O-Ring Production Networks

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Abstract

We study a production network where quality choices are interconnected across firms. High-quality firms are skill intensive and disproportionately source inputs from and sell output to other high-quality firms. Consistent with the theory, we document strong assortative matching of skills in the network of Turkish manufacturing firms. In the data, a firm-specific trade shock from a rich country increases the firm's skill intensity and shifts the firm toward skill-intensive domestic partners. We develop a quantitative model with heterogeneous firms, endogenous quality choices, and network formation. Parameter estimates indicate strong complementarity of quality in production. The effect of firm-specific trade shocks on the demand for skills is about eight times smaller than the average effect of the same shock applied to all firms simultaneously. Aggregate shocks are magnified in general equilibrium because the larger presence of high-quality firms in the network makes it more profitable for other firms to upgrade quality.

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1   Introduction

The space shuttle *Challenger* exploded because one of its innumerable components, the O-rings, malfunctioned during launch. Using this as a leading example, Kremer (1993) studies production processes, in which the value of output may dramatically decrease due to the failure of a single task. In his model, a product may founder from the mistake of a single unskilled worker, even if it aggregates the high value added of many skilled workers. To avoid such losses, a firm that produces complex, higher-quality products hires skilled workers for all its tasks.

Extending this rationale across firm boundaries, the high-quality firm above will source high-quality inputs and sell to high-quality firms that value its output. So, skill-intensive firms match with each other in the network. A firm's decision to upgrade its quality depends critically on the willingness of its trading partners to also upgrade or on its ability to find new higher-quality partners. Shocks to the quality of a few firms may have large general equilibrium effects, because they increase the probability that other firms match with high-quality trading partners. Matching with a high-quality supplier decreases the relative cost from producing high-quality, and matching with a high-quality customer increases the revenue from producing high-quality inputs. So, the new matching increases the incentives for firms not directly impacted by the shock to upgrade their quality. If these firms respond, they themselves improve the quality of matches in the network, further propagating the shock and further increasing the demand for skilled workers used in high-quality production.

This mechanism applies to the quality of products as well as to the quality of inventory controls, research and development, and internal communications. Improvements in these areas generally allow for greater product scope and render the firm more flexible to respond to demand and supply shocks. A firm profits from these improvements if its suppliers also offer scope and flexibility, and if its customers value these same improvements.

We study this interconnection in firms' quality choices theoretically and empirically. Our data comprise all formal Turkish manufacturing firms from 2011 to 2015. We merge value-added tax (VAT) data with matched employer-employee and customs data. We observe the value of trade between each buyer-seller pair of firms; exports by firm, product and destination, and the occupation and wage of each worker in each firm. We develop a quantitative model that accounts for the salient features of the data. We structurally estimate the model and use it to study general equilibrium effects of trade shocks.

We document a novel, strong assortative matching of skills in the network. As an example, Figure 1 graphs the relation between a firm's average log wage (adjusted for

Figure 1: Assortative Matching on Wages
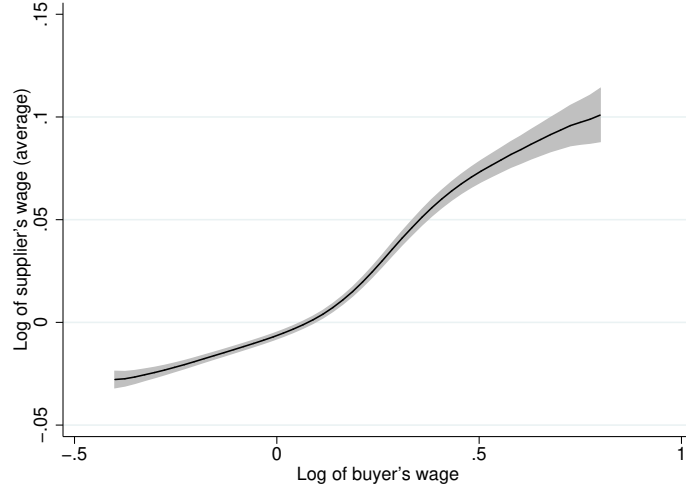
*Notes:*We define the wage of a firm as the firm's wage bill divided by the number of workers. Supplier wage is the average wage across all manufacturing suppliers of a firm, weighted by the firm's spending on each supplier. Both x- and y-axis variables are demeaned from 4-digit NACE industry and region. The fitted curve is obtained from local polynomial regression with Epanechnikov kernel of (residual) wages. The shaded area shows the 95% confidence intervals.

industry-region) against the average wage of its suppliers.[1] A 10 percent increase in a firm's wage is associated with a 2.5 percent increase in its suppliers' wages. This number is large given that the average number of suppliers per firm is 11. This increasing relation between buyer and supplier wage may arise from an extensive margin—high-wage firms match more with each other—or from an intensive margin—high-wage firms spend relatively more on their high-wage suppliers. In a decomposition exercise, we find that the extensive margin accounts for 59% of the relation and the intensive margin accounts for the remainder 41%.

We use shift-share regressions to evaluate firms' responses to shocks and movements along the schedule in Figure 1. Consider a Turkish firm that exports a particular product category to a high-income country, say cotton towels to Germany. An increase in German imports of cotton towels from countries other than Turkey from 2011 to 2015 is associated with an increase in the Turkish firm's wage, and the average wage of its suppliers and customers. The new employees, suppliers and customers that the firm adds over the years, from 2011 to 2015, had on average higher wages than the firms' existing employees and partners in 2011. Our proposed mechanism combined with evidence from the literature

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[1]The figure has only manufacturing firms, later used in our structural estimation but an equally strong pattern emerges in the corresponding figure with all sectors, in Appendix Figure A2.

that high-income countries demand relatively more skill-intensive goods explains these patterns.[2] An increase in the relative demand for high-quality goods, increases a firm's quality and skill intensity. The firm shifts toward skill-intensive trading partners and may prod its existing partners to upgrade.

As explained above, the interconnection in firms' quality choices implies that a shock to a subset of firms may have large general equilibrium effects. To evaluate this claim, we develop a quantitative model with heterogeneous firms, endogenous quality choices and network. Like in Kremer (1993), a firm's quality determines its production function. We assume that higher-quality firms are more skill intensive and allow the marginal product of high-quality inputs to be higher in the production of high-quality output. Firms post costly ads to search for customers and suppliers. Firms may imperfectly direct their search toward customers of specific quality levels. A standard matching function aggregates these ads to form the network of firm-to-firm trade.

The model differs from previous network models (below) in two aspects. First is its use of log-supermodular shifters to generate assortative matching in the network. We follow Teulings (1995) and Costinot and Vogel (2010) for labor, Fieler et al. (2018) for material inputs and apply it anew to directed search.[3] Second, network formation in the model follows a search and matching set up, typically used in labor.[4] This approach facilitates aggregation as the shares of profit, labor and materials in revenue are constant, and revenue is a log-linear function of the firm's productivity for a given quality.

We estimate the model to Turkish manufacturing firms using the method of simulated moments. We focus on manufacturing because the shift-share regressions above, used in the estimation, applies only to tradable goods. The model matches well assortative matching on wages, and the joint distribution of firm sales, wages, number of customers and suppliers. In the data and model, the endogenous elasticity of sales with respect to number of suppliers and with respect to number of customers is about 0.5.

To capture differences in matching across firms with different wages (extensive margin), the model needs relatively little directed search. About nine percent of the ads posted by buyers in the lowest quintile of wages are directed to suppliers in the highest wage quintile, and vice-versa. Differences in marginal productivity capturing the spending patterns (intensive margin) are large. The marginal product of an input in the 90th percentile of the quality distribution is always larger than the marginal product of an input in the 10th

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[2]See Hallak (2006), Brambilla et al. (2012), Manova and Zhang (2012), and Bastos et al. (2018).

[3]See also Milgrom and Roberts (1990) and Costinot (2009) for earlier applications to economics and international trade.

[4]See Mortensen (1986) and Rogerson et al. (2005) for surveys. Eaton et al. (2018) also apply a search-and-matching set up to network formation, but aggregation is very different in their setting.

percentile. But the ratio of these marginal products is 1.48 when producing output in the 90th percentile, and it is 1.13 when producing output in the 10th percentile.

In the data, export intensity is generally higher among high-wage exporting firms than among low-wage ones. This pattern holds in the estimated model because the relative demand for higher-quality is higher abroad. A firm that experiences a 5 percent increase in its export demand, upgrades quality, hires more skilled workers, and its wage increases on average by 0.21 percent. These numbers by construction exactly match the average firm response in the shift-share regressions in the data.

We use a counterfactual to study the general equilibrium effect of this same export demand shock if it occurred not to individual firms but to the whole economy. On average, the aggregate shock increases wages by 1.7 percent for exporting firms, almost an order of magnitude larger than the effect of firm-specific shocks. The direct effect of the shock on some firms' quality decreases the relative cost of producing high-quality and increases the relative revenue from producing high-quality, through the higher-quality of matches in the network. In our counterfactual, changes in demand account for about two-thirds and changes in cost account for one third of the increase in profit from producing high-relative to low-quality goods. The wages of non-exporters, which by definition are not directly affected by the shock, increases by 1.0 percent.

We compare this counterfactual with a special case of the model in which the production of higher quality does not use high-quality inputs intensively, and in which higher-quality firms do not direct their search toward other high-quality firms. By assumption, this special case cannot match the positive assortative matching of firm wages in the data, and firms' quality choices are disconnected from the quality of their trading partners. Because demand for skill-intensive, higher-quality goods is still higher in Foreign, the export shock increases the quality and wages of exporting firms. But the average increase in wage with the aggregate shock is only 0.23 percent for exporters, almost exactly the same as 0.21 percent increase with the firm-specific shocks.

Despite the large magnification effects on demand for skills in the full model, the effect of the export demand shock on aggregate manufacturing output is around 5.7 percent in both counterfactual exercises above. This effect is larger than the Hulten (1978) result, but it is in line with Baqaee and Farhi (2019a) and Huneeus (2018), other network models that like us allow for an elasticity of substitution between varieties larger than one.

The network literature has focused on Hicks-neutral shocks, while quality in our model by definition changes the types of inputs that firms use. To depart from Hicks-neutrality, we abstract from dynamics in Lim (2018) and Huneeus (2018) and from asymmetries in network centrality in Acemoglu et al. (2012), Baqaee and Farhi (2019a), and in market

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distortions in Baqaee and Farhi (2019b), Bigio and La'O (2020) and Liu (2019). The model features roundabout production, technologies with constant elasticities of substitution, and each firm has a continuum of suppliers and customers. Some of these elements and the use of shocks to international trade appear in open economy models as Lim (2018), Tintelnot et al. (2018), Bernard et al. (2019a,b), Eaton et al. (2018), Huneeus (2018).

The estimated model is consistent with previous theories and well-established facts in the quality literature. Namely, the production of higher-quality is intensive in skilled labor, as in Schott (2004), Verhoogen (2008), Khandelwal (2010), and in higher-quality inputs, as in Kugler and Verhoogen (2011), Manova and Zhang (2012), and Bastos et al. (2018). Fieler et al. (2018) combines both of these elements to study, like us, the general equilibrium effect of international trade on demand for skills and quality. None of these firms observe firm-to-firm trade. We complement their findings on prices with direct information on the extent to which skill-intensive, high-wage firms trade with each other. Our main finding on assortative matching is akin to Voigtländer (2014) who shows that skill-intensive sectors use intensively inputs from other skill-intensive sectors in the United States.[5]

The rest of the paper is organized as follows. Section 2 describes the data and the novel empirical facts. To highlight the main features of the model, Section 3 presents a closed-economy version. It presents the set up, its estimation procedure and identification argument. Section 4 extends the model to a small open economy. Section 5 reports our estimation results and connects them to the empirical facts in Section 2. Section 6 experiments with counterfactual aggregate shocks in the estimated model. Section 7 concludes.

2 Data and Empirical Facts

2.1 Data Sources

We combine five data sets from Turkey: (1) value added tax (VAT) data on domestic firm-to-firm trade, (2) data on firms' balance sheet and income statement, (3) firm registry, (4) customs data, and (5) linked employer-employee data. These data sets are all maintained by the Ministry of Industry and Technology. They contain the same firm identifier and comprise all formal firms in Turkey from 2011 through 2015.

The VAT data report all domestic firm-to-firm transactions whenever the total value of

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[5]A related finding is in Carvalho and Voigtländer (2014) who show that firms are more likely to match with the suppliers of their suppliers. They interpret the finding in terms of information frictions.

transactions for a seller-buyer pair exceeds 5,000 Turkish Liras (about US\$1,800 in 2015) in a given year. From the balance sheet and income statement data, we use information on each firm's gross domestic and foreign sales. From the firm registry, we extract the firm's location (province) and industry. The industry classification is the 4-digit NACE, the standard in the European Union. From the customs data, we use information on annual exports by firm, destination country, and 4-digit Harmonized System product code.

The employer-employee data are collected by the Turkish social security administration. We observe the quarterly wage of each worker in each firm. We also observe the worker's occupation (4-digit ISCO classification), age, and gender. The worker identifier is unique, allowing us to track workers across firms and over time.

We restrict most of the analysis to the more tradable, manufacturing sector. Unless otherwise noted, facts about the network refer to trade between firms within manufacturing. Still, for robustness, we verify that assortative matching on wages holds for the whole data, including service firms (Table 1).[6] We drop firms that do not report balance sheets or income statements. These are usually very small firms that use a single-entry bookkeeping system. The cross-sectional facts refer to year 2015. The final sample has 77,418 manufacturing firms in 2015.

Section 2.2 describes the assortative matching in the firm-to-firm network. Section 2.3 associates firm-specific trade shocks to systematic changes in firm outcomes, including wages and network connections. To estimate these trade shocks, we use annual bilateral trade data from BACI, disaggregated at the four-digit Harmonized System product code.[7] Section 2.4 describes other salient features in the data. These features are not novel, but they justify some elements of the model.

2.2   Assortative Matching in the Cross-Section

We document a positive assortative matching in firm wages. At the end of the section, we discuss assortative matching along other dimensions of firm characteristics, including wage residuals. Define $wage_f$ as firm f's total monthly wage bill divided by its number of workers. Define the wage of firm f's suppliers as:

$$\log wage_f^S = \sum_{\omega \in \Omega_f^S} s_{\omega f} \log wage_\omega, \tag{1}$$

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[6]For services, we exclude finance, insurance, utilities and public services.

[7]We aggregate these data from six- to four-digit HS codes for two reasons. First, it is less likely that one source country has significant market power in a given destination at the four-digit product level than at the 6-digit level. Second, the value of trade at the variety (country-product) level is too volatile at the 6-digit product level.

Table 1: Assortative Matching on Wages

Dependent variable: $\log wage_f^S$

	Manufacturing firms			All firms
	(1)	(2)	(3)	(4)
$\log wage_f$	0.294	0.259	0.188	0.241
	(0.013)	(0.012)	(0.009)	(0.013)
$\log employment_f$			0.044	
			(0.003)	
R^2	0.095	0.173	0.199	0.150
N	77,418	77,418	77,418	410,608
Fixed effects		ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. The suppliers' average wage $\log wage_f^S$ is defined in equation (1). Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

where Ω_f^S is the set of suppliers to firm f, and $s_{\omega f}$ is the share of supplier ω in firm f's total spending on inputs.

Table 1 reports the results from the regression

$$\log wage_f^S = \beta \log wage_f + \gamma X_f + e_f, \tag{2}$$

where control variables X_f vary across columns. Columns (1) through (3) contain only the manufacturing firm-to-firm subsample. Column (1) has no control variables. Column (2) includes fixed effects of each industry (4-digit NACE) and province pair. The coefficient decreases from column (1) because firms are more likely to match within province and industry, and some province-industry pairs have higher skill shares. Still, the decrease is modest, from 0.294 to 0.259, suggesting that most of the variation across firms occurs within industry-province pairs. A 10 percent increase in average buyer's wage is associated with a 2.5 percent increase in average supplier wages. This is a large number considering that manufacturing firms have on average 11 manufacturing suppliers.[8]

Column (3) controls for the buying firm's employment. Employment and wages are correlated. So not surprisingly, the coefficient on wages decreases, but its magnitude is still comparable to other columns. Column (4) repeats specification (2) with the sample of all firms. The coefficient of 0.241 is similar to specification (2).

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[8]Turkey is divided into 81 provinces, which vary by size. Each province is further divided into districts, of which the total number is close to 1000. While we have data on each firm's district, we prefer touse province as our geographic unit because a province better represents a local labor market. Still, if we re-estimate the specification in column (2) with district-level fixed effects, the estimated coefficient on the logarithm of $wage_f$ is 0.245 (0.011).

Decomposition into margins   The positive coefficients on Table 1 could arise because high-wage firms have more high-wage suppliers—an extensive margin—or because they spend relatively more on their high-wage suppliers given the same matches—an intensive margin. We decompose the coefficient of our preferred specification (2) into these margins.

Define the extensive margin as the unweighed average of the wage of firm f's suppliers:

$$EM_f^S = \sum_{\omega \in \Omega_f} \frac{1}{|\Omega_f|} \log wage_\omega \tag{3}$$

Define the intensive margin as the difference between $\log wage_f^S$, defined in (1), and the extensive margin:

$$\begin{aligned} IM_f^S &= \log wage_f^S - EM_f^S \\ &= \sum_{\omega \in \Omega_f} (s_{\omega f} - 1/|\Omega_f|)(\log wage_\omega - \sum_{\omega' \in \Omega_f} (1/|\Omega_f|) \log wage_{\omega'}) \end{aligned} \tag{4}$$

The intensive margin of firm f is large if its spending shares $s_{\omega f}$ are particularly large for high-wage suppliers ω.

One at a time, we regress $\log wage_f^S$, EM_f^S and IM_f^S on the wage of firm f and on industry-province fixed effects. The results are in Table 2. The first regression is the same as column (2), Table 1. By construction, the coefficients in the second and third columns add up to the total, 0.259, in the first column. The extensive margin accounts for 59% (= 0.152/0.259) of the partial correlation between the firm's wage and its suppliers' wages, while the intensive margin accounts for 41%. Since these margins are both large, the model will allow for high-wage firms to match more with high-wage suppliers and to spend relatively more on their high-wage suppliers.

Figure 2 illustrates assortative matching using the raw data. We rank firms according to their $wage_f$ and split them into quintiles. Panels (a) and (b) describe firms' upstream links. The height of the bars in panel (a) is the seller quintile's share in the number of suppliers to firms in each buyer quintile. The height in panel (b) is seller quintile's share in the spending by firms in each buyer quintile. So, by construction, the sum of bars with the same color, across seller quintiles, is one for each buyer quintile. Sellers in the highest quintile of wages generally have larger sales and more buyers. So their shares are higher for all buyer quintiles. But in both panels the difference between sellers in quintiles 1 and 5 is much larger when the buyer has high wage. In addition, due to the intensive margin, these differences are more pronounced in panel (b) than (a). In panel (a), high-wage sellers account for 35 percent of suppliers to buyers in the lowest quintile

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Table 2: Assortative Matching on Wages: Decomposition

	total $\log wage_f^S$ (A)	extensive margin EM_f^S	intensive margin IM_f^S
$\log wage_f$	0.259	0.152	0.107
	(0.012)	(0.007)	(0.007)
coeff. / coeff in (A)		*59%*	*41%*
R^2	0.173	0.150	0.089
N	77,418	77,418	77,418
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. The suppliers' average wage $\log wage_f^S$ is defined in equation (1). Ind and prov refer to 4-digit NACE industries and provinces, respectively. Equations (3) and (4) define the extensive (EM_f^S) and intensive margins (IM_f^S). They capture respectively the extent to which firm f matches with high-wage firm or tilts its spending toward high-wage suppliers. Robust standard errors are clustered at 4-digit NACE industry level.

of wages, and they account for 55 percent of suppliers to buyers in the highest quintile. The corresponding numbers of spending in panel (b) are 43 and 83 percent. Panels (c) and (d) describe the corresponding patterns for firms' downstream links. Shares across buyers now add up to one for each quintile of seller. Panels (c) and (d) are almost the mirror images of panels (a) and (b).

Sorting on other dimensions   In extending Kremer (1993)'s rationale across firm boundaries, we predict that skill-intensive firms disproportionately buy and sell goods to other skill-intensive firms. In the results above, we interpret a firm's average wage as a proxy for its skill-intensity under the assumption that firms observe skills better than us econometricians and that wages reflect differences in worker skills.

Appendix Table A3 repeats the regression of column (2) in Table 1 substituting wages with other firm characteristics. Assortative matching on sales is less pronounced than on wages, and it is insignificant for the number of network links.[9] The positive assortative matching on sales could arise because of the correlation between sales and wages. To evaluate their relative importance, Appendix F conducts a horse-race between sales and wages following the literature on marriage markets of agents with multidimensional characteristics. In particular, we use the approach in Johnson and Wichern (1988), in the spirit of Becker (1973). Both wages and sales matter for the positive assortative matching, but wages are about 3 times more important than sales for a firm's downstream linkages
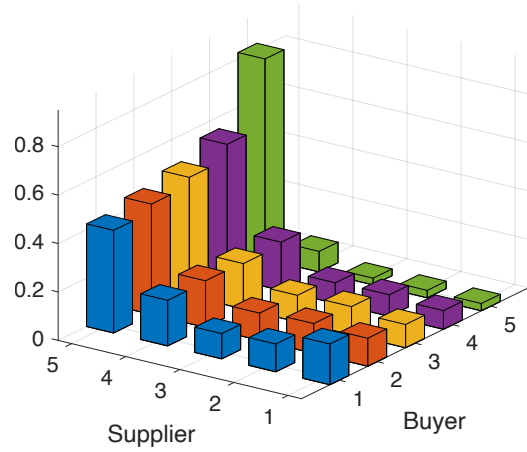
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[9]Lim (2018) also finds assortative matching on sales using data on large firms in the United States (Compustat). This pattern also arises in our estimated model where there is a positive correlation between firm sales and wages.

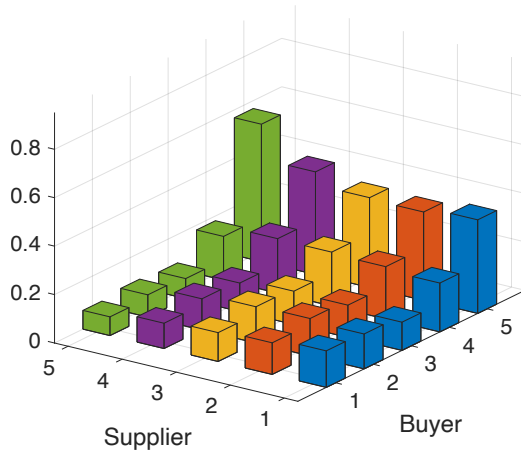Figure 2: Firm-to-firm Trade Links and Values by Quintile

(a) Share of suppliers
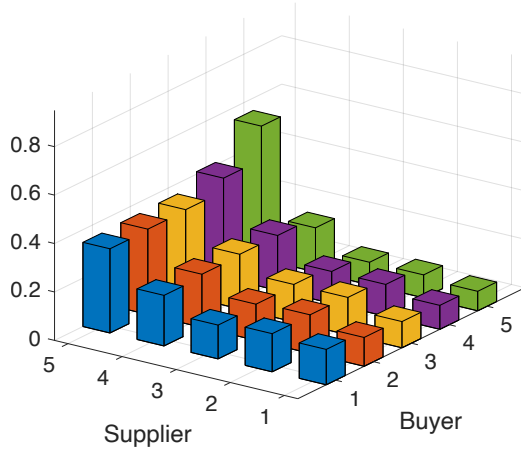
(b) Spending shares

(c) Share of buyers

(d) Sales shares
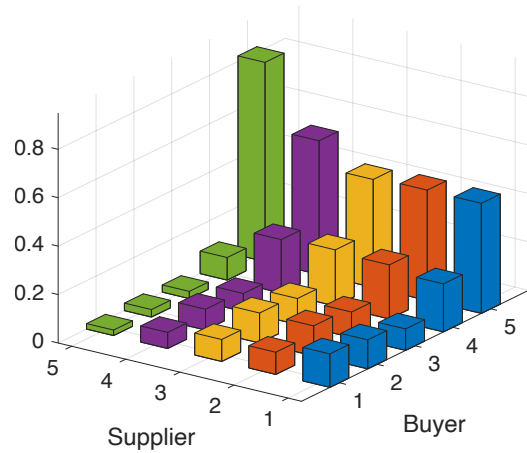
Notes: Sample includes manufacturing buyers and suppliers. Firms are sorted according to the average value of their monthly payments per worker, and grouped into five equal-sized groups. For each buyer (supplier) quintile, expenditures (sales) and number of suppliers (buyers) are aggregated at the level of supplier (buyer) quintile. Buyer and supplier quintiles are shown on x- and y-axis while z-axis shows the corresponding shares. For instance, in panel (a), values on the z-axis show for each buyer quintile on the x-axis the share of suppliers that belong to the wage quintiles on the y-axis.

and 8.5 times more important for its upstream linkages.[10]

Appendix G shows that assortative matching in our data occurs mostly through the unobservable component of wages. The results in Table 1 hold almost unchanged if we construct firm wages from the residual of a regression of wages on workers' observable characteristics and occupation fixed effects.[11] We also construct the measure in Bombardini et al. (2019). First we decompose the variation in firm-worker level wages into firm and worker components as in Abowd et al. (1999) using our employer-employee data from 2014 to 2016. Next, we aggregate the worker-level component at the firm level and repeat the regressions in Table 2 using these firm-specific aggregates. The estimated sorting coefficient is halved compared to our baseline estimate, and its decomposition into extensive and intensive margins remain close to our baseline results.

2.3   Trade Shocks

We use shift-share regressions to study firms' responses to firm-specific trade shocks.[12] Let x_{ckf} be firm f's exports to country c in product category k as a share of the firm's revenue in 2010. Let $\text{Imports}_{ck,t}$ be the total imports of country c in product category k from all countries other than Turkey in year t, and GDP per capita$_{c,t}$ be the income per capita in US\$ of country c in year t. We define two shifters associated with country c and product category k:

$$Z_{ck}^{u} = \Delta \log \text{Imports}_{ck,t} \tag{5}$$
$$Z_{ck}^{a} = (\Delta \log \text{Imports}_{ck,t}) * \log(\text{GDP per capita}_{c,2010})$$

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[10]These numbers are from a canonical correlation analysis first developed by Becker (1973). Similar conclusions arise from multivariate regressions à la Benham (1974).

[11]We regress:

$$\ln wage_{ef} = \beta_1 Age_e + \beta_2 Gender_e + \alpha_o + e_{ef}$$

where $wage_{ef}$ is the wage of employee e in firm f, Age_e and $Gender_e$ is the employee's age and gender, and α_o are occupation fixed effects at the 1-digit ISCO level. We take the wage of firm f as the median residual e_{ef} across its employees. The results in Table A1 are very similar to those of Table 1.

[12]See Bartik (1991) for an early application of these regressions, and Goldsmith-Pinkham et al. (forthcoming) and Adao et al. (2019) for general set ups.

where the operator Δ indicates the change between 2011-2012 and 2014-2015. We construct two measures of export shock to firm f during the period of our data:

$$\text{ExportShock}_f^u = \sum_{ck} x_{ckf} Z_{ck}^u$$

$$\text{ExportShock}_f^a = \sum_{ck} x_{ckf} Z_{ck}^a. \qquad (6)$$

We interpret Z_{ck}^u as a change in the demand for product category k in country c. The underlying assumption is that shocks to imports of product k by country c from countries other than Turkey are uncorrelated to other unobserved shocks to Turkish firms that export k to c. ExportShock$_f^u$ is a standard shift-share shock that, under this assumption, captures the increased demand for firm f's exports. But we are interested in shocks that increase the incentives for firm f to upgrade its quality, and it is well documented that the relative demand for higher-quality, skill-intensive goods is higher in rich countries.[13] Then, export shocks that originate in rich countries should induce larger changes in quality. ExportShock$_f^a$ is an adjusted measure that weights rich countries more.

To compare these measures, we regress

$$\Delta \log wage_f = \delta \text{ExportShock}_f + \alpha_{sr} + \epsilon_f$$

where ExportShock$_f$ may be adjusted or unadjusted as defined in (6), and α_{sr} are industry-province fixed effects.

Columns (1) and (2) of Table 3 reports the results. The unadjusted ExportShock$_f^u$ has an insignificant effect on firm wages, while the adjusted shock has a positive and significant effect. If the production of higher quality is skill intensive, then these two regressions confirm our expectations: An increased demand for a firm's exports only increases a firm's quality if it originates in rich countries.

The mean of ExportShock$_f^a$ is 0.12. To understand the magnitude of the coefficient 0.042 in column (2), consider two firms. They both export a quarter of their sales (the mean export intensity among exporters in the data). One firm exports to a country at the 90th percentile of per capita GDP distribution (US\$41.3 thousand, France), and the other firm exports to a country at the 10th percentile (US\$766, Benin). For the average change in imports over the sample period, $Z_{ck}^u = 5\%$, the implied ExportShock$_f^a$ for the two firms is 13.3 and 8.3 percent, respectively, and the estimated wage increase is 0.56 (=0.042*0.133) and 0.35 percent.

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[13]See footnote 2 for references.

Table 3: Effects of Export Shock

	$\Delta \log \text{wage}_f$ (1)	$\Delta \log \text{wage}_f$ (first stage) (2)	$\Delta \log \text{domestic sales}_f$ (3)	$\Delta \text{export intensity}_f$ (4)	$\Delta \log \text{wage}_f^S$ OLS (5)	$\Delta \log \text{wage}_f^S$ IV (6)
ExportShock_f^u (unadjusted)	0.021 (0.033)					
ExportShock_f^a (adjusted)		0.042 (0.006)	-0.026 (0.022)	0.0146 (0.0023)		
$\Delta \log \text{wage}_f$ (IV = ExportShock_f^a)					0.085 (0.008)	0.434 (0.185)
F-Stat	0.404	43.6	1.409			
N	33,157	33,157	33,157	33,157	33,157	33,157
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Wage_f is the average value of monthly payments per worker in firm f. The suppliers' average wage $\log \text{wage}_f^S$ is defined in equation (1). Δ operator denotes changes between 2011-2012 and 2014-2015. ExportShock_f^u is a weighted average of changes in imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. ExportShock_f^a adjusts these shocks by weighting rich destinations more. See equations (6). Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Given these results, we henceforth use the adjusted export shock in all exercises. In column (3), we replace the dependent variable in column (2) with domestic sales. The insignificant coefficient is reassuring, since we assume that ExportShock_f^a is uncorrelated with domestic shocks to firm f. It is also reassuring that the shock is not spurious but is associated with an increase in the firm's export intensity (export sales divided by total sales) in column (4).

Columns (5) and (6) regress the change in the wage of firm f's suppliers on the change in firm f's own wage:

$$\Delta \log \text{wage}_f^S = \delta \Delta \log \text{wage}_f + \alpha_{sr}$$

In column (6), we instrument the change in the firm's wage $\Delta \log \text{wage}_f$ with the export shock.[14] The coefficient is 0.434 with standard error 0.185. The interpretation is that when a firm's average wage increases by one log point relative to other firms because it experienced a large export shock, then the average wage of its suppliers increase by 0.4 log points. The coefficient in the OLS regression in column (5) is smaller, 0.085. It is difficult to *ex ante* predict the direction of the bias. The OLS coefficient is confounded by unobserved production side shocks that differently affect wage growth of firms in the same industry and province.

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[14]This approach follows Hummels et al. (2014). To study the effect of exports on wages, they use a shift-share variable, similar to ExportShock_f^u, as an instrument for firm exports.

The results in Table 3 are robust to alternative specifications in Appendix Table A4. First, we add adjusted and unadjusted export shocks together in the first stage. Compared to column (2) of Table 3, the coefficient on the income-adjusted export shock retains its sign and magnitude, while the coefficient on unadjusted ExportShock_f^u remains small and insignificant. As the latter is a weak instrument for firm wages, the F-statistic decreases substantially. Second, we add a weighted average of destination GDP per capita measured as of 2010, where the weights are x_{ckf} (without the shocks). The coefficient on the adjusted export shock decreases, but it is still economically and statistically significant. The F-statistic is again smaller than the baseline suggesting that the added variable is a weak instrument for wages.[15] Finally, we add the weighted average of export shocks faced by firm's suppliers to column (6) of Table 3. This exercise checks if the results are driven by a correlation between the foreign demand shocks faced by firms and faced by its suppliers. If they were, then the exclusion restriction on our instrument would be violated. As expected, the foreign demand shocks faced by firm's suppliers raise their wages. But more importantly, the coefficient on the instrumented variable, buyer's wage, is very close to the baseline thus raising our confidence on the instrument.

In sum, Table 3 suggests that the demand for a firm's exports from rich countries increases the firm's wage and its supplier's wage. In Table 4, we investigate whether these increases arise through new workers and network connections or through existing ones. Recall that the export shock is constructed from changes between 2011-2012 and 2014-2015. Using matched employer-employee data, we observe the average wages in 2011-2012 of the workers that the firm hired between 2013 and 2015. In the first column of Panel A, we regress the log difference between these new workers' wages and firm f's average wage in 2011-2012 ($wage_f$) on the ExportShock_f^a. In Panel B, we take the log-difference between new workers and workers that left the firm in the period of our data. In both cases the coefficient is positive and statistically significant. A one standard deviation in ExportShock_f^a, 0.44, is associated with the hiring of workers that have about 0.8 percent higher wages than the firm's existing workers (0.44*0.0189 = 0.008) and 1.1 percent higher wages than the firm's former employees (0.44*0.0247 = 0.011).

The second and third columns repeat the exercise for the firm's new suppliers and

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[15]Recent papers discuss identification in empirical settings similar to ours, which rely on shift-share instruments. Borusyak et al. (2018), and Goldsmith-Pinkham et al. (forthcoming) propose methods to reveal which shares, or shifts, matter the most for the magnitude of the estimator. Adao et al. (2019) discuss inference-related issues. All three papers study empirical settings where the number of shares (weights) is small. Their concerns and proposed methods do not apply to our empirical setting, where the number of weights is larger than the number of observations. It is very unlikely that (i) a single variety (i.e. destination country and 4-digit HS product code pair) or a few varieties dominate our results, and (ii) the correlation of the export shock across product-destination pairs is small.

Table 4: Effects of Export Shock on Composition of Inputs

Panel A Log of	Average wage of new workers relative to all workers at $t = 0$	Average wage paid by new suppliers relative to all suppliers at $t = 0$	Average wage paid by new buyers relative to all buyers at $t = 0$
ExportShock_f	0.0189	0.0241	0.0303
	(0.010)	(0.007)	(0.009)
R^2	0.0531	0.0439	0.0434
Panel B Log of	Average wage of new workers relative to former workers at $t = 0$	Average wage paid by new suppliers relative to former suppliers at $t = 0$	Average wage paid by new buyers relative to former buyers at $t = 0$
ExportShock_f	0.0247	0.0220	0.0305
	(0.009)	(0.012)	(0.009)
R^2	0.0542	0.0662	0.0683
N	33157	33157	33157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. ExportShock_f is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. Time $t = 0$ represents the period before the export shock, 2011-2012. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

new customers.[16] The coefficients are again positive and statistically significant, and they have similar magnitudes as the first column. So, the firm's new suppliers and customers paid on average in 2011-2012 wages that were higher than the wages in the firm's initial trading partners during the same period. Our interpretation is that an increase in rich countries' demand for a firm's output leads the firm to upgrade its quality. The firm then hires more skilled workers, and it adds skill-intensive suppliers and customers to its network connections.

Table 5 verifies that the results in Table 4 is not driven by a few outliers in firms' new connections. We regress the export shock on the share of newly hired workers after the shock, who received higher monthly wages than the firm's average worker before the shock. The second and third columns have the corresponding shares for the firm's new suppliers and new customers. The coefficients are all positive and statistically significant. The shares of new connections with wages higher than the existing workers, suppliers and customers is positively associated with the export shock, after controlling for industry-

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[16]We cannot measure the weights $s_{\omega f}$ that the firm would have placed on new suppliers in the initial year or the equivalent weights of the firm's revenue across new customers. So, we use the unweighted average of equation (3) for all three supplier groups, new, initial, and former, and all three customer groups.

Table 5: Effects of Export Shock on Composition of Inputs: Additional evidence

Share of new	Workers with wages higher than f's avg. wage at $t = 0$	Suppliers with wages higher than f's avg. supplier wage at $t = 0$	Buyers with wages higher than f's avg. buyer wage at $t = 0$
ExportShock$_f$	0.421	0.152	0.169
	(0.154)	(0.0690)	(0.0657)
R^2	0.167	0.0403	0.0394
N	33157	33157	33157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage is defined as the average value of monthly payments per worker. ExportShock$_f$ is a weighted average of changes in (real per capita) income-adjusted imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. Time $t = 0$ represents the period before the export shock, 2011-2012. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

province fixed effects.

2.4 Other Characteristics of the Network

We present three other features of the data governing our modelling choices. First, firm sales is the most important indicator of the number of suppliers and customers of a firm. Table 6 reports the endogenous elasticity of number of customers and suppliers with respect to firm sales. Firm sales explain about a third of variation in the number of buyers, and more than 60 percent of variation in the number of suppliers (R-squared in columns (1) and (4)). Columns (2) and (5) add industry fixed effects, and columns (3) and (6) add also wages. The coefficient on wages is insignificant and does not change at all the R-squared of regressions in columns (2) and (5). In the model, more productive firms post more ads to find suppliers and customers. The number of customers and suppliers to a firm increases log linearly with sales, similar to Table 6.

Second, wholesale, retail and service industries account for almost half of domestic sales of manufacturing firms and their material purchases. But we do not observe the skill intensity of the materials purchased through these service intermediaries. So, we introduce to the model a service sector that aggregates manufacturing inputs into a homogeneous good. The service good is used both as an input in manufacturing production and as a final good. In estimating the model, we match the service shares in manufacturing purchases and sales from our network data.

Third, imports account for only 4 percent of total cost of inputs of a typical manu-

Table 6: Firm Sales and Network Connections

Number of	Customers			Suppliers		
	(1)	(2)	(3)	(4)	(5)	(6)
$\log Sales_f$	0.440	0.462	0.459	0.577	0.593	0.590
	(0.016)	(0.013)	(0.013)	(0.011)	(0.009)	(0.009)
$\log Wage_f$			0.278			0.208
			(0.211)			(0.175)
R^2	0.328	0.472	0.472	0.609	0.645	0.645
N	77,418	77,418	77,418	77,418	77,418	77,418
Fixed effects		Ind	Ind		Ind	Ind

Notes: Wage is defined as the average value of monthly payments per worker. All variables are in logarithms. Ind refers to 4-digit NACE industries and provinces. Robust standard errors are clustered at 4-digit NACE industry level.

facturing firm in our data, compared to a 10 percent share of exports in its total sales. Accordingly, in the open economy model of Section 4, we model manufacturing firms' decisions to export. The estimation uses cross-sectional moments on exporting as well as the wage responses to export shocks in Table 3. But for simplicity, we assume that manufacturing firms cannot directly import, although they indirectly use a foreign service bundle.[17]

We conclude with a brief point on quality measures. Quality in our model is a latent variable that changes the firm's production function, increasing the relative marginal product of skilled workers and skill-intensive inputs. Kremer (1993) refers to this variable intermittingly as quality or complexity. But our emphasis, like his, is in the complementarity between skilled workers in production. Even if we observed prices of transactions in our firm network, it is not clear that standard measures of quality would be superior to wages in capturing the facts above. Since we cannot answer this question with our data, we leave it for future work. Nevertheless, we do observe prices for a small subset of the data: Foreign sales of exporting firms. For these data, Appendix G confirms the positive relation between firm wages and two measures of quality: unit values and the measure by Khandelwal et al. (2013) which uses information on prices and quantities per destination.[18]

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[17]We replicated the moments in Section 2.3 for import shocks, symmetric to export shocks, and found mostly insignificant effects. Possibly, this (lack of) finding arises because only a small share of manufacturing firms import their inputs directly.

[18]These measures are positively correlated. In our estimation, we use moments based on quintiles of firm wages, and the appendix documents a significant overlap between the grouping of firms by these quality measures and by wages.

3   The Closed-Economy Model

The model captures positive assortative matching, at the intensive and extensive margins, in a network endogenously formed through search and matching. To highlight these novel features, we present a closed economy.

There are two sectors: Services and manufacturing. The service sector is perfectly competitive. It produces a homogeneous good with constant returns to scale using manufacturing inputs. The manufacturing sector has heterogeneous firms and free entry.

Each manufacturing firm chooses its quality q from a line segment $Q \subset \mathbb{R}_+$. This choice determines the firm's production function, the marginal product of its labor and material inputs. All tasks performed in a firm of quality $q \in Q$ are also indexed by q. For example, if q is associated with management practices or an integrated computer software, all workers in production or not need to abide by such practices and use the software. Earnings per worker and the marginal product of higher-q inputs may be higher in the production of higher-q output.

Manufacturing firms post ads to find suppliers and customers and are matched to form the firm-to-firm network. Firms may imperfectly direct these ads to other firms' quality levels. Like Lim (2018), each firm is matched with a continuum of suppliers and customers, and it charges the monopolistic-competition markup.

The manufacturing sector is in Section 3.1. Section 3.1.1 sets up the firm's problem, and Section 3.1.2 aggregates firm choices to form the network. The service sector is in section 3.2, and the equilibrium is in section 3.3. Section 3.4 presents key properties of the model. Whenever convenient, we assume functions are continuous, differentiable, and integrable. Parametric assumptions in the estimation ensure these conditions.

3.1   Manufacturing

3.1.1   Entry and the Firm's Problem

The revenue of a firm with quality q, price p and a mass v of ads to find customers (v stands for visibility) is

$$p^{1-\sigma}vD(q) \tag{7}$$

where $\sigma > 1$ is the elasticity of substitution between manufacturing varieties and $D(q)$ is an endogenous demand shifter.

The cost of a bundle of inputs to produce quality q when the firm posts a measure m

of ads to find manufacturing suppliers is

$$C(m,q) = w(q)^{1-\alpha_m-\alpha_s} P_s^{\alpha_s} [m^{1/(1-\sigma)} c(q)]^{\alpha_m} \tag{8}$$

where $\alpha_m, \alpha_s > 0$ are Cobb-Douglas weights with $\alpha_m + \alpha_s \in (0,1)$, P_s is the price of the service good, $w(q)$ is the wage rate per efficiency unit of task q, and $c(q)$ is the cost of a bundle of manufacturing inputs when the firm posts a measure one of ads to find suppliers. The marginal cost of the firm is $C(m,q)/z$ where z is her productivity.

The cost of posting v ads to find customers and m ads to find suppliers is respectively

$$w(q)f_v \frac{v^{\beta_v}}{\beta_v}$$
$$w(q)f_m \frac{m^{\beta_m}}{\beta_m} \tag{9}$$

where f_m, f_v, β_m, and β_v are positive parameters with $\beta_v > 1$, $\beta_m > \alpha_m$.

From (7), the firm charges markup $\sigma/(\sigma-1)$ over marginal cost. Given q, she chooses v, m to maximize profit:

$$\max_{v,m} \frac{vm^{\alpha_m}}{\sigma} \left[\frac{\sigma}{\sigma-1} \frac{C(1,q)}{z} \right]^{1-\sigma} D(q) - w(q)f_v \frac{v^{\beta_v}}{\beta_v} - w(q)f_m \frac{m^{\beta_m}}{\beta_m} \tag{10}$$

Rearranging the first order conditions, the firm's revenue x, mass of ads to find customers v and to find suppliers m, and price p are functions of productivity z and quality q:

$$x(z,q) = \Pi(q) z^{\gamma(\sigma-1)}$$
$$v(z,q) = \left(\frac{x(z,q)}{\sigma f_v w(q)} \right)^{1/\beta_v}$$
$$m(z,q) = \left(\frac{x(z,q)}{\sigma f_m w(q)/\alpha_m} \right)^{1/\beta_m}$$
$$p(z,q) = \frac{\sigma}{\sigma-1} \frac{C(m(z,q),q)}{z} \tag{11}$$

where

$$\Pi(q) = [\sigma w(q)]^{1-\gamma} \left[D(q) \left(\frac{\sigma}{\sigma-1} C(1,q) \right)^{1-\sigma} \left(\frac{f_m}{\alpha_m} \right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v} \right]^{\gamma} \tag{12}$$
$$\gamma = \frac{\beta_v \beta_m}{\beta_v(\beta_m - \alpha_m) - \beta_m} > 1.$$

The elasticity of revenue $x(z,q)$ with respect to productivity z is $\gamma(\sigma-1)$. It is greater than $(\sigma-1)$ because more productive firms post more ads m and v.

Entry and Technology Choice A large mass of entrepreneurs may pay f units of the service good to create a new variety. Upon entry, each entrepreneur draws, independently from a common distribution, a random variable ω that determines her productivity at each $q \in Q$ through a function $z(q,\omega)$. We parameterize $\omega = (\omega_0, \omega_1) \in \mathbb{R}^2$ and

$$z(q,\omega) = \exp\left\{\omega_0 + \omega_1 \log(q) + \overline{\omega}_2[\log(q)]^2\right\} \tag{13}$$

where $\overline{\omega}_2$ is a parameter common to all firms. Since profit (10) is a share $1/(\gamma\sigma)$ of revenue, firm ω chooses q to maximize revenue:

$$q(\omega) = \arg\max_{q \in Q}\left\{x(z(q,\omega),q)\right\}. \tag{14}$$

Function $\Pi(q)$ is by construction (below) continuous in q so that (14) is the maximization of a continuous function in a compact set Q.

Let N be the equilibrium mass of firms, and take total manufacturing absorption as the numeraire. Then, average sales per firm is $1/N$ and free entry implies

$$N = (\gamma\sigma f P_s)^{-1}. \tag{15}$$

3.1.2 Manufacturing firm-to-firm trade

Firm choices above give rise to the measure

$$J(z,q) = N\mathrm{Prob}\left\{\omega : z(q(\omega),\omega) \leq z \quad \text{and} \quad q(\omega) \leq q\right\}. \tag{16}$$

Assume J has a density, denoted with $j(z,q)$. Next we put structure in the model to derive the endogenous terms in $\Pi(q)$ as functions of J and firm outcomes in (11). In this section, manufacturing firm-to-firm trade determines the input cost $c(q)$ and the component of demand $D(q)$ that comes from manufacturing.

Production Function Following Fieler et al. (2018), a firm of quality q matched with a set of suppliers Ω aggregates its manufacturing inputs with a constant elasticity of substitution (CES) function:

$$Y(q,\Omega) = \left[\int_{\omega \in \Omega} y(\omega)^{(\sigma-1)/\sigma}\phi_y(q,q(\omega))^{1/\sigma}d\omega\right]^{\sigma/(\sigma-1)} \tag{17}$$

where $y(\omega)$ is the quantity of input ω and function $\phi_y(q, q')$ governs the productivity of an input of quality q' when producing an output of quality q. We parameterize

$$\phi_y(q, q') = \frac{\exp(q' - \nu_y q)}{1 + \exp(q' - \nu_y q)}, \tag{18}$$

which is increasing in input quality and decreasing in output quality if $\nu_y > 0$. It is also log-supermodular if $\nu_y > 0$. Then, the ratio of the firm's demand for any two inputs 1 and 2 with prices $p(1)$ and $p(2)$ and qualities $q(1) > q(2)$,

$$\frac{y(1)}{y(2)} = \left(\frac{p(1)}{p(2)}\right)^{-\sigma} \frac{\phi_y(q, q(1))}{\phi_y(q, q(2))}, \tag{19}$$

is strictly increasing in the producing firm's quality q. Higher-quality firms spend relatively more on higher-quality firms for any set of input suppliers.

Network We introduce directed search. Buyers can only see the selling ads that are directed to their own q. The ads posted by a seller with quality q' are distributed across buyers' qualities $q \in Q$ according to function $\phi_v(q, q')$ which we parameterize as the density of a normal distribution with variance parameter ν_v and mean q', the quality of the seller posting the ads.[19] In one of the robustness checks on the estimation and counterfactual, sellers choose the direction of search by choosing the mean of ϕ_v, and potentially, the cost of posting ads increases with the distance between the chosen mean of ϕ_v and the seller's quality. Here, we assume an exogenous direction of search for simplicity.

This set up implies that there's a continuum of matching submarkets, one for each buyer quality. In the submarket of buyers with quality $q \in Q$, the total measure of ads posted by buyers and sellers is respectively:

$$M(q) = \int_Z m(z, q) j(z, q) dz \tag{20}$$

$$V(q) = \int_Q \phi_v(q, q') \overline{V}(q') dq' \tag{21}$$

where $\overline{V}(q)$ is the measure of ads posted by sellers of quality q:

$$\overline{V}(q) = \int_Z v(z, q) j(z, q) dz.$$

---

[19]One dimension of directed search, whether from buyers or sellers, is enough to generate assortative matching at the extensive margin.

A standard matching function ([Petrongolo and Pissarides (2001)](#)) determines measure of matches with buyers of quality q:

$$\tilde{M}(q) = V(q)\left[1 - \exp(-\kappa M(q)/V(q))\right]. \tag{22}$$

where parameter $\kappa > 0$ captures the efficiency in the matching market. The success rate of ads is $\theta_v(q) = \tilde{M}(q)/V(q)$ for sellers and $\theta_m(q) = \tilde{M}(q)/M(q)$ for buyers.

Input Costs and Demand Using (21), for each ad posted by a buyer of quality q, the probability of finding a supplier with productivity-quality (z', q') is

$$\theta_m(q)\frac{\phi_v(q,q')v(z',q')j(z',q')}{V(q)} \tag{23}$$

Combining with the CES price associated with production function (17), a bundle of manufacturing inputs used by a firm of quality q posting a measure one of ads to find suppliers costs:

$$c(q) = \left[\frac{\theta_m(q)}{V(q)}\int_Q \phi_y(q,q')\phi_v(q,q'))P(q')^{1-\sigma}dq'\right]^{1/(1-\sigma)} \tag{24}$$

where

$$P(q) = \left[\int_Z p(z,q)^{1-\sigma}v(z,q)j(z,q)dz\right]^{1/(1-\sigma)} \tag{25}$$

takes into account the greater visibility of firms that post more selling ads $v(z,q)$.

We now turn to demand. A firm with quality q posts price p and v selling ads. From (20), the measure of buyers with (z', q') matched to the firm is

$$v\theta_v(q')\phi_v(q',q)\frac{m(z',q')j(z',q')}{M(q')}$$

Conditional on the match, the firm's sales to a buyer with (z', q') is

$$\phi_y(q',q)\left(\frac{p}{c(q')}\right)^{1-\sigma}\frac{\alpha_m(\sigma-1)}{\sigma}\frac{x(z',q')}{m(z',q')}$$

Multiplying these last two expressions and summing over buyers (z', q'), the sales of

the firm to other manufacturing firms is

$$p^{1-\sigma} v D(q)$$

$$\text{where} \quad D_m(q) = \int_Q \frac{\theta_v(q')}{M(q')} \phi_y(q',q) \phi_v(q',q) c(q')^{\sigma-1} X_m(q') dq', \tag{26}$$

$$X_m(q) = \frac{\alpha_m(\sigma-1)}{\sigma} \int_Z x(z,q) j(z,q) dz \tag{27}$$

$X_m(q)$ is the total absorption of manufacturing inputs by buyers of quality q.[20]

3.2 Service Sector and Final Demand

Service firms aggregate manufacturing inputs into a homogeneous good sold in a perfectly competitive market. Their production function is given by $Y(0, \Omega)$ in (17). There's a fixed set of service firms, each endowed with a fixed measure of $\overline{m}$ of manufacturing suppliers.[21] The probability that a service firm matches with a supplier with productivity-quality (z, q) is proportional to the measure of selling ads:

$$\frac{v(z,q) j(z,q)}{V_T}$$

$$\text{where} \quad V_T = \int_Q \overline{V}(q) dq \tag{28}$$

Then, the price index of the service good is

$$P_s = \left[\frac{\overline{m}}{V_T} \int_Q \phi_y(0, q) P(q)^{1-\sigma} dq \right]^{1/(1-\sigma)} \tag{29}$$

Total sales to the service sector by a manufacturing firm with price p, quality q, posting

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[20]We may also derive $D_m(q)$ from (24). The share of spending on materials by buyers of quality q' allocated to a supplier with price p, quality q, and v ads is

$$\theta_m(q') \frac{\phi_y(q',q) \phi_v(q',q) v p^{1-\sigma}}{V(q) c(q')^{1-\sigma}}.$$

Multiplying by domestic spending on materials $X_m(q')$ and integrating over buyers q', demand is

$$v p^{1-\sigma} \int_Q \frac{\theta_m(q')}{V(q')} \phi_y(q',q) \phi_v(q',q) c(q')^{\sigma-1} X_m(q') dq'$$

which is the expression above since $\theta_m(q)/V(q) = \theta_v(q)/M(q)$.

[21]Parameter $\overline{m}$ preserves the log linear form of demand in (7). Ads posted by sellers v would be irrelevant if service firms observed all varieties. Making the service sector more symmetric to manufacturing, with imperfect competition, and costly matches, would complicate the model without new insights.

v ads in Home to find customers is:

$$\frac{v}{V_T}\left(\frac{p}{P_s}\right)^{1-\sigma}\overline{m}\phi_y(0,q)X_s$$

where X_s is total absorption of services. Using (29), these sales are

$$p^{1-\sigma}vD_s(q) \tag{30}$$

$$\text{where}\quad D_s(q) = \phi_y(0,q)\left[\int_Q \phi_y(0,q')P(q')^{1-\sigma}dq'\right]^{-1}X_s$$

They do not depend on $\overline{m}$.

Households consume only the service good. Then service absorption X_s is the share of manufacturing absorption in (10) allocated to service and labor inputs plus profits:

$$X_s = 1 - \frac{(\sigma-1)}{\sigma}\alpha_m.$$

3.3 Equilibrium

The demand shifter faced by a manufacturing firm in (7) is the sum of demand from service (30) and other manufacturing firms (26):

$$D(q) = D_m(q) + D_s(q). \tag{31}$$

We take the supply of efficiency units of labor to produce task q to be an exogenous function $L(q,w)$ where w is the whole wage schedule, $w(q)$ for all $q \in Q$. Labor markets clear if

$$L(q,w) = \frac{1}{w(q)\sigma}\left[(1-\alpha_m-\alpha_s)(\sigma-1)+1-\frac{1}{\gamma}\right]\int_Z x(z,q)j(z,q)dz \tag{32}$$

where the constant is the labor share in manufacturing production in (10). In our empirical application, we assume that average earnings per firm is strictly increasing in q. Using a Roy (1951) model, Teulings (1995) provides a micro foundation for $L(q,w)$ and for this estimation assumption (see Appendix A).[22]

Definition An **equilibrium** is a mass of firms N, a measure function $J(z,q)$, and functions $w(q)$, $\theta_m(q)$, $\theta_v(q)$, $c(q)$, $D(q)$ satisfying the following conditions:

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[22]See also Costinot and Vogel (2010) for an application of Teulings (1995) to international trade.

1. Free entry (15).

2. Labor market clearing (32).

3. Firms maximize profits. Firm ω chooses $q(\omega)$ in (14) and has productivity $z(\omega) = z(q(\omega), \omega)$ at the optimal. Its sales, measure of ads, and prices are $x(z(\omega), q(\omega))$, $m(z(\omega), q(\omega))$, $v(z(\omega), q(\omega))$, and $p(z(\omega), q(\omega))$ in (11).

4. The measure $J(z, q)$ is consistent with firm choices (16).

5. The success rate of ads $\theta_m(q) = \tilde{M}(q)/M(q)$ and $\theta_v(q) = \tilde{M}(q)/V(q)$ where $M(q)$, $V(q)$ and $\tilde{M}(q)$ are in (20), (21) and (22). Functions $c(q)$ and $D(q)$ satisfy (24) and (31).

3.4 Properties of the Network

As mentioned in the introduction, two aspects of the model distinguishes it from previous network models: Its use of log-supermodular functions to capture assortative matching, and the particular search-and-matching set up of network formation. We explain these properties in Sections 3.4.1 and 3.4.2 respectively.

3.4.1 Assortative Matching

Under the assumption that earnings per worker is increasing in firm quality, assortative matching in the model's network arises through buyers' and sellers' quality levels.

For a firm with quality q, the measure of its input suppliers of quality q_1 relative to input suppliers of quality q_2 is (integrating (23)):

$$\frac{\phi_v(q, q_1)\overline{V}(q_1)}{\phi_v(q, q_2)\overline{V}(q_2)} \tag{33}$$

The firm's average spending on its suppliers of quality q_1 relative to its suppliers of quality q_2 is (integrating (19)):

$$\frac{\phi_y(q, q_1)}{\phi_y(q, q_2)} \left(\frac{P(q_1)}{P(q_2)}\right)^{1-\sigma} \frac{\overline{V}(q_2)}{\overline{V}(q_1)} \tag{34}$$

Multiplying these expressions (or using equation (24)), the ratio of the firm's total spending on the two qualities is:

$$\frac{\phi_v(q, q_1)}{\phi_v(q, q_2)} \frac{\phi_y(q, q_1)}{\phi_y(q, q_2)} \left(\frac{P(q_1)}{P(q_2)}\right)^{1-\sigma} \tag{35}$$

25

These expressions summarize the extensive margin (33), intensive margin (34) and total (35) assortative matching in the network. Since the terms $\overline{V}(q)$ and $P(q)$ are common to all buyers, functions ϕ_y and ϕ_v alone govern assortative matching. By definition, a function ϕ is log-supermodular if $\phi(q, q_1)/\phi(q, q_2)$ is increasing in q whenever $q_1 > q_2$ or equivalently $\partial^2 \log(\phi(q, q'))/\partial q \partial q' > 0$. Function $\phi_v(q, q')$ governs the distribution of selling ads posted by suppliers with quality q' across buyers of quality q. If ϕ_v is log-supermodular, then higher-quality firms have relatively more higher-quality suppliers in (33). We parameterize ϕ_v as the density of a normal random variable with variance ν_v. Its derivative $\partial^2 \log(\phi_v(q, q'))/\partial q \partial q' = 1/\nu_v$ tends to zero if firms do not direct their search, $\nu_v \to \infty$. Function $\phi_y(q, q')$ governs the marginal product of an input of quality q' in the production of output quality q. If it is log-supermodular ($\nu_y > 0$ in (18)), then higher-quality firms spend relatively more on each of its higher-quality suppliers in (34).

3.4.2 Search and Matching

We consider a special case of the model to highlight the tractability of our network formation set up and its connection to Melitz (2003), the standard monopolistic competition model with heterogeneous firms. Assume there is only one quality and $\beta_v = \beta_m \equiv \beta$. We set $\phi_v = \phi_y = 1$ without loss of generality and drop the quality arguments from functions.

With $\beta_v = \beta_m$, the ratio of ads to find suppliers and customers in (11) is the same for all firms. Then, the probabilities of success of ads to find suppliers and customers reduce to functions of exogenous variables:

$$\theta_m = \left(\frac{f_m}{\alpha_m f_v} \right)^{1/\beta} \left[1 - \exp\left(-\kappa \left(\frac{\alpha_m f_v}{f_m} \right)^{1/\beta} \right) \right]$$

$$\theta_v = \left[1 - \exp\left(-\kappa \left(\frac{\alpha_m f_v}{f_m} \right)^{1/\beta} \right) \right].$$

With only one quality, price indices c in (24) and P_s in (29) are

$$c = \left(\frac{\theta_m}{V} \right)^{1/(1-\sigma)} P$$

$$P_s = \left(\frac{\overline{m}}{V} \right)^{1/(1-\sigma)} P \tag{36}$$

26

Demand functions D_m in (26) and D_s in (30) become

$$D_m = P^{\sigma-1} \frac{\alpha_m(\sigma-1)}{\sigma}$$
$$D_s = P^{\sigma-1} \left[1 - \frac{\alpha_m(\sigma-1)}{\sigma}\right]$$

So that $D = P^{\sigma-1}$, as in Melitz (2003). Combining this expression with (7) and (25),

$$P = \left(\frac{\Pi}{D} N \mathbb{E}(z^{\gamma(\sigma-1)})\right)^{1/(1-\sigma)}$$
$$\Rightarrow \quad \Pi = \left[N \mathbb{E}(z^{\gamma(\sigma-1)})\right]^{-1}$$

The sales of a firm with productivity z becomes

$$x(z) = \frac{z^{\gamma(\sigma-1)}}{N \mathbb{E}(z^{\gamma(\sigma-1)})}$$

This is exactly the same expression for sales as in Melitz (2003) except for the added parameter $\gamma > 1$. The elasticity of sales with respect to firm productivity is larger than $(\sigma-1)$ because more productive firms post more ads to find suppliers and customers and hence sells more than in a model where all firms sell to all firms and prices are proportional to productivity. But the model captures a key feature of the network that larger firms have more trading partners. The endogenous elasticity of number of customers with respect to sales and of number of suppliers with respect to sales is $1/\beta$ in the model. In the data, both of these elasticities are close to 0.5 (see Table 6).[23]

3.5 Estimation Strategy of the Closed Economy

We calibrate some parameters and propose an indirect inference procedure to estimate the remaining parameters. In this subsection, we provide the basic intuition of the identification of our key structural parameters. We modify the estimation procedure and implement it only in the open economy. An economy is defined by parameters $\{\alpha_m, \alpha_s, \sigma, f_m, f_v, \beta_m, \beta_v, f, \overline{m}, \kappa, \nu_y, \nu_v, \overline{\omega}_2\}$, the bivariate distribution of firms' productivity parameters (ω_0, ω_1) in (13), and labor supply $L(q, w)$.

We calibrate $\{\alpha_m, \alpha_s, \sigma, f_m, f_v, \beta_m, \beta_v, f, \overline{m}\}$. We set $\alpha_m = 0.33$ and $\alpha_s = 0.38$ in (8)

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[23]Appendix B presents the full solution for this special case with two additional assumptions: The labor supply L is exogenous, and the entry cost in (15) uses labor instead of the service good. These assumptions are not essential to obtain closed-form expressions, but they facilitate the derivation of the price index and the comparison to standard models.

to the cost shares of services and manufacturing in the Turkish manufacturing sector. The elasticity of substitution $\sigma = 5$ following Broda and Weinstein (2006). Since search efforts are not observable, we cannot separately identify the cost of one ad, f_m and f_v, from the matching efficiency κ in (22). We then set $f_m = f_v = 1$. We set $\beta_m = 1/0.59$ and $\beta_v = 1/0.46$ to match the endogenous elasticity of number of suppliers and number of customers with respect to firm sales in Table 6.[24] Parameter $\overline{m}$ is not identified because it governs the theoretical price index P_s in (29) but not the observable sales of manufacturing to service firms and consumers in (30). We pick $\overline{m}$ so that equilibrium $P_s = 1$. We observe worker earnings, but not endowments or wage per efficiency unit of labor. In a cross-section we can set $w(q) = 1$ for all q by judiciously picking the measure of efficiency units of labor. We normalize the equilibrium mass of firms $N = 1$. Each firm has a weight $1/100{,}000$, where $100{,}000$ is the number of firms simulated. With $N = P_s = 1$, the entry cost in (15) is $f = (\gamma\sigma)^{-1} = 0.069$.

While we estimate all the model parameters jointly, it is useful to think of our model solution and parameter identification in two distinct blocks. The inner loop takes the equilibrium distribution of productivity-quality $J(z, q)$ as given. It solves the matching and product market equilibrium given firm's optimal search and production decisions. Each firm ω's profitability can then be summarized by $z(q, \omega)^{\gamma(\sigma-1)}\Pi(q)$ in (14), where $\Pi(q)$ is the quality q specific profit shifter solved from the inner block. The outer loop then solves the optimal quality choice for each firm ω. We sample firms from a bivariate distribution $\omega = (\omega_0, \omega_1)$ which determines each firm's productivity at each quality, $z(q, \omega)$ in (13). We will then iterate until firm's optimal quality choice is consistent with the conjectured endogenous equilibrium productivity-quality distribution $J(z, q)$.

3.5.1 Parametrization

We estimate $\kappa, \nu_y, \nu_v, \bar{\omega}_2$, the firm ability $\omega \equiv (\omega_0, \omega_1)$ distribution using the method of simulated moments. (ω_0, ω_1) are bivariate normally distributed, with standard deviation parameters $(\sigma_{\omega_0}, \sigma_{\omega_1})$ and correlation parameter ρ. We simulate the economy for each guess of these seven parameters $\{\kappa, \nu_y, \nu_v, \bar{\omega}_2, \sigma_{\omega_0}, \sigma_{\omega_1}, \rho\}$ and solve for the equilibrium distribution $J(z, q)$.

3.5.2 Inner Loop

We start with a brief discussion of how we solve the inner loop (i.e. matching and product market equilibrium) conditional on the joint distribution $J(z, q)$. The identification of its

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[24]In the data and model, sales are the largest indicator of a firm's number of trading partners so that ignoring wages (or q) provides a good approximation.

related parameters $\{\kappa, \nu_y, \nu_v, \sigma_{\omega_0}, \sigma_{\omega_1}, \rho\}$ follows.

Simulation procedure Start with an initial guess of $c(q) > 0$ and $D(q) > 0$ for all q in the grid and follow steps 1-4:

1. Calculate $C(1, q)$ in (8) and $\Pi(q)$ in (12).

2. Use firm outcomes (11) to calculate aggregate mass of ads $M(q)$ and $V(q)$ in (20) and (21), the mass of matches $\tilde{M}(q)$ in (22) and get the success rates $\theta_m(q)$ and $\theta_v(q)$. Calculate spending on materials $X_m(q)$ in (27) and price indices $P(q)$ in (25).

3. Update the guesses of $c(q)$ and $D(q)$ using (24) and (31).

4. Repeat steps 1-3 until functions $c(q)$ and $D(q)$ converge.

The simulation yields the total demand for efficiency units of labor for all $q \in Q$ in (32). We could always pick the total supply $L(q, w)$ to match demand for each q, and the endowment per worker in firms with quality q to match the earnings per worker in a firm with wage rank in the data equal to the quality rank of q. See Appendix A for this procedure in the Roy model of Teulings (1995).

Moments We match 28 moments. By quintile of firm wage, we match:

1. The mean number of suppliers (5 moments) and mean number of customers (5 moments)

2. The share in total network sales (5 moments) and the standard deviation of sales (5 moments).

3. Average of log-wage of suppliers, unweighted (4 moments) and weighted by spending shares (4 moments).[25]

Identification Although all parameters are estimated jointly, some parameters are associated to some moments more closely. The average number of trading partners per firm identifies κ, the efficiency in transforming ads into matches in (22). Total sales and standard deviation by quintile of quality identifies the parameters σ_{ω_0}, σ_{ω_1}, and ρ.

The third set of moments summarize nonparametrically the total and extensive margins of assortative matching in the network in Table 2. As described in Section 3.4, parameters ν_y govern the intensive margin in (34) through the log-supermodularity of ϕ_y. Parameter ν_v governs the extensive margin (33) through the log-supermodularity of ϕ_v.

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[25]The third set of moments are only four (and not oner per quintile) because we normalize the wages in the lowest quintile to 0 and match the log difference from the lowest quintile in the data and model.

3.5.3 Outer Loop

In the outer loop, firms make optimal quality choice such that the conjectured measure $J(z,q)$ is consistent with all the firm's decisions. The key parameter that governs firm's decision at this stage is the efficiency-quality trade-off $\overline{\omega}_2$. We discuss the identification of $\overline{\omega}_2$ below.

Recall that we parameterize firm productivity in equation (13) as

$$\log z(q,\omega) = \omega_0 + \omega_1 \log(q) + \overline{\omega}_2 [\log(q)]^2$$

where $\overline{\omega}_2$ is a parameter and ω_0 and ω_1 are firm specific. Substituting $z(q,\omega)$ into the firm's quality choice in (14), we have

$$q(\omega) = \arg\max_{q \in Q} \left\{ \gamma(\sigma - 1) \left[\omega_0 + \omega_1 \log(q) + \overline{\omega}_2 [\log(q)]^2 \right] + \log \Pi(q) \right\}$$

Consider any productivity-quality pair (z^*, q^*) with q^* in the interior of Q. The firm ω^* that corresponds to such pair satisfies $z(q^*, \omega^*) = z^*$ and the first order condition:

$$\exp \left[\omega_0^* + \omega_1^* \log(q^*) + \overline{\omega}_2 [\log(q^*)]^2 \right] = z^* \tag{37}$$

$$\gamma(\sigma - 1) \left[\omega_1^* + 2\overline{\omega}_2 \log(q^*) \right] + \frac{\partial \log \Pi(q^*)}{\partial \log(q^*)} = 0 \tag{38}$$

The second order sufficient conditions are

$$2\gamma(\sigma - 1)\overline{\omega}_2 + \frac{\partial^2 \log \Pi(q)}{\partial (\log(q))^2} \leq 0 \qquad \text{for all } q. \tag{39}$$

For any $\overline{\omega}_2$ satisfying (39) and any (z^*, q^*), we can find (ω_0^*, ω_1^*) that satisfies (37) and (38). So, firm ω^* produces output of quality q^* with efficiency z^* in equilibrium.

Two points are in order. First, parameter ω_1 governs the firm's quality choice in (38), and ω_0 governs its productivity at the chosen quality in (37). So, the model needs firm heterogeneity in both dimensions to match the joint distribution of wages (quality rank) and sales.

Second, parameter $\overline{\omega}_2$ is not identified with the cross-sectional distribution of sales and wages. It will need to be informed by the elasticity of firms choices of q with respect to shocks to the economy. Denote the model fundamentals of the economy as Θ and consider a shock that affects an element Θ_i for a single firm ω. The first order condition

(38) implicitly defines the firm's optimal choice $q(\omega)$ as a function of parameter Θ_i:

$$\frac{\partial \log q(\omega)}{\partial \Theta_i} = -\frac{\frac{\partial^2 \log \Pi(q(\omega))}{\partial \log q \partial \Theta_i}}{2\gamma(\sigma-1)\overline{\omega}_2 + \frac{\partial^2 \log \Pi(q(\omega))}{\partial(\log(q))^2}} \tag{40}$$

where the denominator is the second order condition (39) evaluated at the optimal $q(\omega)$. The firm is infinitely elastic to the shock if the second order condition holds with equality and infinitely inelastic as it approaches negative infinity. In the open economy, we interpret the export shocks in Table 3 as such idiosyncratic shocks. Our regression coefficients of how exporter wage responded to the export shocks can be mapped into $\partial \log q(\omega)/\partial\Theta_i$. We can also use our model-based economy to compute the derivatives of $\Pi(q)$. We can then apply (40) to estimate $\overline{\omega}_2$. A key assumption is that the shock does not affect other firms. Otherwise, would affect Π not only directly in the firm's problem, but through other firm's choices in measure J.

4 Open Economy

We embed the model above into a small open economy. The distinctions arise mainly in the manufacturing firm's problem below. Section 4.1 sketches the estimation procedure. Appendix C presents the full model and details the estimation procedure.

Given our empirical focus on exports, we do not model imports of manufactures. Manufacturing firms may export by paying a fixed cost, posting ads abroad and facing an exogenous foreign demand. To produce the final service good, service firms combine the bundle of domestic varieties with the bundle of foreign varieties with a constant elasticity of substitution σ_s. Foreign services cost eP_{Fs} where P_{Fs} is exogenous and e is the real exchange rate, which adjusts to balance trade.

Manufacturing firms A large mass of entrepreneurs may pay a fixed cost f to create a new manufacturing variety. Upon entry, an entrepreneur draws (ω_0, ω_1) determining her productivity $z(q, \omega)$ in (13). The entrepreneur chooses $q \in Q$ and then draws a random fixed export cost f_E units of the service good from a common distribution. She then decides her export status $E \in \{0, 1\}$, posts ads to search for domestic suppliers, for domestic customers, and for foreign customers if $E = 1$. We introduce randomness in the fixed cost of exporting because firms in the data with similar size and wages have different export status. The timing is chosen to facilitate aggregation in the estimation.

By backward induction, we start with the problem of the firm after it has chosen its

quality and export status. The revenue from foreign sales of an exporter with quality q, price p and v ads to find customers in foreign is

$$p^{1-\sigma} v e^\sigma D_F(q) \tag{41}$$

where $D_F(q)$ is an exogenous demand function. The cost of posting v ads in foreign is the same as the domestic cost in (9), $w(q) f_v v^{\beta_v}/\beta_v$. Assuming the same curvature β_v is important to maintain the log linearity in the firm's problem. We assume f_v only to simplify notation since f_v is not identified (Section 3.5).

A firm with quality q, productivity z and export status $E \in \{0,1\}$ chooses the mass of ads to find suppliers m, the mass of ads to find customers v and the share $r_v \in [0,1]$ of the selling ads that are posted domestically:

$$\max_{m,v,r_v} \frac{v m^{\alpha_m}}{\sigma} \left[\frac{\sigma}{\sigma-1} \frac{C(1,q)}{z} \right]^{1-\sigma} [r_v D_H(q) + (1-r_v) E e^\sigma D_F(q)]$$
$$- w(q) f_v [r_v^\beta + (1-r_v)^\beta] \frac{v^{\beta_v}}{\beta_v} - w(q) f_m \frac{m^{\beta_m}}{\beta_m} \tag{42}$$

where $C(1,q)$ is the input cost in (8) and $D_H(q)$ is the endogenous domestic demand shifter, denoted with $D(q)$ in the closed economy (equation (7)). The optimal share of ads r_v does not depend on productivity z:

$$\frac{1 - r_v(q,E)}{r_v(q,E)} = \left(\frac{E e^\sigma D_F(q)}{D_H(q)} \right)^{1/(\beta_v-1)} \tag{43}$$

Given the optimal r_v, problem (42) differs from the closed economy (10) only in the level of demand and cost of posting selling ads v. Then, the profit, labor and input shares are the same as in the closed economy, and the relationship between sales, ads and prices take the form of (11). Total sales is

$$x(z,q,E) = \Pi(q,E) z^{\gamma(\sigma-1)} \tag{44}$$

where

$$\Pi(q,E) = [\sigma w(q)]^{1-\gamma} \left[D(q,E) \left(\frac{\sigma}{\sigma-1} C(1,q) \right)^{1-\sigma} \left(\frac{f_m}{\alpha_m} \right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v} \right]^\gamma \tag{45}$$

$$D(q,E) = \left[D_H(q)^{\beta_v/(\beta_v-1)} + E(e^\sigma D_F(q))^{\beta_v/(\beta_v-1)} \right]^{(\beta_v-1)/\beta_v}.$$

Exporting increases the firm's profit by more than the sum of the profits from operating

separately in each market. The firm uses the same input suppliers for producing all its goods, independent of destination. So, exporting increases the firm's incentives to search for suppliers, which lowers price and in turn increases the firm's incentives to search for customers in both markets. The coefficients in the CES term $D(q, E)$ and γ capture these magnification effects.

The firm exports if its fixed cost parameter $f_E \leq \overline{f}_E(z, q)$ where

$$\overline{f}_E(z, q) = \frac{z^{\gamma(\sigma-1)}}{\gamma \sigma P_s} \left[\Pi(q, 1) - \Pi(q, 0) \right]. \tag{46}$$

Denote with Φ the cumulative distribution function of f_E. After observing its productivity $z(q, \omega)$ but before observing f_E, the firm chooses its quality:

$$q(\omega) = \arg\max_{q \in Q} \left\{ \frac{z(q, \omega)^{\gamma(\sigma-1)}}{\gamma \sigma} \left[\Pi(q, 1)\Phi\left(\overline{f}_E(z(q, \omega), q) \right) + \Pi(q, 0) \left[1 - \Phi\left(\overline{f}_E(z(q, \omega), q) \right) \right] \right] \right.$$
$$\left. - P_s \mathbb{E}(f_E | f_E \leq \overline{f}_E(z(q, \omega), q)) \right\} \tag{47}$$

Aggregation, Network, Equilibrium Appendix C makes exactly the same assumptions on production and network formation as in the closed economy. The only difference is that, because sales, mass of ads and prices depend on export status, aggregation in $M(q)$, $\overline{V}(q)$, $X_m(q)$, and $P(q)$ is over two measure functions:

$$\tilde{J}(z, q, 1) = J(z, q)\Phi\left(\overline{f}_E(z, q) \right)$$
$$\tilde{J}(z, q, 0) = J(z, q) \left[1 - \Phi\left(\overline{f}_E(z, q) \right) \right] \tag{48}$$

where $J(z, q)$ is the measure in (16). The equilibrium is also similarly defined with the additional equilibrium variable e and a trade equilibrium condition.

4.1 Estimation of the Open Economy

Appendix D presents an estimation procedure similar to Section 3.5. This procedure is viable due to the timing of information on the fixed export costs. The exporting threshold in (46) is used to derive measures $\tilde{J}(z, q, E)$ in (48), which are used to aggregate firm outcomes and generate the general equilibrium functions $c(q)$ and $D(q)$ in the inner loop.

The calibrated parameters $\{\alpha_m, \alpha_s, \sigma, f_m, f_v, \beta_m, \beta_v, f, \overline{m}\}$, wage $w(q) = 1$, and labor supply $L(q, \omega)$ are set similarly as in Section 3.5. The export market adds to the definition of the economy the foreign price of services P^*, foreign demand $D_F(q)$, the distribution of fixed costs of exporting f_E, and equilibrium real exchange rates e. The real exchange

rate e is not separately identified from foreign demand in (41). We thus set $e = P^* = 1$. We parameterize the distribution of fixed export cost f_E from a log-normal distribution with mean and standard deviation parameters μ_E and σ_E. We parameterize

$$D_F(q) = b_1 q^{b_2}$$

where b_1 and b_2 are parameters to be estimated.

We use the method of simulated moments to estimate $\{\kappa, \nu_y, \nu_v, \sigma_{\omega_0}, \sigma_{\omega_1}, \rho\}$ (as before) and the additional export-related parameters $\{b_1, b_2, \mu_E, \sigma_E\}$. We add 10 moments to the estimation: The share of firms exporting for each quintile of wage (5 moments), the average export intensity for exporting firms of each quintile of wage (5 moments). In all, there are 10 parameters and 38 moments.

Intuitively, parameter b_1 governs the level of export intensity while b_2 governs how export intensity changes across quintile of firm average wages. If b_2 is large, $D_F(q)/D_H(q)$ is increasing in q and export intensity increases with quintile of wages. Parameter μ_E governs the share of firms exporting and σ_E governs how this share changes across quintiles. If σ_E is large, then the share of firms exporting does not vary much across quintiles because it depends more on firm f_E draws than on wages and sales.

Estimation of $\bar{\omega}_2$ We estimate $\bar{\omega}_2$ using the shift-share regressions of Table 3. Fix a guess of $\bar{\omega}_2$ and the simulated distribution of (ω_0, ω_1). A shock that increases a single firm's export demand $D_F(q)$ by, say 5 percent, in general changes the firm's optimal quality $q(\omega)$ in (47). In particular, if $D_F(q)/D_H(q)$ were increasing in quality as in our estimated model, the firm increases $q(\omega)$. Since each quality in the grid is associated with an average earnings per worker in the data (the ranking is the same), the change in quality is also associated with a change in the firm's average earnings per worker, denoted with $\Delta^{\text{Shock}}(\omega)$.

We sample firms and estimate the expected effect from the idiosyncratic demand shocks in the model as the average $\Delta^{\text{Shock}}(\omega)$ weighted by firms' export probabilities. In the data, a 5 percent increase in a export demand increases the average wage per worker at the firm by 0.21 percent (Column (1) Table 3). We iterate over guesses of $\bar{\omega}_2$ to match this percentage change.

5 Estimation Results

We report the results of the estimation. The estimated parameters are in Table 7 and the targetted moments are in Table 8. Three sets of the parameters govern firm behavior and equilibrium outcomes.

The first set governs network formation: The degree of directed search ν_v, the complementarity of input and output qualities in production ν_y, and the matching friction κ. Parameter ν_v controls the precision of firm's effort in directing their search for the targeted quality segment. Our estimated value is 2.82. Combined with our parametric assumption for ϕ_v of normal density, it indicates that although more selling ads end up in the suppliers' own quality segment, it is far from perfect. For example, buyers of the lowest quintile of quality still get 9 percent of the search ads from the sellers in top quintile. This parameter determines the extensive margin of assortative matching in wages. The estimated complementarity in production $\nu_y = 0.42$. To interpret this parameter consider two suppliers charging the same price, one in the highest quintile of quality and one in the lowest quintile. Conditional on matching, a firm in the top quintile of quality spends 12.04 times more in the high-quality supplier than in the lowest quality, while a firm in the lowest quintile of quality spends only 5.76 times more. This parameter governs the intensive margin of assortative matching. Parameter $\kappa = 8.6 \times 10^{-4}$ indicates a low success rate of finding a business partner given the search effort. This is not surprising given that the mean number of supplier and customer in our data ranges from 5 to 25, a tiny fraction of all the potential partners out there in the manufacturing industry. Although only two parameters ν_v and ν_y govern assortative matching in the model, the model fits well the increasing relation between buyers' and sellers' wages, weighted and unweighted, in Table 8. The model also fits well the increasing number of suppliers and customers per firm by quintile of wages. Parameter κ and the positive correlation between sales and wages govern these moments in the model.

The second set of parameters $\sigma_{\omega_0}, \sigma_{\omega_1}, \rho$ determines the joint distribution of quality and productivity. In the data, firms of the highest wage quintile accounts for 76% of sales in the production network, indicating a large dispersion in sales and its correlation with quality. Parameters $\sigma_{\omega_1} = 0.114$ and $\rho = 0.121$ together governs these patterns in the model. The standard deviation of the conditional distribution, $\sigma_{\omega_0} = 0.12$, fits well the standard deviation of log sales by quintile of wage.

The third set of parameters $\mu_E, \sigma_E, b_1, b_2$ governs exporting behavior. The log of the random export cost has a mean $\mu_E = -3.83$ and a standard deviation $\sigma_E = 1.58$. The export probability is higher among large, high-wage firms. Parameters $b_1 = 101, b_2 = 0.50$

Table 7: Parameter Estimates

	Parameter	Estimate	Standard error
Matching friction	κ	0.00086	-
Directed search	ν_v	2.82	-
Complementarity	ν_y	0.42	-
Sd of quality capability	σ_{ω_1}	0.114	-
Sd of efficiency capability	σ_{ω_0}	0.120	-
Correlation	ρ	0.121	-
Efficiency cost of quality	$\overline{\omega}_2$	-0.106	-
Mean of log export cost	μ_E	-3.83	-
Sd of log export cost	σ_E	1.58	-
Foreign demand shifter	b_1	101	-
Foreign demand curvature	b_2	0.50	-

Notes: This table summarizes the estimated parameters using the method of simulated moments. The first set of parameters are the matching friction parameter (κ), the degree of directed search (ν_v), and the complementarity of input-output qualities (ν_y). The second set are parameters of the joint distribution of firms' initial capability, i.e. the standard deviation of quality capability (σ_{ω_1}), the standard deviation of efficiency capability (σ_{ω_0}), their correlation term (ρ), and the efficiency cost of quality ($\overline{\omega}_2$). The last set are export market parameters including the mean and standard deviation of log export cost (μ_E, σ_E), and the foreign demand shifter and curvature parameter (b_1, b_2).

govern export intensity by quintile of wage. Conditional on exporting, export intensity is increasing in firm wages in the data and the model. That is, the parameter estimates imply that $D_F(q)/D_H(q)$ is increasing in q.

The increasing ratio $D_F(q)/D_H(q)$ matters because a firm-specific shock that increases $D_F(q)$ increases the incentives for the firm (conditional on exporting) to upgrade its quality and thereby increase its wages. This pattern is consistent with the shift-share regressions of Table 3. The empirical estimates imply that a 5% export shock induces a wage increase of 0.21% on average for exporting firms. We pick $\bar{\omega}_2 = -0.106$ so that exporting firms in our model have the same average wage response.[26]

Overall, model-generated moments are similar to the corresponding data moments in the data, in Table 8. As a further validation, Figures A3 and A4 in Appendix E.2 illustrate the predictions of the model for the non-parametric patterns of assortative matching of Figure 2 in Section 2.2 above. These figures are related to targeted moments but they

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[26]Given each guess of $\bar{\omega}_2$, we re-simulate the equilibrium distribution $J(z, q)$. The estimated function $\Pi(q, E)$ is concave in q because all buyers (service and manufacting firms) valuation of quality, ϕ_y in (17), is concave. Then, the quadratic form of $z(q, \omega)$ in (13) with $\bar{\omega}_2 < 0$ together imply that all firms' problem of choosing quality (14) is concave, and quality choices are bounded even for firms that have a comparative advantage in producing higher quality, $\omega_1 > 1$.

Table 8: Model Fit – Targeted Moments

	Quintiles of average wage per worker				
	1	2	3	4	5 (largest)
Mean number of suppliers					
Data	5.8	6.7	5.8	11.4	25.8
Model	5.2	5.2	6.6	9.8	28.3
Mean number of customers					
Data	5.6	7.0	6.7	11.7	25.1
Model	5.9	6.5	8.3	11.5	23.1
Standard deviation of log sales					
Data	1.37	1.34	1.37	1.52	1.79
Model	1.31	1.30	1.32	1.35	1.61
Share of total network sales					
Data	0.03	0.04	0.04	0.10	0.78
Model	0.04	0.04	0.06	0.11	0.74
Fraction of exporters					
Data	0.08	0.18	0.16	0.34	0.57
Model	0.13	0.15	0.21	0.30	0.57
Export Intensity of Exporters					
Data	0.24	0.23	0.23	0.23	0.26
Model	0.20	0.23	0.24	0.24	0.26
Unweighted average log wage of suppliers					
Data	0.00	0.01	0.01	0.04	0.14
Model	0.00	0.02	0.05	0.07	0.11
Weighted average log wage of suppliers					
Data	0.00	0.02	0.02	0.07	0.23
Model	0.00	0.04	0.07	0.11	0.17
Shift-share IV coefficient (5% export shock)					
Data	0.21%				
Model	0.21%				

Notes: This table shows the targeted moments used in the estimation and compares our simulated moments to that from the data. Firms are ranked according to their average wage per worker. We match the following moments by quintile of firm wage: the mean number of suppliers (5 moments), the mean number of customers (5 moments), the share in total network sales (5 moments), the standard deviation of sales (5 moments), the fraction of exporters (5 moments), the export intensity of exporters (5 moments), the unweighted average log wage of suppliers (4 moments), and the average log wage of suppliers weighted by spending share (4 moments), where the latter two are normalized with respect to the first quintile. Besides, we also match the shift-share IV coefficient (1 moment).

were not directly targeted. The model matches well the extent to which firms with similar wages disproportionately transact with each other. It quantitatively fits well in the extensive and intensive margin of trade for each buyer-seller quintile pair. Equipped with these estimates, we investigate the general equilibrium effect of an export demand shock through our O-Ring production network.
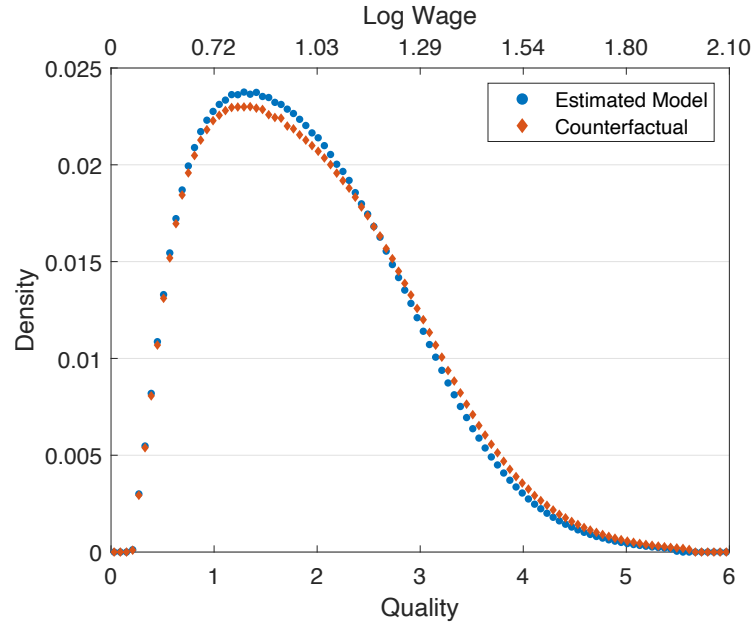
6   Counterfactual Analysis

We conduct a set of counterfactual analyses to understand the interconnection firms' choices of quality. We study how an increase in export demand affects domestic firms' quality choices directly and indirectly through the network.

Our baseline counterfactual increases export market demand $D_F(q)$ by 5%. We maintain the efficiency wages at $w(q) = 1$ for all q, the exchange rate $e = 1$ and price of services $P_s = 1$, and we allow gross manufacturing output to increase with the shock. We believe that this is a good baseline because it captures the direct effect of the shock on manufacturing but shuts down the interaction between manufacturing and the rest of the economy by assuming that (i) labor in and out of manufacturing is perfectly elastic, (ii) the export expansion does not lead to a real exchange rate appreciation, and (iii) the price of the inputs that manufacturing firms use from distributors does not change. Relaxing each of these assumptions, which we plan to do in future robustness counterfactuals, requires out-of-sample assumptions.

Because the ratio $D_F(q)/D_H(q)$ is increasing in q, the direct effect of the shock is to increase the incentives for exporters to upgrade quality just like the responses to the firm-specific shocks in the shift-share regressions. The distinction here is that the shock occurs simultaneously to all firms. As explained in the introduction (and further below), the shock then has a magnifying effect in firms' quality choices. In Figure 3 the density of quality choices in the counterfactual first order stochastically dominates the estimated model qualities. The mapping between quality and wages is the same in the estimated model and in the counterfactual given $w(q) = 1$. Wage increases because higher quality tasks are more skill intensive (Appendix A), and the top x-axis label yields an economic interpretation for the magnitude of quality changes. For example, a firm that produces quality 4 has log wages about 0.25 ($= 1.54 - 1.29$) higher than a firm of quality 3.

In Table 9's upper panel, we report changes in the distribution of quality and its corresponding log wage for exporters and non-exporters. For exporters, the 50th, 75th, 90th, and 95th percentile of the quality distribution increase by 1.9, 2.2, 2.5, and 2.2 percentage points. More interestingly, the quality improvement is not limited to exporters.

Figure 3: Distribution of Quality Choices

Notes: This figure displays the density of the distribution of firms' quality choices in the estimated model (blue circles) and in the counterfactual, after a 5% increase in export market demand (red diamonds). The main x-axis at the bottom shows the quality and the additional x-axis at the top displays the corresponding log wages normalized with respect to the lowest one.

The same percentiles of the quality distribution for non-exporters increase by 1.5, 1.6, 2.1, and 1.9 percentage points respectively. In all, the average wage increases by 1.19 percent (1.73 percent for exporters), an order of magnitude larger than the change induced by idiosyncratic export demand shocks (0.21 percent).

The effect of the export demand shock on firm sales is more heterogeneous than the effect on quality above. The demand faced by lower-quality, non-exporting firms drops for two reasons. First, widespread quality upgrading makes these firms' output less appealing. Second, as exporters' scale of production increases, exporters increase their search effort, they find more suppliers and decrease their prices (for a given quality). The ensuing decrease in the domestic price index decreases domestic demand D_H, which in turn, decreases non-exporting firms' effort to search for suppliers. So, even keeping quality constant, the price of non-exporters increases relative to exporters'. All these forces lead to a drop in sales among lower-quality, non-exporters of 6 to 7 percent. The size distribution then gets more dispersed. In spite of the positive cross-sectional correlation between sales and wages, the counterfactual simultaneously predicts reductions in sales and increases in quality among domestic manufacturers.

The domestic input-output production network plays an important role in explaining

Table 9: Changes in the Distributions

	Percentiles of the distribution				
	25th	50th	75th	90th	95th
ln*(Quality), counterfactual – baseline*					
Exporters	0.014	0.019	0.022	0.025	0.022
Non-exporters	0.000	0.015	0.016	0.021	0.019
All Firms	0.010	0.020	0.023	0.026	0.029
ln*(Wage), counterfactual – baseline*					
Exporters	0.007	0.012	0.018	0.024	0.024
Non-exporters	0.000	0.007	0.009	0.015	0.015
All Firms	0.004	0.010	0.015	0.021	0.027
ln*(Sales), counterfactual – baseline*					
Exporters	-0.035	-0.017	-0.004	0.014	0.026
Non-exporters	-0.066	-0.072	-0.075	-0.070	-0.068
All Firms	-0.034	-0.036	0.009	0.036	0.053
ln*(Number of Suppliers), counterfactual – baseline*					
Exporters	-0.021	-0.010	-0.002	0.008	0.016
Non-exporters	-0.039	-0.043	-0.045	-0.041	-0.040
All Firms	-0.020	-0.021	0.006	0.021	0.031
ln*(Number of Customers), counterfactual – baseline*					
Exporters	-0.016	-0.003	0.009	0.018	0.024
Non-exporters	-0.021	-0.018	-0.016	-0.014	-0.010
All Firms	-0.021	-0.001	0.007	0.025	0.033

Notes: This table shows the changes in the quality, wage, sales, and degree distributions after a 5% increase in export market demand. We calculate the 25th, 50th, 75th, 90th, and 95th percentiles of the distributions in the estimated and the counterfactual equilibrium, and report the changes in log differences for exporters, non-exporters, and all firms separately.

Figure 4: Decomposition of Changes in Domestic Profit Shifter

(a) Baseline Counterfactual

(b) No Complementarity
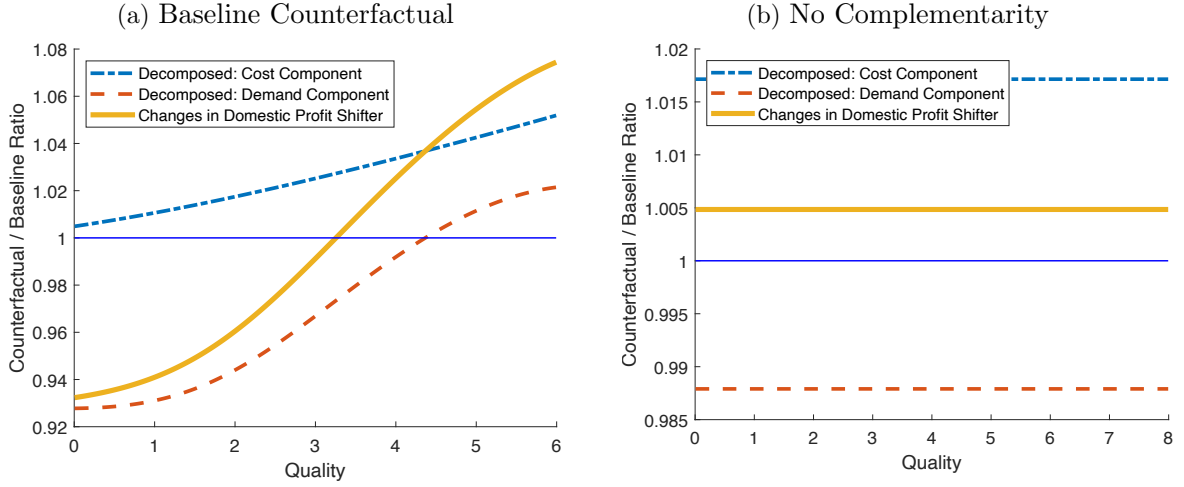
Notes: This figure displays the counterfactual changes in the domestic profit shifter after a 5% increase in export demand relative to the baseline. In particular, the yellow solid line shows the ratio of the domestic profit shifter $\Pi(q,0)$ in the counterfactual and that in the baseline for each quality level q. Recall the domestic profit shifter depends on the network aggregate demand $D(q,0)$ and the aggregate cost index $c(q)$, i.e. $\Pi(q,0) \propto D(q,0)^\gamma \cdot c(q)^{\alpha_m(1-\sigma)\gamma}$. Thus we further decompose the changes in the domestic profit shifter into a demand component $D(q,0)^\gamma$ (red dashed line) and a cost component $c(q)^{\alpha_m(1-\sigma)\gamma}$ (blue dash-dotted line). The baseline counterfactual is shown on the left penel, and the special case with no complementarity is shown on the right panel.

these findings. Profit shifter $\Pi(q,E)$ summarizes the relative benefit of upgrading quality in the model. Figure 4 panel (a), plots the counterfactual changes, relative to the estimated model, of $\Pi(q,0)$ and of its two endogenous components: Demand $D(q,0)$ and cost $c(q)$. First, consider demand $D(q,0)$, represented by the red dotted curve. At the extensive margin, quality upgrading among exporters increases the measure of sourcing ads $M(q)$ in the high-quality segment and improves the success rate of seller ads targeted at those segments. At the intensive margin, conditional on the match, exporters increase their spending on high- relative to low-quality domestic suppliers. Second is the cost component $c(q)$, represented by the blue dotted curve. Quality upgrading among exporters decreases the domestic cost of production because exporters also supply inputs to the domestic market. But because high-quality production is intensive in high-quality inputs (estimated $\nu_y > 0$), this benefit accrues disproportionately to high-quality producers. The more firms respond to these shifts by upgrading their qualities, they reinforce and further augment the effect of the shock. Overall, the profitability in the high-quality segment increases by 6 to 8 percent, while the profitability in the low-quality segment decreases by 4 to 6 percent. Both $c(q)$ and $D(q,0)$ significantly contribute to these changes.

To further probe into these mechanisms, we study a special case of the model without

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the complementarity in matching ϕ_v and in production ϕ_y. We assume that the value of high- and low-quality inputs in production is independent on the quality of the output being produced ($\nu_y = 0$ in (17)), and that all firms' selling ads are uniformly distributed across the quality set Q ($\nu_v \to \infty$). We re-estimated the model with these parameter restrictions. Tables A5 and A6 report the estimates and the model fit. By assumption, the special case cannot match the increasing relation between buyer and supplier wage, weighted or unweighted. For all other moments, the fit of the model in this special case is similar to the general model. In particular, the estimated ratio $D_F(q)/D_H(q)$ is increasing in quality so that exporters upgrade quality when $D_F(q)$ increases.

We then experiment with the same 5% counterfactual increase in export demand $D_F(q)$ in the special case. The average wage increase for exporters is 0.23%, very close to the average firm response to an idiosyncratic export demand shock 0.21%. Figure 4 panel (b) plots the change of $\Pi(q, 0)$ and of its cost $c(q)$ and demand $D(q, 0)$ components. The expansion of the exporters drives down the price indexes in the domestic market. Non-exporters benefit from cheaper inputs but face tougher competition in their output market. Without the complementarity in quality, these changes are independent of quality. Non-exporters experience only a level shift in $\Pi(q, 0)$ by 0.5%. In the model, manufacturing firms make optimal quality choices before their export decision. The flattened $\Pi(q, 0)$ mutes the quality response of all firms in this special case, especially of firms with a low probability of exporting.

Finally, the baseline case and the special case with no complementarity both feature standard firm-to-firm production linkages that potentially magnify the effect of shocks on output. The increase in foreign demand increases exporters' search efforts and decreases their costs relative to non-exporters. Since the elasticity of substitution among inputs $\sigma = 5$ is greater than one, the effects on output are larger than those predicted by the classical Hulten (1978). Baqaee and Farhi (2019a) emphasizes this role of the elasticity of substitution. Output increases by 5.78% in the baseline and by 5.68% in the special case. However, the dramatic differences in quality upgrading between the baseline model and the special case indicate that economies of scale are not sufficient to understand the effect of international trade on developing countries.

7 Conclusion

We document novel facts about firm-to-firm trade using data from Turkey. High-wage firms are more likely to match with each other in the network, and the value of transactions is larger when the trading partners' wages are both high. This positive assortative

matching is robust to numerous checks, and it is much stronger than assortative matching along other firm characteristics. In shift-share regressions, a firm-specific demand shock from a rich export destination is associated with an increase in the firm's wage and in the average wage of its suppliers.

We rationalize these findings in a model where firms' choices of quality and skill intensity are interconnected through the production network. Higher-quality production is intensive in both skilled labor and in higher-quality inputs, and higher-quality firms direct their search toward higher-quality customers. In the estimated model, the average effect of an export shock on manufacturing wages is about eight times larger when it occurs in the aggregate than when it occurs separately to each individual firm.

This magnification effect may shed light on the effects of international trade on developing countries. Goldberg and Pavcnik (2004, 2007) document large increases in demand for skilled workers following trade liberalizations in developing countries. Export promotion was the linchpin of the successful development strategies of many Asian countries, including Singapore, South Korea, Taiwan, Thailand, and more recently, China. The manufacturing sector in these countries grew fast and adopted many skill-biased technologies from developed countries. These patterns are consistent with the widespread adoption of higher-quality, skill-intensive production technologies in our counterfactual.

We see at least three paths of future research. First is explicitly modelling the effects of education on labor supply. Our current analysis presumes perfectly elastic labor supply, but a large inflow of skilled labor into manufacturing in East Asia may not have been possible without massive investments in education. Second is to combine our mechanism with external economies of scale in skill-intensive production, emphasized by recent work on economic geography.[27] Last is explicitly modelling export destinations. Consistent with our shift-share regressions, Goldberg and Reed (2020) find that exporting to developed countries may be particularly beneficial to developing countries.

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[27]See Gaubert (2018), Fajgelbaum and Gaubert (2020), and Davis and Dingel (2020).

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Contents (Appendix)

A   Roy Model of Labor Supply

In the main text, the supply of efficiency units of labor of task q is $L(q, w)$, an exogenous function of the task quality q and the full equilibrium wage schedule $w(q')$ for all $q' \in Q$. This appendix provides a micro foundation for labor supply based on the Roy model in Teulings (1995). It provides sufficient conditions for the ranking of average earnings per firm to equal the ranking of task quality q (also in Teulings (1995)), and it shows that we can construct a set of worker endowments such that labor markets clear and the distribution of earnings per worker across firms exactly matches the data. These claims hold for any fixed continuous and differentiable w, assumptions which hold in the estimation where $w(q) = 1$ for all $q \in Q$.

A measure H of workers have heterogeneous skills, indexed with $s \in [0, 1]$, and distributed in [0,1] according to a density $h(s)$. A worker with skill s is endowed with $e(q, s)$ efficiency units of labor if she works at a firm of quality q. She observes the wage schedule $w(q)$ and chooses task quality q to maximize earnings:

$$\max_{q \in Q}\{w(q)e(q, s)\} \tag{49}$$

Let $s^*(q)$ be the set of skills that choose quality q. To ease notation, assume that $s^*(q)$ is a function or the empty set.[28] The mass of workers supplying task q is $h(s^*(q))$ where we define $h(\emptyset) = 0$.

Then, the supply of efficiency units of labor of task q is

$$L(q, \omega) = H h(s^*(q)) e(q, s^*(q))$$

where we can define $e(q, s^*(q)) = 0$ if $s^*(q) = \emptyset$. Earnings per worker in firms of task q is $w(q) e(q, s^*(q))$.

In the estimation, we assume that earnings per worker is strictly increasing in q. This assumption holds if $e(q, s)$ is increasing in s and strictly log-supermodular. Given this monotonicity, each q in the model is associated with an earnings per worker y in the data where y is such that the share of firms with qualities smaller than or equal to q in the model is equal to the share of firms with earnings per worker less than or equal to y in the data. To show that we can construct a set of endowments $e(q, s)$ that clear the labor market and that deliver the data's distribution of average earnings across firms, it suffices to show that for any quality-earnings pair $(q^*, y^*) \in Q \times \mathbb{R}_{++}$, we can find an endowment function $e(q, s^*)$ such that q^* is the choice and y^* is the maximum in problem (49) when the worker skill is s^*. We parameterize

$$e(q, s^*) = \exp(s_0^* + s_1^* \log(q) + \bar{s}_2 [\log(q)]^2)$$

where $\bar{s}_2$ and $(s_0^*, s_1^*) \in \mathbb{R}^2$ are specific to skill s^*. Sufficient conditions for $e(q, s^*)$ are:

$$y^* = w(q^*) \exp(s_0^* + s_1^* \log(q^*) + \bar{s}_2 [\log(q^*)]^2)$$
$$0 = \frac{d \log[w(q^*)]}{d \log(q)} + s_1^* + 2\bar{s}_2 [\log(q^*)]$$
$$0 > \frac{d^2 \log[w(q)]}{d[\log(q)]^2} + 2\bar{s}_2 \qquad \text{for all } q \in Q.$$

These conditions are analogous to the construction of firm productivity in the second stage of the estimation. The lack of identification of $\bar{s}_2$ is the same as that of $\bar{\omega}_2$.

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[28]Correspondence $s^*(q)$ is a function in the interior or Q assuming that functions $w(q)$ and $h(q)$ are continuous and differentiable, and that $e(q, s)$ is continuous, differentiable and strictly log supermodular.

B   Special Case: One quality, $\beta_v = \beta_m$

We solve for the special case of the model with one quality and $\beta_v = \beta_m$. As mentioned in the main text, we make two additional assumptions: The labor supply L is exogenous, and the entry cost in (15) uses labor. These assumptions facilitate the comparison between the model and Melitz (2003).

With a constant labor supply, the labor market equilibrium delivers wage as a function of exogenous variables:

$$w = \frac{1}{L\sigma}\left[(1 - \alpha_m - \alpha_s)(\sigma - 1) + 1 - \frac{1}{\gamma}\right] \equiv \alpha_L L^{-1}. \tag{50}$$

The cost of entry is wf. Free entry then implies that the number of firms increases in proportion to population as in standard monopolistic competition models

$$N = L(\gamma \sigma f \alpha_L)^{-1}. \tag{51}$$

For easier reference, recall that with one quality,

$$c = \left(\frac{\theta_m}{V}\right)^{1/(1-\sigma)} P$$

$$P_s = \left(\frac{\overline{m}}{V}\right)^{1/(1-\sigma)} P \tag{52}$$

$$\Pi = \left[N\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{-1} \tag{53}$$

Substituting $v(z)$ from (11) into (21), we have

$$
\begin{aligned}
V &= (\sigma w f_v)^{-1/\beta} N^{(\beta-1)/\beta} \frac{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})}{\left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/\beta}} \\
&= (\sigma w)^{-1} f_v^{-1/\beta} (\gamma f)^{(1-\beta)/\beta} \frac{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})}{\left[\mathbb{E}(z^{\gamma(\sigma-1)})\right]^{1/\beta}}
\end{aligned} \tag{54}
$$

Substituting V above, and P_s and c in (52) into $C(1)$ in (8):

$$
\begin{aligned}
C(1) &= w^{1-\alpha_s-\alpha_m} P_s^{\alpha_s} c^{\alpha_m} \\
&= w^{1-\alpha_s-\alpha_m} P^{\alpha_s+\alpha_m} (\overline{m}^{\alpha_s} \theta_m^{\alpha_m})^{1/(1-\sigma)} V^{(\alpha_s+\alpha_m)/(\sigma-1)} \\
&= w^{1-\alpha_s-\alpha_m} P^{\alpha_s+\alpha_m} (\overline{m}^{\alpha_s} \theta_m^{\alpha_m})^{1/(1-\sigma)} (\sigma w)^{-(\alpha_m+\alpha_s)/(\sigma-1)} f_v^{-(\alpha_m+\alpha_s)/[\beta(\sigma-1)]} (\gamma f)^{(1-\beta)(\alpha_m+\alpha_s)/[\beta(\sigma-1)]} \\
&\quad \times \left(\frac{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})}{[\mathbb{E}(z^{\gamma(\sigma-1)})]^{1/\beta}} \right)^{(\alpha_m+\alpha_s)/(\sigma-1)}
\end{aligned}
$$

Substituting $C(1)$ above, $D = P^{1-\sigma}$, Π from (53) into the original expression for Π in (12), we get

$$
\Pi = (\sigma w)^{1-\gamma} \left[D \left(\frac{\sigma}{\sigma-1} C(1) \right)^{1-\sigma} \left(\frac{f_m}{\alpha_m} \right)^{-\alpha_m/\beta_m} f_v^{-1/\beta_v} \right]^{\gamma}
$$

$$
\Rightarrow \frac{P}{w} = \left(\frac{\sigma}{\sigma-1} \right)^{1/(1-\alpha_m-\alpha_s)} (f\gamma f_v^{1/\beta} \sigma w)^{1/(\sigma-1)}
$$
$$
\left\{ [\mathbb{E}(z^{\gamma(\sigma-1)})]^{1/\gamma} \left(\frac{[\mathbb{E}(z^{\gamma(\sigma-1)})]^{1/\beta}}{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})} \right)^{\alpha_m+\alpha_s} (\theta_m^{\alpha_m} \overline{m}^{\alpha_s}) \left(\frac{f_m}{\alpha_m} \right)^{-\alpha_m/\beta} (f\gamma)^{(1-\alpha_s)/\beta} \right\}^{1/[(1-\sigma)(1-\alpha_s-\alpha_m)]}
$$

Using (52) and (54), real wages is

$$
\frac{w}{P_s} = \left\{ \left(\frac{\sigma-1}{\sigma} \right) [\mathbb{E}(z^{\gamma(\sigma-1)})]^{1/[\gamma(\sigma-1)]} \left[\frac{[\mathbb{E}(z^{\gamma(\sigma-1)})]^{1/\beta}}{\mathbb{E}(z^{\gamma(\sigma-1)/\beta})} \left(\frac{f_m}{\alpha_m \gamma f} \right)^{-\alpha_m/\beta} \theta_m^{\alpha_m} \overline{m}^{1-\alpha_m} \right]^{1/(\sigma-1)} \right\}^{1/(1-\alpha_m-\alpha_s)}
$$

The first two terms are standard: The markup $\sigma/(\sigma-1)$ decreases real wages and expected productivity $\mathbb{E}(z^{\gamma(\sigma-1)})$ increases real wages, where productivity is adjusted for the elasticity of sales with respect to productivity. In a standard model, Melitz (2003), these terms are multiplied by a scale effect proportional to $L^{1/(\sigma-1)}$. Here, the scale effect is different because not all agents are matched with all firms. The terms θ_m and $\overline{m}$ captures the efficiency of matching for firms, where θ_m is decreasing in the cost of posting selling ads f_v. An increase in the entry cost f increases the size of each firm, and number of matches per firm.[29] The fraction in expectations, $[\mathbb{E}(z^{\gamma(\sigma-1)})]^{1/\beta}/\mathbb{E}(z^{\gamma(\sigma-1)/\beta}) > 1$, is a measure productivity dispersion. Dispersion increases real wages because the variety gain

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[29]The model can be generalized to eliminate this feature, by allowing search costs to depend negatively on the number of firms in the market.

from having more suppliers and customers accrues disproportionately to large firms.

Search and efficiency. We consider the problem of a planner in investing in ads $m(z)$ and $v(z)$ to maximize consumer welfare. We take as given that the markup decision of the firms is optimal. For simplicity in this exercise, we pick w to be the numeraire since aggregate output does not enter the problem.

From (52), input cost as a function of consumer prices is

$$c = \left(\frac{\tilde{M}}{MV}\right)^{1/(1-\sigma)} P = \left(\frac{\tilde{M}}{mM}\right)^{1/(1-\sigma)} P_s$$

Consumer price is

$$P_s = \left(\frac{\overline{m}}{V}\right)^{1/(1-\sigma)} \left[\int p(z)^{1-\sigma} v(z) dJ(z)\right]^{1/(1-\sigma)}$$

$$= \left(\frac{\overline{m}}{V}\right)^{1/(1-\sigma)} \left(\frac{\sigma}{\sigma-1}\right) P_s^{\alpha_s + \alpha_m} \left(\frac{\tilde{M}}{\overline{m}M}\right)^{\alpha_m/(1-\sigma)} \left[\int z^{\sigma-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{1/(1-\sigma)}$$

$$P_s^{1-\alpha_s-\alpha_m} = \overline{m}^{(1-\alpha_m)/(1-\sigma)} V^{1/(\sigma-1)} \left(\frac{\sigma}{\sigma-1}\right) \left(\frac{\tilde{M}}{M}\right)^{\alpha_m/(1-\sigma)} \left[\int z^{\sigma-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{1/(1-\sigma)}$$

Ignoring the constants, the planner chooses ads $m(z)$ and $v(z)$ to minimize the price index minus the cost of labor used to produce ads.

$$\min_{m(z),v(z)} \left\{ \overline{m}^{(1-\alpha_m)/(1-\sigma)} V^{1/(\sigma-1)} \left(\frac{\tilde{M}}{M}\right)^{\alpha_m/(1-\sigma)} \left[\int z^{-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{1/(1-\sigma)} \right\}^{1/(1-\alpha_s-\alpha_m)}$$

$$+ \lambda \int \left[f_m \frac{m(z)^{\beta}}{\beta} + f_v \frac{v(z)^{\beta}}{\beta}\right] dJ(z)$$

where λ is the marginal product of labor and the planner internalizes V, M and $\tilde{M}$. The first order conditions with respect to $m(z)$ is

$$\frac{\alpha_m}{(1-\sigma)(1-\alpha_m-\alpha_s)} P_s^{\alpha_m+\alpha_s} \left[\int z^{\sigma-1} m(z)^{\alpha_m} v(z) dJ(z)\right]^{\sigma/(1-\sigma)} m(z)^{\alpha_m-1} z^{\sigma-1} v(z) + \lambda f_m m(z)^{\beta-1}$$

$$+ \frac{\alpha_m}{(1-\sigma)(1-\alpha_m-\alpha_s)} P_s^{\alpha_m+\alpha_s} \frac{1}{M} \left(\frac{M}{\tilde{M}} \frac{d\tilde{M}}{dM} - 1\right) = 0 \tag{55}$$

52

The first order conditions with respect to $v(z)$ is

$$\frac{1}{(1-\sigma)(1-\alpha_m-\alpha_s)}P_s^{\alpha_m+\alpha_s}\left[\int z^{\sigma-1}m(z)^{\alpha_m}v(z)dJ(z)\right]^{\sigma/(1-\sigma)}m(z)^{\alpha_m}z^{\sigma-1}+\lambda f_v v(z)^{\beta-1}$$

$$+\frac{1}{(1-\sigma)(1-\alpha_m-\alpha_s)}P_s^{\alpha_m+\alpha_s}\frac{1}{V}\left(\alpha_m\frac{V}{\tilde{M}}\frac{d\tilde{M}}{dV}-1\right)=0 \tag{56}$$

The first line of (55) and (56) are equal at the market solution, from the first order conditions of the firm. Since these are the only terms with firm-specific productivity z, there is no missallocation on ads across firms.

There are four externalities. The first two are the elasticity of $\tilde{M}$ with respect to M in (55) and with respect to V in (56). They both imply a positive externality of ads on the mass of ads, which increase welfare. But ads also create competition. More ads decrease in proportion the probability of success of competing ads. These negative externalities are the negative one term subtracting the elasticities. One can easily show that the two elasticities $\frac{M}{\tilde{M}}\frac{d\tilde{M}}{dM}$ and $\frac{V}{\tilde{M}}\frac{d\tilde{M}}{dV}$ are in (0,1). So, the negative externality is always larger than the positive, which push the planner to posting fewer ads than the market.

C   Open Economy Model

We present the parts of the model that were missing from Section 4. A manufacturing firm with productivity z, quality q and export status E has the following sales x, a measure of ads v to find customers (domestic and abroad) and m to find suppliers, and price:

$$x(z,q,E) = \Pi(q,E)z^{\gamma(\sigma-1)}$$

$$v(z,q,E) = \left(\frac{x(z,q,E)}{\sigma f_v w(q)}\right)^{1/\beta_v}$$

$$m(z,q,E) = \left(\frac{x(z,q,E)}{\sigma f_m w(q)/\alpha_m}\right)^{1/\beta_m}$$

$$p(z,q,E) = \frac{\sigma}{\sigma-1}\frac{C(m(z,q,E),q)}{z} \tag{57}$$

where

$$\Pi(q,E) = [\sigma w(q)]^{1-\gamma}\left[D(q,E)\left(\frac{\sigma}{\sigma-1}C(1,q)\right)^{1-\sigma}\left(\frac{f_m}{\alpha_m}\right)^{-\alpha_m/\beta_m}f_v^{-1/\beta_v}\right]^\gamma \tag{58}$$

$$D(q,E) = \left[D_H(q)^{\beta_v/(\beta_v-1)} + E(e^\sigma D_F(q))^{\beta_v/(\beta_v-1)}\right]^{(\beta_v-1)/\beta_v}.$$

53

With the fixed exporting cost, profit is no longer a constant share of revenue. The expected profit of a firm that draws a productivity parameter ω upon entry is (equation (47)):

$$\pi(\omega) = \max_{q \in Q} \left\{ \frac{z(q,\omega)^{\gamma(\sigma-1)}}{\gamma\sigma} \left[\Pi(q,1)\Phi\left(\overline{f}_E(z(q,\omega),q)\right) + \Pi(q,0)\left[1 - \Phi\left(\overline{f}_E(z(q,\omega),q)\right)\right] \right] \right.$$
$$\left. - P_s \mathbb{E}(f_E | f_E \leq \overline{f}_E(z(q,\omega),q)) \right\}$$

Free entry implies

$$P_s f = \mathbb{E}_\omega(\pi(\omega)) \tag{59}$$

The firm choices give rise to the measure functions:

$$\tilde{J}(z,q) = N\text{Prob}\left\{\omega : z(q(\omega),\omega) \leq z \quad \text{and} \quad q(\omega) \leq q\right\}$$
$$J(z,q,1) = \tilde{J}(z,q)\Phi\left(\overline{f}_E(z,q)\right)$$
$$J(z,q,0) = \tilde{J}(z,q)\left[1 - \Phi\left(\overline{f}_E(z,q)\right)\right] \tag{60}$$

$J(z,q,E)$ is the measure of functions with export status $E \in \{0,1\}$ and productivity-quality pairs less than or equal to (z,q). Denote the density of J as $j(z,q,E)$ for $E = 0,1$.

The production function (17) and network formation are the same as in the closed economy, only expressions for some aggregate variables change. The mass of ads posted by firms of quality q to find suppliers and sellers is respectively:

$$M(q) = \sum_{E=0,1} \int_Z m(z,q,E)j(z,q,E)dz \tag{61}$$

$$\overline{V}(q) = \sum_{E=0,1} r_v(q,E) \int_Z v(z,q,E)j(z,q,E)dz \tag{62}$$

The mass of ads directed at buyers of quality q, $V(q)$, and the mass of matches $\tilde{M}(q)$ are in (21) and (22). The success rate of ads is $\theta_v(q) = \tilde{M}(q)/V(q)$ for sellers and $\theta_m(q) = \tilde{M}(q)/M(q)$ for buyers, as before.

Cost function $c(q)$ and demand function and $D_m(q)$ are in equations (24) and (26) respectively, where now the price index $P(q)$ and spending on manufacturing inputs $X_m(q)$

are:

$$P(q) = \left[\sum_{E=0,1} r_v(q,E) \int_Z p(z,q,E)^{1-\sigma} v(z,q,E) j(z,q,E) dz \right]^{1/(1-\sigma)} \tag{63}$$

$$X_m(q) = \frac{\alpha_m(\sigma-1)}{\sigma} \sum_{E=0,1} \int_Z x(z,q,E) j(z,q,E) dz. \tag{64}$$

The cost of domestic services is defined as before:

$$P_{Hs} = \left[\frac{\overline{m}}{V_T} \int_Q \phi_y(0,q) P(q)^{1-\sigma} dq \right]^{1/(1-\sigma)}$$

where

$$V_T = \int_Q \overline{V}(q) dq.$$

The bundle of services is a combination of domestic and foreign services. It costs:

$$P_s = \left[P_{Hs}^{1-\sigma_s} + (eP_{Fs})^{1-\sigma_s} \right]^{1/(1-\sigma_s)}. \tag{65}$$

We experiment with different assumptions on the response of the trade balance and exchange rate adjustment in our counterfactual. So, we close the equilibrium here in a generic way. Let B be the exogenous trade deficit, i.e., the difference between consumer spending and income. Then, total spending on services is

$$X_s = 1 - \frac{\alpha_m(\sigma-1)}{\sigma} + B \tag{66}$$

where we have taken gross manufacturing output again as the numeraire. Similar to the closed economy, the revenue from sales to service firms of a domestic manufacturing firm posting v ads and price p is

$$p^{1-\sigma} v D_s(q)$$

where

$$D_s(q) = \phi_y(0,q) \left[\int_Q \phi_y(0,q') P(q')^{1-\sigma} dq' \right]^{-1} X_{Hs}$$

$$X_{Hs} = \left(\frac{P_{Hs}}{P_s} \right)^{1-\sigma_s} X_s \tag{67}$$

X_{Hs} is spending on domestic services. Total demand shifter $D(q) = D_m(q) + D_s(q)$ as in

(31).

Trade equilibrium implies that the difference between imports of services and exports of manufacturing equals the exogenous trade deficit B (consumer demand for savings):

$$B = \left(\frac{eP_{Fs}}{P_s}\right)^{1-\sigma_s} X_s - \int_{q \in Q} r_v(q,1) e^{\sigma} D_F(q) \left[\int_z p(z,q,1)^{1-\sigma} v(z,q,1) j(z,q,1) dz\right] dq. \quad (68)$$

So from (66), independently of the trade deficit, spending on domestic services is

$$X_s = 1 - \frac{\alpha_m(\sigma-1)}{\sigma} - \int_{q \in Q} r_v(q,1) e^{\sigma} D_F(q) \left[\int_z p(z,q,1)^{1-\sigma} v(z,q,1) j(z,q,1) dz\right] dq.$$

Labor markets clear if

$$L(q,w) = \frac{1}{w(q)\sigma} \left[(1-\alpha_m-\alpha_s)(\sigma-1) + 1 - \frac{1}{\gamma}\right] \left[\sum_{E=0,1} \int_Z x(z,q,E) j(z,q,E) dz\right] \quad (69)$$

An **equilibrium** is a mass of firms N, an exchange rate e, measure functions $J(z,q,1)$ and $J(z,q,0)$, and functions $w(q)$, $\theta_m(q)$, $\theta_v(q)$, $c(q)$, $D(q)$ satisfying the following conditions:

1. Trade is in equilibrium (68).

2. Labor market clears (69).

3. Firms maximize profits. Firm ω chooses $q(\omega)$ in (47) and has productivity $z(\omega) = z(q(\omega),\omega)$ at the optimal. The firm export status is $E = 1$ if its fixed cost of exporting is less than $\overline{f}_E(q(\omega), z(q,\omega))$, and $E = 0$ otherwise. Its sales, measure of ads, and prices are $x(z(\omega), q(\omega), E)$, $m(z(\omega), q(\omega), E)$, $v(z(\omega), q(\omega), E)$, and $p(z(\omega), q(\omega), E)$ in (57). The direction of selling ads $\mu(q(\omega))$ solves (26).

4. For $E = 0, 1$, the measures $J(z,q,E)$ are consistent with firm choices (60).

5. The success rate of ads $\theta_m(q) = \tilde{M}(q)/M(q)$ and $\theta_v(q) = \tilde{M}(q)/V(q)$ where $\tilde{M}(q)$ is in (22), $V(q)$ is in (21), and $M(q)$ and $\overline{V}(q)$ are in (61) and (62). Cost $c(q)$ satisfies (24) and $D(q)$ satisfies (31), where $P(q)$ and $X_m(q)$ are in (63) and (64).

D Estimation of the Open Economy

E Additional Tables and Figures

E.1 Tables

Table A1: Assortative Matching on Wages: Alternative definition of wages

Dependent variable: $\log wage_f^S$

	Manufacturing firms			All firms
	(1)	(2)	(3)	(4)
$\log wage_f$	0.300	0.262	0.190	0.258
	(0.011)	(0.010)	(0.007)	(0.010)
$\log employment_f$			0.044	
			(0.003)	
R^2	0.092	0.163	0.183	0.128
N	77,418	77,418	77,418	410,608
Fixed effects		ind-prov	ind-prov	ind-prov

Notes: Firm-level wage is calculated as the within-firm median value of the residuals obtained from the following regression:

$$\log wage_{ef} = \beta_1 Age_e + \beta_2 Gender_e + \alpha_o + e_{ef},$$

where $wage_{ef}$ denotres the average value of monthly wage received by each worker in a given firm, and α_o occupation fixed effects at the 1-digit ISCO level. Denoting the set of suppliers of firm f by Ω_f^S, average supplier wage is defined as follows: $\log wage_f^S = \sum_{\omega \in \Omega_f^S} \log wage_\omega s_{\omega f}$, where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Table A2: Assortative Matching on Other Variables

	log market share$_f^S$		log outdegree$_f^S$	
	manuf	all	manuf	all
	(1)	(2)	(3)	(4)
log market share$_f$	0.175	0.154		
	(0.013)	(0.029)		
log indegree$_f$			0.0985	-0.034
			(0.012)	(0.063)
R^2	0.11	0.14	0.09	0.14
N	77,418	410,608	77,418	410,608
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Market share is the share of a firm's sales in total sales of its 4-digit NACE industry, and indegree is the number of domestic suppliers of a firm. Both variables are in logarithms. Denoting the set of suppliers of firm f by Ω_f^S, average supplier market share is defined as follows: $\log \text{market share}_f^S = \sum_{\omega \in \Omega_f^S} \log \text{market share}_\omega s_{\omega f}$, where ω indexes suppliers, and $s_{\omega f}$ is the share of f's purchases from supplier ω. $\log \text{outdegree}_f^S$ is defined similarly using the number of buyers (outdegree) of firm f's each supplier. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Table A3: Assortative Matching on Other Variables (Extensive margin)

	log market share$_f^S$		log outdegree$_f^S$	
	manuf	all	manuf	all
	(1)	(2)	(3)	(4)
log market share$_f$	0.042	0.009		
	(0.009)	(0.025)		
log indegree$_f$			0.009	-0.131
			(0.009)	(0.060)
R^2	0.07	0.12	0.08	0.13
N	77,418	410,608	77,418	410,608
Fixed effects	ind-prov	ind-prov	ind-prov	ind-prov

Notes: Market share is the share of a firm's sales in total sales of its 4-digit NACE industry, and indegree is the number of domestic suppliers of a firm. Both variables are in logarithms. Denoting the set of suppliers of firm f by Ω_f^S, unweighted average of supplier market share is defined as follows: $\log \text{market share}_f^S = \sum_{\omega \in \Omega_f^S} \log \text{market share}_\omega (1/|\Omega_f|)$, where ω indexes suppliers. $\log \text{outdegree}_f^S$ is defined similarly using the number of buyers (outdegree) of firm f's each supplier. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Table A4: Effects of Export Shock: Robustness checks

	$\Delta \log \text{wage}_f$ (1)	$\Delta \log \text{wage}_f$ (2)	$\Delta \log \text{wage}_f^S$ (3)
ExportShock$_f^u$ (unadjusted)	0.015 (0.068)		
ExportShock$_f^a$ (adjusted)	0.041 (0.007)	0.028 (0.008)	
Weighted GDP per capita$_f$		0.007 (0.001)	
$\Delta \log \text{wage}_f$ (IV = ExportShock$_f$)			0.451 (0.224)
ExportShock$_f^{S,a}$ (adjusted)			0.181 0.050
F-Stat	13.3	37.6	
N	33,157	33,157	33,157
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: Wage$_f$ is the average value of monthly payments per worker in firm f. The suppliers' average wage $\log \text{wage}_f^S$ is defined in equation (1). Δ operator denotes changes between 2011-2012 and 2014-2015. ExportShock$_f^u$ is a weighted average of changes in imports at the country (c) and 4-digit HS product (k) level between 2011-2012 and 2014-2015, where weights are constructed as the share of firm f's exports of product k to importer c in its total sales in 2010. ExportShock$_f^a$ adjusts these shocks by weighting rich destinations more. Weighted GDP per capita$_f$ is the weighted average of GDP per capita of firm's destinations in 2010, where the weights are defined as above. See equations (6). Ind and prov refer to 4-digit NACE industries and provinces, respectively. Robust standard errors are clustered at 4-digit NACE industry level.

Table A5: Parameter Estimates (Special Case with No Complementarity)

	Parameter	Estimate	Standard error
Matching friction	κ	0.00089	-
Directed search	$\nu_v \to \infty$	-	-
Complementarity	$\nu_y = 0$	-	-
Sd of quality capability	σ_{ω_1}	0.141	-
Sd of efficiency capability	σ_{ω_0}	0.124	-
Correlation	ρ	0.126	-
Efficiency cost of quality	$\overline{\omega}_2$	-0.119	-
Mean of log export cost	μ_E	-3.95	-
Sd of log export cost	σ_E	1.58	-
Foreign demand shifter	b_1	80	-
Foreign demand curvature	b_2	0.42	-

Notes: This table summarizes the estimated parameters for a special case where we shut down the complementarity in matching ($\nu_v \to \infty$) and in production ($\nu_y = 0$) using the method of simulated moments. The first set of parameter remaining to be estimated is the matching friction parameter (κ). The second set are parameters of the joint distribution of firms' initial capability, i.e. the standard deviation of quality capability (σ_{ω_1}), the standard deviation of efficiency capability (σ_{ω_0}), their correlation term (ρ), and the efficiency cost of quality ($\overline{\omega}_2$). The last set are export market parameters including the mean and standard deviation of log export cost (μ_E, σ_E), and the foreign demand shifter and curvature parameter (b_1, b_2).

Table A6: Model Fit – Targeted Moments (Special Case with No Complementarity)

	Quintiles of average wage per worker				
	1	2	3	4	5 (largest)
Mean number of suppliers					
Data	5.8	6.7	5.8	11.4	25.8
Model	6.6	5.1	5.9	8.3	26.9
Mean number of customers					
Data	5.6	7.0	6.7	11.7	25.1
Model	8.2	6.8	7.5	9.5	21.0
Standard deviation of log sales					
Data	1.37	1.34	1.37	1.52	1.79
Model	1.42	1.33	1.35	1.38	1.72
Share of total network sales					
Data	0.03	0.04	0.04	0.10	0.78
Model	0.07	0.04	0.05	0.09	0.74
Fraction of exporters					
Data	0.08	0.18	0.16	0.34	0.57
Model	0.17	0.15	0.19	0.28	0.56
Export Intensity of Exporters					
Data	0.24	0.23	0.23	0.23	0.26
Model	0.19	0.22	0.24	0.25	0.29
Shift-share IV coefficient (5% export shock)					
Data	0.21%				
Model	0.21%				

Notes: This table shows the targeted moments used in the estimation for a special case where we shut down the complementarity in matching ($\nu_v \to \infty$) and in production ($\nu_y = 0$) and compares our simulated moments to that from the data. Firms are ranked according to their average wage per worker. We match the following moments by quintile of firm wage: the mean number of suppliers (5 moments), the mean number of customers (5 moments), the share in total network sales (5 moments), the standard deviation of sales (5 moments), the fraction of exporters (5 moments), the export intensity of exporters (5 moments). Besides, we also match the shift-share IV coefficient (1 moment).
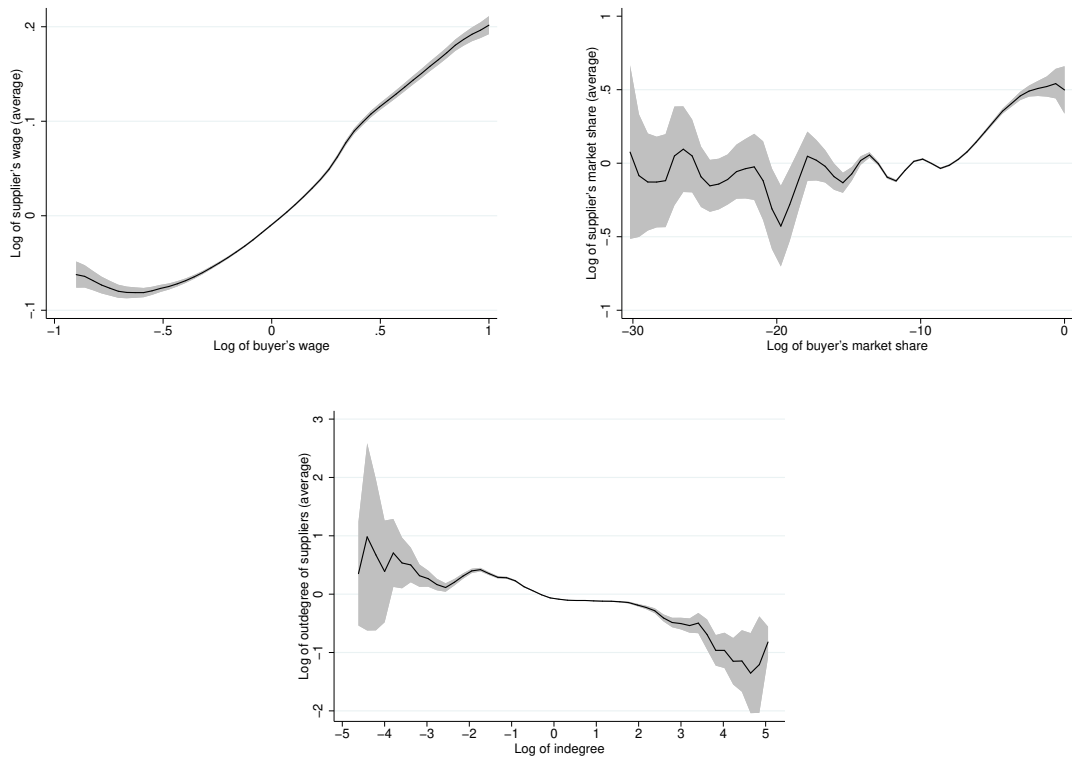
E.2 Figures

Figure A1: Matching on Sales and Network Size (Manufacturing firms)
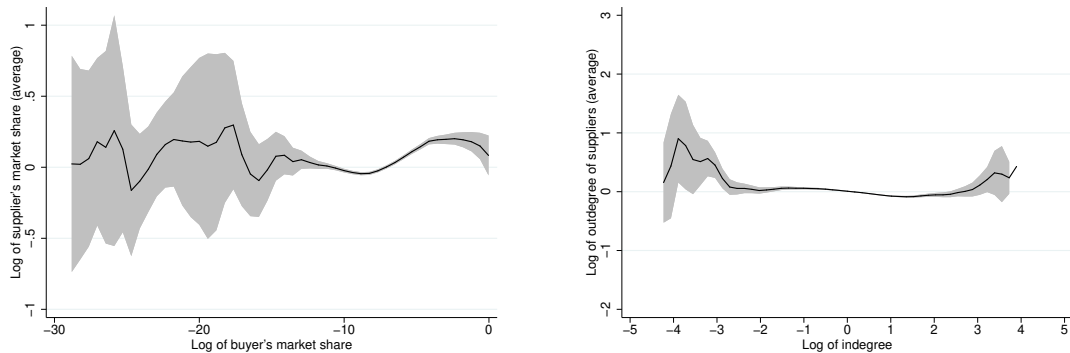
Notes: Sample includes manufacturing firms on both sides of the transaction. Market share is defined as the firm's share in gross sales of its respective 4-digit NACE industry. Indegree and outdegree refer to a firm's number of suppliers and buyers, respectively. Both x- and y-axis variables are demeaned from 4-digit NACE industry averages. The fitted curves are obtained from local polynomial regression with Epanechnikov kernel of the (residual) x-axis variables. The shaded areas show the respective 95% confidence intervals.

Figure A2: Matching on Wages, Sales and Network Size (All firms)

Notes: Sample includes manufacturing and service firms on both sides of the transaction. Wage is the average value of monthly payments per worker. Both buyer and supplier wages are demeaned from their respective industry (4-digit NACE) and region means and adjusted for firm size, i.e. employment. Market share is defined as the firm's share in gross sales of its respective 4-digit NACE industry. Indegree and outdegree refer to a firm's number of suppliers and buyers, respectively. Both x- and y-axis variables are demeaned from 4-digit NACE industry averages. The fitted curves are obtained from local polynomial regression with Epanechnikov kernel of the (residual) x-axis variables. The shaded areas show the respective 95% confidence intervals.

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Figure A3: Untargeted Firm-to-firm Trade Moments for Buyers

(a) Share of Suppliers (Data)
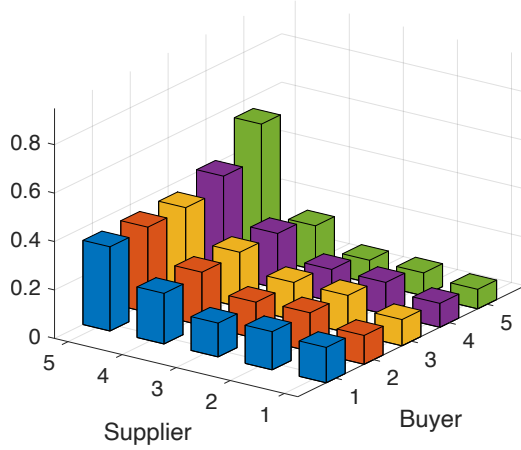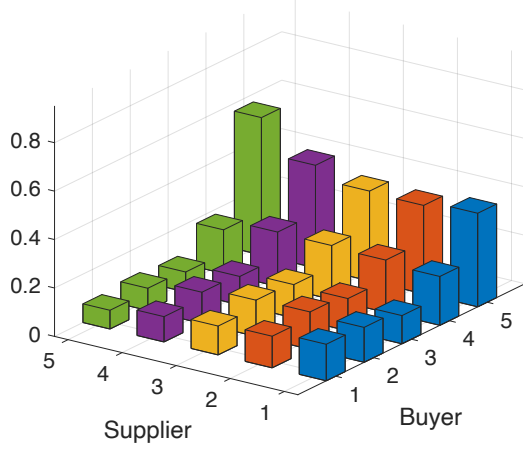
(b) Spending Shares (Data)
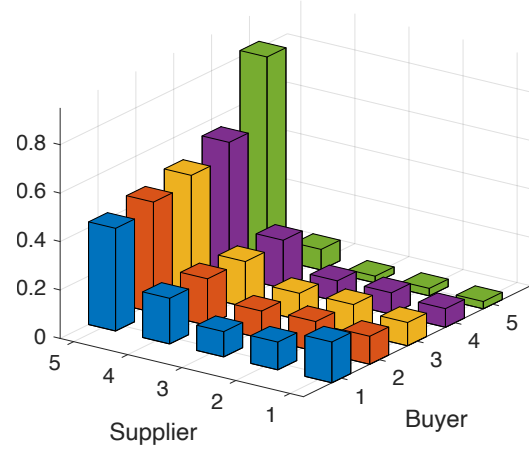
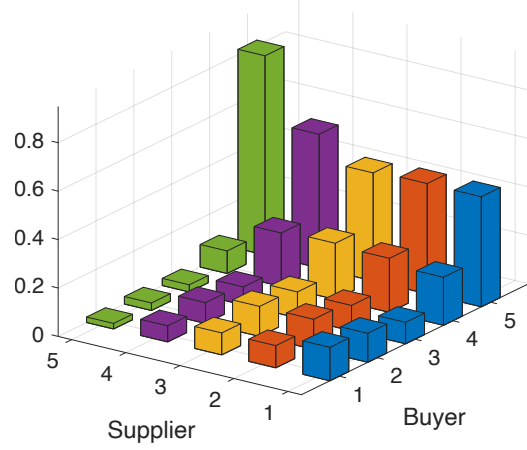(c) Share of Suppliers (Model)
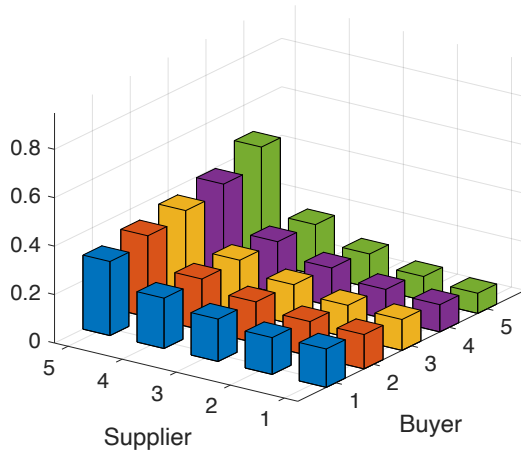
(d) Spending Shares (Model)
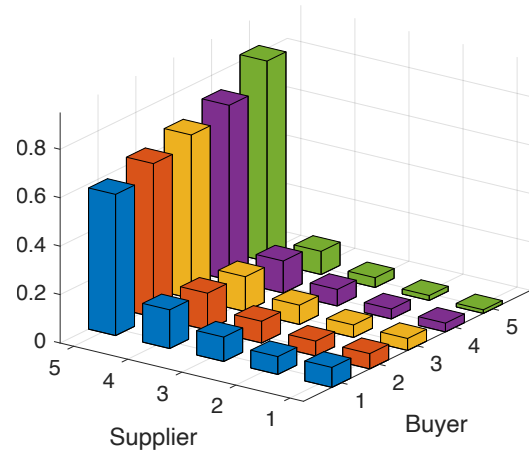
Notes: This figure compares the data moments (top panels) to the untargeted moments implied by the model (bottom panels). Firms are ranked according to their average wage per worker. For each buyer quintile, number of suppliers and expenditures are aggregated at the level of supplier quintile. Buyer and supplier quintiles are shown on the x- and y-axis while z-axis shows the corresponding shares. For instance, panel (a) shows for each buyer quintile the share of suppliers that belong to each wage quintile. Similarly, panel (b) shows for each buyer quintile the spending share on suppliers that belong to each wage quintile.

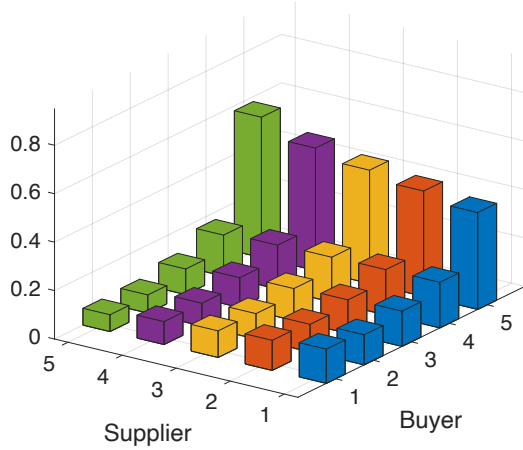Figure A4: Untargeted Firm-to-firm Trade Moments for Suppliers
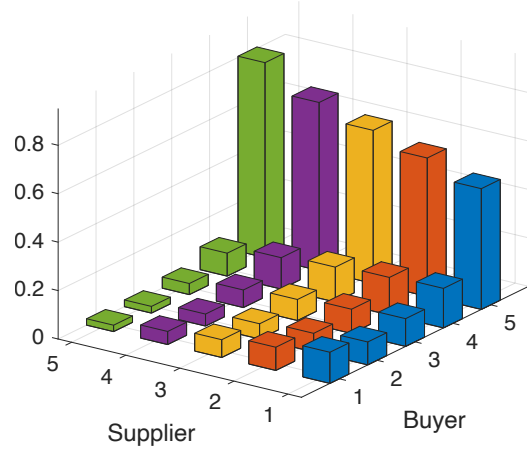
(a) Share of Buyers (Data) (b) Sales Shares (Data)

(c) Share of Buyers (Model) (d) Sales Shares (Model)

Notes: This figure compares the data moments (top panels) to the untargeted moments implied by the model (bottom panels). Firms are ranked according to their average wage per worker. For each supplier quintile, number of buyers and sales are aggregated at the level of buyer quintile. Buyer and supplier quintiles are shown on x- and y-axis while z-axis shows the corresponding shares. For instance, panel (a) shows for each supplier quintile the share of buyers that belong to each wage quintile. Similarly, panel (b) shows for each supplier quintile the sales share to buyers that belong to each wage quintile.

F Canonical Correlation Analysis

To understand the characteristics of the assortative matching pattern between buyers and suppliers in the data, we adopt the Canonical Correlation Analysis used in the literature on marriage market matching as proposed by Becker (1973). The approach, developed by Johnson and Wichern (1988), relies on the assumption that there exists positive assortative matching between buyers and suppliers: more "attractive" buyers match with more "attractive" suppliers. Their attractiveness may depend on a number of characteristics. Here, we will focus on firm size and quality, proxied by average wages. We construct indices that summarize the attractiveness of buyers and suppliers, A_b and A_s, as linear combinations of size and quality:

$$
\begin{aligned}
A_b &= k_1^b \log sales_b + k_2^b \log wage_b \\
A_s &= k_1^s \log sales_s + k_2^s \log wage_s
\end{aligned}
\tag{70}
$$

Since the number of variables is equal to two in both A_b and A_s, the maximum number of (independent) canonical variates pairs is two. The coefficients on sales and wages are estimated by maximizing the correlation between the two attractiveness indices, subject to two normalization restrictions.

More formally, let X_b and X_s denote the vectors of buyer and supplier characteristics, namely sales and wages, and k^b and k^s denote the vectors of respective weights in equation (70). The estimated weights for the first canonical variates pair solve the following maximization problem:

$$
max \ k^{b'} E[X_b X_s'] \ k^s
$$
$$
\text{subject to}
$$
$$
k^{b'} E[X_b X_b'] \ k^b = 1, \ k^{s'} E[X_s X_s'] \ k^s = 1
$$

If the buyer and supplier characteristics have Gaussian distributions, the estimated weights are consistent.[30]

To carry out the analysis, we first demean wage and sales variables from their 4-digit NACE industry averages, and then standardize them, i.e. all four variables ($\ln sales_b$, $\ln wage_b$, $\ln sales_s$, and $\ln wage_s$) have zero mean and unit variance. Therefore, the estimated weights for different variables are directly comparable. Table A7 presents the estimation results. All canonical coefficients are estimated to be positive and statistically

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[30]See Dupuy and Galichon (2015) for a detailed discussion.

Table A7: Results from the Canonical Correlation Analysis

	Canonical coefficients	p-value
$\log sales_b(k_1^b)$	0.29	0.00
$\log wage_b(k_2^b)$	0.80	0.00
$\log sales_s(k_1^s)$	0.11	0.00
$\log wage_s(k_2^s)$	0.94	0.00
First canonical correlation	0.15	0.00
Second canonical correlation	0.04	0.00

Notes: Wage is defined as the average value of monthly payments per worker.

significant at the 1% level. For buyers, the weight associated with the wage variable is larger than the one associated with sales by almost a factor of 3, and for suppliers it is larger by a factor of 8.5. This implies that while firm size increases the attractiveness of both buyers and suppliers, their attractiveness levels are primarily determined by quality. This result is consistent with the size of the bivariate correlations in the raw data: bivariate correlation between wages of buyers and suppliers is 0.15, which compares to a correlation of 0.08 between their sizes. Figure A5 shows a strong positive correlation between the predicted value of the buyer attractiveness index and that of the supplier.

Figure A5: Predicted Attractiveness of Buyers and Supliers
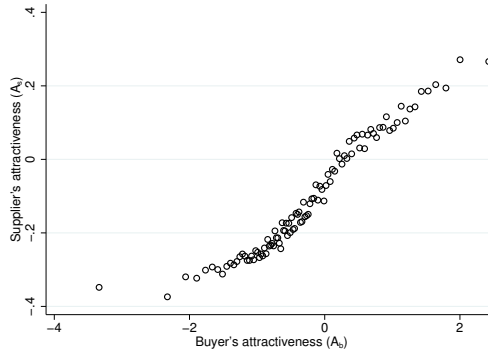
Notes: Sample includes manufacturing firms on both sides of the transaction. A_b and A_s denote the attractiveness indices of buyers and suppliers as defined in (70). Each circle represents the average value of the predicted A_b and A_s within a percentile of A_b.

We augment the Canonical Correlation Analysis with an econometric analysis in the spirit of Benham (1974), where each buyer (and supplier) characteristic, namely size and quality, is regressed on all supplier (buyer) characteristics controlling for industry-region of the buyer and supplier. As before, the variables are standardized so that the coefficient

Table A8: Matching Patterns

	$\log sales_b$ (1)	$\log wage_b$ (2)	$\log sales_s$ (3)	$\log wage_s$ (4)
$\log sales_s$	0.039	-0.026		0.587
	(0.005)	(0.004)		(0.018)
$\log wage_s$	0.018	0.091	0.592	
	(0.007)	(0.004)	(0.012)	
$\log sales_b$		0.593	0.040	0.019
		(0.018)	(0.005)	(0.005)
$\log wage_b$	0.627		-0.028	0.098
	(0.014)		(0.004)	(0.005)
R^2	0.519	0.545	0.503	0.508
N	941,972	941,972	941,972	941,972

Notes: Wage is defined as the average value of monthly payments per worker. All columns include buyer industry-province and supplier industry-province fixed effects. Robust standard errors are clustered at buyer-supplier industry pairs.

estimates are directly comparable to each other.

Table A8 presents the results from the OLS regressions. As expected the correlation between own size and quality for both buyers and suppliers is large – at about 0.6. The estimated coefficients on wages in columns (2) and (4) imply that the correlation between buyer and supplier wages, conditional on their sizes, is large and statistically significant. In particular, a one standard deviation in the wage of the trade partner is associated with almost a 0.1 standard deviation increase in own wage. These results are consistent with the results from the Canonical Correlation Analysis: average wages of buyers and suppliers, after controlling for their sizes, are important determinants of the positive assortative matching pattern observed in the data.

Two points are in order about the analysis above. First, it is useful to draw conclusions about the matching patterns between buyers and suppliers at the extensive margin. In other words, neither the Canonical Correlation Analysis nor the multivariate OLS regressions tell us anything about the intensity of purchases (or sales) within the same quality levels. Second, the estimates presented in Table A8 are not directly comparable to those related to the extensive margin of matching presented in Table 2. The reason is as follows: an observation is a buyer-supplier pair in the former while it is a buyer in the latter where supplier wage is the average of all suppliers of a buyer.

G Additional Robustness Tests

## G.1 Relationship between wages and quality of exports

Our measure of firm's quality is the average value of wage payments per worker. We assume that there is a tight association between ranking of firm-level average wages and quality. An alternative (and a more direct) measure of firm's quality, which is widely used in the literature, is the unit value of exports. To check whether firm's average wages are informative about the quality of its exports, we adopt the approach proposed by Khandelwal et al. (2013). In particular, we use detailed data on the customs records of exporters in our sample (for the year 2015) and estimate the following regression:

$$\ln X_{fpc} + \sigma \ln UV_{fpc} = \alpha_c + \alpha_p + \epsilon_{fpc}, \tag{71}$$

where X_{fpc} is the quantity of exports of product p by firm f to country c, and UV_{fpc} is its unit value. We set $\sigma = 5$. Estimated (logarithm of) quality is given by $\hat{\epsilon}_{fpc}/(\sigma - 1)$. We aggregate it to the firm level by taking its simple average across all varieties (product-country pairs) exported by the firm.

Figure A6 shows the binned scatterplot of product quality estimated from equation (71) against firm-level average wages. Both variables are adjusted for their industry averages (4-digit NACE level). While there is a slightly positive correlation between the two variables for the lower deciles of wages, it becomes steeper for the higher deciles. This implies that firm-level wage is a better predictor of its product quality for relatively high-wage firms. This is also confirmed by the groupings of firms into quintiles based on their wages and product quality. When the quintiles are constructed based on wages, almost half of the firms (45%) in the lowest (highest) quintile fall into the lowest (highest) two quintiles constructed based on product quality. When the middle quintile is included in the calculations, both shares go up to 65%.

Figure A6: Wages and Product Quality
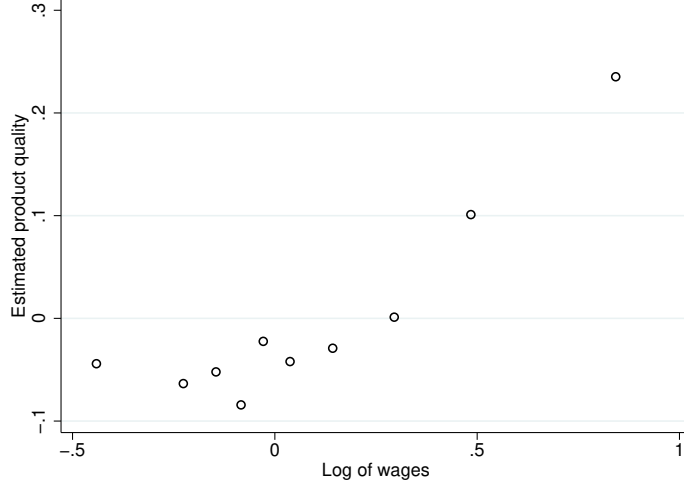
*Notes:*We define the wage of a firm as the firm's wage bill divided by the number of workers. Quality is estimated from equation (71). Both x- and y-axis variables are demeaned from 4-digit NACE industry.

G.2 Alternative measure of worker skills

We construct an alternative measure of worker quality using Turkish linked employer-employee data for the 2014-2016 period. To do so, we follow the approach proposed by Bombardini et al. (2019), which is based on the seminal contribution of Abowd et al. (1999). This approach decomposes the variation in firm-worker level wages into firm and worker components. For the decomposition, we estimate the following standard specification for worker earnings:

$$\ln wage_{eft} = \Gamma X_{eft} + \theta_e + \psi_f + e_{eft}, \tag{72}$$

where the vector X_{eft} includes a number of worker and firm characteristics. For workers, these are age (squared) and dummies for 1-digit ISCO occupation codes. For firms, the controls are dummies for each industry-region-time triplet and size (proxied by gross sales). Controlling for those worker and firm-level characteristics, θ_e and ψ_f capture the unobserved variation in worker earnings due to workers and firms, respectively.

Our sample includes more than 3.2 million firm-worker-year observations. It is well known in the literature that the fixed effects in equation (72) are identified from workers moving between jobs, which creates a connected network of firms. As we focus on manufacturing firms, we use estimated fixed effects obtained only from the largest connected network of manufacturing firms. Given the industry restriction and the short time span

(i.e. 3 years), this sample corresponds to about 65% of all workers.

Using the estimated worker fixed effects $\hat{\theta}_e$, we follow Bombardini et al. (2019) and construct a measure of average worker quality at the firm level:

$$\theta_f = \frac{1}{N_f} \sum_{e \in E_f} \hat{\theta}_e, \tag{73}$$

where N_f denotes the number of workers of firm f, and E_f the set of workers employed by the firm in the year 2015.

There is close overlap between the quintiles of wages and worker skills. In particular, 62% (42%) of firms in the highest (lowest) quintile based on wages fall into the highest (lowest) two quintiles constructed based on average worker skills. When the middle quintile is included in the calculations, the respective shares go up to 85% and 62%.

Table A9: Assortative Matching on Worker Skills

	total	extensive	intensive
	(1)	(2)	(3)
θ_f	0.120	0.080	0.040
	(0.006)	(0.005)	(0.007)
R^2	0.095	0.104	0.045
N	53,601	53,601	53,601
Fixed effects	ind-prov	ind-prov	ind-prov

Notes: θ_f denotes average worker skills for firm f and is defined in equation (73). The suppliers' worker skills are constructed as a weighted average of θ_ω, where weights represent the share of supplier ω in firm f's total spending on inputs. Ind and prov refer to 4-digit NACE industries and provinces, respectively. Equations (3) and (4) define the extensive (EM_f^S) and intensive margins (IM_f^S). They capture respectively the extent to which firm f matches with high-quality firm or tilts its spending toward high-quality suppliers. All specifications include industry-province (ind-prov). Robust standard errors are clustered at 4-digit NACE industry level.

We also re-run our sorting regressions using average worker skills as a proxy for firm quality. Results are presented in Table A9. While the estimated sorting coefficient is halved compared to our baseline estimate, it is still economically and statistically significant. Moreover, its decomposition into extensive and intensive margins remain close to the baseline results.