

# The Welfare Implications of Carbon Price Certainty

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Harvard University

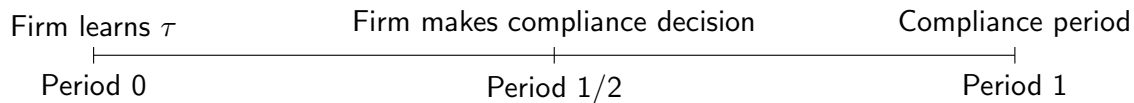
February 27, 2020

# Policy Uncertainty and Investment

- Policy uncertainty affects firm investment:
  - Baker, Bloom, and Davis, 2016; Hassett and Metcalf, 1999; Rodrik, 1991 – building on Arrow, 1959; Bernanke, 1983; and others.
- Distinguish between two types of policy uncertainty:
  - **Uncertainty over policy design**
  - **Uncertainty inherent to policy instrument**
- Inherent uncertainty differs under **price** vs. **quantity** instruments for correcting Pigouvian externalities.

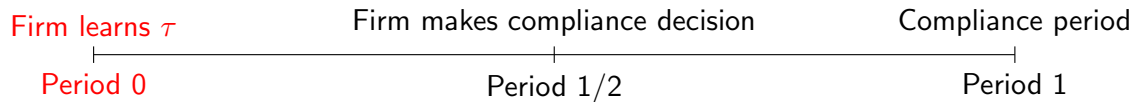
# Inherent Policy Uncertainty

## Price-Based Instrument:



# Inherent Policy Uncertainty

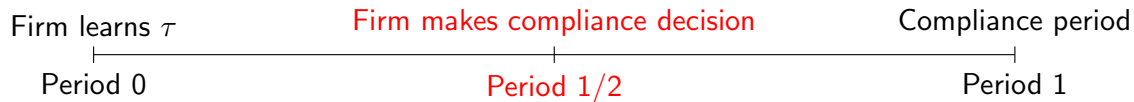
## Price-Based Instrument:



⇒ Firm knows Pigouvian tax  $\tau$  with certainty.

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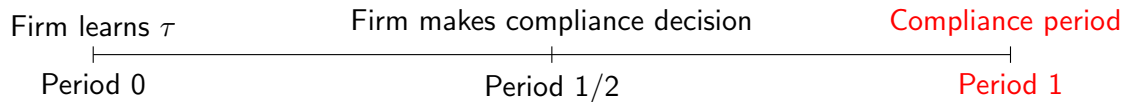
## Price-Based Instrument:



⇒ Firm sets marginal abatement cost equal to  $\tau$ .

# Inherent Policy Uncertainty

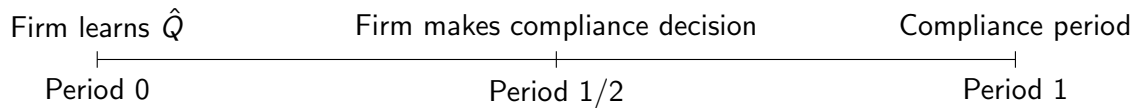
## Price-Based Instrument:



⇒ Regulator enforces firm compliance.

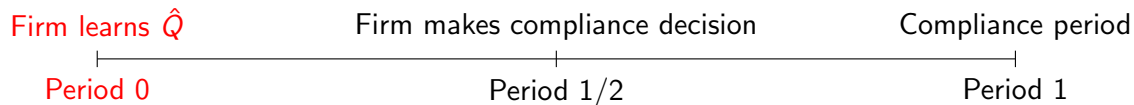
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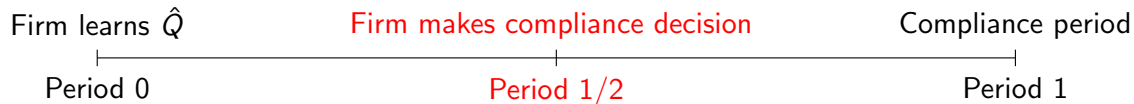


⇒ Firm knows  $\hat{Q}$  with certainty, but not resulting market price.



# Inherent Policy Uncertainty

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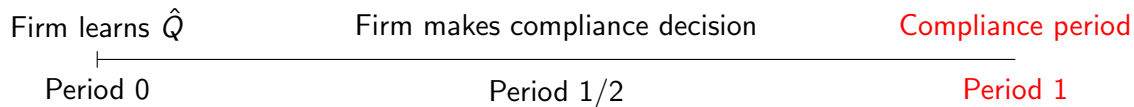


⇒ Firm must form expectation over all other firms' marginal abatement cost curves, output levels, and overlapping policies to estimate market-clearing price.

⇒ Firm then sets marginal abatement cost equal to expected price.

# Inherent Policy Uncertainty

## Quantity-Based Instrument:



⇒ Regulator enforces firm compliance, and market for allowances clears. In general, realized market price does not equal a firm's expected price.

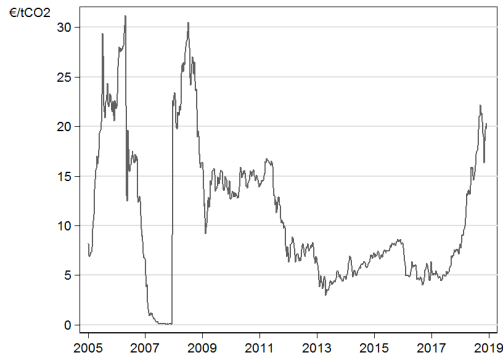
# Long-Lived Abatement Investments

Cost-effective abatement options are often long-lived capital investments:

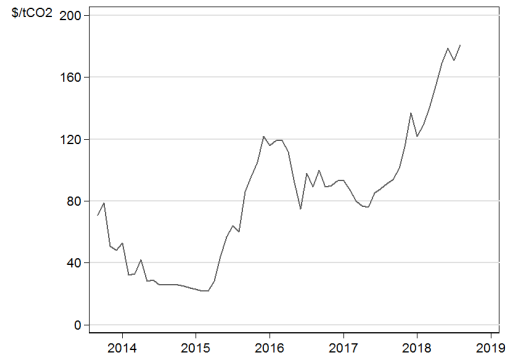
Allowance Market	Abatement Option
<b>SO<sub>2</sub></b>	installing scrubbers, retrofitting plants for low-sulfur coal
<b>NO<sub>x</sub></b>	installing selective catalytic reduction
<b>CO<sub>2</sub></b>	investing in renewables, installing carbon capture and storage
<b>RPS, EEPS</b>	investing in renewables, retrofitting built environment
<b>RFS</b>	investing in biorefineries
<b>Vehicle efficiency</b>	developing new vehicle models

# Allowance Price Volatility

High historical price volatility in allowance and credit trading markets:

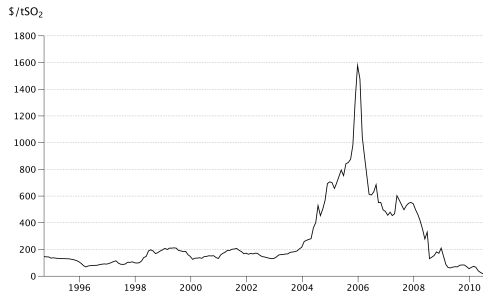


Emissions Trading System CO<sub>2</sub> Allowance Price (EU)



Low Carbon Fuel Standard Allowance Price (California)

# Allowance Price Volatility



SO<sub>2</sub> Allowance Price (US)



NO<sub>x</sub> Allowance Price (US)

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Impact of price volatility is not entirely resolved via financial instruments:

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- Firms do not hedge completely even when financial instruments are available and volatile prices represent substantial business expense.
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- Firms may not know total hedging requirement with certainty, where Total Hedging =  $p \cdot q(p, \theta)$
- Large markets created by regulation face start-up problem.

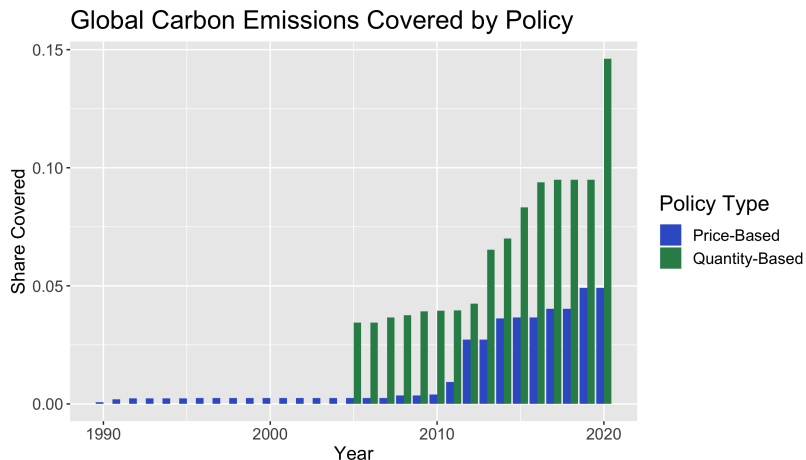
Start-Up Problem

# Evidence of Cost Inefficiency in Cap & Trade

Empirical literature suggests inefficiencies in cap-and-trade programs:

- Carlson et al. (2000): One-half of Phase I units in  $SO_2$  C&T program deviated at some point from least-cost compliance strategies.
- Fowlie (2010), Cicala (2015): Deregulated firms may underinvest in capital-intensive compliance strategies for  $SO_2$  and  $NO_x$  C&T programs, paired with overinvestment by regulated firms.
- Frey (2013), Chan et al. (2018): Overlapping policies further lead to compliance strategies inconsistent with cost minimization.

# Revealed Preference in Environmental Policy Design



Source: World Bank, *State and Trends of Carbon Pricing*

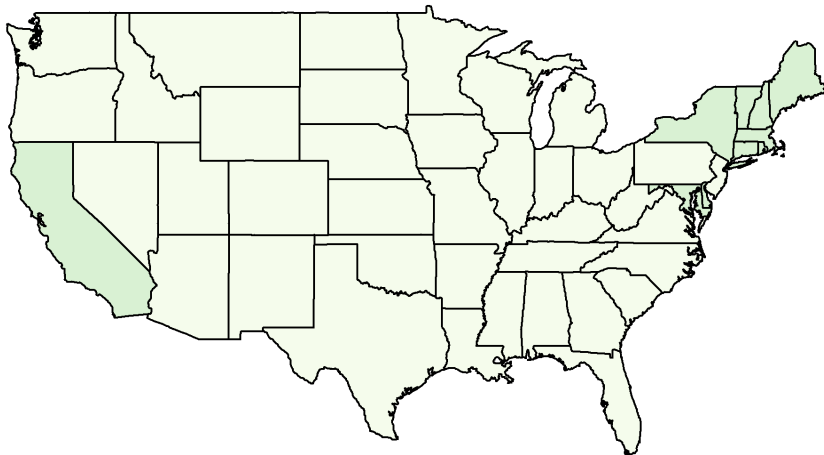
# Revealed Preference in Environmental Policy Design

## **Allowance and credit trading programs in U.S. energy markets:**

- State CO<sub>2</sub> Cap-and-Trade Programs
- State Renewable Portfolio Standard
- Gasoline Sulfur and Benzene Credit Trading
- Renewable Fuel Standard
- Cross-State Air Pollution Rule
- Low-Carbon Fuel Standard

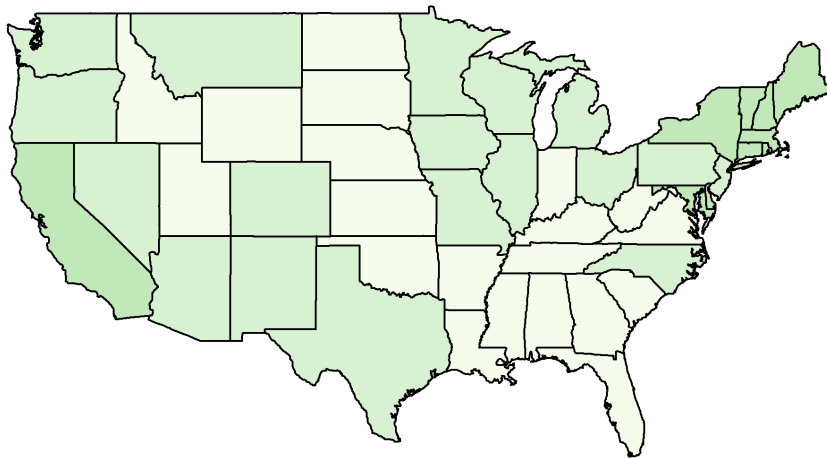
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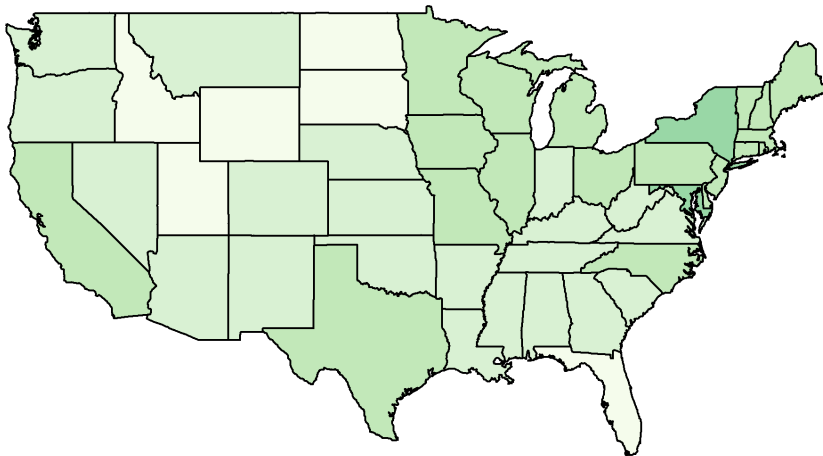
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## State Renewable Portfolio Standards with Credit Trading:



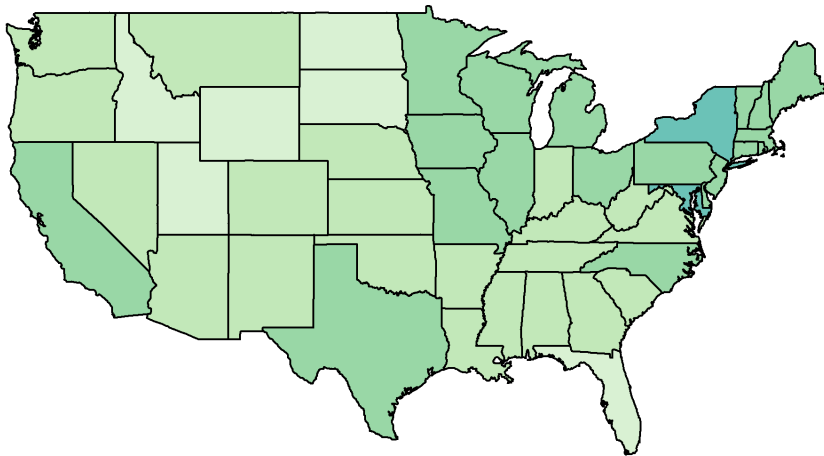
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## Cross-State Air Pollution Rule Allowance Trading:



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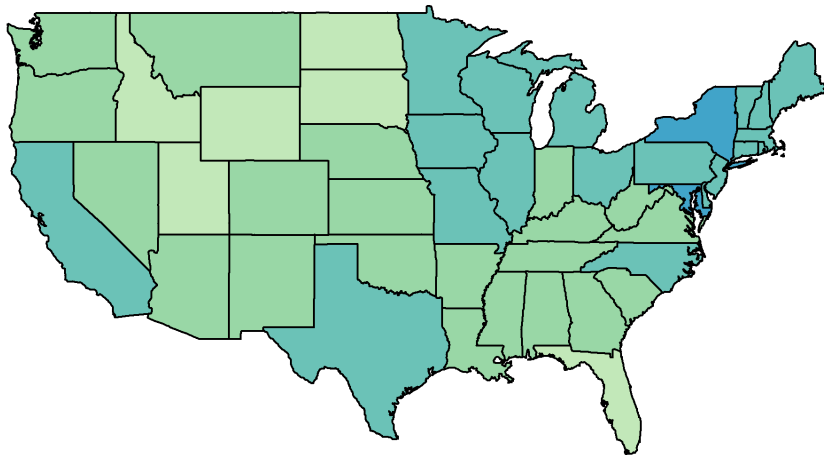
## Renewable Fuel Standard Credit Trading:





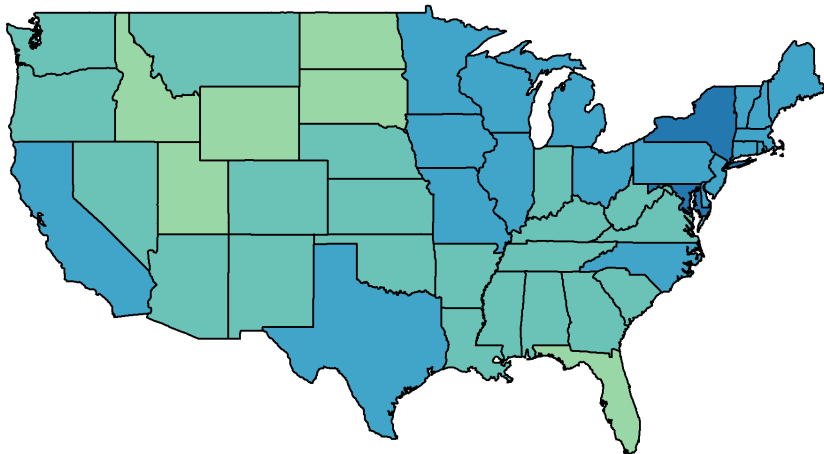
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## Gasoline Benzene Credit Trading:



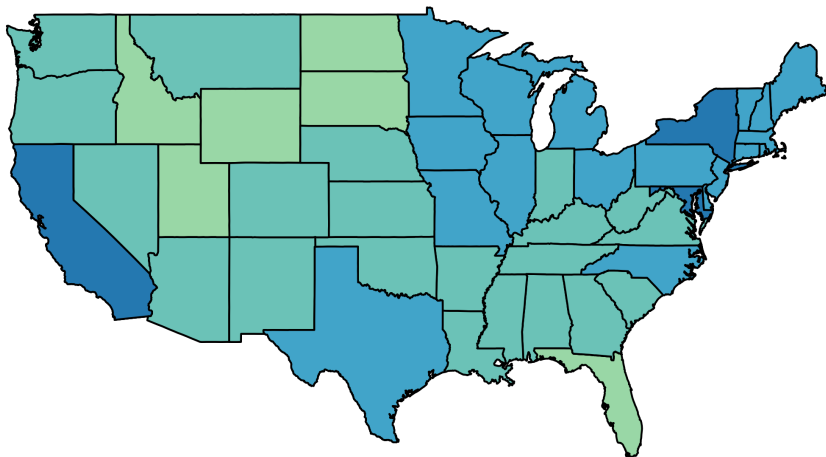
## Revealed Preference in Environmental Policy Design

## Gasoline Sulfur Credit Trading:



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### Low-Carbon Fuel Standard:



**How does this inherent policy uncertainty affect firm behavior in allowance and credit trading markets?**

# Dynamic Model of Firm Investment in Abatement

Adapt Pindyck (1980), Rubin (1996), and Anderson, Kellogg, and Salant (2018) to model emissions trading market:

$$\max_{A,Y} E_0 \left[ \int_0^T e^{-rt} \{ -\psi(A(t)) - P(t)Y(t) \} dt \right]$$

- $A(t)$ : abatement investment
- $\psi(\cdot)$ : investment cost function
- $Y(t)$ : allowances purchased
- $r$ : discount rate
- $P(t)$ : current allowance price, which follows GBM with drift  $\alpha$  and volatility  $\sigma$

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$$A(t) \geq 0, K_0 \text{ given}$$

$$\dot{B} = K(t) + Y(t) - \bar{E}$$

$$B(T) \geq 0, B_0 = 0$$

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Compare to optimality condition when abatement is variable input (Rubin, 1996):

$$\dot{P} = r \cdot P$$

# Optimality Conditions

- 2 However, optimal abatement is now dynamic decision. Assuming firm chooses some unconstrained  $A^* \geq 0$ :

$$(r + \delta)\psi'(A^*) = P + \psi''(A^*)\frac{1}{dt}E_t[dA^*] + \psi'''(A^*)\frac{1}{dt}E_t[(dA^*)^2]$$

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# Model Interpretation

- Banking activity pins down expected price path in equilibrium.
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  - Anticipated cap-and-trade policy reform
  - Changes in economic output
- Modeling long-lived, dynamic investment illuminates impact of price volatility:
  - Firms may have forecast errors in estimating future stream of prices.
  - Firms take into account price volatility in value of smoothing.

**How does this inherent policy uncertainty affect the cost of achieving an emissions target?**

# Simulations

- Model compliance decisions of representative firm given simulated price trajectory:
  - Scenario 1: Firm makes abatement investment decisions given stochastic prices.
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- Both scenarios produce same total emissions reductions, but costs are higher with stochastic prices.
- Price volatility alters effective abatement cost function for quantity-based instruments relative to price-based instruments.

# Model Calibration

- Calibrate abatement cost function to U.S. carbon tax simulations from the Stanford Energy Modeling Forum 32 (Barron et al., 2018), assuming quadratic abatement investment costs.
  - Later work will examine richer specifications of cost function and abatement opportunities.

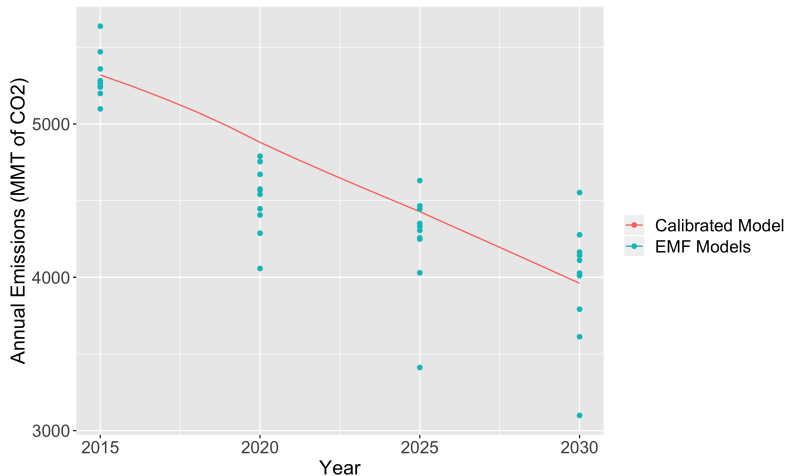
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- Calibrate drift and volatility parameters to historical EU ETS allowance prices for Phases II and III (2008-2018), assuming prices follow geometric Brownian motion.
  - Estimate 5.2% annual expected price growth ( $\alpha = 0.0508$ )
  - Estimate 42.9% annual price volatility ( $\sigma = 0.3925$ )



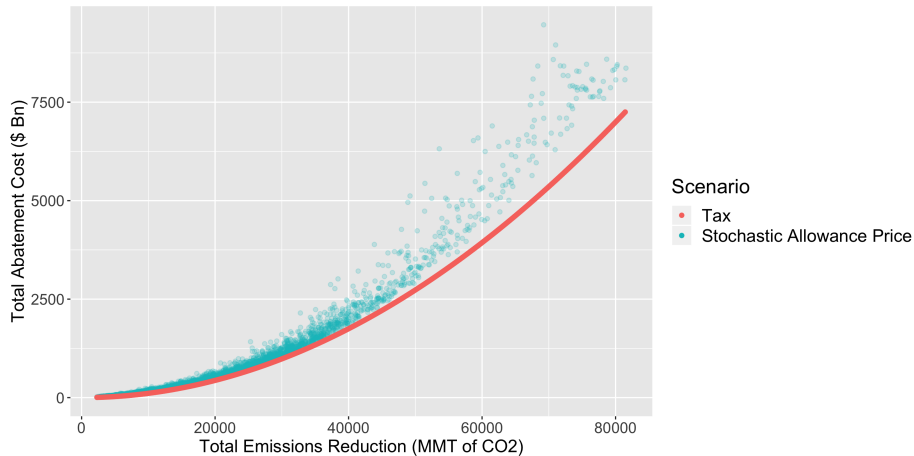
# Model Calibration

## Total Emissions Reduction (10 Years of Abatement Investment):



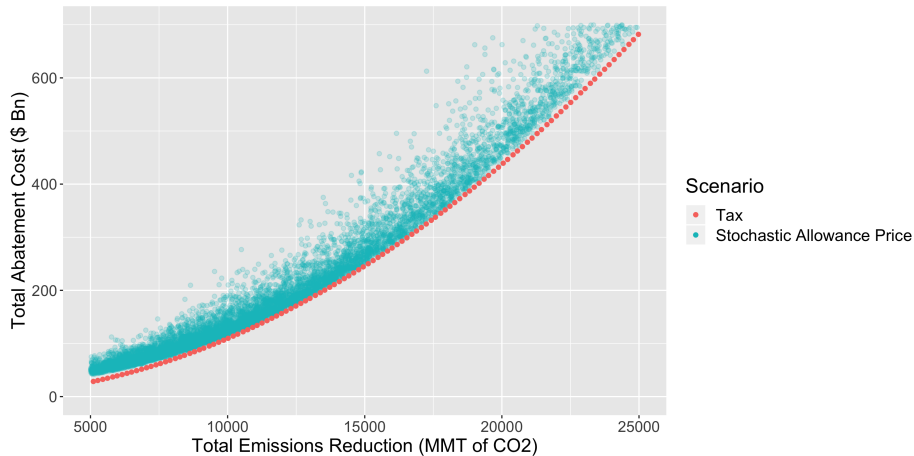
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  - **Optimal price order:**

$$\max_{\tau_1, \tau_2} E \left[ B_1 \left( \sum_{i=1}^N q_1^i(\tau_1, \theta_1^i) \right) - \sum_{i=1}^N C_1^i(q_1^i(\tau_1, \theta_1^i), \theta_1^i) \right. \\ \left. B_2 \left( \sum_{i=1}^N q_2^i(\tau_2, \theta_2^i) \right) - \sum_{i=1}^N C_2^i(q_2^i(\tau_2, \theta_2^i), \theta_2^i) \right]$$

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$$\begin{aligned} \max_{\hat{Q}} E[ & B_1(\sum_{i=1}^N q_1^i(p_1(\hat{Q}, \theta), \theta_1^i)) - \sum_{i=1}^N C_1^i(q_1^i(p_1(\hat{Q}, \theta), \theta_1^i), \theta_1^i) \\ & + B_2(\sum_{i=1}^N q_2^i(p_2(\hat{Q}, \theta), \theta_2^i)) - \sum_{i=1}^N C_2^i(q_2^i(p_2(\hat{Q}, \theta), \theta_2^i), \theta_2^i)] \end{aligned}$$

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- Relative advantage of prices over quantities:

$$\Delta = E\left[\frac{1}{4}\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B''\right)\left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''}\right)^2\right]$$

$C_i''$ : slope of marginal cost function for production unit  $i$

$B''$ : slope of aggregate marginal benefit function

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$C_i''$ : slope of marginal cost function for production unit  $i$

$B''$ : slope of aggregate marginal benefit function

- Compare to Weitzman (2018) result with representative firm: Derivation

$$\Delta = E\left[\frac{1}{4}(C'' - B'') \cdot \left(\frac{\theta_2 + \theta_1}{C''}\right)^2\right]$$

# Revisiting Prices vs. Quantities

- Now assume firm  $i$  sets:

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Mis-allocation across compliance periods

# Welfare Consequences of Uncertainty

- **UNCERTAINTY TYPE #1:** Idiosyncratic forecast errors with no impact on aggregate distribution of quantity across periods.

Relative advantage of prices over quantities: Derivation

$$\Delta = E\left[\frac{1}{4}\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B''\right)\left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''}\right)^2 + \frac{1}{2} \sum_{i=1}^N \frac{\epsilon_1^{i2}}{C_i''}\right]$$

One Period

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- Cost inefficiency arises from failure to allocate quantity optimally across firms.

One Period

# Welfare Consequences of Uncertainty

- **UNCERTAINTY TYPE #2:** Systematic forecast errors that impact aggregate distribution of quantity across periods.

Relative advantage of prices over quantities: Derivation

$$\begin{aligned}\Delta = E[ & \frac{1}{4} \left( \frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left( \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''} \right)^2 + \frac{1}{2} \sum_{i=1}^N \frac{\epsilon_1^i{}^2}{C_i''} \\ & + \frac{1}{2} \left( \frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} \right) \left( \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''} \right)^2 + B'' \left( \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''} \right)^2 + 2B'' \left( \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''} \right) \left( \sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} \right) ]\end{aligned}$$

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- Cost inefficiency arises from failure to allocate quantity optimally across periods.
- Benefit smoothing across periods may increase or decrease, with ambiguous effects for welfare.

# Welfare Consequences of Uncertainty

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# Welfare Consequences of Uncertainty

- Forecast errors asymmetrically affect quantity-based policies, conditional on policy design uncertainty being resolved.
- Both types of firm forecast errors create cost inefficiencies that push regulator to prefer price instruments.
  - Effect on benefit smoothing is ambiguous.
- Given banking and borrowing, uncertainty in one compliance period may continue to create cost inefficiencies in future periods.

## Conclusion & Future Directions

- Evaluations of price versus quantity instruments should take into account asymmetric firm forecast errors.

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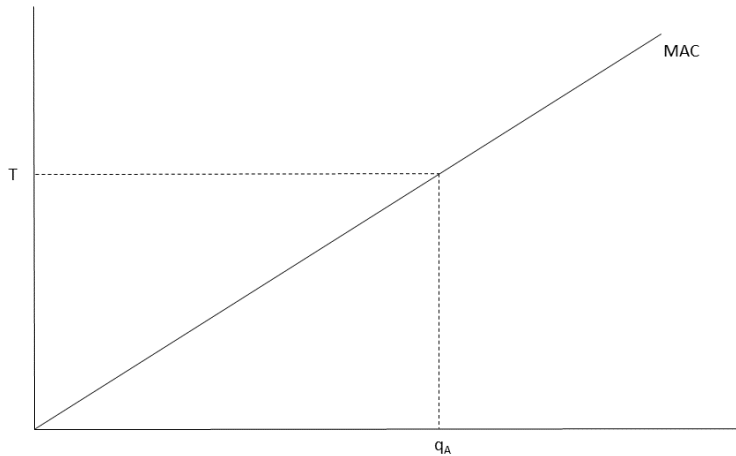
- Evaluations of price versus quantity instruments should take into account asymmetric firm forecast errors.
- Effective abatement cost function depends on policy instrument → in simulations, median percentage difference in costs is 21%.
- Future work will focus on correlation between price uncertainty and abatement cost uncertainty and bringing full dynamics into welfare analysis.

Thank you!

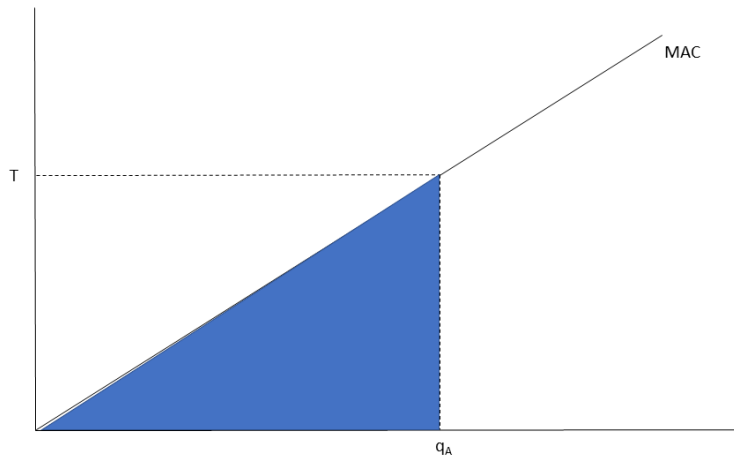
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<https://scholar.harvard.edu/jaldy>

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<https://scholar.harvard.edu/sarmitage>

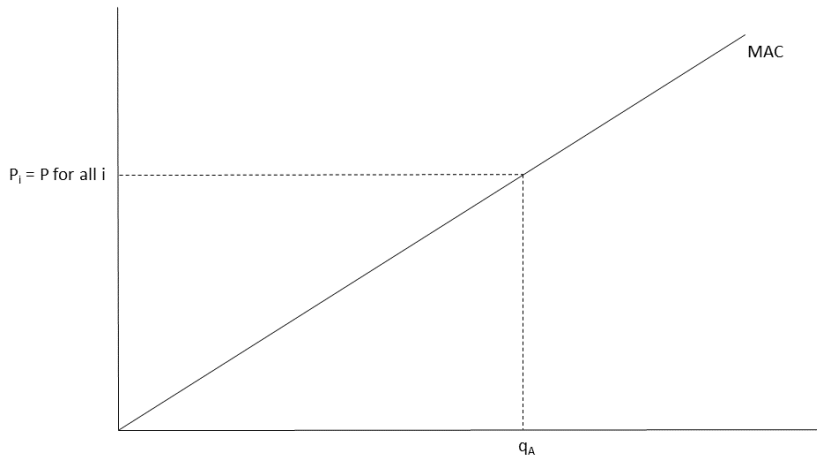
# Graphical Illustration



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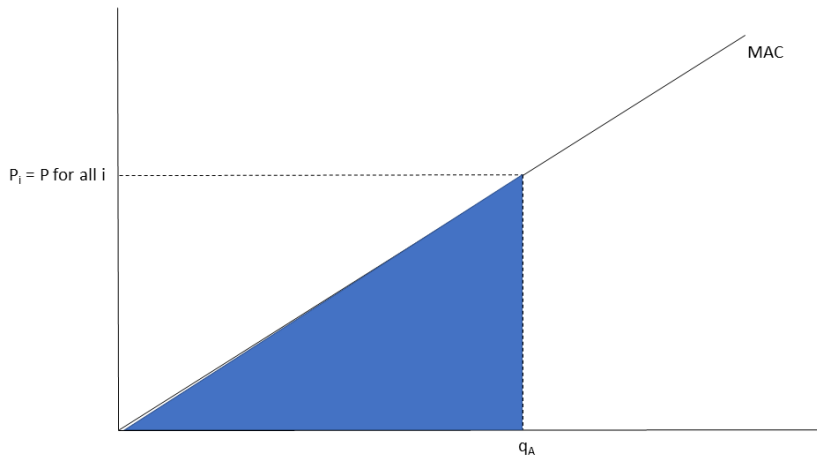


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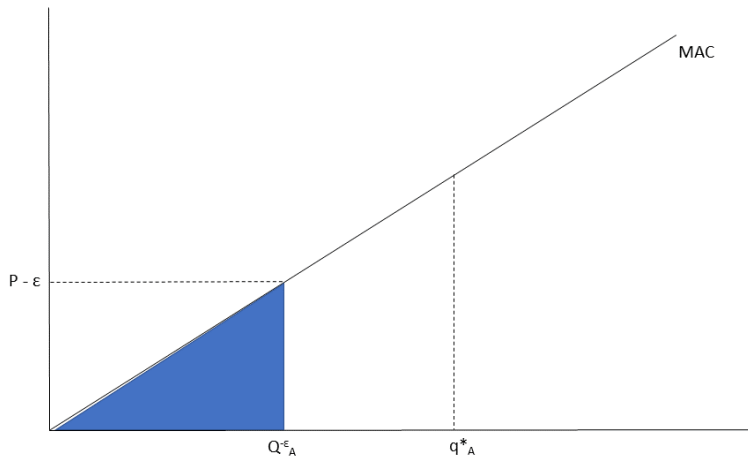




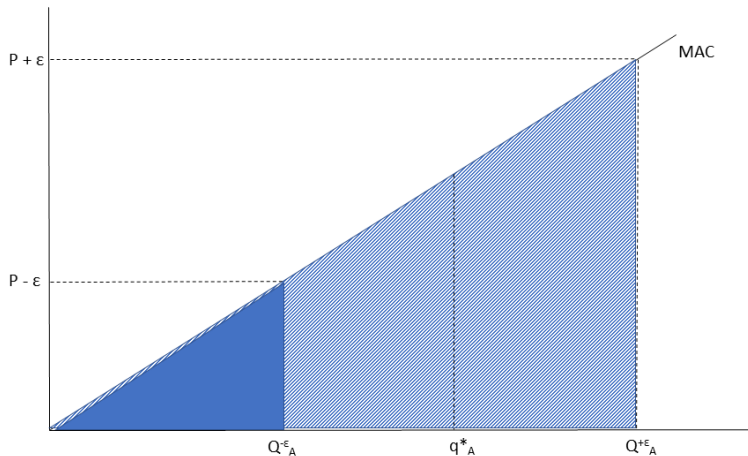
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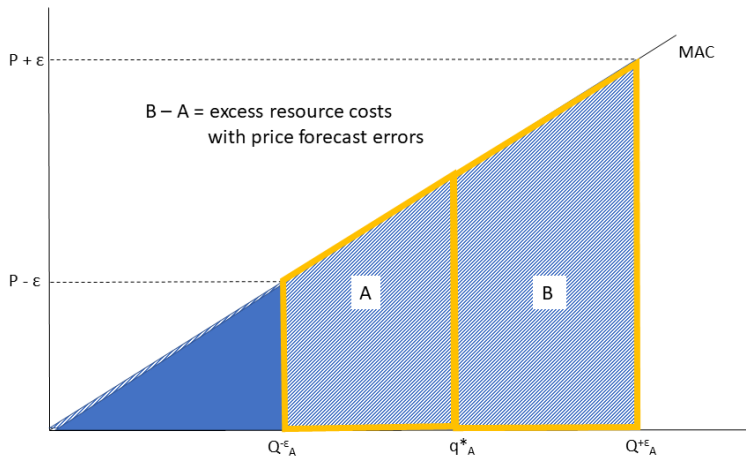
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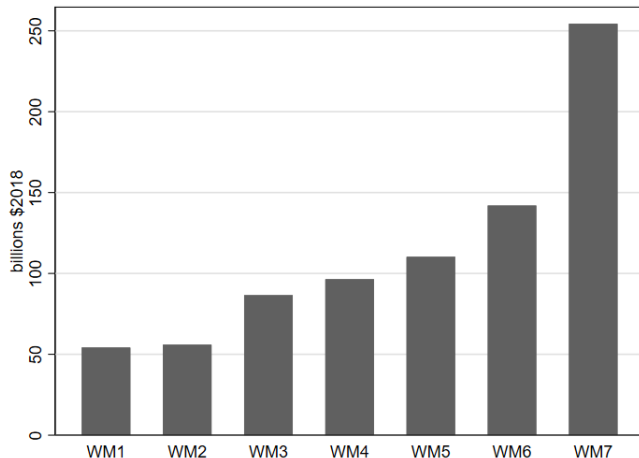


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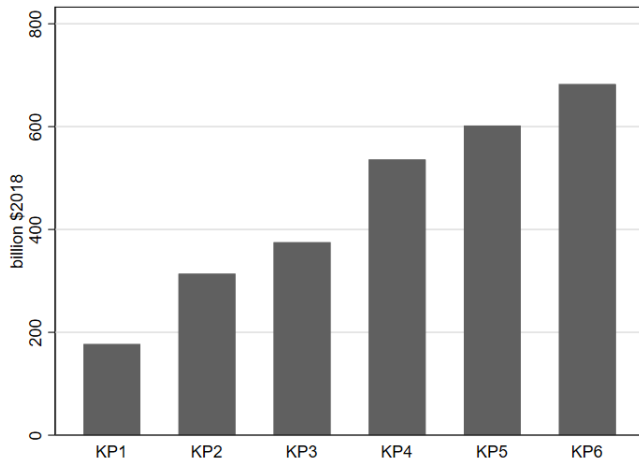
# Start-Up Problem in Allowance Trading Markets

## Value of Allowance Market in Year 1 - Waxman-Markey Bill: [Back](#)



# Start-Up Problem in Allowance Trading Markets

## Value of Allowance Market in Year 1 - Kyoto Protocol: [Back](#)



# Revisiting Prices vs. Quantities Back

- Regulator's optimal price policy:  $\tau_1 = \tau_2 = C'$
- Regulator's optimal quantity policy:  $\hat{Q} = \sum_{i=1}^N \bar{q}_1^i + \sum_{i=1}^N \bar{q}_2^i$  where  $\bar{q}_t^i$  sets  $E[\frac{\partial B_t}{\partial q_t^i}] = E[\frac{\partial C_t^i}{\partial q_t^i}]$ .
- Two key conditions govern market-clearing price: no intertemporal arbitrage and the regulator's quantity limit. Applying these conditions:

$$\hat{p}_1(\hat{Q}, \theta_1, \theta_2) = \hat{p}_2(\hat{Q}, \theta_1, \theta_2) = C' + \frac{\sum_i \frac{\theta_1^i + \theta_2^i}{2C_i''}}{\sum_i \frac{1}{C_i''}}$$

- Quantity response by firm  $i$  in period 1 (for illustration):

$$q_1^i(\hat{p}_1, \theta_1^i) = \frac{\hat{p}_1 - C' - \theta_1^i}{C_i''} + \bar{q}_1^i = \frac{\frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_1^i}{C_i''} + \bar{q}_1^i$$

## Revisiting Prices vs. Quantities [Back](#)

- Quantity response in presence of first-period forecast errors:

$$q_1^i(p_1, \theta_1^i) = \frac{\hat{p}_1(\hat{Q}, \theta_1, \theta_2) + \epsilon_1^i - C' - \theta_1^i}{C_i''} + \bar{q}_1^i$$

- Constraint such that aggregate quantity in first period is unchanged:

$$\sum_{i=1}^N \frac{\epsilon_1^i}{C_i''} = 0$$

- Expected benefits are unchanged, as are expected costs in period 2. Expected costs in period 1:

$$\sum_{i=1}^N E[\theta_1^i (\frac{\frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_{j=1}^N \frac{1}{C_j''}} + \epsilon_1^i - \theta_1^i}{C_i''}) + \frac{C_i''}{2} (\frac{\frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_{j=1}^N \frac{1}{C_j''}} + \epsilon_1^i - \theta_1^i}{C_i''})^2]$$



## Revisiting Prices vs. Quantities [Back](#)

- Aggregate first-period quantity changes:

$$Q_1 = \frac{\hat{Q}}{2} + \sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} + \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}$$

- Second-period price must adjust to ensure regulatory limit is still met:

$$\hat{p}_2'(\hat{Q}, \theta_1, \theta_2, \epsilon_1) = C' + \frac{\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}}{\sum_{i=1}^N \frac{1}{C_i''}}$$

- Aggregate second-period quantity then changes:

$$Q_2 = \frac{\hat{Q}}{2} + \sum_{i=1}^N \frac{\theta_1^i - \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}$$

- Expected benefits and expected costs (first and second periods) all change.

## Revisiting Prices vs. Quantities [Back](#)

- Market-clearing price such that realized quantity is regulated quantity (no intertemporal arbitrage condition no longer applies):

$$\hat{p}(\hat{Q}, \theta) = C' + \frac{\sum_{i=1}^N \frac{\theta_i}{C_i''}}{\sum_{i=1}^N \frac{1}{C_i''}}$$

- Relative advantage of prices over quantities:

$$\Delta_n = \frac{B''}{2} \sum_{i=1}^N \sum_{j=1}^N \frac{\sigma_{ij}^2}{C_i'' C_j''} + \frac{1}{2 \sum_{j=1}^N \frac{1}{C_j''}} \sum_{i=1}^N \sum_{j=1}^N \frac{\sigma_{ij}^2}{C_i'' C_j''}$$

- In the presence of forecast errors, relative advantage of prices over quantities becomes:

$$\Delta_n = \frac{B''}{2} \sum_{i=1}^N \sum_{j=1}^N \frac{\sigma_{ij}^2}{C_i'' C_j''} + \frac{1}{2 \sum_{j=1}^N \frac{1}{C_j''}} \sum_{i=1}^N \sum_{j=1}^N \frac{\sigma_{ij}^2}{C_i'' C_j''} + \underbrace{\sum_{i=1}^N \frac{E[\epsilon_i^2]}{2 C_i''}}_{>0}$$