Optimal Unilateral Carbon Policy*

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Abstract

We consider climate policy in a world with international trade and a global externality from the use of energy. We assume that one region imposes a climate policy and the rest of the world does not, thereby generating concerns about leakage and shifts in the location of production and other activities. We derive the optimal unilateral policy and show how it can be implemented through a tax on extraction, a partial border adjustment on the import and export of energy and on the import of goods, along with an export policy designed to expand the export margin. A novel feature of the optimal policy is that the pricing region exploits international trade in goods to expand the reach of its climate policy. We calibrate and simulate the model to illustrate how the optimal policy compares to more traditional policies.

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1 Introduction

Global climate negotiations have given up trying to achieve a uniform approach to climate change, such as a harmonized global carbon tax. Instead, current negotiations focus on achieving uniform participation, with each country pursuing its own approach and its own level of emissions reductions. As a result, policies to control emissions of carbon dioxide vary widely by country, and are likely to continue to do so for the indefinite future.

Widely varying carbon policies potentially affect patterns of trade, the location of production, the effectiveness of the policies, and the welfare of people in various countries or regions. These effects are of critical importance to the design of carbon policy and to its political feasibility. For example, trade and location effects were central to the design of the European Union Emissions Trading System, the Regional Greenhouse Gas Initiative, and California's carbon pricing system. One of the reasons that the United States did not ratify the Kyoto Protocol was concern about the lack of emissions policies in developing countries and the resulting trade effects. Unless concerns about the effects of differential carbon prices are addressed, it may be difficult to achieve significant emissions reductions.

To address this problem, we develop an analytic general equilibrium model of carbon pricing and trade, where one region imposes a carbon policy and the rest of the world does not. The model is a mix of Markusen (1975) and Dornbusch, Fisher, and Samuelson (1977). Our solution strategy borrows from Costinot, Donaldson, Vogel, and Werning (2015). We solve for the outcomes that are optimal for the region imposing the policy and then show how those outcomes can be implemented in a decentralized equilibrium using taxes and subsidies.

Our solution to the model suggests a novel approach to the problem of unilateral carbon pricing, one where increasing the extent of trade actually improves outcomes rather than making them worse. The approach involves imposing a domestic carbon tax on the extraction of fossil fuels, along with (1) what we call partial or imperfect border adjustments and (2) an export policy designed to expand low-carbon exports from the carbon-pricing region to the rest of the world. The partial border adjustment is a tax on imports of fossil fuels, a rebate of prior taxes paid on exports of fossil fuels, and a tax on imported goods based on the energy used in their production. It is partial in that the border adjustment rate is not the same as (and is typically lower than) the rate of the underlying extraction tax. Furthermore, the border adjustment does not apply to the energy embodied in exported goods. Rather than rebating prior taxes paid, as in a conventional border adjustment, the export policy expands exports via fixed subsidies not determined by prior taxes. In effect, the taxing region maximizes the reach of its carbon tax to include all goods produced domestically (regardless of where they are consumed), all goods consumed domestically (regardless of where they are produced), and, moreover, expands the scope of its exports to further broaden the tax base. As the extent of trade increases, the taxing region is able to expand the tax base further, generating better outcomes.

To understand the quantitative implications of our analysis, we calibrate the model and solve it numerically. In our core calibration, we assume that the OECD countries impose a carbon price and the rest of the world does not. We compare the optimal policy to more conventional policies: (i) a tax on the extraction of fossil fuels (as suggested by Metcalf and Weisbach (2009)); (ii) a tax on the use of energy in production (which is how most cap and trade systems work); and (iii) a tax on production combined with conventional border adjustments (which is the structure of many recent carbon tax proposals). Conventional border adjustments shift a tax on the use of energy in domestic production to a tax on the energy embodied in domestic consumption, so we think of (iii) as a tax on consumption. We also examine optimal combinations of these three conventional policies, focusing on a combination of an extraction tax and a consumption tax, and a combination of a production tax and a consumption tax.

Our quantitative results generate four key lessons. First, it is important to include countries with a large tax base in the taxing coalition. Expanding the tax base by including additional countries makes larger emissions reductions desirable. Second, carbon pricing coalitions may want to enact policies that generate substantial reductions even when the rest of the world does not cooperate. Third, adding border adjustments to a production tax generates only modest gains, and may not be worth the administrative costs. Finally, the hybrid of the extraction tax and the consumption tax performs almost as well as the optimal tax. The likely reason is that they have differing effects on the price of energy, which means that more generally combining taxes with differing effects on the price of energy is likely desirable.

Our core model does not include renewable energy, and stimulating renewables is often seen as a central goal of carbon pricing. To examine this issue, we extend the analysis to include renewable energy. Renewables only require modest adjustments to the optimal policy. Not surprisingly, renewables are exempt from the tax on extraction. If they can be sold in the market at the same price as fossil fuels, this exemption stimulates the production of renewables. In addition, while the optimal policy attempts to limit increases in fossil fuel extraction in other countries, it does not do so for renewables because the additional use of renewables in other countries does not generate harm.

The paper proceeds as follows. The remainder of this section provides additional motivation and reviews the relevant literature. Section 2 lays out the basic elements of the model and characterizes its competitive equilibrium absent taxes, which forms our business-as-usual (BAU) baseline scenario. Section 3 solves the problem of a planner designing an optimal unilateral carbon policy for one region with the other region behaving as in a competitive equilibrium. In Section 4 we derive a set of taxes and subsidies that implement the optimal policy, which we take to be the policy recommendations of this analysis. We explore the quantitative implications of the optimal policy in Section 5, using a calibrated version of the model. Section 6 extends the analysis to include a renewable energy sector. Section 7 concludes.

1.1 Motivation

As noted, the possibility that varying carbon prices in different regions might affect patterns of trade, the location of production and the effectiveness of carbon prices has been a central concern in the design of carbon policy. Our central motivation is to understand these effects and the optimal response to them. A related and second motivation is that there is a virtual zoo of possible responses, and, while there has been extensive work analyzing some of the most prominent responses, so far, it has not been clear how to pick among the full range. That is, the question is not simply whether adding border adjustments to a conventional production tax is desirable, which is the focus of much of the literature. Instead, there are a wide variety of policies, and we need a method of picking among them. To illustrate this latter problem, we describe the range of possibilities here, along the way defining terms that will be useful in understanding the optimal, decentralized solution.

For simplicity, we assume (here and throughout the paper) that the price on carbon is imposed via a tax rather than a cap and trade system. Although there may be differences between taxes and cap and trade systems (e.g., Weitzman (1974)), these differences are not relevant for our purposes. We also only focus on carbon emissions from fossil fuels, which have been the central focus of existing and proposed carbon prices, ignoring agriculture and deforestation, two other major sources of emissions.

Normally, Pigouvian taxes need to be imposed directly on the externality-causing activity rather than on imperfect proxies. There is, however, an almost one-for-one relationship between fossil fuel inputs into the economy and eventual emissions. That is, almost all carbon molecules that enter the economy as fossil fuels are eventually emitted as CO_2 through combustion. This fact means that we can tax carbon at any, or multiple, stages of production without losing accuracy.

Metcalf and Weisbach (2009) exploit this fact to suggest imposing a carbon tax upstream on the extraction of fossil fuels (or very nearly so). They reasoned that there are a small number of large, sophisticated extractors, compared to a much larger number of manufacturers using fossil fuels and a vastly larger number of consumers of products made using fossil fuels. They estimated that the United States could tax essentially all domestic extraction of fossil fuels by taxing just 2,500 entities, compared to, say, the roughly 250 million vehicle tailpipes, among many other items, that would have to be taxed with a direct tax on consumers.

In a closed economy, a tax on extraction would be the same as a tax anywhere else in the chain of production (but for administrative costs). A tax on extraction would be embedded in the price of the fuel, causing manufacturers and consumers (as well as extractors) to internalize climate externalities. This is not true, however, in an open economy. Extraction taxes increase the pre-tax price of fossil fuels. If t_e is the extraction tax and p_e the pre-tax price of energy, extractors receive $p_e - t_e$ after tax. Unless the tax is entirely borne by extractors, p_e will go up. Because the price of fossil fuels goes up, extraction taxes cause foreign extractors, not subject to the tax, to increase extraction, generating what we call extraction leakage. Extraction leakage reduces the effectiveness of an extraction tax. To the extent Foreign emissions go up because of extraction leakage, the taxing region suffers harm that it might otherwise have avoided.

While extraction taxes cause a shift in where extraction occurs, on their own they do not shift where manufacturing and consumption occur. If there is a global price for energy, all manufacturers and all consumers, globally, see the same higher price for energy generated by the extraction tax in the taxing region. They all adjust their production and consumption accordingly, with no particular differentiation between actors in the taxing region and the non-taxing region.

Actual carbon prices are usually imposed on production—that is, on the smokestack—rather than on extraction. For example, the European Union Emissions Trading System is on emissions from industrial use of fossil fuels.

With a production tax at rate t_p, producers pay $p_e + t_p$ for energy, increasing the after-tax price of energy. Once again, in a closed economy, the effects of taxing production would be the same as taxing extraction. In an open economy, however, their effects will not be the same. Production taxes lower the global price of energy because demand will go down: producers will shift to cleaner manufacturing techniques and consumers will demand fewer energy-intensive goods.

To the extent there is a global price of energy, all extractors, globally, see a lower price of energy and extract less. There is no extraction leakage with a pure production tax. That is, shifting away from an extraction tax toward a production tax moderates extraction leakage by moderating the price-increasing effect of an extraction tax (an effect we will see in our optimal solution).

Production taxes however, cause production to shift to untaxed regions because they reduce the comparative advantage of producers in the taxed region. This effect is known as production leakage, or because of the predominance of production taxes, often just leakage. Leakage is generally taken as the central measure of the (in)effectiveness of a carbon policy. It has been called the defining issue in the design of regional climate policies (Fowlie 2009).

If there were no trade costs, production taxes would not affect the location of consumption. All consumers, even those abroad, who purchase goods produced in the taxing region would face a higher price for those goods. And all consumers, even those in the taxing region, would see a lower price for goods produced abroad. Production taxes affect where goods are produced but not where they are consumed. With trade costs, however, taxes on production may shift where consumption takes place because trade costs tie production and consumption together to some extent.

Finally, a carbon tax can be imposed directly on consumption. A tax on consumption would be based on the emissions associated with each good when it was produced. For example, if a consumer buys a toaster, the consumer would pay a tax based on the emissions from the production of the toaster. Because of the very large number of products and consumers, and the difficulty of determining the tax, carbon taxes are not normally proposed to be imposed this way. Gasoline taxes, however, might be thought of as a version of a consumption tax, and these are collected at the gas station pump.

These three "pure" taxes, can be combined. For example, a country that wants to impose a \$100/ton tax on emissions of CO_2 could impose a \$50/ton tax on extraction, a \$30/ton tax on production, and a \$20/ton tax on consumption. As we will suggest, a mix allows the country to moderate the effects of each of the pure taxes. For example, imposing both an extraction tax and a production tax can balance the negative effects on the location of extraction that arise from a pure extraction tax with the negative effects on the location of production from a pure production tax. That is, there are not merely three possibilities, there are an infinite number of combinations of these three that generate different effects.

The last piece of terminology is "carbon border adjustments" or just simply border adjustments.[1] Border adjustments are taxes on imports or rebates of prior taxes paid on exports. They can apply to either fossil fuels or goods, or both. For fossil fuels, the border adjustment is on the carbon content of the fossil fuel. For goods, the border adjustment is on the carbon emissions from the production of the good, what we call the embodied carbon or embodied energy. Kortum and Weisbach (2017) provide a more detailed description of border adjustments.

Border adjustments shift the tax downstream. For example, an extraction tax with border adjustments on the import and export of fuels becomes a tax on domestic production. Any fuel that is extracted domestically but exported has the tax rebated, and any fuel that it extracted abroad but imported has a tax imposed. All fuel used domestically, and only that fuel, bears a tax. Therefore, we can equivalently impose an extraction tax plus a border adjustment or a production tax. They differ only in their nominal description. Similarly, a border adjustment on imports and exports of goods shifts the tax from production to domestic consumption, and we can equivalently impose

[1] The term "border adjustment" is most often used in connection with destination-based VATs, widely used throughout the world. Border adjustments in this context are rebates of prior VAT paid when a good is exported and the imposition of VAT when a good is imported.

The term "carbon border adjustment" is a border adjustment based on the carbon content of goods including the carbon emitted during production, rather than their value (as in a VAT). For simplicity, we shorten the term to just "border adjustment" because the usage is unambiguous here.

a production tax plus a border adjustment or a consumption tax.

Full or perfect border adjustments apply equally to imports and exports and are imposed at the same rate as the underlying tax. Border adjustments can be imposed at a different rate than the underlying tax. If the rate is less than the underlying tax, we can think of the border adjustment as shifting that portion downstream. For example, if an extraction tax is imposed at \$100/ton of CO_2, and the border adjustment is only at \$75/ton, we can think of this as a \$25/ton tax on extraction and a \$75/ton tax on production. Therefore, we can implement combinations of the three pure taxes via border adjustments imposed at different rates than the underlying tax.

Border adjustments can also be imperfect because they apply differently to imports and exports. For example, they can be applied to imports but not exports. A production tax with a border adjustment applied only to imports becomes a tax on all domestic production and on all domestic consumption generating a broader tax base than any of the three pure taxes. More generally, the border adjustment can be applied at different rates to imports and exports (and both those rates might be different than the rate of the underlying tax). We can decompose the effects in the same way as suggested above.

Finally, border adjustments might apply only to a subset of goods, such as only to goods that are particularly energy intensive. Many border adjustment proposals are limited in this way, in large part to minimize administrative costs. Modern economies import and export a vast number of different goods, and computing accurate border adjustments for each of these goods would be difficult. By imposing border adjustments only where their effects are likely to be large, the administrative costs can be reduced.

As can be seen, there are a large number of possible taxes. Our goal is to understand the optimal mix of these possibilities for taxing regions.

1.2 Prior Literature

Because of its prominence, there is a voluminous prior literature studying this problem. The overwhelming majority of studies use computable general equilibrium models to simulate carbon taxes and border adjustments. By our count, there are over 50 CGE studies of the general problem of differential carbon prices in the peer-reviewed literature (and many more in the gray literature) and each study considers multiple different scenarios, which means that there are hundreds of simulations of the problem.[2] For example, Branger and Quirion (2014) perform a meta-analysis of 25 studies of differential carbon taxes (20 of which were CGE studies, 5 of which were partial equilibrium studies). These 25 studies, which make up only a portion of the literature, had 310 different modeled scenarios.

CGE studies almost uniformly use leakage as their measure of the effects of differential carbon prices. Leakage is commonly defined as the increase in emissions in non-taxing regions as a percentage of the reduction in emissions in the taxing region. (Hence, 100% leakage means the policy is totally ineffective in reducing global emissions.)Leakage estimates vary considerably, although within a relatively consistent range. The Branger and Quirion meta-study finds leakage rates between 5% and 25% without border adjustments. They also find that border adjustments reduce leakage by about a third to be within a range of 2% to 12% with a mean value of 8%. Similarly, the Energy Modeling Forum commissioned 12 modeling groups to study the effects of border adjustments on leakage using a common data set and common set of scenarios. Bohringer et al. (2012). They considered emissions prices in the Kyoto Protocol Annex B countries (roughly the OECD) that reduce global emissions by about 9.5%. Without border adjustments, leakage rates were in the range of 5% to 19% with a mean value of 12%. They also find that border adjustments reduce leakage by about a third, with a range between 2% and 12% and a mean value of 8%. Elliott et al (2013) replicated 19 prior studies within their own CGE model, finding leakage rates between 15% and 30% for a tax on Annex B countries that reduced global emissions by about 13%.[3]

We use an analytic general equilibrium model of trade to study the problem. This approach allows us to uncover the underlying economic logic for why some policies perform better than others, although it means that our quantitative analysis is more illustrative than definitive because the model

[2]For surveys of the leakage literature, see Droge et al. 2009, Zhang 2012 and Metz et al. 2007

[3]A smaller number of studies focus on the effects of carbon taxes on particular energy-intensive and trade-exposed sectors. For example, Fowlie et al (2016) consider the effects of a carbon price on the Portland cement industry. They find that a carbon price has the potential to increase distortions associated with market power in that industry. Leakage compounds these costs. They find that border adjustments induce negative leakage because of how industry actors respond, and can generate significant welfare gains at high carbon prices.

is stripped down. There are a small number of studies that precede us in this approach. The classic study, which we build on, is Markusen (1975). Markusen analyzes a two-country, two-good model in which production of one of the goods generates pollution that harms both countries. Writing before climate change was a widespread concern, he considers a simple pollutant, such as the release of chemicals into Lake Erie by polluters in the United States, which harms Canada (as well as the United States). One of the countries imposes policies to address the pollution; the other is passive. Markusen finds that the optimal tax is a Pigouvian tax on the dirty good combined with a tariff (if the good is imported) or a subsidy (if it is exported). The optimal tariff or subsidy combines terms of trade considerations and considerations related to leakage and is generally lower than the Pigouvian tax.[4]

2 Basic Model

We introduce the basic model and its competitive equilibrium here before turning to the planning problem, from which we derive the optimal unilateral carbon policy.

Two countries, Home and Foreign, are endowed with labor, L and L^*, as well as fossil fuel deposited under the ground, G and G^*. The $*$ distinguishes Foreign from Home. Each country has three sectors: energy e, goods g, and services s. Energy is extracted from deposits using labor, goods are produced by combining labor and energy, and services are provided with labor only. Labor is perfectly mobile across the three sectors within a country.[5] As in Dornbusch, Fisher, and Samuelson (1977), goods come in a continuum, indexed by $j \in [0, 1]$.

[4] Hoel (1996) generalizes Markusen's analysis and produces similar results in the context of climate change and carbon taxes. He also considers the case where the country may not impose tariffs. In this case, the optimal policy will involve carbon taxes that vary by sector (even though the harms from emissions do not vary by sector). There are a number of other analytic models of the problem. Other related models include Holladay et al (2018), Hemous (2016), Baylis et al (2014), Jakob, Marschinski and Hubler (2013), Fischer and Fox (2012, 2011), and Hoel (1994).

[5] What we call labor can be interpreted as a combination of labor and capital, which is used to extract energy, produce goods, and provide services.

2.1 Preferences

We denote services consumption by C_s and define an index of goods consumption C_g by:

$$C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)},$$

where $\sigma > 0$ is the elasticity of substitution across the individual goods j. Preferences in Home are:

$$U(C_s, C_g, Q_e^W) = C_s + \eta^{1/\sigma} \frac{C_g^{1-1/\sigma} - 1}{1 - 1/\sigma} - \varphi Q_e^W,$$

where η governs Home's overall demand for goods.[6] The harms from climate change are captured by the last term, where $Q_e^W = Q_e + Q_e^*$ is global energy extraction, and φ is Home's marginal harm from global emissions.[7]

Preferences in Foreign are the same except with η^* in place of η, σ^* in place of σ, and φ^* in place of φ. Throughout we assume $C_s > 0$ and $C_s^* > 0$, a condition that is easily checked. We restrict $\sigma^* \leq 1$ since inelastic Foreign demand simplifies the solution for Home's optimal exports of individual goods.[8]

2.2 Technology

Fossil fuel is deposited in a continuum of fields, characterized by different costs of extraction. We summarize the distribution of costs in Home by an increasing differentiable function $G(a)$, representing the quantity of energy that can be extracted at a unit labor requirement below a. The distribution in Foreign is $G^*(a)$. We assume efficient extraction within each region so that

[6]We follow Grossman and Helpman (1994) in adopting quasi-linear preferences, which greatly simplifies the analysis.

[7]Throughout most of the paper we equate energy with fossil fuel, measured by its carbon content. We introduce green energy in an extension toward the end.

[8]We can relax this assumption by following the strategy in Costinot, Donaldson, Vogel, and Werning (2015). For $\sigma = 1$ preferences simplify to:

$$U = C_s + \eta \int_0^1 \ln c_j dj - \varphi Q_e^W,$$

and likewise in Foreign.

low cost fields are tapped first. Focusing on Home, the labor L_e employed to extract energy Q_e satisfies:

$$L_e = \int_0^{\bar{a}(Q_e)} aG'(a)da, \tag{1}$$

and

$$Q_e = G(\bar{a}(Q_e)), \tag{2}$$

where $\bar{a}(Q_e)$ is the highest-cost field that is tapped.[9] These two expressions apply in Foreign as well, with Q_e^*, G^*, $\bar{a}^*(Q_e^*)$, and L_e^*. The output of the energy sector is in turn used as an intermediate input E and E^* by the goods sector.

Goods $j \in [0, 1]$ are produced with input requirement a_j (in Home) and a_j^* (in Foreign) using a Cobb-Douglas combination of labor and energy:

$$q_j = \frac{1}{\nu a_j} L_j^{\alpha} E_j^{1-\alpha}, \tag{3}$$

where L_j is the labor input, E_j is the energy input, α is the output elasticity of labor, and:

$$\nu = \alpha^{\alpha} (1 - \alpha)^{1-\alpha}. \tag{4}$$

The production function in Foreign is the same, but with a_j^* in place of a_j.[10]

Services are provided in both countries with a unit labor requirement.

[9] An analytically convenient case, reminiscent of Houthakker (1955-56), is:

$$G(a) = Ga^{\beta/(1-\beta)},$$

where G and β are scale and shape parameters of the distribution. Solving (2) and (1) under this parameterization gives a Cobb-Douglas energy extraction function:

$$Q_e = \beta^{-\beta} L_e^{\beta} G^{1-\beta}.$$

Asker, Collard-Wexler, and De Loecker (2019) examine data on costs of extraction across all oil fields in the world. We plot statistics from their data, posted in Asker, Collard-Wexler, and De Loecker (2018), as Figures 6 and 7 in the Appendix. Assuming that oil fields are representative of fossil fuels more generally, it is clear that the constant elasticity form does not fit the data over the entire distribution. We proceed without imposing a particular shape, and when we get to our quantitative illustration we assume only that a constant elasticity applies to the upper end of the cost distributions.

[10] In line with our Ricardian assumptions, we treat α as common across goods and countries. Including the constant ν in the production function leads to simpler expressions for costs, that will appear later.

2.3 International Trade

We assume that energy and services are costlessly traded between Home and Foreign. We take services to be the numéraire, with price 1.[11] Energy is traded at a world price p_e.

Trade in the continuum of manufactured goods follows Dornbusch, Fisher, and Samuelson (1977). Goods are arranged in decreasing order of Home's comparative advantage:

$$\frac{a_j^*}{a_j} = F(j), \tag{5}$$

where we assume that $F(j)$ is a strictly decreasing continuous function.[12]

Goods are traded subject to iceberg costs on Home's exports $\tau \geq 1$ and on Home's imports $\tau^* \geq 1$. The total input requirement for Home to supply good j to Foreign is thus τa_j and for Foreign to supply good j to Home $\tau^* a_j^*$.

2.4 Labor and Energy Requirements

Having described technology and trade for goods, we now introduce a notation for energy and labor input requirements that will prove convenient throughout the rest of the paper. At a given energy intensity:

$$z_j = E_j / L_j$$

we can invert the production function (3) to express the unit labor requirement for good j in Home:

$$l_j(z_j) = \nu a_j z_j^{\alpha - 1}. \tag{6}$$

The corresponding unit energy requirement is:

$$e_j(z_j) = z_j l_j(z_j) = \nu a_j z_j^{\alpha}. \tag{7}$$

Unit labor and energy requirements in Foreign, $l_j^*(z_j)$ and $e_j^*(z_j)$, are the same but with a_j^* in place of a_j.

[11] We will assume that $Q_s^* > 0$ so that given the unit labor requirement for services the wage in Foreign is $w^* = 1$. This outcome is guaranteed with a large enough labor endowment in Foreign.

[12] In order to have well defined integrals in what follows, we also assume that a_j and a_j^* can be treated as continuous functions of j.

So as not to unduly constrain the optimal unilateral policy, the energy intensity for good j may depend not only on where the good is produced but also on where it is shipped. To handle that possibility requires some additional notation.

For each good j we distinguish between Home's exports, $x_j \geq 0$ and Home's production for consumption in Home, $y_j = q_j - \tau x_j$. We also distinguish between Home's imports, $m_j \geq 0$ and Foreign's production for consumption in Foreign, $y_j^* = q_j^* - \tau^* m_j$. (Note that we define exports and imports in terms of the quantity that reaches the destination, taking account of iceberg costs .) For each good j we allow for the possibility of four different energy intensities z_j^y, z_j^x, z_j^m, and z_j^*, one for each of the four lines of production y_j, x_j, m_j, and y_j^*.

Exploiting this notation, we can express labor employed in the goods sector in Home as the sum of the labor used to produce goods consumed domestically and the labor used to produce goods for export:

$$L_g = \int_0^1 \left(l_j(z_j^y) y_j + \tau l_j(z_j^x) x_j \right) dj. \tag{8}$$

We now turn to a more detailed account of how energy is used.

2.5 Carbon Accounting

We set units so that a unit of energy has a unit of carbon. Energy is extracted in both countries. Home may export some of what it extracts or it may import some from Foreign. Carbon is released when the energy is used to produce goods, and these goods embodying carbon emissions are traded. Some goods are exported and others imported before they are consumed by households. We can therefore trace carbon from its extraction through its release into the atmosphere and finally to its implicit consumption by households.

We define I_e as total intermediate demand for energy used by producers in Home and I_e^* by producers in Foreign. Home's net exports of energy, which can be positive or negative, is extraction less intermediate demand:

$$X_e = Q_e - I_e.$$

Defining Foreign's net energy exports X_e^* similarly, the global energy market clears when $X_e + X_e^* = 0$. These expressions account for the first level of trade in carbon.

Table 1: Carbon Accounting Matrix

	Home	Foreign	Total
Home	$C_e^{HH} = \int_0^1 e_j(z_j^y) y_j dj$	$C_e^{HF} = \tau^* \int_0^1 e_j^*(z_j^m) m_j dj$	$C_e = C_e^{HH} + C_e^{HF}$
Foreign	$C_e^{FH} = \tau \int_0^1 e_j(z_j^x) x_j dj$	$C_e^{FF} = \int_0^1 e_j^*(z_j^*) y_j^* dj$	$C_e^* = C_e^{FH} + C_e^{FF}$
Total	$I_e = C_e^{HH} + C_e^{FH}$	$I_e^* = C_e^{HF} + C_e^{FF}$	$I_e^W = C_e^W = Q_e^W$.

The second level of trade in carbon is embodied in goods. The following table depicts the bilateral flows, with rows indicating the location of consumption and columns the location of production:

For example, Home's implicit consumption of carbon C_e is the sum of carbon released by producers in Home serving the local market C_e^{HH} and carbon released by Foreign producers in supplying Home's imports C_e^{HF}.

2.6 Competitive Equilibrium

Before turning to the planning problem from which we derive the optimal unilateral carbon policy, we present the key elements of the competitive equilibrium for this economy. Laying out the competitive equilibrium serves several purposes: (i) it's our business-as-usual (BAU) scenario with no carbon policy, (ii) it describes the behavior of Foreign under a unilateral policy, and (iii) the equilibrium structure helps in interpreting the planner's solution.

In this BAU scenario, trade in services pins down a common wage of $w = w^* = 1$ and trade in energy pins down a common energy price p_e. The model collapses to a nearly text-book version of Dornbusch, Fisher, and Samuelson (1977). Here we briefly state the main outcomes.[13]

[13] The full solution is provided in Appendix A, where it appears as a corollary of the global planner's problem.

2.6.1 Goods Sector

Wherever it comes from, each good is produced at the cost minimizing energy intensity:

$$z_j = z = \frac{1-\alpha}{\alpha p_e}. \tag{9}$$

Applying (6) and (7), if good j is produced in Home the unit cost of production is:

$$p_j = l_j + p_e e_j = a_j p_e^{1-\alpha}.$$

If good j is produced in Foreign a_j^* replaces a_j. These unit costs determine prices via competition in international trade.

Home exports goods for which its cost, including transport, is below Foreign's or $j < \bar{j}_x$, where from (5) the export threshold satisfies:

$$F(\bar{j}_x) = \tau. \tag{10}$$

Likewise, Home imports goods $j > \bar{j}_m$ where the import threshold satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}. \tag{11}$$

These thresholds are invariant to the energy price.

The quantity demanded by Home consumers for a good $j \leq \bar{j}_m$, whose price is thus p_j, is:

$$y_j = \eta p_j^{-\sigma}.$$

Similar expressions apply for y_j^*, m_j, and x_j, taking account of the location of production and transport costs.

2.6.2 Energy Sector

Home's implicit demand for energy is:

$$C_e(p_e) = (1-\alpha)\,\eta\left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(\tau^* a_j^*\right)^{1-\sigma} dj\right) p_e^{-\epsilon_D}.$$

Here:

$$\epsilon_D = \alpha + (1-\alpha)\sigma$$

is Home's elasticity of demand for embodied energy, capturing the elasticity of the unit energy requirement in production α as well as the elasticity of demand σ multiplied by the elasticity of the price of the good with respect to the energy price $1-\alpha$. Foreign's demand for embodied energy is:

$$C_e^*(p_e) = (1-\alpha)\,\eta^* \left(\int_0^{\bar{j}_x} (\tau a_j)^{1-\sigma}\, dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma} dj \right) p_e^{-\epsilon_D^*},$$

where, due to our assumption of $\sigma^* \leq 1$:

$$\epsilon_D^* = \alpha + (1-\alpha)\sigma^* < 1. \tag{12}$$

On the energy supply side, since the wage is 1 the cost of extraction is a. Thus energy is extracted from all fields with $a \leq p_e$. The energy supply curve in Home is thus:

$$Q_e = G(p_e), \tag{13}$$

while in Foreign:

$$Q_e^* = G^*(p_e). \tag{14}$$

The equilibrium energy price equates global supply and demand for energy:

$$C_e(p_e) + C_e^*(p_e) = G(p_e) + G^*(p_e).$$

This energy price determines the size of the goods sector, the energy sector, and the services sector.[14]

With the structure of the model in place, we will turn next to the characterization of Home's optimal carbon policy. Two particular features of the competitive equilibrium play key roles in characterizing the optimal unilateral policy. The first is Foreign's energy demand elasticity ϵ_D^* defined in (12). The second is Foreign's energy supply elasticity:

$$\epsilon_S^* = \frac{dG^*}{dp_e}\frac{p_e}{G^*}. \tag{15}$$

[14]The size of the services sector is determined by the quantity of labor not employed in the production of goods (with labor share α) or the extraction of energy:

$$Q_s = L - L_g - L_e = L - \alpha I_e/(1-\alpha) - \int_0^{p_e} aG'(a)da.$$

The same applied in Foreign with Q_s^* in place of Q_s, I_e^* in place of I_e, and G^* in place of G. We assume that L and L^* are large enough so that Q_s and Q_s^* are both strictly positive. Services are costlessly traded to achieve trade balance:

$$C_s - Q_s = p_e(Q_e - I_e) + \int_0^{\bar{j}_x} \tau p_j x_j dj - \int_{\bar{j}_m}^1 \tau^* p_j^* m_j dj.$$

Although not made explicit in the notation ϵ_S^*, the supply elasticity, unlike the demand elasticity, typically varies with the equilibrium price.

3 Home's Planning Problem

We consider a planner that allocates all resources in Home so as to maximize Home's welfare. Foreign outcomes are determined as a competitive equilibrium given the choices made by the planner. After solving this problem, we'll deduce a set of taxes on extraction, production, consumption, and trade that implement Home's optimal unilateral carbon policy.

3.1 Preliminaries

We start by considering the various constraints faced by the planner, the first of which involves prices.

3.1.1 Prices for International Trade

Home and Foreign trade services at a price of 1. They trade energy at a price p_e, which is chosen by the planner. (Note that choosing a higher price means a policy that restricts energy supply while a lower price means a policy that restricts energy demand.)

For each good j the planner dictates the quantities y_j, x_j, and m_j. It also dictates the corresponding energy intensities of production z_j^y, z_j^x, and z_j^m (even though Home's imports are produced in Foreign, the planner can set the energy intensity for how they are produced).

When Home imports from Foreign, the price is determined by Foreign's cost of production. This cost depends on both labor cost and energy cost in a combination determined by the energy intensity z_j^m that the planner dictates. It also depends on the iceberg cost τ^*. The price of Home's imports of good j, if $m_j > 0$, is thus:

$$p_j^m = \tau^* \left(l_j^*(z_j^m) + p_e e_j^*(z_j^m) \right).$$

When Foreign is producing for its domestic market, its cost-minimizing energy intensity z^* is given by (9) and hence its unit energy requirement is:

$$e_j^*(z^*) = (1-\alpha) a_j^* p_e^{-\alpha}. \tag{16}$$

The price at which it sells to local consumers is:

$$p_j^* = a_j^* p_e^{1-\alpha}. \tag{17}$$

Home is constrained to set the export price of good j weakly below the cost at which Foreign producers can make it themselves and the marginal utility of good j to Foreign consumers:

$$p_j^x \leq \min\left\{p_j^*, \left(c_j^*/\eta^*\right)^{-1/\sigma^*}\right\}.$$

While we don't denote it explicitly, $e_j^*(z^*)$, p_j^*, p_j^m, and p_j^x each depend on the energy price p_e.

We now have expressions for all the prices necessary to evaluate trade balance.

3.1.2 Trade Balance Constraint

The value of Home's net exports of energy, which can be positive or negative, is $p_e X_e$. The value of its exports of services, again positive or negative, is simply:

$$X_s = Q_s - C_s.$$

The value of Home's net exports of goods is:

$$X_g = V_g^{FH} - V_g^{HF},$$

where V_g^{FH} is the value of Home's exports of goods and V_g^{HF} the value of its imports of goods. Net exports in the three sectors is connected via trade balance, which we can express as:

$$X_s = \int_0^1 p_j^m m_j dj - \int_0^1 p_j^x x_j dj - p_e X_e. \tag{18}$$

3.1.3 Energy Constraints

The demand by Foreign consumers for good j is:

$$c_j^* = \eta^* \left(p_j^*\right)^{-\sigma^*}, \tag{19}$$

of which

$$y_j^* = \max\left\{c_j^* - x_j, 0\right\}.$$

is produced in Foreign, at energy intensity z^*. We can then obtain I_e^* (and I_e) by integrating according to the expressions in Table 1.

Foreign's energy-extraction supply curve is given by (14), a function of the energy price p_e as in the BAU scenario. Market clearing for energy implies a constraint on Home's net exports of energy:

$$Q_e^* = -X_e + I_e^*. \tag{20}$$

Likewise, Home's total energy production must satisfy:

$$Q_e = X_e + I_e. \tag{21}$$

We will impose each of these constraints separately in the planner's problem below. Together, they imply the global energy constraint:

$$Q_e^W = I_e + I_e^*. \tag{22}$$

3.1.4 Labor Constraint

Finally, we turn to Home's labor constraint. Assuming efficient exploitation of energy deposits (lowest cost first), the labor L_e required to extract a quantity of energy Q_e is:

$$L_e(Q_e) = \int_0^{G^{-1}(Q_e)} aG'(a)da, \tag{23}$$

where $G^{-1}(Q_e)$ is the highest cost energy deposit used. Home's labor constraint is simply:

$$Q_s = L - L_e(Q_e) - L_g, \tag{24}$$

where L_g is from (8).

3.2 The Planner's Lagrangian

The planner's objective is to maximize Home welfare:

$$U = Q_s - X_s + \frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 \left((y_j + m_j)^{1-1/\sigma} - 1\right) dj - \varphi\left(Q_e + Q_e^*\right),$$

subject to four constraints: (i) Home's labor constraint (24), (ii) trade balance (18), (iii) Home's energy constraint (21), and (iv) Foreign's energy constraint (20).

We can simply substitute the first two constraints into the objective (eliminating $Q_s - X_s$ from the objective). For the third it's easier to apply a Lagrange multiplier λ_e and for the fourth a Lagrange multiplier λ_e^*. The resulting Lagrangian is (dropping constants such as L):

$$\begin{aligned}
\mathcal{L} = & \frac{\eta^{1/\sigma}}{1-1/\sigma} \int_0^1 (y_j + m_j)^{1-1/\sigma} \, dj - \varphi\left(Q_e + Q_e^*\right) \\
& - L_e(Q_e) - \int_0^1 \left(l_j(z_j^y) y_j + \tau l_j(z_j^x) x_j\right) dj \\
& - \int_0^1 \tau^* \left(l_j^*(z_j^m) + p_e e_j^*(z_j^m)\right) m_j dj + \int_0^1 p_j^x x_j dj + p_e X_e \\
& - \lambda_e \left(\int_0^1 \left(e_j(z_j^y) y_j + \tau e_j(z_j^x) x_j\right) dj - Q_e + X_e\right) \\
& - \lambda_e^* \left(\int_0^1 \left(e_j^*(z^*) y_j^* + \tau^* e_j^*(z_j^m) m_j\right) dj - Q_e^* - X_e\right).
\end{aligned}$$

The terms in the Lagrangian are, line by line: (i) Home's welfare less services consumption, (ii) what remains of Home's labor constraint after substituting out the provision of services, (iii) what remains of Home's trade-balance constraint after substituting out services exports, (iv) Home's energy constraint, and (v) Foreign's energy constraint. We want to maximize this Lagrangian by the optimal choice of $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^x\}$, $\{z_j^m\}$, Q_e, X_e, and p_e. (When we get to the point of optimizing over p_e we will be explicit about how Q_e^*, p_j^x, $e_j^*(z^*)$, and y_j^* each depend on p_e.)

We solve this problem, starting with what Costinot, Donaldson, Vogel, and Werning (2015) call the *inner problem*, involving optimality conditions for an individual good given values for Q_e, X_e, λ_e, λ_e^*, and p_e. We then evaluate the optimality conditions for Q_e, X_e, and p_e. The Lagrange multipliers λ_e and λ_e^* will be solved in the process, once we clear the energy market.

3.3 Inner Problem

Solving the inner problem consists of first order conditions with respect to the variables that are specific to some good j: y_j, x_j, m_j, z_j^y, z_j^x, and z_j^m. These first order conditions, and their implications given Q_e, X_e, λ_e, λ_e^*, and p_e, can be considered one good at time. We therefore define a Lagrangian for good j:

$$\begin{aligned}\mathcal{L}_j = & \frac{\eta^{1/\sigma}}{1-1/\sigma}\left(y_j+m_j\right)^{1-1/\sigma} \\ & -\nu a_j\left(y_j\left(z_j^y\right)^{\alpha-1}+\tau x_j\left(z_j^x\right)^{\alpha-1}\right) \\ & -\nu a_j^*\tau^*\left(\left(z_j^m\right)^{\alpha-1}+p_e\left(z_j^m\right)^{\alpha}\right)m_j+p_j^x x_j \\ & -\lambda_e\nu a_j\left(y_j\left(z_j^y\right)^{\alpha}+\tau x_j\left(z_j^x\right)^{\alpha}\right) \\ & -\lambda_e^*\nu a_j^*\left(\max\left\{c_j^*-x_j,0\right\}\left(z^*\right)^{\alpha}+\tau^* m_j\left(z_j^m\right)^{\alpha}\right),\end{aligned}$$

where we have substituted in the expressions for unit input requirements (6) and (7) in Home (as well as their analogs in Foreign). We consider the variables relevant to Home consumers first, then turn to those relevant to Foreign consumers. Many details of the derivation are relegated to Appendix B.

3.3.1 Goods for Home Consumers

The first order condition for z_j^y implies:

$$z_j^y = z^y = \frac{1-\alpha}{\alpha\lambda_e}.$$

The planner requires all Home producers serving the domestic market to produce at the same energy intensity. Similarly, the first order condition for z_j^m implies:

$$z_j^m = z^m = \frac{1-\alpha}{\alpha\left(p_e+\lambda_e^*\right)}.$$

All Foreign producers serving consumers in Home also produce at the same energy intensity, but potentially different from producers in Home.

Due to the inherent corner solutions, the first order conditions for y_j and m_j are more intricate. Yet their implications are easy to distill by defining the good $\bar{j}_m$ for which both first order conditions hold with equality. Applying (5), this cutoff good will satisfy:

$$F(\bar{j}_m) = \frac{1}{\tau^*}\left(\frac{\lambda_e}{p_e+\lambda_e^*}\right)^{1-\alpha}. \tag{25}$$

1. For any good $j < \bar{j}_m$ Home has a comparative advantage, which leads it to produce for itself:
$$y_j = \eta \left(a_j \lambda_e^{1-\alpha} \right)^{-\sigma},$$
while importing nothing, $m_j = 0$.

2. For any good $j > \bar{j}_m$ Foreign has a comparative advantage, which leads Home to import:
$$m_j = \eta \left(\tau^* a_j^* \left(p_e + \lambda_e^* \right)^{1-\alpha} \right)^{-\sigma},$$
while producing nothing for itself, $y_j = 0$.

3. We can ignore the pattern of trade for the measure-zero cutoff good $j = \bar{j}_m$.

Note that the iceberg cost of moving goods from Foreign to Home shrinks the range of goods that Home imports. For any good that Home does import, this iceberg cost also reduces the quantity imported.

3.3.2 Goods for Foreign Consumers

We now turn to Home's exports to Foreign. The first order condition for z_j^x implies:
$$z_j^x = z^x = \frac{1-\alpha}{\alpha \lambda_e}.$$
All Home producers serving the export market produce at the same energy intensity, the same as their energy intensity for serving the domestic market.

From the derivative with respect to x_j, we first obtain an optimality condition that:
$$p_j^x = p_j^*. \tag{26}$$
This result is driven by our assumption that Foreign demand is inelastic.[15]

[15]Suppose not, so that:
$$p_j^x = \left(c_j^* / \eta^* \right)^{-1/\sigma^*} < p_j^*.$$
It follows that $y_j^* = 0$, $c_j^* = x_j$, and:
$$\frac{\partial p_j^x}{\partial x_j} = -\frac{p_j^x}{\sigma^* x_j}.$$

Imposing (26), we can return to the first order condition for x_j to get:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\nu a_j \tau \left(z_j^x\right)^{\alpha-1} - \lambda_e \nu a_j \tau \left(z_j^x\right)^{\alpha} + p_j^x + \lambda_e^* \nu a_j^* \left(z^*\right)^{\alpha}.$$

The terms on the right-hand side are: (i) the marginal value of Home's labor used to produce more of x_j, (ii) the value of the energy used, (iii) the value of the revenue obtained, and (iv) the value of reducing Foreign's energy use. Substituting in the solution for z_j^x, z^*, and p_j^x and then combining the first two terms, this derivative simplifies to:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\tau a_j \lambda_e^{1-\alpha} + a_j^* p_e^{1-\alpha} + a_j^* \lambda_e^* \left(1-\alpha\right) p_e^{-\alpha}. \tag{27}$$

The last term plays a novel role in the solution. It represents the value that the planner places on the energy Foreign would use to produce an additional unit of good j if it didn't import the good from Home.

To distill results about consumption in Foreign, define the good $\bar{j}_x$ such that

$$\left.\frac{\partial \mathcal{L}_j}{\partial x_j}\right|_{j=\bar{j}_x} = 0.$$

Applying (5), this cutoff good satisfies:

$$F(\bar{j}_x) = \frac{\tau \left(\frac{\lambda_e}{p_e}\right)^{1-\alpha}}{1 + (1-\alpha)\frac{\lambda_e^*}{p_e}}. \tag{28}$$

1. For any good $j < \bar{j}_x$ Home has comparative advantage, which leads it to export:

$$x_j = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*},$$

while Foreign produces $y_j^* = 0$ for itself. (Note that Home's export quantity for any such good is at a corner solution, which will become relevant later when we consider optimal p_e.)

In this case, under our assumption that $\sigma^* \leq 1$:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\nu a_j \tau \left(z_j^x\right)^{\alpha-1} - \lambda_e \nu a_j \tau \left(z_j^x\right)^{\alpha} - \left(\frac{1-\sigma^*}{\sigma^*}\right) p_j^x < 0.$$

As a consequence, the planner will reduce x_j, whenever $p_j^x < p_j^*$, which raises p_j^x. The increase in p_j^x ultimately leads to the condition: $p_j^x = p_j^*$.

Table 2: Production and Distribution of a Good

	Home	Foreign
Home	$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$	$m_j = \eta \left(\tau^* a_j^* \left(p_e + \lambda_e^*\right)^{1-\alpha}\right)^{-\sigma}$
Foreign	$x_j = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$	$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*}$

2. For any good $j > \bar{j}_x$ Foreign has a comparative advantage, which leads it to produce for itself:

$$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*},$$

   with $x_j = 0$.

3. We can ignore the pattern of trade for the measure-zero cutoff good $j = \bar{j}_x$.

The iceberg cost τ has no direct effect on Home's goods exports, in contrast to the direct effect of τ^* on Home's goods imports. That's because Home determines the quantity of good j to export based on Foreign's marginal utility evaluated at the cost to Foreign of producing good j itself. Because the iceberg cost appears in (28), however, a higher τ narrows the range of goods that Home exports.

It is useful to compare the terms for each of the four quantities of good j. As in Table 1, the rows indicate the location of consumption while the columns indicate the location of production.

These terms are all as might be expected except for Home's exports, x_j: (i) they reflect the global price of energy p_e rather than Home's shadow price λ_e, (ii) although produced in Home, they reflect Foreign productivity a_j^* rather than Home's productivity a_j, and (iii) they do not reflect the iceberg costs of export τ. That is, $x_j \neq \eta^* \left(\tau a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$ as one would expect. The reason is because if Home is to export it is constrained to price exports no higher than Foreign's cost, and it is never optimal for Home to export at a price below Foreign's cost.

3.4 Outer Problem

We now turn to the optimality conditions for X_e, Q_e, and p_e, rewriting the Lagrangian in terms of aggregate magnitudes:

$$\begin{aligned}\mathcal{L} = &\frac{\eta^{1/\sigma}}{1-1/\sigma} C_g^{1-1/\sigma} - \varphi\left(Q_e + Q_e^*\right) \\ &- L_e(Q_e) - L_g \\ &+ X_g + p_e X_e \\ &- \lambda_e\left(I_e - Q_e + X_e\right) \\ &- \lambda_e^*\left(I_e^* - Q_e^* - X_e\right). \end{aligned} \tag{29}$$

We consider how Q_e^*, L_g, X_g, I_e, and I_e^* depend on the energy price once we get to the first order condition for p_e.

3.4.1 Energy Exports

The first order condition with respect to X_e is:

$$\frac{\partial \mathcal{L}}{\partial X_e} = p_e - \lambda_e + \lambda_e^* = 0,$$

which implies:

$$\lambda_e = p_e + \lambda_e^*. \tag{30}$$

Thus, the optimal energy export policy requires the shadow value of energy in Home λ_e to be the same as the shadow value of energy faced by producers in Foreign that serve customers in Home, $p_e + \lambda_e^*$.

Combined with the results from the inner problem, an implication is that optimal energy intensity is the same for all producers serving consumers in Home (whether the producers are in Home or Foreign) and for all producers in Home (whether serving consumers in Home or Foreign):

$$z^y = z^x = z^m = z = \frac{1-\alpha}{\alpha\lambda_e}, \tag{31}$$

where from the inner problem we established that these energy intensities do not vary by good. The exception to equalization is z^*, the common energy intensity for any goods produced in Foreign for consumption there.

Another implication is that the extensive margin in Home, of what goods are produced locally and what goods are imported, is the same as in the

BAU equilibrium: $F(\bar{j}_m) = 1/\tau^*$. For the goods that Home imports, the price simplifies to:

$$p_j^m = \alpha\tau^* a_j^* \lambda_e^{1-\alpha} + (1-\alpha)\, p_e \tau^* a_j^* \lambda_e^{-\alpha}. \tag{32}$$

Note that this is the price at the port, appropriate for computing the trade balance, not the price that consumers in Home pay.

3.4.2 Energy Extraction

The first order condition with respect to Q_e is:

$$\frac{\partial \mathcal{L}}{\partial Q_e} = -\varphi - \frac{\partial L_e}{\partial Q_e} + \lambda_e = 0.$$

The extra labor in Home to extract a bit more energy is the labor requirement in exploiting the marginal energy deposit, $G^{-1}(Q_e)$.[16] Hence the first order condition simplifies to:

$$Q_e = G(\lambda_e - \varphi). \tag{33}$$

This condition is the same as Home's energy-extraction supply curve (13) in the BAU scenario, but with $p_e + \lambda_e^* - \varphi$ in place of p_e.[17]

We've now determined global energy supply given λ_e^* and p_e, with Home extraction being lower when marginal damages φ are high. A more definitive statement requires solving for λ_e^* and p_e, which we turn to next.

3.4.3 Energy Price

The first order condition with respect to p_e is:

$$\frac{\partial \mathcal{L}}{\partial p_e} = -\varphi\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} + \frac{\partial X_g}{\partial p_e} + X_e - \lambda_e\frac{\partial I_e}{\partial p_e} - \lambda_e^*\left(\frac{\partial I_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e}\right) = 0. \tag{34}$$

[16]Integrating (23) by parts, it becomes:

$$L_e(Q_e) = G^{-1}(Q_e)Q_e - \int_0^{G^{-1}(Q_e)} G(a)da.$$

Differentiating it in this form:

$$\frac{\partial L_e}{\partial Q_e} = G^{-1}(Q_e) + Q_e\frac{\partial G^{-1}}{\partial Q_e} - G(G^{-1}(Q_e))\frac{\partial G^{-1}}{\partial Q_e} = G^{-1}(Q_e).$$

[17]If φ is large enough it's possible that $\partial\mathcal{L}/\partial Q_e < 0$ even at $Q_e = 0$. The corner solution is then $Q_e = 0$ and $\lambda_e \le \varphi$.

To make sense of this condition requires that we clarify how the five aggregate variables Q^*, I_e^*, I_e, L_g, and X_g appearing in (29) depend on p_e.

Dependence on the Energy Price Foreign energy extraction depends directly on the energy price via (14), with elasticity given by (15). The dependence on the energy price is more subtle for the other four aggregates. Home directly chooses z, $\bar{j}_m$, $\bar{j}_x$, $\{m_j\}$, and $\{y_j\}$. This means that we can treat them as fixed when differentiating the Lagrangian with respect to p_e since each satisfies its own first-order condition from the inner problem.[18] Furthermore, we can take as fixed the unit energy requirement for Home producers, whether supplying the domestic or export market. On the other hand $\left\{p_j^x\right\}$, $\{x_j\}$, $\left\{p_j^m\right\}$, $\left\{y_j^*\right\}$, and $\left\{e_j^*(z^*)\right\}$ were not chosen by the planner, or were optimized at a corner solution. They must be considered in the first order condition. We apply (32), (26), (17), (16), and results in the second row of Table 2 to compute the derivatives of the four aggregates.

Energy use by Foreign producers connects to the energy price via:

$$I_e^* = (1-\alpha)\eta^* p_e^{-\epsilon_D^*} \int_{\bar{j}_x}^{1} \left(a_j^*\right)^{1-\sigma^*} dj + \tau^* \int_{\bar{j}_m}^{1} e_j^* m_j dj,$$

so that:

$$\frac{\partial I_e^*}{\partial p_e} = \frac{\partial C_e^{FF}}{\partial p_e} = -\epsilon_D^* \frac{C_e^{FF}}{p_e} < 0, \tag{35}$$

where Foreign's demand elasticity ϵ_D^* is from (12). That is, a change in the energy price effects Foreign's use of energy only through its domestic consumption C_e^{FF} and not through its exports of goods to Home C_e^{HF}. Home has chosen and optimized the determinants of C_e^{HF} ($\bar{j}_m$, m_j, and z^m).

Energy use by Home producers is:

$$I_e = \int_0^{\bar{j}_m} e_j y_j dj + (1-\alpha)\eta^* p_e^{-(1-\alpha)\sigma^*} \int_0^{\bar{j}_x} \tau a_j \left(a_j^*\right)^{-\sigma^*} dj,$$

so that:

$$\frac{\partial I_e}{\partial p_e} = \frac{\partial C_e^{FH}}{\partial p_e} = -(1-\alpha)\sigma^* \frac{C_e^{FH}}{p_e} < 0. \tag{36}$$

[18]Thus, C_g in (29) does not appear in (34) since it depends only on terms that were optimized in the inner problem.

Goods-sector employment in Home is closely tied to energy use via:

$$L_g = I_e/z = \frac{\alpha}{1-\alpha}\lambda_e I_e,$$

so that:

$$\frac{\partial L_g}{\partial p_e} = \frac{\alpha}{1-\alpha}\lambda_e \frac{\partial I_e}{\partial p_e} < 0. \tag{37}$$

Home's net exports are:

$$X_g = \eta^* p_e^{1-\epsilon_D^*} \int_0^{\bar{j}_x} \left(a_j^*\right)^{1-\sigma^*} dj - \alpha\lambda_e^{1-\alpha} \int_{\bar{j}_m}^1 \tau^* a_j^* m_j dj - (1-\alpha)\, p_e \lambda_e^{-\alpha} \int_{\bar{j}_m}^1 \tau^* a_j^* m_j dj,$$

so that:

$$\frac{\partial X_g}{\partial p_e} = \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} = (1-\epsilon_D^*)\frac{V_g^{FH}}{p_e} - C_e^{HF}. \tag{38}$$

Restatement of the Optimality Condition Using these results along with the global energy constraint (22), we can rewrite the first order condition (34) as:

$$\lambda_e^* = \varphi \frac{\partial Q_e^*/\partial p_e}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e} + \frac{\left(Q_e^* - C_e^{FF}\right) - \partial \Pi_g/\partial p_e}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e}, \tag{39}$$

where Π_g is Home's surplus from goods exports:

$$\Pi_g(p_e) = V_g^{FH}(p_e) - \frac{\lambda_e}{1-\alpha} C_e^{FH}(p_e). \tag{40}$$

Using the elasticities demand and supply (local), we can rewrite (39) as:

$$\lambda_e^* = \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} + \frac{p_e\left(Q_e^* - C_e^{FF}\right) - (1-\epsilon_D^*)V_g^{FH} + \sigma^* C_e^{FH}}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}.$$

This equality will generate a locus of combinations of p_e and λ_e^*. The global energy constraint (22) will generate another. Their intersection gives us the solution for p_e and λ_e^*.

3.5 Assessment

We can now compute the optimal policy in principle:

1. The inner problem, together with (30), gives I_e, I_e^*, and X_g in terms of p_e and λ_e^*.
2. Equations (14) and (33) give Q_e^* and Q_e as functions of p_e and λ_e^*.
3. Equation (39) together with the global energy constraint, combining (20) and (21), nails down p_e and λ_e^*.

Along the way we can also solve for X_e, Q_s, and X_s. Before actually computing the solution for a parameterized version of the model, we reinterpret the optimality conditions in terms of a set of taxes and subsidies.

4 Optimal Taxes and Subsidies

We now turn to a set of taxes and subsidies that deliver the unilaterally optimal outcomes in a competitive equilibrium. The taxes must meet the following conditions:

1. By (31), the energy intensity of production is the same for all goods produced in Home or consumed in Home: $z^y = z^m = z^x$.
2. The import margin is the same as in the BAU case (25): $F\left(\bar{j}_m\right) = 1/\tau^*$.
3. The energy price faced by Home producers is λ_e, where by (30): $\lambda_e = p_e + \lambda_e^*$.
4. The energy price faced by producers in Foreign serving customers in Home is also $p_e + \lambda_e^*$.
5. Home's energy extraction must satisfy (33): $Q_e = G\left(p_e + \lambda_e^* - \varphi\right)$.
6. Home's export margin is (28): $F\left(\bar{j}_x\right) = \tau\left(\frac{\lambda_e}{p_e}\right)^{1-\alpha} / \left(1 + (1-\alpha)\frac{\lambda_e^*}{p_e}\right)$.
7. Producers in Home selling goods in Foreign do so satisfying (26): $p_j^x(p_e) = p_j^*(p_e)$.
8. The energy price, p_e, satisfies (39).

While these optimal outcomes are unique, the taxes that deliver them are not. We focus on a set that is easy to describe and potentially practical to administer.[19] In particular, we consider (1) an extraction tax, t_e, set at the standard Pigouvian rate φ, (2) a border adjustment, t_b, and (3) an export policy based on Home's comparative advantage in producing goods. The border adjustment is imperfect or partial in that it need not be at the same rate as the underlying extraction tax (and often it will be at a lower rate) and it applies only to imports and exports of energy and imports of goods, but not to the export of goods. Below we illustrate that this set of taxes implements Home's optimal planning outcome.

4.1 Extraction Tax and Border Adjustments

Consider the following two taxes: a tax on extraction, t_e, equal to marginal harm, φ, and a border adjustment on all imports and exports of energy, and on the energy embodied in imports, all at rate λ_e^*:

$$\begin{aligned} t_e &= \varphi, \\ t_b &= \lambda_e^*. \end{aligned} \tag{41}$$

This set of taxes satisfies conditions 1 through 5.

This border adjustment sets the price of energy in Home to $p_e + t_b = p_e + \lambda_e^* = \lambda_e$. If Home imports energy, the border adjustment adds to the world price p_e, bringing it up to $p_e + t_b$. This price is what producers in Home pay and is also the pre-tax price that Home extractors receive. If Home exports energy the border adjustment is paid to energy exporters so that their pre-tax price remains $p_e + t_b$ even as they sell on the world market at price p_e.

Because the border adjustment applies to the energy embodied in imported goods, Foreign producers of these goods will internalize the cost of energy as being $p_e + t_b$. Their energy intensity of production when shipping to Home is therefore equalized to that of Home producers, giving us $z^y = z^m = z^x$. Moreover, the range of goods that Home imports is invariant to these taxes, so $F\left(\bar{j}_m\right) = \frac{1}{\tau^*}$.

[19] Metcalf and Weisbach (2009) mention ways to ease the administrative burden of a carbon tax while and Kortum and Weisbach (2017) discuss the administrative difficulties of border adjustments, some of which we address with the policy proposed here.

Home extractors receive a pre-tax price of $p_e + t_b$ (whether they sell domestically or export), and they pay an extraction tax, $t_e = \varphi$. The after-tax price received by Home's extractors is thus:

$$p_e + t_b - t_e = p_e + \lambda_e^* - \varphi.$$

Therefore, the energy extraction in Home satisfies (33), inducing the optimal Q_e.[20] While we have not yet determined the optimal level of the border adjustment, we have determined the optimal wedge between the price that Home's producers pay for energy and the price that its extractors receive. That wedge is the extraction tax, which equals the marginal damages from global emissions, φ.[21]

4.2 Export Taxes and Subsidies

4.2.1 Levels

Conditions 1, 6 and 7 determine Home's export policy. Condition 1 requires that producers in Home selling in Foreign face the same energy price as when they sell domestically. This means that the border adjustment should not apply to Home's exports. That is, it is optimal for Home to give its exporters the incentive to produce at lower energy intensity rather than rebating taxes on export.

To meet conditions 6 and 7, Home provides exporters of each good j with a per unit tax or subsidy, depending on Home's comparative advantage in producing that good. To derive the subsidy and tax, first solve for the good $j_0 \in (0, \bar{j}_x)$ that Home exports at a cost exactly equal to the price it charges:

$$\tau a_{j_0} (p_e + t_b)^{1-\alpha} = a_{j_0}^* p_e^{1-\alpha},$$

so that j_0 satisfies:

$$F(j_0) = \tau \left(\frac{p_e + t_b}{p_e} \right)^{1-\alpha}.$$

Exporters could not normally sell any goods, $j \in (j_0, \bar{j}_x]$, because their costs would exceed the price. To meet the optimal export margin, Condition

[20]It's possible to have $p_e + t_b \leq \varphi$ in which case Home's extraction sector shuts down.

[21]Markusen (1975) obtains the same result, refering to it as a production tax rather than an extraction tax as we do. His taxes are ad valorem while ours are specific.

6, Home must subsidize all of these goods in an amount equal to the losses producers would otherwise incur:

$$s_j^x = \tau a_j (p_e + t_b)^{1-\alpha} - a_j^* p_e^{1-\alpha} = \left(\tau \frac{a_j}{a_j^*} \left(\frac{p_e + t_b}{p_e} \right)^{1-\alpha} - 1 \right) p_j^*.$$

For goods $j \in [0, j_0)$, Home has stronger comparative advantage relative to Foreign. Exporters would normally sell at their cost rather than $p_j^*(p_e)$ as required. To induce these exporters to sell at $p_j^*(p_e)$, (Condition 7), Home imposes a per-unit tax at a rate of $t_j^x = -s_j^x$.

4.2.2 Cost

Integrating over all the goods that Home exports, the tax revenue net of the subsidy turns out to be Home's surplus from exporting, which first appeared as (40):

$$\begin{aligned} &\tau \int_0^{j_0} t_j^x x_j(p_e) dj - \tau \int_{j_0}^{\bar{j}_x} s_j^x x_j(p_e) dj \\ &= \eta^* \int_0^{\bar{j}_x} \left(1 - \tau \frac{a_j}{a_j^*} \left(\frac{p_e + t_b}{p_e} \right)^{1-\alpha} \right) \left(p_j^* \right)^{1-\sigma^*} dj \\ &= V_g^{FH} - (p_e + t_b) \frac{C_e^{FH}}{1-\alpha} = \Pi_g. \end{aligned} \tag{42}$$

4.2.3 Crosshauling

The subsidies for exports create the possibility for crosshauling if iceberg costs are sufficiently low. To see why, note that if $t_b > 0$ (which we discuss below),

$$(p_e + t_b)^{1-\alpha} < p_e^{1-\alpha} + (1-\alpha) p_e^{-\alpha} t_b.$$

The right-hand side is the first-order expansion of the left-hand side around $t_b = 0$. Rearranging, multiplying both sides by λ_e, and using $\lambda_e = p_e + t_b$,

$$\frac{\left(\frac{p_e + t_b}{p_e} \right)^{1-\alpha}}{1 + (1-\alpha) \frac{t_b}{p_e}} < 1.$$

If iceberg costs are close enough to 1, this inequality will continue to hold even if the left-hand side is multiplied by the product of the iceberg costs, $\tau\tau^*$. In this case:

$$F(\bar{j}_x) < \frac{1}{\tau^*} = F(\bar{j}_m).$$

Since F is monotonically decreasing, it follows that $\bar{j}_m < \bar{j}_x$. Under these conditions of low trade costs there are goods that Home simultaneously imports and exports. The optimal policy can imply crosshauling.

The economic rationale for crosshauling is that Home controls energy intensity not only for all production in Home, but also for production in Foreign that Home imports. In contrast, goods produced in Foreign, for consumption there, escape Home's climate policy.

Increased trade gives Home more control over the use of energy, helping it to lower global emissions. In particular, if trade costs are low enough, Home is willing to supply a range of goods to Foreign at a price below the shadow price of those goods to Home consumers, which is the rationale for the subsidy s_j^x.

To illustrate, consider a good j for which $a_j = a_j^* = a$, and assume there are no trade costs at all. With $t_b = 0$, Home would be indifferent between exporting this good or having Foreign produce it for itself. But, with $t_b > 0$ global energy use is reduced if the good is produced in Home and exported to Foreign. The energy requirement is $a\,(1-\alpha)\,(p_e + t_b)^{-\alpha}$ which is less than if Foreign produced for itself, with energy requirement $a\,(1-\alpha)\,p_e^{-\alpha}$ per unit produced. On the other hand, Home is indifferent between importing this good or producing it for itself. In either case both the cost of production and the energy content are the same because Home controls the energy content of its imports.

Trade costs mute this effect. With high enough trade costs, $F\left(\bar{j}_x\right) > F\left(\bar{j}_m\right)$. The inherent inefficiency of crosshauling overcomes its advantage in reducing global emissions.

4.3 Optimal Border Adjustment

The final component of the tax system is the level of the border adjustment. We can employ equation (39) for that purpose, substituting in $t_b = \lambda_e^*$, and rearranging to get:

$$t_b = \frac{Q_e^* - C_e^{FF}}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e} + \varphi\frac{\partial Q_e^*/\partial p_e}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e} - \frac{\partial \Pi_g/\partial p_e}{\partial\left(Q_e^* - C_e^{FF}\right)/\partial p_e}, \quad (43)$$

The first term captures terms-of-trade manipulation, the second captures Home's response to shifts in where extraction occurs and the third captures the net cost of Home's export subsidy.

This formula (43) is a generalization of the result in Markusen (1975).[22] We will consider the three terms on the right hand side in turn. It is convenient to first express it in terms of elasticities, as:

$$t_b = \frac{p_e\left(Q_e^* - C_e^{FF}\right)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} + \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} - \frac{-(1-\epsilon_D^*)V_g^{FH} + \sigma^* C_e^{FH}}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}.$$

Note that the denominator is always positive.

4.3.1 No Global Externality or Trade in Goods

With no global externality we have $\varphi = t_e = 0$ and with infinite iceberg costs the last term vanishes. Only the first term in (43) remains. Furthermore, with no trade in goods, the numerator becomes Foreign's net exports of energy, and the denominator is its derivative:

$$t_b = \frac{X_e^*}{\partial X_e^* / \partial p_e} = \frac{p_e X_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}.$$

Equation (43) reduces to the classic inverse elasticity formula for the optimal trade tax.

Suppose the numerator is positive so that Home is a net importer of energy. The optimal policy has Home impose a positive border adjustment $t_b > 0$. The tax on energy imports, lowers the world price of energy p_e, improving Home's terms of trade.

If the numerator is negative so that Home is a net exporter of energy, Home will tax exports with $t_b < 0$. Doing so raises the world energy price, raises demand for energy in Home (which is not subject to the tax), and improves Home's terms of trade. With no global externality there is no presumption that the border adjustment is positive.

[22] He refers to it as an optimal trade tax as opposed to a border adjustment. Without trade in differentiated goods, Markusen's formula doesn't have the last term.

4.3.2 Global Externality but No Trade in Goods

Now consider marginal damages from the global externality of $\varphi > 0$. The first two terms in (43) remain:

$$t_b = \frac{X_e^*}{\partial X_e^*/\partial p_e} + \varphi \frac{\partial Q_e^*/\partial p_e}{\partial X_e^*/\partial p_e} = \frac{p_e X_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} + \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}}.$$

We focus on the second, having discussed above how the trade tax can be used to improve Home's terms of trade. The second term will determine how the optimal border adjustment depends on the Pigouvian extraction tax of t_e, set equal to marginal damages.

This second term, which multiplies the cost of carbon φ, is bounded between zero and one:

$$0 \leq \frac{\partial Q_e^*/\partial p_e}{\partial X_e^*/\partial p_e} = \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} < 1.$$

Thus, the externality on its own (treating $X_e^* = 0$) will lead Home to impose a partial border adjustment in that the rate that is lower than the underlying extraction tax:

$$0 \leq t_b = t_e \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^{FF}} < t_e.$$

The numerator reflects what might be thought of as extraction leakage. Home's tax on extraction increases p_e. In response, Foreign will extract more, as reflected in $\epsilon_S^* Q_e^*$, causing harm to Home of $\varphi \epsilon_S^* Q_e^*$. Increasing t_b moderates this effect because it lowers p_e: a higher t_b gives a higher net-of-tax price to Home's exporters (so expands global energy supply) and makes the use of energy more costly (so lowers global energy demand).

The denominator is the sum of extraction leakage, as just discussed, and demand-side effects. As we increase t_b, the price of energy goes down, resulting in an increase in Foreign consumption of energy: $\epsilon_D^* C_e^{FF}$. (Note that the demand side effect only includes C_e^{FF} because C_e^{FH}, C_e^{HF}, and C_e^{HH} are all controlled through Home's policy.) The border adjustment has to balance these effects.

To illustrate, if extraction in Foreign is very responsive to the price of energy while demand is inelastic, Home's optimal border adjustment will approach the level of the Pigouvian extraction tax. The reason is that a higher border adjustment lowers the world energy price, reducing Foreign's extraction of energy, while in this case not inducing much extra demand in

Foreign. On the other hand, if Foreign extraction is quite inelastic while demand is elastic, Home's border adjustment will approach 0. The reason is that a lower border adjustment raises the world price of energy, reducing Foreign demand, while in this case not inducing much extra extraction in Foreign.

4.3.3 General Case

Allowing for trade in goods means that we need to take account of all three terms in (43), the last of which captures Home's concern with its surplus in exporting goods. Furthermore, the first two terms no longer depend only on Foreign's net energy exports, but on its energy extraction less the energy it uses in supplying domestic goods consumption.

The third term in (43) reflects Home's market power in exporting goods together with its use of goods exports to broaden the scope of its carbon policy. For example, if $\partial\Pi_g/\partial p_e > 0$, Home's surplus from the export market goes up with a higher energy price, so it lowers t_b.

4.4 Recap

Let's summarize what we've achieved by interpreting the optimal policy in terms of this particular set of taxes and subsidies. Home's optimal policy will be an extraction tax $t_e = \varphi$ combined with a border adjustment t_b on energy trade, including the energy content of Home's imports of goods. The border adjustment leaves goods imports unchanged from BAU. The border adjustment is incomplete because Home's energy taxes are not removed on exports of goods. Rather, Home imposes a set of subsidies and taxes per unit exported, without regard to the energy content. Exports are subsidized for goods in which Home's comparative advantage is weak and taxed for goods in which Home's comparative advantage is strong. The subsidy can potentially be large enough that Home exports and imports the same good. The next section proposes a strategy for quantitative evaluation.

5 Quantitative Illustration

We now turn to the quantitative implications of the optimal unilateral policy. We pursue a strategy, based on Dekle, Eaton, and Kortum (2007), in which we calibrate the BAU competitive equilibrium to data on global carbon flows. We then compute the optimal unilateral policy relative to this baseline. Doing so greatly reduces the number of parameters we need to choose.

To proceed we first need to specify the comparative advantage curve $F(j)$ and the energy fields, $G(a)$ and $G^*(a)$. We then calibrate the baseline BAU competitive equilibrium to data on energy extraction and global carbon flows. From this base, we compute the taxes and subsidies that would shift the model economy to the outcomes dictated by the optimal unilateral policy. We also compare the BAU and optimal policies to more conventional policies such as pure extraction, production, and consumption taxes. We start by providing some of the details of the simulation (with most of the derivations relegated to the Appendix), and then present our key results.

5.1 Setup

5.1.1 Functional Forms.

To solve the model numerically we employ several convenient functional forms.

Comparative Advantage We parameterize the efficiency of the goods sector in each country by:

$$a_j = \left(\frac{j}{A}\right)^{1/\theta}, \tag{44}$$

$$a_j^* = \left(\frac{1-j}{A^*}\right)^{1/\theta}, \tag{45}$$

where A and A^* determine absolute advantage in either country, and θ determines (inversely) the scope of comparative advantage. Taking the ratio, we obtain a functional form for Home's comparative advantage curve:

$$F(j) = \frac{a_j^*}{a_j} = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta},$$

where $F(j)$ is continuous and strictly decreasing as specified in (5).

This functional form allows us to easily solve for the import and export thresholds in the BAU. In the BAU baseline a country's average spending per good doesn't depend on the source of the good. Since the share of energy in the cost of any good is the same, the baseline value of the import threshold is:

$$\bar{j}_m = \frac{C_e^{HH}}{C_e} = \frac{A}{A + (\tau^*)^{-\theta} A^*},$$

while the baseline value of the export threshold is:

$$\bar{j}_x = \frac{C_e^{FH}}{C_e^*} = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*}.$$

Energy Supply We parameterize the upper end of the distribution of costs across energy fields, for all $a \geq \underline{a}$, as:

$$G(a) = Ga^{\epsilon_S}, \tag{46}$$

$$G^*(a) = G^* a^{\epsilon_S^*}, \tag{47}$$

where G and G^* are parameters governing the quantity of energy in each region while ϵ_S and ϵ_S^* are the constant elasticities of supply at the upper ends of each distribution.[23]

Normalizing the baseline energy price to 1 in the BAU competitive equilibrium, we have $Q_e = G$ and $Q_e^* = G^*$.

5.1.2 Calibration of BAU Scenario.

We calibrate the BAU scenario to carbon accounting data for 2015 from the Trade Embodied in CO_2 (TECO$_2$) database made available by the OECD.[24] Units are gigatonnes of CO_2. Energy extraction data for 2015 is from the International Energy Agency World Energy Statistics Database. We use emissions factors to convert units of energy to units of CO_2.

For most of our results, members of the OECD form the taxing region, Home, and the non-OECD countries are Foreign. Table 3 provides our calibration.

Note that by this CO_2 metric the OECD represents about one-third of the world. It represents a smaller share of extraction and a larger share of implicit consumption, nearly twenty percent of which is imported.

[23] The distributions are unrestricted for $a < \underline{a}$. The threshold $\underline{a}$, however, must be below the after-tax energy price arising under the optimal unilateral policy.

[24] The values that we take from TECO$_2$ are broadly consistent with those available from the Global Carbon Project.

Table 3: Baseline Calibration for Home as the OECD

	Home	Foreign	Total
Home	$C_e^{HH} = 11.3$	$C_e^{HF} = 2.5$	$C_e = 13.8$
Foreign	$C_e^{FH} = 0.9$	$C_e^{FF} = 17.6$	$C_e^* = 18.5$
Total	$I_e = 12.2$	$I_e^* = 20.1$	$I_e^W = C_e^W = 32.3$
Extraction	$Q_e = 8.6$	$Q_e^* =$23.7	$Q_e^W = 32.3$

Table 4: Parameter Values

α	ϵ_S	ϵ_S^*	σ	σ^*	θ
0.85	0.5	0.5	1	1	4

In addition to the carbon accounting data, we need values for six parameters: θ, ϵ_S, ϵ_S^*, σ, σ^*, and α, the last three of which determine the demand elasticities, ϵ_D and ϵ_D^*.[25] These parameters remain fixed as we compute the optimal policy. We determine values for them using a variety of sources.[26] Table 4 lists our central values for these parameters. (In our simulations, we test for sensitivity to parameter choice, so in some cases, we allow the parameters to vary.) Appendix E provides details on our calibration procedure.

[25]The eight other parameters: A, A^*, G, G^*, η, η^*, τ, and τ^* are all subsumed by calibrating to the carbon accounts.

[26]We choose $\alpha = 0.85$ based on the ratio of the value of energy used in production to value added. (In our model that ratio is $(1-\alpha)/\alpha$.) Values for ϵ_S and ϵ_S^* are estimated using data in Asker, Collard-Wexler, and De Loecker (2018), by fitting the slope of $G(a)$ and $G^*(a)$ among oil fields with costs above the median. Appendix E provides more details. We take $\theta = 4$ based on the preferred estimate in Simonovska and Waugh (2014). The values for σ and σ^* are interim.

5.1.3 From BAU to Optimal

For any endogenous variable x we denote the BAU baseline value by x and the value under the optimal unilateral policy by $x(p_e, t_b)$. In the baseline $t_b = 0$, $t_e = 0$, and the energy price is 1. The magnitude of the tax rates under the optimal policy can be interpreted in the ad-valorem sense, relative to the initial energy price. The optimal unilateral policy requires that we set the extraction tax to $t_e = \varphi$.

For example, under the optimal unilateral policy, the import threshold remains fixed while the export threshold changes to:

$$\bar{j}_x(p_e, t_b) = \frac{\tau^{-\theta} A \left(p_e + (1-\alpha)\, t_b\right)^{\theta}}{\tau^{-\theta} A \left(p_e + (1-\alpha)\, t_b\right)^{\theta} + A^* \left(p_e^{\alpha} \left(p_e + t_b\right)^{1-\alpha}\right)^{\theta}}$$

$$= \frac{\left(p_e + (1-\alpha)\, t_b\right)^{\theta} C_e^{FH}}{\left(p_e + (1-\alpha)\, t_b\right)^{\theta} C_e^{FH} + \left(p_e^{\alpha} \left(p_e + t_b\right)^{1-\alpha}\right)^{\theta} C_e^{FF}}.$$

The second line shows what we achieve by calibrating to the carbon accounts. Energy extraction, for $p_e + t_b - \varphi \geq \underline{a}$, is:

$$Q_e(p_e, t_b) = (p_e + t_b - \varphi)^{\epsilon_S}\, Q_e.$$

Using similar reasoning we can express the values under the optimal unilateral policy for each component of energy demand and for the value of goods trade. See Appendix D for details.

We express the change in welfare relative to Home's baseline spending on goods:

$$W = \frac{U(p_e, t_b) - U}{V_g}.$$

5.1.4 Constrained Optimal Policies

To understand the optimal policy, it is useful to compare it to more conventional policies. We will consider four conventional policies, a pure extraction tax, a pure production tax, a pure consumption tax, and a hybrid consisting of a mix of extraction and consumption taxes. In each case, we find Home's optimal policy assuming it is constrained to a particular choice (e.g., we find Home's optimal extraction tax conditional on Home being constrained to choosing only an extraction tax).

Pure Extraction Tax If Home can only choose an extraction tax, it can only choose Q_e, X_e, and p_e, with all other outcomes determined in a decentralized equilibrium. We solve the Lagrangian for these values and reinterpret the outcome as a decentralized equilibrium. The optimal extraction tax in this case is:

$$t_e = \varphi \frac{\epsilon_D C_e + \epsilon_D^* C_e^*}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*} - \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*}. \tag{48}$$

Ignoring the second term, this rate is below the value of $t_e = \varphi$ in the optimal unilateral policy. How much below turns on the value of $\epsilon_S^* Q_e^*$. If Foreign is a major energy extractor and if its price elasticity of supply is high, then Home will want to choose a lower extraction tax.

Turning to the second term, note that the numerator is the value of Foreign's net exports of energy based on its implicit consumption of embodied energy. Its use of energy in production doesn't matter here. If Foreign is an exporter in this sense then Home wants a lower extraction tax to improve its terms of trade. For the same reason, it will choose a higher extraction tax if Foreign is a large net importer in this sense.

Pure Consumption Tax For a pure consumption tax, we follow the same procedure except now the planner is constrained to choose only: $\left\{z_j^y\right\}$, $\left\{z_j^m\right\}$, $\{y_j\}$, $\{m_j\}$, X_e, and p_e, with all other outcomes determined as in a decentralized competitive equilibrium. Using the same procedure, the optimal consumption tax is

$$t_c = \varphi \frac{\epsilon_S Q_e + \epsilon_S^* Q_e^*}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} + \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{49}$$

We can equivalently impose a consumption tax by imposing an extraction tax at that rate and a full border adjustment, $t_e = t_b$, on all imports and exports of energy and goods. Unlike the optimal unilateral policy, since the border adjustment applies to exports of goods, these goods bear no tax.

Optimal Hybrid In our hybrid tax, we allow Home to combine extraction taxes and consumption taxes. Solving for the optimal mix is the same as for a pure consumption tax, replacing the competitively determined Q_e with the planner's choice. The optimal tax in this case is:

$$t_e = \varphi$$

together with a border tax of:

$$t_b = \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} + \frac{p_e \left(Q_e^* - C_e^* \right)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{50}$$

Since its a partial border adjustment, unlike the pure consumption tax, the net-of-tax price received by Home's energy extractors is $p_e + t_b - \varphi$.

Pure Production Tax and Hybrids We find the optimal pure production tax and hybrids involving production taxes numerically.

5.1.5 Solving for Equilibrium Values

The equilibrium energy price p_e clears the market:

$$C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b) + C_e^{FF}(p_e, t_b) + C_e^{HF}(p_e, t_b) = Q_e(p_e, t_b) + Q_e^*(p_e), \tag{51}$$

while the optimal border adjustment t_b satisfies:

$$t_b = \frac{\varphi \epsilon_S^* Q_e^*(p_e) - p_e C_e^{FF}(p_e, t_b) - (p_e + t_b)\sigma^* C_e^{FH}(p_e, t_b) - (1 - \epsilon_D^*) V_g^{FH}(p_e, t_b)}{\epsilon_S^* Q_e^*(p_e) + \epsilon_D^* C_e^{FF}(p_e, t_b)}. \tag{52}$$

We iterate between (51) and (52) until we find a pair (p_e, t_b) that satisfies both. We can then evaluate any outcome of the model at this pair to explore the optimal unilateral policy. We use a similar procedure for the optimal constrained policies.

Our script is in Matlab, and we use the solving procedure described above rather than a built-in solver. Our code is available at `https://github.com/dweisbach/Optimal-Unilateral-Carbon-Policy`.

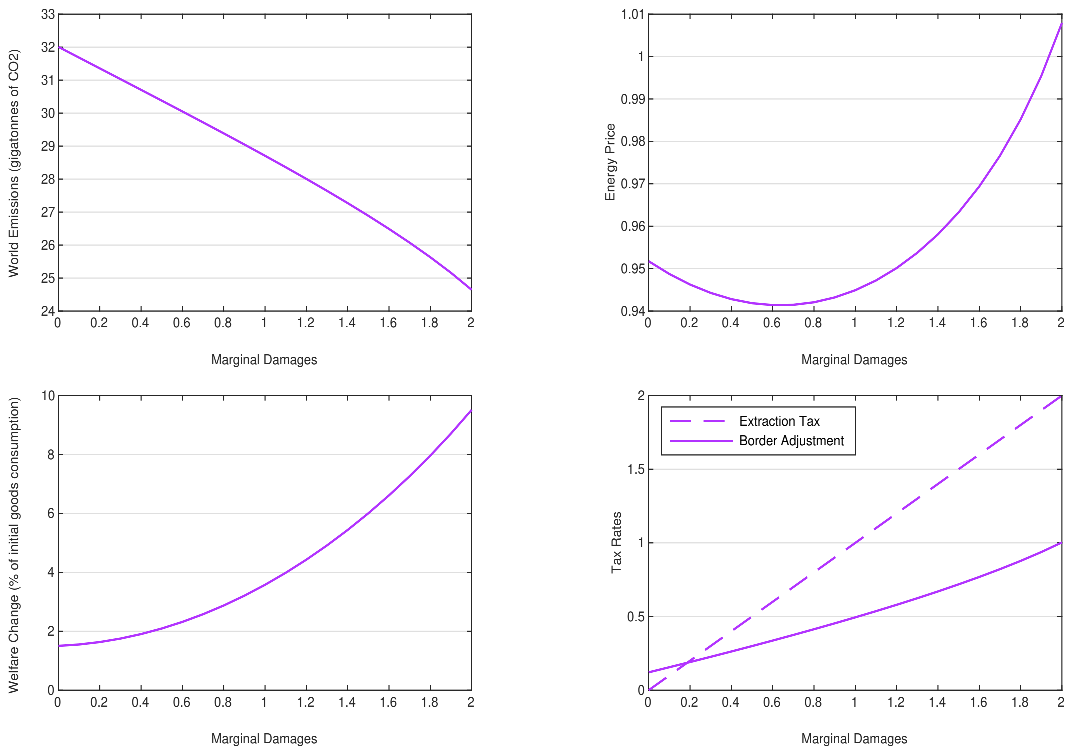
5.2 Simulations

5.2.1 Optimal Policy

We begin with a simulation of the optimal policy in the OECD (Figure 1). We illustrate the policy for the marginal harm ranging from $\varphi = 0$ to $\varphi = 2$. We show the emissions reductions, the change in welfare, the change in the price of energy, and the tax rates under the optimal policy.

Global emissions go down by almost 1/3 with $\varphi = 2$. This is a substantial reduction given that emissions in the OECD are only about 1/3 of global

Figure 1: Optimal Policy in the OECD



emissions. Note that this does not mean that the OECD reduces its emissions to near zero. As we will discuss, some of the reductions arise outside the OECD because of how the optimal policy expands the carbon price to trading partners.

The extraction tax rate (bottom right) is always equal to φ. Recalling that the tax rate can be interpreted in the ad-valorem sense, the optimal tax rates range from 0 to up to twice the initial price of energy.

For values of φ close to zero, the border adjustment is still positive and actually exceeds the extraction tax rate. This policy arises because the OECD has a deficit in energy ($Q_e = 8.6$ and $I_e = 12.2$). The border adjustment hits energy imports, raising the price faced by energy users in the OECD, stimulating energy extraction there, and lowering the price of energy on the world market. The price of energy (pre-border adjustment) falls from 1 to 0.95, an improvement in the OECD's terms of trade. For the same reason, welfare goes up even when there is no need for a climate policy because $\varphi = 0$.

As φ and the extraction tax go up, the two lines cross. When $\varphi = 2$ the border adjustment is just half the value of φ. Energy extractors are thus bearing a large share of the carbon tax. The OECD's policy, however, still pushes the energy price below 1 until φ and the extraction tax approach 2. At this extreme, the net price received by energy extractors in the OECD, $p_e + t_b - t_e$, approaches zero.

To further examine the features of the optimal policy, we present four simulations.

5.2.2 Coalition Size

We start by examining a key factor in global climate negotiations: which countries are in the taxing coalition. As noted, one of the major criticisms of the Kyoto Protocol was that it left major emitters out of the coalition required to adopt carbon policies. The Paris Agreement was, in part, designed to address this criticism.

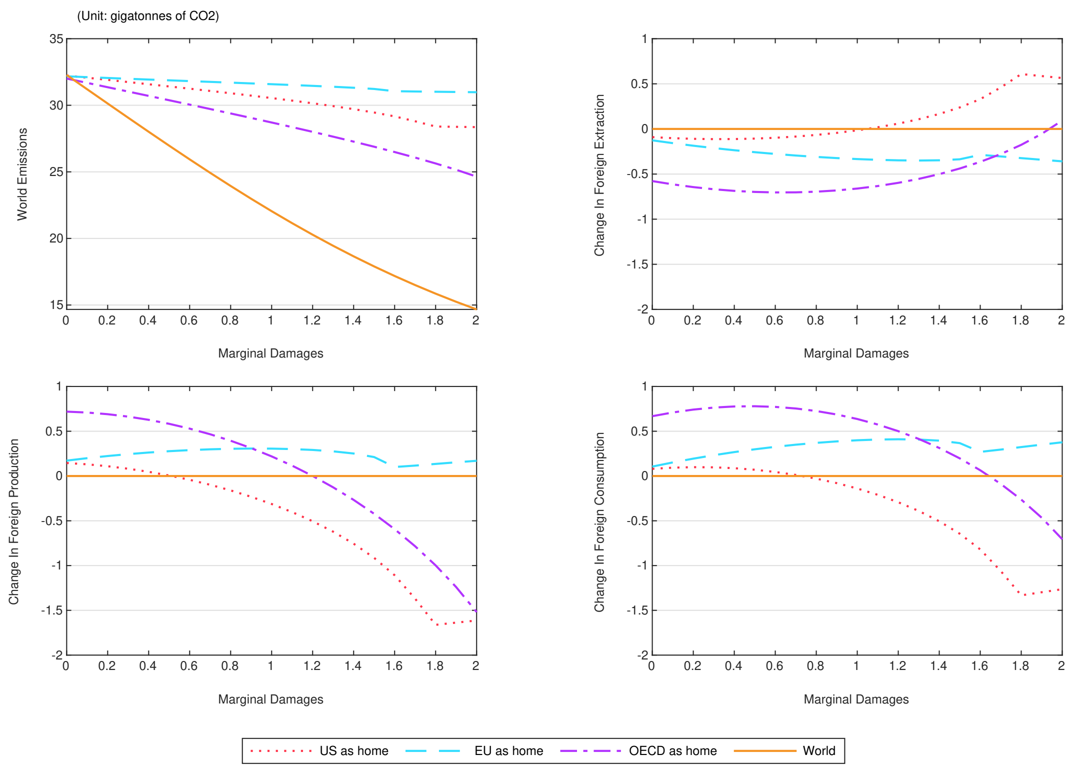
To examine the effects of coalition size, the top left panel of Figure 2 shows the effects on global emissions of optimal unilateral policies assuming that only the European Union (EU), the United States, or the OECD imposed a carbon policy. It compares those three coalitions with the optimal global policy (which reduces to an extraction tax equal to marginal harm).[27] The calibration for the EU is given in Table 5 and for the US in Table 6.All other parameters remain the same across each case.

In each case, the carbon policy is set optimally given the taxing coalition and the marginal harm. Note that because the value of φ scales with the coalition's population, it is misleading to fix φ in comparing results across the different cases. If, for example, $\varphi = 1$ for half the world, it would need to be in the range of 2 for the whole world to represent equal harm per unit of energy. In addition, because Figure 2 shows the optimal policy for each coalition, the tax and subsidy rates for the different coalitions will not, in general, be the same for any given level of marginal harm.

The global policy achieves much larger emissions reductions than the policy in any of the narrower coalitions (even without adjusting for a larger φ appropriate for the whole world). With $\varphi = 2$, the global tax reduces

[27] We treat the global case as the limit of our two-region model as Foreign becomes infinitesimally small. The EU is treated as having 28 members as it had, prior to Brexit, in 2015.

Figure 2: Choice of Pricing Coalition



emissions by 17.6 Gt CO_2, compared to about an 8.7 Gt CO_2 reduction if the carbon policy only applied in the OECD countries. A carbon policy only in the EU is largely ineffective, reducing emissions by only 1.3 Gt CO_2.

These effects are driven at least in part by the size of the coalition, with the size of the coalition measured by the size of the tax base, not by GDP. The GDP of the EU is roughly the same as the GDP of the United States (and its population is larger), but a tax in the United States is much more effective at reducing global emissions because the tax base in the United States is much larger. As can be seen by comparing Tables 5 and 6, the United States extracts much more than the EU (Q_e of 4.5 versus 1.0),uses more energy to produce goods ((I_e of 5.0 versus 3.5) and consumes more energy (C_e of 5.8 versus 4.0).

An additional reason that the size of the coalition might matter is that the larger the coalition, the smaller the possible leakage. At the limit, with

Table 5: Calibration for Home as the European Union

	Home	Foreign	Total
Home	$C_e^{HH} = 3.0$	$C_e^{HF} = 1.0$	$C_e = 4.0$
Foreign	$C_e^{FH} = 0.5$	$C_e^{FF} = 27.8$	$C_e^* = 28.3$
Total	$I_e = 3.5$	$I_e^* = 28.8$	$I_e^W = C_e^W = 32.3$
Extraction	$Q_e = 1.0$	$Q_e^* =$31.3	$Q_e^W = 32.3$

a global coalition, there can be no leakage. The other three panels in Figure 2 explore this issue, showing the change in Q_e^*, I_e^*, and C_e^* for each of the different coalitions. For the most part, Foreign extraction goes down, not up, reflecting what might be thought of as "negative" extraction leakage. (The kinks in the curves for the EU and the US reflect the point at which the policy pushes Q_e to 0.) The change in I_e^* is often positive, indicating production leakage. Note, however, that the change in I_e^* for the EU tax, while positive is less than half a gigaton, which means that production leakage is not the explanation for the poor effectiveness of the EU-only policy.[28] Foreign production goes down for the US and OECD policies for high values of φ, once again reflecting negative leakage, this time with respect to production. Finally, C_e^* goes up, except for the US-only tax for values of φ above about 1 and for the OECD tax for values of φ above about 1.6.

5.2.3 Extraction and Consumption Taxes

Figure 3 compares four different carbon policies in the OECD: the optimal policy, a pure extraction tax (48), a pure consumption tax (49), and the optimal mix of extraction and consumption taxes (50). The extraction tax

[28]It may be possible to argue that the poor performance of the EU reflects the force of leakage, even though the resulting leakage is small, since the optimal policy tries to avoid leakage and in doing so sacrifices some of its effect on global emissions. We find it more direct to simply point out that the optimal policy tries to maximize welfare in the taxing region.

Table 6: Calibration for Home as the United States

	Home	Foreign	Total
Home	$C_e^{HH} = 4.6$	$C_e^{HF} = 1.2$	$C_e = 5.8$
Foreign	$C_e^{FH} = 0.4$	$C_e^{FF} = 26.1$	$C_e^{*} = 26.5$
Total	$I_e = 5.0$	$I_e^{*} = 27.3$	$I_e^{W} = C_e^{W} = 32.3$
Extraction	$Q_e = 4.5$	Q_e^{*} =27.8	$Q_e^{W} = 32.3$

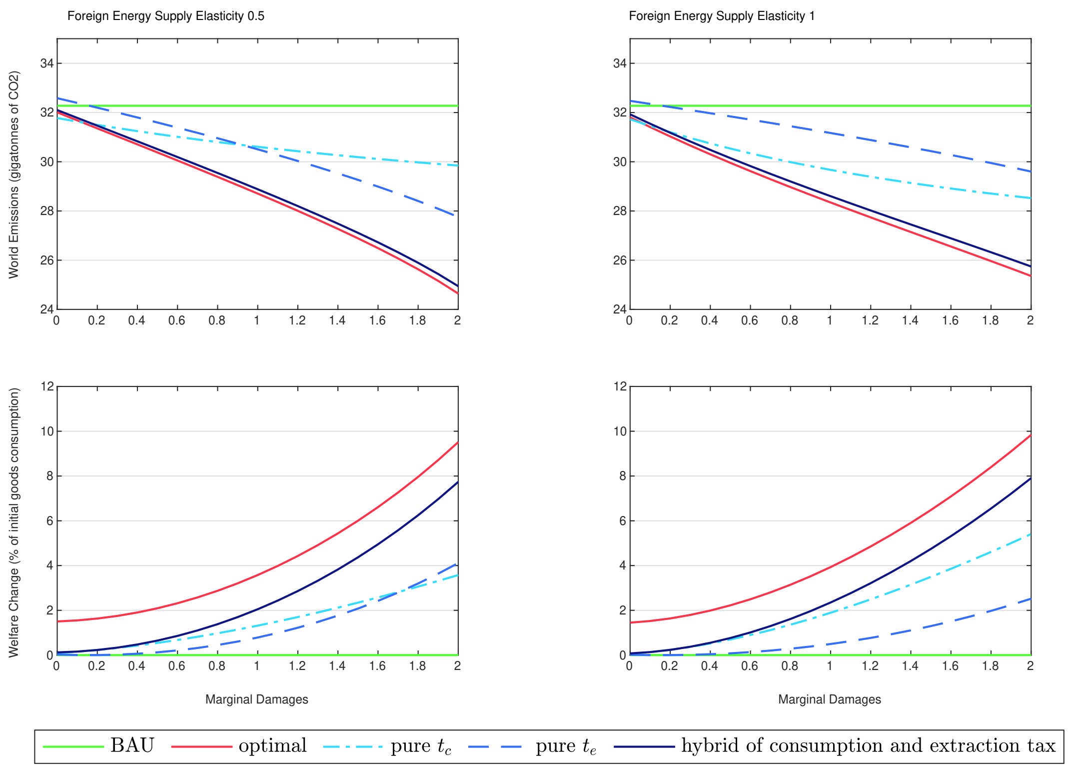
and the consumption taxes represent the two extreme cases: all the way upstream and all the way downstream. Figure 2 shows the results for two values of the supply elasticity in Foreign, our base estimate of $\epsilon_S^* = 0.5$ and an alternative, higher value of $\epsilon_S^* = 1$.

As can be seen, the pure extraction and pure consumption taxes do poorly compared to the optimal tax. They achieve only about half the emissions reductions for any given value of marginal harm. Since they are derived by putting constraints on what the planner can choose, the pure taxes do not perform as well at improving Home's welfare as the optimal tax.

Notably, the OECD would choose to impose a significant carbon policy even when the rest of the world does not. Emissions in the OECD are 12.2Gt CO_2, about a third of global emissions (as reflected in the value of I_e in Table (3)). For $\varphi = 2$, the optimal carbon policy reduces global emissions by 8.7 Gt CO_2, and as Figure 2 shows, only about 1.5 Gt of these reductions are from I_e^*. The OECD would choose similar, though more modest, policies if it is constrained to choosing between a pure extraction or consumption taxes. That the OECD would choose these policies on its own may have important implications for the design of climate negotiations: even if one or more countries hold out, it makes sense for the remaining countries to impose a substantial carbon price.

The choice between pure extraction and consumption taxes depends on Foreign's supply elasticity. For low values, an extraction tax reduces emissions more effectively than a consumption tax does (though they perform

Figure 3: Extraction and Consumption Taxes



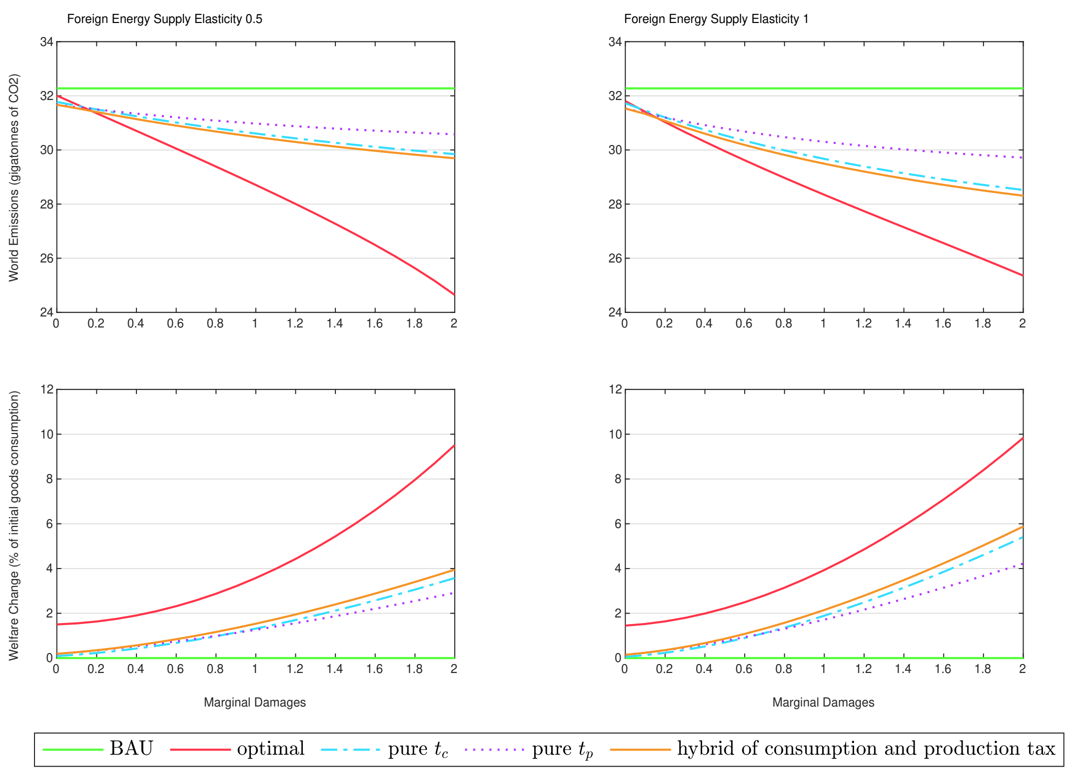
about the same with respect to Home's welfare). With a high elasticity, a consumption tax does better. The reason that the value of ϵ_S^* changes how the taxes perform is that the two taxes have different effects on p_e. Extraction taxes increase p_e while consumption taxes decrease it. When ϵ_S^* is high, increasing p_e generates a larger response in Foreign, which means that the extraction leakage generated by an extraction tax is more of a problem when ϵ_S^* is high. Conversely, because consumption taxes lower p_e, they do better when ϵ_S^* is high.

The hybrid of extraction and consumption taxes does much better than either tax alone. While all three are affected by the value of ϵ_S^*, the hybrid appears less sensitive to ϵ_S^* than either of the pure taxes. While the hybrid tax performs almost as well as the optimal tax in reducing emissions, the cost is higher in the sense that for a given level of emissions reductions, the welfare gains are smaller with the hybrid tax than with the optimal tax. In part this reflects the aggressive pricing of goods exports by Home under the optimal policy.

5.2.4 Production and Consumption Taxes

Actual carbon prices are typically imposed on production. In addition, most studies of unilateral carbon taxes assume that the tax is imposed on production and examine the effects of adding border adjustments. Figure 4 analyzes this set of trade-offs, comparing those taxes to the optimal tax, and an optimally-set hybrid of production and consumption taxes (which we can think of as partial border adjustments). As with Figure 3, Figure 4 shows those results with $\epsilon_S^* = 0.5$ and $\epsilon_S^* = 1$. (Note that the optimal tax and the consumption tax in Figure 4 are the same as in Figure 3.)

Figure 4: Production and Consumption Taxes



Production taxes and consumption taxes perform about the same in this simulation, with consumption taxes having a slight advantage which grows when $\epsilon_S^* = 1$. These results suggest that adding border adjustments to production taxes does not significantly add value, particularly when ϵ_S^* is low. If, as Kortum and Weisbach (2017) suggest, border adjustments are difficult to administer, the gains may not be worth the administrative costs.

The hybrid between production and consumption taxes does not noticeably improve the outcomes. Instead, the gains from a hybrid tax appear to come from adding an extraction tax to the mix. This is likely because extraction taxes operate differently on p_e than either production or consumption taxes. Combining production and consumption taxes does not allow the taxing region to moderate the effects of the tax on p_e, and, therefore, does not generate substantial gains.

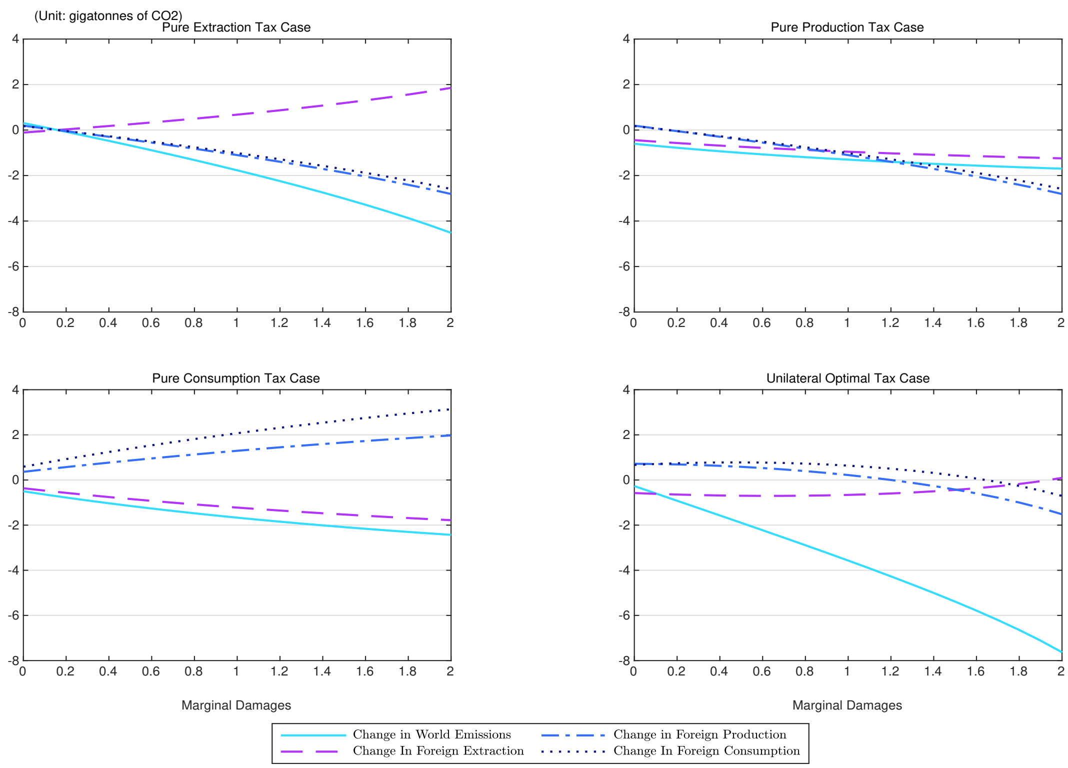
5.2.5 Location

As we noted at the start, a central concern in the design of unilateral carbon prices has been the effects of unilateral taxes on the location of activities, such as causing manufacturing to move abroad. Figure 5 examines this issue, showing how taxes affect location (as measured by activity in the non-taxing region) of extraction, Q_e^*, energy intermediates in production, I_e^*, and implicit consumption of energy, C_e^*. It illustrates these effects for the same taxes used in Figure 3, the optimal tax, an extraction tax, a consumption tax, and a hybrid of those two, with $\epsilon_S^* = 0.5$ (our base calibration). We focus on the effects in Foreign and show, for reference, the change in global emissions. Because we are showing changes from the baseline in Q_e^*, I_e^*, and C_e^*, the units are all carbon dioxide and can be compared.

As expected, due to their differing effects on p_e, the extraction and consumption taxes have diverging effects on Foreign extraction. Extraction taxes increase Foreign extraction while consumption taxes decrease Foreign extraction. Similarly, their effects on production and consumption go in opposite directions. Because they raise p_e (for all producers and consumers), extraction taxes reduce Foreign production and consumption. Because they lower p_e (for all producers and consumers), consumption taxes increase Foreign production and consumption. The effects get larger as marginal harm gets larger.

The hybrid tax and the optimal tax both resemble an extraction tax in that Foreign extraction goes up and Foreign production and consumption go down as marginal harm gets larger. The effects, however, are muted as compared to a pure extraction tax. That is, it appears to be desirable to use features of the different taxes to reduce location shifts to some extent.

Figure 5: Effects on Location



These simulations are preliminary in that our model abstracts from many features of the economy. Nevertheless, they are suggestive of central ideas in tax system design. The key lessons include: (i) it is important to include countries with a large tax base in the taxing coalition; (ii) even if foreign governments do not cooperate, carbon pricing coalitions may want to enact policies that generate substantial reductions; (iii) adding border adjustments to a production tax generates only modest gains, and (iv) combining taxes with differing effects on the price of energy is likely desirable.

6 Renewable Energy

Up to this point we have assumed that all energy is from fossil fuels. We now consider how the optimal policy changes in the presence renewable energy, which we take to be carbon free and a perfect substitute for users.

With renewables, Home's supply of energy is now the sum of fossil fuels, Q_f, and renewables, Q_r:

$$Q_e = Q_f + Q_r,$$

Endowments of fossil fuels are still summarized by $G(a)$ while the quantity of renewable energy that can be generated at a cost below a in Home is $R(a)$. Foreign is endowed with $G^*(a)$ and $R^*(a)$. Foreign's energy supply curve is thus:

$$Q_e^* = G^*(p_e) + R^*(p_e),$$

while their extraction of fossil fuels is:

$$Q_f^* = G^*(p_e).$$

Global emissions are:

$$Q_f^W = Q_f + Q_f^*.$$

We assume that renewable energy is nontradable, so net exports of energy are the same as net exports of fossil fuel: $X_e = X_f$. We also assume that Home continues to use fossil fuels, thus ruling out a situation in which Home chooses $Q_f = 0$ (if marginal damages are very high) while also importing no energy, $X_e = 0$ (if Home's renewable sector is very efficient).

A final assumption, which we explored in Kortum and Weisbach (2017), is that Home cannot influence the size of the renewables sector in Foreign through its import policy.[29] Home may find it impossible to verify the type of energy used in Foreign to produce goods. If it cannot verify the source of energy used, it cannot condition imports on the use of renewables. Moreover, even if Home can verify the type of energy used in production, if Foreign's renewable sector is sufficiently large, it could simply use renewables for its exports to Home and fossil fuels for its domestically consumed goods. In this case, a requirement that Foreign's exports to Home be produced with

[29] Home can influence Foreign's use of renewables via p_e. The share of renewables in Foreign's energy mix, however, could go either up or down with p_e depending on the shape of $G^*(a)$ and $R^*(a)$ in the relevant range around $a = p_e$.

renewable energy would be futile because it would not change Foreign's, or global, emissions.

Under these assumptions introducing renewables into the planner's optimization turns out to be rather simple. Given the price and shadow values of energy (p_e, λ_e, and λ_e^*) the inner problem, which concerns the production and allocation of an individual good j, is unchanged. The outer problem is the same as in our base case (29), except that it must account separately for labor used in the fossil fuels and renewables sectors.

This difference generates only two changes to the first order conditions and the resulting taxes. First, while Home's optimal supply curve for fossil fuels remains the same

$$Q_f = G(\lambda_e - \varphi),$$

the first order condition for Q_r does not include the marginal damages parameter because renewables do not cause harm:

$$Q_r = R(\lambda_e).$$

As a result, while the extraction tax $t_e = \varphi$ continues to apply to Home's fossil-fuel extractors, producers of renewable energy are not taxed at all. They receive the domestic energy price in Home of $p_e + t_b$ for the energy that they produce. This implicit subsidy to renewables is hidden from users of energy who pay $p_e + t_b$ for either type of energy. Similarly, the energy intensity of goods producers is computed as before, without regard to which type of energy they use. We break the connection between carbon and energy.

Second, in the expression for the optimal border adjustment, the term reflecting marginal harm from Foreign extraction applies only to Q_f^* because Foreign's use of renewables does not generate harm in Home:

$$t_b = \frac{Q_e^* - C_e^{FF}}{\partial \left(Q_e^* - C_e^{FF}\right)/\partial p_e} + \varphi \frac{\partial Q_f^*/\partial p_e}{\partial \left(Q_e^* - C_e^{FF}\right)/\partial p_e} - \frac{\partial \Pi_g/\partial p_e}{\partial \left(Q_e^* - C_e^{FF}\right)/\partial p_e}.$$

The interpretation of the border adjustment remains effectively the same as for (43).

The global market-clearing condition for energy becomes:

$$C_e^W = G(p_e + t_b - \varphi) + R\left(p_e + t_b\right) + G^*(p_e) + R^*(p_e).$$

The demand for energy (the left-hand side) remains unchanged by the addition of renewables, given the price of energy and the border adjustment.

7 Conclusion

While the model in this paper is highly stylized, its simplicity yields analytical insights into the features of an optimal unilateral carbon policy. The main new finding is the extent to which international trade can be exploited to broaden the reach of unilateral carbon policy.

To see whether such effects are of first-order importance, it is critical to push the analysis in a more quantitative direction, extending it to multiple countries and perhaps to multiple periods of time as well. For the first extension, the multi-country model of Eaton and Kortum (2002) retains the Ricardian structure of trade in goods used here while the model of Larch and Wanner (2019) is a natural multi-country generalization of the energy sector. On the second extension, the dynamic analysis in Golosov, Hassler, Krusell, and Tsyvinski (2014) appears amenable to nesting within a multi-country world.

References

Asker, John, Allan Collard-Wexler, and Jan De Loecker (2019), "(Mis) Allocation, Market Power, and Global Oil Extraction," *American Economic Review*, 109, 1568-1615.

Asker, John, Allan Collard-Wexler, and Jan De Loecker (2018), "The Welfare Impact of Market Power: The OPEC Cartel," *Working Paper*, Duke University.

Balis, Kathy, Don Fullerton, and Daniel H. Karney. (2014): "Negative Leakage," *Journal of the Association of Environmental and Resource Economists*, 1, 51–73.

Bernard, Alain, Carolyn Fischer, and Alan Fox (2007), "Is There a Rationale for Output-based Rebating of Environmental Levies?," *Resource and Energy Economics,* 29, 83–101.

Bohringer, Christoph, Edward J. Balistreri, and Thomas F. Rutherford. (2012): "The role of border carbon adjustment in unilateral climate policy: Overview of an Energy Modeling Forum study (EMF 29)," *Energy Economics*, 34. The Role of Border Carbon Adjustment in Unilateral Climate Policy: Results from EMF 29, S97–110.

Branger, Frédéric, and Philippe Quiron. (2014): "Climate policy and the 'carbon haven' effect," *Wiley Interdisciplinary Reviews: Climate Change*, 5, 53–71.

Bernard Alain, Carolyn Fischer and Alan Fox (2007), "Is There a Rationale for Output-based Rebating of Environmental Levies?," *Resource and Energy Economics,* 29, 83–101.

Costinot, Arnaud, Dave Donaldson, Jonathan Vogel, and Ivan Werning (2015), "Comparative Advantage and Optimal Trade Policy," *Quarterly Journal of Economics,* 659-702.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum (2007), "Unbalanced Trade," *American Economic Review: Papers and Proceedings,* 97, 351-355.

Dornbusch, Rudiger, Stanley Fischer, and Paul A. Samuelson (1977) "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review,* 67, 823-839.

Eaton, Jonathan and Samuel Kortum (2002) "Technology, Geography, and Trade," *Econometrica*, 70, 1741-1779.

Elliott, Joshua, Ian Foster, Samuel Kortum, Todd Munson, Fernando Perez Cervantes, and David Weisbach (2010), "Trade and Carbon Taxes," *American Economic Review: Papers and Proceedings,* 100, 465-469.

Elliott, Joshua, Ian Foster, Samuel Kortum, Todd Munson, Fernando Perez Cervantes, and David Weisbach (2010), "Trade and Carbon Taxes," *American Economic Review: Papers and Proceedings,* 100, 465-469.

Elliott, Joshua, Ian Foster, Samuel Kortum, Gita K. Jush, Todd. Munson, and David Weisbach. (2013): "Unilateral Carbon Taxes, Border Tax Adjustments and Carbon Leakage," *Theoretical Inquiries in Law,* 14.

EPA (2019): "Inventory of U.S. Greenhouse Gas Emissions and Sinks: 1990-2017. Annex 2 (Methodology for estimating CO2 emissions from fossil fuel combustion), Table A-42 for C coefficient and Table A-52 for heat content," *U.S. Environmental Protection Agency, Washington, DC,* U.S. EPA #430-R-19-001 (PDF).

Fischer, Carolyn, and Alan K. Fox. (2011): "The Role of Trade and Competitiveness Measures in US Climate Policy," *The American Economic Review*, 101, 258–62.

—. (2012): "Comparing policies to combat emissions leakage: Border carbon adjustments versus rebates," *Journal of Environmental Economics and Management*, 64, 199–216.

Fowlie, Merideth, Mar Reguant, and Stephen Ryan. 2014. Market-Based Emissions Regulation and Industry Dynamics," *Journal of Political Economy*, 124, 249–302.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski (2014), "Optimal Taxes on Fossil Fuels in General Equilibrium," *Econometrica*, 82, 41-88.

Grossman, Gene M. and Elhanan Helpman (1994), "Protection for Sale," *American Economic Review*, 84, 833-850.

Hemous, David. (2016). They dynamic impact of unilateral environmental policies, *Journal of International Economics*, 103, 80-95.

Hoel, M. (1996): "Should a carbon tax be differentiated across sectors?," *Journal of Public Economics*, 59, 17–32.

Holladay, J. Scott, Mohammed Mohsin, and Shreekar Pradhan. (2018): "Emissions leakage, environmental policy and trade frictions," *Journal of Environmental Economics and Management*, 88, 95–113.

Houthakker, Hendrik S. (1955-56), "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis," *Review of Economic Studies*, 23, 27-31.

IEA (2020), "World energy balances," *IEA World Energy Statistics and Balances (database)*, https://doi.org/10.1787/data-00512-en.

IPCC (2008), "2006 IPCC Guidelines for National Greenhouse Gas Inventories – A primer," Prepared by the *National Greenhouse Gas Inventories Programme*, Eggleston H.S., Miwa K., Srivastava N. and Tanabe K. (eds). Published: IGES, Japan.

Jakob, Michael, Robert Marschinksi, and Michael Hubler (2013), Between a Rock and a Hard Place: A Trade-Theory Analysis of Leakage Under Production- and Consumption-Based Policies, *Environmental and Resource Economics*, 56, 47–72.

Kortum, Samuel and David Weisbach (2017), "The Design of Border Adjustments for Carbon Prices," *National Tax Journal*, 70, 421-446.

Larch, Mario and Joschka Wanner (2019), "The Consequences of Unilateral Withdrawals from the Paris Agreement," *CESifo Working Paper* 7804-2019.

Markusen, James R. (1975), "International Externalities and Optimal Tax Structures," *Journal of International Economics*, 5, 15-29.

Metcalf, Gilbert E. and David Weisbach (2009),"The Design of a Carbon Tax," *Harvard Environmental Law Review*, 33, 499-556.

Metz et al (2007): Climate Change 2007—Mitigation of Climate Change, Contribution of Working Group III to the Fourth Assessment Report of the IPCC, Cambridge University Press.

OECD (2018), Input-Output Tables. For download at http://oe.cd/i-o. Organisation for Economic Co-operation and Development, Paris.

OECD, Trade in Embodied CO2 Database (TECO2), April 2019.

Simonovska, Ina and Michael Waugh (2014), "The Elasticity of Trade: Estimates and Evidence," *Journal of International Economics*, 92, 34-50.

Weitzman, Martin L. (1974): "Prices vs Quantities," *Review of Economic Studies*, 41, 477–91.

Zhang, ZhongXiang. (2012): "Competitiveness and Leakage Concerns and Border Carbon Adjustments," *International Review of Environmental and Resource Economics*, 6, 225–287.

A Global Planner's Problem

Consider a planner seeking to maximize global welfare:

$$\begin{aligned} U^W = C_s + C_s^* + \frac{\eta^{1/\sigma}}{1-1/\sigma} \int_0^1 \left((y_j + m_j)^{1-1/\sigma} - 1 \right) dj \\ + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*} \int_0^1 \left(\left(y_j^* + x_j\right)^{1-1/\sigma^*} - 1 \right) dj - \varphi^W \left(Q_e + Q_e^*\right). \end{aligned}$$

Here

$$\varphi^W = \varphi + \varphi^*$$

is marginal global damages from global emissions.[30] The planner is constrained by the endowments of labor in each country as well as by a global energy constraint:

$$L_s + L_e + L_g \le L,$$

[30] Since welfare is linear in consumption of services, transfers between countries (as long as both countries still consume services) do not alter global welfare. This indeterminacy has no implications for our objective of determining optimal global energy extraction as well as production and consumption of manufactured goods in each country.

$$L_s^* + L_e^* + L_g^* \leq L^*,$$

and

$$C_e + C_e^* \leq Q_e + Q_e^*.$$

Since $C_s + C_s^* = L_s + L_s^*$ we can substitute the two labor constraints into the objective while applying a Lagrange multiplier λ_e^W to the global energy constraint. The resulting Lagrangian, after dropping L, L^*, and other constants in the objective, is:

$$\begin{aligned}
\mathcal{L} = & \frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 (y_j + m_j)^{1-1/\sigma}\, dj + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\int_0^1 \left(y_j^* + x_j\right)^{1-1/\sigma^*} dj - \varphi^W\left(Q_e + Q_e^*\right) \\
& - L_e(Q_e) - L_e^*(Q_e^*) \\
& - \int_0^1 \left(l_j(z_j^y)y_j + \tau l_j(z_j^x)x_j + l_j^*(z_j^*)y_j + \tau^* l_j^*(z_j^m)m_j\right) dj \\
& - \lambda_e^W\left(\int_0^1 \left(e_j(z_j^y)y_j + \tau e_j(z_j^x)x_j + e_j^*(z_j^*)y_j^* + \tau^* e_j^*(z_j^m)m_j\right) dj - (Q_e + Q_e^*)\right),
\end{aligned}$$

where $L_e^*(Q_e^*)$ is the Foreign analog of (23). The planner chooses Q_e, Q_e^*, $\{y_j\}$, $\{y_j^*\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^*\}$, $\{z_j^x\}$, and $\{z_j^m\}$ to maximize $\mathcal{L}$.

A.1 Solution

Following Costinot, Donaldson, Vogel, and Werning (2015), we first solve the inner problem, involving conditions for an individual good given λ_e^W. We then turn to the outer problem, optimizing over Q_e and Q_e^* while solving for λ_e^W.

A.1.1 Inner Problem

Solving the inner problem consists of evaluating first order conditions with respect to the variables that are specific to some good j: y_j, y_j^*, x_j, m_j, z_j^y, z_j^*, z_j^x, and z_j^m. The Lagrangian for good j is:

$$\begin{aligned}
\mathcal{L}_j = & \frac{\eta^{1/\sigma}}{1-1/\sigma}(y_j + m_j)^{1-1/\sigma} + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*}\left(y_j^* + x_j\right)^{1-1/\sigma^*} \\
& - \nu a_j\left(y_j\left(z_j^y\right)^{\alpha-1} + \tau x_j\left(z_j^x\right)^{\alpha-1}\right) - \nu a_j^*\left(y_j^*\left(z_j^*\right)^{\alpha-1} + \tau^* m_j\left(z_j^m\right)^{\alpha-1}\right) \\
& - \lambda_e^W\left(\nu a_j\left(y_j\left(z_j^y\right)^{\alpha} + \tau x_j\left(z_j^x\right)^{\alpha}\right) + \nu a_j^*\left(y_j^*\left(z_j^*\right)^{\alpha} + \tau^* m_j\left(z_j^m\right)^{\alpha}\right)\right).
\end{aligned}$$

The first order conditions for energy intensities of production imply:

$$z_j^y = z_j^x = z_j^* = z_j^m = z = \frac{1-\alpha}{\alpha\lambda_e^W}.$$

The unit energy requirement in Home is thus:

$$e_j(z) = (1-\alpha)a_j\left(\lambda_e^W\right)^{-\alpha},$$

while in Foreign:

$$e_j^*(z) = (1-\alpha)a_j^*\left(\lambda_e^W\right)^{-\alpha}.$$

The FOC for y_j implies:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq a_j\left(\lambda_e^W\right)^{1-\alpha},$$

with equality if $y_j > 0$. The FOC for m_j implies:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq a_j^*\tau^*\left(\lambda_e^W\right)^{1-\alpha},$$

with equality if $m_j > 0$. The good $\bar{j}_m$ at which the FOC's for y_j and m_j both hold with equality satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}.$$

Thus, if $j < \bar{j}_m$:

$$y_j = \eta\left(a_j\left(\lambda_e^W\right)^{1-\alpha}\right)^{-\sigma}$$

and $m_j = 0$ while for $j > \bar{j}_m$:

$$m_j = \eta\left(a_j^*\tau^*\left(\lambda_e^W\right)^{1-\alpha}\right)^{-\sigma}$$

and $y_j = 0$.

The FOC for y_j^* implies:

$$\left(\left(y_j^* + x_j\right)/\eta^*\right)^{-1/\sigma^*} \leq a_j^*\left(\lambda_e^W\right)^{1-\alpha},$$

with equality if $y_j^* > 0$. The FOC for x_j implies:

$$\left(\left(y_j^* + x_j\right)/\eta^*\right)^{-1/\sigma^*} \leq a_j\tau\left(\lambda_e^W\right)^{1-\alpha},$$

with equality if $x_j > 0$. The good $\bar{j}_x$ at which the FOC's for y_j^* and x_j both hold satisfies:

$$F(\bar{j}_x) = \tau.$$

Since F is monotonically decreasing, it follows that $\bar{j}_x < \bar{j}_m$. For $j < \bar{j}_x$:

$$x_j = \eta^* \left(a_j \tau \left(\lambda_e^W\right)^{1-\alpha}\right)^{-\sigma^*}$$

and $y_j^* = 0$ while for $j > \bar{j}_x$:

$$y_j^* = \eta^* \left(a_j^* \left(\lambda_e^W\right)^{1-\alpha}\right)^{-\sigma^*}$$

and $x_j = 0$.

A.1.2 Implications for Aggregates

Aggregating these results from the inner problem:

$$C_e(\lambda_e^W) = (1-\alpha)\,\eta \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + (\tau^*)^{1-\sigma} \int_{\bar{j}_m}^1 \left(a_j^*\right)^{1-\sigma} dj\right) \left(\lambda_e^W\right)^{-\epsilon_D},$$

$$C_e^*(\lambda_e^W) = (1-\alpha)\eta^* \left(\tau^{1-\sigma^*} \int_0^{\bar{j}_x} a_j^{1-\sigma^*} dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj\right) \left(\lambda_e^W\right)^{-\epsilon_D^*},$$

$$L_g(\lambda_e^W) = \alpha\eta \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj\right) \left(\lambda_e^W\right)^{1-\epsilon_D} + \alpha\eta^* \left(\int_0^{\bar{j}_x} \left(\tau a_j\right)^{1-\sigma} dj\right) \left(\lambda_e^W\right)^{1-\epsilon_D^*},$$

$$L_g^*(\lambda_e^W) = \alpha\eta \left(\int_{\bar{j}_m}^1 (\tau^* a_j^*)^{1-\sigma} dj\right) \left(\lambda_e^W\right)^{1-\epsilon_D} + \alpha\eta^* \left(\int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma} dj\right) \left(\lambda_e^W\right)^{1-\epsilon_D^*},$$

$$C_g(\lambda_e^W) = \eta \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(a_j^* \tau^*\right)^{1-\sigma} dj\right)^{\sigma/(\sigma-1)} \left(\lambda_e^W\right)^{-(1-\alpha)\sigma},$$

and

$$C_g^*(\lambda_e^W) = \eta^* \left(\int_0^{\bar{j}_x} \left(\tau a_j\right)^{1-\sigma} dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma} dj\right)^{\sigma^*/(\sigma^*-1)} \left(\lambda_e^W\right)^{-(1-\alpha)\sigma^*}.$$

These six terms are fully determined by λ_e^W.

A.1.3 Outer Problem

We now turn to the optimality conditions for Q_e and Q_e^* while choosing λ_e^W to clear the global energy market. We can rewrite the Lagrangian in terms of aggregate magnitudes as:

$$\begin{aligned}\mathcal{L} = &\frac{\eta^{1/\sigma}}{1-1/\sigma} C_g^{1-1/\sigma} + \frac{(\eta^*)^{1/\sigma^*}}{1-1/\sigma^*} \left(C_g^*\right)^{1-1/\sigma^*} - \varphi^W (Q_e + Q_e^*) \\ &- \left(L_e(Q_e) + L_e^*(Q_e^*) + L_g + L_g^*\right) \\ &- \lambda_e^W \left((C_e + C_e^*) - (Q_e + Q_e^*)\right).\end{aligned}$$

The first order condition with respect to Home energy extraction implies:

$$Q_e = G(\lambda_e^W - \varphi^W).$$

Likewise for Foreign energy extraction:

$$Q_e^* = G^*(\lambda_e^W - \varphi^W).$$

The global energy constraint determines the Lagrange multiplier as the solution to:

$$C_e(\lambda_e^W) + C_e^*(\lambda_e^W) = G\left(\lambda_e^W - \varphi\right) + G^*\left(\lambda_e^W - \varphi\right).$$

A.2 Decentralized Global Optimum

We can interpret the solution in terms of a decentralized economy with a price of energy:

$$p_e = \lambda_e^W.$$

The global externality can be solved with an extraction tax in both countries equal to global damages:

$$t_e = t_e^* = \varphi^W.$$

Thus, energy extractors in both countries receive $p_e - \varphi^W$.

A.3 Competitive Equilibrium

In a competitive equilibrium agents ignore the global externality. All outcomes other than global welfare are the same as if we simply set $\varphi^W = 0$ in the decentralized global optimum above. We treat this case as our business-as-usual baseline.

B Home Planner's Problem (Missing Steps)

Here we provide missing steps from Section 3 of the text, which derives the unilaterally optimal policy. We focus on the inner problem.

The Lagrangian for good j, repeated here for convenience, is:

$$\begin{aligned}\mathcal{L}_j = & \frac{\eta^{1/\sigma}}{1-1/\sigma}\left(y_j+m_j\right)^{1-1/\sigma} \\ & -\nu a_j\left(y_j\left(z_j^y\right)^{\alpha-1}+\tau x_j\left(z_j^x\right)^{\alpha-1}\right) \\ & -\nu a_j^*\tau^*\left(\left(z_j^m\right)^{\alpha-1}+p_e\left(z_j^m\right)^{\alpha}\right)m_j+p_j^x x_j \\ & -\lambda_e\nu a_j\left(y_j\left(z_j^y\right)^{\alpha}+\tau x_j\left(z_j^x\right)^{\alpha}\right) \\ & -\lambda_e^*\nu a_j^*\left(\max\left\{c_j^*-x_j,0\right\}\left(z^*\right)^{\alpha}+\tau^* m_j\left(z_j^m\right)^{\alpha}\right).\end{aligned}$$

We want to maximize by choice of $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, $\{z_j^y\}$, $\{z_j^x\}$, $\{z_j^m\}$. We consider the variables relevant to Home consumers first, then turn to those relevant to Foreign consumers.

B.1 Goods for Home Consumers

The first order condition for y_j is:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq \nu a_j\left(z_j^y\right)^{\alpha-1}+\lambda_e\nu a_j\left(z_j^y\right)^{\alpha}=\nu a_j\left(z_j^y\right)^{\alpha-1}\left(1+\lambda_e z_j^y\right).$$

Substituting in the optimal $z_j^y = z^y$ and applying (4) this condition reduces to:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq a_j\lambda_e^{1-\alpha}, \tag{53}$$

with equality if $y_j > 0$.

The first order condition for m_j is:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq \nu\tau^* a_j^*\left(z_j^m\right)^{\alpha-1}\left(1+\left(p_e+\lambda_e^*\right)z_j^m\right).$$

Substituting in the optimal $z_j^m = z^m$ this condition reduces to:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq \tau^* a_j^*\left(p_e+\lambda_e^*\right)^{1-\alpha}, \tag{54}$$

with equality if $m_j > 0$.

For good $j = \bar{j}_m$ the right hand sides of (53) and (54) are equal:

$$a_{\bar{j}_m} \lambda_e^{1-\alpha} = \tau^* a^*_{\bar{j}_m} (p_e + \lambda_e^*)^{1-\alpha},$$

yielding, via (5) as in the text:

$$F(\bar{j}_m) = \frac{1}{\tau^*} \left(\frac{\lambda_e}{p_e + \lambda_e^*} \right)^{1-\alpha}.$$

Since F is strictly decreasing, for $j < \bar{j}_m$ we get (53) holding with equality and (54) as a strict inequality. Thus $m_j = 0$ with y_j satisfying (53). For for $j > \bar{j}_m$ we get (54) holding with equality and (53)as a strict inequality. Thus $y_j = 0$ with m_j satisfying (54). This reasoning confirms the results asserted in the text.

B.2 Goods for Foreign Consumers

The derivative with respect to x_j is:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\nu a_j \tau \left(z_j^x \right)^{\alpha-1} - \lambda_e \nu a_j \tau \left(z_j^x \right)^{\alpha} + p_j^x(p_e) + \lambda_e^* \nu a_j^* \left(z^*(p_e) \right)^{\alpha}.$$

Substituting in the optimal $z_j^x = z^x$, this derivative simplifies to:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -a_j \tau \lambda_e^{1-\alpha} + a_j^* p_e^{1-\alpha} + a_j^* \lambda_e^* (1-\alpha) p_e^{-\alpha},$$

whose sign determines the solution for x_j. If

$$a_j \tau \lambda_e^{1-\alpha} < a_j^* p_e^{1-\alpha} + a_j^* \lambda_e^* (1-\alpha) p_e^{-\alpha} \tag{55}$$

then $x_j > 0$. In that case x_j is pushed to the maximum quantity that Foreign will be willing to buy at price p_j^x. That maximum is reached when the marginal utility of Foreign equals that price (which is the price at which Foreign could supply the good for itself):

$$\left(x_j / \eta^* \right)^{-1/\sigma^*} = a_j^* p_e^{1-\alpha}.$$

If inequality (55) is reversed then $x_j = 0$ and y_j^* solves:

$$\left(y_j^* / \eta^* \right)^{-1/\sigma^*} = a_j^* p_e^{1-\alpha}.$$

In this case Foreign produces all of the good j that it consumes.

The good $j = \bar{j}_x$ for which the inequality is () is replacehese results about consumption in Foreign, define the good $\bar{j}_x$ such that

$$\left.\frac{\partial \mathcal{L}_j}{\partial x_j}\right|_{j=\bar{j}_x} = 0.$$

Applying (5), this cutoff good will satisfy:

$$F(\bar{j}_x) = \frac{\tau\left(\frac{\lambda_e}{p_e}\right)^{1-\alpha}}{1 + (1-\alpha)\frac{\lambda_e^*}{p_e}}.$$

1. For any good $j < \bar{j}_x$ Home has comparative advantage, which leads it to export:
$$x_j = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*},$$
while Foreign produces nothing for itself, $y_j^* = 0$. (Home's export quantity for any such good is at a corner solution with $u^{*\prime}(x_j) = p_j^*$.)

2. For any good $j > \bar{j}_x$ Foreign has a comparative advantage, which leads it to produce for itself:
$$y_j^* = \eta^* \left(a_j^* p_e^{1-\alpha}\right)^{-\sigma^*},$$
while demanding no exports from Home, $x_j = 0$. (Foreign's production of any such good is determined by Foreign demand at price p_j^*.)

C Simple Unilateral Policies

Here we derive constrained-optimal policies that focus exclusively on either: (i) Home's extraction of energy, (ii) Home's implicit consumption of energy. We do so by considering a planner whose menu of choice variables is limited in a particular way. All variables not on the menu are determined as in a competitive equilibrium. After solving each planner's problem, we show how it can be implemented with taxes in a decentralized equilibrium, and we present formulas for the optimal tax rates.

In each case the planner's objective is to maximize Home's welfare:

$$U = C_s + \frac{\eta^{1/\sigma}}{1-1/\sigma}\int_0^1 \left(c_j^{1-1/\sigma} - 1\right) dj - \varphi\left(Q_e + Q_e^*\right), \tag{56}$$

subject to four constraints: (i) Home's labor constraint (24), (ii) trade balance (18), (iii) Home's energy constraint (21), and (iv) Foreign's energy constraint (20). While it involves some redundancy, maintaining four constraints facilitates comparison to the unilaterally optimal policy.

C.1 Optimal Pure Extraction Tax

Suppose the planner is constrained to choose only Q_e, X_e, and p_e, with all other outcomes determined in a decentralized competitive equilibrium. We use this problem to derive the constrained-optimal pure extraction tax.

Energy intensities and the intensive and extensive margins of trade are as in a competitive equilibrium, given p_e. Other aggregates depend on p_e as in the competitive equilibrium. In particular, spending on goods by Home's consumers, the term relevant to their welfare, becomes:

$$V_g = \eta^{1/\sigma} {C_g}^{1-1/\sigma} = \eta p_e^{(1-\alpha)(1-\sigma)} \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \int_{\bar{j}_m}^1 \left(\tau^* a_j^*\right)^{1-\sigma} dj \right). \quad (57)$$

Furthermore, this spending term is tightly linked to Home's consumption of embodied energy:

$$p_e C_e = (1-\alpha) V_g,$$

a result that we exploit in what follows.[31] We can also exploit the connection between spending on labor and energy by goods producers in Home :

$$L_g = \frac{\alpha}{1-\alpha} p_e I_e.$$

The use and implicit consumption of energy are connected to the value of Home's net exports of goods via:

$$p_e I_e - p_e C_e = (1-\alpha) X_g.$$

Recall that the trade balance constraint is:

$$X_s = -X_g - p_e X_e.$$

[31] In the absence of border adjustments on goods imports, as is the case with a pure extraction tax, we get the equation: $V_g = V_g^{HH} + V_g^{HF}$. Border adjustments introduce a wedge between the price of goods at the port, relevant for V_g^{HF}, and the price paid by consumers, relevant for V_g. Hence this equation no longer holds.

Home's labor constraint is:

$$Q_s = L - L_e(Q_e) - L_g.$$

Home's energy constraint is:

$$Q_e - I_e - X_e = 0.$$

Foreign's energy constraint is:

$$Q_e^* - I_e^* + X_e = 0.$$

C.1.1 The Planner's Lagrangian

We substitute the first two constraints into the objective (56) to eliminate C_s while attaching Lagrange multipliers λ_e and λ_e^* to the energy constraints. The resulting Lagrangian (dropping constants) is:

$$\begin{aligned}\mathcal{L} = & \frac{\sigma}{\sigma-1} V_g - \varphi\left(Q_e + Q_e^*\right) \\ & - L_e(Q_e) - L_g(p_e) + X_g + p_e X_e \\ & - \lambda_e\left(I_e - Q_e + X_e\right) - \lambda_e^*\left(I_e^* - Q_e^* - X_e\right).\end{aligned}$$

This Lagrangian is similar to the outer problem for the planner choosing the optimal unilateral policy after maximizing the inner problem. The difference is that, by not maximizing the inner problem, we can't invoke the envelope condition. All aggregates, except the three that are directly chosen, depend on the energy price.

C.1.2 Solution

The first order condition for X_e is:

$$\lambda_e = p_e + \lambda_e^*,$$

as in the optimal unilateral policy. The first order condition for Q_e also matches the optimal unilateral policy:

$$Q_e = G(\lambda_e - \varphi).$$

The first order condition for p_e is different. First, exploiting (57) and the equation beneath it:

$$\frac{\partial V_g}{\partial p_e} = \frac{(1-\alpha)(1-\sigma)}{p_e} V_g = (1-\sigma) C_e.$$

Hence, we can write the first order condition as:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_e} &= -\sigma C_e - \varphi \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_g}{\partial p_e} + \frac{\partial X_g}{\partial p_e} + X_e \\ &- \lambda_e \frac{\partial I_e}{\partial p_e} - \lambda_e^* \left(\frac{\partial I_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.\end{aligned}$$

Substituting in the first order condition for X_e and

$$\frac{\partial L_g}{\partial p_e} = \frac{\alpha}{1-\alpha} \left(I_e + p_e \frac{\partial I_e}{\partial p_e} \right),$$

we get:

$$\lambda_e^* \left(\frac{\partial I_e}{\partial p_e} + \frac{\partial I_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = -\sigma C_e - \varphi \frac{\partial Q_e^*}{\partial p_e} + \frac{\partial X_g}{\partial p_e} + X_e - \frac{\alpha}{1-\alpha} \left(I_e + p_e \frac{\partial I_e}{\partial p_e} \right) - p_e \frac{\partial I_e}{\partial p_e}$$

Finally, we can substitute in:

$$\frac{\partial X_g}{\partial p_e} = \frac{1}{1-\alpha} \left(I_e + p_e \frac{\partial I_e}{\partial p_e} \right) + (\sigma - 1) C_e,$$

to get:

$$\lambda_e^* \left(\frac{\partial I_e}{\partial p_e} + \frac{\partial I_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = -\varphi \frac{\partial Q_e^*}{\partial p_e} + Q_e - C_e.$$

We can rewrite this expression as:

$$\lambda_e^* = \varphi \frac{\partial Q_e^* / \partial p_e}{\partial \left(Q_e^* - C_e^W \right) / \partial p_e} + \frac{Q_e^* - C_e^*}{\partial \left(Q_e^* - C_e^W \right) / \partial p_e}.$$

In terms of elasticities, we have:

$$\lambda_e^* = \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*} + \frac{p_e \left(Q_e^* - C_e^* \right)}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*}.$$

C.1.3 Decentralization

In a decentralized equilibrium we can set the extraction tax as $t_e = \varphi - \lambda_e^*$ so that:

$$Q_e = G(p_e - t_e).$$

Using the expression above for λ_e^* we get the optimal value for the extraction tax:

$$t_e = \varphi \frac{\epsilon_D C_e + \epsilon_D^* C_e^*}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*} - \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S^* Q_e^* + \epsilon_D C_e + \epsilon_D^* C_e^*}. \tag{58}$$

Ignoring the second term, this rate is below the value of $t_e = \varphi$ in the optimal unilateral policy. How much below turns on the value of $\epsilon_S^* Q_e^*$. If Foreign is a major energy extractor and if its price elasticity of supply is high, then Home will want to choose a lower extraction tax.

Turning to the second term, note that the numerator is the value of Foreign's net exports of energy based on its implicit consumption of embodied energy. Its use of energy in production doesn't matter here. If Foreign is an exporter in this sense then Home wants a lower extraction tax to improve its terms of trade. For the same reason, it will choose a higher extraction tax if Foreign is a large net importer in this sense.

C.2 Optimal Pure Consumption Tax

Suppose the planner is constrained to choose only: $\left\{z_j^y\right\}$, $\left\{z_j^m\right\}$, $\{y_j\}$, $\{m_j\}$, X_e, and p_e, with all other outcomes determined as in a decentralized competitive equilibrium. We use this problem to derive a constrained-optimal pure consumption tax.

Any good j consumed in Foreign, whether made in Home or Foreign, is produced at energy intensity:

$$z_j^x = z_j^* = z^* = \frac{1-\alpha}{\alpha p_e}.$$

If Foreign produces it the price is:

$$p_j^* = a_j^* p_e^{1-\alpha}$$

while if Home exports it:

$$p_j^x = \tau a_j p_e^{1-\alpha}.$$

Seeking the lowest price, Foreign consumers will import any good $j \leq \bar{j}_x$ and will purchase locally any good $j > \bar{j}_x$, where the cutoff satisfies:

$$F(\bar{j}_x) = \tau.$$

Using these results we can compute some aggregates which depend only on the global energy price. In particular, the value of Home's goods exports is:

$$V_g^{FH} = \eta^* p_e^{1-\epsilon_D^*} \int_0^{\bar{j}_x} (\tau a_j)^{1-\sigma^*} \, dj.$$

A fraction α of this value is paid to labor employed in Home to produce these exports, with the rest going to the energy input:

$$p_e C_e^{FH} = (1-\alpha) V_g^{FH}$$

Foreign's revenue from domestic sales is:

$$V_g^{FF} = \eta^* p_e^{1-\epsilon_D^*} \int_{\bar{j}_x}^{1} \left(a_j^*\right)^{1-\sigma^*} \, dj,$$

with:

$$p_e C_e^{FF} = (1-\alpha) V_g^{FF}.$$

Energy extraction in each country is determined by (13) and (14) as in the competitive equilibrium.

C.2.1 The Planner's Lagrangian

We again substitute the labor and trade balance constraints into the objective (56) to eliminate C_s while attaching Lagrange multipliers λ_e and λ_e^* to the energy constraints. The resulting Lagrangian is (dropping constants):

$$\begin{aligned}
\mathcal{L} = & \frac{\eta^{1/\sigma}}{1-1/\sigma} \int_0^1 (y_j + m_j)^{1-1/\sigma} \, dj - \varphi \left(Q_e + Q_e^*\right) \\
& - L_e(Q_e) - \int_0^1 l_j(z_j^y) y_j dj - \alpha V_g^{FH} \\
& - \int_0^1 \tau^* \left(l_j^*(z_j^m) + p_e e_j^*(z_j^m)\right) m_j dj + V_g^{FH} + p_e X_e \\
& - \lambda_e \left(\int_0^1 e_j(z_j^y) y_j dj + C_e^{FH} - Q_e + X_e \right) \\
& - \lambda_e^* \left(C_e^{FF} + \int_0^1 \tau^* e_j^*(z_j^m) m_j dj - Q_e^* - X_e \right).
\end{aligned}$$

We want to maximize this Lagrangian by the optimal choice of $\{z_j^y\}$, $\{z_j^m\}$, $\{y_j\}$, $\{m_j\}$, X_e, and p_e.

C.2.2 Solution

We start with the *inner problem*, involving conditions for an individual good j given values for X_e, λ_e, λ_e^*, and p_e. We then evaluate the optimal conditions for X_e and p_e while solving for λ_e and λ_e^*.

Inner Problem Solving the inner problem consists of first order conditions with respect to y_j, m_j, z_j^y, and z_j^m. These first order conditions, and their implications given X_e, λ_e, λ_e^*, and p_e, can be considered one good at time. We therefore define a Lagrangian for good j:

$$\begin{aligned}\mathcal{L}_j = &\frac{\eta^{1/\sigma}}{1-1/\sigma}\left(y_j+m_j\right)^{1-1/\sigma} \\ &-\nu a_j y_j \left(z_j^y\right)^{\alpha-1} - \nu a_j^* \tau^* \left(\left(z_j^m\right)^{\alpha-1} + p_e \left(z_j^m\right)^{\alpha}\right) m_j \\ &-\lambda_e \nu a_j y_j \left(z_j^y\right)^{\alpha} - \lambda_e^* \nu a_j^* \tau^* m_j \left(z_j^m\right)^{\alpha},\end{aligned}$$

where we have substituted in the expressions for unit input requirements (6) and (7) in Home (as well as their analogs in Foreign).

The first order condition for z_j^y implies:

$$z_j^y = z^y = \frac{1-\alpha}{\alpha\lambda_e}.$$

Unlike for the optimal unilateral policy or for the case of a pure extraction tax, Home uses a different energy intensity for serving consumers in Home and Foreign.

The FOC for y_j is:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq \nu a_j \left(z^y\right)^{\alpha-1} + \lambda_e \nu a_j \left(z^y\right)^{\alpha} = \nu a_j \left(z^y\right)^{\alpha-1}\left(1+\lambda_e z^y\right).$$

Substituting in the solution for z^y and applying (4) this FOC reduces to:

$$\left(\left(y_j+m_j\right)/\eta\right)^{-1/\sigma} \leq a_j \lambda_e^{1-\alpha},$$

with equality if $y_j > 0$. If this FOC holds with a strict inequality then $y_j = 0$ and Home imports the good.

The first order condition for z_j^m implies:

$$z_j^m = z^m = \frac{1-\alpha}{\alpha\left(p_e + \lambda_e^*\right)}.$$

All producers serving consumers in Home, whether domestic or foreign, produce at the same energy intensity.

The FOC for m_j is:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq \nu a_j^* \tau^* \left(z^m\right)^{\alpha-1}\left(1 + \left(p_e + \lambda_e^*\right) z^m\right).$$

Substituting in the solution for z^m this FOC reduces to:

$$\left(\left(y_j + m_j\right)/\eta\right)^{-1/\sigma} \leq a_j^* \tau^* \left(p_e + \lambda_e^*\right)^{1-\alpha},$$

with equality if $m_j > 0$.

To distill these results about consumption in Home, define the good $\bar{j}_m$ at which the FOC for y and m both hold. Applying (5), this cutoff good will satisfy:

$$F(\bar{j}_m) = \frac{1}{\tau^*}\left(\frac{\lambda_e}{p_e + \lambda_e^*}\right)^{1-\alpha}.$$

1. For any good $j < \bar{j}_m$ Home has a comparative advantage, which leads it to produce for itself:

   $$y_j = \eta\left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma},$$

   while importing nothing, $m_j = 0$.

2. For any good $j > \bar{j}_m$ Foreign has a comparative advantage, which leads Home to import:

   $$m_j = \eta\left(a_j^* \tau^* \left(p_e + \lambda_e^*\right)^{1-\alpha}\right)^{-\sigma},$$

   while producing nothing for itself, $y_j = 0$.

Outer Problem We now turn to the optimality conditions for X_e and p_e while finding the Lagrange multipliers that clear the global energy market. First, we collect results from the inner problem. The price Home pays for imports is:

$$p_j^m = \tau^* l_j^*(z^m) + p_e \tau^* e_j^*(z^m).$$

Since this price depends directly on p_e, so does Home's spending on imported goods:

$$V_g^{HF} = \int_{\bar{j}_m}^{1} p_j^m m_j dj = \int_{\bar{j}_m}^{1} \tau^* l_j^*(z^m) m_j dj + p_e \int_{\bar{j}_m}^{1} \tau^* e_j^*(z^m) m_j dj.$$

By the envelope condition we ignore the dependence of m_j and z^m on p_e, so that the appropriate derivative is:

$$\frac{\partial V_g^{HF}}{\partial p_e} = \int_{\bar{j}_m}^{1} \tau^* e_j^*(z^m) m_j dj = C_e^{HF}.$$

Energy use by Home's producers serving the domestic market is completely determined by the inner problem:

$$C_e^{HH} = \int_0^{\bar{j}_m} e_j(z^y) y_j dj,$$

as is the labor employed:

$$L_g^{HH} = \int_0^{\bar{j}_m} l_j(z^y) y_j dj.$$

We therefore rewrite the Lagrangian in terms of aggregate magnitudes:

$$\begin{aligned}
\mathcal{L} = & \frac{\eta^{1/\sigma}}{1-1/\sigma} C_g^{1-1/\sigma} - \varphi\left(Q_e(p_e) + Q_e^*(p_e)\right) \\
& - L_e(Q_e) - L_g^{HH} - \alpha V_g^{FH}(p_e) \\
& - V_g^{HF} + V_g^{FH} + p_e X_e \\
& - \lambda_e \left(C_e^{HH} + C_e^{FH} - Q_e + X_e\right) \\
& - \lambda_e^* \left(C_e^{FF} + C_e^{HF} - Q_e^* - X_e\right).
\end{aligned}$$

We now turn to the first order conditions for maximizing $\mathcal{L}$ with respect to X_e and p_e.

Home Energy Exports The first order condition with respect to X_e gives:

$$\lambda_e = p_e + \lambda_e^*,$$

as in the optimal unilateral policy. Combined with the inner problem, we get results familiar from the optimal unilateral policy. For $j \leq \bar{j}_m$ Home buys:

$$y_j = \eta \left(a_j \lambda_e^{1-\alpha}\right)^{-\sigma}$$

from domestic producers and for $j > \bar{j}_m$ Home imports:

$$m_j = \eta \left(a_j^* \tau^* \lambda_e^{1-\alpha}\right)^{-\sigma},$$

where the threshold satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}.$$

Finally, we have:

$$L_g = \int_0^{\bar{j}_m} l_j(z_j^y) y_j dj + \int_0^{\bar{j}_x} l_j(z_j^x) x_j dj = \alpha \eta \lambda_e^{1-\epsilon_D} \int_0^{\bar{j}_m} a_j^{1-\sigma} dj + \alpha V_g^{FH}.$$

Optimal Energy Price The first order condition with respect to p_e is:

$$\frac{\partial \mathcal{L}}{\partial p_e} = -\varphi \left(\frac{\partial Q_e}{\partial p_e} + \frac{\partial Q_e^*}{\partial p_e}\right) - \frac{\partial L_e}{\partial Q_e}\frac{\partial Q_e}{\partial p_e} - \alpha \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} + \frac{\partial V_g^{FH}}{\partial p_e}$$
$$+ X_e - \lambda_e \frac{\partial C_e^{FH}}{\partial p_e} + \lambda_e \frac{\partial Q_e}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e}\right) = 0.$$

Subsituting in the first order condition for X_e we get:

$$0 = -\varphi \left(\frac{\partial Q_e}{\partial p_e} + \frac{\partial Q_e^*}{\partial p_e}\right) - \frac{\partial L_e}{\partial Q_e}\frac{\partial Q_e}{\partial p_e} + (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e}$$
$$+ X_e - p_e \frac{\partial C_e^{FH}}{\partial p_e} + p_e \frac{\partial Q_e}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} + \frac{\partial C_e^{FH}}{\partial p_e} - \frac{\partial Q_e}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e}\right).$$

Noting that:

$$\frac{\partial L_e}{\partial Q_e} = p_e,$$

and grouping terms, we can simplify to:

$$0 = (\lambda_e^* - \varphi) \frac{\partial Q_e^W}{\partial p_e} + (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e}$$
$$+ X_e - p_e \frac{\partial C_e^{FH}}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} + \frac{\partial C_e^{FH}}{\partial p_e}\right).$$

To make further progress, from $p_e C_e^{FH} = (1-\alpha) V_g^{FH}$ we have:

$$C_e^{FH} + p_e \frac{\partial C_e^{FH}}{\partial p_e} = (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e}.$$

Substituting into the first order condtion, together with $\partial V_g^{HF}/\partial p_e = C_e^{HF}$, we get:

$$\begin{aligned} 0 &= (\lambda_e^* - \varphi) \frac{\partial Q_e^W}{\partial p_e} + C_e^{FH} - C_e^{HF} + X_e - \lambda_e^* \frac{\partial C_e^*}{\partial p_e} \\ &= (\lambda_e^* - \varphi) \frac{\partial Q_e^W}{\partial p_e} + Q_e - C_e - \lambda_e^* \frac{\partial C_e^*}{\partial p_e}. \end{aligned}$$

Applying the global energy constaint, we finally arrive at:

$$0 = \varphi \frac{\partial Q_e^W}{\partial p_e} + (Q_e^* - C_e^*) - \lambda_e^* \left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e} \right),$$

or:

$$\lambda_e^* = \varphi \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - \partial C_e^*/\partial p_e} + \frac{Q_e^* - C_e^*}{\partial Q_e^W/\partial p_e - \partial C_e^*/\partial p_e}.$$

C.2.3 Decentralization

We can decentralize this outcome by simply imposing an extraction tax, and an equal border tax of:

$$t_e = t_b = \varphi \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - \partial C_e^*/\partial p_e} + \frac{Q_e^* - C_e^*}{\partial Q_e^W/\partial p_e - \partial C_e^*/\partial p_e}.$$

To operationalize the formula, we can rewrite it in terms of elasticites:

$$t_e = t_b = \varphi \frac{\epsilon_S Q_e + \epsilon_S^* Q_e^*}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} + \frac{p_e (Q_e^* - C_e^*)}{\epsilon_S Q_e + \epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{59}$$

The border adjustment is applied to Home's energy imports raising the price of energy in Home to $p_e + t_b$. It is applied to Home's exporters of energy so that they receive a pre-tax price is $p_e + t_b$ wherever they sell. Their net-of-tax price, after paying the extraction tax, is always p_e. The border adjustment is also applied to the energy content of Home's goods imports and it is removed on the energy content of Home's goods exports.

C.3 Optimal Hybrid

Now consider augmenting the pure consumption tax by allowing the planner to choose energy extraction in Home. Doing so should give us a hybrid of the pure extraction tax and the pure consumption tax. We need only tweak the pure consumption case solved above by replacing the competitively determinee Q_e, from (13), with an optimally chosen Q_e.

C.3.1 The Planner's Lagrangian

We can jump directly to the outer problem as the inner problem is unchanged from the pure consumption tax case. The Lagrangian for the outer problem becomes:

$$\begin{aligned}\mathcal{L} = & \frac{\eta^{1/\sigma}}{1-1/\sigma} C_g^{1-1/\sigma} - \varphi\left(Q_e + Q_e^*\right) \\ & - L_e(Q_e) - L_g^{HH} - \alpha V_g^{FH} \\ & - V_g^{HF} + V_g^{FH} + p_e X_e \\ & - \lambda_e \left(C_e^{HH} + C_e^{FH} - Q_e + X_e\right) \\ & - \lambda_e^* \left(C_e^{FF} + C_e^{HF} - Q_e^* - X_e\right).\end{aligned}$$

We now turn to the first order conditions for maximizing $\mathcal{L}$ with respect to Q_e, X_e and p_e.

Home Energy Exports The first order condition with respect to X_e remains:

$$\lambda_e = p_e + \lambda_e^*,$$

as in the optimal unilateral policy.

Optimal Home Energy Extraction The first order condition for Q_e remains as in the optimal unilateral policy:

$$Q_e = G(\lambda_e - \varphi).$$

Optimal Energy Price The first order condition with respect to p_e is now:

$$\frac{\partial \mathcal{L}}{\partial p_e} = -\varphi \frac{\partial Q_e^*}{\partial p_e} - \alpha \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e} + \frac{\partial V_g^{FH}}{\partial p_e}$$
$$+ X_e - \lambda_e \frac{\partial C_e^{FH}}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^{FF}}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.$$

Substituting in the FOC for energy exports and grouping terms, we can write the FOC as:

$$0 = -\varphi \frac{\partial Q_e^*}{\partial p_e} + (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e} - \frac{\partial V_g^{HF}}{\partial p_e}$$
$$+ X_e - p_e \frac{\partial C_e^{FH}}{\partial p_e} - \lambda_e^* \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right).$$

As in the pure consumption case, we have:

$$C_e^{FH} + p_e \frac{\partial C_e^{FH}}{\partial p_e} = (1-\alpha) \frac{\partial V_g^{FH}}{\partial p_e},$$

which together with $\partial V_g^{HF} / \partial p_e = C_e^{HF}$, gives:

$$0 = -\varphi \frac{\partial Q_e^*}{\partial p_e} + C_e^{FH} - C_e^{HF} + X_e - \lambda_e^* \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right).$$

Using the global energy constraint:

$$0 = -\varphi \frac{\partial Q_e^*}{\partial p_e} + C_e^* - Q_e^* - \lambda_e^* \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right),$$

which we can rewrite as:

$$\lambda_e^* = \varphi \frac{\partial Q_e^* / \partial p_e}{\partial \left(Q_e^* - C_e^*\right) / \partial p_e} + \frac{Q_e^* - C_e^*}{\partial \left(Q_e^* - C_e^*\right) / \partial p_e}.$$

C.3.2 Decentralization

We can decentralize this outcome by imposing an extraction tax:

$$t_e = \varphi$$

together with a border tax of:

$$t_b = \varphi \frac{\partial Q_e^* / \partial p_e}{\partial \left(Q_e^* - C_e^*\right) / \partial p_e} + \frac{Q_e^* - C_e^*}{\partial \left(Q_e^* - C_e^*\right) / \partial p_e}.$$

To operationalize this formula for the border tax, we can rewrite it as:

$$t_b = \varphi \frac{\epsilon_S^* Q_e^*}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*} + \frac{p_e \left(Q_e^* - C_e^*\right)}{\epsilon_S^* Q_e^* + \epsilon_D^* C_e^*}. \tag{60}$$

The border adjustment is applied to Home's energy imports raising the price of energy in Home to $p_e + t_b$. It is applied to Home's exporters of energy so that they receive a pre-tax price is $p_e + t_b$ wherever they sell. Their net-of-tax price is always $p_e + t_b - \varphi$. The border adjustment is also applied to the energy content of Home's goods imports and it is removed on the energy content of Home's goods exports.

D Solutions for Quantitative Illustration

Here we provide a list of equations for the parameterized version of the model that we use for the quantitative results in Section 5 of the paper.

D.1 Unilaterally Optimal Outcomes

Imposing (44), (45), (46), and (47) we have:

1. energy intensity (except when Foreign produces for itself):

$$z = \frac{1-\alpha}{\alpha \left(p_e + t_b\right)};$$

2. unit energy requirements (except when Foreign produces for itself):

$$e_j = (1-\alpha) \left(p_e + t_b\right)^{-\alpha};$$

3. export threshold

$$\bar{j}_x = \frac{\tau^{-\theta} A \left(p_e + (1-\alpha) t_b\right)^{\theta}}{\tau^{-\theta} A \left(p_e + (1-\alpha) t_b\right)^{\theta} + A^* \left(p_e^{\alpha} \left(p_e + t_b\right)^{1-\alpha}\right)^{\theta}}$$

4. import threshold:
$$\bar{j}_m = \frac{A}{A + (\tau^*)^{-\theta} A^*},$$

5. energy used by producers in Home to supply Home consumers:
$$C_e^{HH} = \int_0^{\bar{j}_m} e_j(z) y_j dj = \eta (1-\alpha)(p_e + t_b)^{-\epsilon_D} \int_0^{\bar{j}_m} a_j^{1-\sigma} dj$$
$$= \eta (1-\alpha)(p_e + t_b)^{-\epsilon_D} \frac{A^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} (\bar{j}_m)^{1+(1-\sigma)/\theta};$$

6. energy used by producers in Home to supply exports of Home:
$$C_e^{FH} = \tau \int_0^{\bar{j}_x} e_j(z) x_j dj = \tau \eta^* (1-\alpha) p_e^{(\alpha-1)\sigma^*} (p_e + t_b)^{-\alpha} \int_0^{\bar{j}_x} a_j \left(a_j^*\right)^{-\sigma^*} dj$$
$$= \tau \eta^* (1-\alpha) p_e^{(\alpha-1)\sigma^*} (p_e + t_b)^{-\alpha} \frac{(A^*)^{\sigma^*/\theta}}{A^{1/\theta}} B\left(\bar{j}_x, \frac{1+\theta}{\theta}, \frac{\theta - \sigma^*}{\theta}\right),$$

   where and $B(x, a, b)$ is the incomplete beta function;[32]

7. energy used by producers in Foreign to supply Foreign consumers:
$$C_e^{FF} = \int_{\bar{j}_x}^1 e_j^*(z^*) y_j^* dj = \eta^* (1-\alpha) p_e^{-\epsilon_D^*} \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj$$
$$= \eta^* (1-\alpha) p_e^{-\epsilon_D^*} \frac{(A^*)^{(\sigma^*-1)/\theta}}{1 + (1-\sigma^*)/\theta} (1 - \bar{j}_x)^{1+(1-\sigma^*)/\theta};$$

8. energy used by producers in Foreign to supply imports of Home:
$$C_e^{HF} = \tau^* \int_{\bar{j}_m}^1 e_j^*(z) m_j dj = (\tau^*)^{1-\sigma} \eta (1-\alpha)(p_e + t_b)^{-\epsilon_D} \int_{\bar{j}_m}^1 \left(a_j^*\right)^{1-\sigma} dj$$
$$= (\tau^*)^{1-\sigma} \eta (1-\alpha)(p_e + t_b)^{-\epsilon_D} \frac{(A^*)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta} (1 - \bar{j}_m)^{1+(1-\sigma)/\theta};$$

[32]The incomplete beta function is:
$$B(x, a, b) = \int_0^x i^{a-1} (1-i)^{b-1} di,$$
for $0 \leq x \leq 1$, $a > 0$, and $b > 0$.Setting $x = 1$ gives the beta function itself, $B(a, b)$.

9. value of Home exports of goods:

$$V_g^{FH} = \int_0^{\bar{j}_x} p_j^x x_j dj = \eta^* p_e^{1-\epsilon_D^*} \int_0^{\bar{j}_x} \left(a_j^*\right)^{1-\sigma^*} dj$$
$$= \eta^* p_e^{1-\epsilon_D^*} \frac{\left(A^*\right)^{(\sigma^*-1)/\theta}}{1+\left(1-\sigma^*\right)/\theta} \left(1-\left(1-\bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta}\right);$$

10. and value of Home's imports of goods:

$$V_g^{HF} = \int_{\bar{j}_m}^1 p_j^m m_j dj = \left(\tau^*\right)^{1-\sigma} \eta \left(p_e+t_b\right)^{1-\epsilon_D} \left(\frac{p_e+\alpha t_b}{p_e+t_b}\right) \int_{\bar{j}_m}^1 \left(a_j^*\right)^{1-\sigma} dj$$
$$= \left(\tau^*\right)^{1-\sigma} \eta \left(p_e+t_b\right)^{1-\epsilon_D} \left(\frac{p_e+\alpha t_b}{p_e+t_b}\right) \frac{\left(A^*\right)^{(\sigma-1)/\theta}}{1+\left(1-\sigma\right)/\theta} \left(1-\bar{j}_m\right)^{1+(1-\sigma)/\theta};$$

11. intermediate demand for energy (for use in production in Home):

$$I_e = C_e^{HH} + C_e^{FH};$$

12. labor employed in production in Home:

$$L_g = \frac{\alpha}{1-\alpha}\left(p_e+t_b\right) I_e;$$

13. consumption demand for embodied energy in Home:

$$C_e = C_e^{HH} + C_e^{HF};$$

14. value of Home's net exports of goods:

$$X_g = V_g^{FH} - V_g^{HF};$$

15. total spending on goods by Home:

$$V_g = \eta^{1/\sigma} C_g^{1-1/\sigma} = \frac{1}{1-\alpha}\left(p_e+t_b\right) C_e;$$

16. energy extraction by Home:

$$Q_e = \left(p_e+t_b-\varphi\right)^{\epsilon_S} G;$$

17. energy extraction by Foreign:

$$Q_e^* = (p_e)^{\epsilon_S^*} G^*;$$

18. welfare in Home (dropping a constant):

$$U = C_s + \frac{\sigma}{\sigma-1}\eta^{1/\sigma} C_g^{1-1/\sigma} - \varphi Q_e^W = C_s + \frac{\sigma}{\sigma-1} V_g - \varphi \left(Q_e + Q_e^*\right);$$

19. and consumption of services in Home:

$$C_s = Q_s - X_s = L - L_e - L_g + X_g + p_e \left(Q_e - I_e\right).$$

To evaluate welfare it also helpful to have an expression for Home's spending on goods. Combining the expressions above for $\bar{j}_m$, C_e^{HH}, C_e^{HF}, we get:

$$\begin{aligned} V_g = \eta^{1/\sigma} C_g^{1-1/\sigma} &= \frac{1}{1-\alpha}\left(p_e + t_b\right)\left(C_e^{HH} + C_e^{HF}\right) \\ &= \eta \left(p_e + t_b\right)^{1-\epsilon_D} \frac{\left(A + \left(\tau^*\right)^{-\theta} A^*\right)^{(\sigma-1)/\theta}}{1 + (1-\sigma)/\theta}. \end{aligned}$$

D.2 Expression Calibrated to BAU

We now write the key expressions under the optimal unilateral policy in terms of the values we calibrate to under the BAU competitive equilibrium.

1. In the main text, we showed that the export cutoff is:

$$\bar{j}_x(p_e, t_b) = \frac{\left(p_e + (1-\alpha)\, t_b\right)^\theta C_e^{FH}}{\left(p_e + (1-\alpha)\, t_b\right)^\theta C_e^{FH} + \left(p_e^\alpha \left(p_e + t_b\right)^{1-\alpha}\right)^\theta C_e^{FF}},$$

   while in BAU:

$$\bar{j}_x = \frac{C_e^{FH}}{C_e^*} = \frac{\tau^{-\theta} A}{\tau^{-\theta} A + A^*}.$$

2. Energy demand by producers in Home to supply Home consumers becomes:

$$C_e^{HH}(p_e, t_b) = \left(p_e + t_b\right)^{-\epsilon_D} C_e^{HH}.$$

3. Energy demand by producers in Home to supply Foreign consumers requires that we calculate the BAU baseline, in which Home prices at marginal cost:

$$C_e^{FH} = \tau^{1-\sigma^*}\eta^*(1-\alpha)p_e^{-\epsilon_D^*}\frac{A^{(\sigma^*-1)/\theta}}{1+(1-\sigma^*)/\theta}\left(\bar{j}_x\right)^{1+(1-\sigma^*)/\theta}.$$

Under the optimal unilateral policy, we have:

$$\begin{aligned} C_e^{FH}(p_e,t_b) &= \tau^{\sigma^*}\left(1+\frac{1-\sigma^*}{\theta}\right)p_e^{-\epsilon_D^*}\left(\frac{p_e+t_b}{p_e}\right)^{-\alpha}\left(\frac{A^*}{A}\right)^{\sigma^*/\theta}\frac{B\left(\bar{j}_x(p_e,t_b),\frac{1+\theta}{\theta},\frac{\theta-\sigma^*}{\theta}\right)}{\bar{j}_x^{1+(1-\sigma^*)/\theta}}C_e^{FH} \\ &= \left(1+\frac{1-\sigma^*}{\theta}\right)\left(\frac{1-\bar{j}_x}{\bar{j}_x}\right)^{\sigma^*/\theta}p_e^{-\epsilon_D^*}\left(\frac{p_e+t_b}{p_e}\right)^{-\alpha}\frac{B\left(\bar{j}_x(p_e,t_b),\frac{1+\theta}{\theta},\frac{\theta-\sigma^*}{\theta}\right)}{\bar{j}_x^{1+(1-\sigma^*)/\theta}}C_e^{FH}. \end{aligned}$$

4. Energy demand by producers in Foreign to supply Foreign consumers becomes:

$$\begin{aligned} C_e^{FF}(p_e,t_b) &= p_e^{-\epsilon_D^*}\left(\frac{1-\bar{j}_x(p_e,t_b)}{1-\bar{j}_x}\right)^{1+(1-\sigma^*)/\theta}C_e^{FF} \\ &= p_e^{-\epsilon_D^*}\left(\frac{C_e^*\left(p_e^\alpha\left(p_e+t_b\right)^{1-\alpha}\right)^\theta}{C_e^{FH}\left(p_e+(1-\alpha)t_b\right)^\theta+C_e^{FF}\left(p_e^\alpha\left(p_e+t_b\right)^{1-\alpha}\right)^\theta}\right)^{1+(1-\sigma^*)/\theta}C_e^{FF}. \end{aligned}$$

5. Energy demand by producers in Foreign to supply Home consumers becomes:

$$C_e^{HF}(p_e,t_b) = \left(p_e+t_b\right)^{-\epsilon_D}C_e^{HF}.$$

6. The value of Home's exports of goods becomes:

$$V_g^{FH}(p_e,t_b) = p_e^{1-\epsilon_D^*}\frac{1-\left(1-\bar{j}_x(p_e,t_b)\right)^{(\theta+1-\sigma^*)/\theta}}{1-\left(1-\bar{j}_x\right)^{(\theta+1-\sigma^*)/\theta}}V_g^{FH},$$

where we set the baseline value to:

$$V_g^{FH} = \frac{1}{1-\alpha}C_e^{FH}.$$

7. The value of Home's imports of goods becomes:

$$V_g^{HF}(p_e,t_b) = \left(p_e+t_b\right)^{-\epsilon_D}\left(p_e+\alpha t_b\right)V_g^{HF},$$

where we set the baseline value to:

$$V_g^{HF} = \frac{1}{1-\alpha}C_e^{HF}.$$

8. In the main text we showed that Home's extraction is:

$$Q_e(p_e, t_b) = (p_e + t_b - \varphi)^{\epsilon_S} Q_e.$$

9. Foreign's extraction, for $p_e \geq \underline{a}$ is simply:

$$Q_e^*(p_e) = p_e^{\epsilon_S^*} Q_e^*.$$

Using these expressions we can return to (51) and (52), searching for the pair (p_e, t_b) that jointly solves them. Having solved for the optimal border adjustment and the corresponding change in the global energy price we can compute all other outcomes as well.

A key outcome is Home's welfare in moving to the optimal unilateral policy from the BAU competitive equilibrium. The change in Home's welfare is:

$$\begin{aligned} U(p_e, t_b) - U = &-(L_e(p_e, t_b) - L_e) - (L_g(p_e, t_b) - L_g) + (X_g(p_e, t_b) - X_g) \\ &+ (p_e X_e(p_e, t_b) - X_e) + \frac{\sigma}{\sigma - 1}(V_g(p_e, t_b) - V_g) - \varphi(Q_e^W(p_e, t_b) - Q_e^W), \end{aligned}$$

where we denote the value of Home's spending on goods by V_g. Our preferred measure of welfare is normalized by BAU spending on goods:

$$W = \frac{U(p_e, t_b) - U}{V_g}.$$

We have expressions for each of the outcomes required to evaluate this welfare expression:

1. The change in Home's employment in energy extraction is:

$$\begin{aligned} L_e(p_e, t_b) - L_e &= \int_1^{p_e + t_b - \varphi} a G'(a) da \\ &= Q_e \int_1^{p_e + t_b - \varphi} \epsilon_S a^{\epsilon_S} da \\ &= \frac{\epsilon_S}{\epsilon_S + 1}((p_e + t_b - \varphi)^{\epsilon_S + 1} - 1) Q_e. \end{aligned}$$

2. The change in Home's employment in goods production is:

$$L_g(p_e, t_b) - L_g = ((p_e + t_b) M_e(p_e, t_b) - 1) L_g,$$

where the baseline value is:

$$L_g = \frac{\alpha}{1 - \alpha} I_e.$$

3. The value of Home's net exports of goods is:

$$X_g(p_e, t_b) = V_g^{FH}(p_e, t_b) - V_g^{HF}(p_e, t_b).$$

4. The value of Home's net energy exports is:

$$p_e X_e(p_e, t_b) = p_e \left(Q_e(p_e, t_b) - \left(C_e^{HH}(p_e, t_b) + C_e^{FH}(p_e, t_b)\right)\right).$$

5. The value of Home's spending on goods is:

$$V_g(p_e, t_b) = (p_e + t_b)^{1-\epsilon_D} V_g,$$

where the baseline value is:

$$V_g = \frac{1}{1-\alpha} C_e.$$

6. The term that enters the change in Home's welfare is:

$$\frac{\sigma}{\sigma - 1}(V_g(p_e, t_b) - V_g) = V_g \frac{\left((p_e + t_b)^{(1-\alpha)(1-\sigma)} - 1\right)}{(\sigma - 1)/\sigma}.$$

For the case of $\sigma = 1$ this term reduces to:

$$\lim_{\sigma \to 1} V_g \frac{(p_e + t_b)^{(1-\alpha)(1-\sigma)}}{(\sigma - 1)/\sigma} = -(1-\alpha) V_g \ln(p_e + t_b).$$

7. Global emissions are:

$$Q_e^W(p_e, t_b) = Q_e(p_e, t_b) + Q_e^*(p_e).$$

E Data and Calibration

E.1 Calibration

For our quantitative analysis we calibrate the model to fossil fuel extraction and the energy embodied in trade between the region that, in our model, will enact a carbon policy (Home) and the region that will remain with business as usual (Foreign). Our common unit for energy is gigatonnes of CO_2, based on the quantity released by its combustion.

We consider three scenarios for the regions representing Home and Foreign. In the first, the United States is Home and all other countries are Foreign. The alternative scenarios, respectively, are the European Union prior to Brexit (EU28) as Home (and all other countries as Foreign) and the members of the Organization for Economic Cooperation and Development (OECD37) as Home (and all others as Foreign).

Our data source for energy consumption is The Trade in Embodied CO_2 (TECO2) database from OECD. We use their measure of consumption-based CO_2 emissions embodied in domestic final demand and the country of origin of emissions. This database covers 83 countries and regional groups over the period 2005-2015. Carbon dioxide embodied in world consumption in 2015 is 32.78 gigatonnes. We cross-checked the results with a dataset from the Global Carbon Project. The overall difference is less than ten percent.

Extraction data are from the International Energy Agency (IEA), which provides the World Energy Statistics Database on energy supply from all energy sources, including fossil fuels, biofuels, hydro, geothermal, renewables and waste. This dataset covers 143 countries as well as regional and world totals. The data are provided in units of kilotonnes of oil equivalent (ktoe). In order to keep the units consistent with the energy consumption data (gigatonnes of carbon dioxide), we first convert to terajoules (TJ) (1 ktoe = 41.868 TJ) and then apply emission factors to the five fossil fuel types to calculate CO_2 emissions. The five fossil fuel types considered are coal and coal products, natural gas, peat and peat products, oil products, as well as crude, NGL and feedstocks. The emission factors are default emission factors for stationary combustion from the 2006 IPCC Guidelines for National Greenhouse Gas Inventories. To be specific, we convert 1 TJ of crude, NGL and feedstocks to 73,300 kg CO_2, 1 TJ of natural gas to $56,100$ kg CO_2, and 1 TJ of coal, peat and oil products to $94,600$ kg CO_2. Using this calculation, world extraction is 35.96 gigatonnes of carbon dioxide.

To explain the discrepancy between world consumption and world extraction, note that the OECD data for embodied carbon does not include non-energy use of fossil fuels. In other words, some fossil fuels extracted are not combusted to produce energy. Instead, they are consumed directly or as intermediate goods. For example, petroleum can be used as asphalt and road oil and as petrochemical feedstocks for agricultural land. According to EIA (2018), approximately 8 percent of fossil fuels are not combusted in the United States. Applying this rate to the world extraction, we get a number close to world consumption ($35.96 * 0.92 = 33.08$, vs. 32.78).

Given that combusted energy is the source of CO_2 emissions, non-energy use of fossil fuel extraction is excluded in our analysis. We simply re-scale the world extraction data so that world extraction is equal to world consumption. To be specific, the original extraction data is divided by 1.097 (the ratio of world extraction to world consumption). Tables 3, 5, and 6 display the resulting data we use for our calibration.

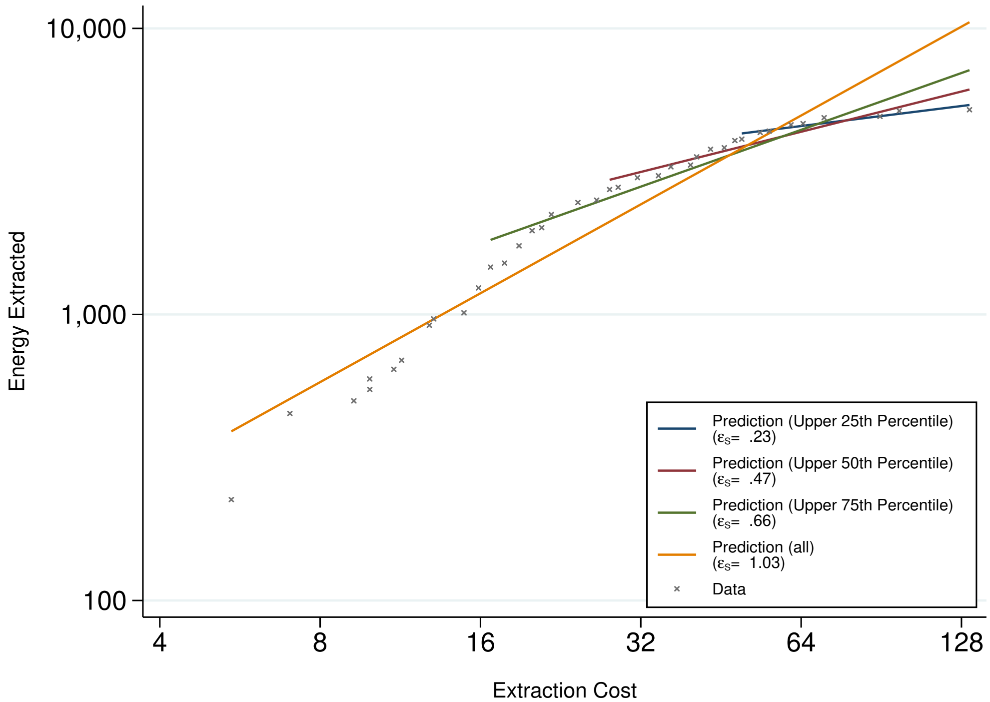
E.2 Parameter Values

For the key parameter in the goods production function α, the output elasticity of labor, we calibrate $(1-\alpha)/\alpha$ to the value of energy used in production $p_e I_e$ relative to the value added.[33] The data from TECO2 records the carbon emissions embodied by sector and country. We can convert to barrels of oil based on 0.43 metric tons of CO_2 per barrel of crude oil (from EPA, 2019). The price per barrel of oil is taken from the average closing price of West Texas Intermediate (WTI) crude oil in 2015, which is \$48.66 per barrel. Value added data comes from OECD Input-Output Tables (2018). We consider three definitions of the goods sector, with both the numerator (value of energy) and the denominator (value added) computed for the same sector definition, either: (i) the manufacturing sector, (ii) manufacturing plus agriculture and construction, and (iii) manufacturing, agriculture, construction, wholesale, retail, and transportation. The values of α that we obtain are, respectively, 0.85, 0.79, and 0.84. Our preferred value is 0.85, very close to two of these three.

For the energy supply elasticities, ϵ_S and ϵ_S^*, we use data from Asker, Collard Wexler, and Jan De Loecker (2018) on the distribution across oil fields of extraction costs. The data come in the form of quantiles (q = 0.05, 0.10, ..., 0.95), separately for the EU, the US, OPEC, and ROW (q% of oil in the US is extracted at a cost below \$$a$ per barrel, for example). We approximate OECD countries by aggregating the EU and US while for the non-OECD region we aggregate OPEC and ROW. To aggregate the quantiles for two regions, we combine them, sort the combination by the cost level, and reassemble after taking account of total oil extraction for each region (available from the IEA). The data are plotted on log scales in Figures 6 and 7, to reveal the supply elasticities.

[33]We think of value added as the closest proxy to labor cost in the model, since we interpret labor in the model as labor equipped with capital.

Figure 6: Calibration of the Extraction Supply Elasticity in Home



The most costly oil fields in either region would be the first to be abandoned under a carbon policy. Thus, the upper end of the cost distribution is the most relevant for calibrating the supply elasticities. Our baseline values of $\epsilon_S = 0.5$ and $\epsilon_S^* = 0.5$ are close to the slope shown in the figures when we consider only costs above the median. Our alternative value of $\epsilon_S^* = 1$ is closer to the slope if we were to use the upper 75% of costs or even all the data.

Lacking this distributional data for coal and natural gas fields, we assume that the distribution for oil extraction is representative of all fossil fuels.

Figure 7: Calibration of the Extraction Supply Elasticity in Foreign