

The Welfare Implications of Carbon Price Certainty

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Harvard University

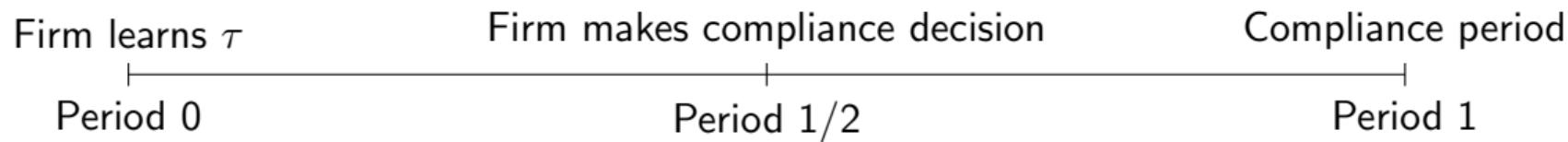
February 27, 2020

Policy Uncertainty and Investment

- Policy uncertainty affects firm investment:
 - Baker, Bloom, and Davis, 2016; Hassett and Metcalf, 1999; Rodrik, 1991 – building on Arrow, 1959; Bernanke, 1983; and others.
- Distinguish between two types of policy uncertainty:
 - **Uncertainty over policy design**
 - **Uncertainty inherent to policy instrument**
- Inherent uncertainty differs under **price** vs. **quantity** instruments for correcting Pigouvian externalities.

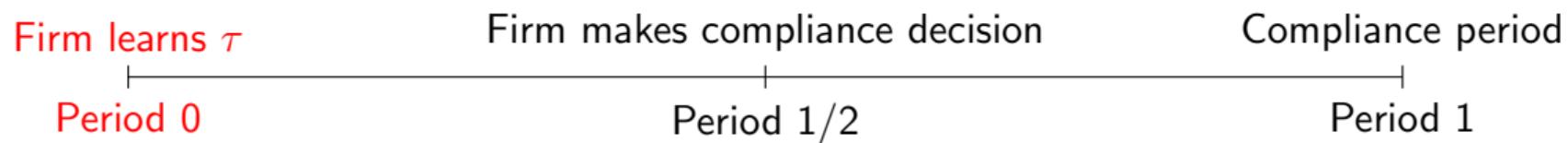
Inherent Policy Uncertainty

Price-Based Instrument:



Inherent Policy Uncertainty

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⇒ Firm knows Pigouvian tax τ with certainty.

Inherent Policy Uncertainty

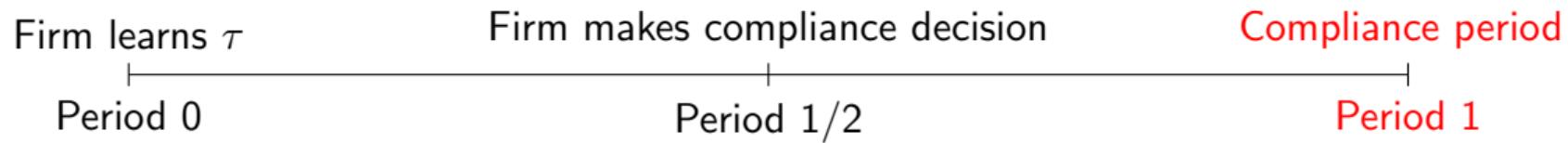
Price-Based Instrument:



⇒ Firm sets marginal abatement cost equal to τ .

Inherent Policy Uncertainty

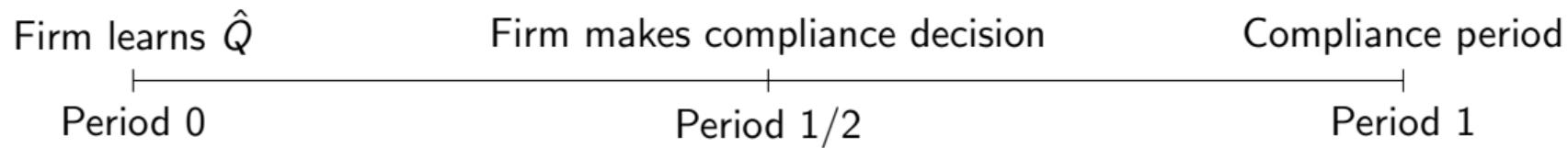
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⇒ Regulator enforces firm compliance.

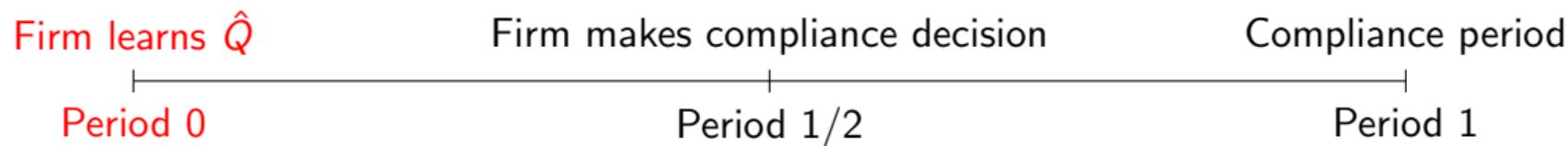
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Quantity-Based Instrument:



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⇒ Firm knows \hat{Q} with certainty, but not resulting market price.

Inherent Policy Uncertainty

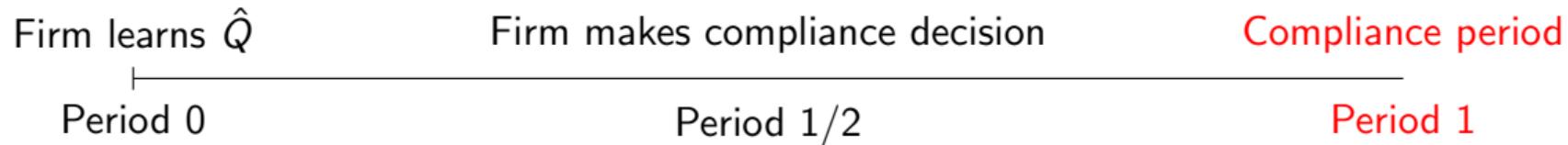
Quantity-Based Instrument:



- ⇒ Firm must form expectation over all other firms' marginal abatement cost curves, output levels, and overlapping policies to estimate market-clearing price.
- ⇒ Firm then sets marginal abatement cost equal to expected price.

Inherent Policy Uncertainty

Quantity-Based Instrument:



⇒ Regulator enforces firm compliance, and market for allowances clears. In general, realized market price does not equal a firm's expected price.

Long-Lived Abatement Investments

Cost-effective abatement options are often long-lived capital investments:

Allowance Market	Abatement Option
SO₂	installing scrubbers, retrofitting plants for low-sulfur coal
NO_x	installing selective catalytic reduction
CO₂	investing in renewables, installing carbon capture and storage
RPS, EEPS	investing in renewables, retrofitting built environment
RFS	investing in biorefineries
Vehicle efficiency	developing new vehicle models

Allowance Price Volatility

High historical price volatility in allowance and credit trading markets:

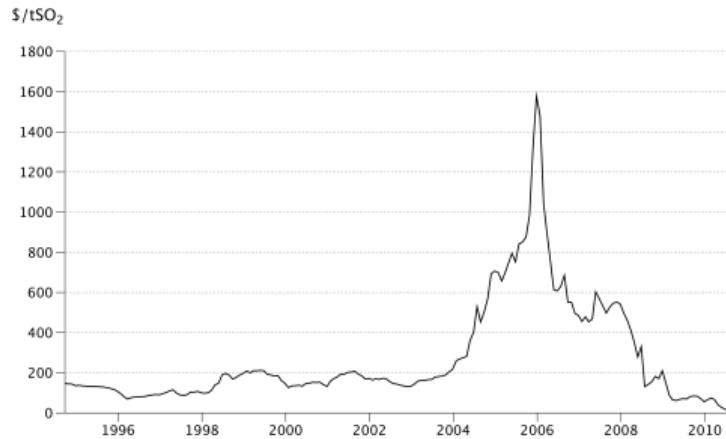


Emissions Trading System CO_2 Allowance Price (EU)

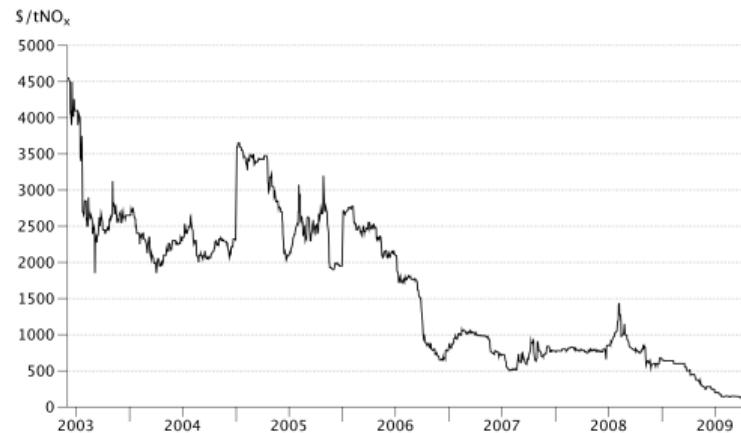


Low Carbon Fuel Standard Allowance Price (California)

Allowance Price Volatility



SO_2 Allowance Price (US)



NO_x Allowance Price (US)

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- Firms do not hedge completely even when financial instruments are available and volatile prices represent substantial business expense.
 - On average U.S. airlines hedged 20% of expected jet fuel expenses over 1996-2003 (Rampini and Viswanathan, 2014).

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- Firms may not know total hedging requirement with certainty, where $\text{Total Hedging} = p \cdot q(p, \theta)$
- Large markets created by regulation face start-up problem.

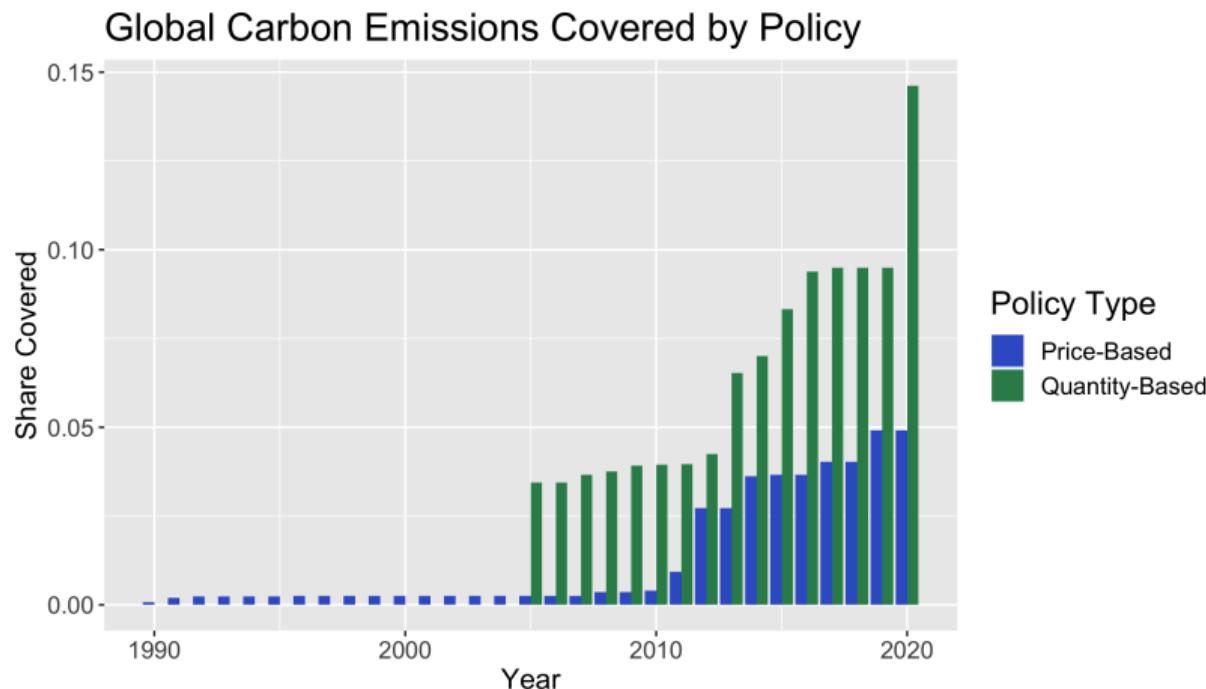
Start-Up Problem

Evidence of Cost Inefficiency in Cap & Trade

Empirical literature suggests inefficiencies in cap-and-trade programs:

- Carlson et al. (2000): One-half of Phase I units in SO_2 C&T program deviated at some point from least-cost compliance strategies.
- Fowlie (2010), Cicala (2015): Deregulated firms may underinvest in capital-intensive compliance strategies for SO_2 and NO_x C&T programs, paired with overinvestment by regulated firms.
- Frey (2013), Chan et al. (2018): Overlapping policies further lead to compliance strategies inconsistent with cost minimization.

Revealed Preference in Environmental Policy Design



Source: World Bank, *State and Trends of Carbon Pricing*

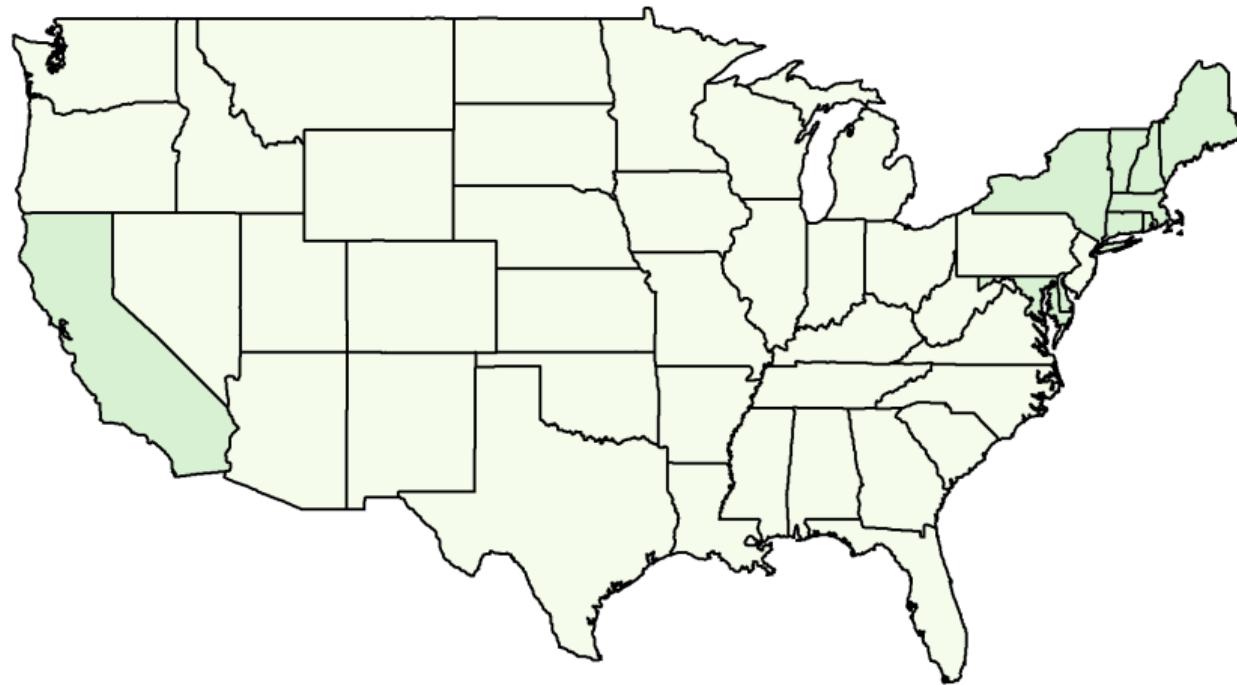
Revealed Preference in Environmental Policy Design

Allowance and credit trading programs in U.S. energy markets:

- State CO_2 Cap-and-Trade Programs
- State Renewable Portfolio Standard
- Gasoline Sulfur and Benzene Credit Trading
- Renewable Fuel Standard
- Cross-State Air Pollution Rule
- Low-Carbon Fuel Standard

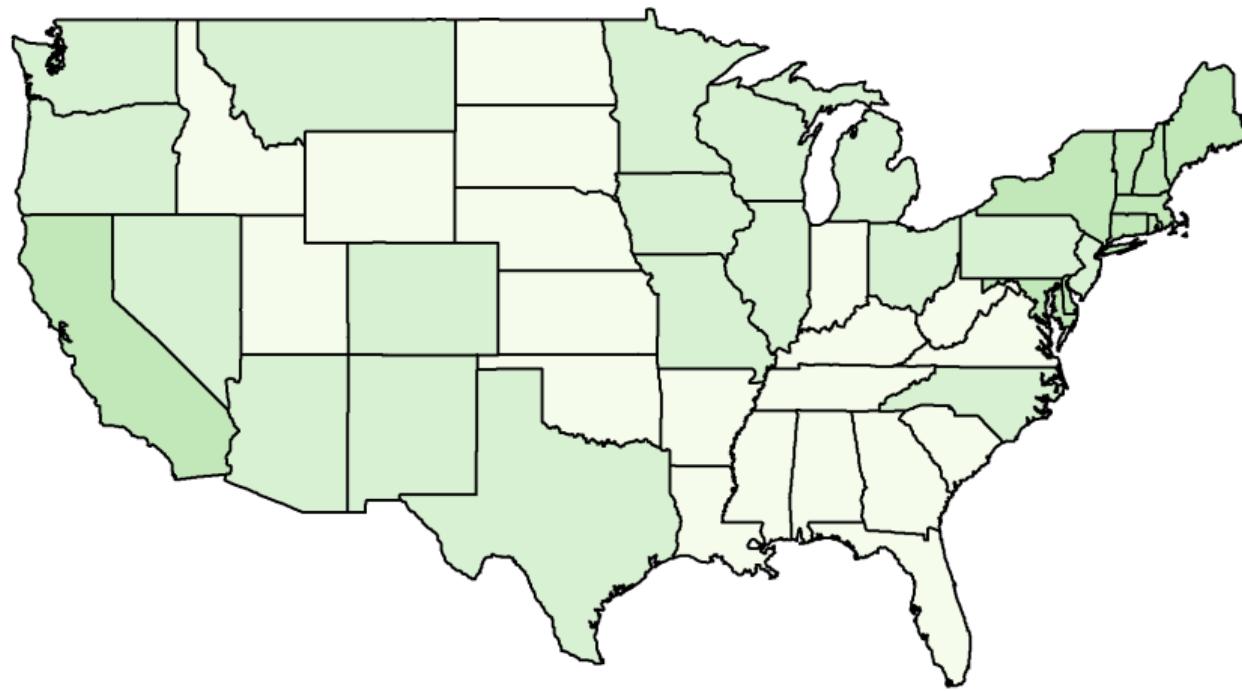
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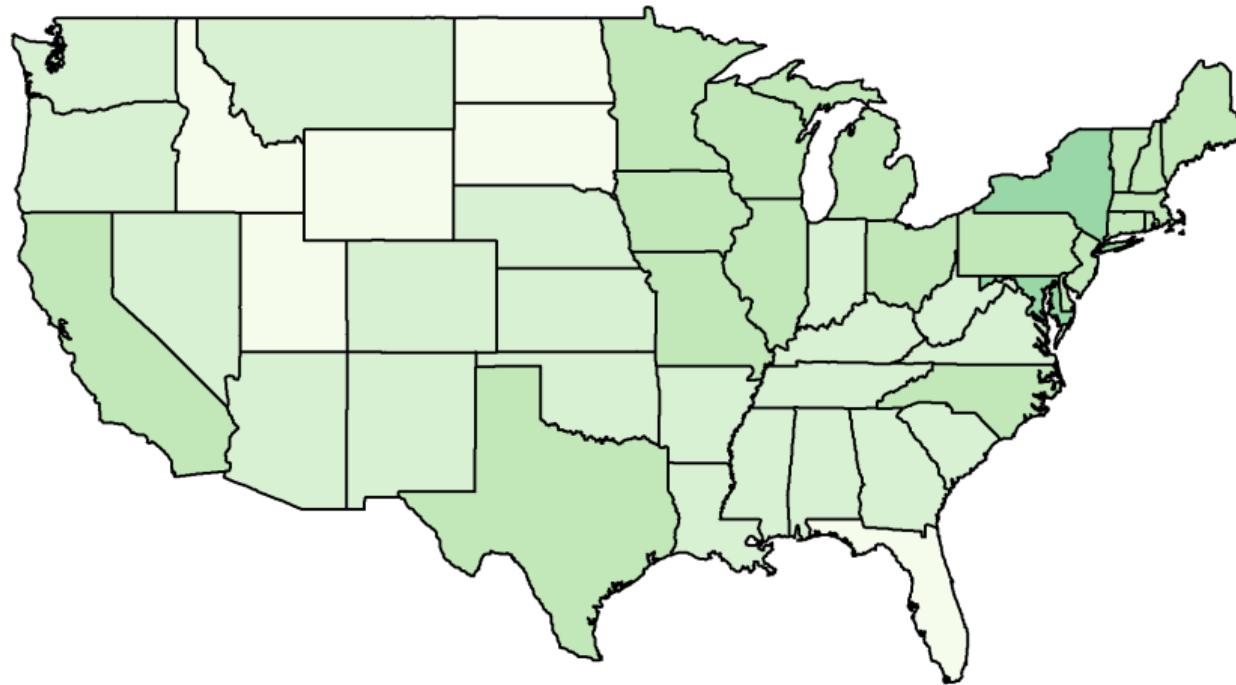
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State Renewable Portfolio Standards with Credit Trading:



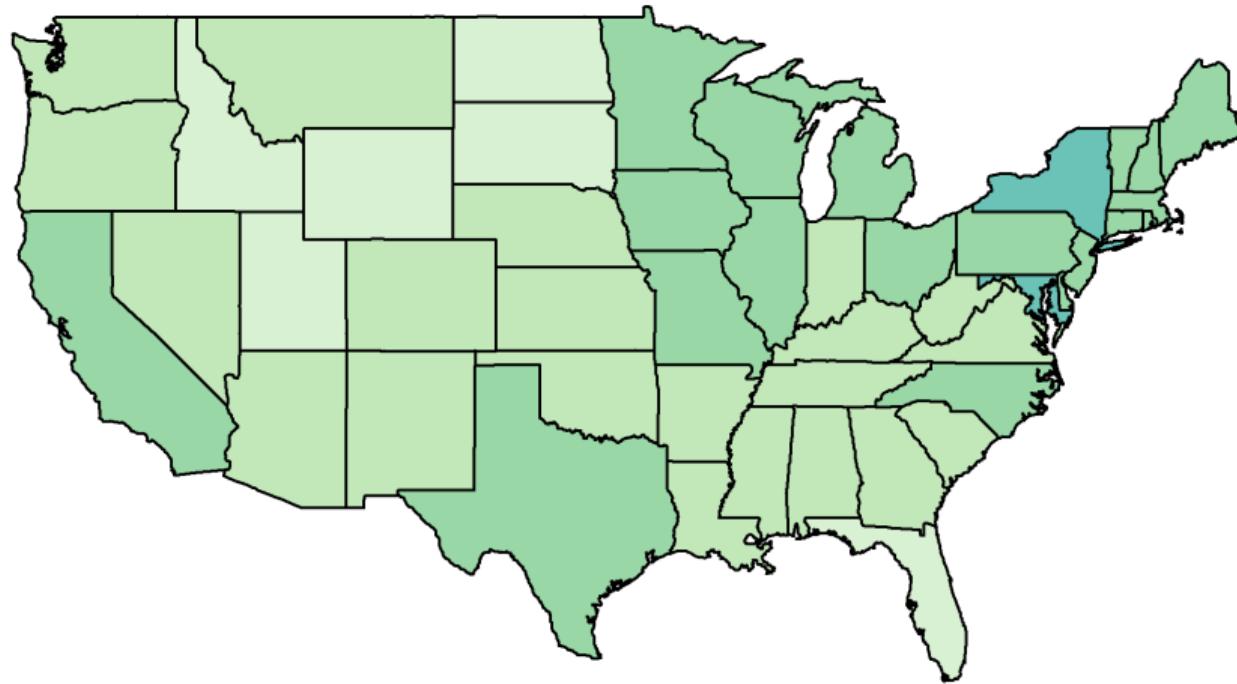
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Cross-State Air Pollution Rule Allowance Trading:



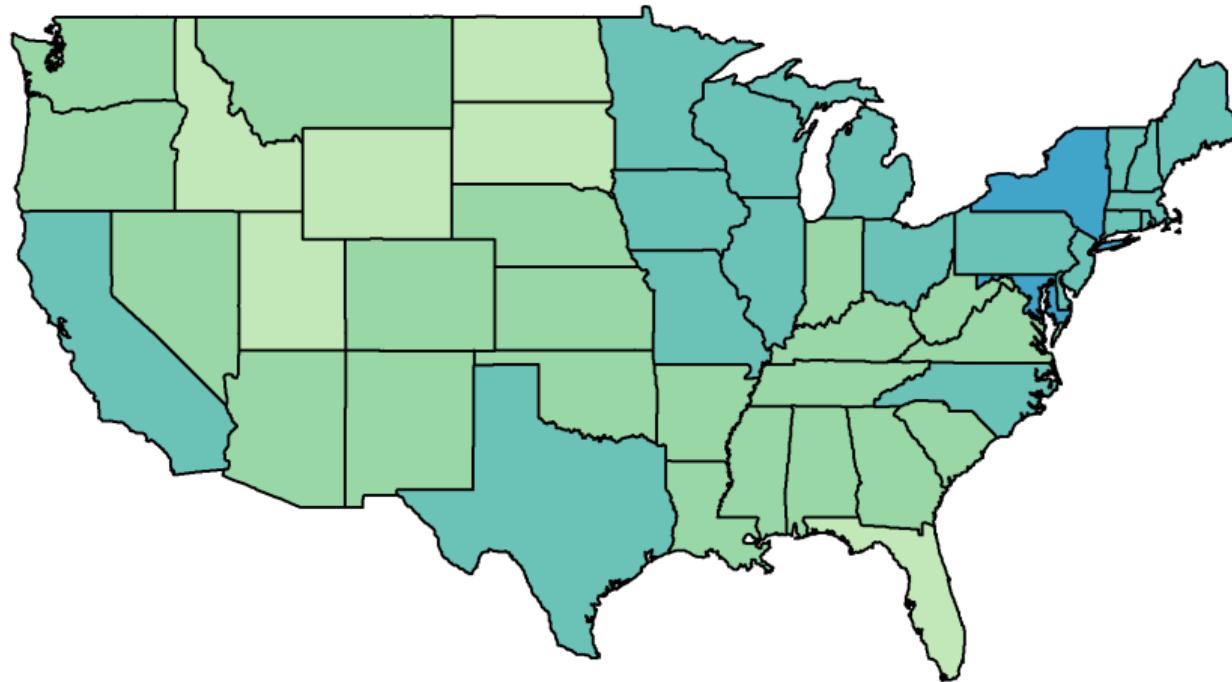
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Renewable Fuel Standard Credit Trading:



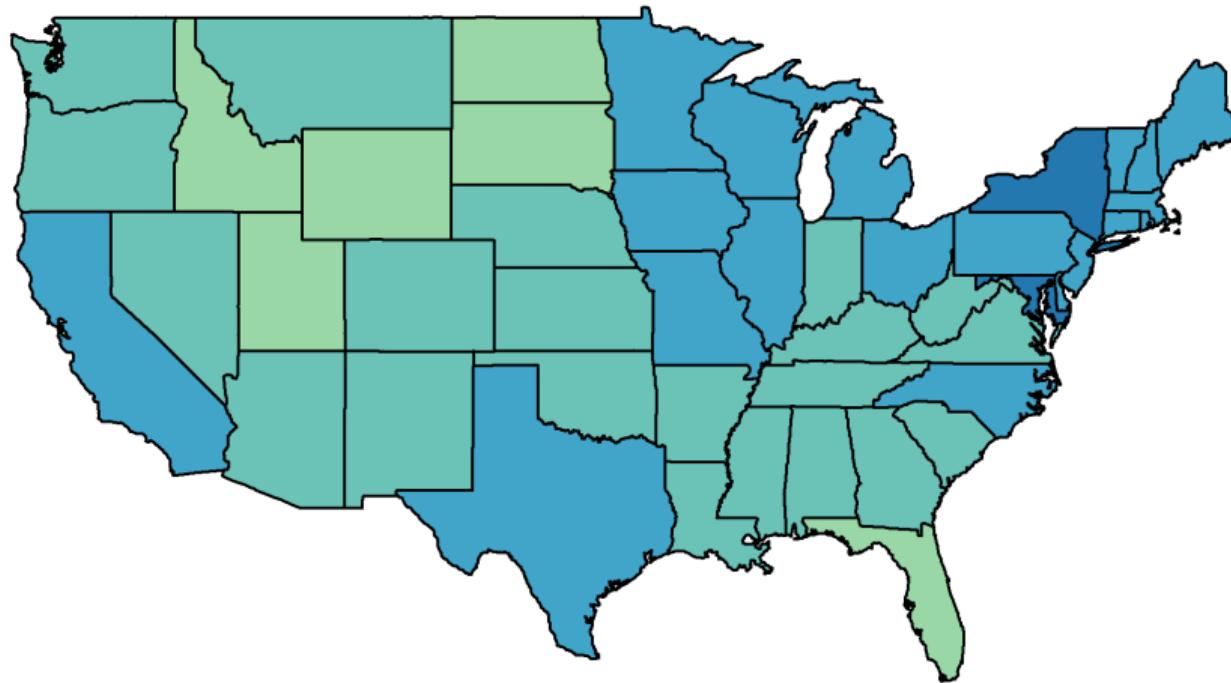
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Gasoline Benzene Credit Trading:



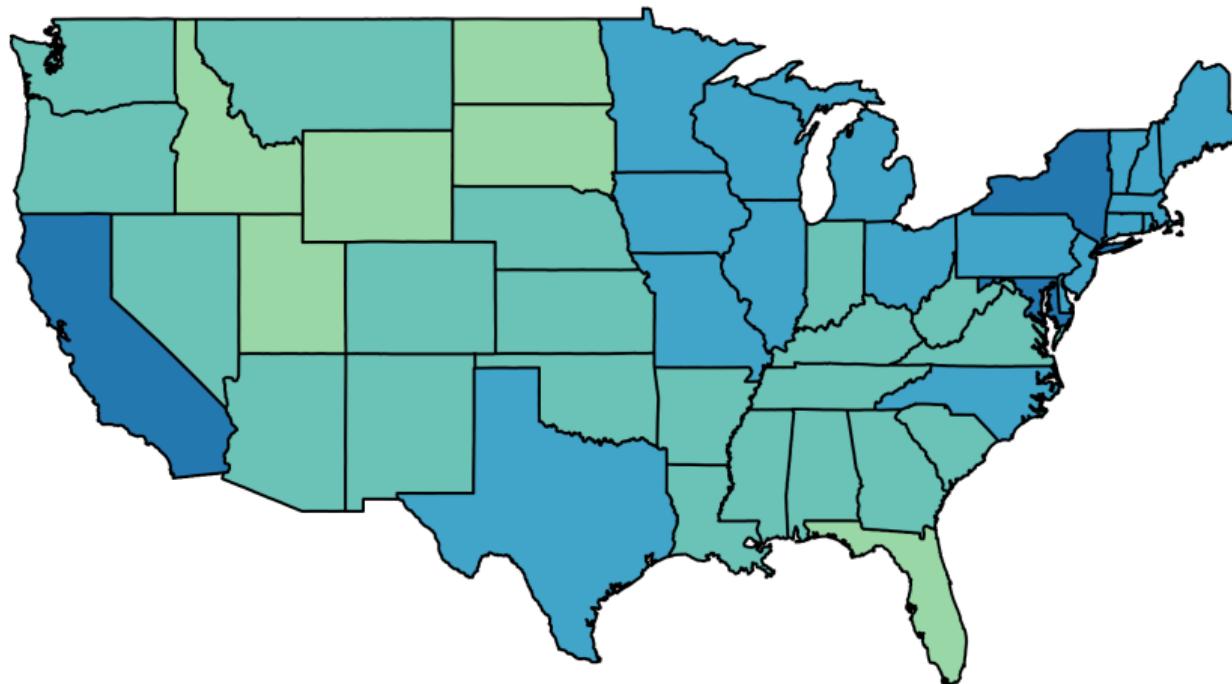
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Gasoline Sulfur Credit Trading:



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Low-Carbon Fuel Standard:



How does this inherent policy uncertainty affect firm behavior in allowance and credit trading markets?

Dynamic Model of Firm Investment in Abatement

Adapt Pindyck (1980), Rubin (1996), and Anderson, Kellogg, and Salant (2018) to model emissions trading market:

$$\max_{A, Y} \mathbb{E}_0 \left[\int_0^T e^{-rt} \{ -\psi(A(t)) - P(t)Y(t) \} dt \right]$$

- $A(t)$: abatement investment
- $\psi(\cdot)$: investment cost function
- $Y(t)$: allowances purchased
- r : discount rate
- $P(t)$: current allowance price, which follows GBM with drift α and volatility σ

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subject to:

$$\dot{K} = A(t) - \delta K(t)$$

$$A(t) \geq 0, \quad K_0 \text{ given}$$

$$\dot{B} = K(t) + Y(t) - \bar{E}$$

$$B(T) \geq 0, \quad B_0 = 0$$

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Optimality Conditions

② However, optimal abatement is now dynamic decision. Assuming firm chooses some unconstrained $A^* \geq 0$:

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Model Interpretation

- Banking activity pins down expected price path in equilibrium.
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 - Changes in economic output
- Modeling long-lived, dynamic investment illuminates impact of price volatility:
 - Firms may have forecast errors in estimating future stream of prices.
 - Firms take into account price volatility in value of smoothing.

How does this inherent policy uncertainty affect the cost of achieving an emissions target?

Simulations

- Model compliance decisions of representative firm given simulated price trajectory:
 - Scenario 1: Firm makes abatement investment decisions given stochastic prices.
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- Both scenarios produce same total emissions reductions, but costs are higher with stochastic prices.
- Price volatility alters effective abatement cost function for quantity-based instruments relative to price-based instruments.

Model Calibration

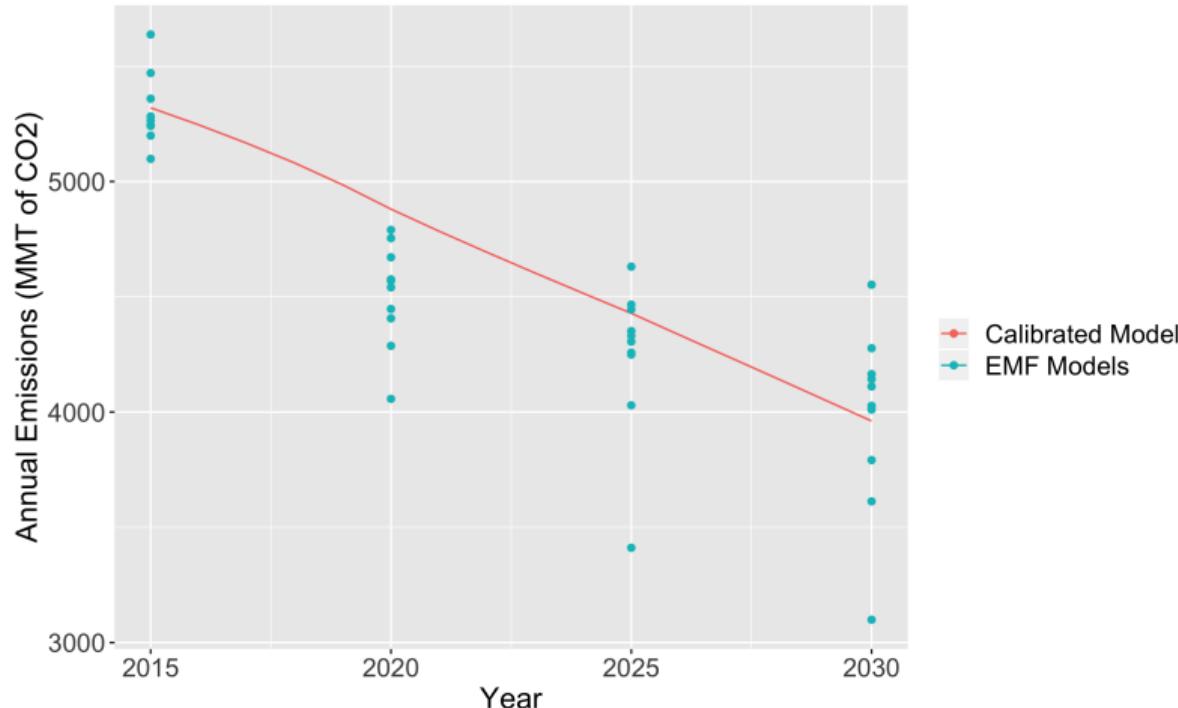
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 - Later work will examine richer specifications of cost function and abatement opportunities.
- Calibrate drift and volatility parameters to historical EU ETS allowance prices for Phases II and III (2008-2018), assuming prices follow geometric Brownian motion.
 - Estimate 5.2% annual expected price growth ($\alpha = 0.0508$)
 - Estimate 42.9% annual price volatility ($\sigma = 0.3925$)

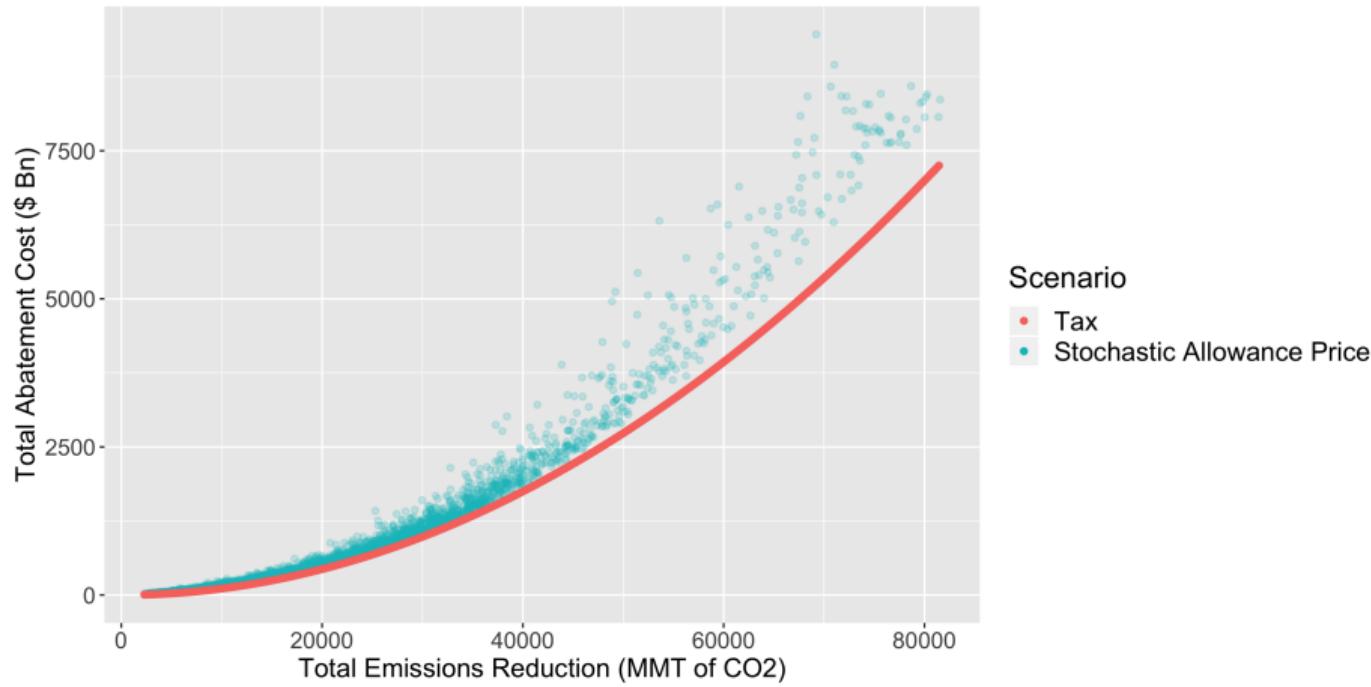
Model Calibration

Total Emissions Reduction (10 Years of Abatement Investment):



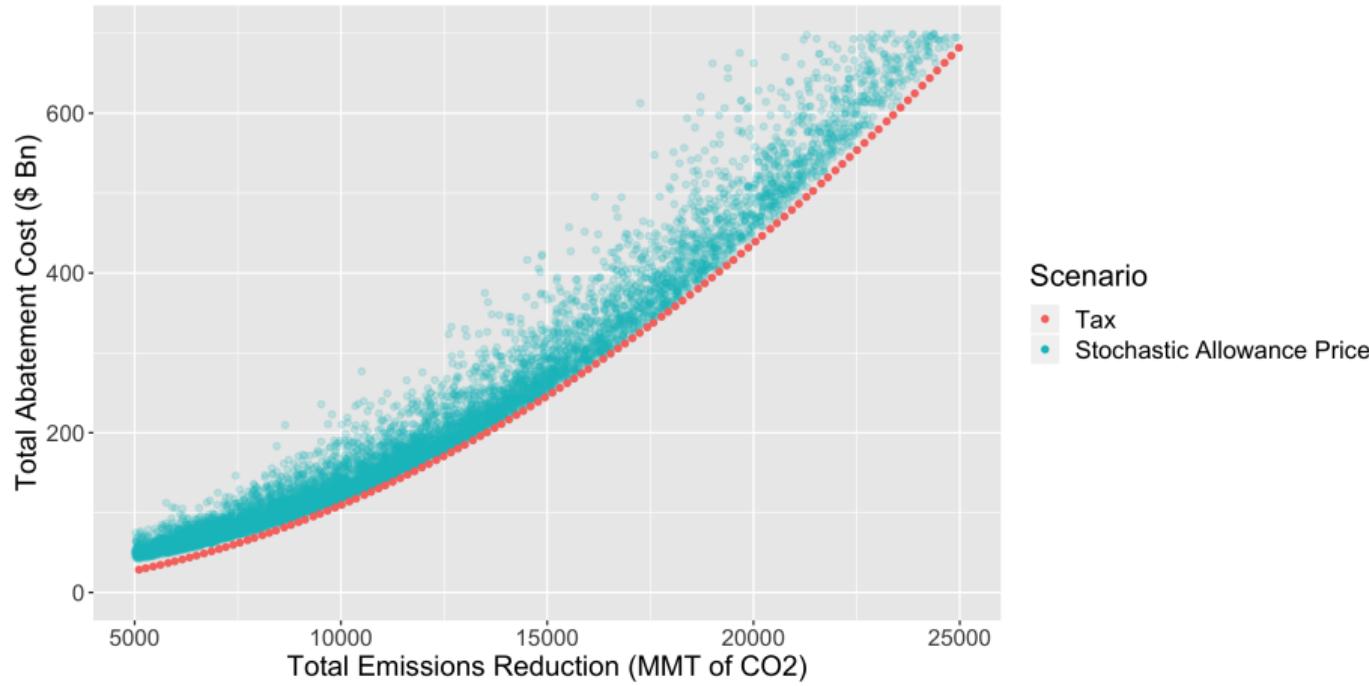
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 - **Optimal price order:**

$$\max_{\tau_1, \tau_2} E[B_1(\sum_{i=1}^N q_1^i(\tau_1, \theta_1^i)) - \sum_{i=1}^N C_1^i(q_1^i(\tau_1, \theta_1^i), \theta_1^i) \\ B_2(\sum_{i=1}^N q_2^i(\tau_2, \theta_2^i)) - \sum_{i=1}^N C_2^i(q_2^i(\tau_2, \theta_2^i), \theta_2^i)]$$

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- Compare to Weitzman (2018) result with representative firm: Derivation

$$\Delta = E\left[\frac{1}{4}(C'' - B'') \cdot \left(\frac{\theta_2 + \theta_1}{C''}\right)^2\right]$$

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Mis-allocation across firms

Revisiting Prices vs. Quantities

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$$\text{Marginal Cost} = \text{Market-Clearing Price} + \text{Forecast Error } \epsilon;$$

- Two types of firm-level forecast errors in multi-period setting:
 - 1 Uncertainty caused by private information \Rightarrow Mis-allocation across firms
 - 2 Uncertainty caused by as-yet unrealized market shocks

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 - 2 Uncertainty caused by as-yet unrealized market shocks \Rightarrow
Mis-allocation across compliance periods

Welfare Consequences of Uncertainty

- **UNCERTAINTY TYPE #1:** Idiosyncratic forecast errors with no impact on aggregate distribution of quantity across periods.

Relative advantage of prices over quantities: [Derivation](#)

$$\Delta = E\left[\frac{1}{4}\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B''\right)\left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''}\right)^2 + \frac{1}{2} \sum_{i=1}^N \frac{\epsilon_1^i{}^2}{C_i''}\right]$$

One Period

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- Cost inefficiency arises from failure to allocate quantity optimally across firms.

One Period

Welfare Consequences of Uncertainty

- **UNCERTAINTY TYPE #2:** Systematic forecast errors that impact aggregate distribution of quantity across periods.

Relative advantage of prices over quantities: [Derivation](#)

$$\begin{aligned}\Delta = & E\left[\frac{1}{4}\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B''\right)\left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''}\right)^2 + \frac{1}{2} \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}^2\right. \\ & \left. + \frac{1}{2}\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}}\right)\left(\sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}\right)^2 + B''\left(\sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}\right)^2 + 2B''\left(\sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}\right)\left(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''}\right)\right]\end{aligned}$$

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[Derivation](#)

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$\underbrace{\quad}_{\geq 0}$

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Welfare Consequences of Uncertainty

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- Cost inefficiency arises from failure to allocate quantity optimally across periods.
- Benefit smoothing across periods may increase or decrease, with ambiguous effects for welfare.

Welfare Consequences of Uncertainty

- Forecast errors asymmetrically affect quantity-based policies, conditional on policy design uncertainty being resolved.

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Welfare Consequences of Uncertainty

- Forecast errors asymmetrically affect quantity-based policies, conditional on policy design uncertainty being resolved.
- Both types of firm forecast errors create cost inefficiencies that push regulator to prefer price instruments.
 - Effect on benefit smoothing is ambiguous.
- Given banking and borrowing, uncertainty in one compliance period may continue to create cost inefficiencies in future periods.

Conclusion & Future Directions

- Evaluations of price versus quantity instruments should take into account asymmetric firm forecast errors.

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Conclusion & Future Directions

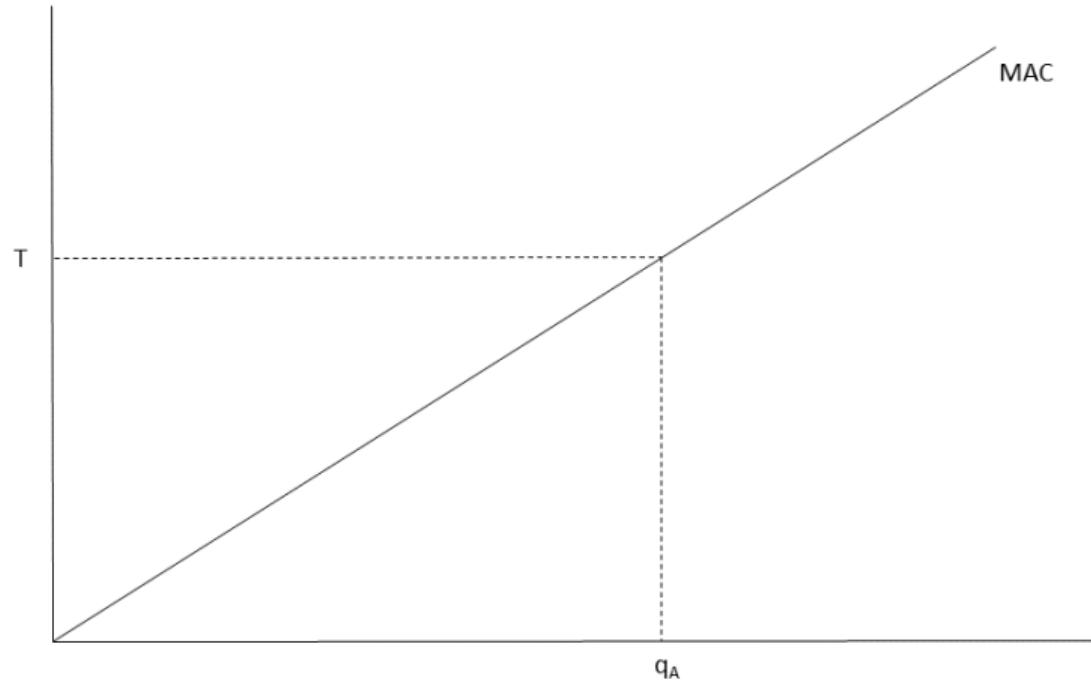
- Evaluations of price versus quantity instruments should take into account asymmetric firm forecast errors.
- Effective abatement cost function depends on policy instrument → in simulations, median percentage difference in costs is 21%.
- Future work will focus on correlation between price uncertainty and abatement cost uncertainty and bringing full dynamics into welfare analysis.

Thank you!

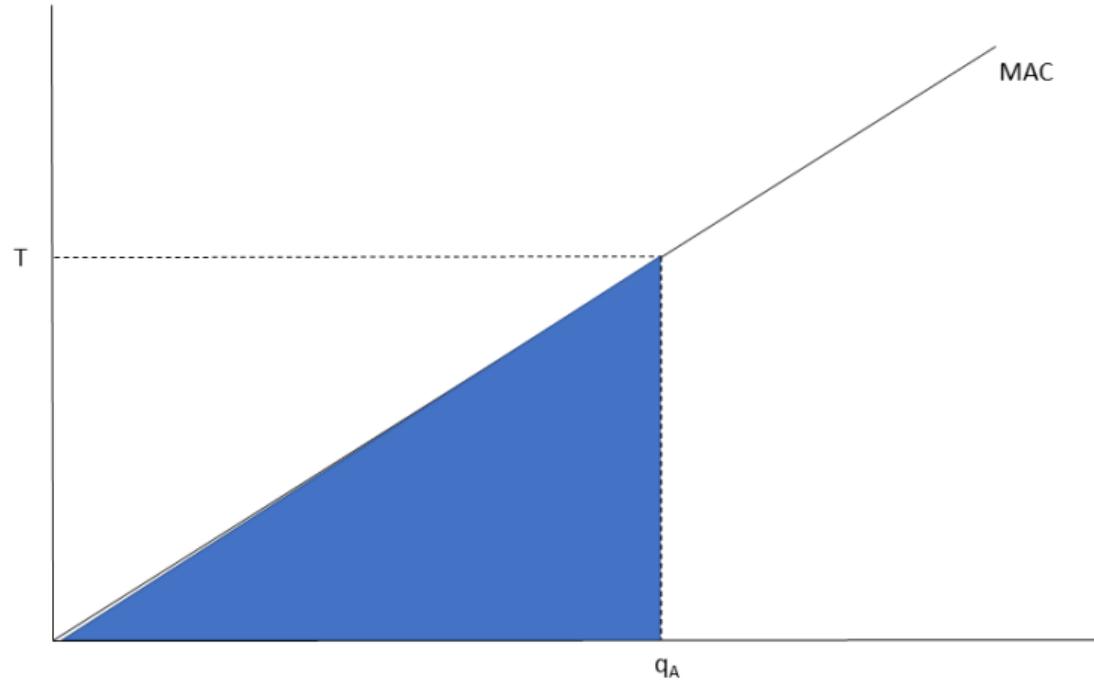
Aldy: joseph_aldy@hks.harvard.edu
<https://scholar.harvard.edu/jaldy>

Armitage: saraharmitage@g.harvard.edu
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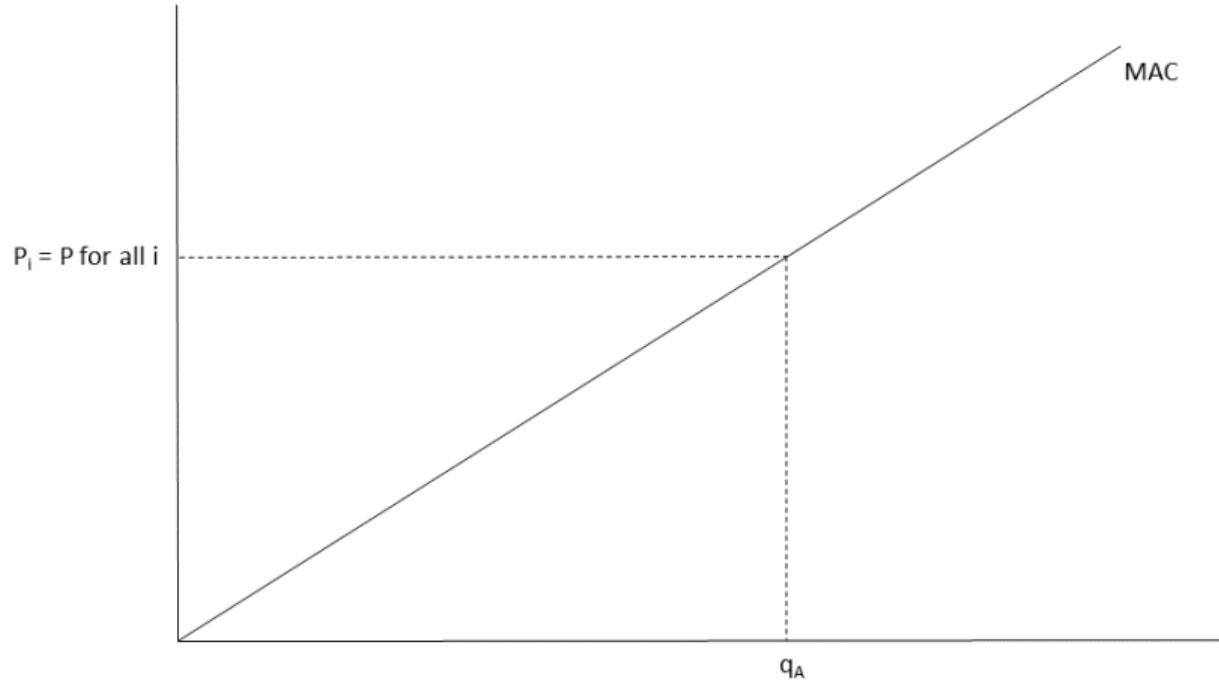
Graphical Illustration



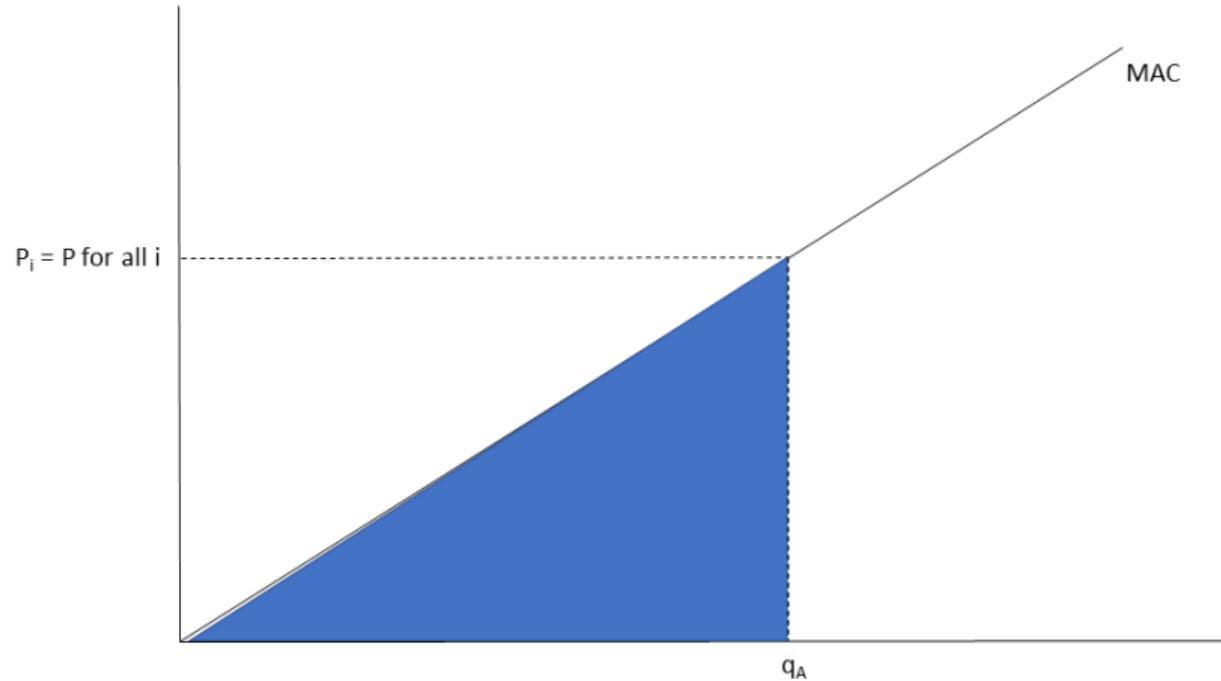
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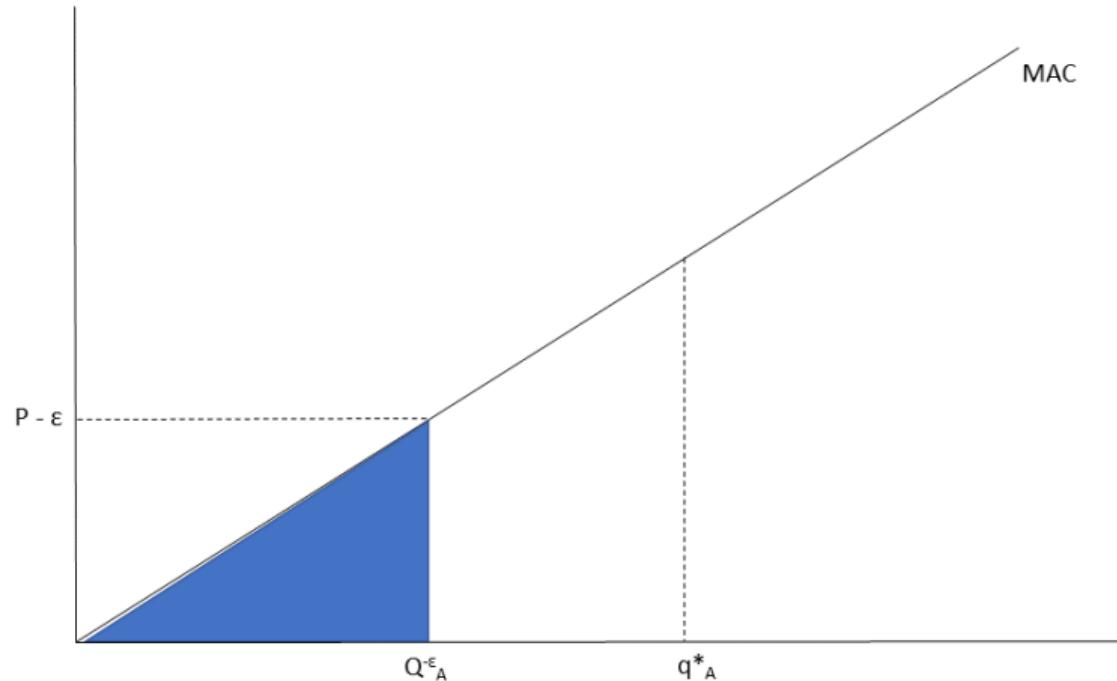
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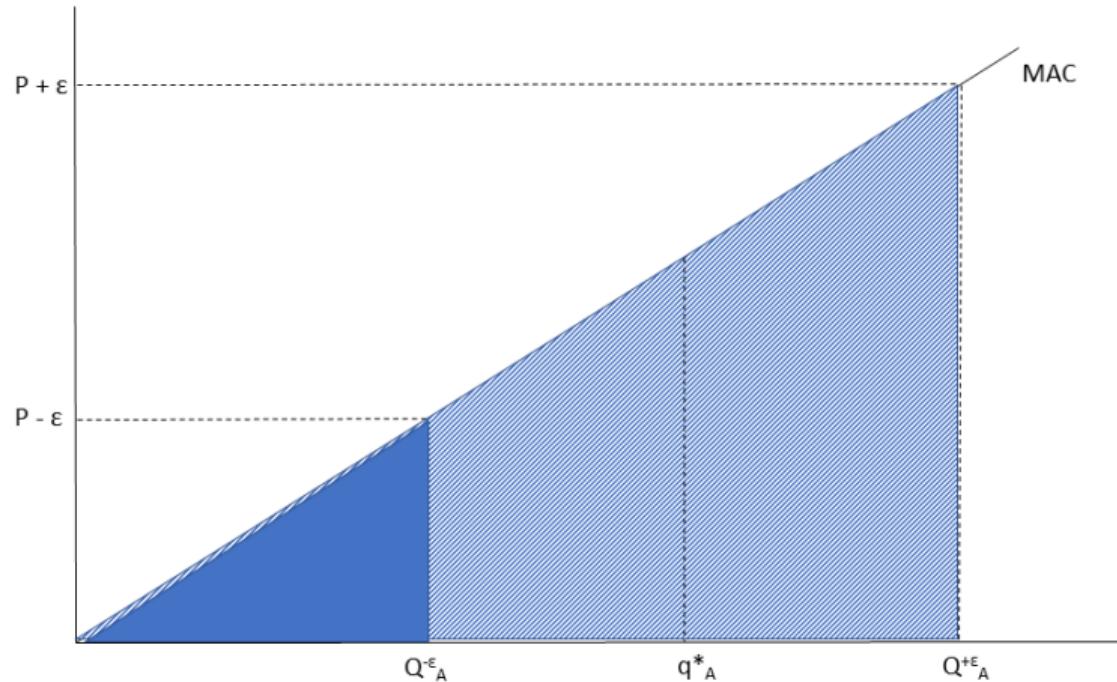
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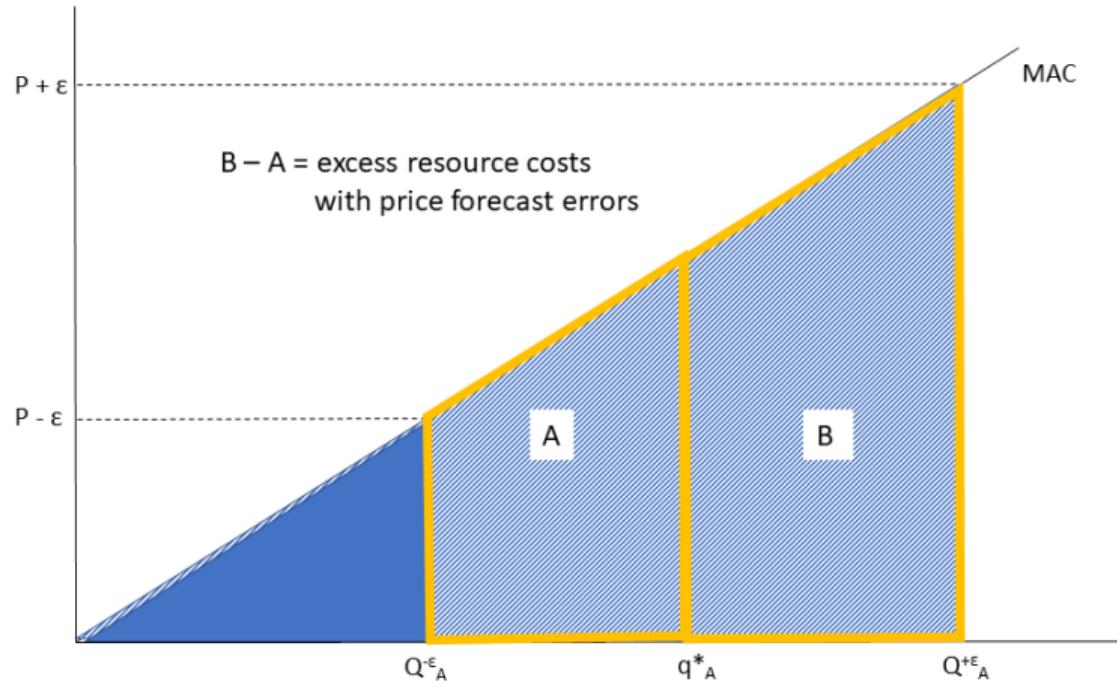
Graphical Illustration



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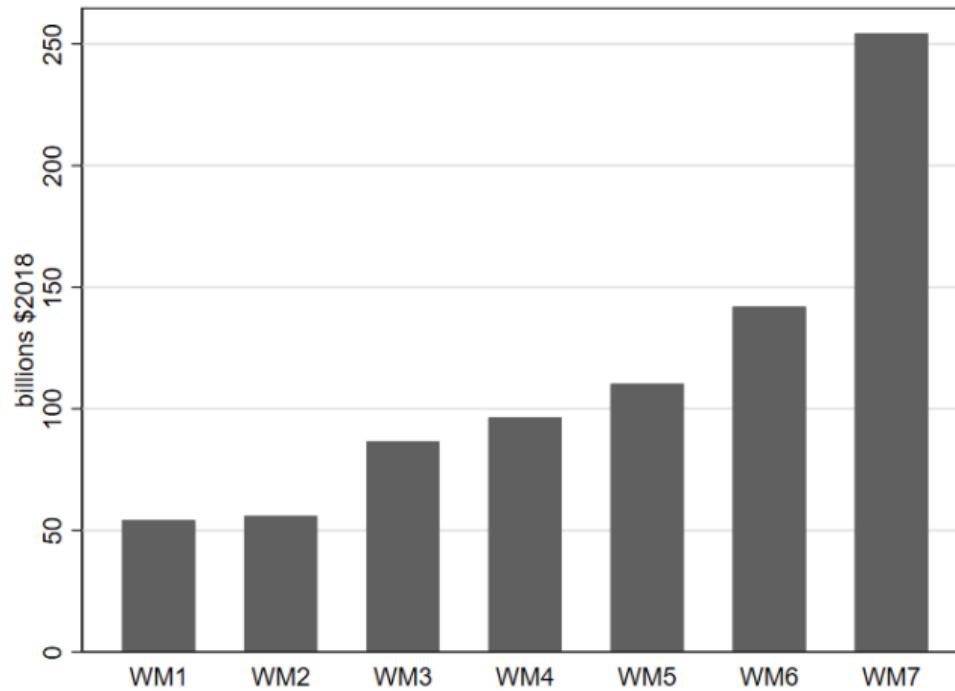


Graphical Illustration



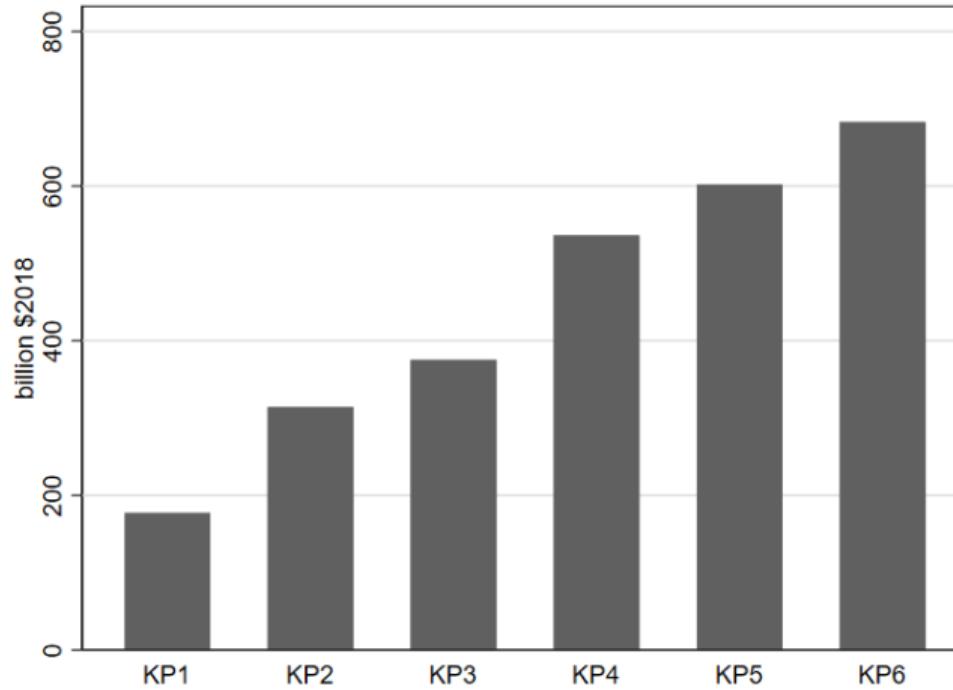
Start-Up Problem in Allowance Trading Markets

Value of Allowance Market in Year 1 - Waxman-Markey Bill: [Back](#)



Start-Up Problem in Allowance Trading Markets

Value of Allowance Market in Year 1 - Kyoto Protocol: [Back](#)



Revisiting Prices vs. Quantities

Back

- Regulator's optimal price policy: $\tau_1 = \tau_2 = C'$
- Regulator's optimal quantity policy: $\hat{Q} = \sum_{i=1}^N \bar{q}_1^i + \sum_{i=1}^N \bar{q}_2^i$ where \bar{q}_t^i sets $E\left[\frac{\partial B_t}{\partial q_t^i}\right] = E\left[\frac{\partial C_t^i}{\partial q_t^i}\right]$.
- Two key conditions govern market-clearing price: no intertemporal arbitrage and the regulator's quantity limit. Applying these conditions:

$$\hat{p}_1(\hat{Q}, \theta_1, \theta_2) = \hat{p}_2(\hat{Q}, \theta_1, \theta_2) = C' + \frac{\sum_i \frac{\theta_1^i + \theta_2^i}{2C_i''}}{\sum_i \frac{1}{C_i''}}$$

- Quantity response by firm i in period 1 (for illustration):

$$q_1^i(\hat{p}_1, \theta_1^i) = \frac{\hat{p}_1 - C' - \theta_1^i}{C_i''} + \bar{q}_1^i = \frac{\frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_1^i}{C_i''} + \bar{q}_1^i$$

Revisiting Prices vs. Quantities

Back

- Quantity response in presence of first-period forecast errors:

$$q_1^i(p_1, \theta_1^i) = \frac{\hat{p}_1(\hat{Q}, \theta_1, \theta_2) + \epsilon_1^i - C' - \theta_1^i}{C_i''} + \bar{q}_1^i$$

- Constraint such that aggregate quantity in first period is unchanged:

$$\sum_{i=1}^N \frac{\epsilon_1^i}{C_i''} = 0$$

- Expected benefits are unchanged, as are expected costs in period 2. Expected costs in period 1:

$$\sum_{i=1}^N E[\theta_1^i \left(\frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_{j=1}^N \frac{1}{C_j''}} + \epsilon_1^i - \theta_1^i \right) + \frac{C_i''}{2} \left(\frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_{j=1}^N \frac{1}{C_j''}} + \epsilon_1^i - \theta_1^i \right)^2]$$

Revisiting Prices vs. Quantities

Back

- Aggregate first-period quantity changes:

$$Q_1 = \frac{\hat{Q}}{2} + \sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} + \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}$$

- Second-period price must adjust to ensure regulatory limit is still met:

$$\hat{p}_2'(\hat{Q}, \theta_1, \theta_2, \epsilon_1) = C' + \frac{\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}}{\sum_{i=1}^N \frac{1}{C_i''}}$$

- Aggregate second-period quantity then changes:

$$Q_2 = \frac{\hat{Q}}{2} + \sum_{i=1}^N \frac{\theta_1^i - \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\epsilon_1^i}{C_i''}$$

- Expected benefits and expected costs (first and second periods) all change.

Revisiting Prices vs. Quantities

Back

- Market-clearing price such that realized quantity is regulated quantity (no intertemporal arbitrage condition no longer applies):

$$\hat{p}(\hat{Q}, \theta) = C' + \frac{\sum_{i=1}^N \frac{\theta_i}{C_i''}}{\sum_{i=1}^N \frac{1}{C_i''}}$$

- Relative advantage of prices over quantities:

$$\Delta_n = \frac{B''}{2} \sum_{i=1}^N \sum_{j=1}^N \frac{\sigma_{ij}^2}{C_i'' C_j''} + \frac{1}{2 \sum_{j=1}^N \frac{1}{C_j''}} \sum_{i=1}^N \sum_{j=1}^N \frac{\sigma_{ij}^2}{C_i'' C_j''}$$

- In the presence of forecast errors, relative advantage of prices over quantities becomes:

$$\Delta_n = \frac{B''}{2} \sum_{i=1}^N \sum_{j=1}^N \frac{\sigma_{ij}^2}{C_i'' C_j''} + \frac{1}{2 \sum_{j=1}^N \frac{1}{C_j''}} \sum_{i=1}^N \sum_{j=1}^N \frac{\sigma_{ij}^2}{C_i'' C_j''} + \underbrace{\sum_{i=1}^N \frac{E[\epsilon_i^2]}{2C_i''}}_{>0}$$