cost parameters $\widetilde{c}_{1,t}$, $\widetilde{c}_{2,t}$, and $\widetilde{c}_{3,t}$ as well as their standard errors from the PPML estimation. The moving cost between two labor markets increases in log distance between the two as expected. This number is also close to the number that has been found in the migration literature, the migration elasticity around -1. As reported in the table, the estimated coefficients are not identical over years, but they are very stable over time.

4.2 Estimation of the Labor Supply Elasticity

The key structural parameter we need to pin down to quantify our model is the labor supply elasticity $1/\nu$. We estimate this parameter by combining the local labor market approach into our structural model. We derive a regression equation similar to Autor, Dorn, and Hanson (2013), where the reduced-form coefficient has a direct link with our structural parameter.[21]

From the equilibrium probability of moving between labor markets m_t^{kl} as derived in (5), the shape parameter of the distribution of worker's idiosyncratic shock on jobs, ν, is essentially the

[20] Alternatively, we can simply have $\mathbf{1}_{R_j \neq R_k}$ instead of D^{jk}, but we use the information on physical distance to back out the region-level mobility friction following the migration literature.

[21] Our estimation strategy is in a similar spirit of Galle, Rodriguez-Clare, and Yi (2017) and Adao, Arkolakis, and Esposito (2019) who estimate the labor allocation elasticity in a static setting. For example, Galle, Rodriguez-Clare, and Yi (2017) also estimate their key structural parameter using the China shock as in Autor, Dorn, and Hanson (2013). In our model, the elasticity is constructed for labor mobility both across regions and sectors. The labor market adjustment in our model is dynamic and subject to mobility friction, while the two aforementioned papers interpret the elasticity in a completely static setting without explicit mobility frictions.

Table 4: Estimated moving cost parameters

Year	$\widetilde{c}_{1,t}$ (log distance)	s.e.	$\widetilde{c}_{2,t}$ (sector)	s.e.	$\widetilde{c}_{3,t}$ (both)	s.e.
2003	1.0775	(0.0008)	1.9983	(0.0048)	-0.2839	(0.0097)
2004	1.0602	(0.0008)	1.9275	(0.0049)	-0.2786	(0.0098)
2005	1.0473	(0.0008)	1.6034	(0.0047)	0.0507	(0.0099)
2006	1.0447	(0.0007)	1.8967	(0.0048)	-0.2808	(0.0093)
2007	1.0667	(0.0007)	2.0184	(0.0048)	-0.4659	(0.0091)
2008	1.0478	(0.0007)	1.9298	(0.0045)	-0.3153	(0.0086)
2009	1.0448	(0.0006)	1.7939	(0.0041)	-0.2755	(0.0079)
2010	1.0355	(0.0006)	1.8190	(0.0042)	-0.2546	(0.0080)
2011	1.0250	(0.0007)	1.8127	(0.0050)	-0.2146	(0.0094)
2012	1.0221	(0.0007)	1.8331	(0.0053)	-0.2194	(0.0098)
2013	1.0290	(0.0006)	1.8491	(0.0044)	-0.2546	(0.0083)
2014	1.0399	(0.0006)	1.9684	(0.0044)	-0.2752	(0.0083)
Average	1.0451		1.8709		-0.2556	

inverse of the labor supply elasticity in our model.[22] Our key results about changes in welfare, option values, and the number of jobs all depend on this key parameter, since the responsiveness of outcomes of interest crucially depends on how elastic labor mobility is.

Once we estimate $\widetilde{V}_t^k$ as the destination fixed effect in the PPML equation, we can calculate the present discounted value of the expected wage as

$$E_t \frac{\beta}{\nu} w_{t+1}^k \quad = \quad \widetilde{V}_t^k + \left(\log \mu_{0,t}^k - \log \mu_{1,t}^k \right) - \beta E_t \left[\widetilde{V}_{t+1}^k - \log \mu_{1,t+1}^k \right].$$

We use the right hand side of the equation above as the dependent variable to estimate ν based on the local labor market approach by estimating

$$\Delta y_t^k = \alpha + \tilde{\beta} \Delta Z_t^k + \lambda_t + \epsilon_t^k, \tag{33}$$

where we set $y_t^k = \widetilde{V}_t^k + \left(\log \mu_{0,t}^k - \log \mu_{1,t}^k \right) - \beta \left[\widetilde{V}_{t+1}^k - \log \mu_{1,t+1}^k \right]$ and $Z_t^k = w_{t+1}^k$, in the equation above. Conditional on $\beta = 0.95$, we back out the elasticity ν from $\tilde{\beta}$.[23]

Since wages can be endogenous we need to use an instrument for wages. In the first stage regression, we regress wages on the instrument $\Delta \bar{Z}_t^k$ discussed in Section 2 for the formal sector. Then, we use the predicted wage as an explanatory variable in the regression equation above. In

[22] More precisely, this parameter is the inverse of the elasticity of labor mobility with respect to frictions, but we will call it as the labor supply elasticity following a convention in the literature.

[23] Our main results are robust to the choice of the discount factor.

summary, our model delivers a simple estimable equation for the labor supply elasticity parameter. We can use the same Bartik-type instrument that we used in the reduced-form analysis, which provides a clean identification of the key structural parameters of the model.

Panel (a) of Table 5 reports the estimates of β/ν as well as the first stage result for a change in wage with the same instrument we used for the reduced-form analysis. If we assume $\beta = 0.95$, then the implied ν is 0.484. Our estimates are similar to what other papers in the literature have found: e.g., Artuç and McLaren (2015) find $\nu = 0.56$ with $\beta = 0.9$ and $\nu = 1.613$ with $\beta = 0.97$.

Table 5: Estimation results for β/ν

A. First stage	
$\Delta\bar{Z}$	0.412
	(0.028)
F-stat	138.480
B. Second stage	
	1.962
	(0.757)

4.3 Welfare Results from the Trade Shock

We revisit the simple regression equation from Section 2 again to estimate the impact of trade on the welfare-related variables as well as the number of job opportunities that are not measured directly from data. We use predicted exports from Section 2 as the explanatory variable Z_t^k in the equation. We replace the dependent variable y_t^k with the welfare expression in equation (13), the number of jobs formula in equation (14), and the option value formulas in equations (12) and (11). We use estimated moving probabilities to construct the welfare, jobs and option value measures, without using any structural parameters, except for the labor supply elasticity ν. With a larger sample or smaller number of choices, it would be possible to plug the data directly into the equations with a simple bin-estimator. The bin-estimators are feasible for μ's in our model. For $\widetilde{m}$'s, on the other hand, we estimate them using the PPML as discussed before, since the bin-estimators are not feasible due to the reasons discussed above. After the estimation, we use the structural elasticity parameter ν to express the estimated numbers in annual average wages.

We regress changes in welfare-related variables on changes in exports with the same instrument as before. This exercise is to study the effect of trade shocks on workers' welfare in a reduced-form way before performing a full general equilibrium counterfactual simulation in the next section. Table 6 reports the estimation results for each welfare-related outcome of interest. Since all dependent variables are divided by ν as shown in each of (12), (11), (13), and (14), we use the estimated ν to back out the implied elasticities of each outcome variable with respect to export revenues of each labor market. For the baseline implied elasticities, we assume $\nu = 0.484$ which is the obtained estimate with $\beta = 0.95$.

Table 6: Export-induced changes in welfare-related variables

	Coefficients	s.e.	Implied elasticities with $\nu = 0.484$
Welfare	0.700	(0.150)	0.339
Job opportunities	0.622	(0.131)	0.301
Internal option value	0.146	(0.022)	0.071
External option values	-0.147	(0.048)	-0.071

The result shows that a positive export shock increases worker's welfare in the corresponding labor market and the number of jobs provided in the labor market. One of the interesting results is that a positive shock increases internal option values but decreases external option values. In existing models such as ACM and CDP, a positive export shock should decrease the option value, as other labor markets become relatively less valuable after a positive export shock in your own labor market. This is captured by the effect on the external option value of our model. On the other hand, in our model with the endogenous number of job opportunities, the internal option value moves towards the opposite direction. As the number of job opportunities increases with a positive export shock, the internal option value increases. Due to this additional effect that our model is able to capture through the number of job opportunities, a positive export shock generates extra positive effects on the total option values compared to the existing models, which will be further confirmed based on simulations in the following section.

5 Model Quantification and Counterfactual Exercises

In this section we first quantify our model by calibrating the remaining parameters that are necessary for quantification. Then we run various counterfactual simulations based on the quantified model. Our model can be quantified to understand the effect of changes in trade environment such as changes in d_{in}^{s} on labor allocation, wages, welfare, the number of job opportunities, option values, and inequality. In this section, we show how we can assess the impact of a trade shock under different policy environments. Specifically, we quantify the role of different degrees of labor mobility friction in transmitting a trade shock to labor market outcomes and worker's welfare. Our rich framework allows us to look at not just different magnitudes of labor mobility frictions, but also potentially different roles of sectoral versus geographical mobility frictions. This exercise is particularly relevant for labor market policies, because it can shed light on which margin to target when it comes to distribution of gains from trade across workers.

A full general equilibrium simulation of a dynamic model of labor adjustment like our model is typically very challenging, because it involves solving for expected values at every period. Caliendo, Dvorkin, and Parro (2019) propose a convenient way of doing counterfactual exercises with this class of models. In a similar spirit with their "dynamic hat algebra" technique, we propose a "flexible dynamic hat algebra" which does not require any restriction on consumer's utility. While the dynamic hat algebra in CDP needs an assumption of log utility, our method can be applied to a similar type of model with any utility function.

5.1 Flexible Dynamic Hat Algebra

We can re-write the model in terms of counterfactual changes in each variable from the initial steady state for convenient counterfactual exercises. Following the spirit of the dynamic hat algebra of CDP, our method allows for a full general equilibrium counterfactual analysis without having to pin down a lot of model parameters that are not of interest in our counterfactual scenarios. In addition, our flexible dynamic hat algebra method does not require any assumption on the utility function. For the main counterfactual exercise, we allow between-labor-market moving cost to vary across periods, while the switching cost between jobs is assumed to be time-invariant. We also assume that the marginal cost of task operation $\tilde{c}$ is time-invariant.

For any variable X of the model, we denote the value of this variable at the initial steady state by X_0. We define the following three operators.

$$\ddot{X}_t \equiv \exp\left(\frac{\beta E_{t-1}X_t - \beta X_0}{\nu}\right)$$
$$\dot{X}_t \equiv \exp\left(\frac{X_t - X_0}{\nu}\right)$$
$$\widehat{X_t} \equiv \frac{X_t}{X_0}$$

Using these operators, the probability of staying in the same job in labor market k at time t can be re-written as

$$\mu_{0,t}^k = \frac{\ddot{V}_{t+1}^k \mu_{0,0}^k}{\ddot{V}_{t+1}^k \left[\mu_{0,0}^k + \left(m_0^{kk} - \mu_{0,0}^k\right)\widehat{N_t^k}\right] + \sum_{l' \neq k} \ddot{V}_{t+1}^{l'} \widehat{N_t^{l'}} m_0^{kl'} \left[\dot{C}(k,l')\right]^{-1}}. \tag{34}$$

The Bellman equation becomes

$$\log \ddot{V}_t^k = \frac{\beta}{\nu} u_0^k (\widehat{u_t^k} - 1) + \beta \log \ddot{V}_{t+1}^k - \beta \log(\frac{\mu_{0,t}^k}{\mu_{0,0}^k}), \tag{35}$$

where u_t^k is the instantaneous utility of workers in labor market k at time t. If we assume that $u_t^k = \log w_t^k$, then the Bellman equation in changes derived in (35) converges to the result of CDP. Between-labor-market moving probability m_t^{kl} can be also written in terms of changes:

$$m_t^{kl} = \frac{\mathbf{1}_{l=k} \ddot{V}_{t+1}^k \left[\mu_{0,0}^k + (m_0^{kk} - \mu_{0,0}^k)\widehat{N_t^k}\right] + \mathbf{1}_{l \neq k} \ddot{V}_{t+1}^l \widehat{N_t^l} m_0^{kl} \left[\dot{C}(k,l)\right]^{-1}}{\ddot{V}_{t+1}^k \left[\mu_{0,0}^k + \left(m_0^{kk} - \mu_{0,0}^k\right)\widehat{N_t^k}\right] + \sum_{l' \neq k} \ddot{V}_{t+1}^{l'} \widehat{N_t^{l'}} m_0^{kl'} \left[\dot{C}(k,l')\right]^{-1}}. \tag{36}$$

For every t, $L_{t+1}^k = \sum_l m_t^{lk} L_t^l$ has to hold.

The trade part of the model can be re-written in terms of changes in our flexible dynamic hat algebra technique. Since the trade part of the model is solved for the period-by-period static equilibrium without transitional dynamics between periods, the hat algebra is relatively simpler only with $\widehat{X} \equiv X_t/X_0$ as in the standard exact hat algebra introduced by Dekle, Eaton, and

Kortum (2008). Changes in sectoral price indices are

$$\widehat{P}^s_{1,t} = \left[\sum_{r''}\lambda^s_{(1,r''),(1,r),0}\hat{T}^{(r'',s)}_{1,t}(\hat{c}^{(r'',s)}_{1,t})^{-\theta}+\sum_{n'\neq 1}\lambda^s_{n',(1,r),0}\hat{T}^s_{n',t}(\hat{c}_{n',t}\hat{d}^s_{n'1,t})^{-\theta}\right]^{-\frac{1}{\theta}} \tag{37}$$

$$\widehat{P}^s_{n,t} = \left[\sum_{r'}\lambda^s_{(1,r),n,0}\hat{T}^{(r'',s)}_{1,t}(\hat{c}^{(r'',s)}_{1,t}\hat{d}^s_{1n,t})^{-\theta}+\sum_{n'\neq 1}\lambda^s_{n',n,0}\hat{T}^s_{n',t}(\hat{c}_{n',t}\hat{d}^s_{n'n,t})^{-\theta}\right]^{-\frac{1}{\theta}}. \tag{38}$$

Equations (25) and (27) can be re-written as follows:

$$\hat{\widetilde{w}}^k_{1,t}\hat{L}^k_{1,t}\widetilde{w}^k_{1,0}L^k_{1,0} = \gamma_l\left(\sum_{r'}\hat{\lambda}^{S_k}_{(1,R_k),(1,r'),t}\lambda^{S_k}_{(1,R_k),(1,r'),0}X^{S_k}_{(1,r'),t}+\sum_{n'\neq 1}\hat{\lambda}^{S_k}_{(1,R_k),n',t}\lambda^{S_k}_{(1,R_k),n',0}X^{S_k}_{n',t}\right) \tag{39}$$

$$\hat{b}^k_{1,t}b^k_{1,0}B^k_1 = (1-\gamma_l-\gamma_m)\left(\sum_{r'}\hat{\lambda}^{S_k}_{(1,R_k),(1,r'),t}\lambda^{S_k}_{(1,R_k),(1,r'),0}X^{S_k}_{(1,r'),t}+\sum_{n'\neq 1}\hat{\lambda}^{S_k}_{(1,R_k),n',t}\lambda^{S_k}_{(1,R_k),n',0}X^{S_k}_{n',t}\right) \tag{40}$$

$$\hat{\widetilde{w}}_{n,t}\hat{\bar{L}}_{n,t}\widetilde{w}_{n,0}\bar{L}_{n,0} = \bar{\gamma}_l\sum_{s'}\left(\sum_{r'}\hat{\lambda}^{s'}_{n,(1,r'),t}\lambda^{s'}_{n,(1,r'),0}X^{s'}_{(1,r'),t}+\sum_{n'\neq 1}\hat{\lambda}^{s'}_{n,n',t}\lambda^{s'}_{n,n',0}X^{s'}_{n',t}\right). \tag{41}$$

Detailed derivation of the flexible dynamic hat algebra is provided in the Appendix.

5.2 Taking Model to the Data

After re-writing the model using the flexible dynamic hat algebra, we take our model to the data first by calibrating it to the base year 2003. We quantify our model for 21 countries including Brazil, plus the rest of the world, thus, $N = 22$.[24] We use the same definition of labor market as in previous sections. We have three sectors–agricultural/resources, manufacturing, and services– and 558 microregions to define a labor market. The service sector is assumed to be non-tradable internationally throughout our counterfactual analyses. As it was assumed in Section 3, domestic trade within a country is frictionless. Using the estimates of ν, $C(k,l)$, and the moving probabilities from Section 4 and the wage data and the initial labor allocation for each labor market of Brazil, we solve for the steady state values across labor markets.[25] We simulate our model for 30 periods ($T = 30$) to see transitional dynamics.

We calibrate the cost share of labor (γ_l and $\bar{\gamma}_l$), the cost share of intermediate inputs (γ_m) to the base year using the World Input-Output database (WIOD) and the share of fixed factor

[24] The list of countries is provided in the appendix.

[25] As in Section 2, we use the residual wage after controlling for observable worker characteristics.

in CDP. We allow these cost shares to be different across sectors to match the data better.[26] In addition, we calibrate labor income of each labor market of Brazil ($\widetilde{w}^k_{1,0} L^k_{1,0}$), GDP of each country other than Brazil ($\widetilde{w}_{n,0}\bar{L}_{n,0}$), trade shares λ's, sectoral expenditure shares in preference ϕ's to exactly match the data of the base year. We further allow for ϕ's to vary by country in the calibrated model. We calibrate the initial share of the informal employment to be 0.38 to match the data. Lastly, we set $\beta = 0.95$ and set the trade elasticity $\theta = 4$ following Simonovska and Waugh (2014).

5.3 Solution Algorithm

Once the model is calibrated to the base year and the steady state moving probabilities, we solve for $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$ by solving the system of market clearing conditions as in equations (39)-(41), where

$$\begin{aligned}
\widehat{\widetilde{\mathbf{w}}}_1 &= \left\{\widehat{\widetilde{w}}^1_{1,1}, \ldots, \widehat{\widetilde{w}}^1_{1,T}, \ldots, \widehat{\widetilde{w}}^k_{1,1}, \ldots, \widehat{\widetilde{w}}^k_{1,T}, \ldots, \widehat{\widetilde{w}}^K_{1,1}, \ldots, \widehat{\widetilde{w}}^K_{1,T}\right\} \\
\widehat{\widetilde{\mathbf{w}}}_{-1} &= \left\{\widehat{\widetilde{w}}_{2,1}, \ldots, \widehat{\widetilde{w}}_{2,T}, \ldots, \widehat{\widetilde{w}}_{n,1}, \ldots, \widehat{\widetilde{w}}_{n,T}, \ldots, \widehat{\widetilde{w}}_{N,1}, \ldots, \widehat{\widetilde{w}}_{N,T}\right\} \\
\widehat{\mathbf{b}} &= \left\{\widehat{b}^1_1, \ldots, \widehat{b}^1_T, \ldots, \widehat{b}^k_1, \ldots, \widehat{b}^k_T, \ldots, \widehat{b}^K_1, \ldots, \widehat{b}^K_T\right\}.
\end{aligned}$$

We denote $\mathbf{L}$, $\ddot{\mathbf{V}}$, $\widehat{\mathbf{N}}$, $\widehat{\mathbf{P}}$, $\mathbf{m}$, and $\boldsymbol{\mu}$ vectors similarly. From the steady state simulation and the data, we have w^k_0, L^k_0, m^{kl}_0, and $\mu^k_{1,0}$ for the initial steady state equilibrium.

The solution algorithm solves for $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$ in the outer iteration and solves for labor dynamics within each iteration of the outer loop. We first guess the initial $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$ and solve for $\widehat{\mathbf{P}}$ using equations (37) and (38). Conditional on the guess of factor prices and the corresponding price changes, we compute $\widehat{\mathbf{N}}$. For the first iteration, we guess labor allocation $\mathbf{L}$. Conditional on the guess of changes in nominal wages and the solution of changes in price indices conditional on the guess, we calculate changes in real wages for each labor market of Brazil. We then solve for implied labor flows $\mathbf{m}$ and $\boldsymbol{\mu}$ using equations (36) and (34). Using the implied labor mobility flows and the guess of $\widehat{\widetilde{\mathbf{w}}}_1$ and the corresponding $\widehat{\mathbf{N}}$, we calculate implied changes in the values $\ddot{\mathbf{V}}$. At the end of the labor loop, we calculate $\mathbf{L}$ which updates the guesses of $\mathbf{L}$. Using the updated labor allocation $\mathbf{L}$, the initial guess of $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$, and the solved $\widehat{\mathbf{P}}$, we calculate the excess demand of each factor following Equations (39)-(41). We update the guess of $\left(\widehat{\widetilde{\mathbf{w}}}_1, \widehat{\widetilde{\mathbf{w}}}_{-1}, \widehat{\mathbf{b}}\right)$ and repeat the algorithm until all excess demands of factors become close to zero.

[26] These cost shares are assumed to be the same across regions within the same sector, because the input-output data are not available at the detailed region level in Brazil.

5.4 Counterfactual Exercises

The benchmark counterfactual shock we explore in this section is a 30% permanent decline in iceberg trade costs d for the manufacturing sector from Brazil to each of its trading partners. This shock can be interpreted as a positive export shock to all manufacturing labor markets in Brazil. We first solve the model conditional on the estimated between-market moving costs to evaluate the effect of a positive export shock on the manufacturing sector on workers' welfare, labor allocation, and transitional dynamics.[27] We then shut down our job opportunities channel by setting the cost of job creation to be infinite and compare this alternative model without job creation externality to the benchmark model. Lastly, we introduce different levels of between-market moving costs in order to understand the role of mobility friction on the transmission of trade shocks on workers' welfare. By changing sector-specific and region-specific components of between-labor-market moving costs separately, our model provides policy implications on which dimension to target when it comes to transmission of trade shocks to worker welfare.

5.4.1 Benchmark Simulation

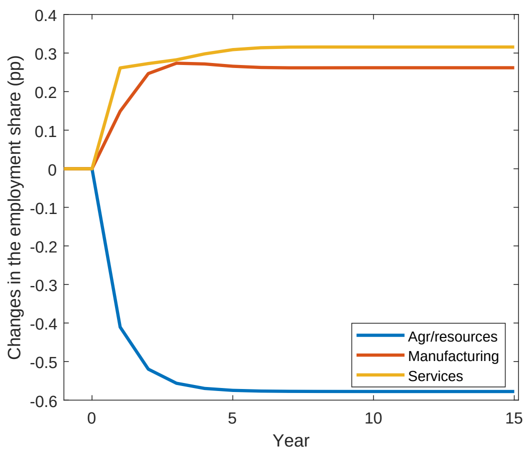
In the benchmark counterfactual scenario, there is effectively a positive export shock on the manufacturing sector. Unlike predictions from static models, transition to the new steady state is not instant in our model with mobility frictions. For example, it takes 5 years for sectoral employment shares to reach about 99% of the level of the new steady state. Figure 2 show the transition paths of sectoral employment shares. The figure on the left panel plots changes in the employment share of each sector within the formal labor market.[28] Changes in the employment share are weighted by the initial employment share of a labor market within each sector. At the new steady state, the employment share increases in the manufacturing and service sectors within the formal labor market, while it decreases in the agriculture sector. The reallocation from the agriculture/resource sector to the manufacturing sector is straight-forward, since the benchmark shock is a positive export shock to the manufacturing sector. The employment share of the service sector which is assumed to be non-tradable increases as well, because real wages increase on average in Brazil due to this positive export shock. At the same time, the share of the residual labor market goes down significantly, because our model does not allow for an expansion

[27] The moving probabilities we solve for the initial steady state equilibrium are conditional on the estimates of mobility frictions. For the baseline counterfactuals, we assume that these mobility frictions do not change.

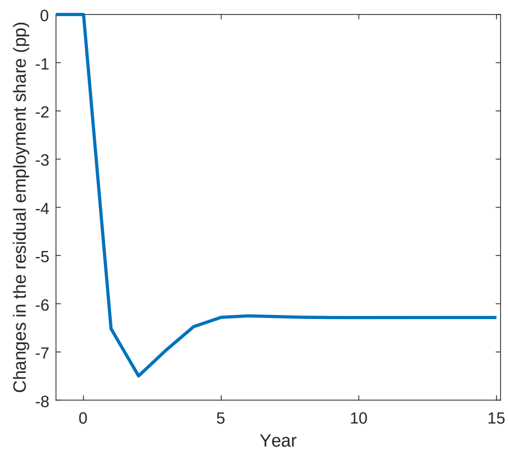
[28] Behind the transition of the sectoral aggregates, the adjustment speed varies by individual labor market as well.

of demand for the residual sector by shutting down production there. In addition, as formal labor markets expand the number of job opportunities, we assume that there is only one job opportunity in the residual sector, which will further attract workers to the formal labor markets.

Figure 2: Transition of employment allocation



(a) Sectoral shares within the formal labor market



(b) Residual labor market

Notes: Panel (a) plots changes in the employment share of each sector within the formal labor market in percentage points. Changes are from the share at the initial steady state. Changes in the employment share of each labor market are weighted by the initial employment of the labor market within each sector. Panel (b) plots changes in the employment share of the residual labor market from its level at the initial steady state. In both figures, the trade cost shock arrives at period 0.

Table 7 summarizes the average changes in between- and within-market mobility between the initial steady state and the new steady state, weighted by the initial employment share of a labor market within each sector. The first column reports changes in the employment share of each sector within the formal labor market between the two steady states as shown in Figure 2. The second column of Table 7 reports the percentage changes of the number of job opportunities for each sector between the two steady states. A 30% decline of trade costs in the manufacturing sector from Brazil to its trading partner countries increases the number of job opportunities that workers can compare by 22.12% in the manufacturing sector in the long run. In the non-tradable service sector which is positively affected by the increase in overall real wages, the number of job opportunities also increases significantly. The agriculture sector also experiences an increase in domestic demand due to the same reason as in the service sector but loses export demand due to the positive manufacturing export shock as it is losing the cost advantage over time. Therefore, the number of jobs increase overall also in the agriculture sector at the new steady state, but the amount of the increase is much smaller compared to the other two sectors. Changes in the number of job opportunities that workers can compare lead to changes in probability of moving between jobs within the same labor market, i.e., internal churning. The last column of the table shows that workers in the manufacturing and service sectors move significantly more between jobs without moving to a different labor market, then they were at the initial steady state. As more job opportunities emerge in those labor markets, it becomes more likely for workers to find a better match so decide to move to a different job.

Table 7: Changes in mobility rates and the number of job opportunities between steady states

	Employment share (pp)	Number of jobs (%)	Internal churning (%)
Agriculture	-0.5776	6.5707	5.2206
Manufacturing	0.2619	22.1192	17.7938
Services	0.3157	22.1220	18.2352

Notes: Table reports the average of changes in mobility-related outcomes between the initial steady state and the new steady state after the benchmark trade shock, weighted by the initial employment share of a labor market within each sector. The first column reports changes in the employment share of each sector within the formal labor market. Changes in the sectoral employment share are in percentage points, and the other two columns are in percentage changes.

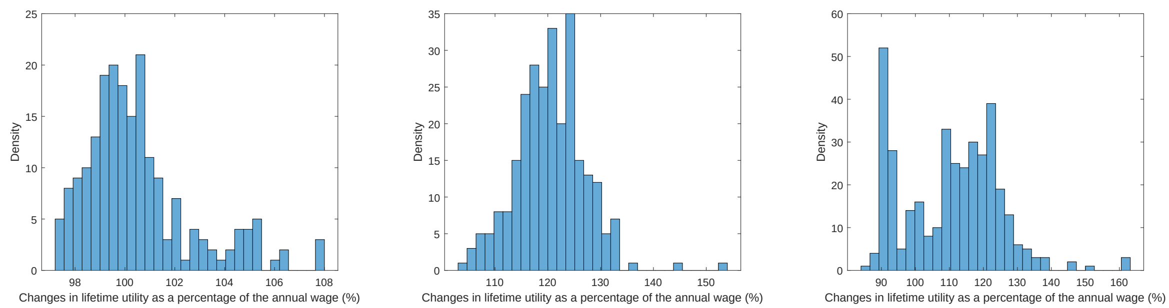
In the first period after the shock, real wages increase in the manufacturing and services sector–by 11.61% and 12.08%, respectively–, but decreases in the agriculture sector by -0.71%. As more workers reallocate from the agriculture sector to the manufacturing and service sectors over time, the general equilibrium downward pressure from a larger labor supply starts playing a role in labor markets of the last two sectors.

These changes in labor market outcomes translate into changes in workers welfare.[29] Our model predicts that the benchmark shock increases the present discounted value of workers' lifetime utility in all labor markets. In aggregate, the increase of lifetime utility is equivalent to a 120.43% one-time and temporary increase of worker's annual wage. While all labor markets enjoy welfare gains, labor markets in the manufacturing and service sectors gain significantly more. Within each sector, welfare gains are heterogeneous across regions as well. Figure 3 shows the within-sector distribution of changes in welfare across regions. On average, workers in the

[29] Welfare in our model is measured by the average value of each labor market V_t^k. Using our flexible dynamic hat algebra, we recover $V_t^k - V_0^k$. We will use welfare, value, and more precisely the present discounted value of workers' lifetime utility interchangeably throughout the rest of the paper.

manufacturing labor markets gain the most (124.39% of the annual wage), and those who in the agricultural labor markets gain the least (100.78% of the annual wage) from the counterfactual shock we introduce.

Figure 3: Within-sector distribution of changes in welfare



(a) Agriculture (b) Manufacturing (c) Service

Notes: Each plot shows the within-sector distribution of changes in average lifetime utility as a percentage of the annual wage across regions.

5.4.2 Mobility Frictions and the Welfare Effect of a Trade Shock

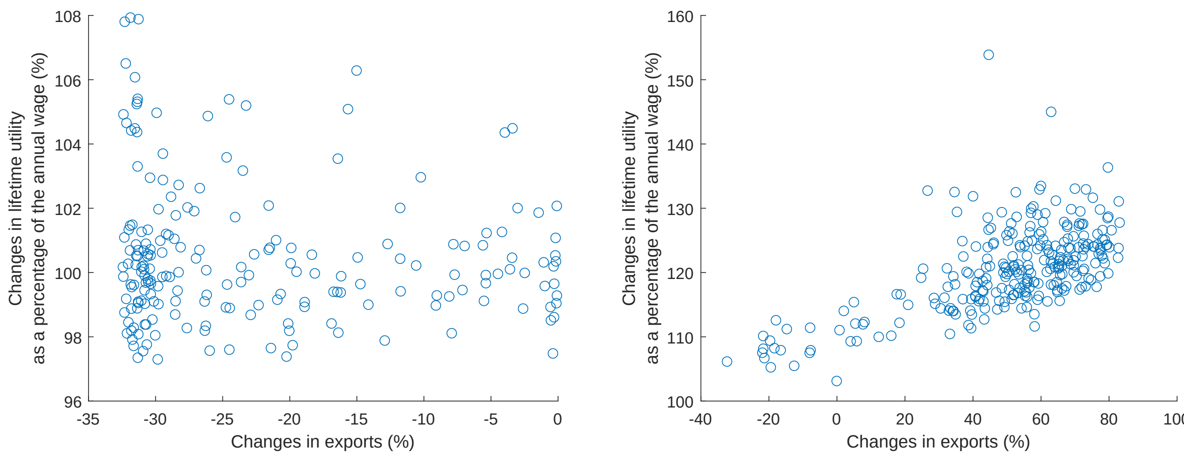
Where are the within-sector heterogeneity of gains from a trade shock shown in Figure 3 originated from? While all labor markets in the same sector face the same trade shock in our counterfactual experiment, some regions gain more than other regions. In Section 2, following the reduced-form literature studying the effect of trade shocks on labor market outcomes, we focused on the regional variation of an *exposure* to changes in export demand. Like many other papers in the literature, this approach uses variation in industry and destination country compositions. We also followed the same approach based on the sufficient statistics of relative value changes in Table 6.

Figure 4 show how much of changes in lifetime utility from our simulated model is explained by changes in exports from each labor market induced by the counterfactual shock. In the manufacturing sector which directly received a positive shock, i.e., a decline in trade cost for exports, the positive relationship between changes in each labor market's own exports and the increase of welfare is clear. In the agriculture sector, on the other hand, there is only a weakly positive relationship between the two.[30] Most agricultural labor markets experience a decrease in exports due to the positive export shock on the manufacturing sector, but all labor markets gain welfare from this shock due to increased domestic demands and option values. There is a

[30] When we pool all labor markets from the two sectors and run a regression similar to what is reported in Table 6, we get the elasticity of 0.244.

weakly negative relationship between the absolute amount of export decrease and the amount of welfare gains. Since changes in exports in the agriculture sector happen only through the general equilibrium interaction, a much weaker explanatory power of changes in its own exports for changes in welfare is not surprising. In our counterfactual scenario, the welfare effect on workers in the agriculture sector is determined by how their labor market is interconnected with other labor markets.

Figure 4: Changes in welfare and changes in exports



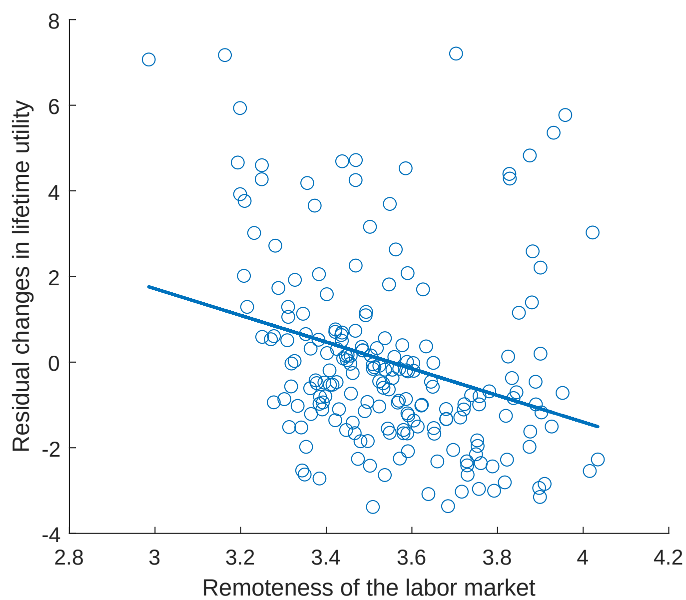
(a) Agriculture (b) Manufacturing

Notes: Each panel plots changes in lifetime utility of each labor market as a percentage of the annual wage against log changes in exports from each labor market.

The interconnectedness between labor markets in our model depends heavily on the mobility frictions that each labor market faces. Given that the own export change per se is not a strong predictor for welfare gains in the agricultural sector from our counterfactual trade shock, we further explore the role of mobility frictions in explaining the heterogeneous welfare effect across labor markets. We first calculate remoteness of each labor market based on our estimate of between-labor-market moving cost $C(k,l)$. For each labor market k, the remoteness measure is defined as the average of $C(k,l)$ across all $l \neq k$ weighted by the initial employment size of the destination market l. This measure describes how remote each labor market is from all other labor markets of the economy.

We then compute residual changes in each labor market's welfare after regressing simulated changes in welfare on simulated changes in exports from each labor market. Figure 5 plots these residual changes in each agricultural labor market's welfare against the remoteness of the labor market. Changes in welfare that are not explained by changes in exports are negatively correlated with the remoteness of the labor market. An agricultural labor market that is more remote from other labor markets gain significantly less from the trade shock. Since the first-order nature of the counterfactual shock is negative for the agriculture sector, workers initially in a more remote agriculture labor market find it more difficult to reallocate to other labor markets which are doing relatively better. Therefore, the degree of mobility frictions faced by workers matters for welfare gains from a trade shock.

Figure 5: Residual changes in welfare and remoteness of the labor market



Notes: This figure plots residual changes in lifetime utility of each labor market as a percentage of the annual wage against the remoteness measure of each labor market.

Mobility frictions also affect option values that workers have after the trade shock. In Section 3, we derived the option value as $-\nu \log(\mu_{0,t}^{k})$. This term implies the utility value of having an option to move to a different job, regardless of whether the new job is in the same labor market or in a different labor market. Option values of each labor market are also heterogeneous across regions within each sector. Figure A6 in the Appendix shows the within-sector distribution of changes in option values due to the benchmark shock.

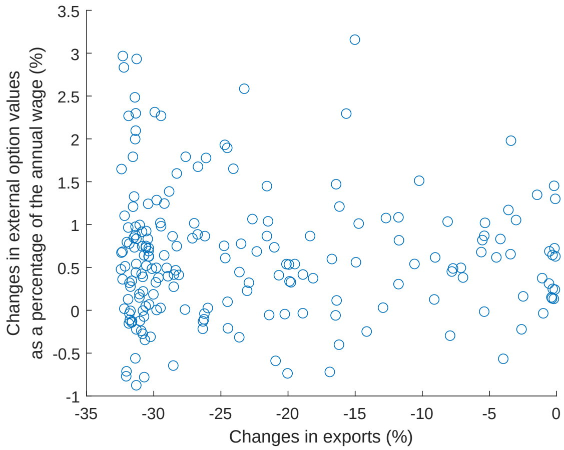
This total option value was then further decomposed into external and internal option values to distinguish utility values of between-market mobility from within-market mobility. In a model without the job opportunity channel, the option value consists of only the external option value. External option values are expected to be negatively affected by a positive shock to the labor market, because potential destinations of moving are doing relatively worse. Internal option values from our model, on the other hand, are expected to be positively affected, because a positive shock to the labor market will increase the number of job opportunities. We showed these relationships in a reduced-form way in Table 6.

Figure 6 plots external and internal option value changes separately against changes in exports of each labor market for agriculture and manufacturing. Similarly to the relationship between changes in welfare and changes in exports, the expected relationship is clearly shown in the manufacturing labor markets which directly receive a positive export shock. In manufacturing, external option value changes are negatively associated with changes in exports from the labor market, while internal option values increase more in the labor market with a larger export increase. In agricultural labor markets, again similarly to the case of values, the magnitude of the decline in its own exports is not a good predictor of changes in external or internal option values.

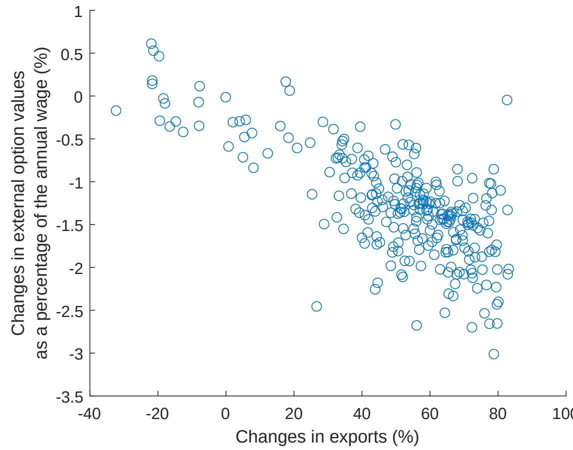
In order to understand changes in external and internal option values in the agriculture sector, we perform a similar exercise we did for values as in Figure 5. We compute residual changes in external and internal option values after regressing them on changes in exports from each agricultural labor market then plot the residual changes against the remoteness measure we calculated. Figure 7 shows that external option values increase more in better-connected agricultural labor markets, while the relationship is exactly the opposite for internal option values. In response to the positive export shock on the manufacturing sector, workers in agricultural labor markets have a higher incentive to move out of the agricultural sector, but having an option of moving between labor markets has a larger utility value for those who face relatively lower between-market mobility frictions. For workers in a more remote labor market, having an option to move between jobs within the same labor market has a larger utility value.

In summary, the welfare effect of a trade shock on each labor market depends significantly on the degree of mobility frictions faced by workers. The job opportunity channel of our model allows us to quantify how utility values of having an option to move between jobs and labor markets are affected by a trade shock, but the magnitude of effect is heterogeneous across labor markets, depending on how well connected each labor market is. This finding motivates policy simulations about lowering mobility frictions across regions and/or sectors, which will be discussed in Section 5.4.4.

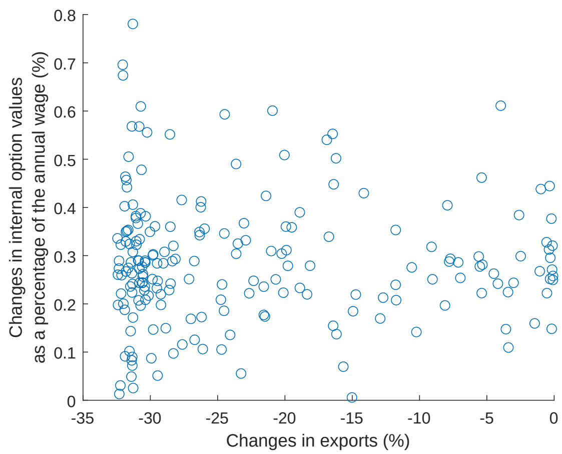
Figure 6: Changes in external/internal option values and changes in exports



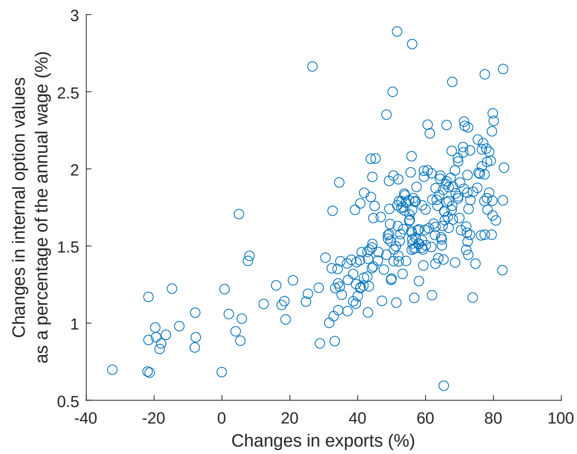
(a) External option values: Agriculture



(b) External option values: Manufacturing



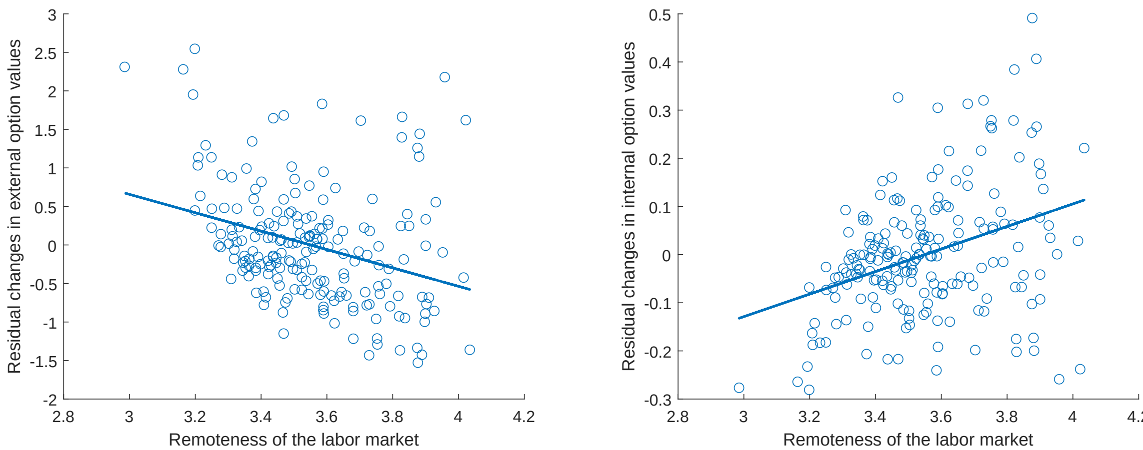
(c) Internal option values: Agriculture



(d) Internal option values: Manufacturing

Notes: Each panel plots changes in external or internal option values of each labor market as a percentage of the annual wage against log changes in exports from each labor market.

Figure 7: Residual changes in external/internal option values and remoteness of the labor market



(a) External option values (b) Internal option values

Notes: Each panel plots residual changes in external or internal option values of each labor market as a percentage of the annual wage against against the remoteness measure of each labor market.

5.4.3 The Role of the Job Opportunity Channel

We can conveniently shut down the job opportunity channel of our model by assuming that the cost of creating tasks is infinite. This limiting assumption will lead to $N_t^k = 1$ in all labor markets for every time period. Table 8 summarizes aggregate and sectoral average changes in present discounted lifetime utility as a percentage of the initial annual labor income. Aggregate welfare increases in both specifications in response to a positive export shock to the manufacturing sector. The amount of increase is 30% larger when we allow for our job opportunity channel to operate. In other words, welfare gains from a positive export shock are magnified through the job opportunity channel.

A larger welfare effect from our benchmark specification compared to the alternative specification without the job opportunity channel can be explained based on the magnitude of labor reallocation and the associated change in option values. The average percentage point change of the sectoral employment share is 0.39% in the benchmark model, while it is 0.33% in the alternative specification without the job opportunity channel. In addition, the baseline model captures the increase of internal mobility induced by the positive export shock. As discussed in previous sections, having more job opportunities with a positive shock increase labor mobility, because it is more likely for workers to be able to find a better match as they are able to compare more options. Therefore, this larger labor mobility induced by the endogenous number of job

Table 8: Average changes in present discounted values as a percentage of the annual labor income (%)

	Baseline	No job opportunity channel
Aggregate	120.43	92.64
Agriculture	100.78	71.75
Manufacturing	124.39	96.83
Services	120.21	92.42

Notes: Table reports for each model specification the average of changes in present discounted lifetime utility as a percentage of the initial annual labor income, weighted by the initial employment share of a labor market.

opportunities translates into a larger welfare effect for workers.

5.4.4 Policy Experiment: Lowering Mobility Frictions

The degree of mobility frictions that workers face is one of the most important determinants of the dynamic welfare consequences of any aggregate shock, including the trade shock that we consider in this paper. Workers face mobility frictions across various dimensions such as regions, sectors, occupations, or firms. In our model, we focus on the effect of the mobility cost across regions and sectors. In the benchmark simulation, we fixed $C_t(k,l)$ to the average of our estimates over the sample period and assumed that it does not change over time. In order to study the role of mobility frictions in transmitting the trade shock to workers' welfare, we run simulations based on two alternative specifications. First, we lower $C_t(k,l)$ for all (k,l) such that $k \neq l$ by 20% together with the same trade shock of a 30% decline of iceberg trade costs for the manufacturing sector from Brazil to each of its trading partner. Second, we lower only the moving cost by the same amount without the trade shock. These two alternative scenarios will allow us to calculate the net effect of having lower mobility costs on the transmission of our trade shock on labor market outcomes and workers' welfare.

Another interesting experiment is to study the role of different dimensions of mobility frictions. For this experiment, we change $C_t(k,l)$ across regions and sectors separately, but by the same amount. In other words, one scenario lowers $C_t(k,l)$ by 20% only across regions, and the other scenario lowers $C_t(k,l)$ by the same amount only across sectors.

Table 9 reports the average counterfactual changes in present discounted lifetime utility across labor markets within each sector, weighted by the initial employment share of a labor market.

Columns (B)-(D) report the net effect of having lower mobility frictions on the welfare effect of the benchmark trade shock, by netting out the effect of lower mobility frictions alone. Regardless of the adjustment margin, the welfare gains from the positive manufacturing export shock are significantly larger when workers face lower mobility frictions. If mobility frictions across both regions and sectors are 20% lower than our estimates, the aggregate welfare gains from the trade shock are on average 16.5% larger. If a policymaker can lower mobility frictions across either regions or sectors, the same amount of the decrease in mobility frictions increase the welfare effect more when the targeted dimension is across regions.

Table 9: Average changes in present discounted values as a percentage of the annual labor income (%)

		Lower mobility frictions		
	Baseline (A)	Both (B)	Regions (C)	Sectors (D)
Aggregate	120.43	140.39	136.81	123.19
Agriculture	100.78	121.83	117.45	104.18
Manufacturing	124.39	144.07	140.61	127.05
Services	120.21	140.20	136.63	122.97

Notes: Table reports for each model specification the average of changes in present discounted lifetime utility as a percentage of the initial annual labor income, weighted by the initial employment share of a labor market.

6 Model Fit and Robustness

In this section we show that the model's predictions are consistent with the empirical patterns observed in the data, and the model can match the flows in the data reasonably well despite having a simple moving cost structure with only three parameter.

6.1 Matching Untargeted Patterns in the Data

We provide additional evidence for the importance of the channels discussed herein and the model's fit in replicating these channels. The model predicts that labor mobility should be highly correlated not only with trade shocks, but with all aggregate shocks. We will show how labor mobility changes over the business cycle consistent with the predictions of the model, which is different from the other structural labor mobility models used in international trade literature.

We first define the stylized "positive labor-market-neutral productivity shock" as a productivity shock that uniformly increases the log number of jobs, $\log N_t^k$, by $z > 0$ and thus values V_t^k by νz in all labor markets, where $\frac{\partial}{\partial x} \log N_t^k = z$ and $\frac{\partial}{\partial x} V_t^k = \nu z$, $\forall k \in \{1, 2, .., K\}$. Using this balanced shock simplifies the algebra, as we do not need to worry about changes in worker flows across labor markets. As we assumed in Section 3, this shock does not change the underlying moving cost structure.

Proposition 1. *A positive labor-market-neutral productivity shock increases the average number of workers changing jobs within a labor market relative to the number of workers staying with the same job.*

Proof. Follows directly from the equation about the number of jobs:

$$\nu \log \mu_{1,t}^k - \nu \log \mu_{0,t}^k = \log N_t^k - \delta_t,$$

thus

$$\frac{\partial}{\partial x} \frac{1}{K} \sum_k \left(\nu \log \mu_{1,t}^k - \nu \log \mu_{0,t}^k \right) > 0.$$

□

Proposition 1 implies that economy-wide positive shocks are expected to increase the average log churning within labor markets.

We could express our main model as a standard discrete choice model (with fixed number of choices), with utility shifters and moving costs that are functions of the number of jobs. We call this alternative isomorphic model as the auxiliary model, and discuss it in the appendix. We denote the moving cost of the auxiliary model as $\bar{C}_t(k, l)$. If we ignore the fact that number of choices can change over time, empirically it would look like the moving costs are changing. In other words, the moving costs estimated using ACM would fluctuate over time as the number of jobs are changing in labor markets.

Proposition 2. *A positive labor-market-neutral productivity shock reduces the average implied moving cost difference between the auxiliary and main models, $\bar{C}_t(k, l) - C_t(k, l)$.*

$$\frac{\partial}{\partial x} \frac{1}{K^2 - K} \sum_k \sum_{l \neq k} \left[\bar{C}_t(k, l) - C_t(k, l) \right] < 0. \tag{42}$$

Proof.

$$\begin{aligned}\frac{\partial}{\partial x}\left[\bar{C}_t(k,l) - C_t(k,l)\right] &= \frac{\partial}{\partial x}\log\left(\frac{N_t^k + \exp(\delta_t)}{N_t^l}\right), \\ &= \frac{N_t^l}{N_t^k + \exp(\delta_t)}\frac{\partial}{\partial x}\left(\frac{N_t^k}{N_t^l} + \frac{\exp(\delta_t)}{N_t^l}\right), \\ &= \frac{N_t^l}{N_t^k + \exp(\delta_t)}\left(0 - \frac{\exp(\delta_t)}{N_t^l}z\right) < 0.\end{aligned}$$

□

Proposition 3. *A positive labor-market-neutral productivity shock increases across-labor-market churning (i.e. inter-labor-market mobility) conditional on formality.*

Proof.

$$\frac{\partial}{\partial x}\log(\frac{\mu_0+\mu_1}{1-\mu_I}) = \frac{\partial}{\partial x}\left(-\log\left(1+\sum_{l\neq k}\exp\left(V_t^l - V_t^k - \bar{C}_t(k,l)\right)\right)\right) < 0$$

because

$$\frac{\partial}{\partial x}\left(V_t^l - V_t^k\right) = 0, \tag{43}$$

by definition due the the nature of the shock and

$$\frac{\partial}{\partial x}\bar{C}_t(k,l) < 0, \tag{44}$$

as it directly follows from the previous proposition since $C_t(k,l)$ is fixed. If

$$\frac{\partial}{\partial x}\log(\frac{\mu_0+\mu_1}{1-\mu_I}) < 0, \tag{45}$$

then

$$\frac{\partial}{\partial x}\log(1-\frac{\mu_0+\mu_1}{1-\mu_I}) > 0. \tag{46}$$

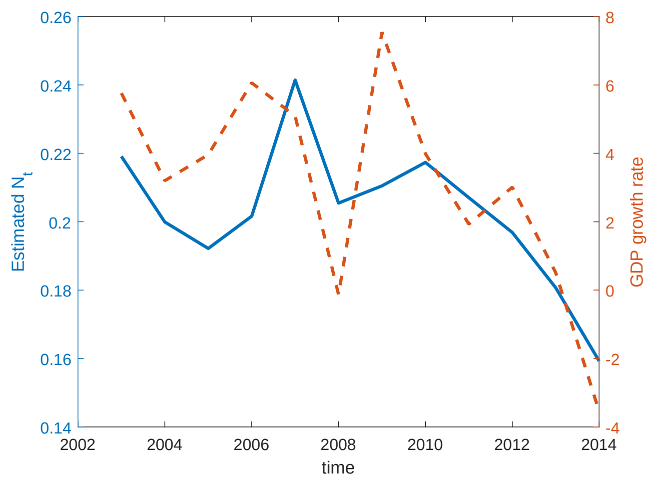
Thus

$$\frac{\partial}{\partial x}\frac{1}{K}\sum_k\log(\frac{\mu_2}{1-\mu_I}) > 0.$$

□

Based on the propositions above, we expect to see positive correlation between aggregate productivity and churning within and across labor markets. Figure 8 shows the GDP growth rate in Brazil between 2003 and 2014, plotted together with the average number of job opportunities N_t^k implied by the model. The correlation coefficient between the two series is equal to 0.68, showing a strong positive correlation as discussed in Proposition 1.

Figure 8: Change in GDP growth and estimated average N_t



Following Proposition 2, Figure 9 shows the inverse of moving costs differences and GDP growh rate, the correlation between the series is equal to 0.62. The figures give us some confidence about the model's ability to capture channels related to the labor mobility. In addition to the evidence from Brazil, we show that $\bar{C}$ is indeed negatively correlated with aggregate shocks, as implied by the model, using data from the U.S. in the appendix Figure A5. Finally, Figure 10 shows the GDP growth rate in Brazil between 2003 and 2014, plotted together with the average number of switchers across labor markets. The correlation coefficient between the two series is also positive as shown in Proposition 4 and equal to 0.8.

Figure 9: Change in GDP growth and $C - \bar{C}$

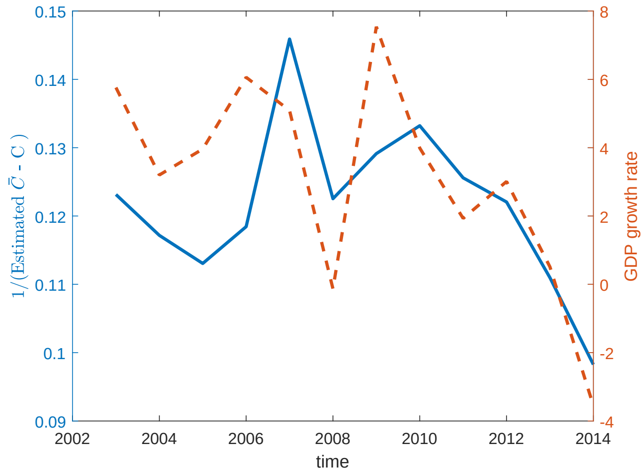
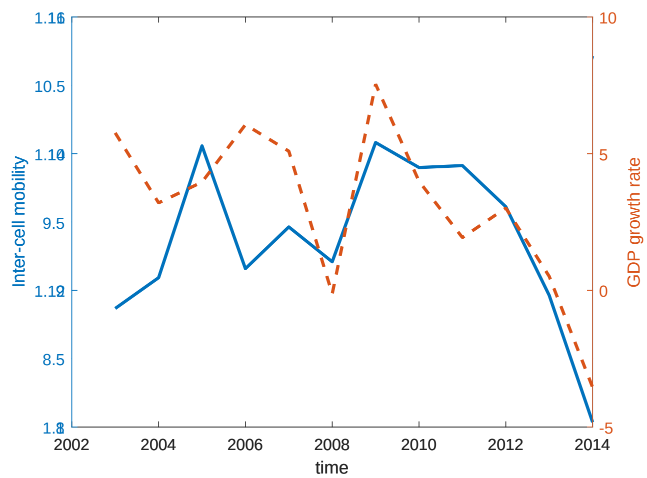


Figure 10: Change in GDP growth and inter-cell mobility

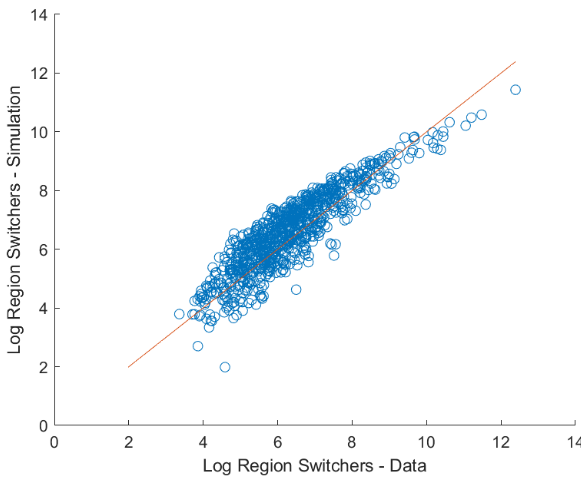


6.2 Data and Simulated Flows

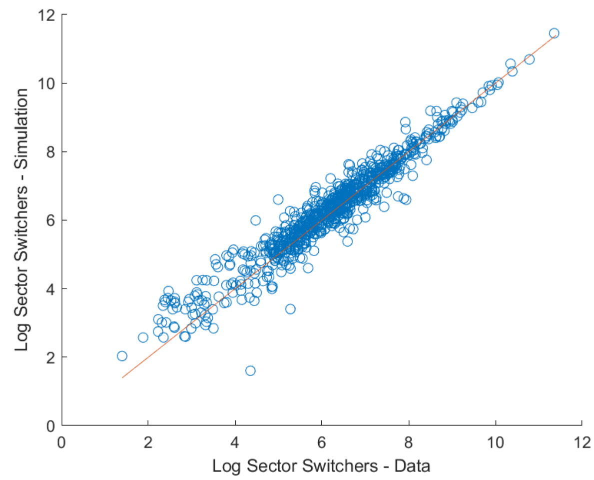
In the subsection 4.1, we use PPML regression to estimate flows, $\widetilde{m}_t^{k,l}$, and the three parameter moving cost function. The proof of equivalence of PPML and Maximum Likelihood for the problem herein is provided in appendix A.3. We use the estimated moving cost function to characterize the initial steady state flows for the counterfactual simulations as elaborated in appendix A.6.1.3. The steady state simulation backs out values to match the labor allocations in the data, and it does not target or try to match flows. We would like to show that despite the simplicity of the moving cost function, the model does a satisfactory job in matching the average flows in the data.

Figure 11 panel (a) shows the average number or workers switching regions in the data versus in the simulations before the counterfactual shocks. The 45 degree line is indicated on the graph. Similarly, panels (b), (c) and (d) show comparison of data and simulations for sector switchers, both sector and region switcher, and workers leaving the residual sector. The simulated flows and actual flows from data are highly correlated for all types of mobility.

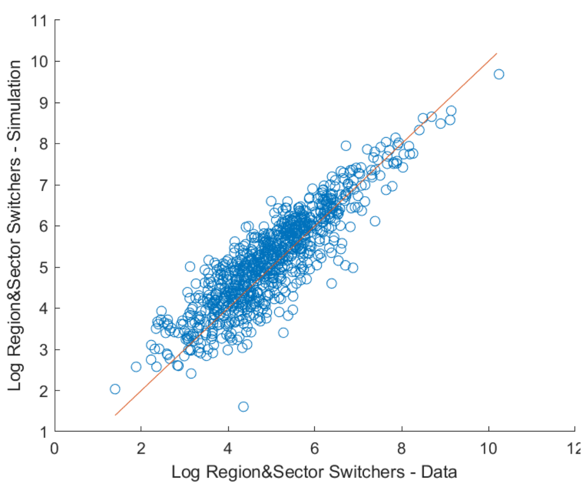
Figure 11: Simulated flows and average flows in the data



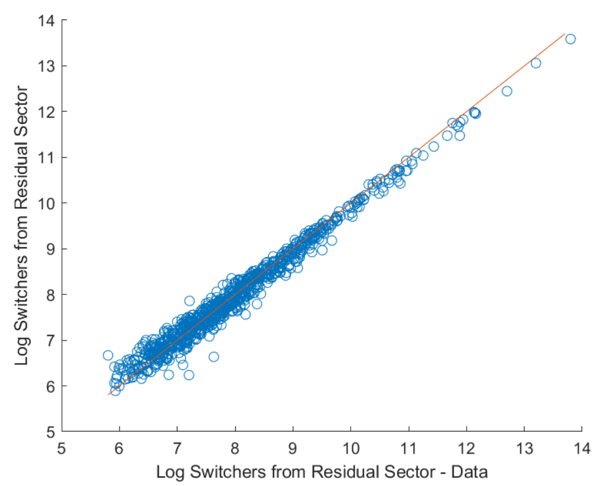
(a) Number of workers changing regions



(b) Number of workers changing sectors



(c) Number of workers changing regions and sectors



(d) Number of workers leaving residual sector

7 Conclusion

We have introduced a new framework to quantify the impacts of trade shocks on labor mobility and worker welfare that combines the advantages of the structural and reduced-form methodologies. Our framework features various drivers of labor mobility across sectors and regions, and identifies how trade shocks impact those determinants endogenously. Models of trade-induced labor mobility have explored wage differentials and idiosyncratic utility as drivers of mobility. We have introduced an additional important motive of mobility: the number of job opportunities provided by different sectors and regions. If a worker can choose her job out of more opportunities, it is more likely that the best one delivers higher welfare. Even when she is hit by a negative labor demand shock in the future, it is more likely that she will be able to find another job without having to move to a different region or sector, which would imply a higher switching cost. Therefore, a labor market experiencing a positive trade shock will attract more workers not just because it provides a higher wage, but also because of the larger number of job opportunities that are created there. This mechanism of dynamic labor adjustment in response to trade shock impacts worker's lifetime welfare.

We have first provided empirical evidence on the causal effects of export shocks on labor markets. The analysis draws on rich employer-employee panel data combined with customs records on export transactions from Brazil. Using changes in external demand directed to the labor market as a source of variation in exports, we documented a positive causal effect of export shocks on employment, residual wages, and job turnover rates in the corresponding labor market.

Motivated by this reduced-form evidence, we developed and structurally estimated a dynamic general equilibrium model of labor mobility. Different labor markets offer different wages and different numbers of job opportunities to workers. A worker chooses the job which gives her the highest utility, where the number of jobs in each labor market is endogenously determined. In a labor market with relatively more job opportunities, workers can choose optimally out of more potential jobs, to each of which workers attach idiosyncratic preference. A job switch requiring a change of labor market implies incurring a higher switching cost compared to a job switch within a labor market. Therefore, a growing labor market with more job opportunities reduces the risk of having to pay a switching cost in the future. The prospect of job switch generates an option value in worker's welfare. Our model further decomposes this option value into the option value associated alternative job opportunities within the current labor market and the option value from having alternative jobs in all other labor markets.

Our model delivers a structural equation of changes in relative worker welfare which is a function of only the estimated probability of moving between labor markets and the labor supply elasticity. The welfare result does not depend on the moving cost structure, observed changes in future wages, or moving probabilities across jobs within a labor market. The effects of a trade shock are fully embedded in the gross flows between labor markets. This is a powerful result which greatly simplifies the analysis of the welfare impacts of trade shocks.

We have structurally estimated the model using the worker-firm data from Brazil. In the first stage of the estimation, we pin down the common value attached to each labor market and the moving cost between labor markets for each worker group using a gravity-like equation. The implied probability of moving between labor markets is then calculated with the estimated value of each labor market and the estimated moving cost. In the second stage of the estimation, we pin down the labor supply elasticity of our model. We first derive an estimable equation describing the relationship between a change in the transformed value of the labor market and a change in wages, with the labor supply elasticity governing the responsiveness of the former with respect to the latter. We instrument the change in residual wages with our Bartik-type instrument, exploiting variation in external demand directed to the region.

Armed with the estimate of the labor supply elasticity, we estimated the causal effect of trade shocks in Brazil on workers' welfare using the same instrument. Our structural IV estimates reveal that a 10% change in exports during the sample period increases the lifetime welfare of a median worker in the formal labor market by 3.39% of the annual wage.

Finally, we use our general equilibrium model in full scope to quantify the effect of trade shocks on workers welfare, labor allocation, real wages, and the number of job opportunities. Using the benchmark shock of a 30% decline in manufacturing trade costs from Brazil to its trading partners, we show that a positive export shock to the manufacturing sector reallocates labor toward the manufacturing and the non-tradable service sectors. We also show that the aggregate lifetime welfare increases in all labor markets, but workers in the manufacturing sector on average experience 23.4% larger welfare increases compared to workers in the agriculture sector.

By comparing our baseline specification to the alternative one without the job opportunity channel, we show that the endogenous number of job opportunities channel magnifies welfare gains from a positive export shock by 30%. Our model can also shed light on predicting the potential benefit of a policy mitigating labor mobility frictions. We show that when workers face

20% lower mobility frictions both across regions and sectors, the welfare increase is 16.5% larger. Reduced mobility frictions across regions have a larger effect on the transmission of a trade shock on workers welfare compared to the reduced mobility costs across sectors.

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A.1 Data sources and description

Here we provide further details about the data sets used in the empirical analysis.

Employer-employee panel data: *Relacao Anual de Informacoes Sociais* (RAIS) is a labor census gathering longitudinal data on the universe of formal workers and firms in Brazil. We use data for the period 2003-2015. RAIS is a high-quality administrative census of formal employees and employers collected every year by the Brazilian Ministry of Labor. These records are used by the government to administer several government benefits programs. Workers are required to be in RAIS in order to receive payments of these programs and firms face fines for failure to report, until they do report. These requirements ensure that RAIS is an accurate and complete census of the formal sector in Brazil (Dix-Carneiro and Kovak, 2017).

RAIS covers virtually all formal workers and provides yearly information on demographics (age, gender, and schooling), job characteristics (detailed 6-digit occupation, wage, hours worked), as well as hiring and termination dates. For each job, the RAIS annual record reports average yearly earnings, as well as the monthly wage in December. We use the information on the December wage, so as to ensure that all labor market outcomes are measured at the same time and avoid potential mismeasurement for workers that did not work full year. RAIS further includes information on a number of establishment-level characteristics, notably number of employees, geographical location (municipality) and industry code, defined according to the 5-digit level of the Brazilian National Classification of Economic Activities (CNAE). Unique identifiers for workers and firms are consistently defined across years and therefore make it possible to follow them over time. The worker unique identifier is the number associated with her registration in *Programa de Inserção Social* (PIS). The establishment unique identifier (*Cadastro Nacional de Pessoa Jurídica*) (CNPJ) is an identification number issued to Brazilian companies by the Secreteriat of the Federal Revenue of Brazil. It consists of a 12 digit number, of which the first 8 digits uniquely identify the firm and the remaining four digits identify the establishment. Therefore, it is possible to identify and track multi-establishment firms. While the RAIS data cover also segments of the public sector, we restrict the analysis to the private sector. The industry classification contain 572 industries at the 5-digit level, of which 42 are in agriculture and natural resources, 286 are in manufacturing and the remainder are in services.

The information on the level of education of the worker is reported in 9 categories: illiterate (corresponding to 0 years of education); primary school dropout (indicating from 1 to 3 years of education), primary school graduate (4 years education), middle school dropout (5 to 7 years of education), middle school graduate (8 years of education), high school dropout (9 to 10 years of education), high school graduate (11 years of education), college dropout (12 to 14 years of education), college graduate (15 years of education), Masters (18 years of education) and PhD (22.5 years of education). To compute average years of education of school dropouts, we consider the mid-point of the interval.

We use the detailed classification of occupations as a measure of the number of different jobs available in a labor market. The Brazilian Classification of Occupations changed in 2002 (CBO-2) and has been reported consistently since 2003. Although the RAIS data are available for earlier years, we restrict the analysis to the post-2003 period in order to ensure that this important variable is defined in a consistent way throughout the period of analysis. CBO-2 aims to portray the reality of professions of the Brazilian labor market. It was established with legal basis in Administrative Rule no. 397 of October 10, 2002. There are 2637 occupation codes at the 6-digit level during this period. The description of each 6-digit code is available at http://portalfat.mte.gov.br/wp-content/uploads/2016/04/CBO2002_Liv3.pdf.

We use information on the establishment's location (municipality) and industry, and worker-level data on gender, age, education and December wage. We focus on workers aged 16 to 64 years old. As in Dix-Carneiro and Kovak (2017), we use the "microregion" concept of the Brazilian Statistical Agency (IBGE) to define regional boundaries. This definition groups together economically integrated contiguous municipalities with similar geographic and productive attributes. The documentation supporting this definition is available at https://biblioteca.ibge.gov.br/index.php/biblioteca-catalogo?id=22269&view=detalhes. We consider a set of 558 consistently defined microregions, grouping the 5571 municipalities in the data. To ensure a consistent definition of microregions over time, when necessary we merge microregions whose boundaries changed over the period of analysis.

Customs records: We also use customs data on export transactions by microregion, industry and destination in each year. These customs records are administrative data collected by Secretaria do Comercio Externo (SECEX) of the Ministry of Development, Industry and Foreign Trade. These data are available since 1997 and contain information on FOB export values and quantities, and are originally defined at the level of the municipality, detailed product category and destination market. The customs records were originally collected by SECEX at the firm-product-destination level. To aggregate up to the municipal level, SECEX attributed each firm-level export transaction to the municipality where the headquarters of the exporting firm are located. The product classification is *Nomenclatura Comum do MERCOSUL* (NCM), at the 8-digit level. For consistency with RAIS, we restricted the analysis to the post-2003 period, and aggregated up to microregion-sector level. To aggregate exports from the NCM 8-digit level to the 5-digit level of the CNAE, we used a concordance made available to us by SECEX.

Industry-level imports of Brazil's destinations: To construct an instrument for exports, we further use yearly data on the industry-level imports of each of Brazil's export destinations. To capture changes in sectoral import demand that are plausibly exogenous to microregions in Brazil, we consider the imports of these countries sourced from all countries other than Brazil (i.e. we exclude imports sourced from Brazil from total imports of each country in a given industry-year). There is a total of 189 destinations reported in the customs data, to which we link information on sectoral import demand from the UN COMTRADE data set.

List of countries for the full quantification: Brazil, Australia, Belgium, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Korea, Mexico, Netherlands, Russia, Spain, Sweden, Switzerland, Turkey, United Kingdom, USA, and the constructed rest of the world.

A.2 Variable definitions and summary statistics

This section describes in detail the variables used in the econometric analysis:

Δ Employment: log change in the number of employees in microregion-sector k between years $t-1$ and t;

Δ Residual wage: log change in average residual wage in the microregion-sector k between years $t-1$ and t;

Δ # of leavers: log change in the number of workers leaving microregion-sector k (i.e., incumbent workers changing either microregion or sector) between years $t-1$ and t;

Δ # of entrants: log change in the number of workers entering microregion-sector pair k between year $t-1$ and t from other microregion and/or sector (thus excluding new entrants to the formal labor force);

Δ # of job switchers: log change in the number of workers that switch jobs (i.e., switch either unique occupation or establishment), while staying in the same microregion-sector between years $t-1$ and t. Unique occupations are defined at the 6-digit level of CBO-2.;

- Δ Exports: log change in the value of the exports originated in the microregion-sector between years $t-1$ and t;
- Δ $\bar{Z}$: log change in the value of import demand directed to the microregion-sector between years $t-1$ and t, as defined in equation (2) in text.

A.3 Estimation of switching probabilities, values and moving costs

Here we show that PPML orthogonality conditions are equivalent to the MLE first order conditions for our model for the estimation of values and moving costs (subject to a normalization ν). We omit the type superscript s. We denote the number of agents moving from j to k with y^{jk}, the expected value (i.e. destination fixed) effect with $\widetilde{V}^k$, the origin fixed effect with $\widetilde{\Gamma}^j$ and the moving cost with $\widetilde{C}(j,k)$.
Consider the following moving cost structure:

$$\widetilde{C}(j,k) = \widetilde{c_1} D^{jk} + \widetilde{c_2} \mathbf{1}_{S_j \neq S_k}, \tag{A1}$$

where $\widetilde{c_j}$ is the distance coefficient (divided by ν) and D^{jk} is the log of distance between l and k, $\widetilde{c_2}$ is the sector switching cost (divided by ν), $\mathbf{1}_{S_l \neq S_k}$ is an indicator function that is equal to one if l and k are associated with different sectors. Note that we impose $D^{ll} = 0$ for every j. We omit the time sub-scripts and the last component of moving cost to simplify exposition.

A.3.1 Maximum Likelihood Estimation (First Order Conditions)

The likelihood function is

$$\mathcal{L} = \prod_j \prod_k \left(m^{jk}\right)^{y^{jk}}, \tag{A2}$$

or alternatively using logarithm

$$\log \mathcal{L} = \sum_j \sum_k y^{jk} \log(m^{jk}). \tag{A3}$$

Note that the moving probability can be expressed as

$$\begin{aligned} m^{jk} &= \frac{\exp\left(\widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k}\right)}{\sum_l \exp\left(\widetilde{V}^l - \widetilde{c_1} D^{jl} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_l}\right)}, \\ &= \exp\left(\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k} - \log \widetilde{L}_t^j\right), \end{aligned}$$

where $\widetilde{\Gamma}^j = -\log\left[\sum_k \exp\left(\widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k}\right)\right] + \log(\widetilde{L}_t^j)$.

The log-likelihood function, then, can be written as

$$\log \mathcal{L} = \sum_j \sum_k y_t^{jk} \left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k} - \log(\widetilde{L}^j)\right],$$

subject to

$$\widetilde{\Gamma}^j = -\log\left[\sum_k \exp\left(\widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k}\right)\right] + \log(\widetilde{L}^j). \tag{A4}$$

The goal is to find $\widetilde{V}^j$, $\widetilde{c_2}$ and $\widetilde{c_1}$ coefficients that maximize the log likelihood function.

Note that

$$\partial \widetilde{\Gamma}^j / \partial \widetilde{V}^k = -m^{jk},$$

$$\partial\widetilde{\Gamma}^j/\partial\widetilde{c}_1 = -\sum_{j\neq k} D^{jk} m^{jk},$$

$$\partial\widetilde{\Gamma}^j/\partial\widetilde{c}_2 = -\sum_{j\neq k} \mathbf{1}_{S_j\neq S_k} m^{jk},$$

and

$$m^{jk} = \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{I_j\neq I_k} - \log(\widetilde{L}^j)\right].$$

Values:

We take the derivative of the log likelihood function with respect to $\widetilde{V}^k$ to find the first order condition

$$\frac{d\log\mathcal{L}}{d\widetilde{V}^k} = \frac{\partial\log\mathcal{L}}{\partial\widetilde{V}^k} + \sum_j \frac{\partial\log\mathcal{L}}{\partial\widetilde{\Gamma}^j}\frac{\partial\widetilde{\Gamma}^j}{\partial\widetilde{V}^k} = 0 \tag{A5}$$

We rearrange the terms:

$$\begin{aligned}
0 &= \frac{\partial\log\mathcal{L}}{\partial\widetilde{V}^k} - \sum_j \frac{\partial\log\mathcal{L}}{\partial\widetilde{\Gamma}^j} m^{jk} \\
&= \sum_j y^{jk} - \sum_j \left(\sum_k y^{jk}\right) m^{jk} \\
&= \sum_j y^{jk} - \sum_j \widetilde{L}^j m^{jk} \\
&= \sum_j y^{jk} - \sum_j \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j\neq S_k}\right] \\
&= \sum_j \left(y^{jk} - \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j\neq S_k}\right]\right)
\end{aligned}$$

thus the first order condition associated with values is

$$\sum_j \left(y^{jk} - \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2}\mathbf{1}_{I_j \neq I_k}\right]\right) = 0. \tag{A6}$$

Distance coefficient:

Then we take the derivative of the log likelihood function with respect to distance coefficient $\widetilde{c_1}$:

$$\frac{d \log \mathcal{L}}{d\widetilde{c_1}} = \frac{\partial \log \mathcal{L}}{\partial \widetilde{c_1}} + \sum_i \frac{\partial \log \mathcal{L}}{\partial \widetilde{\Gamma}^i} \frac{\partial \widetilde{\Gamma}^i}{\partial \widetilde{c_1}} = 0 \tag{A7}$$

$$\begin{aligned}
0 &= -\sum_j \sum_{k \neq j} D^{jk} y^{jk} + \sum_j \sum_l y^{jl} \sum_{k \neq j} D^{jk} m^{jk} \\
&= \sum_j \sum_{k \neq j} D^{jk} y^{jk} \sum_j \widetilde{L}^j \sum_{k \neq j} D^{jk} m^{jk} \\
&= \sum_j \sum_{k \neq j} D^{jk} y^{jk} - \sum_j \sum_{k \neq j} D^{jk} \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2}\mathbf{1}_{S_j \neq S_k}\right]
\end{aligned}$$

thus the first order condition associated with $\widetilde{c_1}$ is

$$\sum_j \sum_{k \neq j} D^{jk} \left(y^{jk} - \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2}\mathbf{1}_{S_j \neq S_k}\right]\right) = 0. \tag{A8}$$

Sector switching coefficient:

Then we take the derivative of the log likelihood function with respect to distance coefficient $\widetilde{c_1}$:

$$\frac{d \log \mathcal{L}}{d\widetilde{c_2}} = \frac{\partial \log \mathcal{L}}{\partial \widetilde{c_2}} + \sum_t \sum_i \frac{\partial \log \mathcal{L}}{\partial \widetilde{\Gamma}^i_t} \frac{\partial \widetilde{\Gamma}^i_t}{\partial \widetilde{c_2}} = 0 \tag{A9}$$

$$
\begin{aligned}
0 &= -\sum_j \sum_{k \neq j} \mathbf{1}_{S_j \neq S_k} y^{jk} + \sum_j \sum_l y^{jl} \sum_{k \neq j} \mathbf{1}_{S_j \neq S_k} m_t^{jk} \\
&= \sum_j \sum_{k \neq j} \mathbf{1}_{S_j \neq S_k} y^{jk} \sum_j \widetilde{L}^j \sum_{k \neq j} \mathbf{1}_{S_j \neq S_k} m^{jk} \\
&= \sum_j \sum_{k \neq j} \mathbf{1}_{S_j \neq S_k} y^{jk} - \sum_j \sum_{k \neq j} \mathbf{1}_{I_j \neq I_k} \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{I_j \neq I_k}\right]
\end{aligned}
$$

thus the first order condition associated with c is

$$
\sum_j \sum_{k \neq j} \mathbf{1}_{S_j \neq S_k} \left(y^{jk} - \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j \neq S_k}\right]\right) = 0. \tag{A10}
$$

A.3.2 PPML (Orthogonality Conditions)

Now we turn to the PPML regression equation. We will show that the orthogonality conditions implied by the PPML regression equation are identical to the ML first order conditions. PPML can be preferable to ML for two reasons: (i) Since is straightforward to take analytical derivatives of the orthogonality conditions, PPML is very low cost computationally. (ii) There are many software packages to estimate PPML. We will prove that PPML and ML estimators are identical for our model.

The PPML equation (without type superscript) is

$$
y_t^{jk} = \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j \neq S_k}\right] + \epsilon_t^{jk}, \tag{A11}
$$

The regression equation can be written in matrix form as

$$
y = \exp\left[XB\right] + \epsilon, \tag{A12}
$$

where y is a vector with elements y_t^{jk}, X is a matrix of destination and origin dummies and switching cost variables, B is the vector of coefficients.

The orthogonality condition of PPML regression is

$$0 \quad = \quad X' \left(y - \exp\left[XB \right] \right). \tag{A13}$$

This matrix operation implies a vector of equations.

We can group the rows (i.e. equations) in of the orthogonality condition matrix above into four categories:

I. Equations associated with the origin coefficients

$$\sum_k \left[y^{jk} - \exp\left(\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k} \right) \right] = 0, \, \forall j, \tag{A14}$$

II. Equations associated with the destination coefficients

$$\sum_j \left(y^{jk} - \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k} \right] \right) = 0, \, \forall k, \tag{A15}$$

III. Equation associated with the distance coefficient

$$\sum_j \sum_{k \neq j} D^{jk} \left(y^{jk} - \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k} \right] \right) = 0, \tag{A16}$$

IV. Equation associated with the sector switching cost coefficient

$$\sum_j \sum_{k \neq j} \mathbf{1}_{S_j \neq S_k} \left(y^{jk} - \exp\left[\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c_1} D^{jk} - \widetilde{c_2} \mathbf{1}_{S_j \neq S_k} \right] \right) = 0. \tag{A17}$$

Note that equation (A6) is same as (A15); equation (A8) is same as (A16); and equation (A10) is same as (A17).

To conclude the proof, we have to show the restriction (A4) of the ML estimation is same as the equation (A14) of the PPML regression.

Consider equation (A14) from above

$$y^{jj} - \exp(\widetilde{\Gamma}^j + \widetilde{V}^j) + \sum_{k \neq j} \left[y^{jk} - \exp\left(\widetilde{\Gamma}^j + \widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j \neq S_k} \right) \right] = 0$$

we can arrange the terms as

$$\begin{aligned}
0 &= \sum_k \left[y^{jk} - \exp\left(\widetilde{\Gamma}^j\right) \exp\left(\widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j \neq S_k} \right) \right] \\
\sum_k y_t^{jk} &= \exp\left(\widetilde{\Gamma}_t^j\right) \sum_k \exp\left(\widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j \neq S_k} \right) \\
\widetilde{L}^j &= \exp\left(\widetilde{\Gamma}^j\right) \sum_k \exp\left(\widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j \neq S_k} \right) \\
\exp\left(\widetilde{\Gamma}^j\right) &= \frac{L^j}{\sum_k \exp\left(\widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j \neq S_k} \right)}
\end{aligned}$$

thus we get

$$\widetilde{\Gamma}^j = \log(\widetilde{L}^j) - \log\left(\sum_k \exp(\widetilde{V}^k - \widetilde{c}_1 D^{jk} - \widetilde{c}_2 \mathbf{1}_{S_j \neq S_k}) \right), \tag{A18}$$

which is equal to the restriction (A4) in the ML estimator. Therefore, solving the first order conditions of the ML estimator is equivalent to solving the orthogonality conditions in PPML.

A.4 Auxiliary model

Consider the following model with the constant number of choices.

The economy has L agents, and each agent is attached to a region and/or sector labor market k, where $k \in \{1, 2, .., K\}$. The number of agents in labor market k is denoted as L_t^k. An agent, who is indexed with h and attached to labor market k, will receive instantaneous utility $\bar{u}_t^h$ at time t defined as

$$\bar{u}_t^h = w_t^k + \eta_t^k + \varepsilon_t^{h,k}, \tag{A19}$$

where $\varepsilon_t^{h,k}$ is distributed Gumbel with mean 0 and scale ν. The labor-market-specific utility shifter η is defined as

$$\eta_t^k = \log\Big(N_t^k + \exp(\delta_t)\Big) - \delta_t, \tag{A20}$$

where $N_t^{s,k}$ and δ_t^s are as defined in the main model section.

The workers pay moving cost

$$\bar{C}_t(k,l) = C_t(k,l) + \log\left(\frac{N_t^k + \exp(\delta_t)}{N_t^l}\right), \tag{A21}$$

where $\bar{C}_t(k,l)$ is the implied moving cost with $\bar{C}_t(k,k) = 0$, and $C_t(k,l)$ is the structural moving cost parameter herein. The auxiliary model above is isomorphic to the model described in Section 3, and as $N_t^k \to 0$ the model becomes equivalent to ACM.

A.5 Sampling rate and asymptotics

In this section, we show how it is possible to use a real number for the number of sampled jobs instead of an integer for simulation purposes. When the number of sampled jobs goes to infinity, welfare and other important variables in the model can still be finite.

Consider $N_t^k = \rho(O_t^k)$, a step function $\rho : \mathbb{R}^+ \to \mathbb{N}^+$ where

$$\vartheta O_t^k < \rho(O_t^k) \leq \vartheta O_t^k + 1. \tag{A22}$$

Imagine that ε is distributed Gumble with mean $\kappa = 0$ and scale parameter ν. The moving cost $\delta = \tilde{\delta} + \nu\log(\vartheta)$. Welfare

$$W \quad = \quad \nu\log\left[\sum N_t^k \exp(\frac{V_t^k - \delta}{\nu})\right], \tag{A23}$$

then

$$W = \nu \log \left[\sum N_t^k \exp(\frac{V_t^k - \tilde{\delta} - \nu \log(\vartheta)}{\nu}) \right], \tag{A24}$$

$$= \nu \log \left[\sum \frac{N_t^k}{\vartheta} \exp(\frac{V_t^k - \tilde{\delta}}{\nu}) \right], \tag{A25}$$

$$= \nu \log \left[\sum \tilde{N}_t^k \exp(\frac{V_t^k - \tilde{\delta}}{\nu}) \right], \tag{A26}$$

$$\tag{A27}$$

where

$$\tilde{N}_t^k = \rho(O_t^k)\frac{1}{\vartheta}. \tag{A28}$$

Note that

$$\lim_{\vartheta \to \infty} \tilde{N}_t^k = O_t^k. \tag{A29}$$

Proof.

$$\lim_{\vartheta \to \infty} \frac{\vartheta O_t^k + 1}{\vartheta} = \lim_{\vartheta \to \infty} \left(O_t^k + \frac{1}{\vartheta} \right), \tag{A30}$$

$$= O_t^k, \tag{A31}$$

and

$$\lim_{\vartheta \to \infty} \frac{\vartheta O_t^k}{\vartheta} = O_t^k. \tag{A32}$$

Note that (squeezing functions with the same limit)

$$\frac{\vartheta O_t^k}{\vartheta} < \frac{\rho(O_t^k)}{\vartheta} \leq \frac{\vartheta O_t^k + 1}{\vartheta}. \tag{A33}$$

thus

$$\lim_{\vartheta \to \infty} \frac{\rho(O_t^k)}{\vartheta} = O_t^k. \tag{A34}$$

A.6 Solution method

A.6.1 Solving Workers' Optimization Problem

A.6.1.1 Substituting out the job switching cost δ

First, we assume that the job switching cost does not vary over time. Note that

$$-\frac{\delta}{\nu} = \left(\log \mu_{1,t}^k - \log \mu_{0,t}^k - \log N_t^k\right),$$

based on equations (6) and (7). Thus

$$\exp\left(-\frac{\delta}{\nu}\right) = \frac{\mu_{1,t}^k}{\mu_{0,t}^k} \frac{1}{N_t^k},$$

Then we can write

$$N_t^k \exp\left(-\frac{\delta}{\nu}\right) = \frac{\mu_{1,0}^k}{\mu_{0,0}^k} \widehat{N_t^k}.$$

This expression will be used in the flow equations to substitute out δ.

A.6.1.2 Operators

Define the following operators for any variable X:

$$\ddot{X}_t \equiv \exp\left(\frac{\beta E_{t-1} X_t - \beta X_0}{\nu}\right),$$
$$\dot{X}_t \equiv \exp\left(\frac{X_t - X_0}{\nu}\right),$$
$$\widehat{X_t} \equiv \frac{X_t}{X_0}.$$

A.6.1.3 Flow equations [to use in the baseline steady state]

$$m_0^{kl} = \frac{\mathbf{1}_{l=k}\lambda_0^l + \mathbf{1}_{l\neq I}\lambda_1^l \exp\left(-\frac{C(k,l)}{\nu}\right) + \mathbf{1}_{l=I}\lambda_I^k}{\lambda_0^k + \sum_{l'\neq I}\lambda_1^{l'} \exp\left(-\frac{C(k,l')}{\nu}\right) + \lambda_I^k}, \tag{A35}$$

where we define

$$\lambda_0^k \equiv \exp\left(\frac{\beta V_0^k}{\nu}\right),$$
$$\lambda_1^k \equiv \exp\left(\frac{\beta V_0^k}{\nu}\right)\frac{\mu_{1,0}^k}{\mu_{0,0}^k}\widehat{N}_0^k,$$
$$\lambda_2^k \equiv \sum_{l'\neq k}\lambda_1^{l'} \exp\left(-\frac{C(k,l')}{\nu}\right)$$
$$\lambda_I^k \equiv \frac{\mu_{I,0}^k}{1-\mu_{I,0}^k}\left(\lambda_0^k + \lambda_1^k + \lambda_2^k\right).$$

Also

$$\mu_{x,0}^k = \frac{\lambda_x^k}{\lambda_0^k + \lambda_1^k + \lambda_2^k + \lambda_I^k}, \tag{A36}$$

where $x \in \{0, 1, 2, I\}$.

A.6.1.4 Flow equation [to use in transition]

$$m_t^{kl} = \frac{\mathbf{1}_{l=k}\ddot{V}_{t+1}^k\left[\mu_{0,0}^k + (m_0^{kk} - \mu_{0,0}^k)\widehat{N_t^k}\right] + \mathbf{1}_{l\neq k}\ddot{V}_{t+1}^l\widehat{N_t^l}m_0^{kl}\left[\dot{C}(k,l)\right]^{-1}}{\ddot{V}_{t+1}^k\left[\mu_{0,0}^k + \left(m_0^{kk} - \mu_{0,0}^k\right)\widehat{N_t^k}\right] + \sum_{l'\neq k}\ddot{V}_{t+1}^{l'}\widehat{N_t^{l'}}m_0^{kl'}\left[\dot{C}(k,l')\right]^{-1}}, \tag{A37}$$

where $\widehat{N_t^I} = 1$ and the residual sector is one of the choices indexed with l and l'.

$$\mu_{0,t}^{k} = \frac{\ddot{V}_{t+1}^{k}\mu_{0,0}^{k}}{\ddot{V}_{t+1}^{k}\left[\mu_{0,0}^{k}+\left(m_{0}^{kk}-\mu_{0,0}^{k}\right)\widehat{N_{t}^{k}}\right]+\sum_{l'\neq k}\ddot{V}_{t+1}^{l'}\widehat{N_{t}^{l'}}m_{0}^{kl'}\left[\dot{C}(k,l')\right]^{-1}}. \tag{A38}$$

A.6.1.5 Bellman equation

Define u_t^k as the instantaneous utility in k at t as a function of wage.

$$\log \ddot{V}_t^k = \frac{\beta}{\nu}u_0^k(\widehat{u_t^k}-1)+\beta\log\ddot{V}_{t+1}^k-\beta\log(\frac{\mu_{0,t}^k}{\mu_{0,0}^k}). \tag{A39}$$

Note that if $u_t^k = \log w_t^k$ the expression above simplifies slightly and becomes equivalent to dynamic hat algebra from CDP.

A.6.1.6 Labor allocation equations

$$L_{t+1}^k = \sum_l m_t^{lk}L_t^l. \tag{A40}$$

A.6.1.7 The number of sampled jobs

The step function we discussed in the previous section of the appendix leads to

$$\widehat{N_t^k} = \widehat{O_t^k}, \tag{A41}$$

and the number of total jobs is defined in the next subsection.

A.6.2 Trade and market clearing

A.6.2.1 Unit cost

We start from a guess of real wage $\widehat{w}$ and multiply it by $\widehat{P}$ to get changes in nominal wage $\widehat{\widetilde{w}}$. This guess after the initial iteration should be the $\widehat{w}$ we gave to the labor loop in the immediately preceeding iteration. We take $\hat{L}$ from the labor loop.

$$\widehat{c}_{1,t}^k = (\widehat{\widetilde{w}}_{1,t}^k)^{\gamma_l}(\widehat{P}_{1,t})^{\gamma_m}(\widehat{b}_{1,t}^k)^{\gamma_b}, \tag{A42}$$

$$\widehat{c}_{n',t} = (\widehat{\widetilde{w}}_{n',t})^{\bar{\gamma}_l} (\widehat{P}_{n',t})^{1-\bar{\gamma}_l} \tag{A43}$$

for $n' \neq 1$, where $\widehat{P}_{n,t} = \prod_s \widehat{P}^{s}_{n,t}{}^{\phi^s}$. Note that the unit cost is used to solve for price indices as below.

A.6.2.2 Price indices

We solve for $\widehat{P}^s_{n,t}$ from the following system of equations. We have NS changes in prices to solve for in each period.

$$\widehat{P}^s_{1,t} = \left[\sum_{r''} \lambda^s_{(1,r''),(1,r),0} \hat{T}^{(r'',s)}_{1,t} (\hat{c}^{(r'',s)}_{1,t})^{-\theta} + \sum_{n' \neq 1} \lambda^s_{n',(1,r),0} \hat{T}^s_{n',t} (\hat{c}_{n',t} \hat{d}^s_{n'1,t})^{-\theta} \right]^{-\frac{1}{\theta}}, \tag{A44}$$

which should hold for any r, because both $\lambda^s_{(1,r''),(1,r),0}$ and $\lambda^s_{n',(1,r),0}$ are equalized across r, repsectively.

$$\widehat{P}^s_{n,t} \quad = \quad \left[\sum_{r'} \lambda^s_{(1,r),n,0} \hat{T}^{(r'',s)}_{1,t} (\hat{c}^{(r'',s)}_{1,t} \hat{d}^s_{1n,t})^{-\theta} + \sum_{n' \neq 1} \lambda^s_{n',n,0} \hat{T}^s_{n',t} (\hat{c}_{n',t} \hat{d}^s_{n'n,t})^{-\theta} \right]^{-\frac{1}{\theta}}, \tag{A45}$$

for $n \neq 1$. After solving for price indices, we calculate the unit cost again using (A42) and (A43). Note that the trade flows we need are only for the initial year $t = 0$.

A.6.2.3 Trade flows

For $n \neq 1$ and $n'' \neq 1$,

$$\hat{\lambda}^s_{(1,r),(1,r'),t} = \hat{T}^{(r,s)}_{1,t} \left(\frac{\hat{c}^{(r,s)}_{1,t}}{\widehat{P}^s_{1,t}} \right)^{-\theta} = \frac{\hat{X}^s_{(1,r),(1,r'),t}}{\hat{X}^s_{(1,r'),t}} \tag{A46}$$

$$\hat{\lambda}^s_{n,(1,r),t} = \hat{T}^s_{n,t} \left(\frac{\hat{c}_{n,t} \hat{d}^s_{n1,t}}{\widehat{P}^s_{1,t}} \right)^{-\theta} = \frac{\hat{X}^s_{n,(1,r),t}}{\hat{X}^s_{(1,r),t}} \tag{A47}$$

$$\hat{\lambda}^s_{(1,r),n,t} = \hat{T}^{(r,s)}_{1,t} \left(\frac{\hat{c}^{(r,s)}_{1,t} \hat{d}^s_{1n,t}}{\widehat{P}^s_{n,t}} \right)^{-\theta} = \frac{\hat{X}^s_{(1,r),n,t}}{\hat{X}^s_{n,t}} \tag{A48}$$

$$\hat{\lambda}^s_{n,n'',t} = \hat{T}^s_{n,t} \left(\frac{\hat{c}_{n,t} \hat{d}^s_{nn'',t}}{\widehat{P}^s_{n'',t}} \right)^{-\theta} = \frac{\hat{X}^s_{n,n'',t}}{\hat{X}^s_{n'',t}}. \tag{A49}$$

Note that any region r' has the same $\hat{\lambda}^s_{(1,r),(1,r'),t}$ and that any region r has the same $\hat{\lambda}^s_{n,(1,r),t}$.

A.6.2.4 Expenditures

Expenditures can be solved from the following system of equations. We have $K + S(N-1)$ expenditures and the same number of equations. Since the closed form of the expenditure equation is quite complicated, it is better to solve for $X^s_{(1,r),t}$ and $X^s_{n,t}$ instead of $\hat{X}^s_{(1,r),t}$ and $\hat{X}^s_{n,t}$.

$$X^s_{n,t} = \phi^s (1 - \bar{\gamma}_l) \sum_{s'} \left(\sum_{r'} \hat{\lambda}^{s'}_{n,(1,r'),t} \lambda^{s'}_{n,(1,r'),0} X^{s'}_{(1,r'),t} + \sum_{n' \neq 1} \hat{\lambda}^{s'}_{n,n',t} \lambda^{s'}_{n,n',0} X^{s'}_{n',t} \right) \tag{A50}$$

$$+\phi^s \left(\hat{\widetilde{w}}_{n,t} \hat{\bar{L}}_{n,t} \widetilde{w}_{n,0} \bar{L}_{n,0} + D_{n,t} \right). \tag{A51}$$

for $n' \neq 1$. For region r of country 1,

$$X^s_{(1,r),t} = \phi^s \gamma_m \sum_{s'} \left(\sum_{r'} \hat{\lambda}^{s'}_{(1,r),(1,r'),t} \lambda^{s'}_{(1,r),(1,r'),0} X^{s'}_{(1,r'),t} + \sum_{n' \neq 1} \hat{\lambda}^{s'}_{(1,r),n',t} \lambda^{s'}_{(1,r),n',0} X^{s'}_{n',t} \right) \tag{A52}$$

$$+\phi^s \left(\sum_{k \in \{k | R_k = r\}} \left(\hat{\widetilde{w}}^k_{1,t} \hat{L}^k_{1,t} \widetilde{w}^k_{1,0} L^k_{1,0} + D^k_{1,t} \right) \right) \tag{A53}$$

We will have to fix deficits either by eliminating it entirely, or as the share of total world GDP.

A.6.2.5 Market clearing and the number of job opportunities

For country 1, we have two separate market clearing conditions. One for labor, and the other for the fixed factor.

$$\hat{\widetilde{w}}^k_{1,t}\hat{L}^k_{1,t}\widetilde{w}^k_{1,0}L^k_{1,0} = \gamma_l\left(\sum_{r'}\hat{\lambda}^{S_k}_{(1,R_k),(1,r'),t}\lambda^{S_k}_{(1,R_k),(1,r'),0}X^{S_k}_{(1,r'),t} + \sum_{n'\neq 1}\hat{\lambda}^{S_k}_{(1,R_k),n',t}\lambda^{S_k}_{(1,R_k),n',0}X^{S_k}_{n',t}\right) \quad \text{(A54)}$$

$$\hat{b}^k_{1,t}b^k_{1,0}B^k_1 = \gamma_b\left(\sum_{r'}\hat{\lambda}^{S_k}_{(1,R_k),(1,r'),t}\lambda^{S_k}_{(1,R_k),(1,r'),0}X^{S_k}_{(1,r'),t} + \sum_{n'\neq 1}\hat{\lambda}^{S_k}_{(1,R_k),n',t}\lambda^{S_k}_{(1,R_k),n',0}X^{S_k}_{n',t}\right) \quad \text{(A55)}$$

However, the first market clearing condition exactly implies the second market clearing condition. Thus, we just need the labor market clearing condition to solve for changes in wages. In addition, the number of job opportunities is $\hat{O}^k_t = \hat{\widetilde{w}}^k_{1,t}\hat{L}^k_{1,t}/\hat{P}_{1,t}$.

For country $n' \neq 1$,

$$\hat{\widetilde{w}}_{n,t}\hat{\bar{L}}_{n,t}\widetilde{w}_{n,0}\bar{L}_{n,0} = \bar{\gamma}_l\sum_{s'}\left(\sum_{r'}\hat{\lambda}^{s'}_{n,(1,r'),t}\lambda^{s'}_{n,(1,r'),0}X^{s'}_{(1,r'),t} + \sum_{n'\neq 1}\hat{\lambda}^{s'}_{n,n',t}\lambda^{s'}_{n,n',0}X^{s'}_{n',t}\right). \quad \text{(A56)}$$

We are not going to look at the effect of $\hat{\bar{L}}_{n,t}$ for other countries $n \neq 1$.

A.6.3 Baseline steady state

Our goal here is to calculate m^{kl}_0 that will give us a baseline steady state consistent with estimated parameters of the model. If we had infinite number of workers, we could use take m^{kl}_0 from data directly. In the data we don't observe m^{kl}_0, but a realization of that from a finite sample. The calculated m^{kl}_0 will feed into the transition solution. We take L^k_0, $\mu^k_{0,0}$, $\mu^k_{1,0}$ and $\mu^k_{I,0}$ from data. Assume that $\Theta = \{\beta, C, \nu\}$ is the set of parameters. We set $\widehat{N^k_0} = 1$ and $\widehat{w^k_0} = 1$ for steady state.

- Using (A36), (A35) and (A40) solve for V^k_0 that gives L^k_0 subject to Θ.
- We calculate m^{kl}_0 using (A35) if k is a formal sector. We take m^{Il}_0 from data when the origin is the residual sector (subject to a simple re-weighting to make sure that inflows and outflows from the residual sector are equal. Otherwise population growth breaks the balance in the data).
- Plug V^k_0 into (A35) and (A36) to recover m^{kl}_0 to use for the transition solution.

A.6.4 Transition

Again, we take L_0^k and w_0^k from data. We use m_0^{kl} and $\mu_{1,0}^k$ from the steady state solution. $\widehat{\mathbf{N}}$ is a set of $\widehat{N_t^k}$'s, $\mathbf{L}$ is a set of L_t^k, $\mathbf{m}$ is a set of m_t^{kl}, μ is a set of μ_t^k, $\mathbf{w}$ is a set of w_t^k, $\mathbf{b}$ is a set of b_t^k and $\ddot{\mathbf{V}}$ is a set of $\ddot{V}_t^k$ for $t \in \{1, 2, 3, .., T\}$, and all possible combinations of k and l. For example $\widehat{\mathbf{N}} = \{\widehat{N_1^1}, \widehat{N_2^1}, ..., \widehat{N_T^1}, \widehat{N_1^2}, ..., \widehat{N_T^K}\}$.

1. Consider an initial guess for $\widehat{\mathbf{w}}$, $\widehat{\mathbf{b}}$, $\mathbf{L}$, $\ddot{\mathbf{V}}$ and $\widehat{\mathbf{N}}$.

2. Solve for $\widehat{\mathbf{P}}$ using (A44) and (A45) with a separate loop that iterates over $\widehat{\mathbf{P}}$.

3. Calculate implied flows, $\mathbf{m}$ and μ using (A37) and (A38).

4. Calculate implied $\ddot{\mathbf{V}}$ via (A39) using $\widehat{\mathbf{w}}$ and $\widehat{\mathbf{N}}$.

5. Calculate implied $\mathbf{L}$ using (A40) with $\mathbf{m}$ from previous step.

6. Update $\widehat{\mathbf{w}}$ and $\widehat{\mathbf{b}}$ to minimize the excess demand of factors which can be calculated by (A54), (A55), and (A56). If the there is an excess factor demand, the algorithm increases the factor prices, otherwise reduces them.

7. Update $\widehat{\mathbf{P}}$ using the updated $\widehat{\mathbf{w}}$ by solving (A44) and (A45) inside a separate loop and calculate implied $\widehat{\mathbf{N}}$

8. Update the guesses of $\mathbf{L}$ and $\ddot{\mathbf{V}}$ using implied numbers.

9. Go to Step 3 and continue iterations.

A.7 Appendix Tables

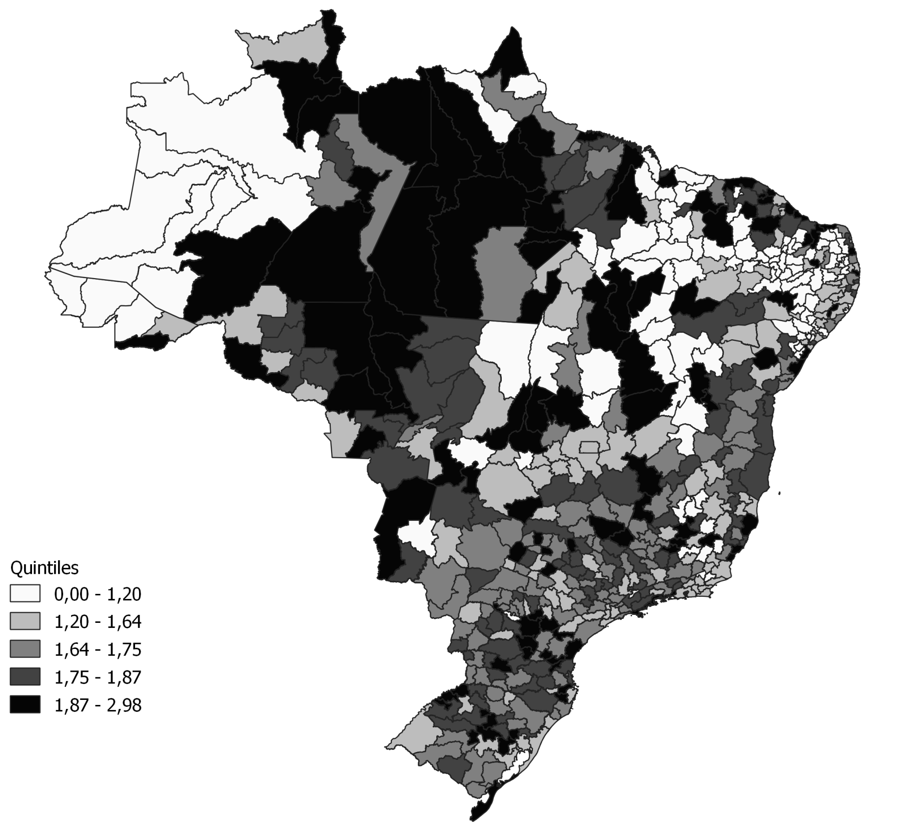
Table A1: Gross job flow rates by sector and year

	Agriculture and mining		Manufacturing		Services		All sectors	
year	inflows	outflows	inflows	outflows	inflows	outflows	inflows	outflows
2004	0.084	0.086	0.066	0.063	0.054	0.076	0.068	0.075
2005	0.091	0.091	0.067	0.094	0.059	0.082	0.072	0.089
2006	0.112	0.091	0.087	0.071	0.060	0.076	0.087	0.079
2007	0.087	0.087	0.069	0.072	0.060	0.079	0.072	0.079
2008	0.083	0.089	0.067	0.074	0.062	0.075	0.071	0.079
2009	0.090	0.102	0.068	0.090	0.067	0.079	0.075	0.090
2010	0.089	0.101	0.075	0.080	0.068	0.082	0.077	0.088
2011	0.094	0.098	0.074	0.082	0.072	0.086	0.080	0.089
2012	0.107	0.094	0.074	0.078	0.072	0.081	0.084	0.084
2013	0.094	0.087	0.076	0.073	0.070	0.073	0.080	0.077
2014	0.092	0.078	0.070	0.065	0.064	0.063	0.075	0.069
Average	0.093	0.091	0.072	0.076	0.064	0.077	0.076	0.082

Notes: Table reports average gross job inflow and outflow rates for each sector and year in Brazilian microregions in 2004-2014

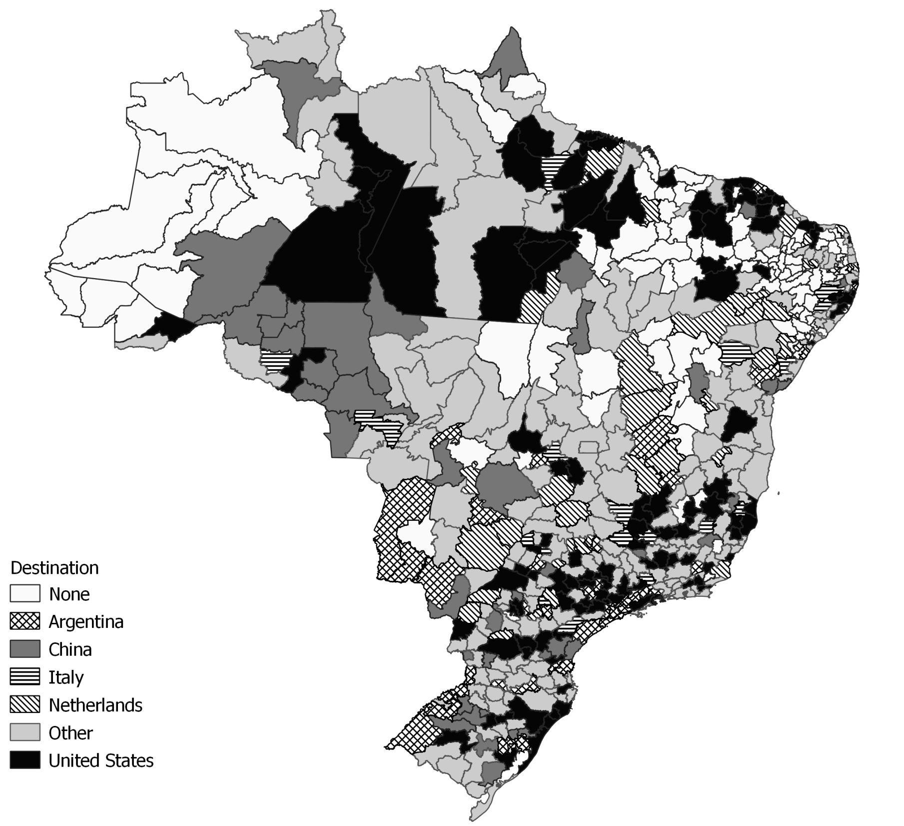
A.8 Appendix Figures

Figure A1: Export revenue per worker, 2003



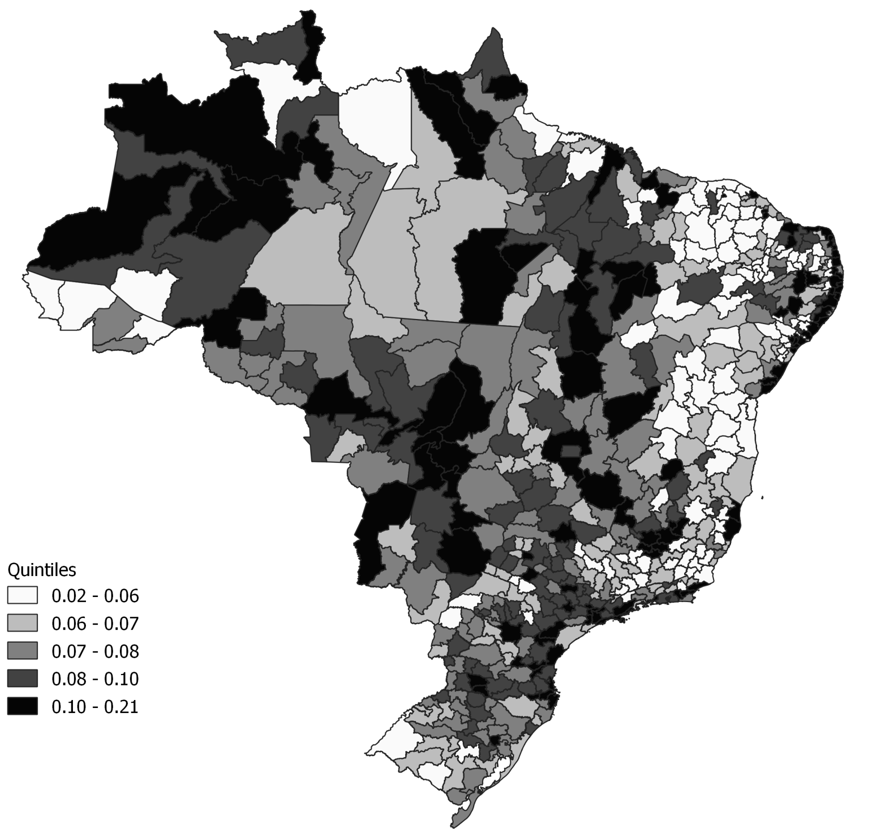
Notes: Figure depicts the log of (1+exports) per worker in Brazilian microregions in 2003.

Figure A2: Top export destinations, 2003



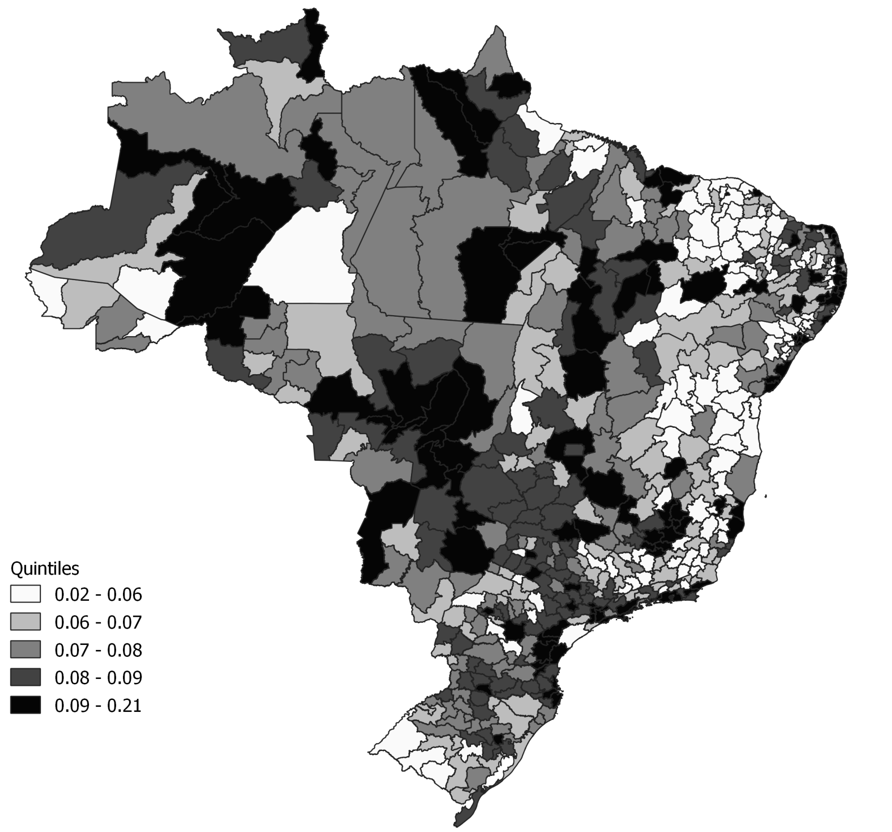
Notes: Figure depicts the top export destination of each Brazilian microregion in 2003.

Figure A3: Gross outflow rates by microregion, 2004-2014



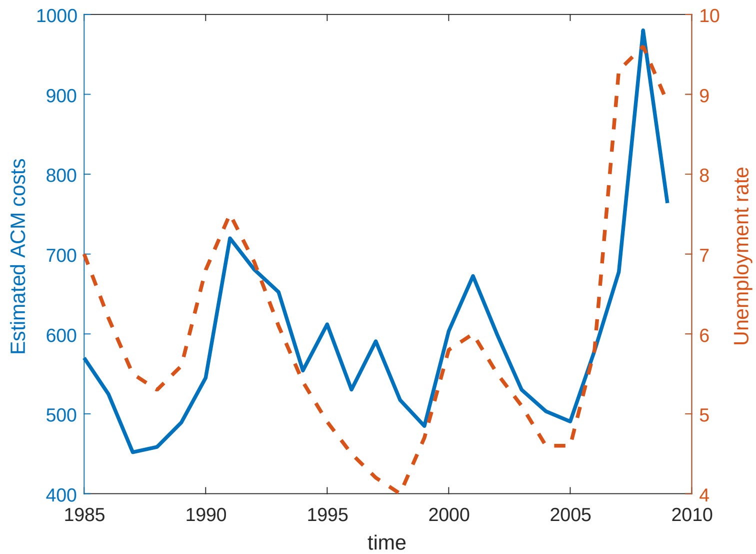
Notes: Figure depicts the average gross job outflow rates observed in each Brazilian microregion in 2004-2014.

Figure A4: Gross inflow rates by microregion, 2004-2014



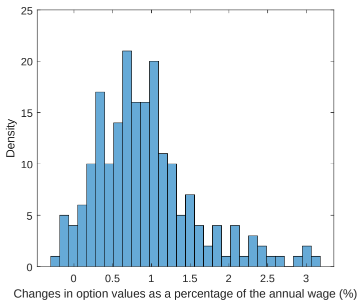
Notes: Figure depicts the average gross job inflow rates observed in each Brazilian microregion in 2004-2014.

Figure A5: Additional evidence on model fit from the USA: change in unemployment rate and estimated time varying ACM moving costs

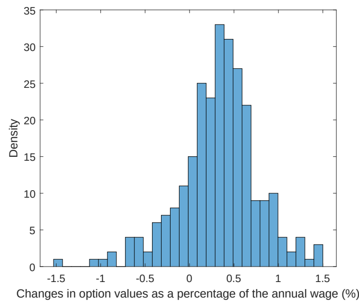


Notes: Figure depicts the correlation between the ACM moving cost and unemployment rate in USA between 1985 and 2009. The correlation coefficeint between the two series is equal to 0.58.

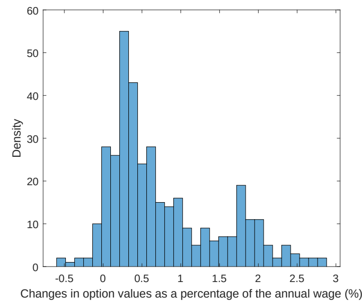
Figure A6: Within-sector distribution of changes in option values



(a) Agriculture



(b) Manufacturing



(c) Service

Notes: Each plot shows the within-sector distribution of changes in option values as a percentage of the annual wage across regions.