Estimating Macroeconomic Models of Financial Crises: An Endogenous Regime-Switching Approach*

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Abstract

We structurally estimate a workhorse open economy model with an occasionally binding borrowing constraint. First, we propose a new specification of the occasionally binding constraint, where the transition between unconstrained and constrained states is a stochastic function of the leverage level and the constraint multiplier, that maps into an endogenous regime-switching model. Second, we develop a general perturbation method for the solution of such a model, showing the importance of approximating at least to second-order. Third, we estimate the model with Bayesian methods to fit Mexico's business cycle and financial crisis history since 1981. The estimated model fits the data well, identifying three sudden stop episodes of varying duration and intensity: the Debt, Tequila, and Global Financial Crises. We find that the crisis episodes generated by our stochastic specification of the borrowing constraint, in addition to the economic dislocations associated with the crisis peak, can display dynamics consistent with the sluggish build-up and recovery phases that are typically seen in the data. In the model, financial crisis events occur when certain cocktails of shocks hit an already prone economy. Different sets of shocks explain Mexico's business cycle and the three historical episodes of sudden stops that we identify in the data.

Keywords: Financial Crises, Business Cycles, Endogenous Regime-Switching, Bayesian Estimation, Occasionally Binding Constraints, Mexico.
JEL Codes: G01, E3, F41, C11.

1 Introduction

The Global Financial Crisis generated a renewed interest in understanding the causes and the dynamics associated with financial crises. In this context, dynamic stochastic general equilibrium (DSGE) models with occasionally binding frictions have proven successful as laboratories to study the anatomy of both business cycles and crises and to explore optimal policy responses to these dynamics. This success is because occasionally binding financial frictions are amplification mechanisms of regular business cycle dynamics. Structural estimation of these models is challenging, yet important as inference on key parameters governing financial frictions, counterfactual policy analysis and structural real-time forecasts all rely on estimation of such models.

In this paper, we structurally estimate a model with an occasionally binding borrowing constraint. We make three main contributions. First, we propose a new specification of the occasionally binding collateral constraint. Second, we device a perturbation solution approach suitable for solving models like ours in a way that permits likelihood-based estimation. Third, we focus on one particular type of crisis, the so-called sudden stop in international capital flows, and apply the proposed approach to the estimation of a medium-scale workhorse DSGE model, investigating sources and frictions of business cycles and sudden stop crises in Mexico since 1981.

As a first step, we propose a new formulation of occasionally binding constraint models. As in models with constraints written as inequalities, our set up has two states or regimes: one in which a given leverage ratio limits borrowing and amplifies regular shocks to explain financial crises as the economy cannot smooth consumption and expand production, and one in which access to financing is plentiful and the economy displays regular business cycles. In our specification, however, transitions between the two regimes occur in a stochastic rather than deterministic manner. Probabilities that depend on the borrowing capacity of the economy and the multiplier associated with a binding collateral constraint govern the transition between these regimes. This formulation maps the modeling of an occasionally binding constraint into an endogenous regime-switching model. The paper focuses on a particular financial friction, but the proposed specification has broader applicability to other types of occasionally binding constraints.

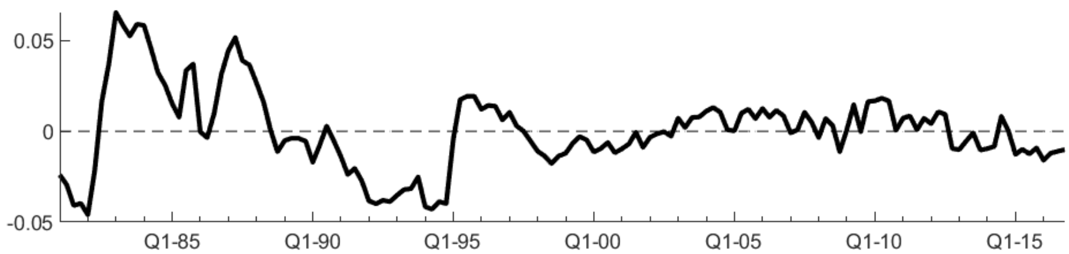
Next, we develop a perturbation-based solution method for solving our model. The perturbation method is fast enough to permit likelihood-based estimation, is readily scalable to models larger than the one we estimate in this paper, and displays typical levels of accuracy. We show that our perturbation approach employing a second-order approximation to the policy functions characterizing optimal behavior captures the effects of endogenous transition probabilities on precautionary behavior, and that these effects would be missed by linear approximations. As in our first contribution, the method applies beyond the scope of the paper, as it can be used with a wide range of models that can be cast as endogenous regime-switching models.

Finally, we apply our borrowing constraint specification and solution method to the Bayesian estimation of a model that characterizes both financial crises and business cycles in emerging market economies. Because our application focuses on emerging markets, we estimate a medium-scale, workhorse model of sudden stops. With the exception of the borrowing constraint specification, the model structure is the same as in Mendoza (2010), but we consider a larger set of exogenous shocks as in Garcia-Cicco et al. (2010). The critical model difference relative to Mendoza (2010) is the specification of the collateral constraint, which as we said earlier can be easily adapted to other model settings. For example, the approach that we propose would be applicable to the formulation and estimation of models with inequality constraints as in Kiyotaki and Moore (1997), Iacoviello (2005), and Liu et al. (2013), Gertler and Karadi (2011), and Gertler and Kiyotaki (2015), Jermann and Quadrini (2012), and Schmitt-Grohe and Uribe (2018), among others.

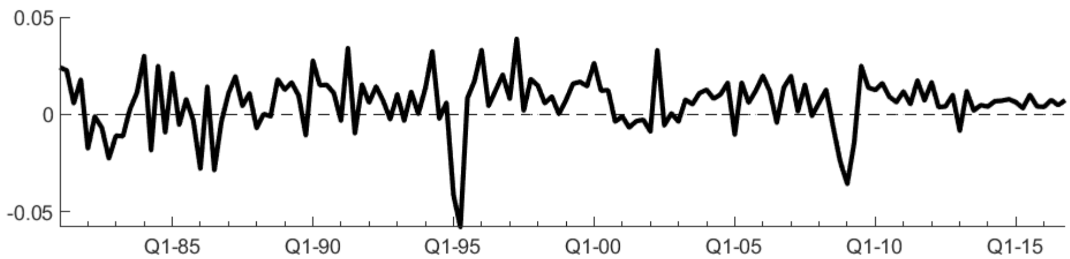
Figure 1 plots two critical variables in our application to Mexico: the current account balance as a share of GDP and the quarterly real GDP growth in deviation from sample mean. The figure illustrates the regular fluctuations in the data as well as multiple episodes of large current account reversals and persistent output growth declines. Large current account reversals and output drops of varying size and persistence are the two main empirical features commonly associated with sudden stops in capital flows. In this paper, we focus on the challenge of fitting a structural model to Mexico's business cycle and sudden stop history; a history that is a shared experience with many other emerging markets exposed to volatile capital flows.

Figure 1: Current Account and Output in Mexico, 1981-2016

(a) Current Account to Output Ratio



(b) Quarterly Output Growth Rate



Note: Panel (a) plots Mexico's current account balance as a share of GDP. Panel (b) shows Mexico's quarterly log-change of real GDP. See the Appendix for data sources. Sample period 1981:Q1-2016:Q4.

Despite the econometric challenges in characterizing data like those displayed in Figure 1, our estimated model fits Mexico's business cycles and sudden stop episodes well, without relying on large shocks to explain crisis periods, but instead letting the structure of the model explain the events. It produces business cycle statistics that match the second moments of the data and evidence on the relative importance of different shocks consistent with the extant literature. Most importantly, it can identify not only acute but short-lived sudden stops periods (which we call peak sudden stop crisis, or peak crisis for brevity), as previously done in the literature with similar models, but also longer-lasting spells of time in which the economy is in the constrained regime (which we call near sudden stop crisis periods, or near crises). Thus, our stochastic specification of the collateral constraint permits identifying sudden stops episodes and estimating crisis dynamics of different duration and intensity, consistent with evidence not only of large economic dislocation during financial crises, but also sluggish build-up and recovery phases surrounding them, as typically associated with these events (Reinhart and Rogoff, 2009; Cerra and Saxena, 2008).

In particular, the estimated model identifies three near sudden stops: the Debt Crisis from 1981:Q3 to 1983:Q2, the Tequila crisis from 1994:Q1 to 1996:Q1, and the spillover effect from the GFC from 2008:Q4 to 2009:Q3, with peak crisis events nested inside them. The identified near sudden stops align well with a purely empirical notion of financial crisis in Mexico (Reinhart and Rogoff, 2009). The peak crisis events nested inside the near crises display dynamics of amplitude comparable to what previously matched in models with occasionally binding constraints, but with more realistic persistence. Moreover, near crisis episodes are preceded by slowly unfolding booms and followed by anemic recoveries in line with empirical evidence of slow risk build-up phases and persistent output losses from sudden stops (Cerra and Saxena, 2008). Finally, we show that while different shocks explain different variables over business cycles, specific shocks matter for historical crisis dynamics.

Related Literature A few papers have already attempted estimation of models with occasionally binding constraints. Bocola (2016), in particular, builds and estimates a model of occasionally occurring debt and banking crises. Notably, estimation is accomplished while solving the model with global methods. This, however, is accomplished at the cost of simplifying the estimation procedure. In particular, the estimation procedure used involves first estimating the model outside the crisis period and then appending the crisis estimates in a second step. While this does not matter for the specific application in Bocola (2016), it would be relevant for countries experiencing serial default or repeated banking crises. Our approach would permit joint estimation of the model inside and outside the crisis state and is potentially scalable to larger and more complex models, while maintaining a satisfactory level of accuracy relative to global solution methods. Our paper relates also to Guerrieri and Iacoviello (2015), who develop a set of procedures for the solution of models with occasionally binding constraints, called OccBin. OccBin is a certainty equivalent solution method that captures non-linearities but not precautionary effects, which are the critical manifestation of the linkages between states of the world in models with occasionally binding collateral constraints. A key feature of our approach is to preserve precautionary saving effects, as agents in the model adjust their behavior due to the presence of the constraint, even when the constraint does not bind and vice versa.

The stochastic specification of the constraint that we propose and the accompanying perturbation solution method could be applied to models with occasionally binding zero-lower bound on interest rates such as for instance Adam and Billi (2006), Adam and Billi (2007), Aruoba et al. (2018), and Atkinson et al. (2018). Existing methods for the estimation of such models may limit scalability due computational costs (Gust et al., 2017). Moreover, the occasionally binding ZLB limit is not comparable to the kind of constraints with endogenous collateral value that we estimate in this paper and is used in the normative literature on macroprudential policies (Benigno et al., 2013, 2016). Indeed, as it is well understood in the literature, endogenous collateral valuation features different amplification mechanisms and entails additional computational complexities (Bianchi and Mendoza, 2018).

In the literature on Markov-switching DSGE models, our paper expands upon the method developed by Foerster et al. (2016), who developed perturbation methods for the solution of exogenous regime-switching models. The perturbation approach that we propose allows for second- and higher-order approximations that go beyond the linearized models studied by Davig and Leeper (2007) and Farmer et al. (2011). In fact, we show that at least a second-order approximation is necessary in order to capture the effects of the endogenous switching.

The paper is also related to the literature that focuses on solving endogenous regime-switching models. Davig and Leeper (2008), Davig et al. (2010), and Alpanda and Ueberfeldt (2016) all consider endogenous regime-switching, but employ computationally costly global solution methods that hinder likelihood-based estimation. Lind (2014) develops a regime-switching perturbation approach for approximating non-linear models, but it requires repeatedly refining the points of approximation and hence it is not suitable for estimation purposes. Maih (2015) and Barthlemy and Marx (2017) also develop perturbation methods for endogenous switching models, but employ a technique that approximates around regime-dependent steady states. In contrast, our generalization of the Foerster et al. (2016) perturbation approach uses a single point of approximation that is well suited for solving models in which the regime-dependency of the steady state is not crucial because of the relatively slow moving nature of state variables such as capital and debt, and the limited frequency and duration of financial crisis episodes.

We also contribute to the literature on likelihood-based estimation of Markov-switching DSGE models initiated by Bianchi (2013), and applied in Bianchi and Ilut

(2017) and Bianchi et al. (2018). Our algorithm differs in two key respects. First, our regime-switching transition matrix reflects the endogenous nature of the switching. Second, conditional on the regime, we have a second order solution, so we employ the Sigma Point Filter (Binning and Maih, 2015) to evaluate the likelihood function in place the modified Kalman filter in Bianchi (2013).

The application of the methodology that we propose relates to the literature on emerging market business cycles, including, among others, Mendoza (1991), Neumeyer and Perri (2005), Aguiar and Gopinath (2007), Mendoza (2010), Garcia-Cicco et al. (2010), Fernandez-Villaverde et al. (2011), and Fernandez and Gulan (2015). Encompassing most shocks previously considered in this literature, we include in our analysis technology shocks, preference, expenditure, interest rate, and terms of trade shocks. Relative to Mendoza (2010), we provide a Bayesian estimation of the model and consider a wider set of structural shocks. While the implied model properties are similar, we find that the estimated values of some of the parameters that are less easily calibrated to the stylized facts of the data differ substantially. Relative to Garcia-Cicco et al. (2010), we evaluate empirically the relative importance of interest rate shocks in a fully non-linear estimated framework, with a fully articulated specification of the financial frictions driving amplifications, and finding that they are quantitatively important for certain features of the data, but not others. Relative to Neumeyer and Perri (2005), we set up a framework that fits the data well without assuming any correlation between the productivity and the interest rate process. Nonetheless, consistent with their main findings, and also with Fernandez and Gulan (2015) and Ates and Saffie (2016), we find that, while we can fit ergodic second moments of the data well with uncorrelated shocks, high and averse short-run correlations are associated with the simulated crisis dynamics in the model. Remarkably, even though the model we specify does not include stochastic volatility as in Fernandez-Villaverde et al. (2011), we do not detect losses of model fit after 1998 when the unconditional volatility of Mexico business cycles declines as visible from Figure 1 above.

Finally, our paper relates to the now large literature on the Bayesian estimation of DSGE models (for example, Schorfheide, 2000; Otrok, 2001; Smets and Wouters, 2007). Our paper extends that successful approach to models with occasionally binding collateral constraints, which have become the benchmark for normative analysis of macro-prudential optimal policy (Bianchi and Mendoza, 2018; Benigno et al., 2013, 2016; Jeanne and Korinek, 2010). Welfare-base analysis of optimal macroprudential policies in model with occasionally binding constraints depends critically on calibrations assumptions and collateral constraint formulations. Structural estimation of these parameters and likelihood based model validation can disciplines model formulation, which in turn is critical for normative policy recommendations.

The rest of the paper is organized as follows. Section 2 describes the model and discusses the proposed formulation of the collateral constraint. Section 3 presents our perturbation solution method for endogenous regime-switching models. Section 4 describes the full information Bayesian procedure we employ. Section 5 reports the empirical results on estimation, model fit, and business cycle properties. Section 6 presents results on financial crises, and Section 7 concludes. The Appendices include additional technical details and empirical results.

2 The Model

The model is a medium scale, workhorse framework for the analysis of business cycles and sudden stop crisis in emerging market economies. The core of the model largely follows Mendoza (2010), although we consider a larger set of shocks as in Garcia-Cicco et al. (2010). It features a small, open, production economy with an occasionally binding collateral constraint, that is subject to temporary productivity, intertemporal preference, expenditure, interest rate, and terms of trade shocks.[1] The collateral constraint that we specify depends on the endogenous variables of the model, including borrowing, capital and its relative price, and hence leverage. Capital and debt choices respond to exogenous shocks, affecting borrowing, which in turn affects the probability of a binding collateral constraint.

Due to the occasionally binding nature of the constraint, this framework can account not only for normal business cycles, but also key aspects of financial crises in both emerging markets advanced economies (for example, Bianchi and Mendoza, 2018). While our application focuses on one particular type of crisis, the so called sudden stop in capital flows, our framework is generally applicable to other macroeconomic models with financial frictions and crises (Kiyotaki and Moore, 1997; Iacoviello,

[1] We omit permanent technology shocks of the type analyzed by Aguiar and Gopinath (2007) because these long-run components cannot be estimated precisely over samples periods of length comparable to ours. Garcia-Cicco et al. (2010) also find that the permanent technology shock is not quantitatively important in a framework with an financial frictions like ours.

2005; Gertler and Karadi, 2011; Jermann and Quadrini, 2012; Liu et al., 2013; Gertler and Kiyotaki, 2015; Bocola, 2016; Schmitt-Grohe and Uribe, 2018).

In the rest of this section, we discuss the representative household-firm and the borrowing constraint. The formal definition of the equilibrium and the full set of equilibrium conditions is reported in Appendix A.

2.1 Preferences, Constraints, and Shock Processes

There is a representative household-firm that maximizes the following utility function

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{1}{1-\rho} \left(C_t - \frac{H_t^{\omega}}{\omega} \right)^{1-\rho} \right\}, \tag{1}$$

where C_t denotes consumption, H_t the supply of labor, and d_t an exogenous and stochastic preference shock specified below. Households choose consumption, labor, capital (K_t), imported intermediate inputs (V_t) given an exogenous stochastic relative price P_t also specified below, and holdings of real one-period international bonds, B_t. Negative values of B_t indicate borrowing from abroad. The household-firm faces the budget constraint:

$$C_t + I_t + E_t = Y_t - \phi r_t \left(W_t H_t + P_t V_t \right) - \frac{1}{(1+r_t)} B_t + B_{t-1}, \tag{2}$$

where Y_t is gross domestic product and is given by

$$Y_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t. \tag{3}$$

Here, A_t denotes an exogenous and stochastic temporary productivity shock. E_t is an exogenous and stochastic expenditure process possibly interpreted as a fiscal or net export shock as in Garcia-Cicco et al. (2010). The term $\phi r_t \left(W_t H_t + P_t V_t \right)$ describes a working capital constraint, stating that a fraction of the wage and intermediate good bill must be paid in advance of production with borrowed funds. The relative price of labor and capital are given by W_t and q_t, respectively, both of which are endogenous market prices, but taken as given by the household-firm. Gross investment, I_t, is subject to adjustment costs as a function of net investment:

$$I_t = \delta K_{t-1} + (K_t - K_{t-1})\left(1 + \frac{\iota}{2}\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)\right). \tag{4}$$

Household-firms can borrow in international markets issuing one-period bonds that pay a market or country net interest rate r_t. The country interest rate between period t and $t+1$ r_t, has three components: an exogenous persistent component, an exogenous transitory component, an endogenous component that depends on the level of debt. The country interest rate is given by

$$r_t = r_t^* + \sigma_r \varepsilon_{r,t} + +\psi_r\left(e^{\bar{B}-B_t} - 1\right), \tag{5}$$

where the persistent exogenous component, r_t^*, follows the process

$$r_t^* = (1-\rho_{r^*})\bar{r}^* + \rho_{r^*} r_{t-1}^* + \sigma_{r^*}\varepsilon_{r^*,t}, \tag{6}$$

while $\varepsilon_{r^*,t}$ and $\varepsilon_{r,t}$ are i.i.d. $N(0,1)$, with σ_{r^*} and σ_r denoting parameters that control the variances of the two components.[2] Moreover, as Mendoza (2010) notes, in our model, the household-firm also faces a second endogenous external financing premium on debt (EFPD), measured by the difference between the effective real interest rate, which corresponds to the intertemporal marginal rate of substitution in consumption, and the market rate r_t. Thus, $EFPD = \mathbb{E}_t[r_t^h - r_t] = \lambda_t/\beta E_t[\mu_{t+1}]$, where $r_t^h = \mu_t/E_t[\mu_{t+1}]$ is the effective real interest rate, μ_t is the Lagrange multiplier on the budget constraint, and λ_t is the multiplier on the collateral constraint. Because of this, the endogenous interest rate component of r_t, $\psi_r\left(e^{\bar{B}-B_t} - 1\right)$ in equation 5 with be calibrated to serves the sole purpose of inducing independence of the model steady state from initial conditions, as in Schmitt-Grohe and Uribe (2003), by setting ψ_r to a very small value.[3] In addition, unlike what is often assumed in the literature (for example, Neumeyer and Perri, 2005), we do not impose any correlation between the innovations to the interest rate process and the productivity process specified below.

[2] While contemporaneous movements in $\varepsilon_{r^*,t}$ and $\varepsilon_{r,t}$ are not identified separately in equations (5) and (A.19), $\varepsilon_{r,t}$ will be identified in the data because of differences in persistence. Including both types of shocks helps fitting the observable counterpart variable in estimation.

[3] Mendoza (2010) uses an endogenous rate of time preference for the same purpose.

The remaining exogenous processes, the preference shock d_t, the temporary productivity shock A_t, the shock to the relative price of intermediate goods P_t, and the domestic expenditure shock E_t, are specified as follows:

$$\begin{aligned} \log d_t &= \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}, & (7) \\ \log A_t &= (1-\rho_A) A^* + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}, & (8) \\ \log P_t &= (1-\rho_P) P^* + \rho_P \log P_{t-1} + \sigma_P \varepsilon_{P,t}, & (9) \\ \log E_t &= (1-\rho_E) E^* + \rho_E \log E_{t-1} + \sigma_E \varepsilon_{E,t}, & (10) \end{aligned}$$

where the starred variables and the $\rho_.$ coefficients denote the unconditional mean value and the persistence parameter of the processes, $\varepsilon_{.,t}$ are assumed i.i.d. $N(0,1)$ innovations, and the $\sigma_{.,t}$ parameters control the size of the process variances.

2.2 The Occasionally Binding Borrowing Constraint: An Endogenous Regime-Switching Specification

The central idea of this paper is to model the occasionally binding nature of a borrowing constraint as an *endogenous* regime-switching process. In one regime, the constraint binds strictly, and in the other it does not; these regimes are denoted with $s_t \in \{0,1\}$, respectively. In the binding regime, total borrowing equals a fraction of the value of collateral:

$$\frac{1}{(1+r_t)} B_t - \phi (1+r_t) (W_t H_t + P_t V_t) = -\kappa q_t K_t, \tag{11}$$

limiting total debt–borrowing plus working capital–to a fraction κ of the market value of capital $q_t K_t$; thus, limiting leverage, consumption smoothing and the purchase of intermediate imported inputs for production purposes. Limited working capital, as in Neumeyer and Perri (2005), Mendoza (2010), Fernandez and Gulan (2015), and Ates and Saffie (2016), amplifies the supply response of the economy to shocks in the constrained state. In the unconstrained regime, lenders finance all desired borrowing.

Given these two regimes representing the occasionally binding nature of the constraint, we assume a *stochastic* characterization of the transition between them that eliminates the non-differentiability of the traditional inequality specification that has appealing empirical properties. The typical inequality specification of the borrowing constraint implies that, for given values of the endogenous and exogenous states, there is one specific level of leverage at which the constraint binds. In contrast, we assume that endogenously increasing leverage and shocks raise the *probability* of switching to the constrained state, but there is no specific level of leverage that coincide exactly with the constrained state. This assumption has the important implication that the run up to a crisis episode, coinciding with the state in which the economy is constrained, the duration of the the crisis phase, and the post-crisis recovery can have varying duration and intensity. We will call the periods right before entering, and right after exiting, the binding regime "near" crisis; and the periods during which the economy is in the binding regime "peak" crisis.

We assume that the probabilities of switching from one regime to the other depend on the endogenous variables of the model. The probability of switching from the non-binding regime to the binding regime is a logistic function of the distance between actual borrowing and the borrowing limit equal to a fraction of the value of collateral. The probability of switching from the binding state back to the unconstrained one, instead, is a logistic function of the collateral constraint multiplier. Therefore the transitions are affected by all endogenous variables in the model and agents have full information with rational expectations on these transitions probabilities.

This regime switching specification of the occasionally nature of the the collateral constraint captures the salient macroeconomic empirical finding that the likelihood of a financial crisis increases with leverage, but high leverage does not necessarily lead to a financial crisis. For example, Jorda et al. (2013) proxy financial leverage by the rate of change of private bank credit relative to GDP. In their database of 14 advanced countries from 1870 to 2008 there are 35 recessions associated with financial crises. Across these episodes, the change in leverage before a crisis is heterogeneous, with the standard deviation of financial leverage twice the mean. This is compelling evidence suggesting that while leverage matters, the exact level of leverage at which a crisis occurs varies considerably across crisis episodes.[4]

In addition, a growing body of microeconomic evidence indicates that a deterministic specification of occasionally binding collateral constraints does not accurately capture lending and borrowing behaviors at the household and firm or bank level. For example, Chodorow-Reich and Falato (2017) and Greenwald (2019) among oth-

[4] The notion of "debt intolerance" discussed by Reinhart and Rogoff (2009) also is also encompassed by our stochastic specification.

ers, show that loan covenants are used to renegotiate credit lines as borrowers approach their limits, rather than simply being cut off from funding as soon as they face financial stress. Campello et al. (2010) provide survey information on the behavior of financially constrained firms and Ivashina and Scharfstein (2010) examine loan level data showing that credit origination dropped during the recent financial crisis because firms drew down from pre-existing credit lines in order to satisfy their liquidity needs. Bank lending standards fluctuating over the cycle could also be consistent with a stochastic specification of the collateral constraint. Thus, in practice, collateral constraints do not seem to bind at any particular leverage ratio.[5]

In the rest of this section, we discuss our stochastic formulation of the slackness condition associated with an occasionally binding borrowing constraint and how this permits casting the occasionally binding constraint model in the form of an endogenous regime-switching framework. We then spell out the assumptions that we make to model the transition between regimes. We conclude the section with some remarks about the implications of our formulation for model dynamics.

2.2.1 The Regime-Switching Slackness Condition

Denote the Lagrange multiplier associated with equation (11) as λ_t and define the "borrowing cushion," B_t^* as the distance of actual borrowing from the debt limit:

$$B_t^* = \frac{1}{(1+r_t)} B_t - \phi (1+r_t)(W_t H_t + P_t V_t) + \kappa_t q_t K_t. \tag{12}$$

. When the borrowing cushion is small, total borrowing is high relative to the value of collateral, meaning that the leverage ratio is high.

Now, the critical step is to modify the typical slackness condition for models with deterministic inequality constraints (i.e., $B_t^* \lambda_t = 0$), so that the two variables, B_t^* and λ_t, are zero if the economy is in the relevant regime. So, we are seeking a representation in which $\lambda_t = 0$ when the economy is in the non-binding regime, and the borrowing cushion $B_t^* = 0$ when we are in the binding regime then.

To accomplish this, and to be consistent with the literature on regime-switching DSGE models in which the *parameters* are the model objects that change state,

[5] Exploring whether the our specification of the borrowing constraint may result from the solution of a limited enforcement problem with renegotiation, hidden liquidity, or random monitoring shocks is beyond the scope of this paper.

we define two auxiliary regime-dependent parameters, $\varphi(s_t)$ and $\nu(s_t)$, such that $\varphi(0) = \nu(0) = 0$, and $\varphi(1) = \nu(1) = 1$.[6] Next, we introduce the following *regime-switching slackness condition*:

$$\varphi(s_t) B_{ss}^* + \nu(s_t)(B_t^* - B_{ss}^*) = (1 - \varphi(s_t)) \lambda_{ss} + (1 - \nu(s_t))(\lambda_t - \lambda_{ss}), \tag{13}$$

where B_{ss}^* and λ_{ss} are the steady state borrowing cushion and collateral constraint multiplier, respectively, defined more precisely in Section 3 below. It is now easy to see that equation (13) implies that, as desired, when $s_t = 0$ then $\lambda_t = 0$, and when $s_t = 1$ then $B_t^* = 0$. Yet, given a regime s_t, equation (13) remains continuously differentiable for any value of B_t^* or λ_t, as no inequality constraint is imposed.[7]

2.2.2 Modelling Endogenous Regime-Switching

To model the transition from one regime to the other, we rely on logistic functions of endogenous variables determined in equilibrium that are tractable and parsimoniously parameterized.[8] Specifically, we assume that the transition from non-binding to binding regime depend on the borrowing cushion, B_t^*:

$$\Pr(s_{t+1} = 1 | s_t = 0, B_t^*) = \frac{\exp(-\gamma_0 B_t^*)}{1 + \exp(-\gamma_0 B_t^*)}. \tag{14}$$

Thus, the likelihood that the constraint binds in the following period depends on the size of the borrowing cushion in the current period. The parameter γ_0 controls the steepness of the logistic function, determining the sensitivity of the probability

[6] In our model these parameters coincide with the regime-switching indicator variable s_t, but in more general settings they may not. The notation provides a general formulation of the slackness condition that is applicable to setups possibly different than the one associated with our specific application. See, for example, the discussion of our stochastic specification in the context of other model settings in Binning and Maih (2017).

[7] Technically, equation 13 "preserves" information in the perturbation approximation that we introduce in Section 3, since, at first order, both variables are constant in the respective regimes. The use of the regime-dependent switching parameters, $\varphi(s_t)$ and $\nu(s_t)$, follows from the Partition Principle of Foerster et al. (2016), which separates parameters based upon whether they affect the steady state or not. Intuitively, $\varphi(s_t)$ captures the *level* of the economy changing across regimes (e.g., capital is lower when the constraint binds), while $\nu(s_t)$ captures the dynamic responses differing across regimes (e.g., the response of investment to shocks changes when the constraint binds).

[8] Bocola (2016) and Kumhof et al. (2015) use a logistic function to model the transition to a default regime, and Davig et al. (2010) and Bi and Traum (2014) use it to study hitting a fiscal limit.

of switching regime to the size of the borrowing cushion. For small values of γ_0, the cushion has a small impact on the probability of a switch to the binding regime. For larger values of this parameter, the probability of a switch to the binding regime increases more rapidly toward 1. Note here that, for certain draws from the logistic function, the borrowing cushion could be negative and the economy could temporarily remain in the non-binding regime.

Similarly, when the constraint binds, the transition probability to the non-binding regime is a logistic function of the Lagrange multiplier, λ_t, according to

$$\Pr\left(s_{t+1}=0|s_t=1,\lambda_t\right)=\frac{\exp\left(-\gamma_1\lambda_t\right)}{1+\exp\left(-\gamma_1\lambda_t\right)}. \tag{15}$$

The probability of switching back from a constrained to an unconstrained regime, therefore, depends on the shadow value of the economy's desired borrowing relative to the limit set by the collateral constraint. As in the case of a switch from the constrained to constrained regime, the parameter γ_1 affects the sensitivity of this probability to the value of the multiplier, with larger values implying a greater sensitivity. A large positive multiplier implies that the constraint binds tightly, and the probability of exiting binding regime is lower. As the multiplier declines, this probability increases. Note again that in the binding regime, it is possible that the desired level of borrowing is less than the level forced upon it by a binding regime, which would manifest itself with a negative collateral constraint multiplier.[9]

Putting equations (14) and (15) together, the regime-switching model has an endogenous transition matrix

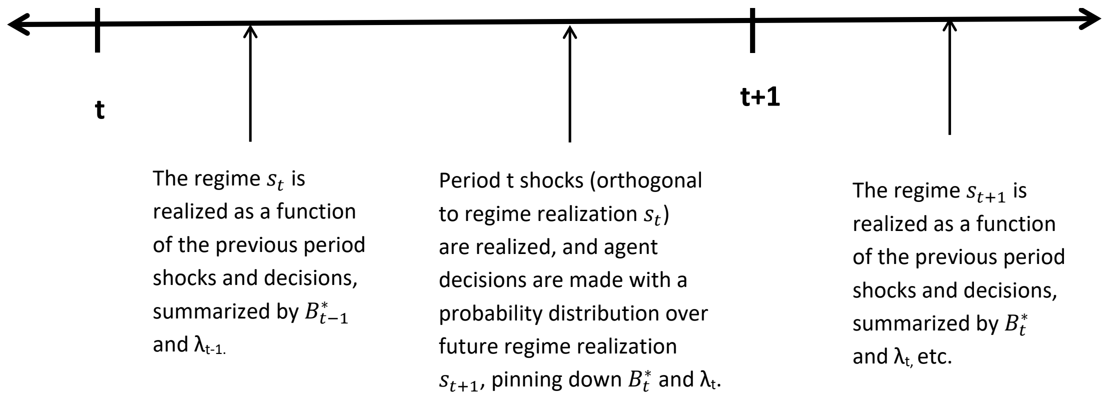
$$\mathbb{P}_t=\begin{bmatrix} 1-\frac{\exp(-\gamma_0 B_t^*)}{1+\exp(-\gamma_0 B_t^*)} & \frac{\exp(-\gamma_0 B_t^*)}{1+\exp(-\gamma_0 B_t^*)} \\ \frac{\exp(-\gamma_1\lambda_t)}{1+\exp(-\gamma_1\lambda_t)} & 1-\frac{\exp(-\gamma_1\lambda_t)}{1+\exp(-\gamma_1\lambda_t)} \end{bmatrix}. \tag{16}$$

2.2.3 Remarks on the Endogenous Regime-Switching Formulation

A few more remarks are useful on how our stochastic formulation of the borrowing constraint works and differs relative to the typical inequality formulation as for instance in (for example, Kiyotaki and Moore, 1997).

[9] By construction, the transition probabilities equal 0.5 when their arguments are zero. In principle, one could relax this assumption by introducing a constant into the arguments of equations (14-15); however, preliminary estimates with this flexibility indicated these parameters were effectively zero, so for simplicity we omit them from the beginning.

Figure 2: Model Timing



First, whether or not the constraint binds in a given period is determined *before* exogenous shocks are realized and economic decisions are made during that period. Figure 2 summarizes the model timing and shows that, at the start of a given period t, the regime outcome s_t is drawn from the logistic distributions as a function of previous period borrowing cushion or collateral constraint multiplier, B^*_{t-1} and λ_{t-1}. Next, exogenous shocks, which are orthogonal to the realization of the regime, are realized, and agents take decisions during period t based on their perfect knowledge of the regime outcome, s_t, as well as a probability distribution over the next period regime realization, s_{t+1}, as in equation (14) or (15). Finally, the regime realization for period $t+1$ is drawn based on exogenous shocks and agents' decisions that pin down B^*_t and λ_t, and so on.

Second, as we have already noted, an implication of our setup is that entry and exit of the economy from the binding regime might be smoother than in models with a traditional inequality specification of the collateral constraint. This is captured by the fact that the multiplier and the borrowing cushion can take negative values in our set up, even though they cannot persistently do so. This captures the ideas that crises approaches more slowly and recoveries are more sluggish than predicted by traditional specifications. Vice versa, there can be instances in which the crisis dynamics can be less persistent than with the traditional specification. For instance, negative values of the borrowing cushion in the non-binding regime are possible if the probability of a binding regime is elevated but such outcome is not realized. Conversely, in the non-binding regime, the logistic function can allow the borrowing constraint to bind in the following period, even if the borrowing cushion in the current period is still positive. How likely these outcomes are depend on the parameter of the relevant logistic function, γ_0. The same logic applies to a probabilistic exit from the binding regime that depends on the multiplier λ_t and the parameter of the logistic function γ_1. Despite the fact that the economy might be stuck in the constrained regime past the time when the collateral constrained multiplier turned negative. In fact, in this case, the economy may be "forced" to borrow the amount set by the constraint, which might be more than desired, until a non-binding realization of the regime is drawn. Conversely, positive values of the multiplier, the economy may end up coming out of the binding regime early.

The third implication of our setup is that by making the transition probability dependent upon endogenous variables, these probabilities that vary over time. In contrast, the exogenous Markov-switching setup (Davig and Leeper, 2007; Farmer et al., 2011; Bianchi, 2013; Foerster et al., 2016) has a constant probability of transitioning between regimes that is independent of the structural shock realizations and the agent decisions. For this reason, our endogenous-switching framework can in principle generate long- or short-lived binding regime episodes depending on the realization of shocks and agents' decisions.

Last but not least, in our set up, agents in the non-binding regime know that higher leverage and borrowing levels increase the probability of switching to a binding regime, and vice-versa. This preserves the interaction in agents' behavior between the two states that gives rise to precautionary behavior, distinguishing this class of models from those in which financial frictions are always binding or are approximated with solution methods that eliminate the interactions across regimes.

3 Solving the Endogenous Switching Model

Having cast our model it in terms of an endogenous regime-switching framework, this Section describes our solution method and our estimation procedure. The model developed in the previous section can in principle be solved using global solution methods, as for example in Davig et al. (2010). However, such an approach would be time-consuming for our model which has two endogenous and five exogenous state variables, the regime indicator, plus six exogenous shocks, and would quickly become prohibitive with larger modes, precluding likelihood-based estimation. Instead, we solve the model using a perturbation approach, which allows for an accurate approximation that is fast enough to permit estimation and potentially applicable to even larger set ups beyond our medium-scale model. We now describe the approximation point and how to define a steady state in this setup, these Taylor-series expansions, and discuss the importance of approximating to at least a second-order in our framework. The competitive equilibrium of the endogenous regime-switching version of the model is defined formally in Appendix A. The derivations of the Taylor-series expansions and other details of the solution method to Appendix B.

3.1 Defining the Steady State

Given the regime-switching slackness condition (13), defining a non-stochastic steady state of an endogenous regime-switching framework is challenging. A steady state in our framework can be defined as a state in which all shocks have ceased and the regime-switching variables that affect the level of the economy ($\varphi(s_t)$) take the *ergodic mean* associated with the steady state transition matrix:

$$\mathbb{P}_{ss} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} & \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}. \tag{17}$$

Note here that, since this matrix also depends on the steady state level of the borrowing cushion and the multiplier, B^*_{ss} and λ_{ss}, which in turn depend upon the ergodic mean of the regime-switching parameters $\varphi(s_t)$ and $\nu(s_t)$, such steady state is the solution of a fixed point problem that is described in more detail in Appendix B.

More specifically, consider the model regime-specific parameters defined above and distinguish between $\varphi(s_t)$, which can affect the level behavior of the economy, and $\nu(s_t)$, which can affect only its dynamics with no effects on the steady state. Then denote with $\xi = [\xi_0, \xi_1]$ the ergodic vector of $\mathbb{P}_{ss}$. Next, apply the Partition Principle of Foerster et al. (2016), to focus only on parameters that affect the level of the economy, and write their ergodic mean as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1), \tag{18}$$

where $\bar{\varphi}$ thus denotes the ergodic mean value of the regime-switching variables–or parameters in a conventional approximation–that affects the steady state.

Defining the steady state as the state in which the auxiliary parameter $\varphi(s_t)$ is at its ergodic mean value implies that the approximation point constructed is a weighted average of the steady states of two separate models: a model in which only the non-binding regime occurs, and one in which only the binding regime occurs; the steady state does not explicitly capture behavior generated by changes in regimes. How close our approximation point is to each of these two other steady state concepts, therefore, will depend on the frequency of being in each of the two regimes. We note here that, since in our application we are modelling binding episodes with limited duration, the ergodic mean is a natural candidate as perturbation point. Given the nature of our application with slow-moving capital and debt state variables, the perturbation point will be in the area of the state space in which the economy operates most frequently. In fact, since the binding regime tends to be self-limiting–that is, being in the binding regime causes the economy to reduce leverage and hence switch back to the non-binding regime–the economy will rarely reach the area around the steady state of the "binding regime only" version of the model.[10]

3.2 The Solution and Its Properties

Equipped with the steady state of the endogenous regime-switching economy, we then construct a second-order approximation to the policy functions by taking derivatives of the equilibrium conditions. We relegate details of these derivations to the Appendix B, but here we provide a summary.

For each regime s_t, the policy functions to our model take the form

$$\mathbf{x}_t = h_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \qquad \mathbf{y}_t = g_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \tag{19}$$

where $\mathbf{x}_t$ denotes predetermined variables, $\mathbf{y}_t$, non-predetermined variables, $\varepsilon_\mathbf{t}$ the set of shocks, and χ a perturbation parameter such that when $\chi = 1$ the fully stochastic model results and when $\chi = 0$ the model collapses to the non-stochastic steady state defined above. Using these functional forms, we can express the equilibrium

[10]Alternative methods for finding solutions to endogenous regime-switching models, such as Maih (2015) and Barthlemy and Marx (2017), advocate using regime-dependent steady states as multiple approximation points. Such a strategy would not be suitable for our purposes because the binding regime steady state is a poor approximation point given that the state is infrequent and usually of shorter duration than normal cycles of expansions and contractions.

conditions conditional on regime s_t as

$$\mathbb{F}_{s_t}\left(\mathbf{x}_{t-1},\varepsilon_t,\chi\right)=0. \tag{20}$$

We then stack the regime-dependent conditions for $s_t=0$ and $s_t=1$, denoting the resulting system of equations with $\mathbb{F}\left(\mathbf{x}_{t-1},\varepsilon_t,\chi\right)$, and successively differentiate with respect to $(\mathbf{x}_{t-1},\varepsilon_t,\chi)$, evaluating at steady state. The systems

$$\mathbb{F}_{\mathbf{x}}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=0,\qquad \mathbb{F}_{\varepsilon}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=0,\qquad \mathbb{F}_{\chi}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=0 \tag{21}$$

can then be solved for the unknown coefficients of the first-order Taylor expansion of the policy functions in equation (19).

A second-order approximation can be found by taking the second derivatives of $\mathbb{F}\left(\mathbf{x}_{t-1},\varepsilon_t,\chi\right)$. In the end, we have matrices $H^{(1)}_{s_t}$ and $G^{(1)}_{s_t}$ characterizing the first-order coefficients, and $H^{(2)}_{s_t}$ and $G^{(2)}_{s_t}$ characterizing the second-order coefficients. Therefore, the approximated policy functions are

$$\mathbf{x}_t\approx\mathbf{x}_{ss}+H^{(1)}_{s_t}S_t+\frac{1}{2}H^{(2)}_{s_t}\left(S_t\otimes S_t\right) \tag{22}$$

$$\mathbf{y}_t\approx\mathbf{y}_{ss}+G^{(1)}_{s_t}S_t+\frac{1}{2}G^{(2)}_{s_t}\left(S_t\otimes S_t\right) \tag{23}$$

where $S_t=\left[\begin{array}{ccc}(\mathbf{x}_{t-1}-\mathbf{x}_{ss})' & \varepsilon_t' & 1\end{array}\right]'$.

Our perturbation method produces a single approximated set of policy function, but cannot be used to guarantee that the solution is unique. This limitation is common to models of occasionally binding constraints that are solved globally with a numerical algorithm that converges but where additional solutions cannot be ruled out. With endogenous regime-switching, we also lack conditions for ensuring stability of the full solution; instead, we check the mean-squared stability of the first-order approximation (Farmer et al., 2011; Foerster et al., 2016), and additionally check for explosive simulations.

Our solution method is fast, and can readily be scaled to handle larger models. In all, we have 23 equations that characterize the equilibrium, and 2 endogenous and 5 exogenous state variables, and a single regime indicator. Our solution method is similar to that in Fernandez-Villaverde et al. (2015). We use Mathematica to take symbolic derivatives, and export the symbolic derivatives so that we can use Matlab to solve the model repeatedly for different parameterizations. The model solves in about a second on a standard laptop.

Lastly, we tested for accuracy of our solution method in our current model as well the smaller model of Jermann and Quadrini (2012) in which we can more easily compare our perturbation method to with global solution methods. We find Euler equation errors for the model we use in this paper on the order of $1 per $1,000 of consumption. This figure is in line with those found for perturbation in fixed probability regime-switching models (Foerster et al., 2016) as well as standard models without regime-switching (Aruoba et al., 2006). When we compare the perturbation method we propose with a standard global method on the endogenous regime-switching model or the version of the model with the traditional inequality constraint, we find that the solution methods produce similar second moments, and model dynamics for key variables of interest. Moreover, the global and perturbation solutions of our endogenous regime-switching model produce very similar Euler equation errors–See Appendix C for more details.

3.3 Approximation Order, Endogenous Switching and Precautionary Saving

Our endogenous regime-switching framework must be solved at least to the second order to capture the effects of endogenous probabilities, which include state-varying precautionary effects. If we were to use only a first-order approximation, our estimation would not capture precautionary behavior associated with rational expectations about the dependency of the probability of a regime change on the borrowing cushion and the multiplier. The following Proposition states this result formally.

Proposition 1 (**Irrelevance of Endogenous Switching in a First-Order Approximation**). *The first-order solution to the endogenous regime-switching model is identical to the first-order solution to an exogenous regime-switching model where the transition probabilities are given by the steady-state value of the time-varying transition matrix.*

Proof. *See Appendix C.*

This Proposition illustrates that using a second-order approximation to the solution is necessary to characterize the model properties associated with the endogenous nature of the regime-switching, including particularly precautionary behavior.[11] This result is similar to the one stating that, in models with only one regime, first-order solutions are invariant to the size of shocks, second-order solutions captures precautionary behavior, and third-order solutions are needed to capture the effects of stochastic volatility (Fernandez-Villaverde et al., 2015).

Unfortunately, the need to use a second-order approximation along with regime-switching creates additional challenges for estimation purposes, and we now turn to our strategy to address them.

4 Estimating the Endogenous Switching Model

We estimate the model with a full information Bayesian procedure. The posterior distribution has no analytical solution and we use Markov-Chain Monte Carlo (MCMC) methods to sample it. Since the Metroplis-Hastings algorithm that we use for sampling is a standard tool used in the literature, we omit a discussion of this step in our procedure. The details of the construction of the state space representation and the filtering steps for the evaluation of the likelihood are reported in Appendix D.

A key obstacle in sampling from the posterior is the evaluation of the likelihood function. We face three difficulties here relative to linear DSGE models. The first is the nonlinearity due to the presence of multiple regimes. The second is the need to approximate to the second-order the model solution that governs the decision rules in each regime. The third is the fact that the transition probabilities are endogenous. Bianchi (2013) develops an algorithm to address the first difficulty. Here we must use an alternative filter to deal with the second order solution and endogenous probabilities in a tractable manner. We use the Unscented Kalman Filter (UKF) to compute approximations to the evaluation of the likelihood function using Sigma Points. This is because, in our application, a regime switch can lead to discarding a large number of simulated particles, lowering accuracy for a given number of particles and greatly increasing the computational cost of obtaining a given level of accuracy. Further, even with a deterministic filter, the filtering step in estimation is relatively costly at about 10 seconds per likelihood evaluation using Matlab; incorporating the Particle

[11] Appendix C provides an illustration of the quantitative importance of using first- versus second-order approximations, examining the impulse response functions to shocks in the non-binding regime.

Filter would increase computing time significantly.[12]

The model's posterior distribution is highly nonlinear, with many local modes due to the complexity of the model. To deal with this issue, we took the following steps: first, we estimated a version of the model without working capital and the occasionally binding constraint, this step helped find approximate estimates for the exogenous processes and non-financial variables; second, conditional on the estimates from the first step, we performed a grid search over the remaining parameters (κ, ϕ, γ_0, and γ_1) to find high posterior regions; third, from the high posterior regions of this grid search, we used a mode-finding routine to find the posterior mode, which forms the basis for our empirical results; lastly, we sampled $500,000$ draws from the posterior with a random-walk Metropolis-Hastings algorithm to explore the parameter space around the mode and characterize credible sets for the estimates.[13] The entire MCMC algorithm took 58 days to complete.

4.1 Observables, Data, and Measurement Errors

The model is estimated with quarterly data for GDP growth (gross output less intermediate input payments), consumption growth, investment growth, and intermediate import price growth, as well as the current account-to-output ratio, and a measure of the country real interest rate. GDP, consumption, and investment are in quarterly, demeaned log differences.[14]

As there are six shocks with six observable series, we do not need measurement errors. However, measurement errors in the observation equation improves performance of the non-linear filter and accounts for any actual measurement error in the data. To limit their impact on the inference, we limit their variance to 5% of the variance of the observable variables. This means that our model will fit the data relatively closely on average; thus, how it performs across cycles and crises and whether it relies on large shocks to fit the data will be important for assessing model performance.

[12]See Binning and Maih (2015) for a comparison between the Sigma Point filter and the Particle Filter (for example, Fernandez-Villaverde and Rubio-Ramirez, 2007) in a regime-switching context, which include degeneracy issues.

[13]For the last MCMC step, we adjusted the scale of the proposal density until we achieved an acceptance rate of 0.25.

[14]See Appendix F for details on variable definitions and data sources. The country interest rate is constructed, following Uribe and Yue (2006), and it is the US 3-Month Treasury Bill minus ex post US CPI inflation rate plus Mexico's EMBI Spread.

Table 1: Calibrated Parameters

Parameter	Description	Value
β	Discount Factor	0.9798
ρ	Risk Aversion	2.0000
ω	Labor Supply	1.8460
η	Capital Share	0.3053
α	Labor Share	0.5927
δ	Depreciation Rate	0.0228
P^*	Mean Import Price	1.0280
E^*	Mean Expenditure	0.2002
ψ_r	Interest Rate Debt Elasticity	0.0010
$\bar{B}$	Neutral Debt Level	-6.1170

4.2 Calibrated Parameters and Prior Distributions

Our objective is to estimate critical parameters governing the model's dynamics in both the binding and non-binding regime, as well as the parameters that govern the transitions between regimes on which we do not have prior information. To make inference on the parameters of interest, we calibrate a subset of parameters on which we have reliable prior information–we discuss the two set of assumptions, shortly.

Table 1 lists the parameters that we calibrate.[15] We set these parameters largely following Mendoza (2010), who calibrated them based upon stylized facts from Mexico's National Accounts. One parameter that does not come from Mendoza (2010) is β, which we set to match the capital-to-output ratio and the debt-to-output ratio, $\bar{B}$. Another important parameter that we calibrate is ψ_r, which is estimated in Garcia-Cicco et al. (2010). We set it to a very small value for the sole purpose of eliminating the dependency of the steady state on initial conditions, while not allowing the parameter to affect the model dynamics (see Schmitt-Grohe and Uribe, 2003).[16] Setting ψ_r very small allows us to evaluate the model's ability to match the behavior of the trade balance and the other key stylized facts of the data without introducing an additional financial friction, in the form of a quantitatively important endogenous component of the interest rate in equations (5)-(A.19).

[15] See Appendix E for more details on the calibration and targeted moments.

[16] Even though we have a borrowing constraint and precautionary savings, the presence of $\psi_r > 0$ serves the same purpose as endogenous discounting in Mendoza (2010). Recall here that our perturbation solution is constructed around a point between the steady state of the "non-binding only" model, which depends on ψ_r, and the "binding only" model.

Table 2 below summarizes our assumptions on the prior distributions. We set two types of priors on the parameters to be estimated. The first type is priors on the parameters. They impose sign restrictions and put lower prior probability on parameter values that generate implausible moments in model simulations.[17] The second type of prior is on a model-implied object: the steady state transition probability of switching from the unconstrained to the constrained regime, given by the steady state value of equation (14), $\Pr(s_{t+1} = 1 | s_t = 0, B^*_{ss})$. We set this prior to be a Beta distribution with mean 0.25 and variance of 0.25. This prior puts lower probability mass on combinations of parameters that either generate extremely infrequent transitions to the constrained regime, or that imply the economy exits the unconstrained regime almost immediately.[18]

5 Empirical Results

Our empirical findings comprise four sets of results. First, we present the estimated parameters, which helps us to characterize the tightness of the working capital and borrowing constraints, and the endogenous transition probabilities. Second, we examine the estimated model's fit to the data. Third, we examine the model's performance from a business cycle perspective, comparing moments in the model and the data and assessing the relative importance of different shocks for regular business cycles. Our fourth set of results focuses on financial crises. We report and discuss the first three sets of results in this section, and present the fourth set in the following separate section.

5.1 Estimated Parameters

For our first set of results focusing on the estimated parameters, Table 2 reports the mode of the posterior distribution of the estimated parameters, together with the median, the 5th and the 95th percentile of the parameter distribution. The parameters

[17] While possible in principle, we do not allow for regime-switching in the shocks processes. This is because we want the collateral constraint to drive regime-switching, rather than changes in the stochastic processes. Allowing for regime change in the shock processes might improve overall fit, but we want the economic features of the model and not changes in the exogenous shock processes to drive fluctuations

[18] The use of priors on model-implied objects has been used by, for example, Otrok (2001) and Del Negro and Schorfheide (2008).

Table 2: Estimated Parameters

Par.	Description	Prior	Posterior			
			Mode	5%	50%	95%
ι	Capital Adj.	N(10,5)	12.703	12.649	12.701	12.724
ϕ	Working Cap.	U(0,1)	0.7113	0.7102	0.7153	0.7207
r^*	Mean Int. Rate	N(0.0177,0.01)	0.0172	0.0115	0.0165	0.216
κ	Leverage	U(0,1)	0.1727	0.1592	0.1756	0.1989
ρ_a	Autocor. TFP	B(0.6,0.2)	0.9796	0.9653	0.9793	0.9881
ρ_e	Autocor. Exp	B(0.6,0.2)	0.9111	0.9066	0.9132	0.9237
ρ_p	Autocor. Imp Price	B(0.6,0.2)	0.9711	0.9609	0.9754	0.9549
ρ_d	Autocor. Pref.	B(0.6,0.2)	0.9810	0.9753	0.9810	0.9843
ρ_{r^*}	Autocor. Persist. Int. Rate	B(0.6,0.2)	0.8929	0.8782	0.8896	0.8995
σ_a	SD TFP	IG(0.01,0.01)	0.0083	0.0066	0.0081	0.0098
σ_e	SD Exp.	IG(0.1,0.1)	0.1806	0.1672	0.1816	0.1892
σ_p	SD Imp. Price	IG(0.1,0.1)	0.0471	0.0382	0.0452	0.0524
σ_d	SD Pref.	IG(0.1,0.1)	0.1123	0.0998	0.1123	0.1194
σ_r	SD Trans. Int. Rate	IG(0.01,0.01)	0.0028	0.0013	0.0025	0.0044
σ_{r^*}	SD, Persist Int. Rate	IG(0.01,0.01)	0.0047	0.0037	0.0047	0.0059
γ_0	Logistic, Enter Binding	U(0,150)	13.552	10.903	13.712	18.014
γ_1	Logistic, Exit Binding	U(0,150)	17.798	15.784	17.800	19.806

Notes: Estimated parameters, with prior distribution and posterior moments. Priors are Normal, Uniform, Beta, or Inverse Gamma; prior distributions show mean and variance, except for uniform where lower and upper bounds are shown. Posterior distribution shows mode (used for model analysis), along with 5-th, 50-th, and 95-th percentiles from MCMC posterior draws.

of the exogenous processes indicate that all shocks have a high degree of persistence, but none have a positive posterior coverage interval near one. The estimated mean interest rate, slightly below 1.75% per quarter, is close to the value estimated by Mendoza (2010). Note that the posterior coverage interval for this variable is fairly diffuse, indicating some uncertainty in its true value. The remaining parameters have tightly estimated posteriors, so we will focus the discussion on posterior modes for the remaining parameters.

Importantly, the model provides precise estimates of critical parameters, namely the investment adjustment cost, working capital, and leverage parameters, and the parameters of the logistic function that help match the time series of the observable variables during both business cycles and financial crises. These parameters cannot be easily measured directly from stylized facts of the data–unlike, for example, capital or labor shares–but are nonetheless important for explaining the behavior of the economy and the amplification of shocks.

The estimate of the investment adjustment cost parameter, ι, which controls investment volatility, is 12.7. Note that this parameter is model dependent and has no real interpretation outside of a particular model; for example, considering an annual frequency, Mendoza (2010) calibrated this parameter to 2.75. The estimate for the working capital constraint parameter indicates that 71% of the wage and intermediate good bill needs to be paid in advance with borrowed funds; this estimate is substantially higher than the 25.79% value set by Mendoza (2010), but much lower than the 100% used by Neumeyer and Perri (2005) or the 125% used by Uribe and Yue (2006). The estimate is close to the 60% calculated by Ates and Saffie (2016) who use interest payments and production costs from Chilean microeconomic data. The estimated value of the leverage parameter in the borrowing constraint (κ) is 0.17, indicating less than a fifth of the value of capital serves as collateral. The estimate is slightly tighter than the benchmark value of 0.20 chosen by Mendoza (2010), which is is right inside the confidence set, and on the low end of the 0.15 to 0.30 range of alternative values considered in that calibration.

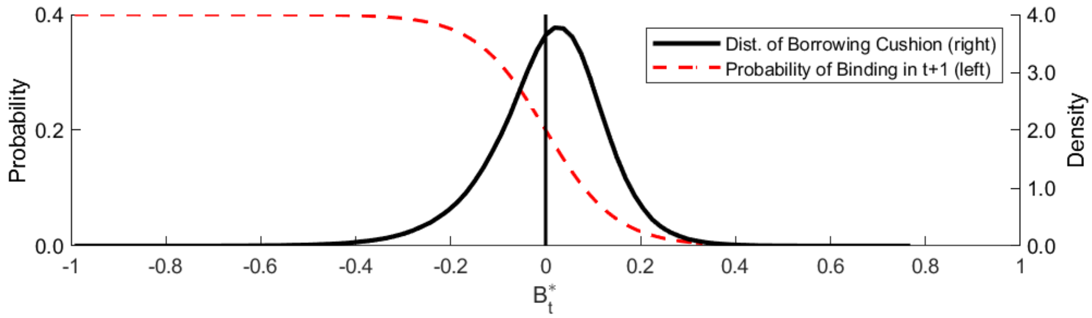
The posterior modes of the logistic parameters in equations (14) and (15) are 13.6 and 17.8, respectively, estimated in a tight range relative to the very loose prior. These estimates are significantly different from zero, thus suggesting that the data reject a model specification in which the transition probabilities are *exogenous*, which is in principle allowed for under the prior distribution.

The logistic parameters matter for transition probabilities through how they interact with the arguments of the logistic distributions. Figure 3 plots the implied probabilities from equation (14) and (15), evaluated at the posterior mode value of γ_0 and γ_1, together with the estimated ergodic distributions of their arguments, the borrowing cushion, B^* and the the constraint multiplier, λ. The figure shows that the ergodic distribution of the borrowing cushion is centered on a positive value, as the economy spends most of its time in the unconstrained regime, above the borrowing limit. As the borrowing cushion falls, the probability of switching to the binding regime increases, and gradually reaches 1 for small negative values, with very little probability mass on large negative realizations of the borrowing cushion.

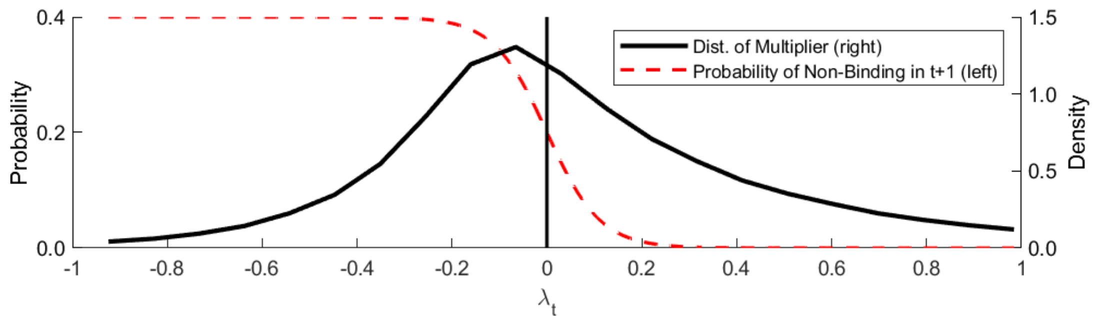
On the other hand, once the economy is in the binding regime, the ergodic distribution of the multiplier is centered on small negative values, with more probability mass on the right tail than the left tail. As λ approaches 0, the probability of switch-

Figure 3: Logistic Functions and Distributions of Their Arguments

(a) Borrowing Cushion and Transition Probability in Non-Binding Regime



(b) Multiplier and Transition Probability in Binding Regime



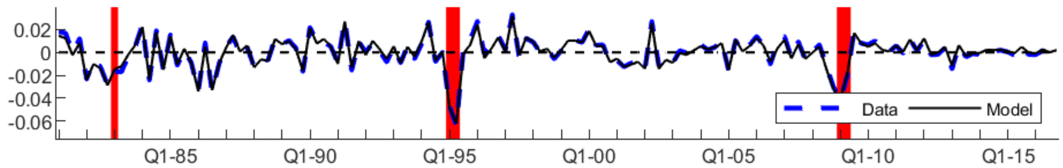
Note: The top panel shows the model-implied distribution of the borrowing cushion B^* in the non-binding regime, and the logistic transition function to the binding regime as in equation (14) implied by our estimates. The bottom panel shows the model-implied distribution of the multiplier, λ, in the binding regime, and the transition function to the non-binding regime as in equation (15) implied by our estimates.

ing to the non-binding regime increases and quickly reaches 1, with a mode on a small negative value. Nonetheless, the there is a significant probability mass for larger negative values. As we explained earlier, negative values of λ reflect instances in which, had the economy been in the non-binding regime, the borrowing cushion would be positive (as a result of the shock realizations and agent decisions as illustrated in Figure 2), but a switch to non-binding regime at has not been yet drawn.[19]

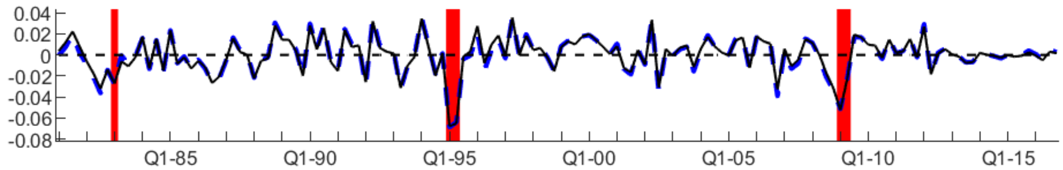
[19]Sufficiently negative values of λ, approximately below -0.2, produce a nearly deterministic switch back to the binding regime. The ergodic distribution of λ in the binding regime (Figure 3b) implies that the probability of exiting that regime exceeds 99% about 1/4-th of the time.

Figure 4: Data and Model Estimates

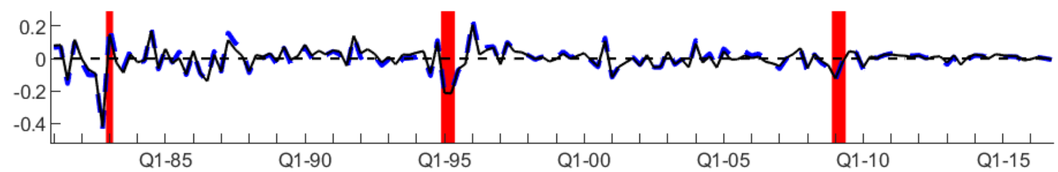
(a) Output Growth



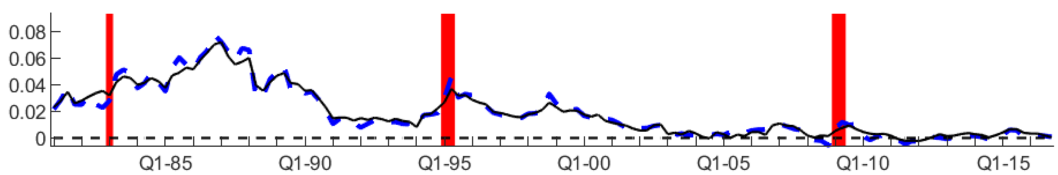
(b) Consumption Growth



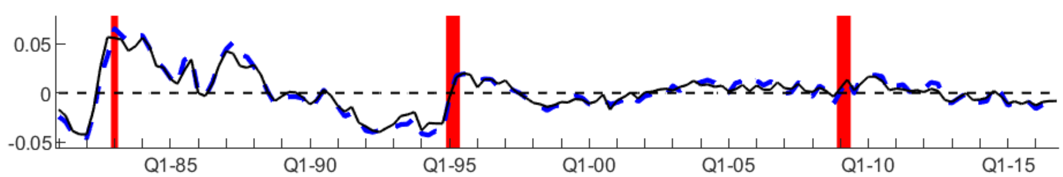
(c) Investment Growth



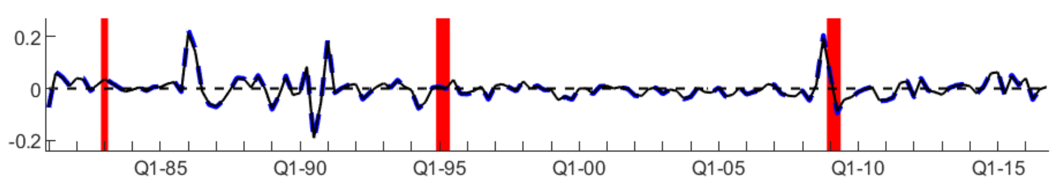
(d) Interest Rate



(e) Current Account to Output Ratio



(f) Import Price Growth



Note: The figure plots observable variables used in estimation (dashed blue lines) and fitted values (i.e., model implied smoothed estimated series based upon the full sample, solid black lines). Red bars indicate model-identified periods of crisis, see text for definition.

5.2 Model Fit

Our second set of results provides evidence on how the estimated model fits the observable variables. The model fit is summarized by Figure 4, which plots observable variables used in the estimation together with the fitted values. The Figure also includes model-identified peak crisis periods (solid red bars); we provide a precise definition shortly, but in brief the economy in the binding regime, output falls, and the current account increase. The fitted series largely follow the actual data. Note that the relatively small role of measurement errors in the estimation procedure implies the structural shocks are driving model fit. Importantly, the model estimates track the data consistently throughout the sample, during both regular business cycle and crisis periods, without loss of fit even in crisis peaks. For example, during the 1995 "Tequila Crisis," the data show large drops and rebounds in output, consumption, and investment growth, and a very sharp reversal in the current account to output ratio. The model tracks crises as well as it tracks regular fluctuations throughout the sample. If, by contrast, one were to observe a loss of fit during crisis episodes, it would suggest that our estimated model finds it difficult to match the data dynamics during these episodes of critical interest in the empirical analysis. In addition, one of the successes of our estimated model is that it achieves this fit without relying on large shocks to explain crisis events (see Appendix G). Instead, it explains crisis dynamics using the model's internal propagation mechanisms that amplify the effects of normally sized shocks–i.e., within two-standard deviations bands.

5.3 The Anatomy of Business Cycles

In our third set of results, we discuss second moments to characterize the estimated model's dynamics and variance decompositions to identify key drivers of the business cycle.[20] All statistics reported are unconditional, rather than conditional on a particular regime.

Table 3 compares data and simulated model second moments, reporting results for three variables used in estimation (output, consumption, investment and the country

[20]All business cycle and crisis statistics relying on simulated data are generated based on the posterior mode estimates. For these simulations, we generate 10,000 samples of 144 quarters long (the length of our data sample), after a burn-in period of 1,000 quarters. We then compute and report median values across these 1,000 runs. We use a pruning method (Andreasen et al., 2018) to avoid explosive simulation paths.

Table 3: Simulated Second Moments: Data and Model

	Relative Std. Dev.		Correlations	
Data Series	Data	Model	Data	Model
Output Growth	1.00	1.00	1.00	1.00
Consumption Growth	1.25	1.92	0.73	0.98
Investment Growth	5.37	5.75	0.53	0.90
Trade Balance to Output Ratio	1.24	0.80	-0.20	-0.21
Country Interest Rate	1.36	0.15	-0.11	-0.03

Notes: The table compares second moments of the data, relative to output growth, with the same moment simulated from the model.

interest rate), and one critical trade variable, the trade balance ratio, not used in estimation. The model describes the business cycle moments quite well, matching the relative volatilities of output, consumption and investment. The volatility ranking is correct, with consumption significantly more volatile than output, which is a robust stylized fact of emerging market business cycles. The model underestimates the relative volatility or the trade balance ratio and, particularly, the country interest rate. The model implied comovements of all variables match the data counterparts remarkably well, again with the exception of the country interest rate, whose correlation is not estimated precisely in the model. The trade balance, in particular, which is not an observable variable used in estimation, is counter-cyclical as in the data, with a model-implied autocorrelation coefficient (not reported) well below one.

Table 4 reports variance decompositions. The table illustrates that all shocks play a quantitatively sizable role in the model, even though different shocks matter more for different variables. Output and consumption are mostly driven by productivity, preference, expenditure, and terms of trade shocks, respectively. Investment is significantly affected by expenditure, preference, productivity, terms of trade, and persistent interest rate shocks. Expenditure and persistent interest rate shocks are the most important drivers of the trade balance, while the country interest rate is clearly driven by persistent interest rate shocks, and to a lesser extent by the temporary component of the cost of borrowing. Demand shocks (expenditure and preference) and interest rate shocks (permanent and temporary components) play a more important role than productivity and terms of trade shocks for financial variables and the multiplier.

While the magnitude of these variance shares are not directly comparable with those estimated by Garcia-Cicco et al. (2010), Fernandez and Gulan (2015), and

Table 4: Estimated Unconditional Variance Decomposition

Variables / Shocks	TFP	Expend.	Import Prices	Pref.	Temp. Int. Rate	Pers. Int. Rate
Output	33.2	17.2	15.7	25.4	2.5	6.0
Consumption	30.3	23.4	14.3	20.6	3.8	7.6
Investment	19.2	29.8	10.3	25.6	4.6	10.5
Trade Bal/Output	9.5	35.2	8.8	17.2	9.2	20.1
Interest Rate	0.0	0.0	0.0	0.0	21.1	78.9
Borrowing Cush.	10.6	32.3	9.9	21.3	9.9	16.0
Debt/Output	15.2	25.5	7.6	40.9	1.4	9.5
Multiplier	9.5	40.5	9.5	18.1	9.6	12.8

Note: The variance decomposition is normalized to sums to 100 by row, estimates may not equal 100 exactly due to rounding. Decomposition computed by removing each shock from full model to compute the marginal impact of the shock; this method ignores nonlinear interactions for ease of comparison with linear models.

Schmitt-Grohe and Uribe (2018), they suggest that both real and financial shocks matter for Mexico business cycles. In particular, we find a lower share for productivity and interest rate shocks than Fernandez and Gulan (2015), although we also consider terms of trade and demand shocks. We also find a share of variance explained by terms of trade shocks that is very close to the structural vector autoregression model estimated by Schmitt-Grohe and Uribe (2018). The estimated share of the variances explained by interest rates shocks is in general smaller than those estimated by Garcia-Cicco et al. (2010), who use a different specification of the financial friction with a debt elastic country premium and a risk premium shock, without amplification mechanism from the financial accelerator (Fernandez and Gulan, 2015), or working capital as in Neumeyer and Perri (2005), Mendoza (2010), Fernandez and Gulan (2015), and Ates and Saffie (2016).

6 The Anatomy of Financial Crises

In this Section, we turn to our fourth and main set of empirical results, which examine the model's ability to describe and interpret financial crises. The defining feature of our model is its ability to characterize dynamics and identify shocks not only over regular business cycles, but also during periods of a particular type of crisis, the so-called sudden stop in capital flows. We start by defining financial crises episodes in a model consistent manner and discuss the inference that we can draw based on the estimated model about when Mexico appeared to be experiencing them. Next, we focus on the model-based dynamics of sudden stop events (henceforth, crisis for brevity). Finally, we investigate the drivers of the three historical episodes of sudden stop that the estimated model identifies in the data: the Debt Crisis of the 1980s, the 1995 'Tequila' Crisis, and the spillover of the Global Financial Crisis (GFC) in 2008-2009.

6.1 Crisis Definition

The estimated model allows us to make inference on whether the economy is in the binding regime, and hence identify periods of sudden stop crisis in a model-consistent manner. In the model, the regime is known by the household-firm, but the estimation procedure does not observe the regime, and it must be inferred based on the information in the data. The estimation results, therefore, can provide a time-varying estimate of the smoothed probability (i.e. based upon the full sample) of being in each regime.[21] Figure 5 plots this estimated probability (solid black line).

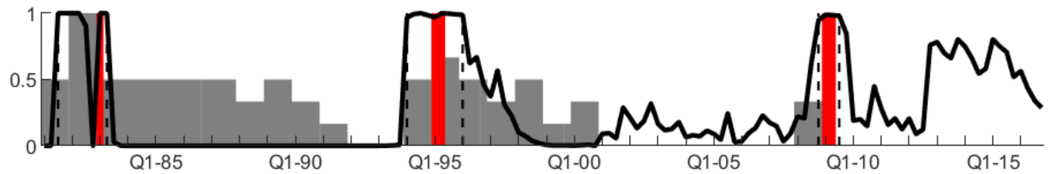
Using the information in Figure 5, we can define a model-consistent concept of sudden stop crisis as a period in which the economy is in the binding regime with probability higher than a certain threshold–say for example at least 90% probability.[22] We will call the sudden stop episodes identified by our estimated model in this manner as "near sudden stops", or near crises" for brevity. For comparison to the extant literature, we also define what we call "peak sudden stop crises", or *peak crises* for brevity (red bars in Figure 5 and 1, as periods in which (i) the smoothed estimate of the probability of being in the binding regime is at least 90%, (ii) the model estimate of output growth (Figure 4, Panel a) is negative by more than one standard deviation, and (iii) the model estimate of the current account ratio (Figure 4, Panel e) increases by more than one standard deviation. This second definition is line with the one employed in the quantitative literature modeling sudden stops with occasionally binding constraints (for example, Mendoza, 2010; Benigno et al., 2013)

[21]The estimated model also provides an estimate of the time-varying transition probability based upon equations (14-15). These are reported in Appendix G and confirm that an exogenous regime switching specification would be rejected by the data.

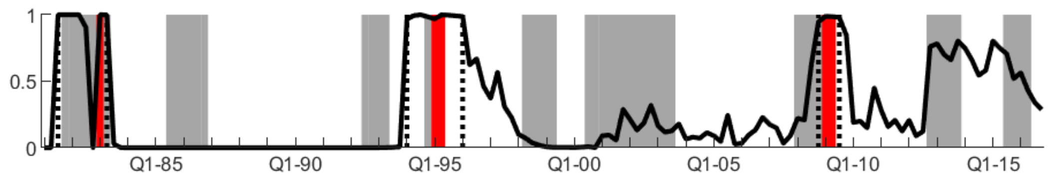
[22]This threshold is intuitive but somewhat arbitrary. The duration of the identified sudden stop episode identified, however, is very robust to using a wide range of values.

Figure 5: Smoothed Probability of Binding Regime

(a) Probability of Binding and Reinhart-Rogoff Dates



(b) Probability of Binding and OECD Recession Dates



Notes: Black line shows model implied smoothed probability of being in the binding regime. Dark gray regions in panel (a) indicates Reinhart and Rogoff (2009) tally index of financial crisis, normalized so that it takes values between 0 (no crisis) and 6 (most severe). Light gray regions in panel (b) indicate OECD recession dates. Red bars indicate model-identified peak crises, vertical blacked dash lines indicate near crisis start and stop dates; see text for details.

and the empirical literature on sudden stops (for example, Calvo et al., 2006).

Figure 5 shows both the estimated *near* and *peak crisis* periods (vertical red bars). The model identifies three peak crisis episodes, each nested inside longer lasting *near crisis* episodes (solid black line close to 1, with start and end quarter marked by vertical dashed lines). The three peak crisis episodes identified are 1983Q1 during the Debt Crisis, 1995Q1-Q2 after the onset of the Tequila Crisis, and 2009Q1-Q2 during the GFC. In each of these three episodes, the economy is in the binding regime before and after the event. In particular, the 1983 peak crisis was preceded by several quarters of near crisis, the Tequila Crisis had a peak in the middle of a relatively longer period of near crisis, while the GFC materialized during a short-lived near crisis period. As Figure 1 shows, the identified peak crises periods corresponds to the trough in output, consumption, investment growth in the data, and the peaks in the current account adjustment and interest rate increases.

Figure 5 also reports a purely empirical definition of financial crisis (dark grey shaded areas in Panel a) and the OECD date of the business cycle of Mexico (light grey shaded areas in Panel b). The empirical notion of financial crisis reported is a normalized version of the *crisis tally* index of Reinhart and Rogoff (2009) (RR).[23] Figure 5 illustrates that our estimated probability of being in a binding regime, which is our model-consistent definition of sudden stop crisis, align quite well with the RR index. Figure 5 also illustrates that our definition of near crisis encompasses and captures the notion of sudden stop typically used in the extant quantitative literature, adding the two criterion on the magnitude of the current account reversal and output drop to the criterion on the smoothed probability. The near crises episodes that our model identifies, i.e., the areas with 90 % probability of being in the constrained regime, track the RR tally index remarkably well around all three episodes identified, even though the crisis signals are less persistent than the tally index.[24] The peak crisis episodes identified by our model are close to highest values of the tally index, even though they don't coincide exactly, especially in the case of the Debt Crisis.

Importantly, our model estimates of these sudden stop crises do not mistake ordinary recessions not associated with spikes in the tally index for near or peak crisis periods. Mexico OECD recessions are illustrated by the light dark shaded areas in Figure 5 Panel (b). The estimated probability of a binding regime is close to 0 during the OECD recessions before the Tequila crisis, during the US recession in 2001, and the Argentine crisis in 2000-2001. The estimated probability of a binding regime does not register stress during the 1998 Russian default and US Long-Term Capital Management debacle that affected only the currency and stock market, without triggering a sudden stop in Mexico.

Overall, Figure 5 shows that our model provides an accurate signal of when the economy is likely to have experienced an actual crisis state (a peak crisis), or a period of fragility and vulnerability (a near crisis), based on external empirical evidence, without mistaking regular recessions or large currency and stock market movements

[23] The RR tally index ranges from 0 to 6, depending on whether a country-year observation is deemed to be in one or more of the following 6 varieties of crisis, assigning the value of one if a variety is present: Currency, Inflation, Stock Market, Sovereign Domestic or External Debt, and Banking Crisis. We follow their methodology to extend the index to cover our full sample. In Figure 5, the index is normalized to range between 0 and 1. See Chapter 1 of Reinhart and Rogoff (2009) for more details.

[24] The model has a harder time tracking the very prolonged crisis period signalled by the tally index after the acute phase of the Debt crisis, and to a lesser extent also the post-Tequila Crisis period. This is to be expected, however, as our model economy is not designed to capture debt overhang or financial intermediation disruptions that drive the classification of RR between 1983 and 1989 and in the mid-1990s. Nonetheless, the model does very well at tracking the consequences of the GFC.

for financial crisis episodes. In the rest of this section, we will open up the model black box and look at both simulated model dynamics around crises episodes and drivers of sudden stop episodes.

Note here, before proceeding, that when we examine the model-simulated crisis properties, as the household-firm making decisions in the model, we know in which regime the economy is without sampling uncertainty. Therefore, when we simulate the model, we do not need to express the binding regime state as a probabilistic statement, as we did from the econometrician's perspective in Figure 5. Near and peak crises in the model simulation, therefore, are going to be redefined as follow: a near crisis period is a quarter in which the economy is in the binding regime; a peak crisis period, instead, is defined as a quarter in which (i) the economy is in the binding regime (ii) output growth is negative by more than one standard deviation, and (iii) the current account to output ratio reverts by more than one standard deviation. Note also that, all simulations in this section are based on 10,0000 sample paths of 144 quarters as in our data sample, after a burn in period of 1,000 quarters.

When we simulate the model from the posterior mode, we find that it generates peak crisis episodes of average duration of one quarter, consistent with the duration of the episodes identified in Mexico data in Figure 5. In contrast, episodes of consecutive near crisis periods have a varying length, ranging from 2-3 quarters to sequences lasting longer than the two-years episodes estimated in the historical cases of Mexico. So we now look at model dynamics around peak crisis episodes and near crisis episodes lasting 8 quarters, like in the case of the Tequila crisis in Figure 5.

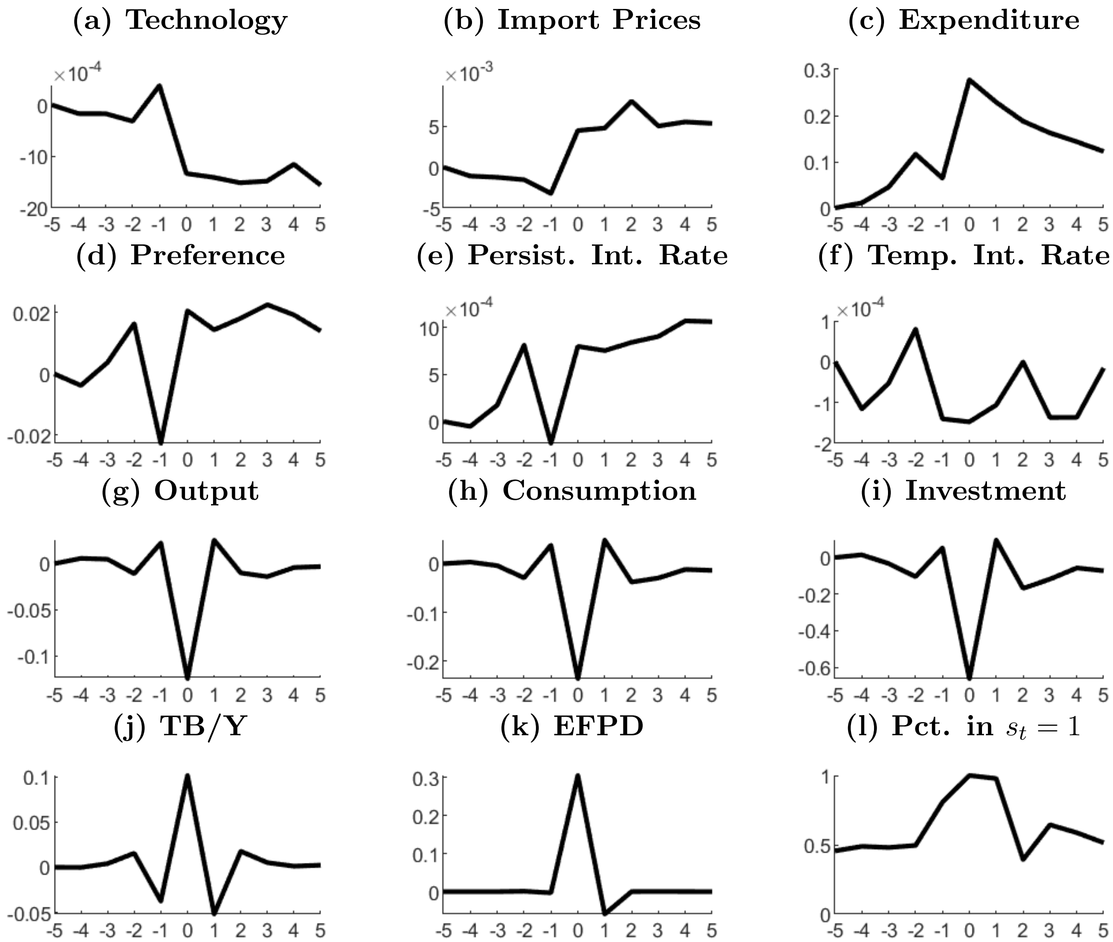
6.2 Simulated Peak Crisis Dynamics

Next, we look at the the dynamics of peak crises episodes through the lens of the model. Figure 6 plots the median model-implied dynamics centered around peak crisis as defined above ($t = 0$) for selected variables.[25]

The figure shows that the typical peak crisis episode is precipitated by a sharp decline in productivity, an increase in the cost of intermediate inputs, combined with an increase in the autonomous component of expenditure the persistent component

[25] We do not compare crisis dynamics to Mexican episodes in the data as we have already discussed model fit, showing that that the estimated model tracks the data well during both cycles and crisis periods alike–see Figure 4. Moreover, as we discuss below and highlight in Table 5, each of Mexico's crisis episodes exhibited distinct dynamics.

Figure 6: Dynamics of Peak Crisis Episodes
(Log-level. $t-5=0$)



Notes: Model-implied peak crisis dynamics. See text for definition of crisis. Crises occur at $t=0$. Plotted dynamics in panels (a)-(k) are medians across all simulated crises episodes, in log-deviations from value at $t=-5$; panel (l) shows percent of simulations in the binding regime each period.

of the country interest rate, which also rises abruptly in the quarter preceding the event. During a peak crisis, output, consumption and investment collapse, and the trade balance and the current account (not reported) revert, swinging abruptly from a deficit to a surplus. Wages and the relative price of capital (also not reported) plummet. The EFPD premium spikes, driven by the positive value of the borrowing constraint multiplier (not reported).

Looking at the run up to and the recovery from peak crisis episodes, 5 quarter before and 5 quarters after the event, we can see that the economy is in the biding regime about 50 percent of the time (panel I). The share of sample draws with the economy in a binding regime also before the peak crisis episodes increases sharply before $t = 0$, and persists for one quarters after the episode, hovering around 60 percent throughout the recovery phase. In the year before the crisis, exogenous processes are also moving together in a distinct manner: productivity is on a downward trend and both components of the country interest rate are increasing. At the same time, the price of intermediate inputs is falling, expenditure is increasing (arguably consistent with a fiscal expansion), while impatience declines as evidenced by a negative realization of the preference process. This cocktail of shocks materializes while the borrowing cushion is already been exhausted (not reported), curtailing consumption smoothing possibilities constraining supply, and keeping the share of samples with pre-crisis periods in the binding regime around 50%.

After the peak crisis, the economy initially rebounds quickly, but the economic environment remains precarious. The sequence of bad economic shocks persists after the event: productivity remains depressed at a lower level than before the crisis peak; the price of imports remains elevated; relative impatience continues; the persistent component of the interest rate increases further; and only expenditure falls gradually after the crisis peak. In the face these shocks, output, consumption, and investment overshoots initially, but then settles at a slightly lower level than before the crisis peak.

An important take-away from Figure 6 is that the typical peak crisis event that the model generates is characterized by shocks that comove in a distinctively averse manner, before, during and after the model identified peak sudden stop episode. This result is in line with the insight from calibrated models of the business cycles in emerging markets that can match the second moments of the data better when the productivity shock is assumed to be negatively correlated with the interest rate shocks.

6.3 Simulated Near Crisis Dynamics

We now turn to the simulated episodes of near crisis as those identified in Figure 5. Figure 7 shows the model dynamics associated with near sudden stop episodes, as well as 5 years (20 quarters) before the beginning of the episode, and 10 years (40 quarters) after the end of the event, setting the initial value of all variables to zero at time -20. Given that that the historical episodes of near crisis identified in Figure 5 in the early 1980s and mid-1990s lasted about 8 quarters, we report results for episodes that meet the near crisis definition, i.e., the economy is in the binding regime, for at least 8 consecutive quarters.[26]

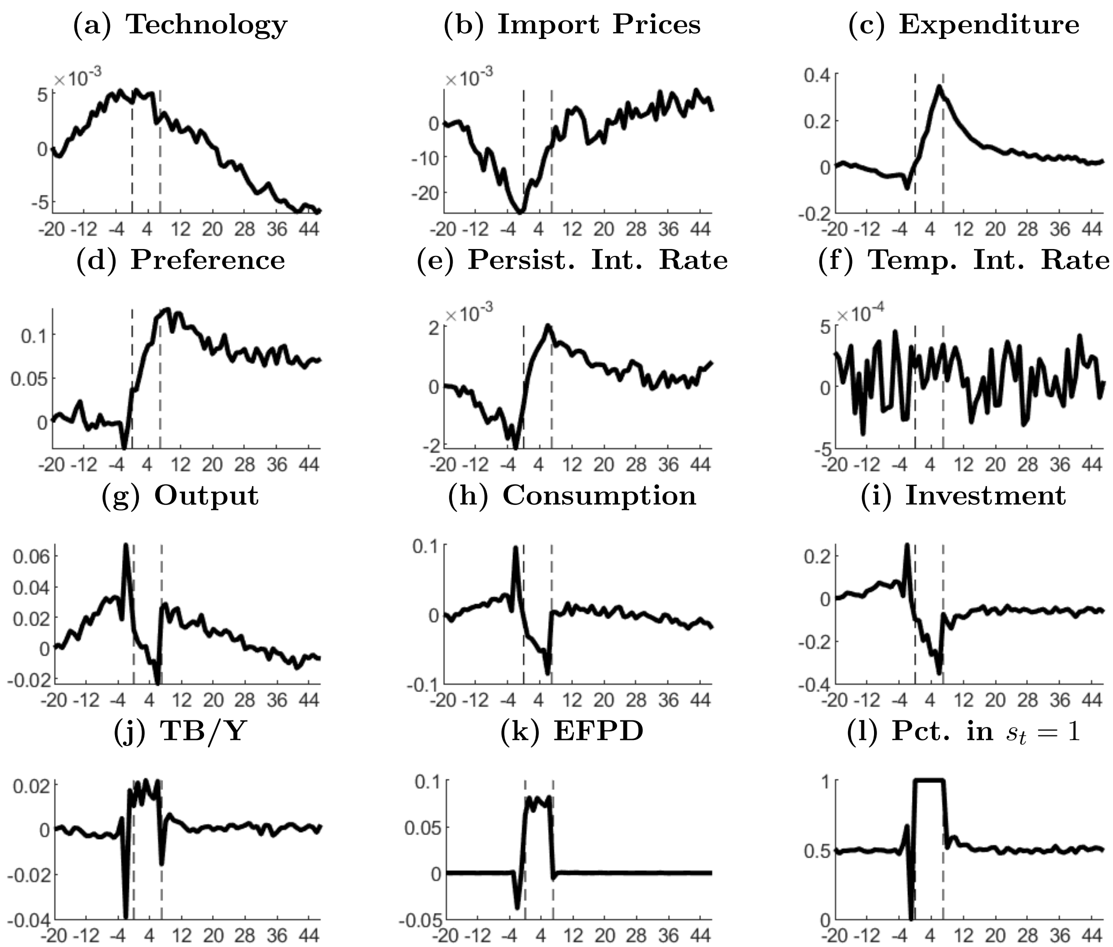
As in the case of peak crisis dynamics, Figure 7 shows that distinctive cocktails of shocks drives the economy before, during and after the event. In the model, near crises episodes are preceded by a "boom" phase cauterized by accelerating productivity, declining import prices and the persistent component of the interest rate, and mildly declining patience. Only the expenditure shock does not seem to be contributing significantly to the boom, up to the period immediately before the beginning of the crisis episode. These external forces gradually stimulate the economy, with increasing output, consumption, and investment. Notably, as in the case of the peak crises, the economy is already in the binding regime in about a fifty percent of the episodes before the beginning of the event.

The economy enters the binding regime at $t = 0$, after a sudden acceleration, with the sudden stop episode starting right before the inception of the crisis. The economy then remains in the near crisis state at least through period $t = 7$ by definition. During the near crisis episode, shocks to productivity, import prices, and the persistent component of the interest rate process revert. However, expenditure continues to increase sharply, despite the changed external environment. The constraint on borrowing limits consumption smoothing and further curtail output supply through the working capital constraint, causing output, consumption, and investment to drop sharply. The trade-balance-to-output ratio, which during the boom phase was close to zero, improves persistently after a sharp deterioration right before the beginning of the crisis, and remains in surplus throughout the rest of the episode. The external finance premium on debt (EFPD) also is elevated throughout the episode.

Even after the economy exits the near crisis event, economic activity continues to decline relative to the initial value, despite a persistent contraction in the exogenous component of expenditure, a falling interest rate, and relatively stable intermediate input prices. Reflecting the persistence of the technology shock, productivity declines throughout the post-crisis period. Reflecting the downward trend in productivity and the persistently counteractive expenditure impulse, output, investment, and to a

[26] As we mentioned earlier near crisis episodes lasting at least 8 quarters are a very small fraction of all binding episodes of at least 2 quarters. However, about 20 percent of the simulated samples contain at least one such episode.

Figure 7: Dynamics of Near Crisis Episodes
(Log-level, $t - 5 = 0$)



Notes: Model-implied near crisis dynamics. Binding regime begins at $t = 0$ until at least $t = 7$. Plotted dynamics in panels (a)-(k) are medians across all crises episodes, in log-levels wit $t - 20 = 0$; panel (l) plots the percent of simulated paths in the binding regime at each period t.

lesser extent consumption decline, and are is still below their initial value at $t = -20$ 10 years after the end of the crisis, even though the economy is no longer in the binding regime.

One important takeaway from Figure 7, therefore, is that our proposed specification of the occasionally binding collateral constraint can generate long lasting boom-bust dynamics. Moreover, several features of these dynamics appear broadly in line with empirical analyses of the long term consequences of financial crisis in Mexico and other emerging markets (Cerra and Saxena, 2008).

6.4 Drivers of Mexico's Crisis History

As as we argued earlier, our estimated model fits well Mexico data (Figure 1, including the peak and near crisis episodes identified in Figure 5. In this last section, therefore, want to examine those historical episodes through the lens of the model, evaluating the relative importance of different shocks driving the economy before, during and after the following peak and near crises: the Debt Crisis of the early 1980s, the Tequila Vrisis of 1994-1995, and the spillover on Mexico from the Global Financial Crisis that originated in the United States in 2008-09.

To do so, we counterfactually recalculate the model likelihood, evaluated at the posterior mode, turning one shock off at the time, while leaving all other shocks at their estimated values, over particular sub-sample periods. We saw earlier that, in the full sample, all shocks play a role in the model. Here, we compute the loss of model fit as measured by the percent log-likelihood change when we turn one particular shock in a given period off compare this statistic across shocks and time periods. We repeat the analysis for all six structural shocks in the model, the three historical episodes of peak crisis, the three episodes of near crisis, as well as two years before the beginning of the near crisis event, and two years after the near crisis event.[27]

Table 5 reports the results. Consider first the Debt Crisis. As the Debt Crisis happened right at the beginning of the sample period, we can only look at the two quarters before the beginning of this near crisis episode. The counterfactual analysis suggests that in the immediate run up to the Debt Crisis, the most important shocks were imported intermediate input prices and interest rate shocks (both temporary and persistent components). The near crisis episode appears driven by the expenditure shock, and to a lesser extent the technology shock. We saw earlier that our estimated model identifies the peak of the debt crisis in 1983:Q1, lasting only one quarter. The counterfactual results in Table 5 suggest that the preference shock and the technology shock were the most important drivers, but also interest rate shocks had a role. Technology and expenditure shocks drive the post-crisis period.

Next, consider the Tequila Crisis. Our estimated model identified a peak crisis lasting for two quarters, in 1995:Q1-Q2. According to our counterfactual results, during the period before the near crisis episode, the most important drivers were the

[27]Because the Debt Crisis and the spillover from the GFC are at the beginning and the end of the sample period, we cannot use the same time-window used in the analysis of the near crisis model generated dynamics.

Table 5: Anatomy of Mexico's Historical Crises

Time Period	TFP	Exp.	Imp. Prices	Pref	Trans Int Rt.	Persist Int Rt.
1983 Debt Crisis						
Two Quarters Prior (81Q1:Q2)	0.4	0.4	**0.7**	-3.2	**0.9**	**0.8**
Near Crisis (81:Q3-83:Q2)	**0.4**	**5.3**	-2.0	-2.8	0.0	-0.8
Peak Crisis (83:Q1)	**1.1**	-2.3	-0.8	**1.5**	**0.4**	**0.2**
Two-years After (83:Q3-85:Q2)	**0.8**	**1.0**	-0.6	0.2	-0.7	-0.7
1995 Tequila Crisis						
Two-years Prior (92:Q1-93:Q4)	-0.1	-1.0	**0.4**	**0.7**	0.1	-0.1
Near Crisis (94:Q1-96:Q1).	-2.2	-0.7	**0.5**	**1.3**	0.2	**0.9**
Peak Crisis (95:Q1-Q2).	**-0.7**	-0.8	-0.2	**1.1**	**0.1**	**0.4**
Two-years After (96:Q2-98Q1)	-0.1	-0.2	0.2	**1.1**	**-0.6**	**-0.4**
2009 Global Fin. Crisis						
Two-years Prior (06:Q4-08:Q3)	-0.7	**2.1**	-0.7	-0.2	-0.7	**0.2**
Near Crisis (08:Q4-09:Q3).	**0.2**	-1.2	**0.3**	**0.5**	**0.2**	0.0
Peak Crisis (09:Q1-Q2)	-0.4	-1.2	**1.2**	**0.3**	**0.2**	-0.1
Two-years After (09:Q4-11:Q3)	-0.4	-1.1	**0.4**	**0.8**	**0.1**	**0.1**

Note: The table shows the relative importance of each shocks during different periods, compared to their importance over the full sample, in percentage point differences. For example, a value of +1 indicates the shock had 1 percentage point greater relative importance than its importance over the full sample. Bold fonts highlight the most important shocks in each period according to this metric. See Appendix H for details. Prior period for 1983 Debt Crisis limited by sample length.

preference shock and the imported input price shock. The importance of these two shocks increases during the near crisis period, even though the shock to the persistent component of the interest rate also becomes more important. During the peak of the crisis, the importance of preference and interest rate shocks declines while the weight in the likelihood of the technology shocks increases. In the post crisis period, the preference shock continues to stand out, while the importance of shocks to both components of the interest rate decline.

Lastly, let us look at the GFC episode which is estimated to peak in 2009:Q1-Q2. The counterfactual likelihood analysis suggests that, before the crisis, expenditure and to a lesser extent a shock to the persistent component of the interest rate were the most important drivers. However, all other shocks become more important during the near crisis period. At the peak of the crisis, the imported price shock becomes more important, the preference shock's role declines, while the temporary interest rate shock continues to play role. In the aftermath of the GFC, the importance of the import price shock and temporary interest rate shock diminishes, while that of preference and persistent interest shocks increases.

7 Conclusions

In this paper we propose a new flexible and scaleable approach to specifying and solving Dynamic Stochastic General Equilibrium (DSGE) models with occasionally binding collateral constraints. We apply this new approach to the analysis of a particular type of crisis, the so-called sudden stop in capital flows that afflicted many emerging market economies, by estimating a medium-scale workhorse model with likelihood based methods.

The critical step in our approach is to specify the occasionally binding nature of the constraint stochastically so that the transition from the unconstrained to the constrained state of the world, and vice versa, can be mapped into an endogenous regime-switching model with transition probabilities depending on the state variables of the model and the collateral multiplier. The perturbation method that we develop to solve the endogenous regime-switching model is suitable for the application of standard non-linear Bayesian estimation procedures. This permits estimating the model using full information methods, allowing us to obtain estimates of critical model parameters and conduct likelihood-based inference and counterfactual experiments.

We apply the framework that we propose to the anatomy of Mexico's business cycle and crisis episodes since 1981, finding that the model fits the data well, critical parameter estimates differ from values previously used in the literature, that different shocks matter for different variables and phases of financial crisis dynamics. In particular, in the model simulations, we find that a specific cocktail of shocks typically drives the economy into the crisis rather than a coincidence of large shocks; this result helps explain why calibrated models of emerging market cycles perform better assuming that productivity and interest shocks are negatively correlated. Finally, we document that our estimated model identifies sudden stops that are longer lasting and more in line with narratives of Mexico's history of financial crises than those typically obtained with traditional inequality specifications of the collateral constraint.

We regard the estimation of larger models–including those with nominal or labor frictions and government intervention, those with permanent and temporary productivity shocks over longer sample periods, and those with financial intermediation or equilibrium default–as important areas of future research.

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Appendix A Model and Competitive Equilibrium Definition

This Appendix derives the model's equilibrium conditions and defines a competitive equilibrium.

A.1 Derivation of Equilibrium Conditions

The household-firm maximizes the utility function

$$U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{1}{1-\rho} \left(C_t - \frac{H_t^\omega}{\omega} \right)^{1-\rho} \right\}, \tag{A.1}$$

subject to

$$C_t + I_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t - \phi r_t \left(W_t H_t + P_t V_t \right) - E_t - \frac{1}{(1+r_t)} B_t + B_{t-1} \tag{A.2}$$

where gross investment follows

$$I_t = \delta K_{t-1} + (K_t - K_{t-1}) \left(1 + \frac{\iota}{2} \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right) \right) \tag{A.3}$$

When binding, the collateral constraint is given by

$$\frac{1}{(1+r_t)} B_t - \phi \left(1 + r_t \right) \left(W_t H_t + P_t V_t \right) = -\kappa q_t K_t \tag{A.4}$$

The first-order conditions of this problem are the following:

$$d_t \left(C_t - \frac{H_t^\omega}{\omega} \right)^{-\rho} = \mu_t \tag{A.5}$$

$$\left(1 - \alpha - \eta \right) A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{-\alpha-\eta} = P_t \left(1 + \phi r_t + \frac{\lambda_t}{\mu_t} \phi \left(1 + r_t \right) \right) \tag{A.6}$$

$$\alpha A_t K_{t-1}^{\eta} H_t^{\alpha-1} V_t^{1-\alpha-\eta} = \phi W_t \left(r_t + \frac{\lambda_t}{\mu_t} \left(1 + r_t \right) \right) + H_t^{\omega-1} \tag{A.7}$$

$$\mu_t = \lambda_t + \beta \left(1 + r_t\right) \mathbb{E}_t \mu_{t+1} \tag{A.8}$$

$$\mathbb{E}_t \mu_{t+1} \beta \begin{pmatrix} 1 - \delta + \left(\frac{\iota}{2} \left(\frac{K_{t+1}}{K_t} \right)^2 - \frac{\iota}{2} \right) \\ + \eta A_{t+1} K_t^{\eta - 1} H_{t+1}^{\alpha} V_{t+1}^{1 - \eta - \alpha} \end{pmatrix} = \mu_t \left(1 - \iota + \iota \left(\frac{K_t}{K_{t-1}} \right) \right) - \lambda_t \kappa q_t \tag{A.9}$$

Market optimal prices for capital and labor are

$$q_t = 1 + \iota \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right) \tag{A.10}$$

$$W_t = H_t^{\omega - 1} \tag{A.11}$$

The borrowing cushion is given by the amount of borrowing over the debt limit

$$B_t^* = \frac{1}{\left(1 + r_t\right)} B_t - \phi \left(1 + r_t\right) \left(W_t H_t + P_t V_t\right) + \kappa q_t K_t \tag{A.12}$$

and the regime-switching slackness condition is given by

$$\varphi \left(s_t\right) B_{ss}^* + \nu \left(s_t\right) \left(B_t^* - B_{ss}^*\right) = \left(1 - \varphi \left(s_t\right)\right) \lambda_{ss} + \left(1 - \nu \left(s_t\right)\right) \left(\lambda_t - \lambda_{ss}\right) \tag{A.13}$$

where λ_t is the multiplier on the international borrowing constraint. The interest rate has a debt elastic component

$$r_t = r_t^* + \psi_r \left(e^{\bar{B} - B_t} - 1 \right) + \sigma_r \varepsilon_{r,t} \tag{A.14}$$

The exogenous processes are given by

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t} \tag{A.15}$$

$$\log E_t = \left(1 - \rho_E\right) \log E^* + \rho_E \log E_{t-1} + \sigma_E \varepsilon_{E,t} \tag{A.16}$$

$$\log P_t = \left(1 - \rho_P\right) \log P^* + \rho_P \log P_{t-1} + \sigma_P \varepsilon_{P,t} \tag{A.17}$$

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t} \tag{A.18}$$

$$r_t^* = (1 - \rho_{r^*})\, \bar{r}^* + \rho_{r^*} r_{t-1}^* + \sigma_{r^*} \varepsilon_{r^*,t} \tag{A.19}$$

A.2 Equilibrium Conditions

A competitive equilibrium of our economy is a sequence of quantities $\{K_t,\ B_t,\ C_t,\ H_t,\ V_t,\ I_t,\ A_t,\ E_t,\ B_t^*\}$ and prices $\{P_t,\ r_t^*,\ r_t,\ q_t,\ w_t,\ \mu_t,\ \lambda_t\}$ that, given the 5 exogenous processes (A.15)-(A.19), satisfy the first-order conditions for the representative household-firm (A.5)-(A.9), the market price equations (A.10)-(A.11), the market clearing conditions (A.2)-(A.3), the debt cushion definition (A.12), regime-switching slackness condition (A.13), and an equation for the interest rate (A.14).

In the paper, we also simulate a number of auxiliary variables as

$$\begin{aligned}
&\text{GDP:} && Y_t = A_t K_{t-1}^{\eta} H_t^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t && \text{(A.20)}\\
&\text{Debt-to-GDP Ratio:} && \Phi_t^{b} = \frac{B_t}{Y_t} && \text{(A.21)}\\
&\text{Current Account-to-GDP Ratio:} && \Phi_t^{ca} = \frac{B_t - B_{t-1}}{Y_t} && \text{(A.22)}\\
&\text{Trade Balance-to-GDP Ratio:} && \Phi_t^{tb} = \frac{Y_t - E_t - C_t - I_t}{Y_t} && \text{(A.23)}\\
&\text{External Fin Premium on Debt:} && EFPD_t = \frac{\lambda_t}{\beta \mathbb{E}_t \mu_{t+1}}. && \text{(A.24)}
\end{aligned}$$

Appendix B Details of the Perturbation Solution Method

This Appendix provides details about two aspects of the solution method: (1) the definition of, and solution for, the steady state of the endogenous regime-switching economy; and (2) the perturbation method that generates second order Taylor expansions to the solution of the economy around the steady state.

B.1 Regime Switching Equilibrium

The 23 equilibrium conditions are written as

$$\mathbb{E}_t f\left(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \mathbf{x}_{t-1}, \chi\varepsilon_{t+1}, \varepsilon_t, \theta_{t+1}, \theta_t\right) = 0. \tag{B.1}$$

We have 7 predetermined variables

$$\mathbf{x}_{t-1} = \left[K_{t-1}, B_{t-1}, A_{t-1}, P_{t-1}, E_{t-1}, d_{t-1}, r^*_{t-1}\right] \tag{B.2}$$

and 16 non-predetermined variables

$$\mathbf{y}_t = \left[C_t, H_t, V_t, I_t, k_t, r_t, q_t, W_t, \mu_t, \lambda_t, B^*_t, Y_t, \Phi^b_t, \Phi^{ca}_t, \Phi^{tb}_t, EFPD_t\right] \tag{B.3}$$

with 6 exogenous shocks

$$\varepsilon_t = \left[\varepsilon_{A,t}, \varepsilon_{E,t}, \varepsilon_{P,t}, \varepsilon_{d,t}, \varepsilon_{r,t}, \varepsilon_{r^*,t}\right] \tag{B.4}$$

and 2 switching variables

$$\theta_t = \left[\varphi\left(s_t\right), \nu\left(s_t\right)\right]. \tag{B.5}$$

These variables are partitioned into some that affect the steady state, $\theta_{1,t}$, and some that do not, $\theta_{2,t}$. The partition in this case is

$$\theta_{1,t} = \left[\varphi\left(s_t\right)\right] \qquad \theta_{2,t} = \left[\nu\left(s_t\right)\right] \tag{B.6}$$

For solving the model, the functional forms are

$$\theta_{1,t+1} = \bar{\theta}_1 + \chi\hat{\theta}_1\left(s_{t+1}\right), \qquad \theta_{1,t} = \bar{\theta}_1 + \chi\hat{\theta}_1\left(s_t\right) \tag{B.7}$$

$$\theta_{2,t+1} = \theta_2\left(s_{t+1}\right), \qquad \theta_{2,t} = \theta_2\left(s_t\right) \tag{B.8}$$

$$\mathbf{x}_t = h_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) \tag{B.9}$$

$$\mathbf{y}_t = g_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \qquad \mathbf{y}_{t+1} = g_{s_{t+1}}\left(\mathbf{x}_t, \chi\varepsilon_{t+1}, \chi\right) \tag{B.10}$$

and

$$\mathbb{P}_{s_t,s_{t+1},t} = \pi_{s_t,s_{t+1}}\left(\mathbf{y}_t\right). \tag{B.11}$$

Using these in the equilibrium conditions and being more explicit about the expectation operator, given $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$ and s_t, then

$$F_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) = \int \sum_{s'=0}^{1} \pi_{s_t,s'}\left(g_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right)\right) f \begin{pmatrix} g_{s_{t+1}}\left(h_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \chi\varepsilon', \chi\right), \\ g_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \\ h_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right), \\ \mathbf{x}_{t-1}, \chi\varepsilon', \varepsilon_t, \\ \bar{\theta} + \chi\hat{\theta}\left(s'\right), \bar{\theta} + \chi\hat{\theta}\left(s_t\right) \end{pmatrix} d\mu\varepsilon' \tag{B.12}$$

Stacking these conditions for each regime produces

$$\mathbb{F}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) = \begin{bmatrix} F_{s_t=0}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) \\ F_{s_t=1}\left(\mathbf{x}_{t-1}, \varepsilon_t, \chi\right) \end{bmatrix} = 0 \tag{B.13}$$

B.2 Steady State Definition and Solution

The model has two features that make it challenging to define a steady state. First, as is common in a regime-switching framework, some auxiliary or structural parameters may be switching. In the case of our application, there is only one auxiliary switching parameter that affects the steady state, $\varphi(s_t)$. Nonetheless, in principle, one could allow for regime switching also for the parameters of the exogenous processes, $a^*(s_t)$ and $p^*(s_t)$, or the structural parameter $\kappa^*(s_t)$, which affect the level of the economy directly, and will thus matter for steady state calculations.[28] Solution methods such as those proposed by Foerster et al. (2016) define the steady state by using the ergodic means of these parameters across regimes. We set $\varepsilon_t = 0$ and $\chi = 0$, which implies a steady state given by

$$f\left(\mathbf{y}_{ss}, \mathbf{y}_{ss}, \mathbf{x}_{ss}, \mathbf{x}_{ss}, 0, 0, \bar{\theta}_{\mathbf{1}}, \theta_2(s'), \bar{\theta}_1, \theta_2(s)\right) = 0 \tag{B.14}$$

for all s', s.

[28]As is well known, over finite periods of time, it is statistically difficult to distinguish between unit root processes and processes with structural break or regime changes. Allowing for regime changes in the process for A_t, therefore, would be a way to accommodate permanent productivity shocks as in Aguiar and Gopinath (2007).

In our case, the transition matrix P is endogenous, making the use of the ergodic distribution problematic, since it depends on economic variables that in turn depend on the ergodic means. The steady state solution method that we propose proceeds in two steps. In the first step, we assume the steady state transition matrix is known and solve for the steady state prices and quantities. In the second step, we use the steady state values of the borrowing cushion B^*_{ss} and multiplier λ_{ss} from Step 1 to update the steady state transition matrix. We iterate these two steps until convergence.

Step 1: Solve by Using a Steady State Transition Matrix. First, assume that the steady state transition matrix at iteration i, $P^{(i)}_{ss}$, is known. Next, let $\xi = [\xi_0, \xi_1]$ denote the ergodic vector of $P^{(i)}_{ss}$. Then, as explained in the paper, define the ergodic means of the switching parameters as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1).$$

The steady state of the economy depends on these ergodic means, we can partially solve for some of the steady state directly

$$A_{ss} = 1, d_{ss} = 1, E_{ss} = E^*, P_{ss} = P^*, q_{ss} = 1, r^*_{ss} = \bar{r}^* \tag{B.15}$$

Suppose we know r_{ss}. Then

$$\Omega_v \equiv \frac{A_{ss} K^\eta_{ss} H^\alpha_{ss} V^{1-\alpha-\eta}_{ss}}{P_{ss} V_{ss}} = \frac{1 + \phi r_{ss} + \phi (1 + r_{ss}) (1 - \beta (1 + r_{ss}))}{1 - \alpha - \eta} \tag{B.16}$$

$$\Omega_h \equiv \frac{A_{ss} K^\eta_{ss} H^\alpha_{ss} V^{1-\alpha-\eta}_{ss}}{W_{ss} H_{ss}} = \frac{1 + \phi (r_{ss} + (1 + r_{ss}) (1 - \beta (1 + r_{ss})))}{\alpha} \tag{B.17}$$

$$\Omega_k \equiv \frac{A_{ss} K^\eta_{ss} H^\alpha_{ss} V^{1-\alpha-\eta}_{ss}}{K_{ss}} = \frac{1}{\eta} \left(\frac{1 - \kappa (1 - \beta (1 + r_{ss}))}{\beta} - 1 + \delta \right) \tag{B.18}$$

$$H_{ss} = \left(\frac{A_{ss}}{\Omega^\eta_k \Omega^\alpha_h (P_{ss} \Omega_v)^{1-\alpha-\eta}} \right)^{\frac{1}{\alpha(\omega-1)}} \tag{B.19}$$

$$V_{ss} \equiv \frac{\Omega_h}{P_{ss} \Omega_v} H^\omega_{ss} \tag{B.20}$$

$$K_{ss} = \frac{\Omega_h}{\Omega_k} H_{ss}^{\omega} \tag{B.21}$$

$$Y_{ss} = \Omega_h H_{ss}^{\omega} - P_{ss} V_{ss} \tag{B.22}$$

$$W_{ss} = H_{ss}^{\omega-1} \tag{B.23}$$

$$I_{ss} = \delta K_{ss} \tag{B.24}$$

$$k_{ss} = K_{ss} \tag{B.25}$$

$$B_{ss} = \bar{B} - \log\left(1 + \frac{r_{ss} - r^*}{\psi_r}\right) \tag{B.26}$$

$$C_{ss} = Y_{ss} - \phi r_{ss}\left(W_{ss} H_{ss} + P_{ss} V_{ss}\right) - E_{ss} + B_{ss}\left(1 - \frac{1}{(1 + r_{ss})}\right) - I_{ss} \tag{B.27}$$

$$\mu_{ss} = \left(C_{ss} - \frac{H_{ss}^{\omega}}{\omega}\right)^{-\rho} \tag{B.28}$$

$$\lambda_{ss} = \left(1 - \beta\left(1 + r_{ss}\right)\right)\mu_{ss} \tag{B.29}$$

$$B_{ss}^* = \frac{1}{(1 + r_{ss})} B_{ss} - \phi\left(1 + r_{ss}\right)\left(W_{ss} H_{ss} + P_{ss} V_{ss}\right) + \kappa K_{ss} \tag{B.30}$$

$$\Phi_{ss}^{b} = \frac{B_{ss}}{Y_{ss}} \tag{B.31}$$

$$\Phi_{ss}^{ca} = 0 \tag{B.32}$$

$$\Phi_{ss}^{tb} = \frac{Y_{ss} - E_{ss} - C_{ss} - I_{ss}}{Y_{ss}} \tag{B.33}$$

$$EFPD_{ss} = \frac{\lambda_{ss}}{\beta \mu_{ss}} \tag{B.34}$$

and then r_{ss} solves

$$\bar{\varphi} B^*_{ss} = (1 - \bar{\varphi}) \lambda_{ss} \tag{B.35}$$

Step 2: Update the Transition Matrix. Step 1 yields the variables B^*_{ss} and λ_{ss}, and hence have a new value of the transition matrix for iteration $i+1$:

$$P^{(i+1)}_{ss} = \begin{bmatrix} p_{00,ss} & p_{01,ss} \\ p_{10,ss} & p_{11,ss} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} & \frac{\exp(-\gamma_0 B^*_{ss})}{1+\exp(-\gamma_0 B^*_{ss})} \\ \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} & 1 - \frac{\exp(-\gamma_1 \lambda_{ss})}{1+\exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}, \tag{B.36}$$

which can be checked against the guess in Step 1. Continue this iterative procedure until

$$\left\| P^{(i+1)}_{ss} - P^{(i)}_{ss} \right\| < tolerance,$$

where we pick a tolerance of 10^{-10}.

B.3 Generating Approximations

To compute a second order approximation to the endogenous regime-switching model solution, we largely follow Foerster et al. (2016), but adapted to the case with endogenous probabilities.

We take the stacked equilibrium conditions $\mathbb{F}(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$, and differentiate with respect to $(\mathbf{x}_{t-1}, \varepsilon_t, \chi)$. In general regime-switching models, the first-order derivative with respect to $\mathbf{x}_{t-1}$ produces a complicated polynomial system denoted

$$\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0. \tag{B.37}$$

Often this system needs to be solved via Gröbner bases, which finds all possible solutions in order to check them for stability. In our case, with endogenous probabilities, the standard stability checks fail, so we will focus on finding a single solution and ignore the possibility of indeterminacy, a common simplification in the regime-switching literature with and without endogenous switching (e.g. Farmer et al., 2011; Foerster, 2015; Maih, 2015; Lind, 2014). In the literature that computes global solutions to non-regime switching occasionally binding constraint models (e.g. Benigno et al. (2013), Mendoza (2010)) there are no proofs of uniqueness, and the focus typically is also on computing a solution checking robustness to initial conditions. To find a solution to our model, we guess at a set of policy functions for regime $s_t = 1$, which collapses the equilibrium conditions $\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0; s_t = 0)$ into a fixed-regime eigenvalue problem, and solve for the policy functions for $s_t = 0$. Then, using this initial solution as guess, we solve for regime $s_t = 0$ under the fixed-regime eigenvalue problem, and iterate on this procedure to convergence. After solving the iterative eigenvalue problems, the other systems to solve are

$$\mathbb{F}_{\varepsilon}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0 \tag{B.38}$$

$$\mathbb{F}_{\chi}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0 \tag{B.39}$$

and second order systems of the form (can apply equality of cross-partials)

$$\mathbb{F}_{\mathbf{i},\mathbf{j}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0,\ \mathbf{i}, \mathbf{j} \in \{\mathbf{x}, \varepsilon, \chi\}. \tag{B.40}$$

Recall the decision rules have the form

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \tag{B.41}$$

$$\mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t, \chi) \tag{B.42}$$

and so the second-order approximation takes the form

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} (S_t \otimes S_t) \tag{B.43}$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} (S_t \otimes S_t) \tag{B.44}$$

where $S_t = \begin{bmatrix} (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' & \varepsilon_t' & 1 \end{bmatrix}'$.

B.4 Proposition 1: Irrelevance of Endogenous Switching in First-Order Solution

To show Proposition 1, we can take the first-order derivatives of (B.13) with respect to its arguments, evaluated at steady state. This differentiation produces

$$\mathbb{F}_{\mathbf{x},s_t}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=\begin{array}{c}\sum_{s'}\pi_{s_t,s',y}\left(\mathbf{y}_{ss}\right)g_{\mathbf{x},s_t}f_{ss}\left(s',s_t\right)\\ +\sum_{s'}\pi_{s_t,s'}\left(\mathbf{y}_{ss}\right)\left[\begin{array}{c}f_{\mathbf{y}_{t+1}}\left(s',s_t\right)g_{\mathbf{x},s'}h_{\mathbf{x},s_t}+f_{\mathbf{y}_t}\left(s',s_t\right)g_{\mathbf{x},s_t}\\ +f_{\mathbf{x}_t}\left(s',s_t\right)h_{\mathbf{x},s_t}+f_{\mathbf{x}_{t-1}}\left(s',s_t\right)\end{array}\right]\end{array}\tag{B.45}$$

$$\mathbb{F}_{\varepsilon,s_t}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=\begin{array}{c}\sum_{s'}\pi_{s_t,s',y}\left(\mathbf{y}_{ss}\right)g_{\varepsilon,s_t}f_{ss}\left(s',s_t\right)\\ +\sum_{s'}\pi_{s_t,s'}\left(\mathbf{y}_{ss}\right)\left[\begin{array}{c}f_{\mathbf{y}_{t+1}}\left(s',s_t\right)g_{\mathbf{x},s'}h_{\varepsilon,s_t}+f_{\mathbf{y}_t}\left(s',s_t\right)g_{\varepsilon,s_t}\\ +f_{\mathbf{x}_t}\left(s',s_t\right)h_{\varepsilon,s_t}+f_{\varepsilon_t}\left(s',s_t\right)\end{array}\right]\end{array}\tag{B.46}$$

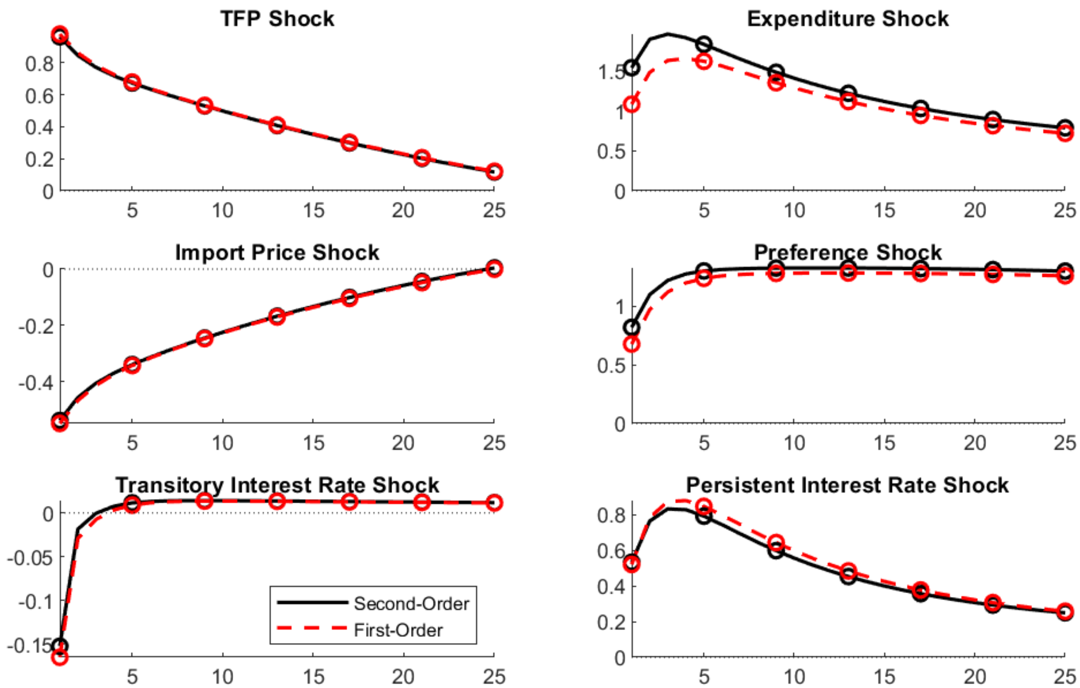
and

$$\mathbb{F}_{\chi,s_t}\left(\mathbf{x}_{ss},\mathbf{0},0\right)=\begin{array}{c}\sum_{s'}\pi_{s_t,s',y}\left(\mathbf{y}_{ss}\right)g_{\chi,s_t}f_{ss}\left(s',s_t\right)\\ +\sum_{s'}\pi_{s_t,s'}\left(\mathbf{y}_{ss}\right)\left[\begin{array}{c}f_{\mathbf{y}_{t+1}}\left(s',s_t\right)g_{\mathbf{x},s'}h_{\chi,s_t}+f_{\mathbf{y}_t}\left(s',s_t\right)g_{\chi,s_t}\\ +f_{\mathbf{x}_t}\left(s',s_t\right)h_{\chi,s_t}\\ +f_{\theta_{t+1}}\left(s',s_t\right)\hat{\theta}\left(s_{t+1}\right)+f_{\theta_t}\left(s',s_t\right)\hat{\theta}\left(s_t\right)\end{array}\right]\end{array}.\tag{B.47}$$

Note, by the definition of a steady state, $f_{ss}\left(s',s_t\right)=0$, and so the first term of each of these expressions equals zero. Hence, we are left with the formulas for a constant probability model from Foerster et al. (2016), with the probabilities given by $\mathbb{P}_{ss}=\pi_{s_t,s'}\left(\mathbf{y}_{ss}\right)$ as stated in the Proposition.

Figure B.1 highlights the implications of the Proposition by showing how the order of approximation matters for the response of one variable, the debt-to-output ratio, to different shocks. The figure shows each of the six shocks we consider, and conditions on remaining in the non-binding regime. While all the shocks have differences between the first- and second-order impulse responses, the magnitude of that difference depends on the shock.

Figure B.1: Comparison of Impulse Responses of Debt-to-Output Ratio in the Non-Binding Regime



Note: Shows impulse responses to a one-standard deviation shock for first- and second-order approximations, conditional on starting and staying in the non-binding regime. Units are log-deviations from the pre-shock period, defined as the regime-specific ergodic mean.

Appendix C Accuracy of the Solution

We assess accuracy by checking the Euler equation error, where

$$EEE_t = 1 - \frac{\lambda_t}{\mu_t} - \beta\left(1 + r_t\right)\mathbb{E}_t \frac{\mu_{t+1}}{\mu_t} \tag{C.1}$$

and the policy functions can be denoted by

$$\lambda_t = \lambda_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right), \quad \mu_t = \mu_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right), \quad r_t = r_{s_t}\left(\mathbf{x}_{t-1}, \varepsilon_t\right), \quad B_t^* = B_{s_t}^*\left(\mathbf{x}_{t-1}, \varepsilon_t\right) \tag{C.2}$$

Table C.1: Comparing Solution Accuracy: Global vs. Perturbation Methods

Comparison	Inequality Constraint	Endo Switch Global	Endo Switch Perturb.
Standard Deviations			
GDP	2.38	2.46	2.28
Hours	1.36	1.46	1.33
Autocorrelations			
GDP	0.94	0.94	0.94
Hours	0.77	0.76	0.77
Euler Eqn Errors ($\log_{10}$)	-10.47	-3.59	-3.41

and so, given $(\mathbf{x}_{t-1}, \varepsilon_t)$

$$EEE_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t) = 1 - \frac{\lambda_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t)}{\mu_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t)} - \frac{\beta(1 + r_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t))}{\mu_{s_t}(\mathbf{x}_{t-1}, \varepsilon_t)} \tag{C.3}$$

$$\times \sum_{s_{t+1}=0}^{1} p_{s_{t,t+1}}(\mathbf{x}_{t-1}, \varepsilon_t) \int_{\mathbb{R}^{\varepsilon}} \mu_{s_{t+1}}(\mathbf{x}_t, \varepsilon_{t+1}) \mu(\varepsilon_{t+1}) d\varepsilon_{t+1}. \tag{C.4}$$

We simulate the model for 10,000 periods, after a 1,000 period burn-in to get sequences of s_t and $(\mathbf{x}_{t-1}, \varepsilon_t)$. For each period in the simulation, we draw 10,000 values of ε_{t+1} to compute the integral. We then average the absolute values across the 10,000 periods, and report the log base-10 of this average.

To compare the our solution method with alternatives such as global projections we solved a smaller-scale occasionally bonding constraint model, (Jermann and Quadrini, 2012) using three different methods. First, we replicate the results in that paper solving the original inequality constraint version of the model with a global projection method. Second, we solve the endogenous regime switching formulation of that model via global projection methods. Third, we solve the endogenous regime switching formulation with our proposed perturbation method.

Table C.1 reports some results and highlights that these three approaches have nearly identical implications for the standard deviations and autocorrelations of GDP and hours, which are two key variables in that model. Further, the Euler equation errors all achieve reasonable levels of accuracy. The traditional inequality constraint had the smallest Euler equation errors. However, the endogenous switching model solved globally and the perturbation solution returned accuracy values of -3.6 and -

3.4 in log-10 points, respectively.[29] The latter two numbers suggest an approximation error of \$2.60 and \$3.90 per \$10,000 of consumption. The higher accuracy comes at a significant computational cost, as the global methods solve in minutes, while the perturbation solution takes less than a second. Moreover, we note here that when Binning and Maih (2017) investigated the properties of our framework with simulated data from other structural model formulations, they found a high degree of accuracy.

Appendix D Estimation Procedure

D.1 State Space

For likelihood estimation, the state space representation is given as follows

$$\mathcal{X}_t = \mathcal{H}_{s_t}\left(\mathcal{X}_{t-1}, \varepsilon_t\right) \tag{D.1}$$

$$\mathcal{Y}_t = \mathcal{G}_{s_t}\left(\mathcal{X}_t, \mathcal{U}_t\right) \tag{D.2}$$

Recall the second-order approximation takes the form

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)}\left(S_t \otimes S_t\right) \tag{D.3}$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)}\left(S_t \otimes S_t\right) \tag{D.4}$$

where $S_t = \left[\begin{array}{ccc} (\mathbf{x}_{t-1} - \mathbf{x}_{ss})' & \varepsilon_t' & 1 \end{array}\right]'$.

Therefore, we can define the state variable as

$$\mathcal{X}_t = \left[\begin{array}{ccccc} \mathbf{x}_t' & \mathbf{x}_{t-1}' & \mathbf{y}_t' & \mathbf{y}_{t-1}' & \epsilon_t \end{array}\right]'. \tag{D.5}$$

The nonlinear transition equations

$$\mathcal{X}_t = \mathcal{H}_{s_t}\left(\mathcal{X}_{t-1}, \varepsilon_t\right) \tag{D.6}$$

[29] Note that this value can be driven lower by optimizing the number of gridpoints in our global solution algorithm.

are given by

$$\begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} \left(S_t \otimes S_t\right) \\ \mathbf{x}_{t-1} \\ \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} \left(S_t \otimes S_t\right) \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} \tag{D.7}$$

The observation equation

$$\mathcal{Y}_t = \mathcal{G}_{s_t}\left(\mathcal{X}_t, \mathcal{U}_t\right) \tag{D.8}$$

is given by

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ r_t \\ \Delta B_t/Y_t \\ \Delta P_t \end{bmatrix} = D \begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} + \mathcal{U}_t \tag{D.9}$$

where D denotes a selection matrix of the form

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \\ r_t \\ \Delta B_t/Y_t \\ \Delta P_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1_{[y_t]} & -1_{[y_t]} & 0 \\ 0 & 0 & 1_{[c_t]} & -1_{[c_t]} & 0 \\ 0 & 0 & 1_{[i_t]} & -1_{[i_t]} & 0 \\ 0 & 0 & 1_{[r_t]} & 0 & 0 \\ 0 & 0 & 1_{[\Phi_t^{ca}]} & 0 & 0 \\ 1_{[P_t]} & -1_{[P_t]} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ \mathbf{y}_t \\ \mathbf{y}_{t-1} \\ \varepsilon_t \end{bmatrix} + \mathcal{U}_t. \tag{D.10}$$

D.2 Filtering

The Unscented Kalman Filter (UKF) uses the unscented transformation to calculate the state mean and covariance matrix. It propagates the deterministically chosen sigma-points through nonlinear functions. The transformed points are used to calculate the mean and covariance matrix. As Julier and Uhlmann (1999) note, the key approximation taken to develop the UKF is that the prediction density and the filtering density are both Gaussian. The filtering and smoothing largely follow Binning and Maih (2015).

The filter starts by combining the state vector and exogenous disturbances into a single vector $\mathcal{X}^a_{t-1} = [\mathcal{X}_{t-1}, \epsilon_t]'$ with the following mean and covariance matrix conditional on $Y_{1:t-1}$ and regime s_{t-1}:

$$\mathcal{X}^a_{t-1}(s_{t-1}) = \begin{bmatrix} \mathcal{X}_{t-1|t-1}(s_{t-1}) \\ 0_\epsilon \end{bmatrix} \tag{D.11}$$

$$P^a_{t-1}(s_{t-1}) = \begin{bmatrix} P^x_{t-1|t-1}(s_{t-1}) & 0 \\ 0 & I \end{bmatrix}. \tag{D.12}$$

The sigma-points $\mathcal{X}^a_{i,t-1}(s_{t-1})$ that consist of the sigma-points for state variables $\mathcal{X}^x_{i,t-1}(s_{t-1})$ and the sigma-points for exogenous shocks $\mathcal{X}^\varepsilon_{i,t-1}(s_{t-1})$ are chosen as follows:

$$\begin{aligned}
\mathcal{X}^a_{0,t-1}(s_{t-1}) &= \mathcal{X}^a_{t-1}(s_{t-1}) && \text{(D.13)}\\
\mathcal{X}^a_{0,t-1}(s_{t-1}) &= \mathcal{X}^a_{t-1}(s_{t-1}) && \text{(D.14)}\\
\mathcal{X}^a_{i,t-1}(s_{t-1}) &= \mathcal{X}^a_{t-1}(s_{t-1}) + \left(h\sqrt{P^a_{t-1}(s_{t-1})}\right)_i \text{ for } i = 1 \ldots L && \text{(D.15)}\\
\mathcal{X}^a_{i,t-1}(s_{t-1}) &= \mathcal{X}^a_{t-1}(s_{t-1}) - \left(h\sqrt{P^a_{t-1}(s_{t-1})}\right)_{i-L} \text{ for } i = L+1 \ldots 2L && \text{(D.16)}
\end{aligned}$$

where $h = \sqrt{3}$ and L denotes the number of state variables and exogenous shocks. The weights for the sigma-points are given by:

$$\begin{aligned}
w_0 &= \frac{h-L}{2h} && \text{(D.17)}\\
w_i &= \frac{1}{2h} \text{ for } i = 1 \ldots 2L && \text{(D.18)}
\end{aligned}$$

The sigma-points and the assigned weights are used to calculate the expected mean and covariance by propagating sigma-points through transition equations and taking weighted average:

$$\mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) = H_{s_t}(\mathcal{X}^x_{i,t-1}(s_{t-1}), \mathcal{X}^\varepsilon_{i,t-1}(s_{t-1})) \tag{D.19}$$

$$\mathcal{X}_{t|t-1}(s_{t-1}, s_t) = \sum_{i=0}^{2L} w_i \mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) \tag{D.20}$$

$$P^x_{t|t-1}(s_{t-1}, s_t) = \sum_{i=0}^{2L} w_i \tilde{\mathcal{X}}_i \tilde{\mathcal{X}}_i^T \tag{D.21}$$

$$\mathcal{Y}_{t|t-1}(s_{t-1}, s_t) = D\mathcal{X}_{t|t-1}(s_{t-1}, s_t) \tag{D.22}$$

where $\tilde{\mathcal{X}}_i = \mathcal{X}_{i,t|t-1}(s_{t-1}, s_t) - \mathcal{X}_{t|t-1}(s_{t-1}, s_t)$. By the above conditions, we get the Gaussian approximation predictive density
$p(\mathcal{X}_t|\mathcal{Y}_{1:t-1}, s_{t-1}, s_t) = N(\mathcal{X}_{t|t-1}(s_{t-1}, s_t), P^x_{t|t-1}(s_{t-1}, s_t))$. The predictions are then updated using the standard Kalman filter updating rule:

$$P^y_{t|t-1}(s_{t-1}, s_t) = DP^x_{t|t-1}(s_{t-1}, s_t)D^T + R \tag{D.23}$$

$$P^{xy}_{t|t-1}(s_{t-1}, s_t) = P^x_{t|t-1}(s_{t-1}, s_t)D^T \tag{D.24}$$

$$K_t(s_{t-1}, s_t) = P^{xy}_{t|t-1}(s_{t-1}, s_t)(P^y_{t|t-1}(s_{t-1}, s_t))^{-1} \tag{D.25}$$

$$\mathcal{X}_{t|t}(s_{t-1}, s_t) = \mathcal{X}_{t|t-1}(s_{t-1}, s_t) + K_t(s_{t-1}, s_t)(\mathcal{Y}_t - \mathcal{Y}_{t|t-1}(s_{t-1}, s_t)) \tag{D.26}$$

$$P^x_{t|t}(s_{t-1}, s_t) = P^x_{t|t-1}(s_{t-1}, s_t) - K_t(s_{t-1}, s_t)P^y_{t|t-1}(s_{t-1}, s_t)K_t^T(s_{t-1}, s_t) \tag{D.27}$$

The updating step gives $p(\mathcal{X}_t|\mathcal{Y}_{1:t}, s_{t-1}, s_t) = N(\mathcal{X}_{t|t}(s_{t-1}, s_t), P^x_{t|t}(s_{t-1}, s_t))$. As a by-product of the filter, we can get the density of $\mathcal{Y}_t$ conditional on $\mathcal{Y}_{1:t-1}$, s_t, and s_{t-1}

$$p(\mathcal{Y}_t|\mathcal{Y}_{1:t-1}, s_{t-1}, s_t; \theta) = N(\mathcal{Y}_{t|t-1}(s_{t-1}, s_t), P^y_{t|t-1}(s_{t-1}, s_t)) \tag{D.28}$$

Since the Unscented Kalman filter with regime switches creates a large number of nodes over each iteration where the filtered mean and covariance matrix need to be evaluated, we implement the following collapsing procedure suggested by Kim and Nelson (1999)

$$\mathcal{X}_{t|t}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:t})}\Big\{\sum_{i=1}^{M} \Pr(s_{t-1} = i, s_t = j|\mathcal{Y}_{1:t})\mathcal{X}_{t|t}(s_{t-1} = i, s_t = j)\Big\} \tag{D.29}$$

$$P^x_{t|t}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:t})}\Big\{\sum_{i=1}^{M}\Pr(s_{t-1} = i, s_t = j|\mathcal{Y}_{1:t})[P^x_{t|t}(s_{t-1} = i, s_t = j) \tag{D.30}$$

$$+(\mathcal{X}_{t|t}(s_t = j) - \mathcal{X}_{t|t}(s_{t-1} = i, s_t = j))(\mathcal{X}_{t|t}(s_t = j) - \mathcal{X}_{t|t}(s_{t-1} = i, s_t = j))^T]\Big\} \tag{D.31}$$

where $\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t})$ and $\Pr(s_t|\mathcal{Y}_{1:t})$ are obtained from the following standard Hamilton filter

$$\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1}) = \Pr(s_t|s_{t-1})\Pr(s_{t-1}|\mathcal{Y}_{1:t-1}) \tag{D.32}$$

$$\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t}) = \frac{p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1})\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1})}{\sum_{s_t}\sum_{s_{t-1}} p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1})\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1})} \tag{D.33}$$

$$\Pr(s_t|\mathcal{Y}_{1:t}) = \sum_{s_{t-1}}\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t}) \tag{D.34}$$

Finally, we can get the conditional marginal likelihood,

$$p(\mathcal{Y}_t|\mathcal{Y}_{1:t-1};\theta) = \sum_{s_t}\sum_{s_{t-1}} p(\mathcal{Y}_t|s_t, s_{t-1}, \mathcal{Y}_{1:t-1})\Pr(s_t, s_{t-1}|\mathcal{Y}_{1:t-1}) \tag{D.35}$$

D.3 Smoothing

Once we run through the UKF for $t = 1, \dots, T$, we can also get $\Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T})$, $\Pr(s_t|\mathcal{Y}_{1:T})$, $x_{t|T}(s_t, s_T)$, and $P^x_{t|T}(s_t, s_T)$:

$$\Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T}) = \frac{\Pr(s_{t+1}|\mathcal{Y}_{1:T})\Pr(s_t|\mathcal{Y}_{1:t})\Pr(s_{t+1}|s_t)}{\Pr(s_{t+1}|\mathcal{Y}_{1:t})} \tag{D.36}$$

$$\Pr(s_t|\mathcal{Y}_{1:T}) = \sum_{s_{t+1}}\Pr(s_t, s_{t+1}|\mathcal{Y}_{1:T}) \tag{D.37}$$

$$\mathcal{X}_{t|T}(s_t, s_{t+1}) = \mathcal{X}_{t|t}(s_t) + \tilde{K}_t(s_t, s_{t+1})(\mathcal{X}_{t+1|T}(s_{t+1}) - \mathcal{X}_{t+1|t}(s_t, s_{t+1})) \tag{D.38}$$

$$P^x_{t|T}(s_t, s_{t+1}) = P^x_{t|t}(s_t) - \tilde{K}_t(s_t, s_{t+1})(P^x_{t+1|T}(s_{t+1}) - P^x_{t+1|T}(s_t, s_{t+1}))\tilde{K}_t(s_t, s_{t+1})^T \tag{D.39}$$

Given the above smoothing algorithm, we implement the collapsing procedures similar to those in the filtering steps:

$$\mathcal{X}_{t|T}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:T})}\left\{\sum_{j=1}^{M}\Pr(s_t = i, s_{t+1} = j|\mathcal{Y}_{1:T})\mathcal{X}_{t|T}(s_t = i, s_{t+1} = j)\right\} \tag{D.40}$$

$$P^x_{t|T}(s_t = j) = \frac{1}{\Pr(s_t = j|\mathcal{Y}_{1:T})}\left\{\sum_{j=1}^{M}\Pr(s_t = i, s_{t+1} = j|\mathcal{Y}_{1:T})[P^x_{t|T}(s_t = i, s_{t+1} = j)\right. \tag{D.41}$$

$$\left.+(\mathcal{X}_{t|T}(s_t = j) - \mathcal{X}_{t|T}(s_t = i, s_{t+1} = j))(\mathcal{X}_{t|T}(s_t = j) - \mathcal{X}_{t|T}(s_t = i, s_{t+1} = j))^T]\right\} \tag{D.42}$$

Appendix E Calibrated Parameters

We largely follow Mendoza (2010) in that the calibrated targets are the same, but adapted to our specification of the model. First, we start by calibrating certain parameters, using the steady state of the model in which there is no working capital constraint and the borrowing constraint does not bind. That is, $\phi = 0$ and $\bar{\varphi} = 0$, the latter implies $\lambda_{ss} = 0$. In addition, we get

$$\beta\,(1 + r_{ss}) = 1 \tag{E.1}$$

and ratios of

$$\Omega_v = \frac{1}{1-\alpha-\eta},\ \Omega_h = \frac{1}{\alpha},\ \Omega_k = \frac{1}{\eta}\left(\frac{1}{\beta} - 1 + \delta\right) \tag{E.2}$$

These imply factor payment ratios are

$$\frac{P_{ss}V_{ss}}{Y_{ss} + P_{ss}V_{ss}} = \frac{1}{\Omega_v} = 1 - \alpha - \eta \tag{E.3}$$

$$\frac{W_{ss}H_{ss}}{Y_{ss}} = \frac{1}{\Omega_h\left(1 - \frac{1}{\Omega_v}\right)} = \frac{\alpha}{\alpha+\eta} \tag{E.4}$$

$$\frac{\left(\frac{1}{\beta} - 1 + \delta\right)K_{ss}}{Y_{ss}} = \frac{\frac{1}{\beta} - 1 + \delta}{\Omega_k\left(1 - \frac{1}{\Omega_v}\right)} = \frac{\eta}{\alpha+\eta} \tag{E.5}$$

Using the National Accounts,

$$\begin{bmatrix} 1-\alpha-\eta = 0.102 \\ \frac{\alpha}{\alpha+\eta} = 0.66 \end{bmatrix} \Longrightarrow \begin{bmatrix} \alpha = 0.59268 \\ \eta = 0.30532 \end{bmatrix} \tag{E.6}$$

We set the depreciation rate to an annual value of 8.8 percent, so

$$(1-\delta)^4 = 1 - 0.088 \Longrightarrow \delta = 0.022766 \tag{E.7}$$

The capital-to-(annual) gross output ratio is 1.758, so in our case the capital-to-(quarterly) gross output ratio Ω_k^{-1} is set to be

$$\Omega_k^{-1} = \left(\frac{1}{\eta}\left(\frac{1}{\beta} - 1 + \delta\right)\right)^{-1} = 4 * 1.758 \Longrightarrow \beta = 0.97977 \tag{E.8}$$

Note that this implies an annualized real interest rate of

$$(1 + r_{ss})^4 = \left(\frac{1}{\beta}\right)^4 = 1.0852, \tag{E.9}$$

which nearly replicates the number in Mendoza (2010) but using different discounting. From the resource constraint

$$\frac{C_{ss}}{Y_{ss}} + \frac{I_{ss}}{Y_{ss}} + \frac{E_{ss}}{Y_{ss}} = 1 + \left(1 - \frac{1}{1 + r_{ss}}\right)\frac{B_{ss}}{Y_{ss}} \tag{E.10}$$

and interpreting E_{ss} as government spending

$$0.65 + 0.172 + 0.11 = 1 + (1-\beta)\frac{B_{ss}}{Y_{ss}} \Longrightarrow \frac{B_{ss}}{Y_{ss}} = -3.3605 \tag{E.11}$$

Note that this implies

$$\frac{B_{ss}}{4Y_{ss}} = -0.840127 \tag{E.12}$$

and we can get

$$Y_{ss} = \frac{\alpha+\eta}{\alpha}\left(\frac{1}{P_{ss}^{1-\alpha-\eta}\left(\frac{1}{\eta}\left(\frac{1}{\beta} - 1 + \delta\right)\right)^{\eta}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha-\eta}\right)^{1-\alpha-\eta}}\right)^{\frac{\omega}{\alpha(\omega-1)}} = 1.8202 \tag{E.13}$$

Which gives

$$E^* = \frac{E_{ss}}{Y_{ss}} Y_{ss} = 0.11 * 1.8202 = 0.20022 \tag{E.14}$$

as well as

$$B_{ss} = -0.840127 * 4 * 1.8202 = -6.11685 \tag{E.15}$$

then, conditional on r^* and ψ_r, $\bar{B}$ is pinned down via

$$\bar{B} = \log\left(1 + \frac{r_{ss} - r^*}{\psi_r}\right) + B_{ss}. \tag{E.16}$$

Appendix F Data Appendix

National accounts are from the National Statistic Office. The data series used in the analysis merge two set of official statistics by updating the level of the accounts based on 1993 constant prices with the quarterly rate of growth of the accounts based on 2008 at constant prices. This merging is necessary as the deflators to splice the accounts in levels were not available at the time of last download of the data (May 2017). The two sets of accounts overlap from 1993:Q1 to 2006:Q4. Over this period, the difference in annual rate of growth is less than 0.01 percent in absolute value for GDP, less than 0.05 percent for consumption, less than 2 percent for investment, and less than 1 and 3 percent for imports and exports, respectively. The correlations between the series are more than 0.9 for all series except investment that is 0.84, pointing to possibly larger measurement errors in this variable. The differences are smaller closer to the end of the sample. For this reason, we choose to update the 1993 accounts rather than backdate the 2008 ones.

The specific sources of the data are as follows:

2006:Q1-2016:Q4 (Labeled 2008 accounts)–Supply and demand of goods and services. Original series (not seasonally adjusted). Constant prices, annual 2008 = 100 (Oferta y demanda de bienes y servicios. Series originales. A precios constantes 2008). Available from http://www3.inegi.org.mx/sistemas/tabuladosbasicos/tabdirecto.aspx.

1980:Q1-2006:Q4 (Labeled 1993 accounts)–Supply and demand of goods and services. Original Series (not seasonally adjusted). Constant prices, annual 1993 = 100. We obtained these from Gabriel (2008).

The data are not seasonally adjusted and show a strong seasonal pattern. To seasonally adjust all series (assumed to be I(1) processes), we adjust the log-difference using the X-12 procedure with the additive option in Eviews. We then use the log of the first observation of the raw series (not seasonally adjusted) and cumulate the seasonally adjusted log-difference.

The net exports to GDP series, used to validate the model externally but not as an observable variable in estimation, is calculated as real exports minus real imports divided by real GDP. The current account as a percentage of GDP is from the balance of payment statistics, obtained from the OECD Economic Outlook Database (Series MEX.CBGDPR.Q, OECD-EO-MEX-CBGDPR-Q). As a proxy for the relative price of intermediate goods, entered as observable in estimation, we use a measure of Mexico's terms of trade obtained from Banco de Mxico (PPI Producer and International Trade Price Indexes, series SP12753).

Mexico's country interest rate is calculated following Uribe and Yue (2006) as

$$r_t = r_t^* + spread_t \tag{F.1}$$

where r^* is the US real interest rate, and *spread* is a proxy for Mexico's country risk or sovereign spread. We compute r^* as 3-month Treasury Constant Maturity Rate adjusted for ex post CPI (annualized) quarterly inflation, using period average data. The source of these data is FRED. For the country spread, as customary, we use the Mexico's component of the JP Morgan EMBI.

Unfortunately, the EMBI spread is available only starting from 1993. In order to estimate the risk premium before 1993, we rely on empirical modeling at the Banco de Mexico using the relation between domestic real interest rates and country risk (Aportela Rodriguez et al., 2001) that estimates a close and stable relation between a measure of the domestic real interest rate and the EMBI spread over the period over which both these two variables overlaps. The only interest series available going back to 1980 is a three-month nominal short-term rate obtained from Banco de Mexico (Average monthly yield on 90-days Cetes, series SF3338).[30] To obtain an estimate of Mexico's country risk before the EMBI was published, we estimate a relationship between this nominal interest rate, i_t, and the EMBI during the period over which the EMBI is observable, adjusting for inflation, π_t, which was an important source of

[30]There are three missing monthly observations for this series: August and September 1986 and November 1988. We fill these gaps using July 1986 for 1986Q3 and the average of October and December 1988 for 1988:Q4.

nominal interest rate variation in the 1980s, and then invert it.

Specifically, we posit the following simplified version of the model that (Aportela Rodriguez et al., 2001) estimate:

$$i_t = \alpha_0 + \alpha_1 \pi_t + \alpha_2 EMBI_t. \tag{F.2}$$

We then invert this relation to obtain an estimate of the country risk component of the domestic real interest rate, which we denote as $E\hat{M}BI_t$. The results of the regression above are (t-statistics in parentheses and $R^2 = 0.883$):

$$i_t = \underset{(-0.42)}{-0.00346} + \underset{(4.46)}{0.397}\pi_t + \underset{(7.37)}{2.770}EMBI_t. \tag{F.3}$$

Appendix G Additional Results

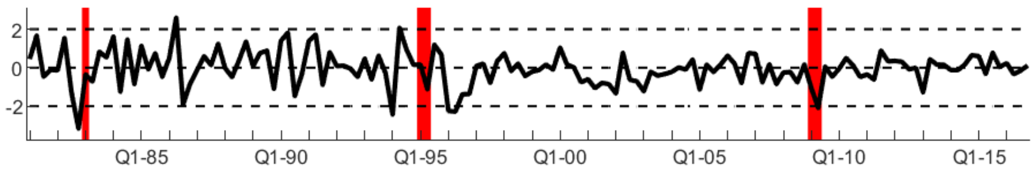
Figure G.1 plots the estimated model implied shocks in standard deviation units, together with a two-standard deviations band. The figure shows that model fit is largely achieved without relying on unusually large shocks, especially during crisis times. Large shocks are needed to fit the import price process, including right before GFC, possibly due the large swings in oil prices during that period. Shocks slightly outside the two-standard divisions band also are estimated right before the 1982 debt crisis. However TFP, expenditure, and preference shocks are all well within the band during the that period.

Figure G.2 plots the pseudo-real-time (i.e. filtered) estimated transition probabilities; panel (a) shows the probability of switching from the non-binding to the binding regime, while panel (b) shows the probability of switching from the binding to the non-binding regime. In other words, they plot the estimated counterpart of the transition probabilities and the identified peak crisis periods defined above. These probabilities provide the odds of switching from one regime to the other as the model travels through the sample. Their behavior is driven by the estimated parameters γ_0 and γ_1 and the estimated values of B^* and λ. Both probabilities are time-varying and hence suggest that a model with exogenous and constant switching probabilities would be misspecified.

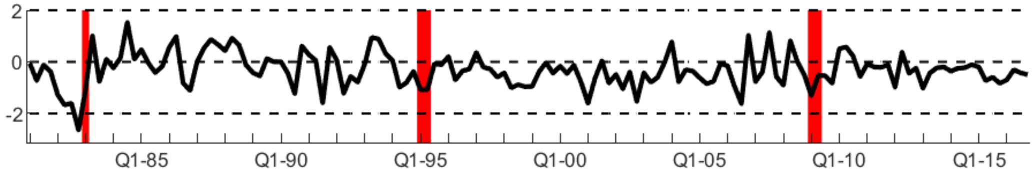
To look at the same important issue from a different model's perspective, we also repeat a similar exercise for the mean estimated crisis probability. Table G.1 provides

Figure G.1: Model Estimated Shocks

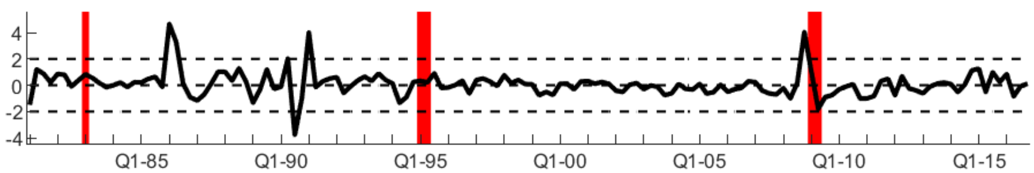
(a) TFP Shock



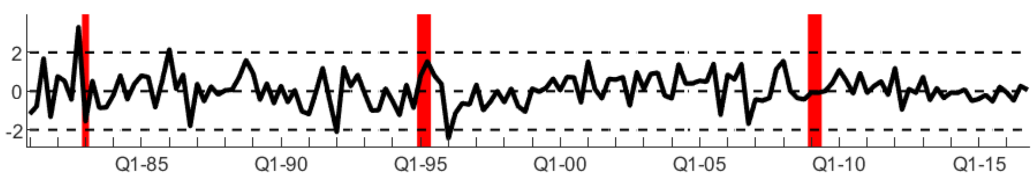
(b) Expenditure Shock



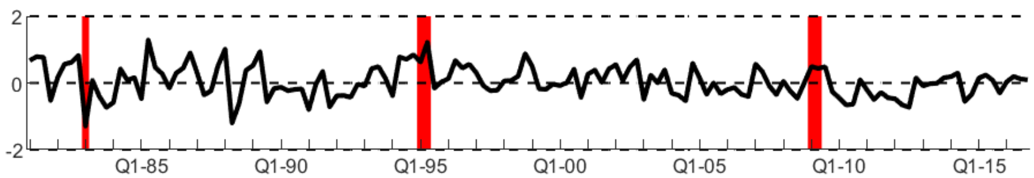
(c) Import Price Shock



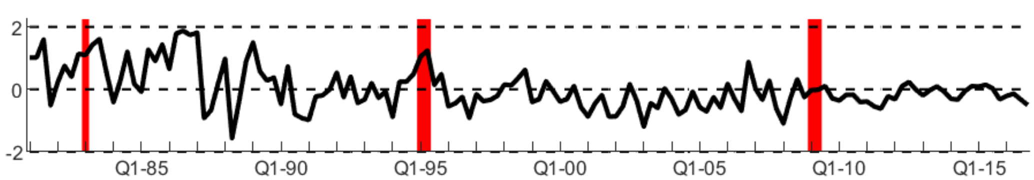
(d) Preference Shock



(e) Transitory Interest Rate Shock



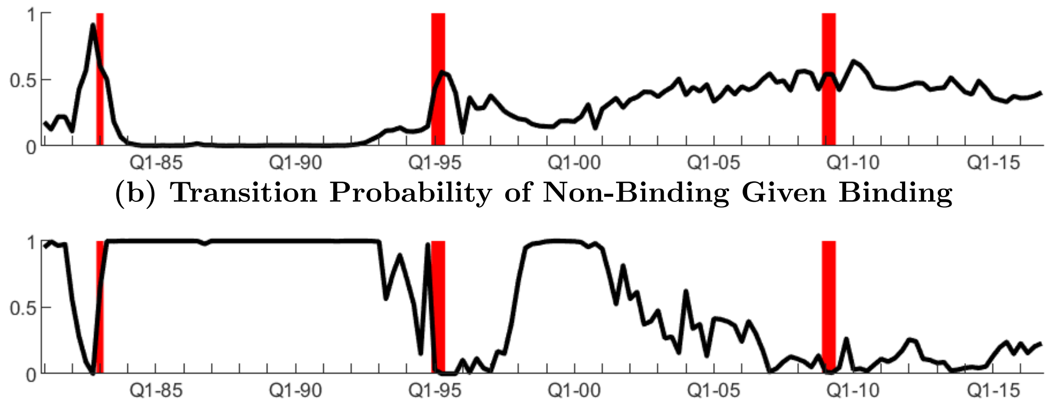
(f) Persistent Interest Rate Shock



Notes: The figure plots the estimated model implied shocks, in standard deviation units, together with a two-standard deviations band (black dashed lines). Red bars indicate model-identified periods of crisis, see text for definition.

Figure G.2: Transition Probabilities

(a) Transition Probability of Binding Given Non-Binding



Notes: Top panel shows model-implied filtered probability of transitioning to binding regime, conditional on being in the non-binding regime, in subsequent period. Bottom panel shows filtered probability of transitioning to non-binding regime, conditional on being in the binding regime, in subsequent period. Red bars indicate model-identified crises, see text for details.

a variance decomposition of the relative frequency with which crisis episodes realize across sample draws, by shutting down one shock at a time. The results are fully consistent with the likelihood based counterfactuals in the main text. In fact, as we can see, turning off the TFP shock, the terms of trade shock, and the transitory interest rate shock has almost no impact on the frequency of crises. Persistent interest rate shocks and preference shocks, and especially expenditure shocks, however, have a much larger impact on the crisis probability. This confirms that the latter shocks may play a relatively larger role in the theoretical model in driving the occurrence of crises.

Appendix H Details on Shock Importance During Crises

To study the importance of shocks around financial crises periods, we compute the marginal impact of each shock. Let LL denote the maximized log-likelihood over the full sample, and let $CLL_{i,t}$ denote the counterfactual full-sample log likelihood when

Table G.1: Decomposition of the Crisis Frequency

	Number of Quarters			Probability		
Variable	Mean	StdDev	Skew	Mean	StdDev	Skew
All Shocks	3.51	2.19	0.92	2.44	1.52	0.92
No TFP Shock	3.55	2.72	1.06	2.47	1.89	1.06
No Expenditure Shock	0.54	1	2.89	0.38	0.69	2.89
No Import Price Shock	3.67	2.77	1.03	2.55	1.93	1.03
No Preference Shock	2.67	2.19	1.08	1.85	1.52	1.08
No Trans. Int. Rate Shock	3.7	2.71	0.95	2.57	1.88	0.95
No Pers. Int. Rate Shock	3.35	2.64	1.14	2.32	1.83	1.14

Note: Model-implied crisis frequencies and its decomposition based upon crisis events in $10,000$ simulated datasets of 144 quarters in length. See text for definition of crisis. 'Number of quarters' refers to counting number of crisis episodes out of 144, 'probability' puts the absolute number as a percentage of 144.

shock i is set to zero in quarter t (i.e. $\varepsilon_{i,t} = 0$). Then the loss in likelihood points,

$$\Delta_{i,t} = LL - CLL_{i,t}, \tag{H.1}$$

is a measure of the importance of $\varepsilon_{i,t}$. The importance of $\varepsilon_{i,t}$ relative to other shocks at time t is given by

$$\Lambda_{i,t} = \frac{\Delta_{i,t}}{\sum_j \Delta_{j,t}}. \tag{H.2}$$

Note that, similar to our variance decomposition results, this marginalizing method ignores possible non-linearities from the second-order solution. Lastly, we report the relative importances when compared to their full sample averages, $\Lambda_{i,t} - \bar{\Lambda}_i$.