The Value of Ignoring Risk: Competition between Better Informed Insurers*

Laura Abrardi†, Luca Colombo‡, Piero Tedeschi§

April 2020

Abstract

We study a competitive insurance market in which insurers have an imperfect informative advantage over policyholders. We show that the presence of insurers privately and heterogeneously informed about risk can explain the persistent profitability, the pooling of risk and the concentration levels observed in some insurance markets. Furthermore, we find that a lower market concentration may entail an increase in insurance premia.

Keywords: Insurance markets, Asymmetric information, Risk assessment, Market concentration.

JEL codes: D43, D82, G22

1 Introduction

In the last few years, the American insurance industry hit repeatedly the headlines both because of a number of reforms that are reshaping it and because of the deep technological

*This paper supersedes a previous version that circulated under the title "Competitive Insurance Markets with Better Informed Principals". For useful comments and suggestions, we thank Andrea Attar, Francesca Barigozzi, Alberto Bennardo, Alessandro Bonatti, Sandro Brusco, Elisabetta Iossa, Marco Li Calzi, Alessandro Pavan and seminar participants at various conferences and institutions where the paper was presented.

†Politecnico di Torino, Dipartimento di Ingegneria Gestionale e della Produzione Corso Duca degli Abruzzi 24, 10129 Torino. E-mail: laura.abrardi@polito.it

‡Università Cattolica del Sacro Cuore, Department of Economics and Finance, Largo A. Gemelli 1, 20123 Milano. E-mail: lucava.colombo@unicatt.it

§Università Cattolica del Sacro Cuore, Department of Economics and Finance, Largo A. Gemelli 1, 20123 Milano. E-mail: piero.tedeschi@unicatt.it

changes that are substantially affecting the traditional business model. In particular, the profitability of insurance companies has been under intense scrutiny, giving rise to a heated debate about the driving forces of insurers' earnings (Cabral *et al.*, 2018). While the matter of profits understandably catalyzed most of the attention, other features of the industry ought to be considered. First, insurance markets appear to be highly concentrated (see e.g. Dafny *et al.*, 2012; Robinson, 2004).[1] Second, insurance contracts often entail premia that purposely ignore risk-relevant information, despite the fact that risk is one of the main cost drivers of insurance policies (e.g. Finkelstein and Poterba, 2014 for the annuity insurance).

These stylized facts challenge the typical characterization of equilibria in competitive insurance markets that builds on the seminal contribution by Rothschild and Stiglitz (1976). Indeed, although Rothschild and Stiglitz's (1976) analysis marks a fundamental advance in the understanding of competition under asymmetric information, the crucial result of that paper is that if an equilibrium exists it must be actuarially fair and separating. The fact that persistent profitability is observed in insurance markets seems to suggest that the classical competition framework may not be the most adequate to address the specificities of the insurance industry. Rather, the combination of both industry concentration and profits points towards the relevance of competitive mechanisms as those addressed by oligopolistic models and/or to the possibility of collusive behavior among insurers. Nonetheless, both hypotheses turn out to be largely discarded by the available empirical evidence (as discussed in Section 2).

The considerations above indicate that there might be a hole in the literature that this paper aims at filling by building a framework in which the key stylized facts reported in the pertinent empirical literature emerge endogenously as a result of information asymmetries and firms' competition. More specifically, in this paper we investigate a competitive insurance market in which insurers are better able than policyholders to estimate risk, although they manage to do so only imperfectly.

Traditional models of asymmetric information in insurance markets (affected by moral

[1] A 2018 report of the American Medical Association reveals that the majority of U.S. commercial health insurance markets are dominated by a small number of players. In 91 percent of metropolitan statistical areas (MSAs), at least one insurer has a commercial market share of 30 percent or greater, and in 46 percent of MSAs, a single insurer's market share is at least 50 percent.

hazard, adverse selection, or both), including Rothschild and Stiglitz (1976), typically assume better informed policyholders. However, the elusive nature of risk and the skills required to estimate it make it plausible that the informative advantage is held not (or not only) by policyholders, but by insurers as well. Insurers are indeed better qualified, for their greater expertise and access to data, to obtain a more precise assessment of risk than policyholders.[2] In this respect, the widespread use of improved and cheap monitoring devices are relentlessly draining the policyholder's informative advantage, whereas the ever-increasing availability of large data sets and advances in big data analytics and artificial intelligence are fueling the informative advantage of insurance companies. Already in 2015, The Economist addressed the problem of "how technology threatens the insurance business"(The Economist, March 16^{th}, 2015).

The difficulties underlying the process of risk estimation, even for experts, may be the cause of heterogeneity among insurers, which may be compounded by insurers' unwillingness (or impossibility) to share their beliefs about risk with competitors.[3]

We contribute to the literature on better informed insurers (e.g., Villeneuve, 2005) by focusing on a model where each insurer estimates policyholders' risk on the basis of a private, imperfect signal. After observing the signal, each insurer offers a menu of contracts to the policyholder, which may depend or not on the received signal, thus generating informative or non-informative equilibria, respectively. We find that in this setup both informative and non-informative equilibria can emerge for different parameter constellations, reconciling the theory with the empirical evidence about risk pooling in insurance markets.[4] Further-

[2] Policyholders' inability to correctly estimate risk has been highlighted by a large number of studies on overconfidence or unrealistic pessimism. The economic literature on overoptimism builds on the seminal contribution of Weinstein (1980). On policyholders' overconfidence, see e.g. Camerer (1997), Fang and Moscarini (2005), Garcia, Sangiorgi and Urosevic (2007), Hoelzl and Rustichini (2005), Kőszegi (2006), Menkhoff *et al.* (2006), Noth and Weber (2003), Sandroni and Squintani (2007), Van den Steen (2004), and Zábojník (2004). As for unrealistic pessimism, refer to the seminal contribution by Kahneman and Tversky (1979).

[3] For instance, many laws require that medical records are not released to outsiders without the consent of the patient. This increases the probability of mistakes in the estimation of risk by insurers. Moreover (see e.g. Fombaron, 1997) companies may learn about the risk of their policyholders by observing claims records and contract choices but will not freely share these private information with rival firms. As a consequence, the rival firms do not have access to accident histories.

[4] Here, risk pooling means that all insurers, regardless of the signals they receive, offer the same (non-informative) contract. The existence of informative and non-informative equilibria depends on the distribution of insurers and policyholders' preferences, as well as on policyholders' out of equilibrium beliefs about

more, we show that the competitive mechanism is not only consistent with, but actually requires, a persistent profitability of the insurance industry (both under informative and non-informative equilibria), overcoming the existence of actuarially fair insurance for all policyholders that is typical of the Rothschild and Stiglitz's (1976) environment. Focusing on informative (separating) equilibria, this result follows from the fact that truthful revelation of insurers' private information requires positive profits in equilibrium, which cannot be achieved under actuarially fair schemes. Insurers' beliefs about the riskiness of the environment in which they operate have important implications on the informational rent needed to induce truthful revelation.[5] More precisely, insurers with a lower assessment of risk may have an incentive to lie, pretending to expect a riskier environment, in order to charge higher premia to customers. Hence, a truthful disclosure of the riskiness of the environment requires a higher informational rent the safer is the insurer's estimation of risk. An even simpler argument holds for non-informative (pooling) equilibria. Indeed, an equilibrium contract must meet the individual rationality constraints of *all* insurers regardless of their assessments of the riskiness of the environment. Hence, given that the participation constraint must be met also for those insurers believing that the environment is very risky, to be an equilibrium a pooling contract must entail positive expected profits for the insurers assessing a safer environment. Due to the implications of truthful revelation (in informative equilibria) and of individual rationality (in non-informative equilibria), positive profits are observed both for pooling and separating equilibria, and in both cases they are shown to be larger for insurers' expecting a safer market environment.

Quite importantly, our results on the existence of pooling equilibria are robust to a specification where we add the information asymmetries typical of the Rothschild and Stiglitz's (1976) approach (i.e. less informed insurer) to those introduced in our setup (i.e. less informed policyholder), and we let the latter become negligible. We show that pooling (non-informative) equilibria may exist also in this limit case – when our model converges to that by Rothschild and Stiglitz (1976) – provided that policyholders' types are not too

insurers' assessments of risk.

[5]Throughout the paper, with riskiness of the environment we indicate insurers' assessment of risk based on observables.

different (i.e. when the assessment of loss probabilities is similar). The intuition for the existence of non-informative equilibria hinges on the informational structure, as contracts offered by more informed insurers constitute a signaling device. Consistently with signaling games, the choice of out-of-equilibrium beliefs allows sufficient degrees of freedom to sustain pooling equilibria for specific parameter sets, impeding the emergence of 'cream-skimming' deviations. Interestingly, this mechanism is robust to the introduction of a second layer of asymmetric information, i.e. the policyholder's private information on the idiosyncratic components of her own risk.

Market concentration plays an important role on the existence of equilibria. Indeed, we prove that there exists an upper bound to the number of firms consistent with the existence of a non-informative equilibrium, and both a lower and an upper bound for the existence of informative equilibria.[6] Furthermore, and somewhat counter-intuitively, it is shown that in the case of non-informative equilibria a larger industry dispersion may entail larger equilibrium profits, consistently with the observed unstable relationship between market concentration and profitability.

In a policy perspective, our results may contribute to the debate about the impact of new technologies in the insurance industry. One major concern in this respect is that – to the extent that new technologies allow a more granular and precise assessment of risk – personalized policies unhinge the pooling of risk that is at the foundation of the existence and efficiency of insurance markets. Conversely, as already noted, the "coming revolution in insurance"(The Economist, March 9^{th}, 2017) is also expected to wreak havoc on a so-far relatively "complacent industry", boosting competition and eroding profits. Our paper suggests that these conclusions might be undeserved, by showing that pooling equilibria and persistent profitability are fully consistent with competition among insurers holding an informative advantage over policyholders.

The rest of the paper is organized as follows. Section 2 investigates the connections between our approach and the pertinent literature. Section 3 describes the baseline model.

[6]In the working paper version (i.e. Abrardi *et al.*, 2019), we report numerical simulations for empirically plausible parameter configurations, indicating that the upper bound is typically fairly small. This suggests that the insurance industry should be quite concentrated, consistently with the available empirical evidence on the structure of the industry.

Section 4 discusses the characteristics and provides existence conditions for non-informative (pooling) equilibria. It also provides (in Sub-section 4.3) an extension of the baseline setup to the case of 'two-sided' asymmetric information – by adding an information asymmetry *à la* Rothschild and Stiglitz (1976). Section 5 focuses on the characterization and existence of informative equilibria. Section 6 extends the baseline model to an arbitrary number of firms, showing the robustness of the results on equilibria characterization, and investigating the correlation between market concentration and profitability in equilibrium. Section 7 concludes. All proofs and technical details that are not crucial for the understanding of the underlying logic of our results are relegated into an Online Appendix.

2 Related Literature

The issue of competition in insurance markets has been largely debated. While the theoretical literature building on Rothschild and Stiglitz (1976) claims that competition rules out the possibility of positive profits for insurance companies even in the presence of information asymmetries, a large body of empirical evidence points to the opposite conclusion, with prices not reflecting insurance costs (e.g. Cummins and Tennyson, 1992; Robinson, 2004; Sommer, 2017).

It is well known that insurance markets are often highly concentrated. In many countries, the commercial health insurance market is dominated by two to three carriers (Robinson, 2004; Fulton, 2017; Dauda, 2018). High levels of concentration are also found for property-liability insurance (e.g. Cummins and Weiss, 1993; Chidambaran *et al.*, 1997), and for private passenger automobile insurance (e.g. Bajtelsmit and Bouzouita, 1998). Nonetheless, there is no evidence of a significant relationship between the rise in insurance prices and market concentration (Dafny *et al.*, 2012; Hyman and Kovacic, 2004). Rather, and quite surprisingly, greater insurer concentration has been shown to depress the prices of health insurance (Dauda, 2018). All this seems to suggest that profitability in insurance markets requires a deeper explanation than the natural correlation between profitability and market concentration.

Also puzzling is the role of risk pooling in insurance. The available empirical evidence points to the existence of observable variables – i.e., attributes of insurance buyers that are correlated with risk – that are not used to price insurance policies. For example, Finkelstein and Poterba (2014) note how insurance companies in the UK annuity market "voluntarily choose not to price on the basis of risk-related buyer information that they collect"such as the annuitant's place of residence, although this may help to predict future mortality. This is starkly at odds with traditional competitive insurance models, showing that risk types must be separated in equilibrium, pointing to yet another hole in the theoretical modeling of insurance markets.

Two streams of recent literature have attempted at reconciling the theory of insurance markets with the empirical literature: the first building on non-exclusive contracts, and the second on exclusive contracts with more informed insurers.

The literature on non-exclusive contracts shows the existence of profitable and pooling equilibria by relying on the possibility for insurers to offer latent contracts that are never chosen in equilibrium. Such contracts are used strategically to prevent deviations by competitors. In a theoretical perspective, a desirable characteristic of latent contracts is that, by inducing non-negative profits if chosen, they are consistent with the logic of sequential rationality. There are however drawbacks. First, latent contracts do not necessarily entail non-negative profits (e.g. the moral hazard context in Attar and Chassagnon, 2009). Second, a large number of latent contracts might be required for an equilibrium to exist (in Attar *et al.*, 2011, an infinite number of latent contracts is offered in equilibrium), which is not that appealing in practice.[7] Third, there is evidence of insurance markets characterized by the presence of *exclusive* contracts that entail positive profits and risk pooling (automobile insurance contracts are almost always exclusive, as shown by Chiappori and Salanie, 2000).

The third issue is of practical importance. Indeed, the existence of exclusive contracts is often the result of regulatory choices. Hence, there is the need for a theory, complementing that on non-exclusive contracts and allowing instead for exclusive contracts, that can serve as guidance for the policy maker. Attar *et al.* (2011) argue that, under exclusive contracts, risk pooling and profitability cannot emerge with more informed policyholders, while Villeneuve

[7]Nonetheless, Attar *et al.* (2011) is extremely elegant in a purely theoretical perspective, achieving both equilibrium existence and uniqueness.

(2005) shows that this is not the case when focusing on competitive insurance markets with more informed insurers.

Also our work focuses on exclusive contracts and is related to that of Villeneuve (2005), although we consider a fundamentally different information structure.[8] Indeed, Villeneuve (2005) assumes that insurance companies observe the policyholder's type, while our analysis is based on insurers receiving only an imperfect signal about the riskiness of the environment.[9] As a consequence, in our model, insurers have no better information about the policyholder's type than the policyholder herself. This is consistent with Harsanyi's approach to incomplete information games entailing that each player has more information about her own type.

Our results depart quite substantially from those obtained by Villeneuve (2005), especially insofar the characterization and existence of informative (separating) equilibria are concerned. We show that informative equilibria must always be profitable in order to induce an insurer who receives a good signal about the riskiness of the environment to reveal his information. This is not the case in Villeneuve (2005), who finds that separating equilibria need not be profitable (although they can occasionally be). Furthermore, in Villeneuve (2005) separating contracts (with full insurance) are an equilibrium, implying that efficiency can always be achieved with more informed insurers. We show, instead, that informative equilibria are typically characterized by severe under-insurance, even when the signals received by insurers are very precise. This seems to indicate a strong reliance of the equilibria characterized by Villeneuve (2005) on the specificities of the information structure he considers.[10] Finally, it is worth noting that, as far as non-informative (pooling) equilibria are concerned, we improve upon Villeneuve (2005) by showing their existence and profitability also for heterogeneous and imperfectly informed insurers.

[8]Our contribution is also linked to Myerson (1983) and Maskin and Tirole (1990 and 1992), although they focus on exclusive contracts and more informed principals in a monopolistic setting. We argue later in the paper that the consideration of a competitive setting substantially alters the information structure of the agency problem.

[9]Note that the analysis in Villeneuve (2005) is a special case of ours when we assume that the signal received by insurers is almost perfect.

[10]While Villeneuve (2005) shows that a separating equilibrium always exists, we find that the existence of informative equilibria is restricted to a specific set of parameter values. Consistently with most literature on exclusive contracts, we cannot provide a general proof of existence of pure strategy equilibria. Nonetheless, in our setup equilibria exist for a wide range of parameters values, as we show in Abrardi *et al.* (2019) by means of numerical simulations based on a CARA specification of policyholders' utility.

3 A Baseline Model

We first present a baseline duopoly model in which insurers are assumed to be better able than policyholders to assess individual risk. Throughout most of the paper, to best highlight the implications of better informed insurers, we also assume that policyholders have *no* private information about their characteristics. This model allows us to convey all fundamental intuitions disposing of the technicalities involved by a generic n- firms oligopoly, to which we turn later in the paper (see Section 6).

This section describes the setup and the timing of the baseline model, it defines the adopted equilibrium concept and the belief systems we allow for, as well as the notions of non-informative and informative equilibria.

3.1 Setup and Timing

We consider an insurance market with two insurers ($i = 1, 2$) and one representative policyholder. The policyholder is endowed with initial wealth W, and faces a possible loss L.

Each insurance contract c defines an insurance premium (T) and a reimbursement (R) in the case of loss. The policyholder's wealth is $W_L = W - L + R - T$ when a loss occurs and $W_N = W - T$ under no loss. Therefore, insurers' contracts can simply be written as $c = (W_L, W_N)$, specifying the policyholder's wealth in the two possible states of the world.

The policyholder is assumed to be risk averse, with preferences represented by the Von Neumann-Morgenstern utility function $U(\cdot)$.[11] The policyholder's expected utility (for a generic loss probability p) when contract $c = (W_L, W_N)$ is implemented can therefore be written as

$$EU_p(c) = pU(W_L) + (1 - p)U(W_N).$$

Insurers are assumed to be risk neutral, so that (for a generic loss probability p) their

[11] When dealing with the issue of equilibrium existence, we specialize our analysis to the case where the preference ordering of the policyholder is represented by a constant absolute risk averse utility function.

profit function when contract $c = (W_L, W_N)$ is implemented is given by

$$E\pi_p(c) = p(W - L - W_L) + (1-p)(W - W_N).$$

Given her specific assessment of the loss probability, the policyholder is willing to accept any contract that is weakly preferred to the no-insurance outside option; i.e. she accepts any insurance contract c satisfying the participation constraint

$$EU_p(c) \geq EU_p(\underline{c}),$$

where $\underline{c} = (W - L, W)$ denotes the no-insurance outside option.

The environment θ in which the insurers and the policyholder operate can be either dangerous (d) with probability P_d, or safe (s) with probability $P_s = 1 - P_d$. The environment $\theta \in \{s, d\}$ affects the loss probability p_θ faced by the policyholder, with $p_s < p_d$. We assume that the value of θ is unobservable, although each insurer i receives independently and privately an imperfect signal $\hat{\theta}_i \in \left\{\hat{s}, \hat{d}\right\}$ about θ. The signal $\hat{\theta}_i$ identifies an insurer's type and it is not observable by the other insurer and by the policyholder. We refer to the vector of signals $(\hat{\theta}_1; \hat{\theta}_2)$ as a signal profile. Each signal $\hat{\theta}_i$ is correct with probability α, so that α indicates signal precision: $Pr(\hat{\theta}_i = \hat{s}|\theta = s) = Pr(\hat{\theta}_i = \hat{d}|\theta = d) = \alpha$ for $i = (1, 2)$, with $1/2 < \alpha < 1$. Since signals are independent and of equal precision, the number of insurers who receive signal $\hat{\theta}_i = \hat{s}$, denoted by $n_{\hat{s}}$, is a sufficient statistics of the loss probability, where obviously $n_{\hat{s}} = (0, 1, 2)$. Namely, $n_{\hat{s}} = 0$ if both firms receive signal $\hat{d}$; $n_{\hat{s}} = 1$ if one firm receives signal $\hat{s}$ and the other receives signal $\hat{d}$; and finally $n_{\hat{s}} = 2$ if both firms receive signal $\hat{s}$. Throughout the paper we summarize, without loss of generality, the signal profile by $n_{\hat{s}}$. We denote with $p_{n_{\hat{s}}} = (p_0, p_1, p_2)$ the loss probability conditional on $n_{\hat{s}}$, i.e.

$$p_{n_{\hat{s}}} = \sum_{\theta = s,d} p_\theta \Pr(\theta|n_{\hat{s}}), \tag{1}$$

where $\Pr(\theta|n_{\hat{s}})$ denotes the probability that the state of the world is θ (and hence the loss probability is p_θ) given that $n_{\hat{s}}$ insurers receive signal $\hat{s}$.

Each insurer i is assumed to offer a *menu*, C_i, of exclusive and non withdrawable contracts, conditional on what is revealed to the policyholder about the signal profile.[12] We denote with $C = \{C_1; C_2\}$ the vector of menus offered by the two insurers. The timing of the strategic interaction between the policyholder and the two insurers is as follows.

0. Nature moves first, choosing the environment θ and drawing – independently and from a common distribution – each insurer's signals on the environment $\hat{\theta}_i$;
1. insurer i, $i = 1, 2$, privately observes $\hat{\theta}_i$ and he updates his prior on θ conditional on $\hat{\theta}_i$;
2. insurers simultaneously make offers consisting of a menu of contracts $C = \{C_1; C_2\}$;
3. the policyholder observes all offers at no cost, updates her beliefs and selects one contract belonging to a specific menu. If she does not accept any contract, she receives her reservation option (no-insurance);
4. the accepted contract is implemented and payoffs are received.

This timing entails that the policyholder and the insurers rely on different estimates of the loss probability. In stage 0, there is no information about the loss probability but for the prior, so that the *ex ante* loss probability can be written as $\bar{p} = p_d P_d + p_s P_s$. The *ex ante* loss probability is updated by insurers at the *interim* stage (stage 1), based on the private signal they receive. We denote by $p_{\hat{\theta}}$ each insurer's *ad interim* estimation of the loss probability, obtained when insurer i observes only his own signal $\hat{\theta}_i$. In the next stage of the game (stage 3), the policyholder gathers market information about the signals received by insurers based on their offers. Let $\tilde{\mu}_i(C) \in [0, 1]$ be the probability (obtained using Bayes rule) that the policyholder assigns to $\hat{\theta}_i = \hat{s}$ given the vector C of contract menus she observes (i.e. $\tilde{\mu}_i(C) = Pr(\hat{\theta}_i = \hat{s}|C)$), so that the vector $\tilde{\mu}(C) = \{\tilde{\mu}_i(C)\}_{i=1,2}$ represents the policyholder's belief about insurers' type (i.e. about the signal profile $\{\hat{\theta}_i\}_{i=1,2}$). Then, based on $\tilde{\mu}(C)$ and using again Bayes rule, the policyholder can estimate the *ex post* loss probability $\tilde{p}(C)$.[13]

[12] The structure of these menus is clarified in Subsection 3.2, as it depends on whether the equilibrium considered is informative or non-informative.

[13] The *ex post* loss probability depends on the characteristics of the emerging equilibrium. If all insurers offer the same contract menu (a pooling uninformative equilibrium), policyholders learn nothing about insurers' signal profiles and need therefore to rely on the loss probability estimated *ex ante*. Conversely, under separating (informative) equilibria, the policyholder infers insurers' type by looking at contract menus, and she can exploit this information to update the *ex ante* loss probability.

3.2 Equilibrium and Beliefs

Given the timing and information structure of the model, the appropriate notion of equilibrium is that of perfect Bayesian equilibrium (PBE). Focusing on symmetric equilibria in which all insurers with the same type offer the same menu, a PBE can be defined as follows.

Definition 1 *A symmetric Perfect Bayesian Equilibrium is defined by (a) a vector of contract menus* $C^e = (C_1^e; C_2^e)$*, where* $C_i^e \in \left\{C_{\hat{\theta}_i}^e\right\}_{\hat{\theta}_i=\hat{s},\hat{d}}$*,* $i = 1, 2$*, depending on the type of insurer* i*, and (b) a belief mapping* $\tilde{\mu}(C)$ *such that:*

1. *insurers' strategies are sequentially rational, so that for any insurer* i *of type* $\hat{\theta}_i$*, the menu* $C_{\hat{\theta}_i}^e$ *is the one maximizing expected profits given (1) the strategy profile of insurer* j*,* C_j^e*,* $j \neq i$*, and (2) the policyholder's strategy;*
2. *for any given information set, the policyholder's equilibrium strategy selects the contract belonging to* C^e *that maximizes her expected utility given her beliefs* $\tilde{\mu}(C)$*;*
3. *beliefs are consistent with Bayes rule when relevant.*

The notion of Perfect Bayesian Equilibrium requires the definition of a belief system based on Bayesian updating on the equilibrium path and arbitrarily defined off the equilibrium path. The pertinent literature on informed insurers uses a variety of out of equilibrium beliefs. That closest to our contribution focuses almost exclusively on two such types of beliefs: pessimistic and optimistic (see e.g. Villeneuve, 2005 and Seog, 2009). The policyholder is defined as optimistic (pessimistic) when she believes that a deviating insurer receives the good signal $\hat{s}$ (bad signal $\hat{d}$), holding constant the equilibrium beliefs about the other. Obviously, beliefs need not be necessarily entirely optimistic or pessimistic: indeed, one could consider a broader array of beliefs, obtained as a convex combination of the two extremes. We focus on this more encompassing view, by defining the probability that the policyholder assigns to the event that a deviating insurer is of type $\hat{s}$ when a deviation is observed as her 'degree of optimism'. Formally,

Definition 2 *The policyholder shows a degree of optimism* $x \in [0, 1]$ *if, for any deviation,* $C_i \neq C_i^e$*,* $\tilde{\mu}_i(C) = x$*. In this case, the policyholder is labeled as* x*-optimistic.*

Note that, as it is standard in the literature, we assume that a deviation by insurer i reveals nothing about the type of the other insurer. Furthermore, the policyholder's degree of optimism x is independent of the specific deviation from the equilibrium operated by insurer i. Allowing for different degrees of optimism x, the properties and implications of alternative belief systems can be investigated. Interestingly, the notion of x-optimistic beliefs includes that of *fully optimistic* ($x = 1$) and *fully pessimistic* ($x = 0$) beliefs, as well as that of *passive* beliefs, which describes a policyholder retaining prior beliefs about the type of the deviating insurer (holding constant her equilibrium beliefs about the other insurer).[14]

It is useful to further discuss the implications of the model information structure, informally anticipating the characteristics of the possible equilibria entailed by such structure. Our setup may give rise to two fundamental asymmetric information problems. The first has the formal structure of a more informed principals problem stemming from the fact that insurers have private information about the riskiness of the environment in which they operate. Two opposite scenarios may emerge depending on whether insurers' information is revealed or not in equilibrium (resulting in informative or uninformative equilibria). In fully informative equilibria, each insurer's menu conveys information about his type: the differences in the menus offered by $\hat{s}$ and $\hat{d}$ insurers imply separating equilibria on the insurers' types in the first stage. Conversely, in non-informative equilibria the insurers' offers do not carry information about the insurers' signals: $\hat{s}$ and $\hat{d}$ insurers offer identical menus, which implies pooling on the insurers' types in the first stage.

The second asymmetric information problem arises exclusively in the context of informative equilibria, and it is originated by the fact that the insurers' menus convey information about their private signals. Thus, the policyholder is able to infer the signal profile by observing the two menus. Such market information is not available to the insurers when they make their offers since they are unaware of their competitor's type. Namely, the policyholder knows the whole signal profile when choosing the contract, while insurers know only their own type when issuing the offer. Despite insurers have private information, the strategic interaction that arises in our competitive setup is very different from that typically investigated by the more informed principal literature. Rather, our problem is analogous to a screening

[14] In our setup, passive beliefs are immediately obtained by setting $x = \alpha P_s + (1 - \alpha) P_d$.

one with better informed agents, where the signal profile constitutes the policyholder's type. Then, insurers offer an incentive compatible menu of different contracts, with one contract for each possible policyholder's type. The resulting equilibrium exhibits strong similarities with the Rothschild and Stiglitz menu, the only difference being that in our setup there are two different types of insurers rather than just one. The implication of the existence of two types of insurers, $\hat{s}$ and $\hat{d}$, is that their offers must necessarily differ. Indeed, any difference in the equilibrium menus of the two types of insurers conveys information about their types, and this is what ultimately makes the offer informative. Therefore, fully informative equilibria must entail separation, both with respect to the insurers' types (through differences in the equilibrium menus) and to the policyholder's types (through the offer of a menu of contracts, with one contract for each signal profile).

Under non-informative equilibria, instead, the insurers' types are not conveyed to the policyholder, which prevents her from making an inference about the signal profile. Indeed, in equilibrium there is only one type of policyholder, who maintains her ex-ante beliefs about risk even after observing the insurers' offers. Thus, given that observationally there is only one type of policyholder, the outcome of non-informative equilibria no longer entails a menu of contracts, but simply a single contract. Moreover, since any difference in the contract offered in equilibrium by the $\hat{s}$ and $\hat{d}$ insurers would convey information about the insurers' types, non-informative equilibria necessarily require that the equilibrium contract coincide for both types of insurers.

4 Non-Informative Equilibria: Characterization and Existence

We now turn to the existence and characterization of equilibrium offers, focusing first on non-informative equilibria.[15] For an equilibrium not to be revealing insurers' information, it must be that both firms offer the same menu of contracts. In terms of strategies, this obviously corresponds to a pooling equilibrium. In order to reduce the burden imposed on our model

[15] Although other types of equilibria may exist under mixed strategies, we restrict our attention to pure strategy equilibria only, consistently with what is typically done in the pertinent literature.

by technicalities, and consistently with the available empirical evidence, throughout the rest of the paper we restrict our attention to contracts entailing under or full insurance.[16] Even in this case, the set of non-informative equilibria is potentially infinite, as it depends on a number of factors, and particularly on the set of beliefs off the equilibrium path. Throughout the paper, we restrict the set of admissible contracts to those implying non-negative profits with positive probability if accepted, hence focusing exclusively on equilibria whose outcomes convey interesting economic insights in terms of profitability.[17]

In this setup, we show that the existence of non-informative equilibria depends fundamentally on insurers holding private information. A similar result has also been obtained by Villeneuve (2005), although in a more restrictive setup where firms *perfectly* know the policyholder's type, while the latter is only imperfectly informed about her own type. Our framework improves over that of Villeneuve (2005) by allowing for a formulation that is fully consistent with Harsanyi's approach to incomplete information games. The existence of pooling equilibria is obviously in stark contrast with Rothschild and Stiglitz (1976), who show that risk pooling cannot emerge when private information is held by policyholders only.

Finally, in Section 4.3 we extend our baseline model to the case of two-sided asymmetric information (i.e. a scenario in which insurers have private information about the environment and policyholders have private information about their own characteristics), in order to compare our results to those emerging in the seminal Rothschild and Stiglitz's (1976) setup.

4.1 Characterization

In non-informative equilibria, no revelation of information occurs as the insurers' offers do not convey information about their signals. This implies that no effective signaling on the insurers' type takes place. Since information would be revealed through differences in the equilibrium offers by the two types of insurers, insurers' offers must be identical, regardless of the signal received, $\hat{s}$ or $\hat{d}$. The observation of the equilibrium offers does not allow the policyholder to gather market information about the realized signal profile. For this reason,

[16] Such restriction is without loss of generality, at least in the case of non-informative contracts.

[17] Abrardi *et al.* (2019) generalize the analysis, showing that any individually rational allocation can be sustained as a non-informative equilibrium, when allowing for (latent) contracts entailing non-positive profits if accepted.

non-informative equilibria imply the existence of only one policyholder's type, who has an *ex ante* estimation of risk. Hence, the insurers do not have to screen among different types of policyholder. Since there is just one type of policyholder, each insurer's offer entails only one single pooling contract. In particular, there cannot exist pooling menus. Thus, $\hat{d}$ and $\hat{s}$ insurers in equilibrium offer the same contract c^e. As no information is conveyed in equilibrium, insurers simply update their beliefs on the environment based on their own signal $\hat{\theta}$. The loss probability estimated by insurers when signal $\hat{\theta}$ is received can thus be written as $p_{\hat{\theta}} = p_s \Pr(s|\hat{\theta}) + p_d \Pr(d|\hat{\theta})$ (*ad interim* estimation of risk). The expected loss probability estimated by the policyholder in equilibrium corresponds instead to the ex-ante loss probability $\bar{p} = p_d P_d + p_s P_s$.

The equilibrium contract c^e must meet three conditions. First, it must be profitable, or fair, for all insurers' types, so that insurers' participation constraint $E\pi_{p_{\hat{\theta}}}(c^e) \geq 0$ must be satisfied for all $\hat{\theta} = \{\hat{s}, \hat{d}\}$. Second, it must be acceptable by a policyholder who has the prior estimation of the loss probability, so that the policyholder's participation constraint $EU_{\bar{p}}(c^e) \geq EU_{\bar{p}}(\underline{c})$ must be satisfied. Third, any deviation acceptable by the policyholder must be less profitable than the equilibrium contract for both $\hat{s}$ and $\hat{d}$ insurers, given the policyholder's beliefs (i.e. there are no profitable deviations).

To better understand the third condition, note that the set of admissible deviations includes single contracts only and not menus (screening of the policyholder's type would not be possible also in deviation), given that there is just one type of policyholder. Nonetheless, insurers can offer different contracts according to their own type, $\hat{s}$ or $\hat{d}$. For all practical purposes, it is sufficient to focus only on the most profitable deviation for a $\hat{\theta}$ insurer, which we denote with $c_{\hat{\theta}}^{dev}$. Note that such deviation is preferred to autarky by the policyholder, given her off-equilibrium risk assessment $\tilde{p}(C)$.[18] Formally, the contract $c_{\hat{\theta}}^{dev}$ can be defined as

$$\begin{aligned} c_{\hat{\theta}}^{dev} \quad &\equiv \quad \arg\max_{c} E\pi_{p_{\hat{\theta}}}(c) \qquad (2) \\ s.t. \quad & \quad EU_{\tilde{p}(C)}(c) \geq EU_{\tilde{p}(C)}(\underline{c}), \end{aligned}$$

[18]With a slight abuse of notation, we consider profits as function of a single contract rather than of a menu.

where c denotes the degenerate menu of contracts C_i offered by insurer i. Note that $c_{\hat{\theta}}^{dev}$ is preferred to autarky, but not necessarily to c^e. Therefore, any deviation from c^e guarantees to a $\hat{\theta}$ insurer a level of expected profits that are at most equal to those induced by $c_{\hat{\theta}}^{dev}$, $E\pi_{p_{\hat{\theta}}}(c_{\hat{\theta}}^{dev})$.

If equilibrium expected profits ($\frac{1}{2}E\pi_{p_{\hat{\theta}}}(c^e)$) are greater than the upper bound of deviation profits ($c_{\hat{\theta}}^{dev}$), i.e

$$\frac{1}{2}E\pi_{p_{\hat{\theta}}}(c^e) \geq E\pi_{p_{\hat{\theta}}}(c_{\hat{\theta}}^{dev}) \text{ for all } \hat{\theta} = \left\{\hat{s}, \hat{d}\right\}, \tag{3}$$

then any acceptable and profitable deviation can be ruled out.

Interestingly, the participation constraint of any insurer is always met if (3) holds, since $E\pi_{p_{\hat{d}}}(c_{\hat{d}}^{dev}) \geq 0$. Therefore, a non-informative equilibrium exists if and only if condition (3) and the policyholder participation constraint are met.

Proposition 1 summarizes the above discussion and it establishes (in Point 3) that for profitable deviations not to exist, policyholder's beliefs must be sufficiently optimistic.[19]

Proposition 1 *If a non-informative equilibrium exists, it is such that:*

1. *insurers' participation constraint $E\pi_{p_{\hat{\theta}}}(c^e) \geq 0$ must be satisfied for all $\hat{\theta} = \{\hat{s}, \hat{d}\}$;*
2. *the policyholder's participation constraint $EU_{\bar{p}}(c^e) \geq EU_{\bar{p}}(\underline{c})$ must hold;*
3. *the policyholder's beliefs are sufficiently optimistic, i.e. $\tilde{p}(C) < \bar{p}$ for any $C \neq C^e$.*

The first two points of Proposition 1 follow directly from the discussion above, while point 3 requires more attention. When the policyholder is overly optimistic about her own risk, she expects larger rebates in the contract than insurers are willing to make. Thus, marginal undercuts from non-informative equilibria are rejected by the policyholder, who demands optimistically a higher discount on the offer. The higher is the policyholder's optimism, the higher is the requested rebate and the lower is the profitability of the deviation. Optimistic beliefs then hinder the effectiveness of the competitive mechanism, thus preventing the emergence of an actuarially fair outcome.

[19] Optimistic beliefs can be easily justified based on the vast literature on overconfidence. See e.g. Kahneman and Tversky (1979), Weinstein (1980), Kreuter and Strecher (1995), and Robb *et al.* (2004) for underestimation of health risk, as well as Groeger and Grande (1996) or Svenson (1981) for overconfidence on driving ability.

The result in point 3 of the proposition is fully consistent with the notion of distant types, which can be found in the literature on better informed insurers (see e.g., Villeneuve, 2005). Following this literature, types are distant if all contracts that are actuarially fair or profitable for a $\hat{d}$ insurer are not acceptable by the policyholder (i.e. they do not satisfy her participation constraint), entailing that a deviation contract cannot be preferred to the equilibrium contract. Since the policyholder's participation constraint crucially depends on the belief system, types are more likely to be distant when the policyholder is sufficiently optimistic off the equilibrium path. In particular, the assumption of distant types is a sufficient condition to prevent deviations by $\hat{d}$ insurers.

Besides characterizing them, we can immediately establish three results about the set of non-informative equilibria.

Result 1. *Equilibrium conditions allow for a multiplicity of equilibrium contracts c^e.*

This result is clearly illustrated in Figure 1, where the policyholder's participation constraint out of equilibrium is represented by the indifference curve $EU_{\tilde{p}}(\underline{c})$.[20] All contracts in the shaded area can emerge as a non-informative equilibrium outcome. Indeed, they satisfy the participation constraint of the policyholder in equilibrium (represented by the indifference curve $EU_{\bar{p}}(\underline{c})$); they meet the insurers' participation constraints, as they lie below the zero isoprofit lines of both $\hat{d}$ and $\hat{s}$ insurers; and they satisfy condition (3), represented by the dashed lines $2E\pi_{p_{\hat{s}}}(c_{\hat{s}}^{dev})$ and $2E\pi_{p_{\hat{d}}}(c_{\hat{d}}^{dev})$.

Result 2. *Non-informative equilibria are always strictly profitable for $\hat{s}$ insurers, due to their lower estimation of risk than $\hat{d}$ insurers.*

It is easy to see that if a contract is profitable for a $\hat{d}$ insurer, then it is profitable for a $\hat{s}$ insurer as well, given the latter's lower expectation of risk. Thus, the constraint $E\pi_{p_{\hat{s}}}(c^e) \geq 0$ is never binding and the condition $E\pi_{p_{\hat{d}}}(c^e) \geq 0$ is sufficient for the insurers' participation constraints to hold.[21] Moreover, also $\hat{d}$ insurers may have strictly positive profits whenever

[20]To lighten the reader's burden, whenever possible, we rely on a graphical analysis to illustrate our main arguments. Note that throughout the paper we disregard all contracts entailing overinsurance, although this does not entail any significant loss of generality in the case of non-informative equilibria.

[21]It is important to highlight that while in Villeneuve (2005) pooling equilibria are not necessarily entailing positive profits, this must be the case in our framework for $\hat{s}$ insurers, which implies that *ex ante* expected profits are always positive in our setup.

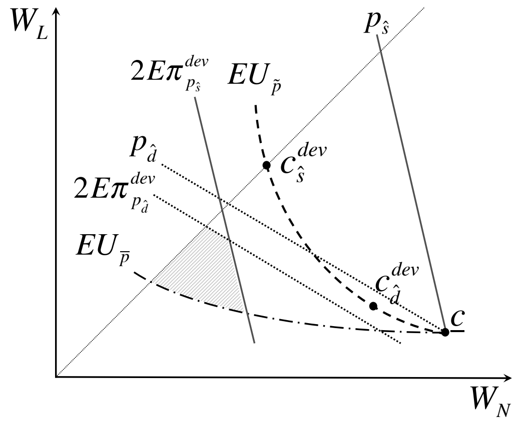

Figure 1: Non-Informative Equilibrium

the r.h.s. of condition (3) is strictly positive. Interestingly, although profitable equilibria may not be robust to refinements such as the Intuitive Criterion, Abrardi *et al.* (2019) show that this is not the case in our setup.

Result 3. *There may exist equilibrium contracts entailing full insurance and hence inducing ex ante Pareto efficient allocations.*

The point is again easily illustrated by focusing on all the contracts along the 45° line in the shaded area of Figure 1. Interestingly and different from what is commonly argued by most of the pertinent literature, Result 3 builds a case in favor of the efficiency of non-informative equilibria (i.e. risk pooling) in insurance markets.[22]

4.2 Existence

In general, the issue of equilibrium existence is demanding if tackled without focusing on specific functional forms. Hence, throughout the paper we limit our analysis of existence to instances where the policyholder's utility function entails constant absolute risk aversion, adopting the following CARA specification

$$U(W_J) = 1 - e^{-\beta W_J}, \quad J \in \{L, N\}, \tag{4}$$

[22]For a notable exception pointing in the same direction of our result, see Diamond (1992).

where β denotes the degree of (absolute) risk aversion. In this case, the existence of non-informative equilibria entailing a positive level of insurance can be easily established under relatively mild conditions on probabilities and on the degree of the policyholder's risk aversion, as summarized in Proposition 2.[23]

Proposition 2 *Given the CARA policyholder's utility specification (4), an (ex-ante) efficient non-informative equilibrium exists if the following conditions hold:*

1. *the probability of the s environment is sufficiently low, P_s close to zero; i.e. both $p_{\hat{d}}$ and $\bar{p}$ are sufficiently close to p_d.*
2. *risk aversion is sufficiently low (β close to zero);*
3. *the loss probability in the s environment is sufficiently low, p_s close to zero, and the precision of the signal is sufficiently high, α close to 1.*

For the sake of simplicity we illustrate the economic content of the three conditions in Proposition 2 by assuming a fully optimistic beliefs system.[24]

It is easy to see that Condition 1 in Proposition 2 guarantees that the insurers and the policyholder's participation constraints are met in equilibrium. As the probability of the safe environment s diminishes, $\bar{p}$ – the loss probability assessment of the policyholder – and $p_{\hat{d}}$ – the loss probability assessment of the $\hat{d}$ insurer – approach the same value p_d.[25] If the policyholder and the insurer have similar assessments of the loss probability, the risk aversion of the former ensures that there exist gains from trade between the policyholder and the $\hat{d}$ insurer. Moreover, recall that the participation constraint of the $\hat{s}$ insurer is always met if that of the $\hat{d}$ insurer holds.

Condition 2 guarantees that no profitable deviations exist for $\hat{s}$ insurers. In fact, the lower is the policyholder's degree of risk aversion, the lower is her willingness to pay for insurance. Then, deviations acceptable by a fully optimistic policyholder (who estimates a loss probability $p_{\hat{s}}$) are such that the expected profits of a $\hat{s}$ insurer (with the same risk

[23] Abrardi *et al.* (2019), focusing on the CARA specification (4), provide numerical simulations confirming that a non-informative equilibrium exists for a wide range of (empirically plausible) parameter values and it is therefore a robust feature of insurance markets.

[24] By continuity, Proposition 2 holds even for less restrictive belief systems, provided they are sufficiently optimistic as required by Proposition 1.

[25] This follows directly from the definition of $\bar{p}$, i.e. $\bar{p} = P_s p_s + (1 - P_s) p_d$ as P_s goes to zero.

assessment) are lower than those he obtains in equilibrium. However, this condition is not sufficient to rule out deviations by a $\hat{d}$ insurer.

Indeed, a $\hat{d}$ insurer can improve his profits by offering an underinsurance contract whenever the policyholder is optimistic. This happens because a $\hat{d}$ insurer estimates a higher risk than the optimistic policyholder, and thus he prefers to reduce insurance coverage. To prevent this deviation by the $\hat{d}$ insurer being profitable, it must be that the policyholder's willingness to pay for insurance is low. This occurs when $p_{\hat{s}}$ is sufficiently low, which in turn requires that the signal is precise and the risk in environment s is negligible, i.e. α is close to 1 and p_s is close to zero as required by Condition 3 in Proposition 2.[26]

4.3 Two Sided Asymmetric Information: A Comparison with Rothschild and Stiglitz (1976)

In the previous section we proved the possibility of profitable pooling equilibria, while the classical Rothschild and Stiglitz (1976) model – where asymmetric information is on the other side of the market – predicts that only separating and actuarially fair contracts may exist. One could conjecture that the existence of pooling equilibria is an implication of the fact that in our setup insurers rather than policyholders have an informative advantage.

In order to check whether our result remains valid when also policyholders have private information, we assume that the risk depends both on the general riskiness of the environment (d or s) and on the specific policyholder's personal characteristics (such as her health or habits), in line with Rothschild and Stiglitz (1976). We further assume that these characteristics are perfectly and privately known by the policyholder, but unobserved by the insurers. Finally, we maintain the assumption that each insurer receives an imperfect private signal about the environment riskiness.

Notice that when the environment is d (or s) with high probability and the precision

[26]Note that if a full insurance deviation is not profitable for a $\hat{s}$ insurer, then it is also not profitable for a $\hat{d}$ one. To see this, observe that the gain of abiding by the equilibrium, relative to a (full insurance) deviation, increases linearly with the assessment of the loss probability. In fact, denoting with W^e the policyholder's constant wealth in the full insurance equilibrium contract and with W^{dev} that in a full insurance deviation, the gain for an insurer of sticking to the equilibrium given a generic loss probability p is given by $\Delta\pi_p = W - pL - W^e - 2\left(W - pL - W^{dev}\right) = -W + pL + 2W^{dev} - W^e$. Since $p_{\hat{s}}$ is lower than $p_{\hat{d}}$ it follows immediately that if no profitable deviation exists for a $\hat{s}$ insurer, then it does not exists for a $\hat{d}$ one as well.

of the signal is high, then our model can be thought of as a perturbation of the original Rothschild and Stiglitz's one. In the following we show that, in this case (and provided that the damage probabilities for different types of the policyholder are not too far away), there exists a set of parameter values guaranteeing the existence of profitable non-informative equilibria that are also pooling on the policyholder's idiosyncratic risk. As already noted in the Introduction, the reason why – differently from Rothschild and Stiglitz (1976) – non-informative pooling equilibria may exist in our setup lies in the signaling content of contracts, which in general makes 'cream-skimming' opportunities more difficult to exploit.

4.3.1 The Model

Under two-sided asymmetric information, the loss probability depends on two factors: the general riskiness of the environment $\theta \in \{d, s\}$ and the idiosyncratic risk $j \in \{h, l\}$ implied by the policyholder's personal characteristics. Based on her idiosyncratic risk, the policyholder may be labeled as high risk (h) or low risk (l). Recall that whether the policyholder is high or low risk $j \in \{h, l\}$ is her private information.

The combination of the general and idiosyncratic sources of risk gives rise to four states of the world, i.e. $\theta j \in \{dh, dl, sh, sl\}$. To ease the notation, we assume that the distribution of the idiosyncratic risk, described by the *ex ante* probabilities P_h and P_l, is independent from that of general risk, which is described by the probabilities P_s and P_d. Both the four probabilities and the fact that they are independent are common knowledge.

We denote with $p_{\theta j} \in \{p_{dh}, p_{dl}, p_{sh}, p_{sl}\}$ the loss probability in the state of the world θj. Given the policyholder's idiosyncratic risk, the d environment is riskier than the s one: $p_{dl} > p_{sl} \geq 0$ and $p_{dh} > p_{sh}$. Given the riskiness of the environment, the h policyholder is riskier than the l one: $p_{dh} > p_{dl}$ and $p_{sh} > p_{sl}$. Each insurer i receives an imperfect private signal $\hat{\theta}_i \in \{\hat{d}, \hat{s}\}$ about the general risk θ, which is correct with probability α, $1/2 < \alpha < 1$. All signals are independent and conditional only on θ. The probability α is common knowledge. The ex-post loss probability depends on the signal profile $n_{\hat{s}} \in \{0, 1, 2\}$ and on the policyholder's specific risk $j \in \{h, l\}$.[27] Therefore, the expected loss probability

[27] Consistently with the one-sided information case, $n_{\hat{s}}$ corresponds to the number of insurers having received signal $\hat{s}$, which can be taken as a sufficient statistic of the signal profile due to insurers'symmetry.

of an agent who could observe all information available in the market would be $p_{n_{\hat{s}}j}$, where $p_{n_{\hat{s}}j} \in \{p_{0h}, p_{0l}, p_{1h}, p_{1l}, p_{2h}, p_{2l}\}$, corresponding to the six possible market states.

The timing of the model is as follows.

1. At the beginning of the game, nature moves and chooses the values of θ and j. Furthermore, it draws independently and from a common distribution the signals $\hat{\theta}_i$ received by each insurer, conditional on θ;
2. insurer i, $i = 1, 2$, privately observes $\hat{\theta}_i$ and, conditional on $\hat{\theta}_i$, he updates his prior on θ; the policyholder observes j and updates her prior given j;
3. each insurer simultaneously offers a menu of contracts;
4. the policyholder observes all menus, updates her beliefs and selects one contract or the reservation option (the no-insurance contract $\underline{c}$);
5. the accepted contract is implemented, and payoffs are received.

In this framework, we define a non-informative pooling equilibrium (or fully pooling) as an equilibrium in which both types of insurers, $\hat{s}$ and $\hat{d}$, offer one identical single contract accepted by both types of policyholder, h and l. Hence, in such equilibrium insurers do not convey any information about the signal $\hat{\theta}$ received and they do not separate the two types of policyholder.

4.3.2 The Existence of Non-Informative Pooling Equilibria

In showing the existence of a non-informative pooling contract c^e, we follow the same approach as in Proposition 2.

First, a non-informative pooling equilibrium must be profitable for both insurers' types; i.e. the insurers' participation constraint $E\pi_{p_{\hat{\theta}}}(c^e) \geq 0$ must hold for $\hat{\theta} = \{\hat{s}, \hat{d}\}$. Second, it must be acceptable by the policyholder given her assessment p_j of the loss probability; i.e. the j policyholder's participation constraint $EU_{p_j}(c^e) \geq EU_{p_j}(\underline{c})$ must hold for all $j \in \{h, l\}$.[28] Third, there must not exist profitable deviations for both $\hat{s}$ and $\hat{d}$ insurers. We characterize the most profitable deviation (and develop the analysis that follows) by focusing on the

[28]Due to the assumption of independence between the distributions of j and θ and the fact that all signals are independent of the distribution of j, we only need to focus on the participation constraint of the l policyholder; i.e. $EU_{p_l}(c^e) \geq EU_{p_l}(\underline{c})$.

case of fully optimistic beliefs. Given that there are two types of policyholders (h and l), a deviating insurer $\hat{\theta}$ must offer a menu $C_{\hat{\theta}}^{dev} = \{c_{\hat{\theta}h}^{dev}, c_{\hat{\theta}l}^{dev}\}$, including one contract for each policyholder's type, that solves

$$\begin{aligned}
& \max_{c_{\hat{\theta}h}^{dev}, c_{\hat{\theta}l}^{dev}} P_h E\pi_{p_{\hat{\theta}h}}(c_{\hat{\theta}h}^{dev}) + P_l E\pi_{p_{\hat{\theta}l}}(c_{\hat{\theta}l}^{dev}) \\
s.t. \quad & EU_{p_{\hat{s}h}}(c_{\hat{\theta}h}^{dev}) \geq EU_{p_{\hat{s}h}}(\underline{c}), \\
& EU_{p_{\hat{s}l}}(c_{\hat{\theta}l}^{dev}) \geq EU_{p_{\hat{s}l}}(\underline{c}), \\
& EU_{p_{\hat{s}l}}(c_{\hat{\theta}l}^{dev}) \geq EU_{p_{\hat{s}l}}(c_{\hat{\theta}h}^{dev}), \\
& EU_{p_{\hat{s}h}}(c_{\hat{\theta}h}^{dev}) \geq EU_{p_{\hat{s}h}}(c_{\hat{\theta}l}^{dev}),
\end{aligned} \tag{5}$$

for all $\hat{\theta} \in \{\hat{s}, \hat{d}\}$. The first and second constraints of problem (5) are the participation constraints of a h and l fully optimistic policyholder, respectively. The third and fourth constraints guarantee the incentive compatibility of the deviation menu. To rule out deviations, expected profits in equilibrium must be greater than the profits in $C_{\hat{\theta}}^{dev}$; i.e.

$$\frac{1}{2} E\pi_{p_{\hat{\theta}}}\left(c^e\right) \geq P_l E\pi_{p_{\hat{\theta}l}}\left(c_{\hat{\theta}l}^{dev}\right) + P_h E\pi_{p_{\hat{\theta}h}}\left(c_{\hat{\theta}h}^{dev}\right) \text{ for all } \hat{\theta} = \left\{\hat{s}, \hat{d}\right\}. \tag{6}$$

Proposition 3 characterizes a set of parameters for which the three conditions above are met and therefore a non-informative pooling equilibrium exists.

Proposition 3 *In a two-sided asymmetric information framework, given the CARA specification (4) of the policyholder's utility, an (ex-ante) efficient non-informative pooling equilibrium with fully optimistic beliefs exists if:*

1. *the signal received by insurers about the riskiness of the environment is very precise (α is close to 1) and the probability of the s environment is sufficiently low (P_s close to 0).*
2. *the policyholder's risk aversion is sufficiently low (β is close to zero);*
3. *the loss probability in environment s is sufficiently low (p_{sl} and p_{sh} are close to 0);*
4. *p_{dl} is sufficiently close to p_{dh};*

Proposition 3 establishes that it is possible to observe an equilibrium pooling the risk of the h and l policyholders. As already noted, Condition 1 – requiring that the s environment is very unlikely and the signal is very precise – implies that this setup can be interpreted as a perturbation of the original Rothschild and Stiglitz's (1976) framework. Condition 2 prevents deviations by a $\hat{s}$ insurer. This condition is analogous to Condition 2 of Proposition 2, entailing that the lower is risk aversion, the lower is the policyholder's willingness to pay for insurance. In the limit, under risk neutrality, profits in the deviation are zero (as in deviation a fully optimistic policyholder has the same risk assessment of an $\hat{s}$ insurer), while profits in equilibrium are strictly positive (since the policyholder assesses a higher loss probability than a $\hat{s}$ insurer, despite her risk neutrality, she is willing to pay a price that is profitable for $\hat{s}$). Condition 3 together with a high level of precision of the signal (α close to 1) excludes deviations by a $\hat{d}$ insurer and it is again equivalent to Condition 3 of Proposition 2. The premium that a fully optimistic policyholder is willing to pay in deviation decreases the lower the loss probability in the s environment, provided that the signal about the riskiness of the environment is very precise (i.e. α is close to one, as required by Condition 1 in the proposition). Condition 4 guarantees that a $\hat{d}$ insurer obtains non-negative profits and a l policyholder accepts the contract offer (i.e. the tightest participation constraints are met). Intuitively, if the s environment is unlikely, the l policyholder estimates a loss probability close to p_{dl}, while a $\hat{d}$ insurer estimates a loss probability between p_{dh} and p_{dl}. If these probabilities are close to each other, then the policyholder's assessment is not far from the insurer's one, and both participation constraints are met. Quite obviously, this condition – focusing on the relative probabilities of the two types of policyholders – has no counterparts in Proposition 2.[29]

5 Informative Equilibria

Having established how our setup would be affected by asymmetric information on both sides of the insurance market, we move back to the case of one-sided asymmetric information on

[29]Abrardi *et al.* (2019) characterize, through numerical simulations based on the CARA specification (4) of policyholder's utility, the parameter regions where non-informative pooling equilibria and standard separating Rothschild and Stiglitz equilibria exist. The numerical analysis confirms the existence of a non-informative pooling equilibrium even when our setup 'converges' to that of Rothschild and Stiglitz (1976), provided that the two types of policyholder are sufficiently close to one another, as implied by Conditions 3 and 4 of Proposition 3.

the type of the insurer and turn to the characterization of informative equilibria in which each insurer's offer reveals his type.[30] In this scenario, by observing all offers, the policyholder is able to infer the set of insurers' signals; i.e. to uncover the number of firms that received signal $\hat{s}$. Hence, as already noted in Section 3, $n_{\hat{s}}$ is a sufficient statistic for insurers' signal. This market information is available to the policyholder when she chooses her contract, while it is not available to the insurers who only know their own type (namely, their private signal) when making their offers. The problem at hands is therefore a screening problem in which the market information about the signal profiles is the analogous of the policyholder's 'type'.

5.1 Characterization

Assuming an informative equilibrium exists, Proposition 4 provides its full characterization.[31] In the following, we denote with $c^e_{\hat{\theta},n_{\hat{s}}}$ – where $\hat{\theta} = \{\hat{s}, \hat{d}\}$ and $n_{\hat{s}} = \{0, 1, 2\}$ – an equilibrium contract offered by a $\hat{\theta}$ insurer when $n_{\hat{s}}$ insurers receive signal $\hat{s}$.

Proposition 4 *If an informative equilibrium exists, it has the following properties:*

1. *the contracts accepted in equilibrium are different for each signal profile;*
2. *if $n_{\hat{s}} = 2$, the accepted equilibrium contract is actuarially fair and it satisfies the incentive compatibility constraint with equality;*
3. *if $n_{\hat{s}} = 1$, the $\hat{s}$ insurer offers contract $c^e_{\hat{s},1}$ defined as*

$$\begin{aligned} c^e_{\hat{s},1} &= \arg\max_c E\pi^e_{p_1}(c) \\ & \quad s.t. EU_{p_1}(c) \geq EU_{p_1}(c^e_{\hat{d},1}); \end{aligned} \tag{7}$$

4. *beliefs are fully optimistic;*
5. *contract $c^e_{\hat{d},0}$ entails non-negative profits while contract $c^e_{\hat{s},1}$ entails strictly positive profits, if accepted; furthermore, the following condition holds*

$$E\pi_{\hat{s}}(c^e_{\hat{s},1}) \geq \frac{1}{2} E\pi_{\hat{s}}(c^e_{\hat{d},0}). \tag{8}$$

[30] To save space, we do not explore the implications of two-sided information on the existence of informative equilibria, as our results in this case are less strikingly different with respect to those emerging in Rothschild and Stiglitz (1976).

[31] Refer to the proof of Proposition 7 for an arbitrary number of insurance firms in the Online Appendix.

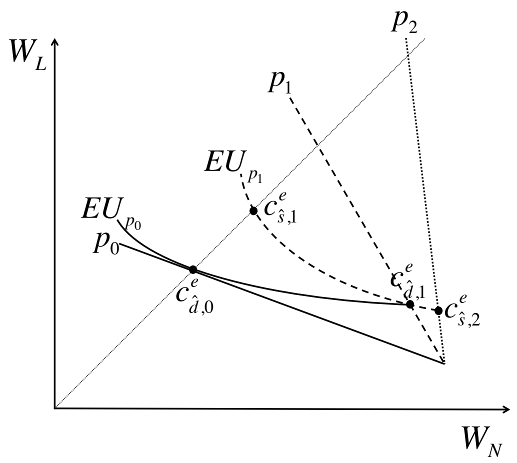

Figure 2: Informative Equilibrium

Figure 2 illustrates a candidate equilibrium that satisfies the conditions in Proposition 4 and in which $\hat{d}$ insurers offer a menu including the two contracts $c^e_{\hat{d},0}$ and $c^e_{\hat{d},1}$; $\hat{s}$ insurers offer a menu including the two contracts $c^e_{\hat{s},1}$ and $c^e_{\hat{s},2}$; the policyholder chooses contract $c^e_{\hat{d},0}$ under signal profile $n_{\hat{s}} = 0$, contract $c^e_{\hat{s},1}$ under signal profile $n_{\hat{s}} = 1$ and contract $c^e_{\hat{s},2}$ under signal profile $n_{\hat{s}} = 2$.

To better convey the main insights and key intuitions behind Proposition 4, it is worth highlighting the role that competition among insurers plays in the contractual design problem. When both insurers receive signal $\hat{s}$ ($n_{\hat{s}} = 2$), their profits are driven to zero by competition. To see why, assume by contradiction that equilibrium profits are positive. In this case, one insurer could profitably deviate by undercutting the other and his deviation would be accepted even by a fully optimistic policyholder, who would believe that the deviating insurer is of type $\hat{s}$. Hence, she would have the same *ex post* assessment of the loss probability as the deviating insurer himself.

A further consequence of competition among $\hat{s}$ insurers is that they offer a fully separating menu of contracts $c^e_{\hat{s},1}$ and $c^e_{\hat{s},2}$ for the signal profiles $n_{\hat{s}} = 1$ and $n_{\hat{s}} = 2$, respectively. Pooling between these two signal profiles cannot be an equilibrium strategy, as there always exists a profitable deviation that captures only the safest policyholder's type (in our case, $n_{\hat{s}} = 2$). Moreover, when both insurers receive signal $\hat{s}$ (i.e. $n_{\hat{s}} = 2$), competition guarantees that the policyholder achieves the highest possible level of expected utility. Hence, the incentive compatibility constraint – requiring that $EU_{p_1}(c^e_{\hat{s},1}) \geq EU_{p_1}(c^e_{\hat{s},2})$ – must necessarily be binding.

Matters become trickier when only one of the insurers receives signal $\hat{s}$. When the signal profile is $n_{\hat{s}} = 1$, the competitor of the $\hat{s}$ insurer must be a $\hat{d}$ insurer offering the menu $\{c^e_{\hat{d},0}, c^e_{\hat{d},1}\}$. In particular, contract $c^e_{\hat{d},1}$ is constrained by incentive compatibility to be an underinsurance contract. There is no such constraint for the $\hat{s}$ insurer, who can therefore maximize his profit, by increasing the insurance coverage with respect to the contract $c^e_{\hat{d},1}$ offered in equilibrium by a $\hat{d}$ insurer. Then, the only possible equilibrium contract offered by a $\hat{s}$ insurer for the signal profile $n_{\hat{s}} = 1$ is the one defined by Condition (7). Therefore, contract $c^e_{\hat{s},1}$ entails full insurance and it is strictly profitable. Moreover, under signal profile $n_{\hat{s}} = 1$, the offer of insurer $\hat{s}$ is accepted with probability 1; i.e. contract $c^e_{\hat{d},1}$ remains latent.

Interestingly, the existence of two different types of insurers creates a wedge with respect to the Rothschild and Stiglitz (1976) model, making it impossible for actuarially fair *equilibrium* outcomes under signal profile $n_{\hat{s}} = 1$. In our framework, $\hat{s}$ insurers possess a competitive edge over $\hat{d}$ types. Such an advantage can be exploited when all competitors receive signal $\hat{d}$; i.e. under the signal profile $n_{\hat{s}} = 1$. It is important to point out that the competitive edge of $\hat{s}$ insurers does not come from a more favorable estimation of risk than that of $\hat{d}$ insurers[32], but rather from the requirement that the contract offered by a $\hat{d}$ insurer under signal $n_{\hat{s}} = 1$ is incentive compatible with the one under signal $n_{\hat{s}} = 0$. Such requirement does not apply to $\hat{s}$ insurers, as they are not in the market under the signal profile $n_{\hat{s}} = 0$.

Note that for informative equilibria to exist beliefs need to be fully optimistic. In order to understand why, suppose that this is not the case. More specifically, let the policyholder's degree of optimism be $x < 1$, and consider the signal profile $n_{\hat{s}} = 1$. In this case, if the $\hat{s}$ insurer deviates, the loss probability estimated by the policyholder is $\tilde{p} > p_1$. It follows that the contract c^{dev} illustrated in Figure 3 is accepted when $n_{\hat{s}} = 1$ and it is profitable. If instead $n_{\hat{s}} = 2$, c^{dev} would be rejected as the policyholder prefers contract $c^e_{\hat{s},2}$ offered by the competitor, which entails no loss with respect to the equilibrium profits (that are equal to zero).

[32]Indeed, under signal profile $n_{\hat{s}} = 1$ all insurers estimate risk by using p_1.

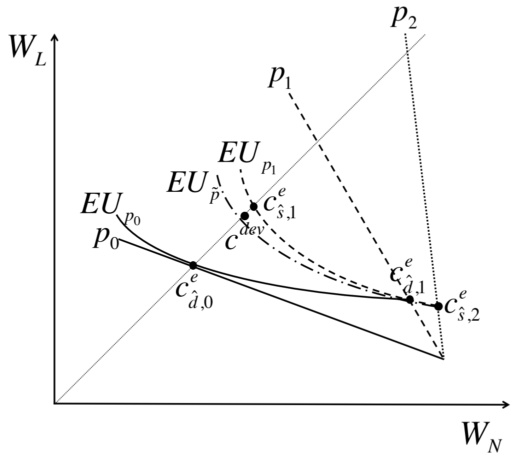

Figure 3: Beliefs in Informative Equilibria

Note also that, due to the fully optimistic beliefs, competition might not work for $\hat{d}$ insurers. The argument is analogous to that we used to sustain non-informative equilibria: if the policyholder is fully optimistic, then she systematically underestimates risk following insurer $\hat{d}$'s deviation; hence, deviations by $\hat{d}$ insurers may be effectively hindered. This implies, analogously to non-informative equilibria, the possible emergence of multiple equilibria, which may entail profitable outcomes in the riskiest signal profile.

Finally, it is worth highlighting the fundamental trade-off underlying a possible deviation by a $\hat{s}$ insurer pretending to be $\hat{d}$. On the one hand, by deviating, the $\hat{s}$ insurer offers a more profitable contract conceived for a riskier signal profile (entailing a higher premium rate) with probability 1/2. On the other hand, sticking to the candidate equilibrium strategy, he sells a less profitable contract (corresponding to the assessment of a safer signal profile) with probability one. More technically, a deviation from the equilibrium arises when insurer $\hat{s}$ pretends to be $\hat{d}$, by offering the menu $\{c^e_{\hat{d},0}, c^e_{\hat{d},1}\}$.[33] To avoid such deviation, one needs to guarantee that $\hat{s}$ insurers truthfully reveal their private signal (yet another reason for $\hat{s}$ insurers enjoying positive profits in equilibrium). On the one hand, in the signal profile $n_{\hat{s}} = 2$, a deviating $\hat{s}$ insurer makes zero profits as it would be considered of type $\hat{d}$ and therefore the policyholder would prefer the competitor's offer. On the other hand, profits are equal to zero also in equilibrium (as $c^e_{\hat{s},2}$ is actuarially fair). Therefore, it suffices to

[33]Note that this type of deviation is not profitable for $\hat{d}$ insurers, as they would suffer a loss by pretending to be $\hat{s}$, hence inducing the policyholder to believe that the signal profile is less risky than it actually is.

compare profits in signal profile $n_{\hat{s}} = 1$. In equilibrium, the $\hat{s}$ insurer would obtain profits $E\pi_{\hat{s}}(c^e_{\hat{s},1})$. Conversely, if he deviates by offering $C^e_{\hat{d}}$, all firms would end up offering the menu $C^e_{\hat{d}}$ to the policyholder, who would then believe that the signal profile is $n_{\hat{s}} = 0$. In this case, the $\hat{s}$ deviating insurer obtains profits $E\pi_{\hat{s}}(c^e_{\hat{d},0})$ with probability 1/2. Hence, Condition (8) guarantees that the $\hat{s}$ insurer has no incentives to deviate. Note finally that an obvious implication of (8) is that, in equilibrium, $c^e_{\hat{d},0} \neq c^e_{\hat{s},1}$.

Furthermore, the contract accepted in $n_{\hat{s}} = 2$ is actuarially fair, while the contract accepted in the riskier signal profile $n_{\hat{s}} = 1$ (i.e. $c^e_{\hat{s},1}$) is not. This suggests that contracts are fully separating, entailing different outcomes for the different signal profiles given that a profitable contract in a signal profile cannot be actuarially fair in a safer one.

It is also instructive to compare informative and non-informative equilibria. Non-informative equilibria exist for a larger set of out-of-equilibrium beliefs than informative ones. Indeed, the former simply requires that beliefs are sufficiently optimistic, while the latter exist only for *fully* optimistic policyholders. In this respect, non-informative equilibria seems to be more 'robust'.

Finally, it is worth connecting our analysis to that of Villeneuve (2005). While he finds that with identical, perfectly informed insurers, informative equilibria can support the first best, the analysis above shows that in our setup informative equilibria always entail an inefficient outcome. More importantly, this result holds also when the precision of the signal received by insurers converges to 1; i.e. when our setup converges to that of Villeneuve (2005), as it is illustrated by the following corollary of Proposition 4.

Corollary 1 *In the limit $\alpha \to 1$, the equilibrium contract $c^e_{\hat{s},2}$ does not converge to full insurance.*

The intuition of Corollary 1 is immediately conveyed by Figure 4. In the limit for α equal to 1, the relevant loss probabilities are $p_1 = \bar{p}$ and $p_2 = p_s$[34]. Given that, from Proposition 4, the contract $c^e_{\hat{s},1}$ offered by an $\hat{s}$ insurer in signal profile 1 must entail full insurance, and that the contract $c^e_{\hat{s},2}$ must be incentive compatible with $c^e_{\hat{s},1}$, then contract $c^e_{\hat{s},2}$ must be inefficient and cannot converge to Villeneuve's full insurance equilibrium outcome c^{AF}_s.

[34]In fact, $p_1 = p_s P_s + p_d P_d$ and $p_2 = \frac{p_s \alpha^2 P_s + p_d (1-\alpha)^2 (1-P_s)}{\alpha^2 P_s + (1-\alpha)^2 (1-P_s)}$.

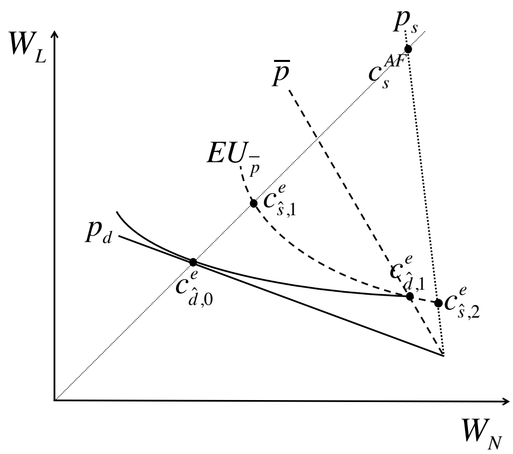

Figure 4: Informative equilibrium in the limit for $\alpha = 1$

5.2 Existence

Proposition 4 fully characterizes informative equilibria but it doesn't help establishing the conditions under which such equilibria exist. As already noted, given the non convexities embedded in the relevant incentive compatibility constraints, a general existence result is very difficult to achieve. Nonetheless, the following Proposition 5, focusing on the CARA specification (4) of the policyholder's utility, provides sufficient conditions for existence by introducing appropriate restrictions on the key parameters of the problem.[35]

Proposition 5 *Given the CARA specification (4) of the policyholder's utility, an informative equilibrium exists when the signal received by insurers is sufficiently precise (α is close to 1), the damage L is sufficiently large and the loss probability in the safe environment (p_s) is sufficiently low.*

The proposition establishes conditions that allow to rule out profitable deviations, at the same time ensuring truthful revelation. More specifically, requiring that the signal is sufficiently precise and the loss probability in the safe state is sufficiently small rules out profitable deviations with cross-subsidies. A cross-subsidizing deviation allows to increase

[35] Abrardi *et al.* (2019) provide a numerical analysis (for the CARA specification (4) of the policyholder's utility) showing that informative equilibria exist for a non trivial range of (empirically plausible) parameter values. Furthermore, and more interesting, it finds that informative and non-informative equilibria may co-exist and that, when this is the case, the latter are typically *ex ante* more profitable than the former. This is also related to the observation that non-informative equilibria can be *ex ante* Pareto-efficient while informative equilibria entail necessarily second best allocations.

profits in safer states, but decreases them in riskier ones[36]. In our setup, this implies that the $\hat{s}$ insurer has an incentive to deviate from the equilibrium by offering more profitable contracts when $n_{\hat{s}} = 2$ and less profitable ones when $n_{\hat{s}} = 1$. Obviously, in a profitable deviation the increase in profits when $n_{\hat{s}} = 2$ should more than compensate the decrease in profits when $n_{\hat{s}} = 1$. If the signal is very precise, the estimated loss probability when $n_{\hat{s}} = 2$ is close to the true loss probability p_s in the safe environment. If p_s is close to zero, the profit when $n_{\hat{s}} = 2$ goes to zero as well, thus eliminating the insurer's incentive to deviate.

The condition on the level of damage L allows instead to meet the truthful revelation condition (8). The larger is the level of damage, the higher is the equilibrium profit that the $\hat{s}$ insurer obtains when $n_{\hat{s}} = 1$. In fact, the level of damage affects positively the policyholder's willingness to pay under signal profile 1[37]. Conversely, a higher equilibrium profit when $n_{\hat{s}} = 1$ decreases the insurer's incentive to deviate by offering the equilibrium contract of $\hat{d}$ for signal profile 0 (i.e. the contract that would be offered in equilibrium when $n_{\hat{s}} = 0$).

6 The $n-$Firm Case

So far our analysis focused on a duopoly model. This allowed us to fully characterize both non-informative (pooling) and informative (separating) equilibria, also providing existence conditions for a CARA specification. It is interesting to investigate whether these results extend to the general case of an oligopolistic industry with more than two firms, hence understanding the role of market concentration for equilibrium existence and industry profitability. While most of the results on the characterization of equilibria remain unaffected when focusing on the $n-$firm case, the analysis of this section shows that the number of firms significantly affect equilibrium existence conditions. In particular, as the number of firms in the industry becomes too large, both informative and non-informative equilibria fail to exist.

[36] Recall that the notion of a cross-subsidizing deviation generalizes that of a pooling deviation in the Rothschild and Stiglitz's (1976) framework.

[37] Indeed, this condition on the level of damage would be analogous to a condition requiring a sufficiently high level of risk aversion, as it is shown in the proof of Proposition 5.

6.1 The Model

We extend the duopoly setup considered so far to investigate an insurance market with n firms ($i = 1, 2, .., n$) and one policyholder. We take the market structure as given, not considering entry or exit from the industry. In this setup, the signal profile $\left\{\hat{\theta}_i\right\}_{i=1,2,...,n}$ is a vector summarizing the n signals received by the firms. The number of insurers who observed $\hat{s}$ is $n_{\hat{s}}$ and it is a sufficient statistic for $\left\{\hat{\theta}_i\right\}_{i=1,2,...,n}$. We denote with $p_{n_{\hat{s}}}$ the loss probability conditional on the signal profile $n_{\hat{s}}$ (see Definition (1)).

The timing and all the assumptions made for the two firm model remain valid in this extended setup. We adapt notation by assuming that each insurer i offers a menu C_i of contracts, and we let $C = \{C_i\}_{i=1,2,...,n}$ be the vector of menus offered by all the insurers. We also continue to assume that all menus offered are exclusive and cannot be withdrawn.

6.2 Equilibrium Characterization and Existence

As in the rest of the paper, we proceed by distinguishing between non-informative and informative equilibria. It is immediate to see that the number of firms doesn't play any substantial role in the characterization of non-informative equilibria. Indeed, only *ex ante* probabilities matter in equilibrium and only *interim* probabilities matter in deviation. Both are unaffected by the number of firms and thus, in characterizing non-informative equilibria, we can immediately rely on the results of Proposition 1. As already discussed in the baseline model with two firms, three conditions are necessary for the existence of non-informative equilibria. This remains true also for the more general framework with n competing firms, with the first two conditions being essentially analogous to their counterparts for the two-firm case. The first condition requires that the insurers' participation constraints, which depend on the *ad interim* loss probabilities $p_{\hat{s}}$ and $p_{\hat{d}}$, must be satisfied. The second condition is the policyholder's participation constraint, which depends on the prior loss probability $\bar{p}$. None of the expressions of $p_{\hat{s}}$, $p_{\hat{d}}$ and $\bar{p}$ depend on the number of firms, as they are estimated only on the basis of the prior or *ad interim* information. Therefore, no substantial difference emerges with the two-firm case with respect to the insurers and policyholder participation constraints. The set of contracts satisfying the participation constraints of the insurers and of the policyholder does not depend on n. The third necessary condition for the existence of non-informative equilibria marks instead a difference with respect to the two-firm case. It requires that the expected profits in equilibrium are larger than those in the most profitable deviation c^{dev} for a $\hat{s}$ insurer. Since each firm sells the equilibrium contract c^e with probability $1/n$, the condition analogous to (6) in the n firms framework is

$$\frac{1}{n} E\pi_{p_{\hat{\theta}}}\left(c^e\right) \geq E\pi_{p_{\hat{\theta}}}\left(c_{\hat{\theta}}^{dev}\right) \text{ for all } \hat{\theta} = \left\{\hat{s}, \hat{d}\right\} \tag{9}$$

From (9), it follows immediately that there exists an upper bound to the number of insurers consistent with the existence of a non-informative symmetric equilibrium c^e, which we denote as $\bar{n}(c^e)$. Note that this upper bound is specific to the proposed equilibrium c^e. By inspection of (9), it is also easy to show that the set of non-informative equilibria shrinks as the industry becomes more dispersed, given that the less profitable contracts are no longer sustainable as equilibria.[38] Since non-informative equilibria are not unique, it is convenient to characterize the equilibrium with the highest possible number of firms $\bar{n}$ among all possible non-informative equilibria:

$$\bar{n} = \max_{c^e} \bar{n}(c^e).$$

From Inequality 9, it follows that the larger are the expected profits associated to the equilibrium contract, the larger is the upper bound.

The above discussion immediately yields the following proposition.

Proposition 6 *There exists an upper bound $\bar{n}$ to the number of firms that is consistent with the existence of a non-informative equilibrium. $\bar{n}$ is increasing in the expected profits corresponding to the equilibrium contract.*

To better understand the nature of the upper bound on the number of firms in a non-informative equilibrium, it is convenient to focus again on condition (9). When the number of competitors in the market increases and all firms offer the same contract, the probability

[38] The characterization of the contract $c_{\hat{\theta}}^{dev}$ in (9) depends on the loss probabilities $p_{\hat{\theta}}$ and $\tilde{p}$, which are unaffected by n. Recall that $p_{\hat{s}} = \frac{p_s \alpha P_s + p_d (1-\alpha)(1-P_s)}{\alpha P_s + (1-\alpha)(1-P_s)}$ and $p_{\hat{d}} = \frac{p_s (1-\alpha) P_s + p_d \alpha (1-P_s)}{\alpha P_s + (1-\alpha)(1-P_s)}$.Hence, an increase of n implies that the profit $E\pi_{p_{\hat{\theta}}}(c^e)$ must increase.

of attracting a customer decreases (i.e., the l.h.s. of (9) is reduced). Hence, the expected profit in equilibrium decreases and the deviation becomes increasingly tempting. It follows that, with a large number of insurers, only contracts with higher premia can be sustained as equilibria. This establishes a non standard, negative relationship between insurance premia and market concentration. Moreover, note that the expected profits generated by the accepted contracts in a non-informative equilibrium remain constant for increasing market concentration, suggesting that an increase in market concentration is not necessarily welfare detrimental for customers; a finding that is consistent with the available empirical literature.

Also the characterization of informative equilibria follows closely the discussion of the two-firm case, although one needs to take into account that increasing the number of firms also increases the number of market states (i.e. the number of signals) and therefore the complexity of the equilibrium. Proposition 7 provides a general characterization of informative equilibria for the $n-$firm case.[39]

Proposition 7 *If an informative equilibrium exists, it is characterized as follows:*

1. *the contracts accepted in equilibrium are different for each signal profile;*
2. *if $n_{\hat{s}} \geq 2$, the contracts accepted in equilibrium are actuarially fair;*
3. *the incentive compatibility constraints for contracts accepted in equilibrium in $n_{\hat{s}} \geq 2$ are binding;*
4. *if $n_{\hat{s}} = 1$, an $\hat{s}$ insurer offers contract $c_1^{\max}$ defined in (7);*
5. *the contracts offered in equilibrium by insurers when all of them receive signal $\hat{d}$, $c^e_{\hat{d},0}$, and that offered by an $\hat{s}$ insurer when he is the only one receiving signal $\hat{s}$, $c^e_{\hat{s},1}$, must entail non-negative profits (if accepted) and guarantee truth-telling, i.e.*

$$E\pi_{\hat{s}}(c^e_{\hat{s},1}) \geq \frac{1}{n} E\pi_{\hat{s}}(c^e_{\hat{d},0}); \tag{10}$$

6. *beliefs are fully optimistic.*

Several of the driving forces behind the equilibrium characterization in Proposition 7 are analogous to those highlighted for the two-firm cases (see the discussion of Proposition 4).

[39] We stick whenever possible to the notation used for the two-firm case.

Notwithstanding there are some specificities of the $n-$firm case that are worth highlighting First, the proposition shows that with more than one insurer receiving signal $\hat{s}$, equilibrium outcomes must be actuarially fair. This is essentially due the fact that insurers compete *à la* Bertrand, exactly as in the two-firm case when $n_{\hat{s}} = 2$. Second, the presence of more than two firms adds an additional reason why equilibrium multiplicity can emerge. Proposition 7 shows that the contracts accepted in states $n_{\hat{s}} \geq 2$ must entail zero profits. In order to sustain an equilibrium it is necessary that at least two firms offer the accepted contract. However, the contracts offered by any other firms are unrestricted provided they are not accepted. Thus, asymmetric equilibria may emerge. Notwithstanding, restricting attention to symmetric equilibria is without loss of generality in terms of equilibrium outcomes. Third, Condition (10) establishes that the $\hat{s}$ insurer must obtain a higher profit in equilibrium rather than in the deviation in which he offers the $\hat{d}$'s equilibrium menu, which introduces a clear difference between the $n-$firm and the two-firm case (see (8)).

Finally, and more important, it is easy to show that under the conditions of Proposition 7, there is an upper bound to the number of firms consistent with an informative equilibrium.[40]

Corollary 2 *There exists an upper bound $\bar{\bar{n}}$ to the number of firms that is consistent with the existence of informative equilibria. $\bar{\bar{n}}$ is decreasing in the expected profits corresponding to the equilibrium contract.*

The existence of an upper bound to the number of firms that is consistent with an informative equilibrium has to do with the fact that, when the number of firms in the industry grows larger, profitable deviations become available via semi-pooling menus. In particular, there exists a menu such that the equilibrium contracts are offered in the states where the number of firms receiving signal $\hat{s}$ – i.e. $n_{\hat{s}}$ – is less than $n/2$, while a pooling contract that is preferred by the policyholder to the equilibrium contracts is offered in states where $n_{\hat{s}} > n/2$. Qualitatively, the existence of a profitable pooling deviation relies on the fact that when the number of firms in the industry grows larger all loss probabilities converge either to p_s, when $n_{\hat{s}} > n/2$, or to p_d, in the opposite case, due to the logic of

[40] It is also immediate to see that Condition (10) implies directly that the maximum number of firms consistent with an informative equilibrium must have a lower bound ($\underline{n}$) as well in order to induce insurers to truthfully disclose their information.

Bayesian updating. This implies that all equilibrium contracts in good states are, on the one hand, very close to each other and, on the other hand, very far from those in bad states. In this situation, with a risk averse policyholder, the same logic that justifies the existence of profitable pooling deviations in the Rothschild and Stiglitz's (1976) framework applies.[41]

Having shown that there is an upper bound to the number of firms that is consistent with the existence of both non-informative and informative equilibria, it is interesting to ask whether such upper bound implies the insurance industry to be concentrated. Although this is clearly an empirical matter, Abrardi *et al.* (2019) show (for the CARA utility specification in Eq. (4) and empirically plausible parameter constellations) that the industry needs to be fairly concentrated for both types of equilibria to exist.[42]

7 Concluding Remarks

The improvements in data collection and storage and the rise of computing power over the last decades have significantly affected the insurance industry, allowing insurance companies to achieve more reliable predictions and risk estimations. In this perspective, the traditional information asymmetry affecting the insurance sector may flip-over to the other side of the market. Indeed, because of their expertise and access to relevant statistics, insurers are likely to be better equipped than policyholders to accurately assess the level of risk associated to a specific environment. Still, a precise assessment of risk is not straightforward even for practitioners, and some degree of heterogeneity in insurers' evaluations is unavoidable, possibly related to the access to different data warehouses, or to the availability of multiple predictive algorithms.

The key result of our analysis is that – when insurers have an imperfect informational advantage over policyholders – equilibria always entail positive profits for some insurers and do not necessarily imply disclosure of the insurers' information despite competition. This

[41] See the proof of Corollary 2 for more details on the nature of profitable pooling deviations.

[42] In all the numerical exercises in Abrardi *et al.* (2019), the number of firms consistent with the existence of an equilibrium never exceeds eight, and it is typically much lower. Although this evidence is not the result of a fully calibrated analysis, it seems to be robust to different parameter specifications. Furthermore, it is consistent with the available empirical evidence showing that actual insurance markets are significantly concentrated.

result is both entirely new to the insurance literature (at least as far as separating equilibria are concerned) and is fully consistent with most of the available empirical evidence that reports the abundance of unused observables in the definition of insurance contracts, as well as the profitability of insurance markets. Intuitively, the emergence of non-informative, profitable equilibria depends on policyholders' overly optimistic attitude about the riskiness of the environment. Indeed, to the extent that optimistic policyholders are willing to accept only 'cheap' contracts (that would excessively curtail insurers' profits), the functioning of the competitive mechanism is effectively hindered. Moreover, equilibrium profits need necessarily be positive also in the case of informative equilibria, as this is required to induce truthful revelation of information by insurers with a low assessment of risk.

The fact that equilibria – both informative and non-informative – can be sustained only if profits are sufficiently high, has direct implications in terms of market concentration. In fact, we show that there is an upper bound on the number of firms consistent with the existence of an equilibrium, and that – quite counter-intuitively – a larger industry dispersion may entail larger equilibrium profits. Also these results are consistent with the available empirical evidence on industry concentration, and on the ambiguous relationship between market concentration and profitability.

In a theoretical perspective, we highlight the role played by the informational structure on the type and features of equilibria. Our contribution to the literature on insurance markets originated by Rothschild and Stiglitz's (1976) seminal paper is twofold. First, our results on non-informative equilibria show that even an infinitesimal (imperfect) informational advantage by insurers might be sufficient for Rothschild and Stiglitz's (1976) equilibria – based on the efficiency of the competitive mechanism – not to survive. Indeed, the failure of the competitive mechanism in our model can be related to that occurring in signaling models, where it originates from the existence of degrees of freedom in the choice of out-of-equilibrium beliefs. The novelty of our contribution is to show that this result is robust to the policyholder having private information on the idiosyncratic components of her own risk (i.e. to the introduction of a second layer of asymmetric information). Second, focusing on informative equilibria, we prove that our equilibria do not converge to those emerging in the signaling models of insurance such as Villeneuve (2005), as a consequence of the fact that insurers might receive heterogeneous and imperfect signals. In particular, the informative equilibria emerging in our setup are always profitable and typically entail under-insurance even when insurers' information is very accurate.

Existing models of insurance markets focus on either insurers, or policyholders, holding perfect information about risk. A more realistic representation should probably allow for both insurers and policyholders holding some amount of private information. Our analysis, by combining features of pure-signaling and pure-screening models, is only a first step in this direction, however enough to suggest that there are novel and unexpected insights to be uncovered.

References

[1] Abrardi, L., Colombo, L. and P. Tedeschi (2019), The Value of Ignoring Risk: Insurance with Better Informed Principals. Working Paper no. 84, *Università Cattolica del Sacro Cuore, Milano.*

[2] American Medical Association (2018). Competition in Health Insurance. A comprehensive study of U.S. markets. Available at *www.ama-assn.org/system/files/*2018-11/*competition-health-insurance-us-markets_1.pdf* (last accessed July 10, 2019).

[3] Attar A. and A. Chassagnon (2009), On moral hazard and nonexclusive contracts. *Journal of Mathematical Economics*, 45(9-10): 511-525.

[4] Attar A., Mariotti T. and F. Salanié (2011), Nonexclusive Competition in the Market for Lemons. *Econometrica*, 79(6):1869-1918.

[5] Bajtelsmit V.L. and R. Bouzouita (1998), Market Structure and Performance in Private Passenger Automobile Insurance. *The Journal of Risk and Insurance*, 65(3): 503-514.

[6] Barseghyan L., Molinari F., O'Donoghue T. and J.C. Teitelbaum (2013), The Nature of Risk Preferences: Evidence from Insurance Choices, *American Economic Review*, 103(6): 2499-2529.

[7] Cabral, M., Geruso, M. and N. Mahoney (2018), Do Larger Health Insurance Subsidies Benefit Patients or Producers? Evidence from Medicare Advantage. *American Economic Review,* 108(8): 2048-2087.

[8] Camerer, C. (1997), Progress in Behavioral Game Theory. *Journal of Economic Perspectives*, 11(4): 167-88.

[9] Chiappori P.-A. and B. Salanie (2000), Testing for Asymmetric Information in Insurance Markets. *Journal of Political Economy*, 108: 56-78.

[10] Chidambaran, N. K., Pugel T.A. and A. Saunders (1997). An Investigation of the Performance of the U.S. Property-Liability Insurance Industry. *The Journal of Risk and Insurance* 64(2): 371-381.

[11] Cohen, A., and L. Einav (2007). Estimating Risk Preferences from Deductible Choice. *American Economic Review*, 97(3): 745-788.

[12] Cummins, J. D. and S. Tennyson (1992), Controlling Automobile Insurance Costs. *Journal of Economic Perspectives*, 6: 95-115.

[13] Cummins, J. D. and M.A. Weiss (1993), Measuring cost efficiency in the property-liability insurance industry. *Journal of Banking & Finance*, 17(2-3): 463-481.

[14] Dafny, L., Duggan, M. and Ramanarayanan, S. (2012), Paying a Premium on Your Premium? Consolidation in the US Health Insurance Industry. *American Economic Review*, 102(2): 1161-85.

[15] Dauda, S. (2018), Hospital and Health Insurance Markets Concentration and Inpatient Hospital Transaction Prices in the U.S. Health Care Market. *Health Services Research*, 53: 1203-1226.

[16] Diamond, P. (1992), Organizing the Health Insurance Market. *Econometrica*, 60(6): 1233-1254.

[17] Fang, H. and G. Moscarini (2005), Morale Hazard. *Journal of Monetary Economics*, 52(4): 749-777.

[18] Finkelstein, A. and J. Poterba (2014), Testing for Asymmetric Information Using ?Unused Observables? in Insurance Markets: Evidence from the U.K. Annuity Market. *Journal Risk and Insurance*, 81: 709-734.

[19] Fombaron, N. (1997), No-Commitment and Dynamic Contracts in Competitive Insurance Markets with Adverse Selection. *Working Paper Thema.*

[20] Fulton B.D. (2017), Health care market concentration trends in the United States: evidence and policy responses. *Health Affairs*, 36: 1530-1538.

[21] Garcia, D., Sangiorgi, F. and B. Urosevic (2007), Overconfidence and Market Efficiency with Heterogeneous Agents. *Journal of Economic Theory*, 30(2): 313-36.

[22] Groeger, J.A and G.E. Grande (1996), Self-preserving Assessments of Skill? *British Journal of Psychology*, 87: 61-79.

[23] Hyman, D. A. and Kovacic, W. E. (2004), Monopoly, Monopsony, and Market Definition: An Antitrust Perspective On Market Concentration Among Health Insurers. *Health Affairs*, 23(6): 25-28.

[24] Hoelzl, E. and A. Rustichini (2005), Overconfident: Do You Put Your Money on It? *Economic Journal*, 115: 305-18.

[25] Insurance Information Institute (2018), Facts + Statistics: Homeowners and renters insurance. https://www.iii.org/fact-statistic/facts-statistics-homeowners-and-renters-insurance, last accessed on October 31, 2019.

[26] Kahneman, D. and A. Tversky (1979), Prospect Theory: An Analysis of Decision Under Risk. *Econometrica*, 47(2), 263-291.

[27] Köszegi, B. (2006), Ego Utility, Overconfidence, and Task Choice. *Journal of the European Economic Association*, 4(4): 673-707.

[28] Kreuter, M.W. and V.J. Strecher (1995), Changing Inaccurate Perceptions of Health Risk. *Health Psychology*, 14: 56-63.

[29] Maskin, E. and J. Tirole, (1990), The Principal-Agent Relationship with an Informed Principal, I: Private Values. *Econometrica*, 58: 379-410.

[30] Maskin, E., and J. Tirole, (1992), The Principal-Agent Relationship with an Informed Principal, II: Common Values. *Econometrica*, 60: 1-42.

[31] Menkhoff, L., Schmidt, U. and T. Brozynski (2006), The Impact of Experience on Risk Taking, Overconfidence, and Herding of Fund Managers: Complementary Survey Evidence. *European Economic Review*, 50(7), 1753-66.

[32] Myerson, R. (1983), Mechanism Design by an Informed Principal. *Econometrica*, 51: 1767-1798.

[33] Noth, M. and M. Weber (2003), Information Aggregation with Random Ordering: Cascades and Overconfidence. *Economic Journal*, 113(484), 166-89.

[34] Robb K.A., Miles, A. and J. Wardle (2004), Subjective and Objective Risk of Colorectal Cancer (UK). *Cancer Causes and Control* 15: 21-25.

[35] Robinson, J. (2004), Consolidation And The Transformation Of Competition. In Health Insurance, *Health Affairs*, 23(6): 11-24.

[36] Rothschild, M. and J.E. Stiglitz (1976), Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *Quarterly Journal of Economics*, 90: 629-649.

[37] Sandroni, A. and F. Squintani (2007), Overconfidence, Insurance and Paternalism. *American Economic Review* , 97(5): 1994-2004.

[38] Seog, S. H. (2009), Insurance Markets with Differential Information. *Journal of Risk and Insurance*, 76: 279-294.

[39] Sommer, D. (2017). The Impact of Firm Risk on Property-Liability Insurance Prices. *The Journal of Risk and Insurance*, 63(3): 501-514.

[40] Svenson, O. (1981), Are we all less Risky Drivers and more Skillful than our Fellow Drivers? *Acta Psychologica*, 47: 143-148.

[41] Van den Steen, E. (2004), Rational Overoptimism. *American Economic Review*, 94(4): 1141-1151.

[42] Villeneuve, B. (2005), Competition Between Insurers with Superior Information. *European Economic Review*, 49(2): 321-340.

[43] Weinstein, N.D. (1980), Unrealistic Optimism About Future Life Events. *Journal of Personality and Social Psychology*, 39: 806-820.

[44] Zábojník, J. (2004), A Model of Rational Bias in Self-Assessments. *Economic Theory*, 23(2), 259-82.

For Online Publication

Appendix A Proofs

Proof of Proposition 1

Parts 1 and 2 of the proposition simply require that the relevant policyholder's participation constraints hold. Point 3 can be proved by contradiction. Assume $\tilde{p} \geq \bar{p}$ and consider that a $\hat{s}$ insurer deviates by adopting an undercutting strategy; i.e. by offering contract $c^{dev} = (W_L^e + \varepsilon, W_N^e + \varepsilon)$, with $\varepsilon > 0$ arbitrarily small. Showing that this deviation is profitable requires that: (i) c^{dev} is always profitable if accepted, (ii) it is preferred to the equilibrium given out-of-equilibrium beliefs, and (iii) it is acceptable in the deviation by the policyholder, i.e. it meets her participation constraint given out-of-equilibrium beliefs $\tilde{p}$. We address each of these conditions in turn.

(i) In the limit for $\varepsilon \to 0$, we have that

$$E\pi_{p_{\hat{s}}}(c^{dev}) = E\pi_{p_{\hat{s}}}(c^e) > \frac{1}{2}E\pi_{p_{\hat{s}}}(c^e),$$

i.e. the deviation c^{dev} is profitable for $\hat{s}$ if it is accepted.

(ii) The policyholder prefers the deviation to the equilibrium contract only if $EU_{\tilde{p}}(c^{dev}) > EU_{\tilde{p}}(c^e)$. Recall that:

$$EU_{\tilde{p}}(c^{dev}) = \tilde{p}U(W_L^e + \varepsilon) + (1-\tilde{p})U(W_N^e + \varepsilon) \quad \text{(A.1)}$$

$$EU_{\tilde{p}}(c^e) = \tilde{p}U(W_L^e) + (1-\tilde{p})U(W_N^e). \quad \text{(A.2)}$$

Given that $\varepsilon > 0$, it must be

$$EU_{\tilde{p}}(c^{dev}) > EU_{\tilde{p}}(c^e). \quad \text{(A.3)}$$

(iii) The deviation is acceptable if it satisfies the policyholder's participation in the deviation, i.e. $EU_{\tilde{p}}(c^{dev}) - EU_{\tilde{p}}(\underline{c}) \geq 0$. By subtracting $EU_{\tilde{p}}(\underline{c})$, on both sides of (A.3), we obtain

$$EU_{\tilde{p}}(c^{dev}) - EU_{\tilde{p}}(\underline{c}) > EU_{\tilde{p}}(c^e) - EU_{\tilde{p}}(\underline{c}). \tag{A.4}$$

Furthermore, note that

$$EU_{\tilde{p}}(c^e) - EU_{\tilde{p}}(\underline{c}) \geq EU_{\bar{p}}(c^e) - EU_{\bar{p}}(\underline{c}). \tag{A.5}$$

Indeed, (A.5) can immediately be rewritten as

$$(\tilde{p} - \bar{p})(U(W_L^e) - U(W - L) + U(W) - U(W_N^e)) \geq 0,$$

which is verified under the assumption that $\tilde{p} \geq \bar{p}$, since $U(W_L^e) \geq U(W - L)$ and $U(W) - U(W_N^e) \geq 0$. Combining (A.4) and (A.5), we have that $EU_{\tilde{p}}(c^{dev}) - EU_{\tilde{p}}(\underline{c}) \geq 0$ holds. Hence, conditions (i)-(iii) hold, and a profitable deviation exists, if $\tilde{p} \geq \bar{p}$. Therefore, for an equilibrium to exist it must be $\tilde{p} < \bar{p}$, which completes the proof. Q.E.D.

Proof of Proposition 2

The relevant loss probabilities read

$$\begin{aligned}
p_{\hat{s}} &= \frac{p_s \alpha(1 - P_d) + p_d(1 - \alpha)P_d}{\alpha(1 - P_d) + (1 - \alpha)P_d} \\
p_{\hat{d}} &= \frac{p_s(1 - \alpha)(1 - P_d) + p_d \alpha P_d}{(1 - \alpha)(1 - P_d) + \alpha P_d} \\
\bar{p} &= p_s P_s + p_d P_d.
\end{aligned}$$

Letting $\alpha = P_d = y$ and considering the limit for $y \to 1$, we have that

$$\begin{aligned}
\lim_{y \to 1} p_{\hat{s}} &= \frac{p_s + p_d}{2} \\
\lim_{y \to 1} p_{\hat{d}} &= p_d \\
\lim_{y \to 1} \bar{p} &= p_d
\end{aligned}$$

Consider the full insurance contract $W_L^e = W_N^e = W^e$ satisfying with equality the participation constraint of the policyholder, as a candidate non-informative equilibrium contract; i.e.

$$e^{-\beta W^e} = (1-\bar{p})\, e^{-\beta W} + \bar{p} e^{-\beta(W-L)},$$

or

$$W^e = -\frac{1}{\beta} \ln \left((1-\bar{p})\, e^{-\beta W} + \bar{p} e^{-\beta(W-L)} \right).$$

For the sake of simplicity, we focus on a fully optimistic policyholder. We first need to check that the proposed equilibrium contract meets the participation constraints of the $\hat{d}$ and $\hat{s}$ insurers. The expected profits for $\hat{d}$ must be positive; i.e. $E\pi_{\hat{d}} = W - p_{\hat{d}} - W^e \geq 0$. In the limit for $y \to 1$, and after straightforward simplifications $E\pi_{\hat{d}}$ can be rewritten as

$$-p_d L + \frac{1}{\beta} \ln \left(1 - p_d + p_d e^{\beta L} \right) \geq 0.$$

This condition holds with equality if $p_d = 0$ and $p_d = 1$, while it is strictly positive for all $p_d \in (0,1)$. In fact, the l.h.s. of the inequality is a strictly concave function in p_d (note that the second derivative w.r.t. p_d is negative; i.e. $-\frac{1}{\beta} \frac{(-1+e^{\beta L})^2}{1-p_d+p_d e^{\beta L}} < 0$ for $\beta > 0$). Then, the participation constraint of a $\hat{d}$ insurer is $E\pi_{\hat{d}} = W - p_{\hat{d}} - W^e > 0$, whenever $p_d \in (0,1)$ and $\beta > 0$.

Given that $p_{\hat{s}} < p_{\hat{d}}$, at the candidate equilibrium also the participation constraint of the $\hat{s}$ insurer is not binding; i.e. $E\pi_{\hat{s}} = W - p_{\hat{s}} - W^e > 0$. Furthermore, by continuity, all participation constraints are also satisfied in a neighborhood of α, P_d close to 1, as required by condition 1 of the Proposition.

We next show that point 2 of the Proposition is sufficient to rule out deviations by a $\hat{s}$ insurer. The most profitable deviation for a $\hat{s}$ insurer, given the policyholder's fully optimistic beliefs, is the full insurance contract lying on the participation constraint of a fully optimistic policyholder; i.e.

$$W_{\hat{s}}^{dev} = -\frac{1}{\beta} \ln \left((1-p_{\hat{s}})\, e^{-\beta W} + p_{\hat{s}} e^{-\beta(W-L)} \right)$$

In our CARA setup, equilibrium condition (3) reads

$$\frac{1}{2}\left(W - W^{e} - p_{\hat{s}}L\right) \geq W - W_{\hat{s}}^{dev} - p_{\hat{s}}L,$$

which can be rewritten as

$$2W_{\hat{s}}^{dev} \geq W - p_{\hat{s}}L + W^{e} \tag{A.6}$$

Therefore, we need to check that A.6 holds in the limit for $\beta = 0$. By Hôpital's rule, we have that

$$\begin{aligned}
\lim_{\beta\to 0} W_{\hat{s}}^{dev} &= \lim_{\beta\to 0} -\frac{-\left(\left(1-p_{\hat{s}}\right)e^{-\beta W}W + p_{\hat{s}}e^{-\beta(W-L)}\left(W-L\right)\right)}{\left(\left(1-p_{\hat{s}}\right)e^{-\beta W} + p_{\hat{s}}e^{-\beta(W-L)}\right)} = \\
&= \left(1-p_{\hat{s}}\right)W + p_{\hat{s}}\left(W-L\right) = W - p_{\hat{s}}L
\end{aligned}$$

and

$$\begin{aligned}
\lim_{\beta\to 0} W^{e} &= \lim_{\beta\to 0} -\frac{-\left(\left(1-\bar{p}\right)e^{-\beta W}W + \bar{p}e^{-\beta(W-L)}\left(W-L\right)\right)}{\left(\left(1-\bar{p}\right)e^{-\beta W} + \bar{p}e^{-\beta(W-L)}\right)} = \\
&= \left(1-\bar{p}\right)W + \bar{p}\left(W-L\right) = W - L\bar{p}.
\end{aligned}$$

Hence for $\beta \to 0$, condition A.6 becomes $2(W - p_{\hat{s}}L) \geq W - p_{\hat{s}}L + W - L\bar{p}$, which holds as $p_{\hat{s}} < \bar{p}$. By continuity, condition (A.6) holds for β sufficiently low.

Finally, we check that no deviations exist for the $\hat{d}$ insurer. Notice that the set of $\hat{d}$'s profitable deviations acceptable by an optimistic policyholder is empty if the policyholder's indifference curve passing through autarky and estimating risk by $p_{\hat{s}}$ runs above the zero-isoprofit line of a $\hat{d}$ insurer for all values. This is the case when the marginal rate of substitution of the policyholder computed in $\underline{c}$, $\frac{1-p_{\hat{s}}}{p_{\hat{s}}}\frac{U'(W)}{U'(W-L)}$, is higher (in absolute value) than the slope of the isoprofit line of an insurer who estimates risk by $p_{\hat{d}}$, $\frac{1-p_{\hat{d}}}{p_{\hat{d}}}$; i.e.

$$\frac{U'(W)}{U'(W-L)} \geq \frac{\frac{(1-p_{\hat{d}})}{p_{\hat{d}}}}{\frac{1-p_{\hat{s}}}{p_{\hat{s}}}}. \tag{A.7}$$

Using the limits for $y \to 1$ under our CARA specification, inequality (A.7) can be written as

$$e^{-\beta L} \geq \frac{1-p_d}{p_d}\frac{p_s+p_d}{2-p_s-p_d},$$

which holds strictly if p_s is sufficiently small (as required by condition 3 in the proposition) and β is close to 0 (as required by condition 2). Q.E.D.

Proof of Proposition 3

It is convenient to first compute all loss probabilities needed to characterize both the equilibrium and the off-equilibrium expected utilities and profits. In equilibrium, the only relevant probabilities for the policyholder are the ex-ante ones; i.e.

$$\begin{aligned} p_h &= p_{sh}(1-P_d)+p_{dh}P_d \\ p_l &= p_{sl}(1-P_d)+p_{dl}P_d. \end{aligned}$$

Focusing on fully optimistic beliefs off the equilibrium path, off-equilibrium probabilities are computed assuming that the deviating insurer receives signal $\hat{s}$. Therefore, by using Bayes rule, we have

$$\begin{aligned} p_{\hat{s}h} &= \frac{p_{sh}\alpha(1-P_d)+p_{dh}(1-\alpha)P_d}{\alpha(1-P_d)+(1-\alpha)P_d} \\ p_{\hat{s}l} &= \frac{p_{sl}\alpha(1-P_d)+p_{dl}(1-\alpha)P_d}{\alpha(1-P_d)+(1-\alpha)P_d}. \end{aligned}$$

The relevant probabilities for the insurer are the interim probabilities – those conditional on the signal received – that are given by

$$\begin{aligned} p_{\hat{s}} &= (p_{sh}P_h+p_{sl}P_l)\frac{\alpha\,(1-P_d)}{\alpha\,(1-P_d)+(1-\alpha)P_d}+(p_{dh}P_h+p_{dl}P_l)\frac{(1-\alpha)\,P_d}{\alpha\,(1-P_d)+(1-\alpha)P_d} \\ p_{\hat{d}} &= (p_{dh}P_h+p_{dl}P_l)\frac{\alpha P_d}{\alpha P_d+(1-\alpha)(1-P_d)}+(p_{sh}P_h+p_{sl}P_l)\frac{(1-\alpha)(1-P_d)}{\alpha P_d+(1-\alpha)(1-P_d)}, \end{aligned}$$

where we made use of the fact that the distribution of j is independent from that of θ. Considering that $p_{sl} = p_{sh} = 0$ (by condition 3 of the Proposition) and $p_{dl} = p_{dh}$ (by condition 4), letting $\alpha = P_d = y$, and considering the limit $y \to 1$, we have that

$$\begin{aligned}
&\lim_{y\to 1} p_{\hat{s}h} = \frac{p_{dh}}{2} \\
&\lim_{y\to 1} p_{\hat{s}l} = \frac{p_{dh}}{2} \\
&\lim_{y\to 1} p_{l} = p_{dh} \\
&\lim_{y\to 1} p_{\hat{d}} = p_{dh} \\
&\lim_{y\to 1} p_{\hat{s}} = \frac{p_{dh}}{2}.
\end{aligned} \tag{A.8}$$

Consider as candidate equilibrium the full insurance contract c^e, which is actuarially fair for $\hat{d}$; i.e $W^e = W - p_{\hat{d}}L$. Note that the participation constraint of a $\hat{s}$ insurer is also satisfied, since $p_{\hat{s}} < p_{\hat{d}}$. For the proposed contract to be an equilibrium one it must be that: i) all relevant participation constraints are satisfied; ii) c^e guarantees higher profits to an $\hat{s}$ insurer than any acceptable deviation; and iii) c^e guarantees higher profits to a $\hat{d}$ insurer than any acceptable deviation. We address each of the conditions in turn.

i) The l policyholder's participation constraint is

$$e^{-\beta(W-p_{\hat{d}}L)} \leq p_l(e^{-\beta(W-L)}) + (1-p_l)e^{-\beta W},$$

which can be rewritten as

$$e^{\beta p_{\hat{d}}L} - p_l e^{\beta L} \leq 1 - p_l.$$

In the limit $y \to 1$ and using (A.8), the above condition becomes

$$e^{\beta L p_{dh}} - 1 \leq p_{dh}(e^{\beta L} - 1) \tag{A.9}$$

Condition (A.9) holds for all βL and p_{dh}. Indeed, the l.h.s. of (A.9) is a convex function in p_{dh}, while the r.h.s. is linear in p_{dh}. This implies that the l.h.s. and the r.h.s. can cross at most twice for $p_{dh} = 0$ and $p_{dh} = 1$. Instead, for all $p_{dh} \in (0, 1)$ condition (A.9) holds as a strict inequality.

ii) Note that a deviation consists of a menu of two contracts, one for the $l-$ and one for $h-$type policyholder, which must be incentive compatible. Therefore, the upper bound on the deviation profits for a $\hat{s}$ insurer is obtained by relaxing the policyholder incentive compatibility constraint on the deviation menu. This implies that the upper bound is characterized by the two full insurance contracts guaranteeing that the policyholder's participation is binding when the loss probabilities are $p_{\hat{s}h}$ and $p_{\hat{s}l}$, respectively; i.e.

$$W_{\hat{s}h}^{dev} = -\frac{1}{\beta} \ln \left(p_{\hat{s}h} e^{-\beta(W-L)} + (1-p_{\hat{s}h}) e^{-\beta W} \right)$$

$$W_{\hat{s}l}^{dev} = -\frac{1}{\beta} \ln \left(p_{\hat{s}l} e^{-\beta(W-L)} + (1-p_{\hat{s}l}) e^{-\beta W} \right).$$

Notice that if the proposed deviation is not profitable, then it is also unprofitable to do screening on one type of policyholder only. The equilibrium profits for $\hat{s}$ are higher than in the given deviation if

$$\frac{W - p_{\hat{s}}L - (W - p_{\hat{d}}L)}{2} \geq P_h (W - p_{\hat{s}h}L - W_{\hat{s}h}^{dev}) + (1 - P_h)(W - p_{\hat{s}l}L - W_{\hat{s}l}^{dev}), \quad \text{(A.10)}$$

i.e.

$$\frac{p_{\hat{d}} + p_{\hat{s}}}{2} \beta L \geq P_h \ln \left(p_{\hat{s}h} e^{\beta L} + (1 - p_{\hat{s}h}) \right) + (1 - P_h) \ln \left(p_{\hat{s}l} e^{\beta L} + (1 - p_{\hat{s}l}) \right). \quad \text{(A.11)}$$

In the limit for $y \to 1$, condition $(A.11)$ becomes

$$e^{\frac{3}{4}\beta L p_{dh}} - 1 \geq \frac{p_{dh}}{2}(e^{\beta L} - 1). \quad \text{(A.12)}$$

It is easy to show that condition (A.12) is met for all p_{dh}. In fact, when $p_{dh} = 0$, (A.12) holds as an equality. Instead, for $p_{dh} = 1$, it becomes

$$e^{\frac{3}{4}\beta L} - 1 \geq \frac{1}{2}(e^{\beta L} - 1).$$

When $\beta L \to 0$ (condition 2 in the proposition), also the latter inequality holds as equality. However, the slope of the l.h.s. is greater than the slope of the r.h.s. in a neighborhood of

$\beta L = 0$. Hence, the inequality holds strictly in a neighborhood of $p_{dh} = 1$ for βL sufficiently close to 0. Therefore, for βL sufficiently low, there exists a $p_{dh} \in (0,1)$ such that the inequality is satisfied with strict sign.

iii) Notice that the set of $\hat{d}$'s profitable deviations acceptable by an optimistic policyholder is empty if the policyholder's indifference curve passing through autarky and estimating risk by $p_{\hat{s}h}$ lays above the zero-isoprofit line of a $\hat{d}$ insurer. This holds true whenever the marginal rate of substitution of the policyholder computed in $\underline{c}$ – i.e. $\frac{1-p_{\hat{s}h}}{p_{\hat{s}h}} \frac{U'(W)}{U'(W-L)}$ – is higher (in absolute value) than the slope of the isoprofit line of an insurer who estimates risk using $p_{\hat{d}}$; i.e.

$$\frac{U'(W)}{U'(W-L)} \geq \frac{\frac{(1-p_{\hat{d}})}{p_{\hat{d}}}}{\frac{1-p_{\hat{s}h}}{p_{\hat{s}h}}}$$

Under the adopted CARA specification and in the limit for $y \to 1$ the above inequality can be rewritten as

$$e^{-\beta L} \geq \frac{1-p_{dh}}{2-p_{dh}}.$$

Noting that the r.h.s. is lower than $\frac{1}{2}$ and the l.h.s. is close to 1 if β is close to 0, immediately allows to conclude that the inequality holds under the condition established in the proposition, which concludes the proof. Q.E.D.

Proof of Corollary 1

From Proposition 4, contract $c^e_{\hat{s},2}$ must be incentive compatible with respect to $c^e_{\hat{s},1}$ and the incentive compatibility constraint must be binding, i.e.

$$EU_{\bar{p}}(c^e_{\hat{s},2}) = EU_{\bar{p}}(c^e_{\hat{s},1}) \tag{A.13}$$

Moreover, the definition of $c^e_{\hat{s},1}$ in point 3 of Proposition 4, $c^e_{\hat{s},1}$ entails full insurance, such that $W^e_{N-\hat{s},1} = W^e_{L-\hat{s},1} = W^e_{\hat{s},1}$. Consider the actuarially fair full insurance contract $(W - p_s L, W - p_s L)$, which is the equilibrium outcome of s insurers in Villeneuve's set up, and assume – by contradiction – that $c^e_{\hat{s},2}$ converges to $(W - p_s L, W - p_s L)$ when $\alpha \to 1$. Then, the incentive compatibility constraint (A.13) can be rewritten as

$$U(W - p_s L) = U(W^e_{\hat{s},1}),$$

which holds if and only if $W - p_s L = W^e_{\hat{s},1}$. However, if $W - p_s L = W^e_{\hat{s},1}$, then $E\pi_{\bar{p}}(c^e_{\hat{s},1}) = (p_s - \bar{p})L < 0$, which violates condition (8). Q.E.D.

Proof of Proposition 5

The proof proceeds in three steps: a) the first characterizing equilibrium contracts, b) the second checking truthful telling, c) the third ruling out deviations with cross subsidies.

a) *Characterization of the equilibrium contracts*

Recall that there are degrees of freedom in the characterization of the equilibrium outcome in signal profile $n_{\hat{s}} = 0$. Suppose that $\hat{d}$ offers the full insurance, actuarially fair contract $c^e_{\hat{d},0}$ in signal profile 0, i.e.

$$W^e_{N-\hat{d},0} = W^e_{L-\hat{d},0} = W - p_0 L.$$

The contract offered by $\hat{d}$ in signal profile $n_{\hat{s}} = 1$ is both incentive compatible with $c^e_{\hat{d},0}$ and actuarially fair by Proposition 4, i.e.

$$1 - e^{-\beta(W - p_0 L)} = p_0(1 - e^{-\beta W^e_{L-\hat{d},1}}) + (1 - p_0)(1 - e^{-\beta W^e_{N-\hat{d},1}})$$

$$E\pi_1(c^e_{\hat{d},1}) = p_1(W - L - W^e_{L-\hat{d},1}) + (1 - p_1)(W - W^e_{N-\hat{d},1}) = 0$$

Using again Proposition 4, we have that the contract $c^e_{\hat{s},1}$ offered by $\hat{s}$ in signal profile $n_{\hat{s}} = 1$ is incentive compatible with $c^e_{\hat{d},1}$ and it entails full insurance ($W^e_{L-\hat{s},1} = W^e_{N-\hat{s},1} = W^e_{\hat{s},1}$), i.e.

$$1 - e^{-\beta W^e_{\hat{s},1}} = p_1(1 - e^{-\beta W^e_{L-\hat{d},1}}) + (1 - p_1)(1 - e^{-\beta W^e_{N-\hat{d},1}}).$$

The contract $c^e_{\hat{s},2}$ offered by $\hat{s}$ in signal profile $n_{\hat{s}} = 2$ is both incentive compatible with $c^e_{\hat{s},1}$

and actuarially fair:

$$1 - e^{-\beta W^e_{\hat{s},1}} = p_1(1 - e^{-\beta W^e_{L-\hat{s},2}}) + (1 - p_1)(1 - e^{-\beta W^e_{N-\hat{s},2}})$$

$$E\pi_2(c^e_{\hat{s},2}) = p_2(W - L - W^e_{L-\hat{s},2}) + (1 - p_2)(W - W^e_{N-\hat{s},2}) = 0.$$

Then, we need to solve for $W^e_{L-\hat{d},1}, W^e_{N-\hat{d},1}, W^e_{\hat{s},1}, W^e_{L-\hat{s},2}, W^e_{N-\hat{s},2}$ the system of equations:

$$\begin{cases} 1 - e^{-\beta(W-p_0L)} = p_0(1 - e^{-\beta W^e_{L-\hat{d},1}}) + (1 - p_0)(1 - e^{-\beta W^e_{N-\hat{d},1}}) \\ p_1(W - L - W^e_{L-\hat{d},1}) + (1 - p_1)(W - W^e_{N-\hat{d},1}) = 0 \\ 1 - e^{-\beta W^e_{\hat{s},1}} = p_1(1 - e^{-\beta W^e_{L-\hat{d},1}}) + (1 - p_1)(1 - e^{-\beta W^e_{N-\hat{d},1}}) \\ 1 - e^{-\beta W^e_{\hat{s},1}} = p_1(1 - e^{-\beta W^e_{L-\hat{s},2}}) + (1 - p_1)(1 - e^{-\beta W^e_{N-\hat{s},2}}) \\ p_2(W - L - W^e_{L-\hat{s},2}) + (1 - p_2)(W - W^e_{N-\hat{s},2}) = 0 \end{cases} \tag{A.14}$$

The system describes two menus, composed by two contracts each. By construction, and given the single crossing property, the four contracts exist and lie between autarky and full insurance.

The second and the last equations in $(A.14)$ can be rewritten as

$$\begin{aligned} W^e_{N-\hat{d},1} &= \frac{W - p_1L - p_1W^e_{L-\hat{d},1}}{1 - p_1} \\ W^e_{N-\hat{s},2} &= \frac{W - p_2L - p_2W^e_{L-\hat{s},2}}{1 - p_2}. \end{aligned} \tag{A.15}$$

Using $(A.15)$, the first equation of system $(A.14)$ can be written as

$$e^{-\beta(W-p_0L)} = p_0e^{-\beta W^e_{L-\hat{d},1}} + (1 - p_0)e^{-\beta\frac{W-p_1L-p_1W^e_{L-\hat{d},1}}{1-p_1}},$$

which implicitly determines $W^e_{L-\hat{d},1}$. The third equation in $(A.14)$ in turn becomes

$$e^{-\beta W^e_{\hat{s},1}} = p_1e^{-\beta W^e_{L-1\hat{d}}} + (1 - p_1)e^{-\beta\frac{W-p_1L-p_1W^e_{L-\hat{d},1}}{1-p_1}},$$

which determines $W^e_{\hat{s},1}$. Finally, the fourth equation in $(A.14)$ can be written as

$$e^{-\beta W^e_{\hat{s},1}} = p_1 e^{-\beta W_L^{2\hat{s}}} + (1-p_1)e^{-\beta \frac{W-p_2L-p_2W_L^{2\hat{s}}}{(1-p_2)}},$$

implicitly determining $W_L^{2\hat{s}}$. Hence, using $(A.15)$, the system $(A.14)$ can be rewritten as:

$$\begin{aligned} e^{-\beta(W-p_0L)} &= p_0 e^{-\beta W^e_{L-\hat{d},1}} + (1-p_0)e^{-\beta \frac{W-p_1L-p_1W^e_{L-\hat{d},1}}{(1-p_1)}} \\ e^{-\beta W^e_{\hat{s},1}} &= p_1 e^{-\beta W^e_{L-\hat{d},1}} + (1-p_1)e^{-\beta \frac{W-p_1L-p_1W^e_{L-\hat{d},1}}{(1-p_1)}} \\ e^{-\beta W^e_{\hat{s},1}} &= p_1 e^{-\beta W_L^{2\hat{s}}} + (1-p_1)e^{-\beta \frac{W-p_2L-p_2W_L^{2\hat{s}}}{(1-p_2)}} \end{aligned} \tag{A.16}$$

Given that the above contracts satisfy, by construction, the relevant participation and incentive compatibility constraints, they constitute an equilibrium if two further conditions are verified: truthful telling, that is the $\hat{s}$ insurer does not have any incentive to mimic a $\hat{d}$ type, and robustness to deviations with cross subsidies.

b) Truthful telling

The truthful telling condition is given by

$$\frac{W-p_1L-(W-p_0L)}{2} \leq W - p_1L + \frac{1}{\beta}\ln\left[p_1 e^{-\beta(W^e_{\hat{s},1})} + (1-p_1)e^{-\beta W^e_{\hat{s},1}}\right],$$

which can easily be written as

$$W^e_{\hat{s},1} \leq W - \frac{(p_0+p_1)L}{2}. \tag{A.17}$$

Note that the first and second equations in (A.16) can be derived from

$$e^{-\beta W^e_{\hat{s},1}} = \tilde{p}e^{-\beta W^e_{L-\hat{d},1}} + (1-\tilde{p})e^{-\beta \frac{W-p_1L-p_1W^e_{L-\hat{d},1}}{(1-p_1)}}. \tag{A.18}$$

More specifically, if $\tilde{p} = p_1$, then $(A.18)$ is equivalent to the second equation of $(A.16)$ and it determines the equilibrium $W^e_{\hat{s},1}$. Instead, if $\tilde{p} = p_0$, $(A.18)$ is equivalent to the first equation of $(A.16)$ and it determines $W - p_0 L$. Moreover, note that

$$\frac{\partial W^e_{\hat{s},1}}{\partial \tilde{p}} = -\frac{1}{\beta}\frac{e^{-\beta W^e_{L-1\hat{d}}} - e^{-\beta\frac{W-p_1L-p_1W^e_{L-1\hat{d}}}{1-p_1}}}{\tilde{p}e^{-\beta W^e_{L-1\hat{d}}} + (1-\tilde{p})e^{-\beta\frac{W-p_1L-p_1W^e_{L-1\hat{d}}}{1-p_1}}} < 0$$

Finally, if $\frac{\partial W^e_{\hat{s},1}}{\partial \tilde{p}} > -\frac{L}{2}$, then $W^e_{\hat{s},1} < W - \frac{(p_0+p_1)L}{2}$. Hence, a sufficient condition for truthful telling to hold is given by

$$-\frac{1}{\beta}\frac{e^{-\beta W^e_{L-\hat{d},1}} - e^{-\beta\frac{W-p_1L-p_1W^e_{L-1\hat{d}}}{1-p_1}}}{\tilde{p}e^{-\beta W^e_{L-\hat{d},1}} + (1-\tilde{p})e^{-\beta\frac{W-p_1L-p_1W^e_{L-\hat{d},1}}{1-p_1}}} > -\frac{L}{2},$$

which is equivalent to

$$(\beta L\tilde{p} - 2)\, e^{-\beta W^e_{L-\hat{d},1}} + (\beta L(1-\tilde{p}) + 2e)^{-\beta\frac{W-p_1L-p_1W^e_{L-1\hat{d}}}{1-p_1}} > 0.$$

The latter inequality holds in the relevant under- or full insurance region if $L > \frac{2}{\beta p_0}$, where $p_0 = \frac{(1-\alpha)^2 p_s P_s + \alpha^2 p_d (1-P_s)}{(1-\alpha)^2 P_s + \alpha^2 (1-P_s)}$, i.e. L is sufficiently large.

c) Robustness to deviations with cross subsidies

Finally, we turn to the robustness of the candidate equilibrium to deviations with cross subsidies by the $\hat{s}$ insurer. In deviation, the insurance company gives up some or all profits in signal profile $n_{\hat{s}} = 1$, but it makes a profit in signal profile $n_{\hat{s}} = 2$. In order to minimize losses in signal profile $n_{\hat{s}} = 1$, the insurer offers a full insurance contract that we denote with $W^{dev}_{\hat{s},1}$. The profit maximizing contract in signal profile $n_{\hat{s}} = 2$ – denoted with $\left(W^{dev}_{L-\hat{s},2}, W^{dev}_{N-\hat{s},2}\right)$ – gives the same level of expected utility as the full insurance contract $W^{dev}_{\hat{s},1}$ – which entails that the relevant incentive compatibility constraint holds with equality – and makes the policyholder indifferent to the equilibrium contract in the same signal profile. Therefore, the deviation with cross subsidies is defined by the system

$$\begin{cases} e^{-\beta W^{dev}_{\hat{s},1}} = p_1 e^{-\beta W^{dev}_{L-\hat{s},2}} + (1-p_1)\, e^{-\beta W^{dev}_{N-\hat{s},2}} \\ p_2 e^{-\beta W^{2\hat{s}}_L} + (1-p_2)\, e^{-\beta W^{2\hat{s}}_N} = p_2 e^{-\beta W^{dev}_{L-\hat{s},2}} + (1-p_2)\, e^{-\beta W^{dev}_{N-\hat{s},2}}. \end{cases} \tag{A.19}$$

By totally differentiating A.19 and simplifying, we obtain

$$\begin{bmatrix} p_1 e^{-\beta W^{dev}_{L-\hat{s},2}} & (1-p_1)\, e^{-\beta W^{dev}_{N-\hat{s},2}} \\ p_2 e^{-\beta W^{dev}_{L-\hat{s},2}} & (1-p_2)\, e^{-\beta W^{dev}_{N-\hat{s},2}} \end{bmatrix} d \begin{bmatrix} W^{dev}_{L-\hat{s},2} \\ W^{dev}_{N-\hat{s},2} \end{bmatrix} = \begin{bmatrix} e^{-\beta W^{dev}_{\hat{s},1}} \\ 0 \end{bmatrix} dW^{dev}_{\hat{s},1}$$

and hence

$$\begin{aligned} \begin{bmatrix} \frac{\partial W^{dev}_{L-\hat{s},2}}{\partial W^{dev}_{\hat{s},1}} \\ \frac{\partial W^{dev}_{N-\hat{s},2}}{\partial W^{dev}_{\hat{s},1}} \end{bmatrix} &= \begin{bmatrix} p_1 e^{-\beta W^{dev}_{L-\hat{s},2}} & (1-p_1)\, e^{-\beta W^{dev}_{N-\hat{s},2}} \\ p_2 e^{-\beta W^{dev}_{L-\hat{s},2}} & (1-p_2)\, e^{-\beta W^{dev}_{N-\hat{s},2}} \end{bmatrix}^{-1} \begin{bmatrix} e^{-\beta W^{dev}_{\hat{s},1}} \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} (1-p_2)\, \frac{e^{\beta W^{dev}_{L-\hat{s},2}}}{p_1-p_2} e^{-\beta W^{dev}_{\hat{s},1}} \\ -p_2 \frac{e^{\beta W^{dev}_{N-\hat{s},2}}}{p_1-p_2} e^{-\beta W^{dev}_{\hat{s},1}} \end{bmatrix}. \end{aligned} \tag{A.20}$$

By totally differentiating the insurer's expected profits with respect to $W^{dev}_{\hat{s},1}$, we can immediately see that the condition for the deviation not to be (locally) profitable reads

$$-\Pr(1|\hat{s}) - \Pr(2|\hat{s}) \left(p_2 (1-p_2) \frac{e^{\beta W^{dev}_{L-\hat{s},2}}}{p_1-p_2} e^{-\beta W^{dev}_{\hat{s},1}} - (1-p_2)\, p_2 \frac{e^{\beta W^{dev}_{N-\hat{s},2}}}{p_1-p_2} e^{-\beta W^{dev}_{\hat{s},1}} \right) =$$

$$= -\Pr(1|\hat{s}) + \Pr(2|\hat{s}) \frac{p_2(1-p_2)}{p_1-p_2} \left(e^{\beta W^{dev}_{N-\hat{s},2}} - e^{\beta W^{dev}_{L-\hat{s},2}} \right) e^{-\beta W^{dev}_{\hat{s},1}} \leq 0 \tag{A.21}$$

Notice that deviating from the equilibrium implies that $W^{dev}_{\hat{s},1}$ increases. This in turn entails that $\left(e^{\beta W^{dev}_{N-\hat{s},2}} - e^{\beta W^{dev}_{L-\hat{s},2}} \right)$ decreases. Since from (A.20), $\frac{\partial W^{dev}_{L-\hat{s},2}}{\partial W^{dev}_{\hat{s},1}} > 0$, $\frac{\partial W^{dev}_{N-\hat{s},2}}{\partial W^{dev}_{\hat{s},1}} < 0$, we have that $\frac{\partial}{\partial W^{dev}_{\hat{s},1}} \left(e^{\beta W^{dev}_{N-\hat{s},2}} - e^{\beta W^{dev}_{L-\hat{s},2}} \right) < 0$.

Hence, it is necessary and sufficient to compute (A.21) at $W^e_{\hat{s},1}$, obtaining

$$W^e_{\hat{s},1} \geq \frac{1}{\beta} \ln \left(\frac{\Pr(2|\hat{s})}{\Pr(1|\hat{s})} \frac{p_2(1-p_2)}{p_1-p_2} \left(e^{\beta W^e_{N-\hat{s},2}} - e^{\beta W^e_{L-\hat{s},2}} \right) \right). \tag{A.22}$$

Recall that the relevant loss probabilities are

$$
\begin{aligned}
p_1 &= p_s P_s + p_d(1-P_s) \\
p_2 &= \frac{\alpha^2 p_s P_s + (1-\alpha)^2 p_d(1-P_s)}{\alpha^2 P_s + (1-\alpha)^2(1-P_s)} \\
P(1|\hat{s}) &= \frac{\alpha(1-\alpha)}{P_s(1-\alpha)+(1-P_s)\alpha} \\
P(2|\hat{s}) &= \frac{P_s\alpha^2 + P_d(1-\alpha)^2}{P_s\alpha + P_d(1-\alpha)}
\end{aligned}
$$

Using these probabilities, we have that

$$
\frac{P(2|\hat{s})}{P(1|\hat{s})}\frac{p_2\,(1-p_2)}{p_1-p_2} = \frac{P_s - 2P_s\alpha + \alpha}{2P_s\alpha + 1 - \alpha - P_s}\cdot
$$
$$
\cdot\frac{\left[(1-\alpha)^2(1-P_s)(1-p_d) + \alpha^2 P_s(1-p_s)\right]\left[p_s P_s\alpha^2 + p_d(1-P_s)(1-\alpha)^2\right]}{(p_d-p_s)P_s(1-P_s)\alpha(1-\alpha)(2\alpha-1)},
$$

which for $p_s = 0$ becomes

$$
\frac{P_s - 2P_s\alpha + \alpha}{2P_s\alpha + 1 - \alpha - P_s}\frac{\left[(1-\alpha)^2(1-P_s)(1-p_d) + \alpha^2 P_s\right]p_d(1-P_s)(1-\alpha)^2}{p_d P_s(1-P_s)\alpha(1-\alpha)(2\alpha-1)}.
$$

Note that the latter expression is 0 for $\alpha = 1$. Since $(A.22)$ is continuous, it must hold also in a neighborhood of $p_s = 0$ and $\alpha = 1$. Noticing that $\lim_{\alpha\to 1} p_0 = p_d$, the truthful telling condition becomes $\beta L p_d > 2$, which requires that L is sufficiently large. Again by continuity the condition holds also in a neighborhood of $\alpha = 1$. Q.E.D.

Proof of Proposition 7

To ease the exposition, the proof is organized as a sequence of Lemmas. We denote with $\Lambda_p(c')$ the set of contracts such that any contract c in the set is (i) preferred to contract c' when the policyholder's loss probability assessment is p – i.e. $EU_p(c) \geq EU_p(c')$ – and (ii) it does not entail negative expected profits for the insurer in signal profile $n_{\hat{s}} = n$ – i.e. $E\pi_{p_n}(c) \geq 0$. Moreover, we denote with Γ_o^e the set of contracts that are accepted with positive probability in equilibrium, and we refer to them as the set of outcomes. Γ_o^e includes two subsets of contracts: the subset of contracts $\Gamma_{\hat{s}}^e$ sold by $\hat{s}$ insurers, and the subset of contracts $\Gamma_{\hat{d}}^e$ sold by $\hat{d}$ insurers. Hence, contracts sold by both $\hat{s}$ and $\hat{d}$ belong to the intersection of $\Gamma^e_{\hat{s}}$ and $\Gamma^e_{\hat{d}}$. Given a contract $c \in \Gamma^e_o$, we denote with N_c the set of signal profiles in which contract c is accepted. Furthermore, we denote with $\tilde{p}_{n_{\hat{s}}}$ the assessment of the loss probability by the policyholder when an $\hat{s}$ insurer deviates and the true signal profile is $n_{\hat{s}}$. Recall that all contracts accepted in equilibrium must be incentive compatible. Indeed, if a contract is not incentive compatible and it is accepted in a different signal profile, it can always be relabeled. We thus denote with $c^e_{\hat{\theta},n_{\hat{s}}}$ the contract that in equilibrium is preferred by the policyholder to the other contracts in the menu $C^e_{\hat{\theta}}$ when the signal profile is $n_{\hat{s}}$.

Our first Lemma establishes that in a specific signal profile – say $n_{\hat{s}}$ – the insurer makes higher profits if he were able to sell the contract of a riskier signal profile rather than the contract designed for $n_{\hat{s}}$.

Lemma A.1 *$E\pi_{p_{n_{\hat{s}}}}(c^e_{\hat{\theta},n_{\hat{s}}-1}) \geq E\pi_{p_{n_{\hat{s}}}}(c^e_{\hat{\theta},n_{\hat{s}}})$ for any pair of full or underinsurance contracts $c^e_{\hat{\theta},n_{\hat{s}}}, c^e_{\hat{\theta},n_{\hat{s}}-1} \in \Gamma^e_o$.*

Proof. Since

$$E\pi_{p_{n_{\hat{s}}}}(c^e_{\hat{\theta},n_{\hat{s}}-1}) = p_{n_{\hat{s}}}(W - L - W_L(c^e_{\hat{\theta},n_{\hat{s}}-1})) + (1 - p_{n_{\hat{s}}})(W - W_N(c^e_{\hat{\theta},n_{\hat{s}}-1}))$$

and

$$E\pi_{p_{n_{\hat{s}}}}(c^e_{\hat{\theta},n_{\hat{s}}}) = p_{n_{\hat{s}}}(W - L - W_L(c^e_{\hat{\theta},n_{\hat{s}}})) + (1 - p_{n_{\hat{s}}})(W - W_N(c^e_{\hat{\theta},n_{\hat{s}}})),$$

the inequality in the statement of the Lemma holds if

$$p_{n_{\hat{s}}}(W_L(c^e_{\hat{\theta},n_{\hat{s}}-1}) - W_L(c^e_{\hat{\theta},n_{\hat{s}}})) + (1 - p_{n_{\hat{s}}})(W_N(c^e_{\hat{\theta},n_{\hat{s}}-1}) - W_N(c^e_{\hat{\theta},n_{\hat{s}}})) \leq 0 \tag{A.23}$$

By incentive compatibility, it must be that $c^e_{\hat{\theta},n_{\hat{s}}} \notin \Lambda_{p_{n_{\hat{s}}-1}}(c^e_{\hat{\theta},n_{\hat{s}}-1})$ and $c^e_{\hat{\theta},n_{\hat{s}}} \in \Lambda_{p_{n_{\hat{s}}}}(c^e_{\hat{\theta},n_{\hat{s}}-1})$. This implies that $W_L(c^e_{\hat{\theta},n_{\hat{s}}}) < W_L(c^e_{\hat{\theta},n_{\hat{s}}-1})$ and that $W_N(c^e_{\hat{\theta},n_{\hat{s}}}) > W_N(c^e_{\hat{\theta},n_{\hat{s}}-1})$. The slope of the indifference curve $EU_{p_{n_{\hat{s}}}}(c^e_{\hat{\theta},n_{\hat{s}}-1})$ evaluated at $c^e_{\hat{\theta},n_{\hat{s}}-1}$ is $-\frac{1-p_{n_{\hat{s}}}}{p_{n_{\hat{s}}}}\frac{U'(W_L(c^e_{\hat{\theta},n_{\hat{s}}-1}))}{U'(W_N(c^e_{\hat{\theta},n_{\hat{s}}-1}))}$, and therefore it must be that $\frac{W_L(c^e_{\hat{\theta},n_{\hat{s}}-1})-W_L(c^e_{\hat{\theta},n_{\hat{s}}})}{W_N(c^e_{\hat{\theta},n_{\hat{s}}-1})-W_N(c^e_{\hat{\theta},n_{\hat{s}}})} > -\frac{1-p_{n_{\hat{s}}}}{p_{n_{\hat{s}}}}\frac{U'(W_L(c^e_{\hat{\theta},n_{\hat{s}}-1}))}{U'(W_N(c^e_{\hat{\theta},n_{\hat{s}}-1}))}$. Given that contract $c^e_{\hat{\theta},n_{\hat{s}}-1}$

entails full or underinsurance, the r.h.s. of the above inequality is larger than $-\frac{1-p_{n_{\hat{s}}}}{p_{n_{\hat{s}}}}$. It follows that $\frac{W_L(c^e_{\hat{\theta},n_{\hat{s}}-1})-W_L(c^e_{\hat{\theta},n_{\hat{s}}})}{W_N(c^e_{\hat{\theta},n_{\hat{s}}-1})-W_N(c^e_{\hat{\theta},n_{\hat{s}}})} > -\frac{1-p_{n_{\hat{s}}}}{p_{n_{\hat{s}}}}$, so that condition (A.23) holds. ■

The next lemma establishes that beliefs cannot be fully pessimistic in equilibrium, so that this case needs no longer be considered in the remaining of the proof.

Lemma A.2 *Equilibrium beliefs cannot be fully pessimistic.*

Proof. The proof proceeds by contradiction. Assume that the $\hat{s}$ insurer deviates by offering the menu of contracts Γ'^e_o accepted in equilibrium, which is obtained by eliminating from Γ^e_o all contracts that make an ex-post loss in equilibrium (note that it has not yet been shown that there do not exists equilibria entailing cross-subsidies). If Γ'^e_o coincides with $\Gamma^e_{\hat{s}}$, then the $\hat{s}$ insurer simply adds an unacceptable contract, so as to signal the deviation to the policyholder. Since Γ^e_o is the union of the menus $\Gamma^e_{\hat{s}}$ and $\Gamma^e_{\hat{d}}$, each of them meeting the participation of the $\hat{s}$ and $\hat{d}$ type respectively, then Γ'^e_o is strictly profitable for $\hat{s}$. Following the deviation, a fully pessimistic policyholder believes that the signal profile is $n_{\hat{s}} - 1$ for any true signal profile $n_{\hat{s}} \geq 1$. In particular, in signal profile $n_{\hat{s}}$, the policyholder chooses the contract $c^e_{\hat{s},n_{\hat{s}}-1} \in \Gamma'^e_{\hat{o}}$ offered by the deviating $\hat{s}$ insurer with positive probability if it is ex-post profitable. Then, by Lemma A.1, the deviation is profitable, which establishes the needed contradiction. ■

The following Lemma shows that a sufficient condition for the incentive compatibility of the set of equilibrium outcomes is that contracts of adjacent signal profiles are incentive compatible.

Lemma A.3 *All incentive compatibility constraints hold for a given system of beliefs if they hold for all pairs of adjacent signal profiles under the same system of beliefs.*

Proof. Consider, without loss of generality, three contracts, $\{c_A, c_B, c_C\}$, in the set of outcomes where the three contracts are the outcomes in the states $n_{\hat{s}} - 1, n_{\hat{s}}, n_{\hat{s}} + 1$, respectively. By assumption, they are incentive compatible for any pair of adjacent signal

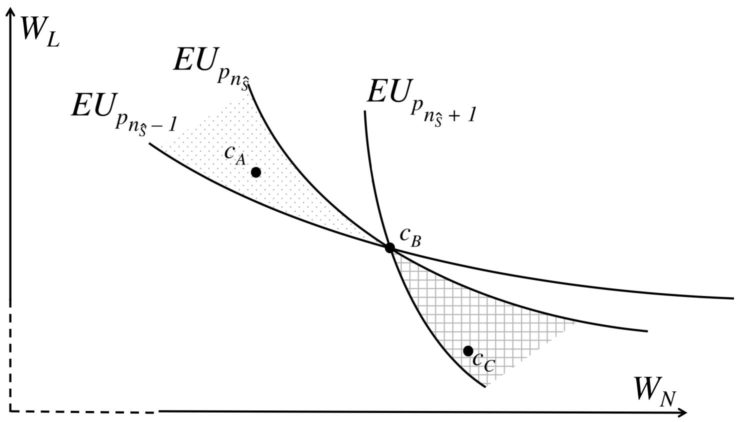

Figure A.1: Graphical intuition of the Proof of Lemma A.3

profiles, i.e.

$$c_A \in \Lambda_{p_{n_{\hat{s}}-1}}(c_B) \tag{A.24}$$

$$c_A \notin \Lambda_{p_{n_{\hat{s}}}}(c_B) \tag{A.25}$$

$$c_C \notin \Lambda_{p_{n_{\hat{s}}}}(c_B) \tag{A.26}$$

$$c_C \in \Lambda_{p_{n_{\hat{s}}+1}}(c_B). \tag{A.27}$$

By the single crossing property, A.24–A.27 imply that $c_A \in \Lambda_{p_{n_{\hat{s}}-1}}(c_C)$ and $c_C \in \Lambda_{p_{n_{\hat{s}}+1}}(c_A)$. Figure A.1 provides a geometric intuition. By induction, the lemma holds also for non-adjacent states.

■

The next lemma characterizes the best reply of the policyholder, proving that an under-insurance contract of an $\hat{s}$ insurer that is accepted in equilibrium is also accepted (meeting both the relevant participation and incentive compatibility constraints) if the policyholder holds more pessimistic beliefs.

Lemma A.4 *If contract* $c^e_{\hat{s},n_{\hat{s}}} \in \Gamma^e_{\hat{s}}$*, then* $c^e_{\hat{s},n_{\hat{s}}}$ *is also accepted with positive probability for any belief* $\tilde{p}_{n_{\hat{s}}}$ *such that* $p_{n_{\hat{s}}-1} > \tilde{p}_{n_{\hat{s}}} \geq p_{n_{\hat{s}}}$.

Proof. The proof proceeds by contradiction and it is based on the construction of a profitable deviation. The subset $\Gamma^e_{\hat{s}}$ must satisfy the ex-ante participation of $\hat{s}$ insurers, and the subset $\Gamma^e_{\hat{d}}$ must satisfy the ex-ante participation of $\hat{d}$ insurers. Then, if $\hat{s}$ offers the menu Γ^e_o – provided that all its contracts are accepted with positive probability under the same equilibrium signal profiles – $\hat{s}$ obtains positive profits.

Consider the menu of contracts Γ'_o, obtained by dropping from the menu of equilibrium outcomes Γ^e_o the contracts that make a loss contingent on the signal profile, i.e. any contract $c \in \Gamma^e_o$ such that $E\left(\pi_{p_{n_{\hat{s}}}}(c)|n_{\hat{s}} \in \Sigma_c\right) < 0$. If $\hat{s}$ offers the menu Γ'_o and its contracts are accepted under the same equilibrium signal profiles, $\hat{s}$ obtains positive profits.

Let $\hat{s}$ deviate by offering the menu Γ'_o (if $\Gamma'_o = \Gamma^e_{\hat{s}}$, then $\hat{s}$ simply adds an unacceptable contract). By assumption, following $\hat{s}$'s deviation, the policyholder assesses the loss probability by $\tilde{p}_{n_{\hat{s}}}$, with $p_{n_{\hat{s}}-1} > \tilde{p}_{n_{\hat{s}}} \geq p_{n_{\hat{s}}}$ for any $1 \leq n_{\hat{s}} \leq n$. Assume that there exists a subset Σ' of signal profiles $n'_{\hat{s}}$ such that, given $\tilde{p}_{n'_{\hat{s}}}$, the policyholder prefers another contract in Γ'_o to $c^e_{n'_{\hat{s}}} \in \Gamma'_o$. In all signal profiles $n_{\hat{s}} \notin \Sigma'$, $\hat{s}$ obtains positive profits with the same (if $c^e_{n_{\hat{s}}} \in \Gamma^e_{\hat{s}}$) or higher (if $c^e_{n_{\hat{s}}} \notin \Gamma^e_{\hat{s}}$) probability than in equilibrium. In all signal profiles $n_{\hat{s}} \in \Sigma'$, $\hat{s}$ makes higher profits by Lemma A.1. Then, Γ'_o is a profitable deviation for $\hat{s}$, which completes the proof of the lemma. ■

The following sequence of Lemmas proves that $\hat{s}$'s equilibrium menu must be fully separating, so that a different contract is offered for each signal profile. The first Lemma of the sequence shows that each contract accepted in equilibrium entails zero expected profits, even if accepted in more signal profiles.

Lemma A.5 *If $c^e \in \Gamma^e_o$ is the equilibrium outcome in a set of signal profiles N_{c^e} such that $n_{\hat{s}} \geq 2$ for any $n_{\hat{s}} \in N_{c^e}$, then $E\left(\pi_{p_{n_{\hat{s}}}}(c^e)|n_{\hat{s}} \in N_{c^e}\right) = 0$.*

Proof. First, we show that a contract belonging to the equilibrium outcome cannot entail negative expected profit. Suppose that this is not the case, assuming that an insurance company offering such a contract in N_{c^e} can drop it from its menu leaving unaffected the choice of the policyholder in all signal profiles by Lemma A.4, thus increasing its profits. Second, we prove that the equilibrium outcome entails no contracts with positive expected profits. Consider that $\hat{s}$ deviates by offering a menu Γ'_o that is obtained by adding to the equilibrium menu an undercutting contract c^{dev} acceptable in signal profile $\overline{n}$, where

$\overline{n} = \max n_{\hat{s}} \in N_{c^e}$. c^{dev} must be such that: 1) $c^{dev} \notin \Lambda_{\tilde{p}_{\overline{n}-1}}(c^e)$ and $c^{dev} \notin \Lambda_{\tilde{p}_{\overline{n}+1}}(c^e)$ (i.e., in signal profiles $\overline{n}-1$ and $\overline{n}+1$, the policyholder must prefer c^e over c^{dev}); 2) $c^{dev} \in \Lambda_{\tilde{p}_{\overline{n}}}(c^e)$ (hence c^{dev} is preferred to c^e for $n_{\hat{s}} = \overline{n}$); 3) c^{dev} must yield higher expected profits than c_e given $n_{\hat{s}} = \overline{n}$.

Given that the deviation contract c^{dev} is incentive compatible by Lemma A.3, single crossing and continuity guarantee that there always exists a contract meeting conditions 1) and 2). For condition 3) to hold as well, it must be that c^{dev} is sufficiently close to c^e. Indeed, by continuity, the two contracts give similar profits, although for $n_{\hat{s}} = \overline{n}$ c^{dev} is accepted with probability 1. By Lemmas A.3 and A.4, the policyholder still prefers $c^e_{n_{\hat{s}}} \in \Gamma'_o$ in signal profile $n_{\hat{s}}$, for any $n_{\hat{s}} \neq \overline{n}$, which concludes the proof. ■

The next Lemma shows that the equilibrium outcome cannot entail pooling contracts on non-adjacent signal profiles.

Lemma A.6 *In non-adjacent signal profiles, the equilibrium outcome cannot encompass pooling contracts.*

Proof. Pooling equilibria in non-adjacent signal profiles cannot be incentive compatible. To see why, focus w.l.g. on insurer $\hat{s}$ (the same argument applies to $\hat{d}$ insurers). Consider three signal profiles $n_{\hat{s}}$, $n'_{\hat{s}}$ and $n''_{\hat{s}}$ such that $c^e_{\hat{s},n_{\hat{s}}} = c^e_{\hat{s},n'_{\hat{s}}} \neq c^e_{\hat{s},n''_{\hat{s}}}$ and $n_{\hat{s}} < n''_{\hat{s}} < n'_{\hat{s}}$, i.e. $\hat{s}$ pools the offer in the non-adjacent signal profiles $n_{\hat{s}}$ and $n'_{\hat{s}}$. For the incentive compatibility of the menu, it must be $c^e_{\hat{s},n''_{\hat{s}}} \notin \Lambda_{p_{n_{\hat{s}}}}(c^e_{\hat{s},n_{\hat{s}}})$, $c^e_{\hat{s},n''_{\hat{s}}} \notin \Lambda_{p_{n'_{\hat{s}}}}(c^e_{\hat{s},n_{\hat{s}}})$ and $c^e_{\hat{s},n''_{\hat{s}}} \in \Lambda_{p_{n''_{\hat{s}}}}(c^e_{\hat{s},n_{\hat{s}}})$, namely contract $c^e_{\hat{s},n''_{\hat{s}}}$ is preferred to $c^e_{\hat{s},n_{\hat{s}}}$ if and only if the signal profile is $n''_{\hat{s}}$. However, the intersection between these three conditions is empty, as $\Lambda_{p_{n''_{\hat{s}}}}(c^e_{\hat{s},n_{\hat{s}}}) \subset \left(\Lambda_{p_{n_{\hat{s}}}}(c^e_{\hat{s},n_{\hat{s}}}) \cup \Lambda_{p_{n'_{\hat{s}}}}(c^e_{\hat{s},n_{\hat{s}}})\right)$ by the single crossing property given that $p_{n_{\hat{s}}} < p_{n''_{\hat{s}}} < p_{n'_{\hat{s}}}$. Note that the proof builds solely on an equilibrium argument, so that beliefs play no role. ■

We are now ready to prove that there cannot be contracts that are accepted with positive probability in more than one signal profile such that $n_{\hat{s}} \geq 2$. Therefore, even if one insurer offers a pooling contract, it can be accepted in only one signal profile.

Lemma A.7 *Consider a contract c^e accepted in equilibrium in a signal profile $n_{\hat{s}} \geq 2$: $c^e \in \Gamma_o^e$. Then c^e is accepted in only one signal profile, i.e. the set N_{c^e} contains only one signal profile for all $n_{\hat{s}} \geq 2$.*

Proof. Assume that the statement in the lemma does not hold, so that the set N_{c^e} contains two or more signal profiles. By Lemma A.5, equilibrium ex-ante expected profits in c^e over N_{c^e} are zero: $E\left(\pi_{p_{n_{\hat{s}}}}(c^e)|n_{\hat{s}} \in N_{c^e}\right) = 0$. Then, the set N_{c^e} of signal profiles such that c^e is accepted must contain at least one signal profile in which $\hat{s}$ obtains strictly positive profits. Such a signal profile must be the safest signal profile in N_{c^e}, and we denote it by $\overline{n}$, i.e. $\overline{n} = \max n_{\hat{s}} \in N_{c^e}$. Consider that $\hat{s}$ deviates by offering a menu Γ_o' that is obtained by adding to the equilibrium menu an undercutting contract c^{dev}, such that it is accepted in the deviation only in signal profile $\overline{n}$. To this aim, c^{dev} must be such that: 1) $c^{dev} \notin \Lambda_{\tilde{p}_{\overline{n}-1}}(c^e)$ and $c^{dev} \notin \Lambda_{\tilde{p}_{\overline{n}+1}}(c^e)$ (i.e., in signal profiles $\overline{n}-1$ and $\overline{n}+1$, the policyholder must prefer c^e over c^{dev}); 2) $c^{dev} \in \Lambda_{\tilde{p}_{\overline{n}}}(c^e)$ (hence c^{dev} is preferred to c^e for $n_{\hat{s}} = \overline{n}$); 3) c^{dev} must yield higher expected profits than c^e given $n_{\hat{s}} = \overline{n}$.

Given that the deviation contract c^{dev} is incentive compatible by Lemma A.3, single crossing and continuity guarantee that there always exists a contract meeting conditions 1) and 2). For condition 3) to hold as well, it must be that c^{dev} is sufficiently close to c^e. Indeed, by continuity, the two contracts give similar profits, although for $n_{\hat{s}} = \overline{n}$ c^{dev} is accepted with probability 1. By Lemmas A.3 and A.4, the policyholder still prefers $c^e_{n_{\hat{s}}} \in \Gamma_o'$ in signal profile $n_{\hat{s}}$, for any $n_{\hat{s}} \neq \overline{n}$, which concludes the proof. ■

Lemmas (A.1) to (A.7) establish part 1 of the proposition for $n_{\hat{s}} \geq 2$. It is now immediate to show parts 2 and 3 (Lemma A.8) of the proposition before moving to the characterization of the equilibrium for $n_{\hat{s}} = \{0, 1\}$ in the subsequent lemmas.

Lemma A.8 *(i) All contracts that are accepted with positive probability in equilibrium are actuarially fair for all $n_{\hat{s}} \geq 2$, so that $E\pi_{p_{n_{\hat{s}}}}(c^e_{n_{\hat{s}}}) = 0$. (ii) Furthermore, in equilibrium, the incentive compatibility constraints of the menu of outcomes Γ_o^e are binding for all $n_{\hat{s}} \geq 2$.*

Proof. The first part of the lemma follows immediately by noting that from Lemmas A.6 and A.7, the set of equilibrium outcomes is fully separating for all $n_{\hat{s}} \geq 2$. Moreover, from Lemma A.5, each contract is actuarially fair.

The proof of the second part proceeds instead by contradiction, showing that a deviation always exists. Suppose that, in signal profile $n'_{\hat{s}} \geq 2$, the incentive compatibility constraint is not binding in equilibrium for at least one signal profile $n'_{\hat{s}}$, i.e. $EU_{p_{n'_{\hat{s}}-1}}(c^e_{n'_{\hat{s}}-1}) > EU_{n'_{\hat{s}}-1}(c^e_{n'_{\hat{s}}})$, where $c^e_{n'_{\hat{s}}-1}, c^e_{n'_{\hat{s}}} \in \Gamma^e_o$.

Consider a contract c^{dev}, such that i) $c^{dev} \in \Lambda_{\tilde{p}_{n'_{\hat{s}}}}(c^e_{n'_{\hat{s}}})$, ii) $c^{dev} \notin \Lambda_{\tilde{p}_{n'_{\hat{s}}-1}}(c^e_{n'_{\hat{s}}-1})$, iii) $c^{dev} \notin \Lambda_{\tilde{p}_{n'_{\hat{s}}+1}}(c^e_{n'_{\hat{s}}})$, and iv) $E\pi_{p_{n'_{\hat{s}}}}(c^{dev}) > 0$ (i.e., c^{dev} is ex-post profitable in signal profile $n'_{\hat{s}}$). Therefore such a contract must lie between the policyholder indifference curves using the probability assessments $\tilde{p}_{n'_{\hat{s}}}$ and $\tilde{p}_{n'_{\hat{s}}-1}$ and the zero isoprofit in $n'_{\hat{s}}$. This area is not empty by the single crossing property and the fact that contract $c^e_{n'_{\hat{s}}}$ is actuarially fair by the first part of the lemma. Assume that an $\hat{s}$ insurer deviates and offers the menu C^{dev} such that C^{dev} is identical to the equilibrium menu of outcomes Γ^e_o but for the fact that, in place of contract $c^e_{n'_{\hat{s}}}$ contract c^{dev} is offered.

In any signal profile $n_{\hat{s}}$ riskier than $n'_{\hat{s}} - 1$, the policyholder chooses contract $c^e_{n_{\hat{s}}} \in \Gamma^e_o$ by Lemmas A.3 and A.4, so that a $\hat{s}$ insurer obtains the same profits as in the equilibrium.

In signal profile $n'_{\hat{s}} - 1$, the policyholder chooses $c^e_{n'_{\hat{s}}-1} \in \Gamma^e_o$ over c^{dev} by construction (condition ii)), so that also in this case a $\hat{s}$ insurer obtains the same profits as in the equilibrium.

In signal profile $n'_{\hat{s}}$, the policyholder chooses c^{dev} over $c^e_{n'_{\hat{s}}} \in \Gamma^e_o$. In fact, given her beliefs $\tilde{p}_{n'_{\hat{s}}}$, with $p_{n'_{\hat{s}}-1} > \tilde{p}_{n'_{\hat{s}}} \geq p_{n'_{\hat{s}}}$, she prefers $c^e_{n'_{\hat{s}}}$ to $c^e_{n'_{\hat{s}}-1}$ by Lemma A.4. Moreover, she prefers c^{dev} to $c^e_{n'_{\hat{s}}}$ by construction (see condition i)). Hence, in signal profile $n'_{\hat{s}} \geq 2$ after the deviation C^{dev} is offered, the policyholder accepts c^{dev} over any other contract offered either by the deviating insurer or by competitors.

In signal profile $n'_{\hat{s}} + 1$, given out of equilibrium beliefs $\tilde{p}_{n'_{\hat{s}}+1}$, the policyholder prefers $c^e_{n'_{\hat{s}}+1}$ to $c^e_{n'_{\hat{s}}}$ by Lemmas A.3 and A.4, and $c^e_{n'_{\hat{s}}}$ to c^{dev} by construction (condition iii)).

Finally, in any signal profile $n_{\hat{s}}$ safer than $n'_{\hat{s}} + 1$, the policyholder chooses contract $c^e_{n_{\hat{s}}} \in \Gamma^e_o$ by Lemmas A.3 and A.4, so that $\hat{s}$ obtains the same profits than in equilibrium.

Since in any signal profile $n_{\hat{s}} \neq n'_{\hat{s}}$ a $\hat{s}$ insurer obtains the same profits as in the equilibrium, while in $n'_{\hat{s}}$ profits are strictly higher, the deviation is profitable, which establishes the contradiction completing the proof ■

The next four lemmas characterize the equilibrium outcome in signal profile $n_{\hat{s}} = 1$.

Lemma A.9 *In signal profile $n_{\hat{s}} = 1$, $\hat{d}$ cannot offer an actuarially fair contract that entails either full or over insurance.*

Proof. From Lemma A.8, the equilibrium outcome entails zero profit in signal profiles $n_{\hat{s}} \geq 2$. Then, the sum of equilibrium profits for a $\hat{d}$ insurer in $n_{\hat{s}} = 0$ and $n_{\hat{s}} = 1$ cannot be negative, else $\hat{d}$'s participation constraint would be violated. One must consider two cases.

Case 1. In $n_{\hat{s}} = 0$ and $n_{\hat{s}} = 1$ a $\hat{d}$ insurer offers a pooling contract. An actuarially fair contract for $n_{\hat{s}} = 1$ would violate the participation constraint of insurer $\hat{d}$, given that the contract would be accepted with positive probability in $n_{\hat{s}} = 0$.

Case 2. In $n_{\hat{s}} = 0$ and $n_{\hat{s}} = 1$ a $\hat{d}$ insurer offers a separating menu. By incentive compatibility, the equilibrium contract offered by $\hat{d}$ in $n_{\hat{s}} = 1$, $c^e_{\hat{d},1}$, must entail underinsurance (recall that over-insurance contracts are ruled out by assumption). ■

Lemma A.10 *In the signal profile $n_{\hat{s}} = 1$, the policyholder accepts the offer of insurer $\hat{s}$ with probability 1.*

Proof. The proof proceeds by contradiction assuming that $EU_{p_1}(c^e_{\hat{d},1}) \geq EU_{p_1}(c^e_{\hat{s},1})$ (in the signal profile $n_{\hat{s}} = 1$ the policyholder chooses the contract $c^e_{\hat{d},1}$) offered by a $\hat{d}$ insurer.

It is easy to show that there always exists a deviation $\tilde{C}$ by a $\hat{s}$ insurer, such that $\tilde{C} = \left\{\tilde{c}, c^e_{\hat{s},n_{\hat{s}}}\right\}_{n_{\hat{s}}=2,\ldots,n}$. $\tilde{C}$ is a menu of contracts obtained from the equilibrium menu $C^e_{\hat{s}} = \left\{c^e_{\hat{s},n_{\hat{s}}}\right\}_{n_{\hat{s}}=1,\ldots,n}$ by substituting contract $c^e_{\hat{s},1}$ with a contract $\tilde{c}$ having the following characteristics: (i) $\tilde{c} \in \Lambda_{p_1}(c^e_{\hat{d},1})$ (i.e., $\tilde{c}$ is preferred to $c^e_{\hat{d},1}$ when the policyholder believes that the loss probability is p_1), (ii) $\tilde{c} \in \Lambda_{p_0}(c^e_{\hat{d},1})$ (i.e., $\tilde{c}$ is preferred to $c^e_{\hat{d},1}$ when the policyholder believes that the loss probability is p_0), and (iii) $\tilde{c}$ lies under the zero-isoprofit line when the signal profile is $n_{\hat{s}} = 1$.

Contract $\tilde{c}$ exists by the convexity of indifference curves and by the fact that, by Lemma A.9, $c^e_{\hat{d},1}$ cannot be an actuarially fair contract. By conditions (i) and (ii) and Lemma A.3, in signal profile $n_{\hat{s}} = 1$, the policyholder prefers $\tilde{c}$ to $c^e_{\hat{d},1}$ regardless of her beliefs ($\tilde{p}_1 \in [p_1, p_0)$). In any other signal profile $n_{\hat{s}} > 1$, she chooses instead the same contracts as in the equilibrium (i.e.implying the outcome Γ_o^e) by Lemma A.4, so that the $\hat{s}$ deviating insurer obtains the same profits as in the equilibrium. This shows that the deviation $\tilde{C}$ is profitable for the $\hat{s}$ insurer, which establishes the contradiction proving the lemma. ■

To complete the characterization of the equilibrium outcome for $n_{\hat{s}} = 1$, we need to introduce some additional notation. Denote with $\overline{c}^e_{\hat{d},1}$ the contract offered by a $\hat{d}$ insurer that is preferred in equilibrium to all other contracts in the menu $C^e_{\hat{d}}$, and strictly preferred to all other contracts in the menu $C^e_{\hat{d}}$ given the out of equilibrium beliefs $\tilde{p_1}$ when the signal profile is $n_{\hat{s}} = 1$ and the $\hat{s}$ insurer deviates (i.e., $p_1 \leq \tilde{p_1} < p_0$).

The contract that maximizes $\hat{s}$'s profits in signal profile $n_{\hat{s}} = 1$ and is preferred to $\overline{c}^e_{\hat{d},1}$ is given by

$$\begin{aligned} \tilde{c}_1^{\max} &= \arg\max_c E\pi_{p_1}(c) \\ &\quad s.t. EU_{\tilde{p}_1}(c) \geq \max\{EU_{\tilde{p}_1}(\overline{c}^e_{\hat{d},1}), EU_{\tilde{p}_1}(\underline{c})\}. \end{aligned} \tag{A.28}$$

The next lemma establishes the uniqueness of such contract.

Lemma A.11 *Contract $\tilde{c}_1^{max}$ is unique.*

Proof. The proof must consider two cases.

Case 1. In signal profile $n_{\hat{s}} = 1$, when the $\hat{d}$ insurer offers menu $C^e_{\hat{d}}$, in equilibrium the policyholder strictly prefers contract $c^e_{\hat{d},1}$ over all other contracts in the same menu (even if, as shown in Lemma A.10, she does not accept it). In this case, the solution of problem (A.28) is unique due to the strict convexity of the indifference curves and the fact that contract $c^e_{\hat{d},1}$ is unique.

Case 2: In signal profile $n_{\hat{s}} = 1$, when the $\hat{d}$ insurer offers the menu $C^e_{\hat{d}}$, in equilibrium the policyholder is indifferent between two or more contracts in the menu (even if, by Lemma A.10, she accepts none of them). Assume that the policyholder is indifferent between two contracts (the same reasoning would hold for three or more contracts), meaning that these contracts provide the same utility $EU_{p_1}(c^e_{\hat{d},1})$ given equilibrium beliefs. Hence, by single crossing, the policyholder would prefer one of the two contracts given the out-of-equilibrium beliefs $\tilde{p_1}$. The utility associated to such contract (given out-of-equilibrium beliefs $\tilde{p_1}$) can be used in the constraint of Problem (A.28). Therefore, since indifference curves are strictly convex and the value of $EU_{\tilde{p_1}}(c^e_{\hat{d},1})$ is unique, it is immediate to conclude that Problem (A.28) admits a unique solution. ■

Finally, Lemma A.12 shows that contract $\tilde{c}_1^{\max}$ is the equilibrium one for $n_{\hat{s}} = 1$.

Lemma A.12 *In signal profile $n_{\hat{s}} = 1$, the equilibrium contract of insurer $\hat{s}$ is $\tilde{c}_1^{max}$ characterized in (A.28).*

Proof. The proof proceeds by contradiction, assuming that $c^e_{\hat{s},1} \neq \tilde{c}_1^{\max}$ and showing that there always exists a deviation by a $\hat{s}$ insurer consisting in the offer of a contract $\tilde{c}_1^{\max}$.

If the signal profile is $n_{\hat{s}} = 1$, following the deviation by the $\hat{s}$ insurer, the policyholder observes the deviation $\tilde{c}_1^{\max}$ and $n-1$ menus $C^e_{\hat{d}}$. She thus estimates the loss probability to be $\tilde{p}_1$, with $p_1 \leq \tilde{p}_1 < p_0$. By Lemmas A.3 and A.4, her preferred contract within the menu $C^e_{\hat{d}}$ is $c^e_{\hat{d},1}$; moreover, $\tilde{c}_1^{\max}$ is preferred to $c^e_{\hat{d},1}$ by definition (see (A.28)). Then, in signal profile $n_{\hat{s}} = 1$, the deviating $\hat{s}$ insurer sells contract $\tilde{c}_1^{\max}$. Since $\tilde{c}_1^{\max}$ solves problem (A.28), while $c^e_{\hat{s},1}$ only satisfies $c^e_{\hat{s},1} \in \Lambda_{p_1}(c^e_{\hat{d},1})$, then $\tilde{c}_1^{\max}$ is more profitable than $c^e_{\hat{s},1}$ in signal profile $n_{\hat{s}} = 1$.

For any other signal profile $n_{\hat{s}} \geq 2$, the $\hat{s}$ deviating insurer's offer (i.e. contract $\tilde{c}_1^{\max}$) is rejected, as the policyholder prefers the competitors' offers. This follows from the fact that the incentive compatibility constraints of the equilibrium menu are binding by Lemma A.8, and that the policyholder by accepting the deviation in $n_{\hat{s}} = 1$ cannot reach an higher utility level than in the proposed equilibrium. This implies that the deviating insurer obtains zero profits under all these signal profiles. Noting that in equilibrium profits are zero by Lemma A.8, for any signal profile $n_{\hat{s}} \geq 2$, the $\hat{s}$ deviating insurer obtains the same profits as in the equilibrium. ■

Before characterizing the equilibrium in signal profile $n_{\hat{s}} = 0$, it is convenient to show that equilibrium beliefs must be fully optimistic, which is done in Lemma A.13 (incidentally, this already establishes part 6 of the proposition).

Lemma A.13 *Equilibrium beliefs must be fully optimistic.*

Proof. Suppose that $EU_{\tilde{p}_1}(\overline{c}^e_{\hat{d},1}) > EU_{\tilde{p}_1}(\underline{c})$. In the solution of problem (A.28), the constraint is binding, i.e. $EU_{\tilde{p}_1}(\tilde{c}^{\max}_1) = EU_{\tilde{p}_1}(\overline{c}^e_{\hat{d},1})$.

Let $\tilde{p}_1 > p_1$. Then, $\tilde{c}^{\max}_1 \notin \Lambda_{p_1}(\overline{c}^e_{\hat{d},1})$, so that in equilibrium the policyholder prefers $\overline{c}^e_{\hat{d},1}$ to $c^e_{\hat{s},1}$, thus contradicting Lemma A.10, according to which $\hat{s}$'s offer is always accepted in $n_{\hat{s}} = 1$.

If $EU_{\tilde{p}_1}(\overline{c}^e_{\hat{d},1}) \leq EU_{\tilde{p}_1}(\underline{c})$, then in equilibrium the participation constraint would not be satisfied, again contradicting Lemma A.10. ■

Note that Lemma A.13 implies that the contract $\tilde{c}^{\max}_1$ characterized by (A.28) is $c^{\max}_1$, which is a full insurance contract.

The next two lemmas characterize the equilibrium in signal profile $n_{\hat{s}} = 0$, establishing part 5 of the proposition.

Lemma A.14 *The profits of insurer $\hat{d}$ in signal profile $n_{\hat{s}} = 0$ are non-negative.*

Proof. The outcome in signal profiles $n_{\hat{s}} \geq 2$ is actuarially fair by Lemma A.8. Moreover, in signal profile $n_{\hat{s}} = 1$, insurer $\hat{d}$ offer is not accepted by Lemma A.10 so that his profits are zero. Then, if in $n_{\hat{s}} = 0$ $\hat{d}$ makes a loss, $\hat{d}$'s participation constraint is not met. ■

Lemma A.15 *Condition (10) holds in equilibrium.*

Proof. Since an insurer $\hat{s}$ could pretend to be $\hat{d}$ by offering the equilibrium menu of the $\hat{d}$ type, it is necessary to guarantee that $\hat{s}$ insurers truthfully reveal their private signal. In signal profile $n_{\hat{s}} = 1$, the $\hat{s}$ insurer in equilibrium would obtain the profit $E\pi_{\hat{s}}(c^e_{\hat{s},1})$. Conversely, if he deviates by offering $C^e_{\hat{d}}$, the policyholder observes that the menu $C^e_{\hat{d}}$ is offered by all firms, and thus believes that the signal profile is $n_{\hat{s}} = 0$, hence choosing contract $c^e_{\hat{d},0}$. In this case, the $\hat{s}$ insurer obtains profits $E\pi_{\hat{s}}(c^e_{\hat{d},0})$ with probability $1/n$. Then, a necessary equilibrium condition for the $\hat{s}$ insurer to truthfully reveal his signal is (10), where the right hand side is strictly positive by construction. Note that in signal profiles $n_{\hat{s}} \geq 2$, condition (10) remains a necessary one whenever the contract offered by the deviating insurer is accepted with positive probability for some $n_{\hat{s}} > 2$, hence entailing positive profits in deviation. It is instead both necessary and sufficient whenever the offer of the deviating insurer is not accepted. ■

Having characterized the equilibrium also for the signal profile $n_{\hat{s}} = 0$, we can finally complete the proof of the first part of the proposition (that so far has been proved just for $n_{\hat{s}} \geq 2$).

Lemma A.16 *The equilibrium outcome Γ_o^e is fully separating for all $n_{\hat{s}}$.*

Proof. From Lemmas A.6 and A.7, the set of outcomes is fully separating for all signal profiles $n_{\hat{s}} \geq 2$. Moreover, it follows immediately from condition (10) that $c^e_{\hat{d},0} \neq c^e_{\hat{s},1}$ – i.e. the outcome in $n_{\hat{s}} = 0$ is different from the outcome in $n_{\hat{s}} = 1$. Finally, as the equilibrium outcome in $n_{\hat{s}} = 2$ is actuarially fair (see Lemma A.8), it must be different from contract $c^e_{\hat{s},1}$ – i.e. the outcome in $n_{\hat{s}} = 1$ is different from the outcome in $n_{\hat{s}} = 2$ – which completes the proof. ■

This completes the proof of the proposition. Q.E.D.

Proof of Corollary 2

Proving the first statement in the corollary requires to prove that there exists a profitable deviation contract menu when n is large enough. In order to characterize the deviation menu, we preliminarily need to determine all relevant loss probabilities. Note first that the loss probability in the signal profile $n_{\hat{s}}$ is

$$p_{n_{\hat{s}}} = \frac{p_s P_s (1-\alpha)^{n-n_{\hat{s}}} \alpha^{n_{\hat{s}}} + p_d (1-P_s)(1-\alpha)^{n_{\hat{s}}} \alpha^{n-n_{\hat{s}}}}{P_s (1-\alpha)^{n-n_{\hat{s}}} \alpha^{n_{\hat{s}}} + (1-P_s)(1-\alpha)^{n_{\hat{s}}} \alpha^{n-n_{\hat{s}}}},$$

which by dividing both numerator and denominator by α^n, can be immediately rewritten as

$$p_{n_{\hat{s}}} = \frac{p_s P_s + p_d (1-P_s) \left(\frac{1-\alpha}{\alpha}\right)^{2n_{\hat{s}}-n}}{P_s + (1-P_s) \left(\frac{1-\alpha}{\alpha}\right)^{2n_{\hat{s}}-n}}.$$

The expression for $p_{n_{\hat{s}}}$ can then be used to write

$$\begin{aligned} p_n &= \frac{p_s P_s + p_d(1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^n}{P_s + (1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^n}, \\ p_{\frac{n}{2}} &= p_s P_s + p_d(1-P_s), \\ p_{\frac{n}{2}-1} &= \frac{p_s P_s \left(\frac{1-\alpha}{\alpha}\right)^2 + p_d(1-P_s)}{P_s\left(\frac{1-\alpha}{\alpha}\right)^2 + (1-P_s)}. \end{aligned}$$

Note that $p_{\frac{n}{2}}$ and $p_{\frac{n}{2}-1}$ do not depend on n. Moreover, in the limit for $n \to \infty$, we have that $\lim_{n\to\infty} p_n = p_s$.

The probability that signal profile $n_{\hat{s}}$ occurs is given by

$$Pr(n_{\hat{s}}) = P_s \alpha^{n_{\hat{s}}}(1-\alpha)^{n-n_{\hat{s}}} + (1-P_s)(1-\alpha)^{n_{\hat{s}}}\alpha^{n-n_{\hat{s}}}$$

that can be rewritten as

$$Pr(n_{\hat{s}}) = \alpha^n \left[P_s \left(\frac{1-\alpha}{\alpha}\right)^{n-n_{\hat{s}}} + (1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^{n_{\hat{s}}} \right].$$

Hence, the probability of the signal profile $n_{\hat{s}}$ conditional on $n_{\hat{s}} \geq n/2$ is

$$Pr(n_{\hat{s}}|n_{\hat{s}} \geq n/2) = \frac{\alpha^n \left[P_s\left(\frac{1-\alpha}{\alpha}\right)^{n-n_{\hat{s}}} + (1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^{n_{\hat{s}}}\right]}{\alpha^n \sum_{t=\frac{n}{2}}^{n}\left[P_s\left(\frac{1-\alpha}{\alpha}\right)^{n-t} + (1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^{t}\right]}.$$

Simplifying for α^n and collecting terms, we obtain

$$Pr(n_{\hat{s}}|n_{\hat{s}} \geq n/2) = \frac{P_s\left(\frac{1-\alpha}{\alpha}\right)^{n-n_{\hat{s}}} + (1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^{n_{\hat{s}}}}{\left[P_s + (1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^{\frac{n}{2}}\right]\frac{1-\left(\frac{1-\alpha}{\alpha}\right)^{\frac{n}{2}+1}}{1-\frac{1-\alpha}{\alpha}}}.$$

Note also that the expected loss probability $\overline{\overline{p}} = \sum_{n_{\hat{s}}=\frac{n}{2}}^{n} p_{n_{\hat{s}}} Pr\left(n_{\hat{s}}|n_{\hat{s}} \geq \frac{n}{2}\right)$ conditional on $n_{\hat{s}} \geq n/2$ reads

$$\overline{\overline{p}} = \frac{p_s P_s + p_d(1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^{\frac{n}{2}}}{P_s + (1-P_s)\left(\frac{1-\alpha}{\alpha}\right)^{\frac{n}{2}}} \tag{A.29}$$

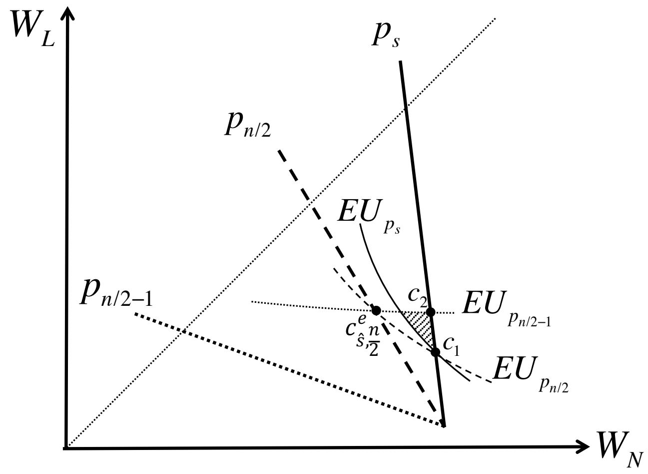

Figure A.2: Deviations

Taking the limit of A.29 for $n \to \infty$ and noting that $1 - \alpha < \alpha$, we have $\lim_{n\to\infty} \overline{\overline{p}} = p_s$.

Having defined all relevant loss probabilities, we can now turn to the characterization of the contracts entering the deviation menu. To do so, it is convenient to define three contracts c_1, c_2, and c_3 as follows. Contract c_1 is such that i) $EU_{p_{\frac{n}{2}}}(c_1) = EU_{p_{\frac{n}{2}}}(c^e_{\hat{s},\frac{n}{2}})$ (c_1 provides the same expected utility as contract $c^e_{\hat{s},\frac{n}{2}}$ given the loss probability $p_{\frac{n}{2}}$), and ii) c_1 is actuarially fair given the loss probability p_s. Contract c_2 is such that i) $EU_{p_{\frac{n}{2}-1}}(c_2) = EU_{p_{\frac{n}{2}-1}}(c^e_{\hat{s},\frac{n}{2}})$ (c_2 provides the same expected utility to the policyholder as contract $c^e_{\hat{s},\frac{n}{2}}$ given the loss probability $p_{\frac{n}{2}-1}$), and ii) it is actuarially fair when the loss probability is p_s. Finally, contract c_3 is such that i) $EU_{p_{\frac{n}{2}-1}}(c_3) = EU_{p_{\frac{n}{2}-1}}(c^e_{\hat{s},\frac{n}{2}})$ (c_3 provides the same expected utility to the policyholder as contract $c^e_{\hat{s},\frac{n}{2}}$ when the loss probability is $p_{\frac{n}{2}-1}$), and ii) $E\pi_{\overline{\overline{p}}}(c_3) = \epsilon > 0$ Note that c_3 allows for arbitrarily small – albeit strictly positive – profits given that the loss probability is $\overline{\overline{p}}$. Note also that c_3 converges to c_2 for $n \to \infty$ since $\lim_{n\to\infty}\overline{\overline{p}} = p_s$. Observe finally that – in defining c_1, c_2, and c_3 – we assume without loss of generality that n is an even number. Contract $c^e_{\hat{s},\frac{n}{2}}$ is then the equilibrium contract offered by the $\hat{s}$ insurer and chosen by the policyholder in the signal profile $n_{\hat{s}} = \frac{n}{2}$. Proposition 7 guarantees that $c^e_{\hat{s},\frac{n}{2}}$ is actuarially fair in signal profile $n_{\hat{s}} = \frac{n}{2}$ and that it entails underinsurance.

Using the definitions of c_1, c_2, and c_3, consider a deviation from the equilibrium by the $\hat{s}$ insurer, in which he offers a menu C^{dev} constituted by the equilibrium contracts $c^e_{\hat{s},n_{\hat{s}}}$ for all signal profiles $n_{\hat{s}} < n/2$, plus the contract c_3, i.e.

$$C^{dev} = \left\{ \{c^e_{\hat{s},n_{\hat{s}}}\}_{n_{\hat{s}} < \frac{n}{2}}, c_3 \right\}.$$

The following Lemma shows that the deviation menu C^{dev} is always profitable and accepted by the policyholder.

Lemma A.17 C^{dev} *is a profitable deviation, which is always accepted by the policyholder.*

Proof. We first show that for C^{dev} to be a profitable deviation if accepted it must be that

1. the contract $c^e_{\hat{s},n_{\hat{s}}} \in C^{dev}$ is chosen when the realized signal profile is $n_{\hat{s}}$, for any $n_{\hat{s}} < n/2$;

2. the contract c_3 is chosen when the realized signal profile is $n_{\hat{s}}$, for any $n/2 \leq n_{\hat{s}} \leq n$.

Recall preliminarily that a separating equilibrium is fully revealing and that the policyholder is fully optimistic, implying that after a deviation of an $\hat{s}$ insurer the policyholder still believes that the signal profile is $n_{\hat{s}}$ (see Proposition 7).

C^{dev} is a profitable deviation as the insurer $\hat{s}$ offering C^{dev} obtains the equilibrium profits in all signal profiles $n_{\hat{s}} < n/2$, and profits $E\pi_{\overline{\overline{p}}}(c_3) > 0$ if $n_{\hat{s}} \geq n/2$ by construction, which is higher than the equilibrium profits. Indeed, the equilibrium contracts $c^e_{n_{\hat{s}}}$ are actuarially fair for $n_{\hat{s}} \geq n/2$ if $n \geq 3$; the only profitable signal profile for $\hat{s}$ in equilibrium is $n_{\hat{s}} = 1$.

We now need to show that conditions 1) and 2) always hold. Focus first on Condition 1) and consider the signal profiles $n_{\hat{s}} < n/2$.

Since $EU_{p_{\frac{n}{2}-1}}(c_3) = EU_{p_{\frac{n}{2}-1}}(c^e_{\hat{s},\frac{n}{2}})$ by definition of c_3, and $EU_{p_{\frac{n}{2}-1}}\left(c^e_{\hat{s},\frac{n}{2}-1}\right) \geq EU_{p_{\frac{n}{2}-1}}(c^e_{\hat{s},\frac{n}{2}})$ by the incentive compatibility of the equilibrium menu, then using the single crossing property it follows immediately that $EU_{p_{\frac{n}{2}-1}}\left(c^e_{\hat{s},\frac{n}{2}-1}\right) \geq EU_{p_{\frac{n}{2}-1}}(c_3)$. Hence, c_3 is not chosen when $n_{\hat{s}} = \frac{n}{2} - 1$ if C^{dev} is offered. Moreover, by the incentive compatibility of the equilibrium menu and by Lemma A.3, the policyholder prefers contract $c^e_{n_{\hat{s}},\hat{s}}$ in any signal profile $n_{\hat{s}} < n/2$. Hence, the equilibrium contracts $c^e_{n_{\hat{s}}}$ are chosen for any $n_{\hat{s}} < n/2$, and the insurer achieves the same profits as in the equilibrium for all these signal profiles.

We now turn to Condition 2). Focus on the signal profiles $n/2 \leq n_{\hat{s}} \leq n$ and denote with $\Lambda_p(c_1)$ the set of contracts c that are strictly preferred to contract c_1 for a given loss probability p, and that are profitable given the loss probability p_s, i.e.

$$\Lambda_p(c_1) = \{c : EU_p(c) > EU_p(c_1), E\pi_{p_s}(c) \geq 0\}.$$

We have that

$$c_2 \in \Lambda_{p_s}(c_1) \subset \Lambda_p(c_1)$$

for all $p > p_s$.

Since $p_{\frac{n}{2}} > p_s$, then it is $c_2 \in \Lambda_{p_{\frac{n}{2}}}(c_1)$.

Consider the equilibrium menu. The following conditions must hold

$$EU_{p_{\frac{n}{2}}}(c_1) = EU_{p_{\frac{n}{2}}}\left(c^e_{\hat{s},\frac{n}{2}}\right) \geq EU_{p_{\frac{n}{2}}}(c^e_{\hat{s},n_{\hat{s}}}) \text{ for any } n_{\hat{s}} \geq n/2,$$

where the first equality comes from the definition of c_1 and the second inequality comes from the incentive compatibility of the equilibrium menu.

This implies that $c^e_{\hat{s},n_{\hat{s}}} \notin \Lambda_{p_{\frac{n}{2}}}(c_1)$ for all $n_{\hat{s}} \geq n/2$. Since $\Lambda_p(c_1) \subset \Lambda_{p'}(c_1)$ for all $p < p'$, then $\Lambda_{p_{n_{\hat{s}}}}(c_1) \subset \Lambda_{p_{\frac{n}{2}}}(c_1)$ for all $n_{\hat{s}} > n/2$. Hence, $c^e_{\hat{s},n_{\hat{s}}} \notin \Lambda_{p_{n_{\hat{s}}}}(c_1)$ for all $n_{\hat{s}} \geq n/2$. By transitivity, the contract c_2 is preferred to $c^e_{\hat{s},n_{\hat{s}}}$ for any $n_{\hat{s}} \geq n/2$. When $n \to \infty$, the contract c_3 converges to c_2, implying that the contract c_3 is preferred to $c^e_{\hat{s},n_{\hat{s}}}$ for any $n_{\hat{s}} \geq n/2$. Hence, C^{dev} is accepted and c_3 is chosen in any signal profile $n_{\hat{s}} \geq n/2$. ∎

Lemma A.17 implies that there exists an upper bound $\bar{\bar{n}}$ to the number of firms that is consistent with the existence of informative equilibria.

To show that $\bar{\bar{n}}$ is decreasing in the expected profits corresponding to the equilibrium contract, it is enough to show that as n increases only those menus associated to lower profits are consistent with an informative equilibrium. To see this, note first that the characterization of informative equilibria leaves degrees of freedom in the contracts offered by the $\hat{d}$ insurers in signal profiles 0 and 1. As a consequence, the position of the contract $c^e_{\hat{s},1} = (W^e_{\hat{s},1}, W^e_{\hat{s},1})$ may vary on the 45 degree line, thus originating multiple equilibria. Note that the wealth in the case of loss, $W_L(.)$, for all possible contracts in the equilibrium menu is a function of $W^e_{\hat{s},1}$. Observe that the equilibrium menu is identified by a set of binding incentive compatibility constraints for all adjacent states (see Lemmas A.3 and A.8 in the proof of Proposition 7), i.e.

$$\begin{aligned}
&EU_{p_1}(c^e_{\hat{s},1}) = EU_{p_1}(c^e_{\hat{s},2})\\
&EU_{p_2}(c^e_{\hat{s},2}) = EU_{p_2}(c^e_{\hat{s},3})\\
&...\\
&EU_{p_{n-2}}(c^e_{\hat{s},n-2}) = EU_{p_{n-2}}(c^e_{\hat{s},n-1})\\
&EU_{p_{n-1}}(c^e_{\hat{s},n-1}) = EU_{p_{n-1}}(c^e_{\hat{s},n})
\end{aligned}$$

Therefore, equilibria with higher $EU_{p_1}(c^e_{\hat{s},1})$ also exhibit higher $EU_{p_{n-1}}(c^e_{\hat{s},n})$, which implies that $\frac{dW_L(c^e_{\hat{s},n})}{dW^e_{\hat{s},1}} > 0$. This in turn entails that a higher $W_L(c^e_{\hat{s},n})$ shifts up the equilibrium menu, which follows directly from the fact that incentive compatibility must hold with equality for all adjacent states.

Given that the expected equilibrium profit for the $\hat{s}$ insurer is

$$E\pi^e_{\hat{s}} = Pr(1|\hat{s})(W - p_1L - W^e_{\hat{s},1}),$$

we obtain that

$$\frac{dE\pi^e_{\hat{s}}}{dW_L(c^e_{\hat{s},n})} = -Pr(1|\hat{s})\frac{dW^e_{\hat{s},1}}{dW_L(c^e_{\hat{s},n})} < 0. \tag{A.30}$$

Consider a deviation in which the $\hat{s}$ insurer gives up some or all profits in signal profile $n_{\hat{s}} = 1$, but he makes a profit in signal profile $n_{\hat{s}} = n$. In order to minimize losses with respect to the equilibrium contract in signal profile $n_{\hat{s}} = 1$, the insurer offers a full insurance contract such that wealth both in the case of loss and in that of no loss is $W^{dev}_{\hat{s},1}$. All deviation contracts offered in signal profiles from $n_{\hat{s}} = 2$ to $n_{\hat{s}} = n-1$ satisfy incentive compatibility with equality, and are actuarially fair. The deviation contract offered in signal profile $n_{\hat{s}} = n$ – i.e. $(W_L(c^{dev}_{\hat{s},n}), W_N(c^{dev}_{\hat{s},n}))$ – meets the incentive compatibility constraint with equality.

The expected profits at the deviation are given by

$$E\pi_{\hat{s}}^{dev} = Pr(1|\hat{s})(W - p_1 L - W_{\hat{s},1}^{dev}) + Pr(n|\hat{s})\left(p_n(W - L - W_L(c_{\hat{s},n}^{dev})) + (1-p_n)(W - W_N(c_{\hat{s},n}^{dev}))\right)$$

Therefore,

$$\frac{dE\pi_{\hat{s}}^{dev}}{dW_{\hat{s},1}^{dev}} = -Pr(1|\hat{s}) - Pr(n|\hat{s})\left(p_n \frac{dW_L(c_{\hat{s},n}^{dev})}{dW_{\hat{s},1}^{dev}} + (1-p_n)\frac{dW_N(c_{\hat{s},n}^{dev})}{dW_{\hat{s},1}^{dev}}\right),$$

i.e.

$$\frac{dE\pi_{\hat{s}}^{dev}}{dW_{\hat{s},1}^{dev}} = -Pr(1|\hat{s}) - Pr(n|\hat{s})\frac{dW_L(c_{\hat{s},n}^{dev})}{dW_{\hat{s},1}^{dev}}\left(p_n + (1-p_n)\frac{dW_N(c_{\hat{s},n}^{dev})}{dW_L(c_{\hat{s},n}^{dev})}\right). \tag{A.31}$$

In the most profitable deviation menu $C_{\hat{s}}^{dev}$ offered by $\hat{s}$, the contract offered in signal profile $n_{\hat{s}} = n$ – $c_{\hat{s},n}^{dev} \in C_{\hat{s}}^{dev}$ – lies on the indifference curve passing through the equilibrium contract $c_{\hat{s},n}^{e}$, i.e.

$$EU_{p_n}(c_{\hat{s},n}^{e}) = p_n U(W_L(c_{\hat{s},n}^{dev})) + (1-p_n)\, U(W_N(c_{\hat{s},n}^{dev})),$$

from which

$$W_N(c_{\hat{s},n}^{dev}) = U^{-1}\left(\frac{EU_{p_n}(c_{\hat{s},n}^{e})}{1-p_n} - \frac{p_n}{1-p_n}U(W_L(c_{\hat{s},n}^{dev}))\right). \tag{A.32}$$

By exploiting (A.32), we obtain

$$\frac{dW_N(c_{\hat{s},n}^{dev})}{dW_L(c_{\hat{s},n}^{dev})} = -\frac{p_n}{1-p_n}\frac{U'(W_L(c_{\hat{s},n}^{dev}))}{U'(W_N(c_{\hat{s},n}^{dev}))},$$

which, substituted into (A.31), gives

$$\frac{dE\pi_{\hat{s}}^{dev}}{dW_{\hat{s},1}^{dev}} = -Pr(1|\hat{s}) - Pr(n|\hat{s})\frac{dW_L(c_{\hat{s},n}^{dev})}{dW_{\hat{s},1}^{dev}} p_n \left(1 - \frac{U'(W_L(c_{\hat{s},n}^{dev}))}{U'(W_N(c_{\hat{s},n}^{dev}))}\right).$$

The previous expression, evaluated at the equilibrium contract, reads

$$\frac{dE\pi_{\hat{s}}^{dev}}{dW_{\hat{s},1}^{dev}} = -Pr(1|\hat{s}) - Pr(n|\hat{s})\frac{dW_L(c_{\hat{s},n}^{e})}{dW_{\hat{s},1}^{e}} p_n \left(1 - \frac{U'(W_L(c_{\hat{s},n}^{e}))}{U'(W_N(c_{\hat{s},n}^{e}))}\right). \tag{A.33}$$

To prevent deviations with cross-subsidies, (A.33) must be negative. Recall that $\frac{dW_L(c^e_{\hat{s},n})}{dW^e_{\hat{s},1}} > 0$, and that $1 - \frac{U'(W_L(c^e_{\hat{s},n}))}{U'(W_N(c^e_{\hat{s},n}))} < 0$ because $W_L(c^e_{\hat{s},n}) < W_N(c^e_{\hat{s},n})$. When n increases, $Pr(1|\hat{s})$ decreases and $Pr(n|\hat{s})$ increases. Then, only equilibria in which the term

$$\frac{dW_L(c^e_{\hat{s},n})}{dW^e_{\hat{s},1}} p_n \left(1 - \frac{U'(W_L(c^e_{\hat{s},n}))}{U'(W_N(c^e_{\hat{s},n}))}\right)$$

is small enough can survive. These equilibria are those in which the underinsurance in the $n_{\hat{s}} = n$ signal profile is low. In order to reduce underinsurance, the wealth in case of loss at the equilibrium $c^e_{\hat{s},n}$, i.e. $W_L(c^e_{\hat{s},n})$, must increase. Using Equation (A.30), this in turn implies that the equilibrium profits $E\pi^e_{\hat{s}}$ must become smaller, which completes the proof of the corollary. Q.E.D.

Not for publication Appendixes

Appendix B Existence and Robustness of Non-Informative Equilibria

In this Appendix we check the existence and robustness of non-informative equilibria by means of latent contracts and equilibrium refinements.

Appendix B.1 Latent contracts

We show that a non-informative equilibrium always exists by relying on the usage of latent contracts. The idea is easily illustrated by Figure B.1. Consider, as a candidate equilibrium, the situation in which both insurers' types offer the menu composed by the two contracts $\left(c^e, c^{lat}\right)$ and the policyholder's out of equilibrium beliefs are fully optimistic. When offered the menu $\left(c^e, c^{lat}\right)$, the policyholder chooses contract c^e. This occurs because, in non-informative equilibria, the policyholder's estimation of the loss probability in equilibrium corresponds to the prior $\bar{p}$. Thus, she always prefers the contract c^e to the contract c^{lat}. Therefore, contract c^{lat} is never chosen in equilibrium (and accordingly it remains latent), thus causing c^e to be the equilibrium outcome. Geometrically, this means that the contract c^{lat} lies below the indifference curve passing through c^e and it is drawn according to the *ex ante* estimation $\bar{p}$ of the loss probability.

Now assume that one insurer wants to deviate from the $\left(c^e, c^{lat}\right)$ menu. A fully optimistic policyholder who observes a deviation believes with probability one that the deviating insurer is of type $\hat{s}$, while she retains her *ex ante* beliefs about the type of the other insurer. Then, a deviation must be preferred to contract c^{lat} when the loss probability is $p_{\hat{s}}$. However, any contract with this property would entail a loss for both $\hat{d}$ and $\hat{s}$ insurers, because c^{lat} is unprofitable by construction (recall that the iso-profit lines through the no insurance contract $\underline{c}$ are those entailing zero profits for the corresponding insurer and they are therefore identifying the insurers' participation constraints). Hence a profitable deviation does not exist.

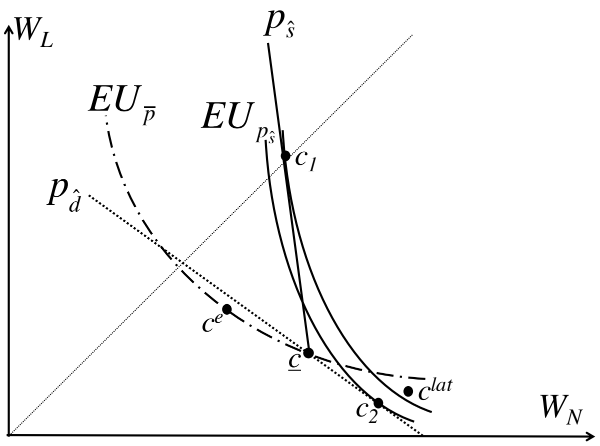

Figure B.1: Non-informative equilibrium with latent contracts

The following proposition formalizes this result by providing the conditions under which a contract c^{lat} exists.

Proposition B.1 *If* $\lim_{Y\to 0} U'(Y) = \lim_{Y\to\infty} U(Y) = +\infty$*, the contract* $c^e \in \Lambda_{\bar{p}}$ *is always an equilibrium outcome.*

To formally prove the existence of at least one contract c^{lat}, it is convenient to introduce the two contracts, c_1 and c_2, represented in Figure B.1.[43] Contract c_1 (resp. c_2) is the contract preferred by the optimistic policyholder among those that can be offered in a deviation by a $\hat{s}$ (resp. $\hat{d}$) insurer. In particular, contract c_1 is the actuarially fair full insurance contract when the loss probability is $p_{\hat{s}}$; i.e. the contract that maximizes the expected utility of the fully optimistic policyholder when she believes that the loss probability is $p_{\hat{s}}$ and the participation constraint of the $\hat{s}$ insurer holds. Contract c_2 maximizes the expected utility of the fully optimistic policyholder when she believes that the loss probability is equal to $p_{\hat{s}}$ and the participation constraint of the $\hat{d}$ insurer holds. Geometrically, c_2 is the contract lying on the tangency between the indifference curve of the policyholder estimating the loss probability to be $p_{\hat{s}}$ and the zero isoprofit line computed with probability $p_{\hat{d}}$.

We let $EU^{lat} = \max\{EU_{p_{\hat{s}}}(c_1), EU_{p_{\hat{s}}}(c_2)\}$, and we define the contract c^{lat} as follows.

Definition B.1 *Contract* c^{lat} *is such that:*

[43]With a slight abuse of notation, we denote indifference curves using the corresponding expected utility levels.

1. *it provides a lower expected utility than the autarky contract* $\underline{c}$, *i.e.*

$$EU_{\bar{p}}\left(c^{lat}\right) < EU_{\bar{p}}(\underline{c}), \tag{B.1}$$

when the loss probability is estimated to be at the ex ante *level* $\bar{p}$*;*

2. *it lies on the highest between the two indifference curves passing through* c_1 *and* c_2*, i.e.*

$$EU_{p_{\hat{s}}}(c^{lat}) = EU^{lat}, \tag{B.2}$$

under the assumption that the loss probability is $p_{\hat{s}}$.

We also define a candidate equilibrium contract c^e as a contract that meets the conditions in the following definition.

Definition B.2 *Contract* c^e *is such that:*

1. *it provides the same expected utility as the autarky contract* $\underline{c}$, *i.e.*

$$EU_{\bar{p}}\left(c^{e}\right) = EU_{\bar{p}}(\underline{c}), \tag{B.3}$$

when the loss probability is estimated to be at the ex ante *level* $\bar{p}$

2. *it lies below the zero isoprofit line of a* $\hat{d}$ *insurer, i.e.*

$$E\pi_{p_{\hat{d}}}(c^e) = 0; \tag{B.4}$$

3. *it entails a positive amount of insurance, i.e*

$$W_N(c^e) < W. \tag{B.5}$$

Conditions (B.3), (B.4) and (B.5) guarantee, respectively, that the policyholder's participation constraint (conditional on the *ex ante* level $\bar{p}$ of loss probability), the $\hat{d}$ insurer's participation constraint, and the $\hat{s}$ insurer's participation constraint hold in equilibrium. Note that at least one contract c^e always exists, namely the no-insurance contract $\underline{c}$.

We are only left to prove that c^{lat} always exists. To do so, we must show that there always exists an intersection between the indifference curve $EU_{\bar{p}}(\underline{c})$ and the highest one among $EU_{p_{\hat{s}}}(c_1)$ and $EU_{p_{\hat{s}}}(c_2)$. In fact, in this case there exists an area of contracts that satisfies simultaneously Conditions $(B.1)$ and $(B.2)$. Note that, under the assumption that $U'(0) \to \infty$, the slope of any indifference curve goes to zero when $W_L \to 0$; i.e. all indifference curves lie in the positive orthant. In other words, if $c^{lat} = (W_L^{lat}, W_N^{lat})$ exists, it must be such that $W_L^{lat}, W_N^{lat} > 0$.

By definition of contract c^{lat}, it must be that

$$
\begin{aligned}
p_{\hat{s}} U(W_L^{lat}) + (1 - p_{\hat{s}}) U(W_N^{lat}) &= EU^{lat}, \\
\bar{p} U(W_L^{lat}) + (1 - \bar{p}) U(W_N^{lat}) &\leq EU_{\bar{p}}(\underline{c}).
\end{aligned}
$$

Using the first equation, $U(W_L^{lat})$ can be written as $U(W_L^{lat}) = \frac{EU^{lat}}{p_{\hat{s}}} - \frac{1-p_{\hat{s}}}{p_{\hat{s}}} U(W_N^{lat})$. By substituting this expression into the second inequality, we obtain

$$
\bar{p}\frac{EU^{lat}}{p_{\hat{s}}} - \bar{p}\frac{1 - p_{\hat{s}}}{p_{\hat{s}}} U(W_N^{lat}) + (1 - \bar{p}) U(W_N^{lat}) \leq EU_{\bar{p}}(\underline{c}),
$$

i.e.

$$
\left(1 - \frac{\bar{p}}{p_{\hat{s}}}\right) U(W_N^{lat}) \leq EU_{\bar{p}}(\underline{c}) - \bar{p}\frac{EU^{lat}}{p_{\hat{s}}}.
$$

Given that $\bar{p} > p_{\hat{s}}$, under the assumption that $\lim_{Y \to \infty} U(Y) = +\infty$, it is always possible to find a W_N^{lat} large enough for the previous inequality to be met, which guarantees the existence of a latent contract as defined in Definition B.1 and hence proves the claim in Proposition B.1.

It is worth stressing that latent contracts are essential to establish the existence result of Proposition B.1. It is widely known that latent policies may be a commitment device for firms to prevent rivals' deviations from equilibrium. Notwithstanding, latent contracts have been criticized along at least two dimensions. First, there is no strong evidence that insurance firms rely on latent contractual schemes. Second, in many cases (such as the one illustrated in Figure B.1) they appear to be inconsistent with the sequential rationality principle, as firms would renege (if they could) the latent contract were it accepted with some (even negligible) probability. For these reasons, in the main text of the paper we restrict the set of admissible contracts to those implying non-negative profits with positive probability if accepted.

Appendix B.2 Equilibrium refinements

The analysis of Section 4 shows that there exist multiple non-informative equilibria. The multiplicity of equilibria arises from the degree of freedom allowed by the off-equilibrium-path beliefs. Here we check whether these equilibria survive when refinements eliminating equilibria supported by somehow unreasonable beliefs are introduced. A useful refinement in the case of non-informative equilibria is the Intuitive Criterion of Cho and Kreps (1987), which requires to identify what type of insurer would profit by deviating from a given equilibrium. Reasonable beliefs should assign zero probability to a type that does not gain anything from deviating.

We apply the definition of the Intuitive Criterion as follows. Consider a PBE: given a type $\hat{\theta}$, some offers of the insurer entail – whatever the subsequent beliefs of the agent –, lower profits (strictly lower for one possible belief at least) than those earned by playing its equilibrium action. A 'reasonable' belief associated with such a dominated offer must put zero mass on type $\hat{\theta}$, provided that the offer gives a higher profit than the equilibrium profits on the remaining type. If this condition is not satisfied, the equilibrium can be eliminated.

Proposition B.2 shows that some non-informative equilibria survive the proposed refinement.

Proposition B.2 *There exists a non-informative equilibrium allocation that entails full insurance and positive profits for the $\hat{d}$ type, which is robust to the Cho-Kreps criterion.*

In order to understand the logic behind Proposition B.2, it is useful to consider the full-insurance, actuarially fair contract illustrated in Figure B.2

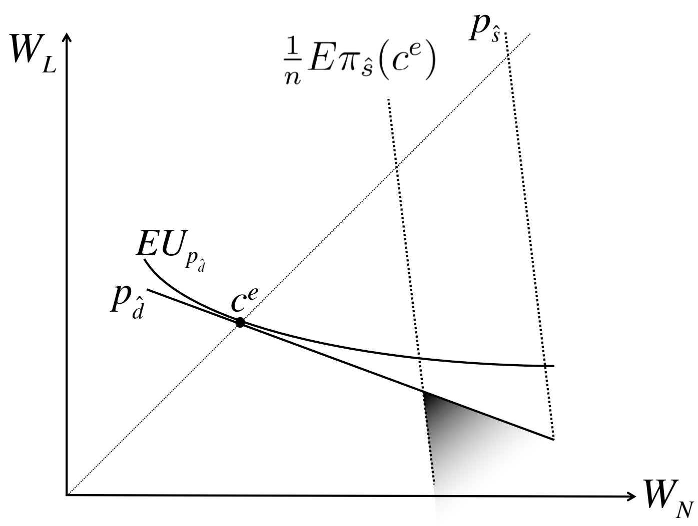

Figure B.2: Refinement of non-informative equilibria

Consider a non-informative Perfect Bayesian Equilibrium in which both insurer's types offer the contract c^e. This contract is actuarially fair for type $\hat{d}$, it entails full insurance, and off the equilibrium beliefs are fully optimistic.

To check whether this equilibrium is robust to the Intuitive Criterion, we identify the contracts that would lead the insurer $\hat{s}$ to earn – whatever the subsequent beliefs of the agent –, lower profits than those obtained by playing his equilibrium action. This set of contracts must lie above the isoprofit line $\frac{1}{n}E\pi_{\hat{s}}(c^e)$ in Figure B.2. A "reasonable" belief associated to this dominated offer must put zero mass on type $\hat{s}$, provided that the offer gives a higher profit than the equilibrium one for type $\hat{d}$. Then, one needs to identify the contracts lying above $\frac{1}{n}E\pi_{\hat{s}}(c^e)$ that yield higher profits than the equilibrium ones to type $\hat{d}$. Given that the $\hat{d}$ insurer makes zero profits in equilibrium, the contracts yielding him positive profits must lie below his zero isoprofit line $E\pi_{\hat{d}}(c) = 0$.

The shaded area in Figure B.2 illustrates all contracts for which an optimistic belief is *not* 'reasonable' according to the Cho-Kreps Intuitive Criterion. Since these contracts are not acceptable by a policyholder holding beliefs $\tilde{p} = p_{\hat{d}}$, the condition is met and the equilibrium is robust to the Intuitive Criterion.

Importantly, non-informative equilibria entailing positive profits can survive the refinement, provided profits are not too large.

Appendix C Numerical Analyses

This appendix provides a numerical analysis aimed at checking that the equilibria characterized in the main text are not of theoretical interest only, but they rather exist for empirically plausible and sufficiently broad parameter constellations, being in this perspective a robust feature of actual insurance markets. All the analysis is carried out for the same CARA specification (4) of the policyholder's utility that we consider in the main text.

Non-informative Equilibria

We first characterize the existence of non-informative equilibria and its robustness to relevant perturbations of our baseline parameter set, focusing on full insurance (efficient) equilibrium contracts such that the policyholder's participation constraint is binding and the her beliefs are fully optimistic.

Figures C.1 and C.2 illustrate how the regions of existence of non-informative full insurance equilibria change as a function of the loss probability in environment s (horizontal axis) and of the precision of the signal α (vertical axis).[44] The three panels in Figures C.1 and C.2 investigate how the region of existence of efficient non-informative equilibria is affected by increasing levels of the probability P_s of the true state of the world being s and of risk aversion β, respectively.[45] Consistently with Proposition 2, an efficient equilibrium

[44]Consistently with the logic of Proposition 2, our baseline parameter set for Figures C.1 and C.2 fixes the values of loss L, initial wealth W and loss probability p_d in the d environment, letting instead P_s, β, α, and p_s vary. In both figures, we let $W = 1000$ and $L = 300$ − corresponding to a loss of 30% of the original wealth − and $p_d = 0.2$. We are well aware that the relevant values of the loss probabilities and of the size of the loss depend largely on the specific insurance markets that are considered. To get some empirical background for our choices, note that Barseghyan *et al.* (2013) find the claim probabilities (a proxy for p_s and p_d in our setup) for home insurance to range from 0.024 (1st percentile) to 0.233 (99th percentile), while those for auto collision to be between 0.026 and 0.139. As for average losses, heterogeneity across lines of insurance is even larger. For example, the Insurance Information Institute (2018) reports that the average claim severity for US home insurance over the period 2013-2017 ranges between 68,000$ for fire and lighting damages to 368$ for credit card losses. Finally, Cohen and Einav (2005) provide a number of different estimates for the risk aversion parameter ranging from values of the order of 10^{-6} to values of the order of 10^{-2}. We put ourselves in the worst possible situation to build our case by setting $\beta = 0.01$. Recall in fact that according to Proposition 2 the existence of a non-informative equilibrium is favored by lower risk aversion. It is important to note that, although our parameter set is far from being properly calibrated, it is broadly consistent with the reported available empirical evidence.

[45]Figure C.1 assumes that $\beta = 0.01$ and considers three possible values of the probability P_s, and namely: $P_s = 0.05$ in panel (a), $P_s = 0.10$ in panel (b), and $P_s = 0.15$ in panel (c). Figure C.2 assumes instead that $P_s = 0.10$ and focuses on alternative values of the degree of risk aversion, and namely: $\beta = 0.01$ in panel

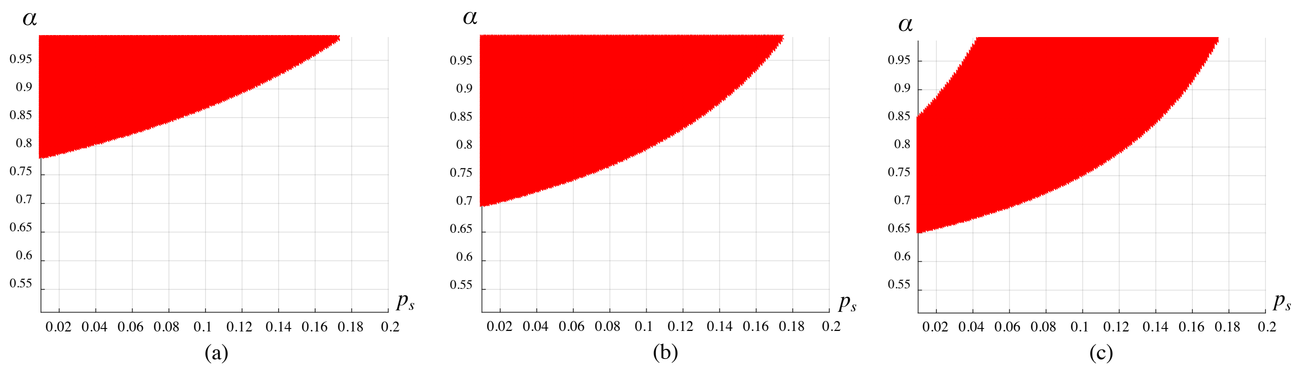

Figure C.1: Existence of non-informative equilibria in the plane α-p_s for increasing levels of P_s: $P_s = 0.05$ in panel (a); $P_s = 0.10$ in panel (b); $P_s = 0.15$ in panel (c)

is more likely to exist when the precision of the signal is high and the loss probability in the environment s is low. A lower probability that the true environment is s increases the likelihood of an equilibrium for high α and low p_s. When P_s becomes too high (such as in panel c of Figure C.1), non-informative equilibria fail to exist, owing to the fact that the participation constraints of the policyholder and of the insurer $\hat{d}$ cannot hold simultaneously. A lower level of P_s eliminates the non existence area in the upper left corner, but at the same time it enlarges the non existence area at the bottom. A lower level of risk aversion is needed to compensate the reduction of the area of existence due to a lower level of P_s. This can be seen in the three panels of Figure C.2, where a lower level of risk aversion is associated to a larger area of equilibrium existence, again consistently with Proposition 2. Indeed, the non existence area at the bottom of both figures is characterized by the presence of profitable deviations that would be accepted by an optimistic policyholder. A lower level of risk aversion helps sustaining the equilibrium by decreasing the profitability of acceptable deviations.

(a), $\beta = 0.05$ in panel (b), and $\beta = 0.10$ in panel (c).

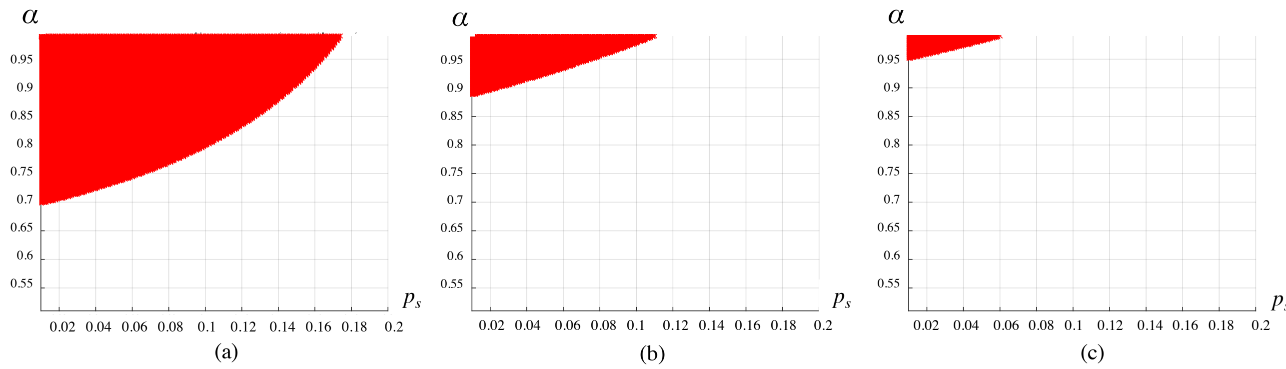

Figure C.2: Existence of non-informative equilibria in the plane α-p_s for increasing levels of risk aversion: $\beta = 0.01$ in panel (a); $\beta = 0.05$ in panel (b); $\beta = 0.10$ in panel (c)

Non-informative Pooling Equilibria in a Two-sided Asymmetric Information Framework

We focus on a setup in which our two-sided asymmetric information framework is as close as possible to that in Rothschild and Stiglitz (1976). Namely, we assume that the probability of the s environment, as well as the loss probabilities p_{sl} and p_{sh} in the s environment, converge to zero.[46] The main goal of our numerical exercise is to characterize and compare the parameter regions where our model admits non-informative pooling equilibria and Rothschild and Stiglitz's (1976) model admits separating equilibria.

Figure C.3 shows the results of such comparison, reporting on the horizontal axis the ratio p_{dl}/p_{dh} of the loss probabilities for the two types of the policyholder in the dangerous state of the world,[47] and on the vertical axis the ex-ante probability P_l that the policyholder is low risk. The red area in the figure represents the region of existence of an efficient non-informative pooling equilibrium for $\alpha = 0.9999$ and $P_s = 0.0001$, while the blue area represents the region of existence of a (separating) equilibrium in the Rothschild and Stiglitz's (1976) framework (i.e., for $\alpha = 1$ and $P_s = 0$).

Figure C.3 suggests that a non-informative pooling equilibrium requires the loss probabilities in the two states dh and dl to be sufficiently close to each other and the probability of

[46] The full parameter constellation used in this numerical exercise is: $\alpha = 0.9999$, $P_s = 0.0001$, $\beta = 0.001$, $p_{sh} = p_{sl} = 0$, $p_{dh} = 0.05$, $W = 1000$, $L = 200$.

[47] Given the assumptions on the loss probabilities in state s, the two policyholder's types coincide when $p_{dl}/p_{dh} = 1$.

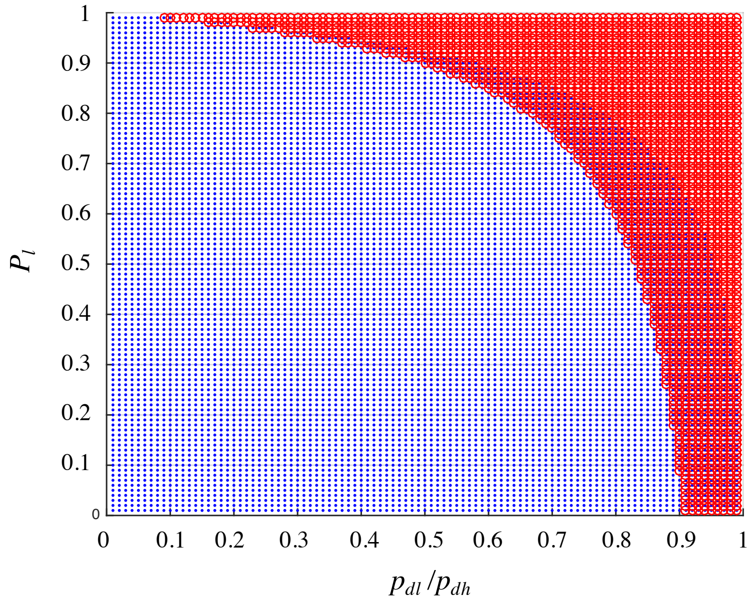

Figure C.3: Existence of non-informative pooling equilibria (in red) and Rothschild-Stiglitz equilibria (in blue) in the plane P_l- p_{dl}/p_{dh}

the policyholder's being of the low risk type to be sufficiently high. Conversely, a separating equilibrium *à la* Rothschild and Stiglitz (1976) seems to exist only if the probability of the l policyholder is sufficiently low and the wedge between the loss probabilities in the two states is sufficiently large. Indeed, for low levels of P_l, the participation constraint of the l policyholder is not consistent with that of the $\hat{d}$ insurer, so that non-informative pooling equilibria cannot emerge. More important, Figure C.3 suggests that non-informative pooling equilibria are possible both when equilibria *à la* Rothschild and Stiglitz (1976) exist and when they do not exist because of the presence of profitable pooling deviations. This indicates that a small perturbation of the Rothschild and Stiglitz's (1976) framework (due to the presence of even a minimal information advantage for insurers about a relevant characteristic of the environment) may result in the existence of non-informative pooling equilibria in insurance markets.

Informative Equilibria and Comparison with Non-informative Equilibria

We now focus again on our baseline one-sided asymmetric information model in order to characterize the existence and robustness of informative equilibria for increasing levels of damage. Proposition 5 establishes that a sufficiently large level of damage L is necessary for ensuring the existence of informative equilibria. Here we show how perturbing the level of damage affects the existence regions of both informative and non-informative equilibria[48].

The green area in Figure C.4 illustrates the changes in the region of existence of informative equilibria (focusing, for given parameter values, on the most efficient equilibrium; i.e. the one in which the $\hat{d}$ insurer offers the full insurance contract under signal profile 0) as a function of the precision of the signal α (on the vertical axis) and of the loss probability in the safe state of the world p_s (on the horizontal axis). The three panels in the figure assess the effects of increasing levels of damage L on equilibrium existence. It is immediate to see from the green area in Figure C.4 that when the loss probability p_s in the safe state is low, for an informative equilibrium to exist it must be that the loss L is sufficiently large. This is indeed what Proposition 5 requires for truthful revelation to hold. Conversely, for higher levels of loss, an informative equilibrium exists only for intermediate levels of signal precision. In fact, when the signal is too precise or too inaccurate, an $\hat{s}$ insurer can profitably deviate (recall that a profitable deviation entails cross subsidization)[49]. When the signal is very precise, a profitable deviation is possible because an $\hat{s}$ insurer believes the signal profile $n_{\hat{s}} = 2$ to be very likely. Indeed, given that the profitability of deviations entailing cross-subsidization originates from the contract offered when $n_{\hat{s}} = 2$, a higher probability of selling this contract makes the deviation profitable. Recall that the loss probability when $n_{\hat{s}} = 2$ (i.e. p_2) belongs to the interval $(p_s, \bar{p})$. Hence, when the signal is very imprecise, the loss probability approximates $\bar{p}$. Given that the policyholder is risk averse, a higher estimated loss probability p_2 implies a larger willingness to buy insurance, which in turn implies higher

[48] The loss probabilities we consider in our numerical exercise are $p_d = 0.05$, $P_s = 0.10$, while the degree of absolute risk aversion is $\beta = 0.01$. The level of loss is $L = 100$ in panel (a) of Figure C.4, $L = 200$ in panel (b) and $L = 500$ in panel (c), whereas the initial level of wealth is $W = 1000$.

[49] Recall also that a profitable deviation with cross subsidies necessarily entails profits larger (lower) than the equilibrium ones if $n_{\hat{s}} = 2$ ($n_{\hat{s}} = 1$).

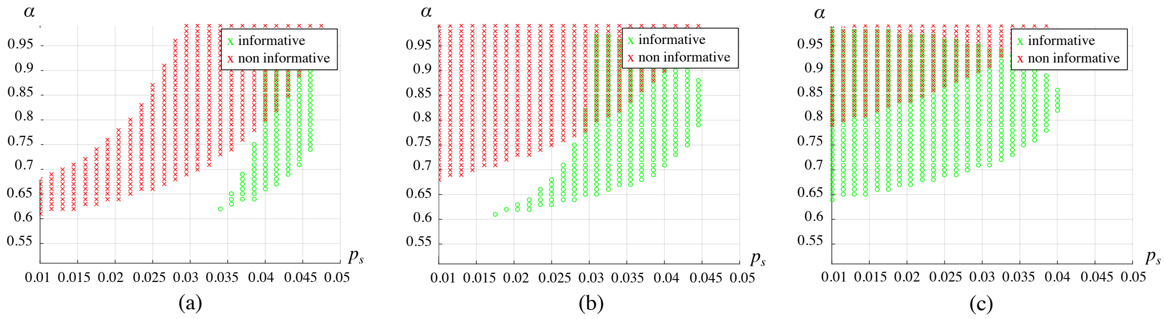

Figure C.4: Range of existence of informative and non-informative equilibria in the plane α-p_s for increasing levels of L: $L = 100$ in panel (a); $L = 200$ in panel (b); $L = 500$ in panel (c)

profits in the deviation contract offered when $n_{\hat{s}} = 2$. This explains why a deviation is more likely to occur when the precision of the signal is low.

By considering at the same time the numerical exercises in this section and those for non-informative equilibria, it is natural to ask whether non-informative and informative equilibria can co-exist for the same parameter constellations. Figure C.4 shows that this is indeed the case for a fairly broad set of parameter values. Given that in equilibrium insurers can end up offering either types of contracts, an issue of equilibrium selection arises. While tackling this problem in details goes behind the scope of the paper, we note that in our numerical exercises non-informative equilibria systematically entail higher *ex ante* profits than informative equilibria.[50]

Market Concentration

We characterize numerically the upper bound on the number of firms in the industry that is consistent with the existence of both informative and non-informative equilibria. As far as non-informative equilibria are concerned, our numerical analysis focuses only on full insurance equilibria based on fully optimistic beliefs. As for informative equilibria, we identify

[50]We conjecture that this numerical result indicates a more fundamental property in the comparison between the two types of equilibria. Indeed, informative equilibria are much more demanding than non-informative ones in terms of the necessary constraints that need to hold. Recall that the non-informative equilibria can be *ex ante* efficient (and those in our numerical simulations are), while the informative ones are necessarily of second best. This may easily translate in a lower profitability of the resulting outcomes in equilibrium. The numerical results supporting the statement in the text are available on request.

the parameters constellations guaranteeing that all the conditions stated in Proposition 7 are met, together with the absence of profitable cross-subsidy deviations for $\hat{s}$ insurers.[51] There exists a large number of different cross-subsidy deviations, involving different combinations of signal profiles. We focus exclusively on cross-subsidy deviations where the $\hat{s}$ insurer marginally reduces his profits in signal profile $n_{\hat{s}} = 1$, to have a marginal gain in signal profile $n_{\hat{s}} = n$. Hence, while we can identify the parameter regions where an informative equilibrium does not exist, we can only provide necessary but not sufficient conditions for existence.[52]

Figure C.5 shows the parameter regions that are consistent with the existence of informative (green) and non-informative (red) equilibria, for increasing levels of precision of the signal (from left to right). Focusing on non-informative equilibria, it is immediate to see that the maximum number of firms $\bar{n}$ consistent with the equilibrium decreases as the level of the damage L grows, which has interesting implications in terms of the level of profits reached in equilibrium. Intuitively, when the damage (or, equivalently, risk aversion) is large, a policyholder is willing to pay higher premia to get insured, owing to her risk aversion. This higher willingness to pay translates into a higher profitability of deviations. Therefore, an equilibrium is possible only if expected profits are sufficiently large in equilibrium. In turn, large profits occur when there is a high probability of selling the contract, i.e. when the number of competitors is sufficiently low.

The maximum number of firms $\bar{\bar{n}}$ consistent with the equilibrium decreases as the level of the damage L increases also for the candidate informative equilibria, although for different reasons than those highlighted above for non-informative equilibria. Intuitively, the larger the damage, the higher is the inefficiency in separating menus, that is the higher is the insurer's incentive to deviate and obtain a profit in the safer market state. To prevent deviations (entailing cross subsidies), the probability of the safer signal profiles must be sufficiently low, which is achieved with a low number of firms. Indeed, if the number of firms is low,

[51]The full parameter set used in the numerical exercises leading to Figure C.5 is: $p_s = 0.01$, $p_d = 0.05$, $P_s = 0.10$, $W = 1000$, and $\beta = 0.01$. The value of α is $\alpha = 0.70$ in panel (a), $\alpha = 0.80$ in panel (b) and $\alpha = 0.90$ in panel (c).

[52]As already noted, the truthful revelation condition is binding for sufficiently low n, while that on the absence of cross-subsidy deviations is binding for n sufficiently large.

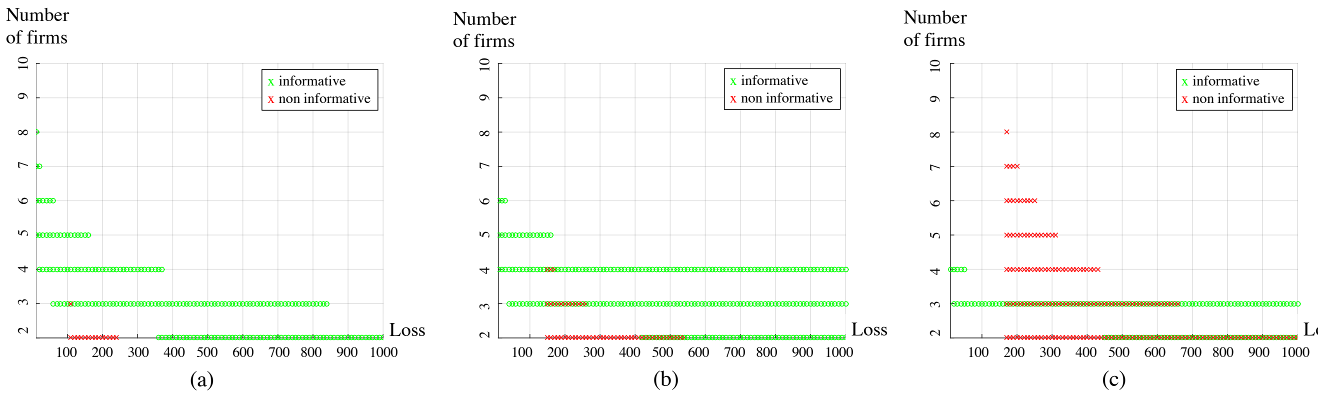

Figure C.5: Range of existence of equilibria in the plane n-L for increasing levels of α: $\alpha = 0.70$ in panel (a); $\alpha = 0.80$ in panel (b); $\alpha = 0.90$ in panel (c)

the estimation of the probability of each signal profile is less precise. Figure C.5 also shows that larger values of α broaden the region of existence of non-informative equilibria, while shrinking that of informative equilibria. Indeed, in the case of non-informative equilibria, signal precision has no impact on the characterization of the candidate equilibrium (that depends on the ex-ante loss probability, which is independent of α), but it does affect the nature of the deviation. In particular, a higher signal precision implies a lower risk assessment by a policyholder who observes a deviation and believes that the deviating insurer received signal $\hat{s}$ (under fully optimistic beliefs). In turn, a lower risk assessment implies a lower willingness to pay for insurance in deviations, reducing their profitability. This makes non-informative equilibria more likely and consistent with a higher number of firms. The opposite occurs for informative equilibria, as an insurer who receives the signal $\hat{s}$ assigns a probability to the safest signal profile that is increasing in the signal precision. When the safest signal profile is more likely, deviations with cross subsidies become more profitable, which entails a shrinking of the parameter region where informative equilibria exist. Overall, although our numerical exercises are far from properly calibrating real insurance markets, the evidence they provide points unambiguously to the fact that the insurance industry must be fairly concentrated for guaranteeing the existence of a (profitable) equilibrium.